Chapter1

Representation of the anomalous magnetic moment of the muons via the novel Einstein-Podolsky-Rosen entanglement

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This paper is dedicated to the memory of Prof. Zbigniew Oziewicz

Abstract In this paper: 1) We review historical and recent verifications of the Einstein-Podolsky-Rosen (EPR) argument that *Quantum mechanics is not a complete theory* supporting the completion of quantum into hadronic mechanics (hm); 2) We review the recently proposed completion of the conventional quantum entanglement of point-like particles without known interactions into the novel *EPR entanglement* of extended particles under non-Hamiltonian interactions quantitatively represented by the novel iso-mathematics of hadronic mechanics; 3) We review and extend the representation via hadronic mechanics achieved in preceding works of all characteristics of muons, including their anomalous magnetic moment, under the assumption that the physical constituents of the muons are ordinary electrons and positrons released free in the spontaneous decay with lowest mode $\mu^{\pm} \rightarrow e^-, e^{\pm}, e^-$, resulting in the structure model $\mu^{\pm} = (e^-_{\downarrow}, e^{\pm}_{\uparrow}, e^-_{\downarrow})_{hm}$ in which the annihilation of the electron-positron pair is confirmed by the muon decay mode $\mu^{\pm} \rightarrow e^{\pm} + 2\gamma$ and explains the instability of the muons; 4) We review and expand preceding works by various authors on the apparent unsettled character of recent measurements of the anomalous magnetic moment of the muons in view of internal non-local and time-irreversible effects; 5) Thanks to the preceding studies, we propose, apparently for the first time, the completion of conventional quantum computers based on 20th century applied mathematics essentially describing isolated components, into the broader *EPR computers* based on iso-mathematics for the description of components in continuous and instantaneous communication, and point out their advantages for increased computational speed, better cybersecurity and increased efficiency.

1. Introduction

Recent accurate measurements conducted at FERMILAB[1] have indicated the following difference between the experimental value of the *muon* g-factor, g^{EXP}_{μ}, and its prediction via quantum electrodynamics, g_μ^{QED},

$$\begin{aligned} g_\mu^{EXP} - g_\mu^{QED} = \\ = 2.00233184122 - 2.00233183620 = \\ = 0.00000000502 > 0. \end{aligned} \tag{1}$$

Additional accurate measurements[2] have shown deviations from quantum mechanical predictions for *atoms* in condensed matter, while measurements[3] have indicated bigger deviations from the predictions of quantum mechanics for *heavy ion.*

The above experiments support:

1) The validity of the historical 1935 argument by A. Einstein, B. Podolsky and N. Rosen that *"quantum mechanics is not a complete theory"* (EPR argument);[4]

2) The significance of historical completions of quantum mechanics, such as the non-linear completion by W. Heisenberg,[5] the non-local completion by L. de Broglie and D. Bohm,[6] and the completion via *hidden variables* by D. Bohm;[7]

3) The validity of the recent verifications of the EPR argument by R. M. Santilli[8,9] based on the completion of quantum mechanics (qm) into hadronic mechanics (hm) according to the EPR argument for the time-invariant representation of extended particles/wavepackets under potential as well as non-linear, non-local and non-potential interactions (see Refs.[10–12] for an outline of the basic methods, Refs.[13,14] for recent overviews and Refs.[15–17] for detailed presentations).

In the preceding Letter,[18] we have outlined the exact and time invariant representation via hadronic mechanics of all characteristics of the muons, including their anomalous magnetic moment, at both the non-relativistic and relativistic levels. In this paper, we outline for self- sufficiency the basic methods used for the verifications of the EPR argument; we review and upgrade the structure model of muons with physical constituents; we review and upgrade the apparently unsettled character of the anomalous magnetic moment of the muons due to internal non-local and time-irreversible effects; and, thanks to the preceding advances we propose, apparently for the first time the completion of conventional quantum computers describing isolated components, into the broader *EPR computers* describing components in continuous and instantaneous communication, and point out their advantages for increased computational speed, better cybersecurity and increased efficiency.

2. Iso-mathematics and iso-mechanics

As it is well known, 20th century applied mathematics is characterized by a universal enveloping associative algebra ξ with conventional associative product $AB = A \times B$ between arbitrary quantities A, B, such as numbers, functions, operators, etc.

Lie algebras L with bracket between Hermitean operators $[A, B] = AB - BA$, then follow as the attached antisymmetric algebra $L \approx \xi^-$, resulting in a unique characterization of Heisenberg's time evolution for an observable A in terms of the Hamiltonian H, $idA/dt = [A, H$, which is solely applicable to point-particles under action-at-a-distance, potential interactions.[4]

The EPR completion of quantum mechanics into hadronic mechanics[17] has been developed to represent *extended* particles in conditions of mutual penetration, as occurring in the nuclear structure, with expected, additional, contact interactions of non-linear, non-local and non-potential type, hereon referred to as *non-Hamiltonian interactions.*

In this section, we outline the rudiments of *iso-mathematics* which is at the foundation of the isotopic branch of hadronic mechanics, known as *iso-mechanics,* for the representation of the extended character of particles and their non-Hamiltonian interactions via the completion of the enveloping associative algebra ξ into the *universal enveloping iso-associative iso-algebra* $\hat{\xi}$ characterized by the *iso-product* (first introduced in Eq. (5), page 71 of Ref.[16] and treated in detail in Ref.[17])

$$A \star B = A\hat{T}B, \quad \hat{T} > 0, \tag{2}$$

where the quantity $\hat{T}(r, p, E, d, \psi, \pi, \tau, ...)$ called the *isotopic element* is positive-definite but possesses otherwise an unrestricted dependence on all needed local variables, such as coordinates r, momenta p, energy E, density d, wave functions ψ, pressure π, temperature τ, etc. (herein tacitly assumed).

For the case of a two-body hadronic bound state, the isotopic element has realizations of the type of the type[12,13]

$$\begin{gathered}\hat{T} = \Pi_{\alpha=1,2} Diag.(\tfrac{1}{n^2_{1,\alpha}}, \tfrac{1}{n^2_{2,\alpha}}, \tfrac{1}{n^2_{3,\alpha}}, \tfrac{1}{n^2_{4,\alpha}})e^{-\Gamma}, \\ n_{\mu,\alpha} > 0, \quad \Gamma > 0,, \quad \mu = 1,2,3,4, \quad \alpha = 1,2,...,N,\end{gathered} \tag{3}$$

by characterizing:

1) The dimension and shape of particles via semi-axes $n^2_{k,\alpha}$, $k = 1,2,3$ (with n_3 parallel to the spin) and the density $n^2_{4,\alpha}$, with all n-characteristic quantities normalized to the value $n^2_{\mu,\alpha} = 1$ for the vacuum.

2) Non-Hamiltonian interactions caused by the mutual penetration of the charge distribution of hadrons via the term e^Γ, where $\Gamma(r, p, \psi, \pi, \tau, ...)$ is a positive-definite quantity with an unrestricted functional dependence on the relative coordinate r, moments p, wave functions ψ, the pressure π, the temperature τ and other characteristics of the medium in which the extended particles are immersed.

Iso-product (2) with realization (3) provides an explicit and concrete realization of Bohm's *hidden variables,*[7] in the sense that *Bohm's variable are hidden in the axioms of quantum mechanics,* such as the universal enveloping associative algebra, and said variables emerge with concrete realizations when said axioms are realized in a way more general than that of the Copenhagen school.

Despite its simplicity, iso-product (2) requires, for consistency, a compatible isotopy of the entire 20th century applied mathematics, with no exception known to the author. In fact, iso-product (2) requires the following completions of 20th century applied mathematics (see Vol. I of Ref.[17] for a general treatment):

A) The compatible, completion of the basic unit $\hbar = 1$ of quantum mechanics into the integro-differential *iso-unit*

$$\hat{I} = 1/\hat{T} > 0, \quad \hat{I} \star A = A \star \hat{I} = A, \tag{4}$$

with ensuing completion of the conventional numeric field $F(n, \times, 1)$ of real $\mathcal{R}$, complex $\mathcal{C}$ and quaternionic $\mathcal{Q}$ numbers n into the *iso-fields* $\hat{F}(\hat{n}, \star, \hat{I})$ of *iso-real* $\hat{\mathcal{R}}$, $\hat{\text{i}}$so-complex $\hat{\mathcal{C}}$ and *iso-quaternionic* $\hat{\mathcal{Q}}$ *iso-numbers* $\hat{n} = n\hat{I}$ and related isotopic operations[20] (see Ref.[21] for an independent study).

B) The completion of conventional Euclidean (and other) coordinates r into *iso-coordinates* and functions $f(r)$ into *iso-functions*[22] (see Ref.[23] for an independent study)

$$\hat{r} = r\hat{I}, \quad \hat{f}(\hat{r}) = [f(r\hat{I})]\hat{I}, \tag{5}$$

as well as the completion of the Newton-Leibnitz differential calculus into the *iso-differential calculus* verifying the basic conditions[22] (see Ref.[24] for comprehensive studies)

$$\hat{d}\hat{r} = \hat{T}d\hat{r}, \quad \frac{\hat{\partial}\hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \hat{I}\frac{\partial\hat{f}(\hat{r})}{\partial\hat{r}}. \tag{6}$$

C) The completion of conventional spaces S over F into *iso-spaces* $\hat{S}$ over iso-fields $\hat{F}$.[22] In particular, the conventional Minkowski space $M(x, \eta, I)$ over $\mathcal{R}$ with spacetime coordinates $x \in \mathcal{R}$, $x^4 = ct$, metric $\eta = Diag(1, 1, 1, -1)$ and unit $I = Diag(1, 1, 1, 1)$, is mapped into the *iso-Minkowski iso-space* $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I})$[25,26] (see Ref.[27] for an independent study) over the iso-real iso-numbers $\hat{\mathcal{R}}$ with iso-coordinates $\hat{x} = x\hat{I} \in \hat{\mathcal{R}}$, iso-metric $\hat{\Omega} = (\hat{\eta})\hat{I} = (\hat{T}\eta)\hat{I}$, and iso-interval

$$\begin{aligned}(\hat{x}^\rho - \hat{y}^\rho)^{\hat{2}} &= (\hat{x}^\rho - \hat{y}^\rho) \star \hat{\Omega}_{\rho\nu} \star (\hat{x}^\nu - \hat{y}^\nu) = \\ &= [\tfrac{(x_1-y_1)^2}{n_1^2} + \tfrac{(x_2-y_2)^2}{n_2^2} + \tfrac{(x_3-y_3)^2}{n_3^2} - \tfrac{(t_x-t_y)^2c^2}{n_4^2}]\hat{I},\end{aligned} \tag{7}$$

where the exponential term $exp\{-\Gamma\}$ is imbedded into the n-characteristic quantities.

D) The compatible completion of all branches of Lie's theory first studied in Ref.[16] (see Vol. II of Ref.[17] for a general treatment and Ref.[29] for an independent study). For instance, an N-dimensional Lie algebra L with Hermitean generators X_k, $k = 1, 2, ...N$ is completed into the infinite family of Lie-Santilli iso-algebras $\hat{L}$ with iso-commutation rules

$$[X_i, X_j]^* = X_i \star X_j - X_j \star X_i = C_{ij}^k X_k. \tag{8}$$

In particular, iso-algebras are called *regular* or *irregular* depending on whether the structure quantities C_{ij}^k are constant or functions of local variables, respectively.[30]

E) The completion of well known space-time symmetries into the *iso-symmetries* of iso-space-time (6), including the completion of the Lorentz symmetry $SO(3.1)$ into the *Lorentz-Santilli iso-symmetry* $\hat{SO}(3.1)$ with iso-transformations[25,26] (see Ref.[27] for independent studies and Ref.[28] for a sample of experimental verifications)

$$\begin{aligned}&x^{1'} = x^1, \quad , x^{2'} = x^2, \\ &x^{3'} = \hat{\gamma}(x^3 - \hat{\beta}\tfrac{n_3}{n_4}x^4), \quad x^{4'} = \hat{\gamma}(x^4 - \hat{\beta}\tfrac{n_4}{n_3}x^3),\end{aligned} \tag{9}$$

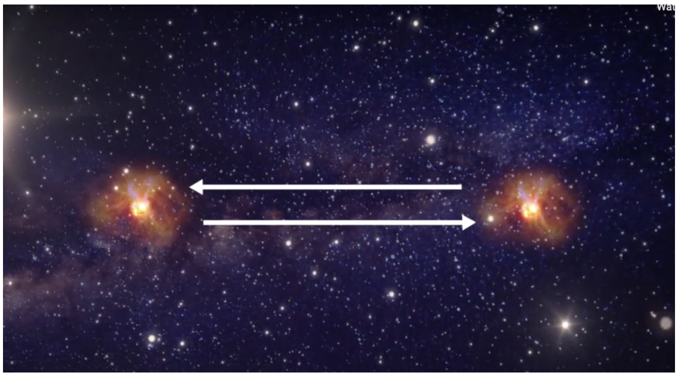

Fig. 1. *In this figure, we illustrate the entanglement of particles with instantaneous mutual actions at a distance, and recall the argument by A. Einstein, B. Podolsky and N. Rosen on the need for superluminal communications to represent said entanglement due to the local character of differentials, potentials and wavefunctions of the Schrödinger equation of quantum mechanics that solely allows a point-like characterization of particles.*[4]

where

$$\hat{\beta}_k = \frac{v_k/n_k}{c_o/n_4}, \quad \hat{\gamma}_k = \frac{1}{\sqrt{1-\hat{\beta}_k^2}}, \tag{10}$$

which iso-symmetry provides the invariance of the local speed of light

$$C = \frac{c}{n_4}, \tag{11}$$

with consequential *iso-renormalization* of the energy (that is, renormalization caused by non-Hamiltonian interactions)

$$E = mc^2 \;\rightarrow\; \bar{E} = mC^2 = \frac{E}{n_4^2}. \tag{12}$$

Additionally, the isotopic representation of the anomalous magnetic moment of the muons requires the completion of the Lorentz-Poincaré symmetry $P(3.1)$ into the *Lorentz-Poincaré-Santilli iso-symmetry* $\hat{P}(3.1)$,[31] and the completion of the spinorial covering of the Lorentz-Poincaré symmetry $\mathcal{P}(3.1)$ into the *iso-spinorial covering of the Lorentz-Poincaré-Santilli iso-symmetry* $\hat{\mathcal{P}}(3.1)$ (in view of the spin 1/2 the muons)[32] (see Ref.[11] for a recent review and Ref.[27] for independent studies).

We should also recall that all aspects of regular iso-mathematics and iso-mechanics can be constructed via the simple *non-unitary transform*[33]

$$UU^\dagger = \hat{I} \neq I, \tag{13}$$

of *all* conventional mathematical or physical aspects, under which: the unit of quantum mechanics is mapped into the iso-unit of iso-mechanics $\hbar = 1 \rightarrow U1U^\dagger = \hat{I}$; the conventional associative product AB is mapped into the iso-product

$$AB \rightarrow U(AB)U^\dagger = (UAU^\dagger)(UU^\dagger)^{-1}(UBU^\dagger) = \hat{A}\hat{T}\hat{B}; \tag{14}$$

and the same holds for the construction of all remaining regular iso-theories.

Finally, we recall that the isotopic element $\hat{T}$ represents physical characteristics of particles. Hence, the invariance of its numeric value is essential for the consistency and experimental verification of any iso-theory. Such an invariance does indeed occur under the *infinite class of iso-equivalence* of isotopic methods which is given by the isotopic reformulation of non-unitary transforms called *iso-unitary iso-transforms* under which we have the numeric invariance of the iso-unit[33]

$$
\begin{gathered}
UU^{\dagger} = \hat{I} \neq I, \quad U = \hat{U}\hat{T}^{1/2}, \quad \hat{U} \star \hat{U}^{\dagger} = \hat{U}^{\dagger} \star \hat{U} = \hat{I}, \\
\hat{I} \rightarrow \hat{U} \star \hat{I} \star \hat{U}^{\dagger} = \hat{I}' \equiv \hat{I}, \\
\hat{A} \star \hat{B} \rightarrow \hat{U} \star (\hat{A} \star \hat{B}) \star \hat{U}^{\dagger} = \hat{A}'\hat{T}'\hat{B}', \quad \hat{T}' \equiv \hat{T}.
\end{gathered} \tag{15}
$$

We should finally indicate that, in view of their invariance under anti-Hermiticity, quantum mechanics and iso-mechanics can only represent systems that are invariant under time reversal, such as stable nuclei.[13] Consequently, in the next sections we study muons during the finite period of their mean lives.

3. The EPR entanglement

According to clear experimental evidence dating back to the early part of the past century, particles that were initially bounded together and then separated, can instantly influence each other at a distance, not only at the particle level, but also classically (see Ref.[34] and papers quoted therein), with intriguing connections to the progressive recovering of Einstein's determinism.[8,9]

The above experimental evidence is generally considered to be represented by quantum mechanics and called *quantum entanglement* (Figure 1). However, Albert Einstein strongly criticized such a view because it would imply superluminal communications that violate special relativity. This occurrence essentially motivated the celebrated 1935 EPR argument on the lack of completeness of quantum mechanics.[4]

One of the EPR arguments is that the Schrödinger equation of quantum mechanics can only represent *a set of isolated point-like particles in vacuum solely under potential interactions.* By contrast, no potential can possibly be introduced to represent the interaction of particle entanglements due to the general lack of charge, ignorable gravitational field, instantaneous interactions at a distance, and other reasons.

In support of Einstein's view, R. M. Santilli pointed out during the *2020 International Teleconference on the EPR Argument*,[13] that, according to quantum mechanics, the sole possible equation for two entangled particles $k = 1, 2$ is given

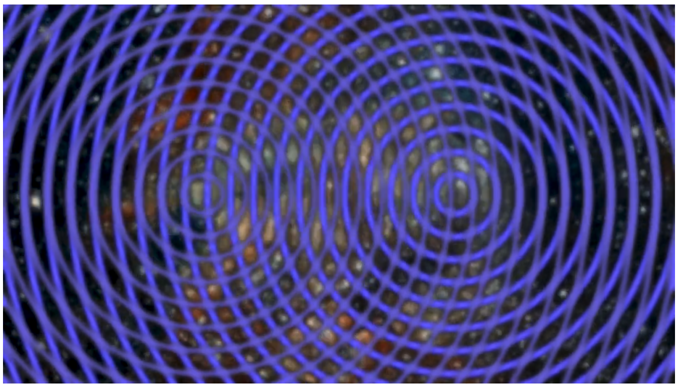

Fig. 2. *In this figure, we illustrate the new Einstein-Podolsky-Rosen entanglement of particles introduced by R. M. Santilli in Ref.*[13] *which entanglement is characterized by contact, therefore instantaneous and non-Hamiltonian interactions originating in the continuous overlapping of the wave packets of particles and represented via isotopic elements of type (3). As such, the EPR entanglement prevents the applicability of Bell's inequality,*[19] *allows an explicit and concrete realization of Bohm's hidden variables*[7] *and permits a progressive recovering of Einstein's determinism in the interior of hyperdense particles, with its full recovering at the limit of gravitational collapse.*[8,9]

by (for $\hbar = 1$)

$$\begin{aligned} &\Sigma_{k=1,2} \tfrac{1}{2m_k} p_k p_k \psi(r_1)\psi(r_2) = \\ &= [\Sigma_{k=1,2} \tfrac{1}{2m_k} (-i \tfrac{\partial}{\partial_{r_k}})(-i \tfrac{\partial}{\partial_{r_k}})] \psi(r_1)\psi(r_2) = E\psi(r_1)\psi(r_2) \end{aligned} \tag{16}$$

which, in the absence of any possible potential, *can only represent two free particles* characterized by the individual wave functions $\psi(r_k)$, $k = 1, 2$, without a dependence on the relative coordinate r.

Santilli then noted that the sole conceivable interaction responsible for the entanglement is that due to the overlapping of the wave packet of particles (Figure 2), thus being non-linear, non-local and instantaneous, by therefore avoiding the need for superluminal communications and being ideally representable with isotopic element (3) of iso-mathematics and iso-mechanics.[13]

By recalling the basic expression of the *iso-linear momentum* of hadronic mechanics characterized by the iso-partial iso-derivative (9) (see Chapter 5, p. 182 on, Vol. II, Ref.[17])

$$\begin{aligned} &\hat{p} \star \hat{\psi}(\hat{r}) = \hat{p}\hat{T}(\hat{r}, ...)\psi(\hat{r}) = \\ &= -i \tfrac{\hat{\partial}}{\hat{\partial}_{\hat{r}}} \hat{\psi}(\hat{r}) = -i\hat{I}(\hat{r}, ...) \tfrac{\partial}{\partial_{\hat{r}}} \hat{\psi}(\hat{r}), \end{aligned} \tag{17}$$

(where the new wave function $\hat{\psi}(\hat{r})$ now depends on the relative iso-coordinate $\hat{r} = r\hat{I}$), the broader notion of *EPR entanglement* proposed in Section 7.2.3, page 61 on of Ref.,[13] is characterized by the *iso-Schrödinger equation for two entangled* *paricles*

$$
\begin{aligned}
&(\Sigma_{k=1,2}\frac{1}{2m_k}\hat{p}_k \star \hat{p}_k) \star \hat{\psi}(\hat{r}) = \\
&= [\Sigma_{k=1,2}\frac{1}{2m_k}(-i\frac{\hat{\partial}}{\hat{\partial}\hat{r}_k})(-i\frac{\hat{\partial}}{\hat{\partial}\hat{r}_k})]\hat{\psi}(\hat{r}) = \\
&= [\Sigma_{k=1,2}\frac{1}{2m_k}(-i\hat{I}\frac{\partial}{\partial\hat{r}_k})(-i\hat{I}\frac{\partial}{\partial\hat{r}_k})]\hat{\psi}(\hat{r}) = \\
&= \{\Sigma_{k=1,2}[(-\frac{\hat{I}^2}{2m_k}(\frac{\partial}{\partial\hat{r}_k})(\frac{\partial}{\partial\hat{r}_k}) - \frac{\hat{I}}{2m_k}(\frac{\partial\hat{I}}{\partial\hat{r}_k})(\frac{\partial}{\partial\hat{r}_k})]\}\hat{\psi}(\hat{r}) = \\
&= \hat{E}\star\hat{\psi}(\hat{r}) = E\psi(\hat{r}) = E\hat{\psi}_1 \star \hat{\psi}_2,
\end{aligned}
\tag{18}
$$

with the following primary characteristics:

3.1. Isotopic equations (18) clearly characterize *a new hadronic interaction represented by the iso-unit* given by the new linear term in the partial derivatives of Eqs. (18), which new term is absent in the quantum equation (16);

3.2. The new hadronic interaction is manifestly *non-linear* (in the wave-function), as desired, yet the theory is *iso-linear,*[17] namely, it is linear on the iso-Hilbert space $\hat{\mathcal{H}}$ over the iso-field $\hat{\mathcal{C}}$, with ensuing verification of the iso-superposition principle $\psi(\hat{r}) = \hat{\psi}_1 \star \hat{\psi}_2$;

3.3. The new hadronic interaction is *non-local* in the sense of occurring in *volumes* characterized by isotopic element (3), namely, by *two overlapping iso-space-time volumes* $V_k = (1/n^2_{1,k}, 1/n^2_{2,k}, 1/n^2_{3,k}, 1/n^2_{4,k},), \ k = 1, 2$, each being on ontological grounds as big as experimental measurements can allow;

3.4. The new hadronic interaction is clearly of *contact-type,* thus being instantaneous by therefore avoiding the superluminal interactions requested by quantum entanglements;[4]

3.5. The EPR entanglement verifies, by conception, the abstract axioms of special relativity although formulated according to their 1983 isotopic realization,[25–28] as it is evident in the relativistic extension of Eq.(18) here omitted for brevity.[4,10–12] We can therefore say that *the EPR entanglement is fully contained in the abstract axioms of special relativity and it has been merely manifested by the above derivation.*

In the author's view, the virtual entirety of the applications in various fields of the hadronic interactions are ultimately reducible to the new notion of EPR entanglement.

4. Non-relativistic representation of the muon structure

As it is well known, the standard model assumes that muons are *elementary particles,* under which assumption, the sole known possibility of representing deviation (1) is the search for new particles and/or new interactions.

In this paper, we study the view presented in pages 849 on of the 1978 paper[35] (see also Section 2.5.5, page 163 of Ref.[12] for a recent update) according to which *muons are naturally unstable, and therefore they are bound states of elementary particles suitable to trigger their decay.*

The well known characteristics of the muon μ^- are the following: mass $105,658\ MeV$; spin 1/2; charge $-\ e$; mean life $\tau = 2.19703 \times 10^{-6}\ s$; and spontaneous decays

$$\mu^- \ \rightarrow \ e^- + \nu + \bar{\nu}, \quad \%\ 10^{-11}, \tag{19}$$

$$\mu^- \ \rightarrow e^- + 2\gamma, \quad \%\ 10^{-11}, \tag{20}$$

$$\mu^- \ \rightarrow \ e^- + e^+ + e^- \quad \%\ 10^{-12}. \tag{21}$$

Therefore, paper[35] assumed that spontaneous decay (21) represents a tunnel effect of the constituents, as a result of which *the muon μ^- is a bound state of two electrons and one positron represented via hadronic mechanics* (Figure 3)

$$\mu^{\pm} = (\hat{e}^-_{\downarrow}, \hat{e}^{\pm}_{\uparrow}, \hat{e}^+_{\downarrow})_{hm}, \tag{22}$$

where the "hat" indicates that the constituents are not ordinary particles, but *iso-particles,* that is, iso-unitary, iso-irreducible iso-representations of $\hat{\mathcal{P}}(3.1)$ over an iso-field $\hat{F}$ in deep EPR entanglement (Figure 3).[32]

By recalling that an electron-positron pair annihilates into 2γ, the presence in the muon structure of an electron-positron pair appears to be confirmed by the two photons of spontaneous decay (21), thus allowing a representation of the instability of the muon, as well as of its main life as shown below.

Note that, since all constituents have point-like charges, model (22) characterizes a mostly flat particle with a charge radius given by the radius of the orbit of the peripheral pair of iso-particles.

To complete model (22), Ref.[35] submitted the hypothesis of the existence of a new electron-positron bound state at short distance of called *hadronic positronium* and denoted $(\hat{e}^+_{\uparrow}, \hat{e}^-_{\downarrow})_{hm}$, which is predicted on grounds of the strongly attractive Coulomb force between the constituents at $10^{-13}\ cm$ mutual distance,

$$F = k\frac{e^2}{r^2} = (8.99 \times 10^9)\frac{(1.60 \times x10^{-19})^2}{(10^{-15})^2} = 230\ N, \tag{23}$$

As it is the case for the conventional positronium, the hadronic positronium is then predicted to have two configurations called *hadronic ortho- and hadronic para-positronium* (Figure 4).

In order to achieve an analytic solution of *three-body* model (22), Ref.[35] conceived the muon as a *two-body* hadronic bound state of an electron and a hadronic positronium, resulting in the structure (Figures 3, 4)

$$\mu^{\pm} = [\hat{e}^{\pm}_{\downarrow}, (\hat{e}^+_{\uparrow}, \hat{e}^-_{\downarrow})_{hm}]_{hm}. \tag{24}$$

Two-body version (24) of the model was then solved analytically in detail in Ref.,[35] pages 632-841 (here omitted for brevity), via the iso-Schrödinger equation of hadronic mechanics with radial component

$$\left[\frac{1}{r^2}(\frac{d}{dr}r^2\frac{d}{dr}) + \tilde{m}(E + W_1\frac{e^2}{r} - W_2\frac{e^{-br}}{1 - e^{-br}})\right] = 0, \tag{25}$$

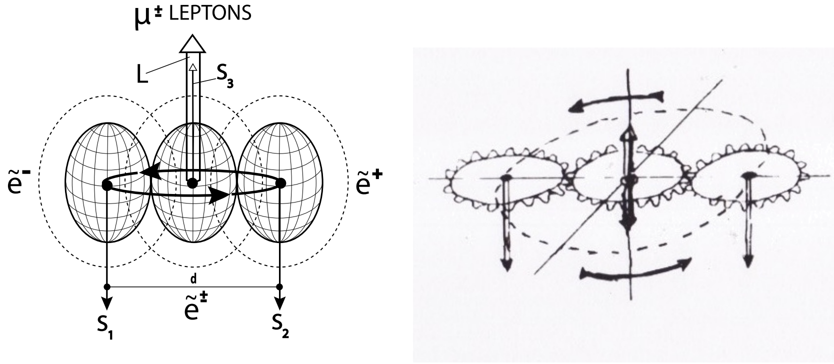

Fig. 3. *The left view illustrates structure model (22) of the muons $\mu^{\pm}$ in their ground state proposed in page 849 of the 1978 paper*[35] *(see also Section 2.5.5, page 163 of the recent update*[12]*). The right view illustrates the stability of structure (22) via a "gear model" also presented in Ref.,*[35] *page 852, due to the singlet coupling of the two pairs of constituents. The muon μ^- (μ^+) is assumed to be a hadronic bound state of an electron-positron pair trapping an electron (a positron) in its center, which constituents are produced free in the spontaneous decay with the lowest mode, Eq. (21). The finite life of the muons is then represented by the annihilation of the electron-positron pair which is confirmed by decay (20). The dashed lines represent the wavepackets of the constituents and their deep EPR entanglement.*

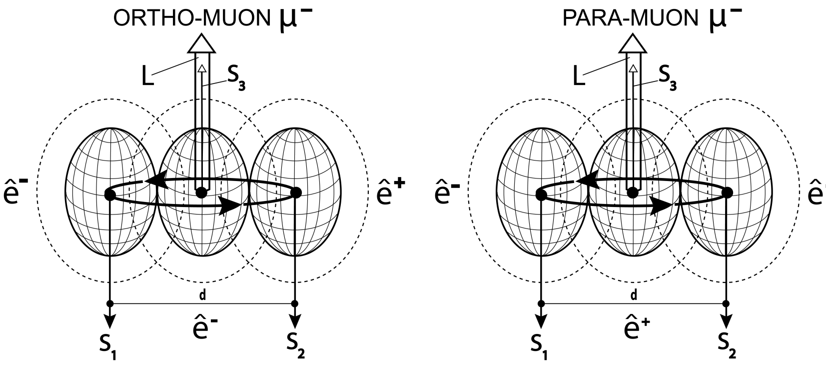

Fig. 4. *In this figure, we illustrate the ortho- and para-configurations of the hadronic positronium contained in model (22) of the muon. These configurations are predicted by hadronic mechanics to exist at short distances from their known existence at atomic distances. The conjugation recommended from particle to antiparticle is the isodual map of Ref.*[36] *representing the conception by P. A. M. Dirac*[37] *that antiparticles have negative energy as a condition to represent particle-antiparticle annihilation into light. According to this view, μ^- is represented by the top two-dimensional part of Eqs. (34), (35) with positive unit $+I_{2\times2}$, while μ^+ is represented by the bottom two-dimensional part with negative unit $-I_{22}$ (see Section 2.3.6, page 118 on of Ref.*[36]*).*

$$E \approx 0, \quad E_\mu = \tilde{m}c^2 = 105 \ MeV, \tag{26}$$

$$\tau^{-1} = 2\pi\lambda^2|\hat{\psi}(0)|^2\frac{\alpha^2 E_e}{\hbar} = 10^{-6}\ s, \tag{27}$$

$$R = b^{-1} = 10^{-13\ cm}, \tag{28}$$

in which Eqs. (26) and (27) are subsidiary constraints.

The above solution achieved a numerically exact and time invariant representation of the following characteristics of the muon: mass $E = 105.658\ MeV$, mean life $\tau = 2.19703 \times 10^{-6}\ s$, charge radius of about $R = d/2 = 10^{-13}\ cm$, spin $S = 1/2$, charge $Q = \pm e$, and parity. According to the knowledge of the time, Ref.[35] assumed that the magnetic moment of the muon was equal to that of the electron.

The representation of the anomalous magnetic moment of the muon requires a relativistic analysis done in the next section. Therefore, in line with experimental value (1), we here merely assume that

$$g_{muon} = 1.00000000502 g_e. \tag{29}$$

Note that model (22) and its quantitative treatment, Eqs. (25), (26), (27), are impossible for quantum mechanics on various grounds, thus mandating the use of hadronic mechanics.

It should be indicated that the above structure model of muons has been extended to all mesons and to all unstable baryons as hadronic structure models of physical constituents produced free in spontaneous decays in a way compatible with the $SU(3)$-color classification.[12]

5. Relativistic representation of the muon structure

An inspection of model (22) (Figure 3) indicates that its relativistic representation requires at least two hadronic completions of the conventional Dirac equation. The first completion is evidently needed for the characterization of the extended shape of the muons which can be represented via the iso-Minkowski iso-space $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I}_{orb})$ over the iso-real iso-field $\hat{\mathcal{R}}$ with common iso-unit

$$\hat{I}_{orb} = 1/\hat{T}_{orb} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2), \quad n_\alpha > 0, \quad \alpha = 1, 2, 3, 4. \tag{30}$$

Recall that central electron of model (22) has a *point-like charge,* but it has an *extended wavepacket* (with radius of the order of 1 fm) in deep EPR entanglement with the peripheral electron-positron pair. Recall also that the peripheral electron-positron pair has null spin, charge and magnetic moment.

It is therefore plausible to assume that the anomalous value (1) of the muon magnetic moment is due to a small deformation (called *mutation*[35]) of the wavepacket of the central electron which can be represented via a two-dimensional, iso-complex iso-Euclidean iso-space $\hat{E}(\hat{z}, \hat{\Delta}, \hat{I}_{spin})$ over the iso-complex iso-field $\hat{\mathcal{C}}$ with iso-unit

$$\hat{I}_{s[in} = 1/\hat{T}_{spin} = Diag.(m_1^2, m_2^2), \quad m_k > 0, \quad k = 1, 2. \tag{31}$$

The above isotopy can be formulated as a realization of Bohm's hidden variable bλ under the uni-modularity condition (see Ref.[9] page 185)

$$Det.\hat{I}_{spin} = I, \quad m_1^2 = 1/m_2^2 = 1/\lambda, \quad \hat{I}_{spin} = Diag.(1/\lambda, \lambda). \tag{32}$$

The iso-Dirac equation resulting from the above two isotopic completions is then given by (for brevity, see the derivation in Ref.,[32] page 189)

$$[\hat{\Omega}^{\mu\nu} \star \hat{\Gamma}_\mu \star \hat{p}_\nu + \hat{m} \star \hat{c}] \star |\hat{\psi}(\hat{x}) >= (-i\hat{I}\hat{\eta}^{\mu\nu}\hat{\gamma}_\mu \partial_\nu + mc)|\hat{\psi}(\hat{x}) >= 0, \tag{33}$$

where $\hat{m} = m\hat{I}_{orb}, \hat{c} = c\hat{I}_{orb}$, with *iso-Dirac iso-matrices*

$$\hat{\Gamma} = \hat{\gamma}\hat{I},\, \hat{\gamma}_k = \tfrac{1}{n_k}\begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \quad \hat{\gamma}_4 = \tfrac{i}{n_4}\begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}, \tag{34}$$

and related *anti-iso-commutation rules*

$$\{\hat{\gamma}_\mu, \hat{\gamma}_\nu\}^* = \hat{\gamma}_\mu \star \hat{\gamma}_\nu + \hat{\gamma}_\nu \star \hat{\gamma}_\mu = 2\hat{\eta}_{\mu\nu}, \tag{35}$$

where one should remember that the iso-metric $\hat{\eta}$ is the most general possible regular and symmetric metric in $(3+1)$-dimensions, thus including the Riemannian metric as a particular case.

The spin of the central electron of model (22) is then characterized by the *iso-Pauli iso-matrices* appearing in iso-equation (34)[8,32]

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & \lambda^{-1} \\ \lambda & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -i\lambda^{-1} \\ i\lambda & 0 \end{pmatrix}, \quad \hat{\sigma}_3 = \begin{pmatrix} \lambda & 0 \\ 0 & -\lambda^{-1} \end{pmatrix}, \tag{36}$$

whose iso-morphism with the conventional Pauli matrices is confirmed by the iso-commutation rules

$$[\hat{\sigma}_i, \hat{\sigma}_j]^* = \hat{\sigma}_i \hat{T}_{spin} \hat{\sigma}_j - \hat{\sigma}_j \hat{T}_{spin} \hat{\sigma}_i = i2\epsilon_{ijk}\hat{\sigma}_k, \tag{37}$$

as well as by the characterization of the conventional spin 1/2

$$\hat{\sigma}_3 \hat{T}_{spin}|\hat{b} >= \pm|\hat{b} >, \quad \hat{\sigma}^{\hat{2}} = (\hat{\sigma}_1 \hat{T}_{spin}\hat{\sigma}_1 + \hat{\sigma}_2 \hat{T}_{spin}\hat{\sigma}_2 + \hat{\sigma}_3 \hat{T}_{spin}\hat{\sigma}_3)\hat{T}_{spin}|\hat{b} >= 3|\hat{b} > . \tag{38}$$

The explicit realization of the hidden variable λ in the spin of the central electron, Eqs. (36), sets the physical foundations for the anomalous magnetic moment of the muon *under the conventional spin* 1/2.

The identification of the numeric values of the characteristic $n-$ quantities of model (22) has been done in Ref.[18] and can be summarized as follows.

According to model (22), the total rest energy of the constituents is $E_{cons} = 3E_e = 1.533\ MeV$, while the muon rest energy is given by the familiar value $E_\mu = 105.7\ MeV$. This implies the *excess energy*

$$\Delta E = 105.7\ MeV - 1.533\ MeV = 104.167\ MeV, \tag{39}$$

under which the Schrödinger equation no longer admits physically meaningful solutions.[12,35]

Isotopic methods provide a *mathematical* representation of excess energy (39) via iso-renormalization (12) resulting in the *numeric value of the density of the muon*

$$n_4^2 = \frac{3E_e}{E_\mu} = \frac{1.533}{105.7} = 0.0145, \tag{40}$$

under which the consistency of the Schrödinger equation is restored at the isotopic level.[12]

Excess energy (39) is *physically* represented by the kinetic energy of the peripheral electron-positron pair. However, the calculation of this kinetic energy for extended particles in deep EPR entanglement is unknown at this writing, thus illustrating the significance at this time of mathematical derivation (40).

It should be indicated that excess energy (39) also occurs in the synthesis of the neutron from the hydrogen in the core of stars,[38,39] as well as, more generally, in the synthesis of hadrons from lighter particles.[12]

The relationship under isotopies between the magnetic moment and the spin of charged particles has been identified in Eq. (6.5), page 190 Ref.[32] with ensuing relation for the g-factors (here reproduced for brevity without its derivation)

$$\hat{g}_\mu^{EXP} = \frac{n_4}{n_3} g_\mu^{QED}, \quad \frac{n_4}{n_3} = 1.00000000502. \tag{41}$$

From value (1), we then have the *numeric value of the characteristic quantity* n_3^2

$$n_3^2 \approx n_4, \quad n_3^2 = 0.0145. \tag{42}$$

According to model (22), the anomalous magnetic moment of the muon is due to a mutation of the charge distribution that, as such, leaves volumes unchanged, thus explaining the sole presence of the space characteristic quantity n_3^2 in expression (41).

The use of the following volume normalization

$$n_1^2 + n_2^2 + n_3^2 = 1, \tag{43}$$

then provides the desired *numeric values of all three space characteristic quantities of the muon*

$$n_1^2 = n_2^2 \approx 0.4926, \quad n_3^2 \approx 0.0145, \tag{44}$$

with similar results for different normalizations.

The above values confirm the expected very prolate character of the assumed muon structure model, Eq. (22), due to the point-like character of the constituents.

6. Apparent unsettled value of the muon magnetic moment

We should recall for due scientific process P. A. M. Dirac's[40] and other authoritative doubts on the final character of the numeric values obtained from quantum electrodynamics due to the divergence of Feynman's and other series (see Ref.[41] for a recent account on QED divergencies).

Additionally, measurements[1] have been done via the assumption that the mean life of muons behaves with speed (or, equivalently, with energy) according to the time dilation law of special relativity

$$t = t_o \sqrt{1 - \frac{v^2}{c^2}}. \tag{45}$$

The exact validity of the above law for the electron and for other *point-like* particles is, nowadays, beyond scientific doubt. However, we should mention the existence of unresolved experiments indicating deviations from law (45) for the behavior of instable hadrons with speed due to expected, internal, non-linear, non-local, and non-potential interactions under which special relativity is inapplicable. In the event said deviations are confirmed, it appears plausible to expect that measurements (1) may in fact need corresponding corrections. The case is treated in detail in Section 6.1.9, page 846 on, Volume IV of Ref.,[42] and can be summarized as follows.

In 1965, D. I. Blokhintsev[43] pointed out the expected inapplicability of special relativity for the interior of hadrons due to their hyperdense character with consequential *non-local internal effects* under which special relativity is inapplicable, and suggested that internal deviations from relativistic quantum mechanics could be measured in the outside via deviations from time dilation law (45).

In 1967, Santilli[44] pointed out that special relativity and relativistic quantum mechanics are reversible over time, thus being inapplicable for time irreversible processes such as spontaneous decays. The origin of time reversibility was identified in the invariance of Lie's theory under anti-Hermitcity. Therefore, Santilli proposed the completion of Lie theory and related physical theories into covering time irreversible theories based on Albert's[45] notion of Lie-admissible/Jordan-admissible algebras[46] to[49] (see Ref.[17] for detailed treatments and Ref.[13] for a recent update).

In 1978, when he was at Harvard University under DOE support, Santilli[25] worked out the Lie-isotopic particular caee of the Lie-admissible formulations by pointing out that the axioms of special relativity remain valid for time reversible processes of extended particles with invariant (7) when realized via the Lie-isotopic formulations, resulting in the axiom-preserving EPR completion of law (45)[25] (see also Ref.,[26] Vol. II of Ref,[47] Lecture Notes[27] from talks delivered in 1991 by Santilli at the ICTP, Trieste, Italy, and Section 8 of the recent update[13])

$$t = t_o \sqrt{1 - \frac{v^2/n_3^2}{c^2/n_4^2}}. \tag{46}$$

The above studies triggered a number of generalizations of law(45), such as those of Refs.[50,51] In 1989, A. K. Aringazin[52] proved that all preceding generalizations of law (45) are particular cases of the isotopic law (46) because they can be obtained via different expansions of the latter law in terms of different parameters and with different truncation, thus restricting the experiments to the test of law (46).

In 1983, S. H. Aronson *et al.*[53] reported the outcome of experiments conducted at FERMILAB showing apparent *deviations* from law (45) for the $K^0 - \bar{K}^0$ system in the energy range from 0 to 100 GeV.

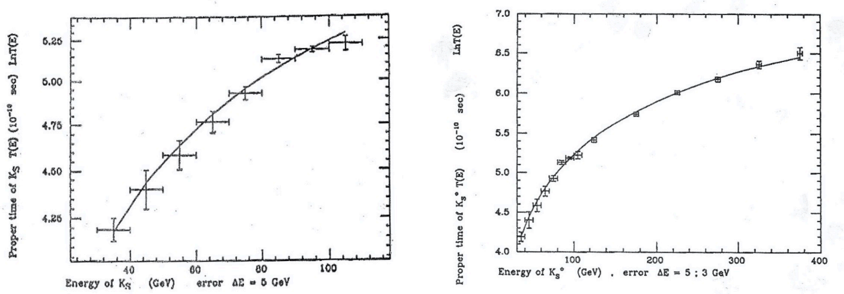

Fig. 5. *In this figure, we reproduce the exact fit*[55] *via isotopic time dilation law (46) of: 1) Deviations*[53] *from the time dilation law (45) for the behavior of the* $K^0 - \bar{K}^0$*-system from* 0 *to* 100 GeV *(left view); and 2) The exact fit of both deviations* 0 *to* 100 GeV[53] *and apparent verification in the range from* 100 *to* 350 GeV[54] *(right view).*

In 1987, N. Grossman *et al.*[54] reported counter-experiments also conducted at FERMILAB showing an apparent *confirmation* of law (45), but in the *different energy range* from 100 to 350 GeV.

In 1992, F. Cardone *et al.*[55] indicated that counter-measurements[54] from 100 to 350 GeV leave basically unresolved the deviations of law (45) from 0 to 100! GeV,[53] and that the isotopic law (46) provides an exact fit for both measurements[53,54] (Figure 5). Finally, in 1998, Yu. Arestov *et al.*[56] pointed out flaws in the theoretical elaboration of the experimental data of measurements.[55]

More recently, Santilli[57] pointed out that, in the event deviations[53] are confirmed, time dilation law (45) should be completed into isotopic law (46) with numeric value of the characteristic quantities of the muons derived from Eq. (41) predicting an *increase* of anomalous value (1)

$$t = t_o\sqrt{1 - 1.00000001004\frac{v^2}{c^2}}, \tag{47}$$

under which conditions a revision of the numeric value of the muon magnetic moment should be expected.

7. Application of the EPR entanglement to computers

It may be of some interest to indicate the expected EPR completion of other branches of physics, such as the completion of conventional quantum computers for the description of isolated constituents into new computers, here suggested under the name of *EPR computers,* for the description of constituents in continuous and instantaneous communications, by therefore approaching the new notion of living organisms attempted in Ref.,[58] with the following expected advances:

1) Faster computations, due to fact that all possible values of the isotopic element (3) are very small according to all available fits of experimental data,[13] with ensuing rapid convergence of isoperturbative series (see also Corollary 3.7.1, page 128 of Ref.[11]). As a confirmation of this expectation, we recall the achievement via iso-mathematics and iso-chemistry of an *attractive* force between the *identical* electrons of valence coupling (see Chapter 4 of Ref.[59]) resulting in a *strong valence bond* that allowed the exact representation of the experimental data for the hydrogen[60] and water[61] molecules with iso-perturbative calculations at least one thousand times faster than their conventional chemical counterparts.

2) Better cybersecurity, due to the formulation over iso-numeric iso-fields, with consequential natural availability of iso-cryptograms equipped with an algorithm changing the numeric value of the isounit with such a frequency to prevent the achievement of a solution within a finite period of time (see Appendix 2C, Vol. I, page 84 of Ref.[17]).

3) Increased efficiency, due to the fact that EPR entanglements are caused by *interactions without potential energy,* thus being more energy efficient than the potential energy dependent quantum computers.

8. Concluding remarks

In the author's view, the most important notion emerging from the preceding study is that of the Einstein-Podolsky-Rosen entanglement representing the instantaneous and continuous communications between extended particles due to the overlapping of their wavepackets, with ensuing non-Hamiltonian interactions represented by Eqs. (2) and (3), whose consistent treatment required the construction of iso-mathematics and iso-mechanics.[17]

In fact, the EPR entanglement has the following important implications:

1) It prevents the applicability of Bell's inequality[19] due to the presence of non-Hamiltonian interactions first studied in Ref.;[8]

2) It provides an explicit and concrete realization of Bohm's hidden variables[7] in terms of the isotopic element first achieved in Ref.;[9]

3) It permits a preliminary, yet numerically exact and time invariant representation of all characteristics of muons, including their anomalous magnetic moment.[1]

In closing, there seems to be grounds for a new physics, with expected corresponding advances in chemistry and biology, via the axiom-preserving completion of the Copenhagen simplest possible realization of quantum axioms into their broadest possible realization suggested by hadronic mechanics.[13]

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