# Representation of the Anomalous Magnetic Moment of the Muons via the Einstein-Podolsky-Rosen Completion of Quantum into Hadronic Mechanics

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In this paper, we briefly review half a century of research by various authors in the axiom-preserving completion of quantum mechanics into hadronic mechanics according to the 1935 Einstein-Podolsky-Rosen argument that "quantum mechanics is not a complete theory" (EPR argument). Said completion is intended to represent extended-particles in conditions of deep EPR entanglement with ensuing potential as well as contact non-Hamiltonian interactions represented by the new operator $\hat{T}$ in the associativity-preserving products $A \star B = A\hat{T}B$ of hadronic mechanics. We recall that muons are unstable and decay spontaneously with a mean live $\tau = 2.19703\times10^{-6}$ s, thus suggesting that they are composite, and therefore extended particles with constituents capable of triggering their decay. We then assume that the physical constituents of the muons are the ordinary electrons released in the spontaneous decay with low mode $\mu^- \rightarrow e^-, e^+, e^-$, resulting in the structure model according to hadronic mechanics (hm) $\mu^- = (\hat{e}^-_\downarrow, \hat{e}^+_\uparrow, \hat{e}^-_\downarrow)_{hm}$ where the "hat" characterizes iso-renormalizations due to non-Hamiltonian interactions. We show that the indicated hadronic structure model achieves an exact representation of *all* characteristics of muons, including rest energy, charge radius, mean life, spin, charge, spontaneous decays and anomalous magnetic moment.

## 1 Introduction

Recent, very accurate measurements [1] have established the following difference between the experimental value *muon* g-factor $g_\mu^{EXP}$ and its prediction via quantum electrodynamics $g_\mu^{QED}$

$$\begin{aligned} & g_\mu^{EXP} - g_\mu^{QED} = \\ & = 2.00233184122 - 2.00233183620 \\ & = 0.00000000502 > 0\,. \end{aligned} \tag{1}$$

Additional accurate measurements [2] have shown deviations from quantum mechanical predictions for *atoms* in condensed matter, and measurements [3] have indicated bigger deviations from the predictions of quantum mechanics for *heavy ion.*

The above experiments support:

1) The validity of the historical 1935 argument by A. Einstein, B. Podolsky and N. Rosen that "quantum mechanics is not a complete theory" (EPR argument) [4];

2) The significance of historical completions of quantum mechanics, such as the non-linear completion by W. Heisenberg [5], the non-local completion by L. de Broglie and D. Bohm [6], and the completion via *hidden variables* by D. Bohm [7];

3) The validity of the recent verifications of the EPR argument by R. M. Santilli [8, 9] based on the completion of quantum mechanics (qm) into hadronic mechanics (hm) according to the EPR argument for the time-invariant representation of extended particles/wavepackets under potential as well as non-linear, non-local and non-potential interactions (see [10–12] for an outline of the basic methods, [13, 14] for recent overviews and [15–17] for detailed presentations).

## 2 Isotopic branch of hadronic mechanics

As it is well known, 20th century applied mathematics is characterized by a universal enveloping associative algebra $\xi$ with conventional associative product $AB = A \times B$ between arbitrary quantities $A, B$, such as numbers, functions, operators, *etc*.

Lie algebras $L$ with bracket between Hermitean operators $[A, B] = AB - BA$, then follow as the antisymmetric algebra attached to $L \approx \xi^-$, resulting in a unique characterization of Heisenberg's time evolution $idA/dt = [A, H]$ for point-particles under action-at-a-distance, potential interactions.

The EPR completion of quantum mechanics into hadronic mechanics has been studied to represent *extended* particles in conditions of mutual penetration, as occurring in the nuclear structure, with expected, additional, contact interactions of non-linear, non-local and non-potential type, hereon referred to as *non-Hamiltonian interactions.*

The axiom-preserving, thus isotopic branch of hadronic mechanics, known as *iso-mechanics,* and its underlying mathematics, known as *iso-mathematics,* represent the extended character of particles and their non-Hamiltonian interactions via the completion of the enveloping associative algebra $\xi$ into the *universal enveloping iso-associative iso-algebra* $\hat{\xi}$ characterized by the *iso-product* (first introduced in Eq. (5), page 71 of [16] and treated in detail in [17])

$$A \star B = A\hat{T}B, \ \hat{T} > 0\,, \tag{2}$$

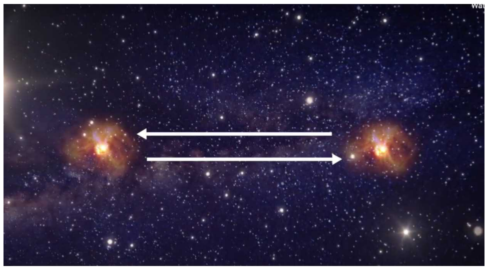

Fig. 1: In this figure, we illustrate the quantum entanglement of particles with instantaneous mutual actions at a distance, and recall the argument by A. Einstein, B. Podolsky and N. Rosen on the need for superluminal communications to represent said entanglement due to the local character of differentials, potentials and wavefunctions of the Schrödinger equation of quantum mechanics which solely allows a point-like characterization of particles [4].

where the quantity $\hat{T}$, called the *isotopic element,* is positive-definite but possesses otherwise an unrestricted dependence on all needed local variables (herein tacitly assumed).

The isotopic element has realizations of the type [12]

$$\hat{T} = \Pi_{\alpha=1,2}\,\mathrm{Diag}\left(\frac{1}{n^2_{1,\alpha}}, \frac{1}{n^2_{2,\alpha}}, \frac{1}{n^2_{3,\alpha}}, \frac{1}{n^2_{4,\alpha}}\right) e^{-\Gamma}\,, \qquad n_{\mu,\alpha} > 0, \quad \Gamma > 0, \quad \mu = 1,2,3,4, \quad \alpha = 1,2,...,N\,, \tag{3}$$

by charaterizing:

1) The dimension and shape of particles via semi-axes $n^2_{k,\alpha}, k = 1,2,3$ (with $n_3$ parallel to the spin) and the density $n^2_{4,\alpha}$, all $n$-characteristic quantities being normalized to the value $n^2_{\mu,\alpha} = 1$ for the vacuum.

2) Non-Hamiltonian interactions via the term $e^{\Gamma}$, where $\Gamma$ is a positive-definite quantity with an unrestricted functional dependence on the wavefunctions as well as the characteristics of the medium in which the particles are immersed.

Iso-product (2) with realization (3) provides an explicit and concrete realization of Bohm's *hidden variables* [7], by therefore supporting the view that quantum mechanics does indeed admit hidden degrees of freedom, provided that quantum axions are realized in a way more general than that of the Copenhagen school.

It should also be noted that iso-product (2) and realizations (3) provide a quantitative representation of the completion of the conventional *quantum entanglement* of *point-like* particles under Hamiltonian interactions, into the covering *Einstein-Podolsky-Rosen entanglement* [13] which is applicable to *extended* particles with non-Hamiltonian interactions due to the deep overlapping of their wavepackets (see Figs. 1 and 2 and references quoted therein).

Despite its simplicity, iso-product (2) requires, for consistency, a compatible isotopy of the entire 20th century applied mathematics, with no exception known to the author [10]. In fact, iso-product (2) requires the following completions of 20th century applied mathematics (see Vol. I of [17] for a general treatment):

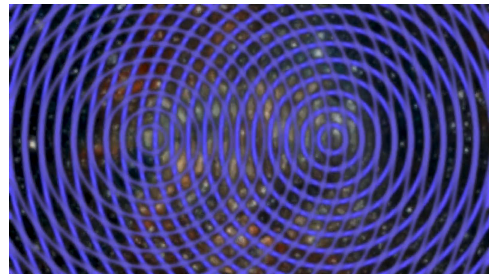

Fig. 2: In this figure, we illustrate the new Einstein-Podolsky-Rosen entanglement of particles introduced in [13], which is characterized by contact, therefore instantaneous and non-Hamiltonian interactions originating in the overlapping of the wavepackets of particles represented via isotopic elements of type (3). As such, the EPR entanglement prevents the applicability of Bell's inequality [18], allows an explicit and concrete realization of Bohm's hidden variables [7], and permits a progressive recovering of Einstein's determinism in the interior of hyperdense particles, with its full recovering at the limit of gravitational collapse [8, 9].

A) The compatible, completion of the basic unit $\hbar = 1$ of quantum mechanics into the integro-differential *iso-unit*

$$\hat{I} = 1/\hat{T} > 0, \quad \hat{I} \star A = A \star \hat{I} = A\,, \tag{4}$$

with ensuing completion of the conventional numeric field $F(n, \times, 1)$ of real $\mathcal{R}$, complex $C$ and quaternionic $Q$ numbers $n$ into the *iso-fields* $\hat{F}(\hat{n}, \star, \hat{I})$ of *iso-real* $\hat{\mathcal{R}}$, *iso-complex* $\hat{C}$ and *iso-quaternionic* $\hat{Q}$ *iso-numbers* $\hat{n} = n\hat{I}$ and related isotopic operations [19] (see [20] for an independent study).

B) The completion of conventional functions $f(r)$ of a local variable $r$ into *iso-functions* that, to have value on an iso-field, must have the structure [21] (see [22] for an independent study)

$$\hat{f}(\hat{r}) = [f(r\hat{I})]\,\hat{I}, \tag{5}$$

and related iso-differential iso-calculus [21] (see [23] for independent studies).

C) The completion of conventional spaces $S$ over $F$ into *iso-spaces* $\hat{S}$ over iso-fields $\hat{F}$ [21]. In particular, the conventional Minkowski space $M(x, \eta, I)$ over $\mathcal{R}$ with spacetime coordinates $x \in \mathcal{R}$, $x^4 = ct$, metric $\eta = \mathrm{Diag}(1, 1, 1, -1)$ and unit $I = \mathrm{Diag}(1, 1, 1, 1)$, is mapped into the *iso-Minkowski iso-space* $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I})$ over the iso-real iso-numbers $\hat{\mathcal{R}}$ [24] (see [25] for an independent study) with iso-coordinates $\hat{x} = x\hat{I} \in \hat{\mathcal{R}}$, iso-metric $\hat{\Omega} = (\hat{\eta})\hat{I} = (\hat{T}\eta)\hat{I}$, and iso-interval

$$(\hat{x}^\rho - \hat{y}^\rho)^{\hat{2}} = (\hat{x}^\rho - \hat{y}^\rho) \star \hat{\Omega}_{\rho\nu} \star (\hat{x}^\nu - \hat{y}^\nu) = = \left[\frac{(x_1 - y_1)^2}{n_1^2} + \frac{(x_2 - y_2)^2}{n_2^2} + \frac{(x_3 - y_3)^2}{n_3^2} - \frac{(t_x - t_y)^2 c^2}{n_4^2}\right] \hat{I}\,, \tag{6}$$

where the exponential term $exp\{-\Gamma\}$ is imbedded into the $n$-characteristic quantities.

D) The compatible completion of all branches of Lie's theory first studied in [16] (see Vol. II of [17] for a general treatment and [26] for an independent study). For instance, an $N$-dimensional Lie algebra $L$ with Hermitean generators $X_k$, $k = 1, 2, ... N$ is completed into the infinite family of Lie-Santilli iso-algebras $\hat{L}$ with iso-commutation rules

$$[X_i \hat{,} X_j] = X_i \star X_j - X_j \star X_i = C^k_{ij} X_k \,, \tag{7}$$

which iso-algebras are called *regular* or *irregular* depending on whether the structure quantities $C^k_{ij}$ are constant or functions of local variables, respectively.

E) The completion of well-known space-time symmetries into the *iso-symmetries* of iso-space-time (6), including the completion of Lorentz's symmetry SO(3.1) into the *Lorentz-Santilli iso-symmetry* $\widehat{\mathrm{SO}}(3.1)$ [24] with iso-transformations

$$\begin{aligned} x^{1'} &= x^1, \quad x^{2'} = x^2, \\ x^{3'} &= \hat{\gamma}(x^3 - \hat{\beta}\tfrac{n_3}{n_4}x^4), \quad x^{4'} = \hat{\gamma}(x^4 - \hat{\beta}\tfrac{n_4}{n_3}x^3)\,, \end{aligned} \tag{8}$$

where

$$\hat{\beta}_k = \frac{v_k/n_k}{c_o/n_4}, \quad \hat{\gamma}_k = \frac{1}{\sqrt{1-\hat{\beta}^2_k}}\,, \tag{9}$$

which provide the invariance of the local speed of light

$$C = \frac{c}{n_4}\,, \tag{10}$$

with consequential *iso-renormalization* of the energy (that is, renormalization caused by non-Hamiltonian interactions)

$$E = mc^2 \;\rightarrow\; \bar{E} = mC^2 = \frac{E}{n_4^2}\,. \tag{11}$$

Additionally, the isotopic representation of the anomalous magnetic moment of the muons requires the completion of the Lorentz-Poincaré symmetry P(3.1) into the *Lorentz-Poincaré-Santilli iso-symmetry* $\hat{\mathrm{P}}(3.1)$ [27], and the completion of the spinorial covering of the Lorentz-Poincaré symmetry $\mathcal{P}(3.1)$ into the *iso-spinorial covering of the Lorentz-Poincaré-Santilli iso-symmetry* $\hat{\mathcal{P}}(3.1)$ (in view of the spin 1/2 of the muons) [28] (see [11] for a recent review and [25] for independent studies).

We should also recall that all aspects of regular iso-mathematics and iso-mechanics can be constructed via the simple *non-unitary transform*

$$UU^\dagger = \hat{I} \neq I\,, \tag{12}$$

of *all* conventional mathematical or physical aspects, under which the unit of quantum mechanics is mapped into the iso-unit of iso-mechanics

$$\hbar = 1 \rightarrow U1U^\dagger = \hat{I}\,, \tag{13}$$

the conventional associative product $AB$ is mapped into the iso-product

$$AB \rightarrow U(AB)U^\dagger = (UAU^\dagger)(UU^\dagger)^{-1}(UBU^\dagger) = \hat{A}\hat{T}\hat{B}\,, \tag{14}$$

and the same holds for the construction of all remaining regular iso-theories.

Finally, we recall that the isotopic element $\hat{T}$ represents physical characteristics of particles. Hence, the invariance of its numeric value is important for the consistency and experimental verification of any iso-theory. Such an invariance does indeed occur under the *infinite class of iso-equivalence* of isotopic methods which is given by the isotopic reformulation of non-unitary transforms called *iso-unitary iso-transforms*

$$UU^\dagger = \hat{I} \neq I, \quad U = \hat{U}\hat{T}^{1/2}, \quad \hat{U} \star \hat{U}^\dagger = \hat{U}^\dagger \star \hat{U} = \hat{I}\,, \tag{15}$$

under which we have the numeric invariance of the iso-unit [29]

$$\hat{I} \rightarrow \hat{U} \star \hat{I} \star \hat{U}^\dagger = \hat{I}' \equiv \hat{I}\,, \tag{16}$$

and of the isotopic element

$$\hat{A} \star \hat{B} \rightarrow \hat{U} \star (\hat{A} \star \hat{B}) \star \hat{U}^\dagger = \hat{A}'\hat{T}'\hat{B}', \quad \hat{T}' \equiv \hat{T}\,. \tag{17}$$

## 3 The structure of muons

As it is well known, the standard model assumes that muons $\mu^\pm$ are *elementary particles,* under which assumption, the sole known possibility of representing deviation (1) is the search for new particles and/or new interactions.

In this paper, we study the view presented on page 849 of the 1978 paper [30] (see also Section 2.5.5, page 163 of [12] for a recent update) according to which *muons are naturally unstable, and therefore they are composite, with a structure suitable to trigger their decay.*

Muons were then represented in [12] as a hadronic bound state of particles produced free in the spontaneous decays with the lowest mode $\mu^- \rightarrow e^-, e^\pm, e^-$ (tunnel effect of physical constituents), resulting in the three-body hadronic structure model with ordinary electrons

$$\mu^- = (\hat{e}^-_\downarrow, \hat{e}^+_\uparrow, \hat{e}^-_\downarrow)_{hm}, \tag{18}$$

in which the presence of positrons was instrumental for the representation of the muon spontaneous decays and its mean life.

Note that the constituents of model (18) are iso-electrons $\hat{e}^\pm$, rather than ordinary electrons $e^\pm$, due to their contact, non-Hamiltonian interactions due to their deep EPR entanglement (Fig. 3), which requires their characterization via an iso-irreducible iso-unitary iso-representation of $\hat{\mathcal{P}}(3.1)$ [28].

Note also that, since all constituents have point-like charges, the charge radius of the model is given by the radius of the orbit of the peripheral iso-electrons.

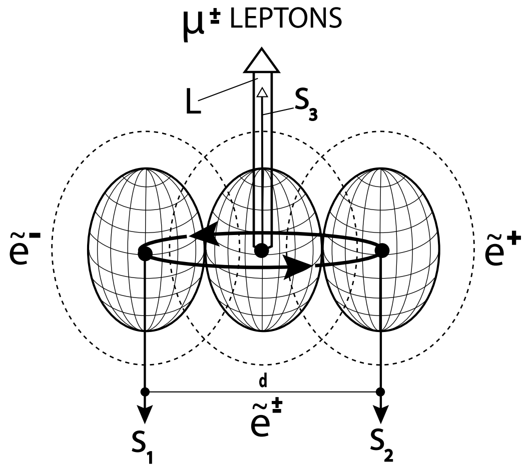


Fig. 3: In this figure, we illustrate structure model (18) of muons in its ground state with $L - 0$, first derived in the 1978 paper [30] (see page 849 on and Section 2.5.5, page 163 of the recent update [12]) as a three-body hadronic bound state of ordinary electrons produced free in the spontaneous decay with the lowest mode. The dashed lines represent the 1 fm wavepackets of the electrons and their overlap represents their deep EPR entanglement. Note the stability of model (18) due to the singlet couplings of the pairs of constituents, and the presence of a positron representing the muon spontaneous decay and its mean life via electron-positron annihilation.

The iso-Schrödinger equation of iso-mechanics, Eq. (5.2.4) of [30] (see also Eqs. (54), page 165, [12]) allowed the non-relativistic representation (here omitted for brevity) of the mass $E = 105.658$ MeV, charge radius of about $R = d/2 = 10^{-13}$ cm, mean life $\tau = 2.19703{\times}10^{-6}$ s, spin $S = 1/2$, charge $Q = \pm e$, and parity.

The 1978 paper [30] provided a representation of the magnetic moment of the muons known at that time, namely, that equal to the magnetic moment of the central electrons. This is due to the lack of contribution to the total magnetic moment from the electron-positron pair in the model which has a null charge and magnetic moment.

## 4 Anomalous magnetic moment of the muons

By keeping in mind that electrons have a point-like *charge* structure (but they have an extended wavepacket with radius of about 1 fm), it appears that the EPR entanglement of the constituents of model (18) (Fig. 3) causes a very small deformation (called *mutation* [30]) of the electrons such as to produce deviation (1). Its quantitative representation can be preliminarily achieved via the following isotopic procedure.

The relationship under isotopies between the magnetic moment and the spin of charged particles has been identified in Eq. (6.5), page 190 of [28], with ensuing relation for the $g$-factors (here reproduced for brevity without its derivation)

$$\hat{g}^{EXP}_{\mu} = \frac{n_4}{n_3} g^{QED}_{\mu} . \tag{19}$$

From value (1), we can then write

$$\frac{n_4}{n_3} = 1.00000000502 . \tag{20}$$

Model (18) for the structure of the muons has been indicated because it is necessary to identify the individual values of the characteristic quantities $n^2_k$, $k = 1, 2, 3$, representing the dimension and shape of the muons, and $n^2_4$, representing its density, with normalization $n^2_\mu = 1$, $\mu = 1, 2, 3, 4$ for conventional electrons and positrons.

Under the assumption of model (18), the total rest energy of the constituents is $E_{cons} = 3E_e = 1.533$ MeV, while the muon rest energy is given by the familiar value $E_\mu = 105.7$ MeV. This implies the excess energy

$$\Delta E = 105.7\,\text{MeV} - 1.533\,\text{MeV} = 104.167\,\text{MeV} , \tag{21}$$

under which the Schrödinger equation no longer admits physically meaningful solutions [12, 30].

Isotopic methods provide a *mathematical* representation of excess energy (21) via iso-renormalization (11) with numeric value of the density

$$n^2_4 = \frac{3E_e}{E_\mu} = \frac{1.533}{105.7} = 0.0149 , \tag{22}$$

under which the consistency of the Schrödinger equation is restored at the isotopic level [12].

Excess energy (21) can be *physically* represented e.g. via the kinetic energy of the peripheral constituents. It should be indicated that missing energy (21) also occurs in the synthesis of the neutron from the hydrogen in the core of stars [31], as well as, more generally, in the synthesis of hadrons from lighter particles [12].

The use of normalization

$$n^2_1 + n^2_2 + n^2_3 = 1 , \tag{23}$$

then provides the desired first approximation of the charge distribution and shape of muons

$$n^2_1 = n^2_2 \approx 0.4926, \quad n^2_3 \approx 0.0149 . \tag{24}$$

The above data confirm the expected very prolate character of structure model (18) due to the point-like character of the constituents.

## 5 Concluding remarks

In the author's view, the most important notion emerging from the preceding study is that of the Einstein-Podolsky-Rosen entanglement representing the instantaneous and continuous communications between extended particles due to the overlapping of their wavepackets, with ensuing non-Hamiltonian interactions represented by (2) and (3), whose consistent treatment required the construction of iso-mathematics and isomechanics [17].

In fact, the EPR entanglement has the following important implications:

1. It prevents the applicability of Bell's inequality [18] due to the presence of non-Hamiltonian interactions first studied in [8];
2. It provides an explicit and concrete realization of Bohm's hidden variables [7] in terms of the isotopic element first achieved in [9]; and
3. It permits a preliminary, yet numerically exact and time invariant representation of all characteristics of muons, including their anomalous magnetic moment [1].

In closing, there seems to be grounds for a new physics, with expected corresponding advances in chemistry and biology, via the axiom-preserving completion of the Copenhagen simplest possible realization of quantum axioms into their broadest possible realization suggested by hadronic mechanics [13].

Received on September 14, 2021

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