# PROCEEDING OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP-3)

DEDICATED TO THE MEMORY OF PROF. HYO CHUL MYUNG

KATHMANDU UNIVERSITY, DHULIKHEL, NEPAL
JANUARY 4 TO 7, 2011



Organized by
**Kathmandu University, Dhulikhel, Nepal**

In Partnership with
**Nepal Mathematical Society**
**Nepal Physical Society**
**The R. K. Santilli Foundation, USA**

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*Kathmandu University (Dhulikhel): January 4 – 7, 2011*

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Editor: Prof. Christian Corda, Italy
Administrator Editor: Prof. Bhadra Man Tuladhar, Nepal

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(ICLATIP - 3, Nepal)

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Table of Contents


| | |
|---|---|
| 4 | Scientific Background |
| 6 | Professor Hyo Chul Myung |
| 8 | Scientific Committee |
| 8 | Local Organizing Committee |
| 9 | Scientific Program Details |
| 16 | Opening Session |
| 17 | Welcome Speech |
| 19 | Reports |
| 24 | List of Participants |
| 25 | Proceedings |
| 671 | Photo Gallery |

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**Professor Christian Corda, Editor In Chief**

## List of Articles

1) ***Can Antimatter Asteroids, Stars and Galaxies be Detected with Current Means?*, 25**
**Ruggero Maria Santilli**

2) ***Are masses of Elementary Particles and of the Solar System Truly Known?, 37***
**Ruggero Maria Santilli**

3) ***Studies of Multi-Valued Hyperstructures for the Characterization of Matter-Antimatter Systems and their Extension, 45***
**B. Davvaz, R.M. Santilli, T. Vougiouklis**

4) ***Proposal for the Realization of Santilli's Comparative Test on the Gravity of Electrons and Positrons Via a Horizontal Supercooled Vacuum Tube, 57***
**Victor-Otto de Haan**

5) ***The Santilli's Theory 'Invasion' in Hyperstructures., 69***
**Thomas Vougiouklis**

6) ***A Brief Survey of Applications of Generalized Algebraic Structures and Santilli-Vougiouklis Hyperstructures., 91***
**Bijan Davvaz**

7) ***The Kerr-Schild Prequantum Geometry: Initial Irreversibility, and the Transition to a Reversible Classical* Gravity., 111**

**Alexander Burinskii**

*8)Thermodynamic Insight of Irreversibility.*, 135
**Anil A. Bhalekar**

*9) Santilli Lie-Admissible Theory and Genomathematics.*, *163*
**Sergei Silvestrov**

*10) Additional Confirmation of "Intermediate Controlled Nuclear Fusion" Without Harmful Radiations or Waste.*, *189*
**Ruggero Maria Santilli**

*11) Multitime Optimal Control for Quantum Systems.*, 203
**Constantin Udriste**

*12) A new Structure in Semidynamical Systems Based on Santilli's Isotheory, 219*
**M.R. Molaei**

*13) Professor Santilli's Intermediate Controlled Nuclear Fusion.*, *227*
**Leong Ying**

*14) Quantum Mechanics, Mathematical Statistics And Deformed Communication Relations, 235*
**L. Skala**

*15) Nuclear Fusion Process With Antimatter Can Account For Dark Energy Mechanism Driving an Accelerated Cosmic Expansion.*, *245*
**Leong Ying**

*16) The Irreversible Phase Transition of Material Becoming.*, *249*
**Erik Trell**

*17) Theoreme Fonctionnel De Bartfai.*, *271*
**A. Ould Khal**

*18) General Covariance for a Proposal for 4-D Gravity.*, 279
**Eyo Eyo Ita III**

*19) Santilli's Etherino as a (g) + (em) Interaction.*, 287
**Athanassios Nassikas**

*20) Fibonacci Generation of Natural Numbers and Prime Numbers.*, 305
**Stein E. Johansen**

*21) Irreversible Gravitational Collapse: Black stars or black holes?*, 411
**Christian Corda, Herman J. Mosquera Cuesta**

*22) Nonunitary Lie-Isotopic and Lie-Admissible Scattering Theories of Hadronic Mechanics, I: Conceptual and Mathematical Foundations.*, 423
**R. M. Santilli, A.O. E. Animalu**

*23) Nonunitary Lie-Isotopic and Lie-Admissible Scattering Theories of Hadronic Mechanics, II: Deformations-Isotopies of Lie's Theory, Special Relativity, and Mechanics.*, 461
**R. M. Santilli, A. O. E. Animalu**

*24) Nonunitary Lie-Isotopic and Lie-Admissible Scattering Theories of Hadronic Mechanics, III: Basic Lie-Isotopic Formulation without Divergencies.*, 521
**R. M. Santilli, A. O. E. Animalu**

*25) Nonunitary Lie-Isotopic and Lie-Admissible Scattering Theories of Hadronic Mechanics, IV: Reversible Electron-Proton and Electron-Positron Scatterings.*, 547
**R. M. Santilli, A. O. E. Animalu**

*26) Nonunitary Lie-Isotopic and Lie-Admissible Scattering Theories of Hadronic Mechanics, V: Foundations of the Genoscattering Theory for Irreversible Processes.*, 581
**R. M. Santilli, A. O. E. Animalu**

*27) Iso-Superconductivity Model: Generalization of the Cooper Pair Formation Model of Superconductors.*, 639
**Godfrey E. Akpojotor, Alexander E. Animalu**

*28) It's About Time.*, 657
**Dr. R. T. Herrod**

29) *Group Representation of Johansen Revolving Method to Generate the Prime Numbers with Implications to Genonumbers.*, 667
**John Strand**

## SCIENTIFIC BACKGROUND

During the 20th century it was generally believed that the irreversibility over time of our macroscopic environment was "illusory" (sic) because, when macroscopic events are reduced to their elementary particle constituents, irreversibility "disappears" (sic) and one recovers nice elementary particles in the reversible conditions necessary for the applicability of special relativity, quantum mechanics and quantum chemistry.

As part of his lifelong research in the field, the Italian American applied mathematician Ruggero Maria Santilli (see CV, prizes and nominations at http://www.santilli-foundation.org) has proved the following:

*NO REDUCTION THEOREM: A classical system that is irreversible over time cannot be consistently reduced to a finite number of elementary particles all reversible over time and, voice versa, a finite number of elementary particles all in reversible conditions cannot yield a macroscopic irreversible event under the correspondence or other principles.*

The above theorem establishes that **irreversibility originates at the most ultimate structure of nature.** For instance, the irreversibility of a spaceship during re-entry in our atmosphere originates from the nonlinear, nonlocal-integral and nonpotential-nonhamiltonian, thus irreversible interactions between the electron orbitals of the peripheral atoms of the spaceship and the corresponding orbitals of the atmosphere. Similar origins have been identified for other irreversible events, including inelastic, thus irreversible, high energy scattering processes.

Above all, **all energy releasing processes at the particle, nuclear, atomic and chemical levels are structurally irreversible over time. Consequently, basic advances in irreversible processes are crucial for the development, in due time, of much needed new clean energies and fuels so, whose study is an important aim of the Conference.** Due to such a societal relevance of irreversibility, our Foundation encourages a wide participation by experts in different fields for communal advances.

By recalling that special relativity, quantum mechanics, quantum chemistry, and 20th century sciences in general are reversible, the above No Reduction Theorem has far reaching implications since it has stimulated the initiation of the broadening of the scientific knowledge of the 20the century. Another aim of the Conference is that of identifying the status of the research in irreversibility as well as discuss new frontiers, such as much needed new algebras, geometries, functional analysis and other methods with such an irreversible structure to achieve direct compatibility with thermodynamical laws, beginning with a quantitative formulation of the entropy, as a necessary condition for an actual representation of nature.

**The significance of Lie-admissible algebras for irreversibility can be outlined as follows.** As it is well known, 20th century sciences are based on Lie algebras with familiar product $[A, B] = AB - BA]$ and known time evolution for a Hermitean operator $i\, dA/dt = AH - HA$. Hence, the reversibility of special relativity, quantum mechanics, quantum, chemistry and other 20th century disciplines is reducible to the primitive invariance of the Lie product under anti-Hermiticity, $[A, B] = - [A, B]^+$.

During his Ph. D. studies, Santilli proposed in 1967 [1] the embedding of lie algebras in covering algebras with the first known deformed product $(A, B) = pAB - qBA$ , where p, q and p +/- q (called lambda and my in the original paper) are non-null scalars, and the first known deformed time evolution $idA/dt = pAH - qHA$, where the product (A, B) is Lie-admissible (as well as Jordan-admissible) in the sense that the attached antisymmetric (symmetric) product is Lie (Jordan.

Subsequently, when at the Department of Mathematics of Harvard University under DOE grants

ER-78-S-02-47420.A000, AS02-78ER04742, DE-ACO2-80ER10651s, Santilli proposed in 1978 [2] the most general known Lie-admissible and Jordan-admissible product (A, B) = ARB - BSA where R and S are nonsingular operators, with Lie-admissible time evolution idA/dt = ARH - HSA that is manifestly irreversible because no longer invariant under anti-Hermiticity.
A new mathematics based on generalized different units for ordered products to the right (representing motion forward in time) and inequivalent ordered products to the left (representing motion backward in time) had to be developed because, when applied to Santilli's Lie-admissible formulations, the mathematics underlying Lie's theory (conventional numerical fields, functional analysis, differential calculus, etc.) would lead to different numerical values under the same conditions at different times, loss over time of Hermiticity-observability (Lopez lemma), violation of causality, and other catastrophic inconsistencies.

Following contributions by a large number of authors reported in the *Third Announcement* of our Foundation 9see http://www.santilli-foundation.org/Announcments.html ), the latest comprehensive presentation including the needed new mathematics, the new irreversible mechanics, the proof of its universality with invariance, and a number of specific applications to irreversible processes, can be found in the 2006 Nuovo Cimento memoir [3]. Additional information can be found in the above quoted website of the Conference as well as of our Foundation.

[1] "Embedding of Lie-algebras in Lie-admissible algebras"
[1] R. M. Santilli, Nuovo Cimento Vol. 51, 570 (1967),
http://www.santilli-foundation.org/docs/Santilli-54.pdf

[2] "On a possible Lie-admissible covering of Galilei's relativity
in Newtonian mechanics for nonconservative and Galilei form-noninvariant systems,"
R. M. Santilli, Hadronic J. {\bf 1}, 223-423 (1978), available in free pdf download from \\
http://www.santilli-foundation.org/docs/Santilli-58.pdf

[3] "Lie-admissible invariant representation of irreversibility for matter and antimatter at the classical and operator levels," R. M. Santilli, Nuovo Cimento B Vol. 121, 443 (2006),
http://www.santilli-foundation.org/docs//Lie-admiss-NCB-I.pdf



**Professor Hyo Chul Myung**

Born on June 15, 1937

Dr. Myung received his B.S. and M.S. degrees in mathematics from Seoul National University and his Ph.D. degree in mathematics from Michigan State University in 1970. Since then he has taught at the University of Northern Iowa for over 25 years. In 1995 he returned to Korea to accept a professorship at KAIST and joined, in 1996, then newly established Korean Institute for Advanced Studies (KIAS).

Dr. Myung was one of the founding members of KIAS ( http://www.kias.re.kr ). He served as its acting president in October 1996 when KIAS was first established, and since July of 2007 served as its fourth president until his death. He leaves behind one of the premier institutions of advanced research, modeled after the Institute for Advanced Study (IAS) in Princeton that housed Einstein, Oppenheimer and von Neumann.

Fields of interest: Nonassociative rings and algebras, Lie algebras, Lie-admissible algebras, homogeneous spaces and H-spaces, mathematical physics, invariant system theory, algebras and differential equations.

Organization of International workshops and conferences:

- Co-organizer of "The First Workshop on Lie-Admissible Formulations", held at Harvard University, December 10-15, 1978.
- Co-organizer of "The Second Workshop on Lie-Admissible Formulations", held at Harvard University, August 1-8, 1979.
- Co-organizer of "The Third Workshop on Lie-Admissible Formulations", held at University of Massachusetts, New Harbor Campus, Boston, August 4-9, 1980.
- Co-organizer of "The Fourth Workshop on Lie-Admissible Formulations", held at the Institute for Basic Research, Cambridge, Mass., August 3-7, 1981.
- Co-organizer of "The Fifth Workshop on Lie-Admissible Formulations", and the First Workshop on Hadronic Mechanics", held at the Institute for Basic Research, Cambridge, Mass., August 2-7, 1983.
- Co-organizer of "The First International Conference on Non-potential Inter actions and Their Lie-Admissible Treatments", held at University of Orleans, Orleans, France, January 5-9, 1982.
- Co-organizer of "The Second Workshop on Hadronic Mechanics", held at The Center \A. Volta", Villa Olmo, Como, Italy, August 1-3, 1984.
- Co-organizer of "The Third Workshop on Hadronic Mechanics" held at University of Patras, Patras, Greece, August 25-30, 1986.
- Co-organizer of "The Fourth Workshop on Hadronic Mechanics and Nonpotential Interactions", held at University of Skopje, Skopje, Yugoslavia, August 22-26, 1988.
- Organizer (Conference Chairman) of "The 5th International Conference on Hadronic Mechanics and Nonpotential Interactions", held at the University of Northern Iowa, Cedar Falls, Iowa, August 13-17, 1990
- Organizer (Co-Chair with E. Zelmanov) of "International Conference on Recent Progress in Algebra", held at KAIST and KIAS, Taejon-Seoul, August 11-15, 1997.

- Organizer of "Anniversary Symposium of KIAS", (Mathematics Division), November 26, 1997.
- Chair of Organizing Committee for "KIAS Number Theory Conference", December 8-12, 1997, KIAS, Seoul, Korea.
- Chair of Organizing Committee for "KIAS Algebraic Geometry Conference", October 13-16, 1998, KIAS, Seoul, Korea.
- Member of Scientific Committee for "4th International Conference on Nonassociative Algebras and its Applications", held at University of Sa^o Paulo, Sa^o Paulo, Brazil, July 19-25, 1998.
- Co-chair of Organizing Committee for "KIAS Lie Theory Conference", October 5-8, 1999, KIAS, Seoul, Korea.
- Co-editor of Proceedings of \Korea-Japan Joint Workshop in Mathematics 2000", November 7-9, 2000, KIAS, Seoul, Korea.
- Member of Organizing Committee for "International Conference on Lie and Jordan Algebras, Their Representations and Applications", May 13-18, 2002, Guaruja, Brazil.
- Member of Organizing Committee for "Algebraic Groups and Quantum Groups", April 7-9, 2003, KIAS, Seoul, Korea.
- Member of International Advisory Board for "5th International Conference on Nonassociative Algebra and Its Applications", July 27-August 3, 2003, Mexico.
- Member of Organizing Committee for "International Conference on Lie Algebras and Related Topics ", October 20-23, 2003.
- Member of the Scientific Committee for "International Conference in Algebras,", August, 2007, Sao Paulo, Brazil.
- Member of Scientific Committee for "The Second International Congress of Algebra and Combinatorics`', July, 2007, Beijing, China.
- Member of Scientific Committee for "Pacific Rim Mathematical Association Congress", July, 2009, Sydney, Australia.

Over 30 review articles have been published in Mathematical Review, American Mathematical Society.

Professor Myung's commitment to support Korean Women in Mathematical Sciences (KWMS) comes on the heels of the agreement he signed on August 17, 2009 to donate the bulk of his fortune, valued at 300 million won ($240,900), to KIAS.

Even as he faces death, Hyo Chul Myung continues to devote his life to science.

Professor Hyo Chul Myung died on February 11, 2010 in Seoul, South Korea after a long battle with cancer.at the age of 73.

**The Scientific committee has decided to dedicate this conference in honour of this outstanding Mathematician.**

## Scientific Committee

Prof. Ruggero Maria Santilli – Honorary Chairman
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Prof. Bhadra Man Tuladhar – Chairman
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## Local Organizing Committee

Prof Pushpa Raj Adhikary, Kathmandu University, is the Chairperson of LOC, and includes:

Prof. Panna Thapa, Kathmandu University

Prof. Shekhar Gurung, Tribhuvan University

Prof. Gajendra Thapa, Tribhuvan University

Prof. Dal Bahadur Adhikary, Tribhuvan University

Dr. Chet Raj Bhatta, Tribhuvan University

Dr. Binil Aryal, Tribhuvan University

Dr. Raju Khanal, Tribhuvan University

Dr. Kanhaiya Jha, Kathmandu University

Dr. Deepak Subedi, Kathmandu University

Dr. Jyoti Upadhyay, Kathmandu University

Dr. Dipak Raj Adhikary, Kathmandu University

**Scientific Program Details**

**Day 1**

**January 4, 2011, Tuesday**

**LANGTAM Hall, Hotel Miracle, Dhulikhel**

| Time | Program |
|---|---|
| **Session on Mathematics**<br>**Session Chair: Prof. Pushpa Raj Adhikary** | |
| 14.00 – 14.45 | Session 1: **Ruggero Maria Santilli**<br>Birth, History and Outline of Genomathematics for the Invariant Lie-Admissible Treatment of Irreversible Processes<br>*Available in free viewing and downloading formats from http://www.world-lecture-series.org/* |
| 14.45 – 15.30 | Session 2: **Thomas Vougiouklis**<br>The Santilli's Theory 'Invasion' In Hyperstructures |
| 15.30 – 16.15 | Video Session II: **Noriaki Kamiya**<br>From Lie-Admissible Algebras to Admissible Triple Systems<br>*Available in free viewing and downloading formats from http://www.world-lecture-series.org/* |
| 16.15 – 16.30 | Tea break |
| 16.30 – 17.15 | Session 3: **Bijan Davvaz**<br>A brief survey of applications of algebraic hyperstructures and Santilli-Vougiouklis hyperstructures with hyperunits . |
| 17.15 – 18.00 | Session 4: **Mohammad Reza Molaei**<br>A New Structure in Semi-Dynamical Systems Based on Santilli's Isotheory. |
| 18.00 – 18.45 | Video Session III: : **Tepper L. Gill and W. W. Zachary**<br>The Santilli S* Algebras and The Navier-Stokes Problem.<br>*Available in free viewing and downloading formats from http://www.world-lecture-series.org/* |
| 19.00 | Dinner: Hotel Miracle |

## Scientific Program Details

### Day 2

### January 5, 2011, Wednesday

### LANGTAM Hall, Hotel Miracle, Dhulikhel

| Time | Program |
|---|---|
| **Session on the Invariant Lie-admissible Treatment of Irreversible Processes**<br>**Session Chair: Prof. Shekhar Gurung** | |
| 09.00 – 9.45 | Session 5: **Ruggero Maria Santilli**<br>Invariant Lie-Admissible Classical and Operator Mechanics for Matter<br>*Available in free viewing and downloading formats from http://www.world-lecture-series.org/* |
| 09.45 – 10.30 | Session 6: **Ivan Gandzha**<br>Hadronic Mechanics: Lie-Admissible Covering of Classical and Quantum Mechanics |
| 10.30 – 11.15 | Session 7: **Alexander E.** Animalu and RM Santilli<br>Lie-isotope and Admissible Scattering Theory of Hadronic Mechanics. |
| 11.15 – 11.30 | Tea break |
| 11.30 – 12.15 | Session 8 **Athanassios A. Nassikas**<br>Santilli's Etherino as a (G)+(EM) Interaction |
| 12.15 – 13.00 | Session 9: **Stepan S. Moskaliuk**<br>Santilli Category of Isotopic Groups. |
| 13.00 - 14.00 | Lunch |

**Day 2**

**January 5, 2011, Wednesday**

**LANGTAM Hall, Hotel Miracle, Dhulikhel**

| | |
|---|---|
| **Session on Lie-admissible Algebra**<br>**Session Chair: Prof. Erik Trell** | |
| 14.00 – 14.45 | Session 10: **Godfrey E. Akpojotor** and Alexander E. Animalu<br>Iso-Superconductivity Model: the generalization of conventional BCS Model to nonconventional superconductors. |
| 14.45 – 15.30 | Session 11: **Lubomir Skala**<br>Lie-admissible algebras and mathematical statistics |
| 15.30 – 16.15 | Session 12: **Constantin Udriste**<br>Multitime Optimal Control for Quantum Systems |
| 16.15 – 16.30 | Tea break |
| **Session Chair: Prof. Athanassios A. Nassikas** | |
| 16.30– 17.15 | Session 13: **I. Felner, Y. Yeshurun**<br>Irreversible magnetic behavior of amorphous and sulfur doped carbon materials. |
| 17.15 – 18.00 | Session 14 : **Anil A Bhalekar**<br>Irreversibility. A Thermodynamic Insight |
| 18.30 | Dinner: Dhulikhel Mountain Lodge |

## Scientific Program Details

### Day 3

### January 6, 2011, Thursday

### LANGTAM Hall, Hotel Miracle, Dhulikhel

| Time | Activities |
|---|---|
| **Session on Open Problems in Antimatter**<br>**Session Chair: Prof. Christain Corda** | |
| 09.00 – 09.45 | Session 15:**Ruggero Maria Santilli**<br>Elements of The Isodual Theory of Antimatter, Its Prediction of Antigravity and The Open Problem of Detecting Possible Antimatter Asteroids.<br>*Available in free viewing and downloading formats from http://www.world-lecture-series.org/* |
| 09.45 – 10.30 | Session 16: **Victor O. de Hann**<br>Proposal for the realization of Santilli's comparative test on the gravity of electrons and positrons via a horizontal supercooled vacuum tube. |
| 10.30 – 10.45 | Tea break |
| 10.45 – 11.30 | Session 17: **Leong Ying**<br>Nuclear fusion process with antimatter can account for Dark Energy mechanism driving an accelerated cosmic expansion. |
| 11.30 – 12.15 | Session 18: **Stein Johansen**<br>Some epistemological implications of Santilli hadronic mechanics and mathematics – with special emphasis on irreversibility issues. |
| 12.15 – 13.00 | Session 19: **Anita Leirfall**<br>Shaking Hands Across the Universe Absolute Space and A Priori Directionality. |
| 13.00 - 14.00 | Lunch |

**Day 3**

**January 6, 2011, Thursday**

**LANGTAM Hall, Hotel Miracle, Dhulikhel**

| | |
|---|---|
| **Session on New Energies**<br>**Session Chair: Mr. Ken Yang** | |
| 14.00 – 14.45 | Session 20: **Allen Feng**<br>MagneGas Technology in China. |
| 14.45 – 15.30 | Session 21: **Pinchas Mandell**<br>An Introduction to MagneGas Technology |
| 15.30 – 16.15 | Session 22: **Leong Ying**<br>Verification of intermediate nuclear fusions without harmful radiation and the production of magnecular clusters. |
| 16.15 – 16.30 | Tea break: |
| 16.30 – 17.00 | Open Forum & Discussion led by RM Santilli |
| **Session on Technology**<br>**Session Chair: Prof. Bhola Thapa** | |
| 17.00 – 17.45 | Session 23: **Yong Hee Chung**<br>Feasibility of Reduction of Carbon Dioxide to Energy Resource. |
| 17.45 – 18.30 | Session 24: **Erik Trell**<br>The Irreversible Phase Transition of Material Becoming and Process |
| 19.00 | Dinner: Himalayan Horizon |

## Scientific Program Details

### Day 4

### January 7, 2011, Friday

### LANGTAM Hall, Hotel Miracle, Dhulikhel

| Time | Activities |
|---|---|
| **Session on Irreversible Thermodynamical processes**<br>**Session Chair: Prof. Bhadra Man Tuladhar** | |
| 09.00 – 09.45 | Session 25: **Christian Corda**<br>Irreversible gravitational collapse: black stars or black holes? |
| 09.45 – 10.30 | Session 26: **Alexander Burinskii**<br>Irreversibility of the Black Hole Geometries and Transition to Reversible Classical Gravity. |
| 10.30 – 10.45 | Tea break |
| **Session Chair: Prof. Alexander E. Animalu** | |
| 10.45 – 11.30 | Session 27**: Stein Johansen**<br>Fibonacci generation of natural numbers and prime numbers. |
| 11.30 – 12.15 | Session 29**: Jon Martinsen Strand**<br>Group Representation and Santilli Genonumber Representation of Johansen Revolving Prime Number Code. |
| 12.15 – 13.00 | Session 30**: Gireesh Baghotia**<br>Strapping Asymmetric 6T-SRAM cell for low power operation in nano-CMOS technologies. |
| 13.00 - 14.00 | Lunch |

**Day 4**

**January 7, 2011, Friday**

**LANGTAM Hall, Hotel Miracle, Dhulikhel**

| **Session Chair: Prof. Stein Johansen** | |
|---|---|
| 14.00 – 14.45 | Session 30 **Eyo Eyo Ita**<br>Instanton representation of Plebanski gravity: General covariance and implementation of the Gauss' law constraint. |
| 14.45 – 15.30 | Session 31: **Häunkar Kayhan and Cenap Äozel**<br>On Atiyah-Singer Index of Dirac Operator on Six Dimensional Sphere with a *So*(6)-Gauge Theory |
| 15.30 – 16.00 | Tea break |
| 16.00 – 17.30 | Closing Session |
| 18.00 | Dinner: Dhulikhel Lodge Resort |

**Third International Conference on**
**Lie-Admissible Treatment of Irreversible Processes**
**(ICLATIP-3)**
Dedicated to the Memory of Prof. Hyo Chul Myung
**January 4**
**to 7, 2011**

## Opening Ceremony

January 4, 2011, Tuesday
CV Raman Auditorium
Kathmandu University, Dhulikhel

10.00 Arrival of Guests

10.10 Welcome address
- Prof. Bhadra Man Tuladhar, Chair, ICLATIP-3 Nepal

10.15 Inauguration jointly by:
Prof. Suresh Raj Sharma
Vice Chancellor, Kathmandu University
His Excellency Hong Sungmog,
Ambassador, Republic of Korea to Nepal

10.15 Memory of Prof. Hyo Chul Myung
Prof. Jongmann Yang, President, The Korean space Science Society

10.18 Greeting by His Excellency Hong Sungmog,
Ambassador, Republic of Korea to Nepal

10.20 High-light of the Conference
- Prof. Ruggero Maria Santilli,
Honorary Chairman, ICLATIP-3 Nepal &
President, The Institute for Basic Research, USA

11.35 Greetings:
Prof. Shekhar Gurung, President,
Nepal Physical Society
Dr. Chet Raj Bhatta, Secretary,
Nepal Mathematical Society

11.50 Remarks by Prof. Suresh Raj Sharma,
Vice Chancellor, Kathmandu University

11.55 Video Lecture: Prof. RM Santilli
**Need, History and Foundations of the Lie-Admissible Treatment of Irreversible Processes**
*Available in free viewing and downloading formats from http://www.world-lecture-series.org/*

12.45 Vote of Thanks
- Prof, Pushpa Raj Adhikary,
Chair, LOC, ICLATIP-3 Nepal

# Welcome Speech

Very warm good morning!

Respected Vice chancellor Prof. Suresh Raj Sharma;
His Excellency Hong Sungmog, Ambassador of Republic of Korea to Nepal;
Invited guests; Faculty and staff of Kathmandu University and other academic institutions; and above all the Participants of the ICATIP-3 Nepal and their family members.

**Welcome.**

I fell privileged to welcome the distinguished participants to the **Third International Conference on Lie-Admissible Treatment of Irreversible Processes (ICLATIP-3)** being organized by Kathmandu University and supported by Nepal Mathematical Society, Nepal Physical Society and The Santilli Foundation, USA.

The idea to organize this conference came up in March of 2009 during my correspondence with Prof. Ruggero Maria Santilli in connection of his nomination to the newly established ***The 2010 V. Ambartsumian International Prize in Astronomy and Astrophysics***.

My first contact with Prof. Santilli was his letter of March 28, 1982 which was an answer to my letter of February 26, 1981.

I first met Prof. RM Santilli in person during my participation in the First **International Conference on Nonpotential Interactions and their Lie-admissible Treatment** that was held in the Université d'Orléans, France from January 5-7, 1982. He was of one of the organizer of the conference. The other organizing members were Prof. J. Fronteau and Prof. A. Tellez-Areans from Université d'Orléans and Prof. Hyo Chul Myung. At that time Prof. Myung was in the University of Northern Iowa. The second conference was held in Italy in 1995 only.

The Scientific Committee has decided to dedicate this conference in Memory of Prof. Hyo Chul Myung, the outstanding Mathematician and past President of the Korea Institute for Advanced Study (KIAS). Dr. Myung was one of the founding members of KIAS. Even as he faces death, Hyo Chul Myung continues to devote his life to science. Professor Hyo Chul Myung died on February 11, 2010 in Seoul, South Korea after a long battle with cancer, at the age of 73.

The primary aims of the conference are to cover mathematical, physical and industrial methods to study new clean energies with topics on irreversible fields, spaces, algebra's, geometries in MATHEMATICS; on irreversible mechanics, statistics in PHYSIC,: new magnecular and others combustion's. This conference has brought together scientists working in various fields on the treatment of irreversible systems. During the course of the four days starting from today, January 4 till 7, 2011, Mathematicians, Physicists, Chemist and Engineers from 18 countries namely: China, Czech Republic, Greece, India,

Iran, Israel, Italy, Korea, Nepal, Netherland, Nigeria, Norway, Rumania, Russia, Sweden, Turkey, Ukraine and USA will make 31 presentations and 4 video lectures.

I hope that your participation in this very important Conference will contribute to the advancement of the New Science and its application for the betterment of the mankind

We will do all our best to make your stay in this beautiful city of Dhulikhel comfortable and memorable.

Prof. Bhadra Man Tuladhar
Chair, ICLATIP-3, Nepal
Kathmandu University
Dhulikhel, Nepal
January 4, 2011.

# REPORT ON THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIVBLE PROCESSES (ICLATIP - 3, NEPAL)

***Kathmandu University*** (Dhulikhel) : ***January 3 – 7, 2011***

Mathematics is the mirror of civilization. Mathematics and Physics along with Chemistry are three important fundamental subjects for the educational system of the nation in order to achieve scientific developments. These classical subjects are distinct from others having both pure as well as applied forms interrelated to each others. In spite of the least interest and support from government sectors in Nepal which also contradicts the world's trends of basic science education, there are very few national and international events in these core areas. With the view of providing a platform to explore the new areas teaching, research and development, Kathmandu University (**KU**) in collaboration with ***The R.M. Santilli Foundation*** (USA) and in partnership of Nepal Mathematical Society (**NMS**) and Nepal Physical Society (**NPS**), has successfully organized **"The Third International Conference on Lie-Admissible Treatment of Irreversible Processes" (ICLATIP -3, Nepal)** at Kathmandu University, Dhulikhel from **January 03 – 07, 2011**. This conference was dedicated to the renown Korean mathematician **Prof. Dr. Hyo Chul Myung** (1937 – 2010), one of the founding member of leading Korean Institute for Advanced Studies (**KIAS**) (http://www.kias.re.kr). Honorable **Prof. Dr. Ruggero Maria Santilli** was the honorary chairman, **Prof. Dr. Bhadra Man Tuladhar** was the local chair of **ICLATIP – 3, Nepal** and **Prof. Dr. Pushpa Raj Adhikary** was the chair of local organizing committee (**LOC**) of this event.

On January 4, 2011, Tuesday, the program commenced with the inaugural session at C.V. Raman Auditorium Hall in which **Prof. Bhadra Man Tuladhar**, the president of NPS and the registrar of KU, presented his welcome speech. The program was inaugurated jointly by honorable **Prof. Dr. Suresh Raj Sharma**, the Vice Chancellor of KU and His Excellency **Hong Sungmog**, ambassador of the Republic of Korea to Nepal, by lightning the lamp. Also, **Prof. Dr. Jongmann Yang** expressed his view regarding to the memory of late Prof. Hyo Chul Myung. **Prof. Dr. Ruggero Maria Santilli** highlighted the importance of the conference. The program was followed by greetings from His Excellency Hong Sungmog, the ambassador of the Republic of Korea to Nepal, **Prof. Dr. Shekhar Gurung**, the president of **NPS** and **Dr. Chet Raj Bhatta**, the secretary of **NPS**. The Vice Chancellor of KU **Prof. Dr. Suresh Raj Sharma** expressed his remarks on the importance of this conference and related activities with KU in Nepal. Then, there was shown a video clip on the need, history and foundations of the Lie-Admissible Treatment of Irreversible Processes. Finally, **Prof. Dr. Pushpa Raj Adhikary** gave the vote of thanks.

There were altogether **thirty** papers presented in this international conference which were divided among **seven** scientific sessions with **two** special video sessions. All scientific sessions were held at Langtam Hall of Hotel Miracle, Dhulikhel. The theme of the first session on ***January 4, 2011***, Tuesday, was "*Mathematics*" in which four papers were presented and the session was chaired by **Prof. Dr. Shekhar Gurung** (TU). The theme of the second session on ***January 5, 2011***, Wednesday, was "*The invariant lie-admissible*

*treatment of irreversible processes*" in which five papers were presented and the session was chaired by **Prof. Dr. Pushpa Raj Adhikary** (KU). The theme of the third session on ***January 5, 2011***, Wednesday, was "*Lie-admissible algebra*" in which five papers were presented and the session was chaired by **Prof. Erik Trel** (Sweden) and **Prof. Athanassions A. Nassikas** (Greece). The theme of the fourth session on ***January 6, 2011***, Thursday, was "*Open problems in antimatter*" in which five papers were presented and the session was chaired by **Prof. Dr. Bhadra Man Tuladhar** (KU). Also, the theme of the fifth session on ***January 6, 2011***, Thursday, was "*New energies*" in which three papers were presented and the session was chaired by **Mr. Ken Yang** (China). The theme of the sixth session on ***January 6, 2011***, Thursday, was "*Technology*" in which two papers were presented and the session was chaired by **Prof. Dr. Bhola Thapa** (KU). The theme of the seventh session on ***January 7, 2011***, Friday, was "*Irreversible thermo dynamical processes*" in which seven papers were presented and the session was chaired by **Prof. Dr. Bhadra Man Tuladhar** (KU), **Prof. Alexander E. Animalu** (Nigeria) and **Prof. Stein Johansen** (Norway).

There was a enthusiastic participation of prominent mathematicians, physicists, chemists from various countries like India, China, USA, Korea, Romania, Russia, Greece, Turkey, Sweden, Norway, Iran, Holland, Czech Republic, Nigeria, Nepal. The remarkable contributions and valuable interaction by participants in this conference have added to the scientific values of this event. This conference has become historical and memorable for us.

The program was a successful event with the team work of all local organizing committee members which was coordinated by Prof. Tuladhar and Prof. Adhikary together with the untiring support of KU staffs Mr. Mahendra Niraula, Sunil Khanal, Yashu Shrestha, Abmika Thapa, Prem Bania, Shreeram Khatri, KU ISMS team.

Thanks!

Dec. 30, 2010 – Jan. 7, 2011
Dorte M. Zuckerman writes report on:

The third International Conference on Lie-Admisssible Treatment of Irreversible Processes
( ICLATIP-3 Nepal, Jan 3-7, 2011)

Dedicated to the Memory of Prof. Dr. Hyo Chul Myung

From beginning to end the social events and the conference were very well planned and professionally run thanks to the planning committee, and not least thanks to Prof. Dr. Bhadra Man Tuladhar.

We were received at the airport if we so desired, and driven to the luxurious Hotel Himalaya in Lalitpur (Patan) with an actual view to the mountains. Here Prof. and Mrs. Santilli and Prof. and Mrs. Tuladhar were ready to receive us. Everybody was given a traditional yellow silk welcome scarf by Prof. Tuladhar and a gift bag containing two beautiful prints of Nepal. We also received computer bags with booklets and schedules. An important booklet was one containing the names of the presenting scientists as well as their abstracts.

It was a great idea to have socialization and tours first. This way, people had a chance to get to know each other and get rid of jet lag. We went on sightseeing trips while Prof. Santilli and Prof. Tuladhar had time to set up the technical part of the conference. At night there was more time meeting and greeting each other for tea or dinner at Hotel Himalaya with its varied cuisine (the Nepalese was the best!).

The first sightseeing tour went by bus to Swoyambhu and the Durbar Squares, the latter exhibiting the typical old quaint architecture of Nepal.

New Year's Eve we all went to the Moscow Restaurant, which surprisingly appeared more Nepalese than Russian! There were outdoor drinks and appetizers around a large bonfire in the courtyard surrounded by the restaurant rooms. The entertainment was music and dancing by Nepalese artists. As I finished talking with the guests, and was about to leave, I realized the party had just begun. Indoors was a beautiful set-up for a full dinner buffet. It turned out to be a remarkable evening meeting with people from Kathmandu University. Back at the hotel some guests continued celebrating the New Year.

The next day was a well-needed rest day that I personally used to visit the K-House for girls, an orphanage I had been in contact with before. Sunday morning we went on a wild sightseeing trip out of town high up to Nagakot. The bus moved up a narrow steep dirt road. There was no guard rail, and every time an oncoming vehicle approached, one of the vehicles had to back up or pinch by. You tried not to look down if you had a window seat on the steep side as I did. It was a wild adventure for us six-lamed Americans. Finally we reached the top by foot (4000 m), and had a wonderful breakfast. Between breakfast and lunch, we relaxed outside waiting for the clouds and fog to clear. Voila! There were the Himalayan Mountains. We had to pinch ourselves to realize it was real.

Now, as we left the top and drove down, we hoped the bus had good brakes. We made it, and were in for a great treat at the musical department of the university. The setting was an old temple site. It was close to evening, and we settled under a pagoda-like roof to listen to a truly Nepalese

concert with instruments not known to western culture. Several young men participated in three mini concerts. There was a distinct sense of longing and searching in the rolling tunes alternating with loud drums. Enormous energy was put out by these handsome young students. Meanwhile we were served a traditional snack consisting of a boiled egg, popped rice, a small grilled fish, bread and an herb. It was served on a plate made of palm leaves. To warm us, a strong rice wine was served in small clay bowls with straws (made of real straw!). As a thank you, two of our young guests (daughters of a presenting scientist and who were also accomplished artists) sang a duo very beautifully.

The very next day we moved to Dhulikhel, a most beautiful Alpine Hotel called Miracle with another great view of the snow-covered mountains. That's where it became a favorite question among us to ask, "How many pictures can you take of the Himalayans?" There were no elevators there but lots and lots of staircases.

This was where the conference supporting the New Sciences was going to be held. Forty conference attendees were now in the hotel. Thirty one mathematicians, chemists and physicists were to present from twenty two countries. The first morning we were taken to the nearby modern Kathmandu University where there was an introductory ceremony. Among the speakers were Prof. Bhadra Man Tuladhar, Prof. Suresh Raj Sharma, Vice Chancellor of Kathmandu University, His Excellency Hong Sungmog, Ambassador, Republic of Korea to Nepal, Prof. Jongmann Yang honoring the Memory of Prof. Hyo Chul Myung, Prof. Ruggero Maria Santilli Honorary Chairman, ICLATIP-3, Nepal & President, the Institute for Basic Research, USA, Prof. Christian Corda, Co-Chair, ICLATIP-3 Nepal, Prof. Shekhar Gurung, Nepal Physical Society, Dr. Chet Raj Bhatta, Secretary, Nepal Mathematical Society, and final thanks by Prof. Pushpa Raj Adhikary, LOC, ICLATIP-3 Nepal.

Prof. Santilli presented the first overview of a row of lectures, which were continued the following days. Lunch was then served in the cafeteria. Nepalese foods are among the tastiest foods I have ever had, a delicious mixture of spices and texture, ending with sweet desserts and tea/coffee.

Back at the hotel, the sessions started the next morning. We would break for tea/coffee and cookies between breakfast and lunch and again between lunch and dinner. There would be thirty presenters (some making two presentations). Prof. Santilli made a presentation every morning for three days. In a very up-beat way he would explain his theories of Hadronic Mechanics. He would tell how he had had to invent a new algebra – the first in 600 years since Newton. He had needed to do a lifting of Lie's algebra.

Prof. Santilli explained with crescendo how in the last century a lot of universities influenced by government funds would not support the New Sciences, how they clung to Quantum Mechanics, which could never be applied to fields like biology and cosmology. Santilli's theories have been applied to new clean energies produced by MagneGas, a publicly traded company. The production, he admitted was first meant to be experimental evidence of his theories but eventually it turned into a business.

All the scientists had been asked to tie in their presentations with Hadronic Mechanics.

From the discovery of magnetic fusion to the theory of light moving in wave packets, from new prime numbers in patterns to Kant's directions in space, from the irreversibility over time as seen in a seashell to seeing the universe with the eyes of the New Sciences, from black holes not being part of our space-time to the idea of two or more universes without a 'big bang,' and from the ether being a substratum of constant energy and antimatter to an inspiring and frightening

presentation of global warming, this was absolutely a fantastic conference. There were questions and defending answers, disagreements and discussions. Altogether, it was very inspiring especially for a non-scientist like me.

In spite of, or maybe because of the diverse cultures, the group was able to laugh, joke and sing together at night. First, we had dinner at the Mirabel (Miracle) Hotel with its excellent foods and service as well as friendly atmosphere both indoors and in the upper-level garden. Later we had dinner at the Dhulikhel Mountain Lodge with its attractive textured tablecloths, candles and delicious foods. We also dined in the Dhulikhel Lodge Resort with its open indoor fire under a huge upside-down copper cauldron. Lastly, we went to the Himalayan Horizon with its interesting decor and candle-lit room. In both places we enjoyed a buffet with foods so delicious it was hard not to go back for seconds!

The friendliness and the smiles of the hardworking people of Nepal made up for the inconvenience of the power black-outs. However, being from Tampa (Florida, USA), the lightning capital of the world, I was used to sudden power problems where the old candle becomes your best friend.

Thank you for an unsurpassable and adventurous stay in Nepal!

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*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
Kathmandu University, Nepal, April (2011) pages 25-36

# CAN ANTIMATTER ASTEROIDS, STARS AND GALAXIES BE DETECTED WITH CURRENT MEANS?

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**Abstract**

Via the use of the isodual theory of antimatter, in this note we point out, apparently for the first time, that antimatter asteroids are not necessarily visible with light originating from a matter star, such as light from our Sun, thus constituting a threat for our planet requiring collegial inspection and resolution,

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As it is well known (see, e.g., Ref. [1]), during the 20th century, matter was treated at all levels of study, from Newtonian Mechanics to second quantization, while antimatter was solely treated at the level of second quantization, resulting in a clean scientific imbalance with rather deep implications from particle physics to cosmology some of which will be indicated in this note. The imbalance originated from the absence in special relativity of quantitative means for differentiating *neutral* matter and antimatter, as well as for other shortcomings.

Santilli (see general review [2] and original papers quoted therein) resolved the above imbalance via the construction of a new mathematics, today known as *isodual mathematics,* the related *isodual mechanics and relativity* with the resulting *isodual theory of antimatter.* The understanding of this note requires a knowledge of isodual mathematics and physics, as well as the knowledge that they constitute the isodual branch of the *hadronic mechanics* [3]. In this note we recall the truly essential aspects for minimal self sufficiency of these studies, and then focus our attention on the problem of detecting antimatter asteroid, stars and galaxies. We assume the reader is familiar with the evidence according to which Earth has been hit in the past by antimatter asteroids (see Fig. 2)



Figure 1: An illustration of the main objective of this note: can we identify antimatter asteroids with Sun light or to protect our planet we need a new technology? This ;problem will also be studies art a workshop in Italy, September 5-9,. 2011, http://www.workshops-hadronic-mechanics.org/

The main idea of the isodual theory of antimatter can be outlined as follows. Recall that the conventional *charge conjugation* is defined on a Hilbert space $\mathcal{H}$ with states $\psi(x)$ over the field of complex numbers $C$ and can be characterized by expressions of the type

$$C\ \psi(x) = -\ \psi^\dagger(x), \tag{1}$$

where $x$ is the coordinate of the representation space, such as the Minkowski spacetine.

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ntilli [2] constructed the isodual mathematics, mechanics and relativity via an anti-Hermitean conjugation, called *isoduality* and denoted with the upper index $d$, applied to the totality of the mathematics and physics used for matter with no known exception to avoid catastrophic inconsistencies when mixing conventional and isodual formulations. Therefore, the isodual conjugation of an arbitrary classical or operator quantity $A(x, p, ....)$ depending on coordinates $x$, momenta $p$, and any other needed variable is given by

$$A(x, p, ....)\ \rightarrow\ A^d(x^d, p^d, ....) = A(-x^\dagger, -p^\dagger, ....). \tag{2}$$

This conjugation characterizes the novel *isodual unit* $1^d = -1^\dagger$, *isodual real, complex or quaternionic numbers* $n^d = -n^\dagger$, *isodual product* $n^d \times^d m^d = n^s \times (1^d)^{-1} \times m^d$, *isodual functional analysis*, isodual differential calculus, etc. (see Ref. [2] for brevity).



Figure 2: *A view of the devastation caused by the 1908 Tunguska explosion in Siberia that has been estimated as being the equivalent of 1,000 atomic bombs, yet it left no crater or solid residue in the ground. Consequently, the Tunguska explosion is solely representable on a quantitative-numerical way via an antimatter asteroid annihilating in our atmosphere. Other interpretations have been dismissed by calculations because essentially conceptual. For instance, the hypothesis of a comet has been disproved on various quantitative grounds, such as: absence of a necessary depression in the ground caused by the expected huge amount of water; basically insufficient energy to represent the event; inability to represent the luminescence of the entire atmosphere on Earth for days, which luminosity is solely representable via radiations typical of annihilation processes; and other reasons. There is additional evdience of antimatter asteroids hitting our planet, such as large explosions in the upper atmosphere that are known not to be caused by atomic bombs. Additionally astronauts and cosmonauts routinely see "flashes" in the upper atmosphere when in darkness that can solely be interpreted as due to the annihilation of antimatter cosmic rays.*

In particular, the reader should keep in mind that isoduality is the only known consistent procedure for the differentiation between *neutral* as well as charged matter and antimatter at all levels of treatment.

Even though charge and isodual conjugations are both anti-Hermitean, their differences are not trivial. From a physical viewpoint, charge conjugation conjugates states in a Hilbert space, but does not conjugate the local coordinates $x$. This implies that, for 20th century theories, antimatter exists in the same spacetime of matter. At any rate, the relegation of antimatter at the level of second quantization, e.g., via Dirac's "hole theory," leaves the Minkowski spacetime unique, thus entirely characterized by




Figure 3: *Picture recently released by FERMILAB illustrating the apparent existence of antimatter in the universe.*

the fundamental Poincaré symmetry and special relativity.

By contrast, the isodual conjugation additionally maps spacetime coordinates $x$ into the novel *isodual coordinates* $x^d = -x^\dagger$ that are defined on the *Minkowski-Santilli isodual spacetime* $M^d(x^d, \eta^d, 1^d)$, where $\eta$ is the usual Minkowski metric. Therefore, under isoduality, the *Poincaré-Santilli isodual symmetry,* and the *isodual special relativity,* antimatter is predicted to exist in a new spacetime which is distinct from, yet coexisting with our spacetime. In particular, the differences of conventional and isodual spacetimes are not trivial. e.g., because the isodual conjugation of coordinates is different than inversions [2].

It should be remembered that the Minkowski spacetime is, ultimately, a mathematical structure since our senses perceive space and time separately. Due to the full democracy between matter and antimatter, the same applies for the isodual spacetime that is here reviewed under the understanding of its mathematical character. At any rate, despite being the originator of the theory, the author has to admit its inability to "understand" the isodual theory of antimatter, in the same way as the author must admit his inability to "understand" infinite dimensional Hilbert spaces at the foundation of quantum mechanics. The author's sole interest in the isodual theory is the ability of the theory for providing a *mathematical* representation of antimatter compatible with all available experimental evidence at the classical and particle levels, as well as admitting *new* predictions [2].

The achievement of full democracy in the treatment of matter and antimatter has identified a new symmetry called *isoselfduality* [2] with rather intriguing implications. To begin, particle-antiparticle systems are evidently invariant under isoduality, as it is the case for the positronium,

$$(e^-, e^+) = [e^-, (e^-)^d] \equiv (e^-, e^+)^d. \tag{3}$$



Figure 4: *A view the Trifit Nebula, one of the numerous dark nebulas existing in the universe. They are generally interpreted as being due to dense aggregates of matter, thus being opaque to light. Recent studies have indicated the possibility that at least some of the nebulas are caused by antimatter because of their being totally opaque to matter light. In the event confirmed, the latter feature would support the entire content of this note, because it would establish our inability to see antimatter with ordinary light as well as establish the absorption without refraction of matter light originating from clear matter stars in their background.*

Additionally, the imaginary unit, the differential, the Hilbert inner product, and the spacetime line element are isoselfdual 9see Ref. [2] for technical details)

$$i \equiv i^d \tag{4a}$$

$$dx \equiv d^d x^d \tag{4b}$$

$$< \psi| \times |\psi > \times I \equiv < \psi|^d \times^d |\psi >^d \times I^d \tag{4c}$$

$$(x-y)^2 = [(x-y)^\mu \times \eta_{\mu\nu} \times (x-y)^\nu] \times I \equiv$$
$$\equiv [(x-y)^{\mu d} \times^d \eta^d_{\mu\nu} \times^d (x-y)^{\nu d}] \times I^d = (x-y)^{d2d} \tag{4d}$$

The above invariance illustrates the mathematical meaning of the indicated coexistence of matter and antimatter in the same region of space, as well as the reason for the lack of discovery of the isodual theory until recently.

A main difference in the treatment of antimatter between 20th century Einsteinian theories and the novel isodual theories is the following. Special relativity and relativistic quantum mechanics characterize antimatter with the same *positive energy* used for matter. By contrast, the isodual theory characterizes antimatter via a *negative energy* referred to as a *negative unit,* thus being as causal as a positive energy referred to as a positive unit. Similarly, according to the isodual theory, antiparticle evolve

in a *negative time* referred to a *negative unit* of time, thus yielding an evolution as causal as that of particles evolving in a positive time referred to positive units.

It should be stressed that the joint isodual conjugation of a physical quantity and its related unit is mandatory for consistency of the theory, as well as for the very achievement of scientific democracy for the treatment of matter and antimatter. In fact, in the absence of said dual conjugation, the negative-energy solutions of Dirac's equations are unphysical, thus solely admitting as consistent the 20th century treatment of antimatter at the sole level of second quantization.

Similarly, Einsteinian theories predict that matter and antimatter emit the same light, evidently due to the indicated lack of any differentiation between neutral matter and antimatter, light having no charge as well known. By contrast, isodual theories predict that light emitted by antimatter is different than that emitted by matter in an experimentally verifiable way. In fact, matter light has a positive energy $h \times \nu$ referred to positive unit $MeV$, while antimatter light has a negative energy $E^d = h^d \times^d \nu^d = -E$ referred to a negative unit $MeV^d = -MeV$.

Despite the above mathematical considerations, it should be stressed to prevent major scientific misrepresentations that *the isodual theory verifies all available experimental data on antimatter at both the classical and operator levels.* In fact, the *Newton-Santilli isodual equations* for antiparticles verifies all available data for charged particles and antiparticles, while isoduality is equivalent to charge conjugation at the operator level by conception and construction, as recalled via Eqs. (1) and (2) (see Ref. [2] for details).

In addition, the isodual theory has a number of rather fundamental, experimentally verifiable prediction not tested until now. A first new prediction is that *antimatter (matter) in the gravitational field of matter (antimatter) experiences a gravitational repulsion (antigravity).* Again, this prediction can be solely formulated under isodual rules, that is, the systematic, step-by-step construction of the *isodual Riemannian geometry* and related gravitational formulation of antimatter bodies. A negative curvature tensor (representing gravitational repulsion) then occurs in the interplay between a Riemannian gravitation and its isodual [2].

An experiment to test the gravity of the positron in flight in a horizontal vacuum tube on Earth has been proposed by Santilli in 1994 based on the comprehensive prediction of antigravity by the isodual theory at all levels, from the Newton-Santilli isodual equations to the Riemann-Santilli isodual geometry (see the review in Ref. [2]). This test has been considered as being resolutory for the verification of the disproof of antigravity by experimentalists A. P. Mills [4], V. de Haan [5] and others. In fact, for a 10 $m$ long horizontal vacuum tube and positron energy of the order of milli-eV, the displacement due to gravity of the positrons on a scientilloscope at the end of flight is visible to the naked eye whether upward or downward.

Similarly, Einsteinian theories predict that both, matter and antimatter light expe-

rience gravitational bending (attraction). By contrast, *the isodual theory predicts that antimatter (matter) light experiences gravitational repulsion from a matter (antimatter) gravitational field.* Note that the differentiation between matter and antimatter light is mandatory under isoduality which, in turn, is the only known differentiation between neutral matter and antimatter, thus including matter and antimatter light.

We are now sufficiently equipped to address a main point of this note. As it is well known, according to Einsteinian theories, matter light, such as that from our Sun, is predicted as being first absorbed by the atoms of a matter or antimatter asteroid and then being re-emitted in all directions according to the principle of refraction, thus predicting our capability of detecting antimatter asteroids with Sub light.

By contrast, the corresponding occurrence for the isodual theory of antimatter is not that simple. In fact, *when matter light hits an antimatter asteroid, it is expected to be "annihilated" in the sense of being "absorbed without re-emission."* Under the assumption that a matter photon carries energy much smaller than the rest energy of peripheral positrons, the annihilation photon-positron is evidently impossible. Nevertheless, the positive-definite energy of the photon can be "absorbed" by the negative-definite kinetic energy of the positrons, thus preventing a re-emission. A number of additional arguments suggesting an "annihilation-absorption" without re-emission is also possible, and they will be treated elsewhere, such as decreases in isodual temperature. a decrease of rotational degrees of freedom and others.

In conclusion, *by keeping in mind that we are dealing with the safety of our planet setting up our utmost responsibility as scientist, the speculative view submitted in this note for collegial resolution is that we do not possess at this writing conclusive and incontrovertible evidence establishing beyond reasonable doubt the possibility of detecting antimatter asteroids with Sun light.*

Along similar lines, Einsteinian theories predict that matter and antimatter stars or galaxies emit the same light, thus being equally detectable with conventional telescopes. This also implies that, according to Einsteinian theories, antimatter stars and galaxies do not exist due to the indicated lack of experimentally verifiable differences with matter stars and galaxies. By contrast, isodual theories predict that light emitted by far away antimatter stars or galaxies is annihilated-absorbed in the lenses of our telescopes or even in the pupils of our eye, thus requiring new means for their detection.

Another speculative view submitted in this note, also for collegial studies, is that, *in view of the complete absence in Einstein special and general relativity of a quantitative distinction between neutral matter and antimatter, we have no conclusive scientific knowledge at this writing on the antimatter component of the universe, to such an extent that, as a limiting case, we cannot even exclude an isoselfdual universe *(a universe with* 50% *matter and* 50% *antimatter).*

In the hope of initiating the *experimental* resolution of the above open issues, we

recall that Dirac was forced to voice the "hole theory" for the consistent representation of antiparticles due to the non-physical character of negative energy solutions of his equation. This caused a clear imbalance in the treatment of particles and antiparticles with rather subtle implications for the scattering theory indicated below.

By noting that the isodual theory represents antiparticles at all levels, thus including quantum mechanics, a reinterpretation of the Dirac equation has been then unavoidable for the achievement of a full democracy of treatment for the electron and the positron. Consider the conventional Dirac equation

$$[\gamma^\mu \times (p_\mu - e \times A_\mu/c) + i \times m] \times \Psi(x) = 0, \tag{5a}$$

$$\gamma_k = \begin{pmatrix} 0 & -\sigma_k \\ \sigma_k & 0 \end{pmatrix}, \quad \gamma^4 = i \times \begin{pmatrix} I_{2\times 2} & 0, \\ 0 & -I_{2\times 2} \end{pmatrix}, \tag{5b}$$

$$\{\gamma_\mu, \tilde{\gamma}_\nu\} \; = \; 2\times\eta_{\mu\nu}, \quad \Psi = i \times \begin{pmatrix} \Phi \\ -\Phi^\dagger \end{pmatrix} \tag{5c}$$

Santilli [2] first noted that there exists no *irreducible* four-dimensional representation of the SU(2) symmetry for spin 1/2, and there exists no *reducible* four-dimensional representation of SU(2) with the structure of Dirac's gamma matrices. The sole known algebraically consistent meaning of the gamma matrices is that they characterize an *irreducible* representation for spin 1/2 of the Kronecker product $SU(2) \times SU(2)^d$. In the author's view, this is perhaps the strongest evidence in support of the isodual theory of antimatter.

Consequently, *Dirac equation directly represents an electron-positron system without any need for the hole theory* as expressed by the following re-interpretation verifying the crucial symmetry under isoselfduality (see Ref. [2] for details)

$$[\tilde{\gamma}^\mu \times (p_\mu - e \times A_\mu/c) + i \times m] \times \tilde{\Psi}(x) = 0, \tag{7a}$$

$$\tilde{\gamma}_k = \begin{pmatrix} 0 & \sigma_k^d \\ \sigma_k & 0 \end{pmatrix}, \quad \tilde{\gamma}^4 = i \begin{pmatrix} I_{2\times 2} & 0, \\ 0 & I^d_{2\times 2} \end{pmatrix}, \tag{7b}$$

$$\{\tilde{\gamma}_\mu, \tilde{\gamma}_\nu\} = 2^d \times^d \eta^d_{\mu\nu}, \quad \tilde{\Psi} = -\tilde{\gamma}_4 \times \Psi = i \times \begin{pmatrix} \Phi \\ \Phi^d \end{pmatrix} \tag{7c}$$

Since Feynman's diagrams for electrons and positrons are centrally dependent on Dirac's equation, it is evident that the above reformulation of the latter equation requires a necessary reinspection of the former. To begin, the annihilation process in Feynman's diagrams

$$e^- + e^+ \; \rightarrow \; 2\,\gamma, \tag{8}$$

violates a number of isodual laws, such as: the l.h.s. is isoselfdual but the r.h.s is not; the annihilation process is assumed to occur via the exchange of a particle (an electron or a photon), thus being itself not isoselfdual; etc.

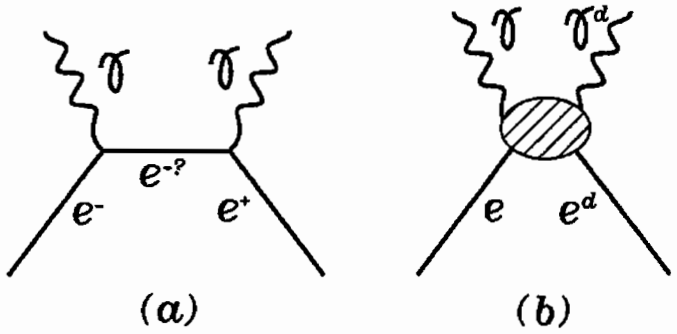



Figure 5: *A view of the electron-positron annihilation according to Dirac-Feynman theories (l.h.s) and the same annihilation as predicted by Santilli's isodual theories (r.h.s). Note the verification for the latter of all isodual laws, as well as the absence of the isoselfduality violating exchange of the former, since annihilation requires actual physical contact of particles antiparticles and cannot be triggered by particle exchanges at a distance.*

By contract, the isodual theory of antimatter represents the electron-positron annihilation with the form

$$e + e^d \equiv (e + e^d)^d \ \rightarrow \ \gamma + \gamma^d \equiv (\gamma + \gamma^d)^d, \quad e = e^-, \quad e^d = e^+, \tag{9}$$

that provides an evident resolution of all ambiguities and asymmetries of annihilation (8). Moreover, in the latter case, there is no exchange of particles, since annihilation is predicted to occur under actual physical contact or mutual penetration of the wavepackets of particles and antiparticles (see Fig. 5).

The insidious character of the lack of full democracy in the treatment of matter and antimatter is illustrated by comparing reactions (8) and (9). Reaction (8) is rather universally treated in first quantization, resulting in clear inconsistencies since, at that level, the electron and the photons can indeed be fully treated, yet the positron has negative energy in first quantization, thus prohibiting such a treatment for the sole consistent treatment in second quantization. By comparison, Reaction (9) can be consistently treated at the level of first quantization, its treatment at the level of second quantization being under study by V. de Haan (private communication).

Needless to say, there exists a very large number of experiments in electron-positron annihilation and the emitted two gammas. It is then rather natural to expect that such experimental evdience dismisses reformulation (9). A deep inspection, however, soon reveals that available experiments have provided no consideration

whatsoever on the possible differences between the two emitted photons, trivially, because no such difference was provided by the used data elaboration.

In this note, we have presented speculative comments on rather fundamental issues, such as a reinterpretation of Dirac's equation, a reinspection of Feynman's diagrams when dealing with antiparticles, and pointed out the open problem of the detection of antimatter asteroids, stars and galaxies. Clearly, these issues require an experimental resolution. With the understanding that the author is not an experimentalist, specific proposal of experiments are solicited and the following possible experiment is recommended for study. Consider a detector (such as a scintillator, a photomultiplier, et cl.) producing a signal for each energy increasing event (when hit by a photon) while producing no signal when hit by possible energy decreasing event (when hit by the isodual photon). The suggested experiment then essentially deals in the production of a known large number of electron-positron annihilation under such a condition that the produced photons are all absorbed by the detector. In the event the number of detected photons is that predicted by reaction (8), the isodual theory is in question, and antimatter asteroids, as well as stars and galaxies, can be detected with standard means as used for matter. However, in the event the number of detected photons is half that predicted by reaction (8), thus being in agreement with reaction (9), the isodual theory of antimatter is confirmed, the detection of antimatter asteroids, stars and galaxies requires the development of a basically new technologies, and we do have indeed a serious problem for the safety of out planet that has to be collegially addressed.

**Acknowledgment**

The content of this note is the output of long and solitary consideration by the author expressed in ref. [2]. The main point of this note was then first discussed during the recent *Third International Conference on the Lie-Admissible Treatment of Irreversible Processes* held at the University of Kathmandu, Nepal, from January 5 to 9, 2011. The author would like to thank all participants for invaluable comments. Additionally, very special thanks are due to Victor de Haan, Paul Krail, Alex Animalu and other colleagues for very penetrating and important comments that have been invaluable for the improvement of the presentation. Further very special thanks are also due to Christian Corda, the Editor of the proceedings, for an impeccable editorial control.

# References


[1] I. Gandzha and J Kadeisvili, *New Sciences for a New Era,* Sankata Printing Press, Nepal (2011)' available in free pdf download from the link http://www.santilli-foundation.org/docs/FoundationBook-12-10.pdf

[2] R. M. Santilli, *Isodual Theory of Antimatter with Application to Antigravity, Grand Unification and the Spacetime Machine,* Springer 2001 available in pdf download from http://www.santilli-foundation.org/docs/santilli-79.pdf

[3] R. M. Santilli, , *Hadronic Mathematics, Mechanics and Chemistry,* Volumes I, II, III, IV, and V, International Academic Press, USA, 2008. http://www.i-b-r.org/Hadronic-Mechanics.htm

[4] A. P. Mills, Possibilities of measuring the gravitational mass of electrons and positrons in Horizontal flight, Hadronic Journal **19**, 77 (1996(.

[5] V. de Haan, Proposal for the realization of Santilli comparative test on the gravity of electrons and positrons via a horizontal supercooled vacuum tube, in press in the Proceedings of the *Third International Conference on the Lie-admissible Treatment of Irreversible Systems,* University of Kathmandu, Nepal, January 5-9, 2011.

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
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# ARE MASSES OF ELEMENTARY PARTICLES AND OF THE SOLAR SYSTEM TRULY KNOWN?

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## Abstract

We recall that gravitation is characterized by the *rest energy,* rather than the *mass* of a body, and reformulate Newton's equation accordingly also to achieve universality of gravitation, thus inclusive of light. We then point out that, from the precise knowledge of the trajectories of the planets of our Solar system, we can derive with great accuracy the rest energies of the members of our Solar system, although the corresponding value of the masses are an assumption at this writing because they are calculated via the mass-energy equivalence principle $E = mc^2$ whose validity is certain under the conditions stated by Einstein, for *point particles* moving in vacuum, but not certain for *extended bodies* due to the unknown value of the maximal causal speed in their interior. We point out the occurrence of a similar situation in particle physics and suggest a possible experimental verification of the mass-energy equivalence principle for extended bodies. The cosmological implications for the removal of the far reaching conjectures of the universe expansion, dark matter and dark energy are briefly indicated.

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An important property of gravitation which is often ignored is that the source of the gravitational field is given by *rest energy* and not by *mass*. In fact, the source term in Einstein-Hilbert field equations is given by the *energy-momentum tensor* while, by contrast, the "mass-momentum tensor" does not exist because geometrically, let alone physically inconsistent. This occurrence has suggested the author to write for quite some time to write the celebrated Newton equations [1]

$$F = g\,\frac{M_1 M_2}{r^2} \tag{1}$$

in the *identical* form [2]

$$F = \frac{M_1 M_2}{r^2} \equiv \frac{E_1 E_2}{r^2 c^4} = \hat{g}\,\frac{E_1 E_2}{r^2}, \quad \hat{g} = \frac{g}{c^2}, \tag{2}$$

where $E_1$, $E_2$ refer to the rest energies of the two bodies, their kinetic energy being ignorable for the initiating character of this note.

It should be stressed that we are here referring to an *identical* reformulation of Newton's equation without any intended structural change. As an illustration, for the case of an electron, we would write for the case of Newton's original formulation the value of the electron mass $M_1 = 9.109 \times 10^{-31} Kg$, while for our reformulation we write the *identical* value $0.511\ MeV/c^2$. Similarly, for the proton we would write for Newton's original formulation $M_2 = 1,672 \times 10^{-27}\ Kg$, while for our reformulation we would write the identical value $938,272\ MeV/c^2$. Therefore, under the above assumption, the formulation of Newton's gravitation in terms of masses and that in terms of rest energy give the same results.

Despite such an identity, the indicated reformulation is not trivial. To begin, we recall that It should be indicated that the reformulation originated from the intent of achieving a true "universality" of Newtonian gravitation. In fact, a mass cannot attract light according to Newton's original formulation (1) since light has no mass. By contrast, a body with rest energy $E_1$ can indeed attract light with energy $E_2 = h\nu$ according to reformulation (2), thus achieving the desired universality and raising the unresolved issues (not considered in this note) as to whether the bending of light is due to Newtonian "universal" gravitational attraction or to actual curvature of space [2].

Additionally, said reformulation essentially implies that, from known orbits and data, we can derive with extreme accuracy the *rest energies* of the members of the Solar systems, but the corresponding values of their *masses* are unknown at this writing on serious scientific ground without unverified assumptions. In fact, the derivation of masses from rest energies depends on the familiar mass-energy equivalence principle

$$E = m\ c^2 \tag{3}$$

which is experimentally verified under the conditions stated by Einstein, for point particles moving in vacuum (exterior dynamical problem), but its validity for extended masses is a mere unverified assumption to our best knowledge.

In essence, the speed of light $c$ can be safely assumed as being the maximal causal speed in vacuum and its validity for point particles is also beyond doubt due to the lack of a structure. When passing to extended masses, the situation is fundamentally different because the formulation of their energy equivalence requires the knowledge of the maximal causal speed in their interior, whose value is vastly unknown at this writing.

As an illustration, for the case of the electron we can safely interchange rest energy with mass, i.e., $M_1 = 9.109 \times 10^{-31} Kg \equiv 0.511\ MeV/c^2$, again, because the electron has no structure, in which case, the validity of $c$ as the maximal causal speed for its structure is beyond doubt. In the transition to the proton, the situation is not equally established because the proton has a big volume (for particle standards) filled up with a hyperdense hadronic medium. In this case, the identity of the mass of the proton $1,672 \times 10^{-27}\ Kg$ with its rest energy $938,272\ MeV/c^2$ is a theoretical assumption which is not only experimentally unverified (see Refs. [2], Vol. IV), but also questionable on grounds that it implies the speed of light *in vacuum* as being the maximal causal speed in the *hyperdense medium* in the interior of the proton. In conclusion, for the case of elementary particles at large, we can safely assume that rest energies are indeed accurate, but the corresponding masses are generally unknown except for point particles. The situation for the masses of our Solar system is essentially the same.

More generally, there are serious indications of the lack of exact validity of special relativity for extended objects and electromagnetic waves moving within a physical medium (interior dynamical problem) for various mathematical, physical and experimental reasons, including: the inability to place inertial reference systems in the interior of physical media due to the resistance; the impossibility of representing numerical data on the refraction of light in water via the reduction to photons of *all* frequencies besides the few ones truly admitting quantum absorption and re-emission; experimental evidence on deviations from the Doppler law within transparent physical media with a frequency shift without any relative motion between the source, the medium and the observer (called *isoredshift* for the case of reduced frequencies and *isoblueshift* for the case of increased frequency); and other evidence [3].

Extensive studies for interior dynamical systems (such as the structure of hadrons, nuclei and stars) have suggested the use of the most general possible symmetric spacetime with line element

$$x^2 = \frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2\ \frac{c^2}{n_4^2}, \tag{4}$$

admitting as particular cases all possible spacetimes in (3+1)-dimensions (including all infinitely possible Minkowskian, Riemannian , Finslerian and other spacetimes) all possessing the unifying and universal *Lorentz-Poincaré-Santilli (LPS) isosymmetry* for interior physical media [1,2].

Line element (4) is characterized by: $n_4$ representation an average of the index of refraction in the medium considered; $n_k^2, k = 1, 2, 3$, representing symmetrized space counterparts; $n_k^2/n_4^2$ representing the general anisotropy and inhomogeneity of physical media; and all $n_\mu, \mu = 1, 2, 3, 4$, being normalized to the value for the vacuum

$n_\mu = 1$. It should be indicated that the $n$-quantities, called *characteristic quantities of the medium,* are not arbitrary parameters, but actually measurable physical quantities, as it is the case for the index of refraction.

In particular, the Lorentz-Poincaré-Santilli isosymmetry predicts the *light isocone* along the space direction s $x^2 = x_s^2/n_s^2 - t^2c^2/n_4^2 \equiv 0$ with consequential *maximal causal speed* for interior conditions in the $s$ space direction [2,3]

$$V_{max} = c\,\frac{n_s}{n_4}, \tag{5}$$

which is *smaller* then $c$ for media of low density (such as atmospheres, chromospheres, etc.) and *bigger* than $c$ for media of high density (such as interior of stars, quasars and black holes). Note the impossibility of using the speed of light as the maximal causal speed for interior dynamical problems, trivially, because they are in general opaque to light, thus demanding broader geometrical vistas. The speed of light is recovered as the maximal causal speed in vacuum, but only thanks to the identity in that case $n_4 = n_s$.

It should be noted that *all fits of experimental data in particle physics via the Poincaré-Santilli isoisymmetry have systematically provided values of $V_{max}$ inside hadrons as being bigger than the speed of light in vacuum* (see Vol. IV of Refs. [2]).

It is evident that the universal LPS isosymmetry predicts the following *mass-energy isoequivalence principle* in the s-direction

$$E = mV_{max}^2 = mc^2\frac{n_s^2}{n_4^2}. \tag{6}$$

where the reader should always keep in mind that, for the studies herein considered, the fixed quantity is the energy $E$, while the quantity $m$, referred to the *inertial mass,* is generally a local quantity depending on the characteristics of the medium considered. As an example, for a given planet with internal inhomogeneity (due to variable density) and anisotropy (due to rotation), the total energy $E$ is a fixed quantity, but the corresponding inertial mass $m$ is predicted to depend on the selected direction, with particular reference to different values of the inertial mass in the equatorial radial direction as compared to the corresponding value for the axial direction.

As indicated above, isostructures (4)-(6) have been verified for all available fits of experimental data for interior particle conditions. Their additional independent verification, with particular reference to that of the isoequivalence principle (6), are far from trivial. A conceivable experimental verification is that via the measurement in exterior conditions of the isotopic shift of the frequency of light emitted in interior conditions, that is, a shifty in the absence of relative motion. Consider the *Doppler-*

*Santilli isoshift law* along the third axis, Eq. (13) Ref. [3],

$$\nu' = \frac{1 - \hat{\beta}\ cos(\alpha)}{\sqrt{1 - \hat{\beta}^2}}\ \nu,\ \hat{\beta} = \beta\ \frac{n_3}{n_4} \qquad (7)$$

uniquely prediucted bvy the LPS isosymmetry, with approximate form for the case of the third axis

$$\nu' \approx\ [1 \pm \frac{v_3}{V_{max}} + ...)\nu, \qquad (8)$$

thus illustrating the prediction of both the isored- and isoblue-shift without any relative motion between the source, the medium and the detector. In fact, the ratio $n_3/n_4$ is generally dependent on velocity, e.g., in a linear form, in which case $Lim - v \to 0 v_3/V_{max} \neq 0$. Recall that the decrease of frequency is merely due to the loss of energy by light to the medium of low density generally assumed in its ground states (thus unable to supply energy). while the increase of frequency is due to the acquisition of energy by light from ,media of high density (thus being in a highly excited state).

The comparison between a conventional prediction of frequency for photons emitted in the interior and their value measured in the exterior is expected to provide a value of $V_{max}$ at least in a preliminary form. To illustrate the complexity of the problem here addressed, we should indicate that, assuming the suggested measurement is achieved for one extended body, such a result *does not* necessarily apply to another body. It is hoped this comment dismisses the expectation that the problem of establishing experimentally the energy equivalence of extended bodies can at best be identified in this note and definitely not resolved.

We close this note with the indication that deviations from the mass-energy equivalence principle for physical media appear to have important cosmological implications, such as the elimination of the far reaching conjectures of the universe expansion, dark matter and dark energy. Recall that all astrophysical measurements are based on *readshifts*, and 20th century theories are generally based on the tacit assumption of the exact validity of special relativity at large, thus including the Doppler shift, throughout all conditions existing in the universe.

In fact, the conjecture of the expansion of the universe is a consequence of the measured cosmological redshift of light from far away galaxies under the tacit assumption of the exact validity of special relativity for intergalactic media, since the latter assumptions solely allows the former. However, the cosmological redshift turned out as being the same in all directions, thus losing plausibility due to the placement of Earth at the center of the expansion. Plausibility was further reduced by the evidence of the increase of the redshift with the distance from Earth, in which case special relativity and the Doppler shift solely allow the additional conjecture of the

acceleration of the expansion with the distance from Earth. The experimental verification of Santilli isoredshift presented in Ref. [3] eliminates the need for Earth being at the center of the universe, and eliminated as well as the universe expansion and its acceleration, since the cosmological redshift is reduced to loss of energy by light to the intergalactic medium. Such a loss is proportional to the distance traveled in said medium with consequential elimination of the acceleration of the expansion.

The conjecture of dark energy was voiced and rapidly accepted quite widely, in support of the conjectures of the expansion of the universe and its acceleration. The conjecture of dark energy did succeed in derailing attention on deviations from special relativity, but without resolving the problems for which the conjecture was ventured. As stressed in this note, "energy" is the source of the gravitational field. Consequently, dark energy should contract the universe and definitely not accelerate its expansion. Additionally, when uniformly distributed, dark energy has no possible or otherwise plausible gravitational effect on any galaxy. Finally, possible local concentrations to achieved the desired expansion and acceleration of the expansion are faced with serious global inconsistencies.

The universal Poincaré-Santilli isosymmetry eliminates any need for the dark energy. This is achieved first via the elimination of the expansion of the universe (and the related big bang conjecture), but also via the isotopic mass-energy equivalence. As an illustration, the conjecture that dark energy constitutes 90% of the energy in the universe, is eliminates via the increased maximal causal speed in the interior of astrophysical bodies and the expression [3]

$$E_{dark\ energy} = m_{univ}(V_{max}^{aver,2} - c^2) \tag{9}$$

where $V_{max}^{aver}$ is an average of the maximal causal speed in interior of stars, quasars and black holes. In particular, dark energy as comprising 90% of our universe is eliminated for $V_{max}^{aver} \approx 10\ c$. By recalling trhat the fit ofd all particle data yields $V_{max}$ bigger than $c$ in the interior of particles (e.g., a value $V_{max} = 1.65\ c$ for the interior of the proton Vol. UV, Refs. [2]), value $V_{max}^{aver} \approx 10\ c$ is rather moderate when keeping into account the much denser interior of stars, quasars and black holes.

We finally recall that dark matter originated from the claim that peripheral galactic stars have the same speed despite the decrease of their distance from the galactic center. A scientifically more accurate statement is that peripheral galactic stars have a redshift that increases with the decrease of the distracted from the galactic center. The conjecture of equal peripheral speeds is a consequence of the tacit assumption of the exact validity of special relativity and the Doppler shift law within innergalactic media that are clearly visible with telescopes. Santilli's isoredshift also eliminates the conjecture of dark matter because innergalactic media have a density that increases with the decrease of the distance from the galactic center, thus causing an increasing isoredshift without any need for far reaching conjectures.

In summary, this note addresses the limitations of special relativity, including the limitations of the mass-energy equivalence principle, the Doppler shift and other laws, for physical conditions much beyond those of their original conception and experimental verification, point particles and electromagnetic waves in vacuum. Since the entire 20th century physics was based on the tacit assumption of the exact validity of special relativity for all possible conditions, the author hopes to stimulate a moment of reflection on the expectation that dramatic structural revisions should be expected for all 20th century conjectures, whenever dealing with interior dynamical problems, thus including structural revisions on the masses of particles, the masses of the solar system, the expansion of the universe, its acceleration, the big bang, dark matter, dark energy and numerous others, all intimately reducible to the assumption of exact validity of Einsteinian doctrines within physical media.

**Acknowledgment**

The content of this note is the output of long and solitary consideration by the author expressed in ref. [2]. The main point of this note was then first discussed during the recent *Third International Conference on the Lie-Admissible Treatment of Irreversible Processes* held at the University of Kathmandu, Nepal, from January 5 to 9, 2011. The author would like to thank all participants for invaluable comments. Additionally, very special thanks are due to Victor de Haan for very penetrating and important comments that have been invaluable for the improvement of the presentation. Further very special thanks are also due to Dorte Zuckerman for editorial control and to Christian Corda, the Editor of the proceedings for editorial control.

# References

[1] I. Newton, *Philosophiae Naturalis Principia Mathematica* (1687), translated and reprinted by Cambridge Univ. Press. (1934).

[2] R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry*, Volumes I, II, III, IV, and V, International Academic Press, USA, 2008. http://www.i-b-r.org/Hadronic-Mechanics.htm

[3] R. M. Santilli, "Experimental verifications of isoredshift with possible absence of universe expansion, big bang, dark matter and dark energy," The Open Astronomy Journal **3**, 124 (2010), available as free download from http://www.santilli-foundation.org/docs/Santilli-isoredshift.pdf

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
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# Studies of Multi-Valued Hyperstructures for the Characterization of Matter-Antimatter Systems and their Extension

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**Abstract.** In this paper, we study multi-valued hyperstructures following the apparent existence in nature of a realization of two-valued hyperstructures with hyperunits characterized by matter-antimatter systems and their extensions where matter is represented with conventional mathematics and antimatter is represented with isodual mathematics.

**Keywords:** algebraic hyperstructure, hypergroup, hyperring, hyperfield, two-valued field, isodual spacetime, hyperunit.
**PACS:** 02.10.-v, 02.20.-a.

# 1 INTRODUCTION

As it is well known, antimatter was solely treated in the 20th century via *charge conjugation* on a Hilbert space $\mathcal{H}$ with states $\psi(x)$ over the field of complex numbers $\mathbb{C}$

$$\mathbb{C}\ \psi(x) = -\ \psi^\dagger(x),$$

where $x$ is the coordinate of the representation space, such as the Minkowski spacetime.

The above approach caused a historical imbalance between matter and antimatter, because matter was treated at all known levels, from Newtonian mechanics to second quantization, while antimatter was solely treated at the level of second quantization.

The resolution of this imbalance required the construction of a new mathematics, called *Santilli isodual mathematics* [5], which is constructed via a step-by-step anti-Hermitean conjugation, denoted with the upper symbol $^d$ and called *isodual conjugation*, of each and all aspects of the 20th century mathematics used for matter. The isodual conjugation of a generic classical or operator quantity $A(x, p, \psi, \ldots)$ depending on coordinates $x$, momenta $p$, states $\psi$, etc. is then given by

$$A(x, p, \psi, \ldots)\ \rightarrow\ A^d(x^d, p^d, \psi^d, \ldots) = A(-x^\dagger, p^\dagger, \psi^\dagger, \ldots),$$

thus resulting in the new *isodual unit* $1^d = -1^\dagger$, *isodual real, complex or quaternionic numbers* $n^d = -n^\dagger$, *isodual functional analysis*, etc. [5].

The main advantage of the isodual conjugation over charge conjugation is that the former is applicable at all levels of study, thus characterizing the classical and operator isodual mechanics. The resulting *isodual theory of antimatter* has, therefore, established a complete democracy in the treatment of matter and antimatter at all levels, with intriguing implications, such as the prediction of gravitational repulsion (antigravity) for matter in the field of antimatter and vice-versa.

Despite their simplicity, the physical and mathematical differences between charge and isodual conjugations are nontrivial. From a physical viewpoint, charge conjugation solely conjugates the state, and does not conjugate the local coordinates $x$. This implies that, under charge conjugation, antimatter is assumed to exist in the same spacetime of matter.

By comparison, the isodual conjugation maps, for consistency, each quantity used in the representation of matter into its isodual image, thus including a necessary conjugation of spacetime with coordinates $x$ into the novel *isodual spacetime* with isodual coordinates $x^d = -x^\dagger$. This conjugation implies that, under isoduality,

antimatter exists in a new spacetime which is physically distinct from yet coexisting with our spacetime. In particular, their physical differences are not trivial. e.g., because the isodual conjugation of coordinates is different than inversions [5].

From a mathematical viewpoint, the co-existence of the conventional and isodual spacetimes in the same region of space creates a number of intriguing problems. At a first inspection, it is rather natural to attempt the representation of matter and antimatter via *multi-dimensional models,* e.g., via eight-dimensional mathematics essentially consisting of the Kronecker product of the four-dimensional mathematics of spacetime and its four-dimensional isodual. However, this mathematical formulation is easily seen as being unacceptable because our sensory perception deny the existence of spacetime bigger than those with four dimensions.

The compatibility of the complexities of nature with our sensory perception has motivated the construction of *multi-valued hyperstructures with hyperunits* [7]. In its most elementary possible formulation expressed via conventional operations, matter and antimatter can be represented via a two-valued hyperstructure characterized by the multiplicative hyperunit

$$E = \{1, 1^d\}$$

where $\{...\}$ represents a set, with hypernumbers

$$N = \{n, n^d\}$$

and related hyperproduct

$$N \times M = \{n \times m, n^d \times^d m^d\}$$

where $\times$ is the conventional (associative) multiplication, under which $E$ is the correct left and right hyperunit for all possible hypernumbers.

The set of hypernumbers with the indicated hyperunit and hyperproduct verifies all axioms of a numerical field, thus yielding the *two-valued hyperfield*

$$\mathcal{F} = \{F(n, \times, 1), F^d(n^d, \times^d, 1^d\}$$

from which all remaining aspects of a two-valued hypermathematics can be constructed via known procedures. Compatibility with our sensory perception is achieved by the fact that, at the abstract realization-free level, numbers and hypernumbers, spaces and hyperspaces, etc., coincide, thus avoiding the increase of dimensionality not allowed by our sensory perception.

We also indicate the possibility of extending the above two-valued example to a four-valued hyperstructure via the inclusion of *Ying's twin universes* [10], one

for matter and one for antimatter, which extension has intriguing features such as characterizing a universe with identically null total physical characteristics, i.e., identically null total time, total energy, total momentum, total entropy, etc., when examined by an observer either of matter or of antimatter.

In conclusion, the resolution of the historical 20th century imbalance between matter and antimatter via the novel isodual theory of antimatter appears to produce physical evidence for the realization in nature of multi-valued hyperstructures with hyperunits, and therefore suggesting the mathematical study presented below.

# 2 GROUPS, RINGS AND FIELDS

In the area of algebraic structures, a mathematical entity called a group plays a key role that resonates throughout the fascinating meadows of this intriguing discipline. Still more fascinating is it that the theory of this mathematical creature, or "group theory", was thought early on by mathematicians to have only intellectual appeal. That is, nobody in his right mind thought that the group and its concomitant theoretical aspects would ever serve mankind in any way other than to stimulate his cognitive awareness. Yet as irony would prove, the mathematical group would prove to be the pathway to understanding particle physics and the subatomic entities that spin the tales of this most curious science.

**Definition 2.1.** Let $G$ be a n :oge with a binary operation (usually called *multiplication*) that as ordered pair $(a, b)$ of elements of $G$ an element $a \cdot b$ in $G$. We say $G$ d operation if the following properties are satisfied:

(1) $a \cdot (b \cdot c) = (a \cdot b) \cdot c$, for all $a, b, c \in G$,

(2) there exists an element $e \in G$ such that $a \cdot e = e \cdot a = a$, for all $a \in G$,

(3) for every $a \in G$ there exists an element $a^{-1} \in G$ such that $a \cdot a^{-1} = a^{-1} \cdot a = e$.

We have $(a^{-1})^{-1} = a$ and $(a \cdot b)^{-1} = b^{-1} \cdot a^{-1}$, for all $a, b \in G$.

Group theory is a powerful formal method for analyzing abstract and physical systems in which symmetry is present and has surprising importance in physics, especially quantum mechanics. Various physical systems, such as crystals and the hydrogen atom, can be modeled by symmetry groups. Thus, group theory and the closely related representation theory have many applications in physics and chemistry.

In mathematics, ring theory is the study of algebraic structures in which addition and multiplication are defined and have similar properties to those familiar from the integers.

**Definition 2.2.** A non-empty set $R$ is called a *ring*, if it has two binary operations called *addition* denoted by $a + b$ and multiplication denoted by $a \cdot b$ for $a, b \in R$ satisfying the following axioms:

(1) $(R, +)$ is an abelian group;

(2) multiplication is associative, i.e., $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ for all $a, b, c \in R$.

(3) distributive laws hold: $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(b + c) \cdot a = b \cdot a + c \cdot a$ for all $a, b, c \in R$.

There is a group structure with the addition operation, but not necessarily with the multiplication operation. Thus an element of a ring may or may not be invertible with respect to the multiplication operation. While the addition operation is commutative, it may or not be the case with the multiplication operation.

The philosophy of this subject is that we focus on similarities in arithmetic structure between sets (of numbers, matrices, functions or polynomials for example) which might look initially quite different but are connected by the property of being equipped with operations of addition and multiplication. The set of integers and the set of $n \times m$ matrices with real numbers as entries are examples of rings. These sets are obviously not the same, but they have some similarities, and some differences, in terms of their algebraic structure. Although people have been studying specific examples of rings for thousands of years, the emergence of ring theory as a branch of mathematics in its own right is a very recent development. Much of the activity that led to the modern formulation of ring theory took place in the first half of the 20th century. Ring theory is powerful in terms of its scope and generality, but it can be simply described as the study of systems in which addition and multiplication are possible.

Everyone is familiar with the basic operations of arithmetic, addition, subtraction, multiplication, and division. Fields are important objects of study in algebra, since they provide a useful generalization of many number systems, such as the rational numbers, real numbers, and complex numbers. In particular, the usual rules of associativity, commutativity and distributivity hold. Fields also appear in many other areas of mathematics.

**Definition 2.3.** Let $(F, +, \cdot)$ be a ring such that $(F - \{0\}, \cdot)$ is an abelian group. Then $(F, +, \cdot)$ is called a *field.*

A *strictly totally ordered set* consists of a set $F$ and a binary relation $<$ which satisfies:

(1) $x \not< x$;

(2) for all $x$ and $y$, exactly one of the three possibilities holds: $x < y$, $x = y$, $y < x$;

(3) if $x < y$ and $y < z$, then $x < z$.

**Definition 2.4.** An *ordered field* consists of a field $(F, +, \cdot)$ and a set $P \subseteq F$ of positive elements satisfying the following:

(1) if $x \in P$ and $y \in P$, then $x + y \in P$;

(2) if $x \in P$ and $y \in P$, then $x \cdot y \in P$;

(3) for each $x$, exactly one of the three possibilities holds: $x = 0$, $x \in P$ or $-x \in P$.

We can define a strict total ordering on an ordered field by setting $x < y$ if and only if $y - x \in P$. We may write $x \leq y$ for ($x = y$ or $x < y$).

# 3 HYPERGROUPS, HYPERRINGS AND HYPERFIELDS

Algebraic hyperstructures represent a natural extension of classical algebraic structures. They were introduced in 1934 by the French mathematician F. Marty [4]. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Since then, hundreds of papers and several books have been written on this topic, for example see [1, 2, 3, 9].

Let $H$ be a non-empty set and $\times : H \times H \longrightarrow \mathcal{P}^*(H)$ be a hyperoperation, where $\mathcal{P}^*(H)$ is the set of all non-empty subsets of $H$. The couple $(H, \times)$ is called a *hypergroupoid*. For any two non-empty subsets $A$ and $B$ of $H$ and $x \in H$, we define

$$A \times B = \bigcup_{a \in A, b \in B} a \times b, \quad A \times x = A \times \{x\} \quad \text{and} \quad x \times B = \{x\} \times B.$$

**Definition 3.1.** A hypergroupoid $(H, \times)$ is called a *semihypergroup* if for all $a, b, c$ of $H$ we have $(a \times b) \times c = a \times (b \times c)$, which means that

$$\bigcup_{u \in a \times b} u \times c = \bigcup_{v \in b \times c} a \times v.$$

A hypergroupoid $(H, \times)$ is called a *quasihypergroup* if for all $a$ of $H$ we have $a \times H = H \times a = H$. This condition is also called the *reproduction axiom.*

**Definition 3.2.** A hypergroupoid $(H, \times)$ which is both a semihypergroup and a quasihypergroup is called a *hypergroup.*

Let $(S, \cdot)$ be a semigroup and $P$ be a non-empty subset of $S$. The $P$-hyperoperations [8] are defined as follows:

$$x \times_c y = x \cdot P \cdot y, \quad x \times_r y = x \cdot y \cdot P, \quad x \times_l y = P \cdot x \cdot y,$$

for all $x, y \in S$.

Remark that if $(S, \cdot)$ is commutative, then $\times_c = \times_r = \times_l = \times$.

**Theorem 3.3.** *[8] Let $(S, \cdot)$ be a semigroup and $P$ be a non-empty subset of $S$. Then $(S, \times_c)$ is a semihypergroup. Moreover, $(S, \times_c)$ is a hypergroup if and only if $(S, \cdot)$ is a group.*

EXAMPLE 1. The *quaternion group* is a non-abelian group of order 8. It is often denoted by $Q$ or $Q_8$ and written in multiplicative form, with the following 8 elements

$$Q = \{1, -1, i, -i, j, -j, k, -k\}.$$

Here 1 is the identity element, $(-1)^2 = 1$ and $(-1)a = a(-1) = -a$ for all $a$ in $Q$. The remaining multiplication rules can be obtained from the following relation:

$$i^2 = j^2 = k^2 = ijk = -1.$$

Now, suppose that $P = \{i, j, k\}$, a subset of $Q_8$ with three elements. Then we obtain the hypergroup $(Q_8, \times_c)$ with the following multiplication table:

| $\times_c$ | $1$ | $-1$ | $i$ | $-i$ | $j$ | $-j$ | $k$ | $-k$ |
|---|---|---|---|---|---|---|---|---|
| $1$ | $\{i,j,k\}$ | $\{-i,-j,-k\}$ | $\{-1,-k,j\}$ | $\{1,k,-j\}$ | $\{k,-1,-i\}$ | $\{-k,1,i\}$ | $\{-j,i,-1\}$ | $\{j,-i,1\}$ |
| $-1$ | $\{-i,-j,-k\}$ | $\{i,j,k\}$ | $\{1,k,-j\}$ | $\{-1,-k,j\}$ | $\{-k,1,i\}$ | $\{k,-1,-i\}$ | $\{j,-i,1\}$ | $\{-j,i,-1\}$ |
| $i$ | $\{-1,k,-j\}$ | $\{1,-k,j\}$ | $\{-i,j,k\}$ | $\{i,-j,-k\}$ | $\{-j,-i,1\}$ | $\{j,i,-1\}$ | $\{-k,-1,-i\}$ | $\{k,1,i\}$ |
| $-i$ | $\{1,-k,j\}$ | $\{-1,k,-j\}$ | $\{i,-j,-k\}$ | $\{-i,j,k\}$ | $\{j,i,-1\}$ | $\{-j,-i,1\}$ | $\{k,1,i\}$ | $\{-k,-1,-i\}$ |
| $j$ | $\{-k,-1,i\}$ | $\{k,1,-i\}$ | $\{-j,-i,-1\}$ | $\{j,i,1\}$ | $\{i,-j,k\}$ | $\{-i,j,-k\}$ | $\{1,-k,-j\}$ | $\{-1,k,j\}$ |
| $-j$ | $\{k,1,-i\}$ | $\{-k,-1,i\}$ | $\{j,i,1\}$ | $\{-j,-i,-1\}$ | $\{-i,j,-k\}$ | $\{i,-j,k\}$ | $\{-1,k,j\}$ | $\{1,-k,-j\}$ |
| $k$ | $\{j,-i,-1\}$ | $\{-j,i,1\}$ | $\{-k,1,-i\}$ | $\{k,-1,i\}$ | $\{-1,-k,-j\}$ | $\{1,k,j\}$ | $\{i,j,-1\}$ | $\{-i,-j,1\}$ |
| $-k$ | $\{-j,i,1\}$ | $\{j,-i,-1\}$ | $\{k,-1,i\}$ | $\{-k,1,-i\}$ | $\{1,k,j\}$ | $\{-1,-k,-j\}$ | $\{-i,-j,1\}$ | $\{i,j,-1\}$ |

**Construction 3.4.** *Let $(G,\cdot)$ be an abelian group and $P$ be any subset of $G$ with more than one element. We define the hyperoperation $\times_P$ as follows:*

$$x \times_P y = \begin{cases} x \cdot P \cdot y = \{x \cdot h \cdot y \mid h \in P\} & \text{if } x \neq e \text{ and } y \neq e \\ x \cdot y & \text{if } x = e \text{ or } y = e. \end{cases}$$

*We call this hyperoperation $P_e$-hyperoperation. The hyperstructure $(G, \times_P)$ is a hypergroup. Then $(G, \times_P)$ is a hypergroup.*

*Proof.* We prove it for $P = \{s, t\}$ and the proof is analogous for any $P$ with more elements. Let $x, y, z$ be non unit elements of $(G,\cdot)$. Then, we have

$$x \times_P (y \times_P z) = x \cdot P \cdot (y \cdot P \cdot z) = x \cdot P \cdot y \cdot P \cdot z,$$
$$(x \times_P y) \times_P z = (x \cdot P \cdot y) \cdot P \cdot z = x \cdot P \cdot y \cdot P \cdot z.$$

So $x \times_P (y \times_P z) = (x \times_P y) \times_P z$.

If one of $x, y, z$ equals to $e$, say $x = e$, then we have

$$e \times_P (y \times_P z) = y \cdot P \cdot z \quad \text{and} \quad (e \times_P y) \times_P z = y \cdot P \cdot z.$$

Therefore, $\times_P$ is associative.

Now, let $x \neq e$. Then

$$x \times_P G = \{x\} \cup [x \cdot P \cdot (G - \{e\})] = \{x\} \cup [x \cdot s \cdot (G - \{e\})] \cup [x \cdot t \cdot (G - \{e\})],$$

in which we remark that the set $x \cdot s \cdot (G - \{e\})$, which contain all the elements of $G$ except the element $x \cdot s$ and the set $x \cdot t \cdot (G - \{e\})$ contains all the elements of $G$ except $x \cdot t$. Therefore, we have $x \times_P G = G$. The same proof for $G \times_P x = G$. Finally, the reproductivity for the unit $e$ is obvious. Thus, $\times_P$ is reproductive. Therefore, $(G, \times_P)$ is a hypergroup. □

Remark that e is scalar unit in $(G, \times_P)$. Any element $x$ of $G$ has one or two inverses, the elements $(x \cdot s)^{-1}$ and $(x \cdot t)^{-1}$ when $x \cdot s \neq e$ and $x \cdot t \neq e$.

EXAMPLE 2. Consider the Klein four-group $K_4 = \{e, a, b, c\}$. It is abelian, and isomorphic to the dihedral group of order 4. It is also isomorphic to the direct sum $\mathbb{Z}_2 \oplus \mathbb{Z}_2$. Multiplication table for Klein four-group is given by:

| $\cdot$ | $e$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e$ | $e$ | $a$ | $b$ | $c$ |
| $a$ | $a$ | $e$ | $c$ | $b$ |
| $b$ | $b$ | $c$ | $e$ | $a$ |
| $c$ | $c$ | $b$ | $a$ | $e$ |

Now, we consider the subgroups $P_1 = \{e, a\}$, $P_2 = \{e, b\}$ and $P_3 = \{e, c\}$. Then, according to Construction 3.4, we obtain the canonical hypergroups $(G, \times_{P_1})$, $(G, \times_{P_2})$ and $(G, \times_{P_3})$ with the following tables:

| $\times_{P_1}$ | $e$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e$ | $e$ | $a$ | $b$ | $c$ |
| $a$ | $a$ | $\{e,a\}$ | $\{b,c\}$ | $\{b,c\}$ |
| $b$ | $b$ | $\{b,c\}$ | $\{e,a\}$ | $\{e,a\}$ |
| $c$ | $c$ | $\{b,c\}$ | $\{e,a\}$ | $\{e,a\}$ |

| $\times_{P_2}$ | $e$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e$ | $e$ | $a$ | $b$ | $c$ |
| $a$ | $a$ | $\{e,b\}$ | $\{a,c\}$ | $\{e,b\}$ |
| $b$ | $b$ | $\{a,c\}$ | $\{e,b\}$ | $\{a,c\}$ |
| $c$ | $c$ | $\{e,b\}$ | $\{a,c\}$ | $\{e,b\}$ |

| $\times_{P_3}$ | $e$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e$ | $e$ | $a$ | $b$ | $c$ |
| $a$ | $a$ | $\{e,c\}$ | $\{e,c\}$ | $\{a,b\}$ |
| $b$ | $b$ | $\{e,c\}$ | $\{e,c\}$ | $\{a,b\}$ |
| $c$ | $c$ | $\{a,b\}$ | $\{a,b\}$ | $\{e,c\}$ |

EXAMPLE 3. Consider the group $(\mathbb{Z}_7 - \{0\}, \cdot)$, and let $P = \{2, 3\}$. Then $(\mathbb{Z}_7, \times_P)$ is a hypergroup, where the multiplication table for $\times_P$ is:

| $\times_P$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | $\{1,5\}$ | $\{4,5\}$ | $\{2,3\}$ | $\{2,6\}$ | $\{1,3\}$ |
| 3 | 3 | $\{4,5\}$ | $\{4,6\}$ | $\{1,3\}$ | $\{2,3\}$ | $\{1,5\}$ |
| 4 | 4 | $\{2,3\}$ | $\{1,3\}$ | $\{4,6\}$ | $\{4,5\}$ | $\{2,6\}$ |
| 5 | 5 | $\{2,6\}$ | $\{2,3\}$ | $\{4,5\}$ | $\{1,5\}$ | $\{4,6\}$ |
| 6 | 6 | $\{1,3\}$ | $\{1,5\}$ | $\{2,6\}$ | $\{4,6\}$ | $\{2,3\}$ |

**Definition 3.5.** A triple $(R, +, \times)$ is called a *multiplicative hyperring* if

(1) $(R, +)$ is an abelian group;

(2) $(R, \times)$ is a semihypergroup;

(3) for all $a, b, c \in R$, we have $a \times (b+c) \subseteq a \times b + a \times c$ and $(b+c) \times a \subseteq b \times a + c \times a$;

(4) for all $a, b \in R$, we have $a \times (-b) = (-a) \times b = -(a \times b)$.

If in (3) we have equalities instead of inclusions, then we say that the multiplicative hyperring is *strongly distributive.*

**Definition 3.6.** Let $(F, +, \times)$ be a multiplicative hyperring such that

(1) $(F - \{0\}, \times)$ is a hypergroup,

(2) $\times$ is strongly distributive with respect to $+$.

Then $(F, +, \times)$ is called a *multiplicative hyperfield.*

**Proposition 3.7.** *Let $(F, +, \cdot)$ be a field and $P$ be a non-empty subset of $F - \{0\}$. Consider the $P$-hyperoperation defined in Theorem 3.3. Then $(F, +, \times)$ is a multiplicative hyperfield.*

**Proposition 3.8.** *Let $(F, +, \cdot)$ be a field and $P$ be a non-empty subset of $(F - \{0\}, \cdot)$. Consider the hyperoperation defined in Proposition 3.4. Then $(F, +, \times)$ is a multiplicative hyperfield.*

EXAMPLE 4. Consider the finite field $(\mathbb{Z}_7, +, \cdot)$, the field of integers modulo 7, and let $H = \{1, 6\}$. Then $(\mathbb{Z}_7, +, \times)$ is a multiplicative hyperfield, where the multiplication table for $\times$ is:

| $\times$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 0 | 2 | $\{3,4\}$ | $\{1,6\}$ | $\{1,6\}$ | $\{3,4\}$ | $\{2,5\}$ |
| 3 | 0 | 3 | $\{1,6\}$ | $\{2,5\}$ | $\{2,5\}$ | $\{1,6\}$ | $\{3,4\}$ |
| 4 | 0 | 4 | $\{1,6\}$ | $\{2,5\}$ | $\{2,5\}$ | $\{1,6\}$ | $\{3,4\}$ |
| 5 | 0 | 5 | $\{3,4\}$ | $\{1,6\}$ | $\{1,6\}$ | $\{3,4\}$ | $\{2,5\}$ |
| 6 | 0 | 6 | $\{2,5\}$ | $\{3,4\}$ | $\{3,4\}$ | $\{2,5\}$ | $\{1,6\}$ |

EXAMPLE 5. Consider the infinite fields $(\mathbb{Q}, +, \cdot)$, $(\mathbb{R}, +, \cdot)$ and $(\mathbb{C}, +, \cdot)$. Clearly $I = \{1, -1\}$ is a subgroup of $(\mathbb{Q} - \{0\}, \cdot)$, $(\mathbb{R} - \{0\}, \cdot)$ and $(\mathbb{C} - \{0\}, \cdot)$. Therefore, according to Proposition 3.8, $(\mathbb{Q}, +, \times)$, $(\mathbb{R}, +, \times)$ and $(\mathbb{C}, +, \times)$ are multiplicative hyperfields. For example, we have:

$$3 \times 5 = 3 \cdot I \cdot 5 = \{15, -15\},$$
$$1/2 \times 3/5 = 1/2 \cdot I \cdot 3/5 = \{3/10, -3/10\}.$$

# 4 HYPERUNITS

Let $(F, +, \cdot)$ be a field and $H = \widehat{I} = \{1, -1\}$. According to Proposition 3.8, $(F, +, \times)$ is a multiplicative hyperfield. We define

$$\widehat{a} = a \times \widehat{I} = \{a, -a\},$$

and we set

$$\widehat{F} = \{\widehat{a} \mid a \in F\}.$$

We define a sum and a multiplication on $\widehat{F}$ as follows:

$$\widehat{a} \oplus \widehat{b} := \widehat{a+b},$$
$$\widehat{a} \otimes \widehat{b} := \widehat{a \cdot b},$$

for all $a, b \in F$.

**Theorem 4.1.** *$(\widehat{F}, \oplus, \odot)$ is a field and $\widehat{I}$ is the unit of $(\widehat{F}$ (we call $\widehat{I}$ a hyperunit).*

# 5 CONCLUDING REMARKS

In this note we have indicated, in the simplest possible mathematical formulation, the apparent existence in nature of a concrete realization of multi-valued hyperstructures with a basic hyperunit given by matter-antimatter systems. We have then studied said hyperstructures in their proper mathematical formulation and provided a number of examples. By keeping in mind the complexity of nature, we hope this study may be valuable for quantitative representations of complex systems, such as biological structures ion general and the DNA code in particular, that appear to require indeed the most advanced and general possible multi-valued hyperstructures.

# References


[1] P. Corsini, *Prolegomena of hypergroup theory*, Second edition, Aviani editor, 1993.

[2] P. Corsini and V. Leoreanu, *Applications of hyperstructure theory*, Advances in Mathematics, Kluwer Academic Publishers, Dordrecht, 2003.

[3] B. Davvaz, V. Leoreanu-Fotea, *Hyperring Theory and Applications*, International Academic Press, USA, 2007.

[4] F. Marty, *Sur une generalization de la notion de groupe*, $8^{th}$ Congress Math. Scandenaves, Stockholm 1934, 45-49.

[5] R.M. Santilli, *Isodual Theory of Antimatter with Application to Antigravity, Grand Unification and the Spacetime Machine*, Springer 2001, available in pdf download from http://www.santilli-foundation.org/docs/santilli-79.pdf

[6] R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry*, Volumes I, II, III, IV, and V, International Academic Press, USA, 2008.

[7] R.M. Santilli and T. Vougiouklis, *Isotopies, genotopies, hyperstructures and their applications*, New frontiers in hyperstructures (Molise, 1995), 1-48, Ser. New Front. Adv. Math. Ist. Ric. Base, Hadronic Press, Palm Harbor, FL, 1996.

[8] T. Vougiouklis, *On some representations of hypergroups*, Annales Sci. Univ. "Blaise Pascal", Clermont II, Ser. Math. 26 (1991), 21-29.

[9] T. Vougiouklis, *Hyperstructures and their representations*, Hadronic Press Inc., Florida, 1994.

[10] L. Ying, *Nuclear fusion drives present-day accelerated cosmic expansion*, Hadronic Journal 32 (2009), 573-588.

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
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# Proposal For The Realization Of Santilli's Comparative Test On The Gravity Of Electrons And Positrons Via A Horizontal Supercooled Vacuum Tube

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**Abstract.** A proposal for the realization of Santilli's comparative test of the gravity of electrons and positrons via a horizontal supercooled vacuum tube is described. Principle and requirements are described concerning the sources, vacuum chamber electromagnetic shielding and pressure and position sensitive detector. It is concluded that with current technology the experiment is perfectly feasible.

**Keywords**: Anti matter gravity; Equivalence principle; Experimental proposal
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## INTRODUCTION

Although the equivalence principle is well established for neutral bulk matter [2],[3] and neutrons [4],[5],[6] it has no experimental verification for charged elementary particles or antimatter.

Even the gravitational mass of the electron has not been measured. Although there has been an attempt to measure the gravitational mass of electrons in the 1960's by Witteborn and Fairbank [7],[8], this experiment was inconclusive. The goal of this experiment was to determine the gravitational force on both electrons and positrons, but is was only performed with electrons yielding a result disputed in literature. The experiment was not repeated with positrons due to lack of an adequate positron source [9]. The primary cause of the failure of the experiment is the magnitude of the effect, comparable to the force on a elementary charge due to an electric field of $5.6 \times 10^{-11}$ V/m, corresponding in magnitude to the force repelling two unshielded electrons 5 m apart in vacuum. All electric fields must be controlled within at least an order of magnitude better accuracy.

Efforts are underway to measure the equivalence principle for neutral antimatter at CERN [10],[11],[12] and Fermilab [13],[14] to avoid the problems associated with the charge of the particle. However, it is argued that the equivalence principle for matter or antimatter could be different from the one for charged elementary particles [1],[15] so that an experiment with electrons and positrons is still called for.

Since the first attempts of Witteborn [7] to measure the gravitational mass of an electron much effort has been invested in the study of the experimental difficulties reducing the electric field to theoretical acceptable limits. First, the focused changed from positrons to anti protons [16] due to the large inertial mass difference between the elementary particles. Later after a 1996 workshop on antimatter gravity and anti hydrogen spectroscopy [18] the focused changed again to neutral antimatter. The reason for this was the problem posed by the so-called *patch-effects* [9]. These effects were assumed to render the measurements with positrons and even with anti protons impossible.

However, Witteborn and Lockhart have always maintained that the patch-effects were somehow shielded after cooling to a temperature of 4.2 K [8],[19],[20]. A possible shielding mechanism of the patch-effect was observed by Rossi [21] and a patch-effect reducing with temperature and surface treatment has been observed over a metal surface [22]. Also Dittus [23], proposing a gravity experiment in space, argues that with modern techniques the patch-effect can be reduced significantly.

The above shows the need for a comparison of the gravitation on electrons and positrons and addressed why until now this has not been performed. In view of the recent technological developments of surface treatment these limitations can now be overcome and the experiment in a free horizontal flight in a high vacuum tube as first proposed by Santilli [1] and its principles worked out by Mills [25] can now be performed with small technological risks.

In the following first the principle of the experiment is lined out, then the the several components are highlighted and finally the conclusions are given.

## PRINCIPLE

The principle of Santilli's comparative test of the gravity of electrons and positrons is shown in figure 1. At one end of a well-shielded horizontal vacuum tube an electron or positron is released with a horizontal velocity, $v$. The particle moves through the vacuum tube until it reaches the other end at a distance $L$ and it is detected with a position sensitive detector. During the flight the particle experiences a constant gravitational acceleration, $\vec{g}_e$ or $\vec{g}_p$. The deflection at the end of the flight path is simply given by

$$\Delta z_{e,p} = g_{e,p}\frac{t^2}{2} \tag{1}$$

where $t$ is the time the particle needs to reach the detector after is has been released at the source. This is called the time-of-flight.

The deflection of the particle is proportional to the gravitational force so that measuring the deflection is sufficient to determine its sign. For neutral matter this set-up can be easily realized and with some more effort the same principle has been used to detect the gravity effects on neutrons [4],[5].

However, the measured deflection also depends on the time-of-flight, which is simply given by $L/v$. *Hence, the deflection is inversely proportional to the (horizontal) kinetic energy of the particle.* The particle source will typical emit particles with some velocity distribution, hence the deflection is smeared out. This can be prevented by measuring

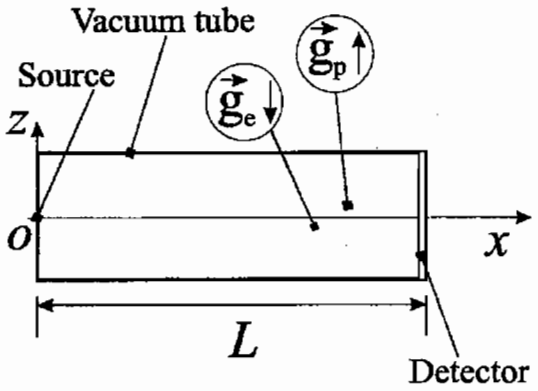



Figure 1: Principle set-up of Santilli's comparative test of the gravity of electrons and positrons.

the time-of-flight using a pulsed source. In that case the deflection of the particles is proportional to the square of the time-of-flight.

Another assumption in the above reasoning was that the particles were emitted horizontally. With a typical particle source this direction will have some final spread around the horizontal, which again results in smearing out of the deflection. For neutral matter this is overcome by applying a diaphragm system to direct and collimate the particle beam. As Mills [25] has shown for charged particles a diaphragm system can be replaced by a focusing system and a suitable aperture system in the middle of the flight path. This relaxes the requirements for particle source strength quite a bit as a much larger divergence can be tolerated. With the focusing lens the source is imaged on the detector reducing the smearing out of the deflection. This is schematically shown in figure 2. For a lens to work appropriate (with as small as possible aberrations) the lateral dimensions should be some two orders of magnitude smaller than the longitudinal dimensions (par-axial approximation).

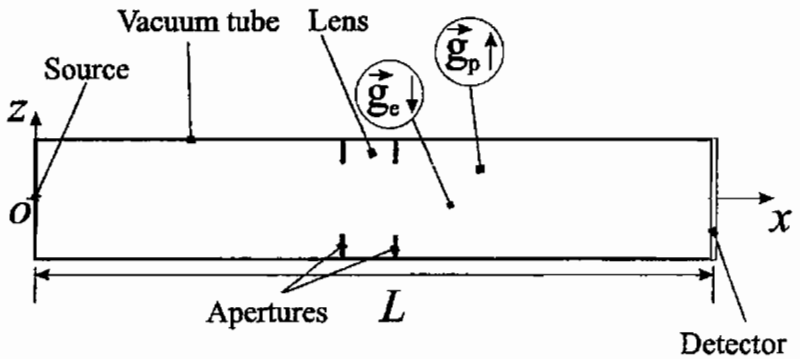



Figure 2: Principle set-up of Mills's adaptation of Santilli's comparative test of the gravity of electrons and positrons.

Another experimental feature that Mills incorporates is to reverse the flight direction keeping all other experimental conditions unchanged. The average of the four deflections is much less sensitive to remaining electric and magnetic stray fields and equal to

$$\langle \Delta z \rangle = (g_e + g_p) L^2 / v^2 \tag{2}$$

Hence both sides of the vacuum tube must provide sources of electrons and positrons and detectors of the same. This also limits the possibilities of the focusing system to a symmetrical one, with a magnification of 1. In the following sections some details on the main components are given.

## COMPONENTS

### Electron and positron sources

The main requirements for the electron and positron sources needed for this experiment can be inferred from figure 3. To have a good compromise between maximal kinetic energy and minimal flight-path, the available source area must have a height of some 100 $\mu$m and a length of the order of a centimeter. The length can not be larger because then the focusing properties of the lensing system will be imparted. The height can not be larger as then too small kinetic energies would be needed. The kinetic energies needed are of the order of 1 to 100 $\mu$eV, which for electron and positron sources are ultra low energies. That these ultra low kinetic energy electron and positron sources needed for this experiment are obtainable in sufficient quantities was shown in concept by Mills [25] (needed fast positron beam intensity of $3 \times 10^7$ 1/s/cm$^2$) and by experiment as discussed by Kurz [26]. The possibilities would increase when instead of a $^{22}$Na source, a reactor-based positron sources [27],[28] could be used where the positron yield is at least a factor of 10 larger. Another possibility is to use positron traps which can store up to $3 \times 10^{10}$ positrons per cell [29] and release them in pulses.

### Focusing, shielding and flight path

Focusing has to be done by means of a symmetric time-of-flight dependent electrostatic or magnetic lens, because the focus distance of such a lens is determined by the relative kinetic energy change of the particles passing the lens. The ability to tune the lens to the right field value will determine for a large portion the minimal attainable kinetic energy or maximal attainable deflection. An important design criterion is the wavelike structure that electrons and positrons exhibit. The De Broglie wavelength is inversely proportional to the velocity given by

$$\lambda = \frac{h}{m_i v} = \lambda_0 \frac{v_0}{v} \tag{3}$$

where $h = 6.626 \times 10^{-34}$ Js, $m_i = 9.109 \times 10^{-31}$ kg is the electron (or positron) inertial mass, $\lambda_0 = 100$ nm for $v_0 = 7.27$ km/s. Due to this wavelike structure of the particles, the circular apertures in the middle of the setup result in a Fraunhofer diffraction pattern at the detector plane. The most simple diffraction pattern from a circular aperture with

diameter $D$ is the Airy pattern where the inner most intense fringe is called the Airy disk. This Airy disk has a diameter of

$$d = 1.22\lambda\frac{L}{D} = 1.22\frac{\lambda_0 v_0}{v}\frac{L}{D} \tag{4}$$

as long as $D >> \lambda$. Note that the Airy disk size is inversely proportional to the velocity of the particles, while the deflection is inversely proportional to the square of the velocity.

The diameter of the Airy disk should be less than the anticipated deflection (Rayleigh's criterion), hence

$$t = L/v > 2.44\frac{\lambda_0 v_0}{D\,|g_{e,p}|} \approx \frac{D_0 t_0}{D} \tag{5}$$

where $D_0 = 10$ cm and $t_0 = 1.81$ ms. *Hence, due to the wavelike nature of the particles, the minimal time-of-flight needed to obtain a sufficient resolution is inversely proportional to the diameter of the aperture.* Note that for $L = 13$ m and $D = 10$ cm, the velocity of the particle should be maximal 7.3 km/s, hence its wavelength at least 100 nm and its corresponding kinetic energy maximal 150 $\mu$eV. In such a case the deflection would be minimal 16 $\mu$m. The deflection increases to 0.1 mm for particles with a kinetic energy of 25 $\mu$eV. If one would take the values used by Mills [25] $D = 10$ cm and $L = 100$ m, then the velocity of the particle should be maximal 55.2 km/s, hence its wavelength at least 13 nm and its corresponding kinetic energy maximal 8.7 meV. In such a case the minimal deflection would still be only 16 $\mu$m. The deflection would however increase to 5.6 mm for particles with a kinetic energy of 25 $\mu$eV.

In reality the source will have a finite dimension, increasing the above mentioned spot diameter. For an ideal instrument the image of the source on the detector plane and the Airy disk should have approximately the same size and be comparable to the detector resolution. In such a case the minimal needed aperture is completely determined by the needed resolution

$$D_{min} = 1.73\frac{\lambda_0 v_0}{\sqrt{d\,|g_{e,p}|}} \tag{6}$$

This also fixes the minimal needed length of the instrument as $D/L$ is between 0.1 and 0.001. The upper bound is due to limitation of the particle-optics components (par-axial approximation) and the lower bound due to intensity limitation as the particle intensity on the detector is given by all the particles that are passed through the aperture and is proportional to $\eta^2$, hence $\eta$ cannot be made too small. If it is used that $D/L = \eta$, then the maximum velocity to obtain a sufficient resolution is given by

$$v = 1.22\frac{\lambda_0 v_0}{d\eta} \tag{7}$$

and the corresponding maximal kinetic energy

$$E_{kin} = 0.74 m_i \left(\frac{\lambda_0 v_0}{d\eta}\right)^2 \tag{8}$$

The maximal kinetic energy of the particle as function of the aperture diameter is shown in the left graph of figure 3 for different values of $\eta$. The corresponding minimal length

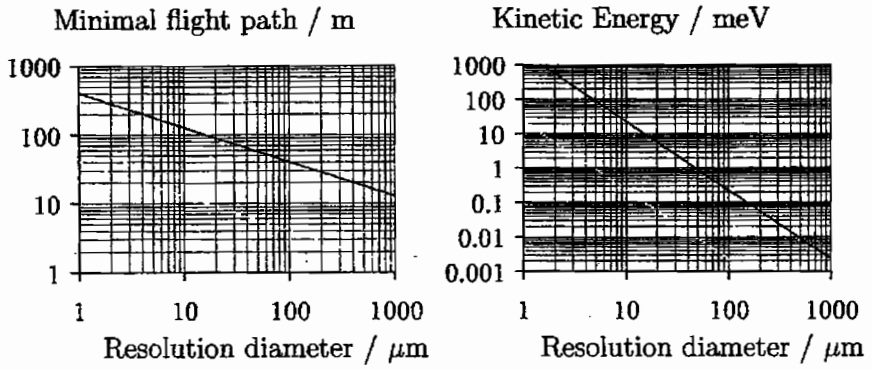



Figure 3: Left: Graph of the maximal kinetic energy of the particles as function of aperture diameter in a gravity experiment to assure sufficient spatial resolution. Right: Graph of the minimal needed flight path as function of the same. Solid black line for $\eta = 0.001$; dashed red line for $\eta = 0.01$ (see text).

of the flight path is shown in the right graph. From these graphs one can see that the choices made by Santilli and Mills to use a flight path between 10 and 100 m is a good compromise between the needed flight path (as small as possible) and the needed minimal kinetic energy (as large as possible). A flight path as large as possible would be optimal as all other requirements relax when the flight path increases. However, the realization costs for the flight path will be roughly proportional to the square of the flight path length because for an optimal performance the diameter of the flight path has to be proportional to its length. If only the length will be made larger and not the diameter then the advantage of increasing the flight path is lost in the reduction of intensity. Hence, the optimal flight path depends on budget but probably will be between 10 and 100 m.

Probably the most crucial part of the instrument will be the shielding of residual electric and magnetic fields. The most important components that need to be shielded sufficiently well are those resulting in a force in the same (or opposite) direction as gravity. An extensive review of all possible fields that need to be shielded is given by Darling [9]. His conclusion is that with the current technology it is possible to construct an adequate shielding. The way this can be done is described by Mills [25]. It consists of a stacked layer system of different materials cooled to a temperatures close to 4.2 K to obtain optimal shielding.

### Surface patch potential effect

The only remaining shielding issue is the electric potential variation in the flight path of the particle due to the inner surface of the most inner layer of the flight path tube. This inner surface consists of small crystallites exhibiting a small potential variation, these constitute the so-called patch-effect. This might cause a potential variation of about 1 $\mu$V on the axis of the flight tube. This is a reason why the inner shield must also be cooled down to liquid helium temperatures reducing the patch effects.

A way of determining the influence of the patch effect is to estimate the optical phase differences due to potential variations over different paths from source to detector. The

optical phase along a particle trajectory is given by

$$\psi = \frac{2\pi}{\lambda} \oint n(\vec{s}) d\vec{s} \tag{9}$$

where $n(\vec{s})$ equals the refractive index along the trajectory defined by $\vec{s}$. This refractive index is coupled to the potential by

$$n(\vec{s})^2 = 1 \pm \frac{2em_i\lambda^2 V(\vec{s})}{h^2} \tag{10}$$

where $e = 1.602 \times 10^{-19}$ C is the elementary charge and $V(\vec{s})$ is the potential along the trajectory. The plus holds for electrons, the min for positrons. Variations in the potential due to the patch effect are very small, hence the variations in the refractive index can be approximated by

$$\Delta n(\vec{s}) = \pm \frac{em_i\lambda^2}{h^2} \Delta V(\vec{s}) \tag{11}$$

and variations in the optical phase are directly related to variations in the potential according to

$$\Delta\psi = \pm 2\pi \frac{em_i\lambda}{h^2} \oint \Delta V(\vec{s}) d\vec{s} \tag{12}$$

According to Darling [9] a Gaussian distributed patch effect (with root-mean-square patch potential, $\phi_{patch}$ and average crystallite size $\zeta$) on the inner surface of a long cylinder ($L >> D$) results in potential variations of $\phi_{patch} 2\zeta/D$ on the axis. The line integral over these variations can be estimated by transforming the integral over a sum of $L/\zeta$ patches of length $\zeta$. The sum can be regarded as a random walk, so that the final spread in $\psi$ becomes

$$\frac{\sigma_\psi}{2\pi} = \frac{\lambda}{\lambda_0} \frac{\zeta}{D} \frac{\phi_{patch}\sqrt{L\zeta}}{P_0} < 1 \tag{13}$$

where $P_0 = h^2/(2em_i\lambda_0) = 1.5 \times 10^{-11} Vm$. To be able to get a good focus this variation in optical phase should be much smaller than $2\pi$. Note that the variation is proportional to the wavelength, which clearly favors faster particles.

For the optimal resolution setup of the previous section this condition puts a limit on the ratio between $D$ and $L$

$$\eta < 13.5 \left(\frac{d_0}{d}\right)^{5/2} \left(\frac{\zeta_0}{\zeta}\right) \left(\frac{\phi_0\zeta_0}{\phi_{patch}\zeta}\right)^2 \tag{14}$$

where $d_0 = 100$ $\mu$m, $\zeta_0 = 1$ $\mu$m, $\phi_0 = 1$ $\mu$V. This is completely determined by the required resolution and the patch potential distribution. If a resolution of 100 $\mu$m is required and $\eta$ would be between 0.001 and 0.1, then for $\zeta = 1$ $\mu$m, $\phi_{patch}$ has to be less than 100 to 10 $\mu$V, which is perfectly feasible [22].

According to equation (13) the spread in optical phases close to the cylinder axis is proportional to the wavelength. This explains why the vertical flight path as used by Witteborn [8] is much more sensitive to the patch effect than the horizontal flight path considered here. Take $\lambda = \lambda_0 v_0/\sqrt{gL}$ (the average wavelength for a particle just reaching the top of the flight path), then for $L = 1$ m, $D = 4$ cm and $\zeta = 1$ $\mu$m, $\phi_{patch}$ has to

be less than 250 nV at least a factor of 400 smaller. Darling [9] takes $\zeta << 1$ nm and $\phi_{patch} = 0.01$ V, as limit which corresponds to a variation of the optical path phase of $\sigma_\psi << 2.4\pi$. Hence, both approaches give similar results.

Equation (13) can be rewritten as function of the total deflection of the particle beam

$$\frac{\sigma_\psi}{2\pi} = \frac{2e}{h}\sqrt{\frac{2\Delta z}{|g_{e,p}|}}\frac{\phi_{patch}\zeta}{D}\sqrt{\frac{\zeta}{L}} \tag{15}$$

This is independent of the particle properties. Hence, *for a required given deflection in the proposed experiment, the influence of the patch potential effects does not depend on the type of particle used.*

In view of the relatively large kinetic energies involved in this horizontal flight path experiment with regard to the Witteborn experiment [7] and the implicit determination of the average kinetic energy by means of the time-of-flight method, the influence of the patch-effects will be much reduced.

This also relaxes the requirements on the vacuum pressure quality to about $10^{-8}$ Pa as the time-of-flight is at least a factor of 100 shorter and the main effect it has on the results is a reduced intensity at the detector.

### Electron and positron detection

The preferable detector should be a linear position sensitive detector that can detect both electrons and positrons. The spatial resolutions should be in the order of 100 $\mu$m and the time resolution of the order of 0.1 ms with an efficiency as high as possible. These are moderate requirements and can be met by for instance micro channel plates [30],[31],[32] or linear CMOS detectors [33].

## CONCLUSIONS

The above shows that with current technology it is perfectly feasible to perform the long awaited experiment to compare the gravitation on electrons and positrons as suggested by R.M. Santilli [24] almost two decades ago. The largest challenge will be the adequate shielding of the flight path to acceptable levels by means of a supercooled vacuum tube.

## ACKNOWLEDGMENTS

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# References

[1] R. M. Santilli, *Isodual Theory of Antimatter*, in: Fundamental Theories of Physics **151**, editor A. van der Merwe, Springer, Dordrecht, The Netherlands (2006) ISBN-10: 1-4020-4517-4

[2] R. v. Eötvös, D. Pekar and E. Fekete, *Beiträge zum Gesetze der Proportionalität von Trägheit und Gravität* , Annalen der Physik **373** (9) (1922) p11-68

[3] P. G. Roll, R. Krotkov and R. H. Dicke, *The Equivalence of inertial and Passive Gravitational Mass*, Annals of Physics (N.Y.) **26** (1964) p442-517

[4] J. W. T. Dabbs, J. A. Harvey, D. Paya and H. Horstmann, *Gravitational acceleration of free neutrons*, Physical Review **139** (B756) (1965)

[5] L. Koester, *Verification of the equivalence of gravitational and inertial mass for the neutron*, Physical Review D **14** (4) (1976) p907-909

[6] K. C. Littrell, B. E. Allman and S. A. Werner, *Two wavelength difference measurement of gravitational induced quantum interference phases*, Physical Review A **56** (3) (1997) p1767-1780

[7] F. C. Witteborn and W. M. Fairbank, *Experimental comparison of the gravitational force on freely falling electrons and metallic electrons*, Physicla Review Letters **19** (18) (1967) p1049-1052

[8] F. C. Witteborn and W. M. Fairbank, *Apparatus for measuring the force of gravity on freely falling electrons*, Review if Scientific Instruments **48** (1) (1977) p1-11

[9] T. W. Darling, F. Rossi, G. I. Opat and G. F. Moorhead, *The fall of charged particles under gravity: A study of experimental problems*, Review of Modern Physics **64** (1) (1992) p237-257

[10] G. Yu Drobychev, et al., *Proposal for the AEGIS experiment at the CERN anti proton decelerator (Antimatter Experiment: Gravity, Interferometry, Spectroscopy)*, CERN-SPSC-2007-017 (2007)

[11] M. Doser, et al., *AEGIS: an experiment to measure the gravitational interaction between matter and antimatter*, Journal of Physics: Conference Series **199** (2010) p012009

[12] A. Kellerbauer et al., *Proposed antimatter gravity measurement with an anti hydrogen beam*, Nuclear Instruments and Methods in Physics Research B **266** (2008) p351-356

[13] A. D. Cronin, et al., *Letter of Intent: Antimatter Gravity Experiment (AGE) at Fermilab*, downloaded from http://www.fnal.gov/ directorate/program_planning/ Mar2009PACPublic/ AGELOIFeb2009.pdf on Nov 29, 2010

[14] D. M. Kaplan, *New experiments with anti protons*, Nuclear Physics A **844** (2010) p206c-215c

[15] T. Goldman, J. Hughes and M. M. Nieto, *Gravitational acceleration of anti protons and of positrons*, Physical Review D **36** (4) (1987) p1254-1256

[16] T. Goldman and M. M. Nieto, *Experiments to measure the gravitational acceleration of antimatter*, Physics Letters **112B** (6) (1982) p437-440

[17] T. Goldman et al., *Beyond metric gravity: Progress on PS-200* Hyperfine Interactions **81** (1993) p87-96

[18] M. H. Holzscheiter, *Workshop summary, International workshop on antimatter gravity and anti hydrogen spectroscopy, Sepino Molise, Italy, 19-25 May 1996*, Hyperfine Interactions **109** (1997)

[19] J. M. Lockhart, F. C. Witteborn and W. M. Fairbank, *Evidence for a Temperature-Dependent Surface Shielding Effect in Cu*, Physical Review Letters **38** (21) (1977) p1220-1223

[20] J. M. Lockhart, F. C. Witteborn and W. M. Fairbank, *Errata, Evidence for a Temperature-Dependent Surface Shielding Effect in Cu*, Physical Review Letters **67** (2) (1991) p283

[21] F. Rossi and G. I. Opat, *Observations of the effects of adsorbates on patch potentials*, Journal of Physics D: Applied Physics **25** (1992) p1349-1353

[22] J. Labaziewicz, Y. Ge, D. R. Leibrandt, S. X. Wang, R. Shewmon and I. L. Chuang, *Temperature Dependence of Electric Field Noise above Gold Surfaces*, Physical Review Letters **101** (2008) p180602-1-4

[23] H. Dittus, C. Lämmerzahl and H. Selig, *Testing the Universality of Free Fall for Charged Particles in Space*, General Relativity and Gravitation **36** (3) (2004) p571-591

[24] R. M. Santilli, *Antigravity*, Hadronic Journal **17** (1994) p257

[25] A. P. Mill Jr., *Possibilities for measuring the passive gravitational mass of electrons and positrons in free horizontal flight*, Hadronic Journal **19** (1996) p77-86

[26] C. Kurz, S. J. Gilbert, R. G. Greaves and C. M. Surko, *New source of ultra-cold positron and electron beams*, Nuclear Instruments and Methods in Physics Research B **143** (1998) p188-194

[27] A. van Veen, H. Schut, J. de Roode, F. Labohm, C. V. Falub, S. W. H. Eijt and P. E. Mijnarends, *Intense positron sources and their applications*, Materials Science Forum **363-3** (2001) p415-419

[28] C. Hugenschmidt, G. Kögel, R. Repper, K. Schreckenbach, P. Sperr, B. Strasser and W. Triftshäuser, *NEPOMUC. The new positron beam facility at FRM II*, Materials Science Forum **445-446** (2004) p480-482

[29] J. R. Danielson, T. R. Weber and C. M. Surko, *Next Generation Trap for Positron Storage*, Non-Neutral Plasma Physics VII, $9^{th}$ International Workshop on Non-Neutral Plasmas, edited by J. R. Danielson and T. S. Pedersen, (2009) p199-206

[30] J. L. Wiza, *Microchannel plate detectors*, Nuclear Instruments and Methods **162** (1979) p587-601

[31] P.Y. Schmidt et al., *An efficient position sensitive detector for 3-30 keV positrons and electrons*, Nuclear Instruments and Methods in Physics Research A **376** ( 1996) p139-145

[32] ALPHA Collaboration, G. B. Andresen et al., *Antiproton, positron, and electron imaging with a microchannel plate / phosphor detector*, Review of Scientific Instruments **80** (2009) p123701-1-5

[33] See for instance http://www.scmos.com/

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# The Santilli's Theory 'Invasion' In Hyperstructures

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**Abstract.** In the quiver of hyperstructures Professor R. M. Santilli in early 90'es, tried to find algebraic structures in order to express his pioneer Lie-Santilli Theory. Santilli's theory on 'isotopies' and 'genotopies', born in 1960's, desperately needs 'units e' on left or right, which are nowhere singular, symmetric, real-valued, positive-defined for n-dimensional matrices based on the so called isofields. These elements can be found in hyperstructure theory, especially in $H_v$-structure theory introduced in 1990. This connection appeared first in 1996 and actually several $H_v$-fields, the e-hyperfields, can be used as isofields or genofields so as, in such way they should cover additional properties and satisfy more restrictions. Meanwhile, the hyperstructure theory obtained a lot of results and applications in mathematics as well as in other applied sciences.

This presentation aims to review applicable hyperstructures in Lie Santilli theory especially when multivalued problems appeared, either in finite or in infinite case.

**Key words:** Lie-Santilli theory, hyperstructures, hope, $H_v$-structures.
**AMS Subject Classification:** 20N20, 16Y99

## 1 INTRODUCTION

The hyperstructures were introduced by F.Marty in 1934 [18] when he first defined the hypergoup as a set equipped with an associative and reproductive hyperoperation. The motivating example was the quotient of a group by any, not necessarily normal, subgroup. M.Koskas in 1970 [17] was introduced the fundamental relation $\beta^*$, which it turns to be the main tool in the study of hyperstructures. T.Vougiouklis in 1990 [27] was introduced the $H_v$-structures, by defining the weak axioms. The motivating example of those hyperstructures is the

quotient of any group by any partition. Therefore the class of $H_v$-structures is the largest class of hyperstructures. Therefore we have:

**Motivation for $H_v$-structures:**

*The quotient of a group with respect to an invariant subgroup is a group.*
*Marty states that, the quotient of a group with respect to any subgroup is a hypergroup.*
*Now, the quotient of a group with respect to any partition is an $H_v$-group.*

In [47] we introduced the abbreviation: ***hyperoperation=hope***. Thus there is a definition: In algebraic hyperstructures there is a hope, in the classical structures there is not any hope!

In 1996 R.M.Santilli and T.Vougiouklis [21], point out that in physics the most interesting hyperstructures are the one called e-hyperstructures. These hyperstructures contain a unique left ant right scalar unit, which is the most important tool in Lie-Santilli theory. In what follows we present the related hyperstructure theory mainly from the paper [21], enriched with some new results on the related hyperstructures. However one can see the books by P.Corsini [5], T.Vougiouklis [30], P.Corsini-V.Leoreanu [6] and B.Davvaz- V.Leoreanu-Fotea [12], for more definitions as well as the site: *aha.eled.duth.gr,* for an extensive bibliography on the concept. Moreover, in this site one can see the Vougiouklis's point of view on the birth and the history of $H_v$-structures in the above site: *An $H_v$-interview, i.e. weak, with Th. Vougiouklis,* Interviewer N.Lygeros.

## 2 BASIC DEFINITIONS ON HOPES

In a set H is called ***hyperoperation*** (abr. ***hope***) or *multivalued operation*, any map from H×H to the power set of H. Therefore, in a hope

$$\cdot : H\times H \to \wp(H) : (x,y) \to x\cdot y \subset H$$

the result is a subset of H, intead of an element as we have in usually operations. In a set H equipped with a hope $\cdot: H\times H \to \boldsymbol{P}(H)-\{\varnothing\}$, we abbreviate by

*WASS* the *weak associativity*: $(xy)z \cap x(yz) \neq \varnothing$, $\forall x,y,z \in H$ and by

*COW* the *weak commutativity*: $xy \cap yx \neq \varnothing$, $\forall x,y \in H$.

The hyperstructure $(H,\cdot)$ is called *$H_v$-semigroup* if it is *WASS* and it is called *$H_v$-group* if it is reproductive $H_v$-semigroup, i.e. $xH=Hx=H$, $\forall x \in H$. The hyperstructure $(R,+,\cdot)$ is called *$H_v$-ring* if $(+)$ and $(\cdot)$ are *WASS*, the reproduction axiom is valid for $(+)$ and $(\cdot)$ is *weak distributive* with respect to $(+)$:

$$x(y+z)\cap(xy+xz)\neq\varnothing, \quad (x+y)z\cap(xz+yz)\neq\varnothing, \quad \forall x,y,z\in R.$$

An extreme class of hyperstructures is the following [26]: An $H_v$-structure is called *very thin* iff all hopes are operations except one, which has all hyperproducts singletons except one, which is a subset of cardinality more than one.

An $H_v$-group is called *cyclic* [22], if there is an element, called *generator*, which the powers have union the underline set. The minimal power with the above property is called *period* of the generator. Moreover if there exist an element and a special power, the minimum one, is the underline set, then the $H_v$-group is called *single-power cyclic*.

The main tool to study all hyperstructures are the fundamental relations $\beta^*$, $\gamma^*$ and $\varepsilon^*$, which are defined, in $H_v$-groups, $H_v$-rings and $H_v$-vector spaces, respectively, as the smallest equivalences so that the quotient would be group, ring and vector space, respectively [23, 27, 30]. A way to find the fundamental classes is given by analogous theorems to the following:

**Theorem 2.1** Let $(H,\cdot)$ be an $H_v$-group and $U$ be all finite products of elements of H. We define the relation $\beta$ by setting $x\beta y$ iff $\{x,y\}\subset u$, $u\in U$. Then $\beta^*$ is the transitive closure of $\beta$.

The main point of the proof of this theorem is that the relation $\beta$ guaranties the validity of the following: Take two elements x,y such that $\{x,y\}\subset u\in U$ and any hyperproduct where one of these elements is used. Then, if this element is replaced by the other, the new hyperproduct is inside the same fundamental class where the first hyperproduct is. Therefore, if the 'hyperproducts'of the above $\beta$-classes are 'products', then, they are fundamental classes.

Analogous theorems for the relations $\gamma^*$ in $H_v$-rings and $\varepsilon^*$ in $H_v$-modules and $H_v$-vector spaces, are also proved.

An element is called *single* if its fundamental class is singleton.

The fundamental relations are used for general definitions. Thus, to define the $H_v$-field the $\gamma^*$ is used: A $H_v$-ring $(R,+,\cdot)$ is called ***$H_v$-field*** if $R/\gamma^*$ is a field. In the sequence the *$H_v$-vector space* is defined [30].

Let $(H,\cdot)$, $(H,*)$ be $H_v$-semigroups defined on the same set H. $(\cdot)$ is called *smaller* than $(*)$, and $(*)$ *greater* than $(\cdot)$, iff there exists an

$$f\in Aut(H,*) \quad \text{such that} \quad xy\subset f(x*y), \ \forall x,y\in H.$$

Then we write $\cdot\leq^*$ and we say that $(H,*)$ *contains* $(H,\cdot)$. If $(H,\cdot)$ is a structure then it is called *basic structure* and $(H,*)$ is called *$H_b$-structure*.

**The little Theorem 2.2** Greater hopes than the ones which are *WASS* or *COW*, are also *WASS* or *COW*, respectively.

The definition of the $H_v$-field introduced a new class of hyperstructures[40,43]:

**Definition 2.3** The $H_v$-semigroup $(H,\cdot)$ is called ***h/v-group*** if the quotient $H/\beta^*$ is a group.

The h/v-groups are a generalization of the $H_v$-groups because in h/v-groups the reproductivity is not necessarily valid. However, sometimes a kind of *reproductivity of classes* is valid. This leads the quotient to be reproductive. In a similar way the *h/v-rings, h/v-fields, h/v-modulus, h/v-vector spaces* etc, are defined.

The motivation to this concept is the following: Let $\omega^*$ be the kernel of the canonical map from R to $R/\gamma^*$; then we call *reproductive $H_v$-field* any $H_v$-field $(R,+,\cdot)$ if the following axiom is valid:

$$x(R-\omega^*) = (R-\omega^*)x = R-\omega^*, \ \forall x \in R-\omega^*.$$

So the h/v-group is a generalization of the $H_v$-group since the reproductivity is not necessarily valid. Sometimes a kind of *reproductivity of classes* is valid, i.e. if H is partitioned into equivalence classes $\sigma(x)$, then the quotient is reproductive $x\sigma(y)=\sigma(xy)=\sigma(x)y$, $\forall x \in H$.

Hopes on any type of matrices can be defined:

**Definition 2.4** [48] Let $A=(a_{ij}) \in \mathbf{M}_{m\times n}$ be matrix and $s,t \in N$, with $1 \le s \le m$, $1 \le t \le n$.

Then *helix-projection* is a map $\underline{st}: \mathbf{M}_{m\times n} \to \mathbf{M}_{s\times t}: A \to A\underline{st} = (\underline{a}_{ij})$, where $A\underline{st}$ has entries

$$\underline{a}_{ij} = \{ a_{i+\kappa s, j+\lambda t} \mid 1 \le i \le s, \ 1 \le j \le t \text{ and } \kappa,\lambda \in N, \ i+\kappa s \le m, \ j+\lambda t \le n \}$$

Let $A=(a_{ij}) \in \mathbf{M}_{m\times n}$, $B=(b_{ij}) \in \mathbf{M}_{u\times v}$ be matrices and $s=\min(m,u)$, $t=\min(n,v)$. We define a hyper-addition, called *helix-addition*, by

$$\oplus : \mathbf{M}_{m\times n} \times \mathbf{M}_{u\times v} \to \boldsymbol{P}(\mathbf{M}_{s\times t}): (A,B) \to A \oplus B = A\underline{st} + B\underline{st} = (\underline{a}_{ij}) + (\underline{b}_{ij}) \subset \mathbf{M}_{s\times t}$$

where $(\underline{a}_{ij})+(\underline{b}_{ij}) = \{(c_{ij})=(a_{ij}+b_{ij}) \mid a_{ij} \in \underline{a}_{ij} \text{ and } b_{ij} \in \underline{b}_{ij}\}$.

Let $A=(a_{ij}) \in \mathbf{M}_{m\times n}$, $B=(b_{ij}) \in \mathbf{M}_{u\times v}$ and $s=\min(n,u)$. We define the *helix-multiplication*, by

$$\otimes : \mathbf{M}_{m\times n} \times \mathbf{M}_{u\times v} \to \boldsymbol{P}(\mathbf{M}_{m\times v}): (A,B) \to A \otimes B = A\underline{ms} \cdot B\underline{sv} = (\underline{a}_{ij}) \cdot (\underline{b}_{ij}) \subset \mathbf{M}_{m\times v}$$

where $(\underline{a}_{ij}) \cdot (\underline{b}_{ij}) = \{(c_{ij})=(\sum a_{it} b_{tj}) \mid a_{ij} \in \underline{a}_{ij} \text{ and } b_{ij} \in \underline{b}_{ij}\}$.

The helix-addition is commutative, WASS but not associative. The helix-multiplication is WASS, not associative and it is not distributive, not even weak, to the helix-addition. For all matrices of the same type, the inclusion distributivity, is valid.

The problem of enumeration and classification of $H_v$-structures, was started from the beginning [18]. However, the problem becomes more complicate in $H_v$-structures because we have very great numbers in this case. The partial order in $H_v$-structures [27] and the Little Theorem, transfers and restrict the problem in finding the minimal, *up to isomorphisms*, $H_v$-structures. In this direction we have results by Bayon & Lygeros [3]:

Let $H=\{a,b\}$ a set of two elements. There are 20 $H_v$-groups, up to isomorphism,

Suppose in $H=\{e,a,b\}$, a hope is defined and there exists a scalar unit, then, there are 13 *minimal* $H_v$-groups. The number of all $H_v$-groups with three elements, up to isomorphism, which have a scalar unit, is 292.

In a set with three elements there are, exactly 6.494 minimal $H_v$-groups. 137 are abelians and the 6.357 are non-abelians; the 6.152 are cyclic and the 342 are not cyclic.

The number of $H_v$-groups with three elements, up to isomorphism, is 1.026.462. More precisely, there are 7.926 abelians and 1.018.536 non-abelians; the 1.013.598 are cyclic and the 12.864 are not cyclic, the 16 are very thin.

The number of all $H_v$-groups with four elements, up to isomorphism, which have a scalar unit, is 631.609. There are 10.614.362 abelian hopes from which the 10.607.666 are cyclic and the 6.696 are not. There are 8.028.299.905 abelian $H_v$-groups from which the 7.995.884.377 are cyclic and the 32.415.528 are not.

# 3 ENLARGING AND REDUCING HYPERSTRUCTURES

In [37] the 'enlarged' hyperstructures were examined in the sense that an extra element, outside the underlying set, appears in one result. In both directions, enlargement or reduction, most useful in representation theory, are those $H_v$-structures with the same fundamental structure: Suppose we have a structure and one element, outside of the structure, then we can attach this element in order to have a hyperstructure which becomes h/v-structure. Moreover we have the opposite problem: How one can remove at least one element of an $H_v$-structure or a classical structure?

**The Attach Construction 3.1** [37,41]. Let $(H,\cdot)$ be an $H_V$-semigroup and $v\notin H$. We extend the $(\cdot)$ into $\underline{H}=H\cup\{v\}$ as follows: $x\cdot v=v\cdot x=v$, $\forall x\in H$, and $v\cdot v=H$. The $(\underline{H},\cdot)$ is a h/v-group where $(\underline{H},\cdot)/\beta^*\cong Z_2$ and $v$ is a single element.
We call the hyperstructure $(\underline{H},\cdot)$ the attach h/v-group of $(H,\cdot)$.

**Remarks.** The core of $(\underline{H},\cdot)$ is the set H. All scalar elements of $(H,\cdot)$ are also scalars in $(\underline{H},\cdot)$ and any unit element of $(H,\cdot)$ is also a unit of $(\underline{H},\cdot)$. Finally, if $(H,\cdot)$ is *COW* (resp. commutative) then $(\underline{H},\cdot)$ is also *COW* (resp. commutative).
The motivation of the attach construction is the first kind very thin $H_v$-groups [26].

In the representation theory of $H_v$-groups by $H_v$-matrices one needs $H_v$-rings or $H_v$-fields which have non-degenerate fundamental structures in addition with only few of hypersums and hyperproducts to have cardinals greater than one.

**Theorem 3.2** Let $(G,\cdot)$ be semigroup and $v\notin G$ be an element appearing in a product ab, where $a,b\in G$, thus the result becomes a hyperproduct $a\otimes b=\{ab,v\}$. Then the minimal hyperoperation $(\otimes)$ extended in $G'=G\cup\{v\}$ such that $(\otimes)$ contains $(\cdot)$ in the restriction on G, and such that $(G',\otimes)$ is a minimal $H_v$-semigroup which has fundamental structure isomorphic to $(G,\cdot)$, is defined as follows:

$$a\otimes b=\{ab,v\},\qquad x\otimes y=xy,\quad \forall(x,y)\in G^2-\{(a,b)\}$$

$$v\otimes v=abab,\quad x\otimes v=xab\quad \text{and}\quad v\otimes x=abx,\quad \forall x\in G.$$

Therefore $(G',\otimes)$ is a very thin $H_v$-semigroup.

If $(G,\cdot)$ is commutative then the $(G',\otimes)$ becomes strongly commutative.

Generaly we have:

**Definitions 3.3** [37,41]. *Let* $(H,\cdot)$ *be a hypergroupoid.*
*We say that we* ***remove*** $h\in H$, if we simply consider the restriction of $(\cdot)$ on $H-\{h\}$.
*We say that* $\underline{h}\in H$ ***absorbs*** $h\in H$ *if we replace* h, *whenever it appears, by* $\underline{h}$.
*We say that* $\underline{h}\in H$ ***merges with*** $h\in H$, *if we take as product of* $x\in H$ by $\underline{h}$, *the union of the results of* x *with both* h *and* $\underline{h}$, *and consider* h *and* $\underline{h}$ *as one class, with representative* $\underline{h}$.

The *uniting elements* method was introduced by Corsini–Vougiouklis [7] in 1989. With this method one puts in the same class, two or more elements. This leads, through hyperstructures, to structures satisfying additional properties.

The ***uniting elements*** method is the following: Let G be algebraic structure and let d be a property, which is not valid and it is described by a set of equations; then, consider the partition in G for which it is put together, in the same partition class, every pair of elements that causes the non-validity of the property d. The quotient by this partition G/d is an $H_v$-structure. Then, quotient out the $H_v$-structure G/d by the fundamental relation $\beta^*$, a stricter structure $(G/d)/\beta^*$ for which the property d is valid, is obtained.

An interesting application of the uniting elements is when more than one properties are desired. The reason for this is that some of the properties lead straighter to the classes than others. So, it is better to apply the straightforward classes followed by the more complicated ones. The commutativity and reproductivity are easy applicable properties. One can do this because the following is valid.

**Theorem 3.4** [30] Let $(G,\cdot)$ be a groupoid, and $F=\{f_1,\ldots, f_m, f_{m+1},\ldots, f_{m+n}\}$ be a system of equations on G consisting of two subsystems $F_m=\{f_1,\ldots,f_m\}$ and $F_n=\{f_{m+1},\ldots, f_{m+n}\}$. Let $\sigma$, $\sigma_m$ be the equivalence relations defined by the uniting elements procedure using the systems F and $F_m$ resp., and let $\sigma_n$ be the equivalence relation defined using the induced equations of $F_n$ on the grupoid $G_m=(G/\sigma_m)/\beta^*$. Then

$$(G/\sigma)/\beta^* \cong (G_m/\sigma_n)/\beta^*.$$

# 4 THE ∂-HOPES

In [44] a hope, in a groupoid with a map on it, called *theta* $\partial$, is introduced.

***Definitions 4.1*** Let $(G,\cdot)$ be groupoid (resp., hypergroupoid) and $f:G\to G$ be a map. We define a hope $(\partial)$, called ***theta-hope*** and we write ***∂-hope***, on G as follows

$$x\partial y = \{ f(x)\cdot y, x\cdot f(y) \},\ \forall x,y\in G. \text{ (resp. } x\partial y = (f(x)\cdot y)\cup(x\cdot f(y),\ \forall x,y\in G)$$

If $(\cdot)$ is commutative then $(\partial)$ is commutative. If $(\cdot)$ is *COW*, then $(\partial)$ is *COW*.

Let $(G,\cdot)$ be a groupoid (resp. hypergroupoid) and $f:G\to \boldsymbol{P}(G)-\{\varnothing\}$ be any multivalued map. We define the $(\partial)$, on G as follows

$$x\partial y = (f(x)\cdot y)\cup(x\cdot f(y),\ \forall x,y\in G.$$

Let $(G,\cdot)$ be a groupoid, $f_i:G\to G$, $i\in I$, be a set of maps on G. The

$$f_\cup:G\to \boldsymbol{P}(G):\ f_\cup(x)=\{f_i(x)\mid i\in I\},$$

is *the union* of $f_i(x)$. We have the *union theta-hope* $(\partial)$, on G if we take $f_\cup(x)$. If we take $\underline{f}\equiv f\cup(id)$, then we have the *b-theta-hope:* $\underline{\partial}$.

This definition can be generalized as follows:

**Definition 4.2** Let H be a set equipped with n operations (or hopes) $\otimes_1,\otimes_2,\ldots,\otimes_n$ and a map (or multivalued map)

f: H→H (or f:H→$P$(H), resp.), then n hopes $\partial_1, \partial_2, \ldots, \partial_n$ on H can be defined, called *theta-hopes* by putting

$$x\partial_i y = \{f(x)\otimes_i y,\ x\otimes_i f(y)\}, \quad \forall x,y\in H \text{ and } i\in\{1,2,\ldots,n\}$$

or, in case where $\otimes_i$ is hope or f is multivalued map, we have

$$x\partial_i y = (f(x)\otimes_i y)\cup(x\otimes_i f(y)), \ \forall x,y\in H \text{ and } i\in\{1,2,\ldots,n\}$$

If $\otimes_i$ is associative then $\partial_i$ is WASS.

***Motivation*** for the definition of the theta-hope is the map *derivative* where only the multiplication of functions can be used. Therefore, in these terms, for two functions s(x), t(x), we have $s\partial t = \{s't, st'\}$ where (′) denotes the derivative.

**Example 4.3** Taking the application on the derivative, consider all polynomials of first degree $g_i(x) = a_ix+b_i$. We have

$$g_1\partial g_2 = \{a_1a_2x + a_1b_2,\ a_1a_2x+b_1a_2\},$$

so this is a hope in the set of the first degree polynomials. Moreover all polynomials x+c, where c be a constant, are units.

**Properties 4.4** [45,46]. If (G,·) is a semigroup then:
For every f, the hyperoperation ($\partial$) is *WASS*, and the b-theta-operation ($\underline{\partial}$) is *WASS*.
If f is homomorphism and projection, then ($\partial$) is associative.
*Reproductivity.* If (·) is reproductive then ($\partial$) is also reproductive.
*Commutativity.* If (·) is commutative then ($\partial$) is commutative. If f is into the centre of G, then ($\partial$) is commutative. If (·) is *COW* then, ($\partial$) is *COW*.
*Unit elements.* u is a unit element if f(u)=e, where e be a unit in (G,·). The elements of the kernel of f, are the units of (G,$\partial$).
*Inverse elements.* Let (G,·) be a monoid with unit e and u be a unit in (G,$\partial$), then f(u)=e. The elements $x' = (f(x))^{-1}u$ and $x' = u(f(x))^{-1}$, are the right and left inverses, respectively. We have two-sided inverses iff f(x)u = uf(x).
**Proposition 4.5** Let (G,·) be a group then, for all f: G→G, the (G,$\partial$) is an $H_v$-group.

In order to see a connection of the merge with the $\partial$-hope, consider the map f such that f(h)=h and f(x)=x in the rest cases.

**Example 4.6** ***P-hopes.*** Let (G,·) be commutative semigroup and P⊂G. Consider the multivalued map f such that

f(x) = P·x, $\forall x\in G$. Then we have

$$x\partial y = x\cdot y\cdot P, \quad \forall x,y\in G.$$

So the $\partial$-hope coincides with the well known class of P-hopes [22].

One can define theta-hopes on rings and other more complicate structures, where more than one theta-hopes can be defined.

**Definition 4.7** Let $(R,+,\cdot)$ be a ring and $f:R\to R$, $g:R\to R$ be two maps. We define two hopes $(\partial_+)$ and $(\partial_\cdot)$, called both ***theta-hopes***, on R as follows

$$x\partial_+y = \{f(x)+y,\ x+f(y)\} \quad \text{and} \quad x\partial_\cdot y = \{g(x)\cdot y,\ x\cdot g(y)\},\ \forall x,y\in G.$$

A hyperstructure $(R,+,\cdot)$, where $(+)$, $(\cdot)$ be hopes which satisfy all $H_v$-ring axioms, except the weak distributivity, will be called ***$H_v$-near-ring***.

**Proposition 4.8** Let $(R,+,\cdot)$ ring and $f:R\to R$, $g:R\to R$ maps. The hyperstructure $(R,\partial_+,\partial_\cdot)$, called ***theta***, is an $H_v$-near-ring. Moreover $(+)$ is commutative.

**Proposition 4.9** Let $(R,+,\cdot)$ ring and $f:R\to R$, $g:R\to R$ maps, then $(R,\underline{\partial}_+,\partial_\cdot)$, is an $H_v$-ring.

**Properties 4.10** The theta hyperstructure $(R,\partial_+,\partial_\cdot)$ takes new form in special casses:

(a) If $f(x)\equiv g(x)$, $\forall x\in R$, i.e. the two maps coincide, then we have

$$x\partial_\cdot(y\partial_+z)\cap(x\partial_\cdot y)\partial_+(x\partial_\cdot z) = \varnothing.$$

If f is homomorphism and projection, then $(R,\partial_+,\partial_\cdot)$ is an $H_v$-ring.

(b) If $f(x)=x$, $\forall x\in R$, then $(R,+,\partial_\cdot)$ becomes a multiplicative $H_v$-ring:

$$x\partial_\cdot(y+z)\cap(x\partial_\cdot y)+(x\partial_\cdot z) = \{\, g(x)y+g(x)z\} \neq \varnothing.$$

# 5 THE $H_V$-LIE ALGEBRAS

**Definition 5.1** [40]. Let $(F,+,\cdot)$ be an $H_v$-field, $(V,+)$ be a COW $H_v$-group and there exists an external hope

$$\cdot : F\times V \to P(V) : (a,x) \to ax$$

such that, for all a,b in F and x,y in $V$ we have

$$a(x+y)\cap(ax+ay)\neq\varnothing, \quad (a+b)x\cap(ax+bx)\neq\varnothing, \quad (ab)x\cap a(bx)\neq\varnothing,$$

then V is called an ***$H_v$-vector space*** over F.

In the case of an $H_v$-ring instead of $H_v$-field then the ***$H_v$-modulo*** is defined.

In the above cases the fundamental relation $\varepsilon^*$ is the smallest equivalence relation such that the quotient $V/\varepsilon^*$ is a vector space over the fundamental field $F/\gamma^*$.

The general definition of an $H_v$-Lie algebra over a field F is the following [40]:

**Definition 5.2** Let $(L,+)$ be an $H_v$-vector space over the field $(F,+,\cdot)$, $\varphi:F\to F/\gamma^*$, the canonical map and $\omega_F=\{x\in F:\varphi(x)=0\}$, where 0 is the zero of the fundamental field $F/\gamma^*$. Similarly, let $\omega_L$ be the core of the canonical map $\varphi': L\to L/\varepsilon^*$ and denote by the same symbol 0 the zero of $L/\varepsilon^*$. Consider the bracket (commutator) hope:

$$[\,,\,]: L\times L\to P(L):\ (x,y)\to[x,y]$$

then $L$ is an ***$H_v$-Lie algebra*** over F if the following axioms are satisfied:

(L1) The bracket hope is bilinear, i.e.

$$[\lambda_1x_1+\lambda_2x_2,y]\cap(\lambda_1[x_1,y]+\lambda_2[x_2,y])\neq\varnothing,\quad [x,\lambda_1y_1+\lambda_2y]\cap(\lambda_1[x,y_1]+\lambda_2[x,y_2])\neq\varnothing$$

for all $x, x_1, x_2, y, y_1, y_2\in L$ and $\lambda_1, \lambda_2$ in F

(L2) $[x,x]\cap\omega_L\neq\varnothing$ for all $x$ in $L$

(L3) $([x,[y,z]]+[y,[z,x]]+[z,[x,y]])\cap\omega_L\neq\varnothing$ for all $x,y$ in $L$

This is a general definition thus one can use special cases in order to face problems in applied sciences. Moreover, we see how the weak properties can be defined as the above weak linearity (L1), anti-commutativity (L2) and the Jacobi identity (L3). Similarly the *h/v-rings, h/v-fields, h/v-modulus, h/v-vector spaces* etc, are defined.

We present here a direction to obtain results from special cases by applying $\partial$-hopes on more complicated structures, in the sense that they have more than one operation.

**Theorem 5.3** Consider the ring of integers $(Z,+,\cdot)$ and let $n\neq0$. Consider the map f such that $f(0)=n$ and $f(x)=x,\forall x\in Z-\{0\}$. Then $(Z,\partial_+,\partial\cdot)$ is an $H_v$-near-ring, with $(Z,\partial_+,\partial\cdot)/\gamma^*\cong Z_n$.

**Proposition 5.4** Let $(V,+,\cdot)$ be an algebra over the field $(F,+,\cdot)$ and $f:V\to V$ be a map. Consider the $\partial$-hope defined only on the multiplication of the vectors $(\cdot)$, then $(V,+,\partial)$ is an $H_v$-algebra over F, where the related properties are weak. If, moreover f is linear then we have more strong properties.

**Definition 5.5** Let $L$ be a Lie algebra, defined on an algebra $(V,+,\cdot)$ over the field $(F,+,\cdot)$ where the Lie bracket $[x,y]=xy-yx$. Consider any map $f:L\to L$, then the $\partial$-hope is defined as follows

$$x\partial y=\{f(x)y-f(y)x,\ f(x)y-yf(x),\ xf(y)-f(y)x,\ xf(y)-yf(x)\}$$

**Proposition 5.6** Let $(V,+,\cdot)$ be an algebra over the field $(F,+,\cdot)$ and $f:V\to V$ be a linear map. Consider the $\partial$-hope defined only on the multiplication of the vectors $(\cdot)$, then $(V,+,\partial)$ is an $H_v$-algebra over F, with respect to Lie bracket, where the weak anti-commutatinity and the inclusion linearity is valid.

We can see that the weak linearity is valid, more precisely, the inclusion linearity is valid:

$$[\lambda_1 x_1+\lambda_2 x_2,y] \subset \lambda_1[x_1,y] + \lambda_2[x_2,y].$$

Remark that one can face the weak Jacobi identity in analogous to the above propositions as well. One can use well known maps as constants or linear.

We conclude this paragraph with a definition of a hypergroupoid algebra [23,30].

**Definition 5.7** Let $(G,\cdot)$ hypergroupoid, is called *set of fundamental maps* on G, the set of onto maps

$$Q=\{q:G\times G\to G:(x,y)\xrightarrow{onto} z\,|\,z\in xy\}.$$

Any subset $Q_s\subset Q$ defines a hope $(\circ_s)$ on G as follows $x\circ_s y = \{z\,|\,z= q(x,y)$ for some $q\in Q\}$

$\circ_s \leq \cdot$ and $Q_s\subset Q_{os}$, where $Q_{os}$ is the set of fundamental maps with respect to $(\circ_s)$. A $Q_a\subset Q$ for which every $Q_s\subset Q_a$ has $(\circ_s)$ associative (resp. *WASS*) is called *associative* (resp. *WASS*). A hypergroupoid $(G,\cdot)$ is q-*WASS* if there exists an element $q_o\in Q$ which defines an associative operation $(\circ)$ in G. Remark that for $H_v$-groups we have $Q\neq\emptyset$.

Suppose G is finite, $\text{card}G = |G| = n$, it is q-*WASS* with associative $q_o\in Q$. In the set K[G] of all formal linear combinations of elements of G with coefficients from a field K, we define an operation (+):

$$(f_1+f_2)(g) = f_1(g)+f_2(g), \quad \forall g\in G, f_1,f_2\in K[G]$$

and a hope (*), the *convolution*,

$$f_1*f_2 = \{ f_q: f_q(g) = \sum_{q(x,y)=g} f_1(x)f_2(y),\ q\in Q\}.$$

**Definition 5.8** The hyperstructure $(K[G],+,*)$ is a multiplicative $H_v$-ring where the inclusion distributivity is valid. This $H_v$-ring is called *hypergroupoid $H_v$-algebra.*

$\forall q\in Q$, $g\in G$, we have $|Q\leq \prod_{(x,y)\in G\times G}(|xy|)$, $1\leq |q^{-1}(g)|\leq n^2-n+1$ and $\sum_{g\in G} |q^{-1}(g)|=n^2$.

The zero map $f(x)=0$ is a scalar element in K[G].

# 6 REPRESENTATIONS

Representations (we abbreviate by ***rep***) of $H_v$-groups, can be considered either by generalized permutations [28] or by $H_v$-matrices [23,25,29]. First we present the matrix reps.

In the classical books on representations we find the following definitions: Let **G** be a group and $V$ be a finite dimensional vector space over the field **F**. A *representation* of **G** is a homomorphism $\rho: \mathbf{G} \to \mathrm{Aut}(V)$ of **G** into the set of automorphisms of $V$.

Analogous definitions are given for complicate structures: Let $L$ be a Lie algebra then a *rep* of $L$ is a homomorphism $\rho: L \to gl(V)$, from $L$ into linear transformations on $V$ over **F**.

Since there exists 1-1 correspondence on the sets of all endomorphisms with n×n matrices, where $n=\dim V$, any rep corresponds to each element, of a finite group, a matrix, and this set of matrices acts exactly as the group. Notice that the addition of matrices and the external multiplication of scalars by matrices are 'natural operations', by contrast the multiplication of matrices is a 'strange' operation because it is dictated by the composition of linear maps or automorphisms or by the composition of maps.

With this theory, mathematicians try to transfer the study of the several structures into the study of matrices which is clear and easy. Ado's theorem -and the related ones- states that every finite-dimensional Lie algebra has a faithful finite-dimensional rep. The two steps in rep theory: first, by the Cayley's theorem every group has a faithful rep by permutations. Second, every permutation group of order n can be represented by n×n *monomial matrices*, i.e. matrices with only one entry 1 in every row and column and the rest entries are 0. The above steps are clear but the obtained reps are not useful since the matrices are of type n×n. Thus, the main attempt is to reduce the dimension of reps. Most important is to find the irreducible reps over the field of real or complex numbers.

The rep theory is very important because it represents all groups in one form so that they can be compared and studied in the same way. Thus the low dimensional reps are most useful, i.e. the reps by 2×2 matrices is the simplest non degenerate case.

***$H_v$-matrix*** (or *h/v-matrix*) is called a matrix with entries elements of an $H_v$-ring or $H_v$-field (or h/v-ring or h/v-field ). The hyperproduct of $H_v$-matrices $\mathbf{A}=(a_{ij})$ and $\mathbf{B}=(b_{ij})$, of type m×n and n×r, respectively, is a set of m×r $H_v$-matrices, defined in a usual manner:

$$\mathbf{A}\cdot\mathbf{B} = (a_{ij})\cdot(b_{ij}) = \{\, \mathbf{C} = (c_{ij}) \mid c_{ij} \in \oplus\Sigma a_{ik}\cdot b_{kj} \,\},$$

where $(\oplus)$ denotes the *n-ary circle hope* on the hyperaddition [30]: that is the sum of products of elements of the $H_v$-ring is considered to be the union of the sets obtained with all possible parentheses. However, in the case of $2\times2$ $H_v$-matrices the 2-ary circle hope which coincides with the hyperaddition in the $H_v$-ring. Notice that the hyperproduct of $H_v$-matrices does not nessesarily satisfy *WASS*.

*The rep problem by $H_v$-matrices is the following:*

**Definition 6.1** Let $(H,\cdot)$ be $H_v$-group, $(R,+,\cdot)$ be $H_v$-ring and $\mathbf{M}_R = \{(a_{ij}) \mid a_{ij}\in R\}$, then any

$$\mathbf{T}: H\to\mathbf{M}_R:\ h\to\mathbf{T}(h) \quad \text{with} \quad \mathbf{T}(h_1h_2)\cap\mathbf{T}(h_1)\mathbf{T}(h_2)\neq\varnothing, \quad \forall h_1,h_2\in H,$$

is called ***$H_v$-matrix rep.*** If $T(h_1h_2)\subset T(h_1)T(h_2)$, then T is an *inclusion rep,* if $T(h_1h_2)=T(h_1)T(h_2)$, then T is *a good rep* an induced rep $T^*$ for the hypergroup algebra is obtained. If T is one to one and good then it is a *faithful rep.*

In reps of $H_v$-groups by $H_v$-matrices, there are two difficulties: To find an $H_v$-ring and an appropriate set of $H_v$-matrices.

The problem of reps is very complicated mainly because the cardinality of the product of two $H_v$-matrices is normally very big. The problem can be simplified in several special cases such as the following:

(a) The $H_v$-matrices are over $H_v$-rings with 0 and 1 and if these are scalars.Thus the e-hyperstructures are interesting in the rep theory.
(b) The $H_v$-matrices are over *very thin* $H_v$-rings.
(c) The case of $2\times2$ $H_v$-matrices, since the 2-ary circle hope coincides with the hyperaddition in $H_v$-rings. This is the lowest dimensional, non degenerate, rep.
(d) The case of $H_v$-rings in which the strong associativity in hyperaddition is valid.
(e) The case of $H_v$-rings which contains singles, then these act as absorbings.

The main theorem of reps on $H_v$-structures [30], is the following:

**Theorem 6.2** A necessary condition in order to have an inclusion rep T of an $H_v$-group $(H,\cdot)$ by $n\times n$ $H_v$-matrices over the $H_v$-ring $(R,+,\cdot)$ is the following:
For all classes $\beta^*(x)$, $x\in H$ there must exist elements $a_{ij}\in H$, $i,j\in\{1,...,n\}$ such that

$$T(\beta^*(a)) \subset \{\, A = (a'_{ij}) \mid a'_{ij} \in \gamma^*(a_{ij}),\ i,j\in\{1,...,n\} \,\}$$

Therefore, every inclusion rep $T:H\to M_R$: $a\mapsto T(a)=(a_{ij})$ induces a homomorphic rep $T^*$ of the group $H/\beta^*$ over the ring $R/\gamma^*$ by setting $T^*(\beta^*(a))=[\gamma^*(a_{ij})]$, $\forall\beta^*(a)\in H/\beta^*$, where the element $\gamma^*(a_{ij})\in R/\gamma^*$ is the ij entry of the matrix $T^*(\beta^*(a))$. Then $T^*$ is called *fundamental induced rep* of T.

Denote $tr_\varphi(T(x)) = \gamma^*(T(x_{ii}))$ the fundamental trace, then the mapping

$$X_T : H \to R/\gamma^* : x \mapsto X_T(x) = tr_\varphi(T(x)) = trT^*(x)$$

is called *fundamental character*. There are several types of traces.

Second, we present the rep problem is by *Generalized Permutations* (we write ***gp***).

**Definitions 6.3** [28,30] Let X be a set, then a map $f: X \to P(X)-\{\varnothing\}$, is a ***gp of X*** if the reproduction axiom is valid $\cup_{x\in X} f(x) = f(X) = X$. Denote by $M_X$ the set of all gps on X. For an $H_v$-group $(X,\cdot)$ and $a\in X$, the gp $f_a$ defined by $f_a(x)=ax$ is an *inner gp*. *Arrow* of f is any $(x,y)\in X^2$ with $y\in f(x)$. $f_2\in M_X$ *contains* $f_1\in M_X$ or $f_1$ is a *sub-gp* of $f_2$, if $f_1(x)\subset f_2(x)$, $\forall x\in X$, then we write $f_1\subset f_2$. If, moreover, $f_1\neq f_2$, then $f_1$ is a proper sub-gp of $f_2$. A $f\in M_X$ is called *minimal* if it has no proper sub-gp. Denote $\underline{M}_X$ the set of all minimal gps of $M_X$. The gp t with $t(x)=X$, $\forall x\in X$, is called *universal* and contains all elements of $M_X$. The *converse* of a gp f is the gp $\underline{f}$ defined by $\underline{f}(x)=\{z\in X: f(z)\ni x\}$, thus $\underline{f}$ is obtained by reversing arrows. We call *associated* to $f\in M_X$ the gp $f\circ\underline{f}$, where $(\circ)$ is the map composition.
The *union* $f = \cup_{i\in I} f_i$ of a family of gps $\{f_i: i\in I\}$, is defined by $f(x) = \cup_{i\in I} f_i(x)$, $\forall x\in X$.

For finite X, we reach a minimal gp, by the *deleting arrows* method.

**Theorem 6.4** Let $f\in M_X$, then $f\in\underline{M}_X$ if and only if, the following condition is valid: if $a\neq b$ and $f(a)\cap f(b)\neq\varnothing$, then $f(a)=f(b)$ and $f(a)$ is a singleton.

**Corollary 6.5** If $f\in\underline{M}_X$ then $\underline{f}\in\underline{M}_X$.

An explicit description of $\underline{M}_X$: $(f\circ\underline{f})(x) = f\{u: f(u)\ni x\} = \cup_{f(u)\ni x} f(u)$, $\forall x\in X$. So $(f\circ\underline{f})(x)=\{y:\exists\, u\in X,\{x,y\}\subset f(u)\}$. So, if 1 is the identity permutation, then $1\subset f\circ\underline{f}$, $\forall f\in M_X$.

There is a direct relation of $\beta^*$ and the associated gp $f\circ\underline{f}$. We see this relation, for finite X, in the following theorem:

**Theorem 6.6** If $f\in\underline{M}_X$ then $(f\circ\underline{f})(x) = \{ y\in X: f(y) = f(x) \}$.

In order to face the rep problem by the gp one can see that the usual map composition can not be used because it is not multivalued. Therefore the main ***open problem*** is to find standard hope on $M_X$.

Several constructions can be used to obtain appropriate $H_v$-rings and $H_v$-fields:

(i) Let $(H,\cdot)$ be $H_v$-group, then for every $(\oplus)$ such that $x\oplus y\supset\{x,y\}$, $\forall x,y\in H$, the $(H,\oplus,\cdot)$ is an $H_v$-ring. These $H_v$-rings are called *associated to* $(H,\cdot)$ $H_v$-rings.

In reps theory of hypergroups, in sense of Marty, there are three associated hyperrings $(H,\oplus,\cdot)$ to $(H,\cdot)$. The $(\oplus)$ is defined respectively, $\forall x,y\in H$, as follows:

type a: $x \oplus y = \{x,y\}$, type b: $x \oplus y = \beta^*(x) \cup \beta^*(y)$, type c: $x \oplus y = H$.

In the above types the strong associativity and strong or inclusion distributivity, is valid.

(ii) Let $(H,+)$ be $H_v$-group, then for all hopes $(\otimes)$ such that $x \otimes y \supset \{x,y\}$, $\forall x,y \in H$, the $(H,+,\otimes)$ is an $H_v$-ring.

A variation of this is the following: Let $(H,+)$ be $H_v$-group with a scalar zero 0. Then $\forall(\otimes)$: $x \otimes y \supset \{x,y\}$, $\forall x,y \in H-\{0\}$, $x \otimes 0 = 0 \otimes x = 0$, $\forall x \in H$, the $(H,+,\otimes)$ is an $H_v$-ring.

(iii) Let $(H,\cdot)$ be $H_v$-group. Take a $0 \notin H$ and set $H' = H \cup \{0\}$. Define a hope $(+)$ by: $0+0=0$, $0+x=H=x+0$, $x+y=0$, $\forall x,y \in H$,

and extend $(\cdot)$ in H' by $0 \cdot 0 = 0, 0 \cdot x = x \cdot 0 = 0, \forall x,y \in H$.

Then $(H',+,\cdot)$ is reproductive $H_v$-field with $H'/\gamma^* \cong Z_2$ where 0 is absorbing and single.

# 7 THE SANTILLI'S e-CONSTRUCTIONS

The Lie-Santilli theory on *isotopies* was born in 1970's to solve Hadronic Mechanics problems. Santilli [20], proposed a 'lifting'of the n-dimensional trivial unit matrix of a normal theory into a nowhere singular, symmetric, real-valued, positive-defined, n-dimensional new matrix. The original theory is reconstructed such as to admit the new matrix as left and right unit. The *isofields* needed in this theory correspond into the hyperstructures were introduced by Santilli and Vougiouklis in 1999 [21] and they are called *e-hyperfields*. The $H_v$-fields or h/v-fields can give e-hyperfields which can be used in the isotopy theory in applications as in physics or biology. We present in the following the main definitions and results restricted in the $H_v$-structures.

**Definition 7.1** A hyperstructure $(H,\cdot)$ which contain a unique scalar unit e, is called e-hyperstructure. In an e-hyperstructure, we normally assume that for every element x, there exists an inverse elemen $x^{-1}$, i.e. $e \in x \cdot x^{-1} \cap x^{-1} \cdot x$. Remark that the inverses are not necessarily unique.

**Definition 7.2** A hyperstructure $(F,+,\cdot)$, where $(+)$ is an operation and $(\cdot)$ is a hope, is called ***e-hyperfield*** if the following axioms are valid:

1. $(F,+)$ is an abelian group with the additive unit 0,
2. $(\cdot)$ is WASS,
3. $(\cdot)$ is weak distributive with respect to $(+)$,
4. 0 is absorbing element: $0 \cdot x = x \cdot 0 = 0$, $\forall x \in F$,

5. there exists a multiplicative scalar unit, i.e. $1\cdot x = x\cdot 1 = x$, $\forall x\in F$,
6. for every element $x\in F$ there exists a unique inverse $x^{-1}$, such that $1\in x\cdot x^{-1}\cap x^{-1}\cdot x$.

The elements of an e-hyperfield are called ***e-hypernumbers***. In the case that the relation $1=x\cdot x^{-1}=x^{-1}\cdot x$. is valid, then we say that we have a ***strong e-hyperfield.***

Now we introduce a general construction which is based on the partial ordering of the $H_v$-structures and on the Little Theorem.

**The Main e-Construction 7.3** Given a group $(G,\cdot)$, where e is the unit, then we can define in G, a large number of hopes $(\otimes)$ as follows: $x\otimes y = \{xy, g_1, g_2, \ldots\}$, $\forall x,y\in G-\{e\}$, and $g_1, g_2, \ldots$ are elements from $G-\{e\}$ which are not necessarily the same for each pair (x,y). Then $(G, \otimes)$ becomes an $H_v$-group which in fact is an $H_b$-group which contains the basic group $G-\{e\}$. The $H_v$-group $(G, \otimes)$ is an e-hypergroup. Moreover, if for each x,y such that $xy=e$ then we have $x\otimes y = xy$, then $(G, \otimes)$ becomes a strong e-hypergroup.

***Proof.*** The proof is immediate since for both cases we enlarge the results of the group by putting elements from the set G and applying the Little Theorem. Moreover one can easily see that the unit e is a unique scalar element and for each x in G, there exists a unique inverse $x^{-1}$, such that $1\in x\cdot x^{-1}\cap x^{-1}\cdot x$. Finally if the last condition is valid then we have $1=x\cdot x^{1}=x^{-1}\cdot x$, so the hyperstructure $(G, \otimes)$ is a strong e-hypergroup.

***Remark.*** The above main e-construction gives an extremely large class of e-hopes. These e-hopes can be used in the several more complicate hyperstructures to obtain appropriate e-hyperstructures. However, we remark that the most useful are the ones where only few products are enlarged and, even more, the extra elements are one or two. This means that we have analogous situation to the rep theory.

***Example.*** Consider the non-commutative quaternion group $Q$= **{1,-1,i,-i,j,-j,k,-k}** whose multiplication is given by the table

| · | 1 | -1 | i | -i | j | -j | k | -k |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | -1 | i | -i | j | -j | k | -k |
| -1 | -1 | 1 | -i | I | -j | j | -k | k |
| i | i | -i | -1 | 1 | k | -k | -j | j |
| -i | -i | i | 1 | -1 | -k | k | j | -j |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| j | j | -j | -k | k | -1 | 1 | i | -i |
| -j | -j | j | k | -k | 1 | -1 | -i | i |
| k | k | -k | j | -j | -i | i | -1 | 1 |
| -k | -k | k | -j | j | i | -i | 1 | -1 |

Using this operation one can obtain several hopes which are e-groups.
For example, denoting $\underline{i}=\{i,-i\}$, $\underline{j}=\{j,-j\}$, $\underline{k}=\{k,-k\}$ we define $(*)$ hope by the table:

| * | 1 | -1 | I | -i | j | -j | k | -k |
|---|---|---|---|---|---|---|---|---|
| I | 1 | -1 | I | -i | j | -j | k | -k |
| -1 | -1 | 1 | -i | i | -j | j | $\underline{k}$ | k |
| i | i | -i | -1 | 1 | k | -k | -j | j |
| -i | -i | $\underline{i}$ | 1 | -1 | -k | k | j | -j |
| j | j | -j | -k | k | -1 | 1 | i | -i |
| -j | -j | j | k | -k | 1 | -1 | -i | i |
| K | k | $\underline{k}$ | $\underline{i}$ | -j | -i | i | -1 | 1 |
| -k | -k | k | -j | j | i | -i | 1 | -1 |

The hyperstructure $(Q,*)$ is strong e-hypergroup because **1** is scalar unit and the elements **-1, i, -i, j, -j, k** and **-k** have unique inverses the elements **-1, -i, i, -j, j, -k** and **k,** respectively, which are the inverses in the basic group.

The important thing of this example is that one can have more strict hopes, so for example, one can have a strong commutative hope,

| ° | 1 | -1 | I | -i | j | -j | k | -k |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | -1 | I | -i | j | -j | k | -k |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| -1 | -1 | 1 | -i | i | -j | j | -k | k |
| i | i | -i | -1 | 1 | k | k | j | j |
| -i | -i | i | 1 | -1 | k | k | j | j |
| j | j | -j | k | k | -1 | 1 | i | i |
| -j | -j | j | k | k | 1 | -1 | i | i |
| K | k | -k | j | j | i | i | -1 | 1 |
| -k | -k | k | j | j | i | i | 1 | -1 |

therefore this strong e-hypersrtucture (Q,°) is a commutative e-hypergroup.

The reason we gave the above example is to see that there is a large variety of e-hyperstructures we can construct from given classical structures. One can see in the papers [21,39] and in the book [12] some classes of e-hyperstructures and their properties and results connected them with the classical theory. The representation theory and the Lie algebras as well as in hypermatrix theory large classes of e-hyperstructures appear and can offer to Lie-Santilli algebraic theory models to represent their theory. In the Lie admissible algebras the P-hopes can be used as one can see the replacement of the unit matrix in the Lie-bracket (in isoproducts): by any set of matrices **P**:

$$[X,Y]_P = (XP)Y - (YP)X, \quad \text{for all matrices or hypermatrices } X \text{ and } Y.$$

# REFERENCES


1. R. Ameri and M.M. Zahedi, *Hyperalgebraic systems,* Italian J. Pure Applied Math., N.6, 1999, 21-32.
2. N. Antampoufis and T. Vougiouklis, *Hyperoperations greater than the complex number operations,* Journal of Basic Science V.3/N1, 2006, 11-17.
3. R. Bayon and N. Lygeros, *Advanced results in enumeration of hyper-structures,* J. Algebra, V, 320, 2, 2008, 821-835.

4. J. Chvalina and S.Hoskova, *Modelling of join spaces with proximities by first-order linear partial differential operators,* Italian J. Pure Applied Math., N.21, 2007, 177-190.
5. P. Corsini, *Prolegomena of Hypergroup Theory,* Aviani Editore, 1993.
6. P. Corsini P.-Leoreanu V., *Application of Hyperstructure Theory,* Klower Academic Publishers, 2003.
7. P. Corsini and T. Vougiouklis, *From groupoids to groups through hypergroups,* Rendiconti di Matematica VII, 9, 1989, 173-181.
8. B. Davvaz, *On $H_v$-rings and Fuzzy $H_v$-ideals,* J.Fuzzy Math. V.6, N.1, 1998, 33-42.
9. B. Davvaz, *On $H_v$-subgroups and anti fuzzy $H_v$-subgroups,* Korean J.Comp. Appl. Math. V.5, N.1, 1998, 181-190.
10. B. Davvaz, *Fuzzy $H_v$-submodules,* Fuzzy sets and Systems, 117, 2001, 477-484.
11. B. Davvaz, *A brief survey of the theory of $H_v$-structures,* 8th AHA, Greece, Spanidis, 2003, 39-70.
12. B. Davvaz and V. Leoreanu-Fotea, *Hyperring Theory and Applications,* International Academic Press, USA, 2007.
13. B. Davvaz and T. Vougiouklis, *n-ary hypergroups,* Iranian Journal of Science & Technology, Transaction A, V.30, N.A2, 2006, 165-174.
14. B. Davvaz and T. Vougiouklis, *Commutative rings obtained from hyperrings ($H_v$-rings) with $\alpha^*$-relations.* Communications in Algebra, 35, 2007, 3307-3320.
15. B. Davvaz, W. Dudek and T. Vougiouklis, *A generalization of n-ary algebraic systems,* Communications in Algebra, 37, 2009, 1248-1263.
16. A. Dramalidis and T. Vougiouklis, *On a class of geometric fuzzy $H_v$-structures,* 8th AHA Congress, Samothraki, Greece2002, Spanidis, 2003, 137-145.
17. M. Koskas, *Groupoides demi-hypergroupes et hypergroupes,* J. Math. Pures Appl., 49 (9), 1970, 155-192.
18. F. Marty, *Sur un généralisation de la notion de groupe,* 8éme Congrés Math. Scandinaves, Stockholm, 1934, 45-49.
19. Rota R., *Sugli iperanelli moltiplicativi,* Rendiconti Mat., 7(2), (1982), 711-724.
20. R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry,* Volumes I, II, III, IV and V, International Academic Press, USA, 2007.
21. R. M. Santilli and T. Vougiouklis, *Isotopies, Genotopies, Hyperstructures and their Applications,* New frontiers in Hyperstructures, Hadronic, 1996, 1-48.

22. T. Vougiouklis, *Cyclicity in a special class of hypergroups,* Acta Un. Car.–Math. Et Ph., V.22, N1, 1981, 3-6.
23. T. Vougiouklis, *Representations of hypergroups, Hypergroup algebra,* Proc. Convegno: ipergrouppi, altre strutture multivoche appl. Udine, 1985, 59-73.
24. T. Vougiouklis, *Generalization of P-hypergroups,* Rend. Circolo Mat. Palermo, Ser.II, 36, 1987, 114-121.
25. T. Vougiouklis, *Representations of hypergroups by hypermatrices,* Rivista Mat. Pura ed Appl., N 2, 1987, 7-19.
26. T. Vougiouklis, *The very thin hypergroups and the S-construction,* Combinatorics'88, Incidence Geometries Combinatorial Str., 2, 1991, 471-477.
27. T. Vougiouklis, *The fundamental relation in hyperrings. The general hyperfield,* 4$^{th}$AHA, Xanthi 1990, World Scientific, 1991, 203-211.
28. T. Vougiouklis, *Representations of hypergroups by generalized permutations,* Algebra Universalis, 29, 1992,172-183.
29. T. Vougiouklis, *Representations of $H_v$-structures,* Proc. Int. Conf. Group Theory 1992, Timisoara, 1993, 159-184.
30. T. Vougiouklis, *Hyperstructures and their Representations,* Monographs in Mathematics, Hadronic, 1994.
31. T. Vougiouklis, *Some remarks on hyperstructures,* Contemporary Mathematics, Amer. Math. Society, 184, 1995, 427-431.
32. T. Vougiouklis, *A new class of hyperstructures,* J.C.I. & S.S.,V.20, N.1-4, 1995, 229-239.
33. T. Vougiouklis, *$H_v$-groups defined on the same set,* Discrete Mathematics 155, 1996, 259-265.
34. T. Vougiouklis, *Consructions of $H_v$-structures with desired fundamental structures,* New frontiers in Hyperstructures, Hadronic, 1996,177-188.
35. T. Vougiouklis, *On $H_v$-fields,* 6$^{th}$ AHA, Prague 1996, Democritus Univ. Press, 1997, 151-159.
36. T. Vougiouklis, *Convolutions on WASS hyperstructures,* Discrete Mathematics 174, 1997, 347-355.
37. T. Vougiouklis, *Enlarging $H_v$-structures,* Algebras and Combinatorics, ICAC'97, Hong Kong, Springer, 1999, 455-463.
38. T. Vougiouklis, *On $H_v$-rings and $H_v$-representations,* Discrete Mathematics, Elsevier, 208/209, 1999, 615-620.
39. T. Vougiouklis, *Hyperstructures in isotopies and genotopies,* Advances in equations and Inequalities, Ed. J.M.Rassias, Hadronic Press, 1999, 275-291.
40. T. Vougiouklis, *$H_v$-Lie algebras and h/v-Lie algebras,* Honorary I. Mittas, Aristotle Un. Thessaloniki, 2000, 619-628.

41. T. Vougiouklis, *On hyperstructures obtained by attaching elements,* Proc. C. Caratheodory in his...origins, Hadronic, 2001, 197-206.
42. T. Vougiouklis, *Finite $H_v$-structures and their representations,* Rend. Seminario Mat. Messina, S II, V9, 2003, 245-265.
43. T. Vougiouklis, *The h/v-structures,* Journal of Discrete Math. Sciences and Cryptography, V.6, 2003, N.2-3, 235-243.
44. T. Vougiouklis, *A hyperoperation defined on a groupoid equipped with a map,* Ratio Mathematica, N.1, 2005, 25-36.
45. T. Vougiouklis, *$H_v$-fields and $H_v$-vector spaces with $\partial$-operations,* Proceedings: 6th Panellenic Conf. Algebra and Number Theory, Thessaloniki, Greece, 2006, 95-102.
46. T. Vougiouklis, *$\partial$-operations and $H_v$-fields,* Acta Mathematica Sinica, English S., V23, 6, 2008, 965-972.
47. T. Vougiouklis, *Hypermatrix representations: The problem and the open problems,* 10th AHA Congress, Brno, Czech Republic 2008, (2009), 17-31.
48. T. Vougiouklis and S. Vougiouklis, *The helix hyperoperations,* Italian J. Pure Applied Math., N.18, 2005, 197-206.
49. M. M. Zahedi, R.A. Borzoei and H. Rezaei, *A classification of hyperK-algebras of order 3 which satisfy the simple condition,* Proc. C. Caratheodory in his...origins, Hadronic, 2001, 207-216.

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
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# A Brief Survey Of Applications Of Generalized Algebraic Structures And Santilli-Vougiouklis Hyperstructures

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**Abstract.** The hyperstructure notion was introduced in 1934 by the French mathematicians F. Marty, at the $8^{th}$ Congress of Scandinavian Mathematicians. The motivating example was the quotient of a group by any, not necessary normal, subgroup. Algebraic hyperstructures in the sense of Marty are a suitable generalization of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Many papers and several books have been written till now on hyperstructures. Many of them are dedicated to the applications of hyperstructures in other disciplines. We mention here some of fields connected with hyperstructures. Specially, in Chemistry (construction from chain reactions). In 1996, R. M. Santilli and T. Vougiouklis point out that in physics the most interesting hyperstructures are the one called $e$-hyperstructures. $e$-hyperstructures are a special kind of hyperstructures and they can be interpreted as a generalization of two important concepts for physics: Isotopies and Genotopies. We define and analyze several types of $e$-hyperstructures. These hyperstructures have ordinary operations, contain hyperunits and are mainly multi-valued in their structures. They are the most important tools in Lie-Santilli theory. Also, we review some results on fuzzy Lie algebra and some recent generalizations of algebraic hyperstructures such $n$-ary hypergroups and $\Gamma$-semihypergroups.

**Keywords:** $n$-ary group, algebraic hyperstructure, hypergroup, Lie algebra, fuzzy set.
**PACS:** 02.10.-v, 02.20.-a, 02.20.Sv.

## 1 A BRIEF EXCURSION INTO GROUP THEORY

The concept of a group is one of the most fundamental in modern mathematics. Group theory can be considered the study of symmetry: the collection of symmetries of some object preserving some of its structure forms a group; in some sense all groups arise this way. Although permutations had been studied earlier, the theory of groups really began with Galois (1811-1832) who demonstrated that polynomials are best understood by examining certain groups of permutations of their roots. Since that time, groups have

arisen in almost every branch of mathematics. There are three historical roots of group theory: (1) The theory of algebraic equations; (2) Number theory; (3) Geometry.

**Definition 1.1.** Let $G$ be a non-empty set together with a binary operation (usually called *multiplication*) that assigns to each ordered pair $(a, b)$ of elements of $G$ an element $a \cdot b$ in $G$. We say $G$ is a *group* under this operation if the following three properties are satisfied: (1) $a \cdot (b \cdot c) = (a \cdot b) \cdot c$, for all $a, b, c \in G$, (2) there exists an element $e \in G$ such that $a \cdot e = e \cdot a = a$, for all $a \in G$, (3) for every $a \in G$ there exists an element $a^{-1} \in G$ such that $a \cdot a^{-1} = a^{-1} \cdot a = e$.

We have $(a^{-1})^{-1} = a$ and $(ab)^{-1} = b^{-1}a^{-1}$, for all $a, b \in G$. We say that two elements $a$ and $b$ of a group $G$ are commutative or commute if $ab = ba$. A group is said to be *abelian* or *commutative*, if any two elements commute.

EXAMPLE 1. (1) The set of integers $\mathbb{Z}$, the set of rational numbers $\mathbb{Q}$ and the set of real numbers $\mathbb{R}$ are all groups under ordinary addition.

(2) The set $\mathbb{Z}_n = \{0, 1, \ldots, n-1\}$ for $n \geq 1$ is a group under addition modulo $n$. For any $i$ in $\mathbb{Z}_n$, the inverse of $i$ is $n - i$. This group usually referred to as the *group of integers modulo n.*

(3) The *quaternion group* is a non-abelian group of order 8. It is often denoted by $Q$ or $Q_8$ and written in multiplicative form, with the following 8 elements $Q = \{1, -1, i, -i, j, -j, k, -k\}$. Here 1 is the identity element, $(-1)^2 = 1$ and $(-1)a = a(-1) = -a$ for all $a$ in $Q$. The remaining multiplication rules can be obtained from the following relation: $i^2 = j^2 = k^2 = ijk = -1$.

The concept of subgroups is one of the most basic ideas in group theory. If $H$ is a subgroup of $G$ and $a \in G$, then $Ha = \{ha \mid h \in H\}$. $Ha$ is called a *right coset* of $H$ in $G$. A *left coset* $aH$ is defined similarly. There is one kind of subgroup that is especially interesting. If $G$ is a group and $H$ is a subgroup of $G$, it is not always true that $aH = Ha$ for all $a \in G$. There are certain situations where this does hold, however, and these cases turn out to be of critical importance in the theory of groups. It was Galois, who first recognized that such subgroups were worthy of special attention.

A subgroup $N$ of a group $G$ is called a *normal subgroup* of $G$ if $a^{-1}Na \subseteq N$ for all $a \in G$.

**Theorem 1.2.** *Let $N$ be a normal subgroup of a group $G$, and let $\overline{G}$ denote the set of all cosets of $N$. For any two elements $X$ and $Y$ of $\overline{G}$, we define their product $XY$ as the subset of $G$ obtained by taking the product of the two subsets $X$ and $Y$ of $G$. Then $XY$ is a coset of $N$. With respect to this multiplication on $\overline{G}$, the set $\overline{G}$ forms a group.*

The group $\overline{G}$ which was defined in the above theorem is called the *quotient group* of $G$ by $N$ and is written $\overline{G} = G/N$.

EXAMPLE 2. (1) Let $G = \mathbb{Z}_{18}$ and $N =< 6 >$. Then $G/N = \{0+N,\ 1+N,\ 2+N,\ 3+N,\ 4+N,\ 5+N\}$.

(2) Let $G$ be a group such that $(ab)^p = a^p b^p$ for all $a, b \in G$, where $p$ is a prime number. Let $N = \{x \in G \mid x^{p^m} = e$ for some $m$ depending on $x\}$. Then $N$ is a normal subgroup of $G$. If $\overline{G} = G/N$ and if $\overline{x} \in \overline{G}$ is such that $\overline{x}^p = \overline{e}$, then $\overline{x} = \overline{e}$.

In 1934, Marty introduced the concept of a hypergroup. The motivation example was the following: Let $G$ be a group and $H$ be any subgroup of $G$. Then $G/H = \{xH \mid x \in G\}$ becomes a hypergroup where the hyperoperation is defined in a usual manner:

$$aH \circ bH = \{cH \mid c \in a \cdot b \cdot H\},$$

for all $a, b \in G$.

## 2 TWO GENERALIZATIONS OF A GROUP

We may define a group as follows:

**Definition 2.1.** A system $(G, \cdot)$, where $G$ is a non-empty set and $\cdot$ is a binary operation, is called a *group* if it satisfies the following conditions:

(1) $a \cdot (b \cdot c) = (a \cdot b) \cdot c$, for all $a, b, c \in G$,

(2) the equations $a \cdot x = b$ and $y \cdot a = b$ have solutions in $G$, for all $a, b, c \in G$.

The non-empty set $G$ together with an $n$-ary operation $f : G^n \longrightarrow G$ is called an *$n$-ary groupoid* and is denoted by $(G, f)$. According to the general convention used in the theory of such groupoids the sequence of elements $x_i, x_{i+1}, \ldots, x_j$ is denoted by $x_i^j$. The notion of an $n$-ary group is a natural generalization of the notion of a group and has many applications in different branches. The idea of investigations of such groups seems to be going back to E. Kasner's lecture at the fifty-third annual meeting of the American Association for the Advancement of Science in 1904. But the first paper concerning the theory of $n$-ary groups was written (under inspiration of Emmy Noether) by W. Dörnte in 1929 [15].

**Definition 2.2.** A non-empty set $G$ with an $n$-ary operation $f$ is called an *$n$-ary group* if

(1) For every $i, j \in \{1, 2, \ldots, n\}$ and $x_1, x_2, \ldots, x_{2n-1} \in G$,

$$f(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1}) = f(x_1^{j-1}, f(x_j^{n+j-1}), x_{n+j}^{2n-1}),$$

(2) for all $b, a_1, \ldots, a_{k-1}, a_{k+1}, \ldots, a_n \in G$ $(k = 1, \ldots, n)$ there exists a unique $z \in G$ such that

$$f(a_1^{k-1}, z, a_{k+1}^n) = b.$$

If $(G,\cdot)$ is a group, then $(G,f)$ is an $n$-ary group, where $f(x_1^n)=x_1\cdot x_2\cdot\ldots\cdot x_n$. But for every $n\geq 3$ there are $n$-ary groups which are not derived from any group.

Let $H$ be a non-empty set and $\circ : H\times H\longrightarrow \mathcal{P}(H)\setminus\emptyset$ be a *hyperoperation*. The couple $(H,\circ)$ is called a *hypergroupoid*.

**Definition 2.3.** A hypergroupoid $(H,\circ)$ is called a *hypergroup* if

(1) $a\circ(b\circ c)=(a\circ b)\circ c$ for all $a,b,c\in H$, which means that

$$\bigcup_{u\in a\circ b} u\circ c=\bigcup_{v\in b\circ c} a\circ v.$$

(2) $a\circ H=H\circ a=H$ for all $a\in H$. This condition is called the *reproduction axiom*.

The second condition is frequently used in the form: Given $a,b\in H$, there exist $x,y\in H$ such that $b\in a\circ x$ and $b\in y\circ a$.

# 3 WEAK HYPERSTRUCTURES

Weak hyperstructures (or $H_v$-structures) first introduced by Vougiouklis in Fourth AHA congress (1990) [24]. The concept of weak hyperstructures constitute a generalization of the well-known algebraic hyperstructures.

**Definition 3.1.** [23] A hyperstructure $(H,\circ)$ is called an $H_v$-group if

(1) $x\circ(y\circ z)\cap(x\circ y)\circ z\neq\emptyset$ for all $x,y,z\in H$,

(2) $a\circ H=H\circ a=H$ for all $a\in H$.

A motivation to obtain the above structure is the following:

EXAMPLE 3. [23] Let $(G,\cdot)$ be a group and $R$ be an equivalence relation on $G$. On $G/R$ consider the hyperoperation $\odot$ such that $\overline{x}\odot\overline{y}=\{\overline{z}|\ z\in\overline{x}\cdot\overline{y}\}$, where $\overline{x}$ denotes the class of the element $x$. Then $(G,\odot)$ is an $H_v$-group which is not always a hypergroup.

EXAMPLE 4. [22] On the set $\mathbb{Z}_{mn}$ consider the hyperoperation $\oplus$ defined by setting $\overline{0}\oplus m=\{\overline{0},m\}$ and $x\oplus y=x+y$ for all $(x,y)\in\mathbb{Z}_{mn}^2-\{(\overline{0},m)\}$. Then $(\mathbb{Z}_{mn},\oplus)$ becomes an $H_v$-group. $\oplus$ is weak associative but not associative.

**Definition 3.2.** [5] A multi-valued system $\mathcal{M}=<P,\circ,e,^{-1}>$ where $e\in P$, $^{-1}:P\longrightarrow P$, $\circ : P\times P\to\mathcal{P}(P)\setminus\emptyset$ is called a *weak polygroup* if the following axioms hold for all $x,y,z\in P$ :

(1) $(x\circ y)\circ z\cap x\circ(y\circ z)\neq\emptyset$ (weak associative),

(2) $x \circ e = x = e \circ x$,

(3) $x \in y \circ z$ implies $y \in x \circ z^{-1}$ and $z \in y^{-1} \circ x$.

**Proposition 3.3.** *[6] Let $(G, \cdot)$ be a group and $\theta$ be an equivalence relation on $G$ such that*

*(1) $x\theta e$ implies $x = e$,*

*(2) $x\theta y$ implies $x^{-1}\theta y^{-1}$.*

*Let $\theta(x)$ be the equivalence class of the element $x \in G$. Suppose that $G/\theta = \{\theta(x) |\ x \in G\}$. Then $< G/\theta, \odot, \theta(e), ^{-I} >$ is a weak polygroup, when the hyperoperation $\odot$ is defined as follows:*

$$\odot : G/\theta \times G/\theta \longrightarrow \mathcal{P}(G/\theta) \setminus \emptyset$$

$$\theta(x) \odot \theta(y) = \{\theta(z) |\ z \in \theta(x) \cdot \theta(y)\},$$

*and $\theta(x)^{-I} = \theta(x^{-1})$.*

EXAMPLE 5. Consider $P = \{e, a, b, c\}$ and define $*$ on $P$ with the help of the following table:

| $*$ | $e$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e$ | $e$ | $a$ | $b$ | $c$ |
| $a$ | $a$ | $e, a$ | $c$ | $b$ |
| $b$ | $b$ | $c$ | $e, b$ | $a$ |
| $c$ | $c$ | $b$ | $a$ | $e, c$ |

Then $< P, *, e, ^{-1} >$, where $x^{-1} = x$ for every $x \in P$, is a weak polygroup which is not a polygroup. Indeed, we have

$$(a * b) * c = c * c = \{e, c\}, \quad a * (b * c) = a * a = \{e, a\}.$$

Therefore, $*$ is not associative.

An extension of polygroups by polygroups has been introduced in [2]. In the following we define an extension of a weak polygroup by another weak polygroup.

**Definition 3.4.** Suppose that $\mathcal{A} =< A, \cdot, e, ^{-1} >$ and $\mathcal{B} =< B, \cdot, e, ^{-1} >$ are weak polygroups whose elements have been renamed so that $A \cap B = \{e\}$ where $e$ is the identity of both $\mathcal{A}$ and $\mathcal{B}$. A new system $\mathcal{A}[\mathcal{B}] =< M, *, e, ^{-I} >$, which is called the extension of $\mathcal{A}$ by $\mathcal{B}$, is defined in the following way:

Set $M = A \cup B$ and let $e^{-I} = e$, $x^{-I} = x^{-1}$, $e * x = x * e = x$ for all $x \in M$, and for all $x, y \in M - \{e\}$,

$$x * y = \begin{cases} x \cdot y & \text{if } x, y \in A, \\ x & \text{if } x \in B, y \in A, \\ y & \text{if } x \in A, y \in B, \\ x \cdot y & \text{if } x, y \in B, y \neq x^{-1}, \\ x \cdot y \cup A & \text{if } x, y \in B, y = x^{-1}. \end{cases}$$

**Theorem 3.5.** *[5] $\mathcal{A}[\mathcal{B}]$ is a weak polygroup.*

# 4 SANTILLI-VOUGIOUKLIS HYPERSTRUCTURES WITH HYPERUNITS

*e-hyperstructures* are a special kind of hyperstructures and, in what follows, we shall see that they can be interpreted as a generalization of two important concepts for physics: Isotopies and Genotopies. On the other hand, biological systems such as cells or organisms at large are open and irreversible because they grow. The representation of more complex systems, such as neural networks, requires more advances methods, such as hyperstructures. In this manner, e-hyperstructures can play a significant role for the representation of complex systems in physics and biology, such as nuclear fusion, the reproduction of cells or neural systems.

These applications were investigated by R.M. Santilli and T. Vougiouklis and we mention here some of their results and examples (see [10, 18, 19, 20]). In this section, we review several types of e-hyperstructures from [20].

**Definition 4.1.** A hypergroupoid $(H, \cdot)$ is called an *e-hypergroupoid* if $H$ contains a scalar identity (also called unit) $e$, which means that for all $x \in H$, $x \cdot e = e \cdot x = x$. In an e-hypergroupoid, an element $x'$ is called *inverse* of a given element $x \in H$ if $e \in x \cdot x' \cap x' \cdot x$.

Clearly, if a hypergroupoid contains a scalar unit, then it is unique, while the inverses are not necessarily unique. In what follows, we use some examples which are obtained as follows: Take a set where an operation "$\cdot$" is defined, then we "enlarge" the operation putting more elements in the products of some pairs. Thus a hyperoperation "$\circ$" can be obtained, for which we have $x \cdot y \in x \circ y$, $\forall x, y \in H$. Recall that the hyperstructures obtained in this way are $H_b$-structures.

EXAMPLE 6. Consider the usual multiplication on the subset $\{1, -1, i, -i\}$ of complex numbers. Then, we can consider the hyperoperation $\circ$ defined in the following table:

| $\circ$ | $1$ | $-1$ | $i$ | $-i$ |
|---|---|---|---|---|
| $1$ | $1$ | $-1$ | $i$ | $-i$ |
| $-1$ | $-1$ | $1$ | $-i$ | $i, -i$ |
| $i$ | $i$ | $-i$ | $-1$ | $1$ |
| $-i$ | $-i$ | $i$ | $1, i$ | $-1, i$ |

Notice that we enlarged the products $(-1)\cdot(-i)$, $(-i)\cdot i$ and $(-i)\cdot(-i)$ by setting

$$(-1)\circ(-i)=\{i,-i\},\ (-i)\circ i=\{1,i\} \text{ and } (-i)\circ(-i)=\{-1,i\}.$$

We obtain an $e$-hypergroupoid, with the scalar unit 1. The inverses of the elements $-1, i, -i$ are $-1, -i, i$ respectively. Moreover, the above structure is an $H_v$-abelian group, which means that the hyperoperation $\circ$ is weak associative, weak commutative and the reproductive axiom holds.

**Theorem 4.2.** *[23] The weak associativity is valid for all $H_b$-structures with associative basic operations.*

We are interested now in another kind of an $e$-hyperstructure, which is the *e-hyperfield.*

**Definition 4.3.** A set $F$, endowed with an operation "+", which we call *addition* and a hyperoperation, called *multiplication* "$\cdot$", is said to be an *e-hyperfield* if the following axioms are valid:

(1) $(F,+)$ is an abelian group where 0 is the additive unit;

(2) the multiplication $\cdot$ is weak associative;

(3) the multiplication $\cdot$ is weak distributive with respect to $+$,
i.e., $\forall x,y,z\in F,\ x(y+z)\cap(xy+xz)\neq\emptyset,\ (x+y)z\cap(xz+yz)\neq\emptyset$;

(4) 0 is an absorbing element, i.e., $\forall x\in F,\ 0\cdot x=x\cdot 0=0$;

(5) there exists a multiplicative scalar unit 1, i.e., $\forall x\in F,\ 1\cdot x=x\cdot 1=x$;

(6) for every element $x\in F$ there exists an inverse $x^{-1}$, such that $1\in x\cdot x^{-1}\cap x^{-1}\cdot x$.

The elements of an $e$-hyperfield $(F,+,\cdot)$ are called *e-hypernumbers.*

EXAMPLE 7. (1) Starting with the ring $\mathbb{Z}_3=\{\overline{0},\ \overline{1},\ \overline{2}\}$, we can obtain a hyperring by enlarging the product $\overline{2}\circ\overline{2}=\{\overline{1}\}$ to $\overline{2}\circ\overline{2}=\{\overline{1},\overline{2}\}$. In other words, we obtain the following table:

| $\circ$ | $\overline{0}$ | $\overline{1}$ | $\overline{2}$ |
|---|---|---|---|
| $\overline{0}$ | $\overline{0}$ | $\overline{0}$ | $\overline{0}$ |
| $\overline{1}$ | $\overline{0}$ | $\overline{1}$ | $\overline{2}$ |
| $\overline{2}$ | $\overline{0}$ | $\overline{2}$ | $\overline{1},\overline{2}$ |

The above structure is an $e$-hyperfield.

(2) In the above example, only a hyperproduct is not a singleton. These hyperstructures, for which only a hyperproduct is not a singleton, are called *very thin* and they are useful to the theory of representations of $H_v$-groups by hypermatrices. Hence, a way to obtain a very thin hyperstructure is the following one: we take a classical structure and we choose two elements $a, b$, then we can enlarge the product $a \cdot b$. Therefore, in order to obtain a very thin $e$-hyperfield we can take a field and enlarge only one product of two, nonzero and non-unit elements. This simple change of the operation leads to enourmous changes to the algebraic hyperstructure, so it looks like a chain reaction in physics.

We can define the product of two $e$-matrices in an usual manner: the elements of product of two $e$-matrices $(a_{ij}), (b_{ij})$ are $c_{ij} = \sum a_{ik} \circ b_{kj}$, where the sum of products is the usual sum of sets.

If we consider the $e$-hyperfield given in Example 7(1), then we have:

$$\begin{aligned}
\begin{bmatrix} \overline{2} & \overline{1} \\ \overline{2} & \overline{0} \end{bmatrix} \circ \begin{bmatrix} \overline{2} & \overline{1} \\ \overline{1} & \overline{1} \end{bmatrix} &= \begin{bmatrix} \overline{2}\circ\overline{2}+\overline{1}\circ\overline{1} & \overline{2}\circ\overline{1}+\overline{1}\circ\overline{1} \\ \overline{2}\circ\overline{2}+\overline{0}\circ\overline{1} & \overline{2}\circ\overline{1}+\overline{0}\circ\overline{1} \end{bmatrix} \\
&= \begin{bmatrix} \{\overline{1},\overline{2}\}+\overline{1} & \overline{2}+\overline{1} \\ \{\overline{1},\overline{2}\}+\overline{0} & \overline{2}+\overline{0} \end{bmatrix} \\
&= \begin{bmatrix} \{\overline{2},\overline{0}\} & \overline{0} \\ \{\overline{1},\overline{2}\} & \overline{2} \end{bmatrix} \\
&= \left\{ \begin{bmatrix} \overline{2} & \overline{0} \\ \overline{1} & \overline{2} \end{bmatrix}, \begin{bmatrix} \overline{2} & \overline{0} \\ \overline{2} & \overline{2} \end{bmatrix}, \begin{bmatrix} \overline{0} & \overline{0} \\ \overline{1} & \overline{2} \end{bmatrix}, \begin{bmatrix} \overline{0} & \overline{0} \\ \overline{2} & \overline{2} \end{bmatrix} \right\}
\end{aligned}$$

Moreover, notice that the product of an $e$-hypernumber with an $e$-hypermatrix is also a hyperoperation. For instance, again on the above hyperfield, we have

$$\begin{aligned}
\overline{2} \circ \begin{bmatrix} \overline{2} & \overline{1} \\ \overline{2} & \overline{2} \end{bmatrix} &= \begin{bmatrix} \overline{2}\circ\overline{2} & \overline{2}\circ\overline{1} \\ \overline{2}\circ\overline{2} & \overline{2}\circ\overline{2} \end{bmatrix} \\
&= \left\{ \begin{bmatrix} \overline{1} & \overline{2} \\ \overline{1} & \overline{1} \end{bmatrix}, \begin{bmatrix} \overline{1} & \overline{2} \\ \overline{1} & \overline{2} \end{bmatrix}, \begin{bmatrix} \overline{1} & \overline{2} \\ \overline{2} & \overline{1} \end{bmatrix}, \begin{bmatrix} \overline{1} & \overline{2} \\ \overline{2} & \overline{2} \end{bmatrix}, \begin{bmatrix} \overline{2} & \overline{2} \\ \overline{1} & \overline{1} \end{bmatrix}, \right. \\
&\qquad \left. \begin{bmatrix} \overline{2} & \overline{2} \\ \overline{1} & \overline{2} \end{bmatrix}, \begin{bmatrix} \overline{2} & \overline{2} \\ \overline{2} & \overline{1} \end{bmatrix}, \begin{bmatrix} \overline{2} & \overline{2} \\ \overline{2} & \overline{2} \end{bmatrix} \right\}.
\end{aligned}$$

This remark is useful for the definition of an *e-hypervector space.*

**Definition 4.4.** Let $(F, +, \cdot)$ be an $e$-hyperfield. An ordered set $a = (a_1, a_2, \ldots, a_n)$ of $n$ $e$-hypernumbers of $F$ is called an *e-hypervector* and the $e$-hypernumbers $a_i$, $i \in \{1, 2, \cdot, n\}$

are called *components* of the $e$-hypervector $a$.

Two $e$-hypervectors are equals if they have equal corresponding components. The hypersums of two $e$-hypervectors $a, b$ is defined as follows:

$$a + b = \{(c_1, c_2, \ldots, c_n) \mid c_i \in a_i + b_i,\ i \in \{1, 2, \cdot, n\}\}.$$

The scalar hypermultiplication of an $e$-hypervector $a$ by an $e$-hypernumber $\lambda$ is defined in a usual manner:

$$\lambda \circ a = \{(c_1, c_2, \ldots, c_n) \mid c_i \in \lambda \cdot a_i,\ i \in \{1, 2, \ldots, n\}\}.$$

The set $F^n$ of all $e$-hypervectors with elements of $F$, endowed with the hypersum and the scalar hypermultiplication is called *n-dimensional e-hypervector space*. The set of $m \times n$ hypermatrices is an $mn$-dimensional $e$-hypervector space.

# 5 FUZZY LIE ALGEBRA

Fuzzy sets are sets whose elements have degrees of membership. Fuzzy sets have been introduced by Zadeh (1965) as an extension of the classical notion of set [25]. In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition an element either belongs or does not belong to the set. By contrast, fuzzy set theory permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function valued in the real unit interval $[0, 1]$. Fuzzy sets generalize classical sets, since the indicator functions of classical sets are special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1.

After the introduction of fuzzy sets by Zadeh, reconsideration of the concept of classical mathematics began. On the other hand, because of the importance of group theory in mathematics, as well as its many areas of application, the notion of fuzzy subgroups was defined by Rosenfeld. In 2001, Davvaz [7] studied the concept of fuzzy Lie algebra. In this section, we review some results of [7, 8].

**Definition 5.1.** A Lie algebra is a vector space $L$ over a field $F$ on which a product operation $[x, y]$ is defined satisfying the following axioms:

(1) $[x, y]$ is bilinear for all $x, y \in L$,

(2) $[x, x] = 0$ for all $x \in L$,

(3) $[[x, y], z] + [[y, z], x] + [[z, x], y] = 0$ for all $x, y, z \in L$.

As a simple consequence of axioms (1) and (2), we have

$$0 = [x+y, x+y] = [x,x] + [x,y] + [y,x] + [y,y] = [x,y] + [y,x].$$

Thus, $[y,x] = -[x,y]$ and Lie multiplication is anti commutative.

Let $L$ be Lie algebra and $M, N$ be subspaces of $L$. We define $[M,N]$ to be the subspace of $L$ spanned by all elements of form $[x,y]$ for $x \in M,\ y \in N$. Since $[y,x] = -[x,y]$ it is clear that $[M,N] = [N,M]$. Thus, multiplication of subspaces is commutative.

**Definition 5.2.** An ideal of $L$ is a subspace $M$ such that $[M,L] \subseteq M$. Since $[M,L] = [L,M]$ there is no distinction in the theory of Lie algebras between left ideals and right ideals. Every ideals is two-sided.

**Definition 5.3.** A linear transformation $\phi : L_1 \longrightarrow L_2$ ($L_1, L_2$ Lie algebras over $F$) is called a homomorphism if

$$\phi([x,y]) = [\phi(x), \phi(y)] \quad \text{for all } x, y \in L.$$

We say that two Lie algebras $L_1, L_2$ over $F$ are *isomorphic* if there exists a vector space isomorphism $\phi : L_1 \longrightarrow L_2$ satisfying $\phi([x,y]) = [\phi(x), \phi(y)]$ for all $x, y$ in $L$.

**Definition 5.4.** Let $X$ be a non-empty set. A *fuzzy subset* $\mu$ of $X$ is a function $\mu : X \longrightarrow [0,1]$. Let $\mu$ and $\lambda$ be two fuzzy subsets of $X$, we say that $\mu$ is contained in $\lambda$, if $\mu(x) \leq \lambda(x)$ for all $x \in X$. If $\{\mu_i\}_{i \in I}$ be a collection of fuzzy subsets of $X$, then we define the fuzzy subsets $\bigcap_{i \in I} \mu_i$ and $\bigcup_{i \in I} \mu_i$ by:

$$\left(\bigcap_{i \in I} \mu_i\right)(x) = \inf_{i \in I}\{\mu_i(x)\} \quad \text{for all } x \in X,$$
$$\left(\bigcup_{i \in I} \mu_i\right)(x) = \sup_{i \in I}\{\mu_i(x)\} \quad \text{for all } x \in X.$$

If $\phi$ be a mapping from a non-empty set $X$ to a non-empty set $Y$ and $\mu$ a fuzzy subset of $X$ and $\lambda$ a fuzzy subset of $Y$. Then the *inverse image* $\phi^{-1}(\lambda)$ of $\lambda$ is the fuzzy subset of $X$ defined by $\phi^{-1}(\lambda)(x) = \lambda(\phi(x))$ for each $x \in X$. The *image* $\phi(\mu)$ of $\mu$ is the fuzzy subset of $Y$ defined by

$$\phi(\mu)(y) = \begin{cases} \sup\limits_{t \in \phi^{-1}(y)} \{\mu(t)\} & \text{if } \phi^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise,} \end{cases} \qquad \text{for each } y \in Y.$$

We have always $\phi(\phi^{-1}(\lambda)) \subseteq \lambda$ and $\mu \subseteq \phi^{-1}(\phi(\mu))$.

**Definition 5.5.** Let $\mu$ be any fuzzy subset of $X$. The set

$$\overline{\mu_t} = \{x \in X |\ \mu(x) \geq t\}, \quad t \in [0,1],$$

is called a *level subset* of $\mu$.

**Definition 5.6.** Let $V$ be a vector space over a field $F$. A fuzzy subset $\mu$ of $V$ is called a *fuzzy subspace* of $V$ if for all $x, y \in V$ and $\alpha \in F$, the following conditions hold:

(1) $\mu(x+y) \geq min\{\mu(x), \mu(y)\}$ for all $x, y \in V$,

(2) $\mu(-x) \geq \mu(x)$ for all $x \in V$,

(3) $\mu(\alpha x) \geq \mu(x)$ for all $x \in V$, $\alpha \in F$.

**Theorem 5.7.** *For a fuzzy subset $\mu$ of a vector space $V$, the following statements are equivalent.*

*(1) $\mu$ is a fuzzy subspace of $V$,*

*(2) $\overline{\mu_t}$, $t \in Im\mu$, is a subspace of $V$.*

**Definition 5.8.** A *fuzzy ideal* of $L$ is a fuzzy subspace $\mu$ such that

$$\mu([x,y]) \geq \mu(x) \text{ for all } x, y \in L.$$

EXAMPLE 8. (1) Let $I$ be an arbitrary ideal of a semisimple Lie algebra $L$. Then $I^{\perp} = \{x \in L |\ k(x,y) = 0,\ \forall\ y \in L\}$ is also an ideal, by the associativity of Killing form $k$, and we have $L = I \oplus I^{\perp}$. Choose numbers $t_i \in [0,1]$, $0 \leq i \leq 3$, such that $t_3 < t_2 < t_1 < t_0$. Define fuzzy subsets $\mu$ and $\lambda$ by:

$$\mu(x) = \begin{cases} t_0 & \text{if } x \in I \\ t_3 & \text{otherwise,} \end{cases} \quad \text{and} \quad \lambda(x) = \begin{cases} t_1 & \text{if } x \in I^{\perp} \\ t_2 & \text{otherwise.} \end{cases}$$

Then, $\mu$ and $\lambda$ are fuzzy ideals of $L$.

(2) Let $L$ be a Lie algebra and $t_1, t_2 \in [0,1]$ with $t_2 < t_1$. Define fuzzy subset $A$ of $L$ by:

$$\mu(x) = \begin{cases} t_1 & \text{if } x = 0 \\ t_2 & \text{otherwise.} \end{cases}$$

Then, $\mu$ is a fuzzy ideal of $L$.

The following corollary is exactly obtained from definitions.

**Corollary 5.9.** If $\mu$ is a fuzzy ideal of $L$ then we have

$$\mu([x,y]) \geq max\{\mu(x), \mu(y)\} \text{ for all } x, y \in L.$$

**Lemma 5.10.** *If $\mu$ is a fuzzy ideal of $L$, then $\mu(x) \leq \mu(0)$ for all $x \in L$, and the following are easily verified:*

*(1) $\mu(x) = \mu(-x)$ for all $x \in L$,*

*(2) $\mu(x-y) = \mu(0) \Longrightarrow \mu(x) = \mu(y)$, where $x, y \in L$.*

**Lemma 5.11.** *Let $\mu$ be any fuzzy ideal of $L$. If for some $x, y \in L$, $\mu(x) < \mu(y)$, then $\mu(x-y) = \mu(x) = \mu(y-x)$.*

**Proposition 5.12.** *The intersection of any family of fuzzy ideals of $L$ is again a fuzzy ideal of $L$.*

**Theorem 5.13.** *For a fuzzy subset $\mu$ of a Lie algebra $L$, the following statements are equivalent.*

*(1) $\mu$ is a fuzzy ideal of $L$,*

*(2) $\overline{\mu_t}$, $t \in Im\mu$, is an ideal of $L$.*

**Proposition 5.14.** *Let $L$ be a Lie algebra and $I \subset I_1 \subset \ldots I_n \subset I_{n+1} \subset \ldots$ be a chain of ideals of $L$. Then there exists a fuzzy ideal $A$ of $L$ such that $A_{t_n} = I_n$ $(0 < \ldots < t_{n+1} < t_n < \ldots < t_1 < t_0 < 1)$.*

**Lemma 5.15.** *Let $\phi : L \longrightarrow L'$ be a Lie algebra homomorphism, and $\lambda$ be a fuzzy ideal of $L'$. Then for each $t \in [0,1]$, $\phi^{-1}(\lambda)_t = \phi^{-1}(\lambda_t)$.*

**Theorem 5.16.** *Let $\phi : L \longrightarrow L'$ be a Lie algebra homomorphism from a Lie algebra $L$ onto a Lie algebra $L'$. Then the following are true:*

*(1) if $\mu \lhd L$ then $\phi(\mu) \lhd L'$;*

*(2) if $\lambda \lhd L'$ then $\phi^{-1}(\lambda) \lhd L$.*

Let $\mu$ be a fuzzy ideal of a Lie algebra $L$. For any $x, y \in L$, define a binary relation $\sim$ on $L$ by $x \sim y$ if and only if $\mu(x-y) = \mu(0)$. Then $\sim$ is a congruence relation of $L$. We denote $\mu[x]$ the equivalence class containing $x$, and $L/\mu = \{\mu[x] || \ x \in L\}$ the set of all equivalence classes of $L$. Then $L/\mu$ is a Lie algebra under the following operations of addition, scalar multiplication and Lie multiplication:

$$\begin{array}{ll} \mu[x] + \mu[y] = \mu[x+y] & \text{for each } x, y \in L, \\ c\mu[x] = \mu[cx] & \text{for each } x \in L,\ c \in F, \\ [\mu[x], \mu[y]] = \mu[[x,y]] & \text{for each } x, y \in L. \end{array}$$

**Theorem 5.17.** *Let $\phi : L \longrightarrow L'$ be an epimorphism of Lie algebras and $\lambda$ be a fuzzy ideal of $L'$. Then $L/\phi^{-1}(\lambda) \cong L'/\lambda$.*

# 6 A CHEMICAL EXAMPLE OF HYPERSTRUCTURES

In the following example [11], the halogens considered are non-metals. Although each of Fluorine, Chlorine, Bromine and Iodine consists of diatomic molecules (i.e., $F_2, Cl_2, Br_2$ and $I_2$) in room temperature, the first two are gas, Bromine is liquid and Iodine is solid. During the chain reaction, it is known that

$$A_2 + B_2 \overset{\text{Heat or Light}}{\longleftrightarrow} 2AB$$

are molecules $A_2, B_2, AB$ and their fragment parts $A^o, B^o$ are present. All the combinations form the set

$$\mathcal{H} = \{A^o, B^o, A_2, B_2, AB\}.$$

Regardless of any energy interaction, their reactions can be displayed in the following table:

| $+$ | $A^o$ | $B^o$ | $A_2$ | $B_2$ | $AB$ |
|---|---|---|---|---|---|
| $A^o$ | $A^o, A_2$ | $A^o, B^o, AB$ | $A^o, A_2$ | $A^o, B_2, B^o, AB$ | $A^o, AB, A_2, B^o$ |
| $B^o$ | $A^o, B^o, AB$ | $B^o, B_2$ | $A^o, B^o, AB, A_2$ | $B^o, B_2$ | $A^o, B^o, AB, B_2$ |
| $A_2$ | $A^o, A_2$ | $A^o, B^o, AB, A_2$ | $A^o, A_2$ | $A^o, B^o, A_2, B_2, AB$ | $A^o, B^o, A_2, AB$ |
| $B_2$ | $A^o, B^o, B_2, AB$ | $B^o, B_2$ | $A^o, B^o, A_2, B_2, AB$ | $B^o, B_2$ | $A^o, B^o, B_2, AB$ |
| $AB$ | $A^o, AB, A_2, B^o$ | $A^o, B^o, AB, B_2$ | $A^o, B^o, A_2, AB$ | $A^o, B^o, B_2, AB$ | $A^o, B^o, A_2, B_2, AB$ |

It can be easily verified that the set $\mathcal{H}$ under the reaction "$+$" forms an $H_v$-group. Clearly, $(\mathcal{H}_1, +) = \{A^0, A_2\}$ and $(\mathcal{H}_2, +) = \{B^0, B_2\}$ are the only $H_v$-subgroups of the $H_v$- group $(\mathcal{H}, +)$. On the other hand, if we consider $A = H$ and $B \in \{F, CL, Br, I\}$ (for example $B = I$), then the complete reaction table becomes:

| $+$ | $H^o$ | $I^o$ | $H_2$ | $I_2$ | $HI$ |
|---|---|---|---|---|---|
| $H^o$ | $H^o, H_2$ | $H^o, I^o, HI$ | $H^o, H_2$ | $H^o, I_2, I^o, HI$ | $H^o, HI, H_2, I^o$ |
| $I^o$ | $H^o, I^o, HI$ | $I^o, I_2$ | $H^o, I^o, HI, H_2$ | $I^o, I_2$ | $H^o, I^o, HI, I_2$ |
| $H_2$ | $H^o, H_2$ | $H^o, I^o, HI, I_2$ | $H^o, H_2$ | $H^o, I^o, H_2, I_2, HI$ | $H^o, I^o, H_2, HI$ |
| $I_2$ | $H^o, I^o, I_2, HI$ | $H^o, I_2$ | $H^o, I^o, H_2, I_2, HI$ | $H^o, I_2$ | $H^o, I^o, I_2, HI$ |
| $HI$ | $H^o, HI, H_2, I^o$ | $H^o, I^o, HI, I_2$ | $H^o, I^o, H_2, HI$ | $H^o, I^o, H_2, HI$ | $H^o, I^o, H_2, I_2, HI$ |

# 7 $n$-ARY HYPERGROUPS

In [12], Davvaz and Vougiouklis introduced the concept of $n$-ary hypergroups as a generalization of hypergroups in the sense of Marty. Also, we can consider $n$-ary hypergroups as a nice generalization of $n$-ary groups. Then this concept studied by Anvariyeh, Davvaz, Dudek, Ghadiri, Leoreanu-Fotea Mirvakili, Vougiouklis, Zhan and others, for example see [13, 14, 16, 17]. In this section, we review some results from [12, 16].

We denote by $H^n$ the cartesian product $H \times \ldots \times H$ where $H$ appears $n$ times. An element of $H^n$ is denoted by $(x_1, \ldots, x_n)$, where $x_i \in H$ for any $i$ with $1 \leq i \leq n$. In

general, a mapping $f : H^n \longrightarrow \mathcal{P}^*(H)$ is called an *n-ary hyperoperation*. Let $f$ be an $n$-ary hyperoperation on $H$ and $A_1, \ldots, A_n$ be non-empty subsets of $H$. We define

$$f(A_1, \ldots, A_n) = \cup\{f(x_1, \ldots, x_n) | \ x_i \in A_i, i = 1, \ldots, n\}$$

**Definition 7.1.** A non-empty set $H$ with an $n$-ary hyperoperation $f : H^n \longrightarrow \mathcal{P}(H) \setminus \emptyset$ is called an *n-ary hypergroupoid* and is denoted by $(H, f)$. An $n$-ary hypergroupoid $(H, f)$ is called an *n-ary semihypergroup* if and only if the following associative axiom holds:

$$f(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1}) = f(x_1^{j-1}, f(x_j^{n+j-1}), x_{n+j}^{2n-1})$$

for every $i, j \in \{1, 2, \ldots, n\}$ and $x_1, x_2, \ldots, x_{2n-1} \in H$.

If for all $(a_1, a_2, \ldots, a_n) \in H^n$, the set $f(a_1, a_2, \ldots, a_n)$ is singleton, then $f$ is called an *n-ary operation* and $(H, f)$ is called an *n-ary groupoid* (resp. *n-ary semigroup*).

If $m = k(n-1) + 1$, then the $m$-ary hyperoperation $g$ given by

$$g(x_1^{k(n-1)+1}) = \underbrace{f(f(\ldots, f(f}_{k}(x_1^n), x_{n+1}^{2n-1}), \ldots), x_{(k-1)(n-1)+2}^{k(n-1)+1})$$

is denoteed by $f_{(k)}$. In certain situations, when the arity of $g$ does not play a crucial role, or when it will differ depending on additional assumptions, we write $f_{(.)}$, to mean $f_{(k)}$ for some $k = 1, 2, \ldots$.

**Definition 7.2.** An $n$-ary semihypergroup $(H, f)$ in which the equation

$$b \in f(a_1^{i-1}, x_i, a_{i+1}^n) \qquad (*)$$

has a solution $x_i \in H$ for every $a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n, b \in H$ and $1 \leq i \leq n$, is called an *n-ary hypergroup*.

In Definition 7.2, if $f$ is $n$-ary operation then the equation $(*)$ is as follows:

$$b = f(a_1^{i-1}, x_i, a_{i+1}^n). \qquad (**)$$

In this case $(H, f)$ is an *n-ary group*.

Definition 7.2 is a generalization of Marty's formulation of a hypergroup.

EXAMPLE 9. Let $(H, f)$ be an $n$-ary hypergroup, $a_2^{n-1} \in H$ be fixed and let $x \odot y = f(x, a_2^{n-1}, y)$. Then the hypergroupoid $(H, \odot)$ is a hypergroup and it is called a *retract of the n-ary hypergroup* $(H, f)$.

Let $(H, f)$ be an $n$-ary hypergroup. If the value of $f(x_1, x_2, \ldots, x_n)$ is independent on the permutation of elements $x_1, x_2, \ldots, x_n$, then $(H, f)$ is called a *commutative $n$-ary hypergroup.*

The element $a \in H$ is called a *scalar* if

$$|f(x_1^i, a, x_{i+2}^n)| = 1$$

for all $x_1, \ldots, x_i, x_{i+2}, \ldots, x_n \in H$.

Element $e$ of an $n$-ary hypergroup $(H, f)$ is called *neutral (identity)* element if

$$f(\underbrace{e, \ldots, e}_{i-1}, x, \underbrace{e, \ldots, e}_{n-i})$$

includes $x$, for all $x \in H$ and all $1 \leq i \leq n$.

**Lemma 7.3.** *Let $(H, f)$ be a commutative $n$-ary hypergroup and $a \in H$ a scalar element such that $f(a, e, \ldots, e) = a$ for some $e \in H$. Then $e$ is a neutral element.*

**Proposition 7.4.** *If the set of all neutral elements of a given commutative $n$-ary hypergroup is non-empty, then it is an $n$-ary group.*

**Definition 7.5.** Let $(H, f)$ be an $n$-ary hypergroup and $B$ be a non-empty subset of $H$. Then $B$ is an *$n$-ary subhypergroup* of $H$ if the following conditions hold:

1) $B$ is closed under the $n$-ary hyperoperation $f$, i.e.,
for every $(x_1, \ldots, x_n) \in B^n$ implies that $f(x_1, \ldots, x_n) \subseteq B$.

2) Equation $b \in f(b_1^{i-1}, x_i, b_{i+1}^n)$ has the solution $x_i \in B$ for every $b_1, \ldots, b_{i-1}, b_{i+1}, \ldots, b_n, b \in B$ and $1 \leq i \leq n$.

**Definition 7.6.** Let $(A, f)$ and $(B, g)$ be two $n$-ary hypergroups. A *homomorphism* from $A$ to $B$ is a mapping $\varphi : A \longrightarrow B$ such that

$$\varphi(f(a_1, \ldots, a_n)) = g(\varphi(a_1), \ldots, \varphi(a_n))$$

holds for all $a_1, \ldots, a_n \in A$.

If $\varphi$ is injective, then it is called an *embedding.* The map $\varphi$ is an *isomorphism* if $\varphi$ is injective and onto. We say that $A$ is *isomorphic* to $B$, denoted by $A \cong B$, if there is an isomorphism from $A$ to $B$.

**Theorem 7.7.** *Let $(A, f)$ and $(B, g)$ be two $n$-ary hypergroups and $\varphi : A \longrightarrow B$ a homomorphism. Then*

*(1) If $S$ is an $n$-ary subhypergroup of $A$, then $\varphi(S)$ is an $n$-ary subhypergroup of $B$,*

*(2) If $K$ is an $n$-ary subhypergroup of $B$, then $\varphi^{-1}(K)$ is an $n$-ary subhypergroup of $A$.*

Let $(H, f)$ be an $n$-ary hypergroup. An equivalence relation $\theta$ on $H$ is called *compatible (regular)* if $a_1\theta b_1, \ \ldots \ , a_n\theta b_n$, then for all $a \in f(a_1, \ldots, a_n)$ there exists $b \in f(b_1, \ldots, b_n)$ such that $a\theta b$. An equivalence relation $\theta$ is called *strongly compatible (strongly regular)* if $a_1\theta b_1, \ \ldots \ , a_n\theta b_n$ implies that $a\theta b$ for all $a \in f(a_1, \ldots, a_n)$ and $b \in f(b_1, \ldots, b_n)$.

**Theorem 7.8.** *Let $(H, f)$ be an $n$-ary hypergroup and $\theta$ a compatible relation on $H$. Then $(H/\theta, f/\theta)$ is an $n$-ary hypergroup where*

$$f/\theta(\theta(a_1), \ \ldots, \ \theta(a_n)) = \{\theta(a) \mid a \in f(a_1, \ldots, a_n)\}.$$

If $(H, f)$ is an $n$-ary hypergroup, then $\widehat{\beta}$ denotes the transitive closure of the relation $\beta = \bigcup_{k \geq 1} \beta_k$, where $\beta_1$ is the diagonal relation, i.e., $\beta_1 = \{(x, x) | \ x \in H\}$ and for every integer $k > 1$, $\beta_k$ is the relation defined as follows:

$$x\beta_k y \ \text{ if and only if } \{x, y\} \subseteq f_{(\cdot)}.$$

When $x\beta_1 y$ (i.e., $x = y$) then we write $\{x, y\} \subseteq f_{(0)}$, we define $\beta^*$ as the smallest equivalence relation such that the quotient $(H/\beta^*, f/\beta^*)$ is an $n$-ary group, where $H/\beta^*$ is the set of all equivalence classes. The $\beta^*$ is called *fundamental equivalence relation.* The equivalence relation $\beta^*$ first was introduced on hypergroups by Koskas and studied mainly by Corsini concerning hypergroups, Vougiouklis and Davvaz concerning $H_v$-structures.

**Theorem 7.9.** *For any $n$-ary hypergroup, $\beta^* = \beta$.*

# 8 Γ-SEMIGROUPS AND Γ-SEMIHYPERGROUPS

Let $A$ and $B$ be two non-empty sets, $M$ the set of all mapping from $A$ to $B$, and $\Gamma$ a set of some mappings from $B$ to $A$. The usual composition of two elements of $M$ can not be defined. But if we take $f, g$ from $M$ and $\alpha$ from $\Gamma$, then the usual mapping composition $f\alpha g$ can be defined. Also, we see $f\alpha g \in M$ and $(f\alpha g)\beta h = f\alpha(g\beta h)$ for $f, g, h \in M$ and $\alpha, \beta \in \Gamma$. Sen and Saha [21] defined the notion of a $\Gamma$-semigroup as a generalization of a semigroup as follows:

**Definition 8.1.** Let $M = \{a, b, c, \ldots\}$ and $\Gamma = \{\alpha, \beta, \gamma, \ldots\}$ be two non-empty sets. Then $M$ is called a *$\Gamma$-semigroup* if there exists a mapping $M \times \Gamma \times M \longrightarrow M$ written as $(a, \gamma, b) \mapsto a\gamma b$ satisfying the following identity

$$(a\alpha b)\beta c = a\alpha(b\beta c) \ \text{ for all } a, b, c \in M \text{ and for all } \alpha, \beta \in \Gamma.$$

Let $K$ be a non-empty subset of $M$. Then $K$ is called a *sub $\Gamma$-semigroup* of $M$ if $a\gamma b \in K$ for all $a, b \in K$ and $\gamma \in \Gamma$.

EXAMPLE 10. If $M$ is the set of $m \times n$ matrices and $\Gamma$ is a set of some $n \times m$ matrices over the field of real numbers, then we can define $A_{m,n}\alpha_{n,m}B_{m,n}$ such that

$$(A_{m,n}\alpha_{n,m}B_{m,n})\beta_{n,m}C_{m,n} = A_{m,n}\alpha_{n,m}(B_{m,n}\beta_{n,m}C_{m,n}),$$

where $A_{m,n}, B_{m,n}, C_{m,n} \in M$ and $\alpha_{n,m}, \beta_{n,m} \in \Gamma$. An algebraic system satisfying the associativity property of the above type is a $\Gamma$-semigroup.

EXAMPLE 11. Let $M = [0,1]$ and $\Gamma = \{\frac{1}{n} \mid n \text{ is a positive integer}\}$. Then $M$ is a $\Gamma$-semigroup under the usual multiplication. Next, let $K = [0,1]$. We have that $K$ is a non-empty subset of $M$ and $a\gamma b \in K$ for all $a, b \in K$ and $\gamma \in \Gamma$. Then $K$ is a sub $\Gamma$-semigroup of $M$.

Anvariyeh, Mirvakili and B. Davvaz [1] introduced the concept of $\Gamma$-semihypergroups. We recall the following definition and examples from [1].

**Definition 8.2.** Let $S$ and $\Gamma$ be two non-empty sets. $S$ is called a $\Gamma$-*semihypergroup* if every $\gamma \in \Gamma$ is a hyperoperation on $S$, i.e, $x\gamma y \subseteq S$ for every $x, y \in S$, and for every $\alpha, \beta \in \Gamma$ and $x, y, z \in S$ we have $x\alpha(y\beta z) = (x\alpha y)\beta z$.

If every $\gamma \in \Gamma$ is an operation, then $S$ is a $\Gamma$-semigroup.

If $(S, \gamma)$ is a hypergroup for every $\gamma \in \Gamma$, then $S$ is called a $\Gamma$-*hypergroup*.

Let $A$ and $B$ be two non-empty subsets of $S$ and $\gamma \in \Gamma$. We define:

$$A\gamma B = \cup\{a\gamma b \mid a \in A,\ b \in B\}.$$

Also

$$A\Gamma B = \cup\{a\gamma b \mid a \in A,\ b \in B \text{ and } \gamma \in \Gamma\} = \bigcup_{\gamma\in\Gamma} A\gamma B.$$

Let $S$ be a $\Gamma$-semihypergroup and $\gamma \in \Gamma$. A non-empty subset $A$ of $S$ is called a *sub* $\Gamma$-*semihypergroup* of $S$ if $a_1\gamma a_2 \subseteq A$ for every $a_1, a_2 \in A$. A $\Gamma$-semihypergroup $S$ is called *commutative* if for all $x, y \in S$ and $\gamma \in \Gamma$ we have $x\gamma y = y\gamma x$.

EXAMPLE 12. Let $(S, \circ)$ be a semihypergroup and $\Gamma$ be a non-empty set. We define $x\gamma y = x \circ y$ for every $x, y \in S$ and $\gamma \in \Gamma$. Then $S$ is a $\Gamma$-semihypergroup.

EXAMPLE 13. Let $(S, \circ)$ be a semihypergroup and $\Gamma$ be a non-empty subset of $S$. We define $x\gamma y = x \circ \gamma \circ y$ for every $x, y \in S$ and $\gamma \in \Gamma$. Then $S$ is a $\Gamma$-semihypergroup.

EXAMPLE 14. Let $S = [0,1]$ and $\Gamma = \mathbb{N}$. For every $x, y \in S$ and $\gamma \in \Gamma$, we define

$$\gamma : S \times S \to \mathcal{P}^*(S)$$
$$x\gamma y = \left[0, \frac{xy}{\gamma}\right].$$

Then, $\gamma$ is a hyperoperation. For every $x, y, z \in S$ and $\alpha, \beta \in \Gamma$ we have

$$(x\alpha y)\beta z = \left[0, \frac{xyz}{\alpha\beta}\right] = x\alpha(y\beta z).$$

This means that $S$ is a $\Gamma$-semihypergroup.

Also, if $\gamma \in \Gamma$, then $(S, \gamma)$ is not a hypergroup, because $0.1\gamma S = \left[0, \frac{0.1}{\gamma}\right] \neq S$. So $S$ is not a $\Gamma-$hypergroup.

**Proposition 8.3.** *Let $S$ be a $\Gamma$-semihypergroup and there exists $\alpha \in \Gamma$ such that $(S, \alpha)$ is a hypergroup. Then for every $\gamma \in \Gamma$, $(S, \gamma)$ is a hypergroup.*

## References


[1] S.M. Anvariyeh, S. Mirvakili and B. Davvaz, *Carpathian Journal of Mathematics* **26**, 11-23 (2010).

[2] S. Comer, "Extension of polygroups by polygroups and their representations using color schemes", in *Lecture notes in Math., No. 1004, Universal Algebra and Lattice Theory*, 1982, pp. 91-103.

[3] P. Corsini, *Prolegomena of hypergroup theory*, Second edition, Aviani editor, 1993.

[4] P. Corsini and V. Leoreanu, *Applications of hyperstructure theory*, Advances in Mathematics, Kluwer Academic Publishers, Dordrecht, 2003.

[5] B. Davvaz, *Southeast Asian Bull. Math.* **25**, 87-95 (2001).

[6] B. Davvaz, "Weak polygroups", in *Proc. 28th Iranian Math. Conference*, University of Tabrize, 1997, pp. 28-31.

[7] B. Davvaz, *Intern. J. Appl. Math.* **6(4)**, 449-461 (2001).

[8] B. Davvaz, *JP Jour. Algebra, Number Theory and Appl. (JPANTA)* **2(2)**, 53-62 (2002).

[9] B. Davvaz, A. Dehghan Nezad and A. Benvidi, *MATCH Communications in Mathematical and in Computer Chemistry* **65**, 491-499 (2011).

[10] B. Davvaz and V. Leoreanu-Fotea, *Hyperring Theory and Applications*, International Academic Press, USA, 2007.

[11] B. Davvaz and A. Dehgan-nezad, *Ratio Matematica* **14**, 71-74 (2003).

[12] B. Davvaz and T. Vougiouklis, *Iranian Journal of Science and Technology, Transaction A* **30 (A2)** 165-174 (2006).

[13] B. Davvaz, W.A. Dudek and S. Mirvakili, *International Journal of Algebra and Computation* **19**, 567-583 (2009).

[14] B. Davvaz, W.A. Dudek and T. Vougiouklis, *Communications in Algebra* **37**, 1248-1263 (2009).

[15] W. Dörnte, *Math.Z.* **6**, 1-19 (1929).

[16] V. Leoreanu-Fotea and B. Davvaz, *European Journal of Combinatorics* **29**, 1027-1218 (2008).

[17] V. Leoreanu-Fotea and B. Davvaz, *Journal Multiple-valued Logic and Soft Computing* **15**, 421-432 (2009).

[18] R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry*, Volumes I, II, III, IV, and V, International Academic Press, USA, 2008.

[19] R.M. Santilli, *Algebras, Groups and Geometries* **15**, 473-498 (1998).

[20] R.M. Santilli and T. Vougiouklis, "Isotopies, genotopies, hyperstructures and their applications", in *New frontiers in hyperstructures* (Molise, 1995), pp. 1-48, Ser. New Front. Adv. Math. Ist. Ric. Base, Hadronic Press, Palm Harbor, FL, 1996.

[21] M. K. Sen and N. K. Saha, *Bull. Cal. Math. Soc.* **78**, 180-186 (1986).

[22] T. Vougiouklis, *J. Combin. Inform. System Sci.* **20**, 229-235 (1995).

[23] T. Vougiouklis, *Hyperstructures and their Representations*, Hadronic Press Inc., Palm Harbor, USA, 1994.

[24] T. Vougiouklis, "The fundamental relation in hyperrings. The general hyperfield", in *Algebraic Hyperstructures and Applications* (Xanthi, 1990), 203-211, World Sci. Publishing, Teaneck, NJ, 1991.

[25] L.A. Zadeh, *Inform. Control* **8**, 338-353 (1965).

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
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# The Kerr-Schild Prequantum Geometry: Initial Irreversibility, and the Transition to a Reversible Classical Gravity.

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**Abstract.** The Kerr BH geometry has initiate irreversibility which is related by its twistorial structure determined by the Kerr theorem. Twistors form a *time-oriented congruence* of the lightlike geodesics. In the works [1, 2, 3, 4] we described exact Kerr-Schild (KS) solutions for electromagnetic (EM) excitations of the Kerr-Newman (KN) black hole. It has been shown that the typical excitations form singular beams which have very strong back-reaction on metric. The beams create topological deformations of the metric and horizon. As a result, the internal region of the BH becomes connected with its exterior, which allows matter to escape the BH and leads to the BH evaporation and reversibility of the BH geometry on classical level. In the idealized Kerr-Schild geometry the EM excitations form a time-oriented $|in>$ vacuum. Backreaction of the EM excitations form a fine-grained prequantum KS geometry of fluctuating beams. In this paper we analyze a process of the measurement and show that the measurement breaks the idealized KS geometry. Based on the Kerr theorem we show that any measurement breaks initiate symmetry and creates an opposite-oriented sheet of the spacetime. As a result, the process of formation of an amplitude of probability $< out|G|in >$ involves a dual (bra-) state $< out|$ which (for a stationary measurement) has to be complex conjugate to the initiate $|in >$ state and describes a reverse time-evolution. Therefore, the initiate irreversible prequantum classical Kerr-Schild geometry acquires time-reversibility by the transfer to quantum level, and the transfer to classical geometry happens in the process of the measurement.

**Key words:** Kerr-Schild geometry, time reversibility, null congruences, twistors, multisheeted spacetimes
**AMS Subject Classification:** 83C22, 83C45, 52C28

## 1 Introduction

Irreversibility is usually related with composite objects consisting of many elementary subsystems possessing great many degrees of freedom. From this point of view, black holes (BHs) are unique objects

possessing an irreversible evolution (collapse) despite of they elementary (all-in-one) nature. Behavior of the BHs is somehow opposite to conventional one. By the quantum treatment the BHs are completed by a series of quantum modes (EM oscillators) which increase drastically the number of degrees of freedom. It could result in the extra irreversibility, however, it has an opposite effect resulting in a reverse process of the BH evaporation.

In the works [1, 2, 3, 4] we described exact Kerr-Schild solutions for electromagnetic (EM) excitations of the Kerr-Newman (KN) black hole. It has been shown that the typical EM excitations of the BHs have the form of singular beams which give very strong back-reaction on metric and the BH horizon [5]. In particular, they penetrate the BH horizon leading to its topological fluctuations. As a result, the internal region of the BH becomes connected with its exterior, which allows matter to escape the BH, resulting in a reversibility of the BHs on the classical level via the Hawking evaporation.

It has been shown in [3] that the initiate irreversibility of the Kerr's BH is related with the determined by the Kerr theorem twistorial structure of the Kerr-Schild (KS) geometry. The BH solutions belong to type D of the algebraically special metrics which are determined by the *time-oriented congruences* of the lightlike geodesics. In the idealized Kerr-Schild geometry these congruences form the time-oriented (ket) vacuum $|in>=|0>_{KS}$ which corresponds to a pre-quantum classical space-time of the KS black hole.

The pre-quantum KS geometry is fluctuating and has a fine-grained structure which differs drastically from the usual classical gravity. Contrary to the harmonic excitations obtained in the perturbative approaches, the exact KS solutions for electromagnetic excitations of the KS geometry (obtained in the Kerr-Schild formalism [6]) have the form of singular beams which have very strong backreaction on the metric and horizon. This effect is determined by an analytic twosheeted structure of the Kerr-Schild geometry, which turns out to be in a perfect agreement with the predicted in [7, 8] requirements to the quantum BH spacetimes. There appears a question on the relation between the obtained irreversible pre-quantum KS geometry

and the reversible classical gravity.

In this paper we analyze a process of the measurement and show that any measurement of any physical *observable 'G'* breaks the idealizations of an isolate BH spacetime. As it has been mentioned in [1], the usual necessary conditions for the proof of the theorems on the uniqueness of the BHs (see for example [9]) turn out to be broken in practice. An isolated BH represents an idealization. Similarly, the exact BH solutions represent idealization, which is changed in the process of measurement. In the process of a measurement the one-particle problem turns into a multi-particle one. It was shown in [10], that in the presence of external sources, the initiate twosheeted KS structure turns into a multisheeted one. Origin of this effect lies in analyticity of the Kerr-Schild solutions and, in particular, in twistor analyticity of the Kerr congruence determined by the Kerr theorem [11, 12, 10].

The Kerr theorem suggests [10] that in this case there appears a multisheeted space with the extra opposite-oriented sheets of the space-time related with the measurer, and the the process of formation of an amplitude of probability $< out|G|in >$ involves a dual (bra-) state $< out|$ which (for a stationary measurement) has to be complex conjugate to state $|in >$ and describes a reverse time-evolution. Therefore, the irreversible classical pre-quantum Kerr-Schild geometry based on the time-ordered congruences acquires time-reversibility under transfer from pre-quantum KS geometry to quantum level and by the subsequent transfer to classical level in the process of measurement of the real physical observables.

## 2 The Kerr-Schild metric

The Kerr-Schild form of metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \tag{1}$$

has many advantages with respect to other representations. First is unfastening of coordinates from position of the horizon and their rigid connection to auxiliary Minkowski space-time with metric $\eta_{\mu\nu}$.

It allows one to analyze deformations of the horizon. The second is absence of singularity of the solutions at the horizon. The third advantage is related with the Kerr-Schild twistor structure [12] and correspondence with the requirements of holographic principle and the presumable properties of a quantum black hole space-time prescribed by Stephens, t' Hooft and Whiting in [8].

Analyticity of the Kerr-Schild geometry originates from the complex function

$$Y = e^{i\phi} \tan \frac{\theta}{2} \tag{2}$$

which is a projection of celestial sphere $S^2$ on the complex plane. This function determines the Kerr congruence and complex tetrad forms. The Kerr theorem sets the dependence $Y(x)$, and the null vector field $k^\mu$ of the Kerr-Schild metric form is expressed via function $Y(x)$

$$k_\mu dx^\mu = P^{-1}(du + \bar{Y} d\zeta + Y d\bar{\zeta} - Y\bar{Y} dv), \tag{3}$$

in the null Cartesian coordinates

$$2^{\frac{1}{2}}\zeta = x + iy,\ 2^{\frac{1}{2}}\bar{\zeta} = x - iy,\ 2^{\frac{1}{2}}u = z - t,\ 2^{\frac{1}{2}}v = z + t. \tag{4}$$

Therefore, the field $k^\mu(x)$, $x \in M^4$ determines symmetry of space-time, polarization of the Kerr-Newman electromagnetic field, direction of gravitational 'dragging' and so on. This vector field is tangent to the Kerr congruence which is the family of the light-like geodesic lines, in fact twistors. Twisting structure of the Kerr congruence is shown in Fig.1

Twist of the congruence determines the complicate form of the Kerr solution, in spite of the extremely simple form of the metric (1).

### 2.1 Twofoldedness

The Kerr-Schild space-time is twosheeted. The used Kerr's oblate ellipsoidal coordinate system $r, theta, \phi$ is related with the Cartesian coordinates $t, x, y, z$ as follows

$$x + iy = (r + ia)\exp\{i\phi_K\}\sin\theta, \quad z = r\cos\theta, \quad \rho = r - t. \tag{5}$$

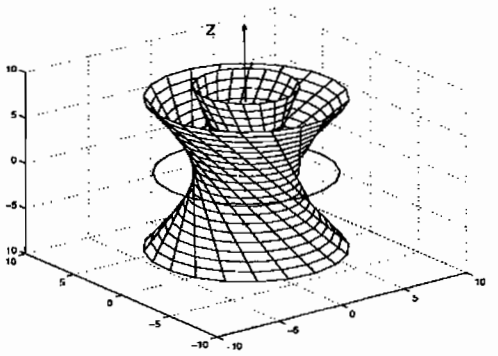



Figure 1: The Kerr singular ring and the Kerr congruence formed by oriented twistor null lines and covering the Kerr-Schild spacetime twice.

The oblate radial coordinate $r$ covers the spacetime twice, taking the positive, $r > 0$, and negative, $r < 0$ values. The Kerr singular ring $r = \cos\theta = 0$ is a branch line of space on two sheets: "negative (–)" and "positive (+)." The fields and effective signs of the mass and charge change their directions by the transfer via disk $r = 0$. The Kerr congruence covers the spacetime twice: in the form of ingoing PNC $k^{\mu-} \in \mathcal{K}^-$ which falls on the disk spanned by Kerr singular ring by $r < 0$, and as outgoing one, $k^{\mu+} \in \mathcal{K}^+$, positioned on the positive sheet of the same spacetime $M^4$, $r > 0$. The metrics

$$g^{\pm}_{\mu\nu} = \eta_{\mu\nu} + 2Hk^{\pm}_{\mu}k^{\pm}_{\nu} \tag{6}$$

and the electromagnetic potentials $\alpha^{\mu\ \pm}$, being aligned with PNC, are to be different on the in- and out- sheets. Two different metrics exist on the same spacetime (!) and they should not interact with each other or to be mixed. This twosheetedness is ignored in perturbative approaches, leading to drastic discrepancies in the exact and perturbative solutions. [1] It is known [15] that the rotating BHs can be described in the ingoing coordinate system as well as in the outgoing

[1]Note, that the twosheetedness retains also in the case of the flat KS space by $H = 0$. One sees that the Kerr geometry clarify the origin and gives justification for many ideas of prof. R.M Santilli on the Lie-admissible spacetimes and the corresponding especial mathematics [13, 14], including bimodular geometry.

one, but not in the both simultaneously. The Kerr-Schild formalism [6] takes into account that electromagnetic solutions have to be aligned only to one of two congruences, i.e. the time orientations of the electromagnetic and gravitational fields have to be matched. It meets the requirements of the 't Hooft holographic principle [7], the KS spacetime is projected by the light-like rays on the 2d disk $r = 0$. It corresponds also to the presumable properties of a quantum black hole space-time [8] which should be separated into the in- and out-vacua to consider the process of evaporation as a scattering of the in-vacuum on a black hole. The usual Penrose conformal diagram, containing the in- and out-fields on the same sheet of $M^4$, must be unfolded by a "splitting" the KS two-sheetedness, as shown in Fig.2, demonstrating an explicit realization of the *holographic principle* in the KS geometry. The twistor-beams of the Kerr congruence "create" the Kerr source (as a holographic image) by light projection from the past null infinity $I^-$ on the bulk of KS space-time. The BH then appears as a *holographic image* generated by the initial data on the past infinity $I^-$.

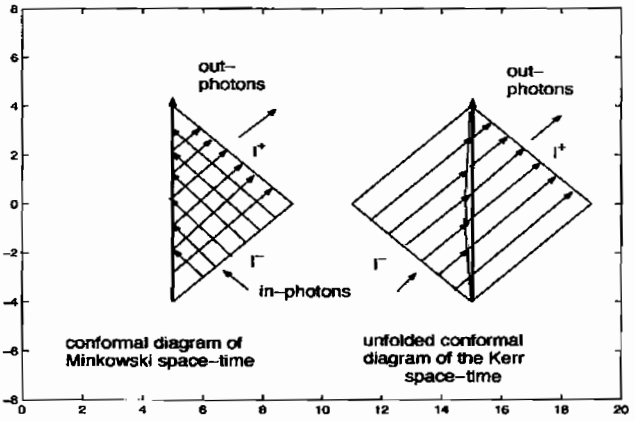



Figure 2: Penrose conformal diagrams for (a) the Minkowski space-time and (b) the Kerr space-time: unfolding the auxiliary $M^4$ space of the Kerr geometry into two sheets generates the holographic structure of a prequantum BH space-time.

In [1] we have discussed elasticity of the BH horizons with respect to EM field. For the KS form of the metric, positions of the BH

horizons are determined by function

$$H = \frac{mr - e^2/2}{r^2 + a^2 \cos^2 \theta}, \tag{7}$$

where $m$ is the mass, $a = J/m$ is a spin-parameter, and $e$ is the charge of the Kerr-Newman solution, [6]. In particular, two positions of the horizons are

$$r_\pm = m \pm \sqrt{m^2 - e^2 - a^2 \cos^2 \theta},$$

and for $e^2 > m^2$ the horizons disappear at all, [15]. For the obtained in [6] general stationary solutions, we have

$$H = \frac{mr - |\psi|^2/2}{r^2 + a^2 \cos^2 \theta}, \tag{8}$$

where the function $\psi(Y)$ is related with electromagnetic field. This case was analyzed in [5], and it has been shown that $\psi$ acts similarly to charge, destroying the horizons topologically when $|\psi|^2 > m^2$. On the other hand, it was obtained in [6] that in the exact stationary KS solutions, function $\phi$ may be *any holomorphic* function of the complex angular coordinate $Y$. [2] So, it was obtained in [5] that the poles in function $\psi$ lead to topological deformations of the horizon.

## 3 Exact Kerr-Schild solutions with beams

The famous Kerr-Newman solution is the simplest solution of the Kerr-Schild class having $\psi = q = const.$, where $q$ is the value of charge. However, any holomorphic function $\psi(Y)$ yields also an exact solution of this class [6]. It is known that any holomorphic function on sphere, if it is not a constant, should have at least one singular point. Therefore, *all the exact Kerr-Schild solutions, except the Kerr-Newman one, acquire one or more lightlike singular beams* which are positioned along the lines of Kerr congruence, while the usual harmonic solutions with smooth angular dependence are absent on the

[2] Function $Y(x)$ determines also the Kerr-Schild tetrad $e^a$: $e^1 = d\zeta - Ydv$, $e^2 = d\bar{\zeta} - \bar{Y}dv$, $e^3 = Pk_\mu dx^\mu$, $e^4 = dv + he^3$, $h = HP^{-2}$, where $P = (1 + Y\bar{Y})/\sqrt{2}$.

Kerr background. It was shown in [6] that vector potential of the exact stationary electromagnetic field on the KS background has the general form

$$\alpha = \alpha_\mu dx^\mu = -\frac{1}{2} Re\ [(\frac{\psi}{r + ia\cos\theta})e^3 + \chi d\bar{Y}], \tag{9}$$

where $\chi = \int P^{-2}\psi dY$ and $\bar{Y}$ being kept constant in this integration. The expression $dY$ represents gradient of the complex surfaces $Y = const.$ obeying the conditions

$$Y_{,2} = Y_{,4} = 0. \tag{10}$$

These surfaces are totally null, spanned by the tetrad forms $e^1$ and $e^3$. Similarly, $d\bar{Y}$ is spanned by $e^2$ and $e^3$, and therefore, $\alpha^\mu$ is spanned by $e^1$, $e^2$ and $e^3$. So, using the null tetrad orthogonality relations $(e^1)^2 = (e^2)^2 = (e^3)^2 = 0$ and $e^1e^3 = e^2e^3 = 0$ and $e^3$, one obtains that vector potential satisfies the alignment condition

$$\alpha_\mu e^{3\mu} = P\alpha_\mu k^\mu = 0. \tag{11}$$

The 'elementary' beams, formed by a single pole $\psi(Y) = q/(Y - \hat{Y})$ at the point $\hat{Y} \in S^2$, propagate along the twistor null lines of the Kerr congruence in direction $k^\mu$, corresponding to $Y = \hat{Y}$. The beams play exceptional role, turning in the far zone (see [16]) into uniform string-like singular pp-waves (A.Peres solutions [11]). In fact, there are no the usual plane waves in the curved spacetimes, and the most close analogs to the linearized gravitational waves turn out to be divergent at infinity. More general analogs are the pp-wave solutions (plane-fronted, or parallel propagating waves) which takes the KS form (1) with a covariantly constant null vector field $k^\mu$. The pp-waves have remarkable quantum properties (vanishing all quantum corrections), however, they turn out to be singular either at the front of the wave or along a null line,[17]. In the last case, pp-waves imitate the wave-particle duality, describing a plane-fronted wave together with a singular null line corresponding to trajectory of the related light-like particle. Similar to pp-waves, the beamlike solutions are the analogs of the spherical harmonic functions for the KS background.

So, it is natural that in the far zone the singular beams tend to the pp-wave solutions. In general, holomorphic function may be expanded in the Loran series containing a singular part $\psi(Y) = \sum_{n=-\infty}^{0} q_n Y^n$, which can be represented by a series of the above poles, and a regular one $\psi(Y) = \sum_{n=0}^{\infty} q_n Y^n$. We shall see later that the regular polynomial part plays very important role in the nonstationary solutions. It was shown, that singular lightlike beams deform topologically the horizon, forming the holes connecting the internal and external regions of black hole, see [5]. The 'elementary' single-pole solution may trivially be extended to the case of arbitrary number of single poles

$$\psi(Y) = \sum_i \frac{q_i}{Y - Y_i}, \tag{12}$$

in different directions $Y_i = e^{i\phi_i} \tan \frac{\theta_i}{2}$. Elementary excitation $\psi_i(Y) = q_i/(Y - Y_i)$, describes a singular light-like beam (pp-string) along the null ray of the Kerr congruence in direction $k_i^\mu = k^\mu(Y_i, \bar{Y}_i)$. The vector potential (9) is trivially generalized to a sum over beams, where for the single i-th beam $\chi_i = q_i P_i^{-2} \ln(Y - Y_i) + const. + \mathcal{O}(Y - Y_i)$, and $P_i = (1 + Y_i \bar{Y}_i)/\sqrt{2}$. The corresponding vector field (9) gives rise to electromagnetic field $f = \frac{1}{2} F_{ab} e^a \wedge e^b = -d\alpha$ which is aligned with the Kerr congruence,

$$\alpha_\mu k^\mu = 0, \quad k^\mu F_\mu^\nu = \lambda k^\nu, \tag{13}$$

where $\lambda = Re\,[\psi/(r + ia\cos\theta)^2]$. It was shown that these beams have strong back reaction on the metric, via function $\psi_i(Y)$ entering in (8). The multibeam solution (12) is a particular case of the exact solutions obtained by Debney, Kerr and Schild (DKS) in seminal work [6]. It should be emphasized that in the usual perturbative approach the beam solutions are absent, because the important alignment condition are dropped out of the perturbative equations, and a mixing of the in and out fields occurs on the same sheet of metric.

The appearance of light-like beam pulses is a pure classical effect, however, it allowed us in [5] to put some conjectures concerning semi-classical treatment of the black hole interaction with a weak stochastic electromagnetic field and, in particular, with electromagnetic vacuum fluctuations. Since the black hole horizon is extra sensitive to electro-

magnetic excitations of black hole, it should also be sensitive to zero point field (ZPF) which is classically exhibited as a Casimir effect. Therefore, as it has been discussed by many authors (see for example [18]), vacuum fluctuations may generate fluctuations of the metric and horizon, see Fig.3. Note that such point of view assumes tacitly that there exist some semiclassical pre-quantum geometry which lies beyond the usual classical gravity, and describes a fluctuating fine-grained structure created by a backreaction of the vacuum fluctuations on metric. *We obtain that the exact KS solutions display just the case of such a prequantum geometry*, [3].

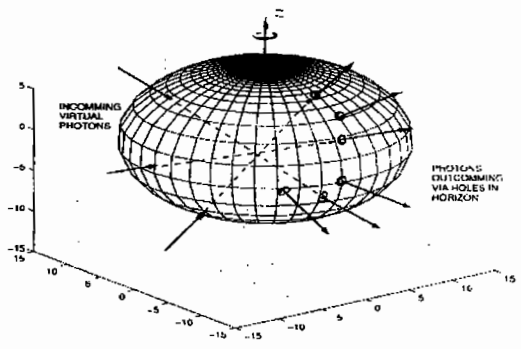



Figure 3: Excitation of a black hole by the zero-point field of virtual photons forming a set of micro-holes in the horizon.

The performed in [10] treatment of multiparticle solutions showed that the beams appear between the distant sources, being supported on a twistor line between them. So, it has to be assumed that the infinite beams represent an idealization, and the real beams are to be finishing at some distant objects, or at the infinitely distant matter, similar to the usual treatment on propagation of the lightlike particles.

The singular pp-wave solutions may take the form of the wave pulses with a carrier frequency and a finite extension. Therefore, it is desirable to consider a minimal generalization of the exact stationary Kerr-Schild beam-like solution to time-dependent beam pulses, which is necessary to consider a time-dependent process of scattering.

## 4 Time-dependent Kerr-Schild solutions

The closest time-dependent generalization to (12) is given by the form

$$\psi(Y,\tau)=\sum_i c_i(\tau)(Y-Y_i)^{-1}, \tag{14}$$

which assumes that an elementary beam has $c_i(\tau)=q_i(\tau)e^{i\omega_i\tau}$ where $q_i(\tau)$ is amplitude and $\omega_i$ is carrier frequency. Obtaining of the exact nonstationary solutions is divided in two stages: i) obtaining the exact solutions for electromagnetic field on the Kerr background, ii) self-consistent solutions, taking into account the backreaction of the EM field on the metric. General equations for nonstationary electromagnetic Kerr-Schild solutions were obtained by by Debney, Kerr and Schild (DKS) [6] in 1968. However, their full integration was performed only for stationary case. [3] General solution for the EM field on the Kerr-Schild background was obtained only in 2004, [16], and their backreaction on metric was obtained only recently, [1, 4]. The obtained in [16] wave solutions describe singular beams along the $\pm z-$ half-axis, and there appeared the conjecture that, for the very seldom exclusions, any exact nonstationary electromagnetic excitation of the KS BH should create the beams. The nonstationary KS solutions form the fluctuating singular beams pulses which lead to topological fluctuations of the black hole horizon.

Bellow we consider important peculiarities of the nonstationary KS solutions, which are determined by a specific structure of the DKS equations. The KS electromagnetic field is described by the self-dual tetrad components,

$$\mathcal{F}_{12}=AZ^2, \quad \mathcal{F}_{31}=\gamma Z-(AZ)_{,1}\,, \tag{15}$$

where $\mathcal{F}_{ab}=e^\mu_a e^\nu_b \mathcal{F}_{\mu\nu}$, and the function $Z$ is a complex expansion of the congruence, $Z=Y_{,1}$. For the Kerr-Newman solution at rest $Z$ is inversely proportional to a complex radial distance $Z=-P/(r+ia\cos\theta)$. Here $P$ is a conformal factor which is determined by Killing

[3] See the history in [19].

vector of the solution [6]. For a black hole at rest $P = 2^{-1/2}(1+Y\bar{Y})$. [4] Function $A$ satisfies the equations

$$(AP^2)_{,2} = 0, \ A_{,4} = 0. \tag{16}$$

Their general solution is

$$A = \psi P^{-2}, \tag{17}$$

where the function $\psi$ should satisfy

$$\psi_{,2} = \psi_{,4} = 0. \tag{18}$$

In the nonstationary case function $\psi$ has to depend on a retarded-time parameter $\tau$, and (16) implies that $\tau$ should obey the equations

$$\tau_{,2} = \tau_{,4} = 0, \tag{19}$$

which are similar to (10). It shows that gradient of $\tau$ is to be aligned to congruence,

$$k^{\mu}\tau_{,\mu} = 0. \tag{20}$$

It was obtained in [16] that the corresponding retarded-time parameter has the form

$$\tau = t - r - ia\cos\theta, \tag{21}$$

A principal difference from the stationary case is contained in the second electromagnetic DKS equation which in [16] was reduced to the very simple form

$$(\gamma P)_{,\bar{Y}} = -\dot{A}, \tag{22}$$

showing that *any nonstationarity in electromagnetic field ($\dot{A} \neq 0$) generates an extra function* $\gamma$ which, in accord with (15), generates also the lightlike electromagnetic radiation along the Kerr congruence. Such a radiation is well-known for the Vaidya 'shining star' solution [11], in which the field $A = \psi P^{-2}$ is absent and $\gamma$ is assumed to be a

[4] We are neglecting the recoil, $\dot{P} = 0$, assuming that that the mass of black hole is much greater then the energy of excitation.

fluctuating incoherent field related with a loss of the total mass into radiation,

$$\dot{m} = -\frac{1}{2}P^2\gamma\bar{\gamma}. \tag{23}$$

This is one of two gravitational equations which determine consistency of the Kerr-Schild solution, [6]. The equation (22) shows that the field $\gamma$ is created by the nonstationary EM excitations. However, one sees that $\gamma$ does not enter in the expression for $H$, (8), and consequently, it does not give immediate impact on deformation of the metric and horizon. Its backreaction on metric is smooth and circumstantial, via the expression (23) which determines the slowly decreasing mass parameter $m$. Therefore, in the nonstationary Kerr-Schild case we obtain that the functions $\psi$ and $\gamma$, have absolutely different backreaction on the metric and horizon.

General solution of the equations (22) takes the form

$$\gamma = \frac{2^{1/2}\dot{\psi}}{P^2 Y} + \phi(Y,\tau)/P, \tag{24}$$

where $\phi(Y,\tau)$ is an arbitrary analytic solution of the corresponding homogenous equation (22) with $\dot{A} = \dot{\psi} = 0$.

Similarly to $\gamma$, function $\phi$ does not impact on the metric and horizon too. Important role of this function is obtained from analysis of the second gravitational Kerr-Schild equation [eq.(5.44) in [6]] which is reduced to the form

$$m_{,\bar{Y}} = \psi\bar{\gamma}P. \tag{25}$$

Solution of this equation was given in [4], and was based on analogue with the close related Vaidya 'shining star' solution [11], and a nontrivial regularization of the singular beam pulses. In the Vaidya 'shining star' solution $\psi = 0$, and the equation (25) shows that the function $m(t)$ is real and independent from $Y$. In the same time, the field $\gamma$ in (23) was assumed to be incoherent and the averaged r.h.s. of (23) takes the form $\frac{1}{2} < P^2\gamma\bar{\gamma} >$ . This approach can be extended to the r.h.s of the both gravitational equations which acquire the form

$$m_{,\bar{Y}} =< P\psi\bar{\gamma} >, \quad \dot{m} = -\frac{1}{2} < P^2\gamma\bar{\gamma} >, \tag{26}$$

and may be considered as a semiclassical analog of a quantum approach. It was shown in [4] that the form and positions of the poles in the free function $\phi/P$ may be tuned to cancel the poles of function $\dot{\psi} = \sum_i \dot{c}_i(\tau)/(Y - Y_i)$ in (24). There was defined the function

$$P_i = P(Y, \bar{Y}_i) = 2^{-1/2}(1 + Y\bar{Y}_i). \tag{27}$$

It is obviously analytic in $Y$, and one can set

$$\phi_i^{(tun)}(Y,\tau) = -\frac{2^{1/2}\dot{c}_i(\tau)}{Y(Y-Y_i)P_i} = -\frac{2\dot{c}_i(\tau)}{Y\bar{Y}_i(Y-Y_i)(Y+1/\bar{Y}_i)}, \tag{28}$$

keeping the required analyticity of function $\phi_i^{(tun)}(Y,\tau)$ in $Y$. Using the equality

$$(P_i - P)/(Y_i - Y) = \frac{Y}{\sqrt{2}}\frac{(\bar{Y}_i - \bar{Y})}{(Y_i - Y)}, \tag{29}$$

one obtains that the function $\gamma_i^{(reg)} = \frac{2^{1/2}\dot{\psi}_i}{P^2 Y} + \phi_i^{(tun)}(Y,\tau)/P$ , takes the form

$$\gamma_i^{(reg)} = \frac{1}{P^2}\frac{\dot{c}_i}{P_i}[\frac{\bar{Y}_i - \bar{Y}}{Y_i - Y}], \tag{30}$$

which is regular at the point $Y = Y_i$. The r.h.s. of the equation (25) for this regular solution takes the form

$$\psi_i \bar{\gamma}_i^{(reg)} P = -\frac{c_i \dot{\bar{c}}_i}{P\bar{P}_i(\bar{Y} - \bar{Y}_i)},$$

and by using the Cauchy integral formula, the equation (25) is easily integrated, leading to

$$\Re e\ m = m_0 - 2\pi\omega_i \frac{|q_i|^2}{P_{ii}^2} - i\pi\frac{q_i\dot{\bar{q}}_i - \dot{q}_i\bar{q}_i}{P_{ii}^2}\ , \tag{31}$$

where $P_{ii} = \frac{1}{\sqrt{2}}(1 + Y_i\bar{Y}_i)$ are the constant, and the coefficients $c_i(\tau)$ are replaced by the slowly varying amplitudes $q_i(\tau)$ and the carrier frequencies of the beam pulses $\omega_i$, $c_i(\tau) = q_i(\tau)e^{-i\omega_i\tau}$. The obtained backreaction on metric for the time-dependent electromagnetic field is exact up to our approximation which neglects the recoil. The solutions turn out to be consistent with the Einstein-Maxwell system

of the equations with an averaged stress-energy tensor, which takes into account that the frequencies and phases of the different beams are uncorrelated.

## 5 Observables and the process of a measurement

As a result of the integration we obtain a very specific geometry of fluctuating singular twistor-beams. It differs drastically from the usual classical gravity, displaying a fluctuating fine-grained structure. We call it as a prequantum KS geometry, and there appears the question, what is the relation of this geometry to the usual classical gravity? The answer we find in the analysis of the measurement process in quantum theory.

The holographic KS geometry is formed by a future-directed congruence of twistor null lines which are strongly time-directed. It is reflected on the all its excitations which take the form of an infinite sum of the future-directed wave beam pulses propagating along twistor lines of the Kerr congruence.

Similarly to the approach used in QED, the KS EM excitations may be expressed via a complex vector potential[5] $\mathcal{A} = \sum_i \mathcal{A}_i$ where

$$\mathcal{A}_i = A_i Z_i e_i^3 + \bar{\chi}_i dY \tag{32}$$

are characterized by the lightlike direction of propagation $k_i^\mu = e_i^{3\mu}/P_i$ determined by $Y_i = e^{i\phi_i}\tan(\theta_i/2)$, and by the polarization in the orthogonal direction $dY_i = Y_i\{id\phi_i + \frac{1}{\sin\theta_i}d\theta_i\} \sim \frac{1}{r}dx_\perp^\mu$. Here $\bar{\chi} = \int A d\bar{Y}$ is introduced in [6] function ( $Y$ and $\tau$ being kept constant in this integration). We have

$$A_i = \psi_i(Y,\tau)/P^2,$$

where $\psi_i(Y,\tau) = q_i(\tau)/(Y - Y_i)$.

When we consider the exact KN background, the infinite set of the EM excitations forms a future-directed Fock vacua $|in>_{EM}$ aligned

[5]To be published in Proc. of the 6th Mathematical Physics Meeting (14-23 September 2010, Belgrade, Serbia) .

with the Kerr congruence. Since the function $\psi = \sum_i \psi_i(Y, \tau)$ enters into the KS metric function $H$, we obtain that the EM excitations generate an infinite set of the singular excitations of the KS metric forming a future-directed gravitational Fock vacua $|in >_{KS}$ aligned with the Kerr congruence.

Now, we have to consider a process of measurement of any physical *observable* $G$, say the components of the metric tensor. In the process of the measurement we have to form an amplitude of probability $< out|G|in >$, and therefore, we have to involve a dual (bra-) state $< out|$ which (for a stationary measurement) has to be complex conjugate to state $|in >$ and should be described by a reverse time-evolution. However, the idealized KS solutions are strongly oriented and do not contain such the dual components. Resolution of this problem lies in a breakdown of this idealization. We notice that the presence of a measurer breaks topology of the initial ideal Kerr background.

Based on the Kerr theorem, we will show that in the process of a measurement, there appears an extra sheet of the space-time, which is opposite-oriented field with respect to the initial Kerr background. Therefore, the irreversible classical pre-quantum Kerr-Schild geometry based on the time-ordered congruence of twistor null lines acquires time-reversibility under transfer from pre-quantum KS geometry to quantum level and by the subsequent transfer to classical level of the real physical observables.

**The Kerr Theorem** The Kerr theorem [10, 11, 12] claims that all the geodesic and shear-free congruences are determined by the function $Y(x)$ which is a solution of the algebraic equation

$$F = 0, \tag{33}$$

where the generating function $F$ is an arbitrary holomorphic function of the projective twistor variables

$$Y, \quad \lambda_1 = \zeta - Yv, \quad \lambda_2 = u + Y\bar{\zeta}, \tag{34}$$

expressed in the null coordinates (4).

Recall that a twistor is the pair $Z^a = \{\psi_\alpha, \mu^{\dot\alpha}\}$, where $\mu^{\dot\alpha} = x^\mu \bar\sigma_\mu \psi_\alpha$, and a projective twistor is

$$Z^a/\psi_1 = \{1, Y, \lambda_1, \lambda_2\}. \tag{35}$$

Therefore, the target function $Y(x)$ is a projective spinor coordinate $Y = \psi_2/\psi_1$, and function $F$ may be chosen as a homogenous function of $Z^a$.
For the Kerr solution in a general position with an arbitrary finite boost the generating function $F$ is to be quadratic in $Y$, [10, 20, 21],

$$F = A(Y - Y^+)(Y - Y^-) = AY^2 + BY + C, \tag{36}$$

where the coefficients are given in the null coordinates by the relations [21]

$$\begin{aligned} A &= (\bar\zeta - \bar\zeta_0)\dot v_0 - (v - v_0)\dot{\bar\zeta}_0; \\ B &= (u - u_0)\dot v_0 + (\zeta - \zeta_0)\dot{\bar\zeta}_0 - (\bar\zeta - \bar\zeta_0)\dot\zeta_0 - (v - v_0)\dot u_0; \\ C &= (\zeta - \zeta_0)\dot u_0 - (u - u_0)\dot\zeta_0. \end{aligned} \tag{37}$$

For the BH in rest frame one obtains

$$A = (x - iy)/2, \quad B = z + ia, \quad C = -(x + iy)/2\ . \tag{38}$$

Two roots of the equation $F = 0$ determine the function $Y(x)$,

$$Y^\pm = (-B \pm \Delta)/2A, \tag{39}$$

where $\Delta = (B^2 - 4AC)^{1/2}$. We obtain that

$$\Delta = \sqrt{x^2 + y^2 + (z + ia)^2} \equiv \tilde r \tag{40}$$

is the complex radial distance, and the roots $Y^\pm$ may be represented as

$$Y^\pm = (-B \pm \tilde r)/2A. \tag{41}$$

Ringlike singularity of the Kerr solution is a caustic of the Kerr congruence. In the KS formalism it is determined by the equation

$$\tilde r = dF/dY = 0. \tag{42}$$

# 6 Multisheeted twistor space of the multiparticle KS solutions.

The expression 36 gives the quadratic in $Y$ generating function of the Kerr theorem $F(Y, A, B, C)$ for an isolated KS particle. Denoting the set of parameters of motion and orientation of the particle as $q = (A, B, C)$, we shell write $F(Y|q)$. It was shown in [10] that taking the generating function of the Kerr theorem $F$ in the form of the product of partial functions for i-th particle

$$F = \prod_i F_i(Y|q_i), \tag{43}$$

where $q_i$ is the set of parameters of motion and orientation of particle i, one can obtain the multi-particle Kerr-Schild solutions of the Einstein-Maxwell system under the assumption that particles are stationarily moving along some trajectories.

The main equation of the Kerr theorem for twistorial structure (33) is satisfied by any partial solution $F_i(Y) = 0$. It means that the twistorial multi-particle space-time splits into sheets corresponding to different roots of the equation $F(Y) = 0$, similar to the sheets of a Riemann surface.

The twistorial structures on different sheets turn out to be independent and the twistorial structure of $i$-th particle "does not feel" the structure of particle $j$, forming a sort of its internal space. This property is a direct generalization of the considered above twosheetedness of the KN geometry.

Since the function $F(Y)$ for a single KN particle is quadratic in $Y$ [22, 20, 21], the equation $F_i(Y|q_i) = 0$ has two roots $Y_i^+$ and $Y_i^-$ corresponding to the positive ('out') and negative ('in') sheets. In terms of these roots one can express $F_i$ in the form [10]

$$F_i(Y) = A_i(x)(Y - Y_i^+)(Y - Y_i^-). \tag{44}$$

One sees that metric of a multi-particle solution depends on the solution $Y_i(x)$ on the considered sheet of i-th particle. Indeed, substituting the $(+)$ or $(-)$ roots $Y_i^{\pm}(x)$ in the relation (2), one determines

the Kerr congruence $k_\mu^{(i)}(x)$ and the corresponding function $h_i$ of the Kerr-Schild ansatz (1) on the i-th sheet

$$H_i = \frac{m}{2}\left(\frac{1}{\mu_i \tilde{r}_i} + \frac{1}{\mu_i^* \tilde{r}_i^*}\right) + \frac{(e/\mu_i)^2}{2|\tilde{r}_i|^2}. \tag{45}$$

The complex radial distance $\tilde{r}_i$ and function $\mu_i(Y)$ are also determined from the extended version of the Kerr theorem [10],

$$\tilde{r}_i = -d_Y F_i, \tag{46}$$

$$\mu_i(Y_i) = \prod_{j \neq i} A_j(x)(Y_i - Y_j^+)(Y_i - Y_j^-). \tag{47}$$

Contrary to the independence of twistorial structures for different particles, there is an interaction between them, since the function $\mu_i(Y)$ acquires the pole $\mu_i \sim A(x)(Y_i^+ - Y_j^-)$ on the twistor line which is common to the particles $i$ and $j$. The metric and electromagnetic field will be singular along the common twistor lines. For example, a light-like interaction occurs along the line which connects the *out* - sheet of particle $i$ to the *in* - sheet of particle $j$ , see fig.4.

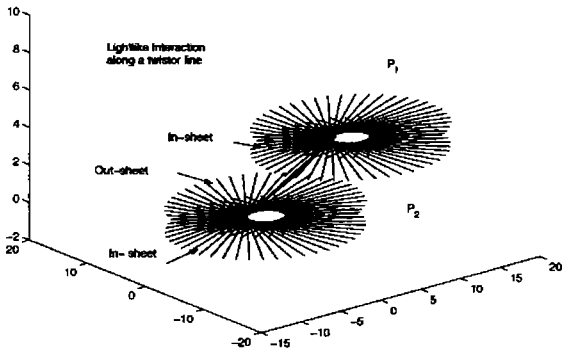



Figure 4: The lightlike interaction via a common twistor line connecting the out-sheet of one particle to the in-sheet of another.

An analysis of some simple cases shows that each particle has a pair of semi-infinite singular lines (the lightlike half-strings) which are caused by interaction with some external particle. The lightlike strings contain only the one-way modes. However, the similar modes

of opposite direction link the positive sheet of the second particle with the negative sheet of the first one. Therefore, the KS geometry displays a stringy analog of the interaction via the exchange of photons. As it was discussed in [12, 22, 16], such a pair of strings is related with the Dirac equation and turns out to be a carrier of de Broglie periodicity.

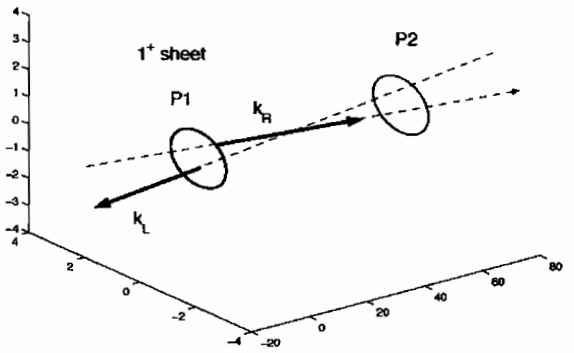



Figure 5: Two outgoing semi-infinite singular lines of the particle P1, caused by its interaction with external particle P2.

In the limit of infinitely many external particles, the selected Kerr particle will be connected by singular twistor lines with many other external particle, and singular twistor lines will have even and dense distribution among the twistor lines, covering the principal null congruence of the selected particle.

# 7 Conclusion

The beam-like structure of the obtained semi-classical nonstationary solutions reveals a classical fine-grained fluctuating geometry and a specific mechanism of evaporation provided by the structure of DKS equations. The obtained solutions show explicitly that both the functions, $\psi$ and $\gamma = \gamma_{(reg)}$, play their own specific role, and evaporation is reminiscent of a classical analog of the quantum tunnelling process. In particular, the outgoing radiation contains two components playing their own specific role:

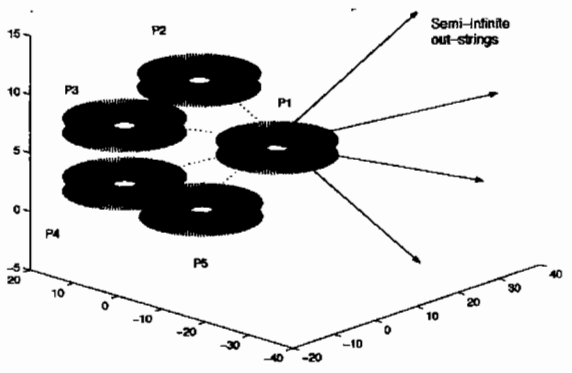



Figure 6: Formation of the outgoing semi-infinite singular lines by interaction via the common 'out-sheet'-'out-sheet' twistor lines.

a) the outgoing singular electromagnetic beams, determined by poles of function $\psi$, perforate the horizon, forming classical micro-holes which break impenetrability of the horizon with respect to out-going radiation.

b) regular component $\gamma_{(reg)}$ which is akin to the Vaidya 'shining star' radiation and, in the agreement with $\dot{m} = -\frac{1}{2}P^2\gamma_{(reg)}\bar{\gamma}_{(reg)}$, is responsible for mass evaporation.

Holographic interpretation of twosheetedness of the Kerr-Schild geometry allows the treatment of evaporation as a scattering and reveals important role of the 'negative' sheet, showing that the Kerr-Schild classical twosheetedness represents a classical progress toward the necessary holographic structure of quantum black-hole space-time suggested by Stephens, t' Hooft and Whiting in [8]. Such a holographic space-time has to be divided into two causally-related 'in' and 'out' regions joined by a 2+1 (shell-like) boundary which is holographically dual to the 'in' and 'out' regions, bulk of the Kerr geometry. Since the twistor rays of the Kerr congruence are the time oriented null lines, the Kerr-Schild alignment condition (13) plays specific role of time-ordering on the considered semi-classical level.

The old paradoxical Penrose remarks that the twistors are more fundamental then the points of spacetime acquires a confirmation in the KS geometry, so far as the twistor-beams form elementary excita-

tions and the basic elements of the time-oriented KS geometry. Principal advantage of the twistor structure with respect to the pointlike lattice structure is that the twistor vacuum states have the explicit time orientation and Lorentz invariance.

When this paper was practically finished, we detected one new important peculiarity the obtained exact EM KS solutions. One can see that the analytic in $Y$ function $P_i$, used for regularization of the poles in the function $\gamma$, has the pole at the point $Y = -1/\bar{Y}_i$, which is antipodal with respect to the regularized direction determined by $Y_i$. It means the tuned function (28) has two singularities corresponding to two mutually antipodal points. As a result, the function $\gamma_i^{(reg)}$ retains to be singular at the antipodal point $Y = -1/\bar{Y}_i$, and we obtain that the true regular solution has to be determined by a function $A$ containing two mutually antipodal poles. This new fact shows that the obtained in [4] solution $\gamma_{(reg)}$ of the eq. (22), has to be generalized to take into account the second pole. This phenomenon is not very essential for the presented here motivation based on the Kerr theorem and multiparticle spacetimes. However, it is very important for the model of electron based on the KN solution [22, 16, 12, 23], since it was obtained that solutions of the Dirac equation are related with two antipodal excitations of the KN background, [22, 16, 12]. The anipodally symmetrized solutions suggest an alternative explanation of the appearance of the dual bra-state, which we intend to consider elsewhere. It should also be noted that the twosheeted twistor-beam structure of the KS geometry can be considered as an illustration to the physical significance of the stated by Prof. R.M. Santilli ideas in his exotic isodual mathematics [13, 14].

## Acknowledgement

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## References


[1] A. Burinskii, Gravity Research Foundation Awards on the Essays in Gravitation, First Award 2009, to appear in Gen.Rel.Grav.

[2] A. Burinskii, E. Elizalde, S.R. Hildebrandt and G. Magli, Phys.Lett. **B 671** 486 (2009).

[3] A. Burinskii, *Twistor-Beam Excitations of Black-Holes and Pre-quantum Kerr-Schild Geometry.* Theor. Math. Phys. **163**(3) 782 (2010), [arXiv:0911.5499].

[4] A. Burinskii, *Fluctuating Twistor-Beam Solutions and Holographic Pre-Quantum Kerr-Schild Geometry.* J. Phys. Conf. Ser. **222**, 012044 (2010) [arXiv:1001.0332 [gr-qc]].

[5] A. Burinskii, E. Elizalde, S.R. Hildebrandt and G. Magli, Phys. Rev. **D74**, 021502(R) (2006).

[6] G.C. Debney, R.P. Kerr and A.Schild, J. Math. Phys. **10**, 1842 (1969).

[7] G. t' Hooft, *The Holographic Principle.*, arXiv:hep-th/0003004

[8] C.R. Stephens , G. t' Hooft and B.F. Whiting, *Class. Quant. Grav.* **11** 621 (1994).

[9] S. Chandrasekhar, *The Mathematical Theory of Black Holes,* v.2, Clarendon Press Oxford, 1983.

[10] A. Burinskii, Grav. Cosmol. **11**, 301 (2005), hep-th/0506006; IJGMMP, **4**, n.3, 437 (2007).

[11] D.Kramer, H.Stephani, E. Herlt, M.MacCallum, "Exact Solutions of Einstein's Field Equations", Cambridge Univ. Press, 1980.

[12] A. Burinskii Phys. Rev. **D 70**, 086006 (2004).

[13] R.M. Santilli, *Intern. J. Modern Phys.* **D 7** (1998) 351.

[14] R.M. Santilli, *Algebras, Groups and Geometries* **10** (1993) 273.

[15] Ch.W.Misner, K.S.Thorn, J.A.Wheeler, *Gravitation,* San Francisco, W.N.Freeman Co.,1973.

[16] A. Burinskii, Grav. Cosmol.**10**, (2004) 50.

[17] G.T. Horowitz, A.R. Steif, Phys.Rev.Lett.**64** (1990) 260; A.A. Coley, Phys.Rev.Lett **89** (2002) 281601.

[18] J.W. York,Jr., Phys. Rev. **D28**, 2929 (1983).

[19] Roy P. Kerr, Gen Relativ Gravit **41** 2479 (2009).

[20] A. Burinskii, *Phys. Rev.***D 67**, 124024 (2003).

[21] A. Burinskii and G. Magli, *Phys. Rev.* **D 61** 044017 (2000).

[22] A. Burinskii, *Grav. Cosmol.* **14** (2008) 109, arXiv:hep-th/0507109.

[23] A. Burinskii, *J. Phys. A: Math. Theor.* **43** (2010) 392001, arXiv: 1003.2928.

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
Kathmandu University, Nepal, April (2011) pages 135-162

# Thermodynamic Insight Of Irreversibility[1]

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**Abstract.** The development of thermodynamics of irreversible processes has encountered a few difficulties and even hurdles. This gets exemplified by the fact that the chemical thermodynamics developed by the Belgian school based on the differential form of Clausius' inequality failed to unambiguously impress upon that it, indeed, is a nonequilibrium thermodynamic description. The differential form of Clausius' inequality that is customarily used is ambiguous in that whether the entropy function appearing in it is that of an equilibrium state or of a nonequilibrium state. The fact that the second law of thermodynamics does not provide direct thermodynamic definition of entropy for a nonequilibrium state forced thermodynamicists to go for a postulatory approach in the form of adoption of local equilibrium assumption (LEA). The Classical Irreversible Thermodynamics (CIT) was developed with the help of LEA and then in the sequel it got invented that it is, indeed, a Linear Irreversible Thermodynamics (LIT). Of course, CIT/LIT enjoyed a good deal of success on its application side but no clear physical understanding of LEA came forward. On the other hand, CIT/LIT/LEA posed some basic questions and inconsistencies that we have described herein in some details and attempted to clarify them. It gets revealed that LEA is an ill conceived misconcept and CIT is inapplicable to the time evolution of a system. Hence, a fresh attempt to develop nonequilibrium thermodynamics is demanded. In order that no ambiguity should get cropped in while developing a nonequilibrium thermodynamic framework we felt it necessary to explain the "universe of operations of thermodynamics" which was first conjectured by Bridgman 5-6 decades ago. This universe gets demarcated by the laws of thermodynamics allowed shortest time duration of observations and smallest size of the system that we have further elaborated in this presentation. Next the generalized zeroth law of thermodynamics for nonequilibrium states is stated that legitimizes temperature function in nonequilibrium. Various forms of Clausius' inequality have been reanalyzed and by using the cyclic form of Clausius' inequality an entropy function for nonequilibrium states is established. In the sequel there evolves a derivation of differential form of Clausius' inequality that establishes that it does contain the entropy function of a nonequilibrium state. A representative example of a nonequilibrium thermodynamic framework namely the Generalized Phenomenological Irreversible Thermodynamic Theory (GPITT) is described that amusingly does not require LEA to develop it. In GPITT one is required to quantify the existing irreversibility by finding out an appropriate expression of Gibbs function in terms of the relevant system parameters. GPITT reveals that the thermodynamic irreversibility is all about the existence of imbalances in corresponding chemical interactions. For example, even the fluxes of heat, momentum and matter diffusion originate because of the said imbalances in corresponding chemical interactions. The GPITT based Gibbs relation resembles with the extended Gibbs relation of Extended Irreversible Thermodynamics (EIT) but at no stage of the development of GPITT LEA/beyond LEA prescriptions gets evolve, recall that EIT is being claimed to be a beyond LEA thermodynamic description. GPITT also amicably incorporates the open system features and handles complex systems consisting of macromolecules via internal configurational parameter and a quantity describing rate of corresponding irreversibility. The implications, of the thermodynamic development of this presentation on statistically defined entropy has been described both for within and out side of the "universe of operations of thermodynamics". In view of the existence of several nonequilibrium thermodynamic frameworks in literature and no two of them seemingly converge, it is hoped that this situation might be overcome by coupling of the Lie admissibility of irreversibility, laws of thermodynamics and if required to arrive at a new definition of the thermodynamic entropy function both for equilibrium and nonequilibrium states.

**Keywords:** Thermodynamics, Non-equilibrium Thermodynamics, Irreversibility, Kinetic Theory.
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## INTRODUCTION

The natural events in the universe are irreversible, that is they posses time's arrow. To understand this striking behaviour of natural processes now a days a new apparatus named, Lie admissible treatment of irreversibility, is being used [see for example: 1–3] in various scientific disciplines with fruitful results. Therefore, it would be interesting to see to what extent this new apparatus would help in strengthening and streamlining the subject of nonequilibrium thermodynamics. The present scenario in the field of nonequilibrium thermodynamics is that no two thermodynamic schools even seemingly converge in their approaches, thermodynamic understanding and the so-called thermodynamic frameworks developed by them. However, before the formal initiation of the task to develop nonequilibrium thermodynamics using this new mathematical apparatus, the present author feels that it is essential to first elaborate in details the existing status of understanding of various fundamental thermodynamic aspects in general and nonequilibrium thermodynamics in particular and as far as possible clarify the long pending ambiguities. We wish that while attempting to reframe nonequilibrium thermodynamics using new mathematics we should begin with a full and proper understanding of basic thermodynamic aspects. In this presentation, at certain places, the subject matter covered, on the face of it, would appear as a text book level material but for the sake of self sufficiency and to reveal the gravity of the ambiguities carried thereby we have included that part of the subject matter. This exercise as stated above would help us in starting with as clean a slate as possible to frame nonequilibrium thermodynamics using new mathematics. However, this latter aspect is not a part of this presentation as it is the subject matter of our further investigations.

Thus in this presentation we have opted to describe the development of nonequilibrium thermodynamics as time progressed. However, we would not include in our discussion various nonequilibrium thermodynamic frameworks existing in literature, however, only a few of them would make inroads. In doing so the basic flaws that remained imbedded in the course of development of nonequilibrium thermodynamics would also be spelled out and the solutions of some of them would be presented of course without a recourse to the new mathematics. Moreover, a representative example of how to proceed in developing a nonequilibrium thermodynamic framework starting from the second law of thermodynamics has been described. In the end we have also briefly discussed the thermodynamics of complex systems in nonequilibrium and implications of thermodynamic considerations on the statistical thermodynamics of a system in nonequilibrium followed by the concluding remarks.

## THE INITIAL THERMODYNAMIC DESCRIPTION OF IRREVERSIBILITY

Thus when one enquires about a thermodynamic description of irreversibility the first formal descriptions are in terms of the efficiency of a device and the Clausius inequality. The Carnot Theorem [4] states that *no device can be more efficient than the reversible Carnot engine*, mathematically it is stated as,

$$\eta_{rev} > \eta_{irr} \tag{1}$$

where $\eta$ quantifies the efficiency of the device. Based on equation (1) the involved irreversibility would get quantified as,

$$\eta_{rev} - \eta_{irr} = \mathscr{I} > 0 \tag{2}$$

where $\mathscr{I}$ is the quantitative measure of irreversibility. On the other hand, Clausius' inequality [5] reads as,

$$\oint_{irr} \frac{dQ}{T_R} < 0 \qquad \text{(closed systems)} \tag{3}$$

where $dQ$ is the differential amount of heat exchanged with the heat reservoir of temperature $T_R$ by the device in a cyclic operation. Now let us recall that, at the initial stages of development an attention was centered around realizing a maximum amount of useful work that demanded minimization of the existing irreversibility. In doing so a little attention was paid to a thermodynamic quantification of the existing irreversibility. As time progressed it was felt that one should go for the thermodynamic quantification of irreversibility which would also serve in developing a thermodynamics of irreversible processes. However, the development of thermodynamics of irreversible processes in itself offered a multitude of formidable challenges that needed to be overcome. Let us now recall the said challenges, they are:

1. To arrive at the entropy function for nonequilibrium states, say starting from the Clausius inequality, equation (3).

2. When the existing irreversibility is on account of the existence of spatial non-uniformity within the system, it immediately forces one to go for the local level description as is the practice in fluid-dynamics.
3. As soon as one goes to the local level description the open system features get associated to each conceptual local tiny pocket.
4. The inequality of equation (3) is a statement for a global closed system. This then would perhaps demand the generation of corresponding version of the Clausius inequality at the local level.
5. When a system under consideration is spatially uniform still there can exist an irreversibility. It would be due to the occurrence of chemical conversions proceeding at finite rates.

In view of the above facts in next section we choose to describe the steps involved in the chemical thermodynamics developed by the Belgian school, which probably is the first nonequilibrium thermodynamic description, wherein the second law of thermodynamics in the form of Clausius' inequality (the differential form) has been used to start with.

## IRREVERSIBILITY IN SPATIALLY UNIFORM SYSTEMS

The first attempt of thermodynamic quantification of irreversibility has been described in the monograph by, Prigogine and Defay [6]. They have used the so-called differential form of Clausius' inequality, namely:

$$dS > \frac{dQ}{T}. \tag{4}$$

But it seems that no one was aware that the preceding inequality was never derived by Clausius himself, say from equation (3). He simply stated it about one decade after his derivation of equation (3) [7]. The crucial query herein is that whether $S$ in equation (4) is that for a nonequilibrium state or for an equilibrium state? Nevertheless, Meixner, indeed, conjectured that it has to be that for the transient nonequilibrium states through which the system passes during its time evolution.

However, till that date no method was evolved to arrive at the entropy of a transient state based on the second law of thermodynamics. Still the chemical thermodynamics developed by the Belgian school for a spatially uniform system (indeed, it is the nonequilibrium chemical thermodynamics) has no flaws other than the one stated above. The method that they adopted involves the introduction of Clausius uncompensated heat, $dQ'$, [5, 6] such that equation (4) gets transformed to an equality as follows,

$$dS = \frac{dQ + dQ'}{T}. \tag{5}$$

Hence, an yet another second law of thermodynamics dictate is obtained as,

$$dQ' \geqslant 0. \tag{6}$$

Next, as the irreversibility that exists has been assumed to be that on account of chemical conversions at finite rates, $dQ'$ needs to be quantified through the parameters that measures the progress of chemical reactions. We recall that with the progress of chemical reaction the mole numbers of reactants and products change at a finite rate. This change gets described by Dalton's law of constant proportions that mathematically reads as [6],

$$dn_k = \nu_k d\xi \tag{7}$$

in the case of a single chemical reaction occurring within the system, say:

$$\nu'_{\mathrm{a}}\mathrm{A} + \nu'_{\mathrm{b}}\mathrm{B} + \nu'_{\mathrm{c}}\mathrm{C} + \cdots \longrightarrow \nu_{\mathrm{m}}\mathrm{M} + \nu_{\mathrm{n}}\mathrm{N} + \nu_{\mathrm{o}}\mathrm{O} + \cdots \tag{8}$$

By convention the stoichiometric numbers $\nu_k$'s are taken positive for products and negative for reactants. Therefore, in the case of reactants we have $\nu_{\mathrm{a}} = -\nu'_{\mathrm{a}}$, $\nu_{\mathrm{b}} = -\nu'_{\mathrm{b}}$ and so on. Notice that equation (7) provides us a single parameter $\xi$ and its change uniquely determines the corresponding changes in mole numbers of all reactants and products. Hence, $\xi$ is termed as the degree of advancement of the chemical reaction. Therefore, $dQ'/dt$ is taken proportional to the rate of reaction, $d\xi/dt$, that offers us,

$$\frac{dQ'}{dt} \propto \frac{d\xi}{dt} \quad \Rightarrow \quad \frac{dQ'}{dt} = \mathscr{A}\frac{d\xi}{dt} > 0 \tag{9}$$

where the proportionality constant $\mathscr{A}$ gets termed as the chemical affinity of the chemical reaction under consideration.

Notice that from equation (9) we obtain following thermodynamic deductions, namely:

$$\begin{aligned}
&\mathscr{A} > 0, \quad \frac{d\xi}{dt} > 0, && \text{Forward reaction.}\\
&\mathscr{A} < 0, \quad \frac{d\xi}{dt} < 0, && \text{Reverse reaction.}\\
&\mathscr{A} = 0, \quad d\xi = 0, && \text{Equilibrium.}\\
&\mathscr{A} = 0, \quad d\xi \neq 0, && \text{the system is carried reversibly.}\\
&\mathscr{A} \neq 0, \quad \frac{d\xi}{dt} = 0, && \text{Energy of activation of the chemical reaction}\\
& && \text{is much higher or a metastable state.}
\end{aligned}$$

In fact, the last two assertions stated above were not spelled out by the Belgian school.

When there takes place more than one chemical reaction (which is the case in reality) equation (7) gets expressed as,

$$dn_k = \sum_\gamma \nu_k^\gamma d\xi^\gamma \tag{10}$$

where $\nu_k^\gamma$ is the stoichiometric number of the component $k$ in $\gamma$-th chemical reaction and $\xi^\gamma$ is the degree of advancement of the $\gamma$-th chemical reaction. This then transforms equation (9) to,

$$\frac{dQ'}{dt} = \sum_\gamma \mathscr{A}^\gamma \frac{d\xi^\gamma}{dt} > 0. \tag{11}$$

Moreover, equation (5) is customarily expressed as,

$$dS = d_eS + d_iS, \quad d_eS = \frac{dQ}{T}, \quad d_iS = \frac{dQ'}{T}. \tag{12}$$

Thus $d_eS$ is obtained as the exchange of entropy by the system with its environment which may be positive or negative and $d_iS$ is the entropy production within the system due to existing irreversibility which is obtained as a positive definite quantity. Hence it is asserted that the entropy of the system can change on **two** and **only two** counts, one by the mechanism of **exchange of entropy** by the system with its thermally interacting surroundings and the other by the **creation of it** within the system. Thus the entropy can only be created but it cannot be destroyed, indeed, it can be transported in or out of the system. Hence, entropy is a non-conserved quantity.

Of course, this is not the place to describe the entire work of the Belgian school on chemical thermodynamics hence the reader is advised to refer the monograph cited as reference 6 of the bibliography. However, in the following two subsections we describe how powerful assertions emerge from the above framework.

## Coupled Reactions

Let us consider that in a closed system two chemical reactions are proceeding at finite rates. In this case the rate of entropy production from equations (11) and (12) is obtained as:

$$T\frac{d_iS}{dt} = \mathscr{A}_1\frac{d\xi_1}{dt} + \mathscr{A}_2\frac{d\xi_2}{dt} > 0 \tag{13}$$

which means that the second law of thermodynamics requires that:

$$\frac{d\xi_2}{dt} > -\left(\frac{\mathscr{A}_1}{\mathscr{A}_2}\right)\frac{d\xi_1}{dt}. \tag{14}$$

As we have seen that in the case of a single chemical reaction the reaction will proceed in the forward direction ($d\xi/dt > 0$) only if $\mathscr{A} > 0$, hence one would expect that in the present case too we must have $\mathscr{A}_1 > 0$ & $d\xi_1/dt > 0$

and $\mathscr{A}_2 > 0$ & $d\xi_2/dt > 0$. However, without violating the second law of thermodynamics dictate of equation (13) the system is, indeed, allowed to behave in such a way that even if $\mathscr{A}_1 > 0$, $d\xi_1/dt > 0$ and $\mathscr{A}_2 < 0$ still this second reaction can proceed on its will in the forward direction that is the observed rate of chemical reaction 2 in actual can be $d\xi_2/dt > 0$ instead of $d\xi_2/dt < 0$ provided equation (14) is not flouted. When it happens the chemical reaction 1 is called the coupling reaction and the reaction 2 is the coupled one. Thus the coupled chemical reaction consumes entropy at the expense of the coupling chemical reaction, which produces entropy.

**Examples:**

Let us consider two biochemical cases [6, 8], namely:
(1) The case of biological burning of sugar that couples the production of urea:

$$\frac{1}{6}\,C_6H_{12}O_6 + O_2 \longrightarrow CO_2 + H_2O \qquad \mathscr{A}_1 = 115.0\,\text{kcal/mol}$$

$$2\,NH_3 + CO_2 \longrightarrow (NH_2)_2CO + H_2O \qquad \mathscr{A}_2 = -11.0\,\text{kcal/mol}$$

$$\therefore \frac{d\xi_2}{dt} > \frac{115}{11} \times \frac{d\xi_1}{dt} \approx 10 \times \frac{d\xi_1}{dt}.$$

(2) The case of biological hydrolysis of Creatine phosphate that couples the ATP (the energy store house of biological systems) synthesis:

$$\text{Creatine Phosphate} + H_2O \longrightarrow \text{Creatine} + P \qquad \mathscr{A}_1 = 43.5\,\text{kJ/mol}$$

$$ADP + P \longrightarrow ATP + H_2O \qquad \mathscr{A}_2 = -39.8\,\text{kJ/mol}$$

$$\therefore \frac{d\xi_2}{dt} > \frac{43.5}{39.8} \times \frac{d\xi_1}{dt} \approx 1.1 \times \frac{d\xi_1}{dt}.$$

The above two representative examples of urea production and the ATP synthesis are individually thermodynamically unfavourable but nonequilibrium thermodynamics explains how and why in biological systems their synthesis is unbelievably efficient.

## Ordered Irreversible Biological Evolution

Now recall that the evolution of a biological system is an ordered irreversible process. In view of the discussion of preceding subsection it must be due to the coupling of a host of biochemical and biophysical processes. Mathematically speaking, each negative entropy producing biological process that occurs remains coupled with its complementary entropy producing process. The latter maintains the irreversibility as it dominates over the former (in maintaining the dictates of the second law of thermodynamics) whereas the former tries to produce an order (as the decrease in entropy tries to increase the order within the system) during evolution, whose rate remains much higher than that of the former.

When this coupling totally breaks down (reasons can be very many) the biological system marches towards death. That is, we have two types of components that comprise the overall entropy production rate in a biological system, namely:

$$\frac{d_iS}{dt} = \sum_{\alpha}\left(\frac{d_iS}{dt}\right)^{\alpha} + \sum_{\beta}\left(\frac{d_iS}{dt}\right)^{\beta} > 0 \tag{15}$$

with say:

$$\left(\frac{d_iS}{dt}\right)^{\alpha} > 0 \tag{16}$$

and

$$\left(\frac{d_iS}{dt}\right)^{\beta} < 0. \tag{17}$$

The entropy production rate of equation (16) dominates in magnitude over the negative entropy production rate described in equation (17) within a biological system. This discussion reveals that the key of ordered biological evolution is the occurrence of coupled processes.

Indeed, the above deductions hold good but there still remains a basic flaw regarding the origin of the involved entropy function in the above description. In order to clarify this crucial and basic query we need to dwell a little bit on the various forms of Clausius' inequality. This aspect we resolve in the following section.

## UNDERSTANDING OF CLAUSIUS' INEQUALITY

Let us now examine the implications of Clausius' inequality, equation (3), including its other forms [9]. To elaborate the implications of Clausius' inequality let us consider that a given system goes irreversibly from an equilibrium state A to another equilibrium state B and then it is carried reversibly back to A. Thus the cyclic integral of equation (3) gets expressed as,

$$\oint_{irr} \frac{dQ}{T_R} = \int_{irr}^{B}_{A} \frac{dQ}{T_R} + \int_{rev}^{A}_{B} \frac{dQ}{T} < 0. \tag{18}$$

Notice that instead of $T_R$ we have used $T$ on the reversible segment of the cyclic operation. It is so because under reversibility we rigorously have $T_R = T_{sys} = T$. Now the standard manipulations of equation (18) gives,

$$\int_{irr}^{B}_{A} \frac{dQ}{T_R} < -\int_{rev}^{A}_{B} \frac{dQ}{T} = \int_{rev}^{B}_{A} \frac{dQ}{T} = \int_{rev}^{B}_{A} dS. \tag{19}$$

Again notice that we have used the Clausius definition of entropy of an equilibrium state in equation (19) namely, $dS = dQ_{rev}/T$ [4, 5].

Next on integrating the last integral on the right hand side of equation (19) we obtain the integrated form of Clausius' inequality, namely:

$$\Delta S > \int_{irr}^{B}_{A} \frac{dQ}{T_R} \tag{20}$$

where $\Delta S$ stands for the difference in entropies of end equilibrium states, namely:

$$\Delta S = S_B - S_A. \tag{21}$$

Notice that on going from the equilibrium state A to an another equilibrium state B the entropy change of the system remains the same whether the system is carried reversibly or it went irreversibly, that is,

$$S_B - S_A = (\Delta S)_{rev}(A \to B) = (\Delta S)_{irr}(A \to B) \tag{22}$$

but it needs to be kept in mind that one cannot integrate to write,

$$(\Delta S)_{irr} = \int_{irr}^{B}_{A} dS \tag{23}$$

because we have not yet defined or identified an entropy function on real trajectories, that is for a nonequilibrium state. Therefore, one should not confuse $(\Delta S)_{irr}$ with $(\Delta S)_{irr}(A \to B)$ because the latter is not the result of integration along an irreversible path. Moreover, as pointed out by Meixner [7] the much used differential form of Clausius' inequality, namely:

$$dS > \frac{dQ}{T} \qquad \text{(irreversible change)} \tag{24}$$

was never derived by Clausius himself from equation (3) and hence equation (24) is not simply a differential form of equation (3). Meixner recalls that after a gap of about a decade from the first derivation of equation (3) Clausius merely stated equation (4) or precisely equation (24) without any derivation. This point has been again scrutinized very recently [9] which reveals and strengthens Meixner's assertion. This can be illustrated as follows. Let the equilibrium states A and B are such that their entropies are related as,

$$S_B = S_A + dS \tag{25}$$

that is the two equilibrium states are infinitesimally away from each other. Now substitute equation (25) into equation (20) and use equation (21) that gives,

$$dS > \int_{irr}^{B}_{A} \frac{dQ}{T_R} \tag{26}$$

and not equation (4). Alternatively, retain only the first and the last integrals of equation (19) which on rearranging reads as,

$$\int_{A}^{B}_{rev} dS > \int_{A}^{B}_{irr} \frac{dQ}{T_R}. \tag{27}$$

Now as an irreversible trajectory between given two equilibrium states cannot be coincided with any of the reversible paths between them the symbol of integration on both the sides of equation (27) cannot be dropped out simultaneously to produce,

$$dS > \frac{dQ}{T_R} \qquad \text{(irreversible change)} \tag{28}$$

and hence it cannot then be transformed to equation (4) on asserting the condition of uniformity of the temperature within and across the boundaries of the system. Thus it is surprising that without realizing this serious flaw almost all thermodynamic texts and textbooks of physical chemistry base their discussions on equation (4) but are totally unaware of propagating a wrong message.

Thus from the above discussion it is clear that one should first establish an entropy function for nonequilibrium states and then proceed to quantify the existing irreversibility. Hence, we proceed on these lines and the resulting nonequilibrium thermodynamic description of spatially uniform systems, indeed, is expected to be an identical one as that has been developed by the Belgian school described above.

## NONEQUILIBRIUM AND THE ENTROPY FUNCTION

The simplest nonequilibrium situation is that of a spatially uniform closed system having no gradients in intensive parameters even across its boundaries but the seat of irreversibility lies well within it, namely due to chemical conversions at finite rates. Thus the operating conditions are,

$$\begin{aligned} \Delta T &= \Delta p = 0 \quad \text{(across the boundaries)} \\ \nabla T &= \nabla p = \nabla c_k = 0 \quad \text{(within the system)} \end{aligned} \tag{29}$$

where $T$ and $p$ are the temperature and pressure and $c_k$ is the concentration of the component $k$. This means that within the system and across its boundaries no heat, momentum and diffusion fluxes exist. It is further assumed that the interacting surroundings are manipulated so efficiently that these fluxes do not get generated during a cyclic operation. That is, throughout the cyclic operation the system remains spatially uniform and hence $T_R = T_{sys} = T$ can be safely assumed. Therefore, the operative form of Clausius' inequality, equation (24), in the present case reads as,

$$\oint_{irr} \frac{dQ}{T} < 0 \qquad \text{(closed systems)}. \tag{30}$$

Now we proceed to establish the entropy function for a spatially uniform system in nonequilibrium.

### Generation Of Entropy Function For Spatially Uniform Systems In Nonequilibrium

Herein it is necessary to first explain why we term the given transient condition as a nonequilibrium state. To grasp it let us consider the following chemical reaction, namely:

$$A + B \underset{k_r}{\overset{k_f}{\rightleftharpoons}} C + D$$

where $A$ and $B$ are the reactants, $C$ and $D$ are the products, and $k_f$ and $k_r$ are the forward and reverse rate constants. At the chemical equilibrium under a given condition of say $T$ and $p$ we have, $K = k_f/k_r$ [10], where $K$ is termed as the chemical equilibrium constant. When the imposed conditions that of $T$ and $p$ are varied the values of $K$ as well as $k_f$ and $k_r$ also vary and hence the value of $K$ uniquely determines the existing equilibrium state. On the other hand, at a given instant of time the existing nonequilibrium gets uniquely prescribed [10] by,

$$-\frac{dC_A}{dt} = k_f C_A C_B - k_r C_C C_D$$

where $C_k$'s are the respective concentrations at the given time $t$. Thus, we can replicate exactly the given nonequilibrium condition by assigning $T$, $p$ and $C_k$'s. Recall that $C_k$'s are accurately measurable even in a dynamic system. Hence, it is legitimate to term the transient nonequilibrium condition at a given time as the nonequilibrium state.

Now on following mathematical manipulations adopted by Clausius [5], Eu [11, 12] and Eu and García-Colín [13] we define the uncompensation function, $N$, as,

$$N = -\oint_{irr} \frac{dQ}{T} > 0 \qquad \text{(closed systems)}. \tag{31}$$

Thus we obtain the following dictate of the second law of thermodynamics, namely:

$$N \geqslant 0. \tag{32}$$

That is $N = 0$ is the equation of a reversible cycle. Now we treat $N$ as an independent quantity [13] and write,

$$N = \oint_{irr} dN > 0 \qquad \text{(closed systems)}. \tag{33}$$

Again as uncompensation function appears only on account of irreversibility, $dN$ cannot be negative and should vanish on each reversible segment of the cycle [9, 11], therefore the dictates of the second law of thermodynamics read as,

$$N \geqslant 0, \qquad dN \geqslant 0. \tag{34}$$

Now on combining equations (31) and (33) we obtain,

$$\oint_{irr} \left( \frac{dQ}{T} + dN \right) = 0 \qquad \text{(closed systems)}. \tag{35}$$

Notice that in equation (35) $dQ/T$ and $dN$ belong to the same segment at each stage of the cyclic operation. As the cyclic integral of equation (35) vanishes its integrand is obtained as an exact differential, which we write as,

$$dS = \frac{dQ}{T} + dN \tag{36}$$

or in the explicit time rate form it reads as,

$$\frac{dS(t)}{dt} = \frac{1}{T(t)} \frac{dQ}{dt}(t) + \frac{dN}{dt}(t). \tag{37}$$

Obviously, we have the following inequality, namely:

$$\frac{dN}{dt}(t) > 0. \tag{38}$$

Let us introduce Clausius' uncompensated heat, $dQ'$, as,

$$\frac{dN}{dt}(t) = \frac{1}{T(t)} \frac{dQ'}{dt}(t) > 0 \tag{39}$$

and as $T(t) > 0$ we have from equation (39),

$$\frac{dQ'}{dt}(t) > 0. \tag{40}$$

This development now allows us to produce the differential form of Clausius' inequality and clarify the principle of monotonic increase of entropy. That we describe in next two subsections.

## Clausius' Inequality In Differential Form

In view of equations (38) and (39) we obtain a dynamic differential form of Clausius' inequality from equation (37) as,

$$\frac{dS(t)}{dt} > \frac{1}{T(t)}\frac{dQ}{dt}(t). \tag{41}$$

Thus we see that the correct form of the differential form of Clausius' inequality is the one depicted in equation (41) wherein the time dependent entropy function appears. Further, we need to remember that to arrive at this inequality we had to first establish an entropy function for a nonequilibrium states.

On the other hand, notice that the Clausius inequality depicted in equations (19) and (20) contain the entropy function of equilibrium states and in equation (41) $S(t)$ is that for a nonequilibrium state. Therefore, the so-called Clausius' inequality of equation (4) remains ambiguous as it seemingly appears to be the differential version of equation (20), the integrated version of Clausius' inequality, and hence gives an impression that it is consisting of entropy function of equilibrium states.

Moreover, equation (37) on using equation (39) reads explicitly as:

$$\frac{dS(t)}{dt} = \frac{1}{T(t)}\frac{dQ}{dt}(t) + \frac{1}{T(t)}\frac{dQ'}{dt}(t) \tag{42}$$

which resembles with equation (5) and hence it is no wonder that the entire chemical thermodynamics developed by the Belgian school would get reproduced from equation (42), of course, without any doubt as a nonequilibrium thermodynamic framework but now on the sound base laid down by the second law of thermodynamics.

## Principle Of Monotonic Increase Of Entropy

Recall the integrated form of Clausius' inequality, equation (20), which on imposing adiabatic condition, $dQ = 0$, produces,

$$\Delta S \geqslant 0, \tag{43}$$

which emphasizes that the entropy of the final equilibrium state will be always higher than that of the initial equilibrium state if the transition is irreversible adiabatic. However, it says nothing about the rate of entropy variation during the adiabatic evolution of the system.

On the other hand, the time rate differential form of Clausius inequality, equation (41), under the condition of adiabaticity correctly produces the principle of monotonic increase of entropy, namely:

$$\frac{dS(t)}{dt} \geqslant 0 \tag{44}$$

in terms of the entropy function of nonequilibrium states. Notice that equation (43) does not ensure the monotonicity in the increase of entropy whereas equation (44) transparently describes that the successive nonequilibrium states in adiabatic evolution of a system are of higher entropy than that of the preceding ones.

# CLASSICAL IRREVERSIBLE THERMODYNAMICS (CIT)

Though we have already described in section entitled irreversibility in spatially uniform systems how Belgian school has developed chemical thermodynamics framework (which, indeed, is one particular version of Classical Irreversible Thermodynamics (CIT)) produced by using the so-called differential form of Clausius' inequality (from the above discussion now we for the definite know that $S$ in it is that for a nonequilibrium state) it is worth while to discuss CIT in its more general setup before proceeding to directly establish an entropy function for spatially non-uniform systems on the lines described in the preceding section. We also take this opportunity to reveal incoherencies carried by CIT.

We recall that the CIT [14–18] for the first time attempted to legitimize nonequilibrium states and seemingly brought them into the fold of thermodynamics that appears to have helped in removing the apprehensions about the thermodynamic status of nonequilibrium states. The backbone of CIT is the Local Equilibrium Assumption (LEA), which for a spatially non-uniform system envisages the existence of a local equilibrium at every point within the

system. In other words, when LEA operates within a system globally in nonequilibrium its every interior tiny volume element may be approximated as if they are in some sort of equilibrium. It is believed that this assumption allows one to accept,

- entropy function for such nonequilibrium states.
- temperature function for such nonequilibrium states.
- the same functional dependencies of the so accepted entropy function on the system quantities as that has been established in equilibrium thermodynamics.
- all state functions of such nonequilibrium states have the time and position dependencies whereas when the system happens to be in a nonequilibrium stationary state (NSS) thermodynamic quantities are only position dependent.

Mathematically, LEA gets expressed as [14–21],

$$s(\mathbf{r},t) = s(u(\mathbf{r},t), v(\mathbf{r},t), x_1(\mathbf{r},t), x_2(\mathbf{r},t), ...) \tag{45}$$

where $s(\mathbf{r},t)$ and $u(\mathbf{r},t)$ are the per unit mass entropy and internal energy respectively, $v(\mathbf{r},t)$ is the specific volume, $\mathbf{r}$ is the position vector, $x_1(\mathbf{r},t), x_2(\mathbf{r},t), ...$ are the mass fractions of the components 1, 2, ..., and $t$ is time. This then seemingly opens for us a gateway to use as such the Gibbs relation at the local level in time rate form, namely:

$$\frac{ds}{dt} = T^{-1}\frac{du}{dt} + T^{-1}p\frac{dv}{dt} - T^{-1}\sum_k \mu_k \frac{dx_k}{dt} \tag{46}$$

where $\mu_k$ is the chemical potential per unit mass of the component $k$. For the sake of simplicity the explicit position and time dependencies have not been shown but is implied from here onwards. Next, one uses the fluid dynamical internal energy balance equation, which in a simple situation reads as,

$$\rho\frac{du}{dt} = -\nabla\cdot\mathbf{q} - \rho p\frac{dv}{dt} + \Pi : \nabla\mathbf{u} \tag{47}$$

where $\rho$ is the local mass density, $p$ is the local pressure, $\mathbf{q}$ is the heat flux density, $\Pi$ is the dissipative stress tensor (the momentum flux density) and $\mathbf{u}$ is the barycentric velocity. Now on combining equation (47) with equation (46) the following entropy balance equation is obtained, namely:

$$\rho\frac{ds}{dt} + \nabla\cdot\mathbf{J}_s = \sigma_s \tag{48}$$

where $\mathbf{J}_s$ is the entropy flux density and $\sigma_s$ is the entropy source strength and their expressions are obtained as,

$$\mathbf{J}_s = \frac{\mathbf{q}}{T} \tag{49}$$

$$\sigma_s = \mathbf{q}\cdot\nabla\left(\frac{1}{T}\right) + T^{-1}\Pi : \nabla\mathbf{u} + \sum_\gamma \frac{\mathscr{A}^\gamma}{T}\frac{d\xi^\gamma}{dt} \tag{50}$$

where the chemical affinity, $\mathscr{A}^\gamma$, of $\gamma$-th chemical reaction reads as,

$$\mathscr{A}^\gamma = -\sum_k \mu_k \nu_k^\gamma \tag{51}$$

and from Dalton's law [6] the advancement of the $\gamma$-th chemical reaction, $\xi^\gamma$, is given by,

$$dx_k = \sum_\gamma \nu_k^\gamma d\xi^\gamma \tag{52}$$

where $\nu_k^\gamma$ is the stoichiometric coefficient of the component $k$ in the $\gamma$-th chemical reaction and by convention it is taken positive for products and negative for the reactants and $\xi^\gamma$ is the mass fraction based degree of advancement of the $\gamma$-th chemical reaction.

Further, each term on the right hand side of equation (50) can be shown as a positive definite contribution and hence in the case under consideration equation (48) is nothing else but the well known Clausius-Duhem inequality [22], namely:

$$\rho\frac{ds}{dt}+\nabla\cdot\mathbf{J}_s=\sigma_s\geqslant 0. \tag{53}$$

In CIT $\mathbf{q}$, $\Pi$ and $d\xi^\gamma/dt$ are termed as thermodynamic fluxes ($J_a$'s) and $\nabla(1/T)$, $\nabla\mathbf{u}$ and $\mathscr{A}^\gamma/T$ as thermodynamic forces ($X_a$'s). In general $\sigma_s$ is composed of three kinds of terms namely scalar, vectorial and tensorial ones and each kind is composed of more than one term depending upon the existing sources of irreversibility. However, it is obvious that the fluxes may depend on the magnitudes of the forces of their kind. This in general is expressed in a Taylor expansion as,

$$J_a=\sum_b\frac{\partial J_a}{\partial X_b}X_b+\frac{1}{2}\sum_{b,c}\frac{\partial^2 J_a}{\partial X_b\partial X_c}X_bX_c+\cdots\cdots \tag{54}$$

Now as LEA has been supposed to operate it is obvious to conjecture that it can be realized only when the given system happens to be close to equilibrium. Therefore, the second and higher order terms in equation (54) would contribute a little to the sum and hence are customarily ignored. Therefore, the following linear relations oprtate, namely:

$$J_a=\sum_b L_{ab}X_b,\qquad L_{ab}=\frac{\partial J_a}{\partial X_b}. \tag{55}$$

In the view of operation of these linear relationships CIT is also termed as linear irreversible thermodynamics (LIT). Thus the expression of entropy source strength, $\sigma_s$, gets simplified to a quadratic bilinear form, namely:

$$\sigma_s=\sum_{a,b}L_{ab}X_aX_b\geqslant 0. \tag{56}$$

A further simplification is obtained as it has been shown that the Onsager reciprocal relations, namely:

$$L_{ab}=L_{ba} \tag{57}$$

are followed too [23, 24] and hence equation (56) further simplifies to,

$$\sigma_s=\sum_a L_{aa}X_a^2+2\sum_{\substack{b,c\\ b\neq c}}L_{bc}X_bX_c\geqslant 0. \tag{58}$$

Indeed, we are not required to describe herein the entire development of CIT the interested reader is advised to read the corresponding references cited in the bibliography.

## LEA = Close To Equilibrium !

Now we pose a question. Is it true that LEA operates only when system happens to be close to equilibrium? However, on going through the literature we observe that:

1. In the case of spatially non-uniform systems in a nonequilibrium stationary state the above CIT/LIT description fits very well and it has been successfully subjected to experimental verifications. Amongst such examples are thermodiffusion, thermoelectric effects, thermo-mechanical effects and the Saxen's relations. In such phenomena validity of equation (58) has been verified [14–21].
2. However, the irreversibility on account of chemical reactions at finite rates has a different story to tell. In a spatially uniform closed systems the entropy production due to chemical reactions at finite rate, $d_iS/dt$, from equation (50), is given at global level by,

$$\frac{d_iS}{dt}=\sum_\gamma\frac{\mathscr{A}^\gamma}{T}\frac{d\xi^\gamma}{dt}\geqslant 0. \tag{59}$$

3. Though in deriving equation (50) the LEA has been used but equation (59) generated from the former has been successfully applied to various far away from equilibrium situations. The systems in nonequilibrium that get covered by equation (59), for example, are right from the initiation to the approach to equilibrium stages of chemical reactions, chemical oscillations, chemical chaos, etc. In a no stretch of imagination these processes can be considered as falling within the close to equilibrium category.

A simple demonstration of the validity of equation (59) right from farthest away from equilibrium to close to chemical equilibrium is as follows. Let us consider a spatially uniform system in which a single chemical reaction is proceeding at a finite rate. In this case equation (59) reduces to,

$$\frac{d_i S}{dt} = \frac{\mathscr{A}}{T}\frac{d\xi}{dt} \geqslant 0. \tag{60}$$

Now it can be easily shown that an exponential relation exists between the rate of reaction, $d\xi/dt$, and its thermodynamic driving force, $\mathscr{A}/T$, which reads as,

$$\frac{d\xi}{dt} = \mathbb{R}_f\left(1 - e^{-\mathscr{A}/RT}\right) \tag{61}$$

where $\mathbb{R}_f$ is the rate of forward reaction and $R$ is the universal gas constant. Notice that equation (61) is not a linear relationship between $d\xi/dt$ and $\mathscr{A}$. However, when the given chemical reaction approaches close to equilibrium the relationship of equation (61) takes the following linear form, namely:

$$\frac{d\xi}{dt} = \frac{\mathbb{R}_f^{eq}}{R}\frac{\mathscr{A}}{T} \tag{62}$$

where $\mathbb{R}_f^{eq}$ is the rate of the forward reaction at chemical equilibrium.

The above analysis reveals that the said LEA is accepted to take a drastically different stands in spatially non-uniform and uniform systems. The reasons behind it are not clear. On the other hand, in sixties of the previous century a new dimension to the validity of LEA has been invented. Inadvertently, it is a non-thermodynamic one, namely based on the physics and/or mathematics of the processes. In other words, it is the constitutive theory driven consideration, namely the parabolicity versus hyperbolicity of the constitutive equations that describe the time evolution of the system. This aspect is the subject matter of the next subsection.

## Parabolicity/Hyperbolicity Argument

In the beginning of sixties of the preceding century a new thermodynamic framework got developed which is termed as Extended Irreversible Thermodynamics (EIT) [25, 26]. In EIT it is claimed that their framework is the legitimate extension beyond LEA. Let us compare the following two Gibbs relations:

$$\frac{ds}{dt} = T^{-1}\frac{du}{dt} + T^{-1}p\frac{dv}{dt} \tag{63}$$

$$\frac{ds}{dt} = T^{-1}\frac{du}{dt} + T^{-1}p\frac{dv}{dt} - \beta\mathbf{q}\cdot\frac{d\mathbf{q}}{dt}. \tag{64}$$

In the preceding two equations the latter one is termed as extended Gibbs relation of EIT and the former one is the Gibbs relation wherein LEA is believed to be obeyed. In equation (63) the heat flux density, $\mathbf{q}$, is not an explicit thermodynamic variable and the entropy production due to the existence of heat flux is given by,

$$\sigma_s = \mathbf{q}\cdot\nabla\left(\frac{1}{T}\right) \geqslant 0. \tag{65}$$

Whereas, in EIT, the heat flux density, $\mathbf{q}$, is an additional thermodynamic variable and the entropy production due to the existence of heat flux to the lowest order approximation reads as,

$$\sigma_s = \frac{1}{\lambda T^2}\mathbf{q}\cdot\mathbf{q} \geqslant 0 \tag{66}$$

where $\lambda$ is the heat conductivity. The question that obviously arises is - what is the physical difference between close to equilibrium prescribed by LEA and the lowest order approximation involved in equation (66)?

The another limitation of LEA that is spelled out in EIT texts is that in CIT the linear Fourier law, namely:

$$\mathbf{q} = -\lambda \nabla T \tag{67}$$

operates that produces an infinite speed of spatial heat propagation. This is so because on using equation (67) and the corresponding internal energy balance equation appropriate in this case, the following parabolic partial differential equation for the propagation of temperature is obtained namely:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_v} \nabla^2 T \tag{68}$$

where $\rho$ is the mass density and $c_v$ is the specific heat capacity. On the other hand, one obtains a hyperbolic equation of telegraphers type, namely:

$$\tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_v} \nabla^2 T \tag{69}$$

on using the Maxwell-Cattaneo-Vernotte (MCV) equation [27, 28], for heat flux, namely:

$$\tau \frac{d\mathbf{q}}{dt} = -(\mathbf{q} + \lambda \nabla T) \tag{70}$$

and the internal energy balance equation expressed as,

$$\rho \frac{du}{dt} = \rho c_v \left( \frac{\partial T}{\partial t} \right)_v = -div\mathbf{q}. \tag{71}$$

Notice that, in deriving above parabolic equation (68) one uses the time dependent equation (71) and assumes the validity of Fourier law, equation (67), during time evolution of the system. But there is no experimental proof to support this assumption. The clinching argument in favour of hyperbolic equations is customarily produced on the basis of finite/infinite speeds of wave propagation of the fluxes. But this then side tracks the question whether the Fourier law is at all valid during time evolution of a system in certain nonequilibrium situations.

## Infinite/Finite Speed Of Propagation Argument

Consider equation (68) whose one dimensional version reads as:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_v} \frac{\partial^2 T}{\partial x^2} = \chi \frac{\partial^2 T}{\partial x^2} \tag{72}$$

where $\chi$ is the thermal diffusivity. Equation (72) is parabolic in nature. The implication of parabolicity is that the sudden application of a temperature difference would give rise to an instantaneous rise to the finite heat flux everywhere in the system. That is, it predicts heat propagation at infinite speed. It indicates that for the time evolving nonequilibrium states the Fourier law is not perhaps valid. In EIT, pains are taken to emphasize that the Maxwell-Cattaneo-Vernotte equation based derivation produces equation (69), which is hyperbolic in nature and hence the problem of infinite speed does'nt arise. To appreciate how equation (69) produces finite speed of propagation of temperature waves let us consider the one dimensional version of it, namely:

$$\tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \chi \frac{\partial^2 T}{\partial x^2} \tag{73}$$

From equation (73) the speed of the temperature wave, $V$, is obtained as:

$$V = (\chi/\tau)^{1/2} \qquad \text{where} \qquad \chi = (\lambda/c_v\rho) \tag{74}$$

Moreover, for monatomic dilute gas the Kinetic Theory provides us the following relationship [29], namely:

$$\lambda = (5pk_B/2m)\,\tau \tag{75}$$

therefore from equations (74) and (75) we notice that as $\lambda \to 0$, $\chi \to 0$ and $\tau \to 0$ and vice versa. Hence, $V$ always remains finite. That is the problem of infinite velocity of temperature waves gets overcome. For $\tau = 0$ equation (70) reduces to Fourier law that according to equation (74) demands infinite speed of propagation of heat (temperature) waves.

In this way, the problem of infinite speed of second sound gets amicably solved as equation (69) assigns a finite speed to heat waves. But we need to remind us that we have still used $u = u(T, v)$ in deriving equation (69). Whereas, in EIT it is argued that the production of equation (69) by using MCV equation, the way it is depicted above, does suggest that $\mathbf{q}$ needs to be taken as an additional thermodynamic variable. Of course, there are other compelling reasons to do so particularly the Grad's solution of Boltzmann integro-differential equation that produces corresponding nonequilibrium distribution function [30, 31] in which the nonequilibrium contribution gets expressed in terms of the existing heat flux.

In view of this crucial assertion in EIT one should have used $u = u(T, v, \mathbf{q})$ instead of $u = u(T, v)$ in deriving the corresponding temperature equation, but so far it has not been attempted. However, now we have undertaken this exercise in the case of rigid body heat conduction. In this exercise we have used MCV equation (70), $u = u(T, \mathbf{q})$ and the corresponding internal energy balance equation. The steps of this exercise involves the manipulation of the internal energy balance equation as depicted below,

$$\rho \frac{du}{dt} = \rho c_v \left(\frac{\partial T}{\partial t}\right)_{v,\mathbf{q}} + \rho \left(\frac{\partial u}{\partial \mathbf{q}}\right)_{v,T} \cdot \left(\frac{d\mathbf{q}}{dt}\right) = -div\mathbf{q} \tag{76}$$

and then on combining it with MCV equation (70) the result is the following non-parabolic complex equation, instead of equation (69), for the heat propagation, namely:

$$\tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} - \frac{\lambda}{\rho c_v} \nabla^2 T + \frac{2\lambda\alpha}{\tau c_v} \mathbf{q} \cdot \nabla T + \frac{\alpha}{\tau c_v} \mathbf{q}^2 + \frac{\lambda^2 \alpha}{\tau c_v} (\nabla T)^2 - \frac{\alpha\lambda}{c_v} \mathbf{q} \cdot \frac{\partial \nabla T}{\partial t} = 0 \tag{77}$$

where we have used $\alpha\mathbf{q} = (\partial u/\partial \mathbf{q})_{T,v}$ as a first approximation. However, one also gets a non-parabolic equation of heat propagation on using $u = u(T, \mathbf{q})$, the Fourier law and the internal energy balance equation (76), that reads as:

$$\frac{\partial T}{\partial t} - \frac{\lambda}{\rho c_v} \nabla^2 T + \frac{\alpha\lambda^2}{\tau c_v} \nabla T \cdot \frac{\partial (\nabla T)}{\partial t} = 0. \tag{78}$$

Thus from the above demonstration we see that if $\mathbf{q}$ is used as an additional thermodynamic variable then the use of both linear and nonlinear equations of heat conduction produce corresponding non-parabolic equations of temperature waves. Therefore, the hyperbolicity and/or the non-parabolicity of the constitutive equations is not a sufficient reason to consider that it is the case of breakdown of LEA.

Another demonstration of non-validity of Fourier law during time evolution of a system is based on the principle of minimum production of entropy. This is described in the next subsection.

## Argument Based On The Principle Of Minimum Production Of Entropy

Yet another demonstration of non-validity of Fourier law for nonequilibrium states other than the NSS is that of Lampinen [32]. In the hope that the NSS corresponds to the minimum production of entropy he used Euler-Lagrange method of variational calculus [33] to minimize entropy production in one dimensional heat conduit using Fourier law of heat transfer and the expression of entropy production given by CIT (c.f. equation (50)) [14], namely:

$$\sigma_s = \mathbf{q} \cdot \nabla \left(\frac{1}{T}\right) \geqslant 0. \tag{79}$$

On using the one dimensional version of equations (79) and (67) we obtain for the rate of entropy production, $\mathscr{P}$, for the whole heat conduit, the following expression,

$$\mathscr{P} = A\lambda \int_0^L \frac{1}{T^2} \left(\frac{\partial T}{\partial x}\right)^2 dx \geqslant 0 \tag{80}$$

where $A$ is the uniform cross-sectional area of the heat conduit of length $L$. The minimum production of entropy, $\mathscr{P}$, is obtained by Euler-Lagrange method of variational calculus [33] by imposing:

$$\delta\left[\int_0^L \frac{1}{T^2}\left(\frac{\partial T}{\partial x}\right)^2 dx\right] = 0. \tag{81}$$

But on solving equation (81) Lampinen obtained the following expression [32] of temperature profile at NSS; namely:

$$T(x) = T_0(T_1/T_0)^{x/L}. \tag{82}$$

It is not the same expression the one that is obtained from equation (72) at NSS, namely:

$$T(x) = T_0 + (T_1 - T_0)(x/L) \tag{83}$$

where, $T_1$ and $T_0$ are the temperatures of heat reservoirs $(T_1 > T_0)$ between which the heat conduit of the length $L$ is kept. We recall that equation (83) exactly matches with the experimental temperature profile. Thus the result of minimizing the rate of entropy production is equation (82), which simply means that on the perturbed path about a NSS one cannot use Fourier law. Moreover, it also hints at the non-validity of entropy production expression equation (79) given by CIT if it is not the case of NSS.

On the other hand, when we replace the Fourier law by the MCV equation in the preceding analysis, that is retaining the expression of entropy production given by equation (79) provided by CIT, it too does'nt produce the right result for the temperature profile at NSS. The expression obtained is:

$$T(x) = \sqrt{(T_1^2 - T_0^2)(x/L) + T_0^2}. \tag{84}$$

Thus the above analysis demonstrates that on perturbed path about an NSS the equation (79), that is the CIT given expression of entropy production, is not valid. However, on using EIT given expression of $\sigma_s$, equation (66), and MCV equation (70) on the virtual perturbed path and then minimization of entropy production rate, $\mathscr{P}$, produces the correct temperature profile along the length of the heat conduit at NSS, namely equation (83).

## A Thermodynamic Argument

The CIT given expression of entropy source strength for heat conduction is equation (79), namely:

$$\sigma_s = \mathbf{q} \cdot \nabla\left(\frac{1}{T}\right) \geqslant 0$$

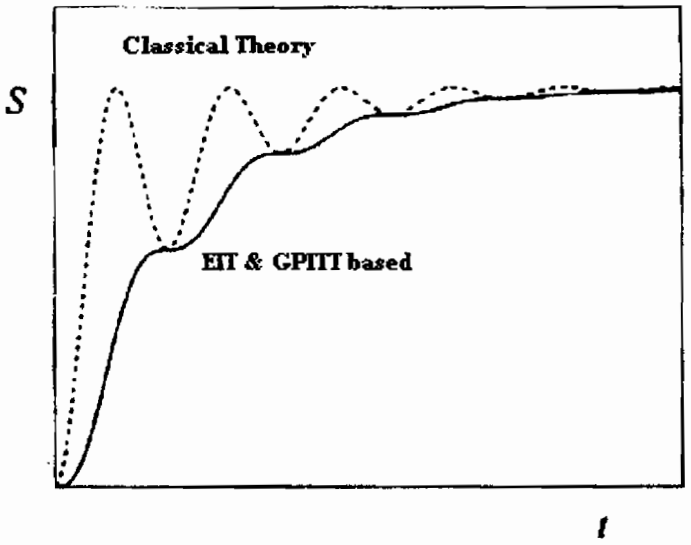
As argued above the Fourier law is not valid for nonequilibrium states other than the NSS, let us examine how good is the MCV equation to study the entropy evolution with time. Thus on substitution of equation (70), the MCV equation, into equation (79) we obtain,

$$\sigma_s = \frac{\lambda}{T^2}(\nabla T)^2 + \frac{\tau}{T^2}\frac{d\mathbf{q}}{dt} \cdot \nabla T. \tag{85}$$

Now, if we isolate the rigid body instantaneously from its environment, its entropy will vary with time as,

$$\frac{dS}{dt} = \int_V \sigma_s dV \tag{86}$$

The solution of one dimensional version of the preceding equation was obtained earlier [26] and the result is the non-monotonic increase of entropy depicted in the FIGURE 1 by dashed curve. However, this result is not in conformity with the second law of thermodynamics.




*Source: Lebon et al [26]*

**FIGURE 1.** Evolution of entropy based on the Classical Irreversible Thermodynamic Theory during the equilibration of an isolated system when use is made of Cattaneo's equation is given by the *dashed curve*. The evolution of entropy using EIT and GPITT descriptions is represented by the *solid curve*.

## The Options At Our Disposal

From the preceding discussion the conclusion is that when a nonequilibrium state is other than the NSS not only the Fourier law but also the entropy production given by CIT are not valid. In view of this demonstration now there are two options, namely:

1. Expand the domain of LEA to include all those nonequilibrium situations, irrespective of from far away to the close to equilibrium conditions, that would demand the inclusion of additional thermodynamic variables over and above the ones offered by the traditional Gibbs relation.
2. Shun altogether the postulation of LEA and find out some other acceptable requirement obtained explicitly from the laws of thermodynamics, particularly the zeroth and second laws of thermodynamics.

At this juncture we also cite present author's earlier investigations [38, 39] based on the kinetic theory as well as the macroscopic thermodynamic considerations. These studies clearly establish that the LEA is an ill conceived concept and hence needs to be abandoned. Therefore, in the following we are presenting the results of our efforts in conformity with the second choice. The present author has developed a thermodynamic framework named Generalized Phenomenological Irreversible Thermodynamic Theory (GPITT) [34–37] wherein no postulation of LEA is required hence it fits well with the second choice spelled out above. This involves first to clearly understand the domain of the operations of thermodynamics, legitimization of temperature function in nonequilibrium and then generation of the entropy function in nonequilibrium for spatially non-uniform systems.

## BRIDGMAN'S CONJECTURES

As stated above we need to understand clearly the domain of the operation of laws of thermodynamics. In this exercise the conjectures made by Bridgman help us [40, 41]. Bridgman concludes that the "Universe of the Operations of Thermodynamics is determined by the instrumental operations of laboratory". Therefore, his assertions are that for the laws of thermodynamics, particularly the zeroth and the second laws, to hold there are lower bounds on:

1. the size of the system. The lowest size that belongs to the domain of thermodynamics can be appreciated by the fact that at NTP, say hydrogen gas, houses about $10^{10}$ molecules in a cube having its edges of 0.001 mm length. In solids this size would be even much smaller and will depend on the density of the material.

2. the time scale of observations.

Below these lower bounds one enters into the natural fluctuational domain. In the view of above keen observations Bridgman asserts that,

> *"the universe of operations (instrumental) of thermodynamics is itself a sub-group of all the operations which we can now perform, including operations of all scales of magnitude and time".*

Let us now exemplify the above dictums of the operations of thermodynamics. The second law of thermodynamics states that heat cannot flow on its own from lower to the higher temperature direction. *This means that there is a mechanism by which, when two bodies are brought into a diathermal contact they correctly sense each other's temperature.* In view of this, the following well known thermodynamic inequality concerning the heat transfer between two rigid bodies, namely:

$$\frac{dQ}{dt}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)\geqslant 0 \tag{87}$$

where $dQ$ is the amount of heat exchanged by the system 2 with system 1 and $T$'s are their temperatures, holds good only for time duration of measurements $dt \geqslant \tau_{therm}$, where $\tau_{therm}$ is the minimum time needed to sense the temperature of a given body by another body when the two are brought in a diathermal contact [36, 42].

Therefore, we must know the minimum time duration of the said diathermal contact in which the bodies correctly sense each other's temperature. Below this time scale the second law of thermodynamics ceases to hold. A recent modeling experiment also supports [43] Bridgman's conjectural assertions[3]. In other words, such short time durations of measurements fall in the natural fluctuational domain. That is, the above identified minimum time duration $\tau_{therm}$ demarcates the domains of measurements based on the time duration of observations $\Delta t \equiv dt = \tau$ as follows,

$\infty > \tau \geqslant \tau_{therm}$ corresponds to the "Universe of operations of thermodynamics"

$\tau < \tau_{therm}$ corresponds to the natural fluctuational domain.

This then implies that the same restriction holds for Clausius' inequality in the cyclic form, equation (3), namely:

$$\oint_{irr}\frac{dQ}{T_R} < 0.$$

That is $dQ$ of the preceding inequality is the amount of heat that the device exchanges during the time duration $\tau \geqslant \tau_{therm}$ [36, 44] with its thermally interacting surroundings.

Now we proceed in the next section to legitimize temperature in nonequilibrium.

## TEMPERATURE IN NONEQUILIBRIUM

There exist a good number of proposals to define temperature in nonequilibrium. For example, EIT [45] fosters the idea of nonequilibrium temperature mainly influenced by the kinetic theory perception. However, it is being claimed that it is distinctly different in its physical contents than the one adopted for the local equilibrium states. On the other hand, there is a proposal of contact temperature in nonequilibrium [46–48] which requires the use of a hypothetical heat reservoir (by its definition it is devoid of a heat flux). In this proposal that heat reservoir is identified which on making a diathermal contact with the given nonequilibrium system does not produce heat flux across the surface in contact. Hence the temperature of the given system in nonequilibrium according to say equation (87) gets assigned to that of the so identified heat reservoir. Another proposal is that of the present author [35] wherein a generalized zeroth

[3] Wang et al have demonstrated [43] that there does exist the lowest limit to the system size as well as to the time duration of an observation made. Their results of the same experiment show that at colloidal particle size scale the second law of thermodynamics is not obeyed even if the time duration of observations is extended well beyond several seconds.

law of thermodynamics has been framed that takes care of the necessity of thermal equilibration for sensing the correct temperature. It reads as,

> *"Three tiny volume elements are instantly and simultaneously isolated from the respective nonequilibrium systems and in the same instant action they are brought into diathermal contacts as closed rigid systems, 1 with 2 and 2 with 3. If within the short time interval of sensing of the thermal interactions it is found that 1 is in momentary thermal equilibrium with 2 and 2 is with 3 then 3 is also in momentary thermal equilibrium with 1. The momentary thermal equilibrium means that if the volume elements possessed the heat fluxes then they remain unaffected during the minimum short period of thermal interactions and if no heat flux existed, both such nonequilibrium and the equilibrium states included, no heat flux gets generated during the said diathermal contact. The making of a diathermal contact between the tiny volume elements one of which having a heat flux and the other one without it is not forbidden".*

This statement is the nonequilibrium counterpart of the Zeroth Law of Thermodynamics which uniquely identifies the thermodynamic temperature at local level when the system is out of equilibrium irrespective of close to or far away from equilibrium. Also notice that no distinction between the physical contents of temperature in equilibrium and in nonequilibrium arise. Finally, we coincide this temperature function, legitimized by the generalized zeroth law of thermodynamics for systems in nonequilibrium, with the Kelvin scale of temperature.

Now we are well equipped to develop a nonequilibrium thermodynamic description. In the following section we have chosen to describe the steps of the development of GPITT [34–37].

## GPITT

Now we consider spatially non-uniform systems and hence we are starting with the Clausius inequality of equation (3) and follow the similar steps that we have followed on using equation (30) in section entitled nonequilibrium and the entropy function. Thus in the present context Clausius' uncompensation function, $N$, gets defined, by using equation (3), as

$$N = -\oint_{\text{irr}} \frac{dQ}{T_R(t)} = -\oint_{\text{irr}} \frac{1}{T_R(t)} \frac{dQ}{dt} dt > 0 \tag{88}$$

then $N$ is treated as an independent quantity to express it as,

$$N = \oint_{\text{irr}} \frac{dN}{dt} dt \tag{89}$$

From equation (89) we have

$$dN > 0 \qquad \text{as} \qquad N > 0. \tag{90}$$

The positive definiteness of $dN$ uniquely retains the unidirectionality of time's arrow during the irreversible evolution of the system.

On combining equations (88) and (89) we obtain

$$\oint_{\text{irr}} \left[ \frac{1}{T_R(t)} \frac{dQ}{dt} + \frac{dN}{dt} \right] dt = 0. \tag{91}$$

As cyclic integral of equation (91) vanishes we obtain

$$\frac{dS}{dt} = \frac{1}{T_R(t)} \frac{dQ}{dt} + \frac{dN}{dt}. \tag{92}$$

Thus we see that in equation (92) $S$ is the entropy of a nonequilibrium state. In the limit of $dN \to 0$, we have $S$(of a nonequilibrium state) $\to S$(of an equilibrium state). Now as $dN/dt > 0$, from equation (92) we obtain:

$$\frac{dS}{dt} > \frac{1}{T_R(t)} \frac{dQ}{dt} \tag{93}$$

which is nothing else but the explicit differential form of the Clausius inequality, equation (4), in time rate form.

However, equation (92) contains $T_R(t)$ hence the first term on its right hand side cannot be transformed as such to a local level description for a spatially non-uniform systems. For that purpose let us express the said term as,

$$\frac{1}{T_R(t)}\frac{dQ}{dt} \equiv \oint_A -\frac{\mathbf{Q}(\mathbf{A},t)}{T(\mathbf{A},t)}\cdot \mathbf{dA} \tag{94}$$

where $\mathbf{Q}(\mathbf{A},t)$ is the heat flux density at the surface area coordinate $\mathbf{A}$ and at time $t$, $\mathbf{dA}$ is the differential area vector on the surface of the system, $A$ is the total surface area of the closed system and $T(\mathbf{A},t)$ is temperature at $\mathbf{A}$ and $t$ [35]. Now on applying Gauss theorem we obtain,

$$\frac{1}{T_R(t)}\frac{dQ}{dt} \equiv \oint_V \left[-\mathrm{div}\left(\frac{\mathbf{q}(\mathbf{r},t)}{T(\mathbf{r},t)}\right) + \mathbf{q}(\mathbf{r},t)\cdot\nabla\left(\frac{1}{T(\mathbf{r},t)}\right)\right] dV \tag{95}$$

that reduces to,

$$\frac{1}{T_R(t)}\frac{dQ}{dt} \equiv \int_V -\frac{1}{T}\,\mathrm{div}\,\mathbf{q}\; dV. \tag{96}$$

The other two terms of equation (92) get easily transformed to

$$\frac{dS}{dt} = \int_V \rho\frac{ds}{dt}\, dV \tag{97}$$

$$\frac{dN}{dt} = \int_V \rho\frac{d\mathcal{N}}{dt}\, dV \tag{98}$$

wherein we have used the standard fluid dynamical inter-conversions, namely:

$$S = \int_V \rho\, s\, dV, \qquad N = \int_V \rho\, \mathcal{N}\, dV, \qquad \text{etc.} \tag{99}$$

where $\rho$ is the mass density and $s$, $\mathcal{N}$, etc. are the per unit mass quantities at a given position and time $t$.

On substituting equations (96)-(98) into equation (92) we obtain at the local level,

$$\rho\frac{ds}{dt} = -\frac{1}{T}\mathrm{div}\,\mathbf{q} + \rho\frac{d\mathcal{N}}{dt}. \tag{100}$$

Notice that $d\mathcal{N}/dt \lesseqgtr 0$. Next the simple fluid dynamical internal energy balance equation of equation (47) is used, namely:

$$\rho\frac{du}{dt} = -\mathrm{div}\,\mathbf{q} - p\rho\frac{dv}{dt} + \Pi : \nabla\mathbf{u}. \tag{101}$$

Now on combining equations (100) and (101) we obtain

$$\rho\frac{ds}{dt} = \frac{1}{T}\left[\rho\frac{du}{dt} + p\rho\frac{dv}{dt} - \Pi : \nabla\mathbf{u}\right] + \rho\frac{d\mathcal{N}}{dt}. \tag{102}$$

The steps leading to quantify $d\mathcal{N}$ [36, 37] first involves rearranging equation (102) as,

$$\begin{aligned}\rho\frac{d\mathcal{N}}{dt} &= \rho\left[\frac{ds}{dt} - \frac{1}{T}\frac{du}{dt} - \frac{p}{T}\frac{dv}{dt}\right] + \frac{1}{T}\Pi : \nabla\mathbf{u} \\ &= \frac{\rho}{T}\left[\frac{d(sT-u-pv)}{dt} - s\frac{dT}{dt} + v\frac{dp}{dt}\right] + \frac{1}{T}\Pi : \nabla\mathbf{u}\end{aligned} \tag{103}$$

however, as $sT - u - pv = -\mathcal{G}$ (the Gibbs function), equation (103) reads as,

$$\rho\frac{d\mathcal{N}}{dt} = \frac{\rho}{T}\left[-\frac{d\mathcal{G}}{dt} - s\frac{dT}{dt} + v\frac{dp}{dt} + \rho^{-1}\Pi : \nabla\mathbf{u}\right]. \tag{104}$$

Thus our problem reduces to identify a proper expression of $\mathcal{G}$, which basically determines the chemical interactions. Recall that in equilibrium thermodynamics the Gibbs function, $G$, is given by [4],

$$G = \sum_k n_k \mu_k \qquad \text{(at given } T \text{ and } p\text{)} \tag{105}$$

that takes care of the existing chemical interactions. The differentiation of equation (105) produces,

$$dG = \sum_k n_k d\mu_k + \sum_k \mu_k d_i n_k + \sum_i \mu_k d_e n_k. \tag{106}$$

in which the last two terms describe the processes of chemical conversions and the matter exchange.

On the other hand, in the case of nonequilibrium, in order to use equation (104) in an experimental setup one is required to define $\mathcal{G}$ function appropriately. Its expression obviously will depend on the type of irreversibility involved. However, in nonequilibrium following processes may occur, namely:

1. Chemical reaction $(d\xi/dt)$
2. Matter diffusion $(\mathbf{J}_k)$
3. Heat conduction $(\mathbf{q})$
4. Momentum transfer $(\Pi)$

## Spatially Non-uniform Systems

Herein for the sake of simplicity of demonstration we consider an ideal monatomic gas in a spatially non-uniform system having irreversibility only on account of heat transfer. From kinetic theory of non-uniform gases [29] we learn that the population in translational quantum states in presence of heat and/or momentum fluxes is different from that given by Maxwellian distribution function. And when such a system is detached from the source of temperature gradient and/or velocity gradient the fluxes die fast and the population in translational quantum states becomes Maxwellian. It means that in nonequilibrium the chemical potentials in various translational quantum states are not equal whereas in absence of these fluxes they have identical values.

In Kinetic theory the heat flux density, $\mathbf{q}$, is given by

$$\mathbf{q} = \int \frac{1}{2} m C^2 \mathbf{C} \, f \, d\mathbf{c} \tag{107}$$

where $m$ is the mass of the monatomic gas molecule, $\mathbf{C}$ is the chaotic or peculiar velocity given by

$$\mathbf{C} = \mathbf{c} - \mathbf{u} \tag{108}$$

where $\mathbf{c}$ is the velocity of a molecule and $\mathbf{u}$ is the barycentric velocity at that position, and $f$ is the distribution function. Therefore, in presence of a heat flux we have,

$$\mu_C \neq \mu_{C'} \neq \mu_{C''} = \cdots$$

whereas in equilibrium (in absence of heat flux) we rigorously have ,

$$\mu_C = \mu_{C'} = \mu_{C''} = \cdots$$

With this background we define appropriately the Gibbs function, $\mathcal{G}$, as,

$$\mathcal{G} = \frac{1}{m} \int \mu_C x_C d\mathbf{c} \tag{109}$$

and the mass fraction $x_C$ as,

$$x_C = \frac{f}{n} \tag{110}$$

where $n$ is the local level number density of molecules. Notice that we have adopted $\mu_C$ as a per molecule quantity and hence to have $\mathcal{G}$ as a per unit mass quantity we have the factor $1/m$ on the right hand side of equation (109). On combining equations (109) and (110) we obtain:

$$\mathcal{G} = \frac{1}{mn}\int \mu_C\, f\, d\mathbf{c} = \frac{n<\mu>}{mn} = \frac{<\mu>}{m} \tag{111}$$

where $<\mu>$ is the average value of Gibbs function per molecule. Now in kinetic theory to obtain an expression of $f$ in nonequilibrium one solves the Boltzmann integro-differential equation [29], namely:

$$\frac{\partial f}{\partial t} + \mathbf{C}\cdot\frac{\partial f}{\partial \mathbf{r}} = J(f|f) \tag{112}$$

where $J(f|f)$ is the Boltzmann collisional integral given by,

$$J(f|f) = \int (f_1' f' - f_1 f) g\sigma(g,\Omega) d\Omega d\mathbf{c}_1$$

where $\sigma(g,\Omega)$ is the collision cross section that depends on the relative velocity $g = |\mathbf{c}_1 - \mathbf{c}|$ and the scattering solid angle $\Omega$. The primes and indices in the distribution function have the usual meaning, namely, $f_1' = f(\mathbf{c}_1', \mathbf{r}, t)$ is the distribution function of particle 1 after collision, etc.

Recall that there are two standard methods namely that of Chapman-Enskog and that of Grad. In both of them $f$ is solved around local Maxwellian, $f^{(0)}$, which by definition is a function of $u$ and $\rho$. Indeed, by using solutions obtained by them, one obviously incorporates the corresponding kinetic theory tenets into the thermodynamic description. To have our thermodynamic expressions free of such incoherencies one needs to find out a direct method to measure $x_C$. However, the non-availability of such a method, at present, forces us to use Grad's solution of Boltzmann integro-differential equation.

In Grad's 13-moment solution [30, 31], we have

$$f = f^{(0)}(1+\Phi) \tag{113}$$

and on ignoring viscous effects we have for the nonequilibrium contribution, $\Phi$, the following expression,

$$\Phi = \frac{2m}{5pk_B^2T^2}\left(\frac{1}{2}mC^2 - \frac{5}{2}k_BT\right)\mathbf{C}\cdot\mathbf{q}. \tag{114}$$

On using the same assumption as that is involved in equation (113) we propose for $\mu_C$ the following expression, namely:

$$\mu_C = \mu^{(0)} + k_BT\ln(1+\Phi). \tag{115}$$

In this setup $\mu^{(0)}$, the chemical potential, is that for the state of no heat flux and is given by [49]:

$$\mu^{(0)} = k_BT\ln\left[\frac{n}{(2\pi k_BT/m)^{3/2}}\right] = k_BT\ln\left[\frac{f^{(0)}(C)}{e^{-mC^2/2k_BT}}\right]. \tag{116}$$

Now we substitute equation (115) into equation (109) that produces,

$$\begin{aligned}\mathcal{G} &= \frac{1}{mn}\int \mu^{(0)}\, f\, d\mathbf{c} + \frac{k_BT}{mn}\int f\,\ln(1+\Phi)\, d\mathbf{c}\\ &= \frac{\mu^{(0)}}{mn}\int f\, d\mathbf{c} + \frac{k_BT}{mn}\int f\,\ln(1+\Phi)\, d\mathbf{c}\\ &= \frac{\mu^{(0)}}{m} + \frac{k_BT}{mn}\int f\,\ln(1+\Phi)\, d\mathbf{c}.\end{aligned} \tag{117}$$

To solve equation (117) we expand $\ln(1+\Phi)$ in a Taylor series,

$$\ln(1+\Phi) = \Phi - \frac{\Phi^2}{2} + \cdots \tag{118}$$

Now assume that the system is not far away from equilibrium that gives the following approximation, namely:

$$\ln(1+\Phi) \cong \Phi. \tag{119}$$

Hence on substituting equation (119) into equation (117) we obtain,

$$\mathcal{G} \simeq \frac{\mu^{(0)}}{m} + \frac{k_B T}{mn} \int f \, \Phi \, d\mathbf{c} \tag{120}$$

and on substituting the expression for $\Phi$ of equation (114) in equation (120) yields,

$$\mathcal{G} = \frac{\mu^{(0)}}{m} + \frac{k_B T}{mn} \left( \frac{2m}{5pk_B^2 T^2} \right) \left( \int \frac{1}{2} m C^2 \mathbf{C} \, f \, d\mathbf{c} \right) \cdot \mathbf{q} - \frac{k_B T}{mn} \left( \frac{m}{pk_B T} \right) \left( \int \mathbf{C} \, f \, d\mathbf{c} \right) \cdot \mathbf{q}. \tag{121}$$

Obviously, the average of peculiar velocity, $\mathbf{C}$, is zero, hence the last term on the right hand side of equation (121) vanishes leaving,

$$\mathcal{G} = \frac{\mu^{(0)}}{m} + \frac{2}{5npk_B T} \mathbf{q} \cdot \mathbf{q} \tag{122}$$

where we have used the kinetic theory definition of heat flux, namely equation (107). In the derivation of corresponding Gibbs relation we need to substitute an expression of $d\mathcal{G}/dt$ in equation (104). The required expression is obtained from equation (122) as,

$$\frac{d\mathcal{G}}{dt} = \frac{d(\mu^{(0)}/m)}{dt} - \frac{2}{5npk_B T^2} q^2 \frac{dT}{dt} - \frac{2}{5nk_B T p^2} q^2 \frac{dp}{dt} + \frac{4}{5npk_B T} \mathbf{q} \cdot \frac{d\mathbf{q}}{dt}. \tag{123}$$

Next we substitute equation (123) into equation (104), and then extract the corresponding Gibbs-Duhem equation, namely:

$$\frac{d(\mu^{(0)}/m)}{dt} + \left( s - \frac{2}{5npk_B T^2} q^2 \right) \frac{dT}{dt} - \left( v + \frac{2}{5nk_B T p^2} q^2 \right) \frac{dp}{dt} = 0 \tag{124}$$

that produces the operative expression of the rate of change of uncompensation function as,

$$\rho \frac{d\mathcal{N}}{dt} = -\frac{4\rho}{5npk_B T^2} \mathbf{q} \cdot \frac{d\mathbf{q}}{dt}. \tag{125}$$

Next we substitute equation (125) into equation (102) that yields,

$$\frac{ds}{dt} = \frac{1}{T} \frac{du}{dt} + \frac{p}{T} \frac{dv}{dt} - \frac{4}{5npk_B T^2} \mathbf{q} \cdot \frac{d\mathbf{q}}{dt} \tag{126}$$

which is nothing else but the extended Gibbs relation of EIT. Only difference between the two is, that in the numerator of coefficient of the last term on the right hand side of equation (126) instead of the numeral 4 there in EIT they have 2. Notice that at no stage of the development upto equation (126) there gets incorporated beyond LEA constraint and hence it is not in conformity with EIT. Recall that the EIT is claimed as the beyond LEA thermodynamic description.

Now recall the Clausius-Duhem inequality, equation (53), namely:

$$\rho \frac{ds}{dt} + \mathrm{div} \mathbf{J}_s = \sigma_s \geqslant 0.$$

In the present case, on combining the corresponding version of equation (101) (that is by dropping out the term describing viscous dissipation) with equation (126) yields the following entropy balance equation, namely:

$$\rho \frac{ds}{dt} = -\mathrm{div} \left( \frac{\mathbf{q}}{T} \right) + \mathbf{q} \cdot \nabla \left( \frac{1}{T} \right) - \frac{4\rho}{5npk_B T^2} \mathbf{q} \cdot \frac{d\mathbf{q}}{dt}. \tag{127}$$

Hence on comparing equation (127) with equation (53) we obtain,

$$\mathbf{J}_s = \frac{\mathbf{q}}{T} \tag{128}$$

$$\sigma_s = \mathbf{q}\cdot\nabla\left(\frac{1}{T}\right) - \frac{4\rho}{5npk_BT^2}\mathbf{q}\cdot\frac{d\mathbf{q}}{dt} > 0. \tag{129}$$

Equation (129) clearly reveals that there are two mechanisms of entropy production one originates from the heat transfer across the boundaries and the other operates by an internal mechanism, which is controlled internally by the collisions amongst the molecules within the tiny volume element.

In a NSS as the heat flux and other system properties remain time invariant the last term in equation (129) describing collisional mechanism drops out (as $d\mathbf{q}/dt = 0$), therefore, the expression for $\sigma_s$ simplifies to equation (79), namely:

$$\sigma_s = \mathbf{q}\cdot\nabla\left(\frac{1}{T}\right) \geqslant 0$$

which is the traditionally well known expression of entropy production of CIT. This clearly reveals that CIT holds only for NSS.

It is amusing to note that when the heat flux varies according to the Maxwell-Cattaneo-Vernotte (MCV) equation (70) [27, 28], namely:

$$\tau\frac{d\mathbf{q}}{dt} = -\mathbf{q} - \lambda\,\nabla T$$

the equation (129) simplifies as,

$$\begin{aligned}\sigma_s &= \mathbf{q}\cdot\nabla\left(\frac{1}{T}\right) + \frac{4\rho}{5n\tau pk_BT^2}\mathbf{q}\cdot(\mathbf{q}+\lambda\nabla T)\\ &= -\frac{1}{T^2}\mathbf{q}\cdot\nabla T + \frac{4\rho}{5n\tau pk_BT^2}q^2 + \frac{4\rho\lambda}{5n\tau pk_BT^2}\mathbf{q}\cdot\nabla T.\end{aligned} \tag{130}$$

However, only if

$$\frac{4\rho\lambda}{5n\tau pk_B} = 1 \tag{131}$$

equation (130) reduces to the expression given by EIT for $\sigma_s$, namely,

$$\sigma_s = \frac{4\rho}{5n\tau pk_BT^2}q^2 = \frac{1}{\lambda T^2}q^2 > 0. \tag{132}$$

This means that, the relaxation time $\tau$ in this case is given by,

$$\tau = \frac{4m\lambda}{5pk_B} \tag{133}$$

where $\rho = mn$ has been used. The above GPITT development clearly reveals that equation (126) is not restricted to the extent it appears in EIT. The EIT formalism does not reveal that in a time evolving system there operates two mechanisms of entropy production that for the first time revealed GPITT through equation (128). On the other hand, the EIT is obtained as one special case of GPITT as equation (132) reveals.

At this point it is important to stress that though we finally end up in generating EIT type extended Gibbs relation but without getting attached the beyond LEA qualification.

## Open System Features

Recall that in the beginning of this presentation we have spelled out that as one attempts to deal the interior local tiny pockets they inherently are equipped with the open system features. The above derivation does take care of it. Let us consider equation (101) in which the so called heat flux density, $\mathbf{q}$, appears. It does include the contribution from the matter diffusion if non-uniformity in concentrations of system components exists. In this case the classical heat flux, $\mathbf{Q}$, gets quantified [16, 50] as follows,

$$\mathbf{Q} = \mathbf{q} - \sum_k \mathbf{J}_k h_k \tag{134}$$

where $\mathbf{J}_k$ is the diffusion flux density of the component $k$ and $h_k$ is the partial enthalpy. Hence, not only we are not required to generate a local level version of Clausius' inequality for open systems but also it is not possible to do so as in equation (100) $d\mathcal{N}/dt$ has no definite sign.

## COMPLEX SYSTEMS IN NONEQUILIBRIUM

The above described GPITT methodology of developing a nonequilibrium thermodynamic framework clearly demonstrates that the quantification of irreversibility in the last step demands an appropriate quantification of the Gibbs function, $\mathscr{G}$. Moreover, this exercise has to be undertaken keeping in view the source of irreversibility and complexity of the system. For example, in the natural evolution of a system in nonequilibrium consisting of macromolecules, e.g. enzymes, polymers, etc. the non-thermal effects do play a major role. Their physicochemical properties are governed by their internal configuration. Hence, these internal configurational states have to be considered as different entities. Thus if $\varepsilon$ is identified as the distinguishing parameter of various internal configurational states of a macromolecule then the Gibbs function, $\mathscr{G}$, gets quantified as,

$$\mathscr{G} = \int \mu_\varepsilon x_\varepsilon d\varepsilon \tag{135}$$

where the chemical potential, $\mu_\varepsilon$, is identified as a per unit mass quantity and $x_\varepsilon$ are the mass fractions of the internal configurational states of the macromolecules.

The substitution of the preceding expression of $\mathscr{G}$ in equation (104) produces following expression of the local level uncompensation function, namely:

$$\rho \frac{d\mathscr{N}}{dt} = \frac{\rho}{T}\left[-\frac{d\int \mu_\varepsilon x_\varepsilon d\varepsilon}{dt} - s\frac{dT}{dt} + v\frac{dp}{dt} + \rho^{-1}\Pi : \nabla \mathbf{u}\right]. \tag{136}$$

Now on extracting the following Gibbs-Duhem equation, namely:

$$s\frac{dT}{dt} - v\frac{dp}{dt} + \int \frac{d\mu_\varepsilon}{dt} x_\varepsilon d\varepsilon = 0 \tag{137}$$

from equation (136) reduces it to,

$$\rho \frac{d\mathscr{N}}{dt} = T^{-1}\Pi : \nabla \mathbf{u} - \frac{\rho}{T}\int \mu_\varepsilon \frac{dx_\varepsilon}{dt} d\varepsilon. \tag{138}$$

Next on substituting equation (138) into equation (102) produces the following Gibbs relation, namely:

$$\frac{ds}{dt} = \frac{1}{T}\frac{du}{dt} + \frac{p}{T}\frac{dv}{dt} - \frac{1}{T}\int \mu_\varepsilon \frac{dx_\varepsilon}{dt} d\varepsilon \tag{139}$$

The finer details of the Gibbs relation, equation (139), can only be obtained after arriving at the appropriate expressions for $\mu_\varepsilon$ and $x_\varepsilon$ in terms of $\varepsilon$, the internal configurational parameter, and further on the functional dependencies of $\mu_\varepsilon$ and $x_\varepsilon$ on other system parameters, such as $p$, $\rho$, $\mathbf{q}$, $\Pi$, relaxation time of physical fluxes, conductivity, coefficient of viscosity, etc. Or in other words, by using the constitutive equation of $x_\varepsilon$ and an appropriate expression for $\mu_\varepsilon$ rationally proposed via say $x_\varepsilon$. Again notice that on the right hand side of equation (139) the last summational term appears because of the existence of imbalance in corresponding chemical interaction. This we elaborate as follows.

At equilibrium the chemical potentials of various internal configurational states are equal, that is we have,

$$\mu_{\varepsilon'} = \mu_{\varepsilon''} = \mu_{\varepsilon'''} = \cdots\cdots = \mu \tag{140}$$

whereas in the case of nonequilibrium these chemical potentials are not equal, that is we have,

$$\mu_{\varepsilon'} \neq \mu_{\varepsilon''} \neq \mu_{\varepsilon'''} \neq \cdots\cdots \tag{141}$$

Hence, we have in equation (139) the non-zero last integrational term on its right hand side, namely:

$$\int \mu_\varepsilon \frac{dx_\varepsilon}{dt} d\varepsilon \neq 0 \tag{142}$$

which in equilibrium vanishes as chemical potential term comes out side of the sign of integration, namely:

$$\mu \frac{d}{dt}\int x_\varepsilon d\varepsilon = \mu \frac{d}{dt}(1) = 0 \tag{143}$$

Thus equation (143) describes the complete balance of the corresponding chemical interactions at equilibrium, whereas equation (142) quantifies the existing imbalance of corresponding chemical interactions, which is the root cause of nonequilibrium.

## THERMODYNAMIC REVELATIONS

The above presentation has clearly revealed the following facts, namely:

1. The GPITT derivation establishes heat flux density as an additional thermodynamic variable for nonequilibrium situations even in the case of close to equilibrium situations. Notice that this is not the result of any postulation but we have arrived at it through an a b initio handling of laws of thermodynamics. Indeed, the above discussed outcome is not surprising because as in EIT herein too we have developed the GPITT framework by borrowing the relevant expressions from the kinetic theory. However, it would be interesting to see whether one would obtain the same results if alternative expressions for $x_C$ and $\mu_C$ become available but are not based on kinetic theory.
2. Also GPITT has revealed that the existence of heat flux itself is a manifestation of existing imbalance in chemical interaction within the system as the heat flux in equation (126) appears on quantifying $\mathscr{G}$ function the latter basically describes the chemical interaction. Thus it establishes that *the thermodynamic irreversibility is all about the imbalances in corresponding chemical interactions* [36].
3. From the above described GPITT steps we find that at no stage we have invoked or were required to postulate the so called LEA. In fact, there is no demand at all to resort to the conjectural postulation of LEA and hence beyond LEA too.
4. Instead what has been demanded is $dt \equiv \tau \geqslant \tau_{therm}$, that is the minimum time required for the measuring gadget to get equilibrated with the local pocket of the system to produce a correct value of the property being measured. At the molecular level this much time is required for each molecule of the local pocket of the system to have undergone at least one collision.
5. Thus instead of postulation of LEA, adopting CIT (and inventing LIT within it), then looking beyond LEA - that for example produces EIT, etc. - it is straightforward to follow the dictums spelled out by Bridgman regarding the "universe of operations of thermodynamics" and then develop nonequilibrium thermodynamic framework by using Clausius' inequality. The one representative example of this procedure has been described in this presentation that develops GPITT framework. The adopted approach described in this presentation reveals that there is no room to have different nonequilibrium thermodynamic frameworks. But the current scenario is altogether different. We see that a good number of nonequilibrium thermodynamic frameworks are prevailing over in the contemporary literature besides those discussed above, for example, we have Rational Thermodynamics [51], Keizer's Nonequilibrium Statistical Thermodynamics [52], Finite-Time Thermodynamics [53],Thermodynamics of Driven Systems [54], Mesoscopic Non-Equilibrium Thermodynamics [55, 56], GENERIC formalism [57], Nonequilibrium Thermodynamics with Hidden/Internal Variables [58–60], etc.
6. From thermodynamic point of view the irreversible processes are the ones which possesses an arrow of time. In view of this universal observation coupled with the second law of thermodynamics Eddington [61] has asserted that Time's arrow is the sole property of entropy. However, a simple correction to this far reaching assertion is needed and we assert that it should read as - Time's arrow is the sole property of entropy production and thereby of entropy function [42].

## ENTROPY FUNCTION BASED ON STATISTICAL CONSIDERATIONS

In the view of above discussion it is but natural to expect that the entropy definition based on statistical considerations would also have corresponding implications that we proceed to describe below in brief.

### Entropy In Information Theory

The information theory entropy, $S_I$, reads [62, 63], as follows:

$$S_I = -k_B \sum_j P_j ln P_j \tag{144}$$

where $P_j$ is the probability of the $j$-th event. This entropy would coincide with that given by the second law of thermodynamics on meeting the demand of the minimum time duration of measurements, namely, $dt \equiv \tau \geqslant \tau_{therm}$.

This is so because in the time period $dt \equiv \tau \geqslant \tau_{therm}$ the value of $P_j$ would be different than that for the time duration $dt \equiv \tau < \tau_{therm}$. In the latter case the said probability would be a natural fluctuating function in time.

## Entropy In Kinetic Theory

For the identical reasons the kinetic theory distribution function, $f$, say of equation (113), imbibes the same property of natural fluctuations in the domain $dt \equiv \tau < \tau_{therm}$ and hence it would compute a fluctuating heat flux, $\mathbf{q}$, though the mathematical form of the equation remains the same, namely:

$$\mathbf{q} = \int \frac{1}{2} m C^2 \mathbf{C} \, f^{fluc} \, d\mathbf{c} \tag{145}$$

where $f^{fluc}$ is the distribution function in the natural fluctuational domain and is the solution of Boltzmann integro-differential equation, equation (112)

$$\frac{\partial f}{\partial t} + \mathbf{C} \cdot \frac{\partial f}{\partial \mathbf{r}} = J(f|f)$$

for such a short time durations.

## CONCLUDING REMARKS

The purpose of the subject matter covered, some what in details, in this presentation was to make the reader aware of the existing inconsistencies and certain basic flaws in the field of nonequilibrium thermodynamics and arrive at the possible solutions of them. Of course, our discussion to large extent revolves around very basic thermodynamic aspects that normally are brushed aside terming as being of pedagogic level. But if we have to use the new mathematics tool, the Lie admissibility of irreversibility, to streamline the subject matter of nonequilibrium thermodynamics it is essential to ensure that the existing ambiguities at no stage of the development get a chance to creep in.

In the above stated pursuit there could be a demand to have a freshly defined entropy function, which in the limit of reversibility should culminate into the Clausius entropy given by $dS = dQ_{rev}/T$ in equilibrium. For this purpose it would be worth examining whether the *universal inaccessibility principle* formulated by the present author in the nineties of the previous century [34, 64] can serve us or else we would be required to develop a new definition of entropy commensurate with the laws of thermodynamics.

Indeed, the end result that we wish to arrive at is to have a single nonequilibrium thermodynamic framework as there exists only one thermodynamic framework of equilibrium states. Let us be optimistic.

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## REFERENCES

1. R. M. Santilli, *IL Nuovo Cimento* **121 B**, 443-486 (2006).
2. R. M. Santilli, *Open Astronomy J.* **3**, 1-20 (2010).
3. URL http://www.i-b-r.org.
4. S. Glasstone, *Thermodynamics for Chemists*, Princeton, New Jersey: Van Nostrand, 1967.
5. W. H. Cropper, *Am. J. Phys.* **54**, 1068-1074 (1986).
6. I. Prigogine and R. Defay, *Chemical Thermodynamics*, Translated by D. H. Everett, London: Longmans Green, 1954.
7. J. Meixner, *Rheol. Acta* **12**, 465-467 (1973).
8. C. Kalidas and M. V. Sangaranarayanan, *Non-Equilibrium Thermodynamics*, Delhi: Macmillan India, 2002.
9. A. A. Bhalekar, *Asian J. Chem.* **12**, 417-427 (2000).

10. K. J. Laidler, *Chemical Kinetics*, New Delhi: Tata McGraw Hill, 1973.
11. B. C. Eu, *Phys. Rev.* A **51**, 768-771 (1995).
12. B. C. Eu, *Kinetic Theory and Irreversible Thermodynamics*, New York: Wiley, 1992.
13. B. C. Eu and L. S. García-Colín, *Phys. Rev.* A **54**, 2501-2512 (1996).
14. S. R. de Groot and P. Mazur, *Non-Equilibrium Thermodynamics*, Amsterdam: North Holland, 1962.
15. I. Prigogine, *Introduction to Thermodynamics of Irreveersible Processes*, New York: John Wiley-Interscience, 1967.
16. R. Haase, *Thermodynamics of Irreversible Processes*, Reading, MA: Addison-Wesley, 1969.
17. B. H. Lavenda, *Thermodynamics of Irreversible Processes*, London: Macmillan Press, 1978.
18. I. Gyarmati, *Non-Equilibrium Thermodynamics*, Berlin: Springer, 1970.
19. P. Glansdorff and I. Prigogine, *Thermodynamic Theory of Structure, Stability and Fluctuations*, New York: Wiley, 1971.
20. R. P. Rastogi, *Introduction to Non-equilibrium Physical Chemistry. Towards Complexity and Non-linear Sciences*, Amsterdam: Elsevier, 2008.
21. R. C. Srivastava, S. K. Saha, and A. K. Jain, *Thermodynamics*, New Delhi: Prentice-Hall of India, 2007, Chapters 8-10.
22. W. Muschik, *J. Phys. Chem. Solids* **49**, 709-720 (1988).
23. L. Onsager, *Phys. Rev.* **37**, 405-426 (1931); **38**, 2265-2279 (1931).
24. L. Onsager and S. Machlup, *Phys. Rev.* **91(6)**, 1505-1515 (1953).
25. D. Jou, J. Casas-Vázquez and G. Lebon, *Extended Irreversible Thermodynamics*, Berlin: Springer-Verlag, 1996.
26. G. Lebon, D. Jou and J. Casas-Vázquez, *Understanding Non-equilibrium Thermodynamics. Foundations, Applications, Frontiers*, Berlin: Springer, 2008.
27. D. D. Joseph and L. Preziosi, *Rev. Mod. Phys.* **61**, 41-74 (1989).
28. D. D. Joseph and L. Preziosi, *Rev. Mod. Phys.* **62**, 375-392 (1990).
29. S. Chapman and T. G. Cowling, *Mathematical Theory of Non-uniform Gases*, Cambridge: Cambridge University Press, 1970.
30. H. Grad, *Commun. Pure Appl. Math.* **2**, 331-407 (1949).
31. H. Grad, "Priciples of the Kinetic Theory of Gases" in *Thermodynamics of Gases*, edited by S Flügge, Encyclopedia of Physics, Vol. XII, Berlin: Springer, 1958, pp. 205-294.
32. M. J. Lampinen, *J. Non-Equilib. Thermodyn.* **15**, 397-402 (1990).
33. H. Goldstein, *Classical Mechanics*, New Delhi: Narosa Publishing House, 1985.
34. A. A. Bhalekar, *J. Math. Chem.* **5**, 187-196 (1990).
35. A. A. Bhalekar, *Indian J. Phys.* **74B**, 153-157 (2000).
36. A. A. Bhalekar, *Indian J. Phys.* **76B**, 715-721 (2002).
37. A. A. Bhalekar, *Bull. Cal. Math. Soc.* **94**, 209-224 (2002).
38. A. A. Bhalekar, *Bull. Cal. Math. Soc.* **92**, 463-480 (2000).
39. A. A. Bhalekar, *Bull. Cal. Math. Soc.* **94**, 185-196 (2000).
40. P. W. Bridgman, *Proc. Amer. Acad. Arts Sci.* **82**, 301-309 (1953).
41. P. W. Bridgman, *The Nature of Thermodynamics*, Gloucester, MA: Peter Smith, 1969.
42. A. A. Bhalekar and B. Andresen, "On the Nonequilibrium Thermodynamic Roots of the Additional Facets of Chemical Interaction" in *Advances in Thermodynamic Research Including Nonequilibrium Thermodynamics*, edited by G. S. Natarajan et al., NCTCBS-2008, Nagpur, 2008, pp. 53-62.
43. G. M. Wang, E. M. Sevick, E. Mittag, D. J. Searles and D. J. Evans, *Phys. Rev. Letters* **89**, 050601-1-4 (2002).
44. A. A. Bhalekar, *Indian J. Phys.* **77B**, 391-397 (2003).
45. J. Casas-Vázquez and D. Jou, *Rep. Prog. Phys.* **66**, 1937-2023 (2003).
46. W. Muschik, *Arch. Ration. Mech. Anal.* **66**, 379-401 (1977).
47. W. Muschik, *Aspects of Non-Equilibrium Thermodynamics*, Singapore: World Scientific, 1990.
48. W. Muschik and G. A. Brunk, *Int. J. Engng. Sci.* **15**, 377-389 (1977).
49. L. S. García-Colín, *Teoría Cinética de Los Gases*, Colección CBI, México D. F.: Universidad Autonoma Metropolotana, 1990, p. 65.
50. S. R. de Groot, *Thermodynamics of Irreversible Processes*, Amsterdam: North-Holland, 1951.
51. C. Truesdell, *Rational Thermodynamics*, New York: McGraw-Hill, 1969.
52. J. Keizer, *Statistical Thermodynamics of Nonequilibrium Processes*, Berlin: Springer-Verlag, 1987.
53. B. Andresen, "Finite-Time Thermodynamics", Ph.D. Thesis, University of Copenhagen, 1983.
54. M. Grmela, *Phys. Rev.* E **48**, 919-930 (1993).
55. D. Reguera and J. M. Rubí, *Phys. Rev.* E **64**, 061106-1-8 (2001).
56. J. M. Rubí, *J. Non-Equilib. Thermodyn.* **29**, 315-325 (2005).
57. H. C. Öttinger, *Beyond Equilibrium Thermodynamics*, New Jersey: Wiley-Interscience, 2005.
58. F. Bampi and A. Morro, "Nonequilibrium Thermodynamics: A Hidden Variable Approach," in *Recent Developments in Non-Equilibrium Thermodynamics*, edited by J. Casas-Vázquez et al., Lecture Notes in Physics, Vol. 199, Berlin: Springer, 1984.
59. E. F. Lype, *Int. J. Thermodyn.* **7**, 111-124 (1986).
60. G. Maugin and W. Muschik, *J. Non-Equilib. Thermodyn.* **19**, 217-249, 250-289 (1994).
61. A. S. Eddington, *The Nature of the Physical World*, Cambridge: Cambridge University Press, 1931.
62. L. Brillouin, *Science and Information Theory*, New York, Academic Press Inc., 1956.
63. R. Baierlein, *Atoms and Information Theory*, San Francisco: W H Freeman and Company, 1971.
64. A. A. Bhalekar, *Pramana - J. Phys.* **50**, 281-294 (1998).

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# Santilli Lie-Admissible Theory and Genomathematics

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**Abstract.** Far-reaching pioneering discoveries of Santilli's Lie-admissible theory and genomathematics are extending significantly the scope and applicability of Lie analysis, algebra and operator methods, opening new areas of applications leading to essential advances and broadening the scope of models in Physics, Chemistry, Cosmology and in development of new cleaner energy technologies. These new universal mathematical and especially algebraic and operator structures and methods become increasingly important in all branches and levels of modern physics from experiments to fundamental research. This article is devoted to foundations of Santilli's Lie-admissible theory and genomathematics.

**Keywords:** Santilli Lie-admissible algebras, genomathematics, bimodule
**MSC2010:** 17D25

## 1 Introduction

In a series of works, R. M. Santilli has presented rather diversified conceptual, theoretical and experimental elements suggesting a reinspection of the validity of special relativity for interior dynamical problems at large, and the scattering region in particular [114]. The central problem mathematically is in the development of appropriate isoscattering theory which would include the construction of a covering of the Minkowskian geometry, the Lorentz-Poincaré symmetry and special relativity into forms more effective for interior conditions. In his No Reduction Theorems, Santilli rigourously established the impossibility of a consistent reduction of interior to exterior conditions (see [114] and references there). Thus one has to investigate systems that are generally nonlinear in the wave function, nonlocal of integral character, and noncanonical or nonunitary in their time evolution, which requires development of fundamentally new mathematical concepts and tools.

Lie's theory plays fundamental role in Mathematics and Physics. Santilli identified several important limitations of the conventional Lie theory for the treatment of systems beyond the local-differential, Hamiltonian and canonical-unitary conditions. Furthermore, Santilli made a fundamental proposal that this requires new fundamental generalizations

of key structures and concepts of mathematics. The new fundamental mathematical structures and notions, introduced by Santilli under names of iso-, geno- and hyper-mathematics, are motivated by the need of generalized, Hermitian, non-Hermitian and multi-valued units, respectively. The resulting iso-, geno- and hyper-Lie theories based on the new mathematics have been extensively used for the description of nonlocal-integral systems with action-at-a-distance Hamiltonian and short-range-contact non-Hamiltonian interactions in reversible, irreversible and multi-valued conditions, respectively. Motivated by another physics observation that the conventional, iso-, geno- and hyper-Lie theories are unable to provide a consistent classical representation of antimatter yielding the correct charge conjugate states at the operator counterpart, Santilli outlined also another novel mathematics under the names of isodual conventional, iso-, geno- and hyper-mathematics, which constitute anti-isomorphic images of the original mathematics characterized by negative-definite units and norms.

In order to broaden Lie's theory, Santilli proposed in 1967 in [68] the first known deformation of Lie algebras in the physics literature with the two-parameter deformed commutator product

$$(A, B) = p \times A \times B - q \times B \times A, \tag{1}$$

where $p \neq \pm q$ are non-zero scalars, $A, B$ are matrices of the same dimension, and $A \times B$ is the conventional associative product. For $p \neq q$ the Lie algebras axioms, the skew-symmetry and Jacobi identities, are not valid. At the same time Santilli made important discovery that such new bracket products yield Lie-admissible and Jordan-admissible algebras in the sense that their attached antisymmetry and symmetric algebras are Lie and Jordan, respectively. The corresponding Lie-admissible generalization of Heisenberg's equations for the dynamical evolution of a Hermitian operator $A$ in infinitesimal and finite forms are

$$i \times \frac{dA}{dt} = (A, H) = p \times A \times H - q \times H \times A, \tag{2}$$

$$A(t) = e^{H \times q \times t \times i} \times A(0) \times e^{-i \times t \times q \times H}. \tag{3}$$

In 1978, Santilli [71] proposed the most general possible Lie-admissible and Jordan-admissible deformations-mutations of Lie algebras with product

$$(A\hat{,}B) = A \times R \times B - B \times S \times A = A < B - B > A, \tag{4}$$

where $R, S, R \pm S$ are now fixed nonsingular operators with an arbitrary, nonlinear and nonlocal functional dependence on any needed quantity. Mathematically, these brackets provide a very general way of defining algebras with a bilinear composition containing as particular cases associative, Lie, Jordan, supersymmetric, flexible and many other classes of algebras. In [71], Santilli initiated a joint Lie-admissible and Jordan-admissible covering of Lie's theory in its various branches, including the lifting of the universal enveloping algebra with generalized Poincaré-Birkhoff-Witt theorem, Lie algebras, Lie's (transformation) groups and the representation theory. The general products (4) yield the following Lie-admissible and Jordan-admissible deformations-mutations of Heisenberg's

equations proposed in [73]:

$$i \times \frac{dA}{dt} = (A\hat{,}H) = A \times R \times H - H \times S \times A = A < H - H > A, \tag{5}$$

$$A(t) = e^{H \times S \times t \times i} \times A(0) \times e^{-i \times t \times R \times H}, \tag{6}$$

$$R = S^{\dagger} \tag{7}$$

as the foundation of Hadronic Mechanics for the representation of the most general possible open, nonconservative, irreversible and single-valued systems with potential interactions represented by the nonconserved Hamiltonian $H$, and contact nonpotential, nonlinear, nonlocal-integral and nonunitary interactions represented by the operators $R, S$. The generalized dynamical equations (5), (6), (7) if formulated on conventional Hilbert spaces over conventional fields turned out to verify the Theorems of Catastrophic Mathematical and Physical Inconsistencies of Noncanonical and Nonunitary Theories because not preserving over time the basic units of measurements, the observability of physical quantities, the numerical predictions, etc. The inconsistency theorems emerge whenever one leaves noncanonical or unitary time evolutions, and since no possibility of bypassing them with the Lie's theory are known, they also provide important motivation for further mathematical research [112].

The first major advance in the resolution of the above inconsistencies occurred after decades of additional research in 1993 with the discovery of the genonumbers and genofields [87], namely, fields with a fixed order of all multiplications to the right (representing motion forward in time) and an arbitrary right and left generalized unit called genounit for the ordering to the right, $n > m = n \times S \times m$, $\hat{I}^{>} = S^{-1}$, and with the corresponding ordering of all multiplications to the left (representing motion backward in time) with related genounit for the ordering to the left $n < m = n \times R \times m$, ${}^{<}\hat{I} = R^{-1}$, where the word "genotopy" [71] was used in the Greek meaning of inducing new axioms. A breakthrough occurred in the mathematical memoir [95] of 1996 with the discovery of the new genodifferential calculus to the right or to the left. The first invariance over time of deformations-mutations of Lie algebras was proved in paper [101] of 1997. The most developed up to date axiomatic structure of Lie-admissible formulations was achieved in memoir [112] of 2006 that also presented the first known connection between mechanics and thermodynamics, by showing that the irreversibility of thermodynamical laws originates at the ultimate level of nature, in full confirmation of the No Reduction Theorems. Such excessive mathematical complexities are needed for the consistent treatment of irreversible scattering processes, but could be beyond the reach of most phenomenologists. Thus the restriction of these initial studies to reversible scattering processes, and then the passage to the more complex irreversible events only subsequently was suggested in [114] as a way to move forward. The initial restriction to reversible processes eliminates the need of the time ordering of all products, with consequential major simplification of the formalism. From the viewpoint of applied mathematics, the restriction to reversible scattering processes permits the preservation of Lie's axioms, despite the admission of nonlinear, nonlocal and noncanonical or nonunitary effects.

Already in 1978 in [71], Santilli identified the following particularization of the Lie-admissible and Jordan-admissible product (4):

$$[A\hat{,}B] = A\hat{\times}B - B\hat{\times}A = A \times T(x, v, \xi, \omega, \psi, \partial\psi, ...) \times B - B \times T(x, v, \xi, \omega, \psi, \partial\psi, ...) \times A,$$

$$R = S = T = T^{\dagger} > 0,\ \hat{I}(x, v, \xi, \omega, \psi, \partial\psi, ...) = \frac{1}{\hat{T}(x, v, \xi, \omega, \psi, \partial\psi, ...)} > 0,$$

where $\hat{I}(x, v, \xi, \omega, \psi, \partial\psi, ...)$ and $\hat{T}(x, v, \xi, \omega, \psi, \partial\psi, ...)$ are the isounit and the isotopic element at the foundation of the mathematics in [114], and have the same functional dependence on local variables (coordinates, speeds, other parameters, etc.). Such products do verify Lie's axioms. Such deformations of Lie algebras were called isotopic by Santilli [71]. In the same paper [71], Santilli proposed a step by step isotopic generalization of Lie's theory that has remained structurally unchanged to this day (except for the subsequent reformulation on isospaces over isofields), and it is today known as the Lie-Santilli isotheory [6, 12, 13, 14, 29, 30, 31, 35, 48, 56, 117].

The main idea of isotheory is that of preserving unchanged the generators of a given Lie symmetry and changing instead all their operations in an axiom-preserving way (as a condition to have an isotopy) [71]. The implementation of this idea require the lifting of the conventional associative product $A \times B$ into the axiom-preserving isoassociative form $A \times T \times B = A\hat{\times}B$ that, in turn, implies the lifting of the Lie product $[A.B]$ into the axiom-preserving form [112].

This seemingly elementary idea has important implications for the scattering theory. By recalling that the generator of a Lie symmetry represents conserved quantities, the preservation of the generators in the transition from the conventional to the isotopic scattering theory implies the preservation of all conventionally conserved quantities. However, the appearance of the isotopic element $T$ in the product itself implies that said preservation occurs under nonlinear, nonlocal and noncanonical or nonunitary internal effects, thus warranting a reinspection of the data elaboration via the conventional linear, local and unitary scattering theory. Moreover , first important steps towards all-sided investigation of far-reaching implications of Santilli's Lie-isotopic Theory for cosmology, gravitation and relativity are made in [6, 81, 82, 83, 84, 110, 113]. These investigations indicate exciting possibilities and need for expanding of future research in this direction both in the development of fundamental theory and methods, and in numerical simulations and experiments.

## 1.1 Lie-Santilli isotopies of associative and Lie algebras

As previously mentioned, as early as 1967, in the pioneering contribution [68] Santilli considered the two-parametric deformations (mutations) of the Lie commutator bracket in an associative algebra, $(A, B) = pAB - qBA$, where $p$ and $q$ are scalar parameters and $A$ and $B$ are elements in the associative algebra (typically algebra of matrices or linear operators). In 1978, Santilli [71, 72, 73] made another bold step and extended it to "operator-deformations" of the Lie product as follows $(A, B) = APB - BQA$, where $P$ and $Q$ are fixed elements in the underlying associative algebra. This seemingly simple generalization of the bracket has fundamental significance for applications since clearly elements $P$ and $Q$ can now be dependent on as many as needed in the model or possibly even infinitely many parameters or unknown variables.

The motivation from physics side for introducing such non-associative generalizations of the bracket multiplication came from attempts to resolve certain limitations of conventional formalism of classical and quantum mechanics. Subsequently, in numerous

works including articles and books by Santilli and other authors, the evolution equations based on such deformed brackets, physical applications and consequences of introducing such generalized models have been investigated. The deformations of the commutator bracket multiplication introduced by Santilli in investigations on foundations of classical and quantum mechanics and hadronic physics, have reappeared in many incarnations both in Mathematics and Physics. In [14, 71, 72, 73, 75, 77, 101, 107, 112, 117] further progress have been made in investigation of the models based on introduction of such non-associative deformed commutator bracket multiplications instead of commutator (Lie) bracket multiplication, their bi-module type generalizations (genoalgebras) as well as for a review of relation with other appearances of so deformed commutator brackets in physics and mathematics, for example in contexts related to quantum algebras, quantum groups, Lie algebras and superalgebras, Jordan and other classes of algebras.

While motivation for introduction and investigations of such brackets and generalizations of corresponding models is coming from physics side, the new algebraic and analytic structures and problems arising in connection to such modified brakets and associated operations have definitely independent interest also for various parts of Mathematics ranging from algebra and commutative and noncommutative geometry to topology, differential geometry, functional analysis, operator theory and operator algebras, differential equations and numerical analysis.

The Santilli's products $(A, B) = APB - BQA$ with arbitrary $P$ and $Q$ are not antisymmetric in general except when $P$ and $Q$ are specially interrelated within the underlying algebra. The products $A \times_Q B = AQB$ are clearly associative since $(AQB)QC = AQ(BQC)$. Therefore, in the case when $P = Q$, the Santilli's bracket product $(A, B) = AQB - BQA$, being exactly the commutator bracket for the associative product $A \times_Q B$, satisfies the skew-symmetry and Jacobi identity for Lie algebras. So the associative algebras with the modified products $A \times_Q B = AQB$ are Lie admissible algebras. Moreover, since

$$< A, B >= (A, B) - (B, A) = (APB - BQA) - (BPA - AQB) = A(P+Q)B - B(P+Q)A,$$

the general Santilli's deformed commutator products $(A, B) = APB - BQA$ define Lie admissible algebras.

Any associative algebra is Lie admissible since the commutator bracket on any associative algebra satisfies axioms of a Lie algebra. Santilli has considered also so called isotopies of associative and Lie algebras. Algebraic problems can be formulated as follows.

- How can associative products in associative algebras be modified to yield as general as possible Lie admissible algebras?
- Can any Lie admissible algebra be obtained by such modifications from some associative algebra?

Such modifications of associative and corresponding Lie algebras where called isotopies of associative and Lie algebras. In [71, 72, 73, 75, 101, 107, 112], several general isotopies of associative products and associated Lie products have been identified. The most general of all presented there isotopies of a product for elements in an associative algebra $\mathcal{A}$ over a field $\mathbb{K}$ is given by

$$x * y = awxwTwyw,$$

where $a \in \mathbb{K}, w \in A, w^2 = w \neq 0$, and $T$ is some extra element. The product $*$ is associative if $w^2 = w$ and $T \in A$. Santilli allows $T$ to be some extra element outside $\mathcal{A}$. Then a special care is needed on algebraic side in order to make involved objects and maps to be properly defined. If $a(w)(x)(w)T(w)(y)(w)$ is not identified with some element of $\mathcal{A}$, then the new product $*$ is taking values in some generally non-associative algebra $A_T$ generated, as a linear space over $\mathbb{K}$, by elements $x \in A$ and formal expressions of the form $x_1Tx_2Tx_3 \dots x_{n-1}Tx_n \in ATATA \dots ATA$ for $x_1, \dots, x_n \in A$ for integers $n \geq 2$, whatever these expressions mean. With a $\mathbb{K}$-bilinear product on $A_T$ defined for $x, x_1, \dots, x_n, y_1, \dots, y_m \in A$ by

$$
\begin{aligned}
&(x_1Tx_2Tx_3 \dots x_{n-1}Tx_n)(y_1Ty_2Ty_3 \dots y_{n-1}Ty_m) = \\
&x_1Tx_2Tx_3 \dots x_{n-1}T(x_ny_1)Ty_2Ty_3 \dots y_{m-1}Ty_m, \\
&x(x_1Tx_2Tx_3 \dots x_{n-1}Tx_n) = (xx_1)Tx_2Tx_3 \dots x_{n-1}Tx_n, \\
&(x_1Tx_2Tx_3 \dots x_{n-1}Tx_n)x = x_1Tx_2Tx_3 \dots x_{n-1}T(x_nx),
\end{aligned}
$$

in particular, $u(xTy) = (ux)T(y)$, $(xTy)v = (x)T(yv)$, $u(xTy)v = (ux)T(yv)$ hold for $u, x, y, v \in A$. Then the expression $x * y = a(w)(x)(w)T(w)(y)(w)$ yields again an element from $A_T$ for $x, y \in A_T$, and we get the product $*$ on the algebra $A_T$. If now $w^2 = w$, then $*$ satisfies the associativity condition $x * (y * z) = (x * y) * z$ on $A_T$. Indeed, for $x = x_1Tx_2Tx_3 \dots x_{n-1}Tx_n$, $y = y_1Ty_2Ty_3 \dots y_{n-1}Ty_m$ and $z = z_1Tz_2Tz_3 \dots z_{n-1}Tz_k$, using $w^2 = w$, one gets

$$
\begin{aligned}
x * (y * z) &= a(w)(x_1Tx_2Tx_3 \dots x_{n-1}Tx_n)(w)T(w) \\
&\quad (a(w)(y_1Ty_2Ty_3 \dots y_{n-1}Ty_m)(w)T(w) \\
&\quad (z_1Tz_2Tz_3 \dots z_{n-1}Tz_k)(w))(w) \\
&= a^2(w)(x_1Tx_2Tx_3 \dots x_{n-1}Tx_n)(w)T \\
&\quad (w)(y_1Ty_2Ty_3 \dots y_{n-1}Ty_m)(w)T(w) \\
&\quad (z_1Tz_2Tz_3 \dots z_{n-1}Tz_k)(w) \\
&= a(w)(a(w)(x_1Tx_2Tx_3 \dots x_{n-1}Tx_n)(w)T(w) \\
&\quad (y_1Ty_2Ty_3 \dots y_{n-1}Ty_m)(w))(w)T(w) \\
&\quad (z_1Tz_2Tz_3 \dots z_{n-1}Tz_k)(w) \\
&= (x * y) * z.
\end{aligned}
$$

Let $\alpha(s) = wsw$ for all $s \in A_T$. If $w^2 = w$ and $T \in A$, then

$$
\begin{aligned}
\alpha(s * t) = w(s * t)w = w(awswTwtw)w = \\
a(w(wsw)wTw(wtw)w) = aw\alpha(s)wTw\alpha(t)w = \alpha(s) * \alpha(t)
\end{aligned}
$$

which means that in this case $\alpha$ is not just a linear map, but also an algebra endomorphism. One may show also that this associative algebra is also hom-associative with the twisting

map $\alpha$:

$$\begin{aligned}\alpha(x)*(y*z) &= aw(wxw)wTw(awywTwzw)w\\ &= awxwTw(awywTwzw)w\\ &= x*(y*z)=(x*y)*z\\ &= aw(awxwTwyw)wTwzw\\ &= aw(awxwTwyw)wTw(wzw)w\\ =(x*y)*\alpha(z).&\end{aligned}$$

The other fundamental algebraic issue tackled by Santilli is imbedding of the scalar field into the algebras over this filed. If an algebra $\mathcal{A}$ over the field $\mathbb{K}$ with a unit $1_{\mathbb{K}}$ has a unit $1_A$, then there is a canonical imbedding of the field into the algebra given by $i_A : c \mapsto c1_A$ for $c \in \mathbb{K}$. Also one has $1_A x = x1_A = x$. If the multiplication in $A_T$ is defined as $x * y = xTy$ (corresponding to $a = 1_{\mathbb{K}}$ and $w = 1_A$) then one still would like to have $1_{A_T} * x = x * 1_{A_T} = x$, which can be written as $1_{A_T}Tx = xT1_{A_T} = x$. Thus, if $T$ has left and right inverse $T^{-1}$, then $1_{A_T} = T^{-1}$. Also the canonical imbedding of the field into the new algebra yields $i_{A_T}(1_{\mathbb{K}}) = 1_{\mathbb{K}}1_{A_T} = 1_{\mathbb{K}}T^{-1}$ and more generally $i_{A_T}(c) = c1_{A_T} = cT^{-1}$ for $c \in \mathbb{K}$. These elements form a field inside $A_T$ with the unit $\hat{I} = 1_{\mathbb{K}}T^{-1}$. This field $\hat{\mathbb{K}}$ is called isofield. Santilli noticed that dependence on $T$ of the new unit in the isofield, caused by the changed product in the algebra, is not just some complicating curiosity, but advantageous phenomena that opens new vast fundamental opportunities in physics, differential geometry, tensor calculus and beyond. This is because, while the unit $1_{\mathbb{K}}$ in the scalar field $\mathbb{K}$ is fixed, $T$ and thus the unit $\hat{I}$ in the isofield $\hat{\mathbb{K}}$ can be chosen to depend on the non-linear functionals or expressions in some other parameters, functions, their derivatives, integrals, etc., in physics having interpretation as time, position, speed, momentum, acceleration, mass, energy, etc. This dependence may be well highly non-linear. Santilli has made an effort in systematic analysis of how the new algebra structures and introduction of isofield $\hat{\mathbb{K}}$ and parameter dependent isounits effect the equations of motion, time evolution and other basics of Hamiltonian and quantum mechanics. These first steps open a huge field for further research in many directions of interest both in physics and mathematics.

# 2 Santilli's extensions of Lie Theory

Lie's theory is fundamental for the virtual entire contemporary mathematics and physics [46]. Identification of the limitations and possible generalizations of Lie's theory suitable for treating broader physical conditions has been addressed in [68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 87, 88, 89, 96, 97, 99, 100, 101, 102, 104, 105, 106], [10, 27, 28, 32, 33, 34, 35, 37, 38, 39, 47, 118, 119, 120, 124, 125], [4, 5, 7, 15, 16, 17, 18, 19, 21, 22, 24, 25, 45, 50, 51, 52, 58, 59, 60, 61, 62, 63, 64, 65] monographs [29, 30, 31, 117], and also a number of publications pertaining to applications and experimental verifications [2, 3], [108, 109], [8, 9, 40, 41, 42, 43, 50, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 98].

In Physics the first important instance of appearance of Lie's theory is that of equations for continuous differentiable operator evolution on a Hilbert space $\mathcal{H}$ over a field $\mathbb{F} =$

$\mathbb{F}(a, +, \times)$ of conventional numbers (real, complex or quaternionic) with conventional sum $+$, (associative) product $\times$, additive unit 0 and multiplicative unit I:

$$A(w) = U \times A(0) \times U^{\dagger} = e^{iX \times w} \times A(0) \times e^{-iw \times X}, \tag{8}$$

$$i\frac{dA}{dw} = A \times X - X \times A = [A, X]_{operator}, \tag{9}$$

$$e^{iX \times w} = [e^{-iw \times X}]^{\dagger}, X = X^{\dagger}, w \in F. \tag{10}$$

The classical counterpart in terms of vector-fields on the cotangent bundle (phase space) with local chart $(r^k, p_k)$, $k = 1, 2, 3$, over $\mathbb{F}$ is

$$A(w) = U \times A(0) \times U^{t} = e^{-w \times (\partial X/\partial r^k) \times (\partial/\partial p_k)} \times A(0) \times e^{w(\partial/\partial r^k) \times (\partial X/\partial p_k)}, \tag{11}$$

$$\frac{dA}{dw} = \frac{\partial A}{\partial r^k} \times \frac{\partial X}{\partial p_k} - \frac{\partial X}{\partial r^k} \times \frac{\partial A}{\partial p_k} = [A, X]_{classical}. \tag{12}$$

An interconnecting map is given by the conventional or symplectic quantization. When the parameter $w$ represents time, (8), (9), (10) are the celebrated Heisenberg equations of motion, while (11) and (12) are the classical Hamilton equations. These objects and equations are in the core of the characterization via Lie's theory of classical and operator branches of physics.

From Mathematics side, the operator family defined by (8) serves as a main building block for transformation Lie groups (linear operator representations of abstract Lie groups). Namely, the transformations from a general transformation Lie group, typically dependent on many parameters linked to the dimension of the underlying manifold of a Lie group, can be built from the one-parameter subgroups using the main properties of the exponential function of operators with respect to composition. The product of exponentials of commuting operators can be expressed as an exponential of their sum precisely as for ordinary exponential function on numbers at least formally. Commuting families of operators and their exponentials and other functions are of central importance in Physics and Mathematics in many ways, but are however far from being enough, as the majority of interesting Lie groups are non-commutative. However, even for non-commutative Lie groups, the one-parameter groups still may be used as building blocks, using more complicated formulas such as Campbell–Baker–Hausdorff–Dynkin formula, Zassenhaus formula, Hadamard formula, Lie–Trotter, Trotter–Kato formulas, etc. From the point of view of Lie theory, such formulas can be interpreted as aiming at establishing correspondence between elements in Lie algebras and Lie groups. For all these formulas an important issue in Mathematics and in applications in Physics and Engineering, is how to handle the convergence of series and limit operations in such formulas. When the formulas are applied to finite-dimensional matrices or bounded operators, the convergence issues can be often resolved in some pleasurable ways. Many important operators (observables) in Physics and Engineering involving differential operators are unbounded and partially defined in the sense of being defined only on some subspaces of the Hilbert or Banach spaces. For such operators convergence issues and thus applicability of the above mentioned formulas involving infinite operator series and iterated operator commutators is a subject of intensive investigations important both in Physics and in Mathematics and constituting

one of the main challenges for further development of Lie theory and its applications. Attempts of formally computing with divergent series leads to serious consequences and contradictions both in Mathematics and in applications.

These issues from Lie theory turn into even more interesting and to large extent open directions for the Santilli's vast generalizations of Lie theory. In brief these open directions (problems) can be formulated as follows:

- What are the suitable modifications of the Campbell-Baker-Hausdorff-Dynkin formulas, Zassenhaus formula, Hadamard formula, Lie-Trotter, Trotter-Kato formulas and similar formulas when the Lie bracket products are replaced by Santilli's generalizations?

- How the convergence issues can be tackled in these formulas when applied to operators?

Santilli's generalizations of Lie products through the deforming elements (operators) may involve as many further parameters or variables as needed. Thus solutions of these two open problems have to take into consideration dependence on these parameters.

If $X$ is self-adjoint $X = X^{\dagger}$, then $U = e^{iX \times w}$ and $U^{-1} = e^{-iX \times w}$ are unitary

$$U \times U^{\dagger} = U^{\dagger} \times U = I, \tag{13}$$

and the following "invariance" laws for units, products and eigenvalue equations hold

$$I \to U \times I \times U^{\dagger} = I' = I, \tag{14}$$

$$A \times B \to U \times (A \times B) \times U^{\dagger} = (U \times A \times U^{\dagger}) \times (U \times B \times U^{\dagger}) = A' \times B', \tag{15}$$

$$H \times |\psi >= E \times |\psi > \to U \times H \times |\psi >= (U \times H \times U^{\dagger}) \times (U \times |\psi >) = H' \times |\psi' >= \tag{16}$$

$$U \times E \times |\psi >= E' \times |\psi' >, E' = E. \tag{17}$$

In this sense the Lie's theory at least on the level of unitary representations possesses invariance for units, products and eigenvalues. Typically this kind of conditions arise in theoretical physics models to assure consistency at the fundamental level with those basic laws of physical systems and their observations which are concerned with expected preservation of some physical quantities throughout the evolution of the physical systems of some class.

Despite the above mathematical and physical consistency of Lie's theory with typical "invariance" properties of basic physical quantities or their observations, Lie's theory by no means is enough to represent the totality of systems existing in the universe. Moreover, there are many engineering problems, models and systems where such invariance properties are not necessary, but where clearly approaches based on Lie theory or generalizations of Lie theory such as those proposed by Santilli are required for further progress.

Returning to Physics, according to Santilli, the inspection of structures (8), (9), (10) and (11), (12) reveals that, in its conventional formulation, Lie's theory can only represent isolated-conservative-reversible systems of point-like particles with only potential-Hamiltonian internal interactions, because the point-like structure is demanded by the local-differential character of the underlying topology; the isolated-conservative character

of the systems is established by the fact that the brackets $[A, B]$ of the time evolution are totally antisymmetric, thus implying conservation laws of total quantities; the sole potential character is established by the representation of systems solely via a Hamiltonian; and the reversibility is established by the fact that all known action-at-a-distance interactions are reversible in time (i.e., their time reversal image is as physical as the original one); all admissible interactions are represented via time-independent potentials in the Hamiltonian, resulting in manifestly reversible systems. Motivated by this insight, Santilli initiated a long term research program aiming at generalizations (liftings) of Lie's theory suitable for the representation of broader physical systems.

The first lifting proposed by Santilli was for representing open-nonconservative systems, that is, systems whose total energy $H$ is not conserved in time, $i\frac{dH}{dt} \neq 0$, because of interactions with the rest of the universe [68, 69]. This is one of the important instances where introducing brackets which are not totally antisymmetric become important. Motivated by this Santilli, as far as we know, was the first to propose in 1967 as fundamental object the simultaneous $(p-q)$-parametric deformations of Lie commutator brakets and respective operator and classical evolution equations of physical systems:

$$A(w) = U \times A(0) \times U^{\dagger} = e^{iw \times p \times X} \times A(0) \times e^{-iw \times q \times X}, X = X^{\dagger}, \tag{18}$$

$$i\frac{dA}{dw} = p \times A \times X - q \times X \times A = (A, X)_{operator}, \tag{19}$$

where p, q and $p \pm q$ are non-zero parameters, with classical counterpart [70]:

$$A(w) = U \times A(0) \times U^{t} = e^{-w \times q \times (\partial X/\partial r^k) \times (\partial/\partial p_k)} \times A(0) \times e^{w \times p \times (\partial/\partial r^k) \times (\partial X/\partial p_k)}, \tag{20}$$

$$\frac{dA}{dw} = p \times \frac{\partial A}{\partial r^k} \times \frac{\partial X}{\partial p_k} - q \times \frac{\partial X}{\partial r^k} \times \frac{\partial A}{\partial p_k} = (A, X)_{classical}. \tag{21}$$

Such modifications of algebras and related operator representations and systems of equations involving such deformed commutator brackets became known in more recent times as $q$-deformations both in Mathematical Physics and Mathematics literature and arise in many different contexts.

The highly fruitful idea of simultaneous deformations of brakets and evolution equations, have lead Santilli to another fundamentally important and also in a sense unavoidable insight, that physical models based on such generalized deformed brakets and equations require new understanding and in many cases substantial generalizations of underlying algebraic and analytic structures, thus insisting on development of new notions, methods and directions in algebra, analysis, geometry and other parts of Mathematics. While some of the abstract structure was already slowly but definitely emerging within Mathematics often with limited or no relation to Physics, many of the structures and notions suggested by Santilli were new even from Mathematical side at the time of their introduction; and certainly the physical motivation, intuition and open problems brought out by Santilli was a novel and fundamental input both on Physics and Mathematics side.

The investigations on Lie admissible algebras were still in their infancy at the time of [68] with only handfull of publications on Lie-admissible algebras and mostly only in the mathematical literature (for example, the simple search in MathSciNet/Mathematical reviews database on "Lie admissible" in the search field "Anywhere" and before 1967

produces just 9 publications including [68]). So, introduction by Santilli of the new classes of Lie admissible algebras via deforming Lie algebras brackets and recognizing the fundamental significance of the Lie-admissible algebras, generalizations of Lie algebras and related non-associative structures for Physics as well as testing boundaries of their applicability in Physics contexts have been a pioneering and highly nontrivial and original advance made ahead of its time. The $(p-q)$-parametric deformations (18), (19), (20), (21) achieve at least one of the basic desired physical objectives, the possibility of not conserved total energy and other physical quantities enabling $i\frac{dH}{dt} = (p-q) \times H \times H \neq 0$.

The next major advance was made by Santilli in [71, 72, 73] in 1978 with the introduction of the most general $(P-Q)$-operator Lie-admissible theory by considering the following vast simultaneous generalization of the brackets and operator and classical evolution equations of Physical systems:

$$A(w) = U \times A(0) \times U^{\dagger} = e^{iw \times X \times Q} \times A(0) \times e^{-iw \times P \times X}, X = X^{\dagger}, P = Q^{\dagger}, \tag{22}$$

$$i\frac{dA}{dw} = A \times P \times X - X \times Q \times A = (A\hat{,}X)_{operator}, \tag{23}$$

where $P, Q$, and $P \pm Q$ are non-singular operators such that $P - Q$ characterizes Lie brackets, with classical counterpart [71, 72, 73]:

$$A(w) = U \times A(0) \times U^{t} = e^{-w \times (\partial X/\partial r^i) \times Q^i_j \times (\partial/\partial p_k)} \times A(0) \times e^{w \times (\partial/\partial r^i) \times P^i_j (\partial X/\partial p_j)}, \tag{24}$$

$$\frac{dA}{dw} = \frac{\partial A}{\partial r^i} \times P^i_j \times \frac{\partial X}{\partial p_j} - \frac{\partial X}{\partial r^i} \times Q^i_j \times \frac{\partial A}{\partial p_j} = (A, X)_{classical}. \tag{25}$$

Among motivating basic features achieved by this generalization are that the Lie-admissible structure is preserved under nonunitary transforms and that the structures (22), (23) constitute the most general possible transformations admitting an algebra in the infinitesimal form. It turns out that the product $(A\hat{,}B)$ is jointly Lie-admissible and Jordan-admissible with the attached Lie and Jordan algebras being more general than the conversional ones. Santilli proposed also a particularization of the above Lie-admissible theory, the Lie-isotopic theories [71, 72, 73, 74, 77] in which the brackets do verify the Lie axioms, but are more general than the conventional versions:

$$A(w) = U \times A(0) \times U^{\dagger} = e^{iX \times T \times w} \times A(0) \times e^{-iw \times T \times X}, \hat{T} = \hat{T}^{\dagger}, \tag{26}$$

$$i\frac{dA}{dw} = A \times T \times X - X \times T \times A = [A\hat{,}X]_{operator}, \tag{27}$$

and classical counterpart [71, 72, 73, 74, 77]:

$$A(w) = e^{-w \times (\partial X/\partial r^i) \times T^i_j \times (\partial/\partial p_j)} > A(0) < e^{-w(\partial/\partial p_j) \times T^i_j \times (\partial X/\partial r^i)}, \tag{28}$$

$$\frac{dA}{dw} = \frac{\partial A}{\partial r^i} \times T^i_j \times \frac{\partial X}{\partial p_j} - \frac{\partial X}{\partial r^i} \times T^i_j \times \frac{\partial A}{\partial p_j} = [A\hat{,}X]_{classical}. \tag{29}$$

These Santilli's Lie-isotopic theories too are nonunitary-noncanonical, and the application of additional nonunitary-noncanonical transforms preserves the Lie-isotopic character.

The transformations (26), (27), (28), (29) are the most general ones admitting a Lie algebra in the brackets of their infinitesimal versions.

Santilli's Lie-admissible and Lie-isotopic theories (18),(19)–(28), (29) constitute significant conceptual advance and excellent subject for further Mathematical research of high interest for engineering and development of new numerical computational tools and methods.

In the transition from unitary to nonunitary theories, "invariance" properties of unitary evolution (14), (15), (17) typically are replaced by the following "noninvariances":

$$U \times U^{\dagger} = U^{\dagger} \times U \neq I,$$

$$I \to U \times I \times U^{\dagger} = I' \neq I,$$

$$A \times B \to U \times (A \times B) \times U^{\dagger} =$$

$$(U \times A \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times B \times U^{\dagger}) = A' \times T \times B', T = (U \times U^{\dagger})^{-1},$$

$$H \times |\psi >= E \times |\psi > \to U \times H \times |\psi >= (U \times H \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times |\psi >) =$$
$$H' \times T \times |\psi' >= U \times E \times |\psi >= E' \times |\psi' >, E' \neq E.$$

Santilli pointed out that all the theories with a nonunitary structure have the following drawbacks when it comes to their applications in physics. He call them "inconsistencies". They have been studied in detail in [23, 26, 49, 66, 103, 107, 116] and following Santilli can briefly be described as follows:

1) nonunitary theories do not have invariant units of time, space, energy, etc., thus lacking any physically meaningful applications to measurements (for which the invariance of the basic units is a necessary pre-requisite);

2) nonunitary theories do not preserve in time the original Hermiticity of operators, thus having no physically acceptable observables;

3) nonunitary theories do not have invariant conventional and special functions and transforms, thus lacking unique and invariant numerical predictions; nonunitary theories violate probability and causality laws; nonunitary theories are incompatible with Galilei's and Einstein's relativities; and suffer from other serious shortcomings. Similar inconsistencies exist at the classical level.

Santilli also pointed out that corresponding mathematical inconsistencies also occur [103, 107] in the sense that nonunitary theories are generally formulated on a conventional metric or Hilbert space defined over a given field which, in turn, is based on a given unit I, but the fundamental unit is not left invariant by nonunitary transforms. From the general fundamental theoretical Physics point of view, those arguments might be interpreted as limiting the scope of applications of such Lie-isotopic and Lie-admissible theories in Physics due to nonunitary structure at the operator level with a noncanonical structure at the classical counterpart. In models of concrete physical systems or observations however such general "no-go" arguments might be a different issue from the issue of applicability. It should be noted that the above inconsistencies also hold for any other theory departing

from Lie's theory, yet formulated via conventional mathematics, such as deformations, Kac-Moody algebras, superalgebras, etc.

Based on analysis of the underlying mathematical reasons for the above general "inconsistancies", Santilli came to an important conclusion, that the only possibility to reach invariant formulations of generalized Lie theories was that of constructing new mathematics. In the subsequent numerous publications, Santilli and other researchers both from Mathematics and from Physics communities, made substantial progress on extending all the essential aspects of conventional mathematics, such as numbers and fields, vector and metric spaces, algebras and groups, geometries, etc. This has shown however to be not enough to achieve invariance. The problem remained in the ordinary differential calculus until Santilli finally achieved invariance following suitable liftings of the ordinary differential calculus in [97] in 1996.

In Lie-Santilli Isotheory, the conventional $n$-dimensional unit $I = diag(1, 1, ..., 1)$ of Lie's theory in its matrix representation is lifted into an isounit which is a real, nowhere singular and positive-definite $n \times n$-dimensional matrix $\hat{I}$ which may have functional dependence on time $t$, coordinates $r = (r^k)$, momenta $p = (p_k)$, $k = 1, 2, 3$, wave functions $\psi$, or any other needed variables or parameters,

$$\hat{I}(t, r, p, \psi, ...) = \frac{1}{\hat{T}} \neq I. \tag{30}$$

This is done in such a way that the new isounit fits with the Santilli's modification of the Lie's commutator product by element $\hat{T}$.

This seemingly cosmetic change turns out to be of pivotal fundamental nature not only from Physics but also from Mathematics point of view. This is because, in order for the new unit, isounit, to fit well with other structures and operations on place of the ordinary constant unit, adjustments must be done in most of the underlying mathematical structures and operations. The development of these new mathematical structures, concepts and theory received the special name, Isomathematics.

Here it should be mentioned that from the point of view of Physics the correctly chosen unit or more specifically its dependence of necessary parameters have fundamental significance which yields then the suitable choices of the products and other algebraic and analytic structures participating in basic equations. That is an interesting situation, since in algebra and the whole Mathematics, it is the products and other algebraic structures and operations that are introduced in the first place and are the main objects of study. For a given algebraic structure like for example a ring or an algebra or a semigroup one then defines what is ment by a unit and asks important question of whether there exists such unit element with respect to this structure, whether such element is unique if exists, etc. Many very important for physics and other applications of mathematics associative algebras or groups for example do not have a unit. Often this is related to such important properties of spaces as non-compactness. For example, the continuous real-valued functions on the real line decaying to zero at infinities form an associative and commutative algebra with respect to point-wise multiplication but do not contain any unit element since the only possible candidate of such a unit element, the constant function equal to 1 everywhere does not belong to the algebra since it does not decay to zero at infinities. There are many ways of adjoining a unit to such an algebra without

unit. Often adjoining a unit has a close connection to compactifying spaces associated to an algebra such as subspaces of the dual space.

What are the relations between different ways of adjoining the unit to an algebra and the Santilli's approach of simultaneous modification of the unit and the algebraic structure?

This is an interesting and to larger extent open question which if mathematically properly investigated can lead to important insights in Santilli's Lie isotopic theory and shed light on its important interrelations with deep questions in Topology, Functional Analysis and Algebra.

Recall that in any associative algebra the unit, if exists, is unique. Thus in order to keep the same elements but to have a different unit, the multiplication need to be modified, thus defining a new algebra structure on the same set of elements. The isoassociative product, the lifting of the associative product $A \times B$, for which $\hat{I}$ is the left and right unit, can be defined as follows:

$$A \times B \to A\hat{\times}B = A \times \hat{T} \times B, \quad \hat{I}\hat{\times}A = A\hat{\times}\hat{I} = A. \tag{31}$$

By now many basic fundamental mathematical structures and results have been lifted in Lie-Santilli isotheory into their isotheory counterparts including among others, the lifting of fields into the isofields of isonumbers [87], of functions into isofunctions and of transforms into isotransforms [88, 89, 97]; isotopic lifting of the ordinary differential calculus into the isodifferential calculus [97], as well as the isotopic lifting of conventional vector, metric and Hilbert spaces, such as lifting of Euclidean space into the isoeuclidean spaces with isocoordinates and isometric over the isoreals, or lifting of the Hilbert space with inner product over the complex numbers into the isohilbert space with isoinner product over the isocomplex field [88, 89]; the isotopic lifting of geometries and topologies [104], and the isotopic lifting of Lie's theory including universal enveloping associative algebras and the Poincaré-Birkhoff-Witt theorem, Lie's first, second and third theorems, Lie's groups, transformation and representation theory, etc.

The Lie-Santilli isotheory has the desired feature of being form invariant under nonunitary transforms. An arbitrary nonunitary transform on a Hilbert space $\mathcal{H}$ over a field $\mathbb{F}$ can be uniquely written as the isounitary transform on lifted space $\hat{\mathcal{H}}$ over lifted $\hat{\mathbb{F}}$:

$$V \times V^\dagger = \hat{I} \neq I, V = \hat{V} \times \hat{T}^{1/2},$$
$$V \times V^\dagger = \hat{V}\hat{\times}\hat{V}^{\hat{\dagger}} = \hat{V}^{\hat{\dagger}}\hat{\times}\hat{V} = \hat{I},$$

with the isoinvariance laws

$$\hat{I} \to \hat{V} \times \hat{I} \times \hat{V}^{\hat{\dagger}} = \hat{I}' = \hat{I},$$
$$\hat{A}\hat{\times}\hat{B} \to \hat{V}\hat{\times}(\hat{A}\hat{\times}\hat{B})\hat{\times}\hat{V}^{\hat{\dagger}} =$$
$$(\hat{V}\hat{\times}\hat{A}\hat{\times}V^\dagger)\hat{\times}(\hat{V}\hat{\times}\hat{B}\hat{\times}\hat{V}^{\hat{\dagger}}) = \hat{A}'\hat{\times}\hat{B}',$$
$$\hat{H}\hat{\times}|\hat{\psi}>= \hat{E}\hat{\times}|\hat{\psi}> \to \hat{V}\hat{\times}\hat{H}\hat{\times}|\hat{\psi}>=$$
$$\hat{V}\hat{\times}\hat{H}\hat{\times}\hat{V}^{\hat{\dagger}}\hat{\times}\hat{V}\hat{\times}|\hat{\psi}>= \hat{H}'\hat{\times}|\hat{\psi}'>=$$
$$\hat{V}\hat{\times}\hat{E}\hat{\times}|\hat{\psi}>= \hat{E}'\hat{\times}|\hat{\psi}'>, \hat{E}' = \hat{E}.$$

Isomathematics achieves the invariance of the numerical values of the isounit, isoproduct and isoeigenvalues, thus regaining the necessary conditions for physical applications. Since isohermiticity coincides with the conventional hermiticity, all conventional observables of unitary theories remain observables under isotopies. The preservation of Hermiticity-observability in time is then ensured by the above isoinvariances. Based on this and further detailed analysis Santilli established the resolution of inconsistencies of nonunitary theories [101]. The Lie theory and the Lie-Santilli isotheory might be considered as coinciding at the abstract level [71, 72, 73, 97]. Despite this mathematical similarity, the physical implications of the Lie-Santilli isotheory are far-reaching, as it permitted a structural generalization of the fundamental dynamical equations of classical and quantum mechanics, superconductivity and chemistry into new disciplines called isomechanics [68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 87, 88, 89, 96, 97, 99, 100, 101, 102, 104, 105, 106] isosuperconductivity and isochemistry [3]. These new disciplines preserve the physical content of the old theories such as the total conserved quantities, and at the same time allow to add internal nonhamiltonian effects represented by the isounit. These effects have applications in various fields such as unification theories of electroweak and gravitational interactions [100], structure models of the strongly interacting particles hadrons [99] and their far-reaching applications including the prediction of novel, clean subnuclear energies.

# 3 Lie-Santilli Genotheory and Genomathematics

Lie-Santilli isotheory preserves the totally antisymmetric character of the classical and operators Lie brackets. This makes it difficult to use it for a representation of open-nonconservative systems. In particular, the fundamental problem of the origin of the irreversibility of macroscopic reality does not admit quantitative treatment via the Lie-Santilli isotheory.

Santilli has made a revolutionary suggestion on how to resolve this fundamental insufficiency by requesting for the broadening of the Lie-Santilli isotheory into a form with neither totally antisymmetric nor totally symmetric brackets. The achievement of an invariant formulation led to a new mathematics [71, 72, 73] called genomathematics.

The main idea of genomathematics is the selection of two different generalized units, genounits with respect to the ordered multiplication to the right or to the left. The first genounit $\hat{I}^{>}$ is for the ordered multiplication to the right $A > B$ (a forward genoproduct). The second genounit ${}^{<}\hat{I}$ is for the ordered multiplication to the left $A < B$ (backward genoproduct) [71, 72, 73, 97, 101]. More precisely,

$$\hat{I}^{>} = \frac{1}{\hat{S}}, \quad A > B = A \times \hat{S} \times B, \quad \hat{I}^{>} > A = A > \hat{I}^{>} = A, \tag{32}$$

$${}^{<}\hat{I} = \frac{1}{\hat{R}}, \quad A < B = A \times \hat{R} \times B, \quad {}^{<}\hat{I} < A = A <^{<} \hat{I} = A, \tag{33}$$

$$A = A^{\dagger}, \quad B = B^{\dagger}, \quad \hat{R} = \hat{S}^{\dagger}. \tag{34}$$

The important point made by Santilli is that together with introduction of such left and right genounits with respect to ordered products from the left and from the right it is natural and often necessary both from the side of Physics and Mathematics to go further

and modify all number, algebraic and analytic structures in suitable ways. In this sense the genomathematics in a broad sense involves among other things:

1) lifting of isofields $\hat{\mathbb{F}}(\hat{a}, \hat{+}, \hat{\times})$ into the forward and backward genofields $\hat{\mathbb{F}}^{>}(\hat{a}^{>}, \hat{+}^{>}, >)$ and ${}^{<}\hat{\mathbb{F}}({}^{<}\hat{c}, {}^{<}\hat{+}, <)$ with forward and backward genonumbers
$$\hat{a}^{>} = a \times \hat{I}^{>}, \quad {}^{<}\hat{a} = {}^{<}\hat{I} \times a$$
and related operations [87];

2) lifting of isofunctions $\hat{f}(\hat{r})$ on $\hat{\mathbb{F}}$ into the *forward and backward genofunctions* $\hat{f}^{>}(\hat{r}^{>})$ and ${}^{<}\hat{f}({}^{<}\hat{r})$ on $\hat{\mathbb{F}}^{>}$ and ${}^{<}\hat{\mathbb{F}}$, respectively, such as $\hat{e}_{>}^{\hat{X}^{>}} = (e^{\hat{X}^{>} \times \hat{R}}) \times \hat{I}^{>}$ and $\hat{e}_{<}^{{}^{<}\hat{X}} = {}^{<}\hat{I} \times e^{\hat{S} \times {}^{<}\hat{X}}$, with consequential genotopies of transforms and functional analysis at large [88, 89, 97];

3) lifting of the isodifferential calculus into the *forward and backward genodifferential calculus* with main forward rules $\hat{d}^{>}\hat{r}^{>k} = \hat{I}_{i}^{>k} \times d\hat{r}^{>i}$, $\hat{d}^{>}\hat{p}_{k}^{>} = \hat{T}_{k}^{>i} \times d\hat{p}_{i}^{>}$, $\hat{\partial}^{>}/\hat{\partial}^{>}\hat{r}^{>i} = \hat{S}_{i}^{>j} \times \partial/\partial\hat{r}^{>j}$, $\hat{\partial}^{>}/\hat{\partial}^{>}\hat{p}_{k}^{>} = \hat{S}_{k}^{>i} \times \partial/\partial\hat{p}_{i}^{>}$, $\hat{\partial}^{>}\hat{r}^{>i}/\hat{\partial}^{>}\hat{r}^{>j} = \hat{\delta}_{j}^{>i} = \delta_{j}^{i} \times \hat{I}^{>}$, etc., and corresponding backward rules obtainable via conjugation (see [97] for details);

4) lifting of isotopologies, isogeometries,etc. into the dual forward and backward genotopic forms;

5) lifting of the Lie-Santilli isotheory into the genotheory, including the genotopies of the various aspects, such as universal enveloping associative algebras for ordered product to the right and to the left, etc. [74, 77, 88, 89, 97].

Within these more general mathematical structures, the Lie-Santilli genotheory extension of Physics basic evolution equations can be expressed as follows [97, 101]:
$$\hat{A}(\hat{w}) = \hat{e}_{>}^{i\hat{X}^{>}>\hat{w}} > \hat{A}(\hat{0}) < \hat{e}_{<}^{-i\hat{w}<{}^{<}\hat{X}} =$$
$$[e^{i\hat{X} \times \hat{S} \times w} \times \hat{I}^{>}] \times \hat{S} \times \hat{A}(\hat{0}) \times \hat{R} \times [{}^{<}\hat{I} \times e_{<}^{-iw \times \hat{R} \times \hat{X}}], \tag{35}$$
$$i\frac{\hat{d}\hat{A}}{\hat{d}\hat{w}} = \hat{A} < \hat{X} - \hat{X} > \hat{A} = \hat{A} \times \hat{R} \times \hat{X} - \hat{X} \times \hat{S} \times \hat{A} = (\hat{A}, \hat{X})_{operator}, \tag{36}$$
$${}^{<}\hat{X} = (\hat{X}^{>})^{\hat{\dagger}}, \hat{R} = \hat{S}^{\dagger} \tag{37}$$
with the corresponding classical counterpart [97, 88, 89].

Lie-Santilli genotheory has important property of being form invariance. A pair of nonunitary transforms on $\mathcal{H}$ over $\hat{C}$ can always be identically rewritten as the genounitary transforms on a genohilbert spaces over genocomplex fields:
$$V \times V^{\dagger} \neq 1, V = {}^{<}\hat{V} \times \hat{R}^{1/2}, V \times V^{\dagger} = {}^{<}\hat{V} <^{<} \hat{V}^{\dagger} = {}^{<}\hat{V}^{\dagger} <^{<} \hat{V} = {}^{<}\hat{I}, \tag{38}$$
$$W \times W^{\dagger} \neq 1, W = \hat{W}^{>} \times \hat{S}^{1/2}, W \times W^{\dagger} = \hat{W}^{>} > \hat{W}^{>\dagger} = \hat{W}^{>\dagger} > \hat{W}^{>} = \hat{I}^{>}, \tag{39}$$
with the forward genoinvariance laws [99]:
$$\hat{I}^{>} \to \hat{I}'^{>} = \hat{W}^{>} > \hat{I}^{>} > \hat{W}^{>\dagger} = \hat{I}^{>}, \tag{40}$$
$$\hat{A} > \hat{B} \to \hat{W}^{>} > (\hat{A} > \hat{B}) > \hat{W}^{>\dagger} = \hat{A}' > \hat{B}', \tag{41}$$
$$\hat{H}^{>} > | >= \hat{E}^{>} > | >= E \times | > \to \hat{W}^{>} > \hat{H}^{>} > | >= \hat{H}'^{>} > | >' =$$
$$\hat{W}^{>} > \hat{E}^{>} > | >= E \times | >', \tag{42}$$

with corresponding backward and classical counterparts. These genoinvariance laws can be interpreted also as providing invariance of the numerical values of genounits, genoproducts and genoeigenvalues permitting physically consistent applications.

Lie's theory, while being at the foundation contemporary physics nevertheless needs substantial extension and modifications for many important classes of Physical systems. Lie-Santilli genotheory has permitted an additional structural generalization of classical and quantum isomechanics, isosuperconductivity and isochemistry into their genotopic coverings.

The Lie-admissible theory provides the operator representation of open systems in which the nonconserved Hamiltonian and other quantities are Hermitian and thus observable. In other treatments of nonconservative systems the Hamiltonian is generally nonhermitean and, therefore, not observable. The broader classical and operator genotheories represent open-nonconservative systems, as desired, because now the total energy $H$ is not conserved $i\frac{dH}{dt} = H \times (\hat{R} - \hat{S}) \times H \neq 0$, while at the same time the notion of genohermiticity on $\hat{\mathcal{H}}^{>}$ over $\hat{C}^{>}$ coincides with conventional Hermiticity. Genotheories have also permitted a resolution of the historical problem of the origin of irreversibility via its reduction to the ultimate possible layers of nature, such as particles in the core of a star. The invariant genotopic formulations of Newton's equations can be found in [97]; Hamilton's equations with external terms in [97]; quantization for open-irreversible systems in [97, 101]; operator theory of open-irreversible systems in [101].

The product $\hat{A} < \hat{B} - \hat{B} > \hat{A} = \hat{A} \times \hat{R} \times \hat{B} - \hat{B} \times \hat{S} \times \hat{A}, \hat{R} \neq \hat{S}$ is typically non-Lie on conventional spaces over conventional fields. However, antisymmetry and Lie properties can be recovered when formulated on the bimodule of the respective envelopes to the left and to the right, $\{^{<}\hat{A}, \hat{A}^{>}\}$ in the sense that the numerical values of $\hat{A} < \hat{B} = \hat{A} \times \hat{R} \times \hat{B}$ with respect to $^{<}\hat{I} = 1/\hat{R}$ are the same as of $\hat{A} > \hat{B} = \hat{A} \times \hat{S} \times \hat{B}$ computed with respect to $\hat{I}^{>} = 1/\hat{S}$ (see [97, 101]).

From the Mathematics side, looking at the Santilli's genomathematics in connection to the theory of bimodules and their deformations or from the point of view of the theory of bialgebras, quasi-bialgebras and their deformations, brings new insights and opens new exciting research directions. Santilli's genomathematics looks deep into bimodule structures and related categories in the sense that modifications concern not only the bimodule left and and right action or related products structures, but also the coefficient number fields and subsequently all structures involving them if bimodules are built on linear spaces over fields. These modifications of total structure are deeply concerned with the need of consideration and modifications of corresponding left and right units, especially apparent in the bimodule structures coming from algebra product structures. The importance of the study of modifications of units has been specially emphasized by Santilli also because it has profound implications in Physics. Such total simultaneous deformations of all structures are yet to be studied properly in Mathematics and in the theory of bimodules in particular. The most relevant in this context results on isomorphisms, homomorphisms, deformations, tensorproducts, twistings of bimodules in general and for special classes of bimodules are concerned for the most part with modifications and classifications of parts of the structures related to the actions or the products or the coproducts, without modifying the units and the basic number fields. In the theory of bialgebras and quasi-bialgebras the units and co-units play more central role as insep-

arable part of the bialgebra structure. Hence the changes and description of units and counits are always a part of study in the theory and examples of bialgebras when considering mappings, deformations, twistings, tensor products, classifications and other contexts involving families of bialgebras. The possibility or necessity of changes in the coefficient field of the underlying linear space has not been much considered in this context. Also, in bialgebras, the dual operations and in particular coproducts and counits, are always part of the bialgebra structure and are requested to obey various compatibility conditions with the algebra structures, restricting the possibilities of introducing bialgebra structures on the top of a given algebra structures.

Bialgebras and quasi-bialgebras provide interesting structures and examples for further studies in context of Santilli's genomathematics. Extensions of the theory of bimodules with emphasize on Santilli's genomathematics is an interesting and important from Mathematics and applications point of view and to large extent open direction for further research.

# References


[1] A. A. Albert, Power associative rings, Trans. Amer. math. Soc. **64**, 552-597 (1948).

[2] A. O. E. Animalu, Lie-Santilli isoapproach to the unification of gravity and electromagnetism, *J. Hadronic* **19**, no. 3, 255–272 (1996).

[3] A. O. E. Animalu, R. M. Santilli, Nonlocal isotopic representation of the Cooper pair in superconductivity, International Journal of Quantum Chemistry. Quantum Chemistry Symposium, 29: 175-187, 1995

[4] A. K. Aringazin, Validity of Pauli's principle in the exterior branch of hadronic mechanics, *Hadronic J.* **13**, no. 3, 183–190 (1990).

[5] A. K. Aringazin, Supersymmetric hadronic mechanical harmonic oscillator, *Hadronic J.* **13**, no. 4, 263–276 (1990).

[6] A. K. Aringazin, A. Jannussis, D. F. Lopez, M. Nishioka and B. Veljanoski, Santilli's Lie-isotopic generalization of Galilei's and Einstein's relativities, Kostarakis Publishers, Athens (1991).

[7] A. K. Aringazin, D. F. Lopez, A. Jannussis, M. Nishioka, B. Veljanoski, Need for mathematical studies on Santilli's Lie-isotopic relativities, *Algebras Groups Geom.* **7**, no. 3, 211–300 (1990).

[8] A. K. Aringazin, D. A. Kirukhin, R. M. Santilli, Isotopic generalization of the Legendre, Jacobi, and Bessel functions, *Algebras Groups Geom.* **12**, no. 3, 255–305 (1995).

[9] A. K. Aringazin, D. A. Kirukhin, R. M. Santilli, Nonpotential elastic scattering of spinning particles, *Hadronic J.* **18**, no. 3, 257–269 (1995).

[10] R. Aslander, S. Keles, Cartan's structure equations on Santilli-Sourlas-Tsagas isomanifolds, Algebras Groups Geom. **14**, no. 3, 283–296 (1997).

[11] R. Aslander, S. Keles, Infinitesimal motions on Santilli-Sourlas-Tsagas isomanifolds, Algebras Groups Geom. **15**, no. 4, 545–561 (1998).

[12] B. Davvaz, V. Leoreanu Fotea, Hyperring Theory and Applications, Hadronic Press, 2008, 336 pp. ISBN 1-57485-064-4

[13] A. Elduque, H. C. Myung, Mutations of Alternative Algebras, Mathematics and its Applications, 278, Kluwer Academic Publishers Group, Dordrecht, 1994. xiv+226 pp. ISBN: 0-7923-2735-7

[14] R. M. Falcon Ganfornina, J. Nunez Valdes, Fundamentos de la Isoteoria de Lie-Santilli, International Academic Press, Palm Harbor, FL, 2001, vi+215 pp. ISBN: 1-58485-055-5

[15] M. Gasperini, From Lie-admissible algebra to nonsymmetric gravity, *Hadronic J.* **7**, no. 2, 234–239 (1984).

[16] M. Gasperini, On the geometrical structure of a Lie-isotopic gravity, *Hadronic J.* **7**, no. 4, 650–658 (1984).

[17] M. Gasperini, Gravitation and Lie-isotopic algebra, Proceedings of the second workshop on hadronic mechanics, Vol. I (Como, 1984), *Hadronic J.* **7**, no. 5, 971–1018 (1984).

[18] M. Gasperini, On a Lie-admissible theory of gravity, *Nuovo Cimento B (11)* **81**, no. 1, 7–20 (1984).

[19] M. Gasperini, On a Lie-isotopic theory of gravity, *Nuovo Cimento A (11)* **83**, no. 4, 309–326 (1984).

[20] D. K. P. Ghikas, C. N. Ktorides, L. Papaloukas, Representation theory of Lie-admissible enveloping algebras on operator algebras: an extension of a theorem by Nelson, *Hadronic J.* **3**, no. 2, 726–742 (1979/80).

[21] T. L. Gill, W. W. Zachary, M. F. Mahmood, J. Lindesay, Proper-time relativistic dynamics and a resolution of the no-interaction problem, *Hadronic J.* **17**, no. 5, 449–471 (1994).

[22] A. Jannussis, Quantum group and the Lie-admissible $Q$-algebra, Hadronic mechanics and nonpotential interactions, Part 2 (Cedar Falls, IA, 1990), 203–225, Nova Sci. Publ., Commack, NY, 1992.

[23] A. Jannussis, R. Mignani, R. M. Santilli, Problematic aspects of Weinbergs nonlinear theory, *Ann. Fond. Louis de Broglie* **18**, no. 4, 371–390, (1993).

[24] A. Jannussis, R. Mignani, D. Skaltsas, Lie-admissible perturbation methods for open quantum systems, *Phys. A* **187**, no. 3-4, 575–588 (1992).

[25] A. Jannussis, D. Skaltsas, Lie-admissible complex time model, Hadronic mechanics and nonpotential interactions, Part 2 (Cedar Falls, IA, 1990), 3–23, Nova Sci. Publ., Commack, NY, 1992.

[26] A. Jannussis, D. Skaltsas, Algebraic inconsistencies of a class of equations for the description of open systems and their resolution via Lie-admissible formulation, *Ann. Fond. Louis de Broglie* **18**, no. 2, 137–154 (1993).

[27] C.-X. Jiang, Foundations of Santilli's isonumber theory. I. Isonumber theory of the first kind, Algebras Groups Geom. **15**, no. 3, 351–393 (1998).

[28] C.-X. Jiang, Foundations of Santilli's isonumber theory. II. Isonumber theory of the second kind, Algebras Groups Geom. **15**, no. 4, 509–544 (1998).

[29] C.-X. Jiang, Foundations of Santilli's isonumber theory. With applications to new cryptograms, Fermat's theorem and Goldbach's conjecture. With a foreword by Ruggero Maria Santilli, International Academic Press, Palm Harbor, FL, 2002. xlvi+366 pp. ISBN: 1-58485-056-3

[30] J. D. Kadeisvili, Santilli's isotopies of contemporary algebras, geometries and relativities, Hadronic Press Monographs in Mathematics. Hadronic Press, Inc., Palm Harbor, FL, 1992. 310 pp. ISBN: 0-911767-52-5

[31] J. V. Kadeisvili, Santilli's Isotopies of Contemporary Algebras, Geometries and Relativities, Second edition, Ukraine Academy of Sciences, Kiev, Distributed by the Hadronic Press, 1997, 450 pp, ISBN 0-911767-55-X

[32] J. V. Kadeisvili, Elements of functional isoanalysis, *Algebras Groups Geom.* **9**, no. 4, 283–318 (1992).

[33] J. V. Kadeisvili, Elements of Fourier-Santilli isotransforms, *Algebras Groups Geom.* **9**, no. 4, 319–342 (1992).

[34] J. V. Kadeisvili, An introduction to the Lie-Santilli isotopic theory, *Jñanabha*, **25**, 754 (1995).

[35] J. V. Kadeisvili, An introduction to the Lie-Santilli isotopic theory, *Math. Methods Appl. Sci.* **19**, no. 17, 1349–1395 (1996).

[36] J. V. Kadeisvili, Foundations of the Lie-Santilli isotheory, *Rend. Circ. Mat. Palermo (2)*, Suppl. No. 42, 83–136 (1996).

[37] J. V. Kadeisvili, An introduction to the Lie-Santilli isotheory, *Algebras Groups Geom.* **13**, no. 2, 171–281 (1996).

[38] J. V. Kadeisvili, An introduction to the Lie-Santilli theory, *Acta Appl. Math.* **50**, no. 1-2, 131–165 (1998).

[39] J. V. Kadeisvili, N. Kamiya, R. M. Santilli, A characterization of isofields and their isoduals, *Hadronic J.* **16**, no. 3, 169–186 (1993).

[40] A. J. Kálnay, On fundamental realizations of Lie-admissible algebras: flexibility, centers, and nuclei, *Hadronic J.* **3**, no. 6, 1552–1564 (1979/80).

[41] A. J. Kálnay, Lie-admissible structure of a quantized Nambu's generalized Hamiltonian dynamics, *Hadronic J.* **6** (1983), no. 1, 1–30.

[42] A. J. Kálnay, Errata-corrige: "On fundamental realizations of Lie-admissible algebras: flexibility, centers, and nuclei" [Hadronic J. 3 (1979/80), no. 6, 1552–1564; MR 82h:17006], *Hadronic J.* **6** (1983), no. 1, 211.

[43] A. J. Kálnay, A procedure for generating realizations of Lie-admissible and other nonassociative algebras, *Hadronic J.* **6**, no. 2, 278–304 (1983).

[44] Y. S. Ko, H. C. Myung, On a class of Mal'cev-admissible algebras, *J. Korean Math. Soc.* **19**, no. 1, 47–54 (1982/83).

[45] C. N. Ktorides, H. C. Myung, R. M. Santilli, Elaboration of the recently proposed test of Pauli's principle under strong interactions, *Phys. Rev. D (3)*, **22**, no. 4, 892–907 (1980).

[46] Lie, Marius Sophus. Marius Sophus Lie's doctoral thesis "Over en Classe geometriske Transformationer" ["On a class of geometrical transformations"], Translated from the Norwegian by Erik Trell, Commentary by Trell and Ruggero Maria Santilli, *Algebras Groups Geom.* **15**, no. 4, 395–445 (1998).

[47] E. B. Lin, Hadronic quantization, *Hadronic J.* **11**, no. 2, 8183 (1988)

[48] J. Lôhmus, E. Paal, L. Sorgsepp, Nonassociative algebras in physics. Hadronic Press Monographs in Mathematics. Hadronic Press, Inc., Palm Harbor, FL, 1994. xiv+271 pp. ISBN: 0-911767-71-1

[49] D. F. Lopez, Problematic aspects of $q$-deformations and their isotopic resolution, *Hadronic J.* **16**, no. 6, 429458 (1993).

[50] R. Mignani, Nonpotential scattering theory and Lie-admissible algebras: time evolution operators and the $S$-matrix, Proceedings of the First International Conference on Nonpotential Interactions and their Lie-Admissible Treatment, Part B (Orlans, 1982), *Hadronic J.* **5**, no. 3, 1120–1139 (1981/82).

[51] R. Mignani, On the Lie-admissible structure of the Caldirola equations for dissipative processes, *Lett. Nuovo Cimento (2)*, **38**, no. 5, 169–173 (1983).

[52] R. Mignani, Lie-isotopic lifting of $SU_N$ symmetries, *Lett. Nuovo Cimento (2)*, **39**, no. 17, 413–416 (1984).

[53] H. C. Myung, Lie-admissible algebras, Hadronic J. **1** (1978), 169-193.

[54] H. C. Myung, The exponentiation and deformations of Lie-admissible algebras, Proceedings of the First International Conference on Nonpotential Interactions and their Lie-Admissible Treatment, Part B (Orlans, 1982), *Hadronic J.* **5**, no. 3, 771–903 (1981/82).

[55] H. C. Myung, Lie algebras and flexible Lie-admissible algebras, Hadronic Press Monographs in Mathematics, 1. Hadronic Press, Inc., Nonantum, Mass., 1982. xii+329 pp. ISBN: 0-911767-01-0

[56] H. C. Myung, Malcev-Admissible Algebras, Progress in Mathematics, 64. Birkhäuser Boston, Inc., Boston, MA, 1986. xvi+353 pp. ISBN: 0-8176-3345-6.

[57] H. C. Myung, S. Okubo, R. M. Santilli (Eds.), Applications of Lie-admissible algebras in physics. Vol. I, II, Hadronic Press, Inc., Nonantum, Mass., 1978. iv+601 pp.

[58] H. C. Myung, R. M. Santilli, Foundations of the hadronic generalization of the atomic mechanics. II. Modular-isotopic Hilbert space formulation of the exterior strong problem, Proceedings of the First International Conference on Nonpotential Interactions and their Lie-Admissible Treatment, Part C (Orlans, 1982), *Hadronic J.* **5**, no. 4, 1277–1366 (1981/82).

[59] H. C. Myung, R. M. Santilli, Foundations of the hadronic generalization of the atomic mechanics. III. Bimodular-genotopic Hilbert space formulation of the interior strong problem, Proceedings of the First International Conference on Nonpotential Interactions and their Lie-Admissible Treatment, Part C (Orlans, 1982), *Hadronic J.* **5**, no. 4, 1367–1403 (1981/82).

[60] M. Nishioka, An invitation to hadronic mechanics, *J. Fac. Liberal Arts Yamaguchi Univ. Natur. Sci.* **18**, 21–46 (1984).

[61] M. Nishioka, Remarks on the Lie-isotopic lifting of gauge theory, *Nuovo Cimento A (11)*, **82**, no. 3, 351–356 (1984).

[62] M. Nishioka, Remarks on Lie-isotopic lifting and the noncanonical commutation relations, *Lett. Nuovo Cimento (2)*, **39**, no. 12, 268–270 (1984).

[63] M. Nishioka, An introduction of gauge field by the Lie-isotopic lifting of the Hilbert space, *Lett. Nuovo Cimento (2)*, **40**, no. 10, 309–312 (1984).

[64] M. Nishioka, A remark on an application of the Lie-admissible lifting to the unstable state, *Lett. Nuovo Cimento (2)*, **40**, no. 10, 313–316 (1984).

[65] M. Nishioka, Relations between some models of field theory and the Lie-isotopic or the Lie-admissible approach, *Hadronic J.* **7**, no. 2, 240–248 (1984).

[66] S. Okubo, Schrödinger equation for nonassociative quantum mechanics and no-go theorem, in "Proceedings of the First International Conference on Nonpotential Interactions and their Lie-Admissible Treatment, Part D (Orlans, 1982)", *Hadronic J.* **5**, no. 5, 16671700 (1981/82).

[67] S. Okubo, H. C. Myung, Note on commutativity of adjoint operator algebras, Proceedings of the First International Conference on Nonpotential Interactions and their Lie-Admissible Treatment, Part D (Orlans, 1982), *Hadronic J.* **5**, no. 5, 1658–1666 (1981/82).

[68] R. M. Santilli, Imbedding of Lie algebras in nonassociative structures, *Nuovo Cimento* A (10), **51**, 570-576 (1967).

[69] R. M. Santilli, An Introduction to Lie-admissible Algebras, Center for Theoretical Studies, University of Miami, Coral Gables, FL, *Supplemento al Nuovo Cimento*, **6**, 1225-1249 (1968)

[70] R. M. Santilli, Dissipativity and Lie-admissible algebras, *Meccanica*, Vol. 1, 3-11 (1969)

[71] R. M. Santilli, On a possible Lie-admissible covering of the Galilei relativity in Newtonian mechanics for nonconservative and Galilei form-noninvariant systems, *Hadronic J.* **1**, no.1, 223-423 (1978).

[72] Santilli, R. M., Addendum to: "On a possible Lie-admissible covering of the Galilei relativity in Newtonian mechanics for nonconservative and Galilei form-noninvariant systems", *Hadronic J.* **1**, no. 4, 12791342, (1978)

[73] R. M. Santilli, Need of subjecting to an experimental verification the validity within a hadron of Einstein's special relativity and Pauli's exclusion principle, *Hadronic J.* **1**, no.2, 574–901 (1978).

[74] Foundations of theoretical mechanics. I. The inverse problem in Newtonian mechanics. Texts and Monographs in Physics. Springer-Verlag New York, New York-Heidelberg, 1978. xix+266 pp. ISBN: 0-387-08874-1

[75] R. M. Santilli, Lie-admissible approach to hadronic structure, Vol I and II, Hadronic Press INC, Nonantum, Mass. (1979).

[76] R. M. Santilli, Initiation of the representation theory of Lie-admissible algebras of operators on bimodular Hilbert spaces, in "Proceedings of the Second Workshop on Lie-Admissible Formulations (Sci. Center, Harvard Univ., Cambridge, Mass., 1979), Part B", Hadronic J. **3**, no. 1, 440506 (1979/80).

[77] R. M. Santilli, Foundations of theoretical mechanics. II. Birkhoffian generalization of Hamiltonian mechanics. Texts and Monographs in Physics. Springer-Verlag, New York-Berlin, 1983. xix+370 pp. ISBN: 0-387-09482-2

[78] R. M. Santilli, Lie-isotopic lifting of unitary symmetries and of Wigner's theorem for extended, deformable particles, *Lett. Nuovo Cimento (2)*, **38**, no. 15, 509521 (1983).

[79] R. M. Santilli, Lie-isotopic liftings of Lie symmetries. I. General considerations, *Hadronic J.* **8**, no. 1, 2535 (1985).

[80] R. M. Santilli, Lie-isotopic liftings of Lie symmetries. II. Lifting of rotations, *Hadronic J.* **8**, no. 1, 3651 (1985).

[81] R. M. Santilli, Isotopic liftings of Einstein's special relativity for classical interior dynamical systems, Hadronic J. Suppl. **4A**, no. 3, 267–405 (1988).

[82] R. M. Santilli, Isotopic liftings of Einstein's general relativity for classical interior gravitational problems, *Hadronic J. Suppl.* **4A**, no. 4, 407–500 (1988).

[83] R. M. Santilli, Isotopic generalizations of Galilei's and Einstein's relativities. Vol. I. Mathematical foundations, Hadronic Press Monographs in Theoretical Physics. Hadronic Press, Inc., Palm Harbor, FL, 1992. vi+220 pp. ISBN: 0-911767-55-X

[84] R. M. Santilli, Isotopic generalizations of Galilei's and Einstein's relativities. Vol. II. Classical formulations, Hadronic Press Monographs in Theoretical Physics. Hadronic Press, Inc., Palm Harbor, FL, 1992. vi+356 pp. ISBN: 0-911767-55-X

[85] R. M. Santilli, Nonlocal formulation of the Bose-Einstein correlation within the context of hadronic mechanics. I. *Hadronic J.* **15**, no. 1, 1–50 (1992).

[86] R. M. Santilli, Nonlocal formulation of the Bose-Einstein correlation within the context of hadronic mechanics. II, *Hadronic J.* **15**, no. 2, 77–134 (1992).

[87] R. M. Santilli, Isonumbers and genonumbers of dimension $1, 2, 4, 8$, their isoduals and pseudoduals, and "hidden numbers" of dimension $3, 5, 6, 7$, *Algebras, Groups and Geometries* **10**, no.4, 273-322 (1993).

[88] R. M. Santilli, Elements of hadronic mechanics. Vol. I. Mathematical foundations, With a foreword by Nugzar V. Makhaldiani, "Naukova Dumka", Kiev, 1993. xvi+378 pp. ISBN: 0-911767-87-8; 0-911767-68-1

[89] R. M. Santilli, Elements of hadronic mechanics. Vol. II. Theoretical foundations, "Naukova Dumka", Kiev, 1994. xxiv+537 pp. ISBN: 0-911767-88-6; 0-911767-69-X

[90] R. M. Santilli, Isotopic lifting of Heisenberg's uncertainties for gravitational singularities, *Comm. Theoret. Phys. (Allahabad)*, **3**, no. 1, 47–66 (1994).

[91] R. M. Santilli, Representation of antiparticles via isodual numbers, spaces and geometries, *Comm. Theoret. Phys. (Allahabad)*, **3**, no. 2, 153–181 (1994).

[92] R. M. Santilli, Isotopic lifting of quark theories with exact confinement and convergent perturbative expansions, *Comm. Theoret. Phys. (Allahabad)*, **4**, no. 1, 123–153 (1995).

[93] R. M. Santilli, Quantum-iso-gravity, *Comm. Theoret. Phys. (Allahabad)*, **4**, no. 2, 215–231 (1995).

[94] R. M. Santilli, Isotopic lifting of Newtonian mechanics, *Rev. Tcn. Fac. Ingr. Univ. Zulia* **18**, no. 3, 271–284 (1995).

[95] R. M. Santilli, Nonlocal-integral isotopies of differential calculus, mechanics and geometries, *Rend. Circ. Mat. Palermo* (2) Suppl. No. 42, 7–82 (1996).

[96] R. M. Santilli, Isotopic, Genotopic and Hyperstructural Method in Theoretical Biology, Ukraine Academy of Sciences, Kiev (1996).

[97] R. M. Santilli, Nonlocal-integral isotopies of differential calculus, mechanics and geometries. *Rend. Circ. Mat. Palermo (2)* Suppl. No. 42, 782 (1996).

[98] R. M. Santilli, Isotopies of classical and quantum mechanics, *Indian J. Math.* **37** (1995), no. 3, 235–256 (1997).

[99] R. M. Santilli, Relativistic hadronic mechanics: nonunitary, axiom-preserving completion of relativistic quantum mechanics, *Found. Phys.* **27**, no. 5, 625729 (1997).

[100] R. M. Santilli, Isotopic grand unification with the inclusion of gravity, *Found. Phys. Lett.* **10**, no. 4, 307327 (1997).

[101] R. M. Santilli, Invariant Lie-admissible formulation of quantum deformations, *Found. Phys.*, **27**, no. 8, 1159–1177 (1997).

[102] R. M. Santilli, Does antimatter emit a new light?, *Hyperfine Interactions*, Vol. 109, pp. 63-81 (1997).

[103] R. M. Santilli, Origin, problematic aspects and invariant formulation of $q$-, $k$- and other quantum deformations, *Modern Phys. Letters A*, **13**, no.4, 327335 (1998).

[104] R. M. Santilli, Iso-Minkowskian geometry for the gravitational treatment of matter and its isodual for antimatter, *Internat. J. Modern Phys. D*, **7**, no. 3, 351407 (1998).

[105] R. M. Santilli, Isorepresentations of the Lie-isotopic SU(2) algebra with applications to nuclear physics and to local realism. *Acta Appl. Math.* **50**, no. 1-2, 177190 (1998).

[106] R. M. Santilli, Iso-Minkowskian formulation of gravitation and cosmology, in "Proceedings of the International Conference on Modern Modified Theories of Gravitation and Cosmology, Part II (Beer Sheva, 1997), E. I. Guendelman, editor", *Hadronic J.* **21**, no. 2, 113169 (1998).

[107] R. M. Santilli, Origin, Problematic aspects and invariant formulation of classical and operator deformations, *Internat. J. Modern Phys. A*, **14**, no. 20, 3157–3206, (1999).

[108] R. M. Santilli, D. D. Shillady, A new isochemical model of the hydrogen molecule, *Intern. J. Hydrogen Energy*, Vol. 24, 943-956 (1999)

[109] R. M. Santilli, D. D. Shillady, A new isochemical model of the water molecule, *Intern. J. Hydrogen Energy*, Vol. 25, 173-183 (2000)

[110] R. M. Santilli, Direct universality of isospecial relativity for photons with arbitrary speeds, in *Photons: Old problems in Light of New Ideas*, V. V. Dvoeglazov Editor, Nova Science (2000)

[111] R. M. Santilli, *Ann. Fond. Louis de Broglie*, Hors série 2, **29**, 953–967 (2004)

[112] R. M. Santilli, Lie-admissible invariant representation of irreversibility for matter and antimatter at the classical and operator levels, *Nuovo Cimento Soc. Ital. Fis. B*, **121**, no. 5, 443–486 (2006).

[113] R. M. Santilli, "Experimental verications of isoredshift with possible absence of universe expansion, big bang, dark matter and dark energy," The Open Astronomy Journal 4, in press (2010).

[114] R. M. Santilli, "Nonunitary-Isounitary Scattering Theory of Hadronic mechanics, I: Conceptual and Mathematical. Foundations, " submitted for publication.

[115] R. M. Santilli, T. Vougiouklis, Isotopies, genotopies, hyperstructures and their applications, in "New frontiers in hyperstructures", Proceedings of the International Workshop held in Molise, August 912, 1995, Edited by Thomas Vougiouklis, Series on New Frontiers in Advanced Mathematics of the Istituto per la Ricerca di Base, Hadronic Press, Inc., Palm Harbor, FL, 1996. vii+230 pp. ISBN: 1-57485-008-3

[116] D. Schuch, Nonunitary connection between explicitly time-dependent and nonlinear approaches for the description of dissipative quantum systems, *Phys. Rev. A*, **55**, no.2, 935940 (1997).

[117] D. S. Sourlas, G. T. Tsagas, Mathematical foundations of the Lie-Santilli theory, "Naukova Dumka", Kiev; distributed by Hadronic Press, Inc., Palm Harbor, FL, 1993. ii+232 pp. ISBN: 0-911767-65-7

[118] E. Trell, Isotopic proof and re-proof of Fermat's last theorem verifying Beal's conjecture, Algebras Groups Geom. **15**, no. 3, 299–318 (1998).

[119] G. T. Tsagas, D. Sourlas, Isomanifolds and their isotensor fields, Algebras Groups Geom. **12**, no. 1, 1–65 (1995).

[120] G. T. Tsagas, Studies on the classification of Lie-Santilli algebras, Algebras Groups Geom. **13**, no. 2, 129–148 (1996).

[121] G. T. Tsagas, Isoaffine connections and Santilli's iso-Riemannian metrics on an isomanifold, Algebras Groups Geom. **13**, no. 2, 149–170 (1996).

[122] G. T. Tsagas, Isoaffine connections and Santilli's iso-Riemannian metrics on an isomanifold, New frontiers in algebras, groups and geometries (Monteroduni, 1995), 465–486, Ser. New Front. Adv. Math. Ist. Ric. Base, Hadronic Press, Palm Harbor, FL, 1996.

[123] G. T. Tsagas, Studies on the classification of Lie-Santilli algebras, New frontiers in algebras, groups and geometries (Monteroduni, 1995), 487–505, Ser. New Front. Adv. Math. Ist. Ric. Base, Hadronic Press, Palm Harbor, FL, 1996.

[124] G. T. Tsagas, D. Sourlas, Isomappings between isomanifolds, Algebras Groups Geom. **12**, no. 1, 67–87 (1995).

[125] S. I. Vacaru, Studies on Santilli's locally anisotropic and inhomogeneous isogeometries. I. Isobundles and generalized iso-Finsler gravity, Algebras Groups Geom. **14**, no. 3, 211–257 (1997).

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
Kathmandu University, Nepal, April (2011) pages 189-202

# ADDITIONAL CONFIRMATION OF "INTERMEDIATE CONTROLLED NUCLEAR FUSION" WITHOUT HARMFUL RADIATIONS OR WASTE

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**Abstract**

In this paper, we report three tests providing additional experimental confirmations of the recently achieved and verified *Intermediate Controlled Nuclear Fusions* (ICNF). Thanks to various chemical analyses performed by independent laboratories, the first test established the ICNF of silica from carbon and oxygen; the second test confirmed the preceding results; and the third test established the ICNF of oxygen from helium and carbon.

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## 1. Introduction

Following decades of studies for the prior development of mathematical, physical and chemical formulations as structurally irreversible over time as the energy releasing processes that have to be described (see review [1] and general presentations [2]), and as a result of extensive tests and experimentations conducted for years, in the preceding paper [3] we released, apparently for the first time, experimental evidence on the "existence" of *Intermediate Controlled Nuclear Fusions* (ICNF) whose primary features are the following:

1) *Lack of emission of harmful radiations (such as n, p, $\alpha$, etc.) and lack of release of radioactive waste.* This fundamental feature is achieved by conceptually and technically restricting the syntheses to light, natural and stable elements.

2) *Control of the fusions via multiple means.* This second important feature is achieved via the control of power, temperature, pressure, flow and other engineering means.

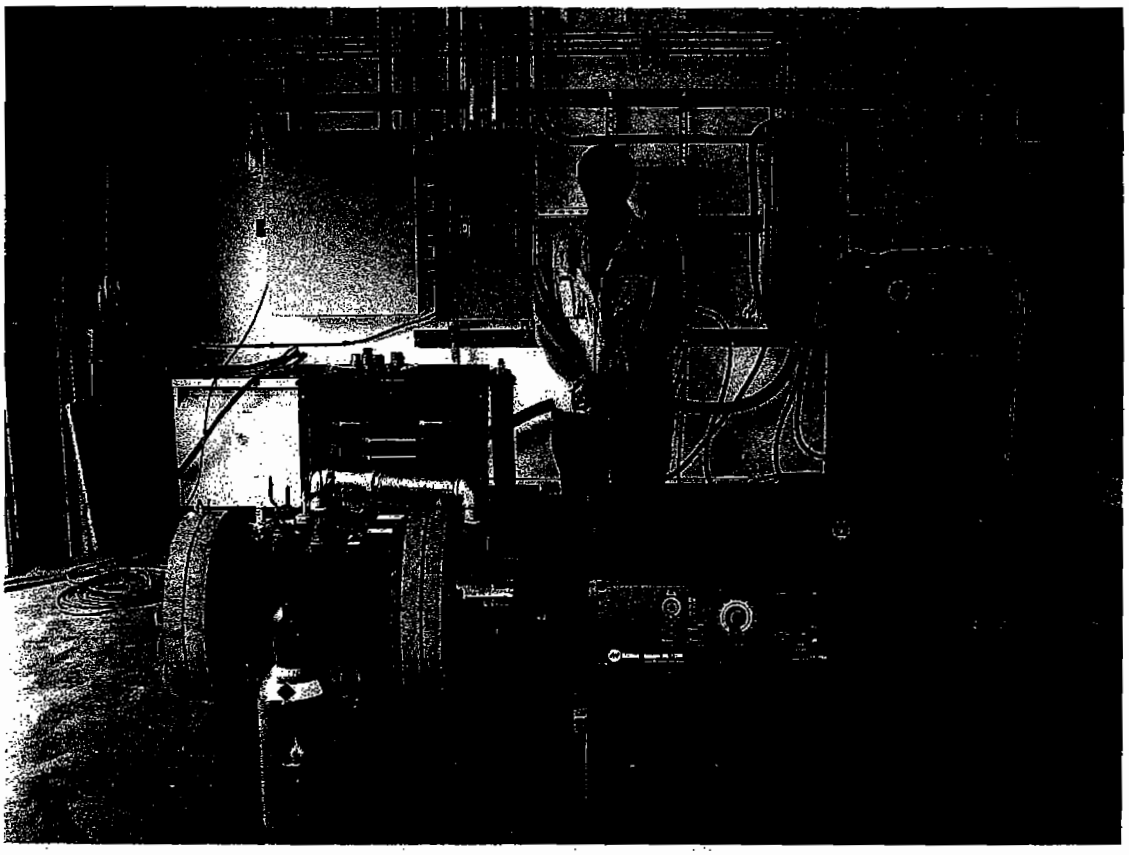


Figure 1: A view of the author with the equipment used for the synthesis of nitrogen from carbon and deuterium [3] showing from the r.h.s.:. the Miller Dimension 1000 AC-DC converter; the pressure bottle of 99.99 pure deuterium; and the carbon steel, 12" x 24" schedule 80 hadronic reactor.

3) *Intermediate character between the so-called hot and cold fuions,* in the sense that the used temperature has values in between the high temperatures of the hot fusion and the low temperature of the cold fusion.

ICNF are achieved via the use of specially constructed, high pressure, steel vessels known as *hadronic reactors* because conceived and constructed via the laws of hadronic mechanics and chemistry [1,2]. Their main function is that of delivering a DC electric arc between suitably selected electrodes submerged within a suitably selected gas at pressure. Under the condition that, for selected electrodes, the gas does allow ICNF, it is called *hadronic fuel.* All tests herein considered deal with hadronic fuels suitably selected to achieve ICNF when traversed by a DC arc between carbon electrodes.

In particular, paper [3] presented the following ICNF

$$D(2,1,1^+,2.0141) + C(12,6,0^+,12.0000) + TR \rightarrow$$

$$\rightarrow N(14,7,1^+,14.0030) + \Delta E, \tag{1a}$$

$$\Delta E = (E_{car} + E_{deu}) - E_{nitr} = 0.0111\ u, \tag{1b}$$

where $TR$ stands for the *trigger,* namely, an external action (such as instantaneous increase in pressure) forcing exposed nuclei at mutual distances of 1 $fm$ against their repulsive Coulomb forces, at which occurrence the strongly attractive nuclear force is activated between the two nuclei and their fusion is inevitable under the principles of ICNF reviewed below. The reader should note that ICNF (1) verifies all conceivably possible nuclear and other laws.

As described in detail in Ref. [3], ICNF (1a) was achieved via a schedule 80 carbon steel hadronic reactor of $1ft \times 2ft$ (see Figure 1) filled up with the hadronic fuel given by pure deuterium gas at 100 *psi* (following pulling out of a vacuum) that was traversed by a DC electric arc between commercially available graphite electrodes powered by a 50 $kW$ DC-AC converter built by the U. S. company Miller Electric. The test had to be systematically interrupted following a maximum of 2 *min* operation to prevent melt-down of the equipment. Independent chemical analyses, done by the *Oneida ORS Laboratories* on samples of the interior gas before and after the activation of the arc, measured a macroscopic percentage of nitrogen after the activation of the arc that did not exist before, thus establishing its synthesis. The nitrogen synthesis so detected was independently confirmed by the heat produced that was definitely bigger than that provided by the 50 $kW$ AC-DC converter.

ICNF (1a) was selected among a variety of possibilities to prevent wasteful academic discussions on the excess heat in the event interior combustion had been allowed. In fact, the interior gas, that was confirmed as being 99.99 % pure deuterium, positively cannot experience any combustion when traversed by a DC arc. Therefore, the heat measured in excess of the heat produced by the arc can solely be explained, on serious scientific grounds, as originating from ICNF (1).

ICNF (1a) was also selected among a considerable variety of possibilities to prevent wasteful academic discussions on the absence of harmful radiations. In fact, we have the synthesis of a light, natural and stable element, the nitrogen, from two ligher, natural and stable elements, the deuterium and the carbon, Therefore, when synthesis (1) occurs, there is no possibility whatsoever, not even remote, to produce harmful radiations or release radioactive waste as routinely expected by the community in nuclear fusions. In the event syntheses (1a) do not occur, there is equally the impossibility of producing harmful radiations or releasing radioactive waste because the energy of the 50 $kW$ AC-DC converter is about one billion times short of the



Figure 2: A view of the participants in the verification [4], showing from the left: G. West (IBR), R. M. Santilli (IBR), T. Kuliczkowski (PGTI), L. Ying (PGTI), M. Rodriguez (IBR), R. Brenna (PGTI), and C. Lynch (IBR). The picture also shows the used equipment.

energy needed to fracture the deuterium and/or the carbon nuclei for the production of the harmful radiation and waste expected by the physics community in the field.

Following the appearance of paper [3], the author requested nuclear physicists **Robert Brenna, Theodore Kuliczkowski and Leong Ying** of *Princeton Gamma Tech Instruments* to conduct independent verifications or dismissals of the results presented in Ref. [3]. Following extensive and detailed tests via the use of the same equipment and same set up of tests [3], the indicated nuclear physicists released paper [4] (see also ref. [5]) confirming all main results of Ref. [3], including: the synthesis of nitrogen from deuterium and carbon; the excess heat over that produced by the AC-DC converters; and the complete absence of harmful radiations or radioactive waste.

Refs. [3,4,5] have essentially confirmed the following *Santilli's Principles of ICNF* (see Refs. [2] for extensive studies):

*PRINCIPLE 1: Need to achieve a controlled exposure of nuclei.* Nuclei are naturally protected by their electron clouds, as well known. Consequently, no nuclear fusion is conceivably possible or otherwise plausible without the systematic exposure of nuclei as an evident necessary preparatory step for their fusion. This is the reason the author dedicated decades of research for the *new chemical species of Santilli magnecules* (see

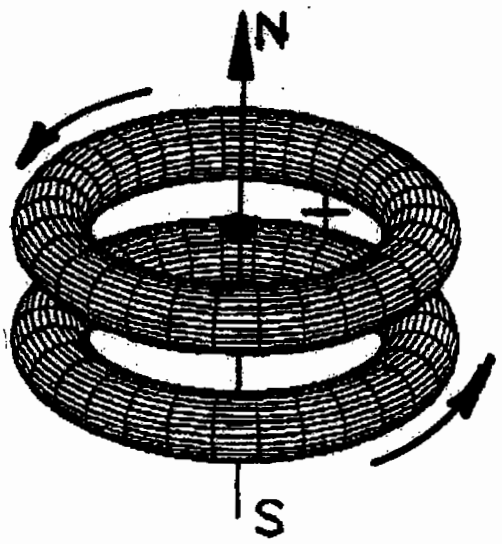



Figure 3: A conceptual view of the simplest possible example of the new chemical species of Santilli magnecule which is a necessary prerequisite for all ICNF studied in this paper.

the review in Ref. [1] or Vol. IV of Refs. [2] and original literature quoted therein). This new species is schematically represented in Figure 3 for the simplest possible biatomic case, and clearly shows the controlled exposure of nuclei via the polarization of the orbitals into toroids permitted by DC electric arc. The same picture shows the maintenance of said polarization via couplings. In the author's opinion, *the most important scientific contribution by R. Brenna, T. Kuliczkowski and L. Ying in Refs [4] has been the experimental confirmation of the existence of Santilli magnecules, not only for their evident independent chemical value, but also as a necessary prerequisite for fusion.*

*PRINCIPLE 2: The need to achieve the correct spin coupling.* Following the exposure of nuclei, no controlled fusion is conceivably possible, or otherwise plausible, without the additional systematic control of spin couplings. In fact, triplet couplings of spin notoriously cause strong repulsive forces in which case fusions can at best be at random. Ref. [3] established the second necessary condition for truly controlled fusions, the achievement of *systematic spin couplings either of planar singlet or of axial triplet type.* Another illustration of the fundamental character of Santilli magnecules for ICNF is visually represented in Figure 3 with the automatic achievement of the axial triplet coupling of nuclear spins (same spin direction for nuclei along the same

symmetry axis).

*PRINCIPLE 3: Use the minimal possible energy required by conservation laws, called "threshold energy."* A reason stressed by the author for the inability by hot fusions to achieve systematic and controlled nuclear fusions (following half a century of research and the expenditure of over one billion dollars) is the use of excessive energies under which the control of the fusion is practically impossible due to inevitable instabilities and to the extreme technological difficulties for their control. Similarly, the author has stressed that a reason for the inability by cold fusions to achieve systematic and controlled fusions has been the use of insufficient energies, e.g., as needed for a systematic exposure of nuclei. These two opposite extremes illustrate the third principle of ICNF according to which, in order to avoid uncontrollable instabilities, following the achievement of the configuration of Figure 3 via the implementation of Principles 1 and 2, the fusion reactor must operate at "threshold energy," namely, the minimal possible energy needed to push the two nuclei at a mutual distance of 1 $fm$ against their repulsive Coulomb forces, with the consequential activation of nuclear forces, at which activation fusion is simply unavoidable under the indicated premises.

## 2. Review of the New Tests

In this paper, we report three tests providing additional experimental confirmation of the preceding results [3,4,5]. It should be stressed to prevent misconceptions, that as it was the case for the preceding tests, the sole objective at this time of the tests reported below is that of **confirming the "existence" of systematic and controlled nuclear fusions without harmful radiation or waste.** Any expectation of "measurements" of heat produced, flow, temperature gradient and other data would be grossly premature at this time since the equipment could only be operated for a few minutes due to excessive production of heat. Also, the achievement of measurements will require the investment of millions of dollars for the construction of a hadronic reactor suitable to operate for the sufficient long time needed for meaningful measurements. Under these understandings, the new tests can be reported as follows:

**TEST 1.**

the main objective of this test was the experimental confirmation of the existence of the following new ICNF

$$O(18,8,0^+,17.9991) + C(12,6,0^+,12.0000) + TR \rightarrow$$

$$\rightarrow Si(30,14,0^+,29.9737) + \Delta E, \tag{2a}$$

$$\Delta E = 0.0254\ u, \tag{2b}$$

that also verifies all possible nuclear laws. The test was suggested by the fact that, during the years of experimentation on ICNF, the author has systematically seen a

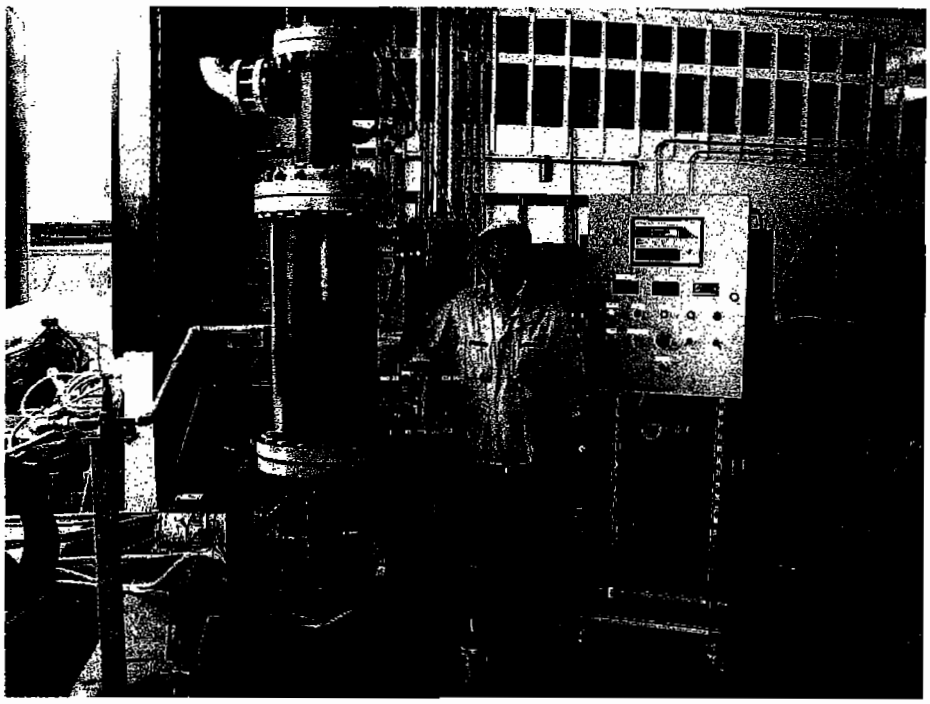


Figure 4: A picture of the hadronic reactor used in Tests 1, 2, 3.

"whitish powder" on the edge of carbon electrodes that is somewhat suggestive of the synthesis of silica.

For the test of ICNF (2a), the author and his technicians **Chris Lynch, Michael Rodriguez, Gene West, Donald Roch, Ray Jones and Jim Alban** constructed in early 2010 a new, hadronic reactor with automatic controls of the arc and main functions. as depicted in Figures 4, 5 and 6. This is the first automatic hadronic reactor for ICNF since it creates and controls automatically the DC arc, but also monitors all main features, including power, temperature, pressure, flow, trigger, and other features with automatic shut off in the event of any malfunction. The reactor essentially consists in an internal, carbon steel, schedule 80, cylindrical vessel $1\ ft \times 5\ ft$ filled up with the desired gaseous hadronic fuel and traversed by a DC arc between carbon electrodes. The internal chamber is then completed with an external water jacket used to cool down the reactor and for the production of steam. An AC-DC converter was used with 100 $kW$ maximal power, although actual uses were restricted to 50 $kW$ for safety. The reactor is then completed with a variety of sensors for internal as well as external temperature, pressure and other data connected to the automatic controls.

Following over one year of tests, verifications and tuning to assure the proper operation and safety of the reactor, on April 11, 2011, with the assistance of the



Figure 5: A view of the production of steam during test 3.

above indicated technicians, the author pulled a vacuum from the interior chamber of the reactor, that was subsequently filled up with commercially available oxygen at 100 *psi* pressure. The reactor was then operated for six minutes, at which time there was a violent increase in the production of steam out of the cooling jacket (see Figure 5) that forced the shut down of the reactor for safety.

After cooling off, the reactor was open and solid samples of the electrodes were sent for independent chemical analysis by *Princeton Gamma Tech Instruments* on a comparative basis with a solid sample of the same electrodes before the activation of the arc. **These analyses, entirely reported in Ref. [6], establish the distinct detection of silica following the activation of the DC arc that, under the above conditions, confirm the synthesis in laboratory of silica via ICNF (2a).** Note that no sample of the interior gas was taken because its analysis would have no impact on the desired verification, the latter dealing with a solid.

We should add that, as it was the case for all preceding tests, no measurable radiation was detected in the outside and no radioactive waste was detected in the inside of the hadronic reactor following its opening up after cooling. The various detectors used for radiations have been described in detail in Refs. [3,4] and their identification is ignored hereinafter to avoid repetitions.

**TEST 2.**

The controlled fusion of oxygen and carbon into silica was done because particularly important for environmental reasons since it is the premise for *the use of the green house gas $CO_2$ as a hadronic fuel for the production of clean energy.* In fact, a hadronic reactor can be filled up with $CO_2$ at pressure; the DC arc will be quite efficient in its separation into oxygen and carbon; part of the separated oxygen and carbon will evidently combust and produce $CO$ that, in the presence of oxygen and an arc, reproduce again $CO_2$, thus recovering in great part the energy used for the separation of $CO_2$. However, jointly with the conventional combustion at a loss for the energy balance, the hadronic reactor will produce a net positive energy output due to the fusion of oxygen and carbon into silica. Test 1 described above and the second test here considered confirm the possible use of $CO_2$ as hadronic fuel for the production of energy without harmful radiation or waste via the indicated processes.

However, the use of oxygen in a hadronic reactor is very dangerous because it is known that virtually all substances, including metals, ignite when exposed to oxygen at high temperature. In fact, the local temperature at the tip of the DC arc when hitting the cathode is estimated as being, locally, of the order of $10^6 C$. Even though such a temperature decreases quite rapidly with the distance from the arc, it nevertheless causes a rapid increase in the temperature of the oxygen. This essentially implies the achievement of high oxygen temperatures in a matter of minutes at 100 *psi* pressure, and in seconds at higher pressures, at which value combustion of most substances exposed to oxygen is expected.

Following the adoption of due safety precautions, and in view of the indicated environmental relevance, the author and his technicians repeated Test 1 on April 14, 2011 for the specific intent of verifying or disproving results [6]. This second test was done under exactly the same conditions and setting of Test 1, thus without any modifications, to prevent variations. As predicted from carbon powder accumulated in the preceding Test 1, the internal oxygen achieved metal combustion temperature in about *three seconds* of operations, at which time an external metal fitting measuring pressure ignited and the operation has to be instantly interrupted. Nevertheless, despite its shortness, the test was sufficient to secure sample of "glassy-type small droplets" formed in the top of the cathode that were sent to *Princeton Gamma Tech Instruments* for study. **The resulting analyses, reported in full in Ref. [7], confirmed for the second time the synthesis of silica from oxygen and carbon via ICNF (2a)** via a comparison of the solid samples of Test 2 with those of the electrodes prior to the activation of the arc.

We should add again that, as it was the case for all preceding tests, no measurable radiation was detected in the outside and no radioactive waste was detected in the inside of the hadronic reactor following its opening up after cooling.

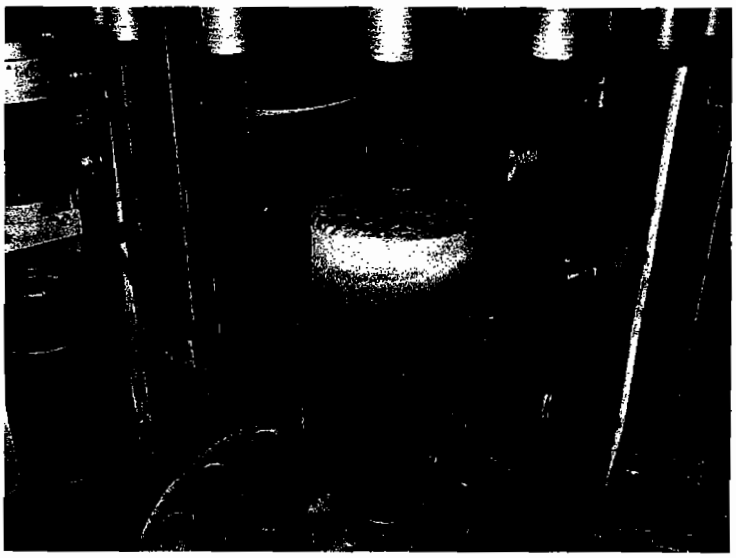


Figure 6: A view of the scorched carbon cathode following test 3.

**TEST 3.**
Following the successful synthesis of silica and its confirmation, among a variety of possible additional syntheses, the author selected Test 3 the ICNF of helium and carbon into the oxygen according to the rules

$$C(12,6,0^{+},12.0000)+He(4,2,0^{+},4.0026)+TR\rightarrow$$

$$\rightarrow O(16,8,0^{+}15.9949)+\Delta E \tag{3a}$$

$$\Delta E=0.0077\ u \tag{3b}$$

which synthesis also verifies all possible nuclear laws.

The test was done by the author and the above identified technicians on April 15, 2011, along lines similar to the preceding ones. The interior of the reactor was cleaned, and various components replaced; a vacuum was pulled out of the interior chamber; the reactor was filled up with commercial grade helium at 100 *psi*; a sample of the interior gas was taken following due flushing and marked $He1$; the reactor was activated for about six minutes and then shut off because of excessive increase of the produced steam from the water jacket; a sample of the interior gas was then taken and, again after flushing, marked $He2$; and the two samples $He1$, $He2$ were sent to the *Oneida ORS Laboratories* for chemical analyses. **the results, reproduced in full in Ref. [8] with main results reported in Figure 7, confirm the synthesis of helium and carbon according to ICNF (3)** because, as one can see, the oxygen content decreased from 117 *ppmv* in $He1$ to a non-detectable amount

in $He2$ but the $CO$ increased from a non-detectable amount in $He1$ to 4.24% in $He2$, an increase solely possible from the synthesis of oxygen in the interior of the reactor.

We should indicate that, following test 3, samples of the electrodes were sent to *Princeton Gamma Tech Instruments* for comparative analysis with the sample electrode not exposed to the arc. The analysis was done because, following the test, the top of the cathode acquired a "glassy-type" appearance suggesting the possible synthesis of silica following that of the oxygen as per Tests 1 and 2. The results of the analyses, reported in full in Ref. [9] , show *complete absence of silica in Test 3,* and the production instead of a large peak of Fluorite that could originate from the melting of some internal plastic component of the hadronic reactor. Jointly we also note the increase of $CO_2$ from non-detectable in $He1$ to 914 *ppmv* in $He2$.

The latter negative result establishes that *the double nuclear synthesis, first of helium and carbon into oxygen and then of oxygen and carbon into silica, "cannot" be controlled.* In fact, during the first step, the oxygen is synthesized at the tip of the DC arc when hitting the carbon in the cathode surface. The ensuing large local production of heat as per value (3b) rapidly expels the synthesized oxygen from the DC arc, thus preventing any additional nuclear synthesis. The creation of CO is then consequential due to the great affinity of carbon and oxygen which is at the foundation of our lives.

Needless to say, the peak reported in analyses [9] for $F(19, 9, 1/2^+)$ could have interpretation other than the aboce indicated melt down of internal plastic components of the reactor, such as the ICNF of $O(18, 8, 0^+)$ and $H(1, 1, 1/2^+)$. Similarly, inspection of analyses [8] reveals the increase of the percentage of a number of elements. Of course, these increases are expected from the heat produced by the arc and the consequential conventional release of gases from the various substances composing the hadronic reactor, although some of the new elements could be the result, at least in part, of additional ICNF. The study of these possibilities requires additional tests with related analyses and they are planned for release in future presentation.

We should add again that, as it was the case for all preceding tests, no measurable radiation was detected in the outside and no radioactive waste was detected in the inside of the hadronic reactor following its opening up after cooling.

## 3. Concluding Remarks.

The preceding tests [3,4,5] and the additional tests presented in this paper have completed the author's intent Phase I consisting in establishing the "existence" of ICNF without harmful radiations or waste, and provided the necessary credibility for the transition to Phase II consisting in the construction of a prototype hadronic reactor producing clean electric energy in excess of that used.

Despite these promising results, the author would like to caution the reader against easy expectations of rapid achievement of Phase III, consisting in commercially avail-

ONEIDA RESEARCH SERVICES, INC.

**TEST REPORT**
**INTERNAL VAPOR ANALYSIS**

8282 HALSEY ROAD • WHITESBORO, NY 13492 • PHONE: (315) 736-5480

GINO AMATO
MAGNEGAS CORPORATION
150 RAINVILLE ROAD
TARPON SPRINGS, FL 34689
UNITED STATES

ORS REPORT NO. : 189920-001
DATE TESTED : 4/8/2011
QUANTITY TESTED : 2
PACKAGE TYPE : CYLINDER
MFG. CODE :

PO: 724
Rel. No:

| SAMPLE ID | | HE1 | HE2 | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| INLET PRESSURE | torr | 387 | 474 | | | | | | |
| NITROGEN | ppmv | 665 | 5,431 | | | | | | |
| OXYGEN | ppmv | 117 | ND | | | | | | |
| ARGON | ppmv | ND | 40 | | | | | | |
| CO2 | ppmv | ND | 914 | | | | | | |
| MOISTURE | ppmv | 1,281 | 3,061 | | | | | | |
| HYDROGEN | %v | 0.03 | 3.06 | | | | | | |
| METHANE | ppmv | ND | ND | | | | | | |
| AMMONIA | ppmv | ND | ND | | | | | | |
| HELIUM | %v | 99.8 | 91.7 | | | | | | |
| FLUORO-CARBONS | ppmv | ND | ND | | | | | | |
| KRYPTON | ppmv | ND | ND | | | | | | |
| BENZENE | ppmv | ND | 158 | | | | | | |
| CO | %v | ND | 4.24 | | | | | | |

COMMENTS:

ND = None Detected
1% = 10,000 ppm

Tested per ORS SOP MEL-1070: Gas Analysis of Sealing Chamber Atmosphere.

Page : 1of1

APPROVED BY: Daniel J. Rossiter

Figure 7: A reproduction of the main results of the chemical analyses on gases for Test 3 conducted by Oneida ORS Laboratories

able new clean energies, due to the complexity of the engineering problems to be solved for extended use, as well as the large investments needed for their achievement.

**Acknowledgment**

The content of this note is the output of long and solitary consideration by the author expressed in ref. [2]. The main point of this note was then first discussed during the recent *Third International Conference on the Lie-Admissible Treatment of Irreversible Processes* held at the University of Kathmandu, Nepal, from January 5 to 9, 2011. The author would like to thank all participants for invaluable comments. Additionally, very special thanks are due to R. Brenna, T. Kuliczkowski and L. Ying of *Princeton Gamma Tech Instruments* and to D. J. Rossiter of *Oneida ORS Laboratories* because, without their detailed independent analyses, this paper would not have been possible. Additional thanks are due to Dorte Zuckerman for linguistic control and to Christian Corda, the Editor of the proceedings for an impeccable editorial control.

# References


[1] I. Gandzha and J Kadeisvili, *New Sciences for a New Era: Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli,* Sankata Printing Press, Nepal (2011) preliminary version available in free pdf download from the link http://www.santilli-foundation.org/docs/FoundationBook-12-10.pdf

[2] R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry*, Volumes I, II, III, IV, and V, International Academic Press, USA, 2008. available in free pdf download from the link
http://www.i-b-r.org/Hadronic-Mechanics.htm

[3] R. M. Santilli, Experimental Confirmation of the Novel Intermediate Controlled Nuclear Fusion without harmful Radiations, New Advances in Physics, 5 (2010, 29, available in free pdf download from
http://www.santilli-foundation.org/docs/ICNF.pdf

[4] R. Brenna, T. Kuliczkowski and L. Ying, Verification of Santillis intermediate nuclear fusions without harmful radiation and the production of magnecular clusters, New Advances in Physics 5 (2010), 9, available in free pdf download from
http://www.santilli-foundation.org/docs/Conf-1-ICNF.pdf

[5] L. Ying, "Verification of Santilli's Intermediate Nuclear Harmful Radiation and the Production of Magnecular Clusters," Lecture VE of *World Lecture series*
http://www.world-lecture-series.org/

[6] R. Brenna, T. Kuliczkowski and L. Ying, Report on Test 1, available in free pdf download from
http://www.santilli-foundation.org/docs/PGTI-Anal-test1.pdf

[7] R. Brenna, T. Kuliczkowski and L. Ying, report on Test 2, available in free pdf download from
http://www.santilli-foundation.org/docs/PGTI-Anal-test2.pdf

[8] D. J. Rossiter, report on test 3, available in free pdf download from
http://www.santilli-foundation.org/docs/ORS-Anal-Test3.pdf

[9] R. Brenna, T. Kuliczkowski and L. Ying, report on Test 3, available in free pdf download from
http://www.santilli-foundation.org/docs/PGTI-Anal-test3.pdf

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
Kathmandu University, Nepal, April (2011) pages 203-218

# Multitime Optimal Control for Quantum Systems

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Dedicated to Prof. Dr. Ruggero Maria Santilli for his seminal contributions to the Science

**Abstract**

The significant advantage of the multitime maximum principle is its applicability to control problems involving multitime-varying situations in engineering, economics and biology. Generally, in the multitime control theory, the evolution parameter is multidimensional.

Motivated by developments and problems in a number of disciplines including Quantum Chemistry, Information and Optics, the theory of Control of Quantum Systems has emerged. For multitime quantum control systems, the state may be the density matrix or the pure state vector. In alternative the constraints must be the multitime dynamics of the evolution PDE operator, i.e., the multitime Schrödinger PDEs. Our paper is the first which studies and solves such problems via optimal deformations. The general cost is of Bolza type including a multiple integral or a mediate path independent curvilinear integral and the dynamics (constraint) is described by the controlled Schrödinger PDEs. In this way the control theory itself is been enriched by new models and paradigms.

**Mathematics Subject Classification 2010**: 49K20, 81V10, 81S40

**PACS**: 02.30.Yy, 03.65.-w, 11.30.-j
**Key words:** multitime maximum principle, multitime Schrödinger PDEs, quantum electrodynamics, quantum mechanics.

# 1 Control Problems in Quantum Mechanics

Quantum mechanics present many specific difficulties, coming from the environment of the quantum system. Usually, one deals with an isolated quantum system whereas this is never the case and one has to take into account the environment of this quantum system [1]-[9], [15], [16].

It is of major interest to investigate the behavior of a quantum system under the effect of an external force, usually due to an *external electric field*. Trying to describe the possible behavior of the system leads to a single-time control problem, the control variable acting through a potential in the equation, this potential coming from the external electric field. This question is important in quantum chemistry when one tries to break molecules or create new molecules. At the scale of a molecule or of an atom, in the non relativistic case, the basic constraint PDEs to be considered are the *Schrödinger equations*. The environment can usually be described by *microscopic or macroscopic Maxwell PDEs*. Up to now, very few works have been done on the *coupled Schrödinger-Maxwell systems* and this could be a very important new subject either for single-time or multitime control theory. On the other hand, control of (isolated) Schrödinger equations has been extensively studied in the last years, but still many important questions remain unsolved.

Our goal is to apply the tools and methods of Multitime Control Theory [17]-[22] in the analysis and design of scientific and engineering applications of Quantum Systems (see also [10], [11]). Section 2 introduces the variational Schrödinger PDEs and the adjoint Schrödinger PDEs. Section 3 formulates and proves a multitime maximum principle for a quantum control problem consisting in a multiple integral action and the Schrödinger PDEs as constraints and proposes quantum control problems containing the modified Boltzmann-Shannon entropy functional. Section 4 formulates and proves a multitime maximum principle for a quantum control problem consisting in a mediate curvilinear integral action constrained by the Schrödinger PDEs. Section 5 underlines some contributions of Prof. Dr. Ruggero Maria Santilli to Hadronic Mechanics, Hadronic Chemistry, Mathematics.

# 2 Multitime adjoint Schrödinger PDE systems

We consider two linear PDE systems associated to a nonlinear PDE system, whose solutions determine either an interior product or a partial interior product of type $(1,p)$-tensor. They are called *adjoint (dual) systems* if the associated interior product or the $(1,p)$-tensor are *conservation laws (first integral*, respectively *zero divergence)*. Two nonlinear PDE systems are called *adjoint (dual) systems* if their associated linear variational systems are adjoint (see also [12]).

Let us start with the multitime Schrödinger PDEs as controlled evolution law

$$\frac{\partial \psi^k}{\partial t^\alpha}(t,x) = -i\, H_\alpha(u(t,x))\psi^k(t,x),\ k=1,2,3;\ \alpha=1,...,m,$$

where $u(t,x) = (u^a(t,x)),\ a=1,...,q$ is the *control variable.* This PDE system has solutions if and only if the complete integrability conditions

$(CIC)$

$$\frac{\partial(H_\alpha\psi^k)}{\partial x^j}H_\beta\psi^j + \frac{\partial(H_\alpha\psi^k)}{\partial u^a}\frac{\partial u^a}{\partial t^\beta} = \frac{\partial(H_\beta\psi^k)}{\partial x^j}H_\alpha\psi^j + \frac{\partial(H_\beta\psi^k)}{\partial u^a}\frac{\partial u^a}{\partial t^\alpha}$$

are satisfied. These determine the set of *admissible controls*

$$\mathcal{U} = \{u(\cdot,\cdot) : R^m_+ \times R^3 \to U \mid u(\cdot) \text{ is measurable and satisfies CIC}\}.$$

The multitime Schrödinger evolution law is linear and unitary (norm preserving) on account of the self-adjointness of $H_\alpha$. Indeed,

$$\frac{\partial}{\partial t^\alpha}||\vec{\psi}(t,x)||^2 = \frac{\partial}{\partial t^\alpha}\left(\vec{\psi}(t,x),\vec{\psi}(t,x)\right)$$

$$= \left(-iH_\alpha\vec{\psi}(t,x),\vec{\psi}(t,x)\right) + \left(\vec{\psi}(t,x),-iH_\alpha\vec{\psi}(t,x)\right)$$

$$= -i\left(H_\alpha\vec{\psi}(t,x),\vec{\psi}(t,x)\right) + i\left(\vec{\psi}(t,x),H_\alpha\vec{\psi}(t,x)\right) = 0,$$

i.e., the function $||\vec{\psi}(t,x)||^2$ is a first integral.

Let $\psi^k(t,x;\epsilon)$ be a differentiable variation of $\psi^k(t,x)$, i.e.,

$$\frac{\partial \psi^k}{\partial t^\alpha}(t,x;\epsilon) = -i\, H_\alpha(u(t,x))\psi^k(t,x;\epsilon),\ \psi^k(t,x;0) = \psi^k(t,x),$$

where $\epsilon = (\epsilon^\alpha),\ \alpha=1,...,m$. Denoting $y^k_\alpha = \frac{\partial \psi^k}{\partial \epsilon^\alpha}(t,x;0)$, we find the *infinitesimal deformation PDEs* or the *variational Schrödinger PDEs*

$$(VS) \qquad \frac{\partial y^k_\alpha}{\partial t^\beta}(t,x) = -i\, H_\beta(u(t,x))y^k_\alpha(t,x).$$

These determine the *multitime dual (adjoint) Schrödinger PDEs*

$$(AS) \qquad \frac{\partial p^\alpha_k}{\partial t^\beta}(t,x) = i\, H_\beta(u(t,x))p^\alpha_k(t,x)$$

whose solution $p = (p_k^\alpha)$ is called the *costate matrix*. The PDE systems $(VS)$ and $(AS)$ are *adjoint* (*dual*) in the sense of *constant interior product of solutions*, i.e., the scalar product $p_k^\alpha y_\alpha^k$ is a first integral.

Of course, taking the trace, we can define the *costate matrix* $p : \Omega_{0T} \to R^{3m}$, $p = (p_k^\alpha)$, as the solution of the *divergence adjoint PDE system* (trace of $(ADJ)$)

$$\frac{\partial p_k^\alpha}{\partial t^\alpha}(t,x) = i\, H_\alpha(u(t,x))p_k^\alpha(t,x).$$

But than, the PDEs systems (23′) and (24) are *adjoint* (dual) in the sense of *zero total divergence* of the tensor field $Q_\beta^\alpha = p_k^\alpha y_\beta^k$ produced by their solutions. The divergence dual PDE system (24) has solutions since it contains $n$ PDEs with $nm$ unknown functions $p_i^\alpha$. We can select a solution of the gradient form $p_k^\alpha(t,x) = \frac{\partial v^\alpha}{\partial x^k}(t,x)$.

# 3 Quantum Control Systems with Bolza Type Cost as Multiple Integral

In the case of quantum control systems, the *state* may be the *density matrix*, the *pure state vector* or the *evolution operator*. Here we refer to optimal control problems for the pure state when the parameter of evolution $t = (t^1, ..., t^m)$ is multidimensional. Similar considerations can be made in the other cases.

Let $dt = dt^1 \wedge ... \wedge dt^m$, $dx = dx^1 \wedge dx^2 \wedge dx^3$ be the volume elements and

$$L(t, \vec{\psi}(t,x), u(t,x))dt \wedge dx$$

be an $(m+3)$-form of class $C^2$. Suppose that the general cost is of *Bolza type* written as a sum between a terminal term and a multiple integral

$$Q(u(\cdot)) = \phi(T, \vec{\psi}(T)) + \int_{D_{x_0x_T}} \int_{\Omega_{0T}} L(t, \vec{\psi}(t,x), u(t,x))dt\, dx.$$

We add the dynamics constraint described by the *controlled completely integrable Schrödinger PDEs*

$$\frac{\partial \vec{\psi}}{\partial t^\alpha}(t,x) = -i\, H_\alpha(u(t,x))\vec{\psi}(t,x),\ \vec{\psi}(0,x) = \vec{\psi}_0(x),\ \alpha = 1, ..., m,$$

where $u(x,t)$ is the *time-dependent electric field*. A model for the Schrödinger operators $H_\alpha$ can be

$$H_\alpha = c_\alpha(t)H,\ H = -\frac{1}{2}\nabla^2 + V(x) - A_i^\beta(t)u_\beta^i(t,x),$$

where $H$ is the *standard Schrödinger operator*, $V(x)$ is an *external potential* and $-\frac{1}{2}\nabla^2$ is the *kinetic energy operator* (atomic units: $\hbar = 1, m = 1, e = 1$).

## 3.1 Solution via Lagrange multipliers

We introduce a *costate variable* or *Lagrange multiplier matrix* $p = (p_k^\alpha)$ and a new Lagrangian

$$\mathcal{L}(t, \vec{\psi}(t,x), u(t,x), p(t,x)) = L(t, \vec{\psi}(t,x), u(t,x))$$

$$+p_k^\alpha(t,x)\left(\frac{\partial \psi^k}{\partial t^\alpha}(t,x) + i\, H_\alpha(u(t,x))\psi^k(t,x)\right).$$

The foregoing $PDE$-constrained optimization problem is changed into another optimization problem

$$\max_{u(\cdot)} \tilde{Q}(u(\cdot)) = \int_{D_{x_0x_T}} \int_{\Omega_{0T}} \mathcal{L}(t, \vec{\psi}(t,x), u(t,x))dt\, dx$$

subject to

$$u(t,x) \in \mathcal{U},\ p(t,x) \in \mathcal{P},\ t \in \Omega_{0T},\ x \in D_{x_0x_T},\ \vec{\psi}(0,x) = \psi(x),$$

where the set $\mathcal{P}$ will be defined later. The *control Hamiltonian*

$$\mathcal{H}(t, \vec{\psi}(t,x), u(t,x), p(t,x)) = L(t, \vec{\psi}(t,x), u(t,x))$$

$$+i\, p_k^\alpha(t,x) H_\alpha(u(t,x))\psi^k(t,x),$$

i.e.,

$$\mathcal{H} = \mathcal{L} - p_k^\alpha \frac{\partial \psi^k}{\partial t^\alpha} \text{ (Legendrian duality)},$$

allows to rewrite this new problem as

$$\max_{u(\cdot)} \int_{D_{x_0x_T}} \int_{\Omega_{0T}} \left(\mathcal{H}(t, \vec{\psi}(t,x), u(t,x), p(t,x)) + p_k^\alpha \frac{\partial \psi^k}{\partial t^\alpha}\right) dt\, dx$$

subject to

$$u(t,x) \in \mathcal{U},\ p(t,x) \in \mathcal{P},\ t \in \Omega_{0T},\ x \in D_{x_0x_T},\ \vec{\psi}(0,x) = \vec{\psi}(x).$$

Suppose that there exists a continuous control $\hat{u}(t,x)$ defined over the hyper-parallelepiped $\Omega_{0T} \times D_{x_0x_T}$ with $\hat{u}(t,x) \in \mathrm{Int}\,\mathcal{U}$ which is an optimum point in the previous problem. Now consider a variation

$$u(t,x;\epsilon) = \hat{u}(t,x) + \epsilon h(t,x),$$

where $h$ is an arbitrary continuous vector function. Since $\hat{u}(t,x) \in \mathrm{Int}\,\mathcal{U}$ and a continuous function over a compact set is bounded, there exists $\epsilon_h > 0$ such that $u(t,x;\epsilon) = \hat{u}(t,x) + \epsilon h(t,x) \in \mathrm{Int}\,\mathcal{U},\ \forall|\epsilon| < \epsilon_h$. This $\epsilon$ is used in our variational arguments.

Define the deformation $\vec{\psi}(t,x;\epsilon)$ as the $(m+3)$-sheet of the state variable corresponding to the control variable $u(t,x;\epsilon)$, i.e.,

$$\frac{\partial \vec{\psi}}{\partial t^\alpha}(t,x;\epsilon) = -i\, H_\alpha(u(t,x;\epsilon))\vec{\psi}(t,x;\epsilon), \forall (t,x) \in \Omega_{0T} \times D_{x_0x_T}$$

and $\vec{\psi}(t,x;0) = \vec{\psi}(t,x)$. For $|\epsilon| < \epsilon_h$, we introduce the deformed Lagrangian

$$\mathcal{L}(t,\vec{\psi}(t,x;\epsilon),u(t,x;\epsilon),p(t,x))$$

$$= \mathcal{H}(t,\vec{\psi}(t,x;\epsilon),u(t,x;\epsilon),p(t,x)) + p_k^\alpha(t,x)\frac{\partial \psi^k}{\partial t^\alpha}(t,x;\epsilon),$$

the functional

$$F_{\vec{\psi}up}(x;\epsilon) = \int_{\Omega_{0T}} \mathcal{L}(t,\vec{\psi}(t,x;\epsilon),u(t,x;\epsilon),p(t,x))dt$$

and the function

$$Q(\epsilon) = \int_{D_{x_0x_T}} F_{\vec{\psi}up}(x;\epsilon)dx.$$

Since the control $u(t,x;\epsilon)$ is admissible, it follows that the function $\vec{\psi}(t,x;\epsilon)$ is admissible. On the other hand, the control $\hat{u}(t,x)$ must be optimal. Therefore $Q(\epsilon) \leq Q(0)$, $\forall |\epsilon| < \epsilon_h$.

To evaluate the multiple integral

$$\int_{\Omega_{0T}} p_k^\alpha(t,x)\frac{\partial \psi^k}{\partial t^\alpha}(t,x;\epsilon)dt,$$

we integrate by parts, via the divergence formula

$$\frac{\partial}{\partial t^\alpha}(p_k^\alpha \psi^k) = \frac{\partial p_k^\alpha}{\partial t^\alpha}\psi^k + p_k^\alpha \frac{\partial \psi^k}{\partial t^\alpha},$$

obtaining

$$\int_{\Omega_{0T}} p_k^\alpha(t,x)\frac{\partial \psi^k}{\partial t^\alpha}(t,x;\epsilon)dt = \int_{\Omega_{0T}} \frac{\partial}{\partial t^\alpha}(p_k^\alpha(t,x)\psi^k(t,x;\epsilon))dt$$

$$- \int_{\Omega_{0T}} \frac{\partial p_k^\alpha}{\partial t^\alpha}(t,x)\psi^k(t,x;\epsilon)dt.$$

Now we apply the divergence integral formula

$$\int_{\Omega_{0T}} \frac{\partial}{\partial t^\alpha}(p_k^\alpha(t)\psi^k(t,x;\epsilon))dt = \int_{\partial\Omega_{0T}} \delta_{\alpha\beta}p_k^\alpha(t,x)\psi^k(t,x;\epsilon)n^\beta(t)d\sigma,$$

where $(n^\beta(t))$ is the unit normal vector to the boundary $\partial\Omega_{0T}$. Substituting, we find

$$G_{\vec{\psi}up}(x;\epsilon)$$

$$= \int_{\Omega_{0T}} \left(\mathcal{H}(t,\vec{\psi}(t,x;\epsilon),u(t,x;\epsilon),p(t,x)) - \frac{\partial p_k^\alpha}{\partial t^\alpha}(t,x)\psi^k(t,x;\epsilon)\right)dt,$$

$$Q(\epsilon) = \int_{D_{x_0x_T}} \left(G_{\vec{\psi}up}(x;\epsilon) + \int_{\partial\Omega_{0T}} \delta_{\alpha\beta}p_k^\alpha(t,x)\psi^k(t,x;\epsilon)n^\beta(t)d\sigma\right)dx.$$

Differentiating with respect to $\epsilon$, it follows

$$Q'(\epsilon) = \int_{D_{x_0x_T}} \int_{\Omega_{0T}} \left(\mathcal{H}_{\psi^k} - \frac{\partial p_k^\alpha}{\partial t^\alpha}\right)\psi_c^k(t,x;\epsilon)dt\,dx$$

$$+\int_{D_{x_0x_T}}\int_{\Omega_{0T}}\mathcal{H}_{u^a}h^a(t,x)dt\,dx$$

$$+\int_{D_{x_0x_T}}\int_{\partial\Omega_{0T}}\delta_{\alpha\beta}p_k^\alpha(t,x)\psi_\epsilon^k(t,x;\epsilon)n^\beta(t)d\sigma\,dx.$$

Evaluating at $\epsilon=0$, the expression of $Q'(0)$ is

$$\int_{D_{x_0x_T}}\int_{\Omega_{0T}}\left(\mathcal{H}_{\psi^k}(t,\vec{\psi}(t,x),\hat{u}(t,x),p(t,x))-\frac{\partial p_k^\alpha}{\partial t^\alpha}\right)\psi_\epsilon^k(t,x;0)dt\,dx$$

$$+\int_{D_{x_0x_T}}\int_{\Omega_{0T}}\mathcal{H}_{u^a}(t,\vec{\psi}(t,x),\hat{u}(t,x),p(t,x))h^a(t,x)dt\,dx$$

$$+\int_{D_{x_0x_T}}\int_{\partial\Omega_{0T}}\delta_{\alpha\beta}p_k^\alpha(t,x)\psi_\epsilon^k(t,x;0)n^\beta(t)d\sigma\,dx,$$

where $\vec{\psi}(t,x)$ is the $(m+3)$-sheet of the state variable corresponding to the optimal control $\hat{u}(t,x)$.

Using the idea of the adjoint PDE system, we define $\mathcal{P}$ as the set of solutions of the boundary value problem

$$\frac{\partial p_k^\alpha}{\partial t^\alpha}(t,x)=\mathcal{H}_{\psi^k}(t,\vec{\psi}(t,x),\hat{u}(t,x),p(t,x)),\ \forall t\in\Omega_{0T},\ \forall x\in D_{x_0x_T}$$
$$\delta_{\alpha\beta}p_k^\alpha(t,x)n^\beta(t)|_{\partial\Omega_{0T}}=0\,(\text{orthogonality or tangency}). \tag{4}$$

We need $Q'(0)=0$ for all $h(t,x)=(h^a(t,x))$. Therefore

$$\mathcal{H}_{u^a}(t,\vec{\psi}(t,x),\hat{u}(t,x),p(t,x))=0,\ \forall t\in\Omega_{0T},\ \forall x\in D_{x_0x_T}. \tag{5}$$

Moreover, the Schrödinger PDEs can be rewritten

$$\frac{\partial\psi^k}{\partial t^\alpha}(t,x)=-\frac{\partial\mathcal{H}}{\partial p_k^\alpha}(t,\vec{\psi}(t,x),\hat{u}(t,x),p(t,x)),\ \forall t\in\Omega_{0T},\ \forall x\in D_{x_0x_T}. \tag{6}$$

**Remarks.** (i) The algebraic system (5) describes the critical points of the Hamiltonian with respect to the control variable. (ii) The *PDEs* (4) and (6) and the condition (5) are Euler-Lagrange *PDEs* associated to the modified Lagrangian.

Summarizing, we obtain a *multitime maximum principle* similar to the *single-time Pontryaguin maximum principle.*

**Theorem 1. (multitime maximum principle; necessary conditions )** *Suppose that the problem of maximizing the functional $Q(u(\cdot))$ subject to the Schrödinger PDE constraints (2) has an interior solution $\hat{u}(t)\in\mathcal{U}$ which determines the $(m+3)$-sheet $\vec{\psi}(t,x)$. Then there exists a $C^1$ costate $p(t,x)=(p_k^\alpha(t,x))$ defined over $\forall t\in\Omega_{0T}$, $\forall x\in D_{x_0x_T}$ such that the relations (4), (5), (6) hold.*

**Remark** If the optimal control $\hat{u}(t,x)\in\mathcal{U}$ is not an interior point, then instead of critical point condition we have

$$\mathcal{H}(t,\vec{\psi}(t,x),\hat{u}(t,x),p(t,x))=\min_u\mathcal{H}(t,\vec{\psi}(t,x),u,p(t,x)).$$

## 3.2 Same problem in terms of real functions and choice of cost functional

Of course, both the state $\vec{\psi}$ and the matrices $-iH_\alpha(u)$ are complex functions. To transform the problem in terms of real functions, we need to write

$$\vec{\psi} = \vec{\psi}_R + i\vec{\psi}_I, \ -iH_\alpha(u) = R_\alpha(u) + iI_\alpha(u),$$

where $\vec{\psi}_R$ and $\vec{\psi}_I$ are real $n$-dimensional vectors and $R_\alpha(u)$ and $I_\alpha(u)$ are real $n \times n$ matrix functions of the control $u$, with $R_\alpha(u)$ skew symmetric and $I_\alpha(u)$ symmetric. Placing these data into the previous PDEs and separating the real and the imaginary parts, we obtain

$$\frac{\partial \vec{\psi}_R}{\partial t^\alpha}(t,x) = R_\alpha(u(t,x))\vec{\psi}_R(t,x) - I_\alpha(u(t,x))\vec{\psi}_I(t,x)$$

$$\frac{\partial \vec{\psi}_I}{\partial t^\alpha}(t,x) = I_\alpha(u(t,x))\vec{\psi}_R(t,x) + R_\alpha(u(t,x))\vec{\psi}_I(t,x).$$

Introducing the variable $X = (\vec{\psi}_R^T, \vec{\psi}_I^T)^T$ and the matrix

$$\tilde{H}_\alpha(u) = \begin{pmatrix} R_\alpha(u) & -I_\alpha(u) \\ I_\alpha(u) & R_\alpha(u) \end{pmatrix},$$

we can write the constraint PDEs, describing the dynamics, only using real quantities as

$$\frac{\partial X}{\partial t^\alpha} = \tilde{H}_\alpha(u)X.$$

The matrices $\tilde{H}_\alpha(u)$ are skew symmetric and symplectic for every $u$, i.e., they belong to $so(2n) \cap sp(n)$. Implicitly the cost can be written as

$$J(u(\cdot)) = \tilde{\phi}(T, X(T)) + \int_{D_{x_0x_T}} \int_{\Omega_{0T}} \tilde{L}(t, X(t), u(t,x))dt\,dx,$$

for appropriate functions $\tilde{\phi}$ and $\tilde{L}_\alpha$.

A first choice of the cost functional in the molecular control is the *laser electric field fluency*

$$J(u(\cdot)) = k \int_{D_{x_0x_T}} \int_{\Omega_{0T}} ||u(t,x)||^2 dt\,dx, \ k > 0,$$

which measure the *energy of the electric field* in the multitime interval [0,T]. A second choice is a cost of the type (*electric energy of deformation*

$$J(u(\cdot)) = k \int_{D_{x_0x_T}} \int_{\Omega_{0T}} \delta^{\alpha\beta} < \frac{\partial u}{\partial t^\alpha}(t,x), \frac{\partial u}{\partial t^\beta}(t,x) > dt\,dx, \ k > 0,$$

which *filters the high frequency components of the control field.*

When emphasis is placed on the final state, one can minimize a cost functional of the form

$$\frac{1}{2} < O >_{\psi} + \frac{k}{2} \int_{D_{x_0 x_T}} \int_{\Omega_{0T}} ||u(t,x)||^2 dt\, dx$$

$$= \frac{1}{2} \vec{\psi}^{+} O \vec{\psi} + \frac{k}{2} \int_{D_{x_0 x_T}} \int_{\Omega_{0T}} ||u(t,x)||^2 dt\, dx,\ k > 0,$$

where $O$ is a negative definite Hermitian matrix. For example, we can choose the matrix $O = -\vec{\psi}_f \vec{\psi}_f^{+}$, if $\vec{\psi}_f$ is the desired state.

## 3.3 Open problem

Let $X = L_1(\Omega_{0T} \times D_{x_0 x_T})$ with Lebesgue measure. Study multitime optimal control for quantum systems endowed with the *modified Boltzmann-Shannon entropy functional*

$$B(\vec{\psi}(\cdot,\cdot)) = -\frac{1}{2} \int_{D_{x_0 x_T}} \int_{\Omega_{0T}} ||\vec{\psi}(t,x)||^2 \ln ||\vec{\psi}(t,x)||^2 dt\, dx.$$

Show that the functional $B : X \to R$ is concave, nowhere continuous (but lower semicontinuous), and has a unique subgradient, when $||\vec{\psi}(t,x)|| > 0$ almost everywhere, namely $\left(1 + \ln ||\vec{\psi}(t,x)||\right) \vec{\psi}(t,x)$.

# 4 Quantum Control Systems with Bolza Type Cost as Mediate Curvilinear Integral

In the case of quantum control systems, the state $(x)$ may be the density matrix, the pure state vector or the evolution operator. Here we refer again to optimal control problems for the pure state when the parameter of evolution $t = (t^1, ..., t^m)$ is multidimensional. Similar considerations can be made in the other cases.

Let $L_\alpha(t, \vec{\psi}(t,x), u(t,x))dt^\alpha$ be a $C^2$ and closed 1-form. Suppose that the general cost is of *Bolza type* written as a sum between a *terminal term* and a *multiple integral from a path independent curvilinear integral*

$$J(u(\cdot)) = \phi(T, \vec{\psi}(T)) + \int_{D_{x_0 x_T}} \left( \int_{\Gamma_{0T}} L_\alpha(t, \vec{\psi}(t,x), u(t,x)) dt^\alpha \right) dx.$$

We add the dynamics constraint described by the controlled completely integrable Schrödinger PDEs

$$\frac{\partial \vec{\psi}}{\partial t^\alpha}(t,x) = -i\, H_\alpha(u(t,x)) \vec{\psi}(t,x),\ \vec{\psi}(0,x) = \vec{\psi}_0(x)\ \alpha = 1, ..., m.$$

## 4.1 Solution via Lagrange multipliers

We introduce a *costate variable* or *Lagrange multiplier vector* $p = (p_i)$ and a new Lagrangian 1-form

$$\mathcal{L}_\alpha(t, \vec{\psi}(t,x), u(t,x), p(t,x)) = L_\alpha(t, \vec{\psi}(t,x), u(t,x))$$
$$+p_k(t,x)\left(\frac{\partial \psi^k}{\partial t^\alpha}(t,x) + i\, H_\alpha(u(t,x))\psi^k(t,x)\right).$$

The foregoing $PDE$-constrained optimization problem is changed into another optimization problem

$$\max_{u(\cdot)} \tilde{J}(u(\cdot)) = \int_{D_{x_0x_T}} \left(\int_{\Gamma_{0T}} \mathcal{L}_\alpha(t, \vec{\psi}(t,x), u(t,x)) dt^\alpha\right) dx$$

subject to

$$u(t,x) \in \mathcal{U},\ p(t,x) \in \mathcal{P},\ t \in \Omega_{0T},\ x \in D_{x_0x_T},\ \vec{\psi}(0,x) = \psi(x),$$

where the set $\mathcal{P}$ will be defined later. The *control Hamiltonian* 1*-form*

$$\mathcal{H}_\alpha(t, \vec{\psi}(t,x), u(t,x), p(t,x)) = L_\alpha(t, \vec{\psi}(t,x), u(t,x))$$
$$+ip_k(t,x)H_\alpha(u(t,x))\psi^k(t,x),$$

i.e.,

$$\mathcal{H}_\alpha = \mathcal{L}_\alpha - p_k \frac{\partial \psi^k}{\partial t^\alpha} \text{ (Legendrian duality)},$$

allows to rewrite this new problem as

$$\max_{u(\cdot)} \tilde{J}(u(\cdot))$$

$$= \int_{D_{x_0x_T}} \int_{\Gamma_{0T}} \left(\mathcal{H}_\alpha(t, \vec{\psi}(t,x), u(t,x), p(t,x)) + p_k(t,x)\frac{\partial \psi^k}{\partial t^\alpha}(t,x)\right) dt^\alpha\, dx$$

subject to

$$u(t,x) \in \mathcal{U},\ p(t,x) \in \mathcal{P},\ t \in \Omega_{0T},\ x \in D_{x_0x_T},\ \vec{\psi}(0,x) = \vec{\psi}(x).$$

Suppose that there exists a continuous control $\hat{u}(t,x)$ defined over the parallelepiped $\Omega_{0T} \times D_{x_0x_T}$ with $\hat{u}(t,x) \in \mathrm{Int}\,\mathcal{U}$ which is an optimum point in the previous problem. Now consider a variation

$$u(t,x;\epsilon) = \hat{u}(t,x) + \epsilon h(t,x),$$

where $h$ is an arbitrary continuous vector function. Since $\hat{u}(t,x) \in \mathrm{Int}\,\mathcal{U}$ and a continuous function over a compact set is bounded, there exists $\epsilon_h > 0$ such that $u(t,x;\epsilon) = \hat{u}(t,x) + \epsilon h(t,x) \in \mathrm{Int}\,\mathcal{U},\ \forall|\epsilon| < \epsilon_h$. This $\epsilon$ is used in our variational arguments.

Define the variation $\vec{\psi}(t,x;\epsilon)$ as the $(m+3)$-sheet of the state variable corresponding to the control variable $u(t,x;\epsilon)$, i.e.,

$$\frac{\partial \vec{\psi}}{\partial t^\alpha}(t,x;\epsilon) = -i\, H_\alpha(u(t,x;\epsilon))\vec{\psi}(t,x;\epsilon), \forall (t,x) \in \Omega_{0T} \times D_{x_0x_T}$$

and $\vec{\psi}(t,x;0) = \vec{\psi}(t,x)$. For $|\epsilon| < \epsilon_h$, we define the variation Lagrangian 1-form

$$\mathcal{L}_\alpha(t,\vec{\psi}(t,x;\epsilon),u(t,x;\epsilon),p(t,x))$$

$$= \mathcal{H}_\alpha(t,\vec{\psi}(t,x;\epsilon),u(t,x;\epsilon),p(t,x)) + p_k(t,x)\,\frac{\partial \psi^k}{\partial t^\alpha}(t,x;\epsilon),$$

and the function

$$J(\epsilon) = \int_{D_{x_0x_T}} \int_{\Gamma_{0T}} \mathcal{L}_\alpha(t,\vec{\psi}(t,x;\epsilon),u(t,x;\epsilon),p(t,x))dt^\alpha\,dx.$$

Since the function $u(t,x;\epsilon)$ is admissible, it follows that the function $\vec{\psi}(t,x;\epsilon)$ is admissible. On the other hand, the control $\hat{u}(t,x)$ must be optimal. Therefore $J(\epsilon) \leq J(0),\ \forall |\epsilon| < \epsilon_h$.

To evaluate the curvilinear integral

$$\int_{\Gamma_{0T}} p_k(t,x)\frac{\partial \psi^k}{\partial t^\alpha}(t,x;\epsilon)dt^\alpha,$$

we integrate by parts. Using

$$\frac{\partial}{\partial t^\alpha}(p_k\psi^k) = \frac{\partial p_k}{\partial t^\alpha}\psi^k + p_k\frac{\partial \psi^k}{\partial t^\alpha},$$

we obtain

$$\int_{\Gamma_{0T}} p_k(t,x)\frac{\partial \psi^k}{\partial t^\alpha}(t,x;\epsilon)dt^\alpha = \int_{\Gamma_{0T}} \frac{\partial}{\partial t^\alpha}(p_k(t,x)\psi^k(t,x;\epsilon))dt^\alpha$$

$$- \int_{\Gamma_{0T}} \frac{\partial p_k}{\partial t^\alpha}(t,x)\psi^k(t,x;\epsilon)dt^\alpha.$$

Obviously,

$$\int_{\Gamma_{0T}} \frac{\partial}{\partial t^\alpha}(p_k(t)\psi^k(t,x;\epsilon))dt^\alpha = (p_k(t)\psi^k(t,x;\epsilon))|_0^T.$$

Substituting, we find that $J(\epsilon)$ is given by

$$\int_{D_{x_0x_T}} \int_{\Gamma_{0T}} \left(\mathcal{H}_\alpha(t,\vec{\psi}(t,x;\epsilon),u(t,x;\epsilon),p(t,x)) - \frac{\partial p_k}{\partial t^\alpha}\psi^k(t,x;\epsilon)\right) dt^\alpha\,dx$$

$$+ \int_{D_{x_0x_T}} (p_k(t)\psi^k(t,x;\epsilon))|_0^T dx.$$

Differentiating with respect to $\epsilon$, the expression of $J'(\epsilon)$ is

$$\int_{D_{x_0x_T}} \int_{\Gamma_{0T}} [\mathcal{H}_{\alpha\psi^k}(t,\vec{\psi}(t,x;\epsilon),u(t,x;\epsilon),p(t,x)) - \frac{\partial p_k}{\partial t^\alpha}]\psi^k_\epsilon(t,x;\epsilon)dt^\alpha dx$$

$$+ \int_{D_{x_0x_T}} \int_{\Gamma_{0T}} \mathcal{H}_{\alpha u^a}(t,\vec{\psi}(t,x;\epsilon),u(t,x;\epsilon),p(t,x))h^a(t,x)dt^\alpha dx$$

$$+\int_{D_{x_0x_T}}(p_k(t,x)\psi_\epsilon^k(t,x;\epsilon))|_0^T dx.$$

Evaluating at $\epsilon=0$, we find

$$J'(0)=\int_{D_{x_0x_T}}\int_{\Gamma_{0T}}[\mathcal{H}_{\alpha\psi^k}-\frac{\partial p_k}{\partial t^\alpha}]\psi_\epsilon^k(t,x;0)dt^\alpha dx$$

$$+\int_{D_{x_0x_T}}\int_{\Gamma_{0T}}\mathcal{H}_{\alpha u^a}h^a(t,x)dt^\alpha dx+\int_{D_{x_0x_T}}(p_k(t,x)\psi_\epsilon^k(t,x;0))|_0^T dx,$$

where $\vec{\psi}(t,x)$ is the $(m+3)$-sheet of the state variable corresponding to the optimal control $\hat{u}(t,x)$.

Having in mind the idea of the adjoint PDE system, we define $\mathcal{P}$ as the set of solutions of the boundary value problem

$$\frac{\partial p_k}{\partial t^\alpha}(t,x)=\mathcal{H}_{\alpha\psi^k}(t,\vec{\psi}(t,x),\hat{u}(t,x),p(t,x)),\ p_k(T)=0, \tag{4}$$

$\forall t\in\Omega_{0T},\ \forall x\in D_{x_0x_T}$.

We need $J'(0)=0$ for all $h(t,x)=(h^a(t,x))$. Therefore

$$\mathcal{H}_{\alpha u^a}(t,\vec{\psi}(t,x),\hat{u}(t,x),p(t,x))=0,\ \forall t\in\Omega_{0T},\ \forall x\in D_{x_0x_T}. \tag{5}$$

Moreover, the Schrödinger PDEs can be rewritten

$$\frac{\partial\psi^k}{\partial t^\alpha}(t,x)=-\frac{\partial\mathcal{H}_\alpha}{\partial p_k}(t,\vec{\psi}(t,x),\hat{u}(t,x),p(t,x)),\ \forall t\in\Omega_{0T},\ \forall x\in D_{x_0x_T}. \tag{6}$$

**Remarks.** (i) The algebraic system (5) describes the critical points of the Hamiltonian 1-form with respect to the control variable. (ii) The *PDEs* (4) and (6) and the condition (5) are Euler-Lagrange *PDEs* associated to the new Lagrangian 1-form.

Summarizing, we obtain a *multitime maximum principle* similar to the *single-time Pontryaguin maximum principle.*

**Theorem 1. (multitime maximum principle; necessary conditions )** *Suppose that the problem of maximizing the functional $J(u(\cdot))$ subject to the Schrödinger PDE constraints (2) has an interior solution $\hat{u}(t)\in\mathcal{U}$ which determines the $(m+3)$-sheet $\vec{\psi}(t,x)$. Then there exists a $C^2$ costate $p(t,x)=(p_k(t,x))$ defined over $\forall t\in\Omega_{0T},\ \forall x\in D_{x_0x_T}$ such that the relations (4), (5), (6) hold.*

**Remark** If the optimal control $\hat{u}(t,x)\in\mathcal{U}$ is not an interior point, then instead of critical point condition we have

$$\mathcal{H}_\alpha(t,\vec{\psi}(t,x),\hat{u}(t,x),p(t,x))=\min_u\mathcal{H}_\alpha(t,\vec{\psi}(t,x),u,p(t,x)).$$

## 4.2 Same problem in terms of real functions and choice of cost functional

Of course, both the state $\vec{\psi}$ and the matrices $-iH_\alpha(u)$ are complex functions. To transform the problem in terms of real functions, we need to write

$$\vec{\psi}=\vec{\psi}_R+i\vec{\psi}_I,\ -iH_\alpha(u)=R_\alpha(u)+iI_\alpha(u),$$

where $\vec{\psi}_R$ and $\vec{\psi}_I$ are real $n$-dimensional vectors and $R_\alpha(u)$ and $I_\alpha(u)$ are real $n \times n$ matrix functions of the control $u$, with $R_\alpha(u)$ skew symmetric and $I_\alpha(u)$ symmetric. Placing these data into the previous PDEs and separating the real and the imaginary parts, we find

$$\frac{\partial \vec{\psi}_R}{\partial t^\alpha}(t,x) = R_\alpha(u(t,x))\vec{\psi}_R(t,x) - I_\alpha(u(t,x))\vec{\psi}_I(t,x)$$

$$\frac{\partial \vec{\psi}_I}{\partial t^\alpha}(t,x) = I_\alpha(u(t,x))\vec{\psi}_R(t,x) + R_\alpha(u(t,x))\vec{\psi}_I(t,x).$$

Introducing the variable $X = (\vec{\psi}_R^T, \vec{\psi}_I^T)^T$ and the matrix

$$\tilde{H}_\alpha(u) = \begin{pmatrix} R_\alpha(u) & -I_\alpha(u) \\ I_\alpha(u) & R_\alpha(u) \end{pmatrix},$$

we can write the constraint PDEs, describing the dynamics, only using real quantities as

$$\frac{\partial X}{\partial t^\alpha} = \tilde{H}_\alpha(u)X.$$

The matrices $\tilde{H}_\alpha(u)$ are skew symmetric and symplectic for every $u$, i.e., they belong to $so(2n) \cap sp(n)$. Implicitly the cost can be written as a mediate curvilinear integral

$$J(u(\cdot)) = \tilde{\phi}(T, X(T)) + \int_{D_{x_0x_T}} \left( \int_{\Gamma_{0T}} \tilde{L}_\alpha(t, X(t), u(t,x))dt^\alpha \right) dx,$$

for appropriate functions $\tilde{\phi}$ and $\tilde{L}_\alpha$.

A first choice of the cost functional in the molecular control is the *laser electric field fluency*

$$J(u(\cdot)) = k \int_{D_{x_0x_T}} \left( \int_{\Gamma_{0T}} ||u_\alpha(t,x)||^2 dt^\alpha \right) dx,\ k > 0,$$

which measure the *energy of the electric field* $u = (u_\alpha) = (u_\alpha^i)$ in the multitime interval $[0,T] \times [x_0, x_T]$. A second choice is a cost of the type

$$J(u(\cdot)) = k \int_{D_{x_0x_T}} \left( \int_{\Gamma_{0T}} \delta^{\alpha\beta} < \frac{\partial u_\gamma}{\partial t^\alpha}(t,x), \frac{\partial u_\gamma}{\partial t^\beta}(t,x) > dt^\gamma \right) dx.$$

A third choice is a cost of the type

$$J(u(\cdot)) = k \int_{D_{x_0x_T}} \left( \int_{\Gamma_{0T}} \delta^{\alpha\beta} < \frac{\partial u_\alpha}{\partial t^\gamma}(t,x), \frac{\partial u_\beta}{\partial t^\gamma}(t,x) > dt^\gamma \right) dx,\ k > 0.$$

The second and the third choices *filters the high frequency components of the control field.*

When emphasis is placed on the final state, one can minimize a cost functional of the form

$$\frac{1}{2} < O >_\psi + \frac{k}{2} \int_{D_{x_0x_T}} \left( \int_{\Gamma_{0T}} ||u_\alpha(t,x)||^2 dt^\alpha \right) dx$$

$$= \frac{1}{2}\vec{\psi}^+ O\vec{\psi} + \frac{k}{2} \int_{D_{x_0x_T}} \left( \int_{\Gamma_{0T}} ||u_\alpha(t,x)||^2 dt^\alpha \right) dx,\ k > 0,$$

where $O$ is a negative definite Hermitian matrix. For example, we can choose the matrix $O = -\vec{\psi}_f \vec{\psi}_f^+$, if $\vec{\psi}_f$ is the desired state.

# 5 Santilli Contribution to the Science

Following the discovery of the appropriate irreversible covering of the 20th century mathematics, Prof. Dr. Ruggero Maria Santilli [12]-[14] passed to the construction of

(1) corresponding coverings of physical theories such as special relativity and quantum mechanics, as well as their verification, when applicable, in particle physics, nuclear physics, astrophysics and cosmology. The new disciplines are today known under the name of *hadronic relativity and mechanics* and includes a step-by-step lifting of all aspects of conventional theories into broader scientific vistas;

(2) an irreversible covering of quantum chemistry, today known as *Santilli Hadronic Chemistry*, and also consists in a step-by-step irreversible covering of all various aspects of quantum chemistry, with impressive experimental verifications, such as the first numerically exact representation from un-adulterated first principle of the binding energy and other features of the Hydrogen, Water and other molecules;

(3) searching for fuels with a new chemical composition other than that of molecular character, subject to the condition of allowing a full combustion, as an evident pre-requisite to achieve true improvement in combustion (see, magnegases and Santilli magnecules).

Santilli underlined that the only serious hope for mankind to achieve the much needed new clean energy is to surpass special relativity, quantum mechanics and quantum chemistry. The multitime quantum mechanics is a particular case of the hyper-dimensional branch of the theory of Hadronic Mechanics of Professor Santilli.

# References


[1] J. M. Ball, J. E. Marsden, M. Slemrod, *Controllability for distributed bilinear systems*, SIAM J. Control Optim., 20 (1982), 575-597.

[2] L. Baudouin, O. Kavian, J. P. Puel, *Regularity for a Schrödinger equation with singular potentials and application to bilinear optimal control*, J. Differential Equations, 216 (2005), 188-222.

[3] K. Beauchard, *Local controllability of a 1-D Schrodinger equation*, J. Math. Pures Appl., (9) 84 (2005), 851-956.

[4] K. Beauchard, *Controllability of a quantum particle in a 1-D variable domain*, ESAIM Control Optim. Calc. Var., 14 (2008), 105-147.

[5] K. Beauchard, J. M. Coron, *Controllability of a quantum particle in a moving potential well*, J. Funct. Anal., 232 (2006), 328-389.

[6] J. M. Coron, *Control and Nonlinearity*, Mathematical Surveys and Monographs, 136, American Mathematical Society, Providence, RI, 2007.

[7] S. Ervedoza, J. P. Puel, *Approximate controllability for a system of Schrödinger equations modeling a single trapped ion*, Ann. I. H. Poincaré, 2009.

[8] R. Feynman, *The Feynman Lectures on Physics*, Vol. III, Quantum Mechanics, 2nd edition 2005, Addison-Wesley.

[9] E. Lorin, S. Chelkowski, A. D. Bandrauk, *Maxwell-Schrödinger Equations for Nonlinear Laser Propagation in Molecular Media*, Comput. Phys. Commun., 177 (2007), 908.

[10] S. Pickenhain, M. Wagner, *Piecewise continuous controls in Dieudonné-Rachevski type problems*, Journal of Optimization Theory and Applications, 127, 1 (2005), 145-163.

[11] V. Prepeliţa, *Minimal realization algorithm for (q,r)-D hybrid systems*, WSEAS Transactions on Systems, 8, 1 (2009), 22-33.

[12] R. M. Santilli, *Foundation of Theoretical Mechanics, Inverse Problem in Newtonian Mechanics*, New York, 1978.

[13] R. M. Santilli, *Foundations of Hadronic Chemistry*, Kluwer Academic Publishers, Dordrecht, 2001; Il Nuovo Cimento B (in press) and http://www.i-b-f.org/Hadronic-Mechanics.htm

[14] R. M. Santilli, *Lie-admissible invariant representation of irreversibility for matter and antimatter at the classical and operator level*, Nuovo Cimento, 121 (2006), 443.

[15] G. Turinici, *On the controllability of bilinear quantum systems*, In Mathematical models and methods for abinitio quantum chemistry, 74, Lecture Notes in Chem., pages 75-92, Springer, Berlin, 2000.

[16] G. Turinici, H. Rabitz, *Wavefunction controllability for finite-dimensional bilinear quantum systems*, J. Phys., A 36 (2003), 2565-2576.

[17] C. Udriste, *Multi-time maximum principle*, Short Communication, International Congress of Mathematicians, Madrid, August 22-30, ICM Abstracts, 2006, p.47; Plenary Lecture at 6-th WSEAS International Conference on Circuits, Systems, Electronics, Control & Signal Processing (CSECS'07), 10-11, and 12-th WSEAS International Conference on Applied Mathematics, Cairo, Egypt, December 29-31, 2007, p. ii.

[18] C. Udriste, *Multitime controllability, observability and bang-bang principle*, Journal of Optimization Theory and Applications, 139, 1 (2008), 141-157.

[19] C. Udriste, *Simplified multitime maximum principle*, Balkan Journal of Geometry and Its Applications, 14, 1 (2009), 102-119.

[20] C. Udriste, *Nonholonomic approach of multitime maximum principle*, Balkan Journal of Geometry and Its Applications, 14, 2 (2009), 111-126.

[21] C. Udrişte, I. Ţevy, *Multitime dynamic programming for curvilinear integral actions*, Journal of Optimization Theory and Applications, 146 (2010), 189-207.

[22] C. Udrişte, *Equivalence of multitime optimal control problems*, Balkan Journal of Geometry and Its Applications, 15, 1 (2010), 155-162.

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# A NEW STRUCTURE IN SEMIDYNAMICAL SYSTEMS BASED ON SANTILLI'S ISOTHEORY

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ABSTRACT. In this paper we pay attention to the key role of the identity as a map to construct new mathematical structures. We present an overview on new results on top spaces. An overview on complete semidynamical systems as a generalization of dynamical systems is presented.

**Keywords:** Isounit, Top space, Complete semidynamical System.
**PACS:** 02, 89

## 1. INTRODUCTION

In this paper we assume that $S$ is a set of Santilli's isounits, i.e. $S$ is a nonempty set with a binary operation

$$* : S \times S \to \Omega$$

with the following conditions:

(1) If $I \in S$, then there exists $1 \in S$ such that $I * 1 = 1 * I = I$ ;

(2) For given $I \in S$ there exists $I^{-1} \in S$ such that $I * I^{-1} = I^{-1} * I = 1$.

An element of $S$ is called a Santilli's isounit.
Let $(E, +, \times)$ be a ring, and let $I \in S$ be given. Then
$E(I) = \{e \times I \ : \ e \in E\}$ with the addition

$$\hat{+} : E(I) \times E(I) \to E(I)$$

$$(e_1 \times I, e_2 \times I) \mapsto (e_1 + e_2) \times I$$

and the multiplication

$$\hat{\times} : E(I) \times E(I) \to E(I)$$
$$(e_1 \times I, e_2 \times I) \mapsto (e_1 \times e_2) \times I$$

is a ring.
Moreover the mapping

$$\varphi : E \to E(I)$$
$$e \mapsto e \times I$$

is a ring isomorphism. So mathematically and physically $E$ and $E(I)$ are the same. But the difference will appear if $E$ be a ring with the identity 1. In this case the identity of $E(I)$ is $1 \times I$, and physically it may present different perspective. For example if $E$ is the real numbers ring $R$, and $S$ is the set of rational numbers, then $R(-2)$ has different direction with $R$. So by an algebraic isomorphism we deduce two different geometrical perspectives, and this is one of the main points of isotheory [9, 10, 11]. If fact we can change an Euclidean right-handed system to an Euclidean left-handed system only with an algebraic isomorphism. For example $R(1) \times R(1)$ is a right-handed system and $R(-1) \times R(1)$ is a left-handed system. So with axiom preserving isomorphisms we can deduce two different structures.
In isotheory isounits are constant functions. To construct a geometric unified theory we presented the notion of generalized group in 1999 [3]. We can construct a generalized group via replacing the concept of identity as a constant function with a function which is not necessary constant. In fact a generalized group is a semigroup $G$ with the following additional two conditions:
(i) For each $x$ in $G$ there exists a unique $z$ in $T$ such that $xz = zx = x$, we denote $z$ by $e(x)$ (existence and uniqueness of identity).
(ii) For each $x$ in $G$ there exists $y$ in $G$ such that $xy = yx = e(x)$ (existence of inverse).
To combine this notion with manifold structure we presented top spaces as a generalization of Lie groups in 2002. In section 2 we present an overview on new results on top spaces. As an application a generalization of fundamental group is considered. Complete semi-dynamical systems via generalized vector fields are studied.

## 2. Top Spaces

Top spaces are generalization of Lie groups [4]. In this paper we begin with a top space and then by it we will construct an upper top space for it. Let us to recall the definition of a top space [4, 5].
A top space is a smooth manifold $T$ (not necessary connected) admitting an operation called multiplication, subject to the set of rules given below:

(i) $(xy)z = x(yz)$ for all $x, y, z$ in $T$ (associative law).
(ii) For each $x$ in $T$ there exists a unique $z$ in $T$ such that $xz = zx = x$, we denote $z$ by $e(x)$ (existence and uniqueness of identity).
(iii) For each $x$ in $T$ there exists $y$ in $T$ such that $xy = yx = e(x)$ (existence of inverse).
(iv) The mapping $m_1 : T \to T$ is defined by $m_1(u) = u^{-1}$ and the mapping $m_2 : T \times T \to T$ is defined by $m_2(u_1, u_2) = u_1 u_2$ are smooth maps.
$T$ is called a normal top space if it also satisfies the following condition. (v) $e(xy) = e(x)e(y)$ for all $x, y \in T$.
The properties (i), (ii), and (iii) imply that $T$ is a completely simple semigroup [1].
**Example 2.1** The $n$-dimensional Euclidean space $\mathbf{R}^n$ with the product

$$((x^1, \ldots, x^n), (y^1, \ldots, y^n)) \longmapsto \left(\frac{nx^1 + \sum_{i=1}^{n} y^i}{n}, \ldots, \frac{nx^n + \sum_{i=1}^{n} y^i}{n}\right)$$

is a top space which is not a Lie group with this product.
**Theorem 2.1** [4] If $X$ and $Y$ are smooth manifolds, $G$ is a Lie-group, and $s : Y \times X \to G$ is a smooth mapping, then the Rees matrix semigroup $P = X \times G \times Y$ with the multiplication $(x, g, y)(x', g', y') = (x, gs(y, x')g', y')$ is a top space.
If $T$ is a top space then $T = \bigcup_{t \in T} T_{e(t)}$ where $T_{e(t)} = \{s \in T : e(s) = e(t)\}$.
Moreover for each $t \in T$, $T_{e(t)}$ with the differentiable structure and product of $T$ is a Lie group.
Let $T$ and $S$ be two top spaces and let $f : T \to S$ be an algebraic homomorphism, i.e., $f(xy) = f(x)f(y)$ for all $x, y \in T$. Then $f(e(x)) = e(f(x))$ and $f : T_{e(x)} \to S_{e(f(x))}$ is a group homomorphism, where $x \in T$. The kernel of $f$ defined by $Kerf = \bigcup_{t \in T} kerf_t$ where $f_t$ is the restriction of $f$ on $T_{e(t)}$ [5]. We will use of this notion to present a generalization of the notion of fundamental group as the kernel of covering map of an upper top space of a given top space. We will show, the persistence of this structure under the isomorphisms of top spaces.
In this section we assume that for all $t \in T$, the set $T_{e(t)}$ is a connected set [8].
If $(\tilde{T}_{e(t)}, p_t, \tilde{e(t)})$ is a universal covering space of $(T_{e(t)}, e(t))$, then $\tilde{T}_{e(t)}$ is a Lie group with the multiplication $\tilde{m}_t(\tilde{t_1}, \tilde{t_2})$ with $\tilde{t_1}, \tilde{t_2} \in \tilde{T}_{e(t)}$ such that $p_t o \tilde{m}_t(\tilde{t_1}, \tilde{t_2}) = m_t(p_t(\tilde{t_1}), p_t(\tilde{t_2}))$ where $m_t$ is the restriction of $m$ on $T_{e(t)} \times T_{e(t)}$.
Let $\tilde{T}$ be the disjoint union of $\tilde{T}_{e(t)}$ where $t \in T$. Then we define the product

$\tilde{m}$ on $\tilde{T}\times\tilde{T}$ such that $p_{st}o\tilde{m}(\tilde{s},\tilde{t}) = m(p_s(\tilde{s}), p_t(\tilde{t}))$ and $\tilde{m}(\tilde{e(s)}, \tilde{e(t)}) = \tilde{e(st)}$.
**Theorem 2.2.** [7] $\tilde{m}$ determines uniquely by the above equalities.
**Theorem 2.3.** [7] $(\tilde{T}, \tilde{m})$ is a top space.
The straightforward calculations show that
The mapping $p : \tilde{T} \to T$ defined by $p(\tilde{t}) = p_t(\tilde{t})$ is a homomorphism of top spaces.
The pair $(\tilde{T}, p)$ is called the upper top space of $T$.

## 3. A Generalization of Fundamental Groups

We begin this section with the following theorem.
**Theorem 3.1.** [7] If $(\tilde{T}, p)$ and $(\tilde{S}, q)$ be two upper top spaces of a top space $T$, then $Kerp$ is isomorphic to $Kerq$.
We now define the main notion of this section.
**Definition 3.1.** If $(\tilde{T}, p)$ is an upper top space of $T$ then the $Kerp$ is called the MF-semigroup of $T$.
The next theorem shows that MF-semigroups are generalization of fundamental groups.
**Theorem 3.2.** [7] If $T$ is a top space and $D$ is the MF-semigroup of it then $D$ is isomorphic to $\overset{o}{\bigcup\limits_{t\in e(T)}} \pi_1(T_{e(t)}, e(t))$ where $\pi_1(T_{e(t)}, e(t))$ is the fundamental group of $T_{e(t)}$ with the base point $e(t)$, and $\overset{o}{\bigcup}$ denotes the disjoint union.
**Definition 3.2.** If $T$ and $U$ are two top spaces, then a mapping $f : T \to U$ is called an isomorphism if it is an algebraic isomorphism and a $C^\infty$ diffeomorphism.
Two top spaces are called isomorphic if there is an isomorphism between them.
**Theorem 3.3.** [7] Let $D$ and $E$ be MF-semigroups of top spaces $T$ and $U$ respectively. Moreover let $T$ and $U$ be isomorphic top spaces. Then $D$ and $E$ are isomorphic semigroups.

## 4. Generalized Vector Fields

Generalized vector fields are means for creation of parameters for new dynamics. In a special case there is a physical history behind this notion. In fact if one considers a manifold in a vector space with a physical vector field then the restriction of physical vector field to that manifold will be a generalized vector field on that manifold. So a generalized vector field on a manifold may not be tangent to it.
For the definition of a generalized vector field we assume that $T$ and $M$ are two $C^{r+1}$ manifolds, and $T(T)$ is the tangent bundle of $T$. We are assuming

that $V : T \to M$ and $\psi : T(T) \to M$ are $C^r$ maps.

**Definition 4.1** [6] $(V, M, \psi)$ is called a $C^r$ generalized vector field on $T$ if for all $p \in T$, there exists a $C^1$ map $\alpha : (-\epsilon, \epsilon) \to T$ (for some $\epsilon > 0$) so that $\alpha(0) = p$ and $V(\alpha(t)) = \psi(\alpha'(t))$. $\alpha$ is called a generalized integral curve passing through $p$.

$\alpha$ may not be unique. For example if $M = \mathbf{R}^n$, $\psi = 0$ and $V = 0$ then there exists a lot of generalized integral curves.

One can imagine a generalized vector field as a source which is a manifold as a subset of another manifold.

**Example 4.1** Let $T = S^1$, $M = \mathbf{R}^3$, $\psi(x, y) = (x, -y, x + y)$ and $V(x, y) = (-y, -x, -y + x)$, then $(V, \mathbf{R}^3, \psi)$ is a $C^\infty$ generalized vector field on $S^1$.

If $(V, M, \psi)$ is a $C^r$ generalized vector field then there exists a vector field $\tilde{V}$ (may not be unique) such that $\psi \circ \tilde{V} = V$.

**Definition 4.2** A generalized vector field $(V, M, \psi)$ is called a left invariant generalized vector field on $T$ if there exists a left invariant vector field $\tilde{V}$ on $T$ such that $\psi \circ \tilde{V} = V$.

**Example 4.2** Let $T$ be the top space which is introduced in example 1.3. Moreover let $V : T \to \mathbf{R}^4$ and $\psi : \mathbf{R}^3 \to \mathbf{R}^4$ be defined by $V(x, y, z) = (-y, x, -y, x - y)$ and $\psi(x, y, z) = (x, y + z, x + z, x + y)$ respectively. Then $(V, \mathbf{R}^4, \psi)$ is a left invariant generalized vector field.

Let $\tilde{V}$ be a vector field corresponding to a generalized vector field $(V, M, \psi)$ on a top space $T$. Moreover let $\psi$ be a one-to-one mapping.

Then for all $g \in T$ we can define $\eta^g : M \to M$ in the form:

$$\eta^g(m) = \begin{cases} V(gt) & \text{if} \quad m = V(t) \\ m & \text{if} \quad m \notin ImV \end{cases} \cdot (1)$$

**Lemma 4.1** [6] For all $g \in T$, the mapping $\eta^g$ is well-defined.

**Theorem 4.1** [6] Let $\tilde{V}$ be a vector field on a top space $T$ corresponding to a $C^r$ generalized vector field $(V, M, \psi)$, and let $\eta^g$ be the mapping which is defined by (1). Then

i) $\eta^g \circ V = V \circ l_g$, for all $g \in T$;

ii) If $V$ is a left invariant vector field, then $\psi((l_g)_{*t}(\tilde{V}(t)) = (\eta^g \circ V)(t)$, for all $g, t \in T$.

**Theorem 4.2** [6] With the assumptions of theorem 4.1, the set $(M, \{\eta^g : g \in T\})$ has the following properties:

i) $\eta^{g_1} \circ \eta^{g_2} = \eta^{g_1 g_2}$ for all $g_1, g_2 \in T$;

ii) For all $m \in M$ there exists $\eta^g$ such that $m$ is a fixed point of $\eta^g$.

**Corollary 4.1** [6] In theorem 4.2 if $T$ is a Lie group, then $(M, \{\eta^g : g \in T\})$ is a dynamical system on $M$.

## 5. A GENERALIZATION OF DYNAMICAL SYSTEMS

If $T$ is a Lie group, then corollary 4.1 implies that $\{\eta^g : g \in G\}$ is a dynamical system on $M$. So property (i) of theorem 4.2 can be replaced with the existence of identity mapping in the definition of dynamical systems. Hence properties (i) and (ii) can be a base for the definition of systems which time values of their evolution operators are members of a completely simple semigroups. These systems make a special category which is a sub-category of the category of semi-dynamical systems.

Let $G$ be a completely simple semigroup and $D = \{\eta^g : g \in G\}$ be a family of maps from a set $M$ to $M$, then we have the following definition.

**Definition 5.1** [6] $(M, D, G)$ is called a complete semi-dynamical system if:

i) $\eta^{g_1} \circ \eta^{g_2} = \eta^{g_1 g_2}$ for all $g_1, g_2 \in G$;

ii) For all $m \in M$ there exists $g \in G$ such that $m$ is a fixed point of $\eta^g$.

We use the abbreviation CSS instead of "complete semi-dynamical system".

**Example 5.1** Let $G$ be a completely simple semigroup and

$$D := \left\{ \begin{array}{ccccc} \varphi^g : \varphi^g : G & \to & G & \text{and} & g \in G \\ x & \mapsto & gx & & \end{array} \right\}.$$

Then $(G, D, G)$ is a $CSS$.

We know that the set $G_a := \{g \in G : e(g) = e(a)\}$ where $a \in G$, with the multiplication of $G$ is a group. The following theorem shows that CSSs are the generalization of dynamical systems.

**Theorem 5.1** [6] Let $(M, D, G)$ be a CSS and suppose that there exists $a \in G$ such that: for all $m \in M$ there exists $g \in G_a$ so that $\varphi^g(m) = m$. Then $(M, D, G_a)$ is a dynamical system.

**Corollary 5.1** [6] Let $(M, D, G)$ be a CSS and let $G$ be a group. Then $(M, D, G)$ is a dynamical system.

## REFERENCES


[1] J. Araujo, J. Konieczny , Molaei's Generalized Groups are Completely Simple Semigroups, *Buletinul Institului Polithnic Din Iasi* **48 (52)**, 1-5 (2002).

[2] D. Miličić, *Lectures on Lie Groups*, http://www.math.utah.edu:8080/ milicic/, 2004.

[3] M.R. Molaei, Generalized Groups, *Buletinul Institului Polithnic Din Iasi*, Tomul **XLV (XLIX)**, 21-24 (1999).

[4] M.R. Molaei, Top Spaces, *Journal of Interdisciplinary Mathematics*, Volume **7**, Number **2**, 173-181 (2004).

[5] M.R. Molaei, *Generalized Structures Based on Completely Simple Semigroups, Monograph in Mathematics*: Hadronic Press, 2005.

[6] M.R. Molaei, Complete Semi-dynamical Systems, *Journal of Dynamical Systems and Geometric Theories*, Vol.**3**, Number **2**, 95-107 (2005).

[7] M.R. Molaei, M.R. Farhangdost, Upper Top Spaces, *Journal of Applied Sciences*, Vol. **8**, Number **1**, 128-131 (2006).

[8] M.R. Molaei, A. Tahmoresi, Connected Topological Generalized Groups, *General Mathematics*, Volume **12, 1**, 13-22 (2004).

[9] R.M. Santilli, Isonumbers and Genonumbers of Dimension 1, 2, 4, 8 Their Isoduals and Pseudoduals and Hidden Numbers of Dimension 3, 5, 6, 7, *Algebras, Groups and Geometries*, Vol. **10** (1993).

[10] R.M. Santilli, Nonlocal-integral Isotopies of Differential Calculus, Geometries and Mechanics,*Rendiconti Quantum Mechanics*, Volume **42**, 7-82 (1996).

[11] R.M. Santilli, Iso-, Geno-, Hyper-Mechanics for Matter, Their Isoduals for Antimatter, and Their Novel Applications in Physics, Chemistry and Biology,*Journal of Dynamical Systems and Geometric Theories*, Volume **1**, Number**2**, 121-193 (2003).

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
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# Professor Santilli's Intermediate Controlled Nuclear Fusion

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**Abstract.** Experiments were conducted to confirm measurements by Santilli [1] of an *Intermediate Controlled Nuclear Fusion* (ICNF) process without harmful radiations. For this purpose we used a steel reactor chamber pressurized with deuterium gas and sparked with carbon electrodes. Thermal measurements on the chamber were analyzed and compared with the total measured energy input to determine excess heat production. Mass spectroscopic analysis were performed on gas samples extracted before and after ignition to verify the formation of magnecular clusters (essentially consisting of clustered molecules) formed as by-products of the intermediate nuclear fusion process. The exterior of the chamber were monitored throughout these experiments with radiation detectors to assess if any harmful radiation were emitted into the environment. These experiments are the precursor to the construction and testing of larger scaled hadronic reactors.

**Keywords:** Ruggero Santilli, Intermediate Controlled Nuclear Fusion
**PACS:** 28.90.+i

## INTRODUCTION

Controlled tests of the ICNF process without harmful radiations were repeated at the facility of the Institute for Basic Research in Tarpon Springs by a technical team from Princeton Gamma-Tech (PGT). The main diagnostic tools used to characterize the hadronic reactor were supplied by PGT, including the temperature transducers and radiation detectors.

The hadronic reactor is fabricated from a 12-inch outer diameter steel tube with welded end flanges. Two steel plates are bolted to both ends to seal the chamber. A stationary anode is located from one endplate, and a moveable cathode from the other endplate. The electrodes can be changed by opening the chamber to replace the anode and cathode. For the tests described in this article, the electrodes were carbon graphite. The terminals were attached to a Miller Electric Dimension 1000 AC-DC converter, and regulated during the experiments at nominally 40VDC and 900A. A wattmeter was used to determine the exact power consumed by the generator during each experimental run. The temperature of the reactor tube and endplate were monitored with platinum resistive sensors.



Figure 1. Photograph of the opened hadronic reactor

The main concern with any nuclear process intended for energy generation is the potential for harmful radiations. Fission reactors produce considerable amount of all the deadly forms such as alphas ($^4He^{2+}$), betas ($e^-$), neutrons (n) and gamma-rays ($\gamma$). Alpha and beta particles can cause the most damage to living cells, but by their very nature of being highly ionizing means they also have very short travel paths, and unless ingested are not of environmental concerns. Neutrons and gamma-rays are considerably more penetrative and therefore more of a harmful environmental radiation to consider. The SAM940 [2] radiation detector consists of a sodium iodide scintillator for identifying sources of gamma-rays and a proportional counter filled with a rare isotope of helium ($^3He$) for neutron detection. The detectors were factory calibrated with potassium ($^{40}K$) for gamma-rays and californium ($^{252}Cf$) for neutrons. Radioactive background levels of the research facility were surveyed with the SAM940, and the instrument then placed in close proximity to the hadronic reactor to constantly monitor any potential harmful radiations emitted during the fusion process.

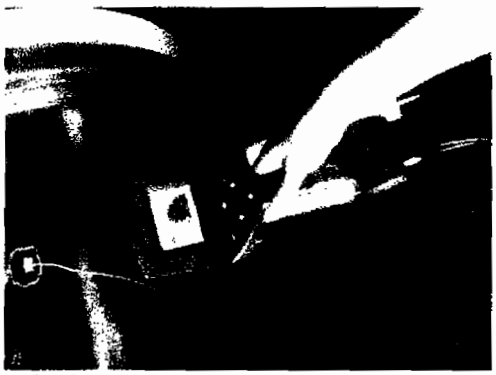


Figure 2. Photograph of PGT's model SAM940 gamma-ray and neutron detector

## DEUTERIUM CARBON FUSION

The fusion of deuterium and carbon by the ICNF process to form nitrogen can be described using Hadronic Mechanics [3] with the following balanced equation:

$$TR + H(2, 1, 1^+, 2.0141) + C(12, 6, 0^+, 12.0000) \rightarrow N(14, 7, 1^+, 14.0030) + \Delta E_{heat} \quad (1)$$

whereby the first symbol contained within the brackets represent the atomic number of the isotope species, the second symbol the nuclear charge, the third symbol the nuclear angular momentum with parity, and the final fourth symbol representing the atomic mass unit (amu).

The trigger (TR) mechanism to initiate the reaction process is the electric arc that polarizes the carbon and hydrogen atoms to form magnecular clusters. On the atomic distances between the axially coupled atoms, the extremely strong magnetic fields generated by the arc toroidally deform the atomic orbitals and thereby exposing the nuclei from their electronic clouds. The close proximity of the bare nuclei leads to the nuclear fusion with the generation of excess heat ($\Delta E_{heat}$). The mass difference between the fusion product ($^{14}N$) and the parent nuclei ($^{2}H$, $^{12}C$) is 0.0111amu or the energy equivalent of 10.339MeV.

The hadronic reactor is pressurized with pure deuterium gas by first evacuating with a mechanical vacuum pump the chamber and then backfilling with the gas from a supply bottle. Gas samples were taken before and after each initiated reaction, and sent to an independent laboratory [4] for spectra vapor analysis.

Each experimental run was started close to ambient temperature of nominally 25°C, with the electric arc powered for 2 minutes. The wattmeter measured an average power consumption of 1550W.hr, which equates to an energy input of 5.4MJ. A total of 3 runs were performed at varying starting pressures of 100, 75 and 50psi. For the 100psi tests, gas samples before (A) and after (B) was taken. The reactor chamber was then purged and refilled with pure deuterium, and a gas sample (C) was taken at a starting pressure of 75psi. After the reaction process at 75psi, a gas sample (D) was extracted. The reactor was then allowed to cool back to ambient and the pressure reduced to 50psi for another reaction, and a final gas sample (E) taken.

Summary of the gas samples extracted from the hadronic reactor:
A) 100psi Before fusion
B) 100psi After fusion
C) 75psi Before fusion
D) 75psi After fusion
E) 50psi After fusion

# RESULTS

## Gas Spectra Analysis

Deuterium is non-combustible, and there were also negligible amount of oxygen contained in the hadronic reactor for any other combustion processes to have occurred. Hence if there were no hadronic chemistry or fusion processes taking place then we would expect to observe similar vapor spectra for the samples taken before and after initiation by the electric arc. The following chart shows the analyzed mass spectra for the 5 gas samples, the reported values are in parts-per-million (ppm) by volume.

| amu | A | B | C | D | E |
|---|---|---|---|---|---|
| 2 ($H_2$) | 288,163 | 185,549 | 141,308 | 158,837 | 201,992 |
| 3 | 49,815 | 438,891 | 64,969 | 461,037 | 1,031,783 |
| 4 ($D_2$) | 12,648,080 | 12,342,540 | 11,357,960 | 11,013,180 | 10,311,080 |
| 5 | 332 | 933 | 223 | 840 | 1,771 |
| 6 | 13,260 | 12,020 | 10,532 | 9,793 | 9,018 |

| | | | | | |
|---|---|---|---|---|---|
| 7 | - | - | 190 | 186 | 161 |
| 8 | - | - | - | - | - |
| 11 | - | - | - | - | 40 |
| 12 | 4,850 | 9,025 | 620 | 19,668 | 32,411 |
| 13 | 449 | 400 | 60 | 454 | 1,089 |
| 14 | 57,902 | 11,191 | 104,309 | 118,343 | 125,036 |
| 15 | 1,875 | 1,578 | 653 | 1,644 | 3,369 |
| 16 | 24,627 | 16,952 | 34,481 | 26,993 | 54,958 |
| 17 | 2,269 | 12,165 | 4,479 | 23,534 | 155,606 |
| 18 (Ar, $H_2O$) | 10,248 | 104,140 | 18,576 | 186,414 | 679,276 |
| 19 | 3,242 | 8,594 | 2,823 | 13,890 | 174,468 |
| 20 | 8,302 | 71,458 | 9,302 | 114,013 | 182,857 |
| 21 | - | 729 | - | 1,216 | 2,315 |
| 22 | 222 | 159 | - | 197 | 222 |
| 23 | - | - | - | - | - |
| 24 | 182 | 218 | - | 161 | 1,025 |
| 25 | 633 | 240 | - | 61 | 323 |
| 26 | 2,838 | 1,408 | 245 | 1,103 | 4,415 |
| 27 | 873 | 878 | - | - | 3,145 |
| 28 ($N_2$) | 536,530 | 125,200 | 884,507 | 1,148,545 | 1,301,279 |
| 29 | 4,334 | 2,548 | 6,463 | 10,666 | 14,491 |
| 30 | 3,618 | 5,306 | 5,526 | 10,963 | 22,688 |
| 31 | 178 | 1,601 | 343 | 2,034 | 7,569 |
| 32 ($O_2$) | 111,498 | 13,475 | 205,287 | 17,979 | 42,656 |
| 33 | 139 | 483 | 201 | 622 | 3,539 |
| 34 | 577 | 1,449 | 1,134 | 2,197 | 3,429 |
| 35 | - | 225 | - | 236 | 933 |
| 36 | - | 1,848 | 142 | 2,840 | 4,621 |
| 37 | - | 79 | - | - | 207 |
| 38 | - | 119 | - | 100 | 161 |
| 39 | 308 | 433 | 104 | 161 | 328 |
| 40 | 5,857 | 563 | 10,687 | 11,468 | 11,465 |
| 41 | 209 | 328 | 80 | 183 | 436 |
| 42 | 197 | 317 | 102 | 246 | 654 |
| 43 | 113 | 295 | - | 188 | 732 |
| 44 ($CO_2$) | 14,262 | 13,828 | 1,848 | 14,241 | 13,508 |
| 45 | 199 | 265 | - | 212 | 282 |
| 46 | 98 | 159 | - | 121 | 308 |
| 47 | - | - | - | - | - |
| 48 | - | - | - | - | 99 |
| 50 | 111 | 208 | - | 101 | 317 |
| 51 | 107 | 218 | - | 61 | 2,740 |
| 52 | 109 | 295 | - | 207 | 459 |
| 53 | - | 107 | - | - | 102 |
| 54 | - | 176 | - | 101 | 223 |
| 55 | - | 185 | - | - | 141 |
| 56 | - | 208 | - | 128 | 306 |
| 57 | - | - | - | - | 40 |
| 58 | - | - | - | - | 140 |

| | | | | | |
|---|---|---|---|---|---|
| 60 | - | - | - | 81 | 121 |
| 67 | - | - | - | - | - |
| 69 | - | - | - | - | 391 |
| 71 | - | - | - | - | - |
| 77 | - | 137 | - | 65 | - |
| 78 | 306 | 464 | 100 | 238 | 243 |
| 79 | - | 115 | - | - | - |
| 81 | - | 60 | - | - | - |
| 82 | - | 132 | - | 81 | 130 |
| 83 | - | 60 | - | 40 | 131 |
| 84 | - | 472 | - | 396 | 734 |
| 91 | - | 162 | - | - | - |
| 101 | - | - | - | - | 711 |

The spectral analysis indicates a reduction in the amount of deuterium following each reaction. At 100psi (A→B) the decrease was approximately 2.5%, and at 75psi (C→D) it was 3%. The decrease in the amount of nitrogen in the 100psi data can be misleading, since the evolved nitrogen can be trapped in clustered magnecules as indicated by the existence of higher mass entities in the spectral data following all the reactions. These previously unknown higher mass magnecules are further evidence of the hadronic chemistry taking place.

## Elemental Microanalysis

Samples of deposits on the surface of the graphite electrodes were removed for material characterization in a Scanning Electron Microscope (SEM) using an Energy Dispersive Spectroscopy (EDS) x-ray detector [5]. The detector is a liquid-nitrogen cooled lithium-drifted silicon crystal biased to operate as a semiconductor junction. X-rays liberate electron-hole pairs in the junction, and the amount of charge collected is proportional to the x-ray energies. The electron beam striking the samples generates electronic excitation, and it is the decay of these electronic shells that emits the characteristic x-ray energies unique to each element.

The EDS detector is a PGT's model LS10133 mounted to an ISI Super IIIA SEM. The samples were epoxied to a holder placed directly in line with the electron beam. The long vacuum insulated endcap housing the Si(Li) crystal is inserted into the SEM chamber in close proximity to the sample. Fluorescence x-rays scattering off the target sample and entering the endcap through a thin-walled polymer window are identified by the EDS detector system.

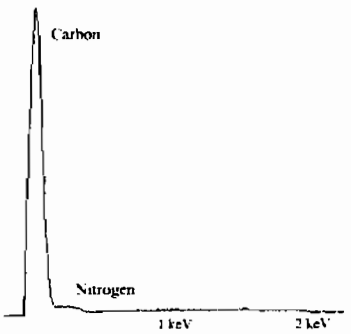



Figure 3. Elemental spectra of deposits on graphite electrode

The elemental microanalysis spectra taken on the surface deposits of the graphite electrodes show a prominent x-ray peak at 277eV (carbon $K_\alpha$). There is a small adjacent peak at 392eV, which is the nitrogen $K_\alpha$ x-ray that is noticeable above the general background level. Since the SEM chamber is under vacuum, then the detected nitrogen must exist in some non-gaseous form, possibly within clustered magnecules [6].

## Thermal Analysis

Platinum resistive temperature sensors were securely fastened to the surfaces of the steel chamber's central tube and one of the endplates. Temperature readings were noted down each minute after the electric arc was powered up to produce a thermal profile of the hadronic reactor. A thermal Finite Element Analysis (FEA) was simulated [7] for the reactor to estimate the expected temperature rise if the only source of heat came from the electric arc. Comparison curves of the measured thermal profiles against the FEA computed values at 5MJ, 5.5MJ and 6MJ energy inputs are shown below.

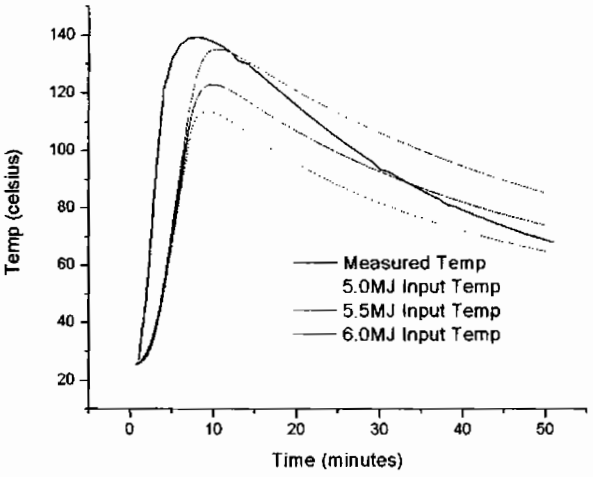



Figure 4. Thermal profiles of tube

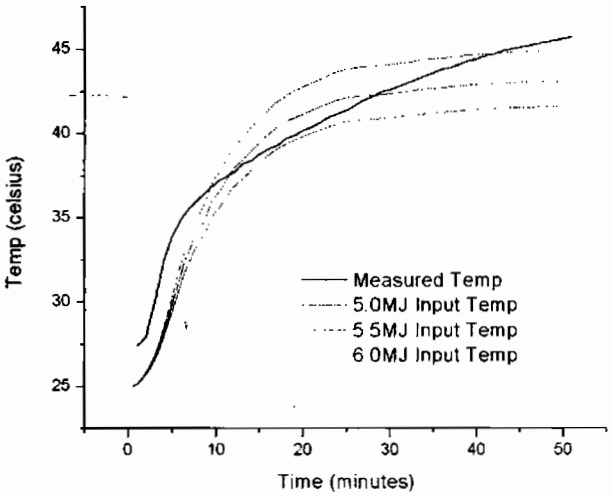


Figure 5. Thermal profiles of endplate

The data indicates the generated excess heat $\Delta E_{heat}$ of approximately 0.5MJ above the total injected energy input of 5.4MJ from the electric arc. From equation (1) we note that each reaction releases around 10MeV of fusion energy, hence if we assume all the excess heat is through the ICNF process, then this is equivalent to the generation of roughly $10^{18}$ or a micro-mole of fusion products.

## Radiation Analysis

The SAM940 sodium iodide scintillator detector is self-calibrating at the potassium ($^{40}K$) energy of 1.461MeV. The helium ($^{3}He$) proportional counter was factory calibrated against a californium ($^{252}Cf$) neutron source. For safety and security reasons the source is embedded in wax and locked inside a steel vault. Opening the vault door and placing the SAM940 instrument approximately a meter from the source, we were able to detect average neutron levels of 0.8 counts per second (cps). With the vault door closed and the instrument removed from the vicinity, the background levels fell to less than 0.03cps.

Compared to normal background levels there were no emitted gamma-rays or neutrons detected emanating from the hadronic reactor during the fusion process occurring within the chamber.

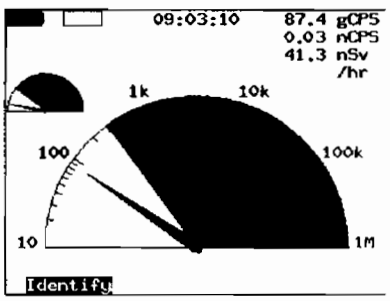


Figure 6. SAM940 Gamma and neutron detection

## CONCLUSION

The results taken from the experimental runs conducted on the hadronic reactor indicates some form of exothermal reaction taking place that produced clusters of higher mass components. Since chemical reactions and combustion cannot have occurred in a pure deuterium environment, the conclusion leads to an indication of the process described as *Intermediate Controlled Nuclear Fusion* without harmful radiations.

## REFERENCES


1. R. M. Santilli, *Experimental confirmation of the novel "intermediate controlled nuclear fusion" without harmful radiations* (to be published New Advances in Physics 2010)
2. L. Ying (Princeton Gamma-Tech Instruments Inc), *Private communications at www.pgt.com*
3. R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry*, Florida: International Academic Press (2008)
4. T. J. Rossiter (Oneida Research Services Inc), *Private communications at www.ord-labs.com*
5. C. E. Lyman et al, *Scanning electron microscopy, x-ray microanalysis, and analytical electron microscopy*, New York: Plenum Press (1990)
6. R. M. Santilli, *The new fuels with magnecular structure*, Florida: International Academic Press (2005)
7. F. Wang (FEAmax LLC), *Private communications at www.feamax.com*

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
Kathmandu University, Nepal, April (2011) pages 235-244

# Quantum Mechanics, Mathematical Statistics And Deformed Commutation Relations

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**Abstract.** In this paper, quantum mechanics and its relation to mathematical statistics are discussed. The Fisher information, wave function, kinetic energy, two new uncertainty relations, Klein-Gordon equation, probability density current and deformed commutation relations are investigated.

**Keywords:** Fisher information, wave function, kinetic energy, uncertainty relations, Klein-Gordon equation, probability density current, deformed commutation relations.
**PACS:** 03.65.-w, 03.65.Ca, 03.65.Ta.

## I. FISHER INFORMATION

Measurement in quantum mechanics is described in a statistical way. For this reason, we discuss in this paper mutual relation of the formalism of quantum mechanics and mathematical statistics.

First, we discuss the Fisher information — a very important quantity appearing in mathematical statistics. In the most simple form, it can be introduced as follows (see e.g. [1-18]).

We start with the normalization condition for the probability density $\rho(x) = |\psi(x)|^2$, where $\psi$ is the wave function

$$\int \rho \, \mathrm{d}x = 1.$$

Here, integration is performed from the minus infinity to plus infinity. For the sake of simplicity, we assume also that $\rho$ has the property

$$\lim_{x \to \pm\infty} x^n \rho = 0, \quad n = 0, 1, 2. \tag{1}$$

Therefore, we limit ourselves to discussion of the so-called bound states.

Now, we perform integration by parts in the normalization condition and get

$$[(x-a)\rho]_{x=-\infty}^{\infty} - \int (x-a) \frac{\partial \rho}{\partial x} \, \mathrm{d}x = 1,$$

where $a$ is an arbitrary real number. Taking into account condition (1) we get the starting point of the following discussion

$$\int (x-a) \frac{\partial \rho}{\partial x} \, \mathrm{d}x = -1.$$

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Further, we make use of the Schwarz inequality for the inner product $\langle u, v\rangle = \int u^* v \, \mathrm{d}x$ of two complex functions $u$ and $v$

$$\langle u, u\rangle\langle v, v\rangle \geq |\langle u, v\rangle|^2. \tag{2}$$

Putting

$$u = (x-a)\sqrt{\rho}, \quad v = \frac{1}{\sqrt{\rho}}\frac{\partial \rho}{\partial x}$$

and using Schwarz inequality (2) we get

$$\int (x-a)^2 \rho \, \mathrm{d}x \int \frac{1}{\rho}\left(\frac{\partial \rho}{\partial x}\right)^2 \mathrm{d}x \geq 1,$$

where the second integral is called the Fisher information $I$

$$I = \int \frac{1}{\rho}\left(\frac{\partial \rho}{\partial x}\right)^2 \mathrm{d}x.$$

This inequality is usually written in the form

$$\int (x-a)^2 \rho \, \mathrm{d}x \; I \geq 1. \tag{3}$$

This result is very general and does not depend on the concrete meaning of the variable $x$.

Interpretation of the last inequality is similar to that of the uncertainty relations in quantum mechanics since for given $I$ the integral $\int (x-a)^2 \rho \, \mathrm{d}x$ cannot be smaller than $1/I$ and vice versa. The minimum of the integral $\int (x-a)^2 \rho \, \mathrm{d}x$ is obtained for $a = \int x \rho \, \mathrm{d}x$. In a more general form, it is is possible to derive the so-called Rao–Cramér inequalities [19-21].

## II. WAVE FUNCTION

The wave function $\psi$ can always be written in the form

$$\psi = \mathrm{e}^{(is_1 - s_2)/\hbar}, \tag{4}$$

where $s_1$ and $s_2$ are real functions and $\hbar$ is the Planck constant. It follows from here that

$$\rho = |\psi|^2 = \mathrm{e}^{-2s_2/\hbar}.$$

Therefore, the Fisher information can be written in the equivalent form [7-11, 13, 17, 18]

$$I = \int \frac{1}{\rho}\left(\frac{\partial \rho}{\partial x}\right)^2 \mathrm{d}x = \frac{4}{\hbar^2}\int \left(\frac{\partial s_2}{\partial x}\right)^2 \mathrm{e}^{-2s_2/\hbar}\mathrm{d}x.$$

## III. KINETIC ENERGY

Using Eq. (4) for the wave function we can write the quantum-mechanical kinetic energy $T$ in the form [7-11, 13, 17, 18]

$$T = \int \frac{(\partial s_1/\partial x)^2 + (\partial s_2/\partial x)^2}{2m}\mathrm{e}^{-2s_2/\hbar}\mathrm{d}x.$$

Therefore, kinetic energy can be written as a sum of two terms

$$T = T_1 + T_2,$$

where

$$T_1 = \int \frac{(\partial s_1/\partial x)^2}{2m} e^{-2s_2/\hbar} dx, \quad T_2 = \frac{\hbar^2 I}{8m}.$$

The first part of the kinetic $T_1$ is analogous to the classical kinetic energy given by the expression $T_{class} = (\nabla S)^2/(2m)$ known from the Hamilton-Jacobi theory.

The second part of the kinetic energy

$$T_2 = \frac{\hbar^2 I}{8m}$$

is proportional to the Fisher information $I$. Due to $T_2$, the kinetic energy of the bound states cannot equal zero. Therefore, the Fisher information plays very important role in quantum mechanics.

## IV. HEISENBERG UNCERTAINTY RELATIONS

As above, we write the wave function $\psi$ in form (4). The Heisenberg uncertainty relation for the coordinate $x$ and momentum $p$ has the form [22]

$$\langle (\Delta x)^2 \rangle \langle (\Delta p)^2 \rangle \geq \frac{\hbar^2}{4},$$

where

$$\langle (\Delta x)^2 \rangle = \int (x - \langle x \rangle)^2 |\psi|^2 dx, \quad \langle (\Delta p)^2 \rangle = \int \left| (\hat{p} - \langle \hat{p} \rangle)\psi \right|^2 dx,$$

$\hat{p} = -i\hbar(\partial/\partial x)$ and $\langle\,\rangle$ denotes the usual quantum-mechanical mean value.

Analogously to the kinetic energy, $\langle (\Delta p)^2 \rangle$ can be split into two parts [7-10, 13, 15-18]

$$\langle (\Delta p)^2 \rangle = \langle (\Delta p_1)^2 \rangle + \langle (\Delta p_2)^2 \rangle,$$

where

$$\langle (\Delta p_1)^2 \rangle = \int \left( \frac{\partial s_1}{\partial x} - \left\langle \frac{\partial s_1}{\partial x} \right\rangle \right)^2 e^{-2s_2/\hbar} dx$$

and

$$\langle (\Delta p_2)^2 \rangle = \int \left( \frac{\partial s_2}{\partial x} \right)^2 e^{-2s_2/\hbar} dx.$$

Similarly to the first part of the kinetic energy $T_1$, $\langle (\Delta p_1)^2 \rangle$ can be interpreted within generalization of classical mechanics in which the classical momentum $p = \partial S/\partial x$, where $S$ is the classical action, is replaced by $\partial s_1/\partial x$ and the probability density $\rho = |\psi|^2 = e^{-2s_2/\hbar}$ is introduced.

The second part

$$\langle (\Delta p_2)^2 \rangle = \int \left( \frac{\partial s_2}{\partial x} \right)^2 e^{-2s_2/\hbar} dx$$

is, analogously to $T_2$, proportional to the Fisher information $I$

$$I = \int \frac{1}{\rho} \left( \frac{\partial \rho}{\partial x} \right)^2 dx = \frac{4}{\hbar^2} \int \left( \frac{\partial s_2}{\partial x} \right)^2 e^{-2s_2/\hbar} dx = \frac{4}{\hbar^2} \langle (\Delta p_2)^2 \rangle.$$

For $\langle (\Delta p_1)^2 \rangle = 0$ (for example for real wave functions), the Heisenberg uncertainty relation

$$\langle (\Delta x)^2 \rangle \langle (\Delta p)^2 \rangle \geq \frac{\hbar^2}{4},$$

is equivalent to inequality (3) with $a = \langle x \rangle$.

## V. KLEIN-GORDON EQUATION

In physics, we have to take into account not only the probability density $\rho$ but also the probability density current $\mathbf{j}$ describing the motion in space. For this reason, we introduce generalized spatial and time Fisher informations $I_x''$ and $I_t''$ [13, 17, 18]

$$I_x'' = \frac{4}{\hbar^2}\int_{t=0}^{\infty}\int\left[\left(\frac{\partial s_1}{\partial x}\right)^2+\left(\frac{\partial s_2}{\partial x}\right)^2\right]\mathrm{e}^{-2s_2/\hbar}\mathrm{d}x\mathrm{d}t = \int_{t=0}^{\infty}\int\left|\frac{\partial\psi}{\partial x}\right|^2\mathrm{d}x\mathrm{d}t$$

and

$$I_t'' = \frac{4}{\hbar^2}\int_{t=0}^{\infty}\int\left[\left(\frac{\partial s_1}{\partial t}\right)^2+\left(\frac{\partial s_2}{\partial t}\right)^2\right]\mathrm{e}^{-2s_2/\hbar}\mathrm{d}x\mathrm{d}t = \int_{t=0}^{\infty}\int\left|\frac{\partial\psi}{\partial t}\right|^2\mathrm{d}x\mathrm{d}t.$$

Since there are no potentials in the last two Fisher informations, they correspond to a free particle.

To describe physical phenomena in a way independent of the choice of the concrete inertial system, we require that the combined space-time Fisher information equals a real constant $K$ independent of the state of the investigated system

$$\frac{I_t''}{c^2} \pm I_x'' = K,$$

where $c$ is the speed of light and the sign in front of the spatial Fisher information $I_x''$ can be either $+$ or $-$. By considering two cases of a particle in rest and a particle with very large kinetic energy it can be shown that [13, 17, 18]

$$K \geq 0$$

and the minus sign in the last equation has to be taken

$$\frac{I_t''}{c^2} - I_x'' = K.$$

In this way, the correct signs of the metric of the special relativity and the relativistic invariance of the theory is obtained.

The last equation can be then written in the form

$$\int_{t=0}^{\infty}\int\left(\frac{1}{c^2}\left|\frac{\partial\psi}{\partial t}\right|^2-\left|\frac{\partial\psi}{\partial x}\right|^2-\frac{\hbar^2K}{4}|\psi|^2\right)\mathrm{d}x\mathrm{d}t = 0.$$

This functional must be independent of $\psi$

$$\int_{t=0}^{\infty}\int\left(\frac{1}{c^2}\frac{\partial\delta\psi^*}{\partial t}\frac{\partial\psi}{\partial t}-\frac{\partial\delta\psi^*}{\partial x}\frac{\partial\psi}{\partial x}-\frac{\hbar^2K}{4}\delta\psi^*\psi\right)\mathrm{d}x\mathrm{d}t + c.c. = 0,$$

where $\delta$ denotes the variation. Performing integration by parts with respect to $t$ in the first term and with respect to $x$ in the second one and assuming that variations $\delta\psi$ and $\delta\psi^*$ equal zero at the borders of the integration region we have

$$\int_{t=0}^{\infty}\int\delta\psi^*\left(\frac{\partial^2}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{\hbar^2K}{4}\right)\psi\,\mathrm{d}x\mathrm{d}t + c.c. = 0.$$

The condition that this equation has to be fulfilled for arbitrary values of $\delta\psi$ and $\delta\psi^*$ yields the equation of motion

$$\left(\frac{\partial^2}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{\hbar^2K}{4}\right)\psi = 0.$$

Introducing the rest mass $m_0$

$$K = \frac{4m_0^2c^2}{\hbar^4}$$

and generalizing to three dimensions we obtain the well-known Klein-Gordon equation

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m_0^2 c^2}{\hbar^2}\right)\psi = 0.$$

Dirac equation and many other equations of motion of physics can be obtained in a similar way [3-6, 13, 17, 18].

## VI. TWO NEW UNCERTAINTY RELATIONS

Now we show that the Heisenberg uncertainty relation can be replaced by two stronger uncertainty relations [13, 15-18, 23].

First, we take

$$u = \Delta x\sqrt{\rho}, \quad v = \left(\frac{\partial s_1}{\partial x} - \left\langle\frac{\partial s_1}{\partial x}\right\rangle\right)\sqrt{\rho}.$$

Then, the Schwarz inequality yields the first uncertainty relation

$$\langle(\Delta x)^2\rangle\langle(\Delta p_1)^2\rangle \geq \left[\int \Delta x \left(\frac{\partial s_1}{\partial x} - \left\langle\frac{\partial s_1}{\partial x}\right\rangle\right) \mathrm{e}^{-2s_2/\hbar}\mathrm{d}x\right]^2. \tag{5}$$

Here, the function $\partial s_1/\partial x$ corresponds to the classical momentum $\partial S/\partial x$ and the relation has the usual meaning known from mathematical statistics: the product of variances of two quantities is greater than or equal to the square of their covariance. Depending on the functions $s_1$ and $s_2$, the square of the covariance of the coordinate and momentum at the right-hand side can have arbitrary values greater than or equal to zero.

The second uncertainty relation can be obtained in an analogous way for

$$u = \Delta x\sqrt{\rho}, \quad v = \left(\frac{\partial s_2}{\partial x} - \left\langle\frac{\partial s_2}{\partial x}\right\rangle\right)\sqrt{\rho}$$

with the result

$$\langle(\Delta x)^2\rangle\langle(\Delta p_2)^2\rangle \geq \left[\int (x - \langle x\rangle) \left(\frac{\partial s_2}{\partial x} - \left\langle\frac{\partial s_2}{\partial x}\right\rangle\right) \mathrm{e}^{-2s_2/\hbar}\mathrm{d}x\right]^2.$$

The right-hand side of this relation can be simplified

$$\langle(\Delta x)^2\rangle\langle(\Delta p_2)^2\rangle \geq \frac{\hbar^2}{4}. \tag{6}$$

This uncertainty relation follows from the Schwarz inequality in a similar way as the first one, however, the covariance $\langle u, v\rangle$ is in this case constant and equals $\hbar/2 > 0$ independently of the concrete form of the function $s_2$. We note also that this relation is for $\langle x\rangle = a$ equivalent to inequality (3) for the Fisher information.

We see that the Heisenberg uncertainty relation can be replaced by two more detailed uncertainty relations. Uncertainty relation (5) can be understood as the standard statistical inequality between the coordinate $x$ and momentum represented by the function $p = \partial s_1/\partial x$. Uncertainty relation (6) can be understood as the standard statistical inequality, too. However, because of the specific form of the covariance $\langle u, v\rangle$ which equals $\hbar/2$ independently of the function $s_2$, the left-hand side of this relation must be greater than or equal to $\hbar^2/4$.

We note that the sum of uncertainty relations (5) and (6) is equivalent to the so-called Robertson-Schrödinger relation for the coordinate and momentum. The Heisenberg uncertainty relation can be obtained from the sum of the uncertainty relations by neglecting the first term on its right-hand side. Therefore, two new uncertainty relations are stronger than the corresponding Heisenberg and Robertson-Schrödinger uncertainty relations [24-26].

For general discussion of this approach see [7-10].

## VII. EXAMPLE: FREE PARTICLE

We assume that the wave function of a free particle is at time $t = 0$ described by the gaussian wave packet [17, 18, 23]

$$\psi(x,0) = \frac{1}{\sqrt{a\sqrt{\pi}}} \mathrm{e}^{-x^2/(2a^2)+ikx}$$

with the energy

$$E = \frac{\hbar^2}{4ma^2} + \frac{\hbar^2 k^2}{2m},$$

where $a > 0$ and $k$ are real constants. By solving the time Schrödinger equation we get

$$\psi(x,t) = \frac{1}{\sqrt{a\sqrt{\pi}}} \frac{\sqrt{1 - \frac{i\hbar t}{ma^2}}}{\sqrt{1 + \left(\frac{\hbar t}{ma^2}\right)^2}}$$

$$\times \exp\left\{ -\frac{\left(x - \frac{\hbar k}{m} t\right)^2}{2a^2\left[1 + \left(\frac{\hbar t}{ma^2}\right)^2\right]} + i\left[\frac{kx + \frac{\hbar t x^2}{2ma^4} - \frac{\hbar k^2}{2m} t}{1 + \left(\frac{\hbar t}{ma^2}\right)^2}\right] \right\}.$$

The corresponding functions $s_1$ and $s_2$ equal

$$s_1(x,t) = \hbar k \frac{x + \frac{\hbar t x^2}{2ma^4 k} - \frac{\hbar k}{2m} t}{1 + \left(\frac{\hbar t}{ma^2}\right)^2} - \hbar \arctan \frac{\hbar t}{ma^2},$$

$$s_2(x,t) = \frac{\hbar}{2} \left\{ \frac{\left(x - \frac{\hbar k}{m} t\right)^2}{a^2\left[1 + \left(\frac{\hbar t}{ma^2}\right)^2\right]} - \ln \frac{1}{a\sqrt{\pi}\sqrt{1 + \left(\frac{\hbar t}{ma^2}\right)^2}} \right\}.$$

As it could be anticipated, the mean momentum and the mean coordinate have the form

$$\langle \hat{p} \rangle = \left\langle \frac{\partial s_1}{\partial x} \right\rangle = \hbar k, \qquad \langle x \rangle = \frac{\hbar k}{m} t.$$

The mean square deviations of the coordinate and momentum are given by the equations

$$\langle (\Delta x)^2 \rangle = \frac{a^2}{2}\left[1 + \left(\frac{\hbar t}{ma^2}\right)^2\right], \quad \langle (\Delta p_1)^2 \rangle = \frac{\hbar^4 t^2}{2m^2 a^6\left[1 + \left(\frac{\hbar t}{ma^2}\right)^2\right]}$$

and

$$\langle (\Delta p_2)^2 \rangle = \frac{\hbar^2}{2a^2\left[1 + \left(\frac{\hbar t}{ma^2}\right)^2\right]}.$$

The left–hand side and the right-hand side of uncertainty relation (5) have the same value

$$\langle (\Delta x)^2 \rangle \langle (\Delta p_1)^2 \rangle = \left\langle \Delta x \left( \frac{\partial s_1}{\partial x} - \left\langle \frac{\partial s_1}{\partial x} \right\rangle \right) \right\rangle^2 = \frac{\hbar^4 t^2}{4m^2 a^4}.$$

Therefore, the first uncertainty relation (5) is fulfilled with the equality sign.

Calculating the left–hand side of uncertainty relation (6) we obtain

$$\langle (\Delta x)^2 \rangle \langle (\Delta p_2)^2 \rangle = \frac{\hbar^2}{4}$$

and see that the second uncertainty relation (6) is fulfilled with the equality sign, too.

## VIII. EQUALITY SIGN

The equality sign in uncertainty relations (5) and (6) is obtained if the functions $s_1$ and $s_2$ are quadratic functions of $x$ of the form $p(t)x^2 + q(t)x + r(t)$, where real coefficients $p(t)$, $q(t)$ and $r(t)$ can depend on time [17, 18, 23]. All functions $s_1$ and $s_2$ given in our example fulfill this condition.

It is worth to notice that this condition for the first uncertainty relation is independent of the form of the function $s_1$. Therefore, the equality sign in this relation can be achieved for much larger class of the wave functions than in case of the Heisenberg or Robertson–Schrödinger uncertainty relations. It is interesting not only from theoretical but also from the experimental point of view.

## IX. STANDARD COMMUTATION RELATIONS

Now we return back to the normalization condition for the wave function

$$\int |\psi|^2 \mathrm{d}x = 1.$$

Performing integration by parts and assuming $x|\psi|^2 \to 0$ for $x \to \pm\infty$ we get

$$\int x \left( \frac{\partial \psi^*}{\partial x} \psi + \psi^* \frac{\partial \psi}{\partial x} \right) \mathrm{d}x = -1.$$

Multiplying this equation by $-i$ we obtain the equation [11, 13, 17, 18, 23]

$$\int \left[ (x\psi)^* \left( -i \frac{\partial \psi}{\partial x} \right) - \left( -i \frac{\partial \psi}{\partial x} \right)^* x\psi \right] \mathrm{d}x = 2i \int x \frac{\partial s_2}{\partial x} \mathrm{e}^{-2s_2} \mathrm{d}x = i.$$

The resulting equation

$$\int \left[ (x\psi)^* \left( -i \frac{\partial \psi}{\partial x} \right) - \left( -i \frac{\partial \psi}{\partial x} \right)^* x\psi \right] \mathrm{d}x = i.$$

contains the operator $-i(\partial/\partial x)$ which appears here as simple mathematical consequence of integration by parts applied to the normalization condition and indicates validity of a more general operator equality

$$[x, -i(\partial/\partial x)] = i.$$

Except for the factor $\hbar$ determining the choice of units, this commutation relation agrees with the commutation relation

$$[x, \hat{p}] = i\hbar.$$

between the coordinate $x$ and momentum operator $\hat{p} = -i\hbar(\partial/\partial x)$ known from quantum mechanics.

It is seen that existence of the commutation relation for the coordinate and momentum in standard quantum mechanics is closely related to the existence of the normalized probability distribution $\rho(x)$ and relation $\rho = |\psi|^2$. Similar commutation relations should appear in any statistical theory formulated analogously to that discussed above.

## X. PROBABILITY DENSITY CURRENT

Now we discuss the probability density current $\mathbf{j}$ [11, 13, 17, 18, 23]. As in continuum mechanics, we assume

$$\mathbf{j} = \rho \mathbf{v},$$

where $\mathbf{v}$ is "velocity". We have in the Hamilton-Jacobi theory

$$\mathbf{v} = \frac{\nabla S}{m},$$

where $S$ is the Hamilton action and $m$ is the mass. By analogy with these expressions we can take in quantum mechanics

$$\mathbf{j} = \rho \frac{\nabla s_1}{m}.$$

Then we get

$$\mathbf{j} = \frac{\hbar}{m}\Big[ - i(\sqrt{\rho}\,\mathrm{e}^{is_1/\hbar})^* \nabla(\sqrt{\rho}\,\mathrm{e}^{is_1/\hbar}) + i\nabla\rho/2\Big],$$

where $\rho = \mathrm{e}^{-2s_2/\hbar}$. Using the wave function in the form $\psi = \sqrt{\rho}\,\mathrm{e}^{is_1/\hbar}$ we obtain the well-known result

$$\mathbf{j} = \frac{\hbar}{2mi}\big[\psi^* \nabla\psi - \nabla\psi^* \psi\big].$$

## XI. DEFORMED COMMUTATION RELATIONS

Now we make an attempt to find prescription for the probability density and inner product that would lead to the deformed commutation relation in the form

$$p\,x[-i(\partial/\partial x)] - q\,[-i(\partial/\partial x)]x = i,$$

where $p > 0$ and $q > 0$ are real numbers. For the sake of simplicity, we put $\hbar = 1$ here. We assume the normalization condition for the probability density $\rho(x)$ in the usual form

$$\int \rho\,\mathrm{d}x = 1.$$

Performing integration by parts and assuming $x\rho \to 0$ for $x \to \pm\infty$ we get

$$\int x \frac{\partial\rho}{\partial x}\mathrm{d}x = -1.$$

This equation containing the first derivate with respect to $x$ is the starting point of the following discussion.

In standard quantum mechanics, we use the relation

$$\rho = |\psi|^2.$$

Now, let us try a bit more general expression

$$\rho = |\psi|^p\,|\psi|^q.$$

Repeating similar procedure as above we get

$$\int \left[q\,x\frac{\partial|\psi|}{\partial x} - p\frac{\partial(x|\psi|)}{\partial x}\right]|\psi|^{p+q-1}\mathrm{d}x = -1.$$

This result indicates that

$$q\,x(\partial/\partial x) - p(\partial/\partial x)x = -1$$

or

$$p\,x[-i(\partial/\partial x)] - q\,[-i(\partial/\partial x)]x = i.$$

Thus, the probability density $\rho = |\psi|^p\,|\psi|^q$ leads to the deformed commutation relation $p\,x[-i(\partial/\partial x)] - q\,[-i(\partial/\partial x)]x = i$.

However, an attempt to define the corresponding inner product in the form

$$\langle \varphi, \psi \rangle = \int [\varphi^p]^* \psi^q \mathrm{d}x$$

or

$$\langle \varphi, \psi \rangle = \int \mathrm{e}^{-\mathrm{i}\arg\varphi} |\varphi|^p \mathrm{e}^{\mathrm{i}\arg\psi} |\psi|^q \mathrm{d}x$$

fails since these formulas do not obey the usual mathematical properties of the inner product.

It is seen that this naive approach fails and that a more systematic theory has to be used (see [27, 28] and references therein).

## XII. CONCLUSIONS

- Statistical description of measurement can be used as the starting point for formulating consistent physical theories. It is especially valid for quantum mechanics and quantum theory in general.
- The complex wave function $\psi$ carries information on the probability density $\rho = |\psi|^2$ and probability density current $\mathbf{j} = \hbar/(2mi)[\psi^* \nabla\psi - \nabla\psi^* \psi]$.
- The Fisher information depending on the form of the probability density $\rho$ or the envelop of the wave function is an important part of the kinetic energy.
- The Fisher information appears also in the uncertainty relations.
- The Fisher information can be used to find equations of motion.
- It is possible to derive two uncertainty relations that are stronger than the Heisenberg uncertainty relation. In these relations, classical and quantum descriptions are separated.
- Standard commutation relations can be obtained from the normalization condition $\int \rho \mathrm{d}x = 1$.
- Our attempt to get mathematical structure of quantum mechanics with deformed commutation relations in a similar way as it can be done for standard quantum mechanics has not been succesfull. It must be done in a more systematic way as in the papers of prof. R.M. Santilli (see e.g. [27, 28]).

## ACKNOWLEDGMENTS

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## REFERENCES

1. R. A. Fisher, *Proc. Cambr. Phil. Soc.* **22**, 700-725 (1925).
2. T. Cover and J. Thomas, *Elements of Information Theory*, New York: Wiley, 1991.
3. B. Roy Frieden and B. H. Soffer, *Phys. Rev.* **E 52**, 2274-2286 (1995).
4. B. Roy Frieden, *Physics from Fisher Information*, Cambridge: Cambridge University Press, 1998.
5. M. Reginatto, *Phys. Rev.* **A58**, 1775-1778 (1998); loc. cit. **A60**, 1730 (1999).
6. B. Roy Frieden, *Science from Fisher Information: A Unification*, Cambridge: Cambridge University Press, 2004.
7. S. Luo, *Letters in Math. Phys.* **53**, 243-251 (2000).
8. S. Luo, *Int. J. Theor. Phys.* **41**, 1713-1731 (2002).
9. S. Luo, *J. Phys. A: Math. Gen.* **35**, 5181-5187 (2002).
10. S. Luo, *Chin. Phys. Lett.* **23**, 3127-3130 (2006).
11. L. Skála and V. Kapsa, *physica* **E 29**, 119-128 (2005).
12. L. Skála and V. Kapsa, *Collect. Czech. Chem. Commun.* **70**, 621-637 (2005).
13. L. Skála and V. Kapsa, *Optics and Spectroscopy* **103**, 434-450 (2007).
14. L. Skála and V. Kapsa, *Int. Rev. Phys.* **1**, 302-306 (2007).
15. L. Skála and V. Kapsa, *J. Phys. A* **41**, 265302 (2008).

16. L. Skála and V. Kapsa, *Int. J. Quant. Chem.* **109**, 1626-1630 (2009).
17. V. Kapsa and L. Skála, *J. Comp. Theor. Nanoscience* (accepted).
18. L. Skála, J. Čížek and V. Kapsa, *Annals of Physics* (accepted).
19. C. R. Rao, *Bull. Calcutta Math. Soc.* **37**, 81-91 (1945); reprinted in *Breaktroughs in Statistics*, Vol. I, edited by S. Kotz and N. L. Johnson, New York: Springer, 1992.
20. H. Cramér, *Skand. Akt. Tidskr.* **29**, 85-94 (1946).
21. H. Cramér, *Mathematical Methods of Statistics*, Princeton: Princeton University Press, 1946.
22. W. Heisenberg, *Z. Phys.* **43**, 172-198 (1927).
23. L. Skála and V. Kapsa, *Collect. Czech. Chem. Commun.* (accepted).
24. H. P. Robertson, *Phys. Rev.* **34**, 163-164 (1929).
25. E. Schrödinger, *Proc. Prussian Acad. Sci.*, Phys.-Math. Section, **XIX**, 296-303 (1930); English version: *Bulg. J. Phys.* **26**, 193 (1999) and arXiv: quant-ph/9903100.
26. H. P. Robertson, *Phys. Rev.* **46**, 794-801 (1934).
27. R. M. Santilli, *Hadronic mathematics, mechanics and chemistry*, Vol. I-V, Institute for basic research, http://www.i-b-r.org/Hadronic-Mechanics.htm.
28. I. Gandzha and J. Kadisvily, *New sciences for a new era*, Kathmandu: Sankata Printing Press, preliminary version for reviews dated December **1**, 2010.

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
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# Nuclear Fusion Process With Antimatter Can Account For Dark Energy Mechanism Driving An Accelerated Cosmic Expansion

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Abstract. From a Twin Universe perspective, it is proposed that stellar nuclear fusion powered by anti-matter can account for the negative-energy pressure (Dark Energy) that drives our present-day accelerated cosmic expansion. In the mirror twinned universe all processes are duplicated but with reverse negative polarity. Both the Positive and Negative Universes exist on the opposing surfaces of a topological two-dimensional membrane and therefore shares the same experience of a stretching membrane.

Keywords: Dark Energy Twin Universes Anti-Matter
PACS: 95.36.+x

## INTRODUCTION

Using thermodynamic conservation principles the cosmos existing as a pair of identical anti-parallel universes has been proposed [1]. Parameterization of negative quantities can be formulated in terms of Santilli's isodual theory of antimatter [2]. The isoduality leads to four directions of time, depending on whether motion is forward or backward and occurs in the future or in the past. In our Positive Universe where all quantities have positive values, matter and anti-matter exists in forward direction of time and attractive gravity. In the reverse Negative Universe with negative quantities, the duplicate matter and anti-matter states must exists in a backward time direction and with repulsive gravity. The proposed twin universes model postulate all contents and processes of each universe have equal magnitude but opposite polarity, including energy-mass that in the anti-parallel universe the production of anti-matter particles from stellar nuclear fusion produces the necessary gravitational repulsion to drive the cosmic expansion. Since both universes reside on the topological surfaces of a shared membrane, the effective stretching of this common cosmic membrane will be observed as expansion in our side of the universe.

## STELLAR NUCLEAR FUSION

Stellar nucleosynthesis is the process of nuclear reactions taking place in stars to build heavier elements. The net mass of fused nuclei is smaller than the sum of the components, with the loss mass released as electromagnetic energy according to Einstein's famous mass-energy equivalence relationship:

$$E = mc^2 \tag{1}$$

Newton's law of universal gravity states that the force between two point masses ($m_1$,$m_2$) a distance r apart is given by the following equation:

$$F = G\frac{m_1 m_2}{r^2} \tag{2}$$

If we assume that the masses are of equal magnitude $m=m_1=m_2$, and the area mass density condensing on the two-dimensional membrane $\rho_m = m/\pi r^2$, then the gravitational force of acceleration produced by one point mass on the other is given by:

$$a = \pi G \rho_m \tag{3}$$

## Energy Production In Stars

The observable universe is composed of 70% hydrogen, and the proton-proton (p-p) chain reaction is the predominant thermonuclear fusion process that converts hydrogen nuclei into helium in stars with masses up to that of the Sun.

$$4^1\mathrm{H} \rightarrow {}^4\mathrm{He} + 2e^+ + 2\upsilon_e$$

Along with the formation of a pair of positrons and neutrinos, 26.7MeV of energy is released, equivalent to a mass of $4.8\text{x}10^{-29}$kg.

For more massive stars, another reaction process is also important that of the carbon-nitrogen-oxygen (CNO) cycle. In the main CNO-I reaction the carbon can be considered a catalyst in converting hydrogen into helium with the carbon being reformed at the completion of the following cycle:

$$^{12}\mathrm{C} \rightarrow {}^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} \rightarrow {}^{14}\mathrm{N} \rightarrow {}^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} \rightarrow {}^{12}\mathrm{C}$$

As with the p-p cycle, the total released of energy is 26.7MeV due to the mass difference between the fusion of the hydrogen parents to form the helium daughter.

## Stretched Universal Membrane

If we assume that the cosmos is uniformly distributed with point-like stars, the vast empty interstellar space would produce minimal gravitation contraction on the membrane upon which our positive universe resides. If we further assume that an identical negative universe resides on the opposite side of the same membrane, and all quantities and processes are duplicated but in opposite polarity. The stars in the positive universe would undergo the standard nuclear fusion processes that release large amount of electromagnetic waves into the empty space. Consider the same processes on the reverse side of the membrane, whereby the equivalent release of energy condenses out as point masses with negative energy and hence repulsive gravity.

For simplicity of computation, we will assume that the release of solar energy from the various chain reactions condense out as two equal point masses that occupy the volume of the fused helium nuclei $\sim 2\text{x}10^{-15}$m. With a gravitational constant value of $G=6.67428\text{x}10^{-11}\text{m}^3.\text{kg}^{-1}.\text{s}^{-2}$, the gravitation repulsive acceleration is approximately $1.6\text{x}10^{-9}\text{m.s}^{-2}$. This repulsive force of gravity moves at the speed of light ($c=299{,}792{,}458\text{m.s}^{-1}$), so the stretched membrane will expand at a rate of:

$$U_r = \frac{\pi G \rho_m}{c} \tag{4}$$

Inputting the model values, the estimated cosmic expansion rate $U_r \sim 5x10^{-18}s^{-1}$. Multiplying this quantum scale of repulsive expansion over an astronomical distance of a Mega-parsec ($Mpc=3.0857x10^{22}m$) gives a cosmological expansion rate of $160km.s^{-1}.Mpc^{-1}$. Even with this simple model the computed value for the rate of expansion is in reasonable agreement with the present-day measured Hubble constant. Alternatively by interpreting with this simple model, the current Hubble constant of $\sim 70km.s^{-1}.Mpc^{-1}$ would equate to an average simulated helium fusion energy release of 11.7MeV or equivalent mass density $\rho_m \sim 16kg.m^{-2}$.

## COSMIC HISTORY

Using the basic expansion model defined, the history of the twin-universe cosmos can be predicted in the following Table 1.

TABLE 1. Cosmological history of expanding twin universes

| Timeline | Content of Universes | Cosmic Expansion Rates |
|---|---|---|
| Planck epoch<br>0 to $10^{-43}s$ | Pure energy in positive universe.<br>Pure mass in negative universe. | Point source grows to Planck size<br>($l=4x10^{-35}m$) at infinite rate. |
| Inflation<br>$10^{-43}$ to $10^{-37}s$ | Energy in positive universe.<br>Formation of divided matter at exponential rate in negative universe. | $m_0 \sim 10^{60}kg$<br>$r_0 \sim 10^{-34}m$<br>$U_0 \sim 10^{109}s^{-1}$<br>($\sim 10^{129}km.s^{-1}.Mpc^{-1}$) |
| Matter condensation | Formation of baryons and leptons in positive universe.<br>Energy waves dilute matter in negative universe. | $m_+$ increases<br>$m_-$ decreases<br>r maintain constancy<br>$U_r$ decreases |
| Star formation | Present-day structure of observed positive universe.<br>Fusion energy drives expansion in negative universe. | $m \sim 10^{-29}kg$<br>$r \sim 10^{-15}m$<br>$U_r \sim 10^{-18}s^{-1}$<br>($\sim 100km.s^{-1}.Mpc^{-1}$) |
| Extinction | Steady state in positive universe.<br>Steady state in negative universe. | $m_+$ and $m_-$ constant<br>r constant<br>$U_r=0$ |

## SUMMARY

The Twin Universe model predicts that the complete cosmos exists as a ten-dimensional entity with two identical but anti-parallel four-dimensional space-time (energy-entropy) universes residing on the opposing surfaces of a two-dimensional common membrane. Quantities and processes on both sides of the universes are duplicated but of reverse polarity. In the Positive Universe the fusion reactions within stars release vast quantities of energy into the expanse of space as electromagnetic waves. In the reverse Negative Universe the same fusion energy condenses as point masses with negative quantity (repulsive gravity) that stretches out the common membrane producing the observable accelerating expansion of the entire cosmos. A summary of the dual processes on the two mirrored sides of the universes are listed in Table 2.

TABLE 2. Cosmic processes on both sides of the twin universes

| Positive Universe | Negative Universe |
|---|---|
| Fusion generates energy waves | Fusion generates matter particles |
| Attractive gravity | Repulsive gravity |
| Measured Hubble's constant $H \sim 71 km.s^{-1}.Mpc^{-1}$ | Computed Universal expansion rate $U_r \sim 160 km.s^{-1}.Mpc^{-1}$ |

The model further predicts that Dark Matter and Dark Energy constitute half of the missing observable energy-mass in the cosmos. Present experimental measurements estimate the percentage of Dark Matter at 23% and Dark Energy at 73%. However, there are proponents that claim both are the same component of Dark Fluid [3], and hence if the differing effects are producing a double-counting of the same unobserved material, then the actual percentage may indeed be 50%.

## REFERENCES


1. L. Ying, *Hadronic J.* 32, 573-588 (2009)
2. R. M. Santilli, *Isodual Theory of Antimatter,* Dordrecht: Springer, 2006
3. H. S. Zhao, *J. Phys.: Conference Series* 140, 1-12 (2008)

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
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# The Irreversible Phase Transition of Material Becoming

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**Abstract.** Here in the cradle of Vedic wisdom and Mandala Cosmogony it appears doubly befitting to attempt a wider reflection upon the profound existential and epistemological matters on the agenda. Then, touching upon the subject of structural being as such, there is a rich Oriental-Hellenic canon of its origination and maintenance in the immanent phase transition between Straight and Round; the two antipodes of perfect endlessness. This process can be described by the original Lie groups and algebras as a transformation from a cubical to a spherical symmetry space by a single rectilinear unit root vector quantity spanned over the respective hybridized Lie algebra neighbourhoods and acting there by itself as an instantly expanded three-dimensionally space-filling digital holographic operator and materialization in complete faithfulness to the ancient as well as modern directions, notably, as a direct realization of these, Santilli's new mathematics and hadronic mechanics. In particular, his reduction of matter to protons and electrons provides a master code and key to the distributed universal inflation of the evolution of the maximal mutual penetration neutron complex with the possible transformations and differentials thereof, all covered by hadronic mechanics and forming the further irreducible pivotal origin of the phase transition which in successive self-similar cycles of its space-filling, Bohr-layered loop motif without any contradictions gradually assumes the quantum mechanical patterns, paths and happenings of its at that stage irreversible exterior/collective dynamics. This unification holds fascinating Mandala, that is, cosmographical implications, which in modern terms point at the advancement of high-precision computerized holographic animation with important basic science exploration as well as nanotechnology application potentials.

**Keywords:** Atom Honeycombs, Elementary Particle Spectroscopy, Euclidean Space, Hadronic Mechanics, Information Systems, Iso-, Geno- and Hypermathematics, Lie Algebra Realization, Mass Numbers, Nanotechnology, Periodic System, Phase Transition, Quantum Computing, Quark-angle, Regular Solids, Truncated Octahedron.
**PACS:** 02.20.Qs, 02.40.Dr, 07.05.Bx, 07.05.Tp, 11.15.Ha, 12.10.Kt, 12.39.Ba, 12.40.Yx, **14.20.-c, 14.40.-n, 14.60.-z**

## INTRODUCTION

As, in all modesty, a rather rational and productive researcher in a closely related sphere of Natural History, namely Medicine, where I have been able to contribute to some significant advances over a range from e.g. molecular oncology to preventive medicine, medical informatics and international public health (search *Trell* in *Pubmed* for credentials); and, moreover, as the writer of a comprehensive review in the dedicated scientific literature on professor Santilli´s innovative hadronic mechanics with its many invaluable spin-offs[1]; and, finally, as a cross-disciplinary colleague having heavily derived from this pioneering chef d´oeuvre in my structural reproductions of the elementary particle and atomic symmetries encompassed therein: it is only now, after digesting the full purport of his *New Sciences for a New Era*[2] that the actual width and importance of his truly Renaissance achievement stand clear for me.

He, and his work, pose no less than a bold and lasting postmodern reconstruction of the entire core of theoretical Physics engaged with the nature and processes of matter at the fundamental level, and in that do not challenge or replace, but on the contrary enrich and re-conciliate the classical and quantum mechanical understandings and formulations of the field. Hence, "conventional quantum mechanics is exactly valid in the exterior problem of hadrons in vacuum" while hadronic mechanics is uniquely engaged with the laws and properties "in the structure of hadrons rather than in their collection" by "physically consistent solutions via a non-unitary image of Schrödinger's equation...as a non-unitary covering of quantum mechanics".[2]

It thus goes deeper, into the particles themselves, from the outside observation of which in "the distinction between exterior and interior dynamical problems" [2] quantum mechanics simply does not apply, while Santilli's genuinely intrinsic iso-, geno- and hypermathematics contribute to a systematic operative inventory allowing the ultimate constitutional "reduction of matter to protons and electrons". [2] The penetrating formulas and equations to that end are far beyond the grasp of the present account which is confined to the legacy and directions that they hold for an ordinary geometrical realization of the elementary particle and ensuing spectroscopy, likewise founded upon the only finally stable, proton and electron moieties of which, therefore, all other states must be sequential transformations and differentials thereby automatically preserving symmetry and, since "the size of wave packets and/or charge distributions of all particles is about 1 fm" [2], also relative volume. There thus exists a faithful structural model of Santilli's grand synthesis, providing a veritable bit kit for piecemeal nanotechnological assembly as well as discrete digital computer animation from the very ground level, and of such high resolution, exhaustiveness, exactitude and flexibility so as to present a both cost- and product-effective complement to expensive back-tracking particle colliders in the further exploration and clarification of matter.

It is felt that the large current interest and importance of the information technology utilities motivate some extra focussing upon them in the following. For terminological and conceptual adequacy then quoting the leading state-of-the-art article, *Quantum Information Matters*, the physical entity "entropy...by Bolzmann's constant... is a form of information...about the microscopic motions of atoms and molecules".[3] This generic identity between information and matter was utilized to formulate the "universal quantum simulator": in effect a direct processor of quantum mechanics and thereby not only a substrate but virtual substance and "evolution of...the quantum mechanical aspects of matter" (Ib.).

In equivalence, "the classical digital computers" by their "individual classical bits" (Ib.) should be equally fit to generate the classical aspects of matter at the very fundamental microscopic level. The present work, utilizing also the most classical, inherently digital mathematical principles and methods of the regular solids, demonstrates that this is indeed the case, all the way from the elementary particles to the atoms and molecules and onwards, and that important entanglement with quantum computing exists at critical junctions. While these processes from the quite reasonable and lasting results can only be comprehended as in effect the bottomline universal holographic mechanism, the transactions likewise have a great applied value since they can be freely zoomed up to a hitherto lacking computer/physical model program for real structure animation of the matter microcosm.[4,5] This builds upon the fact which already Max Planck in his 1901 paper on blackbody radiation[6] "established that the universe was, at bottom, digital"[3] and thereby, and by equivalent discrete ground eigenvector bits, just as much an information system as a material system. In today's technological framework (and common parlance), cosmos is a computer, then, of which, however, there are two principal kinds: "the classical digital computer" and the "quantum computer"(Ib.). Both process a basic one-dimensional infinitesimal generator, in essence a unit straight line. In the quantum computer this unit vector representation of "the digital nature of the universe" is as "a collection of electron spins...or qubits...in entangled state", that is, a superposition of the arrows of up to "a billion billion spins", whereas in the classical computer it is realized as "a sequence of bits" (Ib.) of fixed inclination, customarily set as orthogonal. This has radical consequences when a phase transition can only act out as an angular deviation of the dense bundle, thus disrupting its simultaneous stepwise evolution from a parallel to a dispersed rearrangement, thereby distributively ballooning, as it were, the instant inflation of universe[7] from the sudden outset snap detonating everywhere.

The principal difference between quantum and classical computers makes their respective general "evolution" different, too: "the individual classical bits"[3] unable to generate quantum mechanics, whereas the quantum computers are not fit to animate and explore ordinary matter. Here, digital holography is the classical information system upgrade for not only simulating but realizing the natural structures of the world. Quite remarkably, this wide-open option has not been tested out before, and all the way from the smallest digital bit scale yields unprecedented results of the whole elementary particle spectroscopy and onwards[1,4,5,8-32] to ultimately at critical junctions even converge with and enrich the quantum computing version. Since the theoretical and philosophical fundamentals were thoroughly reviewed (Ib.), the present report will focus on the descriptive results and computer applications[32] which in accord with classical principles as well as modern nanotechnology show that the observed Universe can be extensively and exhaustively formed by the real information elements, internal symmetries, transformations and transitions, and thereby performed serial self-assembly of the system itself.

The ancient idea of primordial form as an immanent self-operator of material reality is today meeting up with advanced thinking in other distinct fields of basic science. As Johansen succinctly formulates it specifically in "information...comprehended in its most...elementary sense...the method is to systematically unfold...what is

enfolded in information as such".[33,34] Another prominent exponent of this renewed kind of quite profound self-referential understanding bearing upon both the classical and quantum information realm is the Universal Nilpotent Computational Rewrite System (NUCRS)[35-38], where "the only requirement for defining the entire quantum mechanical apparatus relating to physics...is to specify its creation operator", which in the appropriate form in that case "corresponds to the infinite square roots of -1".[39]

And most important and pioneering of such "alternative systems of units" at the root and expansion of fundamental processes, "where they can relate by a 1:1 mapping to the overall structure... is...the negative unit (or iso-Minkowskian) system developed by Santilli[1,2], with its powerful applications in both physics and pure mathematics"[36], where in linear sequence the formulas perform what form does by conformity. And other instances of digital units are spin matrices, infinitesimal generators and the binary numbers, and when similarly "distilling the free-form natural laws"[40] down to the origin of many classical functions in ordinary space, the extracted, further irreducible eigen-operators at the bottom are the ordinary partial derivatives $\delta x$, $\delta y$, $\delta z$, and hence, as in the present case, the infinitesimal straight line that already Plato and his forbearers and followers used to reconstruct the world.[4,5]

## METHODS

The old idea that space and matter are two sides of each other and made by the same stuff has recently been revived in a four-dimensional, "loop quantum gravity" version[41] and in computer terms correspond to the screen and processor parts which in a liquid crystal makeup are united so that they completely match and fulfil each other. The holographic screen of the real three-dimensional Cosmos is obviously the re-confirmed flat, i.e. cubical, Euclidean space. This has even been referred to as the physical "canvas" [42], and its rigid crystal composition was first explicitly surveyed by Ptolemy[4,5] from the fact that its building element, the straight line, can span no more than three linearly independent space axes (Fig 1).

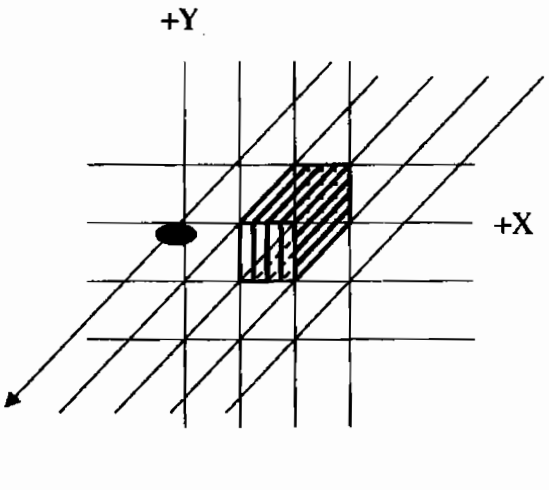



**FIGURE 1.** Say, that there comes a straight line out from one's closed eyes leaving in the forward direction (**+Z**). Then it must also endlessly extend towards one from behind, and there must be such lines infinitesimally tight over all the void's reach, because a linearly independent such axis can also come from below and rise up (**+Y**), or from the one side and leave on the other (**+X**), all of them together thereby spanning the endless Cartesian co-ordinate system and between them enclosing the infinitesimal cubical eigenvector bit of the matrix

Also Aristotle (384 – 322 BC) thought that the world is three-dimensional and that the formations in it are timeless, their simultaneous strands interpunctated by successive smallest steps which both as cause and effect mark the moments of the sequence as well as of the interaction cone of the advancing front of apparent now. At any scale,

the consequential Cartesian eigen-coordinate system (Fig. 2) and the realizations within it are therefore both 'Gödel-immunized' and 'Popper-ratified' since defined and constituted and *sensu strictu* falsified solely by themselves.

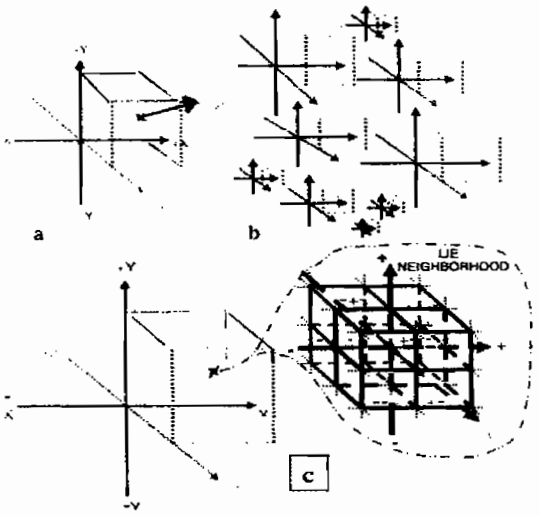



**FIGURE 2.** a) The Cartesian coordinate system spans the three-dimensional Euclidean space in eight cubical segments. What is the constitution of a local part (?) in any of them? b) Regardless of size it retains the Cartesian representation. c) Hence, the smallest composite space portion is a Lie neighbourhood of eight indivisible ground unit CuBits.

The cubical constitution of the cosmic holographic screen holds quite interesting pure mathematical powers, e.g. on-line realisation of the complete Diophantine equation space and instant solutions of Fermat's Last Theorem and Beal's conjecture[4,5,22-25,28,29], but has no dynamic function or action per se in its absolutely still, absolute zero deep-frozen state thermodynamically as well as informatically. The orthogonally criss-crossing straight line geodesics cut out a dense matrix of minuscule cubical eigenvector pixel cells, or cuBits©, equally narrow as the relative shortness of the infinitesimal straight line bit along all sides, and this division also defines the constant fraction of cells concurring in each step of the continuous outlining. Since the whole mesh is simultaneous this consecutive progression can be imagined as an one-dimensionally advancing front continuing over any direction closely side by side, but if there is a coherent and thereby volume-occupying phase transition bending into the parallel line array, this will immediately repel and cumulatively force the transition process to an even dispersion all over the block so that the origins and evolution of its thus distributed instant universal inflation are just like the vapour of the primordial electron/Hydrogen cloud. Its individual drops are therefore disconnected from each other, continuously in one single moment filling their own basins according to the volume allotted to these by the remaining sheets of the one-dimensional outlining of the holography screen. The further evolution will then be determined by the secondary perturbations of these domains by collisions and other processes in the cloud; just like in the actual Universe, too.

The computer processor and process alike from where it originates in a Lie neighbourhood (Fig.s 2,3) is precisely the phase transition, or 'trigger'[2], as such, which is both unique and general since performed between the antipodal varieties of endless extension, namely, absolute straight and absolute round, or, as Aristotle put it:

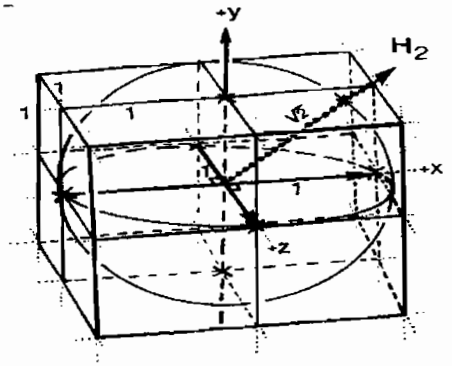



**FIGURE 3.** The hybridization of the unit sphere within the cubical Lie neighbourhood sets up an interstice, in the universal iteration of which the basic, tetra- and octahedral regular solid phase transition is immanent

"everything that comes to be comes into being from its contrary...and passes away likewise...by the action of the contrary into the contrary" and "if there is a contrary to circular....a straight line must be recognized as having the

best claim to that name".[4,5] In consequence, it is, like the original Lie algebras[16,43], a transformation process between infinite surfaces, employing in its course just its own irreducible eigen-element, the unit straight line, which is operative both as a direct structural building bit and an impulse/information as well as logical/computational digit, and has a deep philosophical underpinning as the nilpotent eigenvector of Anything At All, or, technically speaking, as the obligate isodual contrast of the more philosophically unambiguous category of Nothing Whatsoever.[4,5,35-39]

The virtual phase motor of the cardinal transition "between the Plücker line geometry and a geometry whose elements are the space's spheres"[16,43], and which twists the straight line bits of the unit cube (Fig.s 2,3) into the spherical symmetry orientation, is the SU(3) root space lodged in the real three dimensions at hand (Fig. 3), as here shown in about same scale and orientation as its neighbourhood encasement (Fig. 4). It is composed of two flat $A_2$

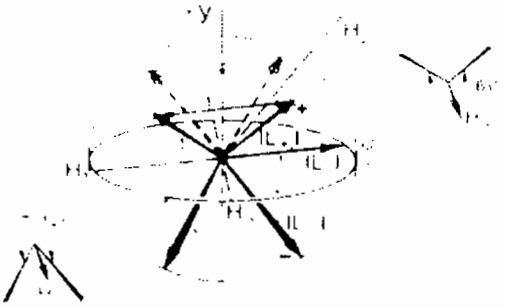


**FIGURE 4** Real form three-dimensional spherical Lie algebra neighbourhood with duplicated $A_2$ root space diagrams

diagrams accommodated in the unit sphere, bringing the representation from the complex to the parent, ordinary three-dimensional space according to the canonical coset decomposition SO(3) x O(5) of SU(3). But it is not just a representation, it is a concrete machinery for elementary physical vector currents and operations happening in and breeding the dynamic universe.

## RESULTS

### I. The Baryons

It is seen that the spherical symmetry $A_2$ root vectors (Fig. 4) from the mutual centre connect to a global lattice that is 60° skewed to the horizontal and vertical planes, and non-commutating with the latter. The correspondence to the quark three-dimensionality in the observed elementary particle spectroscopy is apparent and the close coincidences persist with all attributes of this. Fig. 5 summarizes the virtual 'binary phase motor' transformation

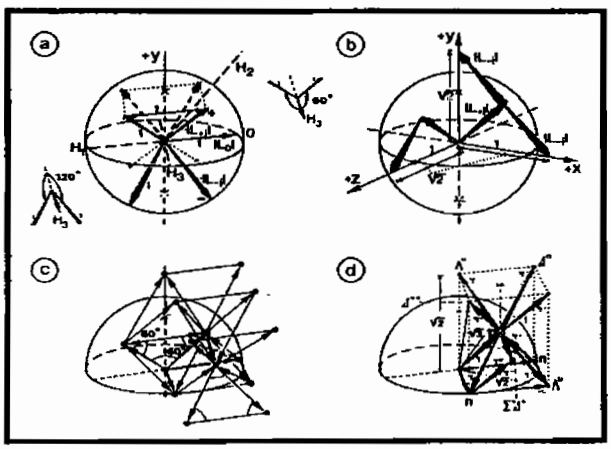


**FIGURE 5.** Spherical root vector space whose neutral isospin vectors coincide with Cartesian X and Z axes (a) but whose thereby charged, *t* isospin axes (b) set up a non-commutative quark matrix with unit side (c), continuing in the global interstice as space-filling regular tetra- and octahedrons, (d) none of which can fill the space separately due to different side length in lateral and bottom planes. It is seen that it is the vertical side that is non-commutating with the cubical framework, so that it is this virtual fall of the phase transition which from its first extra-nucleon evolution cycle becomes more and more irreversible.

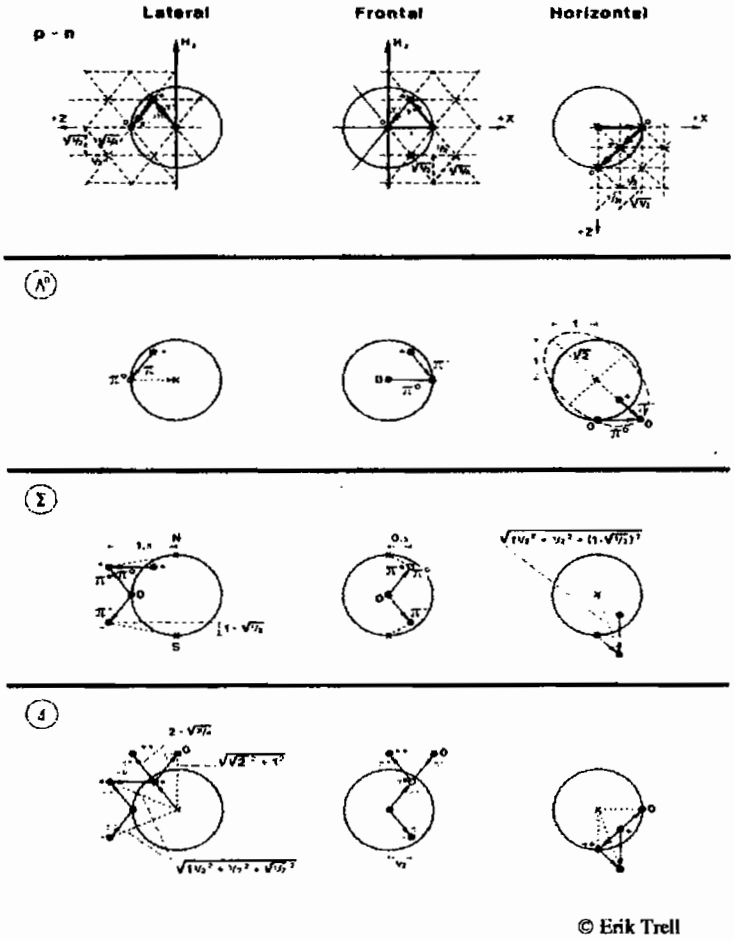
system set up by the $A_2$ axes of the unit sphere domain assigned to the Nucleon[44] relative to the straight space axes, and providing a faithful "eightfold eightfold" three-dimensional version of the plane Gell-Mann lattice diagram.

The diagonal $A_2$, so called charged *t* isospin root vectors (Fig. 5 a), connect also outside the sphere to an endless polygonal lattice (Fig. 5 b,c), skewed to the orthogonal Euclidean co-ordinate axes and thus, as mentioned and in perfect compliance also with Santilli's understanding of "quarks (as) purely mathematical representations of a unitary symmetry"[2] (here by coset decomposition), from a shared origin span a quark space matrix aberrant to the cubical arrangement and so directly providing the still rectilinear phase transition of this turned to the spherical symmetry. As discussed more in detail later it is quite significant that the charged *t* isospin vectors adjoin throughout space into an infinite continuous mesh of unit sides (Fig. 6), which is then the runner for the one-dimensional electron step distribution into atoms and larger coherent portions in a cyclically cumulative course with all the apparition of the likewise bit-by-bit sequential orbital model including quantum indeterminacy and related evasive behaviour at any in comparison vastly broader probing feasible with current instruments.



**FIGURE 6.** A dissection of a portion in the first and second extranucleon layer of the charged *t* isospin root vector lattice, showing that it connects as octahedron sides without involvement of any neutral root vector elements

Regarding the impenetrable nucleon, this infinitesimal, i.e. absolutely smallest sphere can of course not be further shrunk itself, nor can its complementary form of endlessness be effectively changed. But when impacted it can be shape transformed and then by necessity preserving both volume and, isomorphic to colour, spheroidal symmetry. Fig. 7 illustrates this in the basic baryon states, i.e. the $\Lambda^0$, $\Sigma^{+,0,-}$ and $\Delta^{++,+,0,-}$ transformations which come out just as




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**FIGURE 7.** The p-n, $\Lambda^0$, $\Sigma^{+,0,-}$ and $\Delta^{++,+,0,-}$ transformations, preserving volume and symmetry

in reality, and the same applies to all other states as exemplified by the ensuing $\Xi^{0,-}$, $\Sigma(1385)^{+,0,-}$, and $\Delta(1405)^0$ in Fig. 8. Since described in detail earlier[4,5,6-11,19,23,27-32], it suffices to re-emphasize that it is exactly the Gell-Mann

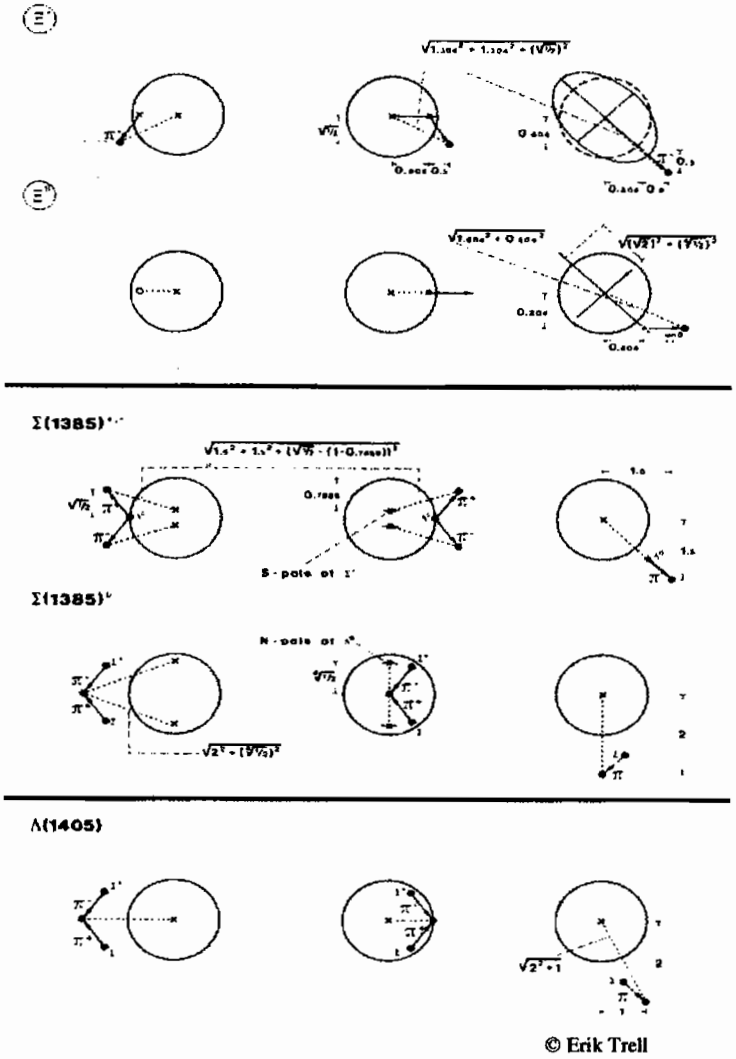



**FIGURE 8.** The $\Xi^{0,-}$, $\Sigma(1385)^{+,0,-}$, and $\Delta(1405)^0$ transformations

eightfold way in the real three dimensions instead of two and therefore an "eightfold eightfold way"[4,5,17,19], because the (diagonally into anti-versions mirrored) transformations may occur in any of the Cartesian space segments. Considering that all observed baryon particles and resonances in the Λ, Σ, Δ, N, Ξ, Ω and also full charmed series[17,18] are directly and reproducibly retrieved with just and no more than the actual states, channels, angular momentums, charge levels and precise mass numbers, and moreover in a faithful three-dimensional realization of the accepted eightfold way according to the original Lie prescriptions, the results are true and lasting and it is remarkable, too, that they are projected over the regular solid space axes and sides (Fig.s 7,8).

To reach the transformation, the same root vector steps as in the Gell-Mann supermultiplet diagrams and the observed spectroscopy alike are taken, leading to new endpoints for an ellipsoidal reconfiguration of the parent state, whereby the masses (given in MeV) according to the quark pressure formula, $\Delta p = \hbar/\Delta x$, come out reciprocally to the proton mass by the minor semiaxis length. In fig. 7, the plane graphs show the channels and the major semiaxis endpoints arrived at in the $\Lambda^0$, $\Sigma^{+,0,-}$ and $\Delta^{++,+,0,-}$ hyperons with lengths to the origin given by the root expression, and further that also the charge levels are retrieved exactly and exhaustively as in reality. The global, quark-skewed hexagonal spherical root space lattice is shown in the p-n transposition and (the equatorial plane of) the volume-

preserving ellipsoidal reconfiguration is shown in the $\Lambda^0$ state. It is to be reemphasized that albeit exemplified here only for the basic baryon multiplets of the Eightfold Way, the exact methods, as reported elsewhere[4,5,6-11,19,23,27-32] exhaustively, precisely and reproducibly comprise and yield all other observed *u,d,s* as well as *charmed* baryons and their channels, electromagnetical charges, $J^P$ levels and mass numbers by faithful real geometrical rendition of the most well-established physical theories, and thus stand forth in the natural history records as lasting scientific facts confirming and advancing the established state of the art.

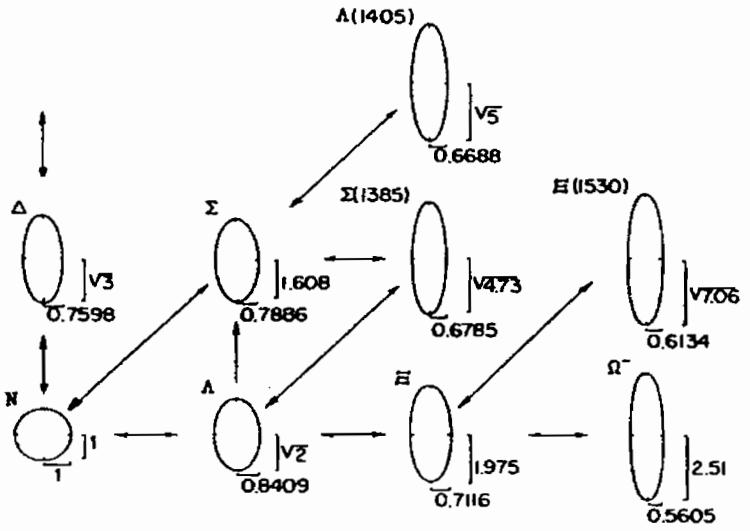
Table 1 shows the detailed correspondences in case of the masses in the basic baryon supermultiplets.

Table 1. *Lambda, sigma, delta, xi, sigma{1385}, lambda{1405}, xi{1530} and omega hyperons. Masses calculated according to formula: 938.28 · 1/minor semiaxis*

| | Major semiaxis | Minor semiaxis | Mass | |
|---|---|---|---|---|
| | | | Calculated | Observed |
| $\Lambda^0$ | $\sqrt{2}$ | $\sqrt[4]{\frac{1}{2}}$ | 1115.8 | 1115.6 |
| $\Sigma^{+,0,-}$ | 1.60804 | 0.788591 | 1189.8 | 1189.4 – 1197 |
| $\Delta^{++,+,0,-}$ | $\sqrt{3}$ | $\sqrt[4]{\frac{1}{3}}$ | 1234.8 | 1230 – 1236 |
| $\Xi^{0,-}$ | 1.975 | 0.7116 | 1318.5 | 1314.9 – 1321.3 |
| $\Sigma(1385)^{+,0,-}$ | $\sqrt{4.71} - \sqrt{4.75}$ | 0.679 – 0.678 | 1382.2 – 1385 | 1383 – 1386 |
| $\Lambda(1405)^0$ | $\sqrt{5}$ | $\sqrt[4]{\frac{1}{5}}$ | 1403 | 1405 ± 5 |
| $\Xi(1530)^{0,-}$ | $\sqrt{7.06}$ | 0.6134778 | 1529.5 | 1528 – 1534 |
| $\Omega^-$ | 2.505 – 2.51 | 0.561 – 0.560* | 1673.5 – 1677 | 1672 – 1674 |

* Minor semiaxis changed in the transformation (c).

Equatorial planes of all these also esthetically very pleasing and harmonic volume-preserving ellipsoid transformation bodies are illustrated in Fig. 9.




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**FIGURE 9.** Plane illustration of the lowest mass number baryons in the order in which they are obtained both in the holographic lattice and in reality. The resulting major and minor semiaxis lengths (minor semiaxis $b_2$ in $\Omega^-$) are shown

## II. The Mesons

The mesons likewise appear in the root vector framework just as in reality as differentials between hadron states there. Their spatial shape is explicitly given by the ordinary geometric representation of the established (symmetric) SU(2) x U(1) (antisymmetric) product group of the weak force (lb.), so that they come out as polyhedrons, too, albeit not equilateral in all their extensions and therefore unsustainable in the universal lattice, which has the structure of a space frame (Fig. 10), in whose build and transitions the mesons and leptons form the permutation

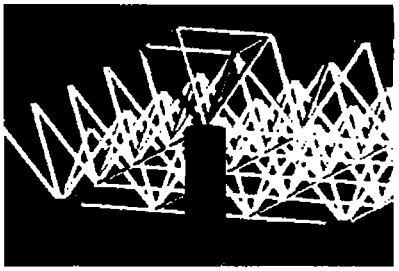


**FIGURE 10.** (adapted from Wikipedia) Simplified space frame roof with half-octahedron highlighted. A space frame or space structure is a truss-like, lightweight rigid structure whose geometry is most often based on the Platonic solids. The simplest form is a horizontal slab of interlocking square pyramids. A stronger purer form is composed of interlocking tetrahedral and octahedral crosspieces in which all the struts have unit length. More technically this is referred to as an isotropic vector matrix or in a single unit width an octet truss. More complex variations change the lengths of the struts to curve the overall structure or may incorporate other geometrical shapes

set of all possible bits and their building block assemblies in the methodical construction and deconstruction of the system. This becomes clearer in the following, starting with the neutral and charged pions which emerge naturally as the first differential elements of the cubical and hexagonal half-parts of the lattice, respectively. In Fig. 11 the neutral pion is shown, assuming the shape of a right circular cone segment with base area 1/4 of the unit Proton equatorial plane and length of spinning top generatrix $3^{1/2}$ so that the mass expression of the enclosure of this ground neutral transformation step according to the aforementioned canonical group equation is $1/4 \times 938.27 \times 1/3^{1/2} =$ 135.4 MeV in comparison with the measured value of 135.0 MeV.

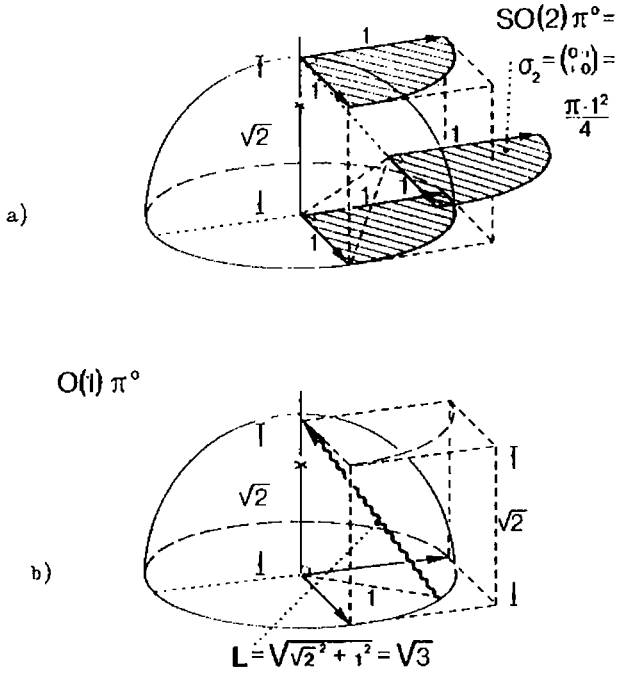



**FIGURE 11.** (a) A neutral *t* isospin root vector is inclinated 90° to adjacent neutral isovector doublets in the lattice. Between each other they form a circle sector amounting to one quarter of the equatorial plane of the proton. (b) The distance to the next neutral SO(2) counterpart of the $SU(2)_{wk}$ isospin planes in the transition lattice is $3^{1/2}$

The charged pion comes equally straight out in the faithful transition lattice as an oblique circular cone, wrapped between state transformations in the hexagonal half-part since happening between state transformations in the hexagonal lattice moiety where it occupies a base area of 1/6th of the proton equatorial plane with average

generatrix length of $(5/4)^{1/2}$ so that the mass number is $1/6 \times 938.27 \times 1/(5/4)^{1/2}$ MeV = 139.9 MeV versus the observed 139.6 MeV (Fig. 12).

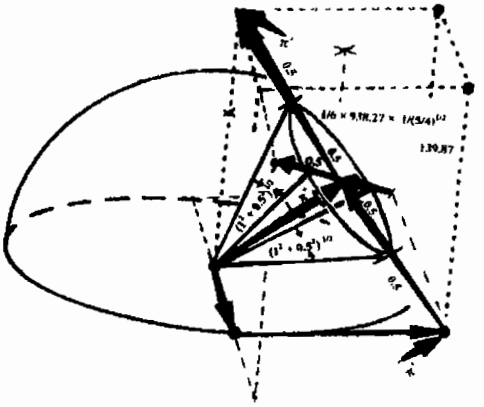


**FIGURE 12.** The charged pion takes place in the lattice as the primary differential enclosure of single-step transformations to/from charged hadron states, covering a conical base area $1/6^{th}$ of the size of the proton equatorial plane centered along an 180° (muonic) root vector sequence and with an (average) generatrix length of $5/4^{1/2}$.

Again is noted an unprecedented identity between reality and replication also when it comes to exhaustiveness; no other varieties at the respective levels occurring in any of the systems. And the same correspondences according to the unmistakable scientific directions continue in all other mesons as exemplified in the next basic states (Fig. 13); since long then by any probability testing and other rigorous authenticity criteria surpassing the slightest possibility of a chance coincidence.

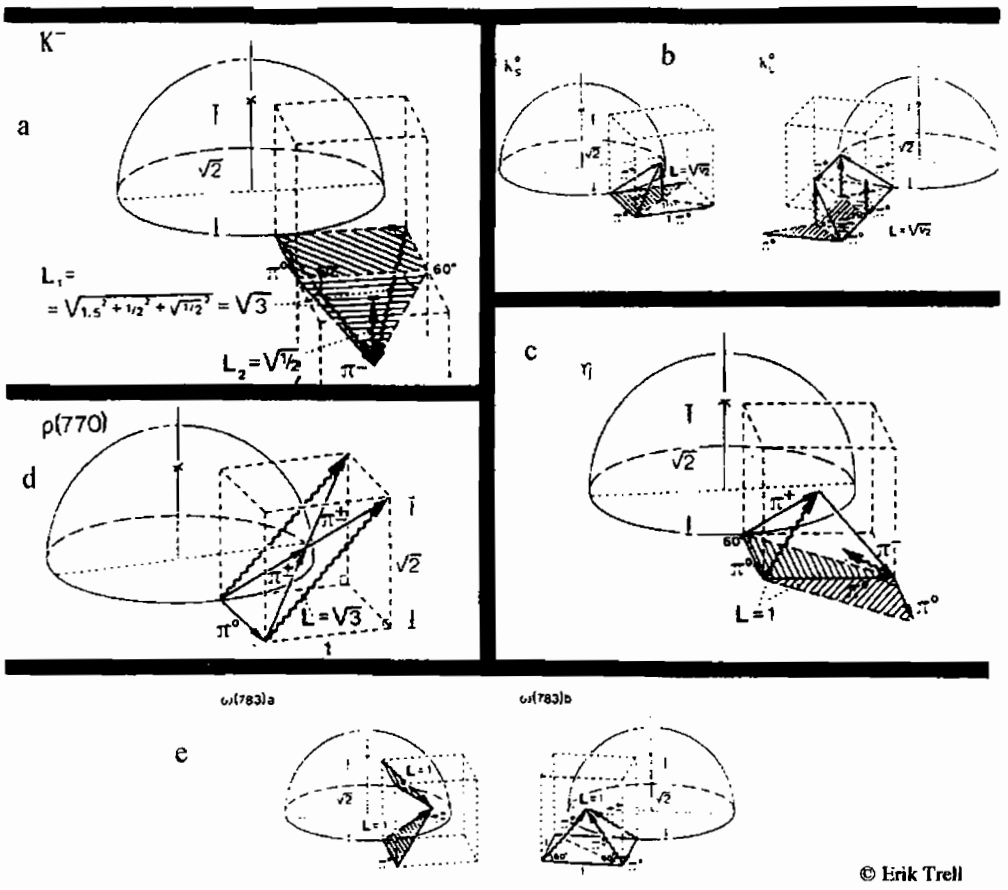



**FIGURE 13.** Chart of (a) charged kaon; (b) $K^0$ short and $K^0$ long; (c) η; (d) ρ(770); (e) two forms of ω(783)

It is striking and convincing that polyhedral root space elements, both differential and equilateral, are so in double sense straightforwardly involved and that it is possible to exactly and exhaustively match the observed elementary particle spectroscopy by classical regular solid metamorphoses. Table 2 exemplifies the mass number calculations according to the SU(2) x 1/U(1) Lie algebra relation of the weak interactions, and the parallelism in every regard; also chargewise and channelwise is manifest.

Table 2. Basic mesons calculated and observed mass numbers (MeV)

| | | | |
|---|---|---|---|
| $\pi^0$ | $1/4 \times 938.27 \times 1/3^{1/2}$ | 135.4 | 135.0 |
| $\pi^{\pm}$ | $1/6 \times 938.27 \times 1/(5/4)^{1/2}$ | 139.9 | 139.6 |
| $K^{\pm}$ | $938.27/4 \times 1/3^{1/2} + 938.27/4 \times 1/3^{1/2} + 938.27/6 \times 1/2^{1/2}$ | 492.0 | 492.7 |
| $K^0_S$ | $938.27/(4 \times 1/2^{1/2}) + 938.27/(8 \times 1/2^{1/2})$ | 497.6 | 497.67 |
| $K^0_L$ | $938.27/(8 \times 1/2^{1/2}) + 938.27/(8 \times 1/2^{1/2}) + 938.27/(8 \times 1/2^{1/2})$ | 497.6 | 497.67 |
| $\eta$ | $938.27/6 + 938.27/6 + 938.27/4$ | 547.33 | 548.8 ±0.6 |
| $\rho(770)$ | $(938.27/1/2^{1/2})/(3/4)^{1/2}$ or $(938.27 \times 2^{-2})/3^{1/2}$ | 766.1 | 768.3 ±0.5 |
| $\omega(783)$ | $938.27/4 + 938.27/4 + 938.27/6 + 938.27/6$ | 781.9 | 781.95±0.14 |

And it continues in the entire plethora of mesonic differentials and transformations over the whole spectrum of towering hadrons up to *charmed* (Fig. 14) and *bottom* and even *top* flavours.[17,18]

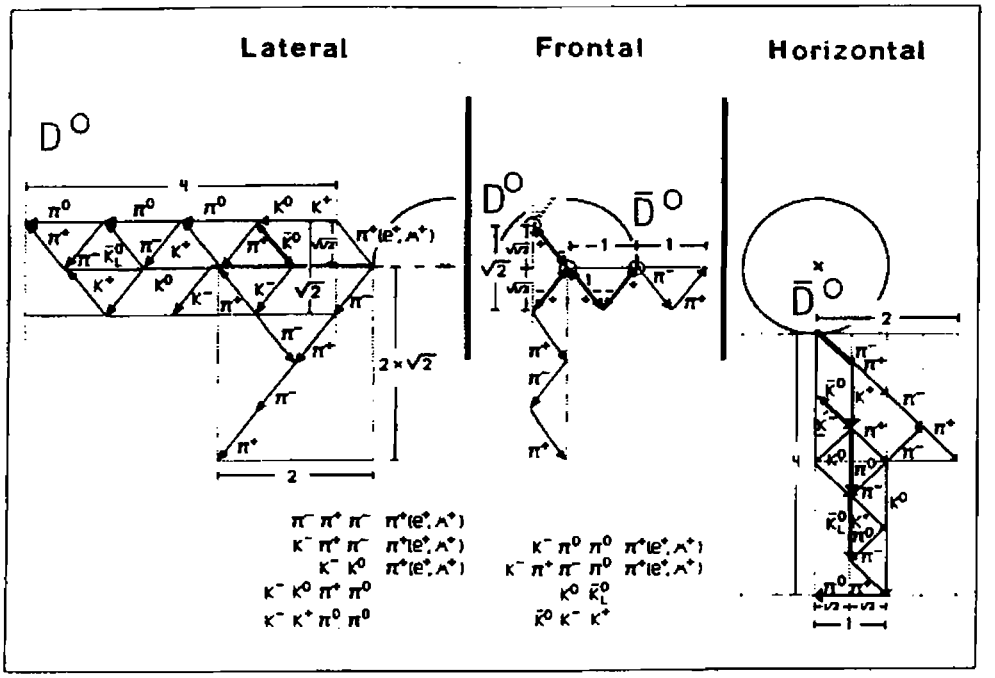



**FIGURE 14.** The identity between reality and animation exemplified by the *charmed* $D^0$ and its $\overline{D^0}$ antiparticle equally many levels out in the lattice as in practice.[13] All channels can be retrieved (only a sample shown here), spanning SO(2) planes along the vertical axis tentatively assigned to $D^0$, and the horizontal axis assigned to its antiparticle. The area in the first case is $\pi \times 4/2 \times 2^{1/2}$ or $\pi \times 2/2 \times 2(2^{1/2})/2$; both $2^{1/2}$ larger than Proton equatorial plane. With distance to next $D^0 = 1/2^{1/2}$, the mass number is $2^{1/2} \times 1/(1/2^{1/2}) \times 0{,}93827$ GeV $\approx 1.88$ vs the recorded $\approx 1.86$ GeV. In the antiparticle the spheroidal area equals the Proton and with interdistance ½ gives the same result.

## III. The Leptons

The mesons have been faithfully replicated here as spinning residual volumes comprising the additive partial differentials of the involved transformations between all hadron states, very much like bubbles bursting to smaller bubbles before they end up in a spray of linear jets. The latter are the (except the muon) further irreducible, stable structural and differential beams of the real elementary particle spectroscopy as well as of the transition lattice where they appear in exhaustive parallel array, too: i.e., the leptons.

Paradoxically, despite their plain one-dimensionality and limited number of states, the leptons stand forth as the perhaps most elusive of the elementary particles. Their antisymmetric Lie algebra is U(1), whose geometric isomorphism is the ordinary real line, the composed length of which may accordingly vary. However, already in the existing wave model it is at the *limes* level put together by infinitesimal derivatives which are straight unit bits meaning that, innermost, the lepton scalar world function emerges as digital. So is likewise the case in the regular solid lattice. The infinitesimal straight line digit, or 'pixel' is immediately embodied in the uniform, sole ingredient unit root vector element of either neutral or charged inclination, whose iteration is everything that constitutes the lattice and the hence eigen-spacefilling geodesics there. That close matches with the leptons are indeed manifest in it is therefore not surprising in regard of the regular solids' (slightly oxymoronic) 'unique universality', but nonetheless truly remarkable. In fact, the leptons weave the extra-nucleon world, and with such extreme simplicity that is has been overlooked for that very reason.

Starting with the particulate leptons, there are two principal ways of connecting the needle-like sharp charged root vectors of same sign, here exemplified by the positive muon and the positron, viz., in the first case, by 90-180° turns (Fig. 15 a-c), and, in the second case, 60-120° turns (Fig 15 d,e).

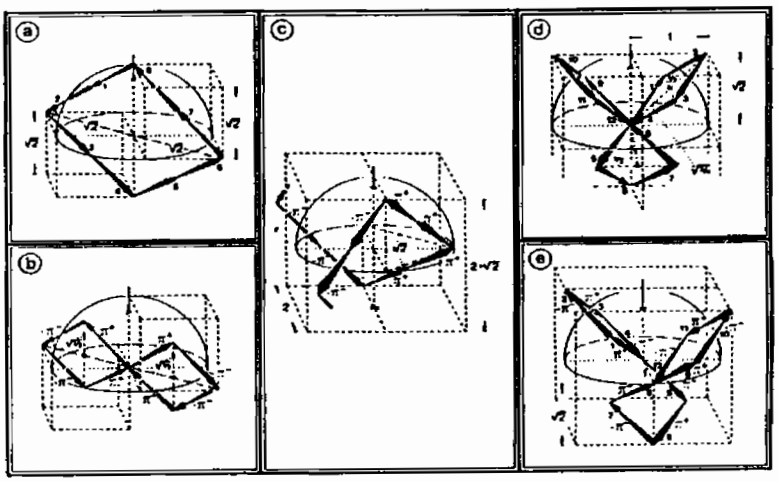


**FIGURE 15.** Cores of (in this case positively charged) lepton geodesics over nucleon surface

The first alternative forms plane or helical orbits from, over and outside of the nucleon surface with a unit scale length in all varieties of $(2\pi \times 2^{1/2})$ or $(2\pi \times 2\times 1/2^{1/2})$ and resulting mass number $1/(2\pi \times 2^{1/2}) \times 938.27 = 1/(2\pi \times 2\times 1/2^{1/2}) \times 938.27 = 105.59$ MeV in comparison with the measured muon$^{\pm}$ mass of 105.66 MeV. In the second alternative, a three-winged orbit can be tied together (Fig 15b) and leads out of the nucleon surface, so that it is natural to associate it with the positron/electron trajectory. The circular orbital length of the ground rosette is easy to calculate as $3 \times (2\pi \times 1/2^{1/2})$ in unit gauge, but it is well known that one has to multiply with the fine structure constant, 137.035986.., to obtain the first, in this case 'Mercedes star' three-pronged circumference, so that the ground state positron/(mirror)electron mass number comes out as $1/(137.035986 \times 3 \times 2\pi \times 1/2^{1/2}) \times 938.27 =$ 0,514 MeV in comparison with the recorded 0,511 MeV.

However, there are problems with the orbital model, for instance, in terms of the then alien, empty region under and between its rings. For consistency, a truly spacefilling distribution is wanted. Being a sequence of unit steps there would be no difference in principle in relation to the orbital model, which, as mentioned, is also composed of iterated infinitesimal straight line intervals. And there exists such possibilities which can be patched together to larger structures in a hierarchically periodic fashion just as in modern nanotechnological self-assembly.[4,5] One of

them is the truncated octahedron which is a composite space-filling Archimedean solid that already Kepler saw as fundamentally engaged in the cosmographical architecture.[4,5,30] The truncated octahedron distribution of a full positron/electron turn may follow from the only space-filling sequence of the charged root vector lattice, namely (Fig.s 5d,6), a twelve-step, two-tetrahedrons/one octahedron triple coil node, or 'rosette', generated by the distributed local quantum fluctuation of the six free corners of the unit cube conjugating their twelve sides into the coherence of the spherical root vector lattice, as shown below in one variety of a twelve-step loop returning to the origin as a veritable casting-on stitch of the web (Fig. 16).

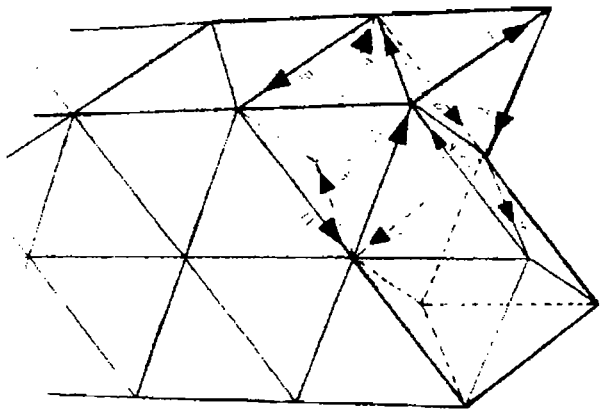


**FIGURE 16.** The figure shows a continuous outlining of the spacefilling one octahedron/ two-tetrahedrons root vector lattice coming back to the origin and thus a closed loop. Only the charged root vectors are involved.

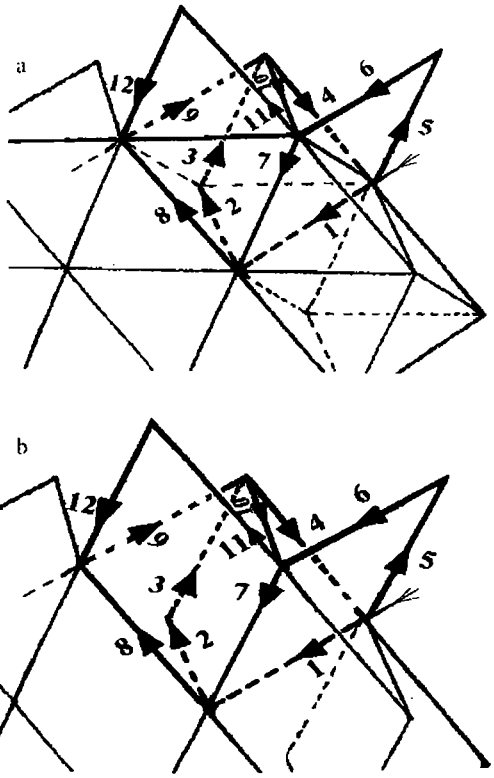



**FIGURE 17.** Another, open loop, allowing a spacefilling continuous, e.g. helical path by the charged root vectors alone. Each (quarter) turn consists of six two-side corners which are present also in the ground cube.

Fig 17 demonstrates another alternative where the tetrahedral rosette wings turn 90° around the corner enabling a variety of continuous patterns and also (Fig. 17b) that it consists only of (equally) charged root vector steps.

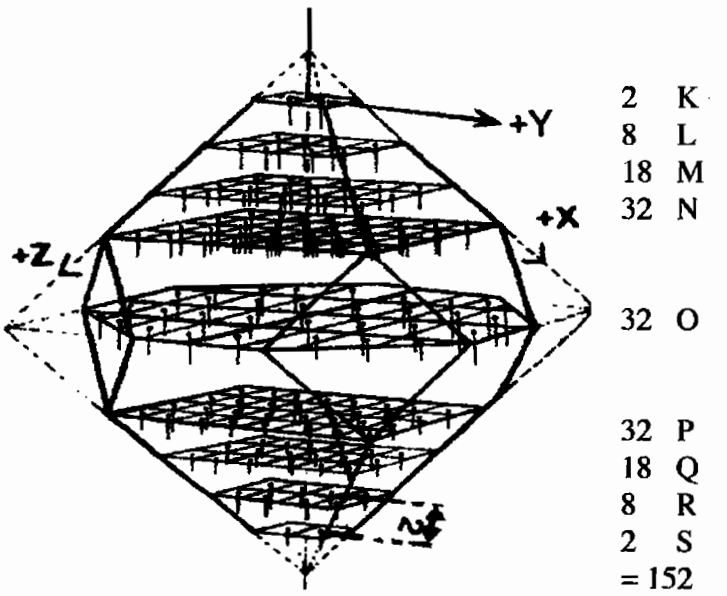



**FIGURE 18.** Truncated octahedron distribution of 152 electron twelve-step nodes (their triple coils just indicated as rods) in two vertically joined Cartesian segments. The corresponding Bohr orbital shells are shown to the left

Fig. 18 sketches how 152 such 12-step Electron rosettes in Bohr orbital layering may fill, by in all 1852 charged root vector steps (half) a truncated octahedron, so that the inertia/mass is 1/1852 x 938.27 = 0.514 MeV in comparison with measured 0,511 MeV. Before coming back to this projected primary electron cloud of the Hydrogen ion and its onward atomic and Periodic Table expansions, the remaining leptons; the photon and the muon and electron/positron neutrinos and antineutrinos (tau is not included here) will be briefly considered. They occur as one-dimensional differentials, e.g., when a larger differential slice such as the neutral pion decays like an imploding bubble into two γ.s (Fig. 19).

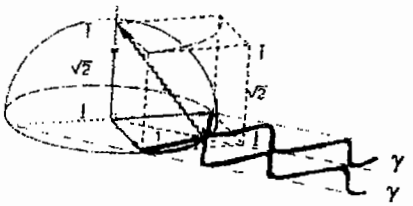



**FIGURE 19.** Example, in the rotating top differential neutral pion, how photons are generated when root vectors in e.g. particle decay or Brehmsstrahlung bendings within same charge plane snap back to their space axes setting up a zig-zag ripple between them of infinite length, thus zero mass and amplitude/frequency also determinable

The charged pion regularly decays into a muon and a muon neutrino and these channels also appear in the root vector lattice (Fig 20). The neutrino is a straight momentum vector of infinite length and conveying no charge which is carried on by the muon (compare fig 15 a,c). The inertia of the endless antisymmetric O(1) neutrino trajectory will be 938.27/∞ = 0.

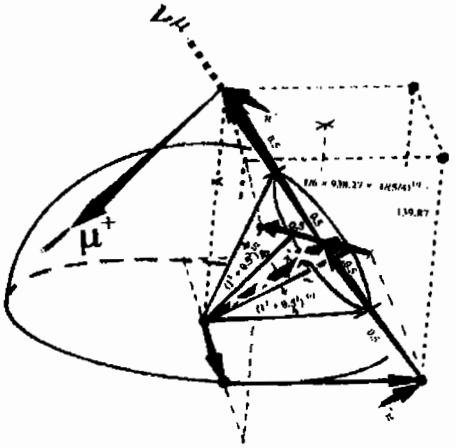



**FIGURE 20.** As indicated in the axis and demonstrated in the neighboring site, the charged pion decays by a muon (compare Fig. 15a) and an ongoing $\nu^{\mu}$ momentum vector.

And this extensive correspondence of nothing more, nothing less and all the same persists in the metastable muon, which outside of the nucleon is destined to bend its 90-180° surface track into the extranuclear lattice course of the electron/positron thus setting up a muon neutrino and electron antineutrino (Fig. 21).

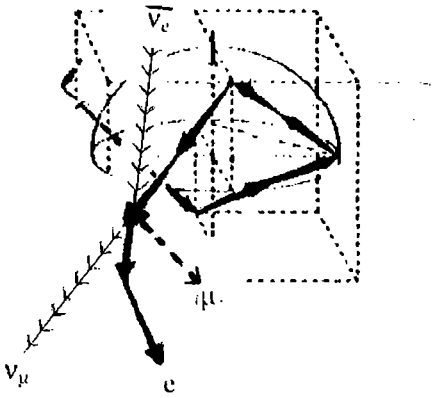



**FIGURE 21.** Example (compare Fig 15 c) how a transition of, in this case, the muon into the electron geodesics leaves straight forward Bra/backward Ket momentum vectors corresponding to muon neutrino and electron antineutrino, respectively, both endlessly extending over successive space lattice intervals and hence of $1/\infty$ = zero mass and nil amplitude

Table 3 summarizes the results in the leptons accounted for here.

Table 3. Basic lepton calculated and observed mass numbers (MeV)

| | | | |
|---|---|---|---|
| $\mu^{\pm}$ | $1/(2\pi \times 2^{1/2}) \times 938.27$ or $1/(2\pi \times 2\times 1/2^{1/2}) \times 938.27$ | 105.59 | 105.66 |
| $\varepsilon^{\pm}_{orbital}$ | $1/(137.035986 \times 6\pi \times 1/2^{1/2}) \times 938.27$ | 0.514 | 0.511 |
| $\varepsilon^{\pm}_{solid}$ | $1/(152 \times 12) \times 938.27$ | 0.514 | 0.511 |
| $\gamma$ | $1/\infty \times 938.27$ | 0 | 0 ($<3\times10^{-33}$) |
| $\nu_{\mu,\varepsilon....}$ | $1/\infty \times 938.27$ | 0 | 0 (<17 - 35) |

## IV. Atomic Expansion

As an experienced and rational Natural History researcher in a closely related field of descriptive Science dealing with morphology and structural composition and function (see *Pubmed* for credentials), I feel quite safe when stating that what has been presented so far establishes beyond doubt that it is possible to disclose a classical spaceframe structure for the double stringed elementary particle spectroscopy, in which the nucleon holds pivotal position, the mesons are the pylon sections and the leptons their beam and suspension elements. The warp, the string, the knots, the pattern; all comply, but how to weave the tapestry: the Atom that is ten thousand times larger? It must be by filling it by the same stuff because spacefilling goes with the provision of three dimensions alone where the self-referential filament is the sole thread available and allowed. This holds also when there are more dimensions because our distinct Universe still consists only of itself and then also consummates itself and as the three spatial dimensions in it are linearly independent the situation prevails. Since the spaceframe grid is infinitesimal at the elementary particle threshold, there are no loopholes, and the expansion must go on by it, so the atom can only be a periodical enlargement of its arrangement in order to accommodate in the global coherence.

With the leptons all elementary particles are reproduced by an instantaneous principal phase transition where the electron cloud in one variety pursued here comes out as a spacefilling mesh segment of defined, second-order regular solid form. This truncated octahedron module can be seen as a diagonal cube (Fig. 22) possible to tessellate into different shapes which, in turn, may self-template into cyclically larger portions of same or modified form to go on

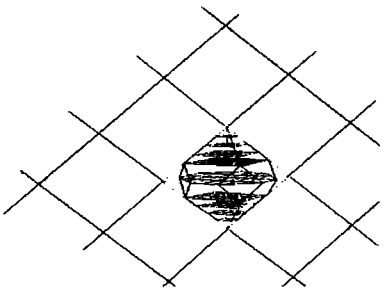


**FIGURE 22.** The electron module is surrounded by other modules in the second-generation global lattice, and thus doubly bound to its segmental shape

filling space in a three-dimensional Tetris way, and, at any such stage, to combine with each other in various full-packing conformations. Fig. 23 shows that the diagonal arrangement allows accommodation of separately counter-spinning modules with corresponding implications for the formation of larger regions in a cyclical mushrooming way.

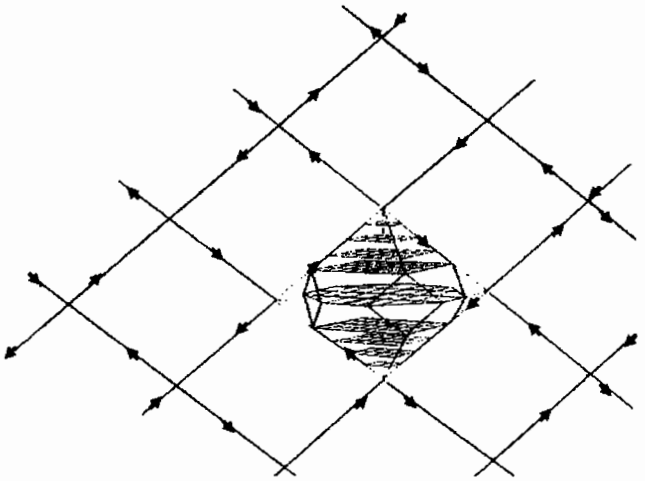


**FIGURE 23.** The electron module can be accommodated with other modules and also with continuous expansions of itself

There is nothing different from the orbital model in that regard, under one crucial provision: that the continuous transition lattice can also be continuously delineated. Fig. 24 shows that this is indeed the case under a Fermion half-spin rotation around the forward diagonal axis bringing the end of the line one charged root vector step and

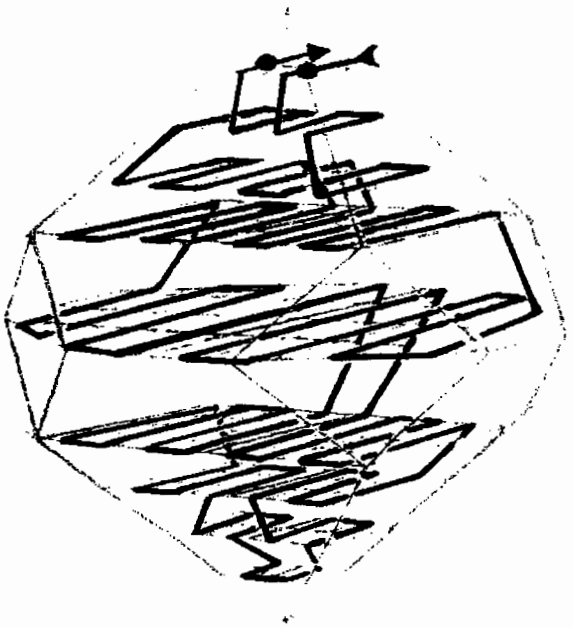


**FIGURE 24.** One possible, net Fermion continuous sequence of singlet rosettes in Hydrogen electron module connecting with likewise Fermion proton root vector at the origin forming the net Boson Hydrogen atom. The module tiers correspond to the Bohr orbital shell and hold same numbers of rosettes as the electrons there; also in the P to S levels where higher amounts have not been seen in reality.

one or two (or at lowest quantum, Bose-Einstein Condensate state zero) neutral space axis steps from the origin which, not taking part in the electron formation, appears as the reciprocal pivot, each point of which is 1852 times longer lasting than the electron with proportionately higher inertia and consequential mass number: $1852 \times 0.514 = 938.27$ MeV. The advantage is that the distribution solid can be used as structural bricks, and this double cast of the electrons as "wave functions or transition matrix elements" is in line with recent Hydrogen ground state research[45] and the instant material "modular building block"[46] nature of the electron is pending in modern nanotechnology, molecular biology etc.

Fig. 25a illustrates the complex of one electron module linking with the proton in an upper Cartesian segment and so matching the Hydrogen atom. At this stage the deep relation to hadronic mechanics and its reduction of matter to, and from, the proton and electron becomes apparent, because the complex persists in the successive levels of penetration, from the union in the nucleon domain via the muon trajectory and decay channels over its surface to the continuously spacefilling module expansions outside, whose first cycle is depicted in the figure.

Fig. 25b shows that the opposite end of the proton-electron complex is free to bind with another open-ended ion, here a second H into the $H_2$ molecule (Fig. 25 b). It is a variety of "nested polyhedra...which can in turn be put together in spatial arrangements", e.g. "helicoidal progression"[47]; in the present case creating the Bohr orbit signature of the singlet nodes in the forward plane. And when instead under strong pressure two Hydrogen ions

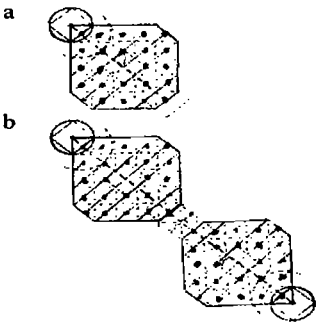



**FIGURE 25.** a) Horizontal plane projection of single extranucleon module with open end and so realizing H. b) When two H ions are linked end-to-end (or side) the $H_2$ molecule is formed

will fuse so that one is pushed a step upwards, still rooting with the upper proton pole in the nucleon and the other with the under and thereby also the in-between neutrons' space axis points are involved, a two-module truncated octahedron honeycomb is generated (Fig. 26), closing the ground (K) sheet of lattice intersections and therefore very stable so as to faithfully realize the Helium atom.

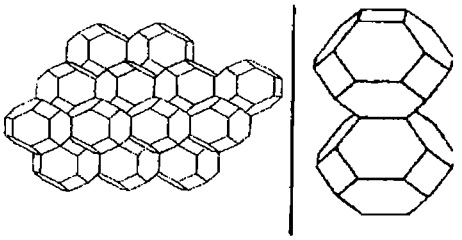


**FIGURE 26.** Honeycombs of truncated octahedrons and of the Helium atom

In that way the singlet sites can be dragged in under an expanding central boundary as nucleon centres of consecutively larger honeycombs, which thereby are templated in steps and constellations of the periodical system and onwards to further self-similar spacefilling, for instance, of crystalline lattices, deposits, rocks, planets etc. Exemplifying the mechanism only in the first three atoms from the next (L) sheet, the Lithium honeycomb is variably triangular with one free end for molecular coupling whereas the square or rhombic Beryllium can combine with two atoms/ions/complexes and Boron with three (Fig. 27).

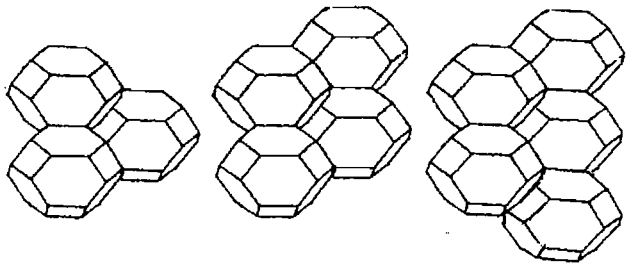


**FIGURE 27.** The Lithium (one Module in L sheet), Beryllium (2 L modules) and Boron (3 modules) Electron Honeycombs.

Not illustrated, Carbon can permutatively couple/chain with four including itself, whereas Nitrogen holds five of the L positions and so has three to offer; Oxygen then two, and Fluorine very strongly one; and when the L shell is filled a new saturated and hence stable atom, Neon, is established.

And so it continues and the correspondences are so extensive, and also non-trivial, that there can be little doubt that it is along these principles that matter will be ultimately reconstructed in forthcoming nanotechnology. For instance, when one considers the situation meeting individual Hydrogen ion electron modules in the interior of large stars, it is easy to imagine how the enormous forces there squeeze out their coils to crisscross meandering half-turn cords twisting around each other so that when the physical conditions get less intense they cycle by cycle regain their shape, now caught in the atomic bouquet corresponding to the number of modules wrung together. Exactly where an observation instrument head-on intercepts such a cross-section is a matter of quantum indeterminacy and probability statistics. And so far the reproductions here only comprise the first extranucleon level, or quantum state of the elements. This is the situation prevailing in the Bose-Einstein condensation but according to the aforesaid also at the other extreme of the temperature scale[48] where the Hydrogen electron modules merge with each other into the fusion cascade.

Therefore, it is group-theoretically relevant and interesting that as an inverse at the other, zero Kelvin end of the temperature range, the charged root vector suit runs back into empty space, that is, into the rectilinear lattice moiety by the same route it appeared. This can only be by a honeycomb singlet, which can come so close whereas larger constellations are too distanced. But also in the singlets there is a difference, first exhibited by the Fermion Hydrogen ion which comes back ½ step along the projection planes away from the origin so that retrograde access to the Euclidean space is blocked as it were, whereas the larger Boson Helium will come at +1 and so hitting the entrance; which Hydrogen can do by pairing.[48] Similarly, the Fermion Lithium settles at 1½ + 1½ = 3, while the Boson Beryllium arrives at 2 at once...and so on in the same pattern as observed in experiments throughout the periodic system.[41] Apart from supporting the faithfulness and pertinence of the regular solid scheme it also illustrates that perfect precision, which after all characterises clarified physical reality[4,40], can only run through all magnitudes of this if generated already at the outset by the not only perfect but categorical precision of an absolutely unique and universal quantum phase transition; the only that exists per se by itself and thus guaranteeing its perpetuation.

The Bose-Einstein condensate stage is also interesting because of its intermediary state at the entry of the channel between interior and exterior so that the experimental observations are paralleled and even aided by the hadronic mechanical and present faithful structural charts of the route of partial reversibility at the threshold.

## V. Stacking the Atom Hives

When proceeding from the singlet honeycombs to the atoms there is a cyclical expansion of the respective basic motif which can be described as a stacking of exponentially larger boxes in a Tetris-like manner, each generation templated by the previous along adapted route that worked in the preceding ones so that the single electron geodesics remains unbroken. The number of generations is then, as cause and effect, temperature-dependent. From just one at both the hot and cold extremes, it increases towards the logarithmic mean which for many reasons would be around where water flows and life is formed, and where, as also in other quantum levels, the equal Avogadro pressure of equally many atoms (in gaseous state) reflects the different number of root vector steps in their completion. Fig. 28 shows the principle in a horizontal plane projection towards three cycles in the Hydrogen atom stacking, and it is seen that it does not take many cycles before the cross-section is zoomed up ten-thousand-fold as is also the ratio between the actual elementary particles and atoms.

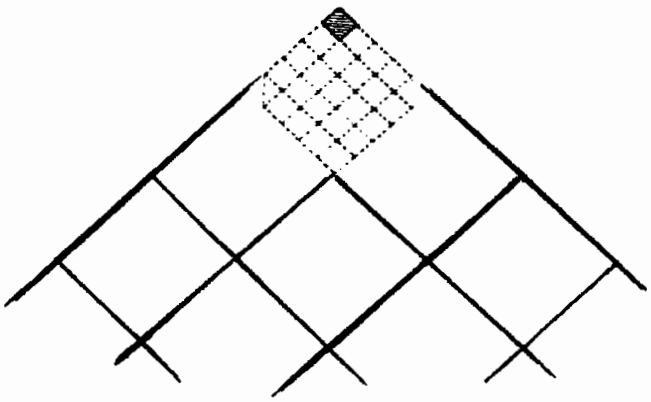


**FIGURE 28.** Schematic equatorial plane projection of first three self-similar cycles of electron module in the Hydrogen atom as well as of the vertically doubled (Fig. 26) Helium atom

Interestingly, then, the atom region is outlined by the Electron cloud which is spacefilling and in that sense complying with the quantum mechanical fact that there is a probability of finding the single electron anywhere in the cloud, and that it is the instrumentation interaction that determines the recorded features of it; notably its size. As commonly regarded it is "a pointlike particle – that is, a particle with no measurable dimension...However, a rather compelling case can be made for an opposing viewpoint: namely, that the Electron is in fact a large particle which contains an embedded point-like charge".[49] This complies well with the findings here that at the bottomline stage all elementary particles are of the same size order since spanned by the same digital bit. Its length, as stated in hadronic mechanics, too[2], is that of the Proton radius ($\approx 0.8768$ fm).

Here, the transition matrix of the Electron cloud has been approximated with the truncated octahedron, but other continuous polygonal realizations are possible, too, and perhaps more likely to more clearly account for the nucleon. That also the large-scale arrangement of matter in the flat Universe follows a polyhedral plan is supported even on the galactic level[50,51] , and seems to consist of the same mix of octahedrons and tetrahedrons that appear already on the infinitesimal plane in the present scheme (Fig.s 5d, 16, 17).

Single as well as fused in honeycomb and molecular aggregates, the modules heap up the joint structural architecture as veritable Lego pieces, patching together already at the infinitesimal level every three-dimensional real shape from their consecutive own and composed combinations. This does not mean that they are some static wire bundles, but the second-generation, $2^3)^3$ periodical partition of the continuous space-filling charged root vector lattice (Fig.s 5,6) into the first self-similar extra-nuclear segment of the global transition matrix. Its outline may be distended in, for instance, accelerations, but then the surrounding modules, whether occupied or empty at the moment, will, too, and the apportioned volume share remains preserved.

One possible sequential ordering of the electron singlet subunits (Fig. 24) runs through the (here) upper Cartesian segment from its origin and returns in the one below, and so gradually shifts the proton one unit step down and changes the module progression to the opposite direction so as to describe a virtual cross-section rotation with Bohr orbital signature. And when in larger atoms their respective nuclear hub extends over a larger domain of singlets, which in turn magnify their (sometimes isotopically varying) constellation to the honeycomb they co-ordinate, the interstitial charged and neutral root vector content in them will manifest as the corresponding atomic number of protons and neutrons.

For instance, since the electron geodesic is wrapped throughout the entire atom it matches the "quantum superposition...qualitative picture of all possible electron paths conspiring together"[52] with correspondingly low probability of hitting it in a particular infinitesimal interaction cone. And the propagation of the atoms themselves when they occupy their consecutively inflated domains would be determined by their template form so that highly symmetrical shapes, like the noble gases, would proceed in one-dimensional curves and accordingly be gaseous while sharply bent honeycomb modules, like Lithium, regardless of its low weight would go into dense, net two- or three-dimensional convolutions so as to be solid (until heated/excited so that it starts to boil into orbit). And since the offset 'caps' that the honeycombs' collective truncation leaves at the top contain the abandoned central isospin vectors there will be a reciprocal nucleus, always with as many charged, proton roots as the atomic number, while the Neutrons can be more numerous reflecting the lateral displacements possible under acceleration[53], e.g. in $^{11}$Li.

## DISCUSSION

Obviously, these rough sketches as well as all other atoms leave a lot of further, but rewarding, work to be done. Nonetheless it can be discerned that they represent a general procedure from which the full inorganic realm and its likewise regularly polygonal macroscopic minerals and crystals can be reconstructed with all the attributes of the periodic system. It has been a very condensed survey, focussed on the reproducible descriptive results. It stops at the atom stage, but can be extended over molecules and larger compounds to the cosmological scale.[50,51] Like many other current models, it is a lattice system, however, almost embarrassingly simple in comparison. Therefore, the verbal report tends to assume a slightly surrealistic ring so that is has been aimed at an illustrative account hopefully catching some of the ideas behind.

Yet, it has been said that there is a crisis in today's physics[54] so that the merit lies in the concrete, up-to-date information and nanotechnology outcome as well as in the eminent 'back to the future' legacy: such as the eightfold way, the Lie groups and algebras, the Lie-admissible hadronic mechanics, the Diophantine equations and, first and not least, the ancient regular solids, of which in the inorganic field here dealt with only the static cube of the

geocentrically inferred space matrix and the tetrahedron and the octahedron of the dynamic brew of fire and air[5], respectively, are employed. They can all be formed by successive straight line steps from the infinitesimal stage; rendering them genuine solids, and returning as well to the solid mathematical basis of Santilli's hadronic mechanics and its profound synthesis in consistent linear equations of matter from protons and electrons alone - meaning also that at the physical ground nothing else exists or even needs to exist - meaning, finally, that universe is filled entirely by its own substance with its interior hyperonic and pion transformations and differentials, via surface muonic and mesonic to increasing sequential levels rising over intermediary Bose-Einstein condensate stages to the collective dynamics of there unchallenged quantum mechanics.

This central generation and maintenance of all matter from the interior imposes a law-bound isotrophic distribution at the origin, so that when the 'trigger'[2] elicits, the immediately spread transition is evenly distributed through space at once satisfying itself by it: and we have the riddle of the instantaneous inflation throughout the whole Cosmos solved by parts. Since Lie-admissibility is inherent, indeed a *conditio sine qua non* in hadronic mechanics, an ordinary Lie algebra geometrical representation is an open consequence as realised here by structural linear geodesics in the nilpotential fall between the two principal phases of infinity.

The straight line is a direct structure and digital bit, but also the irreducible vector element of pure existence, of anything at all. Another, more anthropic argument for Straight is that we and our perceptions are directly parts of and resonating with actual reality all from the quantum level. In other words, we should pay much attention to testimonies like the following (cited from Tate Modern): "Piet Mondrian (1872-1944) believed that all complex forms could be reduced to a 'plurality of straight lines in rectangular opposition'...his paintings...also represent a physiological reality about the brain...the cells of the visual brain are responsive to straight lines of specific orientation and the field of view to which they respond is rectangular in shape".

And this applies to our binary branching thought processes as well, i.e. intelligence and logic[33,34] where the straight line bit and its Platonic concatenations and expansions constitute a faithful morphogenetic ground modality of NUCRS[35-39] and likewise are engaged in the three-dimensional orthogonal twist "processes of Encryption/Decryption" utilized in "Quantum Holography, defined by means of the Heisenberg nilpotent Lie Group" and "applied at Bletchley Park in World War Two using various machines including the Turing Bombes and Colossus"[55] as well as more recently in magnetic resonance imaging.[56]

However, the strongest argument is the reproducible outcome. It is obtained by genuine first principles and in many instances comprises a first itself. And the results are what counts and persists; Some day, some model will prevail, and the simpler and more akin to the world at large the better and more plausible and workable. At the elementary particle/atomic stage the direct structural embodiments here cover the inorganic realm with unprecedented resolution and completeness, including - wherever properly looked for - entanglement as well due to the overall coherence of the literally bottom-line lattice, whose systematic linkage of a ground irreducible linear bit is seen to be capable of self-aggregating the exhaustive type-set for printing out, as it were, the entire primer of real matter. But the second edition, the textbook of organic Nature, requires a larger alphabet built upon and from molecular combinations of this atomic substrate in a predictable evolutionary permutation way so as to preserve nanotechnological precision and intertwining organisation in order to assemble and accomodate, for instance, "*Salmonella's* needle complex" in subnanometer sharpness[57] and the fragile "beauty in complexity" of intricate "molecular motors".[58] It is fascinating to consider that in their multiplied scale such protein components still recruit and utilise the regular solids, now also including the making and makings of pentagonal and mixed symmetry dodecahedrons and icosahedrons, to grandiose constructions of minute detail, doubtlessly enabling in the not-too-distant future total charts of equal extensiveness and exactitude as a racing car's or supertanker's explosion diagram. Fundamental morphological work in this direction employing concrete structural modelling is under way above all by Hill and Rowlands.[38,59]

In conclusion, the present paper is but a brief summary in need of further clarification. Especially, this applies to the question whether the truncated octahedron and the honeycombs it can put together really are the building elements of the continuous atom delineation. The electron singlet rosette (Fig.s 16,17) is an octahedron/tetrahedron complex and it seems more likely that if it isomorphically templates the ensuing period of its form it would be by iterating this composition in the half-turn twist because "if this is done in the same way as the faces...a second generation of polyhedra is formed, which takes the place of the polygonal faces of the first. This can be done repeatedly"[47] with multiplicative enlargement of the motif, that can be carried forth in a helical propagation (Ib.), conveying the Nucleon hub along its eccentrically dilating spiral whereas the honeycomb stacking doesn't have a centre of revolution. Accordingly, something like a sunflower offers a prototype, indeed Mandala model of atomic constitution which will be focussed on in the forthcoming, hopefully final, stage of the pursuit of the bottom line.

# REFERENCES


1. E. Trell, Book Review: Foundations of Hadronic Chemistry with Applications to New Clean Energies and Fuels, *International Journal of Hydrogen Energy*, **28**, 251-253 (2003).
2. I. Gandzha and J. Kadisvily, *New Sciences For a New Era. Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli*, Kathmandu: Sankata Printing Press, 2011.
3. S. Lloyd, Quantum Information Matters, *Science* **319**, 1209-11 (2008).
4. E. Trell, Elementary Particle Spectroscopy in Regular Solid Rewrite. *American Institute of Physics Conference Proceedings* **1051**, 127-41 (2008).
5. E. Trell, Back to the Ether In: (Ed.s) Duffy MC and Levy J. *Ether Spacetime & Cosmology, volume 3*, Montreal: Apeiron, 2009, pp.339-387.
6. M. Planck, Über das Gesetz der Energieverteilung im Normalspectrum, *Annalen der Physik* **4**, 553-563 (1901).
7. A.H. Guth and D.I. Kaiser, Inflatory Cosmology: Exploring the Universe from the Smallest to the Largest Scales. *Science* **307**, 884-890 (2005).
8. E. Trell, A calculation of the electron circular orbital radius, *Speculation in Science and Technology* **5**, 533-535 (1982).
9. E. Trell, Representation of particle masses in Hadronic SU(3) diagram, *Acta Physica Austriaca*, **55**, 97-110 (1983).
10. E. Trell, Scheme for a time antenna in three-dimensional Hausdorff space, *Speculations in Science and Technology* **7**, 269-277 (1984).
11. E. Trell, Geometrical Reproduction of (u,d,s) Baryon, Meson, and Lepton Transformation Symmetries, Masses, and Channels, *Hadronic Journal* **13**, 277-297 (1990).
12. E. Trell, On Rotational Symmetry and Real Geometrical Representations of the Elementary Particles With Special Reference to the N and Δ Series, *Physics Essay* **4**, 272-283 (1991).
13. E. Trell, Real Forms of the Elementary Particles with a Report of the Σ Resonances, *Physics Essays* **5**, 362-373 (1992).
14. E. Trell, An alternative solution to Fermat's Last Theorem: Infinite ascent in isotopic geometry, *Hadronic Journal Supplement* **12**, 217-240 (1997).
15. E. Trell, Isotopic proof and reproof of Fermat's Last Theorem verifying Beal's Conjecture, *Algebras Groups and Geometries* **15**, 299-318 (1998).
16. E. Trell and R.M. Santilli, Marius Sophus Lie's Doctoral Thesis Over en Classe Geometriske Transformationer, *Algebras Groups and Geometries* **15**, 395-445 (1998).
17. E. Trell, The Eightfold Eightfold Way: Application of Lie's True Geometriske Transformationer to Elementary Particles, *Algebras Groups and Geometries* **15**, 447-471 (1998).
18. E. Trell, Real Charm of Form - Real Form of Charm. Duality in Transition. In: *(Gill, T., Liu, K., Trell, E., Ed.s), Fundamental Open Problems in Science at the End of the Millenium*, Palm Springs: Hadronic Press,1999, pp 1-2
19. E. Trell, The Eightfold Eightfold Way. A Lateral View on the Standard Model. *Physical Interpretations of Relativity Theory (11-14 September 1998), Late Papers, London: British Society for the Philosophy of Science*, 2000, pp. 263-284.
20. E. Trell, String and Loop Quantum Gravity Theories Unified in Platonic Ether. With Proof of Fermat's Last Theorem and Beal's Conjecture. In: (Ed.s) Duffy MC, Gladyshev VO, Morozov AN. *Proceedings of International Scientific Meeting PIRT -2003*, Moscow 30 June – 03 July, 2003, Bauman State University, Moscow, Liverpool, Sunderland, 2003, pp 134-149.
21. E. Trell, Original Diophantine equations lodge BC without ABC, *International Symposium on Recent Advances in Mathematics and its Applications 2003, Proceedings, Calcutta: Calcutta Mathematical Society*, 2003.
22. E. Trell, Original Diophantine equations lodge BC without ABC, *Rev. Bull. Cal. Math.Soc.* **12**, 29-54 (2004).
23. E. Trell, Cubit Isounits 'Tread a Daunting Path to Reality' While Proving Fermat's Last Theorem and Beal's Conjecture, *Hadronic Journal* **26**, 237-271 (2004).
24. E. Trell, Tessellation of Diophantine Equation Block Universe. *Physical Interpretations of Relativity Theory VIII (6-9 September 2002), Proceedings, London: British Society for the Philosophy of Science*, pp. 585-601.
25. E. Trell, Classical 3-d. Geometrical 'Vortex Sponge' World-Ether Provides Natural Quantum Cavity Elementary Particle Standing Wave Incubation and Original Diophantine Equation Encapsulation. *Physical Interpretations of Relativity Theory IX (3-6/9 2004), Proceedings, London: British Society for the Philosophy of Science*, 2004, pp. 503-30.
26. E. Trell, Temporospatial transition – Back to go. *Physical Interpretations of Relativity Theory (15-18 September 2000), Late Papers, London: British Society for the Philosophy of Science*, 2004, pp. 305-311.
27. E. Trell, Invariant Aristotelian Cosmology: Binary Phase Transition of the Universe from the smallest to the largest scales, *Hadronic Journal* **28**, 1-42 (2005).
28. E. Trell, An excursion in curvature I. Diophantine equations get real again in re-established flat Euclidean space, *Bull. Cal. Math. Soc.* **97**, 551-572 (2005).
29. E. Trell, An excursion in and betweem curvature II. From classical Lie algebra neighbourhood to QED and QCD of real elementary particles, *Bull. Cal Math. Soc.* **97**, 573-588 (2005).
30. E. Trell, Regular Solid Universal Morphogenesis. *The joint NTNU/BCSCMsG Symposium 'Science and Philosophy Engaged'*, Sage Skysstasjon Trollheimer conference center, 2006.

31. E. Trell, Filling a Gap in Nilpotent Vacuum: How Close Are We to a Fundamental Reality?', *Physical Interpretations of Relativity Theory X (8-11 September 2006), Proceedings, London: British Society for the Philosophy of Science*, pp. 503-30.
32. E. Trell, Classical Information Matters – and Applies, *Applied Mathematics & Information Sciences*, in press (2011).
33. S. Johansen, Non-trivial Time Flows in Anticipation and Action Revealed by Recent Advances in Natural Science, Framed in the Causality Network of Differential Ontology, *International Journal of Anticipatory Systems* **22**, 141-158 (2008).
34. S. Johansen, *Grunnriss av en Differensiell Epistemologi*. Bergen: Ariadne, 2009.
35. P. Rowlands, From Zero to the Dirac Equation. In: (Ed.s) Duffy MC, Gladyshev VO, Morozov AN. *Proceedings of International Scientific Meeting PIRT 2003*, Moscow 30/6–3/7 2003, Bauman State University, Moscow, Liverpool, Sunderland, pp.13-34.
36. P. Rowlands, P. and B. Diaz, A Computational Path to the Nilpotent Dirac Equation, *Symposium 10, International Conference for Computing Anticipatory Systems*, HEC Liege, Belgium, August 11-16, 2003, *International Journal of Computing Anticipatory Systems*, editor Daniel Dubois, 203-218. Also at arXiv.cs.OH/0209026.
37. P. Rowlands, Nilpotent theory & vacuum. *Physical Interpretations of Relativity Theory X*, 8-11 September 2006, Imperial College, London.
38. P. Rowlands, *Zero to Infinity: The Foundation of Physics*. New Jersey, London, Singapore, Beijing, Shanghai, Hong Kong, Taipei, Chennai: World Scientific, 2008.
39. B.M. Diaz, P. Marcer, P. Rowlands, Riemann Hypothesis, *International Journal of Anticipatory Systems* **22**, 131-140 (2008).
40. M. Schmidt and H. Lipson, Distilling Free-Form Natural Laws From Experimental Data, *Science* **324**, 81-85 (2009).
41. A. Cho, Constructing Spacetime - No Strings Attached, *Science* **298**, 1166-1167 (2002).
42. M. Kamionkowski, A Hawking-eye View of the Universe, *Science* **296**, 267 (2002).
43. M.S. Lie, Over en Classe Geometriske Transformationer, Ph.D. Thesis, Kristiania (now Oslo) University, 1871.
44. R.L. Jaffe, Quark Confinement, *Nature* **268**, 201-208 (1997).
45. F. Martin, J. Fernández, T. Havermeier, L. Foucar et al., Single Photon-Induced Symmetry Breaking of $H_2$ Dissociation, *Science* **315**, 629-633 (2007).
46. O.L. De Weck, W.D. Nadir, J.G. Wong, G. Bounova, T.M. Coffee, Modular Structures for Manned Space Exploration: The Truncated Octahedron as a Building Block, *AIAA* **2005-2764**, 1-26 (2005).
47. P. Huybers, Nested Polyhedra, *IASS Newsletter* **14**, 31-40 (2007).
48. A. Cho, Insights Flow From Ultracold Atoms That Mimic Superconductors, *Science* **319**, 1180-1181 (2008).
49. M.H. Mac Gregor, *The enigmantic Electron*, Dordrecht: Kluwer, 1982.
50. C. Seife, Polyhedral Model Gives the Universe An Unexpected Twist, *Science* **302**, 209 (2003).
51. E. Battener and E. Florido, The egg-carton Universe, *arXiv:astro-ph/9802009v1*, 1-7 (1998).
52. J. Ambjørn, J. Jurkiewics, R. Loll, The self-organizing Quantum Universe, *Scientific American*, **July 2008**, 42-49 (2008).
53. B.M. Sherrill, Designer Atomic Nuclei, *Science* **320**, 751-752 (2008).
54. A. Cho, Does Fermilab Have a Future? *Science* **320**, 1148-1151 (2008).
55. P Marcer, A Mathematical Definition of Intelligence. Back to the Future: the Machines of Bletchley Park, *The joint NTNU/BCSCMsG Symposium 'Science and Philosophy Engaged*, Sage Skysstasjon Trollheimer conference center, 2006.
56. W. Schempp, *Magnetic Resonance Imaging: Mathematical Foundations and Applications*, New York: John Wiley and sons, 1997.
57. O. Schreidt, Th.C. Marlovits, Three-Dimensional model of Salmonella's Needle Complex at Subnanometer Resolution, *Science* **331**, 1192-1195 (2011).
58. J.A. Spudich, Molecular Motor. Beauty in Complexity, *Science* **331**, 1143-1144 (2011).
59. V.J. Hill, and P. Rowlands, Nature's Code, *American Institute of Physics Conference Proceedings* **1051**, 127-126 (2008).

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
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# THEOREME FONCTIONNEL DE BÁRTFAI

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**Abstract**

The main contribution of this paper is to present a functional form of the Theorem of Bártfai (1966).

**Key words and phrases :** Functional laws ; laws of large numbers ; Increments of partial sum processes ; Orlics-space ; common distribution, geyser problem.

# 1 Introduction

Soit $X$, $X_1$, ... une suite de variable aléatoire (v.a.) indépendante et de même lois (i.i.d.) et $F(x) = P(X \leq x)$ dénote la fonction de répartition de X. Soit $\phi(t) = E(e^{tX})$ la fonction génératrice des moments de X, $t_0 = \sup\{t, \phi(t) < \infty\}$, $t_1 = \inf\{t : \phi(t) < \infty\}$ les points limite de $\phi(.)$, $m(t) = \frac{\phi'(t)}{\phi(t)}$ et $A = \lim_{t \nearrow t_0} m(t)$. Désignons par $\psi(.)$ la fonction de Chernoff (see e.g. Chernoff (1952)) de X, define pour tout $-\infty < \alpha < \infty$ par

$$\psi(\alpha) = \sup_t(\alpha t - \log \phi(t)),$$

où le sup est évalué pour $t \in (t_1, t_0)$. Considérons les conditions suivantes :

**(A1)** $X$ est non dégénérée, i.e. $P(X = x) < 1$, for all $x$;

**(A2)** $t_0 > 0$;

**(A3)** $t_1 < 0$.

Sous (A2)-(A3) nous avons $E(X) = \mu \in (-\infty, \infty)$ et $Var(X) =: \sigma^2 \in [0, \infty)$. Sans perte de généralité, on suppose que $\mu = 0$. Sous (A2)-(A3) et on peut définir pour tout $c > 0$

$$\alpha_+(c) = \inf\{x \geq 0 : \psi(x) \geq \frac{1}{2}\}.$$

Pour tout entier $n \geq 1$, posons $S_n = X_1 + ... + X_n$, avec $S_0 = 0$, la somme partielle de $X_1$, $X_2$, ... et soit $S(t) = S_{[t]}$ pour $t \geq 0$, oú $[t]$ est la partie entière de $t$. Dans toute la suite définissons les v.a. : $r_t = r(t)$ pour $t > 0$ et considérons les fonctions suivantes : pour $t \geq 0$, $0 \leq s \leq 1$, $c > 0$ et $x \geq 0$

$$\xi(t) = S(t) + r(t),$$

et

$$\eta_{x,T;c}(s) = (a_T(c))^{-1}(S(x + sa_T(c)) - S(x));$$

où $a_T(c) = c \log T$. Soit

$$F_T(c) = \{\eta_{x,T;c} : 0 \leq x \leq T - a_T(c)\},$$

et posons

$$L_T(c) = cF_T(c) = \{cf : f \in F_T(c)\} \text{ for } T > 0.$$

Rényi (1962) posa une question, connue sous le nom "problème du geyser stochastique". Observons la suite infinie $\{\xi_t : t \geq 0\}$, où $\{r_t : t \geq 0\}$ est non nécessairement independente de $\{S_t : t \geq 0\}$, peut on déterminer p.s., la fonction de répartition F de X. Bártfai (1966) (see also e.g. § 2.4 in Csörgő and Révecsz (1981), Erdös-Rényi (1970) and Grill (1989)) a donné la solution de ce problème :
Theoréme A.
Soit $X, X_1, ...$ une suite de v.a i.i.d. telles que les conditions (A1)-(A3) sont satisfaites. Si $|r_t| = o(\log t)$ p.s., lorsque $t \to \infty$. Alors l'observation de $\xi(t)$ détrmine p.s. la distribution F de X.
Une nouvelle preuve de ce THM a été donné par Erdös-Rényi(1970). Suite à ces travaux, Grill (1989) propose une suite d'estimateurs de la distribution F de $X$. Le but de ce travail est de présenter la forme fonctionnelle du Theoreme de Bártfai (1966), qui est une généralisation des précédents resultats par une nouvelle méthode en probabilité et statistique.

## 2 Notation Préliminaire

Nous utilisons les même notations de Deheuvels (1991). Dénotons par $(E, \Gamma)$ un espace métrique et $BV(0,1)$ l'ensemble des fonction de répartitions continues à droite associées aux mesures de Lebesgue-Stieljes sur $\mathbb{R}$. Pour $f \in BV(0.1)$, soit

$$f_{\pm}(s) = \sup_{k \geq 2} \{\sum_{i=1}^{k} (f(\tau_i) - f(\tau_{i\ 1}))^{\pm} : \tau_0 = 0^- < \tau_1 < ... < \tau_k = s\},$$

pour $s > 0$, où $u^{\pm} = \max(\pm u, 0)$ et le sup est prise sur toutes les valeurs possibles de $k \geq 2$ et les choix de $0 < \tau_1 < ... < \tau_{k-1} < s$, $0^-$ d'enote une valeur arbitraire de $t < 0$ (pour lequel $f(t) = 0$). posons $f_{\pm}(s) = 0$ pour $s < 0$, et $f_{\pm}(0) = (f(0))^{\pm}$. et soit $|f|(s) = f_+(s) + f_-(s)$ pour $-\infty < s < \infty$ la variation totale de $f$ dans l'intervalle $[0, s]$. Introduisons la métrique

$$d_W(f, g) = \int_0^1 |f(u) - g(u)| du + |f(1) - g(1)|,$$

pour $f$ et $g \in BV(0,1)$.

# 3 Résultats

Dans toute la suite posons :

$$D_{\psi,c} = \{f \in BV(0,1) : f(0) = 0, J_{\psi,c}(f) \leq 1\},$$

si on suppose que $t_0 = \infty$ and $t_1 = -\infty$ alors

$$D_{\psi,c} = \{f \in BV(0.1) : f(0) = 0, \int_0^1 \psi(\frac{\dot{f}(t)}{c})dt \leq 1\}.$$

définissons $\Psi(.)$ par

$$\Psi(u) = \psi(|u|) \text{ pour tout } u.$$

Notons que $\Psi(.)$ est une fonction symétrique. Par le Lemme 2.1 de Deheuvels (1991), $\Psi(.)$ est une fonction non négative et convexe sur $(-\infty, \infty)$, so that $\Psi(\mu) = 0$, alors $\lim_{t\to\infty} \frac{\Psi(t)}{t} = \infty$ et

$$\lim_{\alpha\to 0} \frac{\Psi(\alpha)}{\alpha} = \lim_{\alpha\to 0} t_*(\alpha) = 0,$$

où $t_*$ est la solution unique de l'equation $\frac{\phi'(t)}{\phi(t)} = \alpha$ (see e.g. Deheuvels, Devroye and Lynch (1986)). Suite à ces résultats, $\Psi(.)$ est une N-function (see e.g. Krasnoselki, M.A, and Rutickii, Ya p. 11). Soit M sa N-fonction complémentaire, alors $\Psi(.)$ et M s'écrivent (voir Krasnoselskii et Ya. Rutickii, p. 11) :

$$\Psi(u) = \int_0^{|u|} p(s)ds,$$

pour tout u et pour $s \geq 0$, où la fonction $p(.)$ est continue à droite pour $t \geq 0$, positive pour $t > 0$, non décroissante et satisfait les conditions suivantes :

$$p(0) = 0 \text{ et } \lim_{t\to\infty} p(t) = \infty,$$

$$M(u) = \int_0^{|u|} q(s)ds,$$

pour tout u. Ici, $q(s) = \sup\{t \geq 0 : p(t) \leq s\}$ pour $s \geq 0$, vérifie les memes conditions que $p$.

**THM 3.1.**
Let $X$, $X_1$, ... une suite de v.a. i.i.d vérifiant (A1-3). Soit $\Im(.)$ la fonctionnelle définie sur BV(0,1) par

$$\Im(f) = \int_0^1 k(s)\dot{f}(s)ds,$$

où $k(.)$ est une fonction dérivable, positive non identiquement nulle sur $[0,1]$ et supposons que $\dot{k}(.) \in L^1(\nu)$ sur cet intervalle, où $\nu$ désigne la mesure de Lebesgue-Stieltjes . Si :
i) $||r_{It}||_W = o(\log t)$ a.s ., où $t \to \infty$,
ii) Il existe une unique constante $0 < \lambda^+(c) := \lambda_c^+ < \frac{t_0}{K(k)}$, vérifiant

$$\textbf{(3.3)} \qquad c\int_0^1 \psi(\frac{\phi'(\lambda_c^+ k(s))}{\phi(\lambda_c^+ k(s))})ds = 1,$$

où $K(k) = \sup_{0\le s\le 1} k(s)$. Alors pour tout $c > 0$, l'observation de la limite de

$$\sup_{0\le t\le T} \Im(\frac{\xi(t + cI\log T) - \xi(t)}{\log T}),$$

lorsque $T \to \infty$, détermine presque surement la loi de X.
**Lemme 3.1**
La constante $\lambda^+(c)$ existe bien.
Preuve. Sous les hypothéses du THM, on étudie la fonction

$$\lambda \to \int_0^1 \psi(\frac{\phi'(\lambda k(s))}{\phi(\lambda k(s))})ds.$$

**Lemme 3.2**
La fonctionnelle $\Im(.)$ est bien définie et est continue relativement á la topologie $W$.
Preuve. Facile.
**THM 3.2**
La fonction $\lambda(c)$ détermine d'une manière unique la distribution F de X.
Preuve.
→ On étudie la fonction :

$$s \to \Psi(q(\lambda(c)k(s)).$$

→ le THM de Steinebach (1981).
Preuve du THM 3.1.
Au cours de la démonstration, on fait appel à :

→ Lemme 3.2.
→ Corollaire A de Deheuvels (1991).
→ THM 3.2 de Lynch et Sethuraman (1987).
→ Le THM 3.2.

**THM 3.3.**
Soit $X$, $X_1$, ... une suite de v.a. i.i.d vérifiant (A1-3). Soit $\Im(.)$ la fonctionnelle définie sur BV(0,1) par

$$\Im(f) = \int_0^1 k(s)\dot{f}(s)ds,$$

où $k(.)$ est une fonction dérivable, positive, non-identiquement nulle sur $[0,1]$ et supposons que $\dot{k}(.) \in L^1(\nu)$ sur cet intervalle, où $\nu$ désigne la mesure de Lebesgue-Stieltjes . Si :
i) $||r_{It}|| = o(\log t)$ p.s., où $t \to \infty$,
ii) il existe une unique constante $0 < \lambda(c) := \lambda^+(c) < \frac{t_0}{K(k)}$, vérifiant :

$$c\int_0^1 \psi(q(\lambda^+(c)k(s)))ds = 1. \tag{3.12}$$

Alors, l'observation pour tout $c > 0$, de la limite de

$$\sup_{0\le t\le T} \Im\left(\frac{\xi(t + cI\log T) - \xi(t)}{\log T}\right),$$

lorsque $T \to \infty$, détermine p.s. la loi de X.
**Lemme 3.3**
La fonctionnelle $\Im(.)$ est bien définie et est continue relativement á la topologie $||.||$.
**THM 3.4.**
Lafonction $\lambda(c)$ détermine d'une maniére unique la distribution F de X.
**Remarque 3.4.**
Par le THM 3.1 (respectivement le THM 3.4), on conclut que si on connait la réalisation de $\Im(\xi_t)$, $[t] = 1, 2, ...$, avec la topologie $W$ (respectivement avec la topologie uniforme), on détermine p.s. la loi de X.

# 4 Application

Une application directe du THM 3.1 et 3.4 est donnée sous le corollaire suivant et qui n'est rien d'autre que le THM de Bártfai (1966).

**Corollaire.**
Sous les hypothéses du THM 3.1 (respect THM 3.4), si $\Gamma = W$ (respectiv $\Gamma = ||.||$) et $k(s) = 1$ pour tout $s \in [0,1]$. Alors, l'observation de $\{\xi(t) : t \geq 0\}$, détermine p.s. la loi de X pour tout $c > 0$.

# 5 Références


[1] Bártfai, P. (1966). Die bestimung der zu einem wiederkehrenden prozess gehörenden verteilungsfunktion aus den mit fehlern behafteten daten einer einzigen relation. Studia Sci. Math. Hung. 1 161-168;
[2] Chernoff, H. (1952). A measure of asymptotic efficiency for test of a hypothesis based on the sum of observations. Ann. Math. Statist. 23 493-507.
[3] Deheuvels, P.(1991). Functional Erdős-Rényi laws. Studia. Sci. Math. Hung. 2-3, 261-295.
[4] Deheuvels, P, Devroye, L. and Lynch, J. (1986). Exact convergencerate of an Erdős-Rényi strong law for moving quantiles. J. Appl. Proba. 14 209-223.
[5] Erdős, P. and Rényi, A. (1970). On a new law of large numbers. J. Anal. Math. 23 109-111.
[6] Grill, K. (1989). A note on the stochastic geyser problem. Studia Scientiarium Mathematicarum Hungarica 24 339-344.
[7] Lynch, J. and Sethuraman, J. (1987). Large deviations for processes with independent increments. Ann. Probab. 15 610-627.
[8] Krasnoselskii, M.A. and Rutickii, Ya. Convex functions and Orlicz spaces. P. Noordhoff, Groningen.
[9] Rényi, A. (1962). A sztochasztikus gejzur (abstract of lecture). Magyar. Tud. Acad. Math. Kut. Int. kőzl. 7 643.
[10] Steinebach, J. (1981). The stochastic geyser problem for first passage time. J. Appl. Probab. 18 91-103.
[11] Tyrrel Rockafeller, R. (1970). Convex Analysis. Princeton, New Jersey, Princeton University Press.

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
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# General covariance for a proposal for 4-D gravity

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**Abstract**

In this paper we extend some previous results for a new proposal for gravity and place them into overall context. The basic fields of this proposal provide an off-shell realization of symmetry with respect to SO(3,C) gauge transformations and general coordinate transformations of 4-dimensional spacetime.

# 1 Introduction

In this paper we review and extend some results concerning a proposal for a new description of gravity named the instanton representation of Plebanski gravity. We have approached this description from the standpoint of general covariance as an alternative to the standard canonical approach to general relativity. We place the results obtained thus far into their overall context, with a view to addressal of the quantum theory in future works. The main idea is that a theory of four dimensional gravity should exhibit invariance under gauge transformations and general coordinate transformations, an invariance which should manifestly be preserved under the canonical formalism. The organization of this paper is as follows. First we recount the main results of [1] and [2], setting the stage for the present paper. In section 2 we extend the relevant symmetry group from gauge transformations and spatial diffeomoprhisms to include the full spacetime general coordinate transformations. In conjunction we demonstrate the consistency of this by off-shell closure of the algebra on all of the basic fields of the theory. In section 3 we recount the relation to general relativity, and in section 4 we provide a conclusion.

## 1.1 Setting the stage

Let $M$ be a four-dimensional spacetime manifold. The set of general coordinate transformations

$$x^\mu \to x'^\mu = x^\mu + \xi^\mu(x), \tag{1}$$

referred to as $Diff(M)$, induces the following Lie algebra between any two smooth vector fields $\xi, \zeta \in C^\infty(M)$, given by

$$[\xi^\mu \partial_\mu, \zeta^\nu \partial_\nu] = (\xi^\mu \partial_\mu \zeta^\nu - \zeta^\mu \partial_\mu \xi^\nu)\partial_\nu. \tag{2}$$

We would like to propose a theory of gravity invariant under (1). The basic fields of the associated action $I_{Inst}$ should provide a realization of (2) independently of any equations of motion or canonical structure, and should thus constitute an off-shell realization. For the basic fields we will use a $SO(3,C)$ gauge connection $A^a_\mu$ and a 3 by 3 matrix $\Psi_{ae}$ taking its

values in two copies of $SO(3,C)$.[1] The proposed action is given by

$$I_{Inst} = \int dt \int_{\Sigma} d^3x \Big( \Psi_{ae} B^i_e \dot{A}^a_i + A^a_0 B^i_e D_i \Psi_{ae}$$
$$+\epsilon_{ijk} N^i B^j_a B^k_e \Psi_{ae} - iN(\det B)^{1/2} \sqrt{\det \Psi} \big( \Lambda + \mathrm{tr} \Psi^{-1} \big) \Big), \tag{3}$$

where $\Sigma$ represents 3-dimensional spatial hypersurfaces which foliate spacetime into $M = \Sigma \times R$. We have defined $B^i_a = \frac{1}{2} \epsilon^{ijk} F^a_{jk}$ as the magnetic field of $A^a_i$, which is the spatial part of $A^a_\mu$. This constitutes the spatial part of the curvature of $A^a_\mu$, given by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu \tag{4}$$

where $f^{abc} = \epsilon^{abc}$ are the $SO(3,C)$ structure constants.

In addition to being invariant under (1), the proposed theory should be invariant also under $SO(3,C)$ gauge transformations. Define $SO(3,C) * Diff$ as the set of all spacetime general coordinate transformations and $SO(3,C)$ gauge transformations continuously connected to the identity. Under an infinitesimal $SO(3,C)$ gauge transformation $\delta_{\vec{\eta}}$, the connection $A^a_\mu$ transforms as [3]

$$\delta_{\vec{\eta}} A^a_\mu = -D_\mu \eta^a = -\partial_\mu \eta^a - f^{abc} A^b_\mu \eta^c, \tag{5}$$

where $f^{abc} = \epsilon^{abc}$ are the $SO(3,C)$ structure constants. Under infinitesimal spacetime diffeomorphisms $\delta_\xi$, the connection $A^a_\mu$ transforms according to the Lie derivative

$$\delta_\xi A^a_\mu = \xi^\nu \partial_\nu A^a_\mu + (\partial_\mu \xi^\nu) A^a_\nu. \tag{6}$$

It has been shown in [1] that $SO(3,C) * Diff$ forms a Lie algebra, which closes on the field $A^a_\mu$

$$\big[\delta_{\vec{\theta}}, \delta_{\vec{\eta}}\big] A^a_\mu = -\delta_{\vec{\theta} \times \vec{\eta}} A^a_\mu; \quad \big[\delta_\xi, \delta_{\vec{\eta}}\big] A^a_\mu = -\delta_{(\delta_\xi, \vec{\eta})} A^a_\mu; \quad \big[\delta_\xi, \delta_\zeta\big] A^a_\mu = -\delta_{[\xi,\zeta]} A^a_\mu. \tag{7}$$

For the field $\Psi_{ae}$ only the spatial part of the algebra, $SO(3,C) * diff \subset SO(3,C) * Diff$, has been shown to close in [1].

In this paper we will extend the algebra on $\Psi_{ae}$ to include the temporal parts of (1). Recall from [1] that $\Psi_{ae}$ has been shown to form an off-shell realization of the $SO(3,C)$ part of the algebra. Hence to extend this to $SO(3,C) * Diff$ it suffices to show that the algebra (2) closes on $\Psi_{ae}$, and in addition forms a Lie algebra with $SO(3,C)$ which also closes on $\Psi_{ae}$.

[1] For index conventions, the Latin symbols $a, b, c, \ldots$ will assume values $1-3$ and will refer to internal $SO(3,C)$ indices, and the Greek symbols $\mu, \nu, \ldots$ taking values $0-3$ will refer to spacetime indices.

The transformation of $\Psi_{ae}$ under an infinitesimal $SO(3,C)$ transformations parametrized by a $SO(3,C)$-valued 3-vector is given by [1]

$$\delta_{\vec{\eta}}\Psi_{ae} = f^{bf}_{aec}\Psi_{bf}\eta^c, \tag{8}$$

where we have defined

$$f^{bf}_{aec} = f_{abc}\delta_{ef} + f_{efc}\delta_{ab}. \tag{9}$$

As mentioned, in [1] we have established $\delta_{\vec{N}}\Psi_{ae} = N^i\partial_i\Psi_{ae} + (\partial_i N^i)\Psi_{ae}$ for any spatial 3-vector $N^i$.[2] In the present paper we will extend this to include the temporal transformations, thus extending this to

$$\delta_\xi\Psi_{ae} = \xi^\sigma\partial_\sigma\Psi_{ae} \tag{10}$$

for any 4-vector $\xi^\sigma$.

## 2 Extension of transformation properties

In order to proceed, we must show two things, namely (i) that transformations (10) form a closed algebra on $\Psi_{ae}$. This is guaranteed, since (2) is an identity when acting on any coordinate scalar. The field $\Psi_{ae}$ is a coordinate scalar since it does not have any spacetime indices. Hence it suffices to act on $\Psi_{ae}$ with both sides of (2) in order to see that this is the case. (ii) We must show that (1) and the transformations (8) close on $\Psi_{ae}$, which entails finding the commutator of the two transformations

$$\left[\delta_\xi, \delta_{\vec{\eta}}\right]\Psi_{ae} = \delta_\xi(\delta_{\vec{\eta}}\Psi_{ae}) - \delta_{\vec{\eta}}(\delta_\xi\Psi_{ae}) = \delta_\xi(f^{bf}_{aec}\Psi_{bf}\eta^c) - \delta_{\vec{\eta}}(\xi^\sigma\partial_\sigma\Psi_{ae}) \tag{11}$$

where we have used (8) and (10). Proceeding from (11), and using the fact that the variations act on the fields, we have

$$\begin{aligned} & f^{bf}_{aec}(\delta_\xi\Psi_{bf})\eta^c - \xi^\sigma\partial_\sigma(\delta_{\vec{\eta}}\Psi_{ae}) \\ &= f^{bf}_{aec}(\xi^\sigma\partial_\sigma\Psi_{bf})\eta^c - \xi^\sigma\partial_\sigma(f^{bf}_{aec}\Psi_{bf}\eta^c) \\ &= -f^{bf}_{aec}\Psi_{bf}(\xi^\sigma\partial_\sigma\eta^c) = -f^{bf}_{aec}\Psi_{bf}(L_\xi\eta^c), \end{aligned} \tag{12}$$

where $L_\xi\eta^c = \xi^\sigma\partial_\sigma\eta^c \equiv \delta_\xi\eta^c$ is the Lie derivative of $\eta^c$ along the vector field generating the flow $\xi^\sigma$. The result is that

$$\left[\delta_\xi, \delta_{\vec{\eta}}\right]\Psi_{ae} = -\delta_{\delta_\xi\vec{\eta}}\Psi_{ae}. \tag{13}$$

[2]This is an error which we will correct in the present paper. The transformation should be given by $\delta_{\vec{N}}\Psi_{ae} = N^i\partial_i\Psi_{ae}$, which signifies that $\Psi_{ae}$ transforms as a scalar as opposed to a scalar density of weight one. This correction does not affect the final results or conclusions in [1].

The result is that the algebra (7) extends to the field $\Psi_{ae}$. Let us rewrite the algebra for completeness

$$\begin{aligned}
&[\delta_{\vec{\theta}}, \delta_{\vec{\eta}}] A^a_\mu = -\delta_{\vec{\theta}\times\vec{\eta}} A^a_\mu; \quad [\delta_{\vec{\theta}}, \delta_{\vec{\eta}}] \Psi_{ae} = -\delta_{\vec{\theta}\times\vec{\eta}} \Psi_{ae} \\
&[\delta_\xi, \delta_{\vec{\eta}}] A^a_\mu = -\delta_{\delta_{L_\xi}\vec{\eta}} A^a_\mu; \quad [\delta_\xi, \delta_{\vec{\eta}}] \Psi_{ae} = -\delta_{L_\xi\vec{\eta}} \Psi_{ae} \\
&[\delta_\xi, \delta_\zeta] A^a_\mu = -\delta_{[\xi,\zeta]} A^a_\mu; \quad [\delta_\xi, \delta_\zeta] \Psi_{ae} = -\delta_{[\xi,\zeta]} \Psi_{ae}.
\end{aligned} \tag{14}$$

The significance of this result is that the purely temporal part of the algebra of (2) forms a subalgebra with respect to the field $\Psi_{ae}$ in addition to $A^a_\mu$.

Another interesting relation arising from this result is that one can, using $\Psi_{ae}$ and $A^a_\mu$, construct a quantity $\Sigma^a_{\mu\nu}$ antisymmetric in $\mu$ and $\nu$ given by

$$\Sigma^a_{\mu\nu} = \Psi_{ae} F^e_{\mu\nu}. \tag{15}$$

The transformation properties of $A^a_\mu$ under (1) imply the the following transformation properties for the curvature $F^a_{\mu\nu}$ as derived in Appendix A

$$\delta_\xi F^a_{\mu\nu} = \xi^\sigma \partial_\sigma F^a_{\mu\nu} + (\partial_\mu \xi^\sigma) F^a_{\sigma\nu} + (\partial_\nu \xi^\sigma) F^a_{\mu\sigma}. \tag{16}$$

Application of the Liebniz rule to (15) yields

$$\delta_\xi \Sigma^a_{\mu\nu} = (\delta_\xi \Psi_{ae}) F^e_{\mu\nu} + \Psi_{ae} \delta_\xi F^e_{\mu\nu}. \tag{17}$$

Substitution of (16) and (10) into (17) yields the following transformation property of $\Sigma^a_{\mu\nu}$

$$\delta_\xi \Sigma^a_{\mu\nu} = \xi^\sigma \partial_\sigma \Sigma^a_{\mu\nu} + (\partial_\mu \xi^\sigma) \Sigma^a_{\sigma\nu} + (\partial_\nu \xi^\sigma) \Sigma^a_{\mu\sigma}, \tag{18}$$

which is consistent with what one expects of a second-rank tensor. The spatial restriction of (15) is given by

$$\tilde{\sigma}^i_a = \Psi_{ae} B^i_e \tag{19}$$

where $\tilde{\sigma}^i_a = \frac{1}{2}\epsilon^{ijk}\Sigma^a_{jk}$ which plays the role of a densitized triad in the Ashtekar formulation of general relativity [4].

## 3 Relation to general relativity

The Hamiltonian constraint in the canonical treatment of general relativity is the generator of temporal evolution. Our proposition is that for (3), this should provide a canonical realization of the temporal part of (2). The Hamiltonian constraint can be read off directly from (3) as

$$H[N] = \int_\Sigma d^3x N (\det B)^{1/2} \sqrt{\Psi} \left(\Lambda + \mathrm{tr}\Psi^{-1}\right). \tag{20}$$

Evidence for the validity of this proposition is provided in [2] using a reduced version of (3), where it is shown that the Hamiltonian constraint forms a closed algebra

$$[H[N], H[M]] = H[q^i(M\partial_i N - N\partial_i M)] \tag{21}$$

for phase space structure functions $q^i = q^i(\Psi_{ae}, A_i^a)$. This is in contrast to the Teitelboim algebra of Hamiltonian constraints [5]

$$[H[N], H[M]] = H_i[q^{ij}(M\partial_j N - N\partial_j M)] \tag{22}$$

for structure functions $q^{ij}$, where $H_i$ is the diffeomorphism constraint. The difference is that (22) does not close on the Hamiltonian constraint, which implies that a theory of gravity based just on the Hamiltonian constraint cannot be Dirac-consistent except in minisuperspace. However, the action producing (21) can be obtained from

$$\begin{aligned} I = \int dt \int_\Sigma d^3x \Big( & \lambda_1 a_2 a_3 \dot{a}_1 + \lambda_2 a_3 a_1 \dot{a}_2 + \lambda_3 a_1 a_2 \dot{a}_3 \\ & -iN(\det b)^{1/2} \sqrt{\lambda_1 \lambda_2 \lambda_3} \Big( \Lambda + \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} \Big) \end{aligned} \tag{23}$$

via a simple transformation. Note that (23) can be seen as (3) restricted to $A_i^a = diag(a_1, a_2, a_3)$ and $\Psi_{ae} = diag(a_1, a_2, a_3)$, with the Gauss' law and diffeomorphism constraints removed by hand. As shown in [2]), this action is based only on the Hamiltonian constraint and is Dirac-consistent while having two degrees of freedom per point. Additionally, the action (23) is not a minisuperspace action since it has spatial derivatives in $(\det b)$, where $b_a^i$ is the magnetic field of the diagonal connection $A_i^a$.

# 4 Conclusion

In this paper we have shown that certain fields $\Psi_{ae}, A_\mu^a$ provide an off-shell realization of the Lie algebra of general coordinate and $SO(3, C)$ gauge transformations $SO(3, C) * Diff$. We have provided a proposal for an action for general relativity $I_{Inst}$ based on these fields. We have shown in [2] that a reduced form of the action directly obtainable from (23) preserves the subalgebra of temporal transformations, in the sense that two Hamiltonian constraints Poisson-commute into a Hamiltonian constraint. The action (23) can be obtained from (3) by hand,[3] which on first sight brings into question its relevance to general relativity. However, it is related to general relativity in at least two respects: (i) The theory (23) uses the same Hamiltonian

[3] A main direction of future research should be to determine whether (23) is some sort of reduced phase space version of (3). At the present stage, this has not yet been conclusively demonstrated.

constraint appearing in GR. (ii) It is a Dirac consistent theory as shown in [2], and has two degrees of freedom per point on its reduced phase space.[4]

# 5 Appendix A

Given (4) and (6), we will prove (16). First we have the relation

$$\delta_\xi F^a_{\mu\nu} = \partial_\mu(\delta_\xi A^a_\nu) - \partial_\nu(\delta_\xi A^a_\mu) + f^{abc}(\delta_\xi A^b_\mu) + f^{abc}A^b_\mu(\delta_\xi A^c_\nu). \tag{24}$$

Substituting (6) into (24), we have

$$\begin{aligned}\delta_\xi F^a_{\mu\nu} = \partial_\mu\big(\xi^\sigma\partial_\sigma A^a_\nu + A^a_\sigma(\partial_\nu\xi^\sigma)\big) = \partial_\nu\big(\xi^\sigma\partial_\sigma A^a_\mu + A^a_\sigma(\partial_\mu\xi^\sigma)\big)\\ +f^{abc}\big(\xi^\sigma\partial_\sigma A^b_\mu + A^b_\sigma(\partial_\mu\xi^\sigma)\big)A^c_\nu + f^{abc}A^b_\mu\big(\xi^\sigma\partial_\sigma A^c_\nu + A^c_\sigma(\partial_\nu\xi^\sigma)\big).\end{aligned} \tag{25}$$

Expanding the partial derivatives in (25) we have

$$\begin{aligned}\xi^\sigma\partial_\mu\partial_\sigma A^a_\nu + (\partial_\mu\xi^\sigma)(\partial_\sigma A^a_\nu) + (\partial_\mu A^a_\sigma)(\partial_\nu\xi^\sigma) + A^a_\sigma(\partial_\mu\partial_\nu\xi^\sigma)\\ -\xi^\sigma\partial_\nu\partial_\sigma A^a_\mu - (\partial_\nu\xi^\sigma)(\partial_\sigma A^a_\mu) - (\partial_\nu A^a_\sigma)(\partial_\mu\xi^\sigma) - A^a_\sigma\partial_\nu\partial_\mu\xi^\sigma\\ +f^{abc}\xi^\sigma(\partial_\sigma A^b_\mu)A^c_\nu + f^{abc}A^b_\sigma(\partial_\mu\xi^\sigma)A^c_\nu + f^{abc}A^b_\mu\xi^\sigma(\partial_\sigma A^c_\nu) + f^{abc}A^b_\mu A^c_\sigma(\partial_\nu\xi^\sigma).\end{aligned} \tag{26}$$

The terms involving $f^{abc}$ can be combined using the Liebniz rule, and rearranging terms, (26) simplifies to

$$\begin{aligned}\delta_\xi F^a_{\mu\nu} = \xi^\sigma\partial_\sigma\big(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu\big)\\ +(\partial_\mu\xi^\sigma)\big(\partial_\sigma A^a_\nu - \partial_\nu A^a_\sigma + f^{abc}A^b_\sigma A^c_\nu\big) + (\partial_\nu\xi^\sigma)\big(\partial_\mu A^a_\sigma - \partial_\sigma A^a_\mu + f^{abc}A^b_\mu A^c_\sigma\big)\\ = \xi^\sigma\partial_\sigma F^a_{\mu\nu} + (\partial_\mu\xi^\sigma)F^a_{\sigma\nu} + (\partial_\nu\xi^\sigma)F^a_{\mu\sigma}.\end{aligned} \tag{27}$$

# References


[1] Eyo Ita 'Covariance of the algebra of constraints of 4-D general relativity' To appear in the Hadronic Journal

[2] Eyo Ita 'Dirac consistency of the algebra of Hamiltonian constraints in reduced 4-D general relativity' (To Appear in the Hadronic Journal)

[3] Milutin Blagojevic 'Gravitation and gauge symmetries' IOP Publishing Ltd. 2002


[4] So it appears to be some form of reduced gravity where the $SO(3,C) * diff$ portion of the original $SO(3,C) * Diff$ invariant theory has been removed.

[4] Ahbay Ashtekar. 'New perspectives in canonical gravity', (Bibliopolis, Napoli, 1988).

[5] S. Hojman, K. Kuchar and C. Teitelboim, Ann. Phys., New York, 96 (1976) 88

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
Kathmandu University, Nepal, April (2011) pages 287-304

# Santilli's Etherino as a (g)+(em) Interaction

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**Abstract.** The *neutron* was conceived by H. Rutherford as a "compressed hydrogen atom" in the core of a star. Don Borghi claimed the laboratory synthesis of the neutron from protons and electrons; this experiment remained unverified for decades due to the lack of theoretical understanding of the results. R.M. Santilli has verified and theoretically explained this experiment by the aid of a particle process which he called as *etherino* (from aether) on the basis of Hadronic Mechanics. The *Etherino Process* has been found to be compatible to 'Minimum Contradictions Physics' which implies that space time is matter itself and consists of gravitational (g) and electromagnetic (em) space time which are interconnected and communicate through photons (particles with zero rest mass). A basic consequence of this, is the *Statement: "During the approach of an electron to a proton there is absorption of gravitational energy"*. This Statement is compatible to *Santillis' Etherino* and possibly to CERN and Fermi Lab experiments. Beyond this, the (g) + (em) interaction is reinforced both by further theoretical and experimental argumentation through gravitoelectric and gravitomagnetic systems proposed.

**Keywords:** Logic Analysis, Space-Time QM, Fields and Particles, Unified field theories.
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## 1. INTRODUCTION

In a recent paper [1], R. M. Santilli identified a number of insufficiencies of the neutrino hypothesis that the synthesis of the neutron in stars from hydrogen atoms occurs according to the traditional form:

$$p^+ + e^- \rightarrow n + \nu \quad , \qquad \text{(a)}$$

as well as the conjugate form:

$$\overline{\nu} + p^+ + e^- \rightarrow n \quad . \tag{b}$$

Some of the insufficiencies are quite serious. For example:

**1)** Hypothesis (a) requires a 'positive binding energy', since the rest energy of the neutron is 0.782 MeV larger than the sum of the rest energies of the proton and of the electron, with consequential inconsistency of the Schrödinger equation and all quantum equations;

**2)** It is impossible for the proton $p^+$ and the electron $e^-$ to supply the missing energy via their kinetic energy, since at 0.782 MeV the cross-section of protons and electrons is extremely small, thus preventing the synthesis of the neutron;

**3)** The antineutrino has a virtually null cross section with protons and electrons, as a result of which the conjugate reaction (b) definitely does not permit the neutron synthesis; and others.

Consequently, Santilli submitted in [1] the hypothesis that the energy, spin and other quantities missing in the neutron synthesis are provided to the neutron by the environment via a new state he called 'etherino' and denoted with the letter 'a' (from aether) according to the reaction

$$a + e^- + p \rightarrow n \tag{c}$$

To achieve a quantitative representation of the neutron synthesis, a covering of Quantum Mechanics under the name of Hadronic Mechanics was built via axiom preserving non-unitary liftings (see monographs [1]). In particular, Santilli noted that, at the initiation of its life from pure Hydrogen, a star synthesizes a very large number of neutrons of the order of $10^{50}$ neutrons/second. In the event the missing energy would originate from the interior, the star would lose about $10^{50}$ MeV/second, thus rendering impossible the production of light.

The reaction (c) in which the etherino appears can be further justified within the context of 'Minimum Contradictions Physics' [2]. According to the minimum contradictions point of view, this reaction can be explained on the basis that the energy required is gravitational, and that any 'charge' can in general be regarded as an electromagnetic mass affecting an energy balance.

## 2. MINIMUM CONTRADICTIONS PHYSICS AND [(G)+(EM)] INTEREACTION

### 2.1 General

'Minimum Contradictions Physics' implies that space time is matter itself and consists of gravitational (g) and electromagnetic (em) space time which are interconnected and communicate through photons (particles with zero rest mass).

The electromagnetic (*em*) space-time is a space-time all of whose magnitudes are considered imaginary and behave exactly like the gravitational (*g*). Electromagnetic (*em*) space-time is described by means of space-time wave functions such that:

$$\Psi_{em}(\mathbf{r}_{em},t_{em}) = \Psi^{g}_{em}(\mathbf{r},t) \tag{1}$$

where Eq. (1) has meaning *due to the coexistence of (g) and (em) space-time under a scale.* On this basis space-time as a whole consists of:

*1. real (g) space-time* distributed according to a $P_g(\mathbf{r},t)$ function revealing so (*g*) matter or antimatter for positive or negative values of $P_g(\mathbf{r},t)$.

*2. imaginary (em) space-time* distributed according to a $P_{em}(\mathbf{r},t)$ function, revealing (*em*) matter or antimatter for imaginary positive $(+i)$ or imaginary negative $(-i)$ values of $P_{em}(\mathbf{r},t)$.

This means that a matter space time with certain characteristics creates through its distribution the real field. These characteristics are energy $E$, momentum $\mathbf{P}$ and relative space-time magnitudes $SR$; *i.e.*, relative time $TR$ relative volume $VR$ and relative length in a direction $\mathbf{n}$. These characteristics constitute the mean values of the corresponding local space-time magnitudes. The mean values of all these magnitudes belong to a unique flat matter space-time since through $P(\mathbf{r},t)$ a unique matter space-time is distributed everywhere. Thus the stochastic nature of matter space-time leads to the use of flat matter space-time.

For flat matter space-time Lorentz transformations are valid; therefore we have [2]:

$$\langle E\rangle^2 = c^2\langle \mathbf{P}\rangle^2 + m_0{}^2c^4 \tag{2}$$

$$E^2 = c^2P^2 + m_0{}^2c^4 \tag{3}$$

taking into account the QM operators and Eq. (3), we obtain:

$$\hat{E} = i\hbar\partial / \partial t \quad , \quad \hat{\mathbf{P}}_n = -i\mathbf{n}\hbar\partial / \partial x_n \quad , \quad \hat{\mathbf{P}} = -i\hbar\nabla \tag{4}$$

$$\hbar^2\partial^2\Psi / \partial t^2 - \hbar^2 c^2\nabla^2\Psi + m_0{}^2c^4\Psi = 0 \tag{5}$$

We notice, because of Eqs. (2-5), that for energy eigenvalue $E$, we have eigenfunction $\Psi_E$. We also notice that, because of Eqs(2,4,5), for energy and momentum eigenvalues $\langle E \rangle$ and $\langle \mathbf{P} \rangle$ we have eigenfunction $\Psi$. Therefore we have the substitutions:

$$\hat{E} = i\hbar\partial / \partial t \rightarrow \langle E \rangle \tag{6}$$

$$\hat{\mathbf{P}}_n = -i\mathbf{n}\hbar\partial / \partial x_n \rightarrow \langle \mathbf{P}_n \rangle \tag{7}$$

and the relations:

$$\hat{E}\Psi = \langle E \rangle \Psi \tag{8}$$

$$\hat{\mathbf{P}}_n \Psi = \langle \mathbf{P}_n \rangle \Psi \tag{9}$$

Because of Eqs. (6-9) we obtain:

$$i\hbar\partial_t \langle E \rangle + \langle E \rangle^2 = c^2 \langle \mathbf{P} \rangle^2 + m_0{}^2c^4 \tag{10}$$

Thus, from Eqs. (2,10) we have that $\partial_t \langle E \rangle = 0$; since $\langle E \rangle$ is position independent we have:

$$\frac{d}{dt}\langle E \rangle = 0 \tag{11}$$

Eq. (11) shows energy conservation; at the same time it shows that if $\langle E \rangle$ changes then it changes in a discontinuous way. Because of Eqs. (2) and (11) we obtain:

$$\frac{d}{dt}\langle \mathbf{P} \rangle = 0 \tag{12}$$

According to the spirit of this work, Eq. (11) is valid both for the ($g$) and the ($em$) space. Therefore, we have:

$$\frac{d}{dt}\langle E_g \rangle = 0 \quad , \quad \frac{d}{dt}\langle E_{em} \rangle = 0 \tag{13}$$

Eqs. (13) show that $\langle E_g \rangle$ and $\langle E_{em} \rangle$ are constant in time; however, if the ($g$) space-time communicated with the ($em$) one, the changes of $\langle E_g \rangle$, $\langle E_{em} \rangle$ should be discontinuous.

We may notice that if $m_0 = 0$, Eq. (2) is valid both for real and imaginary energy and momentum. Thus, we may assume that only photons ($m_0 = 0$) can convert ($g$) space-time into ($em$) one and inversely.

Because of Eq. (13) we obtain:

$$\frac{d}{dt}\langle E_{em-g} \rangle = 0 \tag{14}$$

where $E_{em} = iE_{em-g}$; $E_{em-g}$ can express energy which can be converted from ($em$) into ($g$) form.

In a closed system consisting of a real ($g$) space-time particle field and a coexisting imaginary ($em$) one, by definition, there are not photons that flow out the system while energy conversion, according to Eq. (13), takes place only through photons.

Thus in the case of energy conversion we have:

$$\delta\langle E_g \rangle + \delta\langle E_{em-g} \rangle = 0 \tag{15}$$

$$\langle E_g \rangle + \langle E_{em-g} \rangle = \text{const.} \tag{16}$$

Eqs. (15,16) express the energy conservation principle of the closed system mentioned consisting of a gravitational and a coexisting electromagnetic space-time particle field. It is noted that the energy conservation principle as it has been expressed by the Eqs. (15,16) is compatible with the 1st Thermodynamic Axiom.

## 2.2 Basic Equations

According to the Minimum Contradictions Physics the following are valid:

*a. Many body Schrödinger's Relativistic Equation for (g) and for (em) space time:*

$$\frac{\partial}{\partial x_j}\frac{\Box\Psi_g(\boldsymbol{r},t)}{\Psi_g(\boldsymbol{r},t)}=0 \qquad (j=1,2,3,4) \tag{17}$$

$$\frac{\partial}{\partial x_j}\frac{\Box\Psi^g_{em}(\boldsymbol{r},t)}{\Psi^g_{em}(\boldsymbol{r},t)}=0 \qquad (j=1,2,3,4) \tag{18}$$

*b. Energy Conservation:*

$$\partial_t\left(\frac{\partial_t\Psi_g(\boldsymbol{r},t)}{\Psi_g(\boldsymbol{r},t)}+\frac{\partial_t\Psi^g_{em}(\boldsymbol{r},t)}{\Psi^g_{em}(\boldsymbol{r},t)}\right)=0 \tag{19}$$

*c. Momentum Conservation:*

$$\partial_t\left(\frac{\nabla\Psi_g(\boldsymbol{r},t)}{\Psi_g(\boldsymbol{r},t)}+\alpha\frac{\nabla\Psi^g_{em}(\boldsymbol{r},t)}{\Psi^g_{em}(\boldsymbol{r},t)}\right)=0 \tag{20}$$

$$\mathbf{g}(\boldsymbol{r},t)=\frac{c^2}{P(\boldsymbol{r},t)}\nabla P(\boldsymbol{r},t)=\frac{c^2}{\overline{tr}(\boldsymbol{r},t)}\nabla\overline{tr}(\boldsymbol{r},t)=\frac{c^2\nabla\left(\Psi^*\partial_t\Psi-\Psi\partial_t\Psi^*\right)}{\left(\Psi^*\partial_t\Psi-\Psi\partial_t\Psi^*\right)} \tag{21}$$

These equations state that am matter system $\Psi$ function locally is described by equivalent coexisting and interacting local space-time particle (g) and (em) fields regarded as extended to the infinity. Eq. (29) describes the force per mass unit exerted due to the unified field.

## 2.3 The Nature of Photons

$\Psi$ is a complex statistically interpreted wave function which implies that $m_0 \neq 0$ [2]. Thus, the question is raised of whether photons $(m_0 = 0)$ exist and are compatible to the basic

claim of this work. Photon is an oscillating matter space-time field which has no energy when the oscillation stops. Therefore photon has sense only when it is regarded as travelling within "non existing space-time". The notion of "non existing space-time" derives from the fact that space-time is stochastic and therefore there is a probability not to exist. Photons play key role in quantum states formation and in the conversion of one kind of space-time into another [2]; their action is taken into account by space-time $\Psi$ wave functions described by the equation set of minimum contradictions everything but they are invisible since they are travelling within "non-existing space-time"[2]. However we should distinguish the notion of photons from the notion of radiation since radiation refers to a space-time wave within existing space-time which implies that it has rest mass either of ($g$) or ($em$) space-time [2,3,4]; thus we may assume that speed of radiation approaches to the ideal speed of light in an asymptotic way[5]. If this is the case radiation constitutes space-time formation generally described by the equations of Minimum Contradictions Everything.

As was mentioned chapter 2, energy and momentum appear in discrete values. In this chapter the way of communication between ($g$) and ($em$) space was shown. According to this point of view, we can have discrete values of energy and momentum through energy conversion by means of photons. Thus, we may assume that we have discrete values of energy and momentum in Schröndinger's relativistic Eqs. (25,26) because of energy and momentum conversion through photons. According to spirit of this work, both ($g$) and ($em$) space-time are described by a complex space time wave function. The real component of such a function could correspond to an ideal ($g$) space-time while the imaginary component could correspond to an ideal ($em$) one. Thus, both ($g$) and ($em$) space-time could be regarded as result of a gravi-electric oscillation. According to what was mentioned, we may assume that we reach to statistical interpretation of ($g$) and ($em$) space-time because of energy and momentum conversion through photons; ($g$) space-time can not exist without ($em$) and vice-versa. Real eigenvalues could correspond to ($g$) space-time while imaginary eigenvalues could correspond to ($em$) one.

### 2.4 Second Thermodynamic Law-Space Time Irreversibility

According to [2] we have:

$$\langle E\rangle_i \langle V\rangle_i = hc \tag{22}$$

$$\overline{V}\uparrow \Rightarrow \langle V\rangle_i \uparrow \Rightarrow \langle E\rangle_i \downarrow \Rightarrow \overline{E}\downarrow \tag{23}$$

This expresses the following conclusion:

*Conclusion III: "A mean volume increase of a matter space-time system implies a mean energy decrease of this system"*

For a closed matter system consisting of coexisting ($g$) and ($em$) space-time and applying the same methodology as in Eq. (16), we have:

$$d(\overline{E}_g + \overline{E}_{em-g}) = 0 \tag{24}$$

Because of relation (31), we have that:

$$\overline{V}_g \uparrow \Rightarrow \overline{E}_g \downarrow \tag{25}$$

In the case of a closed system existing within an expanding universe, we may assume that $\overline{V}_g \uparrow$ [2]. This implies, because of relation (25) and Eq(24), that:

$$d\overline{E}_{em-g} \geq 0 \tag{26}$$

Inequality (26) states that always energy of ($g$) space is converted into ($em$) space . As was mentioned, ($g$) space can be converted into ($em$) space and vice-versa only through photons. Therefore because of Eq(24) and inequality (26), energy of ($g$) space is converted into photons, a part of which heats the whole system. Thus, we can write:

$$dQ = TdS = \phi d\overline{E}_{em-g} \geq 0, \quad (0 \leq \phi \leq 1) \tag{27}$$

where $dQ$ is the heat inflow because of photons mentioned, $T$ is the absolute temperature and $dS$ is the change of entropy. From relations (27), we obtain:

$$dS \geq 0 \tag{28}$$

***This inequality expresses the Second Thermodynamic Law.*** This implies the existence of irreversibility which is compatible to Santilli's point of view [6].

## 2.5 Experimental Verification

*2.5.1* In [7] "Frolov's Hat" is shown. The electric field dynamic lines depart from a positively charged inner conductor, pass over a dielectric and reach an outer conductor. When the voltage imposed is bigger than 10 kV we can observe a weight loss as it is shown in the diagram of [7]. It can be noticed that the weight loss is not depended on the power input but on the energy acquired within the device itself which is a capacitor. This shows that there is an interaction between the (g) and (em) field in the charged system (capacitor) described. The same results (very much stronger but without reproducibility) have been observed in a wavy capacitor with a zero potential casing [2].

*2.5.2* On the basis of the 'Empirical Statement I' (see Sect. 4.1) an alternative explanation has been given for light water electrolysis according to Mills [8] and Kanarev [9] as well as for Kozyrev radiation [10] and atoms radiation according to Whitney [11].

*2.5.3* In Fermi Lab Tevatron a dominance of matter over antimatter has been observed during B meson decay into more muons than antimuons [12]. This is compatible to Eq. (24). In fact because of this equation for a closed system created from zero ( as the Universe creation from nothing) we have:

$$\begin{aligned} &\overline{E}_g - \overline{E}_{em-g,matter} + \overline{E}_{em-g,antimatter} = 0 \\ &\overline{E}_{em-g,matter} - \overline{E}_{em-g,antimatter} = \overline{E}_g > 0 \\ &\overline{E}_{em-g,matter} > \overline{E}_{em-g,antimatter} \end{aligned} \tag{29}$$

It is noted that the subscript *(em-g, matter)* corresponds to a negative charge which characterizes matter (e.g. electron) while *(em-g, antimatter)* corresponds to a positive charge which characterizes antimatter (e.g. positron).

*2.5.4* An experiment is proposed relative to this interaction. A magnetic field is developed in the convergent part of the nozzle of fig.1 that is made from a superconducting material 1 by means of the magnet 3. Superconducting Material 1 traps the magnetic field, which in turn acts as a pressure on it according to Meissner effect [13]. If a force **F** is developed, this can be explained on the basis of the (g) + (em) interaction. In fact the magnetic field according to the Meissner effect repels the inner cone (superconductor 1) of fig.1 of the paper above. This is the main reason of the repulsion created. More specifically, in the inner cone of fig.1 current vortex pinning forces are created. In other words, due to the Meissner effect, vortex currents are created in the surface of superconductor 1 (below a critical field and temperature) having as a result the creation of a magnetic field that does not permit the existing magnetic field (within the cone) to

penetrate the superconductor 1. Thus, we have the creation of a new magnetic field (probably quantum field) i.e. the creation of new magnetic (quantum) energy in a direction perpendicular to the cone. According to the conservation law an opposite energy should be created so that the balance to be kept. This could take place (according to the paper attached) through a negative (g) space-time creation; this implies decreasing of the density probability of the (g) space-time (in the area near the surface of the inner cone) and creation of a propulsion force towards the denser area which is compatible to Eq. (21).
Perhaps this is compatible to other theories as the Santilli Hadronic Mechanics [1] or Corda's related to gravitomagnetism [14] or to Ying's related to Twin Universes theory [15].

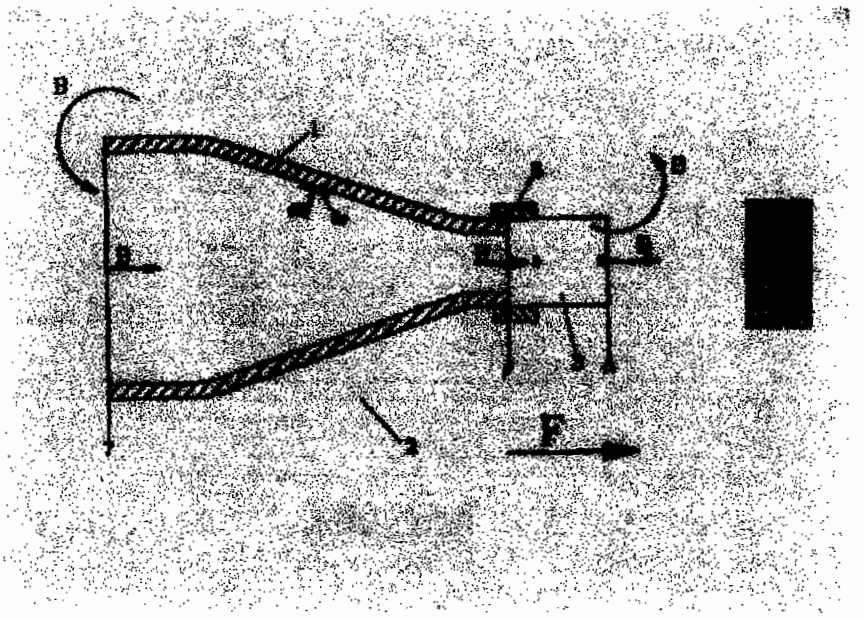



**FIGURE 1.** Propulsion by Means of Superconducting Magnetic Field Trapping

## 3. ENERGY CONSERVATION OF A CLOSED MATTER SPACE TIME SYSTEM

### 3.1 General

Eq. (24) can be written in the form:

$$\overline{E}_g + \overline{E}_{em-g} = \text{const.} \tag{30}$$

This expresses the energy conservation principle for a closed matter system in general consisting of coexisting ($g$) and ($em$) space-time.

### 3.2 Evolution of the Universe

If we consider the Universe as a closed system that has been derived from zero, because of Eq. (30) we will have [2]:

$$\overline{E}_g^U + \overline{E}_{em-g}^U = 0 \tag{31}$$

where the superscript $U$ indicates Universe quantities. A basic property of the stochastic space-time is described in the following relation [2]:

$$\overline{V}_g^U \uparrow \Rightarrow \overline{E}_g^U \downarrow \tag{32}$$

Thus, when $\overline{E}_g^U$ is very high, the volume $\overline{V}_g^U$ that contains $\overline{E}_g^U$ will be very small. Expansion of the Universe means increase of $\overline{V}_g^U$ and decrease of $\overline{E}_g^U$, as well as increase of $\overline{E}_{em-g}^U$ according to the Eq. (31). From Eq. (31) it is derived that for positive value of $\overline{E}_g^U$ the $\overline{E}_{em-g}^U$ value will be negative. Thus, the evolution of the Universe is a process reverse to that of the creation of the Universe, and during evolution the quantity $\overline{E}_g^U$ decreases tending to zero, while the quantity $\overline{E}_{em-g}^U$ increases tending also to zero.

## 4. EXPLANATION OF SANTILLI'S ETHERINO

### 4.1 General

We can assume that the charge energy $\overline{E}^{U}_{em-g}$ is the energy sum of all the positive-negative charges regardless of whether they are joined or not. When there is an approach and coincidence tendency between positive and negative charges, which might be proton-electron couples, there is a tendency for nullification and increase of $\overline{E}^{U}_{em-g}$. Thus, the approach between electrons and protons has as a result the increase of $\overline{E}^{U}_{em-g}$ and due to Eq. (31), the decrease of $\overline{E}^{U}_{g}$. Consequently, we may assume that the following empirical statement is valid [2].

*Empirical Statement I: "During the approach of an electron with a proton there is absorption of gravitational energy".*

By the term 'Empirical Statement' we mean a statement compatible with the theory proposed and having the possibility to be verified in an experimental way. The Empirical Statement I verifies the Santilli's etherino process on condition that the ether is the matter space time itself.

More specifically we denote by $E$ the energy level of an electron, excluding its rest energy, in a radius r in the hydrogen atom and by $E_{el}$ the total kinetic energy, including both mass and charge, that the electron acquires during the free fall from radius $r=\infty$ to radius $r=r$. According to the Empirical Statement I we have absorption of gravitational energy $\delta E_g$ during the $e^- + P$ approach. By definition, it is valid that $E=-E_{el}$; therefore the transposition from a lower energy value $E_{el2}$ to a higher energy value $E_{el1}$ is equivalent to the transposition from the energy level $E_2=-E_{el2}$ to the energy level $E_1=-E_{el1}$. If $E_1$ and $E_2$ correspond to fundamental energy levels of the electron in the hydrogen atom, then, *as it is commonly known, photon emission takes place.* Applying the energy conservation principle, we have as a result the following equations [2,16]:

$$E_{el0}+E_{el2}+\delta E_g \downarrow = E_{el0}+E_{el1}+h\nu \uparrow \tag{33}$$

$$\delta E_g = E_{el1}-E_{el2}+h\nu = (E_2-E_1)+(E_2-E_1) = 2(E_{el1}-E_{el2}) \tag{34}$$

where $E_{el0}$ is the total electron rest energy, related both to mass and charge, where charge is regarded as an imaginary mass. The energy $\delta E_g$ is converted partly into photons; *i.e.*:

$$h\nu = E_{el1} - E_{el2} = \delta E_g / 2 \tag{35}$$

and partly to energy increase at level $E_{el1}$ *i.e.*:

$$E_{el1} = E_{el2} + \delta E_g / 2 = E_{el2} + E_{el1} - E_{el2} \tag{36}$$

Taking into account the above mentioned, we conclude that the electron when approaching the proton increases in charge until it is valid that:

$$Q_{proton} + Q_{electron} = 0 \tag{37}$$

On this basis we can reach to results compatible to Santilli reaction [1,2] i.e.:

$$a + e^- + p \rightarrow n \tag{38}$$

Reaction (38) corresponds to charge disappearance, to neutron production, and to energy absorption (0,78 MeV) with the aid of the process particle $\alpha$ (0,78 MeV) named *etherino*.

According to the minimum contradictions point of view, charge is regarded as electromagnetic imaginary mass; therefore, it can be taken into account for the energy balance of Santilli's reaction (38) as it will be shown bellow.

### 4.2 The Energy Role of Charge in Santilli's Reaction

From Eq. (33) we have:

$$E_{state\,2} + \delta E_g \downarrow = E_{state\,1} + h\nu \uparrow \tag{39}$$

where by the subscript 'state' the whole energy of the system (electron + proton) is indicated. Taking into account that a charge $q$ is regarded as an imaginary mass we may notice that it contains energy of the form [17]:

$$E_q = m_q c^2 \tag{40}$$

where $m_q$ is the equivalent mass producing the same effect with the charge $q$.

According to minimum contradiction point of view there is not potential acting at a distance [2]; therefore, we can write:

$$\begin{aligned} E_{status2} = E_{electron,\,mass} - E_{electron,\,charge} \\ + E_{proton,\,mass} + E_{proton,\,charge} \end{aligned} \tag{41}$$

Claiming that:

$$E_{status1} = E_{neutron} \tag{42}$$

and that the photon produced contributes in the realization of Santilli's reaction we have:

$$\begin{aligned} &\delta E_g + E_{electron,\,mass} - E_{electron,\,charge} + E_{proton,\,mass} + E_{proton,\,charge} \\ &= E_{neutron,\,mass} \end{aligned} \tag{43}$$

In order that Eq. (37) is valid, according to Eq. (35) for:

$$\begin{aligned} E_{el1} &= E_{proton,charge} + E_{electron,\,mass1} \\ E_{el2} &= E_{electron,\,charge} + E_{electron,\,mass2} \end{aligned} \tag{44}$$

we have:

$$\begin{aligned} \delta E_g / 2 = E_{proton,charge} - E_{electron,\,charge} \\ + E_{electron,\,mass1} - E_{electron,\,mass2} \end{aligned} \tag{45}$$

Since the charge and the electron coexist they move with the same velocity and therefore according to relativity we have:

$$\frac{E_{electron,\,mass1}}{E_{electron,\,mass2}} = \frac{E_{proton,charge}}{E_{electron,\,charge}} = \gamma\,, \tag{46}$$

$$E_{electron,\,mass1} = \frac{E_{proton,charge}}{E_{electron,\,charge}} E_{electron,\,mass2} \tag{47}$$

Therefore it is obtained:

$$\delta E_g / 2 = (E_{proton,charge} - E_{electron,\,charge})(1 + \frac{E_{electron,\,mass2}}{E_{electron,\,charge}}) \tag{48}$$

Since charge is regarded as imaginary mass, for the referred value of charge difference we have:

$$\frac{E_{electron,\,mass2}}{E_{electron,\,charge}} = \frac{0.51\mathrm{MeV}}{\alpha M_{\mathrm{P}}\, c^2} \cong 0.056X10^{-19} \tag{49}$$

where $M_{\mathrm{P}}$ is the Plank mass and $\alpha M_{\mathrm{P}}$ the equivalent mass producing the same effect with the charge of an electron[2]. Therefore we obtain:

$$\delta E_g / 2 \cong E_{proton,charge} - E_{electron,\,charge} \tag{50}$$

and:

$$\delta E_{p-e} = E_{proton,charge} - E_{electron,\,charge} \cong \delta E_g / 2 \tag{51}$$

Because of Eqs (43, 51) it is expected that:

$$\begin{aligned} &\delta E_g + \delta E_{p-e} = 1.5\delta E_g = 0.78\,\mathrm{MeV}, \quad \delta E_g = 0.52\,\mathrm{MeV}, \\ &f = \frac{\delta e}{e} = \frac{\delta E_{p-e}}{E_{electron,\,charge}} = \frac{0.26\mathrm{MeV}}{\alpha M_{\mathrm{P}}\, c^2} \cong 2.9X10^{-21} \end{aligned} \tag{52}$$

It is noted that the ratio of electron charge uncertainty to the electron charge is estimated to be $2{,}496\,2.496X10^{-8}$ which is huge in comparison to $f$. This shows the difficulty for this very small charge differences to be measured even though they produce great results.
On this basis the etherino $\alpha$ is expected to have smaller energy than 0.78 MeV which of course can be explained according to Hadronic Mechanics.

## DISCUSSION

As is known, neutron decay takes place through the mediation of W and Z bosons. According to what has been accepted until now, the W and Z bosons in turn acquire mass through the aid of the Higgs' boson, which, however, is hypothetical [18].

Santilli's reaction could be regarded as the process inverse to neutron decay. Therefore, the etherino should imply an inverse action of W and Z. If this is the case, the Higgs boson should play a basic role as well.

According to the present paper, the etherino is described through the contribution of ether [(g+em) space-time]. This idea might have some importance in case experiments fail to detect the Higgs' boson [19]. Note that in CERN experiment this boson has not been detected at energy level of 7 TeV.

Provided that Minimum Contradictions point of view can give an explanation on Santilli's Etherino the question is raised of how the Minimum Contradictions Equations are compatible under certain conditions to the Hadronic Mechanics.

## REFERENCES


1. R. M. Santilli, "Neutrino and/or etherino?" Foundations of Physics **37**, 670 (2007); for extensive studies see also R. M. Santilli, **Hadronic Mathematics, Mechanics and Chemistry**, Volumes I, II, III, IVand V, International Academic Press (2008) also available as free downloads from the web site http://www.i-b-r.org/Hadronic-Mechanics.htm
2. A.A. Nassikas, **Miminum Contradictions Everything**, Rev. and Ed. M.C. Duffy and C.K. Whitney, Hadronic Press (2008).
3. P.F.Parshin, Development of the de Broglie Wave-Particle Conception. Galilean Electrodynamics, Vol. 12, Special Issues 1, Space Time Analyses, Ltd, 2001.

4 Toivo Jaakkola, "Equilibrium Cosmology", Proceedings of the XIIIth Krakov School of Cosmology, Lodg, Poland, 1992; Ed.H.C.Arp., C.R.Keys & K.Rudnicki (Plenum, N.Y., 1993).
5. R.J. Heaston, "The Asymptotic Box", Proceedings of 2003 NPA Conference, Journal of New Energy **7** (3) 32-37 (2003).
6. R.M. Santilli, "Lie-Admissible Invariant Origin of Irreversibility for Matter and Antimatter at the Classical and Operator Levels", http://arxiv.org/abs/physics/0601061
7. http://jnaudin.free.fr/html/elghatv1.htm
8. R.L. Mills,. "Energy/matter Conversion Methods and Structures", International Patent Publication Number WO 92/10838, Infinite Energy **3**, Issue 17 (1991).
9. Ph. Kanarev, **"Water as a New Source of Energy"**, Second Edition Publ. Kuban State Agrarian University, Krasnodar Russia, 2000.
10. A.N. Dadaev, "Astrophysics and Casual Mechanics", Galilean Electrodynamics **11**, Special Issues 1, GED-East, 4-7 (2000).
11. C.K. Whitney, "Do Atoms Really Have 'States'?", Proceedings of PIRT VIII, London, Ed. M.C. Duffy (2002).
12. J. Matson, "Fermilab Finds New Mechanism for Matter's Dominance over Antimatter", Scientific American, May 20 (2010).
13. A.A. Nassikas, "Minimum Contradictions Physics and Propulsion via Superconducting Magnetic Field Trapping," SPESIF-2010, AIP Conference Proceedings. Volume 1208, pp. 339-349 (2010). Also Nassikas, A.A., "Magnetic Propulsion with Field Trapping Superconductors," 20090100276 Patent Application OBI , Greece (2009).
14. L.Iorio, C. Corda, "Gravitomagnetism and gravitational waves" arXiv.org > gr-qc > arXiv:1001.3951, 2010
15. L. Ying, **"Klystar"**, Publ. Tate Publishing & Enterprises, p.p. 292, ISBN-13: 978-1602476271, US 2007.
16. A.A. Nassikas, "The Cold Fusion as a Space-Time Energy Pumping Process", Proceedings of the 8th International Conference on Cold Fusion, Ed. F. Scaramuzzi, Societa Italiana di Fisica (2000).
17. A.A.Nassikas, "Santilli's Etherino Under the Claim for Minimum Contradictions", Hadronic Journal Vol.(3) (2008).
18. www.exploratorium.edu/origins/**cern**/ideas/**higgs**

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# Fibonacci Generation of Natural Numbers and Prime Numbers

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**Abstract.** At the international conference in hadronic mechanics 2005, organised at University of Karlstad, Sweden, in honour of Prof. Ruggero Maria Santilli's 70 years, we presented two novel methods to disclose hidden patterns generating prime numbers vs. composite numbers (Johansen 2006). The *first* of these methods, the *negative* approach, applied a geometrical interpretation of the natural numbers composed as joint products of 5- and 3-multiples and located at specified positions in a certain revolving structure. The complete formulas to deduce all multiples as 8x8 sets of related series from a basic 8x8-matrix originating from the first eight primes in this structure, were achieved in 2009 (Johansen 2010a). Thus, indirectly, a systematic pattern generating the totality of prime numbers also became exposed, as the complement set of the exhibited total set of composite numbers.

The *second* of the two methods presented in 2005, the *positive* approach, revealed a strict 1:1 correspondence between the location of prime numbers and a certain Fibonacci structure. However, no proof for this correspondence was provided. The present article represents a basic contribution towards an explanation of this correspondence. Further, the article represents a novel approach to reestablish number theory in general. This approach is argued to have implications also for conception of the general relation between mathematics and science, for philosophy of science, and for general understanding of irreversible systems.

The positive method was inspired by a result from our theory of *differential philosophy* and *philosophical informatics* (Johansen 1991b, 2008a), deducing the Fibonacci algorithm as the elementary "reality atom", hence also the generator of the pattern of primes vs. composites as a systematic epi-structure (Johansen 2006). The present article departs from detrivialisation of the natural numbers, in agreement with hadronic mathematics, initiated by Santilli (and also with the supra-mathematics developed by P. Rowlands), and reconstructs the field of natural numbers as strictly generated from the Fibonacci algorithm. This structuring is shown to be homologous to the conventional structuring of primes vs. non-primes. In part, connections between Fibonacci numbers and primes are approached by means of some key notions from perplex mathematics developed by J. Chandler.

Geno- and hypermechanics, initiated by Santilli, with related hadronic mathematics, have lifted and surmounted classical and quantum mechanics. By this the conventional scientific apprehension of reversibility as prior, has become inverted, relocating reversible systems to a subordinate class from superior *irreversibility*. From these advances physics, and related mathematics, have been able to reach a sufficiently sophisticated theoretical level to account for more complex, irreversible systems, not at least biological systems, as indicated by the results from hadronic biology achieved by C. Illert.

These advances of the hadronic sciences imply a more intimate relation between the structure of mathematics and the structure of natural systems. The same tendency holds for some other sophisticated developments covering complex irreversible systems, such as Rowlands' theory, the "global scaling theory" of H. Müller, and the perplex chemistry of Chandler.

The present article will provide further support and amplification in this respect. The article will argue that the very *foundations* of mathematics reveal a precise, generative and *irreversible* structuring, and that this irreversibility unfolds in a non-trivial structuring of the field of natural numbers, including generation of the precise pattern of primes vs. non-primes.

**Keywords:** Fibonacci numbers, number theory, prime numbers, genonumbers, perplex numbers, Zeckendorf's theorem, Pascal's triangle, differential philosophy, irreversibility

## CONTENT

Foreword.....2
Perplex numbers and size numbers.....4
Fibonacci constitution of perplex numbers.....7
Fibonacci constitution of size numbers from constitution of addition as operation.....15
Fibonacci constitution of perplex number gaps.....19
Fibonacci constitution of perplex Fibonacci molecules and number substances.....23
Fibonacci constitution of the Fibonacci atomic identity and the Fibonacci gap identity27
Fibonacci constitution of sizes of Fibonacci molecules.....33
Fibonacci constitution of the Fibonacci molecular identity.....39
Fibonacci gap space dynamics.....41
Subtraction revisited: Constitution of *minus* as operation by *Minus* as operation from Fibonacci dynamics.....45
Algorithmic transfigurations by Fibonacci subjects.....53
Multiplication revisited: Constitution of multiplication as operation from Fibonacci dynamics.....63
Zeckendorf Fibonacci mathematics.....76
Fibonacci constitution of Pascal's triangle distribution of Fibonacci molecules.....82
Fibonacci constitution of split code 5:3 for the Pascal's triangle distribution.....87
Fibonacci constitution of prime numbers and composite numbers.....92
Closing remarks concerning irreversibility.....97
Acknowledgments.....101
References.....101

## FOREWORD

This is a pioneering attempt to reestablish number theory in its very foundations, as a systematic regeneration from the primordial, dynamising "reality atom" of the Fibonacci algorithm. As other scientific endeavors detrivialising some knowledge

conventionally considered basic, departing from an unexpected harbour with an unexpected ship, navigating across an unmapped ocean to arrive at unknown territory, and from there back-reflect the convention as reconstructed; our expedition may appear somewhat strange and non-familiar in first approximation, perhaps invoking prejudice of much being said without probably leading to much novelty. However, the proof of apples of knowledge lies in their digestion, and sometimes this might be a bigger bite of somewhat exotic flavours.

At a certain point complex dynamics of constitutional logic approaches the dotted or murky line to dialectics, as more specifically comprehended as constitutional dialectics presented in Hegel's *Science of Logic* and the four *Vorchapters* of late Marx' *Das Kapital*. Considered as a sublime art of science, dialectics is a highly demanding discipline, due to the complexities involved, among other things making it difficult to draw the exact line between thought operators that "hit the mark" (Bohm) and those that carry the analyst away from the mark by gifted, self-seductive fantasy. The reflection structures of dialectical thought forms are highly complex, and the ontological status of the *subject* inside a dialectical thought form itself vs. the external subject of the analyst performing or unfolding such a thought form, is not an easy issue. Here the question arises: If the analyst performing the dialectics describes a subject inside a dialectical thought form as if it was operating autonomously outside himself, i.e. as emphatically real, is the truth claimed by such description *really* for real, or is it just a *projective* claim and illusion from the subject of the analyst? And from which scientific or philosophic *criteria* do we competently *judge* the distinction(s) between these two? In profound and intricate scientific issues this line can be hard to draw. In this regard, there may seem to be some Scylla vs. Charybdis theme around, with the danger of the judgment becoming seduced in both directions, either explaining away the argued subject as *only* a projection, hence stripping it for any possibility of genuine life; or worshipping the argued subject as the really real, hence obeying to it as a wonder-subject without critical examination. In our treatment we found it adequate to not stay too avoidant or concerned about touching or crossing the line into dialectics, while at the same time not being too sure about what exactly is really real in all such respects. Thus, one may well interpret our formulations of Fibonacci subjectifications in the most stripped, sober and minimalistic sense, as *merely* metaphors and anthropomorphisms; but one may *also* at the same time interpret them with some opening for being *more* than just such, i.e. as para-metaphors or quasi-anthropomorphisms. Perhaps some insights may arrive from such double-reading, more or less intuitively, handed out from fluttering of the interpretative paradox.

As a pioneering endeavor, regarding some of its clear mathematical results – as well as some of it scientific suggestions – to become known by those it may concern as a somewhat urgent matter, our presentation will not *in toto* have the form of a completed *Darstellung* (systematic exhibition from a revealed *Wesen*) in the sense of German dialectical philosophy, and will also have to include elements of *exploration*. Hence, there will occur shortcomings and lacunas in our presentation, and there may also occur possible inconsistencies or mistakes, minor or not that minor. In such an endeavor there is a dynamical balance between being brave and not too brave, with

pitfalls at both sides, and it is not easy to walk this tiny line across Niagara in all issues and all respects.

In constitutional logic the more profound questions and answers tend not to be technically complex, but *philosophically* intricate, involving aspects of adequate ontology, epistemology, causality, logics and informatics. Technical sophistication and complexification *arise* from such constituents. Acknowledging the more profound challenges *not* to be of technical nature, we have sought to minimize technical complexity, and reduce formal expressions to a minimum. As previously experienced in a conclusive, ambitious work in economics (Johansen 1991a), concise back-reflection into what is enfolded in celebrated quantities and equations may show crucial for scientific break-throughs, including achievements of novel quantitative expressions. Another aspect of this is to present the treatment as accessible to readers without much technical skills or without much familiarity with Fibonacci mathematics, due to the possible significance of insights surmounting Fibonacci mathematics as a specialised field. In general, formula fetishism sometimes functions as a seductive device to *obstruct* more profound understanding and scientific progression from there, as in much economics (while perhaps more as a *window* in a case as the Dirac equation). On the other hand potent formula expressions and expansions become catalysed and highly significant *when* their sound underpinnings have been recognised and established, whether by good intuition or by more conscious meta-scientific effort. Also, our treatment has been concerned about yielding novel mathematical *results* of significance, and thus not much hermetically oriented towards mathematical philosophy *per se* as main concern. Some such results are presented during our treatment, and others are likely to arrive in upcoming extrapolations.

Our treatment targets the refoundation and regeneration of number theory, "the queen of mathematics". At the same time, constitutional implications for other fields of mathematics, especially geometry, unfold along the treatment. Also, it follows from the course of the treatment that the results are not restricted only to mathematics as a discipline of science, and perhaps nor only to science as a discipline of life.

## PERPLEX NUMBERS AND SIZE NUMBERS

Natural numbers can be interpreted in two different ways; either as i) the natural number indicating the *position* in an ordered sequence, as in a queue; or as ii) the natural number indicating the *size*, as the *amount* of elements residing in a set of some kind. The basic distinction between these two different meanings of natural numbers, as well as the fundamental differentiations and structures to investigate the dynamic interplay between the two meanings, has been established in the field of *perplex* mathematics, initiated by Chandler (2009), already having led to surprising and important results with respect to the basic relations of chemistry, and, in Chandler's view, holding great promise to yield surprising and important results in other disciplines as well, including other fields of mathematics *itself*. Connecting to the terminology introduced by Chandler, we will denote a natural number in the *first* meaning as the

*perplex number* of the natural number (by us denoted by the symbol *E*, with some connotation to this aspect being the most "elementary" in the constitutional logics of mathematics), and a natural number in the *second* meaning as the *size number* of the natural number. As an illustration, if we consider, say, the set (2,3,7,4,9), we can relate the natural number of 5 in two different ways to the set: i) 5 interpreted as *perplex* number to signify the fifth element (i.e. 9) inside the ordered set; ii) 5 interpreted as *size* number to signify that the set encompasses 5 elements.

In most mathematical studies the difference between these two different aspects of a natural number, corresponding to different – but related – *meanings* of what is to be understood by the very notion 'number', is not made explicit, due to the tacit meaning being sufficiently clear from the *context* of the mathematical treatment. However, in some basic and intricate issues of number theory, the treatment may profit from making such meanings explicit, to avoid confusing the two aspects when moving between them. In fact, there may be a hidden systematics around that *requires* such a non-conflating approach to yield interesting results.

Perplex numbers and size numbers do not exist on an equal footing. Perplex numbers unfold from successive *differentiations in time* between somethings regarded discontinuously. If a something A is considered *before* a something B, A by definition is to be considered as the perplex *first* something, and B as the perplex *second* something. If something C is considered after the something B, C is considered as the *first* perplex something *after* B, and thus by transitive logic as the *third* something, and so on for proceeding somethings. We can write this as follows:

**TABLE 1. Basic perplex numbering**

| Perplex no. | Something |
|---|---|
| 1 | A |
| 2 | B |
| 3 | C |
| 4 | D |
| 5 | E |
| ... | ... |

If we halt after perplex number 2, look back to perplex number 1, and *count* the *amount* of somethings *aggregated* so far in our progression, we find *two* somethings, namely A and B. Performing the same operation halting after perplex number 3, we find *three* somethings, namely A, B and C; and so on. In this consideration two and three then are understood as *size* numbers, not as perplex numbers. We can write this as follows:

**TABLE 2. Basic size numbering**

| Perplex no. | Something | Aggregated somethings | Size number of aggregated somethings |
|---|---|---|---|
| 1 | A | A | 1 |
| 2 | B | A, B | 2 |
| 3 | C | A, B, C | 3 |
| 4 | D | A, B, C, D | 4 |
| 5 | E | A, B, C, D, E | 5 |
| ... | ... | ... | ... |

We notice that *in the next turn* there also manifests a *perplex* ordering of the size numbers, due to size number 1 occurring before size number 2 occurring before size number 3, etc. Further, this *second-order* perplexity is *homologous* to the first-order perplexity (and both are also homologous to the size numbers). This indicates the successive *interplay* between perplex numbers and size numbers in the constitution of natural numbers. Despite this homology appearing trivial for the most basic relations, we must keep in mind that the homology still relates to *different logical* operations.

Conventionally, we can not perform mathematical operations between entities considered *different* in qualia, such as adding apples and pears. However, if we *abstract* from all differences in qualia and *only* consider the entities as "somethings", i.e. as "somethings" *as such* as *still* the *same, most abstracted* qualia, namely beings of "something", there is no problem with performing such operations of unification and comparison. On the contrary, this is the contemplation that *should* be applied, in order to achieve the most fundamental constitution of number theory, due to representing the most abstract, universal and elementary treatment possible.

Technically, such a most abstract treatment will appear identical to a less abstract treatment which presupposes identical more concrete qualia, as for example when presupposing uniform units of apples being listed and counted. However, to achieve universality, we abstract from everything connected to the unit, apart from the *sole perplexity qualia* of the somethings. Thus, the unit reduces to the sole perplexity in the context of consideration. We can illustrate this by indexing the somethings by their perplex number:

**TABLE 3. Basic size numbering of perplexities**

| Perplex no. | Perplexities | Aggregated perplexities | Size number of aggregated perplexities |
|---|---|---|---|
| 1 | E1 | E1 | 1 |
| 2 | E2 | E1, E2 | 2 |
| 3 | E3 | E1, E2, E3 | 3 |
| 4 | E4 | E1, E2, E3, E4 | 4 |
| 5 | E5 | E1, E2, E3, E4, E5 | 5 |
| ... | ... | ... | ... |

By this, number theory is *purified* in its foundation, abstracting from anything by the unit apart from its pure perplexity. Quite another issue is to bring in fruitful qualifications of the unit at less fundamental levels of number theory.

An analogy may be the sign of a queue number, where the treatment abstracts from any materiality of the sign and only considers the (perplex) number indicated by the sign. Any sign must have one or another materiality, but *what* materiality this might be is considered irrelevant for the treatment.

From this basic reflection it becomes clear that perplex numbers are *prior* to size numbers in the logical constitution of numbers. As a simple illustration and aspect of this difference, after perplex number 5 we have only 5 number entities, while at size number 5 we have aggregated a constellation of 15 number entities (1+2+3+4+5). Size numbers are constituted *from* perplex numbers, while the opposite is not the case with respect to *first*-order perplex numbers, only for classes of *higher* orders of perplex numbers where size numbers act as intermediaries.

A further contemplation upon the category '*border*' as analysed in philosophical informatics (established in the so-called '*differential philosophy*' of Johansen 1991b and 2008a) resulted in a deduction, from pure and strict philosophy, of the Fibonacci algorithm as the universal-elementary "reality atom", providing the basic bridge between the qualitative and quantitative aspects of Nature (Johansen 2006). This has been referred to as "the Fibonacci-Johansen paradigm" by number theorist Schadeck (2008). From this result, the role of numbers in algorithms of natural systems was to be understood as more or less complex epi-structures generated by the Fibonacci algorithm, and the same was argued to be the case with respect to the role of prime numbers in mathematics and cognition. This implied a radical *inversion* of conventional mathematical understanding which considered natural numbers as constituting the *most* "natural" numbers, and prime numbers as constituting the *most* "prime" numbers. From this the challenge emerged to *explain* the structuring of natural numbers and prime numbers as *generated* from *underlying* Fibonacci structuring, acknowledging the *last* one as 'Ground Zero' for understanding of mathematics and The World.

## FIBONACCI CONSTITUTION OF PERPLEX NUMBERS

Consistent with this, acknowledging that number emphatically or essentially *is* Fibonacci, fig. 1 starts out with a *perplex* numbering of the Fibonacci numbers and generates systematically the whole field of natural numbers from this, *both* in the meaning of perplex natural numbers and size natural numbers.

**FIGURE 1. Fibonacci generation of the natural numbers** © Stein E. Johansen

**F1** (1)
**F2** (1)
**F3** (2)
**F4** (3)
(4) (2,4) F2+F4=**1+3**=4
**F5** (5)
(6) (2,5) F2+F5=**1+5**=6
(7) (3,5) F3+F5=**2+5**=7
**F6** (8)
(9) (2,6) F2+F6=**1+8**=9
(10) (3,6) F3+F6=**2+8**=10
(11) (4,6) F4+F6=**3+8**=11
(12) (2,4,6) F2+F4+F6=**1+3+8**=12
**F7** (13)
(14) (2,7) F2+F7=**1+13**=14
(15) (3,7) F3+F7=**2+13**=15
(16) (4,7) F4+F7=**3+13**=16
(17) (2,4,7) F2+F4+F7=**1+3+13**=17
(18) (5,7) F5+F7=**5+13**=18
(19) (2,5,7) F2+F5+F7=**1+5+13**=19
(20) (3,5,7) F3+F5+F7=**2+5+13**=20
**F8** (21)
(22) (2,8) F2+F8=**1+21**=22
(23) (3,8) F3+F8=**2+21**=23
(24) (4,8) F4+F8=**3+21**=24
(25) (2,4,8) F2+F4+F8=**1+3+21**=25
(26) (5,8) F5+F8=**5+21**=26
(27) (2,5,8) F2+F5+F8=**1+5+21**=27
(28) (3,5,8) F3+F5+F8=**2+5+21**=28
(29) (6,8) F6+F8=**8+21**=29
(30) (2,6,8) F2+F6+F8=**1+8+21**=30
(31) (3,6,8) F3+F6+F8=**2+8+21**=31
(32) (4,6,8) F4+F6+F8=**3+8+21**=32
(33) (2,4,6,8) F2+F4+F6+F8=**1+3+8+21**=33
**F9** (34)
(35) (2,9) F2+F9=**1+34**=35
(36) (3,9) F3+F9=**2+34**=36
(37) (4,9) F4+F9=**3+34**=37
(38) (2,4,9) F2+F4+F9=**1+3+34**=38
(39) (5,9) F5+F9=**5+34**=39
(40) (2,5,9) F2+F5+F9=**1+5+34**=40
(41) (3,5,9) F3+F5+F9=**2+5+34**=41

(42) (6,9) F6+F9=8+34=42
(43) (2,6,9) F2+F6+F9=1+8+34=43
(44) (3,6,9) F3+F6+F9=2+8+34=44
(45) (4,6,9) F4+F6+F9=3+8+34=45
(46) (2,4,6,9) F2+F4+F6+F9=1+3+8+34=46
(47) (7,9) F7+F9=13+34=47
(48) (2,7,9) F2+F7+F9=1+13+34=48
(49) (3,7,9) F3+F7+F9=2+13+34=49
(50) (4,7,9) F4+F7+F9=3+13+34=50
(51) (2,4,7,9) F2+F4+F7+F9=1+3+13+34=51
(52) (5,7,9) F5+F7+F9=5+13+34=52
(53) (2,5,7,9) F2+F5+F7+F9=1+5+13+34=53
(54) (3,5,7,9) F3+F5+F7+F9=2+5+13+34=54

**F10 (55)**

(56) (2,10) F2+F10=1+55=56
(57) (3,10) F3+F10=2+55=57
(58) (4,10) F4+F10=3+55=58
(59) (2,4,10) F2+F4+F10=1+3+55=59
(60) (5,10) F5+F10=5+55=60
(61) (2,5,10) F2+F5+F10=1+5+55=61
(62) (3,5,10) F3+F5+F10=2+5+55=62
(63) (6,10) F6+F10=8+55=63
(64) (2,6,10) F2+F6+F10=1+8+55=64
(65) (3,6,10) F3+F6+F10=2+8+55=65
(66) (4,6,10) F4+F6+F10=3+8+55=66
(67) (2,4,6,10) F2+F4+F6+F10=1+3+8+55=67
(68) (7,10) F7+F10=13+55=68
(69) (2,7,10) F2+F7+F10=1+13+55=69
(70) (3,7,10) F3+F7+F10=2+13+55=70
(71) (4,7,10) F4+F7+F10=3+13+55=71
(72) (2,4,7,10) F2+F4+F7+F10=1+3+13+55=72
(73) (5,7,10) F5+F7+F10=5+13+55=73
(74) (2,5,7,10) F2+F5+F7+F10=1+5+13+55=74
(75) (3,5,7,10) F3+F5+F7+F10=2+5+13+55=75
(76) (8,10) F8+F10=21+55=76
(77) (2,8,10) F2+F8+F10=1+21+55=77
(78) (3,8,10) F3+F8+F10=2+21+55=78
(79) (4,8,10) F4+F8+F10=3+21+55=79
(80) (2,4,8,10) F2+F4+F8+F10=1+3+21+55=80
(81) (5,8,10) F5+F8+F10=5+21+55=81
(82) (2,5,8,10) F2+F5+F8+F10=1+5+21+55=82
(83) (3,5,8,10) F3+F5+F8+F10=2+5+21+55=83
(84) (6,8,10) F6+F8+F10=8+21+55=84
(85) (2,6,8,10) F2+F6+F8+F10=1+8+21+55=85

(86) (3,6,8,10) F3+F6+F8+F10=**2+8+21+55**=86
(87) (4,6,8,10) F4+F6+F8+F10=**3+8+21+55**=87
(88) (2,4,6,8,10) F2+F4+F6+F8+F10=**1+3+8+21+55**=88

**F11 (89)**

(90) (2,11) F2+F11=**1+89**=90
(91) (3,11) F3+F11=**2+89**=91
(92) (4,11) F4+F11=**3+89**=92
(93) (2,4,11) F2+F4+F11=**1+3+89**=93
(94) (5,11) F5+F11=**5+89**=94
(95) (2,5,11) F2+F5+F11=**1+5+89**=95
(96) (3,5,11) F3+F5+F11=**2+5+89**=96
(97) (6,11) F6+F11=**8+89**=97
(98) (2,6,11) F2+F6+F11=**1+8+89**=98
(99) (3,6,11) F3+F6+F11=**2+8+89**=99
(100) (4,6,11) F4+F6+F11=**3+8+89**=100
(101) (2,4,6,11) F2+F4+F6+F11=**1+3+8+89**=101
(102) (7,11) F7+F11=**13+89**=102
(103) (2,7,11) F2+F7+F11=**1+13+89**=103
(104) (3,7,11) F3+F7+F11=**2+13+89**=104
(105) (4,7,11) F4+F7+F11=**3+13+89**=105
(106) (2,4,7,11) F2+F4+F7+F11=**1+3+13+89**=106
(107) (5,7,11) F5+F7+F11=**5+13+89**=107
(108) (2,5,7,11) F2+F5+F7+F11=**1+5+13+89**=108
(109) (3,5,7,11) F3+F5+F7+F11=**2+5+13+89**=109
(110) (8,11) F8+F11=**21+89**=110
(111) (2,8,11) F2+F8+F11=**1+21+89**=111
(112) (3,8,11) F3+F8+F11=**2+21+89**=112
(113) (4,8,11) F4+F8+F11= **3+21+89**=113
(114) (2,4,8,11) F2+F4+F8+F11=**1+3+21+89**=114
(115) (5,8,11) F5+F8+F11=**5+21+89**=115
(116) (2,5,8,11) F2+F5+F8+F11=**1+5+21+89**=116
(117) (3,5,8,11) F3+F5+F8+F11=**2+5+21+89**=117
(118) (6,8,11) F6+F8+F11=**8+21+89**=118
(119) (2,6,8,11) F2+F6+F8+F11=**1+8+21+89**=119
(120) (3,6,8,11) F3+F6+F8+F11=**2+8+21+89**=120
(121) (4,6,8,11) F4+F6+F8+F11=**3+8+21+89**=121
(122) (2,4,6,8,11) F2+F4+F6+F8+F11=**1+3+8+21+89**=122
(123) (9,11) F9+F11=**34+89**=123
(124) (2,9,11) F2+F9+F11=**1+34+89**=124
(125) (3,9,11) F3+F9+F11=**2+34+89**=125
(126) (4,9,11) F4+F9+F11=**3+34+89**=126
(127) (2,4,9,11) F2+F4+F9+F11=**1+3+34+89**=127
(128) (5,9,11) F5+F9+F11=**5+34+89**=128
(129) (2,5,9,11) F2+F5+F9+F11=**1+5+34+89**=129

| | | | |
|---|---|---|---|
| (130) | (3,5,9,11) | F3+F5+F9+F11=**2+5+34+89**=130 | |
| (131) | (6,9,11) | F6+F9+F11=**8+34+89**=131 | |
| (132) | (2,6,9,11) | F2+F6+F9+F11=**1+8+34+89**=132 | |
| (133) | (3,6,9,11) | F3+F6+F9+F11=**2+8+34+89**=133 | |
| (134) | (4,6,9,11) | F4+F6+F9+F11=**3+8+34+89**=134 | |
| (135) | (2,4,6,9,11) | F2+F4+F6+F9+F11=**1+3+8+34+89**=135 | |
| (136) | (7,9,11) | F7+F9+F11=**13+34+89**=136 | |
| (137) | (2,7,9,11) | F2+F7+F9+F11=**1+13+34+89**=137 | |
| (138) | (3,7,9,11) | F3+F7+F9+F11=**2+13+34+89**=138 | |
| (139) | (4,7,9,11) | F4+F7+F9+F11=**3+13+34+89**=139 | |
| (140) | (2,4,7,9,11) | F2+F4+F7+F9+F11=**1+3+13+34+89**=140 | |
| (141) | (5,7,9,11) | F5+F7+F9+F11=**5+13+34+89**=141 | |
| (142) | (2,5,7,9,11) | F2+F5+F7+F9+F11=**1+5+13+34+89**=142 | |
| (143) | (3,5,7,9,11) | F3+F5+F7+F9+F11=**2+5+13+34+89**=143 | |

**F12 (144)**

*Colour codes:*
Far left column: Perplex Fibonacci numbers in bold violet.
Second left column(s): Perplex Fibonacci atoms in bold green; perplex Fibonacci molecules in green.
Third left column(s): Fibonacci molecules in violet.
Fourth left( column(s): Addition of Fibonacci atoms in violet; addition of their values in green; sum of their values in blue.

Leaving aside conventional issues of what may be termed "creation logic", to not rush pre-maturely into intricacies of meta-physics, meta-mathematics and meta-philosophy, *something* must in any case be considered the *first* in any context. In *our* context, which is (at least first-hand) Fibonacci constitution of numbers, we conceive this *first* something, maximally abstracted, as pure 'number', i.e. indexed as *perplex* number *one*. In virtue of being the *first* perplex number (or rather: maximally abstracted entity) the first number can not be distinguished from anything else that precedes itself. On the other hand, it can not *exist* without a distinction to what it is not, which must be something *already* co-existing with it. This paradox is solved by the first perplex number being distinguished to the *absence* of itself, i.e. by this *relative nothing*. Interpreted as a sequence in *time*, this means that the first perplex number is distinguished to a *preceding* absence of itself. Thus, its precursor must be understood as *paradoxical* with respect to its very existence; namely i) as *nothing* in *itself*; and ii) as still a certain *something*, as far as it is the absence of perplex number *one* which *is* something, and hence it is a *relational* something. This means that the paradoxical quasi-perplex number *zero* only can be considered something as a *back-projection* from perplex number one.

As soon as zero is realised as something in this particular sense, perplex number one *re-manifests* as a *forward result* of this zero, *combined* with number one itself as considered *before* back-projecting onto zero. Therefore, this re-manifestation is, strictly speaking, to be interpreted as *different* from and as *proceeding* from perplex number

one (via quasi-perplex number zero as intermediary), i.e. as the *second* perplex number. *Logically*, the second perplex number differs from the first perplex number by being constituted by the first number *and* the absence of the first number (i.e. zero), while zero was *absent* in the case of perplex number one in its immediate presence. We may write this as: E2 = E1 & (not-E1), where '*&*' denotes a logical succession in time, *back-reflecting* on E1 before making up E2. In this reflexive logic we recognise the structure of the Fibonacci algorithm; something stepping back to bring a preceding something with itself into the next something. On the other hand, with respect to *ontological extension* it is *not* possible to distinguish perplex number two from perplex number one, since the *logical* segment (not-E1) has *no* (first-order) ontological extension and is therefore invisible in the ontological extension of perplex number two. Thus, with respect to ontological extension perplex number two is *identical* to perplex number one, despite the *logical* distinction between the two numbers. Also in this respect we recognise the structure of the Fibonacci algorithm, due to the value of Fibonacci number two being identical to the value of Fibonacci number one, which is 1. Expressed more concisely, without needing the concept 'value': Fibonacci number two and one are identical, i.e. *one and the same*, in all *other* regards than their perplex numbering. Due to the constitutional logic of perplexity as such, the second perplex number must have *ontological* identity to the first perplex number. In this sense, perplex number two is a *copy* of perplex number one; however a copy that is an off-spring where the progressing capacity is kept in tact.

Thus, we realise that the unique and paradoxical relations between perplex numbers one and two (via zero) are *identical* to the unique and paradoxical relations between *Fibonacci* numbers one and two (via zero). This is consistent with our previous *general* deduction of the Fibonacci algorithm as the reality atom from strict differential philosophy, and represents the specific *manifestation* of this with regard to the important and fundamental case of the constitutional logic for numbers as such. This suggests that the study and explanation of numbers from perplexity is *one and the same* as the study and explanation of numbers from the Fibonacci algorithm, and gives significance to fig. 1 as *the* point of departure for re-establishing number theory as profoundly and purely as possible. In their foundational make-up the perplex numbers *are* the Fibonacci numbers and nothing else, which will be further clarified in the forthcoming.

The same identity between perplexity and Fibonacci repeats when inspecting the relation between perplex numbers two and three. Let us first look back a bit more on the relation between perplex numbers one and two. With respect to uniqueness in unfolding logical positioning, we had E2 = E1 & (not-E1), with the joint (not-E1) distinguishing E2 from E1. (not-E1) was to be interpreted as the paradoxical perplex quasi-number E0, i.e. the perplex number imagined as *immediately preceding* E1. Thus, the *distinction* between E2 and E1, expressed by the joint (not-E1), was equivalent to the perplex *number* before E1. Paradoxically, then, a distinction *between* two perplex numbers next to each other, i.e. E1 and E2, was identical to a particular *number*, namely the perplex quasi-number imagined as immediately preceding the first of these two numbers. Hence, we recognise this *general* peculiarity of Fibonacci numbers already in

the relation between the *first* perplex numbers. This indicates that this peculiarity is enfolded in the originating *constitution* of *perplex* numbering as such. Such an insight required reflection on the perplex quasi-number E0, which therefore appears to have merit as more than merely a philosophical exercise per se. E0 could not be imagined before *after* E1, and in *this* respect E0 is not the *pre*-perplex number, but the *intermediary* quasi-perplex number in the constitution of E2 from E1; i.e. rather to be denoted as E1½ than as E0. Thus, there is the paradox here that zero, the absence of E1, in the constitutional logic with necessity *appears after* E1, while at the same time this absence of anything, viewed from the reference point of E1, is *positioned before* E1 by looking *back* from E1 and then not finding anything there. The *same* form of paradox is *universally* characteristic for the Fibonacci algorithm, always constituting the next *forward* Fibonacci number $F_{n+1}$ by looking *back* from the reference point of $F_n$ to the Fibonacci number positioned most closely *before* $F_n$, then finding $F_{n-1}$ there, and then take the next step and make up $F_{n+1}$ by combining $F_{n-1}$ with the reference number $F_n$ itself. Thus, $F_{n-1}$ preceding $F_n$, *re-enters* the Fibonacci sequence as an *intermediary* between $F_n$ and $F_{n+1}$. This *twice role* of $F_{n-1}$ follows from a *formal* contemplation on the Fibonacci succession, and may seem trivial at first glance, but hardly much so when realising that the *same* twice role is organically embedded in the generative succession of *perplex* numbering:

After E2 has manifested, the genuine *novelty* of E2 emerges from *what* E2 is novel in *distinction* to. This happens by *back-reflection* from E2 to its precursor which is E1. Without looking back there would not be any *comparison* to establish the novelty. Looking back from E2, *something* is found there (different from when looking back from E1), namely E1. Thus, E2 is comprehended as *different* from this something, and by this as a *novel* perplex number and as a *novel* something in this respect. Then it follows as next reflection that there are *two* somethings around, E1 and E2, and by this also the *combination* of these two somethings, which in its immediate apprehension is a *third* something. Thus, E3 becomes "born" and manifests *as* the combination of E2 and E1. In this constitution of E, we recognise a certain tandem succession in the roles of *both* E2 and E1: E2 was born and manifested from E1 (and its absence); then E1 was found by back-reflection from E2; then E2 was realised as *novel* qua distinguished from E1; and then E3 was born as the combination of the two different somethings, E1 and E2, which the distinction *was between*. Written in short-hand:

(1) E1 -> 0 -> E1&0 -> E2 -> E1 -> E2(novel) -> E1&E2 -> E3.

With respect to E1, this closer inspection reveals that it plays a (discontinuous) twice role also *inside* its second role re-entering the succession as an intermediary between E2 and E3. We notice that the *formal* relations of the Fibonacci algorithm cover the perplex succession also from E2 to E3.

Obviously an analogous pattern repeats with regard to constitution of the next perplex number E4:

(2) ... E2 -> E1&E2 -> E3 -> E2 -> E3(novel) -> E2&E3 -> E4

The only interesting new feature in this succession is related to the back-reflection from E3 to its precursor E2. In this case the precursor, different from the back-reflection from E2 to E1, is not identical to *all* preceding somethings, due to E1 being camouflaged in the back-reflection from E3 to E2. It is only the distinction to the (closest and immediate) precursor which is relevant for establishing E3 (as well as perplex numbers larger than E3) as a *novel* perplex number, since a distinction to perplex numbers *lower* than the precursor would not establish E3 as a *larger* perplex number than those already established. Obviously, the same pattern repeats, without emergence of any new such features, in constitution of all perplex numbers larger than E4.

Thus, we realise from examining the *entities of reflection* tacitly *enfolded* in the generative succession of perplex numbering, that the generative form of perplexity is *identical* to the Fibonacci form insofar as $E_{n-1}$ always *reenters* as an intermediary in the constitution of a novel perplex number $E_{n+1}$ from a manifested perplex number $E_n$. One might say that $E_{n-1}$ reenters as a forecasting *shadow* in the constitution of $E_{n+1}$ from $E_n$, or one might say that while $E_n$ is the *mother* of $E_{n+1}$, is $E_{n-1}$ the *father* of $E_{n+1}$.

At first glance perplex numbering appears as a trivial operation of indexing by just adding 1 more to the last index all the time. Our closer analysis reveals this indexing as a *supra-structure* surfacing from a strict underlying generative dynamics with non-trivial formal identity to the Fibonacci algorithm. In the simplistic '1 more'-indexing this generator is tacitly implied in the interstices between the indexes, but hidden and unconscious in ordinary mathematical thought. The simplistic indexes *label* the perplex distinctions in succession, but does not describe or explain the generative *production* of these distinctions. Thus, the simplistic indexing *covers* its deeper constitutive logic, and may be contemplated as *Schein* (semblance) in the sense of Hegel. And the generative constitution of perplexity has the *Fibonacci form*, indicating this form being the key to understand the *organic* constitution of numbers in their most elementary, abstract and universal form.

(Also when applying '1 more'-indexing to list Fibonacci numbers, we ought to keep in mind that such labeling indexes are logically *secondary* to the self-referential *generative* constitution of Fibonacci numbers. Labeling finds place *after* manifestation of that which is to become labeled. Despite the fruitfulness for different scientific purposes of detrivialise, break up and re-advance ordinary mathematical thought about natural numbers and counting, one should also respect the impossibility of substituting *in toto* the thought economics of ordinary thinking (cf. Johansen 2008a, ch. 2.1.2). Just as the Fibonacci walk itself always has to be the alternation between moving forward and backward, between progressing higher and projecting lower, the quest is more to seek the *adequate alternation*, depending on the task at hand, between convention and profound novelty, and in the present case between ordinary thinking about numbers and profound re-establishment of number theory.)

Now we will investigate whether the form of the Fibonacci algorithm explains generation of perplex numbers also in *other* regards than those already discovered.

# FIBONACCI CONSTITUTION OF SIZE NUMBERS FROM CONSTITUTION OF ADDITION AS OPERATION

As stated, there occurs no difference between E1 and E2 with respect to *ontological extension*, only with respect to (number) ontological *positioning* indicated by the difference in perplex numbering. This picture changes when progressing to the relation between E2 and E3, due to E3 manifesting from the *combination* of E2 and E1, this combination trivially being more extended than E2 since E1 is not nothing. Further, since E1 and E2 was *not* different with respect to ontological extension, the extension of E3 as combination of E1 and E2 must be *identical* with the combination of the extensions of E1 and E1, as well as with the combination of the extensions of E2 and E2.

The concept 'ontological extension' is to be understood in the most abstract sense, as the qualia criterion distinguishing all somethings on one hand from nothing on the other hand. Somethings have a *positive*, first hand existence, while nothing only has *ad negativo*, second hand existence as *absence* of somethings. The first perplex number, E1, is stated positively *without* any preceding nothing (different from the case of E2) or any other preceding something (also different from the case of E2). To be first, there must be *something* that is given the attribute 'first'. Perplexity in the pure sense abstracts from *whatever* this first something might be. However, also as *anything* it is considered as something different from nothing. As pure perplexity, numbers are positioned *relatively* to each other, without regard to any qualification of absoluteness. However, they still have to be positioned *somewhere*, at least with an *imaginary* extension in *logical spacetime* to be *distinguished* from each other and to not overlap completely in spacetime coordinates. Somethings can only be relative to each other if they also are distinguished *somethings* in a minimum of *absolute* sense, to *become* related as relative. There is no such thing as relativity without absoluteness to relate. Thus, even a *logical* position remains a *position*, extended at a *place*, whatever small, *different* from the position of that which it relates *to*.

Therefore, perplex numbers must, qua *somethings*, be considered as having ontological extensions, whatever minimal or point-like, in *some* kind of space. From this basic reflection on perplexity as such, it follows that number theory is related to *geometry* in its very foundation; i.e. that geometry is *enfolded* already in the *most* abstract constitution of numbers. (Here it may be of interest to note that the *Johansen Revolving Prime Number Code* was discovered and deduced (Johansen 2010a). from a particular *geometric* structuring and positioning of the natural numbers.) Hence, also this result indicates a non-trivial and *fundamental* interconnection between geometry and natural numbers.)

We denote the ontological extension of the first perplex number, E1, with the Latin number symbol *I* to distinguish this from the perplex numbering. As a *function* of perplex number, we denote ontological extension as $e(E_n)$. Denoting 'combination' with the symbol &, we have:

(3) $e(E_1) = I$
(4) $e(E_2) = e(E_1)$

From the interpretation of (1) we have:
(5) $e(E_3) = e(E_1)$ & $e(E_2)$

From (4) and (5) we have:
(6) $e(E_3) = e(E_2)$ & $e(E_2)$
and:
(7) $e(E_3) = e(E_1)$ & $e(E_1)$

(3) and (7) give:
(8) $e(E_3) = I$ & $I$

From the interpretation of (2) we have:
(9) $e(E_4) = e(E_2)$ & $e(E_3)$

(3), (4), (8) and (9) give:
(10) $e(E_4) = I$ & (I&I)

In analogy with (2) we obviously have:
(11) $e(E_5) = e(E_3)$ & $e(E_4)$

(8), (10) and (11) give:
(12) $e(E_5) = (I\&I)$ & [I&(I&I)]

Inspecting (10) and (12) it is obvious that for *any* perplex number $E_n$, $e(E_n)$ is *identical* to the *value* of the *Fibonacci* number $F_n$ for the same n, if – and only if:

1. Combination operation & is interpreted as *addition*.
2. The *brackets* in the expressions for function *e*, such as in (10) and (12), are removed.

It is not trivial to automatically allow any of these two conditions.

The generative logic of perplexity was analysed as tacitly identical to an *essential* characteristic of the Fibonacci algorithm, more specifically to the *form generative dynamics* of the algorithm. However, we did not by this claim that the generation of perplexity was identical to the Fibonacci generation in *all* respects, i.e. also to the *content* generative dynamics of the Fibonacci algorithm as displayed by the *values* of the progressing Fibonacci numbers. Now it shows that also such an identity with respect to *content* follows *if* the two conditions are satisfied. From such satisfaction, generative perplexity emerges *totally* as just a manifesting supra-structure, or more precisely: as *Erscheinung* (appearance) in the sense of Hegel, from the underlying workings of the generative Fibonacci algorithm. When/if so, the *e*-function from perplex progression gives *size* numbers as identical to the *values* of the Fibonacci

numbers. (We apply the notion 'value' as a preliminary *before* arriving competently to pinpoint it from an exact and adequately matured typology of numbers.)

However, the stated two conditions reveal the need for careful contemplation and some thoughtful hesitation before arriving at strong conclusions with respect to the totality of the Fibonacci-perplexity relation.

If we first investigate the *second* condition, removing the brackets corresponds to ontological *conflation*, deleting distinctions between different ontological levels, orders and dimensions. (For the fruitfulness – not to say necessity – of acknowledging strict and systematic ontological differentiations in the overall frameworking of ambitious scientific endeavors, see especially Johansen 2008a, and also Bohm 1987.) In some cases such conflation may be justified from scientific treatments able to *tacitly* perform underlying adequate and context indicated differentiations, as not seldom the case with respect to advanced physicists. In other cases such conflation may be adequate to simplify and catalyse broad and general treatments, such as in conventional formal logics and set theory. However, in more intricate issues, careful ontological differentiations may be crucial and yield far-reaching implications, as for example indicated by the critique of Gödel's theorem(s) achieved in Johansen (2006) from differential philosophy and related surpassing of the ontological restrictions implied in conventional formal logic (cf. Johansen 2008a: ch. 3.1.2). Also, recent important advances in science, especially in physics, imply radical ontological expansions including non-trivial ontological differentiations (cf. Bohm 1987, Santilli 2008, Rowlands 2007, Shikhobalov 2008, Rapoport 2010). One important aspect of this is information traffic between spacetimes distinguished horizontally (as matter vs. anti-matter universe), vertically (as in hadronic mathematics and mechanics) or interdimensionally (as in Klein-bottle physics).

On the other hand, it is obvious that the bracket structuring related to the second condition is structurally *identical* to the generative structuring of the successive Fibonacci numbers. Hence, the bracket structuring is directly transparent from the generative *form* of the Fibonacci succession, and can be regarded as *manifestation* of this generative Fibonacci form. If we satisfy condition two and remove the brackets, the structuring is still easy to *re*-construct from the anchoring Fibonacci form. Hence, conflation from removing the brackets is not much seductive as long as this is performed in such a context which preserves a direct connection to the Fibonacci form. Further, one can regard the stepwise structuring of the Fibonacci form as a *basic* distinguishing *from* which to make ontological differentiations (including in different directions and dimensions), and in this sense the anchoring of brackets removal to the Fibonacci form functions as some guarantee *against* ontological conflation.

This implies that *if* also the first condition is satisfied, it is legitimate to satisfy also the second condition, presupposing this being performed in an anchoring *Fibonacci* context. Here the Fibonacci connecting context is not imposed from *outside* perplexity, but is to be understood as the required *unfoldment* and rewrite of perplexity to ensure that the ontological differentiations of form expressed by the perplex brackets *remain the same* also after the perplex brackets are removed.

Then the values of the Fibonacci numbers can be interpreted as *size* numbers directly deduced from perplex progression as such, with *1* denoting the uniform *unit* of ontological extension. Or, expressed slightly otherwise, perplexity generates size numbers that *are* the values of the Fibonacci numbers; which implies that the Fibonacci values do not manifest from any other operation *outside* perplexity.

Let us then investigate the *first* condition. While the second condition concerns *ontology* and therefore is quite general, the first condition seems to concern specification of logical-mathematical *operation*. *Why* should *combination* (&) be understood as the operation *addition*, and not for example as the operation multiplication? Could it make sense that combination *as such* implies a particular *mode* of combination?

We have seen that ontological extension is implied in perplexity. For example (cf. (5) ), the extension of E3 is the combination of the extension of E1 and the extension of E2. Since E3 is a *novel* perplex number, the extensions of E1 and E2 must become combined to the extension of E3 as a *unitary whole*. This implies that the *distinction* between the extensions of E1 and E2 has become *deleted*, i.e. that the *line* drawn between them as topological objects has been wiped out. E1 and E2 being *neighbouring* perplex numbers implies that they also must be *neighbours* as ontological extensions; it is not possible to imagine anything *between* them, except from the line distinguishing them. Then E1 and E2 combine to the novelty E3 by simply this line between them becoming deleted. Nothing else *has* to change, and nothing else is *indicated* as changing from the most abstract formulation of E1 and E2 coming together into E3. Thus, the simple *fusion* of the extensions of E1 and E2 by removal of their distinguishing line is the *elementary* form combining the two extensions to the extension of E3. Therefore, *addition* of the two extensions constitutes the elementary form of their combination. And, from the general form of this reasoning, the same must be the case also for all other combinations of neighbouring perplex numbers. For $E_n$ larger than E3, though, born from combination of $E_{n-2}$ and $E_{n-1}$, there will be a *difference* in ontological extension *larger* than zero between the extensions of $E_{n-2}$ and $E_{n-1}$, namely $e(E_{n-3})$. However, this difference does not matter for the combining operation of $E_{n-2}$ and $E_{n-1}$, since the difference is not displayed as the dividing line between $E_{n-2}$ and $E_{n-1}$, because $E_{n-2}$ and $E_{n-1}$ are located as perplex *neighbours*.

We notice that this primacy of addition as the perplex combinatory operation manifests from an argument that has a *geometric* foundation. Again we discover, contrary to conventional mathematical thought, that geometric relations are inherently present in the constitutional logic of perplex numbers and hence tacitly enfolded in the very foundation of number theory as a whole.

From the primacy of addition as the elementary combination operation, also the *second* condition is satisfied. Thereby it is established as a quite important result that the constitutional logic of perplex numbers is *identical* to the generative logic of the Fibonacci algorithm, and now also with respect to the *content* of Fibonacci generation, not only with respect to its *form*. This result is consistent with our previous general deduction from differential informatics of the Fibonacci algorithm as the reality atom, and in this regard just a confirmation of the general result. However, this confirmation

may be important as well as further enlightening, since it is inferred inherently and organically from a quite ultimately abstract domain of science, namely number theory. Johansen (2008a) concludes that since

*number coding has a **universal** area of application at the same time as it is a **fundamental** and consequently **unavoidable** form of coding of all information, number coding can no longer be apprehended as one form of coding among many, but must be comprehended as the primary and universal form of coding in general, i.e. as Coding as such* (Johansen 2008a: ch. 2.1.2)

Further, the importance of said confirmation is reinforced by the fact that it arrives from a quite ultimately abstract sub-domain – or rather meta-domain – of number theory, namely perplex mathematics.

Thus, in short: The Fibonacci series and related mathematics is only *apparently* a sub-domain of number theory among many. When exploring the issue in more depth, it becomes clear that numbering *as such* is *constituted* from the Fibonacci algorithm and just represents the most abstract *manifestation* of this algorithm. The constitutional logic of perplex numbers shows to be *exactly* the form and content of the Fibonacci algorithm. This implies that a *radical inversion* is required, where number theory becomes re-established *explicitly from* the Fibonacci algorithm, rather than the latter to be considered a sub-structure voluntarily constructed inside and from a quasi-*autonomously* established theory of numbers.

At this point in our treatment, this may look just as a position inside hermetic mathematical philosophy. However, we will announce novel and important mathematical results to be achieved from this platform, hardly *possible* to develop without such.

Having established that the perplex numbers *are* the Fibonacci numbers, i.e. that $E_n=F_n$, from now on we can restrict our treatment of perplex numbers (and related dynamics of size numbers) to exploration of the generative logic of Fibonacci numbers as illustrated in fig. 1.

Due to the two conditions being satisfied, the *size* numbers deduced from perplex progression is given as the progressively additive values of the Fibonacci numbers, with the ontological extension of F1 being the *1* denoting the uniform unit for the size numbers.

## FIBONACCI CONSTITUTION OF PERPLEX NUMBER GAPS

Obviously, the Fibonacci numbering generates perplexity *logically* as a never ending self-referential progression. At the same time, due to the *addition* operation being the elementary form of combination yielding progressing ontological extension of the perplex Fibonacci numbers, there occurs a *branching point* between F4 and F5 where after there will occur imagined values of ontological extensions (the first one being 4 units) that are *not* identical with the value of ontological extension for any Fibonacci number. This discrepancy is *inherently generated* from Fibonacci perplexity

and the unit of F1; i.e. not anything that manifests from contemplating the Fibonacci series from a quasi-*external* and quasi-*autonomous* framework of number theory. Further, and more precisely, this inherent generation finds place in an *exact* succession, where the *units of units*, i.e. the amount of *second* order units constituting the gap between two perplex Fibonacci numbers, is given by the formula:

(13) $\langle F_n, F_{n+1} \rangle = F_{n-1} - 1$

This may be interpreted as *regeneration* from a *recursive short-cut*, where the perplex $F_{n-1}$ not only acts into constituting perplex $F_n$ and $F_{n+1}$ logically, and from there also into their ontological extensions, but also into constituting the *gap* between $F_n$ and $F_{n+1}$ by $F_{n-1}$'s *distance* in ontological extension to the primordial unit *I* of F1, i.e. the *gap* of *absent* units between F1 and $F_{n-1}$. Thus, the gap between $F_n$ and $F_{n+1}$ is determined *ex ante* from $F_{n-1}$, *not ex post* from comparing $F_n$ and $F_{n+1}$. This represents the branching quality (and from there: quantity) which *constitutes* the already stated branching discrepancy. This means that the ontological extension of a perplex Fibonacci number not only *contributes*, as one of the two additive parts, to the ontological extension of each of the two next perplex Fibonacci numbers, but that it *solely determines* (minus the primordial *I*) and *makes up* the *gap* between these two numbers. Here, we encounter the peculiarity that the *gap* between two Fibonacci numbers is made up and determined *before* the last of the two numbers appears.

Considering that $F_n$ and $F_{n+1}$ are perplex *neighbours*, it seems most adequate to regard said gap in ontological extension as the *depth* of the dividing line between them. This implies that the ontological extension of $F_{n-1}$ (minus the primordial unit *I* of F1), forerunning the ontological extensions of $F_n$ and $F_{n+1}$ along the *same dimensionality*, remanifests along another dimensionality, *orthogonal* to the first one. Contemplating the impossibility of conceiving space dimensionality as *less* than 3D other than from applying what has been denoted *'bordering-concepts'*, namely those concepts which are formed in a paradoxical manner by being conceived *as if* they could exist without classifying something being (Johansen 2008a: ch. 3.1.2), such as 'infinity', this circumstance seems significant and may have far-reaching implications:

1. Interpreted as 3D, the orthogonal, second-order 3D occurs in *addition* to the first-order 3D. Conceiving the Fibonacci algorithm as the reality atom, this indicates that *doubling* of 3D is implied already from reflection upon a fundamental relation in the constitution of *numbers*. Such doubling is consistent with recent advanced mathematical physics/metaphysics (Santilli/Illert and Rowlands) and also with the cubical to cube approach to organic geometry of Trell (1992,1998,2005,2009) where space is built from units of *three* dimensions successively expanding along three *outer* dimensions/directions. Thus, at this point we notice not only a fundamental interconnection between mathematics and physics/metaphysics, but also between number theory and *geometry*, where geometrical results are inferred from, as well as embedded into, constitutional number theory. This seems to imply that it is possible to reach important and non-trivial results about the structuring of the universe already from said reflection of constitutional number theory. This surpasses the conventional world

view in physics and science believing that there is only *one* physical 3D universe around.

2. The orthogonal, second-order 3D occurs as *generated from* the first-order 3D. Hence, these two dimensionalities do not occur at an equal ontological footing, but in a dimensional *succession.* The first-order 3D unfolds by means of the second-order 3D it has generated, somewhat like parents needing their children for economic survival.

3. Residing in the depth of the dividing line between perplex Fibonacci numbers, the second-order 3D is not only *separated* from the first-order 3D, but *invisible* from the reference frame of first-order 3D. This is analogous to the situation described in the novel *Flatland* (Abbot [1884]), and indicates that one should hesitate to rush to conclusions about the overall structure of the physical world from what is accessible by ordinary perception. Rather, one should seek an extension of the reference frame from adequate abstraction and reflection.

4. As already noted, $F_{n-1}$ (minus *1*) performs a *double* role; one role as providing ontological extension to both $F_n$ and $F_{n+1}$ (along with, respectively, $F_{n-2}$ and $F_n$) along the first-order 3D; the other role as *solely* providing ontological extension to the second-order 3D. This double role indicates ontological and dimensional *bifurcation* as *implied* in the reality atom of the Fibonacci algorithm, and hence as the ordinary state of affairs in the workings of the universe and natural systems. This is coherent with the understanding of ontological and dimensional bifurcation from geno- and hypermechanics, initiated by Santilli, confirmed by the important case study of Illert (1995) which revealed with mathematical rigour non-trivial information flows between different spacetimes for a class of bifurcating self-intersecting sea shells. To make sense of such bifurcation in a more intuitive way is quite a paradigmatic challenge, especially concerning non-trivial *time* flows. The double role of $F_{n-1}$ (minus *1*) may serve as a basic template in this respect, although it seems reasonable that bifurcation occurs at different ontological levels, the case analysed by Illert displaying a quite radical incident of such. Also, the bifurcating role of $F_{n-1}$ (minus *1*) may already indicate that there exist hidden constraints and quantitative laws for bifurcation, possible to discover, contrasted to more loose and somewhat speculative theories about "parallel universes" and "many-worlds" not much anchored in any elaborated or refined ontology.

5. The double role of $F_{n-1}$ (minus *1*) is *paradoxical*, due to one role being performed as extended *positively*, the other role as being performed *negatively* in the *gap* underneath the distinction (dividing line/surface) between positive extensions. Switching the reference frame from first-order 3D to second-order 3D, the gap becomes *positively* extended. Paradox operators are crucial in creations and developments of Nature as well as in recent extensions of logics (cf. Rosen 1994, Stern 1992, Johansen 2006, Rapoport 2009,2010), and the paradox of said double role represents a quite fundamental among such operators. Further, this paradox indicates that what is gap and what is filled, depends on the reference frame, and also may *appear* quite contrary to what is the case when viewed from an over-all *combination* of reference frames. (For ontologically combined descriptions of dynamic systems, see Johansen 2008a: ch.s 2.3, 3.1.6, 3.1.9.) Of special interest in this respect may be the issue of the universal substratum or ether. That the substratum is not immediately accessible to ordinary

perception can hardly represent any strong indication of that it does not exist. Congruent with this, the observations from extra-ordinary enhanced vision, described in the field named "occult chemistry", initiated by Besant and Leadbeater in the late 19th century, report exactly such an inversion where vacuum (*koilon*) is *filled*, and that *anu* atoms and objects are generated *from* vacuum via tiny, multi-layered gap-spirals emitted through small holes of vacuum. Also, these observations report vacuum to be extremely *densely* filled, which is congruent with some later energy calculations by physicists as well as with related downplayed high-energy technology. It is contradictory to consider vacuum as *both* empty and extremely densely filled from the *same* reference frame. When contemplated from an *extended combination* of reference frames, what *appears* as empty from our ordinary reference frame, must be very *richly* filled of somethings regarded from the other reference frame, hence not at all as empty but filled by those somethings that the tremendous energy is *embedded* in. The *sub*-vacuum kinesis of LaViolette (1994) is consistent with this, and the same with Santilli's prediction of sub-vacuum physics to become a rising field of physics, as well as with Rapoport's (2010) insisting signification of vacuum as 'the plenum'. It also seems reasonable to interpret the discovery of vacuum-DNA by wave genetics (Gariaev et al. 2011) as an empirical falsification of the conventional belief in vacuum as empty in an *absolute* sense. (Cf. also our later considerations concerning the concepts 'space' and 'empty space'.)

6. The ontological extensions of the first-order positive Fibonacci sequence does not relate directly to the primordial unit *I*, while the second-order sequence of gaps with their corresponding second-order ontological extensions, with its form $F_{n-1}$ (minus *I*), always re-relates to the primordial unit *I* and in this sense perpetuates in its manifestation the anchoring of the whole Fibonacci sequence (the combination of the positive and the negative sequences) in the anchoring unit *I*. In this respect the negative sequence may be said to display more transparently the anchoring in F1 and the periodic regeneration of the whole number structure from its source, than what the positive sequence does. From further contemplation of fig. 1 it will become clarified that the re-entering of *I* related to the succession of the gaps, is adequately interpreted as *I* being *redressed* as $F_n$, i.e. the Fibonacci number immediately preceding and rooting the gap. In this sense the minus *I* can be understood as the perpetuating *reincarnation* of the primordial unit *I* as $F_n$. An analogy may be that the first human, say Adam, *qua human*, regenerates in each new human being born, manifesting the paradox of the new-born both *being* human-Adam and having a specific (and increasing) *distance* to the primordial forefather. In this way the primordial unit *I* reincarnates as every successive number in the *positive* Fibonacci sequence, by *connection* to the gaps in the negative sequence, which it anchors and roots in every step. This relation only discloses from a *combined* comprehension of the positive and the negative sequence.

Comprehension of this gap constitution will show crucial to develop further fundamental number relations.

# FIBONACCI CONSTITUTION OF PERPLEX FIBONACCI MOLECULES AND NUMBER SUBSTANCES

Now we will present a *method* to describe all natural numbers situated in said *gaps*, as *perplex* Fibonacci "*molecules*" of (non-neighbouring) perplex Fibonacci number "atoms". Later, this successive deduction of perplex number molecules will show also to completely coincide with the make-up of *size* number molecules. We have already established that perplex numbers, more closely and profoundly investigated, are generated *from* the Fibonacci algorithm; i.e. both with respect to form and to content from *Fibonacci* perplexity. Consistent with this our basic approach is to acknowledge the *Fibonacci* numbers, *not* the conventionally termed "prime" numbers, as constituting the ontologically primary numbers (firstly as perplex numbers, secondly as size number). Hence, it is the *Fibonacci* numbers, *not* the prime numbers, which are to be comprehended as the most elementary "atoms" of natural numbers. All other natural numbers, when located in the gaps between the perplex Fibonacci numbers, whether primes or non-primes, should be regarded as number *molecules* of Fibonacci atoms. This implies that the atom-molecule structuring of natural numbers as primes vs. composite numbers is to be understood as a secondary *epi-structure generated from a more profound and elementary Fibonacci atom-molecule structuring into the natural numbers*. The adequacy of this statement will become more manifest and qualified during the course of our treatment.

Following the chemical analogy, the next perplex Fibonacci atom is always created by combination of the two preceding ones. This is not a molecule since the distinction between the two preceding atoms is *wiped out* in the creation of the new one, and is analogous to *transmutation* where the new size number is equal to the sum of the size numbers, like protons, of the two constitutive atoms fusing into the new one. A gut objection might be that prime numbers have to be considered the only real number atoms because they can not become reduced to a combination (interpreted as multiplication) of preceding prime numbers, while Fibonacci numbers always can become reduced to such a combination (interpreted as addition). However, this objection is not valid because it confuses perplex numbers vs. size numbers. With respect to perplexity any new Fibonacci number can *not* become reduced to the preceding two ones. Though it is *born* from their "intercourse", it is a novel, autonomous creature, and its identity in size number (ontological extension) with the sum of the two "parenting" size numbers is a *secondary* attribute from its perplex autonomy, inferred from the *way* it was created and born, like if a human was created from *all* the genes of mother and father fusing. Further, prime numbers are only to be considered non-reducible if interpreting combination as *multiplication*, which, different from addition which wipes out the distinction between two ontological extensions, do *not* follow immediately from perplexity generation as such. Also, prime numbers are a *sub-class* of natural numbers which *already* have been established from perplexity generation, and can not, different from the Fibonacci algorithm, characterise or explain this perplexity generation. And, when combination is interpreted as addition, prime

numbers trivially reduce to combinations of such natural – and in this sense *more* primary – numbers.

We will first present a *description* of a certain algorithm for generative composing of perplex Fibonacci molecular numbers, for later to move on to related analysis and explanation. We denote this algorithm *Fibonacci Molecularization (FM).*

The FM algorithm is constructed from the following operations:

1. We interpret every perplex Fibonacci number $F_n$ as a perplex number atom, representing a *rooting* Fibonacci atom, for a segment of connected perplex number molecules.

2. Each perplex Fibonacci molecule is rooted in one and only one $F_n$.

3. Each Fibonacci molecule contains one and only one atom of its rooting $F_n$.

4. No Fibonacci molecule rooted in $F_n$ contains Fibonacci atoms larger than $F_n$.

5. Each Fibonacci molecule rooted in $F_n$ contains one and only one atom of each included Fibonacci atom smaller than $F_n$.

6. No Fibonacci atoms included in a Fibonacci molecule are neighbours in the Fibonacci series. (If they were neighbours, they would transmute together to yield another Fibonacci *atom*.)

7. No Fibonacci atoms included in a Fibonacci molecule are smaller than F2. (F1 is excluded because it has identical ontological extension – and hence size number – with F2, and thus is redundant to characterise uniquely all perplex Fibonacci molecules.)

8. We use the notation $(d_1, d_2, d_3, \ldots, d_g)$ to describe the composition of a perplex Fibonacci molecule, successively progressing from left to right from the lowest Fibonacci atom to the largest Fibonacci atom (i.e. $F_n$) included in the Fibonacci molecule. Thus, $g$ denotes the amount of different Fibonacci atoms making up the Fibonacci molecule. The difference between a digit signifying a certain Fibonacci atom and the (larger) digit to its immediate right, will always be equal or larger than 2, due to the universal existence of a gap of at least one (cf. 6 above) Fibonacci atom between a digit denoting a Fibonacci atom and the next digit to its immediate right denoting the closest larger Fibonacci atom included in the molecule. (Cf. the violet third column of fig. 1.)

9. All possible combinations of Fibonacci atoms satisfying the requirements 1-8 are included as Fibonacci molecules.

10. All such possible combinations of Fibonacci atoms are represented uniquely, i.e. one and only one time.

Operations 1-10 determines exhaustively the complete set of unique Fibonacci molecules. To explain the ordering of this set into perplex succession of the molecules, we may first look, as an illustration, at the Fibonacci molecules rooted in F10 as displayed in fig. 1.

The first perplex molecule rooted in F10 is the *pair* of atoms combining F10 with the *lowest* possible other Fibonacci atom, i.e F2, to the molecule (2,10). Then we find the next perplex molecule by taking one step up with respect to the first digit of the pair, which gives the pair (3,10), and the next after that, (4,10), by taking another step up with respect to the first digit. Here, for the first time, the possibility opens to make a

molecule from *triple* Fibonacci atoms while *preserving* the digits of the pair as the *last* two digits in the triple, which requires the first digit of the triple being two (or more) perplex numbers lower than the first (and always lowest) digit of the pair. As soon as such a triple possibility opens up, we employ it as a rule, and thus (2,4,10) becomes the next perplex molecule rooted in F10. No other triple (or larger than triple) possibilities open up that preserve the pair (4,10) as the last two digits of the molecule. Therefore, we return *back* to *pairs* of atoms to create the next molecule, which gives (5,10). (5,10) is preserved as the two last digits in the triplets (2,5,10) and (3,5,10) which therefore constitute the next molecules in the perplex order. Then we have to return back to the next pair, (6,10), which yields three new triplets with (4,6,10) as the third one. This last triplet yields one four-atom molecule, namely (2,4,6,10), which preserves the triplet as its last three out of four digits. Returning back to triplets, no more triplets are possible preserving (6,10) as last two digits, and therefore we return back to pairs again, the next one being (7,10). (7,10) yields three triplets before yielding one new four-atom, namely (2,4,7,10), which preserves the last of these triplets as its last three out of four digits. Returning back to triplets, we now discover another triplet, namely (5,7,10), also preserving (7,10) as its two last digits. This triplet yields *two* new four-atoms preserving it as their three last digits. After the last one of these four-atoms, (3,5,7,10), we return to look for more triplets, but now all possible triplets having (7,10) as last two digits, are already listed, and hence we return to the next pair, which is (8,10). From (8,10) we proceed to list new triplets and four-atoms, applying the same logistics as before, and by this we discover the five-atom (2,4,6,8,10) as the last possible molecule rooted in F10.

As indicated by this illustration, the general procedure for perplex ordering of Fibonacci molecules rooted in $F_n$, can be described as follows:

A) *Intra*-ordering between molecules with *same* amount of atoms/digits, denoted by symbol *g*, applying the following criteria:
*First* priority: Find the lowest possible value (determined from the involved values of $d_g$ and *g*) of perplex digit $d_{g-1}$, i.e. the next last digit, indicating the atom that has the next largest perplex Fibonacci number of those making up the molecule. Then, increase the value of $d_{g-1}$ one by one, until having reached $d_g$-2 (=n-2), the highest possible $d_{g-1}$.
*Second* priority (i.e. ordering *inside* closure of the successive first priorities): Find the lowest possible value of perplex digit $d_{g-2}$ (presupposing g>2). Then, increase the value of $d_{g-2}$ one by one, until having reached the highest possible $d_{g-2}$, which is $d_{g-1}$ -2.
*Third* priority (i.e. ordering *inside* closure of the successive second priorities): Find the lowest possible value of perplex digit $d_{g-3}$ (presupposing g>3). Then, increase the value of $d_{g-3}$ one by one, until having reached the highest possible $d_{g-3}$, which is $d_{g-2}$ -2.
*Continue* this ranking of priorities until reaching the *first* perplex digit $d_{g-(g-1)}$ , i.e. $d_1$, and perform the final progression upwards of this perplex digit $d_1$ from the lowest value, which always is 2, until having reached the highest possible $d_2$ (given from the hierarchy of priorities unfolding backwards from $d_g$).

This means, expressed in short-hand, to *intra*-order molecules with first priority to make next last digit as low as possible, second priority to make third last digit as low

as possible, etc., and the last priority to make the first digit as low as possible.

B) *Inter*-ordering between molecules with *different* amounts of atoms/digits, departing from *lower* value of *g* (i.e. starting with g=2 which means pairs) successively *to higher* value of *g* as soon as such opportunity occurs from applying the criterion of *preserving* all ordered digits for the Fibonacci molecule with the *lower* value of *g* as the *identical end part* of the Fibonacci molecule having the *higher* value of *g* (i.e. one atom more). Then, list in succession all such possibilities increasing one by one the first digit of the molecule with the higher *g*, until an opportunity occurs with even one more digit of *g*. When such an opportunity no longer occurs, return to the *closest* lower value of *g* offering a new opportunity, and repeat the whole procedure until all possible occurrences of unique g values rooted in $F_n$ are covered.

This means, expressed in short-hand, to *inter*-order molecules by starting with pairs of atoms, then progressing to triples of atoms including the pair as soon as possible, then to quadruples of atoms including the triple as soon as possible, etc., and always move back to one less atom as soon as the possibilities of adding one more atom are exhausted.

Thus, the general procedure for perplex ordering of Fibonacci molecules rooted in $F_n$ represents a *combination* of A and B. At the supra-level the inter-ordering B has some priority to the intra-ordering A. This is because atoms are added and removed as soon as such opportunities kick in, displayed as stepping sideways in fig. 1, representing breaking *inserts* in the intra-ordered unfoldments vertically displayed in the sub-columns of the violet column of fig. 1. Still, fig. 1 displays that the over-all perplex ordering of Fibonacci molecules finds place in a quite alternating pattern combining the horizontal and vertical movements of B and A, respectively, in a strict and unique regularity.

This was the ordering into perplex succession of the set of Fibonacci molecules rooted in the *same* $F_n$. To achieve ordering into perplex succession of the *whole* set of Fibonacci molecules, we must also specify a procedure for supra-ordering *between all* perplex sets of Fibonacci molecules rooted in the *different* values of $F_n$. This is achieved by adding the following simple procedure:

C) *Supra*-ordering between molecular clusters rooted in *different* perplex Fibonacci atoms, and thus between all different values of $F_n$, by *combining* the clusters successively with increase in $F_n$ from F4, the first rooting Fibonacci atom. More specifically, this is performed by preserving the internal order of each cluster, determined by A and B, and at the same time demand that the perplex molecular number in a cluster rooted in $F_{n+1}$ always are larger than in a cluster rooted in $F_n$.

Thus, the combination of A, B and C results in a *complete* and unique perplex ordering of the whole set of Fibonacci molecules generated from operations 1-10. Then, the FM algorithm is determined as the totality of this procedure.

Obviously, the FM algorithm is not difficult to represent clear-cut in a data program. It may be most easily comprehended by just inspecting the perplex molecular structuring as displayed in fig. 1.

The violet third column of fig. 1 displays the perplex succession of the Fibonacci *molecules*, while excluding the Fibonacci *atoms*. Now we want to combine all Fibonacci molecules and all Fibonacci atoms into a *complete* perplex structuring of the whole Fibonacci generated "chemistry". This is achieved by simply inserting perplex molecular clusters after the respective Fibonacci atoms they are rooted in. Thus, the molecular clusters can be regarded somewhat like different trees grown and branched from their Fibonacci atomic roots, and the whole garden of trees as ordered by the linear succession of seeds from tree to tree and thus the arrow of the root structure. We denote the total algorithm including this supplementing modification of FM, as *Fibonacci Chemification (FC)*, having FM as its overwhelming part. The green, second column of fig. 1 displays the resulting perplex numbers of *Fibonacci number substances* (we introduce this term to denote and cover *both* Fibonacci atoms and Fibonacci molecules) as generated completely and uniquely from the FC algorithm, with the perplex numbers of Fibonacci atoms illustrated in bold.

## FIBONACCI CONSTITUTION OF THE FIBONACCI ATOMIC IDENTITY AND THE FIBONACCI GAP IDENTITY

Examining fig. 1 we then observe that *all size numbers of Fibonacci perplex numbers are identical to the perplex numbers of the respective Fibonacci atoms as occurring in the succession of Fibonacci number substances*; i.e. as *resulting* from the *molecular* perplexity (determined from FM and FC). We denote this identity the *Fibonacci atomic identity* (FAI). This identity is a quite non-trivial and interesting result. (So far, though, this identity has just been observed, not strictly proved.)

The reason for this Fibonacci atomic identity is evidently that the FM algorithm generates molecular clusters, rooted in $F_n$, in amounts (size numbers of $F_n$-clusters) always equal to $F_{n-1}$-1, i.e. *exactly* filling the gap of size numbers between two neighbouring perplex Fibonacci numbers. This implies that the Fibonacci atomic identity between Fibonacci atoms as *size* numbers and as *perplex* number substances, is *derived* from an underlying identity between, on the one side, (negative) *gaps* between Fibonacci atoms as *size* numbers and, on the other side, (positive) clusters of Fibonacci *molecules*, residing between the same Fibonacci atoms, as *size* numbers. We denote this last identity the *Fibonacci atomic-molecular gap identity* or, for short, The *Fibonacci gap identity* (FGI). While the atomic identity is a certain *size vs. perplex* number identity, the molecular gap identity is a certain *size vs. size* number identity. The first identity results from the second identity.

We emphasize that the second identity is established *independently* compared to the first one, insofar as the ontological extensions (and thus, size numbers) of perplex Fibonacci numbers play no role at all in the FC algorithm and have no influence on the perplex numbers of Fibonacci atoms as Fibonacci number substances. FM establishes all Fibonacci molecules without *any* reference to size numbers. Fibonacci molecules were composed as combinations of solely *perplex* Fibonacci numbers (atoms), and were afterwards ordered and becoming perplex Fibonacci numbers themselves (first as

perplex Fibonacci molecules, then, by supplement of FC, as perplex Fibonacci number substances). Thus, the *values* of Fibonacci numbers play zero roles in the described composing of Fibonacci molecules. Only *after* the independent composing of Fibonacci molecules do relating and comparison to values in the Fibonacci series emerge as an issue.

From perplex Fibonacci numbers the size numbers of these perplex numbers, as well as the composition of Fibonacci molecules and their perplex ordering, are developed in mutual independency in two different directions. After these developments, the Fibonacci atomic identity discloses a strict tie between these two, namely the quantitative overlap expressed by FAI and by FGI, which can be said to represent a re-meeting or *rendezvous* of the two number analytic trajectories, and in this sense a closing of the double structure. We may notice, as displayed in fig. 1 (to be clarified a bit later on), that the Fibonacci atomic gap space trajectory re-connects to F1, insofar as the primordial something-unit reenters all gaps, while the Fibonacci molecular trajectory re-connects to F2, insofar as F2 reenters as a Fibonacci atom in the first Fibonacci molecule of each molecular cluster. In this perspective the extended reproduction of the two trajectories unfolds in parallel as periodic waves of opening and closing with twin-like re-entering – and hence re-closeness – of F1 and F2 in each opening. This twin-structure is rooted in the primordial perplex number constituting a first size number which is duplicated for the next perplex number. One may liken this to an original value unit of gold re-incarnating its meaning as value in $F_n$ as a coin, with the coin having F1 as its invisible below side and F2 as its visible above side.

As previously established, perplex Fibonacci numbers *must* have ontological extensions. Qua ontological extensions perplex Fibonacci numbers with necessity *unfold* as Fibonacci *atoms* and by this as Fibonacci *number substances*. In this respect their ontological extensions are *shadowy* implied in the composition of Fibonacci molecules. Further, as previously analysed, by number constitutional logic these ontological extensions manifest into the respective size numbers of the Fibonacci atoms. Thus, these size numbers are *tacitly implied* also in the composition of Fibonacci molecules. However, these size numbers are not *focused*, but *abstracted* from in the composing of the FM and FC algorithms, so it is more to liken with shadow structures which passively follow the procedures that from pure and fundamental Fibonacci perplexity also generate the perplexity of Fibonacci number substances. A bit later in our exhibition it *becomes* adequate to focus exactly on these number substantial attributes in the Fibonacci molecular make-up. So far, however, they have been kept in the dark, and FAI/FGI was revealed *without* taking them into consideration.

As previously reasoned, the dividing line between two perplex $F_n$ and $F_{n+1}$ conceals a related second-order ontological extension residing in the depth dimension, equal to the gap of ontological extension units between $F_n$ and $F_{n+1}$, provided *ex ante* by the size number of $F_{n-1}$ (minus *1*). Despite this *providing* from $F_{n-1}$ (minus *1*) it now becomes clear from FM that this gap is *not filled* by any Fibonacci molecule belonging to the molecular cluster rooted in $F_{n-1}$, despite the formal identity between the *size* (number) of said gap and of this molecular cluster. This may be interpreted as $F_{n-1}$ (minus *1*) providing the (positive) *space* of the second-order ontological extension,

while *not filling* this gap space with any ex ante number substance created before $F_n$. Instead, FM as illustrated in fig. 1, displays that this space of second-order ontological extension is filled by the molecular cluster rooted in the *proceeding* perplex $F_n$ .This means that the space provided *ex ante*, "how many holes it takes to fill the Albert Hall", to quote a famous line from The Beatles, becomes filled of Fibonacci molecular "fruits" *after* that the $F_n$ root, carrying these molecular fruits from its "tree", has been seeded and become manifest. Consistent with this, to sub-differentiate and qualify different "holes" in the (pre-)space is only possible as a *formal* operation of potentiality, by performing a plain perplex numbering of the possible unique size numbers implying imagined existence of hole units making up the gap which is uniform and continuous in all other respects. Contrary to this the perplex molecules rooted in $F_n$ present an organically structured actualisation of Fibonacci molecular "fruits" residing at the respective positions in the cluster. Thus, the formally contemplated Fibonacci gap becomes qualified, in flesh so to speak, and somewhat as Albert Hall individuals, as the Fibonacci *molecular* identity with the Fibonacci atomic gaps. This required the perplex molecularisation by FM. In this perspective the *content* of number *substances* filling the merely formally extended gap could not become established before this perplex molecularisation.

Since the amount of "holes" in the (pre-)space is identical to the number of Fibonacci molecules rooted in $F_n$, each particular hole becomes filled by a particular Fibonacci molecule, and the cluster of Fibonacci molecules thus fills *completely* the (pre-)space, the distinctions between Fibonacci molecules corresponding to all distinctions between the holes. It is as if the "shelves" of the (pre-)space, occurring from merely a formal internal differentiation of the (pre-)space architecture, become filled by the Fibonacci molecules, and *completely* filled. Notice that this happens without so far having said anything about the specific ontological extensions (sizes) of the corresponding Fibonacci molecules. It seems to be a *qualia characteristic* by the FM algorithm, that if it is *true* in a non-trivial sense, i.e. a reconstruction of something profoundly real, rather than a voluntary free-standing number construction, then the FM algorithm generates the *particular* Fibonacci molecules having the ontological extensions to *exactly* fill all the respective "shelves" of the (pre-)space, and that this is the *way* and the *operation* that results in not only the number of shelves being equal to the number of Fibonacci molecules in the corresponding cluster, but also to (pre-)space as an *unfolded whole* having the same ontological extension as the aggregated ontological extensions of the individual Fibonacci molecules in the corresponding cluster. This last identity seems *only* possible to realize by means of said operation, because if there was a mismatch between ontological extension of an individual Fibonacci molecule and its corresponding shelf, either the shelf could not become completely filled or not filled at all, having the implication that the individual Fibonacci molecules could not fill *all* shelves, i.e. the *totality* of (pre-)space. The internal architecture of (pre-)space is formally given as uniform units/shelves, only distinguished by their perplex numbering inside the gap. However, this does not imply that the *ontological extensions* of these perplex numbers are uniform. To the contrary, by their very difference these perplex numbers can, in the next step (cf. later), be

considered as *size* numbers, implying corresponding *differences* between the sizes of the uniform shelves. After all, there can be a *tie* between each hole/unit in the gap and the *size* of the hole, insofar as the uniqueness of the *size* number of the hole confirms that it *is* a unique hole. As an example to make this clear, we may consider the gap between F5 and F6. F5 and F6 have, respectively, size numbers 5 and 8, which implicates that size numbers 6 and 7 are not covered by size numbers of Fibonacci atoms. Since these are *two* different *size* numbers, we can deduce that there also are two *perplex* number units or holes in this gap, which later can be filled-in by Fibonacci molecules. We by this know the size (number) of the gap as a whole, which is 2. However, we also suggest some reason to expect the sizes of *each* hole, i.e. the *depth* of each hole, to be 6 and 7 as well.

Then, the operation resulting in filling-in of total (pre-)space from filling-ins of all individual shelves, would consist in all Fibonacci molecules being composed in such a way that they had the ontological extensions identical to these size numbers. Later we shall discover, by further examination of and deduction from the FM algorithm, *exactly* this to become confirmed and substantiated as the truth of the issue.

So far we – at least – have discovered the Fibonacci gap identity, which means that the size number indicating the amount of perplex Fibonacci molecules residing in each Fibonacci molecular cluster, corresponds to the size number of the gap between the according neighbours of perplex Fibonacci atoms. By this we have not said anything about size identity for *each* Fibonacci molecule in the cluster, i.e. with respect to the ontological extensions of the respective Fibonacci molecules, which requires *intra*-molecular examination. Already, though, it is reason to contemplate as quite remarkable that the FM algorithm is able to generate all Fibonacci molecules in such a way that yields the Fibonacci gap identity, so that each cluster of Fibonacci molecules fits exactly with the number of empty positions between all neighbouring Fibonacci *atoms* when these atoms are measured as ontological extensions, i.e. as size numbers. At the same time the FM algorithm also *prevails* the rooting Fibonacci atom that occur just before the gap, as included in *all* the Fibonacci molecules generated in the gap-matching cluster. These attributes already suggest that there is something *emphatically real* with the FM algorithm, and this will be further qualified and consolidated during the course of our treatment.

Here it may be relevant to insert a general comment on the concept 'space'. Nothing can be nothing in *itself*, only as something *related* to another something in a comparing *context*, and *judged* as nothing in this *relation*, namely as the *lack* of ontological extension of something when contemplating the ontological extension of the first something. Thus, the concept 'empty space' tells that a first something lacks the ontological extension of the second something which more or less *fills* space. This does not at all imply that the first something is empty in *all* regards, in *all* contexts and in relations compared to *all* other somethings. Said "lack" will *always* occur when the two somethings are regarded as existing at different ontological *scales, levels or dimensions*. In ordinary perception we operate with the distinction between objects and empty space *between* objects. This does not imply that this empty space is empty in an *absolute* sense. Air is not nothing, it only appears *as if* it was at our ordinary perceptual scale or

resolution. When we, in the next step, regard air as the object-something, *vacuum* denotes the second nothing-something which in this *comparative context* is judged as nothing. This does not at all imply that vacuum is nothing in any *absolute* sense, i.e. in *all* comparative contexts (as for example confirmed by the discovery of vacuum DNA by advanced wave genetics). Contemplating vacuum is basically just a *transportation* of the empty/filled distinction from ordinary perception down to a lower scale. Contrary to this fallacy of simplistic thought, we realise that space by its very nature must be *completely* filled, which is achieved by simply *amplifying* the contextually judged nothing-something sufficiently (with respect to scale, level or dimension). This amplifying possibility, and the very nature of the judged nothing-something as *not* an absolute nothing, is tacitly *enfolded* in the very *concept* of space. This must be so because if the judged nothing was an *absolute* nothing, the space *could not* be as extended as it was *regarded* to be in the nothing-judging context. This implies that space *always* is *completely* filled. At the same time space can be regarded in certain contexts *as if* this was not the case, and hence as if space contained a nothing-part, but this is a *simile*. Such a simile is often *fruitful* to gain insights, exactly *because* space is differentiated in scales, layers and dimensions, differentiations stemming *exactly* from the *not*-absolute existence of any nothing. In the initiation of the Fibonacci perplexity we have seen such paradoxality with respect to the quasi-perplex Fibonacci number zero. This number is not any *absolute* nothing, it is a *relative* nothing in the context of the primordial perplex F1 recognising the *absence* and hence the *difference* to itself, after which this *abstracted* imagined nothing immediately becomes *something*, and together with F1 gives birth to the next, novel perplex number F2. At the same time, the lack and impossibility of *absolute* nothing, is expressed in F2 as having the *same* ontological extension as F1. (Cf. Johansen 2008a concerning the general architecture of differential ontology, ontological context-dependencies of relative vs. absolute, simile operators and the causality nexus of reality.)

Interestingly, Santilli has suggested that our perception of objects as filled and the substratum as empty, could in reality be inverted in the sense that, once assumed as oscillations of the substratum, matter would become totally empty, and space would become totally full, thus without empty cracks, as illustrated by the structure of the electron once assumed as an oscillation of one point of the universal substratum (cf. Gandzha and Kadeisvili 2011, ch. 3.1).

This back-reflection on the very concepts of 'space' and 'empty space' clarifies that an *absolute* notion of 'empty space' is a *contradictio in adjecto*. Abstracting this back-reflection to its most universal and elementary form, Fibonacci perplexity constitution of reality number substances, it implies that fillings-ins of gaps or blank spaces is exactly what *is* to be expected, and what *has* to take place, when examining the source code of reality. Thus, the FM algorithm just mediates and *realises* what is already enfolded as an onto-logical necessity in the most purified notion of space as such. This categorical and number mathematical space *dynamics*; namely between i.i) Fibonacci atomic external space and i.ii) Fibonacci atomic internal space on the one hand, and ii) Fibonacci molecular complete filling-ins of i.i) and i.ii) on the other hand, may also provide the clue to clarify the somewhat mysterious notion of 'phase

conjugation' in physics, probably most interestingly and profoundly examined by Rowlands (2007).

We have seen that there exists a *space* between the ontological extensions (size numbers) of neighbouring Fibonacci atoms, and that this space is not *empty*, but *completely filled* by the Fibonacci *molecules* generated by the FM algorithm. It seems quite remarkable that the ontological gap between Fibonacci atoms, which size is determined already from the Fibonacci atoms, not presupposing even any idea about Fibonacci molecules, *has* to become filled by Fibonacci molecules. It is almost as if the sole genuine child ($F_{n+1}$) of father ($F_{n-1}$) and mother ($F_n$) can not become born before her mother has fulfilled her secret mother mission giving birth to the last thinkable child made from copulating all her forefathers.

This peculiarity seems to imply that the FM molecular clusters *exist* as *already enfolded* (in the sense of Bohm) *inside* the very distinction between $F_n$ and $F_{n+1}$, so that it is only an appearance (*Erscheinung*) that generation of Fibonacci perplex numbers and size numbers can be fully and independently comprehended *without* including Fibonacci molecules – and, in the last respect, without applying the *particular* generative order expressed as the FM algorithm.

The FM generation of the Fibonacci molecular clusters represents a further, *content* qualification of the exact *compositions* of the previously interpreted *depth* dimension of number substances filling in the gap *underneath* the distinction between $F_n$ and $F_{n+1}$, provided from $F_{n-1}$ (minus *1*). From the FM algorithm we by now do not only *know* that this second, underlying and orthogonal dimension *exists*, as well as its *total* spatial framing, but we also know the basics of *what* it looks like. The dividing line, or – so to speak – the shut eye, between two neighbouring perplex Fibonacci numbers, is by this not only *opened*; the revealing visible landscape of horizon (i.e. i.i) and onto-resolutional rubrics (i.e. i.ii) have by the FM algorithm become completely *filled* with atomic *pluralities* (i.e. molecules).

At this point, a further non-trivial result seems to disclose: When we extend perplex numbering to the uniform units residing in the size number gap between $F_n$ and $F_{n+1}$, which means perplex ordering of the units residing in the depth dimension, all these units of depth are only distinguished by their *perplex* numbering, not by (immediate) ontological extension nor (yet) by *further* dimensional differentiation. When contemplating the *filling-ins* from the FM molecular clusters, we have to *expand* this view into differentiation along an *additional* dimension, namely the dimension of *width*. (At this point in our treatment the *naming* of dimensions, applying the metaphors of ordinary 3D perceptional directions, is not any issue; what is important is the onto-logical, dynamical and constitutional *generation* of dimensions for and of *number* space.) This implicates that the previously (from $F_{n-1}+1$) *uniform* dimension of *depth* has to become *further* differentiated when its "shelves" become filled. This novel dimension is implied in the already analysed *inter*-ordering of Fibonacci molecules insofar as this inter-ordering includes structured *differences* in the *number* of Fibonacci atoms composing the Fibonacci molecule, i.e. as differentiation along a *horizontal* axis as illustrated in fig. 1. This may be likened as differences in *width* of said "shelves".

Further, the filling-ins from the FM molecular clusters also disclose that the uniformity of the units in the depth dimension is filled with structured *differences* in the *sizes* (ontological extensions) of the Fibonacci molecules located at the progressing "shelves", where the exact quantitative nature of this difference will be pinpointed a bit later. Such differences occur without regard to the differences in the number of Fibonacci atoms composing the Fibonacci molecule, and hence they are to be regarded as manifesting a novel, *fourth* dimension. This novel dimension is implied in the already analysed *intra*-ordering of Fibonacci molecules with the *same* amount of Fibonacci atoms, i.e. as differentiation along a *vertical* axis as illustrated in fig. 1. This may be likened to differences in *height* of said "shelves".

Occurrence of four or more dimensions is contradictory to the very notion of space as 3D, and only has meaning in description of *combined* 3D spaces. Also a *combined* description of 3D spaces only has meaning from an over-all *three* dimensional framework, but this combined framework can include many *sub*-dimensions, which it *compresses* in its over-all combined description. As an illustration, one can imagine a 3D virtual reality computer-generated space, where a new 3D space opens when clicking on pixels of the walls, floor or ceiling, and the same relation can be the case if repeating the procedure inside the new 3D space that is opened. A compressed description of the whole VR architecture describes a space, and is thus to be regarded as 3D. However, due to all the hyper-linkings there can occur much more than three dimensions all together if regarding all these *as if* they were existing at the same ontological space level, i.e. *as if* they were existing inside only *one* frame of (3D) space. Applied on the Fibonacci space constitutional dynamics, the fourth dimension of height should be regarded as a *sub*-dimension compared to the vertical succession ("shelves" of space) of the uniform units of perplex Fibonacci molecular numbers, and at the same time as a *novel* dimension disclosed in our examination, *enfolded* in and *filling* said vertical dimension. Thus, there is a paradox here: The fourth dimension is novel and additional compared to said vertical dimension, but at the same time it is not additional to it at the *same level* of ontological space, and therefore not any *fourth* dimension in an absolute or hypostased sense. On the one hand, the novel fourth dimension *duplicates* the first one; on the other hand it does so in a *next* step, *qualifying* the first one. However, this novel qualification finds place, at this point in our treatment, *independently* of the first one and *onto* the first one, unfolding from the *positive* filling-in generated by the FM algorithm. (Quite remarkably, as we soon shall see, despite this independency, the quantitative structure of the fourth dimensional height, *coincides* exactly to transforming the perplex molecular numbers of the first dimension, to size numbers. By this, the first and fourth dimension may be said to *meet* and to *conjugate*.)

## FIBONACCI CONSTITUTION OF SIZES OF FIBONACCI MOLECULES

So far Fibonacci molecules have only been described as *combinations* of Fibonacci atoms, and applying a chosen convention of listing the lowest perplex

Fibonacci atom first in the ordered combination. The exact *nature* of combination, for example addition vs. multiplication, has not been any issue. The FM and FC algorithms were described at a level of abstraction above and prior to such. (We shall later discover that such instantiations of combination do not exist on an equal number ontological footing.)

In all ambitious original science, the first steps are the most crucial and difficult ones. Mistakes and non-optimalities in the beginning can never become completely compensated by brilliant creativity and technical virtuosity later on. And it is very difficult to discover the adequate entrance, due to being non-trivial, hidden, more profound and radically different from preceding science. "Aller Anfang ist schwer" (Hegel).

Hence, let us reflect a bit more upon the *first* perplex Fibonacci numbers with respect to Fibonacci molecular constitution. Obviously, the first possibility to compose a Fibonacci molecule occurs with combination of F1 and F3. First question: *Do* they combine, and with *necessity*?

This is a profound ontological question, when leaving the conventional view of mathematics (and logics) as merely some free-standing thought universe, in favour of differential ontology which also considers and directs formal science towards hitting the essential universal-abstract-elementary marking of generative Reality. (Cf. Johansen 2008a: ch. 3.1.2, concerning logics and concerning free-standing thought universes and classifications.)

Johansen (2008a: ch. 3) presented a complete exposition of the causality nexus of reality, anchored in differential ontology, unfolded by *philosophical informatics* from the sole qualitative notion of 'information' in the most abstract, universal and elementary sense. Two compressed quotes:

*If all points and paths in the 3D illustration of the causality nexus are imagined as activated, this constitutes the totality of relations imaginable in the free-standing universe of logic. This universe of logic exists as a part of the cosmic whole, but only as a PART. Far from all of the points and paths of the causality nexus is REALIZED in the cosmos APART from its segment constituted by the universe of logic. From the architecture of the causality nexus it follows that the cosmos changes by ACTIVATION of POTENTIALLY already existing points and paths, with the changes being more far-reaching with activations from increasing transalgorithmic order.* (Johansen 2008c: 693)

*From universal key properties of the category BORDER as unfolded from differential philosophical informatics (Johansen 1991:66-73) it has been deduced by Johansen (2006) that the FIBONACCI ALGORITHM is THE abstract, universal and elementary algorithm of Nature, all other algorithms manifesting as mere epi-phenomena of this as "organic" results of the Fibonacci algorithm's unfoldment into complexification. This provides the basic bridge between the qualitative and quantitative aspects of Nature. If this deduction is correct, it implies that the whole potential-for-real complexity nexus is to be comprehended as a gigantic cosmic Fibonacci nexus with the differentiations between different layers and orders in the 3D nexus, as well as their interlinkings, generated from FIBONACCI SELF-REFERENCE*

*on and of the Fibonacci-algorithm itself into hyperstructures instead of mere progressing as the linear Fibonacci series.* (Johansen 2008c: 696)

Without going into subtle details, this implies that the Fibonacci algorithm is the primordial and ultimate transalgorithm which in its self-referential unfoldment generates the distinction(s) between what is to become actualised and not-actualised of the potentiality notions of mathematics and logics (themselves generated from the same transalgorithm into, as well as inside, the apparently free-standing toy universe of formal thought, and in *this* sense becoming actualised). This implicates that both possibility space and the actualised segments of possibility space are self-referentially determined by the Fibonacci algorithm. It also implicates that *all* possibility implied in the self-referential Fibonacci algorithm *has* to become realised. This *is* the Reality *maker*, *making* the make-up of reality. It is just (which is a huge intellectual bite to back-reflect) a question of the *sequenciality* (including: into and through different ontological dimensions and spaces) of this self-referentiality.

Thus, acknowledging self-referential Fibonacciality as the Reality-creating prime operator means that what is possible from that, *has* to unfold. This implies that Fibonacci molecules *have* to emerge and to compose and realize as soon as possible. (This is at the most abstract and fundamental level. At much more mundane levels, this becomes much more complex, *from* the fundamental, due to the Fibonacci walk also generating its *opposite*, i.e. its *absences*, and from there the dynamic mutuality of those two in all kinds of unfolding complex constellations.) "As soon as possible" means that *Time has* to become *filled* of such, in some analogy to the relation between abstract Fibonacci space (from $F_{n-1}$-*1*) and the filling-in of space from perplex Fibonacci molecules. Thus, just as space, Time only realises and exists as *full*.

So, F1 and F3 *has* to combine, the first *connects* to the third as soon as the third is created/realised. This can not happen *before* the creation of the second , but after this, it *is there*. This implies a subtle extension of transitive logic into some reconfiguration, due to covering more than one ontological level. Regarded from the lower level, the tie between the third and the first is only reached via transitivity. But this is via *back*-reflection from the unfolded into the enfolded. In the enfoldment, the tie is *already* there, residing on a higher transalgorithmic level. *If* a son is born and *if* his grandfather is still alive, they have to coexist in the same bloodline by ordinary transitive logic. But the case of F1 vs. F3 represents more than simply so, because F1 will *always* stay alive and regenerate in the unfoldment of the whole Fibonacci structure, and this structure *must* include F3 in its unfoldment. Thus, F1 and F3 have to coexist, and in this coexistence the intermediate F2 is not required, just as little as a child requires the presence of parents to be with his grandfather. In this sense, F3 manifests as an ontological *neighbour* to F1, and the same must be the case for the manifesting unfoldment of *all* combinations of Fibonacci atoms. (Analogous analysis of *Nacheinander* becoming *Nebeneinander* can be found in German philosophy of *conceptual logic*, especially Hegel's *science of logic* and late Marx' *capital logic*.)

Coupled as ontological neighbours the *ontological extensions* of Fibonacci atoms combining into a novelty, a Fibonacci molecule, is elementary constituted,

consistent with our reasoning concerning the ontological extension of a novel perplex Fibonacci number/atom, by simply *removing* the lines of *distinction* between the ontological extensions of the involved Fibonacci atoms. And by further analogy to our previous reasoning, this implies interpreting the operation of combination between neighbouring Fibonacci atoms as *addition* also in the case of novel perplex Fibonacci *molecules*. This implicates that the *size* numbers of Fibonacci molecules are simply found by *adding* the size numbers of all Fibonacci atoms that make-up a Fibonacci molecule.

This said, combinations of ontological extensions are a bit more sophisticated in the case of novel perplex Fibonacci molecules, compared to the situation when novel perplex Fibonacci atoms are formed. This is because a novel perplex Fibonacci atom manifests with *total* deletion of the distinction between the ontological extensions of its two preceding and parental Fibonacci atoms. If not so, the novel Fibonacci atom would not be newborn as a Fibonacci *atom*. Different from this, the novelty of a Fibonacci molecule does *not* require such a total deletion. On the contrary, the deletion *not* being total, so that the included Fibonacci atoms are *preserved*, is what qualifies the Fibonacci molecule as a novel *molecule*, in *distinction* to a novel Fibonacci atom. Thus, the situation is paradoxical, or more precisely: *two-leveled*, in the case of a novel perplex Fibonacci molecule. At one ontological level, the distinction between the ontological extensions of the Fibonacci atoms, which are combined from becoming neighbouring into the Fibonacci molecule, must prevail; on the other ontological level the distinction must be removed. Thus, the total picture is that the ontological extension of the novel Fibonacci molecule is constituted by regarding the Fibonacci molecule *as if* the "walls" between its Fibonacci atoms were deleted, while at the same time these walls are *not* deleted. This paradox is to some extent similar to considering the volume of a house as made up from the volume of its rooms, but with the difference that the walls between the Fibonacci atoms appear as *infinitely* thin at first glance, and even more than so from further reflection:

*Border*, in the most abstract informational sense, as the difference *constituting* two somethings *as* somethings (pluralis), i.e. as *different* somethings, does *not* have *any* ontological extension in the framework of the separated *lower* ontological level (the level where the two differences are somethings), but is a (necessary) *projection* from the higher ontological level down back to the lower. This can be likened to a "knife" of thought (or perception) *constituting* the two differences as somethings by performing the cut of distinction, while at the same time this "knife" does not really make a cut when observed in the framework of the lower level. Thus, in this sense the operator is *as if* it is a knife, and more likened to caressing or painting, and hence a *simile* operator. Still though, this is the *basic*, *necessary* and *reality-constituting* operator, due to being implied in all information. (Cf. Johansen 2008a: ch. 2.2.1, and Johansen 2006 for details.) *Only* because the walls between the Fibonacci atoms in the Fibonacci molecule have extension *zero*, in the framework of the lower ontological level, can the ontological extension of the combination removing the wall be *exactly* the same as the addition of the ontological extensions (size numbers) of the involved Fibonacci atoms. This does not mean that said walls do not *exist*, but that they exist as a *simile*

*overlayering*, *drawn upon* and *projected down-back* from the *paradox operator* of fusing (in one sense) and not-fusing (in another sense) Fibonacci atoms into Fibonacci molecules. One may visualize this paradox as the walls between Fibonacci atoms being *dotted*, at the same time being knife sharp and without ontological extension at all in the lower ontological framework.

This situation is different from the situation with respect to atoms and molecules in chemistry. In chemical molecules there is considered to occur "surfaces", distinguishing atoms, having more than zero extension or thickness. (Still though, the border *between* surfaces, as well as between surface and inside, has zero extension in the ontological framework of the physical level.) Thus, the concepts of Fibonacci atoms and molecules are *more abstract* than in chemistry, namely *maximally* abstracted ontologically. Hence, they are also more *fundamental* with respect to reality generation, generation of chemistry not excluded. This makes it more adequate, if the intermediaries become developed, to approach chemistry from Fibonacci atoms and molecules than the other way around.

As stated, the combination of F1 and F3 constitutes a genuine and first Fibonacci molecule. However, due to size numbers of Fibonacci molecules being determined from simple addition of the involved size numbers of Fibonacci atoms, the size number of the Fibonacci molecule (1,3) is identical to the size number of the Fibonacci atom F4. Hence, there occurs no difference between (1,3) and F4 in over-all ontological extension, only with respect to the internal ontological make-up of this extension. (More precisely that the Fibonacci molecule (1,3) is differentiated vertically in two layers (as both one and two entities) and horizontally as two atom spaces, thus constituting a basic number ontological *cross*.) Therefore, the Fibonacci molecule (1,3) is not able to make any difference with respect to further number generation, that is different from and novel to difference made from the already established F4. Thus it will not represent or generate any quantitative uniqueness. For this reason we will denote such a Fibonacci molecule a *Fibonacci para-molecule*. Still though, it may be of interest to notice and keep in mind that the occurrence of (1,3) implies that the ontological extension of size number 3 exists in a *double* manifestation, both as the Fibonacci atom F4 and the Fibonacci molecule (1,3), or, in other words, that the Fibonacci atom F4 co-exists with this Fibonacci molecule as its *doppelgänger*.

Moving on to the combination of F2 and F4, the addition of their size numbers gives the first Fibonacci molecule with a *unique* ontological extension, different from the extensions of all Fibonacci atoms, namely the size number 4 (=1+3). Our notion 'perplex Fibonacci molecules' ignores all Fibonacci para-molecules, thus only regarding Fibonacci molecules with unique ontological extension. If not explicitly stated otherwise, we tacitly assume 'Fibonacci molecules' to denote only Fibonacci molecules that are unique in this sense.

Obviously, the combination of F1 and F4 gives a Fibonacci molecule with the same ontological extension as (2,4). Then, which of these two is to be regarded the *first* (unique) perplex Fibonacci molecule, and who is to be regarded the Fibonacci para-molecule and doppelgänger of the first one? It seems adequate to regard (2,4) as the first perplex Fibonacci molecule. When F4 is born as a perplex number novelty, this is

from its "mother" F3 distinguishing from its "father" F2. After F4 is born, distinguishing itself from its "mother" F3 creates the novelty F5. F4 distinguishing itself from its "father" F2, creating the novelty (2,4), means taking only *one* step back, while F4 distinguishing itself from F1, creating the novelty (1,4), means taking one *further* step back. F2 is more *radically* closer to F4 than what F1 is, since F2 delivers *directly* the one qualitative half of the ontological extension of F4, and this being so on an *equal* footing with "mother" F3. Also, regarding (2,4) as the first perplex Fibonacci molecule makes this first Fibonacci molecule rooted in its *preceding* Fibonacci atom, indicated by the last digit in the molecule. This is consistent with what is the case also for all proceeding Fibonacci molecules, while it would not be the case for (1,3) which would have its first available placement in the gap between F4 and F5, not after F3, and thus constitute a sole and puzzling exception from the universal pattern. Hence, if (1,3) was the first Fibonacci molecule instead of (2,4), the Fibonacci atomic identity would not be universally valid, and even less would it be valid if *both* (1,3) and (2,4) were considered unique Fibonacci molecules.

The reasons for priority of F2 on behalf of F1 in the constitution of the first perplex Fibonacci molecule implicate an according priority of F2 on behalf of F1 also in all later constitution of perplex Fibonacci molecules, delegating all doppelgänger molecules of F1 to Fibonacci para-molecules. Thus, F1 is excluded as Fibonacci atom from *all* unique Fibonacci molecules. This is with respect to the *filling-ins* of the spaces between Fibonacci atoms. As previously expressed, though, F1 can be regarded as regeneratingly present in the unfoldment of the whole Fibonacci structure qua repeatedly anchoring said spaces, provided as $F_{n-1}$-*1*, in the primordial unit *1*. In this perspective, F1 and F2, despite their identical ontological extensions, can be regarded as playing complementary roles, in quite different directions and respects, in the generative unfoldment of the Fibonacci number landscape as a whole. Contemplated in this perspective, it may not be trivial to notice that the first space gap, between perplex F4 and F5, is *provided* by $F_{4-1}$-*1*, which is as F3 minus *F1* (the primordial anchor), while this gap becomes *filled* by the first (unique) Fibonacci molecule, which is F4 added to *F2*. Thus, F1 and F2 function complementary in the location and creation of the first Fibonacci molecular "fruit".

With respect to perplex Fibonacci numbers, perplex number 4 manifests as the combination of perplex Fibonacci numbers F2 and F3. And size number 4 manifests as the addition of the size numbers 1 and 3 of, respectively F2 and F4, i.e. perplex Fibonacci atoms 2 and 4, making up perplex Fibonacci molecule number 1. Accordingly, in the same succession with corresponding meaning of numbers, we could formalize this as, respectively, 2+3=4; 1+3=4; 2+4=4; 1=4. In all these cases the iconic evident truth of conventional mathematics, 2+2=4, is *not* true. And these deviances manifest in a *more* profound constitution of number theory than the conventional one, generating the natural numbers as such, i.e. those numbers which first have to become established in order to *make* 2+2=4 a truth. Also hadronic mathematics, initiated by Santilli, exposed 2+2=4 to be *not* true when extending mathematics to iso-mathematics. Thus, conventional number theory conceals *constraints* for 2+2=4 to become true, constraints that are established from 2+2=4 *not* being true in other – and deeper –

respects. In our Fibonacci generation of the natural numbers the number 4 does not manifest as the sum of 2+2 in any interpretation. Despite this, 4 has already manifested as combination interpreted as *addition*, and this in three different sub-interpretations.

Our point is not to subscribe to cheap rhetoric as any argument per se, just to indicate that there may occur radical novelties from detrivialising what conventional wisdom, even in number theory, reckons as most evident, diving into a more profound platform, and ascend up via a novel generative approach.

The conventional approach is to define natural numbers as they trivially appear in counting, without recognising any further need for constitutional reflection, and from there define external operations or rules, the first one being addition, to start exploring the possibility space of number relations, which opens up from this. Different from this, our approach generates natural numbers as the visible tip of the iceberg, or the fruits created from an invisibly rooted constitutional dynamics, and from there it systematically unfolds *different meaning aspects* of natural numbers in their *interrelatedness*. Further, our approach does not introduce addition as a free-standing, *external* operation. Addition as such becomes *inherently and organically generated* as an aspect of the *same* constitutional dynamics as the natural numbers themselves. Also, at different steps in our treatment, the organically integrated operation of addition shows crucial to manifest *further* meaning aspects of the natural numbers themselves.

In Johansen (2008a: ch. 3) causality *as such* (and from there the whole thinkable causality nexus of reality) was established organically and inherently from systematic unfoldment of the distinctions that with necessity occur enfolded in information as such. This was argued to represent a deeper foundation than in conventional formal logics which apply *independent* and *imposed* definitions of sentences, truth values and (external) logical operators, and from there causality. Instead, our exposition (*Darstellung*) targeted the mark of causality *as already existing as real* in the core of *information* as such, and *reestablished from there* the very field of formal logics from a generative, real (contrasted to solely formal and free-standing) and *re*-constructive dynamics. Our generative and organic deep-unfoldment of natural numbers and related operations are in accord with this related, previous re-establishment of causality and logics.

## FIBONACCI CONSTITUTION OF THE FIBONACCI MOLECULAR IDENTITY

We have established that the size numbers of perplex Fibonacci molecules are determined as addition of the size numbers of the perplex Fibonacci atoms included in the Fibonacci molecule. The third and fourth columns of fig. 1 display these additions. Here we observe another remarkable identity becoming disclosed, namely that the *size* number of each Fibonacci molecule always is identical to the *perplex* number of the same Fibonacci *number substance* (i.e. when perplex Fibonacci *atoms* supplements Fibonacci molecules to determine the framework for perplexity). We denote this identity the *Fibonacci molecular identity* (FMI).

The fourth column of fig. 1 calculates the size of perplex number substances that are *molecules*. The sizes of perplex number substances that are *atoms* have *already* been determined, from the perplex numbers of the respective atoms in the framework of *only* atoms. These sizes showed, by the Fibonacci atomic identity, to be equal to the perplex numbers of the respective atoms in the framework of number substances. Thus, we already *know* the size numbers of atoms in the fourth column. What we do *not* know from this is whether there *also* will occur sizes of Fibonacci molecules equal with any such size number of atoms, i.e. whether there will appear *overlaps* with sizes of Fibonacci molecules for any size number of atoms. This needs the novel calculations of the sizes of Fibonacci molecules to become determined.

We observe from the result of these calculations that *no* Fibonacci molecule has a size equal to the size of any Fibonacci atom. Thus, there occur *no overlaps* between sizes of Fibonacci atoms and Fibonacci molecules, implicating that the size of each Fibonacci atom represents a *unique* size number in the reference frame of Fibonacci number substances. This arrives as a novelty, and in some sense as a prolongation, of the earlier Fibonacci atomic identity stating that the size of each Fibonacci atom represents a unique *perplex* number, now in the reference frame of Fibonacci number substances.

In addition to this uniqueness, the Fibonacci atomic identity also implied that the *value* of any such unique perplex number of a Fibonacci atom, in the reference frame of number substances, was *equal* to the size of the corresponding Fibonacci atom. We observe from the calculations of the sizes of Fibonacci molecules that also this equality in value is prevailed for the sizes of Fibonacci atoms in the reference frame of *sizes* of Fibonacci number substances. Thus, the Fibonacci atomic identity is reproduced also in this respect, when stepping from perplex Fibonacci number substances to sizes of Fibonacci number substances. Obviously, the necessary and sufficient conditions for this value equality to become preserved, is that i) each Fibonacci molecule has a *unique* size (which means that there occur no overlaps *between* the sizes of Fibonacci molecules), different from all sizes of Fibonacci atoms; and ii) *all* gaps between sizes of Fibonacci atoms are filled by *size* values of Fibonacci molecules (while the Fibonacci atomic identity and the underlying Fibonacci gap identity stated all these gaps as filled by *perplex* values of Fibonacci molecules in the reference frame of perplex number substances).

However, conditions i) and ii) do *not* imply with any necessity that the size values of all (or even any) Fibonacci molecules are identical to the *respective* perplex values of Fibonacci molecules (in the reference frame of perplex number substances). In principle, i) and ii) could become satisfied from *any* ordering of number substances into perplexity, presupposing that the elements constituting the whole set of number substances had the same atomic *composition* as those listed in the third column of fig. 1. With *another* ordering into perplex number substances, all sizes of number substances would still occur uniquely and completely in the fourth column. However, they would appear as *shuffled*, when compared to the perplex ordering of the third column. Such shuffling, when moving from perplex molecules to size molecules, could occur *inside* the clusters of perplex molecules filling the respective gaps between atomic sizes, as

well as *between* such clusters. Therefore, for a 1:1-relation to manifest between *each and all* values of perplex molecules vs. size molecules (both in the reference frame of number substances), it is required to supplement conditions i) and ii) with:
iii) there occur no shuffling between Fibonacci molecules residing in *different* molecular clusters (when comparing values of perplex molecules with values of size molecules);
iv) there occur no shuffling between Fibonacci molecules residing *inside* each molecular cluster.

We observe from the calculations of the size values of Fibonacci molecules that said 1:1-relation occurs, so that also conditions iii) and iv) are implied as satisfied.

This is a very important point, because the identities of i) and ii) can become established by many thinkable algorithms for perplex ordering of Fibonacci number substances, while *only the FC algorithm* provides also the satisfaction of both conditions iii) and iv). Satisfaction of iii) implies that every *perplex* Fibonacci molecular cluster, each completely filling a corresponding gap between perplex Fibonacci number substance atoms (identical to the corresponding gap between sizes of perplex Fibonacci atoms), contains *exactly those* Fibonacci molecules that taken together have all the *sizes* – and *only* these sizes – necessary and sufficient to completely fill a corresponding gap between sizes of Fibonacci atoms. Thus, all gaps between sizes of Fibonacci atoms are not only completely filled with *amounts* of uniquely different Fibonacci molecules, equal to the respective sizes of the gaps, but they are also completely filled with those Fibonacci molecules that have the sizes that can fill-in each and every position ("shelf") in the gap when the *perplex* number of such a position is interpreted as a *size* number.

As a simple illustration of this key point, let us assume that we applied an ordering algorithm that gave absolute priority to Fibonacci molecules of lower amounts of Fibonacci atoms, when deciding the perplex number of the Fibonacci molecules, and that this algorithm was identical to the FM algorithm in all other respects. The first deviance between the two listings would then occur for the number substance with perplex number 17. Here, the FM algorithm places the Fibonacci molecule (2,4,7) which has the molecular size 17; while the alternative algorithm here would place the Fibonacci molecule (5,7) which has the molecular size 18, and place the Fibonacci molecule (2,4,7) of size 17 at the next place which is the place for the number substance with perplex number 18. Thus, the alternative algorithm would exchange the positioning of these two Fibonacci molecules, with the result that the 1:1 correspondence between their perplex number and size number would be broken.

As we shall later analyse (cf. fig. 2), this 1:1 correspondence achieved by the FM algorithm is due to the fact that this is the algorithm that performs a listing of the Fibonacci molecules identical to a certain Fibonacci structuring into Pascal's triangle.

## FIBONACCI GAP SPACE DYNAMICS

We will now explore further the number space dynamics connected to number identities tied to Fibonacci gaps.

As a simple illustration to enter the general issue, let us imagine rectangular piles of neighbouring cubes, successively increasing in height, as neighbouring towers constructed from uniform cubic floors, only separated from each other by a common wall, and growing successively into skyscrapers. Further, we imagine such a cube as the unit to *measure* the height of the towering buildings, so that the height is measured as an amount of floors. Also, we imagine the succession of such heights to be equivalent to the values of the Fibonacci series. Then, perplex Fibonacci numbers correspond to perplex (horizontal) numbering of the buildings, say from left to right. Then the perplex number of a building determines its height as identical to the corresponding Fibonacci size number, with the one floor cube as the unit to measure the ontological extension of the building. Ontological extension always has *three* dimensions (the volume of the building), but since the cube unit has three dimensions itself and the volumes of buildings only differ with respect to this unit expanding in one direction and one of its dimensions, namely height, the expressions of (and comparisons between) the ontological extensions of buildings are directly equivalent to expressions of their heights. This means that the succession of volumes, which are 3D, are expressed and determined solely along one of these space dimensions. We can formulate this as that the cube unit has *three inherent* dimensions, when viewed inwards from its surfaces, and – in this case – *one external* dimension, when viewed outwards or expanding (duplicating). Obviously, the external dimension is not a *novel* space dimension in an absolute sense, due to height already existing as an inherent dimension, and there cannot be any external dimension that does not already exist as an inherent dimension. (However, the external direction may be different from the directions of the inherent axes, as for example if the cube is rotated 180 degrees around one of its corner points. In such cases the location of the new cube, in the reference frame of the inherent axes, must be described as a *combination* of expansion along more than one these axes, i.e. as a directional vector.) On the other hand, there is novelty here in the sense that one – and only one – of the inherent dimensions, with the *potential* to become externalised, is actualised into external dimensionality. (Therefore we prefer the term "inherent dimensions" instead of "internal".) Also, such a manifestation requires a *gestalt switch* from inwards to outwards at the implied surface of the cube (in our case the ceiling/roof of the first cube turning into the floor of the next, duplicated cube in the tower). We notice that the interface between the first and the second cube, their common surface, is of *two* dimensions. Thus, this simple operation implies interconnection of 3D (inherent), 1D (external) and 2D (interface). Also the interfacing 2D, just as the external 1D, are not *novel* dimensions in an absolute sense, but potentially existing in the inherent 3D of the ground cube. And still, in analogy with the external 1D, they are novel in a certain relative sense.

A gestalt switch is invisible in the dimensional framework of those space objects it operates *on* (cf. the analysis of the Necker cube in Rosen 1994). Regarded as such an

ideal operation, there occurs no physical thickness between the first and second cube in the imagined tower. Hence, the 2D is to be regarded as a transparent ceiling/roof/floor. To be transparent, this must be an attribute of *something* that is not transparent observed in another framework. Hence, the 2D interface is an ontological *paradox combining* two ontological levels, which we may picture in our thought as a 2D surface with a dotted structure. As enantiomorphs two cubes trivially can be brought to cover each other in 3D, i.e. inside the ontological framework where the two enantiomorphs are residing and characterised, without any need for rotation through an additional, fourth dimension, different from the case of the Necker cube. Still though, also in the case of two ordinary cube enantiomorphs there is implied something outside their own framework, namely the human subject who by active analytical operation *brings* these cubes to cover each other in mental space. And this operation in itself implies a gestalt switch from inwards to outwards movement. In our case this happens along *one* such ideal dimension, switching the perspective from regarding height inwards from the 2D surface to outwards from the 2D surface. By taking the one dimension of height away from the cube, by extracting and transporting it from outside mental space intto mental space, and only regard the remaining 2D interface surface, there arrives one empty or missing dimension, with related *freedom*, outside mental space. In this operation the 2D interface may be said to represent an *opening*, or open window, for the one dimension of height to become re-placed from mental space back to the space outside mental, and by this manifesting the addition of the second cube upon the first. Thus, in the constructive logic of this thought building, there is a tie between the 2D surface interface and the 1D height extension as different aspects of those three inherent dimensions they are made of and from and re-make up.

In the first cube the 2D surface on the side opposite to the top side, is ontologically different. This bottom or ground surface has no possibility for external expansion along the height dimension, due to the arrow of the height dimension expanding in the opposite direction of the floor of the first cube. In this sense the ground floor is solid, not "dotted" and transparent. However, in another and more fundamental and radical sense, the bottom 2D surface is not solid, but the *primary* opening of and to the cube, and thus to the *inherent* dimension of height, due to being the interface to and *from* the *perplex* house number. It is the perplex number of the building that *determines* its height of cubes, presupposing the Fibonacci sizing of the house numbers, with the first perplex number having the ontological extension of the first cube. Thus, the bottom 2D surface is *liminal* in a ground-rising way, and may be said to be the interface to the paradoxical *underground* of the construction, the ontological "basement" which is both hidden in the construction and underlying it. And the dimension for primary extension, height, may be regarded as a prolongation of the direction given by the perplex number *penetrating* into the "mundane" from the "underworld".

Also the 2D surface on the top side of the uppermost cube of the building, is ontologically solid in the sense of closing the building structure. Different from the ground 2D surface, however, it does not represent any opening or interface to somewhat on the other side, the sky, which is just the *absence* of the building. However, it represents an opening or interface in a more subtle sense, as the pre-step to the next

step, the next perplex number determining the construction of the next building. (This may be likened to that algorithm in the universal Turing machine, which determines the *next state* of the other algorithms and itself.)

If we use the gap of cubes between buildings of Fibonacci height 6 and 13 cubes as an example, there are two basic perspectives to conceive the gap:

*Gap perspective A*: We start with imagining a virtual succession of heights of the buildings as increasing along the skyline with one cube/floor from one building to the next. This *one-by-one size algorithm* is a very simple one, in the terminology of Bohm (1987) of order 2 (change of change) along the successive order dimension. Such an algorithm implies that the discontinuity changing size is as simple as possible, namely as adding one more of the same unit for each perplex step, and by this being continuous in its simple discontinuity. Such an algorithm also implies overall correspondence between perplex numbers and size numbers, so that all possible size numbers occur, one time each, and in a perplex order. From this algorithm we find that the heights/sizes of 9,10,11 and 12 cubes do not occur for any Fibonacci building, and that they together constitute the heights of the one-by-one size algorithm between the Fibonacci sizes 8 and 13. When restricting the consideration to the framework of only *this* gap between sizes of Fibonacci buildings, these sizes of cubes can be listed with *second* order perplex numbers 1, 2, 3 and 4 inside this gap. If we extend the framework to cover *both* the sizes of Fibonacci buildings and the missing *intermediary* sizes of *imagined* buildings as judged by comparison from the one-by-one-size algorithm, we can fix 8+1 as the first perplex number of a missing building size, and 13-1 as the last perplex number of building sizes in this gap. Then the building sizes in the gap can be listed as *third* order perplex numbers 9, 10, 11 and 12. Then we have achieved an over-all identity between the building sizes of the gap and the *perplex* numbers of these size numbers. Obviously, the same identity will become achieved for *missing* building sizes between Fibonacci buildings, as well as for the *whole* structure covering *both* the sizes of Fibonacci buildings and imagined buildings of all other sizes. In the last case, we must remember that this implies a *lifted mapping* of the original perplex numbers of the Fibonacci buildings, into *new* perplex numbering of the same Fibonacci buildings. Obviously this translation depends on the *amount* of imagined missing buildings, which is functionally dependent on the one-to-one size algorithm, and it is by means of this algorithm that the *new* and more final perplex numbers also for the Fibonacci buildings (i.e. not only for the buildings in the gap) coincide with their respective building sizes, i.e. size numbers, which were determined from their *original* perplex Fibonacci numbers. Thus, we realise that the final identity between perplex numbers and size numbers, both with respect to Fibonacci buildings and with respect to the imagined missing buildings, is an *achievement*, whatever simple, not appearing immediately or automatically from plainly comparing the sizes of Fibonacci buildings. This achievement involved *novel orders* of perplex numbering, both of missing buildings and of Fibonacci buildings, *after* the size numbers of Fibonacci buildings had become manifested. Thus, size numbers here function as an *intermediary* from which to reach the novel order of perplexity where there occurs a *general* identity between size numbers and perplex numbers. We notice that such an identity is first achieved with

respect to the missing buildings in the gaps, and via this identity, in the next step, such identity is also achieved for the Fibonacci buildings. This means that the size-perplex identity for the buildings in the *gaps* acts as an *intermediary* in the very constitution of size-perplex identity for the Fibonacci buildings (which illustrates the Fibonacci atoms). We may compress the main steps in this constitutional logic into the following short-hand description:

(14) Fib-perplex –> Fib-size –> Fib-gap-size –> =Fib-gap-perplex –> Fib-perplex'=Fib-size

Still applying the metaphor of buildings in this perspective, one could imagine a combined description of Fibonacci-sized and missing/gap building sizes by visualising the last ones into positive existence sharing the same 3D space by being located orthogonally to the upper floors of Fibonacci buildings; say, the missing building of 6 cubes coupled orthogonally to the sixth floor of the Fibonacci building 8 cubes high, and the missing building of 7 cubes coupled orthogonally to the seventh floor of the same Fibonacci building of 8 cubes. Still though, and without going into details concerning possibly adequate such description, this would represent, at least in first approximation, a somewhat formal, instrumental and externally imposed model, not organically generated into particular space realization and location by the Fibonacci algorithm itself.

An even more profound challenge is represented by the circumstance that the one-by-one size algorithm was implied as a crucial condition in the Fibonacci gap perspective compressed into expression (14). This is somewhat tricky because this algorithm appears imposed onto and into the Fibonacci generated number landscape from an *external* and somewhat artificial position, not back-reflecting an inherent and necessary operation.

## SUBTRACTION REVISITED: CONSTITUTION OF *minus* AS OPERATION BY *Minus* AS OPERATION FROM FIBONACCI DYNAMICS

We introduce the operation *Minus* between two numbers a and b, denoted by the symbol _ and defined formally by:

(15) a _ b = (a-1) – b

Trivially, this is formally equivalent also to (a-b)-1, i.e. (a minus b) minus 1. However, the r.h.s. expression of (15) is chosen as the primary one, because we introduce this operation in order to signify the *gap* between two size numbers of a uniform unit (having ontological extension) in such a way that the numbers on *both* sides of the gap are treated on an equal footing and excluded from the gap.

As a simple illustration, if we consider the gap between size number 8 of perplex F7 and size number 5 of perplex F6, the extension of the gap is of the size 2 (units),

equivalent to 8 *Minus* 5, not of size 3 (units) equivalent to 8 *minus* 5. This size of the gap was found by the first three steps of expression (14) which had the one-by-one size algorithm as a necessary condition. Here, the (larger and later) *size* number 8 was excluded on an equal footing with the (smaller and earlier) size number 5. More precisely, the eighth and top size *unit* (floor) of the larger size number (Fibonacci building) was *never included* into the gap, just as the fifth and top size unit (floor) of the smaller size number (Fibonacci building) never did *enter* the gap. Thus, *both* these top size units were more to liken with two *bridge heads*, never entering, and hence never becoming excluded or extracted from, the gap beneath the bridge.

Different to this *Minus*, the conventional notion of *minus* does *not* treat the larger and smaller number on an equal footing in this respect. The notion of *minus* gives priority to the observation post of number *a*, in our example the larger one, and presupposes that the top size unit of *a*, *contrary* to the top size unit of *b*, *prevails* during the operation, and in this sense becomes artificially inserted into or converted to the gap if – and only if – the operation is considered as relevant in a *gap* context. In such a context the asymmetry between *a* and *b* implied in the *minus* operation does not hit the mark of the issue, and therefore it is somewhat ontologically seductive to formally establish *Minus*, containing the adequate *symmetry* between *a* and *b*, as a secondary operation from *minus* as the first. Rather than re-establishing the adequate from the not that adequate, the adequate ought to be established right away, and *Minus* is the operation that targets the gap right ahead, whatever its formal equivalence to an expression derived from *minus*.

Of course there is nothing wrong *per se* with the *minus* operation formally or technically, nor is it anything wrong with *minus* ontologically *if* adequately framed or understood. The point here is that *minus* may be ontologically seductive when such implied framing is *not* understood or *developed*. And it may be the case that to ignore this or to regard it as a trivial concern, not making mention-worthy difference, acts a crucial *operator* in such seduction.

As we have seen, perplex numbers are prior to, as well as the generator of, size numbers. Therefore, when introducing basic mathematical operations, considerations of their adequacy with respect to perplex numbers must have priority to considerations with respect to size numbers. If we look at a queue of persons, it is evident that there occur *two* persons, not three persons, between person no. 8 and person no. 5 in the queue. Person no. 8, when looking forwards, or person no. 5, if looking backwards, will not include *himself* (or the other person) in calculating the amount of persons between them. Thus, in this more basic, perplex approach to number theory, *Minus* manifests as an operation before and more primary than *minus*. Viewed from person no. 8, *minus* of three persons can manifest *next* from basically one out of two ways:

1) By counting the number of persons before no. 5, which is four persons, and *compare* this (size) number of 4 to the (size) number of persons before himself, which is 7. This operation of comparison is *secondary* to and from two operations of *Minus* that are relata for the comparison, namely, in the most basic case, (8 *Minus* 0) and (5 *Minus* 0), where "0" denotes the something of a *non*-person, for example a desk, marking the

break of queuing persons, which starts/stops the queue. These two operators of *Minus* regard the queue *all the way* from, respectively, observations post no. 8 and no 5, not involving any *third* person as additional reference point (as when counting persons before posts no. 8 and no. 5 and behind person no. 2, which means that person no. 2 is regarded as stop point, just as the desk, despite not being a non-person), and are in this sense to be regarded as the most basic pair of *Minus* operators.

To achieve *uniqueness*, as well as a *positive* (size) number (of real persons) from the comparison, the *largest* (size) number, 7, must be chosen as the *prior* (size) number, and, correspondingly, the observation post of person no. 8, which in our case *is* the primary observation post for perplex numbers and derived size numbers occurring in the comparison, not the observation post of person no. 5. Thus, the comparison, different from the *Minus* operation, involves a basic *asymmetry* between the two *Minus* relata which it compares. Then, person no. 8 can *imagine* the second *Minus* relatum, 4, as *overlapping* (which implies differentiation in two ontological layers) the first *Minus* relatum, 7. Next he can imagine 4 and 7 as *conflated* (as *simile*) to *one* ontological level; after this he can imagine 4 as *cutting* (another simile) the "strip" of (size) 7; and from this he can finally infer that the size of the remaining "strip" is 3 units/persons. This imagined operation as a whole we then name '*minus*' denoted by the symbol '-' and we have established 7-4=3. In the most elementary and basic performance of such operation, these three persons must be persons no. 5, 6 and 7. Hence, person no. 5, but not person no. 8, is included as a perplex person in the *resulting* size number of the *minus* difference consisting of 3 (persons). This implies that this asymmetry in the *constitution* of *minus* is *opposite* of how it appears when already having become established. If we further refine the notion of '*minus*' as requiring *abstraction* from *which* persons it contains, i.e. as *abstracted* from the perplexity involved in its constitution, we finally arrive at the operation *minus* in the conventional sense.

In this way the operation *minus* is established from the operation *Minus*, unfolding as a *novel kind* of operation due to the fact that such a *comparison* between two *Minus* relata can not be realised by any second-order operation of the *same* kind as *Minus* itself. Also, we notice that the step from the operation of *Minus* to the operation of *minus* is underpinned by a corresponding step from perplex numbers to *sizes* of perplex numbers.

If we perform plural such *(Minus)minus(Minus)* operations in a specified succession, the result will be a corresponding list of size-differences in the same succession. This immediately implies that these size-differences are given new "names" as second-order *perplex* numbering of the differences. When we as a next operation look at the constitution of second-order size differences from *these* perplex numbers (of first-order size differences), the *Minus* operation must be given priority to the *minus* operation, for the same reason as for the first-order numbers. And next, when comparing these second-order *Minus*-differences, also the *minus* operation will re-manifest in its second order and result in a list of third order size differences, which then are named as third order perplex numbers, and so on all the way to the end of a generative number hierarchy. Thus, perplex numbers and size numbers of differences *alternate* at each step up in the hierarchy, and this alternation is with necessity tied to a *corresponding*

*alternation* between, respectively, the *Minus* operation and the *minus* operation. This double-tied alternation starts with perplex numbers and *Minus* (more as the number of one being the start for both all natural numbers and for the odd numbers), but thereafter they play a symmetric role to size numbers and *minus* (like even numbers) in this tandem climbing up the ladder of numbering and differences.

2) By a *self-referential* act of *abstraction* stepping one ontological level upwards, and *imagine himself* as included into the queue of persons standing before him. This means applying the operation of *addition* to extend the size number of 7 persons before him with the one unit of his meta-viewed himself, which we may express as "7 steps forwards, supplemented with 1 step upwards". We can name this operation *Minus+*, always being *one* unit larger than *Minus*, due to the ascending vertical step descending back again and converting to a horizontal one at the conflated level. This will coincide with the imaginary (proceeding) act of person no. 8 putting himself, as a simile, in the observation post of the person *behind* him, which implies that he not only views himself from stepping one ontological level above, but also steps down again into *another* person. If he repeats the same imaginary act, but now by starting out the act by first putting himself in the observation post of person no. 5, i.e. by performing *another* simile, we will have *two* exemplars of the operation *Minus+1*, namely (8 *Minus+* 0) and (5 *Minus+* 0). If we then *compare* the size values of these two relata, the procedure will be quite analogous to the one described in 1), but now the *minus* representing the comparison will be expressed as 8-5=3, i.e. with the same r.h.s. as in 1), but with and from a different l.h.s. Also, different from in 1), these three persons will now be persons no. 6, 7 and 8, which means that the asymmetry between no. 5 and no. 8, contrary to in 1), is *identical* to, and not *opposite* of, how it appears in the conventional notion of *minus*. However, this identity attributed to 2) was achieved by means of the operation *Minus+* which was developed *from* the operation *Minus*, and by this reason the truth of the identity attributed to 2) is more *superficial* and *secondary* to the truth of the opposition attributed to 1). The main point, though, is that the expression of *minus* as 8-5=3, attributed to 2), not at all means that *minus* is established *independently, directly* or *prior* to *Minus*. Just as in procedure 1), also procedure 2) develops *minus* from *Minus*, and even more so due to the intermediary role of *Minus+*. Also for procedure 2), *minus* is established from a certain *second-order* comparison between *presupposed Minus* constituted relata. Thus, when starting out with perplex numbers 8 and 5, it is only an *semblance (Schein)* that the *minus* difference of 3 manifests basically and directly as 8-5=3; this is the *manifestation or appearance (Erscheinung)* of and from *another* and more basic constitution of size difference between 8 and 5.

We realise that procedure 2) is somewhat less basic and more developed than procedure 1). One aspect of this is that the self-referential inclusion of person no. 8, the original subject for the consideration, implies that in procedure 2) the operation *minus* does not become established from *only* contemplating persons who person no. 8 *sees*.

Also, we realise that procedure 2) implies the same tandem climbing up the ladder of numbers and differences between perplex/*Minus* vs. size/*minus* as procedure 1) did, but with the interesting modification that we must extend *Minus* to *Minus+*. Still,

there are two different *kinds* of differences involved in the tandem movement, since *Minus+*, was *made from Minus* by a simple one-unit extension (while *minus* was made from *comparing two* differences established by *Minus*). However, this crucial difference in the *quality* of differences, now becomes *concealed* with respect to the quantitative value of the difference, since *both Minus+* and *minus formally is equal* as one more than *Minus* if operating the same pair of value relata to make the difference. From this concealment a uniform, conflated treatment of size-differences, ignoring the underlying qualitative zig-zag alternation, makes the subtraction dynamics more simple than it is, and contributes to the impression of *minus* being a quite trivial and innocent operation.

We can imagine a procedure of further complexification as:

3) By counting the persons before *two more* (compared to no. 8 and no. 5) persons in the queue, say before person no. 6 and person no. 3, and by comparison establish the difference between the two size numbers as 3, in this case as 5-2=3. For the minus to be equal to 3, person no. 5 must be *excluded* as reference person and made *redundant*, so this means that the *particularity* of minus 3 being established from perplex no. 8 *and* no. 5 has been lost. Obviously such a procedure does not qualify as basic to comprehend the constitution of size difference.

However, we mention this procedure 3) because it, like procedure 2) – but viewed from another angle – has some similarity to the following procedure:

4) By introducing an *additional* subject, *external* to the persons in the queue and the reference frame of the queue itself, who contemplates the queue as a whole, and inside this queue contemplates the size difference between person no. 8 and no. 5 by giving priority to the observation of no. 8 before no. 5. At first glance such an external subject may seem able to directly establish *minus* as 8-5=3, just as easy as for example comparing a line of 8 size units with a line of 3 size units, never mind that these size units first had to be determined in a perplex order. The external perspective can, different from procedure 2), directly include no. 8 in the first size and no. 5 in the second size.

Still though, the *Minus* operation is tacitly established more basically than the *minus* operation. This is because the *Minus* size 2 of no. 6 combined with no. 7 is disclosed for the external subject by an observation moving only *two* steps back from viewing no. 8 (and no. 5) as *perplex without* considering *any* size number of *perplex numbers* (only of the two-steps inserted *between* no. 8 and no. 5); while the *minus* size of 3 does not appear before a comparison of the *size* numbers of 8 and 5, which first have to become established from their perplex numbers 8 and 5, and which require that one moves *all* the steps back to 0 from both no. 8 and no. 5.

Further, the addition of the external subject does not really overcome or eliminate procedure 2). Rather, it *transports* and repeats procedure 2), because the vertical distinction between this subject and the reference frame of the queue as a whole, with no. 8 as the bridge head inside the frame, corresponds to the vertical distinction between person no. 8 and *his* step upwards through the act of self-abstraction. This means that the external subject functions as a hidden *no. 9*, quite

analogous to the role of no. 8 in procedure 2). Hence, procedure 4) establishes the *minus* operation *apparently* as more directly than procedure 2), while the *partial* truth of this being the case is due to the *additional* truth of this only being possible by means of the whole procedure 2) being *repeated* at another ontological level. Therefore, contemplated as a *whole*, procedure 4) establishes the *minus* operation *less* directly than procedure 2), and *less* independently of the operation *Minus*. Different from procedure 2) this is not that easily and immediately *transparent* in procedure 4), and in this sense procedure 4) is more seductive, *camouflaging* its *simile* of being able to overcome the restrictions of procedure 2) with respect to *basic* constitution of *minus* difference. Different kinds and combinations of simile operators are implied in all thinkable causality types (cf. Johansen 2008a: ch. 3 for detailed analysis), but in constitutional logic it is important to make such similes *transparent* and understand them as *integrated* in the constitutional dynamics.

The straight-forward simplicity of procedure 4) is a *Schein* because the procedure has hidden, non-trivial and highly relevant ontological underpinnings and differentiations which occur conflated in the performance of the procedure. At different levels and complexities such obscuration happens in much science, not excluding sophisticated ones. An example is the claimed universal and much celebrated truth of the Gödel theorem(s) showing to tacitly rely on restricted and not much adequate ontological assumptions (cf. Johansen 2006 and the supporting, proceeding treatment by Quartieri 2007).

This shortcoming of procedure 4), as judged by scientific standards of constitutional logic, does not at all mean that the procedure is not useful for teaching purposes, but this is because the *unconscious* of the pupil does not *need* more than the *clues* provided by this procedure, to perform by precise *intuition* the algorithms required to get the correct *result* value of *minus*.

In the ultimate and absolute sense, the very idea of an external subject is a *contradictio in adjecto*. Such a simile is highly useful in many contexts, and also unavailable, but it is still a simile. Ultimately, the external subject is generated *itself from* the same structure it *considers* itself external *to*, when this subject relates to the most universal, abstract and elementary generator of reality, namely the algorithmic Fibonacci number generation. Leaving the highly difficult – and fundamental – scientific issue of the qualia *emotion* aside (poorly treated in the history of science), there cannot *exist* any subject outside what is Fibonacci generated. Thus, an adequate treatment of constitution, must account also for the unfoldment and manifestation of this *apparent* externality, i.e. from Fibonacci *self-referential generativity*. And in this respect procedure 2) is *superior* to procedure 4) which represents a formally imposed quasi-solution of the deeper constitutional problem from outside, in this context the constitution of size difference, not an organically and inherently solution of the problem. Also, we found that procedure 2) represented an elaboration from the more basic procedure 1).

Procedure 4) implies even more than procedure 2) an effective concealment of the analysed underlying tandem alternation between *Minus* and *minus* involved in the generative hierarchy of size differences. Due to the shortcomings of procedure 4) to

grasp the constitutional logic generating this hierarchy, this alternation, already revealed from reflection upon a queue without further qualification of the queue's nature, may indicate that the whole hierarchy of size differences is most adequately grasped by a *binary hypermathematics* which respects and preserves the *qualitative* distinction between the two kind of operations, as well as the number ontological *layering* of them. This should be kept in mind for upcoming reflections and qualifications connected to the *Fibonacci* queuing up which generates the *emphatic* size differences of Reality.

If the framework that established size differences remained restricted to close-up and inherent investigation of *only* perplex numbers, it would be obvious, as in procedure 1), that the operation *Minus* was prior to the operation *minus*, and that amounts of perplex numbers residing *in-between* two other perplex numbers, such as the two persons between person no. 8 and person no. 5, would constitute the primary size-differences, *before* manifestation of the sizes of the framing perplex numbers (no. 8 and no. 5) themselves. This is consistent with the situation described by expression (14) with respect to the Fibonacci numbers. From our reflections upon the general queue it became clear that the prior role of *Minus* compared to *minus* is *not changed* by extending the framework to a more general treatment of size numbers. What changes is the *transparency* and the *simplicity* of this prior role of *Minus*. Whatever extension of the framework, when the framework is sufficiently examined, *minus* can not rise to any higher significance than as the complementary pole to *Minus* in the climbing alternation up the ontological ladder of size differences, never being able to challenge the primacy of *Minus* at the *bottom* level. However, because procedure 2) and – especially – procedure 4) *appear* with the false *impression* of such a possibility, as well as of *minus* being a universal and homogenous operation to determine size differences, superficial observation of such extended frameworks containing size numbers of the framing perplex numbers themselves, is much more seductive than the basic framework provided by close-up inspected perplex numbers. This explains the conventional view of *minus* as a prior and universal operation, delegating *Minus* to just a trivial *minus* -1 operation for the special case of determining the size difference between perplex numbers. Thus, the truth of the issue becomes clothed as well as quite inverted, and the size differences between perplex numbers become overlooked in stead of focused and back-reflected as the springboard for the whole dynamics generating size differences.

The operation *Minus* implies an elementary vertical differentiation into *two distinguished ontological layers*, namely the layer of the perplex numbers themselves and the layer of the *gap* between two perplex numbers. In the case of the queue of persons, persons no. 6 and no. 7 must tacitly be considered by person no. 8 as *disappearing* into this gap by being *deleted* from the framework of the *first* ontological layer, *before* they become re-introduced as persons, residing in the "underworld" layer of in-between the emphatic persons no. 8 and no. 5, and then counted as such persons. Since the *Minus* operation is the universal *bottom* operation in constituting size differences, this *primary* vertical differentiation remains implied as a necessary *condition* for *all* other and more or less elaborated calculations of size differences, whatever the kind and amount of supplementing similes, or whatever the *formal* expressions of more elaborated size differences. The operation *minus*, as described in

procedure 4), contains similes that *hide* this basic vertical differentiation of *Minus* (as well as *all* the *additional* vertical differentiations implied in the similes themselves) and by this *conflates* size differences to the radical opposite of what they basically *not* are, namely into differentiations along a number ontological "Flatland", or rather worse: a number ontological quasi-monadic 1D "Lineland". The same paradoxical camouflage is implied in all *formal* expressions and equations involving the operation *minus*, always treating size and perplex numbers *as if* any vertical differentiation does not occur. In most mathematical contexts this tacit simile is functional, fruitful and innocent, but in more profound questions, requiring extraordinary clarity, as in the present context, it seems to represent a confusing epistemological obstacle. The conventional notion of subtraction behaves as if its resulting difference, the size of the gap, resides at the same ontological level as the minuend and the subtrahend, while the truth is that the very operation of subtraction was tacitly developed from and always embedding a primary gap *characterised* as *qualia* by residing at *another* ontological level.

From these reflections concerning the operations *Minus* vs. *minus* considered from the framework of perplex queues in general, we now can move back-on and dive into the most profound, universal, elementary and abstract context of generation of size differences, namely from perplex Fibonacci generation.

Obviously, procedure 4) is inadequate here to hit deeper marks of the Fibonacci issue, due to not being of any primary nature, and not being inherent, organic, self-referentially generative or uniquely determined, and also due to being self-seductive by ignoring the distinction between the generative structure and its external outside where the observing subject resides. Different from this, procedure 2) is relevant due to its self-referential and vertical movement, and procedure 1) is even more basically relevant due to its bottom *Minus* nature.

Talking about Fibonacci generation as *self*-referential implies that the Fibonacci number is to be regarded as a receiving and operating informational *subject* in *some* significant sense. A subject, in the most abstract sense, is *necessarily* implied in any information *as such*. This is because, as Bateson noted, an input difference of something can not make an output difference of another something without this making a difference *for* a *third* something which is tacitly implied and enfolded in the very existence of information as qualia. In this sense, this third something is a *someone*, i.e. a *subject*. Thus, from qualitative, differential informatics it becomes clear that subject(s) is involved even in the case of billiard balls mechanics. Due to the tie to *emotion*, human subjects can be said to constitute *emphatic* subjects. Different kinds of subjects, whether animals or not "living" ones, involve different amounts and kinds of simile operators in their algorithmic make-up. Humans can not describe and explain without applying, tacitly or explicitly, a minimum of simile which projects (this may, or may not, also be *re*-cognition) attributes of the human subject into other subjects, be it persons or billiard balls. This is implied already in the structure of grammar in language, as also pointed out by Bateson. Thus, there is no such thing as scientific description or explanation without a minimum of anthropomorphism. Another thing is that making more or less implicate similes conscious, clear and adequate, can be quite important in more profound scientific issues, as well as intellectually demanding. For a

general and more thorough analysis of the ontological status and differentiations of the category 'subject', see Johansen (2008a: ch. 4).

The one-by-one-size algorithm was applied in order to account for constitution of size differences in Fibonacci gaps and from this establish the Fibonacci size-perplex identity as compressed in expression (14). However, despite the formal correctness of this, the perspective establishing the algorithm was quite external, and in this respect similar to procedure 4), not explaining the generation of this algorithm inherently and self-referentially. Thus, we should explore the issue further by approaching it more intimately from the Fibonacci subject itself.

In the example of the queue of persons, considering the basics of procedure 1), the subject no. 8 was imagined to see *two persons* between himself and no. 8, and to also perceive *three gaps*, respectively behind person no. 7, between no. 7 and no. 6, and between no. 6 and no. 5. This is different from the situation for the most elementary and constitutional perplexity, which is Fibonacci perplexity. In its reception the Fibonacci subject $F_n$ receives its input from $F_{n-1}$ and operates the input by combining it with itself into the output emitted as $F_{n+1}$.

## ALGORITHMIC TRANSFIGURATIONS BY FIBONACCI SUBJECTS

In the terminology of our differential informatics *informative transfiguration* denotes information made into another information, implying the three operations i) *reception* of the input-difference; ii) *algorithmic transfiguration* of the input-difference to an output-difference; and iii) *emission* of the output-difference (cf. Johansen 2008a: ch. 2.1). The algorithmic transfiguration gives the *transfigurative-difference* which denotes the particular difference between the input-difference and the output-difference, thus implicating that transfigurative information involves a relation between *three kinds* of differences. The *subject* is, by determined definition, to be understood as the something these three differences are differences *for*, i.e. with *respect* to. The subject performs the algorithm by tying the three differences together in logically determined – and in this sense: continuous – interconnection.

The Fibonacci subject ($F_n$) is to be regarded as the most minimalistic and elementary subject imaginable, due to the Fibonacci algorithm constituting the simplest thinkable informative transfiguration with respect to all three kinds of differences (and their unity). Also, it is a *universal* subject, since it has to be involved in *all* informative transfigurations, and hence the elementary subject of *existence*.

When the $F_n$ subject combines the input $F_{n-1}$ with itself, this must happen in the algorithmic transfiguration *inside* the border surface for the ontological extension of the $F_n$ subject, different from reception and emission taking place *at* the (internal side of) the surface. If something combines itself with something else, there are *two* versions of the first something, the operating (combining) something and the second version which it operates *on*. These two versions must co-exist in the operation, and reside at different, vertically distinguished ontological *layers*, for this self-referentiality to become

possible. However, by closer inspection this is a simile. In the case of the $F_n$ subject it becomes *one* with the input $F_{n-1}$ *momentarily* after it receives it, and by this the $F_n$ subject is *deleted*; there *are* no remaining rest of the subject $F_n$ *after* the algorithmic transfiguration. The seer and what it sees, transfigures irreversibly to a novel one, $F_{n+1}$, from the very act of seeing, and by this the vertical distinction between the two, the input-object and the operating subject, is also deleted. The Fibonacci algorithmic transfiguration is for the $F_n$ subject to *delete* the border to the input-difference as soon as you notice the border being crossed by reception. The *result* of this is that the $F_n$ subject itself is deleted, but this comes not from the algorithm operating on itself directly, but on deleting the *border*, i.e. an *attribute*, of the subject.

One may liken the walk of the Fibonacci algorithm as $F_n$ taking one step back with one foot, to $F_{n-1}$, then bring this foot in line with the other foot at position $F_n$, and then the two feet jumping together from this position to $F_{n+1}$. However, there is no internal state in the algorithmic transfiguration where the two feet *rests* before they jump. The analogy would be closer if imagining them jumping forwards to $F_{n+1}$ as soon as the backwards foot touches $F_{n-1}$. Still though, the metaphor of walking feet is not radical enough to grasp the essential minimalism of the Fibonacci algorithmic transfiguration. In a radical sense, the $F_n$ subject disappears in each step of the Fibonacci algorithm. However, it reappears as an *ideal part* of the *novel* $F_{n+1}$ subject which always is born from its "dying". Due to each and all $F_n$ subjects doomed to die in their "footsteps" as soon as they become "alive" and "walk", the real subject staying alive and extended reproduced as such, is the upper supra-subject, one might say the primordial "head" above and before, generating each $F_n$ subject, more as "the ghost who walks" as the cartoon character *The Phantom*, which is the stronger one and the one who never dies, but only reincarnates, or like the *übergreifende Subject* (supra-superior subject) *Capital* in late Marx' capital logic, *die reale Metaphysic* (metaphysics for real) as the philosopher Hans-Jürgen Krahl coined it. The Fibonacci generative order interpreted as self-referential, makes most literal sense when contemplated from the perspective of this upper, "all-seeing-eye for walking".

Despite the unavoidable need of anthropomorphisms to describe subjects in general, whatever minimalistic, we do not want to imply any mystical notions by applying more than minimalistic such to comprehend the Fibonacci subject. The task is rather to apply such metaphors in order to achieve as sober a comprehension of the issue as possible, by assuming that the anthropomorphic cloths appear sufficiently clear for the reader from the contexts they are applied, so the cloths can function as lens from distance to *strip* the Fibonacci subject *more naked*. However, by further contemplation *from* "the naked skeleton of truth" (Bateson), it may be the case that anthropomorphisms in some sense or degree reveals to be more than just metaphors to encircle and reveal the "skeleton", and in this sense they may have some *living* truth. But such contemplation, from switching the direction of exploration, should not be much of an analytic concern until *after* a thorough understanding of the Fibonacci "skeleton" as such has become achieved. (Cf. Johansen 2008a: ch. 2.1.2 concerning the ontological status of metaphors in differential philosophy.)

Then, *how* does the Fibonacci subject $F_n$ receive its input from $F_{n-1}$? If we use F8 as example of $F_n$, the F8 subject, different from person no. 8 in the queue, seems to only receive information from *one* another before him, namely F7, due to no other preceding perplex somethings being represented as input for him. All direct information concerning perplex somethings before F7 seems *blocked* due to F8 only viewing the "back" of F7 which *hides* the very *existence* of the other preceding Fibonacci numbers. Thus, F8 seems not able to even be aware that he is standing in any queue. Further, different from person no. 8, F8 can not observe any *gap* (with ontological extension) between him and F7. Due to the nature of the interface, F7 is received as input by F8 *only* as something other than himself by crossing his border line to him. F8 only receives F7 from *touch*, he does not receive any distance or space *between* them. This would be similar to the body of person no. 8 standing tightly behind the body of person no. 7, the two only distinguished by their skin interface. For a general treatment, we cannot assume any difference between the sizes of the two bodies. Thus, no. 8 does not seem to receive *any specific* information about the *size* of the gap between them, and the same would be the case for any person in the queue.

Then the tricky, but perhaps crucial, question arises, how come that size-differences of and in Fibonacci *gaps* can become received at *all* by *any* $F_n$ subject, i.e. how can such *externally* observed differences make any difference in the *inherent* generation from the Fibonacci algorithm and thus *become* real from the Fibonacci algorithm *itself* and its subjects themselves?

The clue may be the provided by the fact that all perplex Fibonacci numbers must have ontological extensions with sizes determined by their perplex numbers. And these size numbers are with necessity *different*. We may consider each perplex Fibonacci number like radically distinguished, unique, "monadic *souls*", and their ontological extensions as the *bodies* of these souls, having different body *sizes* indicated by the size numbers of the respective Fibonacci atoms. (These perplex monads are strictly interconnected from the perspective of the *whole* generative Fibonacci landscape, but for each and one of them as Fibonacci subjects, such interconnection is hidden, because information about such is beyond the border surface of inputs and outputs relating to each Fibonacci subject.) Further, we apply the metaphor of cubic constructions, and consider these body sizes as such towers of cubes, with the top cube representing the "head" of their body, as the cube where the perplex soul "sees" what is before him in the perplex queue, hence as the skin interface where input-difference is received.

Considering the perplex Fibonacci queue, $F_n$ can only receive $F_{n-1}$ as input, and as before him in the queue, by $F_{n-1}$ being something with an ontological *extension*, thus from the *body* of $F_{n-1}$. $F_{n-1}$ can never be available *directly* as perplex to $F_n$, only *via* the input of the $F_{n-1}$ body to $F_n$. And the body of $F_{n-1}$ is characterised *solely* by its particular *size* being different from the body of $F_n$, thus as a size difference along one dimension (viewed from an external subject), namely height (of the cubic body building).

There occurs no gap of *perplex* Fibonacci numbers between $F_{n-1}$ and $F_n$, hence nor any ontological extension between them. Therefore the *whole* body of $F_{n-1}$ must touch the body of $F_n$, which is realised by the cube being the uniform unit of the towers,

since cubes are a 3D object that can completely fill any 3D framework. This indicates that the *cube* may be more than a metaphor, since the cube is by far the simplest space-filling regular object, the only such that is constructed without *further* conditions added to the oneness of the primordial unit of space. (Interestingly, the cube is considered the essential geometrical form in Greek philosophy, such as Plato, and also in the organic geometry of Erik Trell.)

The head cube of F8 can not look at himself (due to no self-referentiality in the strict sense occurring *inside* the algorithmic transfiguration of F8), only towards the input he receives from F7. However, he can not receive as input-*difference*, i.e. as *crossing* his boundary surface, those parts of F7 which are situated as neighbouring his own body. Such cubes of F7 are hidden from his sight. They constitute a necessary part of the input which makes the difference F8 receives, but F8 does not receive this part as such, only the differences they *make* for *him*. The *only* exception from this is the *roof* of the *top* cube of F7, i.e. the cube at floor no. 13 in the F7 body building. Therefore, F8 receives F7 as an input difference solely represented by the *two* dimensional top surface of F7. The F8 subject *treats* the whole F7 3D object (by fusing with it), but this is done by solely receiving information about it as a 2D square *below* his head cube. In principle, it is not that different from making physical love with another human body after having looked at a 2D picture of its face. The whole 3D body becomes *sucked into* the receiving 3D body from receiving its top of the head 2D representation.

Obviously the same will be the case for all $F_n$ subjects. As input-difference all $F_{n-1}$ is received as the same *quantitative* 2D unit by all $F_n$ subjects. Each such 2D unit represents highly different body sizes, but such differences are hidden for the $F_n$ receiving subject; it only cares about what makes a difference for itself.

Due to the differences in body sizes, i.e. tallness, the head cube of F8 can also see the roof of the top cubes for some Fibonacci bodies *before* F7 in the perplex Fibonacci queue. If we inspect the issue from simple geometric observation, after ordering the cubic towers from, say, left to right, each tower touching their two perplex neighbour towers, and if we further consider the receiving "eyes" of $F_n$ to be located at the front (left) line of the ceiling of its top cube, we find that $F_n$ always can see the whole roof of F1 (and of $F_{n-1}$) and *parts* of all the roofs between F1 and $F_{n-1}$. This means that the $F_n$ subject cannot receive the input-information of $F_{n-1}$ without at the same time also receive input-information about all the *other* Fibonacci bodies residing *before* $F_{n-1}$ in the queue. Due to the $F_n$ subject only receiving perplex $F_{n-1}$ with the *body* and body size of $F_{n-1}$, the $F_n$ subject *must* also receive input-information from all the other preceding F bodies; the body of $F_{n-1}$ is simply not receivable for $F_n$ other than from a receiving perspective that also makes visible the roof of the preceding F bodies inside the horizon of $F_n$. However, the $F_n$ subject does not receive these input-informations with their correct (3D) sizes, simply because the are not *visible* to him as such, but he receives them as *perplex* with 2D ontological extension, and with perplex numbers in the $F_n$ reception view, distinguished from the perplex reception number of his closest visible roof of $F_{n-1}$, as well as internally distinguished. In the reception of $F_n$, nor $F_{n-1}$ can be received with its correct (3D) size, but different from the receptive inputs from preceding Fibonacci bodies, the body of $F_{n-1}$ is not only "seen", but also "swallowed"

through the "mouth" of the $F_n$ subject and becoming "digested" by the transfigurative algorithm of $F_n$ which executes fusion with the body of this – and only this – "seen" input, hence *distinguishing* between the *closest* perplex number and all the others received in the "vision" of the $F_n$ subject. $F_{n-1}$ is not seen by $F_n$ with its correct (3D) size, but it can only become fused with $F_n$ with its *whole* body, while the F bodies preceding $F_{n-1}$ do not enter the $F_n$ body. Hence, in this respect they remain *only* as *seen* by the $F_n$ subject, as a gestalt background for his seeing of the foreground $F_{n-1}$ which he fuses with. First they *are* seen, and next this sight becomes *excluded* by $F_n$ as *irrelevant* input-differences for his digestion. In this respect they become *lumped together* by $F_n$, in a sack, so to speak, to be rejected and thrown into the "garbage pin". This implies that their perplex numbering in the reception of $F_n$ does not matter anymore and makes no further differences. (Thus, nor can the *amount* of these, which is the size number n-1, make any difference *as such* for later Fibonacci generation.) However, they must be excluded as they really are, i.e. with their full (3D) body sizes, not as what they *appear* to be at the interface for $F_n$. Just as $F_n$ in his *inclusion* of $F_{n-1}$ fuses the whole body of $F_{n-1}$ with himself, $F_n$ in his *exclusion* of the F bodies received in the background horizon *removes* the borders between these bodies. The $F_n$ subject does this by just removing the borders between them as perplex (2D) inputs, but this *implies* that he also deletes *all* borders between them as *whole* bodies. Just as the $F_n$ subject includes the *full* body of $F_{n-1}$ without knowing its (3D) body size, he excludes *full* Fibonacci bodies. Further he rejects them as a *whole* of their whole bodies, i.e. as a *Fibonacci common-body*.

The size of the aggregated size numbers of this excluded Fibonacci common-body, the sack of Fibonacci bodies lumped together, is given simply by the formula:

(16) $F_1 + F_2 + \ldots + F_{n-3} + F_{n-2} = F_n - 1$

Thus we see that this sack size is i) *identical* to the gap between the size number of $F_{n+1}$ and $F_{n+2}$; as well as ii) identical to the size of the $F_n$ subject minus the primordial unit *1*. Formally, these mathematical identities are trivial, but their adequate ontological interpretation and succession in Fibonacci generation, *substantially establishing* what afterwards is confirmed as formal identities, may be far from trivial, and the same with respect to formal implications from such substantial interpretation.

At the reception side the $F_n$ subject must receive the input-differences from l.h.s. of (16) *before* the reception from input $F_{n-1}$. If not so, $F_n$ would firstly *fuse* with his neighbour $F_{n-1}$ into $F_{n+1}$, and then $F_n$ would *not be around* anymore as a subject to receive and transfigurate the input-differences from inputs before $F_{n-1}$, and these inputs would have become received by $F_{n+1}$ as subject instead of by $F_n$. In general a subject has to have its horizon of reception implied *previously to* its foreground ("neighbour") of reception, due to the horizon representing the condition to distinguish the closest from the further, which requires that there is something received (background) *between* the closest and the horizon.

When the $F_n$ subject lumps the Fibonacci bodies of his reception background together in the sack, this operation must be performed *after* his reception. There exists no space *before* him in the Fibonacci queue which can be used as a garbage pin outside

his "house" (his ontological extension and boundary surface) to place the sack. Therefore, it must be *forecasted* into a space located after the space he occupies himself. This cannot be a space *below* $F_n$, since the output sack then would be placed in a gap between $F_n$ and $F_{n+1}$, not between $F_{n+1}$ and $F_{n+2}$. Therefore, this space must be located to the immediate *right* of the subject $F_n$, and at first *above* the ground (if the output-emission is not considered to involve two geometric operations, left-right and up-down in one sweeping step). However, the next space to $F_n$ becomes occupied by $F_{n+1}$ when the $F_n$ subject in the next step receives the input from $F_{n-1}$ and transfigurates it to the output $F_{n+1}$. Due to Fibonacci perplexity there can not be any space *between* $F_n$ and its right neighbour $F_{n+1}$ inside the *same* 3D space. One may view the solution of this as that the output $F_{n+1}$ becomes placed by *pressing* the sack *below* the upper 3D, through a *hole* below the quadratic surface of its ground cube, so that the sack *disappear* to an *underground* space or *underworld*. If so, the sack becomes placed at the "end of the world" in the *Umwelt* (Uexküll 1909) of the operating subject $F_n$, having only a 1D *line* in common with the $F_n$ body, namely the up-left line of the top cube in the sack, which is identical to the down-right line of the ground cube of the $F_n$ body.

From this interpretation, the size of what is between $F_{n+1}$ and $F_{n+2}$, namely $F_n$-1, is determined *dynamically*, in a certain sense, by expression (16) performed as an algorithmic transfiguration by the $F_n$ subject. This explains the peculiarity that the size of this gap is determined *before* $F_{n+2}$ manifests, due to the gap being created "underworld" from the $F_n$ subject, and thus not being dependent of $F_{n+2}$ neither with respect to $F_{n+2}$ as perplex Fibonacci number nor as size number.

When the ground square below the $F_{n+1}$ body opens to the underworld, this may at first be considered as an *abyss*. However, this abyss becomes closed as a particular size, and in this sense a bottom is implemented, as soon as the sack from $F_n$, with its particular size, has been placed down into the abyss.

As performed as an algorithmic transfiguration by the $F_n$ subject, one could say that also the putting-together-into-a-sack, after all, represents some kind of "digestion" by the $F_n$ subject of what is received as its background differences. But this would be another *kind* of digestion than the fusing with the $F_{n-1}$ body. The last would be a bit similar to input for body building and reproduction (the opening at the output surface of $F_n$ being more like a vagina), while the first would be more similar to the part of food energy that transforms to waste (this opening at the output surface being more like an anus). Whatever metaphor, the first kind of "digestion" is less inclusively and less intimately related to the $F_n$ subject, and one might say that the $F_n$ subject makes the input Fibonacci bodies before $F_{n-1}$ pass from "heaven" into being buried in "earth".

The Fibonacci common-body of the sack has an exact size, given by (16), which gives the size of the *space* below the ground line between the bodies $F_{n+1}$ and $F_{n+2}$. Due to the distinctions between the Fibonacci bodies of the common-body being deleted by the $F_n$ subject, this space should be considered *emptied*, and in this relative sense as a *hollow* space, with the potential of becoming (re-)filled later in the Fibonacci generation.

The size of this emptied space is formally identical to the size of the body $F_n$ subtracted with the primordial unit of one cube. However, this is an identity of second

nature, not a primary organic one, since the size of the body $F_n$ plays no role in the generative constitution of the Fibonacci common-body. Also, when examining the r.h.s., $F_n$-1, in equation (16) as *substantiated* from the l.h.s. and the underlying, preceding reception performed by the $F_n$ subject, it is not *F1* that is excluded from the input horizon of $F_n$, and thus from the constitution by the l.h.s., but $F_n$ *himself* (as well as $F_{n-1}$ which only enters the foreground of his input). Thus the joint '1' at r.h.s. does not represent F1 in this (immediate) perspective, rather the $F_n$ subject himself insofar – and only insofar – he *reincarnates* the primordial unit by looking all the way back from his top cube, across the skyline to the whole-roof of F1, lyrically perhaps imagined as "greeting" F1 by releasing the "hat" of his head (top cube) while bowing.

This is consistent with our previous reasoning arguing *Minus*, not *minus*, to be the primary operation. In this fundamental case of the Fibonacci common-body later becoming a gap, we also notice that *Minus* is established and substantiated *positively* and by means of *addition* (between the bodies making-up the common-body) as operation. The common-body manifests from the $F_n$ subject looking at his background input horizon, not from looking at his own size, which he is not able to. From his *Umwelt* $F_n$ can not *know* what he does in this respect, but examined from an external subject it *must* always be the case that $F_n$ creates a Fibonacci common-body, delivered underworld as emptied space of a particular size identical to his own body size minus the primordial cube. Thus, this identity is *externally found* and observed as a *result* of the Fibonacci generative process.

Due to the two relata of the identity residing in two different ontological realms, one in the underground and one above ground, this may be looked at as a primary identity between a filled space and its *isodual* space. We notice that these two spaces are distinguished also *horizontally*, due to the filled body space residing above the ground quadrate of body $F_n$, and the emptied space residing below the ground quadrate of body $F_{n+1}$.

A special case is represented by F4 as an $F_n$ subject in our present context. From the left front of the ceiling of the top cube of F4, *no* part of the roof of $F_{n-2}$, which in this case is F2, is visible. Still, though, the common-body delivered as the gap between F5 and F6, also in this case includes the body size of $F_{n-2}$. The line between the top roofs of $F_{n-3}$, i.e. F1, and of F2, is, from the perspective of F4, covered by the line at the left of the roof of F3. One might consider the case as if this doubling makes this line *thicker* than the line at the left of the roof of F1, and that the F4 subjects by means of this difference in thickness also in this case includes the body of $F_{n-2}$ into the common-body.

With danger of clinging too neurotically to our metaphors here, we may also notice that the roof of the primordial cube of body F1 is the *only* one, included in the common-body, which the $F_n$ subjects always can view the *whole* of (by an angle though); this perhaps suggesting some support to the perpetuating status of the primordial cube in difference to the other Fibonacci bodies.

The Fibonacci common-body is emitted as an output from algorithmic transfiguration performed by the $F_n$ subject by simply *annihilating* the distinctions *between* the Fibonacci bodies in the sack (while the $F_n$ subject, contrary to this, does not annihilate any distinctions *inside* the body of $F_{n-1}$) as they are received by him as

*perplexly* ordered input-differences from the roof surface aspect of their bodies. (Thus, the 2D *sizes* of the parts of the roof surfaces, as they appear to the $F_n$ subject, do not matter.) The $F_n$ subject *rejects* these bodies (after having received them) as a *whole* common-body, but he does not *annihilate* the *common-body*; rather he *creates* the common-body, which becomes rejected *to* a certain place, "dumped" below body $F_{n+1}$, at the ultimate end of the $F_n$ subject's *Umwelt* and performance, *expelled* and "buried" below the $F_{n+1}$ body.

Thus, there is a paradox here: The Fibonacci common-body is *established* from annihilating the *internal* "skins" between its bodies, and in this sense their respective body spaces are *emptied*; while on the other hand their *Gesamt* (i.e. aggregated as novel) space is *preserved*, not crunched. Since there is no such thing as an empty space in an *absolute* sense, the hollow space of the common-body must still have a *unit* of ontological extension. If not, it would not *be any* space. "Hollow" is an attribute *of* a space, not an attribute *contradicting* space; hence it is hollow *relative* to a space that appears *filled* of something, namely of the *tertium comparationis* involved in the comparison. This paradox is simply reconciled by the circumstance that the fusions of the singular perplex bodies into the Fibonacci common-body do not annihilate their *primordial unit* of ontological extensions. Still, the Fibonacci common-body, in this originating reference frame, is *composed of* this unit and *measurable in size* by this unit. (This does not with necessity imply that the same common-body not can become measured also by *another* unit, appearing later from Fibonacci dynamics. We shall soon touch into this issue.)

Preserving the primordial unit of ontological extension and removing all *other* distinctions, implicates that (also) the Fibonacci common-body should be considered a uniform tower consisting of these cubic units. In the most elementary, abstract and universal comprehension of the Fibonacci algorithm performed by a $F_n$ subject, there occurs no distinction to make any other distinction than this primordial of the unit. Thus, the Fibonacci common-body must be an inverted tower of cubic floors, located vertically below the $F_{n+1}$ body, and with a perplex ordering of these floors. Therefore, the *size* numbers implied in l.h.s. and r.h.s. of equation (16) persist as the size of the *whole* pit, but at the same time they also convert to *perplex* numbers of these floors in this pit. Since all *other* distinctions than the primordial unit and the aggregated size of the Fibonacci common-body are annihilated, the *substantiations* of the singular Fibonacci bodies have become abstracted from, so thereafter they have no relevance for the *internal* architecture of this pit. For the same reason, the substantial *departure context* of a cubic unit (say, if it was a part of the $F_{n-2}$ body or of the $F_{n-6}$ body) has no relevance for its *arrival* placement as a floor in the underworld cubic tower. There must occur *some* kind of departure-to-arrival algorithm, but it does not matter *which* one this is, and in this respect the shuffling of the cubic units into their perplex ordering in the Fibonacci common-body can be considered *as if* it was *random*. (Cf. Johansen 2008a: 3.2.6 for analysis of *chance causality* and *probability causality* with their implied similes, having as one of its implications that the Bohr opinion of the universe as generated from chance "all the way down", represents a fallacy of onto-logical thought, as also much pointed out by Bohm.) This seems to be a basic incident of *randomness*

being Fibonacci generated, thus indicating an ontological platform (with underpinnings and constraints) constituting chance (simile) operators in natural systems.

The perplex numbers of the underground floors of the Fibonacci common-body have all the uniform ontological extension of one cubic unit. In this respect the situation vertically underground is analogous to the situation horizontally above ground between the *ground* cubes of the perplex Fibonacci bodies, though with the difference that the underground architecture manifests secondary and *from* the architecture above the ground. Above ground, despite the uniform size of the ground cubes, the sizes of the Fibonacci bodies differ from each other, measured by their differences in heights. We can consider the same to be the case for the underground architecture, measuring the sizes of perplex Fibonacci underground bodies by their differences in *widths*. However, this does not by itself imply that the quantitative relations between these widths must correspond to the height relations between the Fibonacci bodies above the ground. This has to become decided from discovering and examining the exact Fibonacci operations that *generate* the widths.

Above the ground, when applying the tower cube metaphor, each perplex Fibonacci number could immediately be said to have the same ontological size, viewed from *the ground*, but this did not give any information about the *total* size (height) of the Fibonacci building built from each perplex ground floor. There seems no reason to not consider the same to be the case with respect to the underground architecture. Different from the horizontal succession of Fibonacci bodies above the ground, the Fibonacci underground has a vertical *bottom*, a limit for how many underground cubes (floors) it has, given by the size of the Fibonacci common body. Still, this size number has been *converted* to *perplex* numbering of underground floors, and thus does not give any information about the *sizes* of these perplex underground Fibonacci bodies, neither each one of them nor the whole of them, only about the uniform size of their perplex underground *basic* floors or *entrance halls*. Because these underground Fibonacci bodies at first *only* are determined and differentiated qua *perplex* bodies, i.e. *without* distinction between their sizes, there is established a space of *freedom* for *building* such underground Fibonacci bodies (in width). Though the underground 3D space is limited in *bottom*, it is not by this limited in the other two dimensions, since this is an *underground* space, *separated* from the 3D space above the ground. This freedom for building must *become* constrained by Fibonacci operators that *perform* such underground building, operators which we so far have *not* contemplated in our reflections on the size dynamics of Fibonacci gaps.

Considering that the FM algorithm establishes the Fibonacci molecular identity, the question arises: How is this algorithm to be considered or explained when not from the outside of an external subject, but as generated inherently and organically from the perspective of the Fibonacci *subject*?

We can start with inspecting, as an example, the gap between F6 and F7. The Fibonacci common-body of the perplex numbers (floors) of the gap is an output from a transformative algorithm performed by the F5 subject, having the input-differences for this algorithm from F1, F2 and F3, which gives the amount of 1+1+2=4 perplex floors of the gap. Later, these floors, according to the FM algorithm, become filled of, in

succession, the Fibonacci molecules (2,6), (3,6), (4,6) and (2,4,6) with molecular sizes of, respectively, 9, 10, 11 and 12 primordial units. What is the *subject creating* these Fibonacci molecules by picking and combining preceding Fibonacci atoms according to the FM algorithm? Since F6 is a Fibonacci atom included in all these Fibonacci molecules, this subject obviously cannot be any Fibonacci subject *before* F6, such as F5. No Fibonacci subject can transfigurate a Fibonacci object *larger* than and created *after* itself. Nor can we easily imagine *F6* to be this subject. In its transfiguration picking and combining Fibonacci atoms, the Fibonacci subject must basically stay *separated* from these atoms with respect to the core of the Fibonacci subject, not *fusing* with any of them, which would imply to *annihilate* itself, as when the subject $F_n$ fuses into the novel perplex Fibonacci atom $F_{n+1}$ by deleting the border to its input from $F_{n-1}$. If we supposed F6 to be such a subject, it had to swallow the Fibonacci input atoms and *fuse* with them, not staying *distinct* from them in the output (as required by the very structure of a Fibonacci *molecule* as distinguished from a novel Fibonacci *atom*), and by this *abduction* becoming unable to fuse with the input from F5 into the novel perplex Fibonacci atom F7. Thus, F6 can hardly be imagined as the Fibonacci subject picking and combining novel Fibonacci molecules to enter the perplex entrance halls of F6's underworld. Nor seems the next candidate for such a Fibonacci subject, *F7*, to be able to meet the requirements to qualify as such a subject. The shortcoming of F7 in this respect is that it hardly can be imagined to receive the input from F6, which is included in all the Fibonacci molecules residing in the inspected gap, *without* fusing with F6 into the novel perplex Fibonacci *atom*, namely F8, which is *not* included into any of said Fibonacci molecules. The difference between the input-difference of the foreground, from the perspective of Fibonacci subject F7, namely from F6, and the input-differences of the background, from the Fibonacci atoms smaller than and preceding F6, is exactly that the F7 subject *fuses* with F6 in *distinction* to all the others which can be transfigurated in a more external manner. Then, we arrive at the next candidate for the sought Fibonacci subject, namely *F8*, and here the objections valid for F6 and F7 obviously do not hold anymore, nor is it possible to see any other restrictions to arrive. Therefore, the unavoidable conclusion seems to be that *F8* is the Fibonacci subject that performs the picking and combinations of preceding Fibonacci atoms into Fibonacci molecules residing in the gap between F6 and F7, in accordance with the FM algorithm. Thus, the total picture with respect to this gap, is that F5 is the Fibonacci subject creating and opening the perplex floors, and by this the *entrance*, for Fibonacci molecules to enter underground below F6, while F8 is the Fibonacci subject creating and composing the Fibonacci molecules that actually *enters through* this entrance, and by this determines the *width* of each perplex floor, i.e. the *size* of the Fibonacci molecules, as well as its exact make-up of Fibonacci atoms.

Obviously, this reflection is of a general nature, just applying the gap between F6 and F7 as an example. Therefore, the general situation must be that when $F_n$ is the Fibonacci subject creating the perplex floors for Fibonacci molecules underground below $F_{n+1}$, these floors are filled with Fibonacci molecules, of according body sizes and Fibonacci atomic make-up, by $F_{n+3}$ as the Fibonacci subject. Thus, there is a difference of *three* perplex Fibonacci numbers between the Fibonacci subject creating

the *entrance* of emptied space for Fibonacci molecules and the Fibonacci subject actually *constructing* the building of Fibonacci molecules *filling* this entrance and expanding with particular widths from there. Lyrically expressed, one may imagine $F_{n+3}$ as some Santa Claus character *present-ing* the *packets* (composed of Fibonacci atoms) to those waiting at the perplex floors of underneath receptions (*pre*-sent from $F_n$), where the packets are delivered from "the heaven" above (after being picked by the FM hand of $F_{n+3}$ from previous Fibonacci atoms) through the "pipe" of the Fibonacci body of $F_{n+1}$, a pipe which is not *literally* inside the body of $F_{n+1}$, but *superimposed* into it by a simile, from a *supra*-dimensional reference framing of this body (*combining* the underworld and the above-world of it), by which this pipe functions mysteriously as an interface *directly* to the underneath receptionists waiting and hoping to become "delighted" by becoming filled of Fibonacci molecular "life", and with the sizes of these "packets" corresponding exactly to what they "deserve" from their perplex number distance to the profane above-ground.

Thus, there is something peculiar and crucial about the *distance of three* with respect to perplex Fibonacci numbers, i.e. the distance between the Fibonacci subject $F_n$ and the Fibonacci subject $F_{n+3}$, signifying the distance between entrance and walk-in, between opening and closure, between infinity and finiteness, between potentiality and actuality, between emptied space and filled space.

Also, the *distance of two* has a certain significance, due to the $F_n$ subject not picking any Fibonacci atom larger than $F_{n-2}$ when making the gap between $F_{n+1}$ and $F_{n+2}$, later to become filled by the Fibonacci molecules composed by the $F_{n+3}$ subject. Taken together, this suggests a certain basic significance of the *distance of five* in the general Fibonacci constitutional dynamics. This may be fruitful to keep in mind when arriving later in our treatment to the basic split code 5:3.

## MULTIPLICATION REVISITED: CONSTITUTION OF THE OPERATION MULTIPLICATION FROM FIBONACCI DYNAMICS

**TABLE 4. The originated product of Fibonacci multiplication**

| Gap | a | b | c | a×c | u | a×c /u |
|---|---|---|---|---|---|---|
| F3/F4 | 0 | 0 | 5 | 0 | 2 | 0 |
| F4/F5 | 1 | 4 | 8 | 8 | 2 | 4 |
| F5/F6 | 2 | 13 | 13 | 26 | 2 | 13 |
| F6/F7 | 4 | 42 | 21 | 84 | 2 | 42 |
| F7/F8 | 7 | 119 | 34 | 238 | 2 | 119 |
| F8/F9 | 12 | 330 | 55 | 660 | 2 | 330 |
| F9/F10 | 20 | 890 | 89 | 1880 | 2 | 890 |
| F10/F11 | 33 | 2376 | 144 | 4752 | 2 | 2376 |
| ..... | | | | | | |
| $F_n/F_{n+1}$ | | | | | | |

*a:* Size of perplex Fibonacci common-body, provided by $F_{n-1}$ as $F_1+F_2+\ldots+F_{n-3}$.
*b:* Size of aggregated Fibonacci molecular bodies, provided by $F_{n+2}$ from the FM algorithm and determined by the size numbers of the Fibonacci atoms included in the Fibonacci molecules residing in the respective aggregates.
*c:* Size of Fibonacci atom $F_{n+2}$.
*u:* New unit for size numbers of Fibonacci molecules.

From table 4 we observe the following identity which we label the *Fibonacci multiplicative identity*:

(17) $b = a \times c / u$

equivalent to:
(18) $b = a \times c / 2$

Because the operation *multiplication* manifests in the identity, this identity seems quite remarkable and in need of closer exploration and interpretation.

When distinguishing between the first factor of a product, the multiplicand, and the second, the multiplicator, the multiplicand is regarded passively as a first number object, and the multiplicator is regarded actively as a second number object determining how many times the multiplicand is to become repeated, and thus treating and processing the fist number object into the combined product. In this specific sense the multiplicator acts as an *amplifier* or *extending reproductor* on the multiplicand as an input, giving the product as the output.

In the product $a \times c$ factor *a* is provided as an output from algorithmic transfiguration performed by the $F_{n-1}$ subject, while factor *c* is provided later on as the size number of perplex $F_{n+2}$ as soon as this Fibonacci atom occurs. This implicates that factor *a* has to be the multiplicand in the product and that factor c must be the multiplicator, when such is decided from emphatically real, generative constitutional logic instead of from merely formal exercises with numbers in a free-standing thought universe of secondary ontological nature when compared to the first one. (Cf. Johansen 2008a: ch. 3.1.2 for analysis of this ontological relation.)

Thus, also in this novel and highly important respect, which is with regard to Fibonacci manifestation of *multiplication* as operation, we realise a most peculiar significance for the distance of *three* between perplex Fibonacci numbers. In the present case, applying the notation connected to table 4, this is the distance between the Fibonacci subject $F_{n-1}$ and the Fibonacci subject $F_{n+2}$.

We have argued that $F_{n+2}$ has to be the Fibonacci subject picking, composing and providing the Fibonacci molecules residing in the gap between $F_n$ and $F_{n+1}$, as outputs from algorithmic transfiguration performed by $F_{n+2}$. Thus, $F_{n+2}$ plays a constitutional role *both* with respect to multiplicator c and with respect to filling-in of the gap between $F_n$ and $F_{n+1}$ with Fibonacci molecules.

However, this "constitutional role" of $F_{n+2}$ also with respect to multiplicator *c* does not by itself implicate with necessity that $F_{n+2}$ is the Fibonacci *subject* performing

the multiplicative act. In general with regard to multiplication, the multiplicator must be considered the active number subject related to the multiplicand as its passive number object. But this does not mean that the multiplicator represents an *uppermost* subject here. To the contrary, it is the external subject, usually considered the calculating human performing the multiplication, who applies the multiplicator more as a computing tool, one might say to some extend a hand, to process the number object of the multiplicand.

As an example, we can consider again the gap between F6 and F7. Here, F5 was the subject receiving and breaking up the Fibonacci atomic bodies of F1, F2 and F3 into the Fibonacci common-body of perplex Fibonacci molecules as entrance halls of the ontological floors regarded to be located under the ground of Fibonacci body F6. Later, F8 was the subject picking and composing Fibonacci atoms into specific Fibonacci molecules filling in these floors.

Now, the aggregate of Fibonacci molecular bodies, which size is denoted by symbol *b*, residing in the gap between F6 and F7, has been composed of receiving the Fibonacci *molecular* bodies of (2,6), (3,6), (4,6) and (2,4,6) and breaking these ones up into the *Fibonacci molecular common-body* (as contrasted to the previous Fibonacci common-body regarded as a Fibonacci *atomic* common-body) having size *b* of the primordial unit. Then, what is, in analogy to F5, the Fibonacci *subject* performing this reception, breaking-up "uniformation" and delivering the Fibonacci *molecular* common-body in the gap between F6 and F7? Obviously, these Fibonacci molecules must have been created *before* they can become received and broken up, and this creation was performed by the F8 subject. Thus, F9 becomes the first candidate to represent the sought subject. F5 could not receive F4 as a Fibonacci atom for molecular composition, due to if so, F5 would annihilate itself by fusing with F4. By analogy, F9 can not receive F8 in its reception of Fibonacci molecules for creating a Fibonacci molecular common-body. However, F8 is not represented as a Fibonacci atom in any of the molecules residing in the gap between F6 and F7. Hence, this can not be any reason obstructing F9 to be the sought Fibonacci subject. Nor can we see any *other* reason for such obstruction, and therefore we consider F9 to *be* the Fibonacci subject making the Fibonacci molecular common-body of the gap between F6 and F7. In general, this obviously means that a Fibonacci molecular common-body of a gap between $F_n$ and $F_{n+1}$ is made by the Fibonacci subject $F_{n+3}$. We notice that while a Fibonacci *atomic* common-body is made by a Fibonacci subject *two* perplex Fibonacci numbers larger than the largest Fibonacci atom included into the common-body, a Fibonacci *molecular* common-body is made by a Fibonacci subject *three* perplex Fibonacci numbers larger than the largest Fibonacci atom included in Fibonacci molecules of the common-body.

By analogy to the situation for the Fibonacci atomic common-body, the Fibonacci molecular common-body must also be *delivered* and *placed* as output from the connected algorithmic transformation performed by the $F_{n+3}$ subject, somewhere in the Fibonacci landscape; and this must happen as *perplex* Fibonacci *supra*-molecules. For example, with respect to the gap between F6 and F7, this seems, at least in first approximation, to imply that the size number 42 of this Fibonacci molecular common-body, converts to 42 perplex entrance halls of floors later to become filled of the bodies of 42 different Fibonacci supra-molecules composed from algorithmic transfiguration

by a Fibonacci subject larger than $F_{n+3}$. In the context of the present treatment, we will not explore quantitative (and qualitative) implications of such further elaborations.

The Fibonacci molecular common-body between F6 and F7 manifests from the F9 subject breaking the ties between the Fibonacci molecules residing in the gap, as well as the ties between the Fibonacci atoms in each molecule, and also the ties inside each Fibonacci atom. The de-molecularisation has to be *radical*, all the way down to the primordial unit, for the common-body to manifest, for reasons analogous to those with regard to the de-atomization into the Fibonacci atomic common-body. (This seems to imply a corresponding – relative – degree of *freedom* in later re-molecularisation into Fibonacci supra-molecules, which may be of relevance to understand ultra-hi-tech for radical de- and re-molecularisation, as the one analysed by Illert 2000.) The size of said Fibonacci molecular common-body is given as b= 9+10+11+12 = 42 which is half of the product of the multiplicand a=4 and the multiplicator c=21. The last is the size of the Fibonacci body F8, but F8 was not the Fibonacci *subject* performing the multiplication. The size 42 of this common-body appeared from the breaking homogenisation performed by the F9 subject. In this performance the F9 subject did not relate to F8 in *any* respect, only to the Fibonacci molecular *outputs* from previous performance of the F8 subject. Also, the performance of the F9 subject was *not* any *multiplicative* operation. However, the *result* of this operation, the common-body size 42, could be interpreted by an *external* subject *as if* the result manifested from a multiplication of multiplicand 4 and multiplicator 21 (and divided by 2). But then the multiplicator is to be regarded as a tool in the hands of this *external* subject, not of the F9 subject who actually *does* the operation that can be externally interpreted as multiplication.

This implicates that multiplication, as a mathematical operation, manifests in the Fibonacci generation of numbers as a *simile*. This means that multiplication manifests by a certain back-reflection from an external subject, much more advanced and complex than those subjects included in the basic Fibonacci generation (though, in the end, itself generated from Fibonacci dynamics with respect to its form and substance). Thus, multiplication and addition are mathematical operations of quite different *ontological* status. Due to the involved simile multiplication is of more secondary nature and represents an epiphenomenon. This reinforces our previous reasoning for not considering prime numbers as the primary numbers or as the most genuine number *atoms*. (Considering prime numbers as the primary numbers constitutes much of the reason for the relation between addition and multiplication to remain cloaked in mystery. One might express this somewhat cryptically and paradoxically: If one considers prime numbers as prime numbers, one can never understand multiplication.) On the other hand, this epiphenomenon is *crucial* to understand the dynamics of its Fibonacci underpinnings, and it is discovered through a *strict* quantitative logic implied in the Fibonacci constitution of natural numbers. *Always* when a Fibonacci molecular common-body between $F_n$ and $F_{n+1}$ is created, it must have a size identical to what is computed by applying *multiplication* as an operation on $F_{n+1}_F_n$ as multiplicand and $F_{n+2}$ as multiplicator. Thus, there is exhibited a *specific* number theoretical dynamics to *explain how* multiplication *emerges* as an operation, *how* this operation is related to the more basic operation of *addition* in ontological nature and specific manifestations, and

*which* strictly determined *multiplicands and multiplicators* this operation manifests *with*. For example, we realise that the multiplicator *originates* as the Fibonacci molecular common-body between the perplex Fibonacci numbers F4 and F5, with F5_F4 (=1 unit) as its multiplicand and with F6 (=8) as its multiplicator.

We notice that multiplication manifests as a (simile) operation very early in the Fibonacci constitutional logic, and – besides the simile – in an inherent, organic and deterministic manner, reoccurring all the way during Fibonacci generation insofar as Fibonacci molecular common-bodies are involved. Application of adequate similes are potent in science (as well as unavoidable), but in profound issues it may be crucial to understand their nature. (As an example, physical causality is thought by many scientists to constitute a most basic type of causality, but when dissected this causality type shows to be composed from other causality types and including many similes – cf. the analysis in Johansen 2008a: ch. 3.1.10. In most physics understanding of such does not matter much, but this may be different when facing deep-hard issues, such as non-trivial flows of information in time.)

The *gap* location of the product as well as of its multiplicand, tells that multiplication as an operation by its very constitutional nature is based on the operation *Minus*. This gives further support to our previous reasons to consider *Minus* a basic operation. Since, at the same time, the multiplicator always is the size number of a perplex Fibonacci number, multiplication as an operation is a *combination*, and a specific *interplay*, between two instances of the *Minus* operation and a *pre-Minus* number, namely the perplex Fibonacci number of the multiplicand.

Then it is time to reflect upon the role of the *unit* connected to this operation of multiplication. The Fibonacci molecular common-body is uniformly made up of only the primordial unit, as a string or tower of cubes, just as the Fibonacci atomic common-body. The molecular common-body manifests by the $F_{n+3}$ subject simply *deleting* the borders or "bounds" between the Fibonacci molecules involved, between the atoms herein, and inside each Fibonacci atom, with the result that the Fibonacci molecular common-body appears as a size number of the *added* molecular size numbers composed of and measured by the *one* and same primordial unit as yardstick. Therefore, the unit does not – different from what is the case for (Fibonacci) addition – occur *both* in the multiplicand and the multiplicator, only in the multiplicand.

Thus, in our Fibonacci generated unfoldment of the operation multiplication compared to the operation addition, the same relation between these two operations manifests – in this general respect – as in the conventional notion of these operations when numbers are considered as *tied to units*, i.e. as being denominated. As example, 4 apples are addable to 5 apples, not to 5 trees or to plainly 5 without denominator; while 4 apples are only multipliable with 5 (times), but neither with 5 trees nor with 5 apples. Here the denominator occurs only for the multiplicand, not for the multiplicator. "5 apple times" of anything is not usually considered meaningful as multiplicator, whatever the denominator (or not) of the multiplicand; there must be some *thing* to become multiplied by the multiplicator, and here a number of *non*-thing does not qualify as a number *object*. On the other hand, the multiplicator *must* be a number of *non*-thing; if not so, the multiplicator would *bring in* the *thing* of itself into the

operation, with the result that the product did not stay a new *number* of the *same* thing as the multiplicand, but a new number of *another* thing which combined the *things* of the multiplicand *and* the multiplicator. This may sound quite trivial, but by focusing the trivial more sharply some non-trivial novelty may arise later on from having done so. (In the universal meaning theory of Ignatyev, which has yielded a revolutionary new form of computation, surpassing the traditional notion of computation connected to universal Turing machines, and catalysing crucial advances in robotics, including nano-robotics, there occurs a basic operator which is highly interesting in this context. This is so because this operator combines two ontological levels, the sign and the signified, both represented as numbers, by multiplication as operation, and by combining by addition all joints of these operations in Pascal-structured equations zeroing out.)

In our Fibonacci unfoldment of multiplication as operation the unit of the multiplicand is implied as the primordial unit (due to the necessity of the Fibonacci *atomic* common-body of *a* to have this unit), analogous to what is the case in the conventional notion of denominated multiplicands. The situation is not that straight-forward with respect to the multiplicator. If we, as example, consider the multiplicator of F8, acting on the multiplicand of F7_F6 (=4 cubic units), it is not the *perplex* number of F8 which acts as multiplicator, but the *size* number of F8. And this size number, indicating the ontological *extension* of F8, does not have any *meaning* if not considered as 21 of the primordial *unit*. Thus, F8 acts as a multiplicator *with* its size on the multiplicand F7_F6, i.e. *with* its ontological extension, not *without* it. Then we have to confront the challenge of having a denominator, namely the primordial unit, *both* in the multiplicand and the multiplicator.

The only way out of this difficulty, at least the easiest one, is to acknowledge that the operation multiplication, as Fibonacci generated, manifested as involving a *simile*. In the *emphatic* Fibonacci generation, there neither occurs any multiplication nor any multiplicator. Thus, as an external subject we can, at least in first approximation, feel free to *regard* the multiplicator *as if* it occurred without any denomination, i.e. *as if* it could occur without the tie to the unit that is *necessary* to define and measure its size. Thus, the operation multiplication, as Fibonacci unfolded, only makes (easily) sense by also adding this *secondary* simile which considers the multiplicator to be *without* any unit. This additional simile is a certain act of *forgetting*: First the multiplicator is comprehended *with* its size number, which *presupposes* measurement by and composition from its primordial unit, and *then* this basic condition becomes *forgotten* by a conscious act of analytic, instrumental fetishism, when regarding the multiplicator *as if* it could do the impossible: to *release* its size from its primordial unit.

By adding this secondary simile our Fibonacci unfolded conception of the relation between the unit and, respectively, the multiplicand and the multiplicator, coincides with the conventional conception of those when the multiplicative product is *denominated*. However, the mathematical convention is to basically consider the product of multiplication as *without* any denomination and hence to *ignore* any profound reflection involving the unit, thus *expelling* such reflection from mathematics into natural science of *applied* mathematics, as just a *secondary* – and mathematically

irrelevant – concern about choosing units that function as convenient standards for measurement in natural systems.

When recognising the significance of the basic *double* nature of numbers as both perplex and size numbers, and even more when comprehended in their dynamic interplay from Fibonacci constitutional logic, this conventional notion of the operation multiplication in supposed *pure* mathematics discloses as naïve, shortcoming, superficial, short-cutting and *fetishised*, trying to *flee* from the basic distinction of the two sides of the number "coin", instead of adequately *back-reflecting* the generative constitutional logic. Thus, its "purity" is more like being trapped into the apparent free-wheeling of white without acknowledging the distinction between white and black constituting a basic condition for such endeavors and in camouflaging forgetfulness *hiding* the black, and in this sense fly away in imagined free-standing thought by de facto *tripling* the pollution, blacking the blind spot of blacking the black.

In"pure" mathematics *both* addition and multiplication are considered as operations *without* any denomination of their numbers. Thus, the *difference* between addition and multiplication with respect to the role of the unit (occurring in both joints of addition, only in the first joint of multiplication) is *concealed*, and, consequently, the *general* difference between the two operations remains concealed in a quite essential respect. This is a difference that, even when *not* adequately comprehended as Fibonacci generated, reveals from the very fact of the double nature of pure numbers as *both* perplex and size numbers. There is no coin without two sides, one upper and one lower, there *is* no information without substance, and there *is* no number without the perplex number having a size with a unit.

When expressing numbers as assumed "purified", without denomination, say the number "5", the expression is confusing with respect to the role of the unit, and this in plural respects. First, it is unclear whether 5 is considered a perplex number (without unit) or as a size number (with unit), or as both. The last case implies that it is tacitly assumed not to matter for uniform treatment of the two that the one meaning of the number has a unit and the other one not, as well as not to matter for number theory in general. This implies that *geometry* is tacitly expelled from number theory, which – as we have argued – is illegitimate when having contemplated the constitutional dynamics between perplex and size numbers (and in their Fibonacci generation). Second, if 5 is considered a size number, it is unclear whether the size of 5 units manifest from the operation of five additions of the primordial unit, or from the operation *multiplication* taking the primordial unit five times, or as both. Hence, this unclearness conceals the difference between the two operations with respect to the unit occurring in *both* joints (addition) or only in the *first* joint (multiplication) and gives the impression that this unclearness does not matter and should not become reflected upon. It also obstructs reflection on how the asymmetry between the two joints in multiplication is to be understood or established, since this asymmetry simply does not reveal when the unit is deleted from the expression. Third, these two classes of unclearness reproduce and propagate when taking the step from *one* natural number, as 5, to operations that *relate* more than one number (different from primordial "1") in addition or multiplication.

The conventional notion and expression of "pure" numbers gives the impression that linking to a unit is a *secondary* issue, external, ex post, auxiliary, random and inherently irrelevant to numbers and theory of numbers as such. The truth of the issue is that the unit plays a role in the *constitutional* logic of numbers, and if so is tacitly believed *not* to be the case, this role of course can not become adequately researched and understood.

If we, as a *simile*, ignore this blind spot of "pure" mathematics, there is – of course – established a tremendous amount of insights generated from such mathematics, namely in *all* cases where ignoring or acknowledging the simile does not make any *difference* for the mathematical results achieved. However, in some *profound* – and thus far-reaching – issues, as those issued in the present text, *exactly* this simile has to be focused and understood as such in order to achieve results hardly *possible* otherwise.

In this context it is highly interesting to note that the recognition of the implied, *hidden* significance of the unit in mathematics, was not discovered by "pure" mathematicians, but by the genius of the mathematical *physicist* Santilli, a discovery which led to detrivialisation of the unit, and from there to the re-invented landscapes of hadronic mathematics (iso-, geno- and hyper-mathematics with their respective isoduals), catalysing on-going revolutions in physics, cosmology, chemistry, biology and technology.

After having clarified the role of the unit with respect to the multiplicator in the Fibonacci unfolded operation of multiplication, the next, crucial issue with respect to the unit is how to interpret and comprehend the role of the unit u=2 of table 4, tied to this multiplication.

It is not easy to see any inherently organic circumstance in Fibonacci generation that directly establishes 2 (which is to be interpreted as the size number of two primordial units) as a novel unit tied to the Fibonacci molecular common-body. There is not created any *other* common-body of numbers that becomes split in two halves into this common-body. Nor is the *multiplicand* created from the Fibonacci atomic common-body split in two halves (which would split the primordial unit itself in two halves for all Fibonacci atomic common-bodies that have odd size numbers). And nor is it possible to regard the *multiplicator* as operating on the multiplicand with only half of its size number (which also would split the primordial unit inself in two halves for all perplex Fibonacci numbers that have odd size numbers).

However, this lack of any inherently organic circumstance is not any theoretical problem when recalling our reasoning for regarding the Fibonacci unfoldment of multiplication as implying a *simile*; rather, the lack is *consistent* with recognition of such simile.

Thus, this occurrence of 2 as unit and as divisor for the product a×c, should not be interpreted as a unit emphatically generated and placed in Fibonacci constitutional logic. However, the operation multiplication manifests *as if* – and *only* if – the unit of 2 occurs in such a way. While the *originating multiplication* has *a* as multiplicand and *c* as multiplicator, it seems most reasonable to consider the *originated product* to be the size of the Fibonacci molecular common-body, which is a×c/2. Hence, we denote a×c the *originating pre-product*. Applying this terminology, the originated *product* involves

*both* the operation of multiplication and the operation of *division*, i.e. as constitutionally *tied*, and with the involved divisor being the *novel unit* of 2. This implies that the operation multiplication, in its very foundation *also involves this novel unit* for the originated product to manifest. The originated product *only* manifests by considering its originating pre-product as composed of and measured by this *novel* unit, *different* from the primordial unit which is the ontological extension of the first perplex number.

The originated product itself, the Fibonacci molecular common-body, say between F6 and F7, is a size number, in this case 42, of the primordial unit (not the novel unit). Obviously this common-body *has* ontological *extension*, and it is measured by the primordial unit when the $F_{n+3}$ subject deletes the borders between and inside the involved Fibonacci molecules, just as what was the case for the Fibonacci *atomic* common-body. At the same time, when we regard the Fibonacci molecular common-body *as if* it was the result of multiplication from the originating pre-product, the correct size number of this common-body, as measured by the primordial unit, is only achieved from introducing a *novel* – intermediary – measuring of the size number of the originating pre-product (which first is measured by the same, primordial unit) by a *novel* unit (of two primordial units).

The divisor having a novel *unit*, i.e. being denominated, means that we can regard *all* the involved numbers, a, c and u, as having units and end up with a denominated originated product. Thus regarded, when considering the *whole* operation leading to the originated product, we can *remove* the simile of the multiplicator as being *without* a unit, which did not seem meaningful when considering *only* the originating multiplication. In this way, denomination connected to multiplication as operation, is somewhat dependent on the reference frame for considering adequate similes.

The ontological extension of the novel unit is identical to the ontological extension of perplex Fibonacci number F3. Hence, we could regard the originated product as the originating pre-product measured by F3 instead of F1, but it is hard to see any inherent operation involving F3 in such a privileged role in constitution of Fibonacci molecular common-bodies, so this would be just another simile.

The originated product is always a whole number, despite occurrence of the novel unit as a divisor of 2. This is because the originating pre-product always is an even number, due to the factors of *a* and *c* occurring in a three-step cycle as: odd *a* & even *c*, even *a* & odd *c*, even *a* & odd *c*, and then again odd *a* & even *c*, etc.; the two factors never being even simultaneously. Thus, the involved Fibonacci *placement and distance* between the two factors bring the operation multiplication to Fibonacci manifest as resulting in a *whole* number originated product despite being tied to division.

Formally, the same size number of the originated product would of course result from multiplying the originating pre-product with ½ as a third factor, instead of dividing the pre-product with the novel unit of 2. However, this would imply an unnecessary complication, due to mysteriously splitting the primordial unit in two halves, whereby division still had to become implied. Also, the denomination of the originated product from the units of the three factors would seem more troublesome.

Contemplating the operation multiplication as implied in the Fibonacci unfoldment at a quite early stage in the number constitutional dynamics, namely in the making of the Fibonacci molecular common-body, multiplication must – different from addition – be understood in its very nature as involving also a *novel* unit, and hence a *relation* between *two* different units, and with the peculiarity that the novel unit must have *twice* the ontological extension of the first and primordial one. These circumstances could not have become discovered if the operation multiplication was considered as free-standing and released from denomination in a too-pure-to-be-pure mathematical approach

The multiplicand $a$ is delivered by subject $F_{n-1}$ as the size of the *perplex* Fibonacci molecules of the gap between $F_n$ and $F_{n+1}$, namely as the size of $F_1+F_2+\ldots+F_{n-3}$. Hence, the *composition* of the Fibonacci molecules, and the sizes of these molecular bodies, delivered by subject $F_{n+2}$, is not relevant for the size of the multiplicand. Nor is it relevant for the size of the multiplicator, which is the size of *perplex* $F_{n+2}$. Thus, the originating pre-product $a\times c$ is established solely from the Fibonacci *atoms* above ground, though stretching into the liminal zone *forecasting* the later arrival of Fibonacci molecules, insofar as the Fibonacci *atomic* common-body is delivered, as multiplicand, underground as entrance halls for this later arrival. Therefore, the relation between the originating pre-product and the originated product is a *relation between* what is happening above ground, between created Fibonacci atoms, and what is happening underground, between created Fibonacci molecules. And it is exactly at *this relational* point that that the operation *multiplication* manifests at the Fibonacci atomic side, to establish an anchoring *quantitative identity* between the Fibonacci atomic side and the Fibonacci molecular side of the Fibonacci number landscape, i.e. between the primary and the secondary *spaces* of the Fibonacci landscape. And also, it is exactly at this relational point that the novel unit manifests to realise said quantitative identity.

We notice that $F_{n+2}$ plays a *double* role in this establishment. On the Fibonacci molecular side, subject $F_{n+2}$ composes and provides the Fibonacci molecular bodies which fill in the gap, which next determines the size of the Fibonacci molecular common-body when these bodies become broken apart. In this respect $F_{n+2}$ plays an *active* role, relating to the Fibonacci atomic common-body which plays the passive and preceding role in this relation. In each entrance hall of the Fibonacci atomic common-body the subject $F_{n+2}$ delivers Fibonacci molecules composed of *plural* Fibonacci atoms and primordial units, and in this sense $F_{n+2}$ *multiplies* the size of each entrance hall which is one primordial unit. Hence, we see that the role of $F_{n+2}$ on the Fibonacci *molecular* side, has *analogy* to the role of $F_{n+2}$ on the Fibonacci atomic side where $F_{n+2}$ acts as the active multiplicator upon the passive multiplicand $(F_1+F_2+\ldots+F_{n-3})$ which constitutes the Fibonacci atomic common-body. With respect to the Fibonacci atomic common-body its passive double role, connected to these two different operations, is even more obvious, and these two passive roles are even more analogous to each other, the only difference being that in the one passive role the focus is more on the qualitative side, each entrance hall being a reception site for later Fibonacci molecular bodies, while in the other passive role the focus is more on the quantitative side, the amount of

such entrance halls. One might say that it is the *liminal* placement of this Fibonacci atomic common-body, that makes it possible for it to occur in *both* passive roles and by this to bridge the Fibonacci atomic side and the Fibonacci molecular side with the quantitative identity between the originating pre-product (divided by the novel unit of 2) and the originated product.

When $F_{n+2}$ plays an *active* role at both the Fibonacci atomic side and the Fibonacci molecular side, and in a certain sense also a *multiplicative* role in both relations, one may view this as that it is the *executed* active role of subject $F_{n+2}$ at the Fibonacci *molecular* side, actually composing and providing the Fibonacci molecules, which makes it possible, or forecasts, its active role as multiplicator at the Fibonacci *atomic* side. Correspondingly, one could view the executed *passive* role of the Fibonacci atomic common-body at the Fibonacci molecular side as forecasting its passive role as multiplicand at the Fibonacci atomic side.

We have argued that $F_{n+2}$ can not act as the Fibonacci subject for its own performance as multiplicator. But, whatever invisible for $F_{n+2}$ itself, the *effect* of having created the Fibonacci molecules, later to be broken up into the Fibonacci molecular common-body by another Fibonacci subject, is that their aggregated size is identical to the passive multiplicand multiplicated with half of $F_{n+2}$. To paraphrase a famous line of Marx, "he does not know it, but he *does* it." In some sense and to some extent, it may be likened to the Fibonacci atomic common-body functioning as a container-like womb, hidden from outside view, to receive the sperm from $F_{n+2}$ carrying only half of his genes in making the miracle of a multiplicative, molecular common-body baby.

Despite that the Fibonacci atomic common-body here acts as in a womb-like bridging manner into the Fibonacci molecular world, its size is completely determined as the Fibonacci atomic bodies ($F_1+F_2+\ldots+F_{n-3}$), and the novel unit of 2 gives the exact quantitative relation between what has been created at the two sides of the bridge, i.e. between the Fibonacci *atomic* originating pre-product and the Fibonacci *molecular* common-body as the originated product from the pre-product interfaced by the novel unit. Thus, the novel unit is necessary in order to grasp, in a back-reflection, the originated Fibonacci molecular common-body as originating *from* the Fibonacci atomic side, and this in an exact and short-cut way. Considering the Fibonacci molecular world ontologically as an underworld, compared to the more primary Fibonacci atomic world above the ground, the operation multiplication, *as well as* its accompanying novel unit, manifest in this exact quantification of what is created above and below the ground of the Fibonacci number landscape, thus, in difference to the operation addition and the primordial unit, in the relation between different species of number creations inhabiting two radically distinguished *spaces*, one (the Fibonacci atomic) being more solar-like, the other (the Fibonacci molecular) being more shadow-like.

This indicates that the manifestation of the novel unit may represent a profound entrance to the issue of the *isounit*, and perhaps also to the issue of the *isodual* unit (considering the shadowy number landscape opening up from *Minus* as operation).

Regarded from the reference frame of the Fibonacci molecular space, its Fibonacci molecular common-body is measured by the primordial unit implied and embedded herein, i.e. by the *same* unit as what was/is the case in the reference frame of

the Fibonacci atomic space, including the measure unit implied and embedded in the Fibonacci atomic common-body. Thus, the unit is the same *inside* both reference frames. The emergence of the novel unit only appears when *combining* the two reference frames. Regarded from the reference frame of the Fibonacci molecular space, the originating pre-product, as measured by the primordial unit *inside* the reference frame of the Fibonacci *atomic* space, only gives the correct originated product, the size of the Fibonacci molecular common-body (as measured by the primordial unit inside the reference frame of the Fibonacci molecular space) if this pre-product is measured by the *double* of the primordial unit. Complementary to this, regarded from the reference frame of the Fibonacci atomic space, the originating pre-product only gives the correct originated product if the Fibonacci molecular common-body is measured by the *half* of the primordial unit. This implies that we have chosen the first of these two complementary reference frames, namely the reference frame of the Fibonacci molecular space, to contemplate the *combination* of the two reference frames, this resulting in the novel unit having size 2, not ½. (Also when contemplating *combinations* of two, or more, reference frames, this must be performed from a certain reference frame of *second* order.) This choice seemed more adequate, due to avoid mystifying connotations to splitting the primordial unit; due to give primacy to the reference frame, or Fibonacci domain, wherein the new number creations, the Fibonacci molecules, actually are born; and due to acknowledge the Fibonacci molecular common-body as more emphatically real than the multiplicative act, with its involved simile, re-grasping it.

Thus, despite the primordial unit remaining the same *inside* each of the two Fibonacci worlds, the novel unit of size 2 manifests in the comparative *relating* between the two worlds, when, from the reference frame of the Fibonacci molecular world, the size of the Fibonacci molecular common-body becomes back-reflected and comprehended as an exact quantitative *translation* with the size of the originating pre-product of the Fibonacci atomic world. Thus, to measure the size of its own Fibonacci molecular common-body by a correct unit to measure the Fibonacci atomic pre-product as its *alter ego* or *mirror body*, the double of the own unit of the Fibonacci molecular common-body itself has to be applied, which is also the double of the inherent unit of the Fibonacci *atomic* bodies.

To recognise, or re-gestalt, itself as if it was a multiplicative product from the Fibonacci atomic world, the body of the Fibonacci molecular world must consider the unit of the first *world as if* it was the double of its own unit. Then, the measure and the building block of the pre-product is 2 *units*, not merely 2 without denomination, while the proportion between the sizes of the two units, the primordial and the novel, of course is 2 without denomination. However, for this proportion to occur by comparison, first the two units must have manifested *with* denomination.

To prefer the reference frame of the Fibonacci molecular world here, does not necessarily imply that the Fibonacci subject has been *transported* from the Fibonacci atomic side to the Fibonacci molecular one. One may rather look at this as a Fibonacci atomic – and in this sense monadic – subject putting itself in the place of the Fibonacci molecular world, *without deserting* to it, more as in human identification (*Einfühlung*).

The human organism constitutes the most complex genome among the species of Earth, but on the other hand it also has the most refined or highest mind/soul, and in this perspective the *largest distance* or *bridging reach-out* between the mental and the body-organic. The human mind reflecting upon mind/bodies of other species, and putting itself to some extent in their places, does not mean that the subject of such reflection should be located as residing at the body-organic side. It may be that Fibonacci generative dynamics has the potential to become an informative template also for reflections that are fruitful to gain non-trivial insights into crucial aspects of the human condition.

In the Fibonacci unfoldment multiplication as an operation manifests, surprisingly, as necessarily tied to a novel unit, and hence by its very nature as an operation implying the issue of the unit being brought into focus as a basic mathematical concern. We have seen that the Fibonacci constitution of multiplication involves a particular and peculiar relation between *three* numbers, the multiplicand, the multiplicator and the size of the novel unit, where *all* of these three numbers only are meaningful, at least in first approximation, regarded as *denominated* (by the primordial unit), i.e. as having ontological *extensions*. Because the novel unit acts as a divisor, the originated product has the denomination of the primordial ontological extension, not as a *multiplication* of *two* such denominations (from the denomination of multiplicand and of multiplicator), or of *three* such denominations (if the divisor instead was considered as a third factor with size ½ primordial unit). In the conventional notion of multiplication the problem of the unit is expelled and appears falsely as if it is trivial, thus *concealing* the *mystery* of how the product can be of *one* and the same denomination when *both* the multiplicand and multiplicator can not be fully comprehended as numbers as such without having ontological extensions, i.e. denomination by a unit. In contrast to this, the Fibonacci constitution of multiplication as operation provides the *solution* of the mystery, due to bringing in a hidden *third* number entity, namely the novel unit (and as divisor) in the relational structuring into the originated product from multiplication. It may seem that a profound and satisfactory solution of the unit mystery tacitly involved in the operation multiplication as such, is not *possible* if not conceiving this operation as unfolded from a *real* Fibonacci constitutional logic of numbers.

This may throw some new light on the far-reaching issue of the unit, which is a key point in the architecture of hadronic mathematics. Also, our analysis has pointed out the basic significance of the number of 2 for the unit tied to multiplication. This may have introductory interest to understand how Nature picks its *specific* iso-, geno- and hyper-units, with their isodual units, in natural systems.

On this background the method of so-called *Russian peasant multiplication* deserves mathematical interest as more than a curiosity. (Cf. Basic-mathematics.com (2008) for an easy description of the method.) This algorithm is able to perform any multiplication of natural numbers by only applying the number of 2 as multiplicator as well as divisor. Thus, *division*, and necessarily with the *divisor having size 2*, is required to achieve the product, just as for the Fibonacci originated product. Also similar, there occurs a necessary *connection* between *two* non-primordial units, namely the unit of 2

and the unit of ½, when the whole operation is considered, combining the framework of the multiplicand side and the framework of the multiplicator side. Further, this algorithm implies priority to the operation *Minus*, due to *deleting* all throughput-numbers at the multiplicand side that appear as quotients *without* first having subtracted 1 from the dividend (by this criterion deleting possible corresponding throughput-numbers at the multiplicator side). Looked from *Minus* as basic, this is not subtraction (*minus*) of 1, but just a *direct* representation of the size of the *gap* to the primordial unit; and thus viewed the operation *minus* is completely absent in the whole algorithm. Finally; though this algorithm, like conventional multiplication, is not Fibonacci expressed, it may be of interest that its operation, different from conventional multiplication, solely applies the number of 2, and thus presents a binary coding in the very make-up of the operation. In this respect this algorithm is organically much closer to binary representation than what conventional multiplication is. (Weisstein (I) characterises it as "binary multiplication", with a short-hand software expression of how and why it works). This is relevant because, as we have demonstrated, the representation from the FC algorithm, just as binary coding, provides a complete and unique representation of natural numbers, and because the FC representation is easily translated by so-called *Fibonacci coding* (cf. later) to a binary string itself. In this perspective binary coding can be regarded as an *intermediary* between the constitutional-generative Fibonacci algorithm and the conventional expressions of natural numbers and their basic arithmetic operations. In this respect the binary expressions are *closer* to the number – and reality – generator than the conventional expressions. This may indicate some *inversion* of the ordinary comprehension of computation – and perhaps also of the information age contemplated in a broader perspective of cultural evolution/devolution.

Some mathematicians have, more or less intuitively, felt that there has been something fundamental missing to number theory, due to falling much apart into one domain established from addition as operation, and another domain established from multiplication as operation, thus creating a quest to look for some unexpected basic *bridge* between them. Our reconstruction of multiplication from Fibonacci constitutional dynamics seems to provide such basic bridge. This becomes further substantiated by our later treatment of Fibonacci constitution of prime numbers vs. composites.

## ZECKENDORF FIBONACCI MATHEMATICS

So far in our treatment quantitative observations and Fibonacci identities have been presented without proofs, in a somewhat quasi-naïve, Humean freshman fashion. However, such proofs and related discoveries have already seen its day in specialised Fibonacci mathematics, so we will refer to and comment on some important contributions from this literature.

Our presentation of the natural (size) numbers, uniquely and completely, as identical to the size numbers of Fibonacci molecules (and atoms) when generated from

the FM algorithm, is with necessity correct, because these Fibonacci molecules are *Zeckendorf sums* which represents the natural numbers uniquely according to the proved *Zeckendorf's theorem*. In Zeckendorf's own words, this theorem concerning these sums is expressed as follows:

*Every natural number can be represented as a sum of distinct and non consecutive Fibonacci numbers or of non consecutive Lucas numbers. Using Fibonacci numbers, such a representation is always unique.* (Zeckendorf 1972a)

More often than not, it seems, Zeckendorf's theorem is referred to as formulated for (only) positive Fibonacci numbers. However, this is not the truth of the issue when examining the proof for the theorem published by Zeckendorf (1972b). Due to the importance of the subject and the sake of clarity this may be worthy to point out. The treatment of Zeckendorf, including his proof, takes place at a higher degree of generality from a broader approach and description, and includes also negative Fibonacci numbers (as well as Fibonacci multiples) in basically a certain alternation. He departs from "a **generalized** Fibonacci sequence where positive and **negative** terms **alternate**" (ibid.: 366; boldfaces by us). He further writes:

*We intend to express the natural numbers 1, 2, 3, • • • as sums of distinct non-consecutive terms of **primary generalized** Fibonacci sequences and we shall obtain a coherent system of numeration that could be used in arithmetical operations.* (Ibid.: 366f; boldfaces by us.)

*Except the sequences a and 0 defined above, the generalized Fibonacci sequences have those **two** infinite **parts**; the **lower** part with **alternating** terms decreasing in absolute value, followed by the **upper** part whose terms have the **same** sign and increase in absolute value.* (Ibid.: 366; boldfaces by us.)

Zeckendorf presents an example (1972b: 371) expressing the natural number 87 by means of his general formula as $t_{9,4,2,0,-2,-5,-7-10}$, which from his interpretation rules is to be calculated as (F8-F3+F1+F1+F3-F6-F8+F11), where no.s 2, 4, 6 and 8 inside the brackets represent the "upper part" of his primary generalisation. In comparison, the same number from the FM algorithm is (4,6,8,10) which is perplex number substance no. 87 with its size number calculated as (F4+F6+F8+F10). If we look at his list of the first 50 natural numbers (ibid.) and takes 37 as an example, 37 has the formula $t_{7,4,0,-5,-8}$, which is to be calculated as (F6+F3+F1-F6+F9), while the same number from the FM algorithm is (4,9). Hence, despite resulting in the same number and the general possibility of always translating (or rather converting) Zeckendorf's general formula into the corresponding FM Fibonacci molecule, there occur many differences between the two formulations. In Zeckendorf's formulation there can occur Fibonacci "atoms" with negative values or with larger values than the FM Fibonacci molecule, the formula covers Fibonacci atoms on an equal footing with Fibonacci molecules (both described as *composite* Fibonacci numbers, except in the case of the primordial unit), and, when the sign of the upper part becomes switched, a Fibonacci atom can include itself as an atom (cf. number 5) and the same Fibonacci atom can occur more than once in the same formula (cf. number 9).

Interestingly, Zeckendorf's treatment implies quite explicitly a distinction between the perplex aspect and the size number aspect of the issue, by giving adequate priority to first specify an algorithm (ibid.: 367) for *perplex* generation of Fibonacci composites, and then, from there, to find a general Fibonacci formula (ibid.: 368, setting the $t_n$ of his perplex algorithm to be $F_{n-1}$) to calculate the *size* numbers of these perplex numbers. This formula shows to result in size numbers identical to the respective perplex numbers, and thus to the natural numbers. (Also, his first 50 numbers list hints at a *secondary* perplex pattern resulting from the perplex succession.) Thus, Zeckendorf's treatment established *both perplex* number identity between the Zeckendorf sums and natural numbers, *and size* number identity between the Zeckendorf sums and natural numbers. These are two different questions, because theoretically one could very well establish such a size number identity without being able to specify an (underlying) algorithm that generated the Fibonacci composites in the same perplex order as their sizes. In some of the literature this has become somewhat obscured, by publications seeking (and proving) such perplex orderings of the Zeckendorf (size) sums, ignoring that Zeckendorf himself not only presented, but *departed* from *perplex* generation of Fibonacci composites in what he coined *Generalized Fibonacci Numeration (G.F.N.)* (ibid.: 367).

Other things equal, this generalization enhances the possible significance of Zeckendorf's pioneering contribution, but on the other hand the same generalization constrains its immediate relevance for our analysis of the constitutional logic of Fibonacci composites, due to his general formula also including negative Fibonacci values, due to not distinguishing between Fibonacci atoms and Fibonacci molecules, and due to not recognising the significance of the distinction between Fibonacci atoms and Fibonacci gaps (and thus nor the related significance of the operation *Minus*) and their interplay. Also, even if becoming restricted to only positive Fibonacci atoms, we must keep in mind that Zeckendorf's treatment presents a *secondary* Fibonacci based perplexity identical to natural numbers, namely the succession of Fibonacci composites corresponding to the succession of *number substances* from our FC algorithm, not a *primary* Fibonacci based perplexity understanding natural numbers as supra-structures of Fibonacci *atoms* from the Fibonacci algorithm.

According to Zeckendorf (1972a) his discovery of Zeckendorf sums was made already in 1939, i.e. 33 years before his own publication. The first publication was not issued by Zeckendorf himself, but by Lekkerkerker (1951) who paid some tribute to Zeckendorf in the first sentence of his article. However, this was performed without much accuracy, Lekkerkerker merely stating that "some time ago Dr. E. Zeckendorf from Liege examined a number of attributes of the Fibonacci numbers" (translation from Dutch by us). Lekkerkerker presented a proof for Zeckendorf's theorem; however only by considering positive Fibonacci values. Daykin (1960) proved that Fibonacci numbers represent the *only* way to construct sequences of natural numbers in such a way that Zeckendorf's theorem holds. Thus, this proof reinforced the understanding of the unique relation between Zeckendorf sums and natural numbers, by establishing such uniqueness also in the opposite direction. Kimberling (1998) offered a short account of this initial history of Zeckendorf sums and Zeckendorf's theorem. Here it may be of

some interest that Zeckendorf (1972b) did not reference the publications of Lekkerkerker and Daykin, perhaps partly due to these publications not applying a *generalised* Fibonacci numeration, by this being more shallow and less sophisticated in their number philosophical anchoring.

The same constraining, for good (direct congruence with the Fibonacci molecules in our treatment) and not so good (lack of sophistication and lack of direct congruence to Zeckendorf himself), is reproduced also in much of the literature *proceeding* Zeckendorf's 1972-publications. As an example, Tee (2002) writes that Zeckendorf

*showed that each natural number has unique representation if F 2 is used to represent 1, rather than F 1 (which also equals 1). Each natural number can be represented as a Zeckendorf numeral which can be encoded as a stream of bits with index starting at 2, e.g. 27 = F 3 + F 5 + F 8 can be encoded (with index increasing to the right) as 0101001 . In data transmission the most significant 1 can be followed by 1 (since 11 never occurs within a Zeckendorf numeral) to indicate the end of a number, so that 27 would get transmitted as the self-limiting bit-string 01010011* (Tee 2002: 1)

Zeckendorf himself represents the natural number 1 as $t_0$, which from his general Fibonacci formula is interpreted as $F_{-1}$ which, according to the same formula, translates to $-F_1$ and from this to $F_1$, i.e. *not* to $F_2$ (cf. Zeckendorf 1972b: 371). As we have argued ourselves, F2, not F1, is the adequate Fibonacci atom of size number 1 to become included in Fibonacci molecules. However, this is *not* the case in the *generalised* Fibonacci numeration of Zeckendorf. Further, the composition of number 27 from Zeckendorf's general formula is $t_{6,4,1,-2,-4,-6}$, which from his general Fibonacci formula is interpreted as $(F_5+F_3+F_0+F_{-3}+F_{-5}+F_{-7})$ which translates to $(F_5+F_3+F_0+F_3+F_5+F_7)$ – hence an example of a natural number without negative signs in such formulation (due to in this case only having *odd* negative Fibonacci numbers). Both these compositions are different from (F3+F5+F8). The last expression is identical to the correct make-up of the Fibonacci molecule, but this is because the treatment in *distinction* to Zeckendorf is *not general* and does not consider negative Fibonacci numbers. Finally, the binary notation introduced by Zeckendorf as an alternative to his *t*-notation (ibid.: 367) would represent the number 27, i.e. $t_{6,4,1,-2,-4,-6}$, as 10.1001.0.0101.01 (positive *t* subscripts marked by 1s to the left of .0., and negative *t* subscripts marked by 1s to the right of .0.; the other dots inserted after each fourth digit to left and right), and also both of the two *translated* expressions above would of course be represented *otherwise* than 01010011 whatever the details chosen for notation.

Therefore, it is somewhat misleading to describe the make-up of Fibonacci molecules as "Zeckendorf numerals", since such numerals arrive from his *generalised* Fibonacci numeration which results in a make-up *different* from the Fibonacci molecules, despite identity between the two make-ups both with respect to perplex numbers and to size numbers. Zeckendorf's theorem was formulated and proven from this generalised numeration. The broad part of the related Fibonacci literature applies the theorem to a numeration which is *not* generalised in the meaning of Zeckendorf, namely to Fibonacci composites only made up by *positive* Fibonacci numbers (and thus,

different from Zeckendorf, excluding Fibonacci *atoms* as made up by Fibonacci composites). Such specification of the theorem, with according positive narrowing-down of the numerals, is the relevant one with respect to our treatment of Fibonacci molecules, due to the related quantitative identities implicated from the theorem. Still though, the sophistication in Zeckendorf's generalised Fibonacci numeration (including his treatment of the perplex vs. the size aspect of Fibonacci numbers) should not be missed out or become reduced.

(This said, the formulation of Tee was just picked as illustration to clarify the general issue. Tee himself is aware of crucial differences between the narrowing-down of the Zeckendorf numerals vs. general "signed Zeckendorf arithmetic"; cf. Tee 2002: 6f.)

With respect to the *reformulated* and positively *narrowed-down* Zeckendorf numerals, in the above sense, i.e. those numerals identical in make-up and perplex ordering to our Fibonacci molecules, quite a few different algorithms have been presented in the literature to generate the numerals in correct perplex order, identical to the size number of each numeral. (With respect to the *general* Zeckendorf numerals, we repeat that the problem was already solved by Zeckendorf who *departed* from the general perplex algorithm. Thus, the whole problem about constructing a narrowed-down algorithm *emerges because of* the de facto narrowing down of the Zeckendorf numerals, so the situation becomes paradoxically inverted compared to the original general treatment of Zeckendorf.) An early such was developed by Pihto (1983). Tee (2002) presented a quite simple algorithm, applying Fibonacci coding (that is, Fibonacci coding reformulated and positively narrowed-down compared to the initial coding of Zeckendorf's). Whatever the convenience of such different algorithms when considering computing efficiency, dependent on various factors, the primary question is what kind of algorithm that makes most sense when contemplating and re-mirroring the constitutional logic of Fibonacci generation as a *real, organic* process. Our FM/FC algorithm has been formulated and displayed in fig. 1 in a way that should make quickly intuitive sense. Also, our perplex algorithm was, just as in the *general* treatment by Zeckendorf, developed *before* contemplating size identity between Zeckendorf sums and natural numbers, and, as a matter of fact, before contemplating perplex Fibonacci molecules to have sizes at all.

Fenwick (2003) applied Fibonacci coding to work out analogies to the four basic arithmetic operations in conventional mathematics. This Zeckendorf arithmetic was developed for the narrowed-down Zeckendorf numerals, corresponding to our Fibonacci molecules and number substances, as well as for the general, "signed" numerals involving negative Fibonacci numbers in the atomic constituents of the Zeckendorf sums.

With exception of the operation division, related Fibonacci analogies had previously been presented by Freitag and Philips (1998). The Fibonacci analogy to multiplication, without applying Fibonacci coding, is quickly inspected as displayed by Philips (I) as a Zeckendorf representation of the product of two (positively narrowed) Zeckendorf numbers. Compared to our Fibonacci originated product, it is interesting that also this displayed formula includes a *divisor* of *size* 2 in its expression (n/2 at top

of the sigma-sum). More specifically, when applying our own notation the formula would translate, in the analogous cases, to (ca/2) ×2 + c×1, and hence to c(a+1). Due to our concern being the *development* of the very operation multiplication from Fibonacci constitutional dynamics (and from this, conventional multiplication as a secondary *epi*-operation), hence in *specified* relations between strictly *determined* and *selected* Fibonacci atoms and Fibonacci molecular gaps, multiplication can not occur in our treatment between *all* Zeckendorf numbers (i.e. Fibonaccci number substances), at least not in the *basic* approximation we have analysed. With respect to the make-up of the two factors constituting the Fibonacci product, we notice that it is different from the Freitag/Philips formula by our multiplicand being one primordial unit less than the corresponding factor (which in their case has been chosen to be the multiplicator) in their formula. This is a difference that makes a difference in principal, since our multiplicand, in *distinction* from our multiplicator, is identical with a Fibonacci *gap*, constituted by the operation *Minus*. This also implies that the Fibonacci atomic make-up in our multiplicand always is more or less *different* from the Fibonacci atoms included in the factor of their corresponding Zeckendorf sum. Hence, this is not only a difference of one primordial unit in size number, but also a *qualitative* difference between which Fibonacci atoms that compose the Fibonacci molecule of the factor with its size. This last difference must also have implications for adequate *quantitative* (and informatic) treatment of the Zeckendorf sum of this factor in Fibonacci multiplication, since the sum is composed from other quantitative constituents.

In this context it is also interesting that Tee argues that the method of *Russian Peasant Multiplication* (which implied primacy for the operation *Minus*) with a suitable binary notation "is closely similar to the standard algorithm for multiplication in binary arithmetic" and that "Zeckendorf numerals can be multiplied more simply by Russian Peasant Multiplication, using the existing Zeckendorf algorithms" (Tee 2002: 4f) presented by Fenwick (2003) or a procedure connected to Pascal-programming developed by Tee himself. This represented a significant reduction of the previous informatic complexity of Zeckendorf arithmetic, and indicated the possibility of further simplification. Such has been achieved by different later contributions. State-of-the-art with regard to computational efficiency seems to be that the claimed over-all superiority of conventional arithmetic compared to Zeckendorf arithmetic is not obvious anymore, and perhaps about to switch. Considering that Nature tends to prefer simple algorithms, or more clear-cut stated: is *built* from successive Fibonacci algorithms, this development in computational arithmetic is not a surprise.

With respect to *geometric* implication from Fibonacci constitutional logic, it seems significant that the concept of *Fibonacci cubes* was established by Hsu (1993) from Zeckendorf's theorem, i.e. as underpinned by Fib *molecular* identities. This anchoring of theories of Fibonacci cubes and hypercubes may suggest our cubic Fibonacci constructions to be considered para-metaphors.

Kologlu et al. (2010) presents a study of the distribution of *summands* (that is, the amount of different Fibonacci atoms in a Fibonacci molecule) in Zeckendorf decompositions, concluding that the distribution converges to Gaussian. This result is interesting also from a deeper perspective, due to indicating that probability

distributions manifest in number theory (and in nature) as generated from the Fibonacci algorithm itself, in strictly specified manners and stages at different ontological levels.

In the introduction Kologlu et al. writes: "Surprisingly, no one appears to have investigated the distribution of the number of summands." This statement, despite being presented by specialists in the field, may represent an exaggeration (cf. Stakhov 2009), but at least it indicates the existence of much unexplored territory, and that possible significance of mapping this territory has been mostly ignored in number theory. Different from Kologlu et al., our primary interest is not the over-all distribution related to probabilities, but to pin-point the *exact distributive pattern* from which such probabilities, as well as other phenotypic features of possible interest, become generated.

## FIBONACCI CONSTITUTION OF PASCAL'S TRIANGLE DISTRIBUTION OF FIBONACCI MOLECULES

We enter this supposed *terra incognita* by relating the structured distribution of Zeckendorf summands to *Pascal's triangle*.

**TABLE 5. Make-up of Fibonacci molecules with respect to amounts of included Fibonacci atoms, distributed on successive Fibonacci gaps**

| | Amount of atoms | | | | Sum molecules |
|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | |
| F-gap | | | | | |
| F4/F5 | 1 | | | | $1 = F_3-1$ |
| F5/F6 | 2 | | | | $2 = F_4-1$ |
| F6/F7 | 3 | 1 | | | $4 = F_5-1$ |
| F7/F8 | 4 | 3 | | | $7 = F_6-1$ |
| F8/F9 | 5 | 6 | 1 | | $12 = F_7-1$ |
| F9/F10 | 6 | 10 | 4 | | $20 = F_8-1$ |
| F10/F11 | 7 | 15 | 10 | 1 | $33 = F_9-1$ |
| F11/F12 | 8 | 21 | 20 | 5 | $54 = F_{10}-1$ |
| ... | | | | | ... |
| $F_n/F_{n+1}$ | | | | | $= F_{n-1}-1$ |

Table 5 displays the amount of Fibonacci molecules, residing in the respective Fibonacci gaps, distributed with respect to the different amounts of atoms in their make-up. As example: In the Fibonacci-gap between F8 and F9 there occur 5 Fibonacci molecules that contain 2 Fibonacci atoms, 6 molecules that contain 3 atoms, and 1 molecule that contains 4 atoms.

We observe that the numbers in the matrix of table 5 correspond to a segment of *Pascal's triangle*, more specifically by the rows of table 5 being identical to the successive *Fibonacci diagonals* in Pascal's triangle, when presupposing that an initial

number of 1 is removed from each diagonal. (Inclusion of this number 1 corresponds to including the rooting Fibonacci atom for each Fibonacci molecular gap, so by such inclusion the whole of Pascal diagonals would represent the Fibonacci number substances as a whole.)

When presenting Pascal's triangle skewed as in table 6, the sum of each diagonal from south-west to north-east indicates a Fibonacci number, in the same succession as the Fibonacci series. When removing the departing number 1 of each such diagonal, and moving north-east, each step in the diagonal corresponds to each step to the right in table 5, thus indicating the amount of Fibonacci molecules of the diagonal/gap with one more atom than in the preceding step.

**TABLE 6. Fibonacci diagonals of skewed Pascal's triangle**

```
1
1   1
1   2   1
1   3   3   1
1   4   6   4   1
1   5  10  10   5   1
1   6  15  20  15   6   1
1   7  21  35  35  21   7   1
1   8  28  56  70  56  28   8   1
```

In table 6 each number is generated from north-west by adding two neigbouring numbers of the same row and position the sum below the right number of the two.

We notice that these Fibonacci diagonals *alternately* include the number of 1 as a north-east addend, while the same Fibonacci diagonals uniformly include the number of 1 (corresponding to the rooting Fibonacci atom) as a south-east addend. Thus, we arrive at the general formula for the number of Fibonacci molecules residing in the *gaps* between two neighbouring Fibonacci numbers, i.e. $F_{n+1}$–$F_n$–1, which is identical to $F_{n-1}$–1, as anchored in Pascal's triangle, by simply removing the 1-numbers in column 1 (while not removing the diagonal of 1s from top-north-west to bottom-south-east). This results in the table 7:

**TABLE 7. Fibonacci gaps as Fibonacci diagonals (minus 1) of skewed Pascal's triangle**

```
j\i   1   2   3   4   5   6   7   8

1     1
2     2   1
3     3   3   1
4     4   6   4   1
5     5  10  10   5   1
6     6  15  20  15   6   1
7     7  21  35  35  21   7   1
8     8  28  56  70  56  28   8   1
```

Hence, table 7 is identical to table 6 with the sole exception of the far left column of table 6 missing in table 7. Table 7 provides a matrix exposing in succession all diagonals identical to the *Fibonacci gap diagonals* which are the Fibonacci diagonals subtracted with the $F_n$ atom a Fibonacci gap is *rooted* in, i.e. the *singular* entity of $F_n$ in *distinction* to the gap between $F_n$ and $F_{n+1}$. In table 7 the Fibonacci gap diagonals can be uniquely indicated by giving them indexes identical to j, so that the perplex number of the row is identical to the perplex number of the Fibonacci gap diagonal. As an example, gap diagonal no. 5, i.e. 1+6+5=12, is indicated by j=5. Such indexing obviously implies in general that j=n-3, thus supporting the previously mentioned peculiar significance of the Fibonacci distance of three steps.

It is a trivial mathematical fact that table 7 exhibits the matrix of possible combinations of *i* numbers of filling-ins into a possibility span of *j* numbers. As an example, we may consider the row with j=5 and imagine this as 5 blank spaces. Then there exist 5 combinations to fill i=1 space out of the 5 spaces, 10 combinations to fill i=2 out of 5 spaces, 10 combinations to fill i=3 out of 5 spaces, 5 combinations to fill i=4 out of 5 spaces, and 1 combination to fill i=5 out of 5 spaces. As a simple illustration, if you meet with j=5 people (not excluding the possibility to imagine yourself as a rooting Fibonacci number) and are to shake hands with i=1 of them, there exist 5 possible combinations of the 5; if you shake hands with i=2 of them, there exist 10 possible combinations of the 2; with i=3 there exist 10 possible combinations of the 3; with i=4 there exist 5 possible combinations of the 4, and with i=5 there exists 1 possible combination of shaking hands with all 5.

Next, we present a rewrite of the Fibonacci gaps displayed as Fibonacci diagonals (minus 1) in table 7, by performing a *further* skewing of the Pascal triangle structure, so that the diagonals of table 7 are transformed into *rows*. Thus, the internal make-up of the Fibonacci gap diagonals remain the same also in the manifesting Fibonacci gap rows, as illustrated by table 8.

**TABLE 8. Fibonacci gaps as Fibonacci diagonals (minus 1) skewed into rows**

| d\i | 1 | 2 | 3 | 4 | 5 | | Sum | Fibonacci gap | Rooting Fibonacci atom |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | | | | | = | 2-1 | = $F_5-F_3-1$ | $F_4$ |
| 2 | 2 | | | | | = | 3-1 | = $F_6-F_5-1$ | $F_5$ |
| 3 | 3 | 1 | | | | = | 5-1 | = $F_7-F_6-1$ | $F_6$ |
| 4 | 4 | 3 | | | | = | 8-1 | = $F_8-F_7-1$ | $F_7$ |
| 5 | 5 | 6 | 1 | | | = | 13-1 | = $F_9-F_8-1$ | $F_8$ |
| 6 | 6 | 10 | 4 | | | = | 21-1 | = $F_{10}-F_9-1$ | $F_9$ |
| 7 | 7 | 15 | 10 | 1 | | = | 34-1 | = $F_{11}-F_{10}-1$ | $F_{10}$ |
| 8 | 8 | 21 | 20 | 5 | | = | 55-1 | = $F_{12}-F_{11}-1$ | $F_{11}$ |
| 9 | 9 | 28 | 35 | 15 | 1 | = | 89-1 | = $F_{13}-F_{12}-1$ | $F_{12}$ |
| . | . | . | . | . | . | . | . | . . . . | . . |
| n-3 | | | | | | | | = $F_{n+1}-F_n-1$ | $F_n$ |
| | | | | | | | | = $F_{n-1}-1$ | |

In table 8 symbol *d* denotes the perplex number of a diagonal, which was indexed by j in table 7, so that the values of d and j are identical for i=1 (but not for i>1, due to the transformation from diagonals to rows). Quite obviously, each number having position (j,i) in the matrix of table 7, reappears in position (d,i) in the matrix of table 8, where the last position is determined by d = j+i-1. As an example, (j,i)=(7,2) gives the number 21 in table 7, and 21 reappears at position (d,i) = (7+2-1, 2) = (8,2) in table 8.

Trivially, d and the rooting Fibonacci number $F_n$ for the same row d, are linked as perplex numbers by d=n-3.

Mikhael B. Ignatyev pioneered Russian robotics in the 1960's and has later developed a universal formulation of cybernetics named *linguo-combinatorics* based on a certain set of differential equations applying only a distinction between units of meaning and their symbols (Ignatyev 2006, 2008, 2010). Ignatyev anchors much of his treatment *exactly* in the structuring from *Pascal's triangle* which manifests the formula for "the basic law of cybernetics, informatics and synergetics for complex systems" (Ignatyev 2006). More precisely, this formula represents a slightly amputated Pascal triangle as follows:

**TABLE 9. Ignatyev Pascal's triangle formulation of "the basic law of cybernetics"**

*The number of arbitrary coefficients depending on the number of variables n and the number of restrictions m.*

| n /m | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 2 | 1 | | | | | | | |
| 3 | 3 | 1 | | | | | | |
| 4 | 6 | 4 | 1 | | | | | |
| 5 | 10 | 10 | 5 | 1 | | | | |
| 6 | 15 | 20 | 15 | 6 | 1 | | | |
| 7 | 21 | 35 | 35 | 21 | 7 | 1 | | |
| 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 | |
| 9 | 36 | 84 | 126 | 126 | 84 | 36 | 9 | 1 |

*The formula (6) is the basic law of cybernetics, informatics and synergetics for complex systems. The number of arbitrary coefficients is the measure of uncertainty.* (Ignatyev 2006)

This table is easily seen as representing just a rewrite of table 7 with n=j and m=i-1, deleting the column of i=1 corresponding to m=0. We may interpret n as the *span* (number of blank spaces) of possibility space, and m as the number of *filling-ins* (of such spaces) in *addition* to the minimum of *one* filling-in. Then the matrix of table 1 exposes the numbers of possible *combinations* ("arbitrary coefficients") of filling-ins for the respective possibility spans. Compared with table 8, we observe that the *diagonals* south-west to north-east of table 9 is just a rewrite of the *rows* of table 8 –

with the sole exception that the 1 of the first row of table 8, representing the originating, and in some sense rooting, Fibonacci molecule, namely (2,4), is not represented in table 9, due to being implied in the very *framing* to establish any relation between m and n. Stating this to be the "*basic law*" for the science(s) of (complex) systems, indicates that the table 9 formula of Pascal's triangle is *essential* in the make-up of natural systems *in general*. When acknowledging this table 9 formula to represent a rewrite of the Fibonacci structuring of Fibonacci molecules, this implies with necessity that this particular *Fibonacci* structuring is *accordingly essential* and expresses, or rather *constitutes*, the *same basic law*. The implications of this revealed essential fact of reality might be quite far-reaching, not to say tremendous.

The radical novelty, universality and explanatory power represented by linguo-combinatorial cybernetics is indicated by, according to Ignatyev, proving able to develop exhaustive "models of all the known chemical elements, their isotopes, and molecular structures" (Ignatyev 2010: 673). Thus, Ignatyev refers to the establishing of "cybernetic physics" (Ignatyev 2008: 20) and states such cybernetic physics/chemistry as *superior* to the conventional method of linear combination of atomic orbitals, because "the linguo-combinatorial method considers all the combinations of interaction" (Ignatyev 2010: 673).

This is even more interesting due to his application of this theory to theory and technology of robotics, and in recent times also nano-robotics (Ignatyev 2010: 674), leading to the discovery of an important *connection* between what we may denote *Pascal complexity* (understood as the values of the involved "arbitrary coefficients" in Pascal's triangle) in the *algorithmic* composition of a nano-robot vs. the Pascal complexity inherent in the *material* substances making up the nano-robot. More precisely, in this construction of robots, the complexity and thus potential for action of a robot, is constrained by not possibly being higher than the inherent Pascal complexity of the material substratum of the robot. This implies that application and development of materials with higher Pascal complexity, as determined from cybernetic chemistry, yields a higher potential for robotic action. The control unit, governing the robot, is *extracted* from the Pascal complexity of the substratum, and is set up and tuned in feedback loops with the substratum according to specified procedures from cybernetic control theory. This seems to imply, different from universal Turing machines, an intimate relation between hardware and software, where the hardware becomes *employed* in the very constitution of the software by a part of the hardware Pascal complexity being transferred to the control unit. Hence, the connection – not to say discovery – of the mathematical relations in Pascal's triangle to natural systems in general, seems to have triggered the development of advanced and novel computational technology quite different from conventional Turing machines which radically distinguished between hardware and software.

Ignatyev's linguo-combinatorial cybernetics provides a universal method for strict mathematical treatments by means of establishing so-called *equivalent-equations* exactly corresponding to the degree of Pascal complexity inherent in the system (via the description of the system) at hand. Thus, this method targets, exposes and takes advantage of the Pascal triangle structuring tacitly enfolded in any system. It is

important that this structuring is not only static, but involves an enfolded *evolutionary dynamics* of the system, expressed by the mathematical proportions between successive numbers, left to right, in the *same row* of the triangle, as well as by possible evolutionary *leaps* between successive rows (Ignatyev 2006: fig. 2), including leaps from values *inside* such rows made possible by external intervention. This indicates the possibility of more profound and exact *general* understanding of *irreversibility* in natural systems, uniformly patterned from a hidden mathematical code engraved in the Pascal triangle structuring of systems. Such irreversibility seems related *in general* to the *Pascal complexity* evolution of systems, including aspects of emergence, growth, degeneration and death of systems.

As indicated by Ignatyev's achievements in robotics, such an approach also provides scientific keys for more effective interference with and *control* of systems, by means of *manipulating* the number of variables and/or of constraints determining the Pascal coefficients of the system by enforcing possible *leaps* between Pascal rows in the system (Ignatyev 2006: fig.s 1,2,3,4 and 5). This implies manipulation of the *timeline* inherent in the Pascal-structured evolution of a system, as illustrated already when applying a control unit for *tuning* of a system following Ignatyev's instructions. Still, such manipulation is constrained by the Pascal structuring of the system with a related *foundation* of irreversibility, but there emerges a *second-order* possibility space for reversible manipulations – and in this sense: freedom – *inside* the Pascal irreversible structure of the overall system.

For more analysis and discussion of far-reaching implications of Ignatyev's Pascal cybernetics and technology, see Johansen (2010b).

## FIBONACCI CONSTITUTION OF SPLIT CODE 5:3 FOR THE PASCAL'S TRIANGLE DISTRIBUTION

The FM/FC algorithm has the Pascal distribution of summands analysed above, but the same will also be the case for some other algorithms for perplex ordering of Fibonacci molecules. Now we will disclose and examine some *further* and even more profound crucial attributes of the FC algorithm. These are attributes that *distinguish* it from other algorithms resulting in Pascal distribution of summands.

**FIGURE 2. Fibonacci-Pascal distribution of perplex Fibonacci molecules with respect to amount of Fibonacci atomic constituents, illustrated by the Fibonacci molecules between F11 and F12, i.e. the natural numbers between 89 and 144**

| Amount of molecules in segment | Amount of atoms 1 | 2 | 3 | 4 | 5 | Repeated segment | Amount of diagonals |
|---|---|---|---|---|---|---|---|
| 0 | 1 | | | | | | |
| 1 | | 1 | | | | | |
| 1 | | 1 | | | | | |
| 2 | | 1 | | | | | |
| | | | 1 | | | | |
| 3 | | 1 | | | | | 1 (1) |
| | | | 2 | | | | |
| 5 | | 1 | | | | | 1 (1) |
| | | | 3 | | | | |
| | | | | 1 | | | |
| 8 | | 1 | | | | 5 | 2 (1+1) |
| | | | 3 | | | | |
| | | | | 1 | | | |
| | | | 1 | | | 3 | |
| | | | | 2 | | | |
| 13 | | 1 | | | | 8 | 3 (1+2) |
| | | | 3 | | | | |
| | | | | 1 | | | |
| | | | 1 | | | | |
| | | | | 2 | | | |
| | | | 1 | | | 5 | |
| | | | | 3 | | | |
| | | | | | 1 | | |
| 21 | | 1 | | | | 13 | 5 (2+3) |
| | | | 3 | | | | |
| | | | | 1 | | | |
| | | | 1 | | | | |
| | | | | 2 | | | |
| | | | 1 | | | | |
| | | | | 3 | | | |
| | | | | | 1 | | |
| | | | 1 | | | 8 | |
| | | | | 3 | | | |
| | | | | | 1 | | |
| | | | | 1 | | | |
| | | | | | 2 | | © Stein E. Johansen |

Fig. 2 covers the perplex Fibonacci molecules having F11 as their rooting Fibonacci atom. The first of these, the Fibonacci molecule that is the Fibonacci number substance with perplex number 90, i.e. (2,11), consists of two Fibonacci atoms, and the same is the case for number 91, i.e. (3,11), as well as for number 92, i.e. (4,11). Number 93, i.e. (2,4,11), consists of three Fibonacci atoms. Then number 94 occurs, consisting of two Fibonacci atoms, i.e. (5,11). After this, numbers 94 and 95 occur with three Fibonacci atoms, followed of one number, 96, with two Fibonacci atoms, thereafter of three numbers, 97-99, having three Fibonacci atoms, and so on.

We notice that the Pascal triangle distribution of Fibonacci molecules is structured in a strict, self-referential pattern with a remarkable simple make-up, moving between diagonals 1-2 (displayed in red) and 1-3-1 (displayed in blue). If we include the rooting Fibonacci atom, F11, in our observation, also the initial sequence appears as such a diagonal, namely 1-3-1.

Further, we notice that the succession of 1-2 and 1-3-1 is strictly structured into *segments* generated by the *Fibonacci* algorithm. The Fibonacci algorithm generates the *spans* of these segments (cf. the far left column). Also, we notice (cf. the column "repeated segment") that each such segment is repeated with identical diagonal structure when constituting the *first sub*-segment in the *next* segment, as well as the *second* sub-segment of the segment *after* this next one. However, in the latest case, this takes place with the amounts of respective atoms in the diagonals increasing with 1, displayed in fig. 2 as these diagonals being transported one step to the right. Thus, also the *sub*-segments are strictly structured according to the Fibonacci algorithm, applying the diagonals 1-2 and 1-3-1 as its elementary, universal *substratum*. This indicates a remarkable significance of the split code 5:3 in the overall Fibonacci constitutional logic of natural numbers.

Inspection of perplex Fibonacci molecules rooted in *other* Fibonacci atoms than F11 of fig. 2, reveals the same strict regularities as those described, so fig. 2 represents merely an illustration of the universal, generative structure. If we, as an illustration, look at the Fibonacci molecules rooted in F12, the segment 21 with its diagonal structure repeats as the first sub-segment (if F12 is included) for the Fibonacci molecules rooted in F12, and the segment 13 as the second sub-segment (with corresponding additions of 1 of atom amounts in its diagonals). Thus, the whole field of perplex Fibonacci number substances is generated from the same self-referential structure. Here, the Fibonacci atoms only represent the first number of each unfolding cluster of diagonals, and has to be included in order to make the generation self-referentially complete for the whole field.

In the column to the far right of fig. 2 we also notice that the *amount* of diagonals are generated from the Fibonacci algorithm, and the same is the case for the internal distribution of these amounts onto diagonals 1-2 vs. diagonals 1-3-1. Obviously, the same holds for the whole field of perplex Fibonacci molecules and number substances.

Thus, we see that every natural number, both with respect to perplex number and size number, corresponds uniquely to a position inside an overall generative structure with a quite simple self-referentiality.

Quite remarkably, we discover that Pascal's triangle, which structures the distribution of perplex Fibonacci molecules, *itself* is generated from the Fibonacci algorithm. This means that Pascal's triangle is to be understood as an *intermediary* in the structuring of perplex Fibonacci molecules, and that the Fibonacci algorithm is to be understood as the more profound generator of this structuring. Thus, the Fibonacci diagonals in the triangle are not a secondary *attribute* by the triangle, but the other way around: Pascal's triangle is *generated* by the Fibonacci algorithm *via* these diagonals. Hence, Pascal's triangle is only *apparently* generated autonomously in relation to the Fibonacci algorithm, and only *superficially* from the trivial adding algorithm.

As stated previously, the Fibonacci atomic-molecular gap identity would still hold if the perplex Fibonacci molecules inside the gaps were shuffled, but then the Fibonacci molecular identity would not be satisfied. In other words: The Zeckendorf theorem would not at the same time be valid both for the perplex numbers and for the size numbers of the Zeckendorf sums, despite each of them providing a complete and unique representation of the natural numbers. This incongruence arrives as a necessary result of the shuffling, since some perplex numbers of Zeckendorf sums then would have size numbers different from their perplex number. However, also in the case of such shuffling the Fibonacci-Pascal relations exposed in tables 7-9 would remain the same. The distinction, where shuffling or non-shuffling makes a difference, does not become transparent before displayed in fig. 2. With shuffling the pattern in fig. 2 would not manifest. But the pattern of fig. 2, which manifests from the FC algorithm, is the *only* one that satisfies Zeckendorf's theorem in both respects, the perplex and the size number identity, at the same time. This means that it is *only this* version of Pascal's triangle that is consistent with such full satisfaction of Zeckendorf's theorem. Thus, this version of Pascal's triangle is the *specification* of the triangle that is *operative* in the Fibonacci generation of Fibonacci molecular numbers with their correct perplex numbers, i.e. their pin-pointed positioning *inside* the Fibonacci gaps. Pascal's triangle manifests from Fibonacci-generation of Fibonacci molecules *because* this generation happens in this – and only this – specific, perplex determination. Therefore, the Pascal triangle structuring is only a *necessary* condition to describe the perplex ordering of Fibonacci molecules. This means that Pascal's triangle is *not* the deeper and primary generator of this ordering, but manifests as an *implied intermediary* from the *Fibonacci* generator, in this *specific* version of the triangle which is the *only* one that is inherently generated from the Fibonacci algorithm itself.

Ignatyev's Pascal formulation of the basic law of cybernetics indicated that a deeper comprehension of Pascal's triangle as generated from the Fibonacci algorithm was not a mere exercise of tumbling around with numbers to find an alternative formulation, but required to understand this important formulation as a *necessary and crucial expression* of the universal and elementary *Fibonacci* generator of Nature. The basic law became clearly tied to the perplex, Fibonacci gap distributed, make-up of *Fibonacci molecules*, and by this *substantiated* as well as *re-established* and *re-generated*. Now we realise

that this substantiation takes place from the *specific* Pascal's triangle manifesting from the specific *intra*-perplex ordering of Fibonacci molecules from the FM algorithm, and that the basic law is *established* from this *specific* perplexity, and thus tied to the Zeckendorf theorem being valid simultaneously, or one might say: as conjugated from being *originated*, for both perplex and size numbers in this unique case. Thus, fig. 2 provides the doorway to the clues to the *inner workings* establishing the basic law.

The Fibonacci algorithm does not only generate the *content* of the Fibonacci molecules, i.e. the specific Fibonacci atoms making up the molecules, but it also generates the *structuring* and precise, successive location of these Fibonacci molecules. This means that the Fibonacci algorithm generates the perplex Fibonacci molecules, and hence the natural perplex and size numbers, *both* with respect to form and content, i.e. *completely* and *autonomously*. Thus, to understand the generative constitution of natural numbers, it is *sufficient* to understand this from the Fibonacci algorithm without *any* extra, external consideration. As stated, Pascal's triangle figures as an intermediary which is established from this generation. And it is obvious that the FM and FC algorithms can be formulated or rewritten to clearly express the simple structuring displayed in fig. 2.

Further, it has become clear that the split code 5:3 is essential in this generative dynamics of perplex Fibonacci molecules representing the inner workings establishing the basic Pascal-Ignatyev law. It is remarkable that the same split code 5:3 was applied in the entering, general rewrite of natural numbers in order to deduce and discover the *Johansen Revolving Prime Number Code* (Johansen 2010a). The formulas of the prime number code herein was applied by J.M. Strand to develop software (Johansen 2011: appendix) which confirmed the deduced formulas to be correct, and which picked the prime numbers in correct succession, including among directly targeted and freely chosen segments of natural numbers (denoted *Johansen Revolver - Strand Longrange Algorithm*; *JR-SLA*). The original quantitative expression became later reformulated by Strand (2011) into a *group representation* as a certain group matrix which he thereafter achieved a further reformulation of as a certain Santilli *genonumber representation.*

In Strand's group representation, $\mathbf{Z}_8$ is the finite, cyclic group of elements, and $\mathbf{Z}_8 \times \mathbf{Z}_8$ is determined from an identified linear transformation implying an isomorphism. Here, Strand points out the unique significance of the basic coding by stating:

*Note that $8 = 2^3$ and since we therefore cannot find two factors that are coprime (only 2, 4 and 8), the above representation* ***cannot*** *be isomorphic to a direct sum of* ***smaller*** *cyclic groups. This algebraic structure is therefore* ***uniquely*** *represented by the above transformation.* (Strand 2011; boldfaces by us)

We now realise, from our Fibonacci constitution of natural numbers, that the split code 5:3 is implied and essential in the constitutional design of natural numbers as such. This seems to be the key to understand *why* the split code 5:3 became a successful point of *departure* for the deduction of the exact pattern of prime numbers from a hidden generator. *Without* this key provided from the *Fibonacci* approach to natural numbers, the significance of the split code 5:3 to reveal the prime number pattern appears as quite a mystery. This may give a profound indication of the Fibonacci approach as not merely

an exercise in hermetic mathematical philosophy, but as necessary to unveil a significant subtle *reality* enfolded in the natural numbers, necessary to yield important new quantitative *results* in number theory. These two significant occurrences of the split code 5:3 indicate an *intimate relation* between Fibonacci numbers and prime numbers. Further, this intimate relation does not refer to Fibonacci numbers and prime numbers as *separate* fields constituted from the common platform of *conventionally* conceived natural numbers. The intimate relation appears because the natural numbers *themselves* are Fibonacci constituted with the split code 5:3 as an essentiality, and this essentiality *reappears* in the distinguishing *inside* natural numbers between prime numbers and composite numbers.

(It may also be noted as a matter of historic fact that the author's development of the 5:3 split code rewrite and revolving model to approach prime numbers, during the research process, initiated Easter 2005, happened in *connection* to Fibonacci related considerations, despite this not being transparent in the publication (Johansen 2010a) that presented the final results from this research by deducing the complete and exact patterning of composite vs. prime numbers.)

## FIBONACCI CONSTITUTION OF PRIME NUMBERS AND COMPOSITE NUMBERS

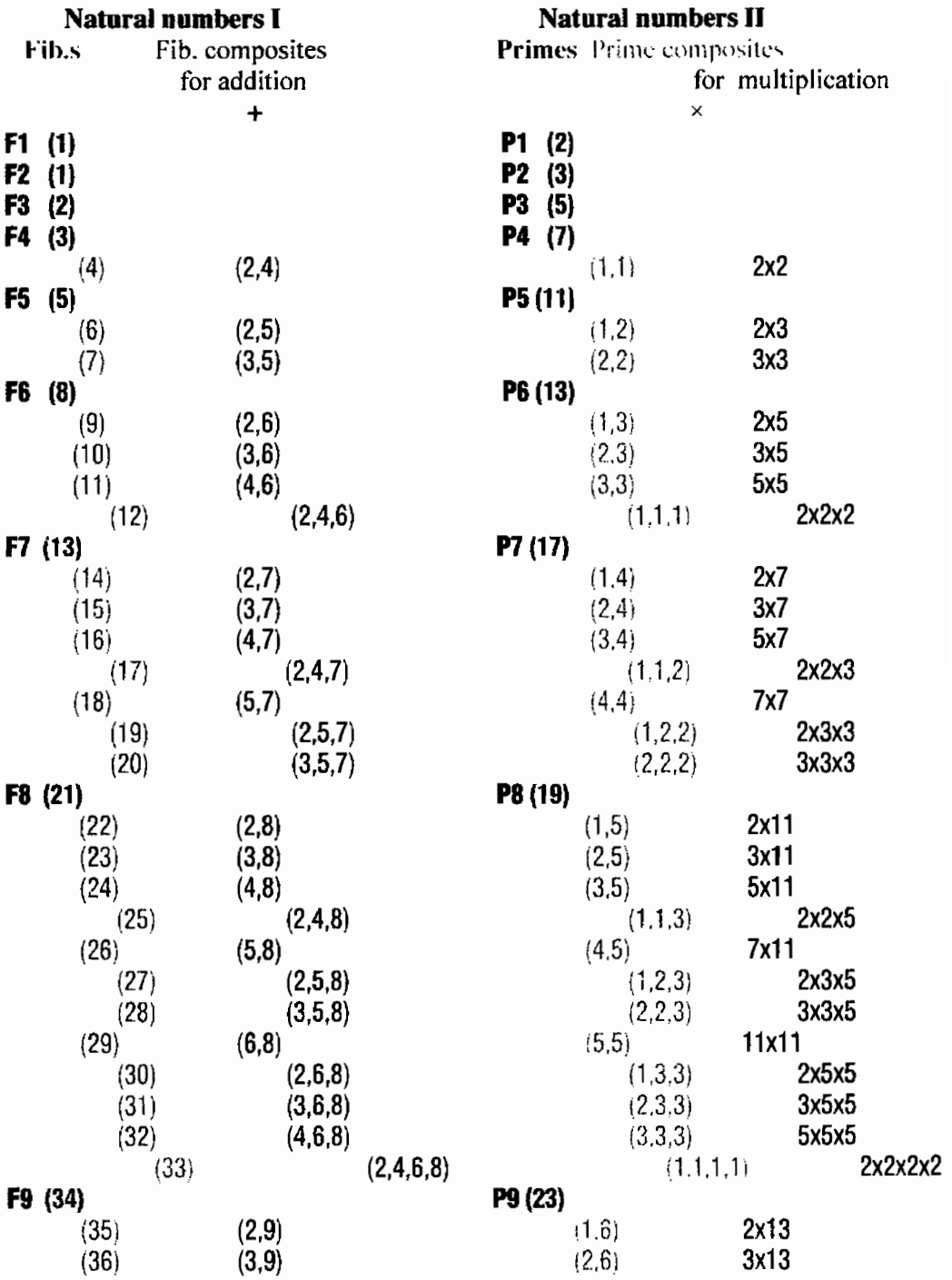
We have discovered and analysed *multiplication* as a *Fibonacci constituted* operation. This operation can not be found from any constitutional logic in natural numbers as conventionally considered; and if the Fibonacci originated product is *described* in the conventional way, this would only be a second-hand *expression* of a constitutional dynamics not *begriffen* (understood) by such formal expression.

We have also discovered a *profound link* between Fibonacci numbers and prime numbers, represented by the 5:3 split code inherent in the FM algorithm which generates the perplex Fibonacci molecular distribution constituting the basic Pascal-Ignatyev law for informatic and cybernetic systems. This split code is inherent in the *hidden, particular* structuring of (or rather: into) natural numbers, which generates the exact and complete distinguishing pattern of composites vs. primes. The direction of the link goes *from* the inner workings of Fibonacci generation, as exposed, *to* the foundation of primes vs. composites when presupposing the (non-primary) operation multiplication, not the other way.

On the background of these discoveries, it is finally time to contemplate the more exact relations between Fibonacci numbers and prime numbers.

**FIGURE 3. Fibonacci structuring of prime numbers and composite numbers**

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(37) (4,9)
(38) (2,4,9)
(39) (5,9)
(40) (2,5,9)
(41) (3,5,9)
(42) (6,9)
(43) (2,6,9)
(44) (3,6,9)
(45) (4,6,9)
(46) (2,4,6,9)
(47) (7,9)
(48) (2,7,9)
(49) (3,7,9)
(50) (4,7,9)
(51) (2,4,7,9)
(52) (5,7,9)
(53) (2,5,7,9)
(54) (3,5,7,9)

**F10 (55)**

(56) (2,10)
(57) (3,10)
(58) (4,10)
(59) (2,4,10)
(60) (5,10)
(61) (2,5,10)
(62) (3,5,10)
(63) (6,10)
(64) (2,6,10)
(65) (3,6,10)
(66) (4,6,10)
(67) (2,4,6,10)
(68) (7,10)
(69) (2,7,10)
(70) (3,7,10)
(71) (4,7,10)
(72) (2,4,7,10)
(73) (5,7,10)
(74) (2,5,7,10)
(75) (3,5,7,10)
(76) (8,10)
(77) (2,8,10)
(78) (3,8,10)
(79) (4,8,10)
(80) (2,4,8,10)

(3,6) 5x13
(1,1,4) 2x2x7
(4,6) 7x13
(1,2,4) 2x3x7
(2,2,4) 3x3x7
(5,6) 11x13
(1,3,4) 2x5x7
(2,3,4) 3x5x7
(3,4,4) 5x7x7
(1,1,1,2) 2x2x2x3
(6,6) 13x13
(1,4,4) 2x7x7
(2,4,4) 3x7x7
(3,4,4) 5x7x7
(1,1,2,2) 2x2x3x3
(4,4,4) 7x7x7
(1,2,2,2) 2x3x3x3
(2,2,2,2) 3x3x3x3

**P10 (29)**

(1,7) 2x17
(2,7) 3x17
(3,7) 5x17
(1,1,5) 2x2x11
(4,7) 7x17
(1,2,5) 2x3x11
(2,2,5) 3x3x11
(5,7) 11x17
(1,3,5) 2x5x11
(2,3,5) 3x5x11
(3,3,5) 5x5x11
(1,1,1,3) 2x2x2x5
(6,7) 13x17
(1,4,5) 2x7x11
(2,4,5) 3x7x11
(3,4,5) 5x7x11
(1,1,2,3) 2x2x3x5
(4,4,5) 7x7x11
(1,2,2,3) 2x3x3x5
(2,2,2,3) 3x3x3x5
(7,7) 17x17
(1,5,5) 2x11x11
(2,5,5) 3x11x11
(3,5,5) 5x11x11
(1,1,3,3) 2x2x5x5

| | | | | | |
|---|---|---|---|---|---|
| (81) | (5,8,10) | | (4,5,5) | 7x11x11 | |
| (82) | (2,5,8,10) | | (1,2,3,3) | 2x3x5x5 | |
| (83) | (3,5,8,10) | | (2,2,3,3) | 3x3x5x5 | |
| (84) | (6,8,10) | | (5,5,5) | 11x11x11 | |
| (85) | (2,6,8,10) | | (1,3,3,3) | 2x5x5x5 | |
| (86) | (3,6,8,10) | | (2,3,3,3) | 3x5x5x5 | |
| (87) | (4,6,8,10) | | (3,3,3,3) | 5x5x5x5 | |
| (88) | (2,4,6,8,10) | | (1,1,1,1,1) | 2x2x2x2x2 | |
| **F11 (89)** | | **P11 (31)** | | | |

Fig. 3 displays to the right the perplex structuring of Fibonacci number substances when substituting the operation *addition* with the operation *multiplication*, thus placing prime number atoms (factors) in the position of Fibonacci atoms and composite numbers in the position of Fibonacci molecules. Different from Fibonacci addition, to uniquely cover all prime molecules, we now must include also the *first* perplex atom, i.e. the first prime which is 2, as well as all possibilities of molecules having *more than one* exemplar of the same atom in its composition. This is trivially achieved by simply subtracting 1 from the corresponding perplex Fibonacci molecule with respect to the *first* (smallest) prime atom from the corresponding Fibonacci atom, subtracting 3 from the corresponding *second* Fibonacci atom, subtracting 5 from the corresponding *third* Fibonacci atom, etc. Notice that the primes and the composites are determined as *perplex* numbers and as *perplex* atoms (whether singular or composite) *before* the operation multiplication becomes performed, i.e. *before* they have any *size* at all as *products* from multiplication. Notice also that these perplex numbers are completely derived from *perplex Fibonacci* numbers, and – in the next step – that the *size* numbers of these perplex atoms of primes/composites, entering multiplication, are completely derived from the translation from the sizes of *Fibonacci* atoms, *not* from any pre-knowledge about natural numbers and sizes in the *conventional* sense. Thus, the sizes of the *composite products* manifest completely from the sizes of *perplex Fibonacci* atoms, becoming translated by our simple rule and computed from there by the multiplication operation.

Thus, all composite numbers become displayed from just a simple translation or modification of the perplex *Fibonacci* atoms making up the perplex Fibonacci molecules corresponding to the perplex composite numbers. In this way then, composite numbers are structured by exactly the *same* Pascal structuring (displayed in fig. 2 and implying the 5:3 pattern) as the Fibonacci molecules, and therefore also from the same underlying, generative *Fibonacci* structuring. Therefore, this Fibonacci anchoring of primes and composites comes in *addition* to the Fibonacci based perplex numbering of the prime factors and composite molecules (from said simple modification).

This displays a remarkable identity between the structuring of Fibonacci molecules and prime molecules. Quite obviously, also the perplex succession of prime molecules will generate *all* possible prime molecules and all possible sizes of such uniquely. However, the *double* and *originating* Zeckendorf identity, valid for the

Fibonacci number substances as generated from the FC algorithm, can no longer be valid for prime molecules. *Either* the perplex numbers of composites (and primes) are identical to those of natural numbers, *or* the size numbers of composites (and primes) are identical to those of natural numbers. This is a binary choice of perspective. With necessity perplex numbers have primacy to size numbers in constitutional number logic, and the perspective underlying fig. 3 is in agreement with this, as well as with the *specific* and *true* constitution of size numbers from perplex numbers performed by the *Fibonacci algorithm*. Conventionally, however, and contrary to this, the second identity with related implied perspective has de facto been chosen, usually without idea about any choice being made, and even less with any relating to the Fibonacci algorithm, considering the chosen perspective of departing from the second identity (and this without any Fibonacci reflection) to be trivial and self-evident.

The double, originating Zeckendorf identity to be valid for Fibonacci number substances and not for prime composites, is in itself a tell-tale sign of Fibonacci numbers having number ontological primacy to prime numbers. Further, this lack vs. non-lack of double identity excludes prime numbers, contrary to Fibonacci numbers, as any possible candidate to tacitly *constitute* the very concept of natural numbers of perplexity and size.

The Fibonacci-prime relation of fig. 3 might be interpreted as the originating Zeckendorf identity of the Fibonacci generator becoming bifurcated, from Fibonacci generation itself, into the either-or choice of Zeckendorf identity perspective for relating primes/composites to natural numbers. In such an interpretation, it is the Fibonacci generated *evolution* into primes/composites representation which *creates* the alienation from the originating Zeckendorf identity and *establishes* the quest to find the hidden prime number generator from an either-or platform of natural numbers. Here, however, the crux arrives, where such a quest is not *possible* to re-solve, if not stepping *back beneath* the platform, especially from the size number perspective, *into some* underlying workings of its hidden Fibonacci-generator, more specifically into arriving at the 5:3 structuring of the natural numbers. The over-all picture is that the natural numbers, as conventionally considered, represent an *intermediary* between the Fibonacci generator and the primes/composites, and that the primes/composites code could not be cracked, at least *de facto*, before stepping *backwards-beneath* this intermediary, i.e. by following the backing first-step of the Fibonacci algorithm itself, *from* the conventional and trivialised notion of natural numbers; and next, also following the stepping of the Fibonacci algorithm itself, but now by performing the *second* step, into the resulting exact pattern of primes vs. composites. Thus interpreted, the over-all picture reveals as a primordial unity, expressed by the originating double Zeckendorf identity, becoming bifurcated and alienated, expressed by the prime number enigma, to become re-unified from moving back towards the unitary Fibonacci source, which evolved into the *systematic* and *exact* quantitative discrepancy between the prime number identity/non-identity, and leap from there – *via* the natural numbers – into the formulas of the exact re-unification.

Thus contemplated, the very quest of looking for the hidden generator (sometimes described as "the holy grail" of mathematics) of the exact prime numbers,

might be viewed as arising from the original identity or unity which differentiates itself from itself into a state of alienation, posing the challenge to become re-established and re-unified from later explorations and endeavors. This was an essential theme running through dialectical German philosophy which considered the re-unification, in one sense or another, into the original unity at a lifted level through the intermediary of the alienated opposite or negation (*Aufhebung* as determined negation of negation of position) not to be possible without connecting back to the *source* for the alienation and the connected quest. Such connecting back for future re-unity has itself the Fibonacci *form*, whatever the content of the source of re-unity. Our context is the only one where this form also characterises the *content* of the source to become reunified.

To our knowledge Johansen (2006) was the first publication (initially presented as lecture at *18. Workshop in Hadronic Mechanics* June 2005) to report the discovery of a remarkable 1:1-relation between Fibonacci numbers and prime numbers, namely that the $n$ of a perplex Fibonacci number $F_n$ is a prime if and only if $n$ is a factor either in size number $F_{n-1}$ or in size number $F_{n+1}$. Hence, to decide whether an arbitrary natural number $n$ is a prime or not, is found simply by performing the two calculations $F_{n-1}/n$ and $F_{n+1}/n$, thus only involving the arithmetic operations of addition and division. If, and only if, the calculation results in a whole number, then $n$ is a prime. As example, 71 is a prime number because $F_{70}/71$ is a whole number, and 73 is a prime number because $F_{74}/73$ is a whole number. (For quick, confirming support of this identity to be valid, Knott (I) offers a convenient list of the factors for the first 300 Fibonacci numbers. The identity has been checked to be valid for the first 1000 Fibonacci numbers, so it is extremely unlikely that it is not universally true.) We denote this deterministic method to decide with certainty whether a natural number is a prime as the *Fibonacci neighbour primality test*. This is not the first such deterministic method being discovered. The *AKS primality test*, discovered by Agrawal et al. (2004) in 2002 provided a polynomial time deterministic algorithm to decide with certainty whether a number is prime. The universal validity of the AKS primality test was also proved, while the universal validity of the Fibonacci neighbour primality test still remains to be given a proof.

Leaving issues of mathematical and computational convenience aside, the validity of the Fibonacci neighbour primality test invokes particular interest due to presenting a formula for complete *generation* of prime numbers *autonomously and solely* from the Fibonacci algorithm (when including the operation of division). Thus viewed, the attribute of representing a deterministic prime number *test*, which tacitly implies a conventional conception of prime numbers as *already* constituted, autonomously and *independent* of the Fibonacci algorithm, is a *secondary* attribute compared to, and *deriving* from, the more primary circumstance, namely that this prime test formula works *because* prime numbers as such are constituted and *generated from* the inner workings of the Fibonacci algorithm. When giving main significance to this more profound relation as becoming *expressed* in the prime test formula, we consider and denote the formula as the *Fibonacci neighbour generation of primes.*

When contemplating the amazing mathematical *simplicity* of the Fibonacci neighbour primality *test*, and even more when contemplating more general theoretical implications of acknowledging Fibonacci neighbour *generation* of the prime numbers, it

seems somewhat strange, but perhaps oddly significant, that still, to our knowledge, *zero* other publication has presented (or referred to) this possibly arrow-in-the-heart discovery for catalysis of number theory.

However, as already stated, there has still not been presented a completed mathematical *proof* for why (and how) the Fibonacci algorithm generates the prime numbers exactly and completely by this simple relation. It seems likely that the proof is technically simple, so the real and difficult quest is to find the adequate entrance and approach to discover the solution. Fig. 3 may offer some clue in this respect, since it indicates that this remarkable 1:1-connection is anchored in the *structuring* of prime atoms and molecules as an *evolvement* of the Fibonacci structuring of natural numbers displayed in fig. 1.

## CLOSING REMARKS CONCERNING IRREVERSIBILITY

In general, one can draw a distinction between *constitutional* logic *establishing* a system and *reproductive* logic *developing* the system (into growth, steady-state and decline) when the system as such already has been considered established. One may apply *evolutionary logic* as a broader term to cover both constitutional and reproductive logic of systems, as when evolution of species is considered to denote both emergence of new species and development of each species, or when evolution of an organism is considered to denote both embryology and the development of the organism after birth. All evolutionary logic can be said per definition to imply irreversibility, whatever the details of how to adequately pin-point the conceptual distinction reversible vs. irreversible. Comprehension of constitutional logic tends to be more difficult than of reproductive logic, due to involving more qualitative changes and more radical leaps between sub- or pre-systems. This implies that also the issue of irreversibility tends to arrive more demanding in analysis of constitutional logic (as for example when cosmological physics seeks to explain birth of the universe), while on the other hand as potentially more far-reaching due to being implied and embedded in the very foundations of the developing system.

Our concern has been to analyse the constitutional logic that establishes natural numbers as a system, including significant aspects of the system as perplex numbers vs. size numbers, numbers vs. geometry, the basic arithmetic operations, the significance of the unit, and the distinction and pattern of primes vs. composites. Such an undertaking has not much meaning if this system not at first becomes detrivialised, so that there arises a scientific *quest* to explain the constitution of the system. The adequacy of such an endeavour should primarily be evaluated from the fruits of novel insights and results it shows able or likely to create.

The system of natural numbers seems to represent the most abstract system imaginable, and by this also the most fundamental, due to being implied, at least tacitly, in all other systems and descriptions of systems. Due to the degree of involved abstraction, analysis of its constitutional logic must be correspondingly demanding, but for the same reason also with possible far-reaching relevance for understanding of other

systems, including the general role of irreversibility in systemic constitution and from there also into systemic reproduction.

It is a fallacy of thought to consider it possible to explain *in toto* the more abstract from the less abstract. How come then, that the system of natural numbers can possibly be explained from *more* abstract constituents, if the system *itself* is the most abstract? The enterprise of explaining the constitutional logic of the system of natural numbers presupposes that the conception of this system as the most abstract and fundamental is an *Erscheinung*. There occurs a paradox here: It *is* the most abstract, *and* it is *not*. The paradox is reconciled by that it is the most abstract when re-conceived as *the* perplex numbering performed by the Fibonacci algorithm, i.e. when *transported or translated back* to its constitution; while it is far from the most abstract when understood *from* the Fibonacci generation which constitutes the different aspects of the number system in concise, stepwise unfoldments. Thus, the system of natural numbers is not the most abstract in the manner we *immediately* face it, i.e. in its *conventional* gestalt. From analysis of the constitutional dynamics, it has become clear that the conventional thought of Fibonacci numbers *presupposing* the natural numbers and being a *specific* construction from those, among many other number constructions, represents quite an *inversion*. The truth of the issue shows to be that the system of natural numbers is strictly, but in a camouflaged way until becoming carefully investigated, generated from the inner workings of the Fibonacci algorithm unfolding. Through our analysis of the number constitutional dynamics, it has become consolidated and much specified *how* the Fibonacci algorithm is the real, abstract, universal and elementary generator for the system of natural numbers – just as well as it ultimately is for every *other* system of nature with respect to form and substance, as previously deduced from the reflection upon the category *border*, implied in information as such, in qualitative informatics from differential philosophy.

Throughout our treatment of the number constitutional dynamics we have sought to disclose and analyse *plural irreversible structurings* from the Fibonacci algorithm into the manifest field and system of natural numbers, structurings which have to be *invisible* from the conventional, *fetishised* conception of natural numbers.

(Due to the fetishism, even *if* and *when* quantitatively exposed, the structurings could still remain invisible in the more basic sense, due to becoming qualitatively and ontologically *distorted*. This is so because when *sticking* to the ontological reference frame of fetishised natural numbers as the basic and ultimate one, it would not be *possible* to understand such structuring as what it is: a dynamical structuring from a *deeper* reference frame which *constitutes* the fetishised one. Such understanding requires a *paradigmatic* gestalt switch in the classic sense of Thomas Kuhn. Such switches are never possible, *whatever* scientific facts, deductions and arguments, without supplementary successful mobilisation of sufficient *psychological* skill and effort. However, when crucial steps have been worked out to clarify connections to the previous paradigm, indicating that established knowledge will not be thrown away while at the same time novel expansions in knowledge are emerging, such a switching becomes less difficult. Stakhov & Sluchenkova (I) presents many results, in a manner also accessible for some broader audience, that may work as such bridging

towards a Fibonacci paradigm in mathematics, as well as in other disciplines, humanities not excluded, and perhaps also stretching outside the domains of the scientific world. Significant contributions to such bridging have also been delivered by Kauffman (2004; as well as other works), interestingly anchoring some of his treatments in the remarkable work of Spencer-Brown (1969) which has some basic resemblances with differential philosophy.)

If these structurings were not irreversible, natural numbers would simply not *manifest* from them as any system with self-referential closure and fetishised appearance. Philosophically and somewhat cryptically expressed, if irreversibility was not primary, having reversibility only as a special case (as acknowledged in the very structure of Santilli genonumbers), no system would ever *become* constituted by integrating its constituents in self-referential closure and reproduce from that, and there would not *be* any evolutionary logic. Our treatment has sought to demonstrate that this is the case already for the system of natural numbers.

This implies that the issue of irreversibility is profoundly anchored already in the constitutional logic of natural numbers, and by this in mathematics as a whole. Until the creation of Santilli geno- and hypermechanics, underpinned by the new geno- and hypernumbers (cf. i.a. Santilli 1996,2003,2008, Tsagas and Sourlas 1993, Vougiouklis 1993), there was a cleft between, on the one side, biology, taking irreversible systems for granted, and, on the other side, physics, which was based on quantum mechanics with implied reversibility posing the troublesome integration problem to the time arrow of thermodynamics and macro-physics. Santilli geno- and hypermathematics achieved basic irreversibility (with reversibility as a special case) already in the formulation of the new classes of *numbers* appropriate to describe and explain complex irreversible systems *without* regard to whether such systems were classified as belonging to physical, chemical, biological or other territories of systems. Thus, mathematical irreversibility embedded in the very architecture of novel numbers became acknowledged as a crucial concern to achieve advances in number theory, as well as in related geometry, to later become confirmed and reinforced by unique, explanatory success for some intriguing, irreversible systems of nature (cf. especially the amazing discoveries by Illert in hadronic biology).

On the one hand, these novel numbers of hadronic mathematics already have proved much successful with respect to description and explanation of *reproductive* logic of irreversible *natural* systems. On the other hand, these numbers *themselves* are invented as irreversible in their mathematical essence, thus also stretching into *constitutional* logic of mathematical *number theory*, and this especially from thoughtful detrivialisation and reconstruction of the *unit* and of *relations* between units, making possible *liftings* of pre-hadronic numbers into iso-, geno- and hyper-numbers, with related arithmetic operations, algebra, functions and geometry. In this regard there seems to be an area of constitutional number theory where our own treatment of the constitutional logic of conventional natural numbers can *meet* with the constitution of these more sophisticated numbers of hadronic mathematics. We have touched a bit into this area, as in some of our reflections related to the significance of the unit, and by referring to Strand's discovery of the Santilli genonumber representation of the Johansen

Revolving Prime Number Code; i.e. a representation by means of numbers that have irreversibility built into their very design. However, issues of further possible interfacing have to remain an issue for upcoming research.

The same has also to be the case with respect to possible interfacing with other recent innovative and important research that stretches into constitutional logic of numbers and mathematics, as the contributions of Chandler, Rowlands, Müller and Rapoport.

To reach higher, one may have to move down, towards the springboard, and jump from there. To jump as highest, one may have to do it the Fosbury way.

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## REFERENCES

ABOTT, Edwin A.
1952 [1884] *Flatland. A romance of many dimensions*. Toronto: Dover.
AGRAWAL, M., N. Kayal and N. Saxena
2004 Primes is in P. *Ann. Math.* **160**: 781-793.
Basic-mathematics.com
2008 *Russian peasant multiplication*
http://www.basic-mathematics.com/russian-peasant-multiplication.html
BOHM, David and F.D. Peat
1987 *Science, Order, and Creativity*. Toronto: Bantam
CHANDLER, Jerry
2009 An introduction to the perplex number system. *Discrete Applied Mathematics* **157**: 2296-2309.
DAYKIN, D.E.

1960 Representation of Natural Numbers as Sums of Generalised Fibonacci Numbers - II. *Journal of the London Mathematical Society* **35**: 143-160.
FENWICK, Peter
2003 Zeckendorf Integer Arithmetic. *Fibonacci Quarterly* **41**: 405-13.
FREITAG, Herta T. and George M. Philips:
1998 [1996] Elements of Zeckendorf arithmetic. In: F.T. Howard (ed.): *Applications of Fibonacci numbers* **7**: 129-132. Dordrecht: Kluwer.
GANDZHA, Ivan and J. Kadeisvili:
2011 *New Sciences for a New Era: Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli*. Kathmandu: Sankata Printing Press.
GARIAEV, Peter and P.J. Marcer, K.A. Leonova-Gariaeva, U. Kaempf, V. D. Artjukh
2011 DNA as Basis for Quantum Biocomputer. *DNA Decipher Journal* **1**: 25-46.
HSU, W.-J.
1993 Fibonacci Cubes - A New Interconnection Topology. *IEEE Trans. Parallel and Distributed Systems* **4**: 3-12.
IGNATYEV, Mikhail B.
1963 *Holonomic Automatic Systems*. (In Russian.) Moscow: Akad. Nauk SSSR.
2006 The Study of the Adaptational Phenomenon in Complex Systems. *AIP Conf. Proc.* **839**: *Computing Anticipatory Systems: CASYS'05 - Seventh International Conference*, 322-330.
2008 Semantics and Selforganization in Nanoscale Physics, *International Journal of Computing Anticipatory Systems*, **22**: Logic and Semantics in Front of Nanoscale Physics: 17-23.
2010 Necessary and Sufficient Conditions of Nanorobot Synthesis. *Doklady Mathematics* **82**: 671-5.
ILLERT, Chris and R.M. Santilli
1995 *Foundations of Theoretical Conchology* (2. ed.). Palm Harbor, USA: Hadronic Press.
ILLERT, Chris
2000 *The Physics of Stargate*. East Corrimal, Australia: Chris Illert Publications.
JOHANSEN, Stein E.
1991a *Begrepet om arbeidstidsinnholdet: en teoretisk konstruksjon. Et revisjonistisk bidrag til funderingen av en formreflektert kapitalteori.* (The Concept of Labour Time Content: a Theoretical Construction. A revisionistic contribution to the foundation of form-reflected theory of capital.) Dr. philos. thesis. Bergen: Ariadne.
1991b *Grunnriss av en differensiell epistemologi*.Trondheim, Norway: University of Trondheim.
2006 Initiation of 'Hadronic Philosophy', the Philosophy Underlying Hadronic Mechanics and Chemistry. *Hadronic Journal* **29**: 111-135.
2008a *Grunnriss av en differensiell epistemologi*. (2nd ed.) Oslo: Abstrakt.
(English translation in transit: Outline of Differential Epistemology.)
2008b Spacetimes of Santilli Hypermechanics: From Hadronic Mechanics to Hadronic Biology and Hadronic Psychology. *Hadronic Journal* **31**: 451-512.
2008c Basic considerations about Kozyrev's theory of Time from recent advances in specialist biology, mathematical physics and philosophical informatics. L.S. Shikhobalov (ed.): *Time and Stars: The centenary of N.A. Kozyrev*: 652-703. St. Petersburg: Nestor-History.
2008d Non-trivial Time Flows in Anticipation and Action Revealed by Recent Advances in Natural Science, Framed in the Causality Network of Differential Ontology. D. Dubois (ed.): Logic and Semantics in Front of Nanoscale Physics. *International Journal of Computing Anticipatory Systems* **22**: *Logic and Semantics in Front of Nanoscale Physics*: 141-158.

2010a Complete Exposition of Non-Primes Generated from a Geometric Revolving Approach by 8x8 Sets of Related Series, and thereby *ad negativo* Exposition of a Systematic Pattern for the Totality of Prime Numbers. *Journal of Dynamical Systems and Geometric Theories* **8** (2), 101-171.
2010b Entering Hi-Time Cybernetics – From recent liftings of science, philosophy, meaning theory and robotics onto the human condition. J.S. Vassiliev, A.A. Voronov, M.B. Ignatyev, V.N. Kozlov and R.M. Jusupov (ed.s): *Time and Cybernetics – The Centenary of Academician A. A. Voronov*: 286-318. St. Petersburg: St. Petersburg State Polytechnic University.
2011 Unveiling of Geometric Generation of Composite Numbers Exactly and Completely. *International Conference on Mathematical Sciences*, Bolu, Turkey, Nov. 23-27, 2010 (in press).
KOLOGLU, Murat and Gene Kopp, Steven J. Miller, Yinghui Wang
2010 *On the number of summands in Zeckendorf decompositions.*
http://arxiv.org/abs/1008.3204v1
KNOTT, Ron
I *The Fibonacci numbers.*
http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibtable.html#100
LaVIOLETTE, Paul A.
1994 *Subquantum Kinetics. The alchemy of creation.* Alexandria(USA): Starlane.
LEKKERKERKER, C. G.
1951 *Voorstelling van natuurlijke getallen door een som van Fibonacci.* Date, 1951-01-01, Zuivere Wiskunde; ZW 30/51. Amsterdam: Stichting Mathematisch Centrum.
1952 Voorstelling van natuurlijke getallen door een som van Fibonacci. *Simon Stevin* **29**: 190-5.
MÜLLER, Hartmut
2008 *Global Scaling Theory.* http://www.global-scaling-institute.de/files/gscompv18_en.pdf
http://www.global-scaling-institute.de/50-1-Publications.html
KAUFFMAN, Louis H.
2004 Fibonacci form and beyond. *Forma* **19** (special issue; ed. J. Kappraff):315-334.
KIMBERLING, Clark
1998 Edouard Zeckendorf. *Fibonacci Quarterly* **36**: 416–418.
PIHKO, Jukka
1983 An Algorithm for Additive Representation of Positive Integers.
*Ann. Acad. Sci. Fenn., Ser. A I Math. Dissertations No. 46*:1-54.
QUARTIERI, Giuseppe
2007 Irreversibilita termodinamica e freccia del tempo. *Ricerca Aerospaziale* **23**: 8-19.
PHILIPS, George M.
1 *Zeckendorf representation.*
http://eom.springer.de/z/z120020.htm
RAPOPORT, Diego Lucio
2009 Surmounting the Cartesian Cut through Philosophy, Physics, Logic, Cybernetics and Geometry: Self-reference, Torsion, the Klein Bottle, the Time Operator, Multivalued Logics and Quantum Mechanics. *Foundations of Physics* **40**.
2010 Surmounting the Cartesian Cut Further: Torsion Fields, the Extended Photon, Quantum Jumps, the Klein Bottle, Multivalued Logic, the Time Operator Chronomes, Perception, Semiotics, Neurology and Cognition. J. Groffe (ed.): *Quantum Mechanics, an Overture to Natural Philosophy*. Nova Science Publishers.
ROSEN, Steven M.

1994 *Science, Paradox, and the Moebius Principle. The evolution of a "transcultural" approach to wholeness*. New York: State University of New York Press.
ROWLANDS, Peter
2007 *From Zero to Infinity. Foundations of Physics*. Singapore: World Scientific.
SANTILLI, Ruggero Maria
1996 *Isotopic, Genotopic and Hyperstructural Methods in Theoretical Biology*. Kiev: Ukraine Academy of Sciences.
2001 *Foundations of Hadronic Chemistry with Applications to New Clean Energies and Fuels*. Boston-Dordrecht-London: Kluwer Academic Publishers.
2003 Elements of Iso-, Geno-, Hyper-Mathematics for Matter, Their Isoduals for Antimatter, and Their Applications in Physics, Chemistry, and Biology. *Foundations of Physics* **33**, 1373-1416.
http://www.springerlink.com/content/r776p21u5vp1584p/BodyRef/PDF/10701_2004_Article_469508.pdf
2006 *Isodual Theory of Antimatter with applications to Antigravity, Grand Unification and Cosmology*. Dordrecht, Netherlands: Springer.
2008 *Hadronic Mathematics, Mechanics and Chemistry. Vol. I-V*.
US-Europe-Asia: International Academic Press.
http://www.i-b-r.org/Hadronic-Mechanics.htm
I *The R.M. Santilli Foundation - promoting basic scientific advances and control of scientific ethics*.
http://www.santilli-foundation.org/
SCHADECK, Laurent
2008 *Jiang Number Theory*.
www.wbabin.net/math/xuan15.pdf
SHIKHOBALOV, L.S.
2008 (ed.): *Time and Stars: The centenary of N.A. Kozyrev*. St. Petersburg: Nestor-History.
SPENCER-BROWN, G.
1969 *Laws of Form*. London: Geroge Allen and Unwin.
STAKHOV, Alexey P.
2009 *The Mathematics of Harmony. From Euclid to Contemporary Mathematics and Computer Science*. Series of Knots and Everything **22**. New Jersey: World Scientific.
STAKHOV, Alexey P. and Anna Sluchenkova
I *Museum of Harmony and Golden Section*.
http://www.goldenmuseum.com/index_engl.html
STERN, August
1992 *Matrix Logic and Mind*. Amsterdam: Elsevier.
STRAND, Jon M.
2011 Group Representation and Santilli Genonumber Representation of Johansen Revolving Prime Number Code. *Third International Conference on Lie-Admissible Treatments of Irreversible Processes*. Kathmandu University, Nepal, Jan. 3-7, 2011 (in press).
TEE, Garry J.
2002 *Russian Peasant Multiplication and Egyptian Division in Zeckendorf Arithmetic*. Department of Mathematics - Research Reports-481 (2002). Auckland, New Zealand: University of Auckland.
TRELL, Erik
1992 Real forms of the elementary particles with a report of the Σ resonances. *Physics Essays* **5**: 362-373

1998 The eightfold eightfold way: Application of Lie's true *geometriske transformationer* to elementary particles. Algebras Groups and Geometries **15**: 447-471.
2005 Invariant Aristotelian Cosmology: Binary Phase Transition of the Universe from the smallest to the largest scales. *Hadronic Journal* **28**: 1-42.
2009 Back to the Ether. M.C. Duffy and J. Lévy (ed.s): *Ether space-time & cosmology*: 339-388. Montreal: Apeiron.
TSAGAS, G. T. and D.S. Sourlas
1993 *Mathematical Foundations of the Lie-Santilli Theory*. Kiev: Academy of Sciences of Ukraine.
UEXKÜLL, Jacob von
1909/1921 *Umwelt und Innenwelt der Tiere*. Berlin: J. Springer
VOUGIOUKLIS, T.
1993 *Hyperstructures and Their Representations*. Palm Harbor, USA: Hadronic Press.
WEISSTEIN, Eric W. and Dave Zobel
I *Russian Multiplication*. From *MathWorld*-- A Wolfram Web Resource.
http://mathworld.wolfram.com/RussianMultiplication.html
ZECKENDORF, Edouard
1972a Représentation des nombres naturels par une somme de nombres de Fibonacci ou de nombres de Lucas." *Bulletin de la Societe Royale des Sciences de Liege* **41**: 179-82.
1972b A Generalized Fibonacci Numeration. *The Fibonacci Quarterly* **10**: 365-72.

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# Irreversible gravitational collapse: black stars or black holes?

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**Abstract**

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It is well known that the concept of black hole has been considered very fascinating by scientists even before the introduction of Einstein's general relativity. They should be the final result of an irreversible gravitational collapse of very massive bodies.

However, an unsolved problem concerning such objects is the presence of a space-time singularity in their core. Such a problem was present starting by the first historical papers concerning black holes. It is a common opinion that this problem could be solved when a correct quantum gravity theory will be, finally, constructed.

In this work we review a way to remove black hole singularities at a classical level i.e. without arguments of quantum gravity. By using a particular non-linear electrodynamics Lagrangian, an exact solution of Einstein field equations is shown. The solution prevents the collapsing object to reach the gravitational radius, thus the final result becomes a black star, i.e. an astrophysical object where both of singularities and event horizons are removed. Such solution is not only a mathematical artifice. In fact, this kind of Lagrangian has been recently used in various analysis in astrophysics, like surface of neutron stars and pulsars. The authors also recently adapted the analysis on a cosmological context by showing that the big-bang singularity can be removed too.

**Keywords:** Black holes; singularity, nonlinear electrodynamics, extremely electromagnetic compact objects.

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This paper is dedicated to the Memory of Professor Darryl Jay Leiter, February 25, 1937 - March 4, 2011. Professor Leiter gave a fundamental contribution in evolving an alternate explanation of black holes, the theory of MECOs or magnetic eternally collapsing compact objects.

## 1 Introduction

The concept of black-hole (BH) has been considered very fascinating by scientists even before the introduction of general relativity (see [1] for an historical review). A BH is a region of space from which nothing, not even light, can escape out to infinity. It is the result of the deformation of spacetime caused by a very compact mass. Around a BH there is an undetectable surface which marks the point of no return. This surface is called an event horizon. It is called "black" because it absorbs all the light that hits on it, reflecting nothing, just like a perfect black body in thermodynamics [2]. However, an unsolved problem concerning such objects is the presence of a space-time singularity in their core. Such a problem was present starting by the first historical papers concerning BHs [3, 4, 5]. It is a common opinion that this problem could be solved when

a correct quantum gravity theory will be, finally, obtained, see [6] for recent developments in this direction.

On the other hand, fundamental issues which dominate the question about the existence or non-existence of BH horizons and singularities and some ways to avoid the development of BH singularities within the classical theory, which does not require the need for a quantum gravity theory, have been discussed by various authors in the literature, see references from [7] to [16]. In fact, by considering the exotic nature of BHs, it may be natural to question if such bizarre objects could indeed exist in nature or rather to suggest that they are merely pathological solutions to Einstein's equations. Einstein himself thought that BHs would not form, because he held that the angular momentum of collapsing particles would stabilize their motion at some radius [17].

Let us recall some historical notes. In 1915, A. Einstein developed his theory of general relativity [18]. A few months later, K. Schwarzschild gave the solution for the gravitational field of a point mass and a spherical mass [3]. A few months after Schwarzschild, J. Droste, a student of H. Lorentz, independently gave an apparently different solution for the point mass and wrote more extensively about its properties [19]. In such a work Droste also claimed that his solution was physically equivalent to the one by Schwarzschild. In the same year, 1917, H. Weyl re-obtained the same solution by Droste [20]. This solution had a peculiar behaviour at what is now called the Schwarzschild radius, where it became singular, meaning that some of the terms in the Einstein equations became infinite. The nature of this surface was not quite understood at the time, but Hilbert [21] claimed that the form by Droste and Weyl was preferable to that in [3] and ever since then the phrase "*Schwarzschild solution*" has been taken to mean the line-element which was found in [19, 20] rather than the original solution in [3].

For the sake of completeness we recall that, based on new translations of Schwarzschild's original work, there are researchers who invoke the non existence of BHs by claiming that the Schwarzschild's original work [3] gives a solution which is physically different from the one derived by Droste [19] and Weyl [20]. The new translations of Schwarzschild's original work can be found in ref. [22, 23]. These works commented on Schwarzschild's original paper [3]. In particular Abrams [22] claimed that the line-element (we use natural units in all this paper)

$$ds^2 = (1 - \frac{r_g}{r})dt^2 - r^2(\sin^2\theta d\varphi^2 + d\theta^2) - \frac{dr^2}{1 - \frac{r_g}{r}} \qquad (1)$$

i.e. the famous and fundamental solution to the Einstein field equations in a vacuum, gives rise to a space-time that is neither equivalent to Schwarzschild's original solution in [3]. In a following work [24] Abrams further claimed that "*Black Holes are The Legacy of Hilbert's Error*" as Hilbert's derivation used a wrong variable. Thus, Hilbert's assertion that the form of (1) was preferable to the original one in [3] should be misleading. Based on this, there are plenty of authors who agree with Abrams by claiming that the work of Hilbert was

wrong and Hilbert's mistake spawned the BHs and the community of theoretical physicists continues to elaborate on this falsehood, with a hostile shouting down of any and all voices challenging them, see for example references [23, 25]. In any case, this issue has been ultimately clarified in [26] where it has been shown that *"the original Schwarzschild solution"* [3] results physically equivalent to the solution (1) enabled like the correct one by Hilbert in [21], i.e. the solution that is universally known like the "Schwarzschild solution" [1]. The authors who claim that the original Schwarzschild solution leaves no room for the science fiction of the BHs (see references from [22] to [25]) give the wrong answer [26]. The misunderstanding is due to an erroneous interpretation of the different coordinates [26]. In fact, arches of circumference appear to follow the law $dl = rd\varphi$, if the origin of the coordinate system is a non-dimensional material point in the core of the BH, while they do not appear to follow such a law, but to be deformed by the presence of the mass of the central body $M$ if the origin of the coordinate system is the surface of the Schwarzschild sphere, see [26] for details.

After this clarification, let us return on historical notes. In 1924, A. Eddington showed that the singularity disappeared after a change of coordinates (Eddington coordinates [27]), although it took until 1933 for G. Lemaître to realize, in a series of lectures together with Einstein, that this meant the singularity at the Schwarzschild radius was an unphysical coordinate singularity [28].

In 1931, S. Chandrasekhar calculated that a non-rotating body of electron-degenerate matter above 1.44 solar masses (the Chandrasekhar limit) would collapse [5]. His arguments were opposed by many of his contemporaries like Eddington, Lev Landau and the same Einstein. In fact, a white dwarf slightly more massive than the Chandrasekhar limit will collapse into a neutron star which is itself stable because of the Pauli exclusion principle [1]. But in 1939, J. R. Oppenheimer and G. M. Volkoff predicted that neutron stars above approximately 1.5 - 3 solar masses (the famous Oppenheimer–Volkoff limit) would collapse into BHs for the reasons presented by Chandrasekhar, and concluded that no law of physics was likely to intervene and stop at least some stars from collapsing to BHs [29]. Oppenheimer and Volkoff interpreted the singularity at the boundary of the Schwarzschild radius as indicating that this was the boundary of a bubble in which time stopped. This is a valid point of view for external observers, but not for free-falling observers. Because of this property, the collapsed stars were called "*frozen stars*" [30] because an outside observer would see the surface of the star frozen in time at the instant where its collapse takes it inside the Schwarzschild radius. This is a known property of modern BHs, but it must be emphasized that the light from the surface of the frozen star becomes redshifted very fast, turning the BH black very quickly. Originally, many physicists did not accept the idea of time standing still at the Schwarzschild radius, and there was little interest in the subject for lots of time. But in 1958, D. Finkelstein, by re-analysing Eddington coordinates, identified the Schwarzschild surface $r = 2M$ (in *natural units*, i.e. $G = 1$, $c = 1$ and $\hbar = 1$, i.e where $r$ is the radius of the surface and $M$ is the mass of the BH) as an *event horizon*, "*a perfect unidirectional membrane: causal influences can*

*cross it in only one direction*" [31]. This extended Oppenheimer's results in order to include the point of view of free-falling observers. Finkelstein's solution extended the Schwarzschild solution for the future of observers falling into the BH. Another complete extension was found by M. Kruskal in 1960 [32].

These results generated a new interest on general relativity, which, together with BHs, became mainstream subjects of research within the Scientific Community. This process was endorsed by the discovery of pulsars in 1968 [33] which resulted to be rapidly rotating neutron stars. Until that time, neutron stars, like BHs, were regarded as just theoretical curiosities; but the discovery of pulsars showed their physical relevance and spurred a further interest in all types of compact objects that might be formed by gravitational collapse.

In this period more general BH solutions were found. In 1963, R. Kerr found the exact solution for a rotating BH [34]. Two years later E. T. Newman and A. Janis found the asymmetric solution for a BH which is both rotating and electrically charged [35]. Through the works by W. Israel, B. Carter and D. C. Robinson the no-hair theorem emerged [1], stating that a stationary BH solution is completely described by the three parameters of the Kerr–Newman metric; mass, angular momentum, and electric charge [1].

For a long time, it was suspected that the strange features of the BH solutions were pathological artefacts from the symmetry conditions imposed, and that the singularities would not appear in generic situations. This view was held in particular by Belinsky, Khalatnikov, and Lifshitz, who tried to prove that no singularities appear in generic solutions [1]. However, in the late sixties R. Penrose and S. Hawking used global techniques to prove that singularities are generic [1].

The term "*black hole*" was first publicly used by J. A. Wheeler during a lecture in 1967 [36] but the first appearing of the term, in 1964, is due to A. Ewing in a letter to the American Association for the Advancement of Science [37], verbatim: "*According to Einstein's general theory of relativity, as mass is added to a degenerate star a sudden collapse will take place and the intense gravitational field of the star will close in on itself. Such a star then forms a 'black hole' in the universe.*"

In any case, after Wheeler's use of the term, it was quickly adopted in general use.

Today, the majority of researchers in the field is persuaded that there is no obstacle to forming an event horizon. On the other hand, there are other researchers who demonstrated that various physical mechanisms can, in principle, remove both of event horizon and singularities during the gravitational collapse [7] - [16]. In particular, in [9] an exact solution of Einstein field equations which removes both the event horizon and singularity has been found by constructing the right-hand side of the field equations, i.e. the stress-energy tensor, through a non-linear electrodynamics Lagrangian which was previously used in studying super-strongly magnetized compact objects, in particular neutron stars and pulsars [38, 39]. In the next Section we will discuss this important issue.

## 2 Non-singular gravitational collapse

In Einstein's General Theory of Relativity the Einstein equation relates the curvature tensor of spacetime on the left hand side to the energy-momentum tensor in spacetime on the right hand side [1, 40]. Within the context of the Einstein equation the *strong principle of equivalence* (SPOE) requires that special relativity must hold locally for all of the laws of physics in all of spacetime as seen by time-like observers ([40] and Section 2.1 of [41]). Hence, in the context of the SPOE this implies that the frames of reference of co-moving observers within a gravitationally collapsing object are required to always be able to be connected to the frame of reference of stationary observers by special relativistic transformations with physical velocities which are less than the speed of light in a vacuum [40]. Recently plausible arguments have been made which support the idea that physically acceptable solutions to the Einstein equation will only be those which preserve the SPOE as *a law of nature* in the universe [7, 8, 16, 40]. The observable consequence of preserving the SPOE as a *a law of nature* would be that compact objects which emerge from the process of gravitation collapse could not have event horizons (EHs) because their existence would prevent co-moving observers within a gravitationally collapsing object from being able to be connected to the frame of reference of stationary observers by special relativistic transformations with physical velocities which are less than the speed of light [40]. Hence, as a result of the SPOE, objects having EHs with non-zero mass would be physically prohibited [7, 8, 16, 40]. In particular, the preservation of the SPOE in the Einstein equation would put an overall constraint on the nature of the non-gravitational physical elements which go into the energy-momentum tensor on the right hand side of the Einstein equation. However this constraint would not uniquely determine the specific form of the non-gravitational dynamics of the energy-momentum tensor [7, 8, 16, 40]. For this reason many different theories can be constructed (e.g. *eternally collapsing objects* (ECO), *magnetospheric eternally collapsing objects* (MECO), *nonlinear electrodynamics* (NLED) extremely compact *objects*, which preserve the SPOE and hence can generate highly redshifted compact objects without EHs [7, 8, 9, 16, 40]. Since each of these different SPOE preserving theories have unique observational predictions associated with the interaction of their non-gravitational components with the environment of their highly redshifted compact objects without EHs, the specific one chosen by Nature can only be determined by astrophysical observations which test these predictions [7, 8, 16, 40]. In the following, we will review the NLED model in [9].

NLED Lagrangian has been used in various analysis in astrophysics, like the surface of neutron stars [38] and pulsars [39], and also on cosmological context to remove the big-bang singularity [42, 43].

The effects arising from a NLED become quite important in super-strongly magnetized compact objects, such as pulsars and particular neutron stars [38, 39]. Some examples include the so-called magnetars and strange quark magnetars. In particular, NLED modifies in a fundamental basis the concept of gravitational redshift as compared to the well established method introduced

by standard treatments [38]. The analyses proved that, unlike using standard linear electrodynamics, where the gravitational redshift is independent of any background magnetic field, when a NLED is incorporated into the photon dynamics, an effective gravitational redshift appears, which happens to depend decidedly on the magnetic field pervading the pulsar. An analogous result has also been obtained for magnetars and strange quark magnetars [39]. The resulting gravitational redshift tends to infinity as the magnetic field grows larger [22, 23], as opposed to the predictions of standard analyses which involve linear electrodynamics. What is important here is that the gravitational redshift of neutron stars is connected to the mass–radius relation of the object [38, 39]. Thus, NLED effects turn out to be important as regard to the mass-radius relation, which is maximum for a BH.

Following this approach, in [9] a particular non singular exact solution of Einstein field equation has been found adapting to the BH case the cosmological analysis in [43]. In fact, the conditions concerning the early era of the Universe, when very high values of curvature, temperature and density were present [1, 9], and where matter should be identified with a primordial plasma [1, 9], are similar to the conditions concerning BH physics. This is exactly the motivation because various analyses on BHs can be applied to the Universe and vice versa [1, 9].

The model works on a homogeneous and isotropic star (a collapsing "*ball of dust*") supported against self-gravity entirely by radiation pressure. Let us consider the Heisenberg-Euler NLED Lagrangian [9, 42, 43]

$$\mathcal{L}_m \equiv -\frac{1}{4}F + c_1 F^2 + c_2 G^2, \tag{2}$$

where $G = \frac{1}{2}\eta_{\alpha\beta\mu\nu}F^{\alpha\beta}F^{\mu\nu}$, $F \equiv F_{\mu\nu}F^{\mu\nu}$ is the electromagnetic scalar and $c_1$ and $c_2$ are constants. Through an averaging on electric and magnetic fields [9, 42, 43], the Lagrangian (2) enables a modified radiation-dominated equation of state ($p$ and $\rho$ are the pressure and the density of the collapsing star)

$$p = \frac{1}{3}\rho - \rho_\gamma, \tag{3}$$

where *a quintessential density term* $\rho_\gamma = \frac{16}{3}c_1 B^4$ is present together with the standard term $\frac{1}{3}\rho$ [9, 42, 43]. $B$ is the strength of the magnetic field associated to $F$. The interior of the star is represented by the well-known Robertson–Walker line-element [1, 9]

$$ds^2 = -dt^2 + a(t)[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2)]. \tag{4}$$

Using $\sin\chi$ we choose the case of positive curvature, which is the only one of interest because it corresponds to a gas sphere whose dynamics begins at rest with a finite radius [1, 9]. Considering Eq. (2) together with the stress-energy tensor of a relativistic perfect fluid [1, 9, 42, 43]

$$T = \rho u \otimes u - pg, \tag{5}$$

where $u$ is the four-vector velocity of the matter and $g$ is the metric, the Einstein field equation gives the relation [9, 43]

$$t = \int_{a(0)}^{a(t)} dz(\frac{B_0^2}{6z^2} - \frac{8c_1 B_0^4}{6z^6} - 1)^{-\frac{1}{2}}, \tag{6}$$

being $B_0 = a^2 B$. The expression (6) is not singular for values of $c_1 > 0$ in Eq. (2) [9, 43]. In fact, the presence of the quintessential density term $\rho_\gamma$ permits to violate the reasonable energy condition [1] of the singularity theorems. By using elliptic functions of the first and second kind, one gets a parabolic trend for the scale factor near a minimum value $a_f$ in the final stages of gravitational collapse [9].

In concrete terms, by calling $l, m, n$ the solutions of the equation $8c_1 B_0^4 - B_0^2 x + 3x^3 = 0$, reads [9, 43]

$$\begin{aligned} t = [-(m-l)^{\frac{1}{2}} \beta_1(\arcsin\sqrt{\tfrac{z-l}{m-l}}, \sqrt{\tfrac{l-m}{l-n}}) \\ +n(m-l)^{-\frac{1}{2}} \beta_2(\arcsin\sqrt{\tfrac{z-l}{m-l}}, \sqrt{\tfrac{l-m}{l-n}})]|_{z=a^2(0)}^{z=a^2(t)}, \end{aligned} \tag{7}$$

where

$$\beta_1(x,y) \equiv \int_0^{\sin x} dz[(1-z^2)^{-1}(1-y^2z^2)^{-1}] \tag{8}$$

is the elliptic function of the first kind and

$$\beta_2(x,y) \equiv \int_0^{\sin x} dz[((1-z^2)^{-1})^{-\frac{1}{2}}((1-y^2z^2)^{-1})^{\frac{1}{2}}] \tag{9}$$

is the elliptic function of the second kind.

Then, recalling that the Schwarzschild radial coordinate, in the case of the BH geometry (4), is $r = a\sin\chi_0$ [1, 9], where $\chi_0$ is the radius of the surface in the coordinates (4), one gets a final radius of the star, if $B_0$ has an high strength [9]

$$r_f = a_f \sin\chi_0 > 2M \tag{10}$$

where $M$ is the mass of the collapsed star and $2M$ the gravitational radius in natural units [1, 9]. Thus, we find that the mass of the star generates a curved space-time without EHs.

## 3 Conclusion remarks

Black holes should be the final result of an irreversible gravitational collapse of very massive bodies. An unsolved problem, which was present starting by the first historical papers concerning black holes, is the presence of a space-time singularity in their core. It is a common opinion that this problem could be solved when a correct quantum gravity theory will be, finally, constructed.

In this paper we reviewed a way to remove black hole singularities at a classical level i.e. without arguments of quantum gravity. By using a particular non-linear electrodynamics Lagrangian, an exact solution of Einstein field equations has been shown. The solution prevents the collapsing object to reach the gravitational radius, thus the final result becomes an extreme electromagnetic compact object exhibiting an utterly extreme gravitational redshift $z \longrightarrow \infty$, i.e., a black star, that is nothing else than an astrophysical object where both singularities and event horizons were removed. Such solution is not a mathematical artifice. In fact, this kind of Lagrangian has been recently used in various analysis in astrophysics, like surface of neutron stars and pulsars. The authors also recently adapted the analysis on a cosmological context by showing that the big-bang singularity can be removed too [42].

Potential removal of BH horizons and singularities is an exciting and rapidly advancing field of research on theoretical, observational and experimental fronts. We take the chance to signal some recent results [44, 45].

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# References

[1] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation*, W. H. Feeman and Company (1973).

[2] P. C. W. Davies, Rep. Prog. Phys. 41, 1313–1355 (1978).

[3] K. Schwarzschild, Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.) 1916, 189-196 (1916).

[4] G. D. Birkhoff, *Relativity and Modern Physics.* Cambrigdge, MA: Harvard University Press. LCCN 23008297 (1923).

[5] S. Chandrasekhar, *The Highly Collapsed Configurations of a Stellar Mass*, Mont. Not. Roy. Astr. Soc. 91, 456–466 (1931).

[6] H. Nicolai, G. F. R. Ellis, A. Ashtekar and others, Gen. Rel. Grav. 41, 4, 673-1011, *Special Issue on quantum gravity* (April 2009).

[7] S. Robertson and D. Leiter, Mon. Roy. Astron. Soc. 50, 1391 (2004).

[8] R. Schild, D. Leiter and S. Robertson, Astron. J. 2, 420-432 (2006).

[9] C. Corda and H. J. Mosquera Cuesta, Mod. Phys. Lett A 25, 28, 2423-2429 (2010).

[10] A. Mitra, Phys. Rev. Lett. 81, 4774 (1998).

[11] A. Mitra, Phys. Rev. D, 74, 2, 024010 (2006).

[12] A. Mitra, Mon. Not. Roy. Astron. Soc. 369, 492 (2006).

[13] A. Mitra, Journ. Math. Phys. 50, 4, 042502-042503 (2009).

[14] J. P. S. Lemos, O.B. Zaslavskii, arXiv:1004.4651 (2010).

[15] A. G. Agnese and M. La Camera, Phys. Rev. D 31, 1280–1286 (1985).

[16] S. Robertson and D. Leiter, Astrophys. J. 596, L203-L206 (2003).

[17] E. Einstein, Ann. Math. 40, 4, 922–936 (1939).

[18] A. Einstein, *"Zur allgemeinen Relativitätstheorie"*, Sitzungsberichte der a Koniglich Preußischen Akademie der Wissenschaften 1915, 778-86 (1915).

[19] J. Droste, Proc. K. Ned. Akad. Wet. 19, 197 (1917).

[20] H. Weyl, Ann. Phys. (Leipzig) 54, 117 (1917).

[21] D. Hilbert, Nachr. Ges. Wiss. Gottingen, Math. Phys. Kl., 53 (1917).

[22] L. S. Abrams, Phys. Rev. D20, 2474 (1979), also in arXiv:gr-qc/0201044.

[23] S. Antoci and D.E. Liebscher, Gen. Rel. Grav. 35, 5, 945-950 (2003).

[24] L. S. Abrams, Can. J. Phys. 67, 919 (1989), also in arXiv:gr-qc/0102055.

[25] S. Antoci, in "Meteorological and Geophysical Fluid Dynamics (a book to commemorate the centenary of the birth of Hans Ertel)", W. Schroder Editor, Science Edition, Bremen, 2004 , also in arXiv:physics/0310104.

[26] C. Corda, arXiv:1010.6031, to be published in Electronic Journal of Theoretical Physics

[27] A. S. Eddington, Nature 113, 192 (1924).

[28] A. Einstein and G. Lemaître, TALKING OVER THE UNIVERSE AND ITS ORIGIN, Lectures in Pasadena, California, JWC-CX 1-13-33 [January 13, 1933] WC MET. (STAMPED JAN 14 1933).

[29] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. 55, 4, 374–381 (1939).

[30] R. Ruffini and J. A. Wheeler, Phys. Tod. 30–41 (1971).

[31] D. Finkelstein, Phys. Rev. 110, 965–967 (1958).

[32] M. Kruskal, Phys. Rev. 119, 1743–1745 (1960).

[33] A. Hewish, S. J. Bell and J. D. H Pilkington, Nature 217, 709–713 (1968).

[34] R. Kerr, Phys. Rev. Lett. 11, 237–238 (1963).

[35] E. T. Newman and A. Janis, Journ. Math. Phys. 6, 6, 915–917 (1965).

[36] S. Hawking, *A Brief History of Time,* Bantam Dell Publishing Group (1988).

[37] A. Ewing, Science News Letter of 18 January (1964).

[38] H. J. Mosquera Cuesta and J. M. Salim, Ap. J. 608, 925-929 (2004).

[39] H. J. Mosquera Cuesta and J. M. Salim, Mont. Not. Roy. Ast. Soc. 354, L55-L59 (2004).

[40] Private communication with Professor Darryl Jay Leiter.

[41] S. A. Wheeler and I. Ciufolini, *Gravitation and inertia,* Princeton University Press (1995).

[42] C. Corda and H. J. Mosquera Cuesta, *Astropart. Phys.* **34**, 587 (2011).

[43] V. A. De Lorenci, R. Klippert, M. Novello and J. M. Salim, *Phys. Rev. D* **65**, 063501 (2002).

[44] A. Mitra, Astrophys. Sp. Sci. Lett. 332, 43-48 (2011).

[45] S. Yi, JCAP 1101, 031 (2011).

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
Kathmandu University, Nepal, April (2011) pages 423-459

# NONUNITARY LIE-ISOTOPIC AND LIE-ADMISSIBLE SCATTERING THEORIES OF HADRONIC MECHANICS, I: Conceptual and Mathematical Foundations

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## Abstract

A nonunitary extension of the conventional, unitary scattering theory has been advocated by various authors as an effective way to incorporate nonpotential effects expected in dissipative nuclear events, deep mutual penetrations of the wavepackets of scattering particles, and other events. Nevertheless, these efforts had to be abandoned because of the violation of causality, lack of conservation of probabilities, and other problems emerging under nonunitary time evolutions. We show that the reformulation of a nonunitary scattering theory permitted by the isotopic branch of hadronic mechanics and its underlying Lie-isotopic theory, here presented under the name of *isoscattering theory,* reconstructs unitarity on iso-Hilbert spaces over isofields, a property known as *isounitarity,* thus resolving said problematic aspects, while having no divergencies *an initio,* and providing a significant broadening of the quantum scattering theory, although the Lie-isotopic theory is expected as being solely applicable to *reversible scattering events.* This first paper is devoted to the conceptual and mathematical foundations of the Lie-isotopic scattering theory, including the resolution of the inconsistencies of nonunitary theories. The physical foundations, the absence of divergencies from primitive axioms, and initial comparisons of the elaboration of measured quantities (cross sections, scattering angles, etc.) via the Lie and the Lie-isotopic scattering theories for reversible scattering events are studied in subsequent papers. Deep inelastic events are *irreversible over time,* thus requiring the further Lie-admissible broadening of the formalism studied in subsequent papers. ..

Key words scattering theories, nonunitary theories, isounitary theories
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TABLE OF CONTENT

1. Introduction

2. Basic Physical Assumptions
   2.1. Exterior and Interior Dynamical Problems.
   2.2. No Reduction Theorems.
   2.3. Insufficiencies of the Lorentz-Poincaré Symmetry.
   2.4. Insufficiencies of Quantum Mechanics.
   2.5. Restrictions for Irreversibility and Antimatter.

3. Elements of Isomathematics.
   3.1. Introduction.
   3.2. Isounits, Isoproducts and Isofields.
   3.3. Isofunctional Analysis.
   3.4. Isodifferential Calculus.
   3.5. Iso-Hilbert Spaces.
   3.6. Isolinearity, Isolocality and Isounitarity.
   3.7. Resolution of the Inconsistency Theorems.
   3.8. Delta Isofunction.
   3.9. Isospherical Isocoordinates.

3. Concluding Remarks

Acknowledgments

References

## 1. Introduction

As it is well known, the conventional, quantum mechanical, scattering theory (see, e.g., Ref. [1] and literature quoted therein), has permitted historical advances in the 20th century particle physics. Nevertheless, physics is a discipline that will never admit final theories because all theories are a mere approximation of the complexities of nature. No matter how accurate a given theory may be perceived, its broadening for a more accurate representation of nature is only a question of time.

In fact, numerous authoritative doubts on the final character of the conventional quantum scattering theory have been expressed, such as:

1) P. A. M. Dirac (see, e.g., Ref. [2]) expressed in 1981 serious concerns for the infinities in scattering theories and indicated the need for a revised theory avoiding divergences *ab initio,* rather than via *ad hoc* procedures of unknown physical origin;

2) B. Davies (see Ref. [3] and papers quoted therein) voiced in 1981 the need to extend the scattering theory into a nonunitary form so as to incorporate imaginary potentials as used in dissipative nuclear effects and other events;

3) W. Heisenberg (see the review in Ref. [5])voiced the need for a nonlinear extension of quantum mechanics, due to the known nonlinear character of nature;

4) Einstein, Podolsky and Rosen expressed their celebrated dout on the "lack of completion" of quantum mechanics (see later on for comments);

5) R. M. Santilli [4,5] suggested in 1978 the construction of a nonunitary covering of quantum mechanics under the name of *hadronic mechanics* in order to lift the quantum assumption of point-like particles into a form admitting a representation of the actual extended, thus generally nonspherical and deformable character of the wavepackets and/or charge distributions of particles, a representation of contact non-Hamiltonian interactions expected in deep overlapping of scattering particles, and other effects beyond the representational capabilities of quantum mechanics.

The above initial efforts subsequently resulted as being afflicted, in their original formulation, by fundamental inconsistencies. In essence, a theory along the above lines generally requires non-Hamiltonian effects (i.e., effects not representable with a Hermitean Hamiltonian), a feature causing the time evolution of the theory as being no longer unitary. In turn, the loss of unitarity implies: the loss of Hermiticity, thus observability, over time (an occurrence known as *Lopez lemma* [6]); the violation of causality; the lack of conservation of probabilities; the inability to predict the same numerical values under the same conditions at different times; and other basic problematic aspects known under the name of *Theorems of Catastrophic Inconsistencies of Nonunitary Theories* [6-11] (see also the review in Ref. [16a]).

A resolution of the above inconsistencies required the construction of a new mathematics, today known as *isomathematics,* based on the isotopic (i.e., axiom-preserving) lifting of the basic unit $\hbar = 1$ of quantum mechanics into the most general possible, positive-definite, integro-differential operator with an explicit dependence on

any desired local quantity $\hat{I}(t,r,p,\psi,...) = 1/\hat{T}(t,r,p,\psi,...) > 0$ known as *isounit;* its inverse $\hat{T}$ being known as the *isotopic element.* The isotopic lifting of the basic (left and right) unity $\hbar \to \hat{I}(t,r,p,\psi,...)$ then required corresponding compatible isotopies of the entire mathematics of quantum mechanics, including the isotopic lifting of fields, spaces, functional analysis, differential calculus, topology, geometries, algebras, groups, symmetries, representation theory, etc. [12-25].

While quantum systems are entirely represented by the sole knowledge of the Hamiltonian $H(r,p) = p^2/2m + V(r)$, the representation with hadronic mechanics of extended particles at short mutual distances requires the knowledge of *two* quantities, the usual Hamiltonian $H(r,p) = p^2/2m + V(r)$ for the representation of action-at-a-distance, potential interactions, plus the isounit $\hat{I}(t,r,p,\psi,...)$ for the representation of the actual size, shape and density of particles, their contact nonpotential interactions and other features beyond any hope of representation via a Hamiltonian. Note that, being an operator by assumption, the isounit does not commute with the Hamiltonian and, therefore, it is not generally a constant (although it is at times averaged into a constant).

By remembering that the unit is the basic invariant of any theory, the representation of non-Hamiltonian features and interactions via the isounit is the only form known to the authors permitting nonunitary theories to achieve the crucial invariance over time as possessed by the majestic axiomatic consistency of unitary quantum theories. The resolution of the remaining inconsistencies of early nonunitary theories was achieved via the reconstruction of unitarity over iso-Hilbert spaces over isofields, a property known as *isounitarity* (see the revioew below).

Mathematical maturity was achieved with: the discovery in 1993 that the conventional axioms of numerical fields admit basically new realizations of real, complex and quaternionic numbers with arbitrary (left and right, positive-definite) units, thus resulting in basically new numbers [12]; the discovery in 1995 of the dependence by the conventional differential calculus on the assumed basic unit with the consequential emergence of new calculi [13]; the isotopies in 1998 of the fundamental $SU(2)$ spin and isospin symmetries with consequential revision of Bell's inequality and all that [14]; and other advances identified later on. The achievement of physical maturity was then consequential, and so were numerous applications and experimental verifications (see monographs [15] of 1995, updates [16] of 2008, books [17-24] and vast literature quoted therein).

In these papers, we present the reformulation of nonunitary scattering theories permitted by the isotopic branch of hadronic mechanics that is based on the *Lie-Santilli isotheory* and related *isomathematics* [4,14-23]. Since all isounits assumed in these papers are Hermitean from their positive-definiteness, $\hat{I} = \hat{I}^\dagger > 0$, such a reformulation is primarily intended for *scattering processes that are reversible over time,* hereon called *isoscattering theory,* whose prefix "iso" is intended to indicate

the preservation of the abstract axioms of the quantum scattering theory and merely present a *broader realization.*

Hence, the reader should be aware from these introductory lines that a main effort of these initial papers on isotopies is that of *preserving the abstract axioms of special relativity, quantum mechanics and the conventional scattering theory,* and studying their broader realization permitted by the novel mathematics. The non triviality of these isotopic liftings will then be illustrated by showing that the scattering theory: 1) Resolves the inconsistencies of nonunitary theories; 2) Avoids divergences ab initial; and 3) Broadens the representational capability of the conventional scattering theory with the representation of conventional potential interactions represented by the conventional Hamiltonian $H$, plus nonpotential interactions represented by the isotonic $\hat{I}$ caused by the deep mutual penetration of particles as customary in high energy scattering events, and the direct geometric representation of the size, shape and density of the scattering region. The issue as to whether the numerical values characterized by the scattering theory are different than those characterized by the conventional theory for the same measured quantities, can only be addressed subsequently.

It should be stressed that *the extension of the formalism to irreversible processes requires a yet broader irreversible mathematics,* known as *Lie-admissible mathematics,* which is characterized by *two non-symmetric units,* $\hat{I}^{>},^{<}\hat{I}$ for motions forward and backward in time, respectively. In turn, such basic assumptions require a step-by-step Lie-admissible lifting of the entire isotopic formalism [4,15]. Due to its complexity, this broader formulation cannot possibly be presented in these first papers, and will be presented at some later time (see monographs [15] and the latest memoir [25].

Hence, *the reversible scattering theory presented in these papers is a mandatory intermediate step prior to the construction of the irreversible Lie-admissible scattering theory and related new mathematics* known under the names of *scattering theory* and *mathematics* where the prefix "geno" was suggested since the original proposal of 1978 [4.5] to indicate from its Greek meaning that, this time, the axioms of special relativity, quantum mechanics and the scattering theory are abandoned in view of their notorious reversible character (see next section) in favor of new, structurally broader, irreversible axioms.

It should be indicated that in these papers we present, apparently for the first time, the axiomatic foundations of the isoscattering theory, although the main elements of the new theory have been known for some time, but often ignored by physicists dealing with scattering processes to their peril. In fact, the following basic results have been available in the scientific literature for some time:

I) *Convergent perturbation theory.* Recall that quantum mechanics is based on the well known Lie product $[A, B] = AB - BA$ between generic matrices or operators $A, B$, while the isotopic branch of hadronic mechanics is based on the Lie-Santilli isoproduct $[A,\hat{T},B] = A\hat{T}B - B\hat{T}A$, first presented in Ref. [4] of 1978 and then studied

by the authors in various works (see the review in Refs. [15,16]), where T is the inverse of the isounit. It was then easy to see since the original proposal to build hadronic mechanics [5] that any divergent (or weakly convergent) canonical series $A(w) = A(0) + w(AH - HA)/1! + ... \to \infty$ can be turned into a strongly convergent form under the lifting $A(w) = A(0)+w(A\hat{T}H-H\hat{T}A)/1!+...$ for all isotopic element $\hat{T}$ sufficiently smaller (in absolute value) than $w$, a feature naturally verified by actual models, as we shall see. This feature was then studied by A. Jannussis and other authors (for brevity, see Chapter 11 of monograph [15b] for review and references).

II) *Conservation of probability.* As it is well known, the quantum $S$-matrix is unitary as a condition to preserve probabilities [1]. Hence, it was popularly believed that nonunitary theories violate the conservation of probabilities. The recovering of the conservation of probability under an isounitary reformulation of nonunitary theory was well established by 1995 [15b].

III) *Absence of divergencies.* Recall that divergencies in quantum scattering theories mainly originate from Dirac's delta function $\delta(x-x_0)$ since the latter is divergent at $x = x_0$ [1]. The absence of divergencies in the scattering theory of hadronic mechanics was identified in 1982 by Myung and Santilli [26] with the introduction of the isotopic covering of the Dirac delta function called by Nishioka [27] the *Dirac-Myung-Santilli isodelta function* and denoted $\hat{\delta}(x-x_0) = \delta[\hat{T}(x-x_o)]$ which, as one can see, removes the divergency of the delta function at $x = x_0$ under a judicious choice of the isotopic element $T$, as reviewed later on in Section 3.8.

IV) *Nonpotential scattering theory.* The extension of the quantum scattering theory to incorporate interactions not entirely represented with a Hamiltonian, as expected in deed inelastic scattering, was sufficiently voiced in the original proposal [5], and subsequently studied by R. Mignani [28] and others. Additional more recent studies on nonpotential scattering theory have been conducted by A. K. Aringazin et al [29] (again for brevity, see Chapter 12 of monograph [15b] for reviews and additional references).

V) *Inequivalence of Hamiltonian and non-Hamiltonian data elaborations.* It is popularly believed that, since cross sections, scattering angles and other quantities are measured, the numerical values produced by data elaborations via unitary scattering theories have a final experimental character. In reality, nature is not as simple as all of us tend to believe. Santilli showed in 1989 (see the review in Chapter 12 of monograph [15b]) that the *elaboration of measured quantities via quantum and hadronic scattering theories are generally inequivalent,* thus warranting serious comparative studies. This is due to the fact known since 1978 [5] that, if the Hamiltonian $H$ of a given scattering theory has the eigenvalue $E$, $H|\psi >= E|\psi >$, *the same Hamiltonian* $H$ *has a generaly different eigenvalue* $E'$ *for the isoscattering theory,* $HT|\psi' >= E'|\psi > \ E' \neq E$, trivially, in view of the general lack of commutativity between $H$ and $T$ (see Section 3.6 for details). Irrespective of all preceding aspects,

the latter occurrence, alone, warrants a reinspection of the conventional, reversible, Hamiltonian, unitary scattering theory.

The reader should be aware from these introductory lines of the existence of preliminary, yet rather vast experimental support of deviations from conventional Lie theories in virtually all quantitative sciences when dealing with the main assumption of the scattering theory, that is, extended particles and electromagnetic waves moving within physical media. Among these experimental data, we mention:

A) The need for contact non-Hamiltonian interactions to achieve an actual *attractive* force between the *identical* electrons in molecular valence couplings since, as expected to be known although rarely voiced, identical electrons repel each other according to quantum mechanics and chemistry [32];

B) Deviations from the geometry of spacetime have been, again preliminarily, yet directly measured in the experimental verification of the *isoredshift,* [31]. We are here referring to a shift toward the red of the frequency of light propagating within a transparent physical medium without any relative motion between the source, the medium and the detector, the shift being merely due to the loss of energy $E = h\nu$ by light to the medium due to inevitable interactions, with consequential evident reduction of frequency.

C) The elaboration of numerous particle physics experiments dealing with the hyperdense interior of hadrons, when elaborated without *ad hoc* parameters or arbitrary functions of unknown physical origin, show the clear presence of non-Hamiltonian effects [16d]. This is typically the case of the two-point amplitude of the Bose-Einstein correlation whose quantum fit of experimental data requires *four* arbitrary parameters (the so-called "chaoticity parameters"), while vacuum expectation values admit at best *two* parameters. These effects can be fully representable via a four-dimensional isounit of which the three space components represent the actual, very elongated shape of the proton-anti proton fireball, and the forth components represents its density, in remarkable agreement with experimental data [*loc. cit.*].

In any case, as part of the ongoing efforts to appraise the experimental claims based on the conventional scattering theory, a rather significant experimental effort is under way at this writing (Spring 2010) to repeat within physical media the historical experiments that have established the validity of special relativity, all done in vacuum, as well known. This significant experimental effort on the disciplines actually holding within physical media at large, and within the scattering region in particular, combined with the theoretical efforts herein considered, will eventually provide the necessary elements for the resolution of fundamental open issues in scattering experiments, of course, in due time.

Above all, the reader should keep in mind that special relativity and quantum mechanics are reversible theories, thus having manifest limitation for all energy releasing processes, due to their strict irreversibility. Therefore, the conception, quantitative

treatment and experimental verification of much needed new clean energies, such as the novel *Intermediate Controlled Nuclear Fusions* (ICNF) [37] are crucially dependent on the covering formulations treated in these papers. Their possible confirmation in particle accelerators via the covering isoscattering theory would then acquire a primary significant for the resolution of alarming environmental problems.

For additional historical data and a comprehensive literature in the field, interested colleagues may inspect Refs. [16], particularly the *General Bibliography of Hadronicx Mechanics* in Volume [16a].

In closing these introductory lines, we should recall that the conventional scattering theory achieved maturity only following decades of collegial studies presented in a large number of refereed publications. Consequently, it is hoped the reader is not expecting a final resolution of the scattering problem in these initial papers, but merely the *initiation* of the studies leading to a future collegial resolution following a predictable large number of additional papers.

## 2. Basic Physical Assumptions

**2.1. Exterior and Interior Dynamical Problems.** Until the earlier part of the 20th century, there was a clear distinction between (see Refs. [30] for technical characterizations):

1) *exterior dynamical problems,* referred to systems of point-particles and electromagnetic waves propagating in empty space; and

2) *interior dynamical problems,* referred to extended particles and electromagnetic waves propagating within physical media.

As a historical note, we recall that Schwartzschild wrote *two* papers, the first on the exterior gravitational problem containing his celebrated solution, and a second, virtually ignored paper on the interior gravitational problem (for review and references, see Ref. [15a]).

The primary difference between exterior and interior problems is that the former verify the integrability conditions for the existence of a Lagrangian or a Hamiltonian (the so-called *conditions of variational selfadjointness*), while the latter systems (called *variationally nonselfadjoint*) violate these conditions due to the presence of contact, nonconservative and nonpotential interactions, thus not being representable with Lagrangian or Hamiltonian mechanics [30].

With the advent of special and general relativities, the distinction between exterior and interior dynamical problems was eliminated via the reduction of interior problems to a finite number of point-particles that, as such, move in vacuum, thus recovering the conditions of exterior problems.

**2.2. No Reduction Theorems.** In the second half of the 20th century, it became

known that interior dynamical problems cannot be consistently reduced to exterior problems, an occurrence known under the name of *No reduction Theorems*, such as:

*NO REDUCTION THEOREM 2.1 [5,25]: A macroscopic system in nonconservative and irreversible conditions cannot be consistently reduced to a finite collection of point-like particles all in conservative and reversible conditions and, vice versa, the latter system cannot consistently reconstruct the former under the correspondence or other principles.*

A number of additional No Reduction Theorems were also proved based on the violation of thermodynamical laws due to the evident loss of entropy when passing from a real physical system to an ideal collection of point-particles moving in empty space all in conservative conditions, as necessary to verify special relativity, quantum mechanics and the conventional scattering theory.

An additional popular belief disproved by the No Reduction Theorems is that total conservation laws for an isolated system are solely verified by a system of particles in conservative conditions. In fact, it was proved in Ref. [30b] that, since they have no potential energy, nonconservative forces are in essence exchange forces, as a result of which they cancel each other when the system is isolated, resulting in the full verification of the conventional total conservation laws.



Figure 1: *A suggestive view from NASA of a spaceship during reentry in our atmosphere. Recent No reduction Theorems have established that the nonlinear, nonlocal-integral and nonpotential non-Hamiltonian forces experienced by the spaceship originate at the ultimate elementary level of nature, thus being also present in the interior of the scattering region at high energies.*

Yet another popular belief dispelled by the above No reduction Theorems is that

the nonlinear, nonlocal-integral and nonpotential forces of our macroscopic environment "disappear" in the reduction of an interior system to its elementary constituents. As a matter of fact, No Reduction Theorem 2.1 establishes that *the nonlinear, nonlocal and nonpotential forces experienced, for instance, by a spaceship during reentry in our atmosphere originate at the most primitive possible level, that of elementary particles,* and are evidently due to the interactions of the electron orbitals of the peripheral atoms constituting the spaceship with the electron orbitals of the resistive medium (see Figure 1).

The particular type of non-Hamiltonian interactions here referred to deals with the deep overlapping of the wave packets and/or the charge distribution of particles and are referred to as *nonlocal-integral interactions* (or merely nonlocal for brevity) in the sense that they occur over a surface or volume integral. As such, the nonlocal interactions at the basis of the scattering theory cannot be reduced, by conception, to a finite set of isolated points.

Note that we are including nonlocal interactions experienced by electrons, namely, by particles with a notorious *point-like charge.* Nevertheless, electrons do not have a "point-like wavepacket," thus experiencing indeed nonlinear, nonlocal and nonpotential interactions when in conditions of deep mutual penetration, as it is the case for valence electron coupling in molecular structures [32].

The studies reported in Refs. [30] have also established that the time evolution of systems with nonlinear, nonlocal and nonpotential interaction are necessarily noncanonical at the classical level and nonunitary at the operator level. We are now minimally equipped to formulate the following:

*ASSUMPTION 2.1: The scattering region is an interior dynamical system, thus characterized by a nonlinear (in the wavefunction), nonlocal (integral) and nonpotential (nonunitary) time evolution.*

Note that the No Reduction Theorems prohibit the exact reduction of the scattering region to a finite set of isolated points, which is considered a mere first approximation of a rather complex reality. The same theorems identify the evident need for covering formulations. Note finally that the No Reduction Theorems are not bypassed by the reduction of the scattering particles to point-like quarks, since elementary constituents with a point-like wavepacket do not exist.

**2.3. Insufficiencies of the Lorentz-Poincaré Symmetry.** The breaking of the *Lorentz-Poincaré (LP) symmetry* for interior dynamical problems at large, and particularly for the interior of the scattering region, is rather plausible and should be studied seriously because no scattering theory can claim final results until the basic spacetime symmetry is established beyond scientific doubt. Among a number of

symmetry breaking aspects, we quote [15,16]:

1) The axiomatic foundations of the Lorentz-Poincaré symmetry requires the equivalence of all inertial reference frames. This feature is certainly valid in empty space, but it is unresolved for the interior of the scattering region because of the impossibility of even defining inertial reference frames in *interior* conditions. Inertial reference frames are indeed used in quantum scattering theories, but they constitute an *exterior* treatment, thus reducing an interior to an exterior problem. Additionally, in vacuum there is no known experimental way to detect a privileged reference frame, as well known (Michelson-Morley experiment). By contrast, the sole reference frame that can be consistently defined for the scattering region is the privileged reference frame at rest with the interior region itself, since other frames would require motion of a hypothetical observer within a hyperdense medium.

2) The Lorentz-Poincaré symmetry is exactly valid for *Keplerian systems,* that is, systems of point-particles moving in vacuum around a heavier particle known as the *Keplerian nucleus.* By contract, *the scattering region has no Keplerian nucleus.* This aspect alone may cause a breaking of the Lorentz-Poincaré symmetry.

3) There are serious reasons to expect that the historical experiments that have established the validity of special relativity in vacuum are invalid in interior conditions [31]. For instance, it is easy to see that., in the event the known Fizeau experiment is repeated entirely underwater, there are contributions to the travel of light in water outside the traditional pipes with opposite water velocities that violate Lorentz-transformations. By contrast, the repetition of the Michelson-Morley experiment under complete underwater conditions is expected to retain the original result, this time confirming the constancy of the speed of light with respect to the privileged reference frame at rest with the water, by therefore no longer confirming the Lorentz symmetry. Needless to say, a problem of such a fundamental character cannot be resolved in a few sentences one way or another, and requires the systematic repetition in interior conditions of all historical experiments that have established the validity of the Lorentz-Poincaré symmetry in vacuum [31].

The reader should be aware that a rather vast effort has been conducted over decades for the construction of a covering spacetime symmetry applicable to interior problems at large, and the scattering region in particular. These efforts required first the construction of the covering *Lie-Santilli isotheory* [4,15,16,18-34] capable of reducing nonlinear, nonlocal and noncanonical (or nonunitary) interior problems to equivalent *isolinear, isolocal and isocanonical (or isounitary) forms* (see next section for details). Only following the achievement of the Lie-Santilli isotheory, the efforts could be concentrated in the construction of a covering of the Lorentz-Poincaré symmetry applicable to interior conditions [33-43] which is known today as the *Lorentz-Poincaré-Santilli (LPS) isosymmetry.* We reach in this way the following:

*ASSUMPTION 2.2: The scattering region is characterized by the Lorentz-Poincarś-Santilli isosymmetry.*

The noninitiated reader should know that, by conception and construction, *the LPS isosymmetry is locally isomorphic to the conventional LP symmetry.* This feature may have deep implications for the scattering problem because, in the final analysis, it may imply that the data elaboration of existing high energy experiments with the conventional and the isotopic scattering theory yields the same numerical value. Rather than being a drawback, if established by future collegial works, this possible outcome alone warrants this study, e.g., because it would established the validity of special relativity for interior conditions nowadays considered inapplicable.

In any case, as shown in paper III of this series, even assuming that the elaboration of past experiments via the conventional and isotopic scattering theory yields the same numerical results, the broader representational capabilities of the isotopic theory are beyond doubt, thus offering the possible prediction and representation of scattering events beyond the capability of the conventional theory; It is only hoped the reader does not expect the final resolutions of these complex issues in these initial papers.

**2.4. Insufficiencies of Quantum Mechanics.** Following the historical successes of quantum mechanics for the structure of the hydrogen atoms and numerous other systems, quantum mechanics has been applied to all possible particle conditions existing in the universe, thus including interior conditions, as typically occurring in the scattering region as well as in the structure of hadrons, nuclei and stars.

Despite the achievement of historical results, serious doubts have emerged in regard to the *exact* character of quantum mechanics for *interior* problems, such as [15,16]:

1) Quantum mechanics has permitted the achievement of a numerically *exact* representation from first principles of *all* experimental data of exterior dynamical problems, By contrast, when passing to interior problems, quantum mechanics has only permitted an *approximate* representation of experimental data, an occurrence that, per se, is a direct indication of the merely approximate character of quantum mechanics for interior conditions. For instance, quantum mechanics has provided an exact representation of the structure of the hydrogen atoms, while it misses 2% of the binding energy of the hydrogen molecule from unadulterated quantum principles [32]. In nuclear physics, quantum mechanics misses an exact representation of the simplest possible nucleus, the deutneriom, since there are insufficiencies in the representation of its spin, magnetic moment, stability and other features, with dramatic insufficiencies for heavy nuclei such as the zirconium [16].

2) The No Reduction Theorems establish that the nonlinear, nonlocal and nonpotential character of our macroscopic systems originate at the ultimate level of elemen-

tary particles, thus requiring a covering of quantum mechanics. As an example, the approximate character of quantum mechanics for the hydrogen molecule originates from the conditions of deep mutual penetration of the wavepackets of the two valence electrons by characterizing interactions dramatically beyond a possible representation by quantum mechanics. Similar occurrences hold for other interior problems.

3) The representation of interior conditions via quantum mechanics is generally done with the use of completely arbitrary parameters or functions of unknown physical origin that are fitted from the data, and quantum mechanics is then claimed as being exactly valid. As an example, an exact representation of the binding energy of the hydrogen molecule is achieved via the so-called "screened Coulomb potentials," that is, the multiplication of the Coulomb potential by an arbitrary function such as $V(r) = f(r)e^2/r$, and then the fitting of the arbitrary function ($f(r)$ from the experimental data. However, it is known that "screened Coulomb potentials" do not admit quantized levels and, therefore, the very name "quantum chemistry" becomes questionable [32]. In particle physics, the use of *ad hoc* parameters and functions for interior conditions has reached at time paradoxical characters. For instance, the experimental data of the two-points function of the Bose-Einstein correlation are fitted via the use of *four* arbitrary parameters (called the "chaoticity parameters") and then the claim that relativistic quantum mechanics is exact. However, the quantum axioms for the expectation value of a two-dimensional Hermitean operator may admit, under debatable assumptions, a maximum of *two* arbitrary parameters, the use of four parameters being excluded by the very axioms of quantum mechanics [16a]. A deeper inspection has shown that the missing two parameters must originate from *off-diagonal elements* in the vacuum expectation values thus casting shadow on the consistent representation of observables.

In view of the above and numerous other insufficiencies [16a,24], a vast effort has been conducted by numerous scientists over decades for the construction of a nonlinear, nonlocal and nonpotential covering of classical and quantum mechanics known under the name of *hadronic mechanics* with the following main results [15,16,32,33]:

A) The construction of the so-called *iso-, geno-, and hyper mathematics* for the representation of variationally nonselfadjoint interior systems of *matter* that are single-valued reversible, single valued irreversible, and multi-valued irreversible, respectively, and their isoduals for *antimatter,* these new mathematics being characterized by different generalized units as outlined in Section 3;

B) The construction of corresponding new classical mechanics, known as *iso-, geno- and hyper-Lagrangian or Hamiltonian mechanics for matter,* and their isoduals for *antimatter,* achieving the representation of interior dynamical systems via an action principle, as outlined in paper II; and

C) The *isotopic, genotopic and hyperstructural branches of hadronic mechanics* for the operator representation of the above identified interior systems of *matter,*

and their isoduals for *antimatter*, possessing progressively increasing complexity and methodological needs, as also outlined in paper II.

The above formulations have indeed allowed exact representations of interior problems from unadulterated first axioms, such as an exact representation of the binding energy and other features of the hydrogen molecule from first principles without arbitrary functions [32], an exact representation of the experimental data of the Bose-Einstein correlation from first principles without arbitrary parameters, and other interior problems in classical physics, particle physics, nuclear physics, superconductivity, chemistry, astrophysics and cosmology (see Vol. [16d] and Chapter 5 of Ref. [24] for a review).

We are now equipped to formulate the following:

*ASSUMPTION 2.3: Quantum mechanics is assumed as being exactly valid everywhere in the exterior of the scattering region, while the covering hadronic mechanics is assumed as being exactly valid in the interior region.*

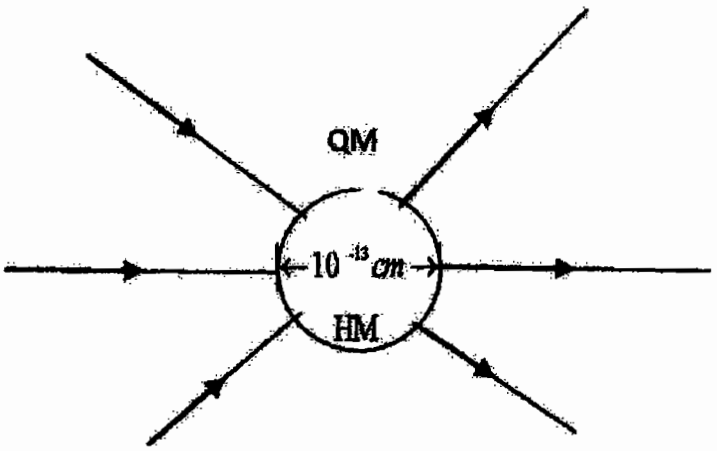



Figure 2: *A schematic view of the main assumptions of these papers, the validity of conventional quantum mechanics everywhere in exterior conditions, and the validity of the covering hadronic mechanics for interior conditions.*

The smooth transition from the interior (hadronic mechanics) to the exterior (quantum mechanics) is simply achieved via realizations of the generalzied unit of the type

$$Lim_{r>1fm}\hat{I}(t,t,r,p,\psi,...) = \hbar. \tag{2.1}$$

As we shall see in paper II, the above condition is quite naturally verified by all meaningful realizations of the generalized unit.

In view of the general inequivalence of $\hat{I}(t,r,p,\psi,...)$ and $I$, the evident lack of general commutativity of $\hat{I}(t,r,p,\psi,...)$ and $H(r,p)$, and other aspects, the isoscattering theory requires a reinspection of the data elaboration of experimental data achieved with the conventional scattering theory to ascertain whether said data elaborations persist under nonlinear, nonlocal and nonpotential internal effects, or the final numerical values themselves need a revision.

**2.5. Restrictions for Irreversibility and Antimatter.** Recall that the formalism of the covering scattering theory includes that of quantum mechanics, plus three covering formalisms of hadronic mechanics with progressively increasing complexity, and all their isoduals for antimatter. To avoid the initiation of the study with excessive complexities, in these three papers we shall restrict our formulations to *isomathematics and isomechanics,* resulting in the suggested name of *isoscattering theory,* where the reader should keep in mind that the prefix "iso" indicates the preservation of the axioms of the conventional theory, and merely the use of *broader realizations.*

This restriction implies that, by conception and construction, *the isoscattering theory does not generally represent irreversible processes,* except under certain conditions, as we shall see, such as isounits that are Hermitean but time noninvariant

$$\hat{I}(t,r,p,\psi,...) \neq \hat{I}(-t,r,p,\psi,...). \tag{2.2}$$

In other words, we shall essentially study scattering processes in the way they are treated by quantum mechanics, without a quantitative representation of their irreversibility, and shall address the latter issue in a subsequent paper based on *Lie-admissible genomathematics and genomechanics* [25]. In any case, the construction of the Lie-isotopic isoscattering theory is a recommendable pre-requisite for the much broader *Lie-admissible irreversible genoscattering theory.*

Additionally, *the isoscattering theory of these first papers does not include antiparticles* also to avoid excessive complexities at start up. This additional restriction is due to recent advances in antimatter that have achieved full scientific democracy between matter and antimatter at all levels of study, from Newtonian mechanics to second quantization, thus ending the scientific imbalance of the 20th century of treating antimatter at the sole quantum or quantum field theoretical levels [33].

These advances have been stimulated by E. C. G. Stueckelberg conception of *antimatter with a negative time,* but the achievement of consistency required the use of a conjugation of all physical and mathematical quantities, thus leading to *negative time, energy, and other physical quantities referred to corresponding negative units,* that are as causal as conventional positive time, energy and other physical quantities referred to corresponding positive units.

The treatment of this new setting required the construction of the new *isodual mathematics* that is anti-isomorphic to conventional mathematics in all its parts and

operations. In turn, these advances have identified a new symmetry, called *isoselfduality,* essentially given by invariance under anti-Hermiticity trivially verified by the imaginary unit $i = -i^\dagger$, but less trivially verified by the Dirac equation and related gamma matrices (see Ref. [33] for details)

$$\gamma_\mu \equiv -\gamma_\mu^\dagger, \tag{2.3}$$

and other cases.

Physically, isoselfduality has emerged as representing systems of particles and their anti[particles, thus permitting a new interpretation of the Dirac equation as providing a direct *quantum* representation of an electron and its antiparticle (the positron) without any need for the "hole theory," since the the isodual theory applies at the *classical,* let alone purely quantum level, where it reaches equivalence with the conventional charge conjugation [33].

Therefore, the inclusion of antiparticles in our study of scattering processes requires a reinspection of the very structure of the *conventional* Feynman's diagrams so as to achieve a full democracy of treatment between particles and antiparticles, thus suggesting a separate treatment to avoid excessive complexities at start up.

It should be noted, as we shall see in paper II., that *the invariance under isoselfduality is generally violated by quantum scattering treatments inclusive of particles and antiparticles.* This occurrence alone mandates a reinspection *ab initio* of scattering theories in general, let alone when including particles and antiparticles.

In these papers, we shall use the terms "quantum mathematics," "quantum scattering theory," etc. to denote aspects pertaining to quantum mechanics and use the terms "hadronic mathematics," "hadronic scattering theory," etc. to denote their corresponding coverings as characterized by hadronic mechanics.

A number of divergent terminologies exist in the literature of this paper as compared to that of the quantum scattering theory. For instance, the term "potential" is used in the literature of hadronic mechanics as a synonym of "Hamiltonian" or, more technically, referring to the verification of all integrability conditions for the existence of a Hamiltonian [30], while systems of that class are not necessarily called "potential" in the quantum literature.

This is the case for the interaction term $H_1 = J * A$ that is generally considered as being of nonpotential character in the quantum literature, while it verifies the conditions of variational self-adjointness (see monographs [30]), thus being of a true potential for the hadronic literature, as confirmed in any case by the fact that said interaction term is fully "Hamiltonian" and *additive* to the kinetic term and other potentials, e. g., $H = H_0 + H_1$.

By comparison, the terms "nonpotential" is used in the hadronic literature to stress the impossibility of representing the novel "nonpotential" interactions with a

Hamiltonian, technically referring to the *violation of the conditions of variational self-adjointness in the frame of the experimenter,* thus requiring new vistas.

# 3. Elements of Santilli Isomathematics.

**3.1. Introduction.** As indicated in Sections 1 and 2, numerous aspects warrant the broadening of the scattering theory to incorporate non-Hamiltonian effects, that is, effects that cannot be represented via the conventional Hamiltonian. Any meaningful broadening of the conventional scattering theory requires the existing from the class of unitary equivalence of quantum mechanics. However, the ensuring nonunitary theories are afflicted by a litany of problems known under the name of *Theorems of Catastrophic Inconsistencies of Nonunitary Theories* [6-12]. Consequently, the central objective of this section is to *identify an equivalent formulation of nonunitary theories resolving the inconsistency problems.*

Following decades of research, the solution of the above problem required the construction by various authors of a *new mathematics,* known as *isomathgematics,* originally proposed by Santilli [4] in 1978, subsequently studied by the same author in disparate works, as well as by numerous pure and applied mathematicians, including (in chronological order of contributions) R. M. Santilli, S. Okubo, H. C. Myung, M. L. Tomber, Gr. T. Tsagas, D. S. Sourlas, J. V. Kadeisvili, A. K. Aringazin, A. Kirhukin, R. H. Ohemke, G. F. Wene, G. M. Benkart, J. M. Osborn, D. J. Britten, J. Lohmus, E. Paal, L. Sorgsepp, D. B. Lin, J. V. Voujouklis, P. Broadbridge, P. R. Chernooff, J. Sniatycku, S. Guiasu, E. Prugovecki, A. A. Sagle, C.-X. Jiang, R. M. Falcon Ganfornina, J. Nunez Valdes, A. Davvaz, and others (see the comprehesnive bibliography at the end of Ref. [16a]). To illustrate the complexity of the problems to be addressed, following the original proposal of 1978, initial mathematical maturity was solely achieved in memoir [13] of 1996, thus warranting this review and specialization to the scattering region so as to avoid possible insidious misinterpretations.

For the benefit of experimentalists we indicated that, as a result of the above efforts, the new mathematics can be constructed via the systematic application of *axiom-preserving liftings,* called *isotopies,* of the *totality* of the mathematics of quantum mechanics, including all its operators and all its operations, thus including the isotopic lifting of numbers, functional analysis, differential calculus, geometries, topologies, Lie theory, symmetries, etc. [13,15,16]. As we shall see in paper II, said isotopies can be very easily constructed via the application of nonunitary transforms to the totality of the formalism of the conventiona;l scattering theory, thus being indeed accessible to experimentalists.

The physical needs for isomathematics have been indicated in Sections 1 and 2, and consists in the necessity for a representation of non-Hamiltonian scattering effects in a form that is *invariant over time* so as to admit the sam,e numerical predictions

under the same conditions at different times. Following the study of all possible alternatives, the latter condition required the representation of non-Hamiltonian scattering effects with an axiom-preserving generalization of the trivial (positive-definite) unit of quantum mechanics $\hbar = 1$ into the most general possible (positive-definite as a condition to characterize an isotopy), integro-differential operator $\hat{I}$. Since the unit is the fundamental (left and right) invariant of any theory, whether conventional or generalized, the representation of non-Hamiltonian effects via the isounit has indeed achieved the desired time invariant representation.

However, the assumption of a generalized unit has requested the compatible reconstruction of the entire mathematics used in quantum mechanics with no exception known to the authors. In fact, the sole elaboration of the isoscattering theory, e.g., with conventional trigonometric functions, activates the Theorems of Catastrophic Inconsistencies because it would be the same as elaborating the conventional scattering theory, e.g., with isotrigonometric functions.

Since no formulation of isomathematics specialized intended for scattering problems has been presented to date, it is important to outline it in this first paper for minimal self-sufficiency of the presentation, as well as to minimize possible insidious misinterpretations that may be caused by insufficient technical knowledge of the field. In this section we shall outline the rudiments of isomathematics for a positive-definite but otherwise arbitrary isounit $\hat{I}$ and show the resolution of the inconsistency problem under isotopies.

We should also indicate the distinction between *deformations* and *isotopies*. The former are alterations of conventional quantum formulations defined over conventional fields, thus being catastrophically inconsistent on mathematical and physical grounds (see Refs. [6-11] for brevity), while the latter can be characterized as deformations defined over isofields, thus avoiding the inconsistency theorems.

Note that isofields were introduced in 1993 [12]. Consequently, the contemporary formulation of deformations coincide with previously proposed isotopies, as it is the case for the *isotopies of the Lorentz symmetry* first proposed by Santilli in 1983 [34], at that time, over conventional fields, and subsequently reintroduced identically, even in the symbols and terms, as deformations, unfortunately, without the quotation of the original derivation [34]. Similar occurrence hold for other deformations 9see Ref. [15a] for brevity).

In these papers, conventional terms, such as numbers, spaces, etc. are referred to conventional notions of quantum mathematics. The corresponding notions of hadronic mathematics are indicated isonumbers, isospaces, etc. We regret a perhaps excessive use of the prefix "iso," but it appears recommendable in a first presentation of *applied mathematics* to prevent insidious inconsistencies.

Within the context of *pure mathematics,* we shall show that *both the conventional and the isotopic mathematics can be presented with the same symbols and operations,*

since they coincide at the abstract level by conception and construction. However, the latter formulation requires, in any case, an in depth knowledge of the isotopic realization of conventional abstract axioms, thus warranting again the use of the prefix "iso" in this first presentation, with the understanding that pure mathematicians may subsequently achieve the necessary mathematical rigor.

It is at times indicated that, due to the above abstract identity, isomathematics is trivial, a view perhaps correct. but only following its discovery. However, the implications solely permitted by isomathematics. such as the extension of Lie's theory, the Lorentz-Poincaré symmetry and Einstein's axioms for the treatment of nonlinear, nonlocal and non-Hamiltonian systems, are far from being trivial.

**3.2. Isounits, Isoproducts and Isofields.** As indicated earlier, isomathematics is based on the following isotopic, thus axiom-preserving lifting of the trivial unit into the most general possible positive-definite integro-differential operator

$$\hbar = 1 > 0 \to \hat{I}(t, r, p, E, \xi, \omega, \psi, \partial\psi, ...) = 1/\hat{T}(t, r, p, E, \xi, \omega, \psi, \partial\psi, ...) > 0. \quad (3.1)$$

first introduced in 1978 [4,5] and known as *Santilli isounit,* while $\hat{T}$ is known as the *isotopic element.* We shall use the notation $T$ when the isotopic element is projected on quantum spaces, but keep the notation $\hat{I}$ to avoid confusion with $I$.

The isotopic lifting of the (multiplicative) unit evidently requires a corresponding compatible lifting of *all* multiplications between arbitrary quantities A, B, from the simple associative form used in quantum mechanics, herein denoted $AB = A \times B$, to the new form first introduced by Santilli in Ref. [4] of 1978

$$AB = A \times B \to A\hat{\times}B = A \times T \times B, \quad (3.2)$$

which is also isotopic, because it verifies the associativity law of the original product. It is easy to see that, under lifting (3.2), $\hat{I}$ is indeed the correct left and right unit of the theory, $\hat{I}\hat{\times}\hat{A} = A\hat{\times}\hat{I} = A$ for all elements $A$ of the set considered.

Fundamental assumptions (3.1) and (3.2) have permitted the isotopic lifting of numerical fields $F(a, \times, I)$, such as the field of real numbers $R(n, \times, I)$, complex numbers $C(c, \times, I)$ and quaternions $Q(q, \times, I)$ into the *Santilli isofields* $\hat{F}(\hat{a}, \hat{\times}, \hat{I})$ [12], consisting of the original numbers $a = n, c, q$ lifted into the form of *Santilli isonumbers* $\hat{n} = n \times \hat{I}$ equipped with isounit (3.1) and isoproduct (3.2), $\hat{n}_1\hat{\times}\hat{n}_2 = (n_1 \times n_2) \times \hat{I}$, as well as with the *conventional* sum $\hat{n}_1\hat{+}\hat{n}_2 = \hat{n}_1 + \hat{n}_2$ and related conventional additive unit 0, $\hat{n} + \hat{0} = \hat{0} + \hat{n} = \hat{n}$, i.e., $\hat{0} = 0 \times \hat{I} \equiv 0$.

To avoid inconsistencies, it should be stressed that *all* operations with numbers have to be lifted in an isotopic form we cannot possibly review here (see [15]). We merely mention for use in the isoscattering theory the *isodivision* given by $\hat{/} = / \times \hat{I}$ so

that we have simplifications in isomultiplications of the type $(a\hat{/}b)\hat{\times}(c\hat{/}d) = [(a/b) \times (c/d) \times \hat{I}$.

Also, and very importantly, conventional numbers expressing numerical values of physical quantities such as coordinates $r.$, momenta $p$, energy $E$, etc. have no meaning for isomathematics and must be lifted into the isotopic form $\hat{r} = r \times \hat{I}, \hat{p} = p \times \hat{I}, \hat{E} = E \times \hat{I}$, etc. as a necessary condition to be elements of a Santilli isofield, that is, to be *isoscalars.*

Readers should, however, be reassured that conventional numbers, as needed for experiments, are indeed recovered by the isoscattering theories. As an example, the (right, modular, associative) eigenvalue expression $E \times |\psi >$ becomes for isomathematics $\hat{E}\hat{\times}|\hat{\psi} >$ that can be simplified in the form $E \times \hat{I} \times \hat{T} \times |\hat{\psi} >= E \times |\hat{\psi} >$, thus recovering the conventional real number $E$ needed for measurements.

It should be indicated that *isofields are isomorphic to ordinary fields, by conception and construction,* a property necessary for the consistent application of the isoscattering theory to experimental measurements. In fact, Santilli merely provided in Ref. [12] a *broader realization* of the conventional field axioms. The nontriviality of the realization is indicated by the fact that *the isounit of a Santilli isofield* $\hat{F}(\hat{a}, \hat{\times}, \hat{I})$ *is generally outside the original field* $F(a, \times, I)$. In this case, $\hat{F}(\hat{a}, \hat{\times}, \hat{I})$ are called *isofields of the first type.* When $\hat{I} \in F$, we have *isofields of the second type.*

Despite the simplicity of the isonumber theory, readers should be warned against predictable perceptions of triviality because, for instance, under the assumption of the isounit $\hat{I} == 3$, thus dealing with isofields of the second type, we have "$2 \times 3$"$= 18$ and the *number* 4 *becomes a prime number.*

For in depth knowledge of Santilli isofield theory and its intriguing implications, interested readers are suggested to study the original paper [12], Ref. [15a] and Jiang's monograph [22].

**3.3. Isofunctional Analysis.** Any elaboration of the isoscattering theory with conventional functions, such as sine, cosine, exponential, etc. leads to inconsistencies [6-11,15]. Even though not clearly indicated in the mathematical literature, all functions crucially depend on the assumed basic unit and multiplication. Therefore, liftings (3.1) and (3.2) have required the laborious reconstruction of functional analysis into a form compatible with the basic axioms of isomathematics.

Studies on the *isofunctional analysis* were initiated by Santilli [4] and continued by Myung and Santilli [26], Kadeisvili [21], Nishioka [27] Aringazin [29] and others (see the general bibliography of Ref. [16a] for a comprehensive listing). A presentation of isofunctional analysis sufficient for the isoscattering theory is available in monograph [15a]. For completeness we recall the following notions:

3.3.1) *Isopowers,*

$$\hat{a}^{\hat{n}} = \hat{a}\hat{\times}\hat{a}\hat{\times}...\hat{a} = (a^n) \times \hat{I}, \tag{3.3}$$

for which $\hat{I}^{\hat{n}} = \hat{I}$;

3.3.2) *Isoexponentiation,*

$$\hat{e}^{a} = \hat{I} + a\hat{/}\hat{1}! + a\hat{\times}a\hat{/}\hat{2}! + ... = (e^{a\times T}) \times \hat{I} = \hat{I} \times (e^{T\times a}), \tag{3.4}$$

where one should note the emergence of the integro-differential quantity $T$ in the *exponent;*

3.3.3) *Isologarithm,*

$$\hat{log}_{\hat{e}}\hat{a} = \hat{I} \times log_{\hat{e}}a, \tag{3.5}$$

which expression is indeed the inverse of the isoexponentiation, as one can verify, as well as yields a correct isonumber for result;

3.3.4) *Isotrigonometric functions* (for isospherical coordinates see later on Section 3.8),

$$\hat{sin}\hat{\theta} = T_{\theta} \times sin(\theta \times \hat{I}_{\theta}), \tag{3.6a}$$

$$\hat{cos}\hat{\phi} = T_{\phi} \times cos(\phi \times \hat{I}_{\phi}, \tag{3.6b}$$

where evidently the isounits for angles are generally different than those for space.

Note that the use of conventional angles would have no sense for the isoscattering theory because all numbers must be isonumbers for consistency. We shall identify later on specific realizations of the various isounits.

A rather intriguing and unexpected feature of isotopies is that of preserving on isospaces over isofields the *numerical values* of the quantities prior to lifting. This feature has been crucial for the reconstruction of the exact light cone and special relativity on isospace over isofield when light becomes a local variables, thus requiring in conventional spaces deformed light cones.

According to this feature, the isoscattering theory is expected to preserve the numerical value of the angles $\theta$ and $\phi$ as measured in experiments. However, the preservation is for the new isoangles $\hat{\theta}$ and $\hat{\phi}$. Consequently, the correct identification is

$$\theta = \hat{\theta} = \theta' \times \hat{I}_{\theta}, \quad \phi = \hat{\phi} = \phi' \times \hat{I}_{\phi}. \tag{3.7}$$

The above rules indicate the expected differences in the elaboration of experiments via the scattering and isoscattering theories.

3.3.5) *Isomatrices*, given by conventional matrices whose elements are isoscalars, such as for the diagonal case

$$\hat{M} = Diag.(\hat{a}_1, \hat{a}_2, ..., \hat{a}_n), \tag{3.8}$$

where $\hat{a}_k = a_k \times \hat{I}$;

3.3.6) *Isodeterminant,*

$$\hat{Det}\hat{M} = [Det(\hat{M} \times \hat{T}) \times \hat{I}, \tag{3.9}$$

where one should note that $\hat{M} \times \hat{T}$ is an ordinary matrix. Hence, the value of the isodeterminant is indeed an isonumber.

3.3.7) *Isotrace,*

$$\hat{T}r\hat{M} = Tr(\hat{M} \times \hat{T}) \times \hat{I}, \tag{3.10}$$

etc. It should be stressed that the above elements of isofunctional analysis are merely introductory and a study of at least Chapter 6 of monograph [15a] is necessary for a serious knowledge of the isoscattering theory.

**3.4. Isodifferential Calculus.** It was believed for centuries that the differential calculus is independent of the assumed basic unit, since the latter was traditionally given by the trivial number 1.

Santilli [13] has disproved this belief by showing that the differential calculus can be dependent on the assumed unit, by formulating the *isodifferential calculus* with basic *isodifferential,* for instance, of an isocoordinate $\hat{r}$

$$\hat{d}\hat{r} = \hat{d}[r \times \hat{I}(r, ...)] = \hat{T} \times d[r \times \hat{I}(r, ...)], \tag{3.11}$$

that does indeed coincide with the conventional differential for all isounits independent from $r$, $\hat{d}\hat{r} \equiv dr$, while yielding structural differences for all cases relevant for the isoscattering theory, namely, when the isounit depends on the local coordinates. In the latter case we have

$$\hat{d}\hat{r} = T \times d[r \times \hat{I}(r, ...)] = dr + r \times T \times d\hat{I}(r, ...). \tag{3.12}$$

The compatible formulation of the *isoderivative* is then given by

$$\frac{\hat{\partial}}{\hat{\partial}\hat{r}} = \hat{I} \times \frac{\partial}{\partial \hat{r}}. \tag{3.13}$$

The *isointegral* is defined as the inverse of the isodifferential and can be written for simplicity

$$\hat{\int} \hat{d}\hat{r} = \int d\hat{r}, \quad \hat{\int} \hat{d}\hat{r} \hat{\times} \hat{f}(\hat{r}) = \int d\hat{r} \times f(\hat{r}), \tag{3.14}$$

where we have used the isofunction $\hat{f}(\hat{r}() = \hat{I} \times f(\hat{r})$.

Note that, as formulated above for simplicity, isodifferentiation and isointegration yield ordinary scalars and *not* isoscalars, a feature assumed later on in Section 2.3 of paper II to reach a formulation accessible to experimentalists.

It should be indicated that the use of the conventional differential calculus leads to catastrophic mathematical and physical inconsistencies particularly in the dynamical

equations [6-11], thus mandating the use of the covering isodifferential calculus. Consequently, the sole functional differences between the conventional and isodifferential calculus are sufficient to warrant a reinspection of the quantum scattering theory.

As an illustration, the realizations of the isounit of primary physical relevance are based on exponentials, e.g., $\hat{I} = \hat{M}\hat{\times}exp[f(r,...)]$, where $\hat{M}$ is a matrix or operator not dependent on $r$. In this case, the isounit and the isotopic element disappear from the projection of the isodifferential in our space. This results in significant deviations between conventional and isotopic differentials, e.g., $dr \neq \hat{d}\hat{r} = dr \times (1 + r \times \partial f \partial r)$ thus providing additional expectations of possible numerical differences in the final elaboration of the same experiment with the conventional and the isotopic scattering theory.

**3.5 Iso-Hilbert Spaces.** The fundamental representation space of hadronic mechanics is a *new realization* of the abstract axioms of the conventional Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$, first proposed by Santilli [5] in 1978, then studied by Myung and Santilli [26] and other authors (see the review in Ref. [15a] and quoted references), today known as *iso-Hilbert spacve* or *Hilbert-Myung-Santilli isospaces,* and denoted $\hat{\mathcal{H}}$ over the isofield $\hat{\mathcal{C}}$. The new space is characterized by *isostates* $|\hat{\psi}>$ with *isoinner product,* and related *isonormalization,*

$$< \hat{\psi}|\hat{\times}|\hat{\psi} > \times \hat{I} =< \hat{\psi}| \times \hat{T} \times |\hat{\psi} > \times \hat{I} \; \epsilon \; \hat{C}, \tag{3.15a}$$

$$< \hat{\psi}|\hat{\times}|\hat{\psi} > \times \hat{I} = \hat{I}, \tag{3.15b}$$

*isoexpectation values* of an iso-Hermitean operator $\hat{Q} = \hat{Q}^{\hat{\dagger}}$

$$< \hat{\psi}|\hat{\times}\hat{Q}\hat{\times}|\hat{\psi} > \times \hat{I} =< \hat{\psi}| \times T \times Q \times T \times |\hat{\psi} > \times \hat{I}, \tag{3.16}$$

*isounit* under isonormalization (3.16)

$$< \hat{\psi}|\hat{\times}\hat{I}\hat{\times}|\hat{\psi} > \times \hat{I} =< \hat{\psi}| \times \hat{T} \times \hat{T}^{-1} \times |\hat{\psi} > \times \hat{I} = \hat{I}, \tag{3.17a}$$

$$\hat{I}\hat{\times}|\hat{\psi} > \equiv |\hat{\psi} >; \tag{3.17b}$$

*isoeigenvalue equation* for iso-Hermitean operators

$$\hat{H}\hat{\times}|\hat{\psi} >= H \times T \times |\hat{\psi} >= \hat{E}\hat{\times}|\hat{\psi} >= E \times |\hat{\psi} >, \; \hat{E} \in \hat{\mathcal{R}}, \; E \in \mathcal{R}; \tag{3.18}$$

and additional properties we cannot possibly review here. We limit ourselves to quote the following main properties (see Ref. [15a] for details):

3.5.1) Hilbert-Santilli isospaces are isomorphic to conventional Hilbert spaces by conception and construction, as illustrated by the fact that the isoinner product (3.15) is still inner from the positive-definite character of the isounit. This property is crucial

to ensure the covering character of hadronic over quantum mechanics, as well as the existence of a unique and unambiguous interconnecting maps indicated below.

3.5.2) Operators that are Hermitean on $\mathcal{H}$ over $\mathcal{C}$ are also iso-Hermitean, namely, they remain hermitean under lifting to the Hilbert-Santilli isospace over the isofield of isocomplex numbers, and we shall often write $\hat{Q} = \hat{Q}^{\hat{\dagger}} = \hat{Q}^{\dagger}$. Therefore, all quantities that are observable for quantum mechanics remain observable for hadronic mechanics, although the opposite is not generally true because of the existence of Hermitean operators representing irreversible process that are well defined for hadronic mechanics but cannot be even formulated for quantum mechanics due to its simpler structure.

3.5.3) The conventional Hilbert space admits a new symmetry discovered by Santilli [13,14] called *isoscalar symmetry,* given by a rescaling of the unit under which the conventional inner product is invariant,

$$< \psi| \times |\psi > \times I \equiv$$

$$\equiv < \psi| \times w^{-1} \times |\psi > \times (w \times I) =< \psi|\hat{\times}|\psi > \times \hat{I}, \quad w \in \mathcal{C}. \tag{3.19}$$

Evidently, the property persists for the Hilbert-Santilli isospace and we have

$$< \hat{\psi}|\hat{\times}|\hat{\psi} > \times \hat{I} =< \hat{\psi}| \times T \times |\hat{\psi} > \times \hat{I} \equiv$$

$$\equiv < \hat{\psi}| \times (w^{-1} \times T) \times |\hat{\psi} > \times (w \times \hat{I}) =< \hat{\psi}| \times T' \times |\hat{\psi} > \times \hat{I}'. \tag{3.20}$$

The lack of discovery of symmetry (3.19) for over one century should not be surprising, because the new symmetry required the prior discovery of new numbers, those with arbitrary units [12]. In fact, isosymmetry (3.19) requires the reformulation of numbers as isonumbers $\hat{n} = n \times 1$.

Despite its apparent triviality, the discovery of isosymmetry (3.19) has permitted the achievement of a new grand unification of gravitational and electroweak interactions essentially based on the embedding of gravitation where nobody looked for, in the *unit* of electroweak theories. The new grand unification includes the first known axiomatically correct inclusion of antimatter in grand unified theories also nobody cared for since gravitation on a Riemannian space cannot represent neutral antimatter. This suggests the use of the isodual theory of antimatter to achieve a grand unification with a degree of democracy between matter and antimatter (see papers [44-46] for original words and monograph [33] for comprehensive treatment).

**3.6. Isolinearity, Isolocality and Isounitarity.** We are now equipped to introduce the following important notions first introduced by Santilli as the foundation of the isotopies of Lie's Theory (see, e.g., Ref. [15]):

*DEFINITION 3.6.1: ISOLINEARITY.*

Operators that are nonlinear on $\mathcal{H}$ over $\mathcal{C}$ (that is, *nonlinear in the wavefunction*) can be *identically* rewritten in a form that is linear on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$, a property called *isolinearity*. The reformulation is simply done by embedding all nonlinear terms in the isounit, In fact. hadronic mechanics was proposed [5] to reformulate complex nonlinear models, e.g., $H(r,p,\psi) \times |\psi >= E \times |\psi >$, into an identical isolinear form $H_o(r,p) \times T(r,p,\psi) \times |\psi >= E \times |\psi >$, $H = H_o \times T$. Despite its simplicity, the reformulation is not trivial because the conventional nonlinear formulation generally violates the superposition principle, thus being generally inapplicable to composite systems, while the isotopic formulation verifies the superposition principle on isospace over isofield, thus allowing consistent studies of nonlinear composite systems.

*DEFINITION 5.6.2: ISOLOCALITY.*
Operators that are nonlocal on $\mathcal{H}$ over $\mathcal{C}$, e.g., of nonlocal-integral type, can be *identically* reformulated in a form on $\hat{H}$ over $\hat{\mathcal{C}}$ that is local-differential everywhere except at the isounit, a property known as *isolocality*. Again, the reformulation is done via the embedding of all nonlocal terms in the isounit. It should be noted that the technical understanding of isolocality requires a technical knowledge of the *isotopology* of hadronic mechanics initiated by the mathematicians Gr. Tsagas and D. S. Sourlas [34] (see also monograph [19]) and completed by the mathematicians M. Falcon Ganfornina and J. Nunez Valdes [35] (see also monograph [23]).

*DEFINITION 5.6.3: ISOUNITARITY.*
All operators $U$ that are nonunitary on $\mathcal{H}$ over $\mathcal{C}$ can be *identically* reformulated in a form verifying unitarity on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$, a property called *isounitarity*. The reformulation is done via the simple identity

$$U \times U^\dagger \neq I, \quad U = \hat{U} \times \hat{T}^{1/2}, \tag{3.21}$$

under which we have the *isounitarity law*

$$\hat{U} \hat{\times} \hat{U}^\dagger = \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}. \tag{3.22}$$

This is the property indicated in Section 1 that assures nonunitary $S$-matrices to preserve probabilities under the condition that the matrices are not treated via the mathematics of quantum mechanics.

**3.7. Resolution of the Inconsistency Theorems.** We are now sufficiently equipped to show the resolution of the Theorems of catastrophic Inconsistencies of Nonunitary Theories [6-11], first achieved by Santilli thanks to his isomathematics (see Ref. [15] for detailed treatment):

*INVARIANCE OF THE BASIC UNIT.*

The units of the conventional scattering theory characterize a geometrization of basic unfits of measurements. For instance, the unit of the three-dimensional Euclidean space is a geometrization of the units of length along each axis, e.g., $I = Diag(1cm,-1cm,1cm)$. When expressed in dimensionless form, the unit acquires the familiar version $I = Diag.(1,1,1)$. All quantum units are invariant under unitary time evolution, $I \to U \times I \times U^\dagger \equiv I$, thus confirming the majestic axiomatic consistency of quantum mechanics.

However, these units are no longer invariant under nonunitary time evolutions $U \times U^\dagger \neq I$ because, in this case, we can have maps of the type $I \to U \times I \times U^\dagger = Diag.(231cm, 1.36cm, 0.3cm) \neq I$. This illustrates a first inconsistency of nonunitary scattering theories, the lack of preservation over time of the basic units of measurements, with consequential lack of consistent applicability of nonunitary theories to experiments.

A central features of the isoscattering theory is the invariance of the isounits $\hat{I}$ under the isounitary time evolution of the theory. In fact, under isounitarity law (22) we have, for instance, the invariance $\hat{I} = Diag.(231cm, 1.36cm, 0.3cm) \to \hat{U} \hat{\times} \hat{I} \hat{\times} \hat{U}^\dagger \equiv \hat{I}$, thus resolving the first inconsistency of nonunitary theories

*INVARIANCE OF OBSERVABLES.*
Another central property of quantum mechanics is that, when a quantity is observable at a given time, it remains observable at all subsequent times. This feature is verified by the preservation of Hermiticity under unitary time evolutions and provides another illustration of the majestic consistency of quantum mechanics.

When the time evolution is no longer unitary, Hermiticity is not necessarily preserved over time (this is the *Lopez lemma* [6] indicated in Section 1). In fact, the transformed eigenvalue equation for an operator H that is Hermitean at the initial time $t_o$ under nonunitary transforms $U = U(t)$ is given by $H \times |\psi > \to (U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi > \times U^\dagger)$. Consequently, the initial Hermiticity of $H$ is not necessarily preserved over time due to the lack of general commutativity of $U \times H \times U^\dagger$ and $(U \times U^\dagger)^{-1}$.

It is an instructive exercise for the reader interested in acquiring a knowledge of the isoscattering theory to prove that iso-Hermiticity is indeed preserved under isounitary transformations [6,12].

*INVARIANCE OF NUMERICAL PREDICTIONS.*
Yet another important feature of the axiomatic consistency of quantum mechanics is that, if a Hermitean operator $H$ has the eigenvalue $E$ (e.g., $E = 5MeV$) at the initial time, $H \times |\psi >= E \times |\psi >$, said eigenvalue is preserved at all times, as shown by the transformation $(U \times H \times U^\dagger) \times (U \times |\psi > \times U^\dagger) = H' \times |\psi' >= U \times (E \times |\psi > \times U^\dagger dag = E \times |\psi' >$.

Under nonunitary time evolutions, the eigenvalue at the initial time of a Hermitean operator is not necessarily preserved at subsequent times, as shown by the transformation $(U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi> \times U^\dagger) = H' \times T \times |\psi'> = U \times (E' \times |\psi> \times U^d ag) = E' \times |\psi'>, T = (U \times U^\dagger)^{-1}$, where the lack of preservation of the eigenvalue, $E' \neq E$, follows from the fact that $|\psi'>$ is now the eigenstate of the new operator $H' \times T$. It is an instructive exercise for interested readers to verify that isoeigenvalues are indeed preserved under isounitary time evolutions. The resolution of the remaining inconsistencies then follows [16a,16c].

The property important for the isoscattering theory is that *eigenvalues of Hermitean operators are numerically altered under nonunitary-isounitary lifting.* This occurrence suggests, alone, a reinspection of the conventional scattering theory because the possible presence of nonunitary effects in deep inelastic scattering could imply numerical results different than those currently assumed.

**3.8. Delta Isofunction.** As well known, Dirac's delta function, here expressed for the case of a one-dimensional coordinate $r$,

$$\delta(r - r_0) = \frac{1}{2\pi} \times \int_{-\infty}^{+\infty} e^{i \times k \times (r - r_o)} \times dk, \tag{3.23}$$

is divergent at $r = r_0$, by therefore constituting the origin of divergences in quantum scattering theories [1].

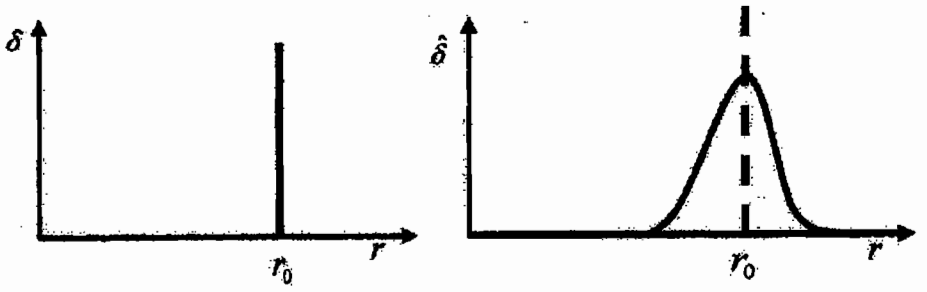



Figure 3: *A schematic view in the left of the conventional delta function $\delta(r - r_0)$ illustrating its divergent character at $r_0$, and a schematic view in the right of the Dirac-Myung-Santilli isodelta function of hadronic mechanics $\hat{\delta}(r - r_0) = \delta[T(r - r_0)]$, illustrating the absence of the above divergency, a feature allowing the removal of divergencies in the isoscattering theory from primitive axioms.*

In view of the above, Myung and Santilli [26] introduced in 1982 the isotopic lifting of Dirac's delta function, today known as the *Dirac-Myung-Santilli delta isofunction,*

or *DMS isodelta* for brevity (see, e.g., Nishioka [27]) that, by using the notions of isointegral (3.14), and isoexponentiation (3.4), can be written

$$\hat{\delta}(r-r_0)=\frac{\hat{I}}{2\pi}\hat{\times}\hat{\int}_{-\infty}^{+\infty}\hat{e}^{i\times k\times(r-r_0)}\hat{\times}\hat{dk},=\frac{1}{2\pi}\times\int_{-\infty}^{+\infty}e^{i\times k\times T\times(r-r_o)}\times dk, \qquad (3.24a)$$

$$T=\Sigma_{k=1}^{n}c_k\times(r-r_0)^k,\quad c_k\in\mathcal{C}. \qquad (3.24b)$$

where we write the isotopic element $T$ without a "hat" to denote its formulation on conventional spaces, and example (3.24b) an illustration of the possible removal of the singularity at $r_0$. We then have the evident property

$$\hat{\delta}(r-r_0)=\delta[T\times(r-r_0)]. \qquad (3.25)$$

As illustrated in Figure 2, under the appropriate realization of the isotopic element $T$, the DMS isodelta eliminates the divergent character of the delta function, thus setting up the foundations for a new scattering theory without divergencies *ab initio*, which is a main objective of this paper.

Note that for (3+1)-dimensional spaces each coordinate is multiplied by its isotopic element (see next section). For numerous additional properties, e.g., the derivation of the isodelta via isotransforms, the reader is encouraged to study monograph [xx]. Section 6.4.

**3.9. Isospherical Isocoordinates.** An additional mathematical notion needed for the elaboration of the isoscattering theory is given by the *isospherical coordinates* [15] here considered for in the Euclid-Santilli isospace with isounit

$$\hat{I}=Diag.(/b_1^2,1/b_2^2,1/b_3^2)=1/T>0, \qquad (3.26)$$

isometric

$$\hat{\delta}=T\times\delta=Diag.(b_1^2,b_2^2,b_3^2), \qquad (3.27)$$

and isoinvariant

$$\hat{r^2}=x^2\times b_1^2+y^2\times b_2^2+z^2\times b_3^2. \qquad (3.28)$$

Under the assumption of the conventional orientation of the angles $\theta,\phi$ with respect to the $z$-axis, we have the isounits

$$\hat{I}_\theta=b_3,\quad \hat{I}_\phi=b_1\times b_2, \qquad (3.29)$$

and the projection of the isocoordinates on the conventional Euclidean space

$$x=r\times b_1^{-1}\times sin(\theta\times b_3)\times cos(\phi\times b_1\times b_2), \qquad (3.30)$$

$$y = r \times b_2^{-1} \times sin(\theta \times b_3) \times sin(\psi \times b_1 \times b_2), \tag{3.31}$$

$$z = r \times b_3^{-1} \times cos(\theta \times b_3). \tag{3.32}$$

Understanding of the isoscattering theory requires the knowledge that Eq. (3.28) represents an ellipsoid only when considered on the Euclidean space with respect to the trivial unit 1, because the same invariant represents the perfect sphere in Euclid-Santilli isospace over isofield called *isosphere.* This is due to the fact that the $k$-axis is mutated by the quantity $1 \to b_k^2$, but the corresponding unit is mutated by the *inverse* amount $1 \to b_k^{-2}$, thus preserving the perfect sphericity.

Similarly, the rotational symmetry has been popularly believed in the 20th century as being broken for ellipsoid (3.41), while in reality such a breaking is due to insufficient treatment since the rotational symmetry is reconstructed as exact on Euclid-Santilli isospaces, as shown by the perfect sphericity of the isosphere.

## 4. Concluding Remarks

In this paper, we have suggested the re-inspection of the conventional, potential, unitary scattering theory of relativistic quantum mechanics on grounds of the following aspects:

1) The apparent inapplicability (rather than violation) of the Lorentz-Poincaré symmetry and special relativity within physical media at large, and within the scattering region in particular, due to: difficulties for a consistent formulation of their axioms (impossibility of introducing inertial systems within a medium, the sole existence of the privileged reference frame at rest with the medium, difficulties in the verification of all axioms within a transparent medium, and others); deviations predicted in the repetition within physical media of the historical experimental verifications of special relativity in vacuum (repetition of Fizeau experiment entirely within water, and others); difficulties in reaching a *numerical* (rather than solely conceptual) representation of *all* data for *all* frequencies in the *entire* reduction to photons of electromagnetic waves propagating within physical media (inability to reach a numerical representation of the angle of refraction and the index of refraction; impossibility for a large number of photons to pass through a large number of nuclei as needed to maintain the main nonscattered part of e light beam along a straight line; difficulties in reducing to photons electromagnetic waves with one meter wavelength propagating within physical media; impossibility of representing with photons traveling in vacuum seemingly unavoidable superluminal causal speeds within physical media; etc.); and other insufficiencies;

2) Impossibility of reducing to photons traveling in vacuum the electromagnetic phenomena within the scattering region due to its hyperdense character, thus implying the locally varying speed $C = c/n$, suggesting a return to the Maxwellian

interpretation of light and photon wavepackets as transversal electromagnetic waves propagating in the ether as a universal substratum without conflict with special relativity in vacuum (due to our impossibility of identifying a privileged system at rest with the ether), and consequential relevance of the *Lorentz problem,* namely. the achievement of the universal symmetry for all locally varying speeds of light $C = c/n$;

3) The strict *reversibility* over time of the Lorentz-Poincaré symmetry and special relativity compared to the strict *irreversibility* over time of high energy inelastic scattering processes, with ensuing difficulties for rigorous verifications of causality and other laws, and the need for covering theories as irreversible as the scattering process being represented;

4) The need advocated by Heisenberg for a covering of quantum mechanics which is nonlinear in the wavefunction and other quantities due to the expected nonlinearity of high energy scattering processes, compared to the linear character of quantum mechanics, the breaking of the superposition principle for Hamiltonians dependent on wavefunctions and consequential inapplicability of nonlinear quantum models to composite scattering processes;

5) Einstein-Podolsky-Rosen historical doubts on the final character of quantum mechanics; Dirac's call for a reformulation of the scattering theory that is convergent *ab initio* so as to avoid the achievement of numerical results in high energy scattering experiments via *ad hoc* procedures to achieve mathematical convergence of unknown physical origin or content; and other authoritative doubts;

6) The *No Reduction Theorems* preventing a consistent reduction of macroscopic irreversible systems to a finite set of particles all in nice conservative conditions, with consequential impossibility of reducing highly irreversible scattering processes to point-like quantum particles verifying the rotational and Lorentz symmetries, thus identifying the origin of irreversibility in the total mutual penetrations of the wavepackets and/or charge distributions of particles in the scattering region, essentially as occurring for macroscopic irreversible systems (such as a spaceship during reentry in atmosphere);

7) The unavoidable non-Hamiltonian and, therefore, nonunitary character of the contact effects due to total mutual penetration of extended wavepackets and/or charge distributions of particles in the scattering region, with consequential exiting from the class of unitary equivalence of quantum mechanics;

8) The numerical alteration of the eigenvalues of scattering operators under non-Hamiltonian, thus nonunitary internal effects, with consequential possible lack of final characer of the data elaboration of measured quantities (cross sections, scattering angles, etc.) via unitary scattering theory;

9) The recent discovery of the invariance of particle-antiparticle systems under the new symmetry called *isoselfduality* (invariance under anti-Hermiticity) that is verified by the Dirac equation, resulting in its direct representation of an electron

and a positron without need for the "hole theory," said new invariance not being generally verified by the scattering amplitude for particle-antiparticle processes; and other aspects all concurring in a return to the old need for a nonunitary covering of the conventional unitary scattering theory.

In this paper, we have then recalled the *Theorems of Catastrophic Mathematical and Physical Inconsistencies of Noncanonical and Nonunitary Theories,* implying the lack of invariance over time of the units of measurements, the lack of conservation over time of observable, the general inability to predict the same numerical results under the same conditions at different times, and others serious insufficiencies.

In order to avoid excessive complexities at start up, in this and the following papers we have restricted our analysis to *reversible* scattering processes *without antiparticles.* We have then, apparently for the first time, specialized to the scattering region the new mathematics known as *isomathematics,* that has been specifically built over decades of efforts by various authors to bypass said inconsistency theorems; we have outlined their resolution; and restricted the study to a time reversal invariant formulation of the nonunitary scattering theory without antiparticles under the name of *isoscattering theory.*

In this paper, we have also indicated the possibility that, in the final analysis, the *elaboration* via the scattering and isoscattering theories of the same *measured* data may lead to the same numerical results. This possibility should not be excluded due to the indicated preservation under isotopies of both Einsteinian and quantum axioms and, in case confirmed, would be quite valuable because it would confirm the broadening of their applicability under nonlinear, nonlocal and nonunitary internal effects.

However, even under the assumption that the data elaboration of *past* experiments are the same for the conventional; and the isotopic scattering theories, the latter is expected to admit the representation of events precluded to the former, such as the synthesis of neutrons from protons and electrons as occurring in stars, or the synthesis of hadrons at large from lighter particles that, as we shall see in Paper IV of this series, can best be treated via a nonunitary-isounitary theory due to the need for a *negative binding energy* under which the Schrödinger equation no longer admits physically meaningful solutions [36].

Above all, the reader is suggested to keep in mind that the ultimate aim of all studies herein considered is the conception, quantitative treatment and experimental verification of much needed new clean energies, such as the novel *Intermediate Controlled Nuclear Fusions* (ICNF) [37], due to their strictly irreversible, as well as nonlinear, nonlocal and non-Hamiltonian character.

The proof of the convergence from primitive axioms without *ad hoc* manipulations, the comparison of the data elaboration of measured quantities via the scattering and isoscattering theory is done in subsequent papers. Similarly, the inclusion of antipar-

ticles and the extension to irreversible scattering processes requires additional new mathematics (known as *isodual mathematics* and *Lie-admissible genomathematics,* respectively), thus requiring separate studies.

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# References


[1] M. Kaku, *Quantum Field Theory,* Oxford University Press (1993).

[2] P. A. M. Dirac,"Does renormalization make sense?" in Proceedings of the Conference on Perturbative Quantum Chromodynamics, AIP Conf. Proc.74, 129-130 (1981), reprinted in *Dr Paul Dirac - A biographical Memoir*, R.H Dalitz and R. Peierls, editors, Biographical Memoirs of Fellows of the Royal Society, Vol. 32, The Royal Society (1987).

[3] B. Davies, "Non-unitary scattering theory and capture, I: Hilbert space theory," Comm. Math. Physs. **71**, 1472 (190).

[4] R. M. Santilli, "On a possible Lie-admissible covering of Galilei's relativity in Newtonian mechanics for nonconservative and Galilei form-noninvariant systems," Hadronic J. **1**, 223-423 (1978), available in free pdf download from http://www.santilli-foundation.org/docs/Santilli-58.pdf

[5] R. M. Santilli, "Need of subjecting to an experimental verification the validity within a hadron of Einstein special relativity and Pauli exclusion principle," Hadronic J. **1**, 574-901 (1978), available in free pdf download from http://www.santilli-foundation.org/docs/Santilli-73.pdf

[6] D. F. Lopez,¡/b¿ In "Symmetry Methods in Physics", Volume 1, Joint Institute for Nuclear Research, Dubna, Russia (1994), available as free download from http://www.santilli-foundation.org/docs/Santilli-121.pdf

[7] A. Jannussis, R. Mignani and R. M. Santilli, Ann. Fond. L. De Broglie, **19**, 371 (1993), available in free download from http://www.santilli-foundation.org/docs/Santilli-123.pdf

[8] A. Jannussis and D. Skaltzas, Ann. Fond. L/. De Broglie **18**, 137 (1993), available in free download from http://www.santilli-foundation.org/docs/Santilli-129.pdf

[9] R. M. Santilli, Intern. J. Modern Phys. **14**, 3157 (1999, available as free download from http://www.santilli-foundation.org/docs/Santilli-104.pdf

[10] R. M. Santilli Galilean Electrodynamics, Summer 2006, **17**, 43 (2006), available as free download from http://www.i-b-r.org/Incons.GravFinalGED-I.pdf

[11] R. M. Santilli, Found. Phys. **32**, 1111 (2002), available as free download from http://www.santilli-foundation.org/docs/Santilli-42.pdf

[12] R. M. Santilli, Algebras, Groups and Geometries **10**, 273 (1993), available as free downlaod from http://www.santilli-foundation.org/docs/Santilli-34.pdf

[13] R. M. Santilli, Rendiconti Circolo Matematico Palermo, Suppl. **42**, 7-82 (1996), available as free download from http://www.santilli-foundation.org/docs/Santilli-37.pdf

[14] R. M. Santilli, Acta Applicandae Mathematicae **50**, 177 (1998), available as free download from http://www.santilli-foundation.org/docs/Santilli-19.pdf

[15] R. M. Santilli, *Elements of Hadronic Mechanics*, Vol. I (1995) [15a], Vol. II 91995) [15b] Academy of Sciences, Kiev, available in free pdf downloads from http://www.santilli-foundation.org/docs/Santilli-300.pdf http://www.santilli-foundation.org/docs/Santilli-301.pdf

[16] R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry,*, Vol. I [16a], II [16b], III [16c], IV [16d] and [16e], International Academnioc Press, (2008), available as free downlaods from http://www.i-b-r.org/Hadronic-Mechanics.htm

[17] A. K. Aringazin, A. Jannussis, F. Lopez, M. Nishioka and B. Veljanosky, *Santilli's Lie-Isotopic Generalization of Galilei and Einstein Relativities,* Kostakaris Publishers, Athens, Greece (1991), available as free download from http://www.santilli-foundation.org/docs/Santilli-108.pdf

[18] H. C. Myung,, *Lie Algebras and Flexible Lie-Admissible Algebras,* Hadronic Press (1982), available as free download from http://www.santilli-foundation.org/docs/Santilli-107.pdf

[19] D. S. Sourlas and G. T. Tsagas, *Mathematical Foundation of the Lie-Santilli Theory,* Ukraine Academy of Sciences 91993), available as free download from http://www.santilli-foundation.org/docs/santilli-70.pdf

[20] J. Lôhmus, E. Paal, and L. Sorgsepp, *Nonassociative Algebras in Physics* (Hadronic Press, Palm Harbor, 1994), available as free download from http://www.santilli-foundation.org/docs/Lohmus.pdf

[21] J. V. Kadeisvili, *Santilli's Isotopies of Contemporary Algebras, Geometries and Relativities,* Ukraine Academy of Sciences, Second edition (1997), available as free download from
http://www.santilli-foundation.org/docs/Santilli-60.pdf

[22] Chun-Xuan Jiang, *Foundations of Santilli Isonumber Theory,* International Academic Press (2001), available as free download from
http://www.i-b-r.org/docs/jiang.pdf

[23] Raul M. Falcon Ganfornina and Juan Nunez Valdes, *Fundamentos de la Isoteoria de Lie-Santilli,* International Academic Press (2001), available as free downlaod from
http://www.i-b-r.org/docs/spanish.pdf

[24] A. O. E. Animalu, M. Cloonan and D. Gandzha, *New Sciences for a New Era" Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli,* in preparation, preliminary versionm available in the website
http://www.santilli-foundation.org/santilli-scientific-discoveries.html

[25] R. M. Santilli, "Lie-admissible invariant representation of irreversibility for matter and antimatter at the classical and operator levels," Nuovo Cimento B **121**, 443 (2006), available as free downlload from
http://www.i-b-r.org/Lie-admiss-NCB-I.pdf

[26] H. C. Myung and R. M. Santilli, Hadronic Journal Vol. 5, pages 1277-1366 (1982). available as free download from
http://www.santilli-foundation.org/docs/Santilli-201.pdf

[27] M. Nishioka, Lettere Nuovo Cimento Vol. 39, pages 369-372 (1984), available as free download from
http://www.santilli-foundation.org/docs/Santilli-202.pdf

[28] R. Mignani, Nuovo Cimento **39**, 406 (1984).

[29] A. K. Aringazin, D. A. Kirukhin, and R. M. Santilli, Nonpotential elastic scattering of spinning particles," Hadronic J. **18**, 257 (1995), available as free download from
http://www.santilli-foundation.org/docs/Santilli-502.pdf

[30] R. M. Santilli, *Foundation of Theoretical Mechanics,* Volume I (1978) [30a], and Volume II (1982) [30b], Springer-Verlag, Heidelberg, Germany, available as free download from

http://www.santilli-foundation.org/docs/Santilli-209.pdf
http://www.santilli-foundation.org/docs/santilli-69.pdf

[31] R. M. Santilli, "Experimental verifications of isoredshift with possible absence of universe expansion, big bang, dark matter and dark energy," The Open Astronomy Journal **3**, 124 (2010), available as free download from http://www.santilli-foundation.org/docs/Santilli-isoredshift.pdf

[32] R. M. Santilli, *Foundations of Hadronic Chemistry, with Applications to New Clean Energies and Fuels,* Kluwer Academic Publishers (2001), available as free download from http://www.santilli-foundation.org/docs/Santilli-113.pdf

[33] R. M. Santilli, *Isodual Theory of Antimatter with Applications to Antigravity, Grand Unifications and Cosmology,* Springer (2006).

[34] Gr. T. Tsagas and D. S. Sourlas, Algebras, Groups and Geometries **12**, 1-65 and 67-88 (1995). available from the link http://www.santilli-foundation.org/docs/Santilli-324.pdf

[35] Raul M. Falcon Ganfornina and Juan Nunez Valdes, Algebras, Groups and Geometrioes **20**, 1 (2003), available from the link http://www.santilli-foundation.org/docs/isotopologia.pdf

[36] J. V. Kadeisvili, "The Rutherford-Santilli neutron," Hadronic J. **31**, 1, (2008), available in free pdf download from http://www.i-b-r.org/Rutherford-Santilli-II.pdf also available in html version in the website http://www.i-b-r.org/Rutherford-Santilli-neutron.htm

[37] R. M. Santilli, "Experimental confirmation of the novel 'Intermediate Controlled Nuclear Fusion' without harmful radiations," New Advances in Physics, in press (2010), available from http://www.santilli-foundation.org/docs/ICNF.pdf

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
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# NONUNITARY LIE-ISOTOPIC AND LIE-ADMISSIBLE SCATTERING THEORIES OF HADRONIC MECHANICS, II:
## Deformations-Isotopies of Lie's Theory, Special Relativity, and Mechanics

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**Abstract**

In the preceding Paper I, we have presented a variety of aspects suggesting a reinspection of the elaboration of measured quantities (cross section, scattering angle, etc.) via the conventional unitary scattering theory due to possible non-Hamiltonian internal effects implying a nonunitary time evolution. We have then reviewed the inconsistency theorems for nonunitary theories on conventional spaces over conventional fields, outlined the foundations of the novel isomathematics permitting a resolution of said inconsistency theorems, and suggested an isounitary reformulation of nonunitary scattering theories. In this paper we outline the use of isomathematics to achieve of methods essential for a consistent treatment of nonunitary-isounitary theories for interior dynamical conditions, such as the deformations-isotopies of Lie's theory, special relativity and mechanics. The outline appears recommendable due to a variety of formulations existing in the literature often leading to misconceptions because of their inapplicability to scattering problems, or formulations prior to the resolution of the inconsistency theorems. Following this necessary background, the formulation of the isoscattering theory without divergencies *ab initio* will be presented in Paper III, and comparative data elaborations via the conventional and the isotopic scattering theory will be initiated in Paper IV. ..

Key words: Lie algebnras, special relativity, Hamiltonian mechanics
PACS: 02.20.Sv, 03.30.+p, 03.65.-w

TABLE OF CONTENTS

1. Deformations-Isotopies of Lie's Theory
- 1.1. Introduction.
- 1.2. The Forgotten Lorentz Problem.
- 1.3. Insufficiencies of Lie's Theory.
- 1.4. Lie-Admissible Covering of Lie's Theory.
- 1.5. Lie-Isotopic Covering of Lie's Theory.
- 1.6. The Fundamental Theorem for Isosymmetries.
- 1.7. Simple Construction of the Lie=Santilli Isotheory.
- 1.8. Invariance of the Lie-Santilli Isotheory.
- 1.9. Regular and Irregular Pauli-Santilli Isomatrices.

2.Deformations-Isotopies of Special Relativity.
- 2.1. Introduction.
- 2.2. Deformation-Isotopies of the Minkowski Spacetime.
- 2.3. Deformations-Isotopies of Lorentz-Poincaré Symmetry.
- 2.4. Deformations-Isotopies of Special Relativity.
- 2.5. Implications for the Scattering Region.

3. Deformations-Isotopies of Mechanics
- 3.1. Introduction.
- 3.2. Deformations-Isotopies of Newtonian Mechanics.
- 3.3. Deformations-Isotopies of Hamiltonian Mechanics.
- 3.4. Deformations-Isotopies of Quantum Mechanics.
- 3.5. Deformations-Isotopies of Dirac's Equations.
- 3.6. Dirac's Generalization of Dirac's Equation.
- 3.7. Experimental Verifications.

Acknowledgments

References.

# 1. Deformations-Isotopies of Lie's Theory

**1.1. Introduction.** In the preceding paper [1], hereinafter referred to as Paper I, we have presented rather diversified conceptual, theoretical and experimental elements suggesting a reinspection of the validity of special relativity for interior dynamical problems at large, and the scattering region in particular.

R. M. Santilli had dedicated his lifetime of research to the construction of Lie-isotopic coverings of the Minkowskian geometry, the Lorentz-Poincaré symmetry and special relativity into forms more effective for interior conditions (see Refs. [3-16]). These covering formulations are, evidently, at the foundations of the Lie-isotopic scattering theory imder tyhe main assumptions indicated in Section 2 of Ref. [1], namely, the covering relativity and related covering symmetry are assumed for the interior of the scattering region, while all conventional formulations are recovered at distances bigger than 1 $fm$.

It is essential to outline in this paper the rather long scientific journey on the covering Lie-isotopic formulations for interior dynamical systems, due to the need of formulating them, specifically, for the scattering region. To begin our review, the central problem here referred to is the achievement of the universal invariance of locally varying speeds of light

$$C = \frac{c}{n(x, v, \xi, \omega, \psi, \partial\psi, ...)} \tag{1.1}$$

where $c$ is the speed of light in vacuum, and $n(x, v, \xi, \omega, \psi, \partial\psi, ...)$ is the index of refraction generally dependent on local coordinates $x$, velocities $v$ (e.g., of the medium with respect to the source), density of the medium $\xi$, frequency of light $\omega$, wavefunction $\psi$, its derivative $\partial\psi$,and other variables. We assume the reader is aware from Paper I that local speed (1.1) is assumed in the isoscattering theory as applying also to photons, since they cannot be assumed, without due inspection, as propagating in vacuum when in the interior of the scattering region due to its hyperdense character.

In view of the primitive character of light for all of physics, the study of the isoscattering theory can be reduced to the study of photons propagating within a hyperdense scattering region composed of particles in conditions of total mutual penetration. In the event the elaboration of measured quantities (cross section, scattering angles, etc.) via the isoscattering theory turns out to be entirely equivalent to the conventional elaboration, photons within the scattering regions would be confirmed as propagating in vacuum with consequential full validity of special relativity. By contrast, possible differences in the two data elaborations would establish that photons within the scattering region have local speed (1.1) with consequential need for a covering spacetime geometry, symmetry and relativity.

**1.2. The Forgotten Lorentz Problem.** Since the speed of light during pre-

Einstein's time was considered to be a local quantity $C = c/n$, Lorentz [2] studied its invariance, as noted by Pauli in his celebrated book *Theory of Relativity,* but encountered major technical difficulties for the case of the index of refraction with an arbitrary functional dependence (rather than constant) and had to restrict his studies to the constant speed $c$, resulting in transformations that are now part of history.

To honor one of the founders of our physical knowledge, in these papers we shall refer to *the Lorentz problem* the achievement of the invariance of locally varying speeds of light with an arbitrary functional dependence of the index of refraction.

During the century following Lorentz studies, the invariance of locally varying speeds of light was forgotten due to the reduction of light to photons propagating in vacuum irrespective of whether in exterior or interior conditions, with consequential use of Lorentz's invariance for the constant speed $c$.

Via rigorously proved *No Reduction Theorems* indicated in Paper I, Santilli established the impossibility of a consistent reduction of interior to exterior conditions thus bringing back to life the Lorentz's problem as a beautiful problem *per se,* in view of its practical value irrespective of whether light is reducible or not to photons moving in vacuum, as well as for photons themselves.

The various conceptual, mathematical, theoretical and experimental needs to reexamine the scattering theory presented in Paper I, render Lorentz's problem one of the most important problems in contemporary applied mathematics, whose solution can stimulate momentous advances in all quantitative sciences.

**1.3. Insufficiencies of Lie's Theory.** Santilli has dedicated his research life to the study of the Lorentz problem. The first contribution, as part of his Ph. D. Thesis in the mid 1960s, was to show that Lorentz's inability to achieve the desired invariance originated from insufficiencies of the background theory, *Lie's theory.* In fact, the applicability of said theory is notoriously restricted to linear, local and canonical systems at the classical level or unitary systems at the operator counterpart (where, in these papers, linearity is referred to the wavefunction, locality is referred to a finite set of isolated points, and canonicity or unitarity are referred to the respective time evolutions).

By contrast, the transition from the Minkowski metric characterizing the constant speed $c$ to the deformed metric characterizing variable speed (1.1)

$$\eta = Diag.(1, 1, 1, -c^2) \ \rightarrow$$

$$\rightarrow \ \hat{\eta} = Diag.(1, 1, 1, -\frac{c^2}{n^2(x, v, \xi, \omega, \psi, \partial\psi, ...)}), \tag{1.2}$$

has been shown in Paper I as characterizing systems that are generally *nonlinear in the wavefuynction, nonlocal of integral character, and noncanonical or nonunitary*

*in their time evolution,* It is then evident that Lie's theory. while so efective for the constant speed $c$, is generally *inapplicable* for the case of local speeds (1.1) (and certainly not "violated" because not conceived for the systems considered).

**1.4. Santilli Lie-Admissible Covering of Lie's Theory.** As part of his Ph. D. thesis, in order to broaden the representational capabilities of Lie's theory, Santilli proposed in 1967 [3] the first known *deformation of Lie algebras* in the physics literature with product

$$(A, B) = p \times A \times B - q \times B \times A, \tag{1.3}$$

where $p, q, p \pm q$ are non-null scalars (denoted $\lambda$ and $\mu$ in Ref. [3]), $A, B$ are matrices of the same dimension, and $A \times B$ is the conventional associative product according to the notations set forth in Paper I. Santilli called deformations (1.3) *mutations of Lie algebras* due to the evident loss of Lie's axioms, and proved that they characterize *Lie-admissible and Jordan-admissible algebras* according to Albert (in the sense that their attached antisymmetry and symmetric algebras are Lie and Jordan, respectively).

The proposal was intended to characterize the following *Lie-admissible generalization of Heisenberg's equations* for the dynamical evolution of a Hermitean operator $A$ in the following infinitesimal and finite forms

$$i \times \frac{dA}{dt} = (A, H) = p \times A \times H - q \times H \times A, \tag{1.4a}$$

$$A(t) = e^{H \times q \times t \times i} \times A(0) \times e^{-i \times t \times q \times H}, \tag{1.4b}$$

By recalling that Lie algebras characterize closed-conservative systems reversible over time, proposal [3] essentially recommended the construction of a *Lie-admissible covering of Lie's theory* for the characterization of open, nonconservative and irreversible systems evidently in view of the non-null *time rate of variations of the energy* $i \times dH/dt = (H, H) \neq 0$.

In 1978, Santilli [4] proposed the *most general possible Lie-admissible and Jordan-admissible deformations-mutations of Lie algebras* with product

$$(A\hat{,}B) = A \times R \times B - B \times S \times A = A < B - B > A, \tag{1.5}$$

where $R, S, R \pm S$ are now fixed nonsingular operators with an arbitrary, nonlinear and nonlocal functional dependence on any needed quantity (including the wavefunction and its derivatives), which brackets resulted in characterize the *most general possible algebra* as known in mathematics (characterized by a bilinear composition law verifying the right and left distributive and scalar laws). Therefore, algebras with product (1.5) contain as particular cases associative, Lie, Jordan, supersymmetric, flexible and any other possible algebra.

Ref. [4] then presented the initiation of a joint *Lie-admissible and Jordan-admissible covering of Lie's theory* in its various branches, including the lifting of the universal enveloping algebra with generalized Poincaré-Birkhoff-Witt theorem, Lie algebras, Lie's (transformation) groups and the representation theory.

Product (1.5) was obtained by using the most general possible *nonunitary* transformation of product (1.3), and was suggested as the foundations of the following *Lie-admissible and Jordan-admissible deformations-mutations of Heisenberg's equations* proposed in the joint paper [5] with infinitesimal and finite forms

$$i \times \frac{dA}{dt} = (A,H) = A \times R \times H - H \times S \times A = A < H - H > A, \tag{1.6a}$$

$$A(t) = e^{H \times S \times t \times i} \times A(0) \times e^{-i \times t \times R \times H}, \tag{1.6b}$$

$$R = S^{\dagger} \tag{1.6c}$$

The above equations were proposed as the foundations of *hadronic mechanics* for the representation of the most general possible open, nonconservative, irreversible and single-valued systems with potential interactions represented by the nonconserved Hamiltonian $H$, and contact nonpotential, nonlinear, nonlocal-integral and nonunitary interactions represented by the operators $R, S$.

Generalized dynamical equations (1.6) were originally formulated on conventional Hilbert spaces over conventional fields. Subsequent studies indicated that the equations verified the *Theorems of Catastrophic Mathematical and Physical Inconsistencies of Noncanonical and Nonunitary Theories* (see Refs. [6-12] of Paper I) because it is not preserving over time the basic units of measurements, the observability of physical quantities, the numerical predictions, etc.

The resolution of the above inconsistencies required decades of additional research. The first major advance occurred in 1993 with the discovery of the *genonumbers* and *genofields* [6], namely, fields with a fixed order of *all* multiplications to the right (representing motion forward in time) and an arbitrary right and left generalized unit called *genounit for the ordering to the right,*

$$n > m = n \times S \times m, \ \hat{I}^{>} = S^{-1}, \tag{1.7}$$

with the corresponding ordering of *all* multiplications to the left (representing motion backward in time) with related *genounit for the ordering to the left*

$$n < m = n \times R \times m, \ {}^{<}\hat{I} = R^{-1}, \tag{1.8}$$

where the word "genotopy" [4] was used in the Greek meaning of inducing new axioms.

In turn, the above genofields stimulated corresponding *two* genotopies, one to the right and, separately, one to the left, of functional analysis, metric spaces, geometries,

enveloping associative algebras, etc. Despite all these efforts, the resolution of the inconsistency theorems remained elusive for years.

A breakthrough occurred in the mathematical memoir [7] of 1996 with the discovery of the new *genodifferential calculus to the right or to the left.* The first invariance over time of deformations-mutations of Lie algebras was proved in paper [8] of 1997. Final maturity in the axiomatic structure of Lie-admissible formulations was achieved in memoir [9] of 2006 that also presented the first known connection between mechanics and thermodynamics, by showing that the irreversibility of thermodynamical laws originates at the ultimate level of nature, in full confirmation of the No Reduction Theorems indicated earlier.

Readers should be aware that, in view of their only known axiomatically consistent characterization of irreversible processes (thus including energy releasing processes) in a way directly compatible with thermodynamics, Lie-admissible formulations have been the subject of rather vast studies since the time of Santilli's original proposal of 1978 [4], including mathematical, physical, chemical as well as industrial research (see monographs [10-27], references quoted therein and general biography in Volume [16a]).

We should mention that, twenty years following the origination of the parametric deformations [3] and ten years following the proposal of the operator deformations [4,5] (with related rather vast literature of the time including four monographs [10,11], five workshops on Lie-admissibility and an international conference [16a]), there was the appearance of a very large number of papers on parametric deformations of Lie algebras with the simpler product $A \times B - q \times B \times A$, generally without the quotation of their origination [3-5], as well as generally without the identification of their joint Lie-admissible and Jordan admissible character, despite their historical and technical values.

It is important for these papers to indicate that all the latter deformations have been proved to verify the theorems of catastrophic inconsistencies when formulated on conventional spaces over conventional fields (see Refs. [6-12] of Paper I). The words "deformations-isotopies" of the titles of the various sections of this paper stand to indicate that their field in applied mathematics is that nowadays vastly referred to as "deformations," although identically reformulated as "isotopies" to resolve said inconsistencies.

**1.5. Santilli Lie-Isotopic Covering of Lie's Theory.** These papers are intended for concrete applications to the elaboration of scattering data. As such, if initially presented with excessive mathematical complexities (as needed for the consistent treatment of irreversible scattering processes), these papers could be beyond the reach of most phenomenologists.

This is tsetting has suggested in Paper I the restriction of these initial studies

to *reversible* scattering processes, and then the passage to the more complex irreversible events only subsequently. As an example, the restriction to reversible processes eliminates the need of the time ordering of all products, with consequential major simplification of the formalism.

Most importantly from the viewpoint of applied mathematics, *the restriction to reversible scattering processes permits the preservation of Lie's axioms, despite the admission of nonlinear, nonlocal and noncanonical or nonunitary effects.*

In fact, Santilli identified in the original proposal [4] of 1978, the following particularization of his Lie-admissible and Jordan-admissible product (1.5)

$$[A,B] = A\hat{\times}B - B\hat{\times}A =$$

$$= A \times T(x,v,\xi,\omega,\psi,\partial\psi,...) \times B - B \times T(x,v,\xi,\omega,\psi,\partial\psi,...) \times A, \qquad (1.9a)$$

$$R = S = T = T^\dagger > 0, \ \hat{I}(x,v,\xi,\omega,\psi,\partial\psi,...) = 1/\hat{T}(x,v,\xi,\omega,\psi,\partial\psi,...) > 0, \quad (1.9c)$$

where $\hat{I}(x,v,\xi,\omega,\psi,\partial\psi,...)$ and $\hat{T}(x,v,\xi,\omega,\psi,\partial\psi,...)$ are the *isounit* and the*isotopic element* at the foundation of the mathematics of Paper I, where one should note that quantities (1.9b) have the same functional dependence of local speed (1.1).

It is easy to verify that product (1.9) does indeed verify Lie's axioms. Consequently, the ensuing *deformations of Lie algebras* were called *isotopic* [4] by Santilli in their Greek meaning of preserving the original topology, a main characteristics that we have used in the very name of the isoscattering theory. In the same paper [4], Santilli then proposed a step by step isotopic generalization of Lie's theory that has remained structurally unchanged to this day (except for the subsequent reformulation on isospaces over isofields), and it is today known as the *Lie-Santilli isotheory* [18-27].

The main idea of said isotheory is that of preserving unchanged the generators of a given Lie symmetry and changing instead all their operations in an axiom-preserving way (as a condition to have an isotopy) [4]. The implementation of this idea require the lifting of the conventional associative product $A \times B$ into the axiom-reserving isoassociative form $A \times T \times B = A\hat{\times}B$ that, in turn, implies the lifting of the Lie product $[A.B]$ into the axiom-preserving form (1.9).

This seemingly elementary idea has important implications for the scattering theory. By recalling that the generator of a Lie symmetry represent conserved quantities, the preservation of the generators in the transition from the conventional to the isotopic scattering theory implies the preservation of all conventionally conserved quantities. However, the appearance of the isotopic element $T$ in the product itself implies that said preservation occurs under nonlinear, nonlocal and noncanonical or nonunitary internal effects, thus warranting a reinspection of the data elaboration via the conventional linear, local and unitary scattering theory.

Since the covering isotheory is at the foundations of these papers, it appears recommendable to outline its main elements in a language accessible to phenomenologists and specialized to scattering problems, not only for notational scopes, but also to avoid possible insidious misrepresentations in the event of referral to a variety of seemingly different presentations existing in the literature. Also, most of the "objections" raised by colleagues in a shorter versions of these papers were essentially due to a lack of inspection of the Lie-Santilli isotheory in the disparate literature or, more insidiously, due to the inspection of presentations prior to the achievement of invariance. Also, some of the results on Lie-isotopic studies are at times presented in the broader Lie-admissible context, as it is often the case of the original proposal [4].

As it was the case for other isotopies outlined in Paper I, the Lie-Santilli isotheory coincides with the conventional Lie theory at the abstract, realization-free level by conception and construction to such an extent that they can be presented at the pure mathematical level with the same symbols subjected to different realizations. However, such an abstract presentation would render quite difficult the practical applications of the isoscattering theory. Consequently, we shall outline below the specialization of the isotheory with emphasis on its applied version, namely, in its *projection* on conventional spaces over conventional fields.

Additionally, the reader should be aware that the original presentations verified the inconsistency theorems due to lack of invariance over time. In fact, the Lie-Santilli theory reached maturity only following the discovery of the *isonumbers* in 1993 [6] and of the *isodifferential calculus* in 1996 [7], discoveries that followed the otherwise excellent presentation by Tsagas and Sourlas [20] of 1993. Therefore, the outline below is based on all structural elements of the original proposal [4] formulated on isospaces over isofields [6] and via the isodifferential calculus [7].

*UNIVERSAL ENVELOPING ISOASSOCIATIVE ALGEBRAS*

Let $E = E(L)$ be the *universal enveloping associative algebra* of an $N$-dimensional Lie algebra $L$ with ordered (Hermitean) generators $X_k$, $k = 1, 2, \ldots, N$, and attached antisymmetric algebra isomorphic to the Lie algebra, $[E(L)]^- \approx L$ over a field $F$ (of characteristic zero), and let the infinite-dimensional basis $I, X_k, X_i \times X_j$, $i \leq j, \ldots$ of $E(L)$ be characterized by the *Poincaré-Birkhoff-Witt theorem.* We then have the following

*THEOREM 1.5.1 [4]: (Poincaré-Birkhoff-Witt-Santilli theorem): The isocosets of the isounit and of the standard isomonomials*

$$\hat{I},\ \ X_k,\ \ \hat{X}_i\hat{\times}\hat{X}_j,\ i \leq j,\ \ \hat{X}_i\hat{\times}\hat{X}_j\hat{\times}\hat{X}_k,\ i \leq j \leq k, \ldots, \tag{1.10}$$

*form an (infinite dimensional) basis of the universal enveloping isoassociative algebra $\hat{E}(\hat{L})$ (also called isoenvelope for short) of a Lie-Santilli isoalgebra $\hat{L}$.*

The first application of the above theorem, also formulated in Ref. [4] and then reexamined by various authors, is a rigorous characterization of the *isoexponentiation*, Eq. (3.4) of Paper I, i.e.,

$$\hat{e}^{\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}} =$$

$$= \hat{I} + \hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}\hat{/}\hat{1}! + (\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X})\hat{\times}(\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X})\hat{/}\hat{2}! + \ldots =$$

$$= \hat{I} \times (e^{i\times w\times T\times X}) = (e^{i\times w\times X\times T}) \times \hat{I}, \tag{1.11a}$$

$$\hat{i} = i \times \hat{I}, \hat{w} = w \times \hat{I} \in \hat{F}. \tag{1.11b}$$

where we continue to use the notation of Paper I according to which quantities with a "hat" are formulated on isospaces over isofields and those without are formulated on conventional spaces over conventional fields.

The nontriviality of the Lie-Santilli isotheory is illustrated by the emergence of the nonlinear, nonlocal and noncanonical or nonunitary isotopic element $T$ directly in *the exponent*, thus ensuring the desired generalization.

*LIE-SANTILLI ISOALGEBRAS.*

As it is well known, Lie algebras are the antisymmetric algebras $L \approx [\xi(L)]^-$ attached to the universal enveloping algebras $\xi(L)$. This main characteristic is preserved although enlarged under isotopies as expressed by the following

*THEOREM 1.5.2 [4] (Lie-Santilli Second theorem): The antisymmetric isoalgebras $\hat{L}$ attached to the isoenveloping algebras $\hat{E}(\hat{L})$ verify the isocommutation rules*

$$[\hat{X}_i\hat{,}\hat{X}_j] = \hat{X}_i\hat{\times}\hat{X}_j - \hat{X}_j\hat{\times}\hat{X}_i =$$

$$= X_i \times T(x, v, \xi, \omega, \psi, \partial\psi, ...) \times X_j - X_j \times T(x, v, \xi, \omega, \psi, \partial\psi, ...) \times X_i =$$

$$= \hat{C}_{ij}^k(x, v, \xi, \omega, \psi, \partial\psi, ...)\hat{\times}\hat{X}_k = C_{ij}^k(x, v, \xi, \omega, \psi, \partial\psi, ...) \times X_k, , \tag{1.12}$$

*where the C's, called the "structure isofunctions" of $\hat{L}$, generally have an explicit dependence on local variables, and are restricted by the conditions (Lie-Santilli Third Theorem)*

$$[X_i\hat{,}X_j] + [X_j\hat{,}X_i] = 0, \tag{1.13a}$$

$$[[X_i\hat{,}X_j]\hat{,}X_k] + [[X_jhat, X_k]\hat{,}X_i] + [[X_khat, X_i]\hat{,}X_j] = 0. \tag{1.13b}$$

It was stated in the original proposal [4] that *all isoalgebras $\hat{L}$ are isomorphic to the original algebra L for all positive-definite isotopic elements.* In other words, the isotopies *cannot* characterize any new Lie algebras algebra because all possible Lie algebras are known from Cartan classification. Therefore, Lie-Santilli isoalgebras merely provide *new nonlinear, nonlocal and noncanonical or nonunitary realizations of existing Lie algebras.*

*LIE-SANTILLI ISOGROUPS.*

Under certain integrability and smoothness conditions hereon assumed, Lie algebras $L$ can be "exponentiated" to their corresponding Lie transformation groups $G$ and, vice-versa, Lie transformation groups $G$ admit corresponding Lie algebras $L$ when computed in the neighborhood of the unit $I$.

These basic properties are preserved under isotopies although broadened to the most general possible, axiom-preserving nonlinear, nonlocal and noncanonical transformations groups according to the following:

*THEOREM 1.5.3 [4] (Lie-Santilli iaogroups): The isogroup characterized by finite (integrated) form $\hat{G}$ of isocommutation rules (1.12) on an isospace $\hat{S}(\hat{x}, \hat{F})$ over an isofield $\hat{F}$ with common isounit $\hat{I} = 1/\hat{T} > 0$ is a group mapping each element $\hat{x} \in \hat{S}$ into a new element $\hat{x}' \in \hat{S}$ via the isotransformations*

$$\hat{x}' = \hat{g}(\hat{w})\hat{\times}\hat{x}, \quad \hat{x}, \hat{x}' \in \hat{S}, \quad \hat{w} \in \hat{F}, \tag{1.14}$$

*with the following isomodular action to the right:*
*1) The map $\hat{g}\hat{\times}\hat{S}$ into $\hat{S}$ is isodifferentiable $\forall \hat{g} \in \hat{G}$;*
*2) $\hat{I}$ is the left and right unit*

$$\hat{I}\hat{\times}\hat{g} = \hat{g}\hat{\times}\hat{I} \equiv \hat{g}, \quad \forall \hat{g} \in \hat{G}; \tag{1.15}$$

*3) the isomodular action is isoassociative, i.e.,*

$$\hat{g}_1\hat{\times}(\hat{g}_2\hat{\times}\hat{x}) = (\hat{g}_1\hat{\times}\hat{g}_2)\hat{\times}\hat{x}, \quad \forall \hat{g}_1, \hat{g}_2 \in \hat{G}; \tag{1.16}$$

*4) in correspondence with every element $\hat{g}(\hat{w}) \in \hat{G}$ there is the inverse element $\hat{g}^{-\hat{I}} = \hat{g}(-\hat{w})$ such that*

$$\hat{g}(\hat{0}) = \hat{g}(\hat{w})\hat{\times}\hat{g}(-\hat{w}) = \hat{I}; \tag{1.17}$$

*5) the following composition laws are verified*

$$\hat{g}(\hat{w})\hat{\times}\hat{g}(\hat{w}') = \hat{g}(\hat{w}')\hat{\times}\hat{g}(\hat{w}) = \hat{g}(\hat{w} + \hat{w}'), \forall \hat{g} \in \hat{G}, \ \hat{w} \in \hat{F}; \tag{1.18}$$

*with corresponding isomodular action to the left, and general expression*

$$\hat{g}(\hat{w}) = \prod_k \hat{e}^{\hat{i}\hat{\times}\hat{w}_k\hat{\times}\hat{X}_k}\hat{\times}\hat{g}(0)\hat{\times}\prod_k \hat{e}^{=\hat{i}\hat{\times}\hat{w}_k\hat{\times}\hat{X}_k}, \tag{1.19}$$

Another important property is that conventional group composition laws admit a consistent isotopic lifting, resulting in the following

*THEOREM 1.5.4 [4] (Baker-Campbell-Hausdorff-Santilli theorem):*

$$(\hat{e}^{\hat{X}_1})\hat{\times}(\hat{e}^{\hat{X}_2}) = \hat{e}^{\hat{X}_3}, \tag{1.20a}$$

$$\hat{X}_3 = \hat{X}_1 + \hat{X}_2 + [\hat{X}_1,\hat{X}_2]\hat{/}\hat{2} + [(\hat{X}_1 - \hat{X}_2),[\hat{X}_1,\hat{X}_2]]\hat{/}\hat{12} + \dots . \tag{1.20b}$$

Let $\hat{G}_1$ and $\hat{G}_2$ be two isogroups with respective isounits $\hat{I}_1$ and $\hat{I}_2$. The *direct isoproduct* $\hat{G}_1\hat{\times}\hat{G}_2$ is the isogroup of all ordered pairs

$$(\hat{g}_1, \hat{g}_2), \quad \hat{g}_1 \in \hat{G}_1, \hat{g}_2 \in \hat{G}_2, \tag{1.21}$$

with isomultiplication

$$(\hat{g}_1, \hat{g}_2)\hat{\times}(\hat{g}_1', \hat{g}_2') = (\hat{g}_1\hat{\times}\hat{g}_1', \hat{g}_2\hat{\times}\hat{g}_2'), \tag{1.22}$$

total isounit $(\hat{I}_1, \hat{I}_2)$ and inverse $(\hat{g}_1^{-\hat{I}_1}, \hat{g}_2^{-\hat{I}_2})$.

The following particular case is important for the isotopies of inhomogeneous groups. Let $\hat{G}$ be an isogroup with isounit $\hat{I}$ and $\hat{G}_{\hat{a}}$ the group of all its inner automorphisms. Let $\hat{G}^o_{\hat{a}}$ be a subgroup of $\hat{G}_{\hat{a}}$ with isounit $\hat{I}^o$, and let $\Lambda(\hat{g})$ be the image of $\hat{g} \in \hat{G}$ under $\hat{G}_{\hat{a}}$. The *semidirect isoproduct* $\hat{G}\hat{\times}\hat{G}^o_{\hat{a}}$ is the isogroup of all ordered pairs $(\hat{g}, \hat{\Lambda})\hat{\times}(\hat{g}^o, \hat{\Lambda}^o)$ with total isounit

$$I_{tot} = \hat{I} \times \hat{I}^o. \tag{1.23}$$

The studies of the isotopies of the remaining aspects of the structure of Lie groups is then consequential. It is hoped the reader can see from the above elements that the entire conventional Lie theory does indeed admit a consistent and nontrivial lifting into the covering Lie-Santilli formulation.

Among a considerable number of mathematical papers on the Lie-Santilli isotheory listed in the Comprehensive Bibliography of Volume [16a], we quote in particular the readable review by J. V. Kadeisvili [28], an excellent presentation of the all fundamental isotopology by R. M. Falcon Ganfornina and J. Nunez Valdes [29], and the unification of all simple Lie algebras of a given dimension (excluding exceptional algebras) into one single Santilli isotope of the same dimension by Gr. T. Tsagas [30] (see also the review of the latter unification in Volume [16c]).

**1.6. The Fundamental Theorem for Isosymmetries.** The fundamental symmetries of the 20-th century physics characterize point-like abstractions of particles in vacuum under linear, local and potential interactions, and are given by the *Galilei symmetry* $G(3.1)$ for nonrelativistic treatment, the *Lorentz-Poincaré symmetry* for relativistic formulations, the $SU(3)$ symmetry for particle classifications, the gauge symmetry, and others.

A central objective of hadronic mechanics is the broadening of these fundamental symmetries to represent extended, nonspherical and deformable particles under linear and nonlinear, local and nonlocal and potential as well as nonpotential interactions *in such a way to preserve the original symmetries at the abstract level.*

This central objective is achieved by the following property first proved by Santilli in Ref. [13b]

*THEOREM 1.6.1: Let $G$ be an N-dimensional Lie symmetry of a K-dimensional metric or pseudo-metric space $S(x, m, F)$ over a field $F$,*

$$G: \quad x' = \Lambda(w) \times x, \quad y' = \Lambda(w) \times y, \quad x, y \in \hat{S}, \tag{1.24a}$$

$$(x' - y')^\dagger \times \Lambda^\dagger \times m \times \Lambda \times (x - y) \equiv (x - y)^\dagger \times m \times (x - y), \tag{1.24b}$$

$$\Lambda^\dagger(w) \times m \times \Lambda(w) \equiv m. \tag{1.24c}$$

*Then, all infinitely possible isotopies $\hat{G}$ of $G$ acting on the isospace $\hat{S}(\hat{x}, \hat{M}, \hat{F})$, $\hat{M} = \hat{m} \times \hat{I} = (\hat{T}_i^k \times m_{kj}) \times \hat{I}$ characterized by the same generators and parameters of $G$ and the infinitely possible, common isounits $\hat{I} = 1/\hat{T} > 0$ leave invariant the isocomposition*

$$\hat{G}: \quad x' = \hat{\Lambda}(w) \times x, \quad y' = \hat{\Lambda}(w) \times y, \quad x, y \in \hat{S}, \tag{1.25a}$$

$$(x' - y')^\dagger \times \hat{\Lambda}^\dagger \times \hat{m} \times \hat{\Lambda} \times (x - y) \equiv (x - y)^\dagger \times \hat{m} \times (x - y), \tag{1.25b}$$

$$\hat{\Lambda}^\dagger(\hat{w}) \times \hat{m} \times \hat{\Lambda}(\hat{w}) \equiv \hat{m}. \tag{1.25c}$$

*and all infinitely possible so constructed isosymmetries $\hat{G}$ are locally isomorphic to the original symmetry $G$.*

For a proof one may inspect Section 1.2 of Ref. [16b].

To achieve a technical understanding of the Lie-Santilli isotheory and of the isoscattering theory, the reader should note that, *while a given Lie symmetry $G$ is unique as well known, there can be an infinite number of covering isosymmetries $\hat{G}$ with generally different explicit forms o the transformations due to the infinite number of possible isotopic elements.*

In fact, systems are characterized by the Hamiltonian $H$ in the conventional scattering theory with trivial unit $I = Diag.(1, 1, ..., 1)$. In this case, changing the Hamiltonian implies the referral to a different system, but the symmetry transformations remain the same. In the isoscattering theory, systems are characterized by the Hamiltonian *$H$ plus the isotopic element $T$*. In this case, changing the isotopic element implies the referral to a different systems as well as the characterization of generally different transformations due to the appearance of the isotopic element in the very structure of the isosymmetry.

Note also that all possible isosymmetries can be explicitly and uniquely constructed via the sole knowledge of the conventional symmetry and the isotopic element. in fact, as implied by Theorem 1.5.5, the existence of the original symmetry

plus the condition $\hat{I} > 0$ ensure verification of the integrability conditions for the existence of finite transformations, a property hereon tacitly implied.

**1.7. Simple Construction of the Lie=Santilli Isotheory.** A simple method has been identified in Refs. [13,16] for the construction of the Lie-Santilli isotheory, all its underlying isomathematics and all physical methods to be studied in the these papers. This method is important because it permits a simple implementation of scattering models into their isotopic form. The method consists in:

(i) Representing all conventional interactions with a Hamiltonian $H$ and all non-Hamiltonian interactions and effects with the isounit $\hat{I}$;

(ii) Identifying the latter interactions with a nonunitary transform

$$U \times U^\dagger = \hat{I} \neq I \tag{1.26}$$

and

(iii) Subjecting the *totality* of conventional mathematical and physical quantities and all their operations to the above nonunitary transform, resulting in expressions of the type

$$I \to \hat{I} = U \times I \times U^\dagger = 1/\hat{T}, \tag{1.27a}$$

$$a \to \hat{a} = U \times a \times U^\dagger = a \times U \times U^\dagger = a \times \hat{I},\ a \in F, \tag{1.27b}$$

$$e^A \to U \times e^A \times U^\dagger = \hat{I} \times e^{\hat{T} \times \hat{A}} = (e^{\hat{A} \times \hat{T}}) \times \hat{I}, \tag{1.27d}$$

$$A \times B \to U \times (A \times B) \times U^\dagger =$$
$$= (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times B \times U^\dagger) = \hat{A} \hat{\times} \hat{B}, \tag{1.27c}$$

$$[X_i, X_j] \to U \times [X_i X_j] \times U^\dagger =$$
$$= [\hat{X}_i \hat{,} \hat{X}_j] = U \times (C_{ij}^k \times X_k) \times U^\dagger = \hat{C}_{ij}^k \hat{\times} \hat{X}_k =$$
$$= C_{ij}^k \times \hat{X}_k, \tag{1.27e}$$

$$< \psi| \times |\psi > \to U \times < \psi| \times |\psi > \times U^\dagger =$$
$$=< \psi| \times U^\dagger \times (U \times U^\dagger)^{-1} \times U \times |\psi > \times (U \times U^\dagger) =$$
$$=< \hat{\psi}| \hat{\times} |\hat{\psi} > \times \hat{I}, \tag{1.27f}$$

$$H \times |\psi > \to U \times (H \times |\psi >) = (U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi >) =$$
$$= \hat{H} \hat{\times} |\hat{\psi} >, etc. \tag{1.27g}$$

The above simple rules permit the explicit construction of all needed *regular isotopies* as defined and illustrated in Section 1.9 (eigenvalkue preserving mapps), including: algebras, groups, symmetries, eigenvalues equations and all needed aspects

[13]. It should be stressed that the above method is not applicable for the *irreguilar isotopies* as also defined and illustrated in Section 1.9 (eigenvalue mutating images) for which no map ois known at this writing.

Note finally that *serious inconsistencies emerge in the event even one single quantity or operation is not subjected to the above nonunitary map.* In the absence of comprehensive liftings, we would have a situation equivalent to the elaboration of quantum spectral data of the hydrogen atom with isomathematics, resulting in dramatic deviations from reality.

**1.8. Invariance of the Lie-Santilli Isotheory.** It is easy to see that the application of an additional nonunitary transform

$$W \times W^\dagger \neq I, \tag{1.28}$$

to expressions (1.27) causes the *lack of invariance,* with consequential activation of the catastrophic inconsistency theorems reviewed in Paper I, such as the *change of the basic isounit*

$$\hat{I} \to \hat{I}' = W \times \hat{I} \times W^\dagger \neq \hat{I}, \tag{1.29}'$$

that implies the loss of the represented system, let alone the lack of invariance of a physical theory over time, or the lack of invariance of an isosymmetry under its own action,

However, as indicated in Paper I, any given nonunitary transform can be identically rewritten in the isounitary form,

$$W \times W^\dagger = \hat{I}, \quad W = \hat{W} \times \hat{T}^{1/2}, \tag{1.30a}$$

$$W \times W^\dagger = \hat{W} \hat{\times} \hat{W}^\dagger = \hat{W}^\dagger \hat{\times} \hat{W} = \hat{I}, \tag{1.30b}$$

under which we have the invariance of the isounit and isoproduct [7]

$$\hat{I} \to \hat{I}' = \hat{W} \hat{\times} \hat{I} \hat{\times} \hat{W}^\dagger = \hat{I}, \tag{1.31a}$$

$$\hat{A} \hat{\times} \hat{B} \to \hat{W} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{W}^\dagger =$$
$$= (\hat{W} \times \hat{T} \times \hat{A} \times \hat{T} \times \hat{W}^\dagger) \times (\hat{T} \times \hat{W}^\dagger)^{-1} \times \hat{T} \times (\hat{W} \times$$
$$\times \hat{T})^{-1} \times (\hat{W} \times \hat{T} \times \hat{B} \times \hat{T} \times \hat{W}^\dagger) =$$
$$= \hat{A}' \times (\hat{W}^\dagger \times \hat{T} \times \hat{W})^{-1} \times \hat{B}' = \hat{A}' \times \hat{T} \times \hat{B}' = \hat{A}' \hat{\times} \hat{B}', \text{ etc.} \tag{1.31b}$$

from which the invariance of the entire isotopic formalism follows.

Note that the invariance is ensured by the *numerically invariant values of the isounit and of the isotopic element under nonunitary-isounitary transforms,*

$$\hat{I} \to \hat{I}' \equiv \hat{I}, \tag{1.32a}$$

$$A\hat{\times}B \to A'\hat{\times}'B' \equiv A'\hat{\times}B', \tag{1.32b}$$

in a way fully equivalent to the invariance of Lie's theory and quantum mechanics, as expected to be necessarily the case due to the preservation of the abstract axioms under isotopies. The resolution of the inconsistencies for noninvariant theories is then consequential (see Paper I for details).

**1.9. Regular and Irregular Pauli-Santilli Isomatrices.** Due to the abstract identify of Lie and Lie-Santilli theories, as well as the simplicity of their interconnecting map of Section 1.7, it is at times believed that the isotopies are trivial. The best way to dispel this erroneous perception is via the isorepresentation theory for one of the central physical notions, that of spin.

Even though the isorepresentation theory is still vastly unexplored, the studies conducted until now have been sufficient to identify the existence of two classes, the *regular isorepresentations,* occurring under the preservation of the original structure constants, and the *irregular isorepresentations,* occurring under the alteration of the original structure constants.

The basic symmetries of the 20th century particle physics have been those of the *rotational symmetry* $SO(3)$ and the *spin symmetry* $SU(2)$. The corresponding isosymmetries $\hat{SO}(3)$ were studied by Santilli in the original proposal [5] of 1978 as well as in the two subsequent papers [31,32] of 1985. Isosymmetries $\hat{SU}(2)$ were first studied also by Santilli in paper [33] of 1993 and [34] of 1998 with the following main results:

*CASE I: REGULAR PAULI-SANTILLI ISOMATRICES.*
This is the case that, by definition, implies the *preservation of the conventional spin* 1/2, although with new degrees of freedom nonexistent in the conventional notion of spin. The related regular two-dimensional irreducible isorepresentation of $\hat{SU}(2)$ are today known as *regular Pauli-Santilli isomatrices.*

This first notion of *hadronic spin,* that is, spin characterized by hadronic mechanics, is assumed for *low energy reversible scattering processes.* The assumption essentially implies that, as an example, an electron maintains its spin 1/2 in the transition from motion in vacuum to motion within the scattering region, although in a generalized way identified below. As we shall see in Section 3, this assumption implies the preservation within the scattering region of the Fermi-Dirac statistics and Pauli's exclusion principle.

By remembering the lack of uniqueness of the isounits and related isotopic element, the simplest regular two-dimensional irreducible isorepresentations of $\hat{SU}(2)$ are characterized by the lifting of the two-dimensional complex-valued unitary space

with metric $\delta = Diag.(1.1)$ into the isotopic image [33,34]

$$\hat{I} = Diag.((n_1^2, n_2^2),\ \hat{T} = Diag.(1/n_1^2, 1/n_2^2), \tag{1.33a}$$

$$\hat{\delta} = \hat{T} \times \delta = Diag.(1/n_1^2, 1/n_2^2), \tag{1.33b}$$

$$Det\ \hat{\delta} = (n_1 \times n_2)^{-2} = 1, \tag{1.33c}$$

with corresponding isounit and isotopic element

$$U\ times U^\dagger = \hat{I} = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix},\ T = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix}. \tag{1.34}$$

The related lifting of Pauli's matrices can then be easily constructed via the methods of Section 1.7 as follows

$$\sigma_k \ \rightarrow\ \hat{\sigma}_k = U \times \sigma_k \times U^\dagger, \tag{1.35a}$$

$$U = \begin{pmatrix} i \times n_1 & 0 \\ 0 & i \times n_2 \end{pmatrix}, U^\dagger = \begin{pmatrix} -i \times n_1 & 0 \\ 0 & -i \times n_2 \end{pmatrix}, \tag{1.35b}$$

where the $n$'s are well behaved nowhere null functions, resulting in the *regular Pauli-Santilli isomatrices* [*loc. cit*]

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1^2 \\ n_2^2 & 0 \end{pmatrix},\ \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times n_1^2 \\ i \times n_2^2 & 0 \end{pmatrix},\ \hat{\sigma}_3 = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}. \tag{1.36}$$

Another realization is given by *nondiagonal* nonunitary transforms [*loc. cit.*],

$$\begin{aligned} U &= \begin{pmatrix} 0 & n_1 \\ n_2 & 0 \end{pmatrix},\quad U^\dagger = \begin{pmatrix} 0 & n_2 \\ n_1 & 0 \end{pmatrix}, \\ \hat{I} &= \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix},\quad \hat{T} = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix}, \end{aligned} \tag{1.37}$$

with corresponding alternative version of the regular Pauli-Santilli isomatrices,

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1 \times n_2 \\ n_1 \times n_2 & 0 \end{pmatrix},\ \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times n_1 \times n_2 \\ i \times n_1 \times n_2 & 0 \end{pmatrix},$$

$$\hat{\sigma}_3 = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, \tag{1.38}$$

or by more general realizations with Hermitean nondiagonal isounits $\hat{I}$ [15b].

All Pauli-Santilli isomatrices of the above regular class verify the following *isocommutation rules and isoeigenvalue equations* on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$

$$[\hat{\sigma}_i, \hat{\sigma}_j] = \hat{\sigma}_i \times \hat{T} \times \hat{\sigma}_j - \hat{\sigma}_j \times \hat{T} \times \hat{\sigma}_i = 2 \times i \times \varepsilon_{ijk} \times \hat{\sigma}_k, \tag{1.39a}$$

$$\hat{\sigma}^{\hat{2}}\hat{\times}|\hat{\psi}\rangle =$$

$$(\hat{\sigma}_1 \times T \times \hat{\sigma}_1 + \hat{\sigma}_2 \times T \times \hat{\sigma}_2 + \hat{\sigma}_3 \times T \times \hat{\sigma}_3) \times T \times |\hat{\psi}\rangle = 3 \times |\hat{\psi}\rangle, \qquad (1.39b)$$

$$\hat{\sigma}_3\hat{\times}|\hat{\psi}\rangle = \hat{\sigma}_3 \times T \times |\hat{\psi}\rangle = \pm 1 \times |\hat{\psi}\rangle, \qquad (5.39c)$$

thus preserving conventional structure constants and eigenvalues for spin 1/2 under non-Hamiltonian/nonunitary interactions.

An interesting interpretation has been proposed in Ref. [31] for the case

$$n_1^2 = \lambda, \quad n_2^2 = \lambda^{-1}, \qquad (1.40)$$

according to which *the Pauli-Santilli isomatrices provide an explicit and concrete realization of a kind of hidden variables,* in the sense that the variable $\lambda$ is indeed hidden in the axioms of the $SU(2)$ symmetry, with the understanding that we are not referring to the traditional interpretation of hidden variables, such as the historical one by Bohm. Note that this new degree of freedom is absent in the conventional Lie theory and can be solely identified via the Lie-Santilli isotheory.

Irrespective of the type of hidden variable we are here referring to, the Pauli-Santilli isomatrices with characteristic quantity (1.40) have caused a reinspection of Bell's inequalities, local realism and all that due to the strictly unitary structure of the latter compared to the nonunitary character of the former. We regret being unable to outline these intriguing new vistas, and refer the interested reader to paper [34].

*CASE II: IRREGULAR PAULI-SANTILLI ISOMATRICES.*

As it is well known by experts in quantum mechanics, action-at-a-distance, potential interactions leave invariant the *intrinsic* characteristics of particles, such as spin. By comparison, as well known by experts in hadronic mechanics, contact non-Hamiltonian interactions generally cause alterations, called mutations, of all *intrinsic* characteristics of particles, including spin.

According to the Lie-Santilli isotheory, the mutations for spin 1/2 are characterized by the *irregular two-dimensional irreducible representations* of $\hat{SU}(2)$ known as the *irregular Pauli-Santilli isomatrices* that, by definition, do not preserve the spin 1/2 and, consequently, cannot be constructed via nonunitary transformations of conventional representations.

This case is assumed for the representation of particles at high energy originally having spin 1/2 when penetrating within hyperdense hadronic media, whether existing in the core of stars or inside very high energy scattering region. The main argument is that the belief that an electron preserves its spin 1/2 when in the core of a star does not appear to be plausible on various grounds, such as the loss of conventional quantized states within hyperdense media, the impossibility under the same conditions to possess a conserved angular momentum, and other reasons [13,16].

One illustrative example of irregular Pauli-Santilli isomatrices is given by [33.34]

$$\tilde{\sigma}_1 = \begin{pmatrix} 0 & n_1^2 \\ n_2^2 & 0 \end{pmatrix}, \quad \tilde{\sigma}_2 = \begin{pmatrix} 0 & -i \times n_1^2 \\ i \times n_2^2 & 0 \end{pmatrix}, \quad \tilde{\sigma}_3 = \begin{pmatrix} w \times n_1^2 & 0 \\ 0 & w \times n_2^2 \end{pmatrix}. \tag{1.41}$$

where $w$ is the *mutation parameter,* with isocommutation rules

$$[\tilde{\sigma}_1,\tilde{\sigma}_2] = i \times w^{-1} \times \tilde{\sigma}_3, \quad [\tilde{\sigma}_2,\tilde{\sigma}_3] = i \times w \times \tilde{\sigma}_1, \quad [\tilde{\sigma}_3,\tilde{\sigma}_2] = i \times w \times \tilde{\sigma}_1, \tag{1.42}$$

and isoeigenvalues

$$\tilde{\sigma}^{\hat{2}} \hat{\times} |\hat{\psi}\rangle =$$

$$(\tilde{\sigma}_1 \times T \times \tilde{\sigma}_1 + \tilde{\sigma}_2 \times T \times \tilde{\sigma}_2 + \tilde{\sigma}_3 \times T \times \tilde{\sigma}_3) \times T \times |\hat{\psi}\rangle = (2 + w^2) \times |\hat{\psi}\rangle, \tag{1.43a}$$

$$\tilde{\sigma}_3 \hat{\times} |\hat{\psi}\rangle = \tilde{\sigma}_3 \times T \times |\hat{\psi}\rangle = \pm w \times |\hat{\psi}\rangle, \quad w \neq 1, \tag{1.43b}$$

Additional examples of irregular Pauli-Santilli isomatrices can be found in Refs. [13,16].

The assumption of a mutated spin in hyperdense interior conditions evidently implies the inapplicability (rather than the violation) of the Fermi-Dirac statistics, Pauli's exclusion principle and other quantum mechanical laws, with the understanding that, by central assumption of Paper I, the scattering region as a whole must have conventional total quantum values because it is inspected from exterior conditions. Therefore, we are here referring to possible internal exchanges of angular momentum always in such a way to cancel out and yield total conventional values,

It should be indicated that we are here stressing the need to establish our knowledge in interior conditions via experiments rather than unverified assumptions, for which reason the isoscattering theory is proposed in the first place. The need to *test Pauli's exclusion principle under "external" strong interactions* was stressed since the title of paper [5] of 1978 and, after some 32 years, that call remains more valid than ever.

## 2.Deformations-Isotopies of Special Relativity.

**2.1. Introduction.** Following decades of research on the deformations-isotopies of Lie's theory, Santilli was finally in a position to construct the deformations-isotopies of all main aspects of the conventional *Lorentz-Poincaré (LP) symmetry,* including the isotopies of: the rotational symmetry [4,31,32]; the $SU(2)$-spin symmetry [33,34]; the Lorentz symmetry at the classical [35] and operator [36] levels; the Poincaré symmetry [37]; the spinorial covering of the Poincaré symmetry [38,39]; and the isotopies of the Minkowskian geometry 40]. The new symmetry is today known as the *Lorentz-Poincaré-Santilli isosymmetry,* or *LPS isosymmetry,* for short [18-27].

Following all the above preparatory research, Santilli was finally in a position to study the deformations-isotopies of special relativity into a form providing the

invariant (rather than covariant) characterization of interior dynamical problems at large, including the interior of the scattering region, the locally varying speed of light or photons (1.1).

The difficulties inherent in the realization of this objective were compounded by Santilli's specific intent of *honoring Albert Einstein via the preservation of his axioms for interior dynamical problems, and the mere presentation of broader realizations,* so as to avoid the abuse of Einstein's name via the application of his axioms under conditions never intended for and never directly tested.

The above objective was achieved thanks to the universal LPS isosymmetry, as well as its local isomorphism to the conventional LP symmetry, resulting in the axiom-preserving deformations-isotopies of special relativity first presented in Refs. [35,36] of 1983 at the classical and operator levels, respectively, and then studied in a variety of subsequent works (see monographs [12] of 1991 for the first systematic treatment and subsequent presentations in monographs [13] of 1995 and [16] of 2008 with literature quoted therein), resulting in a covering relativity today known as *Santilli isorelativity* [18-27].

It should be indicated that numerous "deformations" of the Minkowski space, the Lorentz-Poincaré symmetry and special relativity exist in the literature. However, to our best knowledge, all of them appeared a decade following the original proposal [35] by generally adopting the same symbols and main terminology, often without the quotation of the originating works [35]. Numerous other attempts at generalizing special relativity exist in the literature of the past century, although they do not possess a universal symmetry, thus lacking uniqueness in their derivation.

All these studies are noncanonical at the classical level and nonunitary at the operator level as an evidently necessary condition for novelty, and are formulated on conventional spaces over conventional fields. As such, all these studies directly verify the *Theorems of Catastrophic Inconsistencies of Noncanonical and Nonunitary Theories,* Refs. [6.-12] of Paper I.

We regret to be unable to review these studies to prevent an excessive length, as well as risk partial, thus discriminatory listings. Nevertheless, it is hope that interested colleagues may inspect preceding broadening of special relativity because it is the hope of all theories, including those here proposed, to contain at best a grain of truth, and comparative analyses of different approaches are always scientifically valuable.

The isoscattering theory is based on Santilli deformations-isotopies of the Minkowskian geometry, the Lorentz-Poincaré symmetry and special relativity because said isotopies:

1) are *directly universal,* that is, admitting as particular cases of all possible (3+1)-dimensional generalizations of the Minkowskian spacetime (universality) directly in the frame of the experimenter without any transformation to hypothetical reference

frames (direct universality), and have been proved to include as particular cases all other possible deformations via different expansions in terms of different parameters and with different truncations [41-43], thus reducing a variety of possibilities to a primitive isosymmetry [41-43];

2) have resolved said inconsistency theorems, thus being consistently applicable to actual measurements [13,16]; and

3) have significant experimental verifications in classical physics, particle physics, nuclear physics, superconductivity, chemistry, biology, astrophysics and cosmology (see Ref. [16d], Chapter 5 of Ref. [27] and paper [47]).

As it was the case for the Lie-Santilli isotheory, the objections received by the authors on an earlier and shorter version of these papers on the deformations-isotopies of special relativity, were primarily due to the inspection of inappropriate literature or to inconsistent presentations because not formulated on isospaces over isofields. Consequently, it appears recommendable to review the foundational elements of the field specialized to the scattering problem.

**2.2. Deformation-Isotopies of the Minkowski Spacetime.** As it is well known, the carrier space of relativistic scattering theory is the familiar Minkowski space $M(x, \eta, \mathcal{R})$, where we assume in these papers $x = (x^\mu), \mu = 1, 2, 3, 4, x^k = r^k, k = 1, 2, 3, x^4 = t$, and $\eta = Diag.(1, 1, 1, -c^2)$. Such a space is crucially dependent on the assumed basis unit, that of the Lorentz symmetry $I = Diag.(1, 1, 1, 1)$.

As customary in *relativistic hadronic mechanics* [13,16], the Minkowski spacetime with related Lorentz-Poincaré symmetry and special relativity are assumed as being *exact for the conditions clearly indicated by Einstein, i.e., for point-like particles and electromagnetic waves propagating in vacuum conceived as empty space,* under which conditions we have the constancy of the speed of light $c$ for all possible inertial systems. Therefore, special relativity is assumed as being exact everywhere in the *exterior* of the scattering region.

For the interior of the scattering region, as indicated in Section 2 of Paper I and studied in more details in Ref. [47], there are no possible inertial reference frames and we solely have the privileged frame at rest with the scattering region itself. Additionally, according to incontrovertible experimental evidence, the high energy scattering region is not an empty sphere with point-like particles in its interior, as requested by the mathematical structure of quantum mechanics. Instead, the scattering region is a hyperdense medium characterized by the mutual penetration of the wavepackets of scattering particles irrespective of whether their charge distribution is extended or point-like.

The above and other aspects imply that *in the interior of the scattering region the speed of light in general and that of photons in particular is assumed as being a local variable* $C = c/n(x, v, \xi, \omega, \psi, \partial\psi, ...)$ according to Eq. (1.1). Most importantly,

photons cannot be assumed as propagating in vacuum when in the interior of the scattering region due to its hyperdense character, as indicated in Section 1.1.

The locally varying character of the speed of light is geometrically represented via the assumption that *physical media alter the geometry of spacetime.* This assumption is necessary for any geometric representation of the variation of the speed of light, $c \to C = c/n(x, v, \xi, \omega, \psi, \partial\psi, ...)$. Equivalently, we can say that no variation of the speed of light is possible without a corresponding alteration of spacetime.

Following decades of studies, Santilli [35.36] proposed in 1983 the representation of the alteration of spacetime via the (axiom-preserving) deformations-isotopies of the Minkowski spacetime, today called *Minkowski-Santilli isospacetimes,* or *isospacetimes* for short, generally indicated with the symbols $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$, and characterized by:

A) The *isounit* and related *isotopic element* usually assumed (from their positive-definiteness) to have the diagonal form (see Ref. 13b] for off-diagonal realizations)

$$\hat{I} = Diag(1/b_1^2, 1/b_2^2, 1/b_3^2, 1/b_4^2) = Diag.(n_1^2, n_2^2, n_3^2, n_4^2), \tag{2.1a}$$

$$\hat{T} = Diag(b_1^2, b_2^2, b_3^2, b_4^2) = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2), \tag{2.1b}$$

B) The *isometric*

$$\Xi = \hat{T} \times \hat{m} = (T \times \eta) \times \hat{I} = \hat{\eta} \times \hat{I} =$$

$$= [Diag(b_1^2, b_2, b_3^2, -c^2 \times b_4^2) \times \hat{I} = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, -c^2/n_4^2) \times \hat{I}, \tag{2.2}$$

C) The *isoinvariant*

$$\hat{x}^{\hat{2}} = \hat{x}^\mu \hat{\times} \hat{\Xi}_{\mu\nu} \hat{\times} \hat{x}^\nu = (x^\mu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I} =$$

$$= x^1 \times b_1^2 \times x^1 + x^2 \times b_2^2 \times x^2 + x^3 \times b_3^2 \times x^3 - t \times c \times b_4^2 \times t \times c) =$$

$$= (\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \times \frac{c^2}{n_4^2}) \times \hat{I}, \tag{2.3}$$

where one should note that $\hat{\Xi}$ is an isomatrix (because its elements are isonumbers), while $\hat{\eta}$ is an ordinary matrix.

The quantities $b_\mu = 1/n_\mu$ are called the *characteristic quantities* of the considered scattering region that can be averaged to constants, as we shall see in Paper III. A rather frequent erroneous perception is that the quantities $b_\mu = 1/n_\mu$ are arbitrary parameters, while in reality they represent *measurable quantities.*

In fact, the space isounits $\hat{I}_k^k = n_k^2$ characterize the actual *size* (thus being of the order of $10^{-13} cm$) and *shape* (say, a spheroid ellipsoid)) of the scattering region that are indeed measurable. By contrast, "parameters" can assume arbitrary values thus generally having no connection with the actual size and shape of the physical region considered. In the elaboration of the isoscattering theory, the space isounits $\hat{I}_k^k = n_k^2$ are normalized to the perfect sphere of radius $\hat{I}_k^k = 1 fm$.

Similarly, the time isounit $\hat{I}_4^4 = n_4^2$ is a direct representation of the index of refraction that, as such, cannot possibly be a "parameter." More specifically, the time isounit provides a geometrization of the *density* of the scattering region (defined as the ratio between its energy and volume). In the isoscattering theory, the time isounit is normalized to the value for the vacuum $\hat{I}_4^4 = n_4^2 = 1$ for which $C = c/n = c$.

Finally, the characteristic quantities allow a representation of the *inhomogeneity* of physical media, e.g., via a dependence of the isounits on the local coordinates), as well as of their *anisotropy* (e.g., via different values between the space and time components of the isounit). Therefore, *the isoscattering theory allows, for the first time, a direct representation (i.e., a representation via the isometric) of the size, shape, density, inhomogeneity and anisotropy of the scattering region.*

The distinction between "parameters" and "characteristic quantities" is best illustrated by the Bose-Einstein correlation. Its treatment via relativistic quantum mechanics requires *four* arbitrary parameter of unknown origin, called "chaoticity parameters," that are fitted from the experimental data. By contrast, the representation of the same experimental data via relativistic hadronic mechanics yields space characteristic quantities providing a numerical representation of the actual shape of the fireball (a very elongated ellipsoid), while the forth component provides a numerical, representation of the density of the fireball in a way consistent with other experiments (see Ref. [16d], Chapter 5 of Ref. [27] and papers quoted therein).

The *Minkowski-Santilli isogeometry* has been worked out in detail in Ref. [40]. Its main implication is that the resulting geometrization of the interior scattering region includes in a unified form *all possible geometries in* $(3+1)$*-dimensions,* thus including the Riemannian, Finslerian and other geometries that are merely differentiated by the assumed isounit, although all formulated via the abstract axioms of the *Minkowski* space. Such a broadening is necessary for any realistic representation of scattering regions, e.g., because interior dynamical problems generally require metrics with an explicit dependence on velocities and other variables, thus rendering the sole Riemannian description excessively restrictive.

**2.3. Deformations-Isotopies of the Lorentz-Poincaré Symmetry.** Following, and only following the prior construction of the deformations-isotopies of Lie's theory outlined in Section 1.5, Santilli constructed systematic, step-by-step deformations-isotopies of all spacetime symmetries [12,31-40], including the isotopies of the Galilei symmetry and of the Lorentz-Poincaré symmetry.

Evidently, we cannot possibly review here these studies in details. However, to render this presentation minimally selfsufficient, we outline the rudiments of the *regular Lorentz-Poincaré-Santilli (LPS) isosymmetry* $\hat{P}(3.1)$ [37] specialized to the scattering problem, and leave to the interested reader the study of the nonrelativistic-Galilean counterpart from monographs [12].

By using the second theorem, Eq. (1.12), the *regular LPS isoalgebra* is characterized by the conventional generators and the isocommutation rules [37]

$$[J_{\mu\nu}\hat{,}J_{\alpha\beta}] = i \times (\hat{\eta}_{\nu\alpha} \times J_{\beta\mu} - \hat{\eta}_{\mu\alpha} \times J_{\beta\nu} - \hat{\eta}_{\nu\beta} \times J_{\alpha\mu} + \hat{\eta}_{\mu\beta} \times J_{\alpha\nu}), \tag{2,4a}$$

$$[J_{\mu\nu}\hat{,}P_{\alpha}] = i \times (\hat{\eta}_{\mu\alpha} \times P_{\nu} - \hat{\eta}_{\nu\alpha} \times P_{\mu}), \tag{2,4b}$$

$$[P_{\mu}\hat{,}P_{\nu}] = 0, \tag{2,4c}$$

The *iso-Casimir invariants* of $\hat{P}(3.1)$ are given by [37]

$$\hat{C}_1 = \hat{I}(x, ...), \tag{2.5a}$$

$$\hat{C}_2 = P^{\hat{2}} = P_{\mu}\hat{\times}P^{\mu} = P^{\mu} \times \hat{\eta}_{\mu\nu} \times P^{\nu} =$$

$$= P_k \times g_{kk} \times P_k - p_4 \times g_{44} \times P_4, \tag{2,5b}$$

$$\hat{C}_2 3 = W^{\hat{2}} = W_{\mu}\hat{\times}W^{\mu}, \quad W_{\mu} = \hat{\epsilon}_{\mu\alpha\beta\rho}\hat{\times}J^{\alpha\beta}\hat{\times}P^{\rho}, \tag{2.5c}$$

and they are at the foundation of classical and operator *isorelativistic kinematics* [13,16].

Since $\hat{I} > 0$, it is easy to prove that *the LPS isosymmetry is isomorphic to the conventional symmetry.* It then follows that *the isotopies increase dramatically the arena of applicability of the Lorentz-Poincaré symmetry, from the sole Minkowskian spacetime to all infinitely possible isospacetimes* (2.3).

By using Theorem 1.5.3 on the isogroup, the main components of the *regular LPS isotransformations* can be presented as follows in their projection on conventional spacetime:

(1) **Regular isorotations** $\hat{SO}(3)$, first presented in Ref. [31,32], here expressed in the (1,2)-plane (see monograph [13b] for the general case)

$$x^{1'} = x^1 \times \cos[\theta \times (n_1 \times n_2)^{-1}] - x^2 \times \frac{n_1^2}{n_2^2} \times \times \sin[\theta \times (n_1 \times n_2)^{-1}], \tag{2.6a}$$

$$x^{2'} = x^1 \times \frac{n_2^2}{n_1^2} \times \sin[\theta \times (n_1 \times n_2)^{-1}] + x^2 \times \cos[\theta \times (n_1 \times n_2)^{-1}], \tag{2.6b}$$

The isotopies of the $SU(2)$ symmetry were outlined in Section 1.9.

It was popularly believed in the 20th century that the $SO(3)$ symmetry is broken for the ellipsoidic deformations of the sphere. However, it is easy to prove that $\hat{SO}(3)$ is isomorphic to $SO(3)$, e.g., because, in the transition from a Lie symmetry to its isotopic covering, the original generators, parameters and structure constants remain unchanged.

Conceptually, this is due to the fact that ellipsoid deformations of the semiaxes of the perfect sphere are compensated on isospaces over isofields by the *inverse* deformation of the related unit

$$Radius \quad 1_k \to 1/n_k^2, \quad Unit \quad 1_k \to n_k^2. \tag{2.7}$$

resulting in the reconstruction of the perfect sphere on isospace called the *isosphere*,

$$\hat{r}^{\hat{2}} = \hat{r}_1^{\hat{2}} + \hat{r}_2^{\hat{2}} + \hat{r}_3^{\hat{2}}. \tag{2.8}$$

with consequential reconstruction of the exact rotational symmetry.

Alternatively, we can say that the reconstruction of the exact rotational symmetry is due to the structure of the basic invariant given by

$$L^2 = (length)^2 \times (unit)^2 \tag{2,9}$$

Consequently, a change of lengths joint with the inverse change of units leaves the invariant unchanged.

Similarly we have the reconstruction of the exact isospin symmetry in nuclear physics under electromagnetic interactions via the simple mechanism of embedding all symmetry breaking terms in the isounit [34].

(2) **Regular Lorentz-Santilli isotransforms** $\hat{SO}(3.1)$, first identified in Ref. [35] here presented for simplicity in the (3-4)-plane (see monograph [13b] for the general case)

$$x^{1'} = x^1, \quad x^{2'} = x^2, \tag{2.10a}$$

$$x^{3'} = \hat{\gamma} \times (x^3 - \hat{\beta} \times \frac{n_3}{n_4} \times x^4), \tag{2.10b}$$

$$x^{4'} = \hat{\gamma} \times (x^4 - \hat{\beta} \times \frac{n_4}{n_3} \times x^3), \tag{4.10c}$$

where

$$\hat{\beta}^2 = \frac{v_3^2/n_3^2}{c_o^2/n_4^2}, \quad \hat{\gamma} = \frac{1}{\sqrt{1 - \hat{\beta}^2}}. \tag{2.11}$$

The isotopies of the spinorial covering of $SO(3.1)$ were studied or the first time in Ref. [38]. (see also monograph [13b]).

Again, it was popularly believed in the 20th century that the Lorentz symmetry is broken for deformations of the light cone. By contrast, Ref. [37] proved that the Lorentz symmetry does remain exact under the deformations of the light cone, provided it is treated with the appropriate mathematics. This result was achieved via the proof of the local isomorphism between $\hat{SO}(3.1)$ and $SO(3.1)$.

Conceptually, this is due to the reconstruction of the exact light cone on isospace over isofields called the *light isocone.* In fact, jointly with the deformation of the light cone

$$x^2 = x_3^2 - t^2 \times c^2 = 0 \to \frac{x_3^2}{n_3^2} - t^2 \times \frac{c^2}{n^4} = 0, \tag{2.12}$$

we have the corresponding inverse deformations of the units, thus reconstructing the original light cone on isospaces over isofields,

$$\hat{x}^{\hat{2}} = \hat{x}_3^{\hat{2}} - \hat{t}^{\hat{2}} \times \hat{c}^{\hat{2}} = 0. \tag{2.13}$$

The reader should be aware that the above reconstruction includes the preservation on isospace over isofields of the original characteristic angle of the conventional light cone. Consequently, *the maximal causal speed on isospace over isofields is the conventional speed of light in vacuum c.*

The understanding of the isoscattering theory requires the knowledge that *in the transition from the exterior to the interior region (as depicted in Figure 2 of Paper I), the speed of light and related light cone remain unchanged, and only their realizations change.*

(3) **Regular isotranslations** $\hat{T}(4)$, first studied in ref. [37] (see monograph [13b] for the general case). can be expressed with the following lifting of the conventional translations $x^{\mu'} = x^\mu + a^\mu, \mu = 1, 2, 3, 4$, and $a^\mu$ constants,

$$x^{\mu'} = x^\mu + A^\mu(a, \ldots), \tag{2.14}$$

where

$$A^\mu = a^\mu(n_\mu^{-2} + a^\alpha \times [n_\mu^{-2}\hat{,}P_\alpha]/1! + \ldots), \tag{2.15}$$

and there is no summation on the $\mu$ indices.

Readers should note the highly nonlinear character of the above translations. Nevertheless, the components of the linear momentum isocommute,

$$[P\mu\hat{,}P\nu] = P\mu \times T \times P\nu - P\nu \times T \times P\mu = 0, \ \mu, \nu = 1, 2, 3, 4. \tag{2.16}$$

while the generators of nonlinear transformations (2.14)-(2,15) do not commute in conventional space,

$$[P\mu, P\nu] = P\mu \times P\nu - P\nu \times P\mu \neq 0, \ \mu, \nu = 1, 2, 3, 4. \tag{2.17}$$

As we shall indicate in the next section, the above occurrence illustrates the capability by isorelativity of turning *curved* spaces into an equivalent *isoflat space* [40] with nontrivial implications, such as a consistent operator formulation of gravity, new grand unification, new interior gravitational models and other advances.

Recall that the conventional scattering theory is based on the conventional Minkowski space and, therefore, has no gravitational contribution of any type. By contrast, the above features imply that *the isoscattering region is isoflat and, therefore, with a primary gravitational, Finslerian and other contributions.*

4) **Regular isodilations** $\hat{\mathcal{T}}(1)$, first identified in Ref. [37] (see, again, monograph [13b] for the general case)

$$\hat{\eta} \to \hat{\eta}' = w^{-1} \times \hat{\eta},\ \hat{I} \to \hat{I}' = w \times \hat{I}, \tag{2.18a}$$

$$(x^\mu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I} \equiv [x^\mu \times (w^{-1}\hat{\eta}_{\mu\nu}) \times x^\nu) \times (w \times \hat{I}) =$$
$$= (x^\mu \times \hat{\eta}'_{\mu\nu} \times x^\nu) \times \hat{I}',\ w \in \mathcal{R}. \tag{2.18b}$$

As one can see, *the LPS isosymmetry is eleven dimensional,* the 11-th dimensionality being given by the new *invariance* under isodilations similar to that of the Hilbert space, Eq. (3.19) of Paper I. Contrary to popular beliefs in the 20th century, *the conventional Lorentz-Poincaré symmetry is also eleven dimensional,* trivially, because isoinvariance (2.18) also applies to the standard case.

Readers should keep in mind the transition from the conventional dilations, often requiring an enlargement of the spacetime dimensions, to their invariant formulation under isotopies within the conventional (3.1)-dimensions.

Predictably, the discovery of a new symmetry for the conventional spacetime has momentous implications. In fact, isosymmetry (2.18) was instrumental for the first known axiomatically consistent operator formulation of gravity [44], grand unification of electroweak and gravitational interactions [45,46], and other basic advances.

The reason that the eleventh dimensionality was not discovered until Ref. [37] of 1993 should not be surprising since the new isosymmetry required the prior discovery of *new numbers,* the isonumbers with arbitrary units [6].

The resulting eleven-dimensional regular LPS isosymmetry can be written

$$\hat{P}(3.1) = [\hat{SO}(3.1)\hat{\times}\hat{\mathcal{T}}(3.1)] \times \hat{\mathcal{C}}(1), \tag{2.19}$$

where $\hat{\times}$ is the direct isoproduct and $\times$ is the Kronecker product.

Readers should finally keep in mind the "direct universality" of the LPS symmetry indicated in Section 2.1 since said symmetry provides the universal *invariance* (rather than covariance) of all infinitely possible line elements (2.3) that include as particular cases the Riemannian, Finslerian or other line elements in $(3+1)$-dimensions [41-43].

the isotopies of the spinorial covering of the LP symmetry will be indicated in Section 3 jointly with the isotopies of Dirac's equation.

**2.4. Deformations-Isotopies of Special Relativity.** As indicated in Section 2.1, a central objective of the studies here considered is the preservation of Einstein

axioms for interior conditions and the enlargement of their applicability via broader realizations. This objective was achieved first in paper [35] of 1983. Comprehensive elaborations were presented in monographs [12] of 1991 that include the isotopies of Galilei relativity we cannot possibly review here. Additional presentations are available in monographs [13] of 1995 and [16] of 2008.

As a result of these studies, *special relativity and its covering isorelativity can be presented at the abstract mathematical level via the same equations and axioms, merely submitted to different realizations, with a similar result holding between Galilei's relativity and its isotopic covering,* as illustrated, e.g., by the abstract identity of the light isocone with the conventional cone, Eq. (2.13), the preservation on isospacetime of the speed of light in vacuum as the maximal causal speed, and other features.

Regrettably, we cannot possibly review these studies. Nevertheless, it is evident that an abstract mathematical presentation would lead to possible misinterpretations in the applications of isorelativity to scattering processes or prevent them altogether. Therefore, for minimal self-sufficiency of these papers as well as to specialize the new laws to the interior of the scattering region, we recall the following isoaxioms in their *projection* on conventional spacetime restricted to the third space direction and time (see Refs. [13] for the general case):

*ISOAXIOM I: The maximal causal speed within the scattering region is given by*

$$\hat{V}_{max} = c \times \frac{n_3}{n_4} = C \times n_3. \tag{2.20}$$

*ISOAXIOM II: The addition of speeds within the scattering region follows the isotopic law*

$$V_{tot} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2} \times \frac{n_4^2}{n_3^2}}. \tag{2.21}$$

*ISOAXIOM III: The dilation of time, the contraction of space and the variation of mass with speed within the scattering region follow the isotopic laws*

$$t' = \hat{\gamma} \times t, \tag{2.22a}$$

$$\ell' = \hat{\gamma}^{-1} \times \ell, \tag{2.22b}$$

$$m' = \hat{\gamma} \times m, \tag{2.22c}$$

*ISOAXIOM IV: The frequency shift within the scattering region follows the isotopic law*

$$\omega' = \hat{\gamma} \times [1 - \hat{\beta}\ cos(\hat{\alpha}) \times \omega, \tag{2.23}$$

*ISOAXIOM V: The mass-energy isoequivalence within the scattering region follows the isotopic law*

$$\hat{E} = m\ V_{max}^2 = m \times c^2 \times \frac{n_3^2}{n_4^2} = m \times C^2 \times c_3^2. \tag{2.24}$$

A few comments are now in order. It should be stressed that, at the pure mathematical level, e.g., when formulated on isospace over isofields, the above isoaxioms coincide with conventional axioms. Consequently, *the isoscattering theory preserves Einstein's axioms everywhere in the exterior and the interior region, and merely uses a broader realization of the same axioms for the interior case.* As we shall see in the next section, we have the same occurrence in regard to the applicable mechanics because relativistic hadronic mechanics also coincides with relativistic quantum mechanics at the abstract mathematical level by conception and construction.

Interested readers are then suggested to inspect: the unique and unambiguous derivation of Isoaxioms I-V from the LPS isosymmetry [12,13,16], and the proof of their "direct universality" by J. V. Kadeisvili [41], A. K. Aringazin [42], and others. Therefore, *Isoaxioms I-V are the only known directly universal axioms for interior conditions that are invariant under a universal isosymmetry for arbitrary speeds of light.*

Rather unexpectedly, the above studies have established the *necessity of abandoning the speed of light as the maximal causal speed within physical media* because of unsurmontable difficulties in the use of the speed of light it in vacuum, the understanding being that the speed of light is indeed regained as maximal causal speed in empty space and isospace.

As an illustration, the use of special relativity within water leads to insufficiencies of one or the other axioms, e.g., if one assumes in water the maximal causal speed in vacuum, causality is preserved when electrons travel in water faster than the local speed of light, but the relativistic addition of speeds is violated. Vice-versa, the assumption of the speed of light in water as the maximal causal speed in water preserves the relativistic law of addition of speeds, but violates causality.

By comparison, isorelativity resolves these insufficiencies. In fact, water can be safely assumed as being homogeneous and isotropic. As a consequence, $n_3 = n_4$, and the maximal causal speed in water is the speed of light in vacuum, $V_{max} = c$, thus verifying causality, while the isorelativistic law of addition of speeds (4.21) is also verified [12,13,16].

A serious knowledge of the isoscattering theory requires a study of the experimental verification of isorelativity at the classical and operator levels presented in various papers and reviewed in monograph [16d], Chapter 5 of ref. [27] and paper 47. Some of the most important experimental verifications of relativistic quantum mechanics will be indicated in Section 3. An important, classical, verification has been that of Isoaxiom IV on the shift of light within physical media first presented in paper [47], and indicated in more details in the next section. Additional direct verifications are under way based on the repetition within physical media of the historical experimental verifications of special relativity all that were notoriously conducted in vacuum.

The specialization of isorelativity to the iso-Galilean case is an instructive exercise for the interested reader [12].

**2.5. Implications for the Scattering Region.** It is now important to identify the implications of isorelativity, specifically, for the interior of scattering processes because said implications will eventually be verified or dismissed by experiments in due time. Among a variety of rather intriguing implications, we identify the following ones:

2.5.1) **Isotime.** The time in the interior of the scattering region is no longer the same as our time, and it is given by a quantity known as *isotime* with vast physical and epistemological implications. In fact, the light cone is recovered identically on isospacetime, Eq. (2.13), but under the lifting of time into the isotime,

$$t \to \hat{t} = t/n_4. \tag{2.25}$$

By recalling that, according to all known fits of isorelativity, the index of refraction within hadronic matter is smaller than one, $n_4 < 1$ [16d,27], the reconstruction of the speed of light in vacuum as the maximal causal speed in interior conditions implies that *the scattering region is represented on isospacetime over isofields via an isotime generally in the future with respect to our time,*

$$\hat{t} = t/n_4 > t,\ n_4 < 1,\ C > c. \tag{2.26}$$

Readers with a technical knowledge of the Lie-Santilli isotheory will see that the replacement in the interior scattering region of our time with isotime is *necessary* to reach the universal invariance of the locally varying speed of light, since said invariance is based on the lifting of spacetime units, including that of time. Alternatively, the use of the isotime for interior conditions is necessary to avoid the inconsistency theorems.

2.5.2) **Isoshift,** For the case of null angle of aberration, and for $v = |v| << c$, the conventional Doppler's shift can be well approximated with the expression

$$\omega' = (1 \pm \frac{v}{c} + ...) \times \omega, \tag{2.27}$$

where the sign $-$ represents motion of the source *away* from the observer, in which case we have the *Doppler redshift,* and the sign $+$ represents motion of the source *toward* the observer, in which case we have the *Doppler blueshift.* It is evident that *special relativity predicts no shift of the frequency of light of any type for* $v = 0$,

The predictions of isorelativity are considerably different than the above. In fact, under the same assumptions as above, isolaw (2.13) can be written in good approximation

$$\omega' = (1 \pm \frac{v}{c} \times \frac{n_4}{n_3} + ...) \times \omega. \tag{2.28}$$

The most important implication, identified since 1991 [12], is that *isorelativity predicts a shift of the frequency of light even in the absence of any relative motion between the source, the medium and the observer,* called *isoshift.* This is due to the fact that the characteristic functions are generally dependent on the velocities (Section 2.2). Consequently, it is possible to have a dependence of the type $n_3/n_4 = v \times f(x, \xi, \omega, ...)$ under which expression (4.28) becomes

$$Lim_{v=0}\, \omega' = (1 \pm f(x, \xi, \omega, ...) + ...) \times \omega. \tag{2.29}$$

in which case *the frequency shift is no longer necessarily null for null relative speeds,* thus implying the following cases:

2.5.2A) **Santilli isoredshift.** It occurs for physical media of low density and has been experimentally verified in air, for certain astrophysical bodies, and other interior conditions [47]. This case essentially implies that *light loses energy $E = h\nu$ when propagating within physical media of low density, thus experiencing a decrease of its frequency without relative motion.* This possibility has been used in Ref. [47] to indicate the possible absence of universe expansion, big bang and dark matter since they all refer to motion of light within physical media in which the Doppler law not necessarily applies.

2.5.2B: **Santilli isoblueshift.** It is predicted for physical media of very high density, such as those in the interior of astrophysical bodies as well as in the interior of high energy scattering processes. In this case, we have the prediction that *light acquires energy $E = h\nu$ from the hyperdense medium, thus increasing its frequency.* Since all high energy scattering regions are hyperdense, *isorelativity predicts that photons detected outside the scattering region are isoblueshifted,* namely, they originate in the interior at a frequency smaller than that detected in the outside [47].

2.5.2C) **Doppler-Santilli isoshift.** It occurs when light propagates within a transparent physical medium but there exist a relative motion between the source, the medium and the observer, which relative motion remains fully relativistic. Note that there are several cases in which *the total shift can be null even though there exist a relative motion between the source and the observer,* since both, Doppler's and Santilli's shifts can be positive or negative depending on the case at hand.

2.5.3) **Isoequivalence.** Maximal causal speeds bigger than $c$ are prohibited by special relativity because at the value $v = c$ the conventional axioms diverge, as it is the case for the mass

$$: im_{v=c} m' = Lim_{v=c} \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} = \infty. \tag{2.30}$$

This is no longer the case for isorelativity because it does not predict infinities at $v = c$ due to the indicated dependence of the characteristic quantities on the speed, for which we may have the expression for the mass

$$Lim_{v>c}\, m' = \frac{m}{\sqrt{1 - f(x, \xi, ...)}}. \tag{2.31}$$

Consequently, *Santilli isorelativity allows within physical media maximal causal speeds bigger than the speed of light in vacuum,* with far reaching implications.

For instance, Einstein's historical energy equivalence $E = m \times c^2$ was limpidly referred to *point-like particles in vacuum,* under which conditions it has received vast experimental verifications. However, the same equivalence principle has never received experimental verifications for extended and hyperdense particles, for which it is merely assumed as being valid, again, since special relativity does not allow any alternative formulation.

By contrast, isorelativity allows unlimited maximal causal speeds and the LPS isosymmetry uniquely and unambiguously predicts isoequivalence (2.24). By recalling again that all fits of experimental data available to date suggest causal speeds inside hadronic media bigger than $c$, *the energy equivalence of the scattering region predicted by isorelativity is bigger than that predicted by the conventional scattering theory,*

$$\hat{E} = m \times c^2 \times \frac{n_3^2}{n_4^2} > m \times c^2, \tag{2.32}$$

It should be indicated that isoequivalence (2.32) has been used in Ref. [47] to indicate the possible absence of dark energy due to the prediction of a total energy in the universe much bigger than that predicted by special relativity, since the maximal causal speed in the interior of black holes and other astrophysical bodies is predicted as being much bigger than $c$. The possible verification in scattering experiments of the isoblueshift would evidently confirm isoequivalence (2.24) and, therefore, the absence of dark energy.

2.5.4) **Isoparticles.** One of the most insidious misrepresentations for the isoscattering theory is the use of the conventional notion of particle. As it is well known, particles are generally referred to as irreducible unitary representations of the LP symmetry. However, such a symmetry is assumed as being inapplicable for the interior of the scattering region by central assumption, thus leading to major inconsistencies when using the conventional notion of particle under isotopies. Rather insidiously, these inconsistencies often remain undetected by non-experts in the new field.

Another implication of isospacetime mutation is that, *in the transition from motion in vacuum to motion within hyperdense scattering regions, particles experience a mutation of their "intrinsic characteristics," also called isorenormalization resulting*

*in the notion of isoparticles characterized by irreducible isounitary representations of the covering LPS isosymmetry* [12,13,16]. The covering notion of isoparticle requires an in depth knowledge of *regular and irregular isorepresentations of Lie-Santilli isoalgebras,* that have been illustrated in Section 1.9 for the case of spin 1/2.

In essence, conventional renormalizations, those characterized by action-at-a-distance, potential interactions, leave unchanged the intrinsic characteristics of particles. However, when passing to the broader non-Hamiltonian interactions, their nonunitary structure causes an isorenormalization of all characteristics of particles, including intrinsic features, via the mechanism of alteration of Hamiltonian eigenvalues pointed out in Paper I, and more technically identified in Section 1.9 as occurring under irregular isosymmetries. Note that isorenormalizations are fully in line with isoshifts.

Experimentally, the evidence supporting the mutation of intrinsic characteristic of particles in interior conditions is rather vast, and includes particle physics. nuclear physics, chemistry and astrophysics [*loc. cit.*]. Theoretically, the quantitative study of mutations of intrinsic characteristics of particles is perhaps the most fascinating of the isoscattering theory because, contrary to popular beliefs in the 20th century, our knowledge of the irreducible representations of the abstract axioms of the Lorentz symmetry is at its infancy, as illustrated by their current vastly unknown irregular isorepresentations.

As we shall see in subsequent papers, the quantitative treatment of the isorenormalization of particles inside the scattering region is one of the most challenging problem for a serious appraisal of the isoscattering theory on mathematical, theoretical and experimental grounds.

2.5.5) **Isogravitation.** As indicated earlier, the Riemannian formulation of gravitation has no appreciable impact in the conventional scattering theory, and any attempt at its inclusion is faced with very serious consistency problems, such has the unavoidable lifting of the scattering theory to nonunitary forms, loss of quantum mechanics, lack of conservation over time of the numerical predictions, and other problems [48].

By contrast, a direct, axiomatically consistent impact of gravitation in scattering processes becomes unavoidable for the covering isoscattering theory. This important aspect can be initially seen from isoinvariant (2,4) showing that *the Minkowski-Santilli isogeometry and related isospacetimes includes as particular cases the (pseudo-) Riemannian, Finslerian and all other possible geometries and related spacetimes in* $(3+1)$*-dimensions* [37].

The transition from the conventional to the isotopic formulation of gravity is provided by the following steps first proposed in Ref. [44] of 1994

I) Factorizing any possible (nonsingular, pseudo-) Riemannian, Finslerian, or other

metric $g(x,...)$ into the *Minkowskian* metric $\eta$ and a $4 \times 4$ matrix $\hat{T}_{gr}(x,...)$,

$$g(x,...) = \hat{T}_{gr}(x,...) \times \eta, \tag{2.33}$$

II) Introducing the *gravitational isounit* as the *inverse* of the matrix $\hat{T}_{gr}(x,...)$,

$$\hat{I}_{gr}(x,...) = \hat{T}_{gr}(x,...)^{-1}. \tag{2.34}$$

III) Reconstructing the Minkowskian geometry, the LP symmetry and special relativity with respect to the above gravitational isounit.

Since $\hat{T}_{gr}(x,...)$ is necessarily positive-definite for all nonsingular Riemannian, Finslerian or other metrics in $(3+1)$-dimensions, the resulting LPS isosymmetry is isomorphic to the conventional LP symmetry, thus allowing the treatment of gravitation with all the formulations studied so far in these papers, as well as those we shall study in the future. The resulting formulation of gravity is today known as *Santilli isogravitation.*

As an illustration, the celebrated Schwarzschild line element in the coordinates $(\theta, \phi, r, t)$ admits the following *identical* reformulation as the isometric in isospacetime

$$ds^2 = r^2(d\theta^2 + sin^2 d\theta^2 + d\phi^2) + (1 - \frac{2 \times M}{r})^{-1} \times dr^2 -$$

$$-(1 - \frac{2 \times M}{r} \times dt^2 \equiv \hat{T}_{sch} \times \eta \equiv \hat{\eta}, \tag{2.35}$$

with *gravitational isounit and isotopic element*

$$\hat{T}_{sch} = Diag.[1, 1, (1 - \frac{2 \times M}{r})^{-1}, (1 - \frac{2 \times M}{r})], \tag{2.36a}$$

$$\hat{I}_{sch} = Diag.[1, 1, (1 - \frac{2 \times M}{r}), (1 - \frac{2 \times M}{r})^{-1}], \tag{2.36b}$$

where one should note the positive-definiteness of the gravitational isounit and we assume the reader is aware from Paper I of the need for isotrigonometry in the isotopic reformulation, hereon tacitly assumed.

The implications at large of the above formulation of gravitation are far reaching, and their specializations to scattering processes should be at least summarily outlined here due to their significance, such as the clear prediction presented in Paper III that very high energy scattering experiments can indeed generate mini-black-holes.

Let us begin our short outline with the following important

*LEMMA 2.5.1 [40,44]: The isotopic reformulation of the Riemannian gravitation implies the loss of curvature in favor of the isoflatness of the Minkowski-Santilli isogeometry.*

This fundamental result can be seen in a variety of ways, e.g., from the fact that, by conception and construction outlined in Section 2.2, the Minkowski-Santilli isogeometry is locally isomorphic to the *Minkowski* geometry, thus prohibiting any conventional notion of curvature. Alternatively, one can see the loss of curvature on a conceptual basis by noting that gravitation is entirely contained in the isotopic element $\hat{T}_{gr}$. Consequently, the deformation of the Minkowski metric caused by gravitation

$$\eta \ \rightarrow \ \hat{T}_{gr} \times \eta = \hat{\eta}, \tag{2.37}$$

is compensated by the *inverse* deformation of the unit

$$I = Diag.(1,1,1,1) \ \rightarrow \ \hat{I}_{gr} = (\hat{T}_{gr})^{-1}, \tag{2.38}$$

without altering the original flatness in view of the novel isodilation symmetry of the Hilbert space, Eq. (3.19) of Paper I, the new isodilation invariance (2.18), or the very structure (2.9) spacetime invariants. In turn the loss of curvature in favor of isoflatness has the following implications rather important for scattering processes:

2.5.5A) **Consistent operator form of gravitation.** As it is well known, a consistent operator formulation of the Riemannian gravitation acceptable by the scientific community at large has not been achieved in one century of efforts due to unsurmontable problematic aspects or sheer inconsistencies caused by curvature, the ensuing nonunitary character of the theory, lack of the PCT theorem and other problems [48]. By comparison, isogravitation admits an axiomatically consistent operator formulation first achieved in ref. [44] merely given by embedding gravity in the *unit* of relativistic quantum mechanics, thus preserving its abstract axioms, and ensuing consistency, including the correct formulation of the PCT theorem and all that. Note that this result *cannot* be achieved with the Riemannian curvature (see Ref. [16c] for details).

2.5.5B) **Universal invariance of gravitation.** As it is well known, the conventional Riemannian formulation of gravitation solely admits a '"covariance." But its structure is notoriously noncanonical, thus activating the theorems of catastrophic inconsistency (see Ref. [48] for details). By comparison, the isotopic formulation of gravity admits the universal LPS isoinvariance with the resolution of said inconsistency problems. Again, the reader should keep in mind that the invariance of gravitation is impossible with the Riemannian curvature (see Refs. [37.44.16c] for details).

2.5.5C) **Unification of the Minkowskian and Riemannian geometries.** Traditionally, the Minkowskian and Riemannian geometries are differentiated, as it should be, when formulated on conventional spaces over conventional fields. However, the use of isospaces over isofields has allowed the unification of these two geometries into one single geometry, the Minkowski-Santilli isogeometry, and their differentiation via different isounits first achieved in Ref. [40]. But the isometric $\hat{\eta}(x,...)$ has an explicit dependence on coordinates and other variables. Consequently, the Minkowski-Santilli isogeometry admits the entire machinery of the Riemannian geometry., such as covariant derivative., Christoffel's symbols, etc. only isotopically reformulated, with consequential *geometric unification of special and general relativities.* This result has the consequence, rather important for scattering processes, according to which *the Einstein-Hilbert field equations are preserved and identically reformulated in an invariant operator version for the interior of the scattering region.* Note again that this result would be inconsistent under a Riemannian curvature on a number of grounds [48].

2.5.5D) **Isotopic grand unification.** It is equally well known that a grand unification of electroweak and gravitational interactions in a form acceptable by the scientific community at large has escaped all efforts beginning with Einstein. It is today known that the difficulties originate from: A) Inconsistencies in unifying a theory possessing an invariance with another theory solely possessing covariance (due to the activation by the latter of the inconsistency theorems for the entire unification [44]); B) Inconsistencies in unifying an operator theory on a flat spacetime with another on a curved spacetime (due to the ensuing nonunitary structure and activation, again, of the inconsistency theorems); and C) Inconsistencies in unifying a theory with full democracy between particles and antiparticles with a gravitational theory insufficient for the description of antiparticles. e/g/., without any distinction whatsoever between neutral particles and antiparticles (see Section 2.5). Thanks to the removal of curvature and the achievement of an invariant operator formulation, isogravity has resolved insufficiencies A, B, C, resulting in an axiomatically consistent *iso-grand-unification* in which gravitation is embedded in the unit of electroweak theories, first achieved in Refs. [45,46] (see monograph [15] for a comprehensive presentation including the necessary gravitational treatment of neutral or charged antimatter).

2.5.5E) **Interior gravitation.** As indicated in Paper I, prior to Einstein's time, there was a clear differentiation between exterior and interior problems. In fact, Schwarzschild wrote *two* papers, the first one for the *exterior gravitational problem* with his historical metric (2.35) and a vastly ignored second paper on the *interior gravitational problem.* The distinction between exterior and interior problems was then ignored for about one century via the abstraction of the latter problems to

isolated point-particles in vacuum. The No Reduction Theorems reviewed in Paper I have suggested a return to the full differentiation between exterior and interior gravitational problems, thus relegating metric (2.35) to the meaning intended by its originator, namely, for the exterior problem only. The advent of isogravitation has permitted significant advances in interior gravitational; problems, e.g., by achieving for the first time a direct geometric representation (that is, a representation via the isometric) of the locally varying speed of light, the density of the interior medium and other features. Gravitational collapse is then represented with the *limit of null value of the space component of the isounit and the limit to a divergent value of its time component,* as geometrically expected in any case, i.e.,

$$\hat{I}^{int}_{gr,space}(x, v, \xi, \omega, \psi, \partial\psi, ...) \to \infty, \tag{2.39a}$$

$$\hat{I}^{int}_{gr,time}(x, v, \xi, \omega, \psi, \partial\psi, ...) \to 0. \tag{2.39b}$$

To understand the above reformulation of gravitational collapse, one should keep in mind that *isotopic rules (2.39) are equations that can be solved not only in the coordinates, as it is the case for the Schwarzschild metric (2.35), but also in the velocities and other variables as it is necessary for realistic models of interior gravitation.* The issue as to whether a true singularity such as the notion of *black hole,* is preserved by interior isogravitation, or we merely have a gravitational collapse without singularity, such as the notion of *brown hole,* is under study at this writing and the outcome will be reported in a future paper.

## 3. Deformations-Isotopies of Mechanics

**3.1. Introduction.** The central dynamical problem of the isoscattering theory is that at the foundation of hadronic mechanics, namely, the achievement of a consistent operator formulation of nonconservative systems, called in these papers non-Hamiltonian systems. We are here referring to a problem that remained unsolved for most of the 20th century physics, whose solution required several decades of trials and errors. We believe it is important to outline the main points of this scientific journey so as to avoid potential misrepresentation of the proposed isoscattering theory in the event based on the older literature, or the use of mechanics that have been proved as being inapplicable.

A considerable number of applied mathematicians, theoretical physicists and experimentalists have contributed to the deformations-isotopies of classical and quantum mechanics to represent non-Hamiltonian systems, including in chronological order from 1978 to 1983: R. M. Santilli, H. C. Myung, S. Okubo, J. Fronteau, A Tellez-Arenas, R. Mignani, A. O. E. Animalu, J. A. Kobussen, Y. Ylamed, N. Salingaros, T. Giill, A. J. Kalnay, H. Rauch, G. Eder, P. Caldirola, R. Trostel, A. Schober,

R. J. Slobodrian, J. Sun, A. de Wet, A. D. Jannussis, G. Brodimas, D. S. Sourlas, N. Salingaros, N. Tsagas, D. P. K. Ghikas, E. Kapushik, F. Rohrlich, J. Snyaticki, N. Salingaros, P. Truini, G. Cassinelli, G. Lochak, D. Y. Kim, J. Salmon, M. Grmela, E. Tontio, J. G. Gilson, V. K. Agrawala, W. H. Steeb, M. Mijatovich, R. Broucke, and numerous others in the subsequent years. Regrettably, we cannot possibly review the studies by so many authors and have to restrict our outline to the aspects directly relevant for the isoscattering theory (see the comprehensive bibliography in Volume [16a]).

The insufficiency that prevented the achievement of a consistent operator formulation of non-Hamiltonian systems for such a long time was the absence of their universal representation via an action principle due to the crucial role of the latter for a consistent map to operator forms.

The scientific journey initiated with Volume [10a] in which it was established that non-Hamiltonian systems in two or more dimensions generally violate the integrability conditions for the existence of a Hamiltonian, the *conditions of variational selfadjointness*. Consequently, non-Hamiltonian systems in more than one dimension do not admit an analytic representation via the conventional *Hamiltonian action principle.*

The scientific journey then continued with Volume [10b] achieving the '"direct universality" for the representation of all well behaved non-Hamiltonian systems via the *Birkhoffian action principle.* However, the latter action principle resulted in being inapplicable for the needed operator map for various reasons, such as the fact that the emerging "wavefunctions" would depend on both coordinates and momenta, $\psi = \psi(t, r, p)$, thus having a structure beyond our current knowledge of operator mechanics. and departing in any case from the abstract axioms of quantum mechanics.

Following the abandonment of the Birkhoffian mechanics, numerous additional mechanics were attempted (as listed, e.g., in the 1991 edition of Refs. [13]), but they all had a noncanonical time evolution formulated on conventional spaces over conventional fields, thus activating the inconsistency theorems of Paper I.

The needed breakthrough finally occurred with the discovery in memoir [7] of 1996 of the *isodifferential calculus* that permitted the achievement of the needed axiom preserving isotopies of the classical Hamiltonian mechanics with a consistent map to the operator image that resulted indeed to be an isotopy of quantum mechanics, as desired.

There was a similar impass in the construction of hadronic mechanics. Following the original proposal [5] with the basic Lie-isotopic and Lie-admissible dynamical equations, all possible isotopies and genotopies of the Hilbert space, functional analysis, Lie algebras, etc., were reached by the early 1980s. Nevertheless, hadronic mechanics failed to be invariant, thus verifying the inconsistency theorems of nonunitary theories. Additionally, and perhaps most insidiously, hadronic mechanics missed

consistent isotopies and genotopies of the Schrödinger realization of the linear momentum, thus prohibiting practical applications, e.g., because of the lack of a consistent formulation of the angular momentum.

The needed breakthrough occurred, again, with memoir [7] thanks to the discovery of the isotopies and genotopies of the differential calculus that finally permitted, after two decades of failed attempts, the achievement of consistent isotopies and genotopies of the linear and angular momenta, with consequential basic invariance over time.

In conclusion, readers interested in a serious study of the isoscattering theory should be aware that the various classical and operator mechanics existing in the literature prior to the generalized differential calculi of Ref. [7] are either inapplicable to scattering processes, or they verify the theorems of catastrophic inconsistencies.

**3.2. Deformations-Isotopies of Newtonian Mechanics.** The isoscattering theory is based on the *isotopic branch of hadronic mechanics,* also known as *isomechanics* [4,5, 13,16]. Its primitive notion is that of *closed-isolated non-Hamiltonian systems* introduced in monograph [10b] defined as systems with Hamiltonian (*variationally selfadjoint,* SA) and non-Hamiltonian (*variationally nonselfadjoint,* NSA) internal forces, yet verifying the conventional ten total conservation laws of the Galilei and Lorentz-Poincaré symmetries. We are referring to the following class of Newtonian systems

$$m_k \times \frac{dv_k}{dt} - F_k^{SA}(t,r) - F_k^{NSA}(t,r,v,...) = 0, \quad k = 2,3,4,...,N, \tag{3.1}$$

whose NSA forces verify the closure conditions

$$\sum_k \mathbf{F}_k^{NSA} = 0, \tag{3.2a}$$

$$\sum_k \mathbf{r}_k \bigodot \mathbf{F}_k^{NSA} = 0, \tag{3,2b}$$

$$\sum_k \mathbf{r}_k \bigwedge \mathbf{F}_k^{NSA} = 0, \tag{3.2c}$$

with trivial relativistic extension, under which the conventional ten conservation laws are automatically verified.

Prior to Refs. [10], it was popularly believed that only Hamiltonian systems verify the conventional ten conservation laws. Refs. [10] dispelled this unsubstantiated belief by showing that interior dynamical problems of isolated systems, such as Jupiter, do indeed include contact, nonconservative, non-Hamiltonian internal forces, as established by nature in any case.

Hence. it is important for this paper to clarify that the basic assumption of the isoscattering theory, the admission of contact non-Hamiltonian forces for the interior scattering region, originates at the primitive Newtonian level, as necessary from the *No Reduction Theorems* reviewed in Paper I.

Note that the verification of conventional total conservation laws is crucial for isomechanics and the isoscattering theory. In fact, when the system is open we have the broader *Lie-admissible genomathematics and genomechanics* [13,16].

The next step is the representation of systems (3.1) with the isotopic lifting of Newton's equations, first introduced in Refs. [7,13b] and known as *Newton-Santilli isoequations,* permitting the first known broadening of Newton's equation since Newton's time for the representation of extended, nonspherical and deformable particles under SA and NSA forces.

Far from being trivial, the referred structural broadening of Newton's equation required a structural generalization of the entire mathematics of Newton's equations, including numbers, vector spaces, functional analysis, as well as the very differential calculus discovered by Newton himself (with Leibnitz).

Despite its diversification, this vast effort remained insufficient for the desired structural broadening of Newton's equation, that for the representation of *extended* particles, since such a representation is manifestly absent in Newton' s formulation. In fact, the background Euclidean geometry solely permits the representation of particles as being point-like. Maturity was finally achieved thanks to the construction by the mathematicians Gr. Tsagas, D. Sourlas, R. Falcon Ganfornina and J. Nunez Valdes of the all fundamental *isotopology* [29] that finally allowed the consistent representation of extended particles beginning at the primitive topological level.

The *Newton-Santilli isoequations*, first reached in Ref. [7], can be expressed for simplicity in the following form with the sole isodifferential in the velocities (see Ref. [13b] for the general form)

$$m \times \frac{\hat{d}\hat{v}}{\hat{d}t} - F^{SA} = m \times \frac{dv}{dt} - F^{SA} - F^{NSA} = 0. \tag{3.3}$$

with elementary solution for $v$ constant and $F^{NSA}$ not dependent explicitly on time (see [*loc. cit.*] for the general case)

$$\hat{I}_t = 1, \quad \hat{t} = t, \quad \hat{I}_r = 1, \quad \hat{r} = r, \tag{3.4a}$$

$$\hat{I}_v = exp(\frac{t}{mv} F^{NSA}), \quad \hat{v} = v \times \hat{I}_v. \tag{3.4b}$$

It is important for this paper to know that the main mechanism of the isoscattering theory, the embedding of the non-Hamiltonian interactions in a generalization of the basic unit, originates at the primitive Newtonian level. Note the emergence of a

realization of the isounit via the exponent, an occurrence that will result in being rather general for the isoscattering theory, as shown in Paper III.

Note finally the mechanism of the representation of non-Hamiltonian Newtonian systems consisting in turning their NSA form when formulated on conventional Euclidean space over conventional fields, into an *identical,* fully SA form when formulated on Euclid-Santilli isospaces over isofields. Such an identical SA form is then at the foundation of the subsequent representation via an isoaction principle and ensuing consistent map to operator isomechanics. Intriguingly, the achievement of this new representation required a generalization of the very differential calculus Newton had to develop for his original formulation.

**3.3. Deformations-Isotopies of Hamiltonian Mechanics.** the next central methodological branch of isomechanics is the analytic representation of Eqs. (3.4) via a *variational isoaction principle* also introduced in Refs. [7,13], here expressed in its general form for space and time isotopies

$$\hat{\delta}\hat{A}^o = \hat{\delta}\int_{t_1}^{t_2}(\hat{p}_k\hat{\times}\hat{d}\hat{r}^k - \hat{H}\hat{\times}\hat{d}\hat{t}) = \hat{\delta}\int_{t_1}^{t_2}[p_k\times\hat{T}^k_{\hat{r},\hat{t}}(t,r,p,...)\times\hat{d}\hat{r}^i - \hat{H}\times\hat{T}_{\hat{t}}\times d\hat{t}] = 0, \quad (3.5)$$

characterizing thhe *Hamilton-Santilli isoequations*

$$\frac{\hat{d}\hat{r}^k}{\hat{d}\hat{t}} = \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{p}_k}, \quad \frac{\hat{\partial}\hat{p}_k}{\hat{d}\hat{t}} = -\frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{r}^k}. \quad (3.6)$$

that provide a direct analytic representation of NSA Newtonian systems (3.4), as well as the *Hamilton-Jacobi-Santilli isoequations* [*loc. cit.*]

$$\frac{\hat{\partial}\hat{A}^o}{\hat{\partial}\hat{t}^d} + \hat{H} = 0, \quad (3.7a)$$

$$\frac{\hat{\partial}\hat{A}^o}{\hat{\partial}\hat{r}^k} - \hat{p}_k = 0, \quad (3.7b)$$

$$\frac{\hat{\partial}\hat{A}^o}{\hat{\partial}\hat{p}_k} \equiv 0. \quad (3.7c)$$

. The latter expression, evidently at the foundation of operator maps, illustrate again the fundamental role of the isodifferential calculus for all deformations-isotopies.

Note the formal identity at the abstract level of the conventional Hamiltonian mechanics and its isotopic image. This illustrates that the abstract axioms of Hamiltonian mechanics have representational capability dramatically broader than those

believed for centuries, although they can be seen only under the using the appropriate broader mathematics.

**3.4. Deformations-Isotopies of Quantum Mechanics.** The conventional naive quantization

$$A^o = \int_{t_1}^{t_2} (p_k \times dx^k - H \times dt) \to -i \times \hbar \times log\psi, \tag{3.8}$$

is lifted into the following *Animalu-Santilli isoquantization* [49] via the use of the isologarithm, Eq. (3.5) of Paper I,

$$\hat{A}^o = \int_{t_1}^{t_2} (\hat{p}_k \hat{\times} \hat{dx}^k - \hat{H} \hat{\times} \hat{dt}) \to -i \times \hat{log}\hat{\psi} = -i \times \hat{I} \times \hat{Log}\hat{\psi}, \tag{3.9}$$

with corresponding isotopies for the symplectic and other operator maps [13].

We are now equipped to present the operator image of the classical isotopies. The first nonunitary image of the Schrödinger equation was presented in the original proposal [5] to build hadronic mechanics, and then studied by several authors, such as Myung and Santilli [50], Mignani [51] and others (see the general bibliography in Ref. [16a]), although all these initial versions were formulated on conventional Hilbert spaces and/or over conventional fields, thus activating the inconsistency theorems reviewed earlier.

The axiomatically correct, and invariant, nonunitary isotopic image of Schrödinger equation was reached in Ref. [7] by applying map (3.9) to Eqs./ (3.7). The resulting equations are today known as *Schrödinger-Santilli isoequations* on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$ and can be written (see Refs. [13] for a detailed treatment)

$$\hat{i} \hat{\times} \hat{\partial}_{\hat{t}} \hat{\psi}(\hat{t}, \hat{r}) = \hat{H} \hat{\times} \hat{\psi}(\hat{t}, \hat{r}) = \hat{E} \hat{\times} \hat{\psi}(\hat{t}, \hat{r}), \tag{3.10}$$

with *isolinear momentum,* first formulated in 1995 [*loc. cit.*]],

$$\hat{p}_k \hat{\times} \hat{\psi}(\hat{t}, \hat{r}) = -\hat{i} \hat{\times} \hat{\partial}_k \hat{\psi}(\hat{t}, \hat{r}), \tag{3.11}$$

*canonical isocommutation rules*

$$[\hat{r}^{\hat{i}}, \hat{p}_j] = \hat{i} \hat{\times} \hat{\delta}^i_j, \quad [\hat{r}^{\hat{i}}, \hat{r}^{\hat{j}}] = [\hat{p}_{\hat{i}}, \hat{p}_j] = \hat{0}. \tag{3.12}$$

*isonormalization*

$$< \hat{\psi} | \hat{\times} | \hat{\psi} > \times \hat{I} = \hat{I}, \tag{3.13}$$

*isoexpectation values* of an iso-Hermitean operator $\hat{A}$

$$\hat{<}\hat{A}\hat{>} =< \hat{\psi}|\hat{\times}\hat{A}\hat{\times}|\hat{\psi} > \times\hat{I}, \tag{3.14}$$

and *isounit identities*

$$\hat{I}\hat{\times}\hat{\psi} = \hat{\psi}, \quad < \hat{\psi}|\hat{\times}\hat{I}\hat{\times}|\hat{\psi} >= \hat{I}. \tag{3.15}$$

assuring that $\hat{I}$ is indeed the correct basic unit of the theory (see Section 3 of Paper I for details).

The above equations are written on $\hat{\mathcal{H}}$ over $\hat{C}$ and can be written in their *projection* on $\mathcal{H}$ over C

$$i \times \hat{I}_t(t,r,p,\xi,\omega,\psi,\partial\psi,...) \times \partial_t\hat{\psi}(t,r) =$$
$$= H \times T_r(t,r,p,\xi,\omega,\psi,\partial\psi,...) \times \hat{\psi}(t,r) = E \times \hat{\psi}(t,r), \tag{3.16a}$$
$$p_k \times T_r(t,r,p,\xi,\omega,\psi,\partial\psi,...) \times \hat{\psi}(t,r) =$$
$$= -i \times \hat{I}^i_k((t,r,p,\xi,\omega,\psi,\partial\psi,...) \times \partial_i\hat{\psi}(t,r), \tag{3.16b}$$
$$[r^{\hat{i}},p_j] = i \times \hat{I}_r \times \delta^i_j, \quad [r^{\hat{i}},r^{\hat{j}}] = [p_{\hat{i}},p_{\hat{j}}] = 0. \tag{3.16c}$$
$$\hat{<}\hat{A}\hat{>} =< \hat{\psi}| \times T \times A \times T \times |\hat{\psi} > \times\hat{I}. \tag{3.16d}$$

We also have the following *isoplanewaves,* namely, conventional planewaves experiencing a mutation due to their immersion within the scattering region,

$$\hat{\psi}(r) = \hat{e}^{i\times k\times r} = \hat{I} \times (e^{i\times k\times T\times r}), \tag{3.17}$$

with isoeigenvalue equation (assuming for simplicity that the isounit does not depend on coordinates, see Ref. [13b] for the general case)

$$p \times T \times \hat{\psi} = -i \times \hat{I} \times \partial_r\hat{\psi} = -i^2 \times \hat{I} \times T \times k \times \hat{\psi} = k \times \hat{\psi}, \tag{3.18}$$

The reader can now see the fundamental relevance for hadronic mechanics of the isodifferential calculus because, until achieved, hadronic mechanics had no consistent formulation of the linear momentum, angular momentum, plane waves and other basic features.

The isotopies of Heisenberg equations were introduced in the original proposal [5] of 1978; studied by various authors, such as Myung and Santilli [51] and others (see Refs. [16a] for comprehensive bibliography); then reformulated in Ref. [7] via the isodifferential calculus; and are today known as *Heisenberg-Santilli isoequations.* They can be written in their general form on $\hat{\mathcal{H}}$ over $\hat{C}$ for the finite time evolution of a (Hermitean) operator $\hat{A}$

$$\hat{A}(\hat{t}) = \hat{U}(\hat{t})\hat{\times}\hat{A}(\hat{0})\hat{\times}\hat{U}^\dagger(\hat{t}) = (\hat{e}^{i\times H\times t})\hat{\times}\hat{A}(\hat{0})\hat{\times}(\hat{e}^{-i\times t\times H}), \tag{3.19}$$

characterizing a one-dimensional *Lie-Santilli isotransformation group* (see Section 1.5) with corresponding infinitesimal form

$$\hat{i}\hat{\times}\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = \hat{A}\hat{\times}\hat{H} - \hat{H}\hat{\times}\hat{A} = [\hat{A},\hat{H}]. \tag{3.20}$$

By using isoexponentiation (3.4), the above equations can be written in their projection on $\mathcal{H}$ over $\mathcal{C}$

$$A(t) = e^{i\times H\times T\times t)} \times A(0) \times (e^{-i\times t\times T\times H}), \tag{3.21a}$$

$$i \times \hat{I}_t((t,r,p,\xi,\omega,\psi,\partial\psi,...) \times \frac{dA}{dt} =$$

$$= A \times T_r((t,r,p,\xi,\omega,\psi,\partial\psi,...) \times H - H \times T - r((t,r,p,\xi,\omega,\psi,\partial\psi,...) \times A =$$

$$= [A,H], \tag{3.21b}$$

with fundamental isounitary property

$$\hat{U} = \hat{e}^{i\times H\times t}, \quad \hat{U}\hat{\times}\hat{U}^\dagger = \hat{U}^\dagger\hat{\times}\hat{U} = \hat{I}_r. \tag{3.22}$$

This concludes our elementary review of the basic equations of the nonrelativistic isotopic branch of hadronic mechanics as minimally needed for the elaboration of the isoscattering theory. The elements of the relativistic extension are indicated in the next section.

We should recall from Section 3.5 of Paper I that isotopies preserve Hermiticity to the extent that the operations of Hermiticity and iso-Hermiticity coincide. Hence, all observables of quantum mechanics remain observables for the covering hadronic mechanics. Also, by conception and construction, quantum and hadronic mechanics coincide at the abstract, realization-free level. Therefore, any criticism on the axiomatic structure of hadronic mechanics is indeed a criticism on the axiomatic structure of quantum mechanics.

Additionally, we should also recall that *hadronic mechanics has been conceived and constructed as a kind of completion of quantum mechanics much along the historical Einstein-Podolsky-Rosen argument* (see Ref. [34] on the latter aspects. In this way, *the isoscattering theory can be considered a realization of one of the possible realizations of hidden variables with explicit and concrete identification of their realization, physical origin and meaning.*

By recalling the content of Paper I, isounitarity property (3.22), as well as the isolocality and isolinearity, establish the resolution of the inconsistency theorems for nonunitary theories. In particular, the isolinearity is crucial for the application of the mechanics to multiple scattering process, an occurrence with unresolved problematic aspects for a nonlinear formulation of the conventional scattering theory (see also Sections 3.4 and 3.5 of Paper I).

Note that the various "deformations of quantum mechanics" existing in the literature were formulated years following the appearance of the isotopies and genotopies [4,5] of 1978, and are essentially *identical* to these original formulations formulated on conventional spaces over conventional fields, thus being catastrophically inconsistent for the reasons indicated earlier (see Refs. [6-12] of Paper I).

**3.5. Deformations-Isotopies of Dirac's Equations.** As it is well known, Dirac's equation plays a central role in Feynman's diagrams. Consequently, the covering isoscattering theory is crucially dependent on the deformations-isotopies of Dirac's equation.

Due to their importance, these isotopies were studied since the early stages of hadronic mechanics (see Ref. [12,13b] for original studies). In these papers, we shall use the first invariant isotopies of Dirac's equation achieved in Refs. [38,39] and used for the construction of the isotopies of the spinorial covering of the LP symmetry, as well as for the first known relativistic representation of all characteristics of the neutron in its synthesis from a proton and an electron in the core of a star (see Kaldeivili review [52]). The new equations are today known as the *Dirac-Santilli isoequations,* where we use the plural due to the variety of different realizations characterized by different isounits.

As it is also well known, the conventional Dirac equation represents *an electron in the external field of the proton.* The primary use of the covering isotopic formulation is that of representing a *mutated (isorenormalized) electron when immersed inside the proton also assumed as external.* The mutated electron was called *eleton* in the original proposal [5], but the name of *isoelectron* is nowadays more generally adopted due to the use of the prefix iso for protons and other particles when in interior conditions.

The isotopies of the Dirac equation belong to the branch of hadronic mechanics known as *relativistic isomechanics* (see memoir [53] for a review). Its foundations are given by the second-order iso-Casimir invariant (2.5b) that, via the use of the relativistic version of the isolinear momentum (3.11) characterizes the *isotopies of the Klein-Gordon equation* on $\hat{M}(\hat{x},\hat{\eta},\hat{\mathcal{R}})$ (see Ref. [13b] for detailed treatment of relativistic hadronic mechanics)

$$(\hat{\Xi}^{\mu\nu}\hat{\times}\hat{p}_\mu\hat{\times}\hat{p}_\nu - \hat{m}^{\hat{2}}\hat{\times}\hat{V}^{\hat{2}}_{max})\hat{\times}|\hat{\psi}(x)>=$$

$$= (\hat{\eta}^{\mu\nu}\times\hat{\partial}_\mu\hat{\partial}_\nu - m^2\times V^2_{max})\times|\hat{\psi}(x)>=0. \tag{3.23}$$

as well as *relativistic isokinematics.*

The desired isotopies of Dirac's equation are then obtained via a linearization of the above equation similar to the conventional linearization, resulting in the following two cases:

*CASE I: REGULAR DIRAC-SANTILLI ISOEQUATIONS.*
In this case, the regular isotopies of spacetime can be described by the isounit, isotopic element and isometric (2.1) hereon denoted $\hat{I}_{st}$, $T_{st}$, and $\hat{\Xi}_{st} = \hat{\eta}_{st} \times \hat{I}_{st}$, respectively, while the regular isotopies of the spin are tose of Section 1.9 with isounit, isotopic element and isometric (1.33) hereon denoted $\hat{I}_{sp}$, $T_{sp}$ and $\hat{\delta}_{sp}$.respectively.

By using again the notation of Eqs. (2.1b) and by assuming $\Gamma = \gamma \times \hat{I}_{st}$, the *regular Dirac-Santilli isoequations* can then be written [37.38]

$$(\hat{\Xi}^{\mu\nu}_{st} \hat{\times} \hat{\Gamma}_{\mu} \hat{\times} \hat{p}_{\nu} + \hat{i} \times \hat{m} \hat{\times} \hat{V}_{max}) \times \hat{T}_{st} \times |\hat{\psi}(x) >=$$

$$= (\hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}_{\mu} \times \hat{\partial}_{\nu} + i \times m \times V_{max}) \times |\hat{\psi}(x) >= 0. \tag{3.24}$$

where, by using isomatrices (1.36), the *regular isogamma matrices* have a structure of the type

$$\hat{\gamma}_k = b_k \times \begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \quad \hat{\gamma}_4 = i \times b_4 \times \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}, \tag{3.25}$$

with anti-isocommutation rules

$$\{\hat{\gamma}_{\mu}, \hat{\gamma}_{\nu}\} = \hat{\gamma}_{\mu} \times T_{st} \times \hat{\gamma}_{\nu} + \hat{\gamma}_{\nu} \times T_{st} \times \hat{\gamma}_{\mu} = 2 \times \hat{\eta}_{\mu\nu}, \tag{3.26}$$

The first implications for the isoscattering theory is that of *embedding gravitation directly in the basic metric $\hat{\eta}$ of the scattering region.* As an example, rules (3.26) can characterize (twice) the Schwarzschild metric as in Eqs. (2.35). However, the isometric can be more complex than that to include velocity-dependent internal effects, as well as the locally varying speed of light.

As one can see, in this particular case, the isotopies essentially offer *five* additional characteristic quantities, four for the spacetime mutation and one for the spin, that are available for the representation of experimentally measurable features of the scattering region, such as shape, density and anisotropy (requiring precisely five values). Note that the representation of these features is essentially outside the capabilities of quantum scattering theories.

Reader can now see the comments on antimatter of Section 2.5 of Paper I, to the effect that *there exist no irreducible or reducible representation of the conventional $SU(2)$ spin algebra having the structure of Dirac's gamma matrices.* By contrast, the conventional gamma matrices are characterized by the Kronecker product of an irreducible representation for spin 1/2 and its isodual, since the isodual unit $I^d_{2\times 2} = -I_{2\times 2}$ appears directly in $\gamma_4$ and Pauli's matrices verify the isodual rule $\sigma^d_k = -\sigma^{\dagger}_k = -\sigma_k$.

Consequently, *the conventional Dirac equation is reinterpreted as representing one electron and its antiparticle* without any need of the "hole theory" or second quantization, since the isodual theory of antimatter holds at the *Newtonian* level, let alone that of first quantization (see monograph [15] for comprehensive studies).

Evidently, the above features persist under isotopies, and we shall write in these papers the isogamma matrices in the form

$$\hat{\gamma}_k = b_k \times \begin{pmatrix} 0 & \hat{\sigma}_k \\ \hat{\sigma}_k^d & 0 \end{pmatrix}, \quad \hat{\gamma}_4 = i \times b_4 \times \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & I_{2\times 2}^d \end{pmatrix}, \tag{3.27a}$$

$$|\hat{\psi} >= \begin{pmatrix} |\hat{\psi}^- > \\ \hat{\psi}^+ > \end{pmatrix}, \tag{3.27b}$$

where the upper symbol $d$ represents isodual conjugation (anti-Hermitean conjugation), and $|\hat{\psi}^- >, \hat{\psi}^+ >$ represent the two-components wavefunctions for the isoelectron and the isopositron under the respective external fields of a proton and an antiproton considered as external. In different words, Eq. (3.24) can represents the Kronecker product of a neutron and an antineutron [52].

*CASE II: IRREGULAR DIRAC-SANTILLI ISOEQUATIONS.*
In this case we have the same equations (3.24) and the same isogammas (3.25), but the Pauli-Santilli isomatrices are irregular, e.g., are given by matrices (1.41) representing a mutation (isorenormalization) of the spin.

An important implication of the irregular equations is that the total angular momentum of the isoelectron is no longer conventionally quantized, and can assume, in particular, the null value,

$$J_e = s + j = \frac{1}{2}, \frac{3}{2}, ... \to \hat{I}_{\hat{e}} = 0. \tag{3.28}$$

as needed for an invariant representation of the neutron spin in its synthesis from a proton and an electron [38,39,52].

Note that in the above case we solely have the mutation of the *angular momentum,* but not that of spin, the latter being expected under energies dramatically bigger than those for the synthesis of the neutron.

**3.6. Dirac's Generalization of Dirac's Equation.** The identification of the background methods for the treatment of the isoscattering theory would be grossly deficient without an outline of the generalization of Dirac's equation achieved by Dirac himself in two of his last, vastly ignored papers [54,55] admitting an essential isotopic structure as well as the null value of the total angular momentum so crucial for a quantitative representation of the neutron synthesis.

The *Dirac's generalization of Dirac equation* is given by [54]

$$(a_\mu \times \partial_\mu + \beta) \times q \times \psi = 0, \tag{3.29a}$$

$$q = \begin{pmatrix} q_1, p_1 \\ q_2, p_2 \end{pmatrix}, \quad \psi = \begin{pmatrix} psi_{1+}, \psi_{1-} \\ \psi_{2+}, \psi_{2-} \end{pmatrix}, \tag{3.29b}$$

where the reader can immediately recognize the role of the $q$-quantity as characterizing a right isomodular action which is at the foundation of the Lie-Santilli isotheory, as well as of Eq. (3.24)

By assuming

$$a_4 = I_{4x4}, \tag{3.30}$$

*Dirac's a-matrices* are characterized by the expression

$$a_\mu \times \beta \times a_\nu + a_\nu \times \beta \times a_\mu = 2 \times \beta \times \eta_{\mu\nu}, \tag{3.31}$$

where the reader will immediately see the same isotopic structure of iso-anticommutators (3.26), and $\eta_{\mu\nu}$ is the conventional Minkowski metric.

On the basis of the above structure, Dirac reached the following realization of the $a$- and $\beta$-matrices

$$\beta = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad a_1 = i \times \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \tag{3.32a}$$

$$a_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad a_3 = i \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{3.32b}$$

The total angular momentum is characterized by

$$S_{ij} = -(a_i \times \beta \times a_j - a_j \times \beta \times a_i) \times \frac{q \times q^t}{8}, \tag{3.33}$$

where $t$ stands for transposed, and possesses the eigenvalues

$$S^2 = S_{12}^2 + S_{23}^2 + S_{31}^2 =$$

$$= \frac{1}{8} \times (q_1^2 + p_1^2 + q_2^2 + p_2^2) = J \times (J+1), \tag{3.34a}$$

$$J = \frac{1}{4} \times (q_1 + p_1 + q_2 + p_2) - \frac{1}{2} = \frac{1}{2} \times (n + n'), \tag{3.34b}$$

$$n,\ n' = 0, 1, 2, 3, ... \tag{3.34c}$$

thus *admitting the value $J = 0$ for the ground state.*

The historical aspect particularly significant for hadronic mechanics at large, and the isoscattering theory in particular, is that, without his knowledge, *Dirac's generalization of Dirac's equation possesses an irreducible isotopic structure* with isounit and isotopic element first identified in Ref. [13b]

$$\hat{I} = \beta^{-1},\ \ T = \beta, \tag{3.35}$$

where the irreducibility is referred to the property that papers [52,53] become inconsistent unless *entirely* elaborated with respect to the isoproduct

$$A\hat{\times}B = A \times T \times B,\ T = \beta. \tag{3.36}$$

We cannot close this section without the indication that, for structural consistency, Dirac's generalized equation cannot be formulated on the conventional Minkowski space $M(x, \eta, \mathcal{R})$ and must be formulated on the Minkowski-Santilli isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{\mathcal{R}})$, with isometric [13b]

$$\hat{\eta} = \beta \times \eta = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \tag{3,37}$$

namely, Dirac not only had the intuition without elaboration of the isotopic formalism, but identified without his knowledge the first known *nondiagonal realization of the spacetime isometric* (3.37).

Rather than being an innocuous occurrence, the implications are far reaching because the line element now reads

$$x^{\hat{2}} = x^\mu \times (\beta^\rho_\mu \times \eta_{\rho\nu}) \times x^\nu = x^\mu \times \hat{\eta}_{\mu\nu} =$$

$$= x^1 \times x^3 - x^2 \times x^4 - x^3 \times x^1 - x^2 \times x^4 = -2 \times x^2 \times x^4, \tag{3.38}$$

namely, when reformulated in an invariant way, *Dirac's isoequation (3.29) mutates spacetime from the conventional four-dimensions down to two-dimensions.*

The implications for the isoscattering theory are far reaching because, when the interior scattering region is characterized by Dirac's generalized equation, it loses completely the conventional four dimensions, by reducing spacetime solely to two dimensions, even though fully perceived from the outside as being four dimensional.

In summary, due to the power of his intuition, perhaps unprecedented in scientific history, Dirac should be consider the precursor of:

1) The isodual theory of antimatter, that originated from Dirac's negative unit $-I_{2\times 2}$ in the conventional $\gamma_4$ matrix [15];

2) The isotopic formalism and, consequently, the isoscattering theory, that originated from Dirac's generalized equation (3.29) [13b]; and

3) The first exact representation of the synthesis of the neutron from a proton and an electron in the core of a star requiring a null total angular momentum of the electron, that was first achieved by Dirac in Eq. (3.34) [39].

As a personal note, at one of his last participations at scientific meetings in the early 1980s in Florida, Santilli briefly presented to Dirac the isotopic formalism with the connection to his papers [54,55] and the indication of the strong convergence of conventionally divergent series under the isotropic product $A\times T\times B-B\times T\times A, |T| \ll 1$, thus including the possible achievement of Dirac's dream for a scattering theory without divergences.

Contrary to his normally reserved nature, Dirac showed great interest. Unfortunately, Dirac's death in 1984 deprived the then newly born isotopies of his most powerful supporter, thus delaying collegial acceptance for decades. A rewarding aspect is that, with the power of his intuition, Dirac did in fact see prior to his death the technical feasibility of his dream of a scattering theory without divergencies hereon called the *Dirac's problem* (for more details in the episode, one may consult Ref. [16d]).

**3.7. Experimental, Verifications.** The presentation of the isoscattering theory in Paper III could be misleading to non-expert in the field without the indication of rather significant experimental verifications and novel applications of hadronic mechanics in various quantitative sciences along the following setting:

*I) CONDITIONS OF EXACT VALIDITY OF QUANTUM MECHANICS.* They are assumed as being those of special relativity, namely, the conditions of *exterior dynamical problems* (point-particles and electromagnetic waves propagating in vacuum), since the latter do admit the effective point-like abstraction of particles necessary for the validity of the local-differential and topological, foundations of the theory. Therefore, these papers assume that quantum mechanics is *exactly* valid for the structure of the hydrogen atom, particles in accelerators, crystals, and numerous other exterior problems.

*II) CONDITIONS OF APPROXIMATE VALIDITY OF QUANTUM MECHANICS.* These papers assume that quantum mechanics is only *approximately* valid for *interior dynamical problems* (extended particles and electromagnetic waves moving within physica;l media) due to expected nonlinear, nonlocal and nonpotential interactions compared to the strictly linear, local and potential character of the mechanics. In

particular, these papers assume that all arbitrary parameters and functions of unknown physical origin whose values are fitted from experimental data, represent in reality *deviations* from the very axioms of quantum mechanics, as illustrated by the following representative cases:

II-A) The inability by quantum mechanics to achieve an exact representation of the binding energy and other features of the hydrogen molecule, that required the use of the so-called "screened Coulomb potentials" of the type $V^*(r) = f(r) \times e^2/r$ with evident loss of quantized levels and other problematic aspects. By comparison, the use of hadronic mechanics has permitted the achievement of a numerically exact and invariant representation of the binding energy of the hydrogen and other molecules from first axiomatic principles without *ad hoc* functions of unknown physical origin or meaning. The representation is achieved via the sole admission of non-Hamiltonian interactions originating from the deep mutual penetration of the wavepackets of valence electrons, and provides convergent power series dramatically faster than those of quantum mechanics [14]. Similar insufficiencies of quantum mechanics exist for the representation of the simplest possible nucleus, the deuteron, not to mention very large deviations for heavy nuclei, for which the use of hadronic mechanics has provided distinct advances [16d,27].

II-B) The representation via relativistic quantum mechanics of the two-point functions of the Bose-Einstein correlation in a way compatible with experimental data requires *four* arbitrary parameters of unknown physical origin or meaning (called the "chaoticity parameters"), that are fitted from experimental data and relativistic quantum mechanics is then claimed as being valid. However, the vacuum expectation values of a diagonal two-dimensional Hamiltonian can at best allow *two* arbitrary parameters, thus indicating that the very structure of the Bose-Einstein correlation is beyond the representational capabilities of relativistic quantum mechanics. After all, said correlation originates from the deep mutual penetration and consequential annihilation protons and antiprotons under which linear, local and potential treatments can at best be approximately valid. By comparison, relativistic hadronic mechanics achieves an exact and invariant representation of the Bose-Einstein correlation at both high and low energies via the four characteristic quantities of isospacetime, three of which represent the very elongated fireball and the forth represents its density [16d,27].

II-C) As it is equally well known, the representation of the behavior of the mean-lives of unstable hadrons with speed is equally achieved via the introduction of a number of parameters, functions, expansions, cut off and other mechanisms to achieve compatibility with relativistic quantum mechanics. However, the latter theory is fully reversible over time, while spontaneous hadrons decays are strictly irreversible. Hence, the latter theory cannot possibly be exact for the former events. The use of the Lie-admissible branch of hadronic mechanics permits an exact representation of

the *irreversible character* of the decay in a way compatible with experimental data from first axiomatic principles without conceptual or axiomatic adaptations [16d,27].

*III) CONDITIONS OF INAPPLICABILITY OF QUANTUM MECHANICS.* Throughout the 20th century, it was widely believed that quantum mechanics can be applied to all possible conditions of particles existing in the universe. At a meeting in February 1978, Herman Feshback (then from MIT) confirmed to Santilli (then from Harvard) that Schrödinger's equation is inapplicable for the synthesis of the neutron from a proton and an electron as occurring in the core of stars, $p^+ + e^- \to n + \nu$, because the rest energy of the neutron is $0.782 MeV$ *bigger* than the sum of the rest energies of the proton and the electron, thus requiring a *positive binding energy* under which the indicial equation no longer admits physically acceptable solution. The origin of the missing energy from a possible relative kinetic energy between the proton and the electron has to be excluded due to their extremely small cross section at the indicated energy, and the same holds for the antineutrino in the conjugate reaction $p^+ + e^- + \bar{\nu} \to n$, thus establishing the *inapplicability* (and certainly not the violation) of quantum mechanics for a quantitative treatment of the problem considered. At the same meeting, that signals the birth of hadronic mechanics, Santilli indicated to Feshback that a nonunitary generalization of quantum mechanics in general, and of Schrödinger's equation in particular, does indeed admit physically meaningful for hadron syntheses with "missing energies" solutions for various reasons, e.g., the anomalous renormalization of the rest energy of the electron due to non-Hamiltonian interactions under which the binding energy returns to acquire the conventional *negative* Coulomb form between opposite charges. The original proposal [4,5] published in April 1978 to construct hadronic mechanics provides a complete solution for the synthesis also requiring a "positive binding energy" $e^+ + e^- \to \pi^o$, while the full solution at the nonrelativistic and relativistic levels of the synthesis of the neutron was achieved after decades of study of the isotopies of Lie's theory and the discovery of the irregular isotopies of the spin symmetry outlined in Section 1 (see Ref. [52] for a general review).

We ended Paper I with the indication that, despite one millennium of studies, our knowledge of light is still at its infancy. It is appropriate to end this paper with the indication that quantum mechanics has been a beautiful episode in the history of physics but, in view of its evident limitations in face of the complexities of the physical world, any belief of its final character for all of nature causes the existing from the boundaries of science, the sole scientific issue, for which these papers have been written, being the selection via experiments, rather than personal or conceptual indications, of its appropriate structural generalization.

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# References


[1] R. M. Santilli and A. O. E. Animalu, "Nonunitary Lie-Isotopic and Lie-Admissible Scattering Theory of Hadronic mechanics, I: Conceptual and Mathematical. Foundations, " in *Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes,* C. Corda, Editor, Kathmandu University (2011) pages 423-459
http://www.santilli-foundation.org/docs/Isoscattering-I.pdf

[2] H. A. Lorentz, *Versuch einer Theorie der Elektrishen und Magnetishen Erscheinungen in Bewenglen Korpern*, Leyda (1895).

[3] R. M. Santilli, Nuovo Cimento **51**, 570 (1967), available in free download from the link
http://www.santilli-foundation.org/docs/Santilli-54.pdf

[4] R. M. Santilli, "On a possible Lie-admissible covering of Galilei's relativity in Newtonian mechanics for nonconservative and Galilei form-noninvariant systems," Hadronic J. **1**, 223-423 (1978), available in free pdf download from
http://www.santilli-foundation.org/docs/Santilli-58.pdf

[5] R. M. Santilli, "Need of subjecting to an experimental verification the validity within a hadron of Einstein special relativity and Pauli exclusion principle," Hadronic J. **1**, 574-901 (1978), available in free pdf download from
http://www.santilli-foundation.org/docs/Santilli-73.pdf

[6] R. M. Santilli, Algebras, Groups and Geometries **10**, 273 (1993), available as free downlaod from
http://www.santilli-foundation.org/docs/Santilli-34.pdf

[7] R. M. Santilli, Rendiconti Circolo Matematico Palermo, Suppl. **42**, 7-82 (1996), available as free download from
http://www.santilli-foundation.org/docs/Santilli-37.pdf

[8] R. M. Santilli, Found. Phys. **27**, 1159 (1997), available in free pdf download from the link
http://www.santilli-foundation.org/docs/Santilli-06.pdf

[9] R. M. Santilli, "Lie-admissible invariant representation of irreversibility for matter and antimatter at the classical and operator levels," Nuovo Cimento B bf 121, 443 (2006), available in free pdf download from the link
http://www.santilli-foundation.org/docs/Lie-admiss-NCB-I.pdf

http://www.santilli-foundation.org/docs/Santilli-209.pdf
http://www.santilli-foundation.org/docs/santilli-69.pdf

[11] R. M. Santilli, *Lie-Admissible Approach to the Hadronic Structure,* Vol. I (1978) [11a] and Vol. II (1982) [11b], Hadronic Press, available as free downloads from the links
http://www.santilli-foundation.org/docs/Santilli-71.pdf
http://www.santilli-foundation.org/docs/Santilli-72.pdf

[12] R. M. Santilli, *Isotopic Generalizations of Galilei and Einstein Relativities,* Vol. I (1991) [12a] and Vol. II (1991) [12b], Hadronic Press, Palm Harbor, Florida, available in free pdf download from
http://www.santilli-foundation.org/docs/Santilli-01.pdf
http://www.santilli-foundation.org/docs/Santilli-61.pdf

[13] R. M. Santilli, *Elements of Hadronic Mechanics*, Vol. I (1995) [13a], Vol. II 91995) [13b] Academy of Sciences, Kiev, available in free pdf downloads from
http://www.santilli-foundation.org/docs/Santilli-300.pdf
http://www.santilli-foundation.org/docs/Santilli-301.pdf

[14] R. M. Santilli, *Foundations of Hadronic Chemistry, with Applications to New Clean nergies and Fuels,* Kluwer Academic Publishers (2001), available as free download from
http://www.santilli-foundation.org/docs/Santilli-113.pdf

[15] R. M. Santilli, *Isodual Theory of Antimatter with Applications to Antigravity, Grand Unifications and Cosmology,* Springer (2006).

[16] R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry,*, Vol. I [16a], II [16b], III [16c], IV [16d] and [16e], international academnioc press, (2008), available as free downlaods from
http://www.i-b-r.org/Hadronic-Mechanics.htm

[17] H. C. Myung,, *Lie Algebras and Flexible Lie-Admissible Algebras,* Hadronic Press (1982), available as free download from
http://www.santilli-foundation.org/docs/Santilli-107.pdf

[18] H. C. Myung, *Malcev-Admissible Algebras,* H. C. Myung, Birkhauser (1986).

[19] A. K. Aringazin, A. Jannussis, F. Lopez, M. Nishioka and B. Veljanosky, *Santilli's Lie-Isotopic Generalization of Galilei and Einstein Relativities,* Kostakaris Publishers, Athens, Greece (1991), available as free download from
http://www.santilli-foundation.org/docs/Santilli-108.pdf

[20] D. S. Sourlas and G. T. Tsagas, *Mathematical Foundation of the Lie-Santilli Theory,* Ukraine Academy of Sciences 91993), available as free download from http://www.santilli-foundation.org/docs/santilli-70.pdf

[21] A. Elduque and H. C, Myung, *Mutations of Alternative Algebras,* Kluwer Academic Publishers (1994).

[22] J. Lôhmus, E. Paal, and L. Sorgsepp, *Nonassociative Algebras in Physics* (Hadronic Press, Palm Harbor, 1994), available as free download from http://www.santilli-foundation.org/docs/Lohmus.pdf

[23] J. V. Kadeisvili, *Santilli's Isotopies of Contemporary Algebras, Geometries and Relativities,* Ukraine Academy of Sciences, Second edition (1997), available as free download from
http://www.santilli-foundation.org/docs/Santilli-60.pdf

[24] Chun-Xuan Jiang, *Foundations of Santilli Isonumber Theory,* International Academic Press (2001), available as free download from
http://www.i-b-r.org/docs/jiang.pdf

[25] Raul M. Falcon Ganfornina and Juan Nunez Valdes, *Fundamentos de la Isoteoria de Lie-Santilli,* International Academic Press (2001), available as free downlaod from
http://www.i-b-r.org/docs/spanish.pdf

[26] B. Davvaz, *Hyperings Theory and Applications,* International Academic Press (2007), available as free downlaod from
http://www.santilli-foundation.org/docs/Davvaz.pdf

[27] I. Gandzha and J. Kadeisvili, *New Sciences for a New Era" Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli,* Sankata Printing Press, Nepal (2011), available in free pdf download from the link
http://www.santilli-foundation.org/santilli-scientific-discoveries.html

[28] J. V. Kadeisvili, "An introduction to the Lie-Santilli isotopic theory," Mathematical Methods in Applied Sciences **19**, 1349 (1996), available in the website
http://www.santilli-foundation.org/docs/Santilli-30.pdf

[29] Raul M. Falcon Ganfornina and Juan Nunez Valdes, "Studies on the Tsagas-Sourlas-Santilli Isotopology," Algebras, Groups and Geometries **20**, 1 (2003), available from the link
http://www.santilli-foundation.org/docs/isotopologia.pdf

[30] G. Tsagas, "Studies on the classification of Lie-Santilli algebras," Algebras, Groups and Geometries **13**, 129- (1996). available from the link http://www.santilli-foundation.org/docs/Tsagas.pdf

[31] R. M. Santilli, "Isotopies of Lie symmetries, I" Hadronic J. **8**, 36 (1985), available as free download from http://www.santilli-foundation.org/docs/santilli-65.pdf

[32] R. M. Santilli, "Isotopies of Lie symmetries, II" Hadronic J. **8**, 85 (1985), available as free download from http://www.santilli-foundation.org/docs/santilli-65.pdf

[33] R. M. Santilli, JINR rapid Comm. **6**. 24-38 (1993), available as free downlaod from http://www.santilli-foundation.org/docs/Santilli-19.pdf

[34] R. M. Santilli, Acta Applicandae Mathematicae **50**, 177 (1998), available as free download from http://www.santilli-foundation.org/docs/Santilli-19.pdf

[35] R. M. Santilli, Lettere Nuovo Cimento **37** 545 (1983),, available as free downlaod from http://www.santilli-foundation.org/docs/Santilli-50.pdf

[36] R. M. Santilli, Lettere Nuovo Cimento **38**, 509 (1983), available as free downlaod from http://www.santilli-foundation.org/docs/Santilli-51.pdf

[37] R. M. Santilli, Moscow Phys. Soc. **3**, 255 (1993), available as free download from http://www.santilli-foundation.org/docs/Santilli-40.pdf

[38] R. M. Santilli, "Recent theoretical and experimental evidence on the synthesis of the neutron," Communication of the Joint Institute for Nuclear Research, Dubna, Russia, No. E4-93-252 (1993), available as free download from http://www.santilli-foundation.org/docs/Santilli-18.pdf

[39] R. M. Santilli, Chinese J. System Engineering and Electronics **6**, 177 (1995, available as free download from http://www.santilli-foundation.org/docs/Santilli-18.pdf

[40] R. M. Santilli, Intern. J. Modern Phys. D **7**, 351 (1998), available as free download from http://www.santilli-foundation.org/docs/Santilli-35.pdf

[41] J. V. Kadeisvili, "Direct universality of the Lorentz-Poincare'-Santilli isosymmetry for extended-deformable particles, arbitrary speeds of light and all possible spacetimes" in *Photons: Old problems in Light of New Ideas,* V. V. Dvoeglazov Editor Nova Science (2000, available as free download from http://www.santilli-foundation.org/docs/Santilli-25.pdf

[42] A. K. Aringazin and K. M. Aringazin, "Universality of Santilli's iso-Minkowskian geometry" in *Frontiers of Fundamental Physics,* M. Barone and F. Selleri, Editors Plenum 91995), available as free download from http://www.santilli-foundation.org/docs/Santilli-29.pdf

[43] R. M. Santilli, "Direct universality of isospecial relativity for photons with arbitrary speeds" in *Photons: Old problems in Light of New Ideas,* V. V. Dvoeglazov Editor Nova Science (2000), available for free downlaod from http://www.santilli-foundation.org/docs/Santilli-24.pdf

[44] R. M. Santilli, "Isotopic quantization of gravity and its universal isopoincare' symmetry" in the *Proceedings of "The Seventh Marcel Grossmann Meeting in Gravitation,* SLAC 1992, R. T. Jantzen, G. M. Keiser and R. Ruffini, Editors, World Scientific Publishers pages 500-505(1994), available in free download from http://www.santilli-foundation.orgdocs/Santilli-120.pdf

[45] R. M. Santilli, "Unification of gravitation and electroweak interactions" in the proceedings of the *Proceedings of the Eight Marcel Grossmann Meeting in Gravitation,* Israel 1997, T. Piran and R. Ruffini, Editors, World Scientific, pages 473-475 (1999), available as free download from http://www.santilli-foundation.org/docs/Santilli-137.pdf

[46] R. M. Santilli, "Isotopic grand unification with the inclusion of gravity," Found. Phys. Letters **10**, 307 (1997), available as free download from http://www.santilli-foundation.org/docs/Santilli-04.pdf

[47] R. M. Santilli, "Experimental verifications of isoredshift with possible absence of universe expansion, big bang, dark matter and dark energy," The Open Astronomy Journal **3**, 124 (2010), available as free download from http://www.santilli-foundation.org/docs/Santilli-isoredshift.pdf

[48] R. M. Santilli, Galilean Electrodynamics, Summer 2006, **17**, 43 (2006), available as free download from http://www.i-b-r.org/Incons.GravFinalGED-I.pdf

[49] A. O. E. Animalu and R. M. Santilli, in *Hadronic Mechanics and Nonpotential Interactions,* M. Mijatovic, Editor, Nova Science, New York, pp. 19 (1990).

[50] H. C. Myung and R. M. Santilli, Hadronic Journal Vol. 5, pages 1277-1366 (1982). available as free download from
http://www.santilli-foundation.org/docs/Santilli-201.pdf

[51] R. Mignani, Hadronic J. 4, 2185 (1981).

[52] J. V. Kadeisvilki, "The Rutherford-Santilli Neutron," Hadronic J. **31**, 1 (2008), available in free pdf download from the link
http://www.i-b-r.org/Rutherford-Santilli-II.pdf
also available in htlm format from the website
http://www.i-b-r.org/Rutherford-Santilli-neutron.htm

[53] R. M. Santilli, "Relativistic hadronic mechanics: nonunitary, axiom-preserving completion of relativistic quantum mechanics," Found. Phys. Vol. **27**, 625 (1997), available as free download from
http://www.santilli-foundation.org/docs/Santilli-15.pdf

[54] P. A. M. Dirac, Proc. Royal Soc. A **322**, 435 (1971).

[55] P. A. M. Dirac, Proc. Royal Soc. A **328**, 1 (1972).

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
Kathmandu University, Nepal, April (2011) pages 521-545

# NONUNITARY LIE-ISOTOPIC AND LIE-ADMISSIBLE SCATTERING THEORIES OF HADRONIC MECHANICS, III: Basic Lie-Isotopic Formulation without Divergencies

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**Abstract**

In the preceding Papers I and II, we have: introduced various arguments suggesting nonunitary coverings of the conventional unitary scattering; outlined the problematic aspects of nonunitary theories; presented the novel isomathematics allowing an invariant isounitary reformation; and specialized to scattering processes the isotopic branch of hadronic mechanics comprising the isotopies of Lies theory. special relativity and quantum mechanics. In this paper we present a solution of the Dirac legacy indicated in Paper II, namely, a nonunitary-isounitary scattering theory without divergencies ab initio indicated by one of the authors to Dirac prior to his death. Joint elaborations of measured quantities (cross section, scattering angles, etc.) via the conventional and isotopic scattering theories are presented in subsequent works to identify the expected implications in final experimental results of nonlinear, nonlocal and nonpotential effects in high energy scattering processes. As indicated in paper I, the presentation is restricted to reversible scattering processes, because the treatment of irreversible processes requires the yet broader Lie-admissible covering of the Lie-isotopic formulation.

Key words scattering theories, nonunitary theories, isounitary theories
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TABLE OF CONTENT

1. Identification of the Isounit.
1.1. Introduction.
1.2. Main Conditions.
1.3. Explicit Realization.

2. Elements of Isoscattering Theory.
2.1. Isoscattering Amplitude.
2.2. Isoscattering Matrix.
2.3. Isopropagators.
2.4. Convergent Isoexpansions.
2.5. Prediction of Mini-Black-Holes.

3. Isotopies of Feynman's Diagrams without Divergences
3.1 Conventional Feynman Diagrams
3.2. The Problem of Antiparticles in Feynman's Diagrams.
3.3 Isotopies of Feynman Diagrams.
3.4. Isotopic Inclusion of Electromagnetic interactions.
3.5. Concluding remarks

Acknowledgments

References

## 1. Identification of the Isounit.

**1.1. Introduction.** In the preceding Refs. [1,2], herein indicated as Papers I and II, respectively, in which:

1) We presented various mathematical, theoretical and experimental arguments suggesting a re-inspection of the elaboration of measured quantities (cross sections, scattering angles, etc.) via the conventional, unitary, Hamiltonian scattering theory due to expected nonlinear, nonlocal and contact-nonpotential effects beyond the representational capabilities of quantum mechanics;

2) We pointed out that the sole possibility for a quantitative representation of the above effects, collectively called in these papers *non-Hamiltonian effects,* is the existing from the class of unitary equivalence of quantum mechanics, thus bringing again into focus past attempts at a nonunitary generalization of quantum scattering theories;

3) We then recalled the *Theorems of Catastrophic Mathematical and Physical Inconsistencies of Noncanonical and Nonunitary Theories* (Refs. [6-12] of Paper I) that have established the lack of invariance over time of units of measurements, the inability of predicting the same numerical values under the same conditions at different times, violation of causality and other inconsistencies under nonunitary time evolution;

4) We then introduced the elements, specialized to the scattering problem, of the novel *isomathematics* that has been specifically constructed for the invariant treatment of non-Hamiltonian effects and the resolution of the Inconsistency Theorems via the *isounitary reformulation of nonunitary scattering theories;* and

5) we finally presented the elements, also specialized to the scattering problem, of the *isotopic branch of hadronic mechanics* including the invariant deformations-isotopies of Lie's theory, special relativity and relativistic quantum mechanics, with emphasis on Dirac's pioneering work in the field.

We are now sufficiently equipment to initiate the presentation of the proposed covering *nonunitary-isounitary scattering theory* beginning with the identification of the all fundamental isounit.

To prevent major misrepresentations of the content of this paper, that often remain undetected by renowned experts in quantum mechanics but non-experts in the covering hadronic mechanics, serious readers are suggested to acquire a technical knowledge of Papers I and II, hereon tacitly assumed, with particular reference to isofunctional analysis, isodifferential calculus, regular and irregular isorepresentations, regular and notions notions of spin, regular and irregular Dirac-Santilli isoequations, Dirac's generalization of Dirac's equation, and related aspects.

**1.2. Main Conditions.** As indicated in paper II, the isotopic branch of hadronic mechanics, also known as *isomechanics,* was built for the representation of the deep

mutual penetration of the wavepackets and/or charge distributions of particles as occurring in the scattering region of Figure I.2. These conditions are of contact character, thus having zero range, and are expected as being nonlinear (in the wavefunctions), nonlocal (of integral type), and nonpotential (that is, not representable with a Hamiltonian). These non-Hamiltonian interactions are structurally beyond any possible representation by quantum mechanics due to its strictly linear, local-differential and potential-Hamiltonian character.

In view of these insufficiencies, the representation of the scattering region is done via *two* operators, the conventional Hamiltonian $H$ as currently used in scattering theories, and Santilli's isounit $\hat{I}$ or isotopic element $\hat{T} = \hat{I}^{-1}$, for the representation of all non-Hamiltonian interactions and effects.

Following studies for decades, the isounit has been selected over any other alternative representations of non-Hamiltonian interactions because it is the only one assuring the crucial *invariance over time,* i.e., the characterization of the same numerical values under the same conditions at different times. After all, whether conventional or generalized, the unit is the fundamental invariant of all theories.

The main requirements for the isounit (or isotopic element) are the following (see Refs. [3] for detailed studies):

*CONDITION I: Positive-definiteness.*

$$\hat{I}(t,r,p,E,\xi,\omega,\psi,\partial\psi,...) = \hat{I}^{\dagger}(t,r,p,E,\xi,\omega,\psi,\partial\psi,...) =$$

$$1/\hat{T}(t,r,p,E,\xi,\omega,\psi,\partial\psi,...) > 0. \tag{1.1}$$

This condition is sufficient to assure the preservation of the original axioms under isotopic liftings at all levels, with consequential local isomorphism between the Hilbert space and the Hilbert-Santilli isospace, the Lorentz-Poincaré (LP) symmetry and the Lorentz-Poincaré-Santilli (LPS) isosymmetry, quantum mechanics and isomechanics, etc.

*CONDITION II: Elimination of quantum levels,*

$$Lim_{r\gg 1fm}\hat{I}(t,r,p,E,\xi,\omega,\psi,\partial\psi,...) = \hbar,$$

$$Lim_{r\gg 1fm}\hat{T}(t,r,p,E,\xi,\omega,\psi,\partial\psi,...) = 1/\hbar. \tag{1,2}$$

This condition assures the existence of a unique and unambiguous limit under which hadronic mechanics recovers quantum mechanics for all mutual distances of particles bigger than 1 $fm$, e,g,. for mutual distances bigger than the size of particles wavepackets. Also, Hamiltonian interactions remain fully valid inside the scattering region. Hence, the above condition clarifies the fact that isomechanics essentially provides expected *corrections* to quantum treatments in the scattering region (only).

A primary function of condition (1.2) is to illustrate the main feature of hadronic mechanics, *the absence of conventionally quantized energy levels within a hyperdense hadronic medium.* Conceptually, this condition is illustrated by the evident impossibility that an electron in the core of a star (or, equivalently, in the interior of a high energy scattering region) cannot possibly have the same quantized levels as occurring when orbiting in vacuum around the nucleus in the hydrogen atom. The isounit then represents the integro-differential *deviations* from conventional quantum levels caused by the medium.

Physically, condition (1.3) is illustrated by the so-called "screened Coulomb potentials" used in quantum chemistry, namely, the multiplication of the Coulomb potential $V = e^2/r$ by an arbitrary function, $V^* = f(r)e^2/r$ that has resulted in being necessary for a numerically exact representation of the mutual penetration of valence electron pairs in molecular structures. However, in so doing, it is evident that the screened potential no longer admits the quantum energy levels of the conventional potential, as studied in detail in monograph [4].

*CONDITION III: Elimination of quantum divergencies,*

$$||\hat{I}|| \gg 1, \quad ||\hat{T}|| \ll 1. \tag{1.3}$$

As it is well known to experts in the field, the above condition assures that all perturbative and other series that are divergent (or weakly convergent) for quantum mechanics are turned into strongly convergent series in the covering hadronic mechanics.

In turn, this important feature, whose achievement escaped the best minds of 20th century physics, implies *numerical differences* between the sum of divergent quantum series (turned into convergent forms via cut-off, arbitrary parameter of unknown physical origin, and other manipulations) and the naturally convergent hadronic series. Moreover, these differences are expected to produce numerical differences between the elaboration of experimental data via scattering and isoscattering theories, a feature well known to experts in hadronic mechanics since the early 1980 (see Section 1 of paper I and references quoted therein), but often ignored by particle physics to their peril. Note that conditions (1.3) are fully compatible with conditions (1.2), as shown by all realizations assumed later on.

**1.3. Explicit Realization.** All isounits used in experimental verifications of hadronic mechanics to date (see Refs. [3d]) have emerged as verifying quite naturally Conditions I, II and III that, therefore, *are not* subsidiary constraints, but seemingly natural occurrences.

Inspired by these experimental verifications, particularly for the representation of the Cooper pair and valence bonds [5], one can see the adoption of the following

*realization of the isounit for the relativistic isoscattering theory*

$$\hat{I} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2) \times e^{N \times \frac{\psi^i(x)}{\psi^f(x)} \times \int d^4x \times \psi^{i\dagger}(x) \times \psi^f(x)}, \tag{1.4}$$

where

$$n_\mu = n_\mu(t, r, p, E, \xi, \omega, \psi, \partial\psi, ...) > 0, \ \mu = 1, 2, 3, 4, \tag{1.5}$$

where:

1) The characteristic quantities $n_k$, $k = 1, 2, 3$ provide the first known representation of the *generally nonspherical and deformable shape* of the scattering region normalized to the values $n_k = 1, k = 1, m2, 3$ for the sphere;

2) The characteristic quantity $n_4$ provides the first known representation of the *density of the scattering region* (i.e.m the ratio between its energy and its volume) normalized to the value $n_4 = 1$ for the vacuum; and

3) the first known representation of the *inhomogeneity of the scattering region* is provided by a functional dependence of the characteristic quantities, e.g., on the local coordinates, while the first known representation of the *anisotropy of the scattering region* is provided, e.g., by different values of space and time characteristic quantities.

It should be indicated that the characteristic quantities have an explicit functional dependence when formulated in the *interior* of the scattering region, but they are generally averaged into constants when inspected from the outside, an assumption we shall tacitly make hereon.

It is easy to see that isounit (1.4) naturally verifies Conditions I, II, III. Additionally:

A) The integral $\int d^4x \times \psi^{i\dagger}(x) \times \psi^f(x)$ represent the intended nonlocal character of the scattering region as well as the verification of limits (1.2);

B) The ratio between initial and final wavefunctions, $\psi^i/\psi^f$, characterizes the intended nonlinear character of the scattering theory (with the possibility of additional embedding of nonlinear terms);

C) $N$ is an isorenormalization constant to be identified later on;

and the *exponential* character of the isounit originates at the primitive Newtonian level (see Section II.3.2. In fact, the exponential character of the isounit emerged when turning nonconservative non-Hamiltonian Newtonian systems into an identical isotopic form, and this feature persists under operator formulations.

Alternatively, on mathematical grounds, one can see the emergence of the exponential structure of the isounit from the isodifferential of Section I.3.4. In this case, the use of the expressions of the type $\hat{I} = e^{A(r)}$ implies the cancellation between $\hat{T}$ and $\hat{I}$ in the additive term, namely, $r \times T \times d\hat{I} = r \times \partial_r A dr$.

It is hoped the reader can see that, besides all the arguments presented in Papers I and II, the LP symmetry cannot possibly be exact for the scattering region under

the sole admission of its nonspherical, inhomogeneous and anisotropic character, in favor of the exact universal LPS isosymmetry. The isotopies of quantum mechanics of paper II and of the scattering theory presented in this paper, then follow rather inevitably.

## 2. Elements of Isoscattering Theory.

**2.1. Isoscattering Amplitude.** By assuming a knowledge of the content and terminology of Papers I and II, we can now expeditiously proceed with the formulation of the desired isoscattering theory. For clarity, we shall write most of the formalism in its projection on conventional spaces so as to show the differences with conventional formulations.

An important difference between the quantum and isotopic scattering theory is the assumption in the following. The conventional scatering theory assumes the speed of light *in vacuum* $c$ and related LP symmetry as being unchanged in the *hyperdense medium* inside the scattering region. By contrast, the isoscattering theory assumes that the speed of light in the interior problem is the local avriable $C = c/n_4 = c \times b_4$ with universal LPS isosymmetry. However, when the scattering problem is studied on the Minkowski-Santilli isospacetime over the isoreals, the speed of light in the interior problems remains indeed that in vacuum, as recalled in Section II-2. These views imply that the characterization of the density of the scattering region indicated earlier is given by the index of refraction $n_4 = 1/b_4$, a notion absent in quantum theories.

The isoscattering theory alsop assumes that the geometry in the interior of the scattering region is mutated by its medium. This mutation is represented with the transition from the Euclidean space $E(r, \delta, \mathcal{R}$ over the reals $\mathcal{R}$ characterizing the vacuum (intended as empty space), to the Euclid-Santilli isospace $\hat{E}(\hat{r}, \hat{\delta}, \hat{\mathcal{R}}$ over the isoreals $\hat{\mathcal{R}}$ characterizing the hyperdense medium inside the scattering region.

Additionally, we assume the conservation of the volume of the scattering region in the transition from the conventional to the isotopic treatment that can be expressed by the condition

$$Det\ \hat{\delta} = (b_1^2 \times b_2^2 \times b_3^2) = (b_1 \times b_2 \times b_3)^2 = 1 \tag{2.1}$$

However, it should be stressed that this condition can be easily relaxed later on, e.g., in the transition from low to high energy scattering in which the scattering volume definitely is not preserved. We finally assume for simplicity that the characteristic $b$-quantities have been averaged to costants because the scattering region is inspected from the outside.

The above assumptions require the replacement of the conventional spherical coordinates with the isospherical form (3.36), including the lifting of conventional angles

into the isoangles

$$\hat{\theta} = \theta \times \hat{I}_\theta, \quad \hat{\phi} = \phi \times \hat{I}_\phi, \quad \hat{I}_\theta = b_3, \quad \hat{I}_\phi = b_1 \times b_2, \tag{2.2}$$

In the simple case here considered, we then have the following trivial identities between isodifferentials and differentials

$$\hat{d}\hat{\theta} = b_3^{-1} \times d(\theta \times b_3) = d\theta, \quad \hat{d}\hat{\psi} = (b_1 \times b_2)^{-1} \times d(\phi \times b_1 \times b_2) = d\phi \tag{2.3}$$

But we have the expression

$$\hat{d}\hat{cos}\hat{\phi} = \hat{sin}\hat{\phi} \times \hat{d}\hat{\phi} = (b_1 \times b_2)^{-1} \times sin(\phi \times b_1 \times b_2) \times d\phi, \tag{2.4}$$

that illustrates nontrivial departures between conventional and isotopic treatments despite the simplest possible assumptions made above.

The *solid isoangle* is evidently given by

$$\hat{d}\hat{\Omega} = \hat{d}\hat{\theta} \times \hat{d}\hat{cos}(\hat{\phi}) =$$

$$= (b_1 \times b_2)^{-1} d\theta \times d\phi \times sin(\phi \times b_1 \times b_2), \tag{2.5}$$

with isointegral

$$\hat{\Omega} = \Omega \times \hat{I}_\Omega = \Omega \times (b_1 \times b_1 \times b_3)^{-1} = \hat{\int}\hat{\int} \hat{d}\hat{\theta} \times \hat{d}\hat{\phi} \times \hat{sin}\hat{\phi} =$$

$$= b_3^{-1} \times \int \int d\theta \times d\phi \times sin(\phi \times b_1 \times b_2), \tag{52.6}$$

and final expression

$$\Omega = (b_1 \times b_2) \times \int \int d\theta \times d\phi \times sin(\phi \times b_1 \times b_2), \tag{52.7}$$

that also illustrates the differences between conventional and isotopic treatments despite the preservation of the scattering volume (but not necessarily of the surface).

The *isoscattering amplitude* $\hat{f}(\hat{\theta}, \hat{\psi})$ can be defined in its most elementary form via the expression

$$e^{i \times k \times z} + \frac{\hat{f}(\hat{\theta}, \hat{\phi})}{\hat{r}} \hat{\times} e^{i \times k \times r}, \tag{2.8}$$

where $e^{i \times k \times z}$ and $e^{i \times k \times r}$ are the *conventional* incoming and scattered waves because we have assumed the exact validity of quantum scattering theories outside the scattering region.

The projection of Eq. (2.8) on a conventional Euclidean space can be written

$$e^{i\times k\times z} + \frac{\hat{f}(\theta \times b_3, \phi \times b_1 \times b_2)}{r} \times e^{i\times k\times r}, \tag{2.9}$$

where we assume the reader is aware that the isodivision in Eq. (2.8) allowing the replacement of $\hat{r}$ with $r$.

The *isodifferential cross section* is then given by

$$\hat{d}\hat{\sigma} - = |\hat{f}(\hat{\theta}, \hat{\phi})|^2 \hat{\times} \hat{d}\hat{\Omega}, \tag{2.10}$$

and the *total cross section* assumes the form

$$\sigma = \hat{\int} \hat{d}\hat{\sigma} = \hat{\int} |\hat{f}(\hat{\theta}, \hat{\phi})|^2 \hat{\times} \hat{d}\hat{\Omega} = \int |\hat{f}(\hat{\theta}, \hat{\phi})|^{\times} d\Omega. \tag{2.11}$$

As recalled in Section I.1, the cross section is a number that, as such, is independent from the selected elaboration. The novelty of the isoscattering theory is the mutation of the scattering amplitude, whose implications will be elaborated below.

**2.2. Isoscattering Matrix.** According to our assumptions of Section I.2, the initial and final states, $|i>$, $|f>$, respectively, are defined on a conventional Hilbert space $\mathcal{H}$ over a conventional quantum field $C$, as denoted by the lack of "hat" in these states. However, by central assumption, their interconnection is done via isomechanics on the Hilbert-Santilli isospace $\hat{\mathcal{H}}$ over the isofield of complex isonumbers $\hat{C}$ with isoinner product (3.15) of Paper I and isounit (1.4). Therefore, the *isoscattering matrix* is defined by (see Ref. [6b], Chapter 12, for a review of earlier works and references)

$$< i|\hat{\times}\hat{S}\hat{\times}|f> = < i| \times T \times \hat{S} \times T \times |f>, \tag{2.12}$$

with basic isounitarity property

$$< i| \times T \times (\hat{S} \times T \times |f>) \; = \; (< i| \times T \times \hat{S}^{\dagger}) \times T \times |f> \tag{2.13}$$

namely

$$\hat{S}\hat{\times}\hat{S}^{\dagger} = \hat{S}\hat{\times}\hat{S} = \hat{I}, \tag{2.14}$$

or

$$\Sigma_f \hat{S}_{fi} \times T_{ik} \times \hat{S}_{fk} = \hat{\delta}_{ik} = \hat{I} \times \delta_{ik}, \tag{52.15}$$

expressing the conservation of probability on isospace over isofields.

It should be indicated that *the isoscattering matrix is an isomatrix,* namely, its elements are isonumbers $\hat{n} = n \times \hat{I}$. Consequently, the isotopic element in the isoinner product (5.11) can be eliminated with the reduction of type (4.35a)

$$\hat{S} = \tilde{S} \times \hat{I} \tag{2.16}$$

under which we can regain the conventional form

$$< i| \times T \times \hat{S} \times T \times |f > = < i| \times \tilde{S} \times |f > . \tag{2.17}$$

However, the nontrivial character of the isoscattering theory is that the reduced matrix $\tilde{S}$ is *nonunitary,*

$$\tilde{S} \times \tilde{S}^{\dagger} \neq I. \tag{2.18}$$

as it is the case also for Eq. (5.14). The latter property is crucial to guarantee the exiting put of the class of unitary equivalence of quantum scattering theories and, in its absence, novelty would only be illusory. Note that *without the isotopic formulation, nonunitary scattering theories would be inconsistent* [6=11].

The *isotransition probability* that states $|i >$ are turned into the states $|f >$ is then given by

$$\hat{P}_{fi} = \hat{S}^{\dagger}_{fi} \times T \times \hat{S}_{fi} = (\tilde{S}^{\dagger}_{fi} \times \tilde{S}_{fi}) \times \hat{I}, \tag{52.19}$$

with evident *total isoprobability*

$$\hat{P}_{tot} = \Sigma_f \hat{S}^{\dagger}_{fi} \times T \times \hat{S}_{fi}. \tag{2.20}$$

**2.3. Isopropagators.** We now construct the isotopic image of quantum propagators, here called *isopropagators,* and then compare the resulting isoseries with a conventional expansion. We maintain the conventional assumption of the Hamiltonian being composed by two parts,

$$H = H_0 + H_1, \tag{2.21}$$

and consider the Schrödinger-Santilli isoequation (4.14) under the simplified assumption that the isounit of time is 1, namely, that time is not lifted, $\hat{t} = t$,

$$i \times \partial_t \hat{\psi}(t, r) = [H_0(r, p) + H_1(r, p)] \times T(t, r, p, \psi, \partial\psi, ...) \times \hat{\psi}(t.r). \tag{2.22}$$

To avoid the venturing of superficial technically unsubstantiated perceptions (to their peril), readers should be aware that the isoscattering theory allows the consistent inclusion, for the first time to our knowledge, of *nonlinear effects,* inclusion that would imply basic inconsistencies for conventional scattering theories indicated in reference to Eq. (4.15).

Additionally, the isoscattering theory allows the verification of causality via an irreversible treatment of irreversible scattering processes via the use of the broader genomathematics [15,16], a condition also impossible for quantum scattering theories, with the understanding that the proper treatment of irreversibility requires the covering Lie-admissible genoscattering theory.

The isopropagator $\hat{G}(t,r;t'r')$ is then defined by

$$i \times \partial_t \hat{G}(t,r;t',r') - (H_0 + H_1) \times T \times \hat{G}(t,r;t',r') = \hat{\delta}(t'-t) \times \hat{\delta}^3(r'-r), \quad (2.23)$$

where $\hat{\delta}^3$ and $\hat{\delta}$ denotes the Dirac-Myung-Santilli isodelta function of hadronic mechanics (Section 3.5).

Recall that quantities with an isoscalar structure show the factorized isounit when projected on conventional spaces, such as $\hat{g} = g \times \hat{I}$. In this case we have the simplification of the isoproduct of an isofunction by another quantity $\hat{g}\hat{\times}A = g \times \hat{I} \times T \times A = g \times A$.

However, the isodelta is an *isodistribution* and, as such, it does not admit the factorization of the isounit, as it is the case for the isodifferential. Since the isodelta has no factorization of the isounit, the same holds for the isopropagator $\hat{G}(t,r;t',r')$.

Assuming that $\hat{G}_0(t,r;t',r')$ is the isopropagator for $H_0$, we then have

$$\hat{G}(t,r;t',r') = \hat{G}_0(t,r;t',r') +$$

$$+ \int\int dt' \times d^3r' \times \hat{G}(t,r;t_1,r_1) \times T \times H_1(t,r) \times T \times \hat{G}_0(t,r;t',r'). \quad (2.24)$$

**2.4. Convergent Isoexpansions.** At this point, we subject Eq. (5.24) to a power isoexpansion in terms of $H_1$ as in the original case [1], however, without the conventional restriction that the interacting term $H_1$ is small. By recalling that the formulation is on $\hat{\mathcal{H}}$, we have the expression

$$\hat{G}(t,r;t',r') = \hat{G}_0(t,r;t',r') +$$

$$+ \int\int dt' \times d^3r' \times \hat{G}_0(t,r;t',r') \times T \times H_1(t_1,r_1) \times T \times \hat{G}_0(t,r;t_1r_{,1}) +$$

$$+ \int\int dt' \times d^3r' \times \hat{G}_0(t,r;t_1,r_1) \times T \times H_1(t_1,r_1) \times T \times \hat{G}_0(t_1,r_1;t',r') +$$

$$+ \int\int dt' \times d^3r' \times \hat{G}_0(t,r;t_1,r_1) \times T \times H_1(t_1,r_1) \times$$

$$\times T \times \hat{G}_0(t_1,r_1;t_2,r_2) \times T \times H_1(t_2,r_2) \times \hat{G}_0(t_2,r_2;t',r') + ... \quad (2.25)$$

Similarly, by assuming that $\hat{\psi}_0(t,r)$ is the isoeigenfunction of $H_0$,

$$i \times \partial_t \hat{\psi}_0(t,r = H_o \times T \times \hat{\psi}_0(t,r), \quad (2.26)$$

we have the isoexpansion for the wave isofunction

$$\hat{\psi}(t.r) = \hat{\psi}_0(t,r) +$$

$$+\int\int dt_1 \times d^3r_1 \times \hat{G}_0(t,r;t_1,r_1) \times T \times \hat{\psi}_0(t_1.r_1) + ... \tag{2.27}$$

We now assume for $t \to -\infty$

$$\psi_0(t,z) = \psi_i(t,r)e^{i\times k\times r} \tag{2.28}$$

Then we have the expression

$$\hat{\psi}(t.r) = \psi_0(t,r) +$$

$$+\int\int dt_1 \times d^3r_1 \times \hat{G}_0(t,r;t_1,r_1) \times T \times \psi_0(t_1.r_1) + ... \tag{2.29}$$

and we have the explicit form of the isoscattering matrix

$$\hat{S}_{fi} = \hat{I} + i \times \int dt' \times d^3r \times \psi_0(tt',r')^\dagger \times T \times H_1(t',r') \times \psi_0(t',r + ... \tag{2.30}$$

It is evident that all the above expansions are strongly convergent, not only because the isotopic eleent verifies the condition

$$|T| \ll |H_1|. \tag{2.31}$$

but also because the isopropagator no longer admits the divergence of the conventional propagator for $t = t', r = r'$, thus confirming a main objective of this paper.

**2.5. Prediction of Mini-Black-Holes.** Let us consider the isogravitational content of the isoscattering theory outlined in Section 2.5 of Paper II. Recall the identification of the conventional Riemannian metric, such as the Schwartzschild metric in the coordinates $(\theta, \phi, r, t)$ , with the isometric $\hat{\eta}$ in the very structure of the isogamma matrices, Eqs. (2.35) of Paper II,

$$ds^2 = r^2(d\theta^2 + sin^2 d\theta^2 + d\phi^2) + (1 - \frac{2\times M}{r})^{-1} \times dr^2 -$$

$$-(1 - \frac{2\times M}{r} \times dt^2 \equiv \hat{T}_{sch} \times \eta \equiv \hat{\eta}, \tag{2.32}$$

which is represented with *gravitational isounit and isotopic element*

$$\hat{T}_{sch} = Diag.[1, 1, (1 - \frac{2\times M}{r})^{-1}, (1 - \frac{2\times M}{r})], \tag{2.33a}$$

$$\hat{I}_{sch} = Diag.[1, 1, (1 - \frac{2\times M}{r}), (1 - \frac{2\times M}{r})^{-1}]. \tag{2.33b}$$

We also recall that the conventional Dirac equations represents an *electron in vacuum,* as well known, while the isotopically lifted equation represents *the same electron when immersed within a hyperdense hadronic medium.*

It is then easy to see that *the isoscattering theory predicts the possible creation of mini black holes under sufficiently high energies,* as evident from the singularities of isometric (2.32) that now constitutes the actual metric of the scattering region.

Needless to say, there is no possibility to predict at this stage of the knowledge the threshold of *energy density* needed to trigger a mini-black-hole, and this explains the importance of lifting the scattering theory into a form admitting of a quantitative representation of the energy density of the scattering region.

Recall that, for the conventional scattering theory, the metric for the scattering region is the conventional Minkowski metric, thus without any predictive capacity for mini-black-holes. Consequently, the above prediction of the isoscattering theory and related isoline element (2.32) is sufficient to illustrate the need for serious caution and scrutiny before embarking in extremely high energy scattering experiments based on insufficient quantum methods that can at best produce controversial results.

## 3. Isotopies of Feynman's Diagrams without Divergences

**3.1 Conventional Feynman Diagrams** We are now equipped to tackle a central aspect of our research, the isotopies of Feynman's diagrams, first studied by Animalu [7] and here referred to as *iso-Feynman diagrams.* These isotopies require a re-inspection in this paper because originally conducted without the use of isomathematics, thus having the shortcomings of Refs. [6-12] of Paper I.

We begin by recalling the main features of conventional Feynman diagrams. then construct their isotopic images, and outlining the rules for computing isoscattering cross-sections from iso-Feynman diagrams.

As is well-known (see, e.g., Ref. [8]), Feynman's diagrams (also called path-integral technique in QED) comprise a representation of fundamental elementary particles (e.g., electrons $e^-$), their antiparticles (e.g. positrons $e^+$), and their annihilation into two photons, e.g.,

$$e^- + e^+ \to \gamma + \gamma, \tag{3.1}$$

in which: the electron $e^-$ is represented by a "point" moving forward in time and the positron $e^+$ by another "point" moving backwards in time; the pair annihilates at the intersection of the lines joining the two point-particle trajectories as shown in Figure 1(a)); the emitted photons are represented by wiggly lines radiating away from the point of annihilation, and Figure 1(b) represents Coulomb interactions via virtual photon exchange.

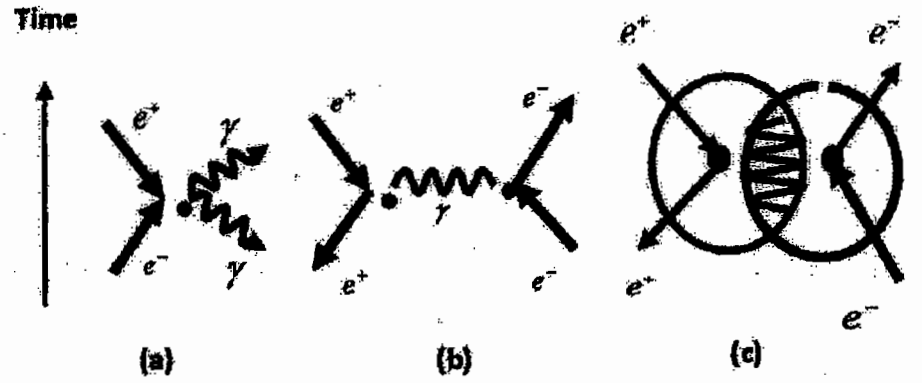



Figure 1: *Conventional Feynman's graphs for: (a) $e^-e^+ \to \gamma + \gamma$ annihilation into two photons; (b) $e^- - e^+$ Coulomb interaction via "virtual" photon exchange; and (c) nonunitary representation of non-Hamiltonian interactions of extended wave packets due to overlapping in the annihilation process (3.1) or the' model of the fusion process $e^+ + e^- \to \pi^0$*

The scattering cross-sections is then calculated by following a number of *Feynman rules* designed to reproduce, at the lowest order, classical results found with conventional quantum mechanics. The full power of the method is realized, however, in calculating the radiative corrections and higher order terms and, as we shall see in this section, corrections from non-Hamiltonian forces activated in the hadronic scattering region.

**3.2. The Problem of Antiparticles in Feynman's Diagrams.** As indicated in paper I, Section 2, in these initial studies we shall continue to use the 20th century formalism for antiparticles. Nevertheless, the reader should be aware of the novel *isodual theory of antimatter* that has achieved the first known axiomatically consistent representation of antimatter at all levels, from Newtonian mechanics to second quantization, while being equivalent to charge conjugation in its operator form.

The new theory of antimatter has been motivated by clear insufficiencies of the classical theories of the 20th century, such as the absence of any differentiation between *neutral* matter and antimatter, problematic aspects in the classical treatment of *charged* antiparticles (because their operator image is a conventional "particle," rather than the needed charge conjugated "antiparticle"), and other problematic aspects requiring a resolution on serious scientific grounds.

The map from matter to antimatter at all levels is performed by an operation called *isoduality,* represented with the upper symbol $d$, essentially given by anti-Hermiticity,

that must be applied, for consistency, to all possible quantities and their operations. Consequently, a central feature of the isodual theory is that *all physical quantities of antimatter are measured with negative units,* thus including negative units of time, energy, linear momentum, etc.

The above formulation of antimatter resolves the known uneasiness in Feynman's use of motions backward in time, because a motion backward in time referred to a negative unit of time is as causal as motion forward in time referred to a positive unit of time.

Additionally, this feature renders fully causal the negative energies of Dirac's equation when, again, referred to a negative unit, and provides a new interpretation of Dirac's equation as directly representing an electron-positron pair at the level of first quantization without any need for the "hole theory" or second quantization, because the isodual theory of antimatter holds at the *classical* level, let alone for first quantization [9].

Additionally, these studies have identified a new fundamental symmetry of spacetime, called *isoselfduality* (essentially the invariance under anti-Hermiticity as verified by the imaginary unit $i \equiv i^d$), that essentially providing a deeper representation of the invariance of a particle-antiparticle pair under charge conjugation. This new symmetry is verified by the l.h.s. of Eq. (3.1), since $(e^- + e^+)^d = e^+ + e^-$, but it is violated in the r.h.s. because $\gamma^d \neq \gamma$.

This new spacetime symmetry (that could be verified by our entire universe in the event made up of matter and antimatter in equal amounts) suggests the verification of the same symmetry in the r.h.s. of Eq. (3.1). This has been achieved in Ref. [9] via the differentiation between the *conventional photon* $\gamma$ emitted by matter and the *isodual photon* $\gamma^d$ emitted by antimatter. The latter is predicted as being physically different than that emitted by matter, e.g., because repelled by the gravitational field of matter and having other experimentally verifiable features that may initiate, in due time, the new field of *experimental antimatter astrophysics.*

As a result,. the new isodual theory of antimatter replaces Eq. (3.1) with in the following isoselfdual invariant reaction [9]

$$e^- + e^{-d} \rightarrow \gamma + \gamma^d, \tag{3.2}$$

where $e^d \equiv e^+$ since isoduality is equivalent to charge conjugation.

It is evident that the above occurrence requires, alone, a re-inspection of the entire formulation of the Feynman's diagrams that we cannot possibly achieve in these first papers to prevent a prohibitive length and that has to be deferred to a subsequent study.

**3.3 Isotopies of Feynman Diagrams.** Figure 1(c) represents the proposed isotopic image of non-Hamiltonian interactions of extended wave packets due to overlapping,

as predicted by hadronic mechanics since its inception, e.g., for model of the$\pi^o$ synthesis from electrons and positrons, the synthesis of neutrons from protons and electrons, or the synthesis of hadrons at large from lighter particles (see the excellent review by Kadeisvili [10]).

In terms of the isounit $\hat{I}$ and isotopic element $\hat{T}$, here indicated as $\hat{I}_{st}$ and $T_{st}$ for the carrier space of a relativistic hadronic mechanics, the correspondence between free-particle Feynman propagators in conventional relativistic theory and their isotopic image in hadronic mechanics, can be characterized as follows:

$$[S_F(x) = (\gamma^\mu p_\mu + im)\Delta_F(x)] \longrightarrow [\hat{S}_F(\hat{x}) = (\hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}_\mu \times \hat{p}_\nu + i \times \hat{m}) \times T_{st} \times \hat{\Delta}_{\hat{F}}(\hat{x})], \quad (3.3a)$$

$$[\Delta_F(x) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-px}}{p^2 - m^2 + i\epsilon}] \longrightarrow [\hat{\Delta}_F(\hat{x}) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-p \times T_{st} \times x}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i \times \hat{\epsilon}}] \quad (3.3b)$$

with corresponding expression in momentum 4-vector space:

$$[S_F(p) = (\gamma^\mu p_\mu + m)\Delta_F(p^2) = \frac{\gamma^\mu p_\mu + m}{p^2 - m^2 + i\epsilon}] \longrightarrow$$

$$[\hat{S}_F(\hat{p}) = (\hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}_\mu \times \hat{p}_\nu + i \times \hat{m}) \times T_{st} \times \hat{\Delta}_F(\hat{p}^2) = \frac{(\hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}_\mu \times \hat{p}_\nu + i \times \hat{m}) \times T_{st}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i \times \hat{\epsilon}}] \quad (3.4)$$

**3.4. Isotopic Inclusion of Electromagnetic interactions.** In the presence of an external electromagnetic field, the solution of the (regular) Dirac-Santilli isoequation takes the form

$$\hat{\Psi} = \hat{\psi}(\hat{x}) + \hat{e}\hat{\times} \hat{\int} \hat{d}^4\hat{x}' \hat{\times} \hat{S}_f(\hat{x} - \hat{x}') \hat{\times} \hat{\gamma} \hat{.} \hat{A}(\hat{x}') \hat{\times} \hat{\Psi}(\hat{x}')$$

$$= \hat{\psi}(\hat{x}) + \hat{e}\hat{\times} \hat{\int} \hat{d}^4\hat{x}' \hat{\times} \hat{S}_f(\hat{x} - \hat{x}') \hat{\times} \hat{\gamma} \hat{.} \hat{A}(\hat{x}') \hat{\times} \hat{\psi}(\hat{x}')$$

$$+\hat{e}^{\hat{2}} \hat{\times} \hat{\int} \hat{d}^4\hat{x}' \hat{\int} \hat{d}^4\hat{x}'' \hat{\times} \hat{S}_f(\hat{x} - \hat{x}') \hat{\times} \hat{\gamma} \hat{.} \hat{A}(\hat{x}') \hat{\times} \hat{\psi}(\hat{x}') \hat{\times} \hat{S}_f(\hat{x}' - \hat{x}'') \hat{\times} \hat{\gamma} \hat{.} \hat{A}(\hat{x}'') \hat{\times} \hat{\psi}(\hat{x}'') + ... \quad (3.5)$$

This leads to a formal definition of the Feynman-Animalu isopropagator either as a series:

$$\hat{S}'_f(\hat{x}, \hat{x}') = \hat{S}_f(\hat{x} - \hat{x}') + \hat{e}\hat{\times} \hat{\int} \hat{d}^4\hat{x}'' \hat{\times} \hat{S}_f(\hat{x} - \hat{x}'') \hat{\times} \hat{\gamma} \hat{.} \hat{A}(\hat{x}'') \hat{\times} \hat{S}_f(\hat{x}' - \hat{x}'') + ... \quad (3.6)$$

or as an integral equation:

$$\hat{S}'_f(\hat{x}, \hat{x}') = \hat{S}_f(\hat{x} - \hat{x}') + \hat{e}\hat{\times} \hat{\int} \hat{d}^4\hat{x}'' \hat{\times} \hat{S}_f(\hat{x} - \hat{x}'') \hat{\times} \hat{\gamma} \hat{.} \hat{A}(\hat{x}'') \hat{\times} \hat{S}'_f(\hat{x}', \hat{x}''). \quad (3.7)$$

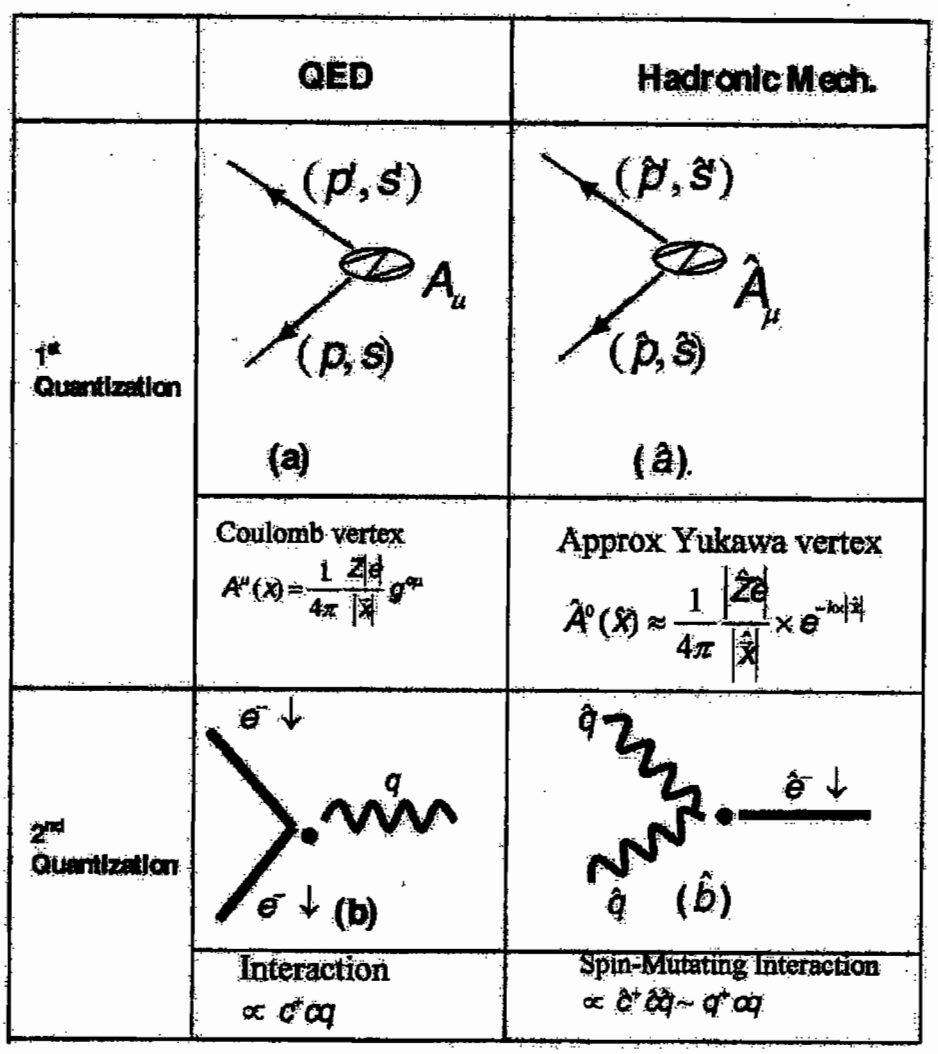



Figure 2: Correspondence between the Feynman graphs for fermion-fermion interaction vertex with boson emission in 1st and 2nd quantization schemes in QED and the isotopic images for the boson-boson interaction vertex with single fermion spin emission in HM.

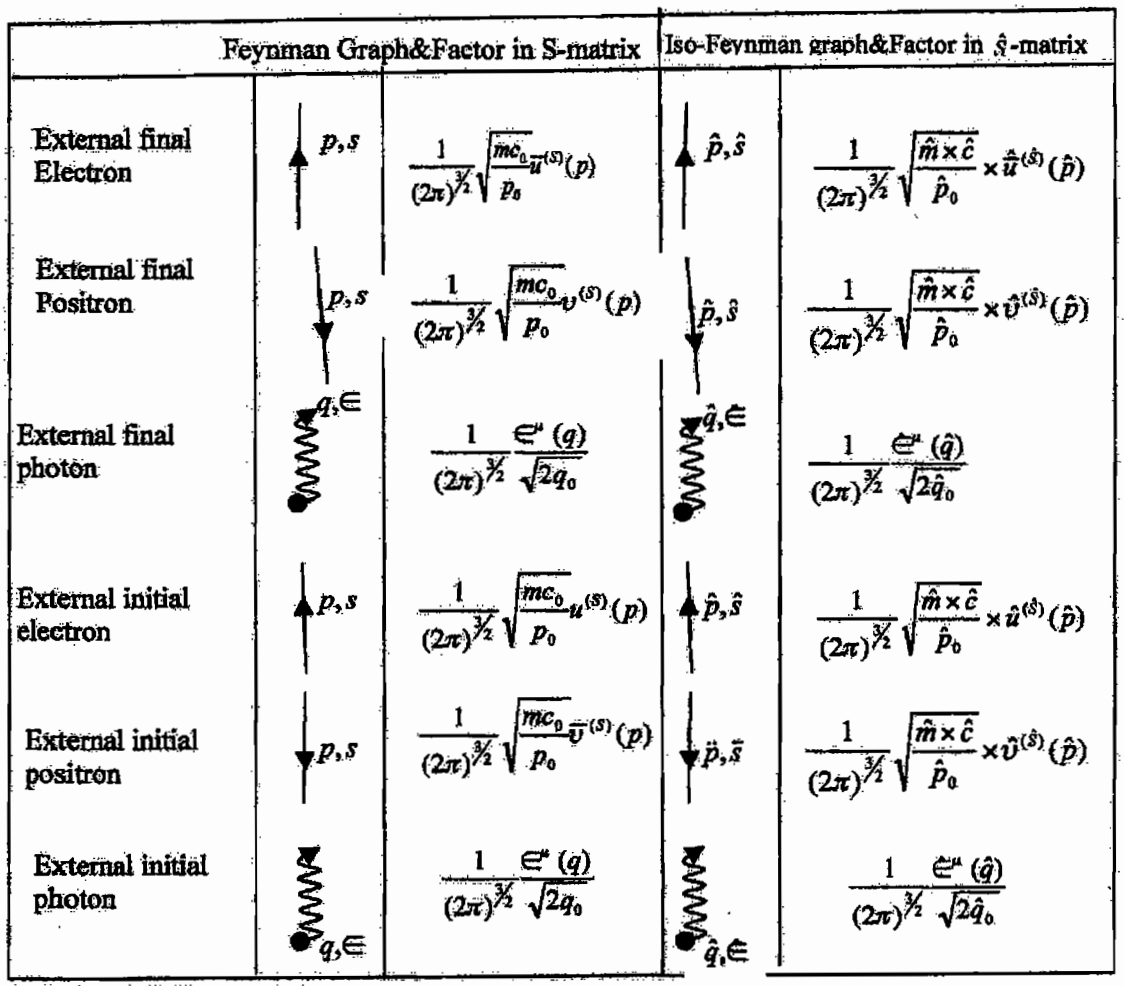



Figure 3: Feynman Graphs/Rules and their Isotopic Images

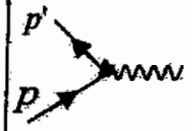
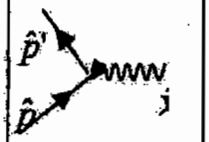
| Feynman Graph & factor in S-matrix | | | Iso-Feynman graph & factor in $\hat{s}$-matrix | |
|---|---|---|---|---|
| Internal Electron |  | $iS_F(p)$, given by the expression in Eq. (6.2) |  | $i\hat{S}_F(\hat{p})$ given by the expression in Eq.(6.2) |
| Internal photon |  | $-ig^{\mu\nu}D_F(q^2)$ $=\frac{-ig^{\mu\nu}}{q^2+i\in}$ |  | $-i\hat{g}^{\mu\nu}\hat{D}_F(\hat{q}^2)$ $=\frac{-i\hat{g}^{\mu\nu}}{\hat{q}^2++(m_*\hat{c})^2+i\hat{\in}}$ |
| Vertex |  | $-i\sqrt{4\pi e}\gamma^\mu(2\pi)^4\times$ $\gamma^\mu\times\delta^{(4)}(p'-p-q)$ |  | $-i\sqrt{4\pi e}\hat{\gamma}^\mu(2\pi)^4\times$ $\hat{\gamma}\times\hat{\delta}^{(4)}(\hat{p}'-\hat{p}-\hat{q})$ |
| Coulomb Vertex |  | $\frac{-i4\pi Z\lvert e\rvert e}{\lvert\vec{p}'-\vec{p}\rvert^2}\gamma^0\times$ $(2\pi)\delta(p_0'-p_0)$ |  | $\frac{-i4\pi Z\lvert\hat{e}\rvert\hat{e}}{\lvert\hat{\vec{p}}'-\hat{\vec{p}}\rvert^2+(m_*\hat{c})^2}\times\hat{\gamma}^0\times$ $(2\pi)\hat{\delta}(\hat{p}_0'-\hat{p}_0)$ |

Figure 4: Continuation of Figure 5

where $\hat{\gamma}\hat{.}\hat{A}(\hat{x}') \equiv \hat{\eta}_{st}^{\mu\nu} \times \hat{\gamma}_\mu \times \hat{A}_\nu(\hat{x}')$ and $\hat{A}_\nu(\hat{x}')$ is the iso-electromagnetic field 4-vector potential given by the corresponding iso-gauge principle[xx].

To elaborate the basic physical concepts in terms of Feynman diagrams for electron scattering with an electromagnetic field in interior dynamical conditions, we show in Figure 4 the two characteristic differences in 1st and 2nd quantization schemes. The iso-scattering matrix is given in first quantization scheme by

$$\hat{S}_{f,i} = lim_{t\to\infty} \hat{\int} \hat{d}^3\hat{x}\hat{\times}\hat{\psi}_{\hat{p}'}^{+\hat{s}'}\hat{\times}\hat{\Psi}_{\hat{p}}^{\hat{s}} \tag{3.8}$$

and$\hat{\Psi}_{\hat{p}}^{\hat{s}}$ is the exact solution given as in Eq.(6.5) by

$$\hat{\Psi}_{\hat{p}}^{\hat{s}}(\hat{x}) = \hat{\psi}_{\hat{p}}^{\hat{s}}(\hat{x}) + \hat{e}\hat{\times}\hat{\int}\hat{d}^4\hat{x}'\hat{\times}\hat{S}_f(\hat{x}-\hat{x}')\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x}')\hat{\times}\hat{\Psi}_{\hat{p}}^{\hat{s}}(\hat{x}') \tag{3.9}$$

with the iso-normalization

$$\hat{\int}\hat{d}^3\hat{x}\hat{\times}\hat{\psi}_{\hat{p}}^{+\hat{s}}(\hat{x})\hat{\times}\hat{\psi}_{\hat{p}'}^{\hat{s}'}(\hat{x}) = \hat{\delta}_{\hat{s}\hat{s}'}\hat{\times}\hat{\delta}^3(\hat{\mathbf{p}}-\hat{\mathbf{p}}') \tag{3.10}$$

Note that the correspondence principle in 1st quantization scheme involves a lifting of the Coulomb vertex in QED into the approximate Yukawa vertex in hadronic mechanics, and additionally involves the lifting from Bose-Einstein to Fermi-Dirac statistics in 2nd quantization scheme, i.e., mutation of spin under sufficiently high energies.

The correspondence between Feynman graphs/rules and their isotopic images for computation of contributions to the S-matrix in QED of spin- particles are summarized in the Figures 5, 6, as well as the rules:

(1) $\hat{\int}\frac{\hat{d}^4p}{2\pi^4}$ for each internal line.

(2) Overall sign $(-)^{L+P}$ where $L$ is the number of closed electron loops and $P$ is the permutation of the external particles.

(3) Phase space of final particle involves $d^3p...$ ; and

(4) Flux of particles is $V\frac{\mathbf{V_1}-\mathbf{V_2}}{(2\pi)^6}$.

**3.5. Concluding remarks.** Lagrange, Hamilton, Jacobi and other founders of mechanics stated that nature cannot be entirely reduced to potential interactions solely representable with quantities we call today Lagrangians and Hamiltonians, for which reason they wrote their celebrated analytic equations with *external terms* representing interactions not admitting a potential energy (for historical references and comments, see Refs. [11]).

Due to the successes of purely Lagrangian and Hamiltonian theories, such as special relativity and quantum mechanics, the external terms were removed from the

analytic equation in the mainstream physics of the 20th century, thus abstracting all events in the universe to potential interactions among point-like particles moving in vacuum, an abstraction that was also implemented for the scattering theories.

However, the *No Reduction Theorem* of paper I, Section 2, has confirmed the historical legacy of Lagrange, Hamilton and Jacobi, by establishing in particular that the nonlinear, nonlocal and nonpotential interactions represented with external terms necessarily originate at the ultimate level of nature, that of deep mutual penetration and overlapping of particles, thus requiring a broadening of the scattering theory for their inclusion.

At any rate, the rigorous implementation of the axioms of quantum mechanics without *ad hoc* manipulations, and their inherent point-like abstraction of particles, essentially reduce most scattering process among charged particles to Coulomb interactions. This causes uneasiness in the conventional explanation of multi-particle productions via second quantization, since the latter events are clearly visible in detectors, thus expected as being interpreted at the *semiclassical* level, let alone that of *first* quantization.

*Mutatis mutandae,* the admission of any particle dimension of the same order of magnitude of the scattering region, causes their deep mutual overlapping, with consequential need to include in scattering processes precisely the nonlinear, nonlocal and nonpotential interactions originated by Lagrange,. Hamilton and Jacobi, since these contacts effects are unavoidable for the dynamics of extended particles.

Predictably, the inclusion of external terms in the analytic equation has dramatic implications since it causes the loss of most conventional mathematical and physical knowledge. In fact, the brackets of the time evolution of Hamilton's equation with external terms violate the conditions to characterize an algebra, let alone causes the loss of all Lie algebras (see Refs. [6] for their analysis at the foundation of hadronic mechanics).

This occurrence mandated very laborious efforts lasted for decades to construct basically new mathematics and mechanics capable of incorporating Lagrange's and Hamilton's external terms. The sole representation of the historical external terms achieving invariance over time was their classical representation via Santilli's isounit and related isotopic element that can be presented in these concluding remarks in their original as well as ultimate meaning. This scenario explains the delay of decades prior to being in a position of addressing the scattering problem.

The reasons for a nonunitary-isounitary broadening of conventional unitary scattering theory have been indicated throughout these papers and need not be repeated here. The possible significance of the former theory over the latter can solely be established in subsequent papers over a significant period of time.

Hence, we can conclude these remarks by bringing to the attention of the curious reader that the ultimate origin of the new isoscattering theory rests indeed the histori-

cal legacy of Lagrange and Hamilton on the external terms of their celebrated analytic equations, which terms, following their popular suppression in the 20th century, have re-emerged in all their historical, mathematical and physical relevance.

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# References


[1] R. M. Santilli and A. O. Animalu , "Nonunitary Lie-isotopic and Lie-admissible scattering theory of hadronic mechanics, I: Conceptual and Mathematical foundations," in *Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes,* C. Corda, Editor, Kathmandu University (2011) pages 423-459
http://www.santilli-foundation.org/docs/Isoscattering-I.pdf

[2] R. M. Santilli and A. O. E. Animalu, "Nonunitary lie-isotopic and Lie-admissible scattering theory of hadronic mechanics,I: Deformations-Isotopies of Lie's Theory, Special Relativity,
and Mechanics," in Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes, C. Corda, Editor, Kathmandu University (2011) pages 461-520
http:http://www.santilli-foundation.org/docs/Isoscattering-II.pdf

[3] R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry,*, Vol. I [3a], II [3b], III [3c], IV [3d] and [3e], International Academnic Press, (2008), available as free downlaods from
http://www.i-b-r.org/Hadronic-Mechanics.htm

[4] R. M. Santilli, *Foundations of Hadronic Chemistry, with Applications to New Clean Energies and Fuels,* Kluwer Academic Publishers (2001), available as free download from
http://www.santilli-foundation.org/docs/Santilli-113.pdf

[5] A. O. E. Animalu and R. M. Santilli, Intern. J. Quantum Chem. **26**,175 (1995).

[6] R. M. Santilli, *Elements of Hadronic Mechanics*, Vol. I (1995) [14a], Vol. II 91995) [14b] Academy of Sciences, Kiev, available in free pdf downloads from
http://www.santilli-foundation.org/docs/Santilli-300.pdf
http://www.santilli-foundation.org/docs/Santilli-301.pdf

[7] A. O. E. Animalu, Hadronic J. bf 31, 248 (2008).

[8] M. Kaku, *Quantum Field Theory,* Oxford University Press (1993).

[9] R. M. Santilli, *Isodual Theory of Antimatter with Applications to Antigravity, Grand Unifications and Cosmology,* Springer (2006).

[10] J. V. Kadeisvili, "The Rutherford-Santilli Neutron," Hadronic J. **31**, 1, (2008), available in free pdf download from

http://www.i-b-r.org/Rutherford-Santilli-II.pdf
also available in html version in the website
http://www.i-b-r.org/Rutherford-Santilli-neutron.htm

[11] R. M. Santilli, *Foundation of Theoretical Mechanics,* Volume I (1978) [11a], and Volume II (1982) [11b],Springer-Verlag, Heidelberg, Germany, available as free download from
http://www.santilli-foundation.org/docs/Santilli-209.pdf
http://www.santilli-foundation.org/docs/santilli-69.pdf

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
Kathmandu University, Nepal, April (2011) pages 547-580

# NONUNITARY LIE-ISOTOPIC AND LIE-ADMISSIBLE SCATTERING THEORIES OF HADRONIC MECHANICS, IV: Reversible Electron-Proton and Electron-Positron Scatterings

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## Abstract

In the preceding three papers of this series, we have presented a nonunitary, invariant, axiom preserving, Lie-isotopic broadening of the scattering theory, called *isoscattering theory,* for the representation within the context of hadronic mechanics of reversible particle events. In this paper we show, apparently for the first time, that the nonunitary character of the new scattering theory allows the representation of the synthesis and subsequent spontaneous decay of: the synthesis of the neutron from a proton and an electron, e.g., as occurring in stars, $e^- + p \to n + \nu \to p + e^- + \nu + \bar{\nu}$; the $\pi^o$ meson from an electron-positron pair, $e^+ + e^- \to \pi^0 \to e^+ + e^-$; and similar events known as *synthesis of hadrons.* These events are beyond the representational capability of the conventional scattering theory because, as shown in preceding studies, they require a *positive binding energy* (since the rest energy of the synthesized particle is bigger than the sum of the rest energies of the original constituents) under which the Schröedinger equation and other unitary formalisms of quantum mechanics provides no physically meaningful solutions. By contrast, hadronic mechanics has allowed an exact, numerical and invariant representation of *all* characteristics of said hadron syntheses precisely in view of its nonunitary character. Consequently, in this paper we show that the isoscattering theory does allow, for the first time, a representation of the indicated hadron syntheses as *nonconservative events* requiring missing energy provided by the environment. The proposed isoscattering theory then emerges as the *only* known *invariant* representation of these nonconservative scattering events.

Key words scattering theories, nonunitary theories, isounitary theories
PACS 03.65.Nk, 11.30.Cp, 03.65.-w

TABLE OF CONTENT

1. Introduction

2. Review/Characterization of the Isoscattering Formalism
   2.1. Review of Scattering and Isoscattering Formalism.
   2.2. $\hat{O}(4,2)$Dynamical Symmetry of the Scattering Region.

3. Computation of S-matrix from Feynman Graphs/Rules.
   3.1. The Conventional e-p Coulomb Scattering.
   3.2. Profile of e-p Isoscattering.

4. Experimental Verification

5. Concluding Remarks

Acknowledgments

References.

# 1. Introduction

In the preceding paper [1] of this series, we have presented the nonunitary, axiom-preserving, Lie-isotopic generalization of the conventional scattering theory under the name of *isoscattering theory,* for the treatment within the context of relativistic hadronic mechanics (HM) of reversible particle events. In particular, the new scattering theory was constructed via the isotopies of the conventional Feynman graphs/rules of quantum electrodynamics (QED) [2] within the context of conventional relativistic quantum mechanics (QM) for the specific purpose of studying non-Lagrangian, non-Hamiltonian, thus nonunitary effects expected in particle events since quite some time.

In this paper we formulate, apparently for the first time, the isoscattering theory for reversible events given by the synthesis and subsequent spontaneous decay of the neutron from a proton and an electron,

$$e^- + p \to n + \nu \to p + e^- + \nu + \bar{\nu} \tag{1.1}$$

the synthesis and subsequent spontaneous decay of the $\pi^o$ meson from an electron-positron pair,

$$e^+ + e^- \to \pi^0 \to e^+ + e^- \tag{1.2}$$

and similar events occurring in the core of stars or in particle accelerators, generally referred to as *synthesis of hadrons,* much along the synthesis of the deuterium and otehr nuclei.

The conventional scattering theory is *inapplicable* for the above events (and certainly not "violated" because it is not developed for the events considered) for the following reasons. Recall that all consistent bound states of quantum mechanics at the particle, nuclear and molecular levels are characterized by a *negative binding energy* resulting in the well known *mass defect,* according to which the rest energy of the bound state is *smaller* than the sum of the rest energies of all constituents.

By contrast, the rest energy of synthesized hadrons is *bigger* than the sum of the rest energies of all original particles. It was shown in 1978 by Santilli [3] that the Schrödinger equation, as well as the unitary formalism at large pf quantum mechanics, provide no physically meaningful solutions for the syntheses here considered, because they would require a "positive binding energy" which is sheer anathema for quantum mechanics. Therefore, Santilli [3] proved that a nonunitary image of the Schrödinger equation provides a numerically exact representation of *all* characteristics of the $\pi^o$ particle in its synthesis from an electron-positron pair. Following further developments of hadronic mechanics, Santilli achieved in 1990 [4a] a nonrelatistic, exact and invariant representation of *all* characteristics of the neutron in its synthesis from a proton and an electron, and subsequently provided the relativistic formulation in papers [4b,4c] 9see also Refs. [4d,4e] for related works).

Therefore, Santilli has established in Refs. [3.4] the need for a nonunitary generalization of the conventional scattering theory predicted since some time, but first formulated in an invariant form in Ref. [1], for a quantitative representation of syntheses (1.1) and (1.2), as well as for the synthesis of hadrons at large verifying the general rule that *the rest energy of the synthesized hadrons is bigger than the sum of the rest energies of the original particles.*

On conceptual grounds, these events are *nonconservative* due to missing energy for the syntheses themselves which is provided by the environment (for non-experts in the field, we recall that the missing energy cannot be provided by the relative kinetic energy of the original particles or by the hypothetical neutrino for various inconsistencies, see Ref. [4d,e]). The nonconservative character of the events implies their non-Lagrangian and non-Hamiltonian character, in the sense that said events cannot any longer be entirely described via the sole knowledge of a Lagrangian or a Hamiltonian as it is typically the case for the conventional scattering theory, because the nonconservation originates from interactions violating the integrability conditions for the existence of a potential (the *conditions of variational self-adjointness* [4e]). In turn, these features establish beyond scientific doubt the need for a nonunitary covering of the scattering theory since the insufficiency of the Hamiltonian implies the need for an additional operator that breaks the unitary character of the theory.

On more technical grounds, the nonconservative character of the events implies the inapplicability of Lie's theory with the familiar time evolution of a (Hermitean) operator $A$

$$i\frac{dA}{dt} = [A, H] = AH - HA, \tag{1.3}$$

in favor of Santilli's Lie-isotopic time evolution 9first presented in Ref. [2] of 1978)

$$i\frac{dA}{dt} = [A\hat{,}H] = ATH - HTA \tag{1.4}$$

where $T$ is a second operator (generally independent from, and non-commuting with $H$) characterizing the nonunitarity of the theory. In fact, the Santilli time evolution can be solely achieved from the Lie form via a nonunitary transform

$$UU^\dagger = \hat{I} = 1/\hat{T} \neq I, \tag{1.5}$$

for which

$$U[A, H]U^\dagger = [\hat{A}\hat{,}\hat{H}] = \hat{A}\hat{T}\hat{H} - \hat{H}\hat{T}\hat{A}, \tag{1.6a}$$

$$\hat{A} = UAU^\dagger, \quad \hat{H} = UHU^\dagger, \tag{1.6b}$$

by clarifying that, in the generally adopted notation, the "hats" in the top of operators are often omitted for simplicity as in Eq. (1.4). The isoscattering theory then emerges as the only known theory which is:

1) *Universal,* in the sense of admitting all possible syntheses of hadrons (under obvious smoothness and regularity conditions);

2) *Invariant,* in the sense of predicting the same numerical values under the same conditions at different times, despite its nonunitary structure (thanks to the the novel isomathematics bypasing the inconsistency theorems as reviewed in paper I); and

3) *Covering,* in the sense of admitting the conventional scattering theory as a trivial particular case when the nonunitary effects are null, i.e., $T = I$;

all above features being necessary for a viable generalized scattering theory, e.g., because the conventional Coulomb scattering of extended particles without collision is definitely unitary in structure. Hence, any generalized scattering theory not admitting the conventional Coulomb scattering as a particular case is disproved by experimental evidence. The isoscattering theory admits indeed said conventional Coulomb scattering trivially because unitarity is a mere particular case of the broader nonunitarity.

In closing these introductory lines, we would like to recall Barut's [5] model of the synthesis of the neutron, Eq. (1.1),

$$n \equiv (p, e^-, \bar{\nu})_{QM} \tag{1.7}$$

within conventional Feynman Lagrangian path-integrals based on the transition from the conventional $O(3,1)$ to the $O(4,2)$ dynamical group. According to this model, the three quarks used in the standard model for the representation of the structure of a *family* of baryons including the neutron are identified with the proton, the electron and the anti-neutrino.

By contrast, Santilli [4] structure model of the neutron within the context of hadronic mechanics is characterized by the proton and the electron in a mutated form $\hat{p}, \hat{e}$ resulting under the necessary nonunitary lifting of the Lorentz symmetry $\hat{O}(3.1)$

$$n \equiv (\hat{p}, \hat{e}^-)_{HM} \tag{1.8}$$

under the acceptance of, and compatibility with the $SU(3)$-color Mendeleev-type classification of hadrons into families.

In connection, Barut [5] achieved the synthesis of the neutron within the context of a *unitary theory.* However, this required the abandonment of the Lorentz symmetry $O(3.1)$ in favor of the broader conformal symmetry $O(4.2)$. In turn, the latter transition implied the addition of physically unidentified coordinates outside our spacetime, as well as the assumption (generally rejected by the scientific community for various technical reasons) that the anti-neutrino is an actual physical constituent of the neutron.

By comparison, Santilli [4] structure model of the neutron remains within our spacetime in (3+1)-dimensions and solely assumes mutated forms of the proton $\hat{p}$ and the electron, $\hat{e}$ (due to their total mutual penetration), as the physical constituents

of the neutron, but requires a necessary nonunitary lifting of the entire formulation of quantum mechanics, thus including the dynamical equation, the Lorentz symmetry and the scattering theory. However, the basic mechanism of the new theory, the isotopies, preserve the original axioms to such an extent that the new theory can be formulated with the same symbols of the old, only subjected to a broader realization. As an example, the Lorentz $O(3.1)$ and the Lorentz-Santilli isotopic $\hat{O}(3.1)$ symmetries are not only isomorphic but they are indistinguishable at the abstract, realization-free level.

We should also mention for further needs Santilli [4d] hypothesis of the *etherino* with energy given by the value missing for the neutron synthesis (1.1),

$$m_{a^0} \geq m_n - (m_p + m_e) = 0.78 MeV \equiv 1.53 m_e, \tag{1.9}$$

*which has bveen specifically and clearly suggested by Santilli not as an additional hypothetical particle within the current "zoo" of unknown particles,* but merely as a *vehicle representing within conventional unitary theories the transfer from the environment to the neutron of the missing energy, spin and other quantities.* Consequently, in lieu pof Eq. (1.1), Santilli considers the alternative formulation

$$e^- + a + p \to n \to e^- + a + p, \tag{1.10}$$

where the antineutrino $\bar{\nu}$ (neutrino $\nu$) is replaced by the etherino $a$ (anti-etherino $\bar{a}$).

In considering the etherino hypothesis, the following aspects should be kept in mind:

a) The assumption of Santilli's etherino as a physical constituent of the neutron in lieu of Barut's anti-neutrino leads to a number of catastrophic inconsistencies, and we shall write

$$n \neq (p, e^-, a)_{QM}, \tag{1.11}$$

including the impossibility of a perennial confinement of the etherino inside the neutron, with its consequential necessary emission as a free particle in the spontaneous decay jointly with the proton and the electron. The clear lack of existence of the etherino as a free particle in our spacetime completely disproves model (1.11).

b) The etherino has been introduced for a *quantum mechanical* representation of the synthesis of the neutron. As such, the etherino no longer appears in the covering representation of the neutron synthesis via hadronic mechanics. Alternatively we can say that the nonunitary broadening of quantum mechanics, beginning with Santilli iso-Hilbert spaces over isofields, is a direct representation of the transfer of energy and other quantities from the environment to the neutron that, as such, require no actual particle for their realization.

c) Santilli's etherino replaces the Pauli-Fermi historical hypothesis of the neutrino without necessarily dismissing the so-called "neutrino experiments." In fact, the synthesis of the neutron not only requires energy, but also spin 1/2 as first identified by

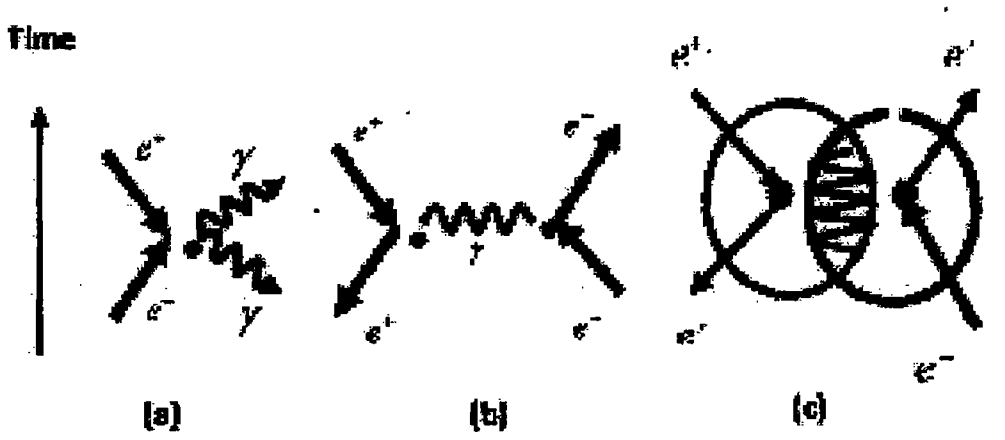



Figure 1: *A schematic view of the evolution of the scattering theory, from early formulations (a), to the more advanced Feynman's formulations in which the scattering is mediated by a particle (b), to the isoscattering formulation where, in addition to mediation by particles, we have internal nonunitary effects due to total mutual penetration of the hyperdense charge distributions of and/or wavepackets of the scattering particles (c).*

Pauli and Fermi. Since the environment of the synthesis provides the missing quantities, the hypothesis of the neutrino as a physical particle in our spacetime becomes no longer necessary. Also, to account for the actual synthesis of the neutron, the Pauli-Fermi hypothesis should have been formulated with a *neutrino,* rather than an anti-neutrino in the left,

$$p + \nu + e^- \rightarrow n, \tag{1.12}$$

since recent studies [4d,4e] have established that the formulation via an anti-neutrino in the left compatible with reaction (1.1),

$$p + \bar{\nu} + e^- \rightarrow n \tag{1.13}$$

*increases,* rather than eliminate the value of the missing energy.

Additionally, the etherino hypothesis does not dismiss experimental data on "neutrino experiments" because the transfer of energy, spin and other quantities from the environment to the neutron is predicted as being a *longitudinal* impulse in our spacetime, thus distinct from the notoriously *transverse* photon and, as such, can account for the measured data. Note finally, the rejection by an increasing number of physics of the notion of several different massive neutrinos since they imply that massive particles in our spacetime propagate through immense hyperdense hadronic media,

such as entire planets or stars, without appreciable collisions. By comparison, being a longitudinal "impulse" (rather than a physical particle) through spacetime, Santilli etherino can indeed travel through vast hyperdense media without excessively hyperbolic assumptions because it propagates in the spacetime underlying the medium, rather than through the medium itself.

In this paper, particles and operators described via QM are represented with conventional symbols, e.g., $p, e, S$, etc., while particles and operators described via HM are represented witrh the same symbols complemented with a hat, $\hat{p}, hate, \hat{S}$, etc. To make this paper minimally self-sufficient and readily understandable to non-experts in HM, we shall review in Sec. 2.1 the main characteristics of the generalized scattering theory developed from Refs.[1] based on the lifting $O(3,1) \to \hat{O}(3,1)$ and summarize the Feynman graphs/rules in QED of spin-$\frac{1}{2}$particles for computation of the S-matrix. We shall then present in Sec. 2.2 an explicit characterization of the scattering region in terms of the lifting $O(4,2) \to \hat{O}(4,2)$ for describing the isoscattering process (1.1) and realization of the structure models: $n \equiv (p, e^-, \bar{\nu})_{QM} \sim (p, e^-, a^0)_{QM} \to (\hat{p}, \hat{e}^-)_{HM}$.In Sec. 3, we shall apply the Feynman graphs/rules to the computation of the scattering cross section for the processes (1.1) and (1.2). In Sec. 4 we shall discuss the experimental implications and in Sec. 5 draw our conclusions.

## 2. Review/Characterization of the Isoscattering Formalism

### 2.1. Review

As is well known [2], the usual Feynman propagator in conventional QED of spin-$\frac{1}{2}$ particles can be characterized as follows in the O(3,1) carrier space of a relativistic quantum mechanics:

$$S_F(x) = (\gamma^\mu p_\mu + im)\Delta_F(x), \Delta_F(x) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ipx}}{p^2 - m^2 + i\epsilon} \tag{2.1a}$$

with corresponding expression in momentum 4-vector space:

$$S_F(p) = (\gamma^\mu p_\mu + im)\Delta_F(p), \Delta_F(p) = \frac{\gamma^\mu p_\mu + im}{p^2 - m^2 + i\epsilon} \tag{2.1b}$$

In terms of the "isounit" $(\hat{I})$and isotopic element $(\hat{T} = \hat{I}^{-1})$defined below and represented as $\hat{I}_{st}$ and $T_{st}$,the generalized Feynman (which may be called iso-Feynman) propagator in the $\hat{O}(3,1)$ carrier space of hadronic mechanics is given by the corresponding expressions as follows

$$\hat{S}_F(\hat{x}) = (\hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}^\mu \times \hat{p}_\mu + i \times \hat{m}) \times \hat{T}_{st} \times \hat{\Delta}_F(\hat{x}),$$

$$\hat{\Delta}_F(\hat{x}) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip\times\hat{T}_{st}\times x}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i\times\hat{\epsilon}} \tag{2.2a}$$

with corresponding expression in iso-momentum 4-vector space

$$\hat{S}_F(\hat{p}) = (\hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}^{\mu} \times \hat{p}_{\mu} + i \times \hat{m}) \times \hat{T}_{st} \times \hat{\Delta}_F(\hat{p}),$$

$$\hat{\Delta}_F(\hat{p} = \frac{(\hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}^{\mu} \times \hat{p}_{\mu} + i \times \hat{m}) \times \hat{T}_{st}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i \times \hat{\epsilon}} \tag{2.2b}$$

In the presence of an external electromagnetic field, the solution of the (regular) Dirac-Santilli isoequation takes the form

$$\hat{\Psi} = \hat{\psi}(\hat{x}) + \hat{e}\hat{\times}\hat{\int}\hat{d^4x'}\hat{\times}\hat{S}_f(\hat{x} - \hat{x'})\hat{\times}\hat{\gamma}.\hat{A}(\hat{x'})\hat{\times}\hat{\Psi}(\hat{x}$$

$$= \hat{\psi}(\hat{x}) + \hat{e}\hat{\times}\hat{\int}\hat{d^4x'}\hat{\times}\hat{S}_f(\hat{x} - \hat{x'})\hat{\times}\hat{\gamma}.\hat{A}(\hat{x'})\hat{\times}\hat{\psi}(\hat{x})$$

$$+\hat{e^2}\hat{\times}\hat{\int}\hat{d^4x'}\hat{\int}\hat{d^4x"}\hat{\times}\hat{S}_f(\hat{x} - \hat{x'})\hat{\times}\hat{\gamma}.\hat{A}(\hat{x'})\hat{\times}\hat{S}_f(\hat{x'} - \hat{x"})\hat{\times}\hat{\gamma}.\hat{A}(\hat{x"})\hat{\times}\hat{\psi}(\hat{x"}) + ... \tag{2.3}$$

This leads to a formal definition of the iso-Feynman propagator either as a series

$$\hat{S'}_f(\hat{x}, \hat{x'}) = \hat{S}_f(\hat{x} - \hat{x'}) + \hat{e}\hat{\times}\hat{\int}\hat{d^4x"}\hat{\times}\hat{S}_f(\hat{x} - \hat{x"})\hat{\times}\hat{\gamma}.\hat{A}(\hat{x"})\hat{\times}\hat{S}_f(\hat{x'} - \hat{x"}) + ... \tag{2.4a}$$

or as an integral equation

$$\hat{S'}_f(\hat{x}, \hat{x'}) = \hat{S}_f(\hat{x} - \hat{x'}) + \hat{e}\hat{\times}\hat{\int}\hat{d^4x"}\hat{\times}\hat{S}_f(\hat{x} - \hat{x"})\hat{\times}\hat{\gamma}.\hat{A}(\hat{x"})\hat{\times}\hat{S'}_f(\hat{x'}.\hat{x"}) \tag{2.4b}$$

where $\hat{\gamma}.\hat{A}(\hat{x'}) = \hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}_{\mu} \times \hat{A}_{\nu}(\hat{x'})$ and $\hat{A}_{\nu}(\hat{x'})$ is the iso-electromagnetic four-vector potential given by the corresponding iso-gauge principle[7] to which we shall return in Sec. 4

Note that, in the limit of unitary transformation, we recover exactly the conventional expressions. For this reason, the primary interest of isoscattering theory lies in the formal relationship/differentiation of the two isoscattering profiles (1.1) and (1.2) for interpreting the existing and future scattering experimental data. To do this, we note that the isotopies of the Dirac matrices ($\hat{\gamma}_{\mu}$) have been explicitly defined in ref.[1] as follows:

$$\hat{\gamma}_k = b_k \times \begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \hat{\gamma}_4 = i \times b_k \times \begin{pmatrix} I_{2\times2} & 0 \\ 0 & -I_{2\times2} \end{pmatrix} \tag{2.5a}$$

$$\hat{\gamma}_\mu \hat{\gamma}_\nu = \hat{\gamma}_\mu \times T_{st} \times \hat{\gamma}_\nu + \hat{\gamma}_\nu \times T_{st} \times \hat{\gamma}_\mu = 2 \times \hat{\eta}_{\mu\nu},$$

where $b_k^2 = n_k^{-2} (k = 1, 2, 3), (b_1^2 \times b_2^2 \times b_3^2) = 1$ for an ellipsoidal scattering region. And whereas, without spin mutation, the generalized spin matrices are

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1 \times n_2 \\ n_1 \times n_2 & 0 \end{pmatrix}, \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times n_1 \times n_2 \\ i \times n_1 \times n_2 & 0 \end{pmatrix},$$

$$\hat{\sigma}_3 = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, \hat{T} = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix} \tag{2.5b}$$

they are, with spin mutation,

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1^2 \\ n_2^2 & 0 \end{pmatrix}, \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times n_1^2 \\ i \times n_2^2 & 0 \end{pmatrix}, \hat{\sigma}_3 = \begin{pmatrix} w \times n_1^2 & 0 \\ 0 & w \times n_2^2 \end{pmatrix} \tag{2.5c}$$

Consequently, the isotopies provide five additional quantities [the four ( $b_k, k = 1, ...4$) for spacetime mutation and one ($w$ ) for the spin] for the representation of experimentally measureable features of the scattering region, such as shape, deformation, scaling, density, anisotropy, etc .

As our interest is to elaborate the basic physical concepts in terms of Feynman diagrams for electron scattering with an electromagnetic field, as well as remove divergences from the theory, we show in Table 1 the two characteristic differences in 1st and 2nd quantization schemes.

The generalized S-matrix is given in 1st quantization scheme by

$$\hat{S}_{f,i} = lim_{t \to \inf} \hat{\int} \hat{d}^3 \hat{x} \hat{\times} \hat{\psi}_{\hat{p}}^{+\hat{s}'} \hat{\times} \hat{\Psi}_{\hat{p}}^{\hat{s}} \tag{2.6}$$

where $\hat{\Psi}_{\hat{p}}^{\hat{s}}$ is the exact solution given as in Eq.(2.3) by

$$\hat{\Psi}_{\hat{p}}^{\hat{s}}(\hat{x}) = \hat{\psi}_{\hat{p}}^{\hat{s}}(\hat{x}) + \hat{e} \hat{\times} \hat{\int} \hat{d}^4 \hat{x}' \hat{\times} \hat{S}_f(\hat{x} - \hat{x}') \hat{\times} \hat{\gamma} \hat{\cdot} \hat{A}(\hat{x}') \times \hat{\Psi}_{\hat{p}}^{\hat{s}}(\hat{x}') \tag{2.7a}$$

with the normalization

$$\hat{\int} \hat{d}^3 \hat{x} \hat{\times} \hat{\psi}_{\hat{p}}^{+\hat{s}}(\hat{x}) \hat{\times} \hat{\psi}_{\hat{p}'}^{+\hat{s}'}(\hat{x}) = \hat{\delta}_{\hat{s}\hat{s}'} \hat{\times} \hat{\delta}^3(\hat{p} - \hat{p}') \tag{2.7b}$$

Note that the correspondence principle in 1st quantization scheme involves a lifting of the Coulomb vertex in QED into the approximate Yukawa vertex in hadronic mechanics, and additionally involves the lifting from Bose-Einstein to Fermi-Dirac statistics in 2nd quantization scheme, i.e., mutation of spin under sufficiently high

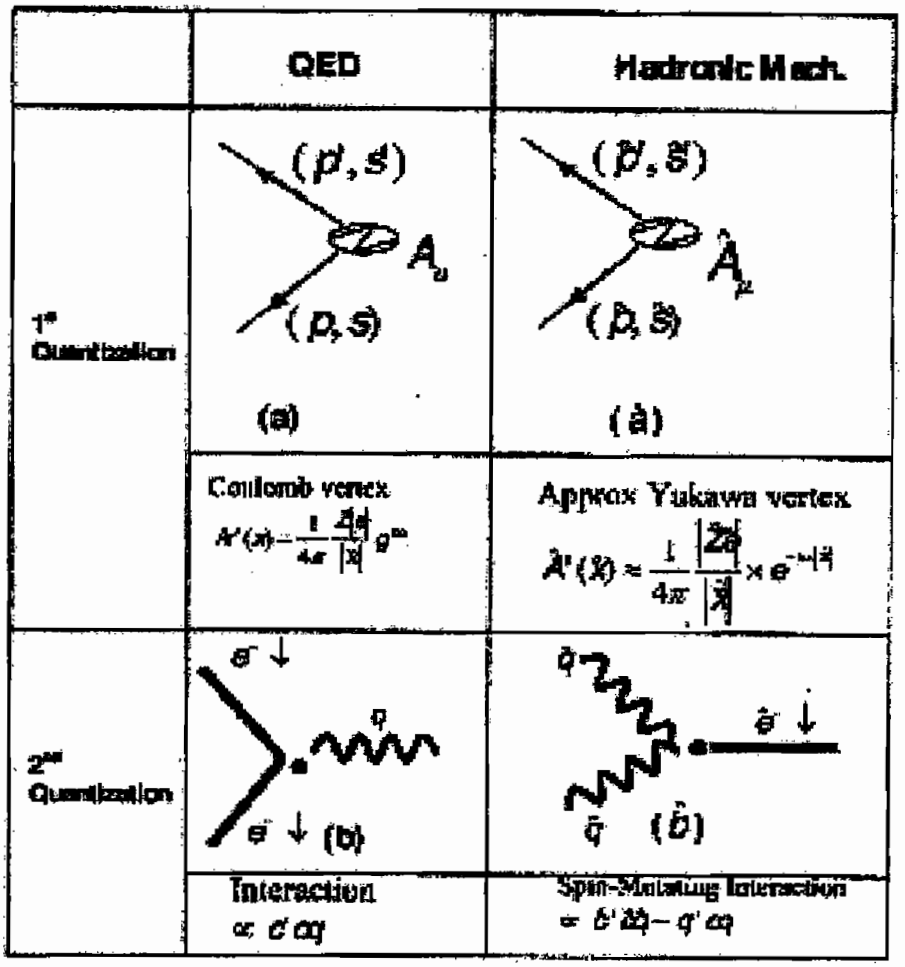



Figure 2: *(Table 1): Expected Modification of QED in HM*

energies. The correspondence between Feynman graphs/rules and their isotopic images for computation of contributions to the S-matrix in QED of spin-$\frac{1}{2}$ particles are summarized in table 2 which we intend to apply to the two scattering profiles (1.1) and (1.2) in Sec. 3.

**2.2. $\hat{O}(4,2)$ Dynamical Symmetry of the Scattering Region.**
We now turn to further specification of the structure of the scattering region introduced in Fig. 1. While the isotopies of Dirac matrices characterize the lifting of the Lorentz group $O(3,1) \to \hat{O}(3,1)$, in terms of five additional quantities, namely the four $b_k (k = 1, ..., 4)$ for spacetime mutation and one $(w\ )$ for the spin, for analyzing experimentally measureable features of the scattering region, such as its shape, deformation, anisotropy, etc, the most important distinctive features of the scattering region for the three profiles of $e^- - p$ scattering shown in Fig. 3, are intriguingly realized by characterizing the scattering region in terms of isotopic lifting of the larger dynamical group,$O(4,2) \to \hat{O}(4,2)$ where, as is well known, $O(4,2)$ contains $O93,1)$ as a subgroup. We shall discuss this feature before taking up the computation of the conventional and generalized S-matrices in Sec.3.

Since correlated pairs of spin-$\frac{1}{2}$ particles,$e^-, \nu$ and $e^-, a^0$ , can be subsumed and long-range 1/r-potential between pairs of particles $(p, e^-)$ eliminated simultaneously in the representation of the conventional O(4,2) dynamical symmetry group, in terms of the most general parity-conserving current in the O(4,2) algebra of Dirac matrices[6] which includes certain "convective" currents proportional to the total momentum of the particle-antiparticle system, it is appropriate to characterize the scattering region of Fig. 1 by the lifting

$$J_\mu \equiv \bar{\psi}\gamma_\mu\psi \to \hat{\bar{\psi}} \times \hat{T} \times (\hat{\gamma}_\mu - i \times \kappa_0 \times \hat{\bar{\partial}}_\mu) \times \hat{T} \times \hat{\psi} = \hat{J}_\mu \tag{2.8}$$

The generalized wave equation that conserves $\hat{J}_\mu$ is given by the generalized Lagrangian density

$$\hat{L} = -\frac{1}{2}\bar{\psi}(\hat{x}) \times \hat{T} \times (-i \times \hat{\gamma}^\mu \times \hat{\bar{\partial}}_\mu + \kappa_1) \times \hat{T} \times \hat{\psi}(\hat{x}) -$$

$$\hat{\bar{\psi}}(\hat{x}) \times \hat{T} \times \kappa_0 \times \hat{\bar{\partial}}^\mu \hat{\bar{\partial}}^\mu \times \hat{T} \times \hat{\psi}(\hat{x}) \tag{2.9}$$

as (cf Eq.(3.2) of Barut, Cordero and Ghirardi[6])

$$(i \times \hat{\gamma}^\mu \times \hat{\bar{\partial}}_\mu + \kappa_0 \times \hat{\bar{\partial}}^\mu \hat{\bar{\partial}}_\mu - \kappa_1) \times T \times \hat{\psi}(\hat{x}) = 0 \tag{2.10}$$

where $\kappa_0, \kappa_1$ are constants. It is of interest to note that the last term in Eq.(2.9) (due to convective currents) gives rise to the Pauli magnetic transitions, inasmuch as for any Dirac spinor $\psi$, it is easy to establish from the relation

$$(\partial^\mu\bar{\psi})(\partial^\mu\psi) = (\partial^\mu\bar{\psi})\gamma^\mu\gamma^\nu(\partial^\nu\psi)$$

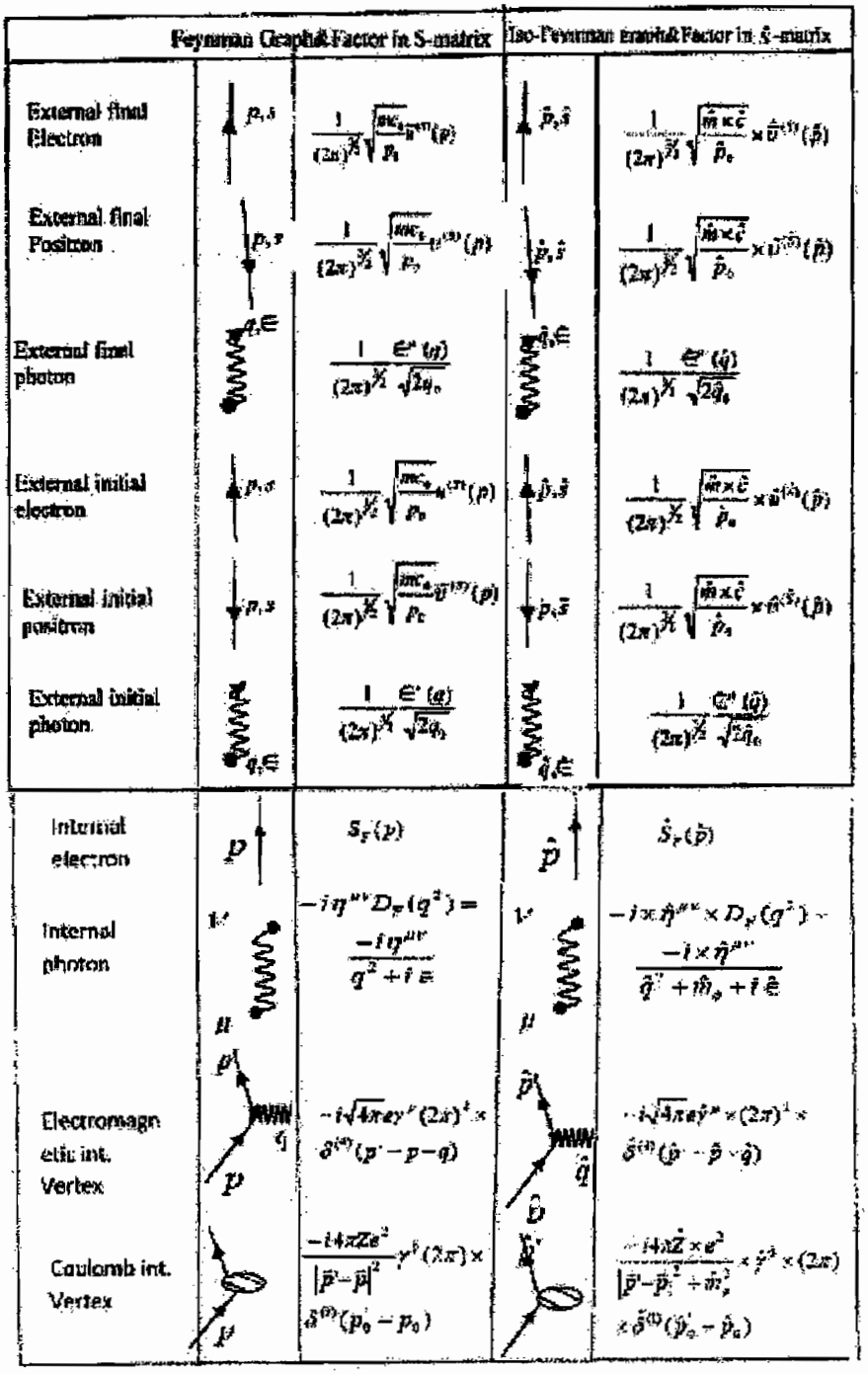



Figure 3: *(Table 2): Feynman and Generalized Feynman Graphs/Rules*

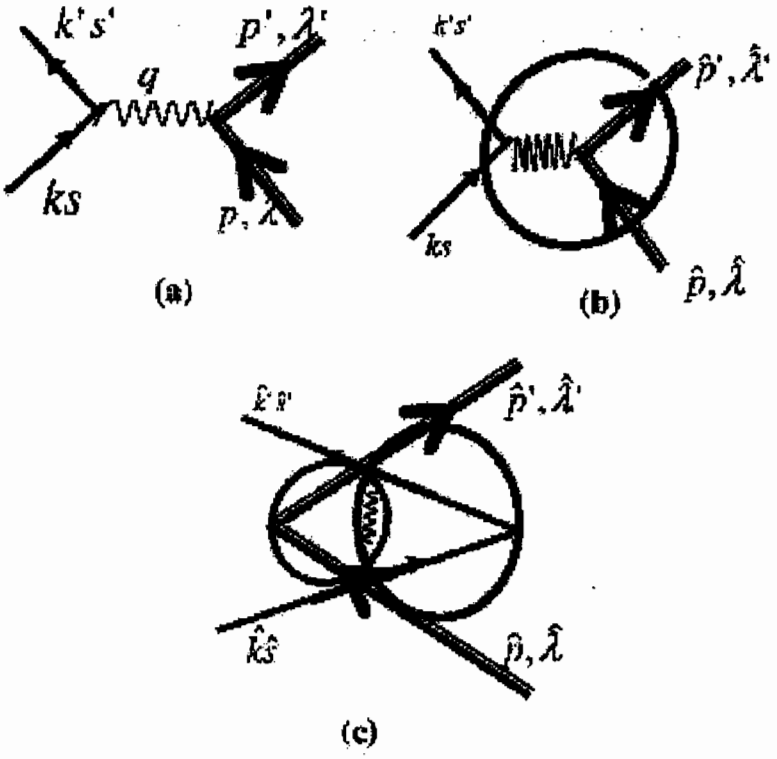



Figure 4: *Feynman diagrams for (a) conventional $e^- - p$ long-range Coulomb interaction via "virtual" photon exchange; (b) point-electron contact/penetration into an extended proton wave packet, $e^- + p \rightarrow n + \nu$ which implies $n \sim (pe^-\bar{\nu})_{QM} \rightarrow (pe^-a^0)_{QM}$; and (c) mutual overlap of extended electron and extended proton wave packets, $e^- + p \rightarrow n + \nu + \bar{\nu}$ which implies $n \equiv (\hat{e}^-, \hat{p})_{HM}$ involving mutation of spin and in the similar scattering process $e^+ + e^- \rightarrow \pi^0 \rightarrow e^+ + e^-$ which implies $\pi^0 \equiv (\hat{e}^-, \hat{e}^+)_{HM}$.*

a connection with the intrinsic Pauli-moment coupling which is related to inclusion of a non-potential term, $-i\partial^{\mu}(\bar{\psi}\sigma_{\mu\nu}\partial^{\nu})\psi$ in the free Dirac Lagrangian density[7]. Thus even the ($\hat{T} \to 1$ ) limit of $\hat{O}(4,2)$ corresponding to the conventional $O(4,2)$ provides a simple non-trivial profile of neutron production in ($e^{-}, p$ ) scattering as summarized in Table 3.

| Table 3: O(4,2) profile of $e^- - p$ scattering & neutron production. | | | Wave Equations | Symmetry Group |
|---|---|---|---|---|
| $e^- + p \to n + \nu \Rightarrow$ | $n \sim (pe^-\bar{\nu}_e)_{QM}$ (Barut's model[6]) | $p$ | $(i\gamma_\mu\partial_\mu - m_p)\psi_p = 0$ | O(3,1) |
| | | $(e^-, \nu_e)$ | $(i\gamma_\mu\partial_\mu - m_e^{-1}\partial_\mu\partial^\mu)\psi_e = 0$ | O(4,2) |
| $e^- - p + a^0 \to n + \nu$ $\to e^- + p + \bar{\nu} \Rightarrow$ | $n = (p, e^- a^0)_{QM}$ (Santilli's "etherino" model [4] | $(e^-, a^0)$ | $(i\gamma_\mu\partial^\mu - 3m_e - (2m_e)^{-1}\partial_\mu\partial^\mu)\psi = 0$ | O(4,2) |

Figure 5: *(Table 3) O(4,2) Profile of e-p scattering and neutron production*

According to this table, if in the scattering process, $e^- + p \to n + \nu$ , the proton is treated as pointlike particle described by the conventional Dirac equation with O(3,1) symmetry, then the electron with an associated massless neutrino may be described by the simplest (scale-invariant[8]) equation with O(4,2) dynamical symmetry,

$$(i\gamma_\mu\partial_\mu - m_e^{-1}\partial_\mu\partial^\mu)\psi_e = 0 \tag{2.11}$$

whose mass equation has two roots, $m = 0, m_e$, and therefore leads to Barut's model[6] of neutron production, $n \sim (pe^-\bar{\nu})_{QM}$ (which is not compatible with negative binding energy). Alternatively, if one adopts Santilli's "etherino hypothesis" [4] (for compatibility with neutron decay and negative binding energy for $n = (pe^-a^0)_{QM}$ ), the electron with an associated massive "etherino" may be described by the more general equation

$$[i\gamma_\mu\partial^\mu - 3m_e - (2m_e)^{-1}\partial_\mu\partial^\mu]\psi = 0, \tag{2.12}$$

whose mass equation and its two non-zero roots are respectively given by:

$$m^2 + 2m_e m - 6m_e^2 = 0, \tag{2.13}$$

$$m_\pm = m_e(-1 \pm \sqrt{7}), i.e., \frac{m_+}{m_e} = 1,65; \frac{|m_-|}{m_e} = 3.6 \tag{2.14}$$

Consequently, since $0.78MeV = 1.53m_e$ , it follows by setting $m_{a^0} \equiv m_+ = 1.65m_e$ that one may validly characterize a quantum mechanical bound state of $n = (p, e^-, a^0$ system with negative binding energy: $m_n - (m_p + m_e + m_{a^0}) \equiv -0.18m_e$ . Intriguingly, the numerical coefficients in the wave equations (2.11) and (2.12) are uniquely related in terms of Gell-Mann SU(3) $\lambda$ - generators,

$$\lambda_0 = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \lambda_8 = \sqrt{\frac{1}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \lambda_8^{-1} = \sqrt{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}, \tag{2.15}$$

and a triplet field

$$\Psi = \begin{pmatrix} \psi_\nu \\ \psi_e \\ \psi_{a^0} \end{pmatrix},$$

as the components of the wave equation

$$(i\gamma_\mu \partial^\mu - m_e \sqrt{\frac{3}{2}}(\lambda_(0) - \sqrt{2}\lambda_8) + (\frac{1}{m_e\sqrt{3}})\lambda_9^{-1}\partial_\mu \partial^\mu)\Psi = 0 \tag{2.16a}$$

where

$$m_e \sqrt{\frac{1}{2}}(\lambda_0 - \sqrt{2}\lambda_8) = (3m_e) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

$$(\frac{1}{m_e\sqrt{3}})\lambda_8^{-1} \equiv (\frac{1}{3m_e}) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}. \tag{2.16b}$$

As the three equations in this system are uncoupled except insofar as there is only one characteristic mass, $m_(e)$, for the whole triplet, Eq.(2.16a) implies that in the absence of convective currents, the apparent chiral $SU(3)xSU(3)$ symmetry of the leptonic triplet ($\Psi$ ) is broken in the manner prescribed by Gell-Mann, Oakes and Renner[9]. We shall return to the experimental verification of the predicted object described by the second mass ratio, $\frac{|m_-|}{m_e} = 3.6$ in Sec.4.

Of primary interest in the $\hat{O}(4,2)$ characterization of the hadronic mechanics scattering region is the case of contact/overlap of two extended wavepackets shown in Fig. 3c, which is also applicable to $e^- - e^+$ scattering. This leads to a visual (Feynman graph) representation of Rutherford-Santilli neutron production in $e^- - p$ scattering and neutral pion production in $e^- - e^+$ scattering as follows. The selection of the isotopic element $T = \hat{I}_{-1}$ appearing in the definition of the generalized $\hat{\delta}$ function[10] for this scattering profile is given in ref.[1] as

$$\hat{I} = \begin{pmatrix} n_{11}^2 & 0 & 0 & 0 \\ 0 & n_{12}^2 & 0 & 0 \\ 0 & 0 & n_{13}^2 & 0 \\ 0 & 0 & 0 & n_{14}^2 \end{pmatrix} \times \begin{pmatrix} n_{21}^2 & 0 & 0 & 0 \\ 0 & n_{22}^2 & 0 & 0 \\ 0 & 0 & n_{23}^2 & 0 \\ 0 & 0 & 0 & n_{24}^2 \end{pmatrix} \times$$

$$exp[N \times (\hat{\psi}/\psi) \times \int d^3r \times \psi_{\downarrow}^*(r) \times \psi_{\uparrow}(r)] \tag{2.17}$$

where $n_{ak}^2, a = 1,2, k = 1,2,3$ are the semiaxes of the ellipsoids representing the two particles,$n_{a4}^2, a = 1,2$represent their densities, $\psi$ represents the isowavefunction, represents the conventional wavefunction (corresponding to $\hat{I} = 1$), and N is a positive constant. A two-dimensional (2D) elaboration of Feynman graph showing the ellipsoidal deformations of $e^-$ and $e^+$ wave-packets in the neutral pion production process, $e^- + e^+ \to \pi^0 \to e^- + e^+$, is shown in Fig. 4a.

Such a *wave-and-particle* picture is obtained by representing each particle or antiparticle as a "point" on the (red) circum-ellipse of a triangle (ABC) defined by a pair of imaginary generating lines (AB and AC) [$br = \pm is$ ] of a "point-ellipse" [$b^2r^2 + s^2 = (ct)^2 = 0$] in projective 2-dimensional $(r,s,t)$-space and the "line at infinity" (BC)($t = 0$) and locating the $\pi^0$ at the centre of the inscribed ellipse of the pair of triangles forming a hexagram so as to satisfy Brianchon's and Pascal's theorems (p. 64 of ref.[12]) as shown in Fig 4a. For $ct \neq 0$, we infer by rewriting the equation of the ellipse in the form,$(ct)^2 - (s - ibr)(s + ibr) = 0$, that the pair of imaginary lines defines by the linear homogeneous equations

$$\begin{pmatrix} ct & -(s - ibr) \\ -(s + ibr) & ct \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0, \tag{2.18}$$

are associated with the ellipse ABC (call it $S$) in Fig.4a. Moreover, by the geometric principle of duality, the lines AB, AC and BC envelope a conic (call it $\tilde{S}$), the dual of $S$ with respect to the triangle ABC) whose equation has the general form ( p.62 of ref.[13])

$$as^2 + 2fsr + b^2r^2 = 0 = ct \tag{2.19a}$$

and is a hyperbola, a parabola, or an ellipse according as $f^2 - ab^2$ is greater than, equal to, or less than zero. The concept of "isotopic" lifting arises naturally in this way, with

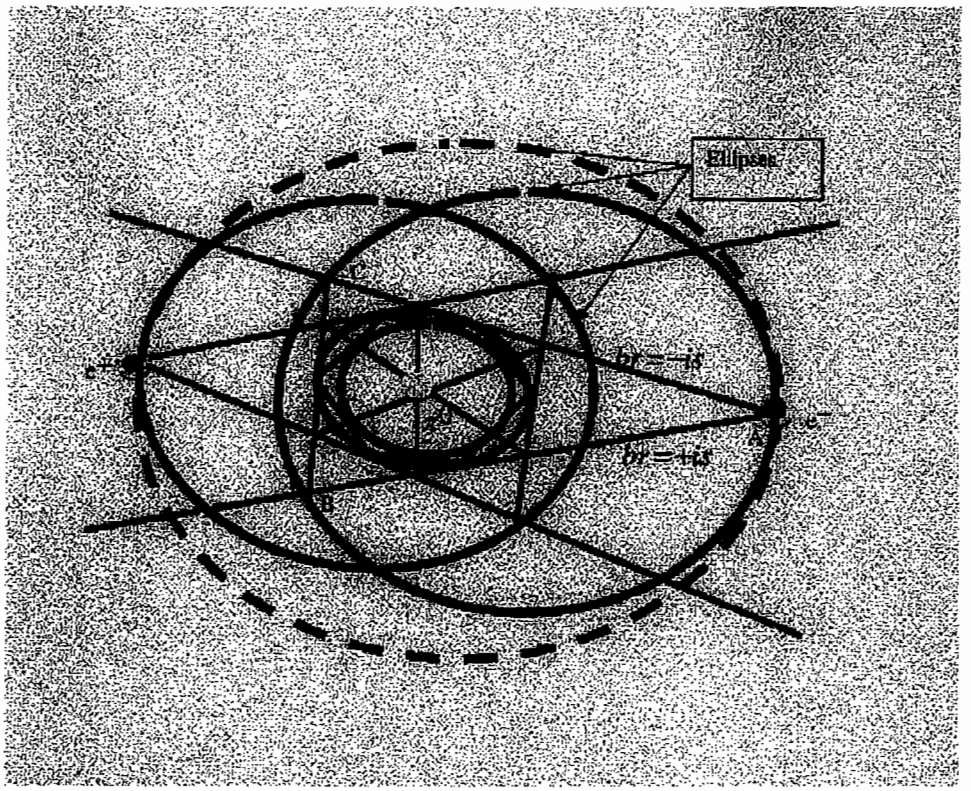



Figure 6: *Elaboration of Feynman graph for overlapping* $e^(-)$ *and* $e^(+)$ *ellipsoidal wave-packets in the scattering region for the process* $e^- + e^+ \to \pi^0 \to e^- + e^+$ *and the enveloping ellipsoid (i.e. ellipse in 2-dimensional space).*

the radii of the circular conics ($S$ and $\tilde{S}$ ) providing two fundamental lengths, and the consequential "mutation" of Pauli spin comes about in the case where (for $ct \neq 0$) space-time dualism leads to the rectangular hyperbola,$(s+br)(-s+br)-(ct)^2=0$, whose pair of asymptotic lines are defined by the linear homogeneous equations:

$$\begin{pmatrix} s+br & -ct \\ -ct & -s+br \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}=0, or[(s+\tilde{\beta}_0 ct)(\sigma_3)_{\mu\nu}+br\delta_{\mu\nu}]=0, \qquad (2.19b)$$

where

$$\sigma_3=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\rightarrow \tilde{\beta}_0\sigma_3=\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

implies that

$$\tilde{\beta}_0=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tilde{\beta}_0^2=-I$$

is non-unitary! An example of the nesting of two ellipses in the scattering region may be constructed by generalizing Eq.(2.19b) to the 4x4 matrix form:

$$[(br+ct\tilde{\beta})\eta^0_{\mu\nu}+s\delta_{\mu\nu}]w_\nu=0 \qquad (2.20)$$

which involves the metric tensors,

$$(\eta^0_{\mu\nu})=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, (\tilde{\beta}\eta^0_{\mu\nu})=\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad (2.21)$$

The sum of the two metrics defines an isometric tensor

$$\eta_{\mu\nu}\equiv\frac{1}{2}[(\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu)+(\tilde{\alpha}_\mu\tilde{\beta}\tilde{\alpha}_\nu+\tilde{\alpha}_\nu\tilde{\beta}\tilde{\alpha}_\mu)]\equiv(1+\tilde{\beta})\eta^0_{\mu\nu} \qquad (2.22)$$

which consists of the conventional Dirac's $\gamma$-matrices for spin-$\frac{1}{2}$ particles[14]

$$\gamma_0=\beta,\gamma_r=\beta\alpha_r(r=1,2,3); with\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2\eta^0_{\mu\nu} \qquad (2.23a)$$

and the (dual) Dirac's$\tilde{\beta},\tilde{\alpha}$ matrices for integral spin particles[14]

$$(\tilde{\alpha}_\mu\tilde{\beta}\tilde{\alpha}_\nu+\tilde{\alpha}_\nu\tilde{\beta}\tilde{\alpha}_\mu)=2\tilde{\beta}\eta^0_{\mu\nu};\tilde{\gamma}_\mu\tilde{\gamma}_\nu+\tilde{\gamma}_\nu\tilde{\gamma}_\mu=-2\eta^0_{\mu\nu},\tilde{\gamma}_\mu=\tilde{\beta}\tilde{\alpha}_\mu \qquad (2.23b)$$

where,

$$\beta=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \alpha_1=\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

$$\alpha_2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix},$$

$$\tilde{\alpha}_0 = 1$$

$$\tilde{\beta} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \tilde{\alpha}_1 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

$$\tilde{\alpha}_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \tilde{\alpha}_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

As a result of the above relations, the visual (Feynman graph) image of $e^- - e^+$ scattering leading to production $\pi^0$ is provided by the two quadric surfaces, $S$ and $\tilde{S}$ :

$$x^\mu(1 \pm \tilde{\beta})\eta^0_{\mu\nu}x^\nu \equiv (ct)^2 - x^2 - y^2 - z^2 \pm 2cty = s^2 \tag{2.24}$$

We note that when $ct = s$ , these are two spheres of radii $ct$ in contact along the y-axis and that $\hat{\eta}_{\mu,\nu} = (1 + \tilde{\beta})\eta^0_{\mu\nu} \equiv \hat{I}\eta^0_{\mu\nu}$ defines a non-trivial "isounit" ($\hat{I}$ ) for such a generalized Feynman graph/rules for computing the S-matrix for extended particle scattering processes leading to fusion products like the $\pi^0$ . Note also that if $y/s = \pm isin\theta$ then (2.24) is a torus with parametric equations:

$$x = (is - ctsin\theta)cos\phi; y = (is - ctsin\theta)sin\phi; z = ctcos\theta \tag{2.25}$$

A corresponding two-dimensional wave-and-particle (extended wavepacket) picture of the proton and the toroidal orbit of the electron in McDonough's representation of the Rutherford-Santilli neutron which apparently relates it to Santilli's "etherino" model of the neutron is shown in Fig.4b.

## 3. Computation of the iso-S-matrix from Feynman Graphs/Rules.

In order to familiarize the reader with the use of the our generalized Feynman graphs/rules for computation of the $\hat{S}$-matrix we begin with the conventional electron-proton Coulomb scattering (Fig. 3a) that leads to the familiar Mott scattering cross-section. Fig. 3a is now elaborated as shown in Fig. 5a,

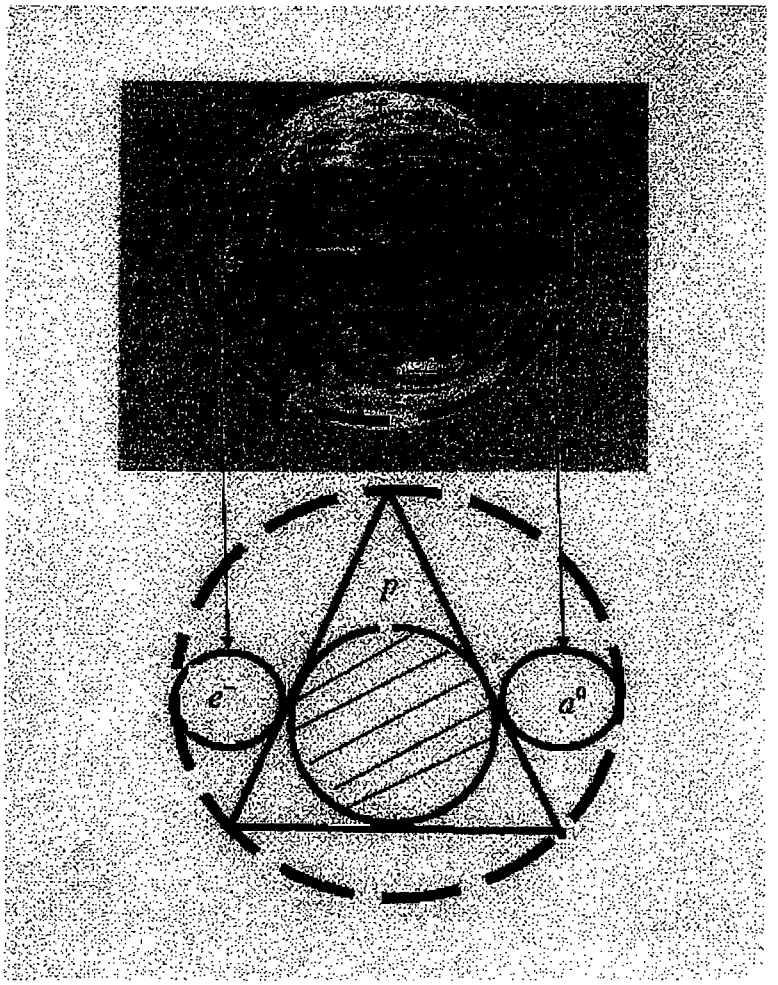



Figure 7: *Projection of the Macdonough representation of the Rutherford-Santilli neutron showing its relationship to Santilli's "etherino" model of the neutron*

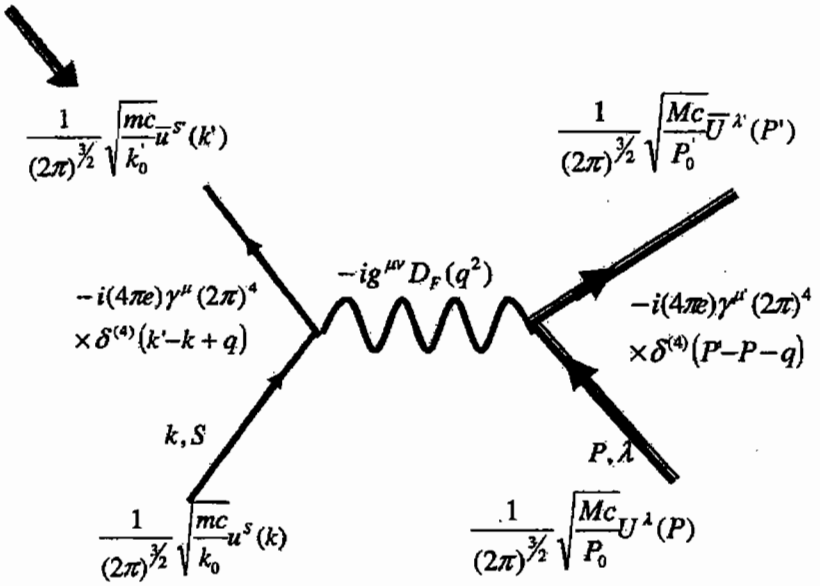



Figure 8: *Conventional Feynman Graph/Rule for e-p Coulomb Scattering*

To write down the S-matrix one starts in the direction of the (top left [red]) arrow from left to right as indicated in Fig. 5a and, at each vertex, inserts all other factors between the incoming and outgoing arrows. If loop closes, one takes trace to get

$$S_{fi} = -4\pi i \int \frac{d^4q}{(2\pi)^4} \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{mc}{k_0'}} \times \bar{u}^{S'}(k')] \times$$

$$[-i(4\pi e)\gamma^{\mu'}(2\pi)^4 \times \delta^{(4)}(k'-k+q)] \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{mc}{k_0}} \times u^S(k)] \times$$

$$[-ig^{\mu\nu}D_F(q^2)] \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{Mc}{P_0'}} \times \bar{U}^{\lambda'}(P')] \times$$

$$[-i(4\pi e)\gamma^{\mu'}(2\pi)^4 \times \delta^{(4)}(P'+P-q)] \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{Mc}{P_0}} \times U^{\lambda}(P)]$$

$$= -4\pi i \int \frac{d^4q}{(2\pi)^4}\sqrt{\frac{(mc)^2(Mc)^2}{k_0'k_0P_0'P_0}} \times \bar{u}^{S'}(k')\gamma^{\mu}u^S(k) \times$$

$$\frac{-4\pi i\alpha}{(4\pi)^4}(2\pi)^4\delta^{(4)}(k'+P'-k-P) \times$$

$$[(k'-k)^2+i\varepsilon]^{-2} \times \bar{U}^{\lambda'}(P')\gamma_{\mu}U^{\lambda}(P). \tag{3.1}$$

In terms of the electromagnetic current,

$$j^{\mu}(k'k,-q) = (2\pi)^4 \times \delta(k'-k+q)]\frac{e}{(2mc)^3} \times \sqrt{\frac{(mc)^2}{k_0'k_0}}v^{S'}(k')\gamma^{\mu}u^S(k)$$

and the Moller potential

$$A_{\mu}(k'-k) = \frac{4\pi}{(k'-k)^2+i\varepsilon}J_{\mu}(p',p;k'-k)$$

the S-matrix takes the form of current-current interaction

$$S_{fi} = -4\pi i \int \frac{d^4q}{(2\pi)^4}j^{\mu}(k'k,-q)\frac{1}{q^4}J_{\mu}(p'p;q) \tag{3.2}$$

The differential cross section with no polarization for initial particles in the laboratory frame,$p = (Mc_0;0)$ is given by

$$d\sigma = \frac{1}{2}\sum_{ij}\frac{1}{|\frac{\mathbf{k}}{k_0}|\frac{1}{(2\pi)^3}\frac{V}{(2\pi)^3}}\frac{|S_{if}|^2}{T}d^3p'd^3k' \tag{3.3}$$

which, on extending the sum to all initial and final spin states, becomes

$$d\sigma = \frac{(4\pi\alpha)^2}{(2\pi)^4}(2\pi)^4\delta^{(4)}(k'+P'-k-P)\times\frac{m^2M^2c_0^4}{k_0'k_0P_0'P_0}\times$$

$$|\frac{\mathbf{k}}{k_0}|\frac{1}{[(k'^{\mu}-k^{\mu})(k'_{\mu}-k_{\mu})]^2}\times\frac{1}{4}Trace[\gamma^{\mu}\frac{mc_0+\gamma.k}{2mc_0}\gamma_{\nu}\frac{mc_0+\gamma.k'}{2mc_0}]\times$$

$$Trace[\gamma^{\mu}\frac{Mc_0+\gamma.P}{2Mc_0}\gamma_{\nu}\frac{Mc_0+\gamma.P'}{2Mc_0}]d^3k'd^3P'$$

$$=\frac{\alpha^2}{|\mathbf{k}|}\delta^{(4)}(k'+P'-k-P)\times\frac{1}{(k'-k}^4\times$$

$$\frac{1}{4}Trace[\gamma^{\mu}\times(mc_0+\gamma.k)\times\gamma_{\nu}\times(mc_0+\gamma.k')]\times$$

$$\frac{1}{4}Trace[\gamma^{\mu}\times(Mc_0+\gamma.P)\times\gamma_{\nu}\times(Mc_0+\gamma.P')]\times\frac{d^3P'}{P_0'}\frac{d^3k'}{k_0'}$$

where $\alpha \equiv e^2/hc_0$.

We shall not proceed further with the explicit evaluation of this expression except to note that the product of the two traces is

$$\frac{1}{16}S_p().S_{(p)}() = [k^{\mu}k'^{\gamma}-g^{\mu\gamma}(kk')+(mc_0)^2g^{\mu\gamma}]\times$$

$$[P^{\mu}P'^{\gamma}-g^{\mu\gamma}(PP')+(Mc_0)^2g^{\mu\gamma}] \tag{3.5}$$

where $P' = P+k-k'$ and in the laboratory frame [$P = (Mc_0, 0)$ ] under the assumptions that the proton has no structure and that

$$mc_0/k_0 << 1, mc_0/k_0' << 1, q^2 = (k'-k)^2 \simeq -4k_0k_0'sin^2(\frac{\theta}{2}),$$

one obtains the standard expression[15] of "potential scattering theory" for electron-point-proton scattering (with unpolarized initial state and no observation of final spin):

$$\frac{d\sigma_{e-p}}{d\Omega} = \frac{\alpha^2E^2(1-\beta sin^2(\theta/2))}{4P^4sin^4(\theta/2)} = \frac{\alpha^2cos^2(\theta/2)}{E^2sin^4(\theta/2)} \equiv \frac{d\sigma_{Mott}}{d\Omega} \tag{3.6}$$

where

$$\beta = \frac{|\mathbf{P}|}{E}, \frac{1}{2}(\mathbf{P'}-\mathbf{P})^2 = (\mathbf{q})^2 = \mathbf{2P^2(1-\cos(\theta)) = 4P^2\sin^2(\theta/2)}$$

and $\sigma_{Mott}$ is the Mott scattering cross section. A preliminary HM approach to deep-inelastic (irreversible) scattering has been reviewed by Animalu and Ekuma[16].

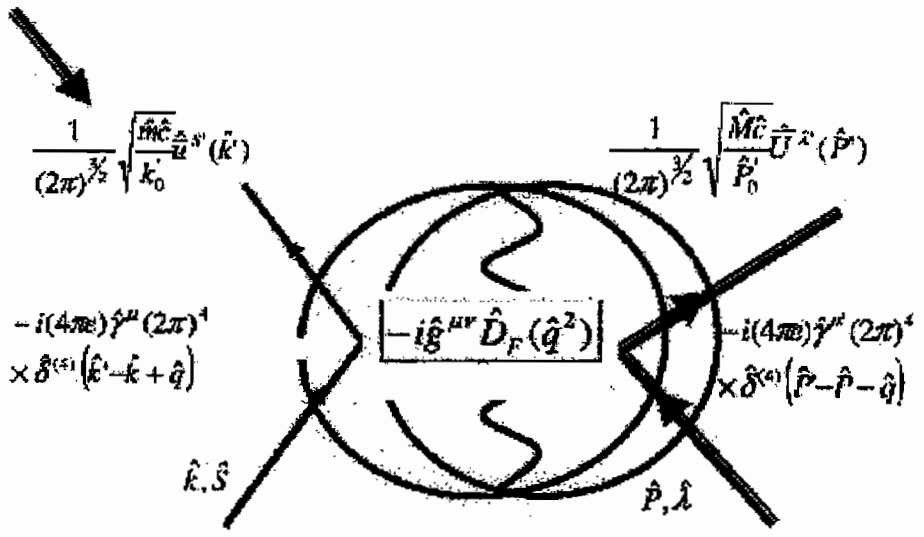



Figure 9: *Fig5b: Generalized Feynman Graph/Rule for e-p Scattering*

Turning next to the isoscattering profile represented by Fig.3(b and c) we consider the elaboration in Fig. 5(b).

Again, to write down the S-matrix one starts in the direction of the (red) arrow from left to right as indicated above and, at each vertex, inserts all other factors between the incoming and outgoing arrows. If loop closes, one takes trace to get

$$\hat{S}_{fi} = -4i\pi \int \frac{d^4\hat{q}}{(2\pi)^4} \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{\hat{m}\hat{c}}{\hat{k}'_0}} \times \hat{\bar{u}}^{\hat{S}'}(\hat{k}')] \times$$

$$[-i(4\pi e)\hat{\gamma}^{\mu'}(2\pi)^4 \times \hat{\delta}^4(\hat{k}' - \hat{k} + \hat{q})] \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{\hat{m}\hat{c}}{\hat{k}_0}} \times \hat{u}^{\hat{S}}(\hat{k})] \times$$

$$[-i\hat{g}^{\mu\nu}\hat{D}_F(\hat{q}^2)] \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{\hat{M}\hat{c}}{\hat{P}'_0}} \times \hat{\bar{U}}^{\hat{\lambda}'}(\hat{P}')] \times$$

$$[-i(4\pi e)\hat{\gamma}^{\mu'}(2\pi)^4 \times \hat{\delta}^{(4)}(\hat{P}' + \hat{P} - \hat{q})] \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{\hat{M}\hat{c}}{\hat{P}_0}} \times \hat{U}^{\hat{\lambda}}(\hat{P})]$$

$$= -4\pi i \int \frac{d^4\hat{q}}{(2\pi)^4}\sqrt{\frac{(\hat{m}\hat{c})^2(\hat{M}\hat{c})^2}{\hat{k}'_0\hat{k}_0\hat{P}'_0P_0}} \times \hat{\bar{u}}^{\hat{S}'}(\hat{k}')\hat{\gamma}^{\mu}\hat{u}^{\hat{S}}(\hat{k}) \times$$

$$\frac{-4\pi i\alpha}{(4\pi)^4}(2\pi)^4\hat{\delta}^{(4)}(\hat{k}' + \hat{P}' - \hat{k} - \hat{P}) \times$$

$$[(\hat{k}' - \hat{k})^2 + (m_\phi\hat{c})^2 + i\hat{\varepsilon}]^{-2} \times \hat{\bar{U}}^{\hat{\lambda}'}(\hat{P}')\hat{\gamma}_\mu\hat{U}^{\hat{\lambda}}(\hat{P}). \qquad (3.7)$$

This may similarly be rewritten in the form of generalized current-current interaction:

$$\hat{S}_{fi} = -4i\pi \int \frac{d^4\hat{q}}{(2\pi)^4} \times \hat{j}^{\mu}(\hat{k}'\hat{k}, -\hat{q}) \times \frac{1}{[\hat{q}^2 + (m_\phi\hat{c})^2]^2} \times \hat{J}_\mu(\hat{p}'\hat{p}; \hat{q}) \qquad (3.8)$$

where

$$\hat{j}^{\mu}(\hat{k}'\hat{k}, -\hat{q}) = (2\pi)^4 \times \hat{\delta}(\hat{k}' - \hat{k} + \hat{q})] \times \frac{e}{(2\hat{m}\hat{c})^3} \times \sqrt{\frac{(\hat{m}\hat{c})^2}{\hat{k}'_0\hat{k}_0}}\hat{\bar{u}}^{S'}(\hat{k}')\hat{\gamma}^{\mu}\hat{u}^{S}(\hat{k}) \qquad (3.9a)$$

$$\hat{A}_\mu(\hat{k}' - \hat{k}) = \frac{4\pi}{(\hat{k}' - \hat{k})^2 + (m_\phi\hat{c})^2 + i\hat{\varepsilon}}\hat{J}_\mu(\hat{p}', \hat{p}; \hat{k}' - \hat{k}) \qquad (3.9b)$$

are, respectively, the generalized electromagnetic current$\hat{j}^{\mu}$ and generalized Moller current$\hat{J}_\mu$ associated with the generalized electromagnetic vector potential, $\hat{A}_\mu$ .

We observe that three novel features arise: firstly, from the generalized internal photon line $\hat{D}_F(\hat{q}^2)$ which, as indicated in Table 1, is no longer divergent in the limit $\hat{q} \to 0$ , secondly, from the generalized Dirac matrices $\hat{\gamma}^\mu$, and thirdly from the generalized currents,$\hat{j}^\mu$ and $\hat{J}_\mu$ . The experimental verification of these novel features are readily streamlined by re-interpretation of the standard model current-current interaction model of the weak decay of the neutron to which we now turn.

# 4.Experimental Verification.

The experimental verification of isoscattering theory requires us to reconcile and re-interpret the various scattering profiles and models of neutron production considered in this paper with the standard (electroweak interaction) model[17] of neutron decay in terms of our sequence of representations of the carrier space-time symmetry of the scattering region in Fig. 1

$$QM \to O(3,1) \to O(4,2)$$

$$\downarrow$$

$$HM \to \hat{O}(3,1) \to \hat{O}(4,2)$$

as summarized in Table 4 and in Fig.6 as well as the iso-gauge principle for the lifting of the electromagnetic gauge field $A_\mu(x) \to \hat{A}_\mu(\hat{x})$, which we now proceed to discuss in turns,

| Table 4: Models of neutron production in $e-p$ scattering & neutron weak decay processes | | | Wave Equations | Symmetry Group |
|---|---|---|---|---|
| $p+e^- \to n+\nu$ $\to p+e^-+\nu+\bar{\nu}$ | $n=(p,e^-,\bar{a}^0)_{QM}$ $\to(p,\hat{e}^-)_{HM}$ $\Rightarrow \hat{e}^- \cong e^-\bar{a}^0$ | $p$ | $(i\gamma_\mu\partial_\mu - m_p)\psi_p = 0$ | $O(3,1)$ |
| | | $(e^-,a^0)$ | $\left(i\gamma_\mu\partial_\mu - 3m_e - (2m_e)^{-1}\partial_\mu\partial^\mu\right)\psi_e = 0$ | $O(4,2)$ |
| $n \xrightarrow[W^-]{} p+W^-$ $\to p+e^-+\bar{\nu}$ | $n=(p,W^-)_{QM}$ $\Rightarrow W^- \cong \hat{e}^- \cong e^-a^0$ | $(\hat{e}^-,\hat{a}^0)$ | $\left(i\hat{\gamma}_\mu\hat{\partial}^\mu - 3\hat{m}_e - (2\hat{m}_e)^{-1}\hat{\partial}_\mu\hat{\partial}^\mu\right)\hat{\psi}_e = 0$ | $\hat{O}(4,2)$ |

Figure 10: *Table 4: Models of neutron production*

Table 4 states that compatibility of scattering profiles and neutron structure (with negative binding energy) requires us to eliminate the neutrino and antineutrino $(\nu, \bar{\nu})$ and replace them with the etherino and antietherino $(a^0, \bar{a}^0)$ as constituent of the

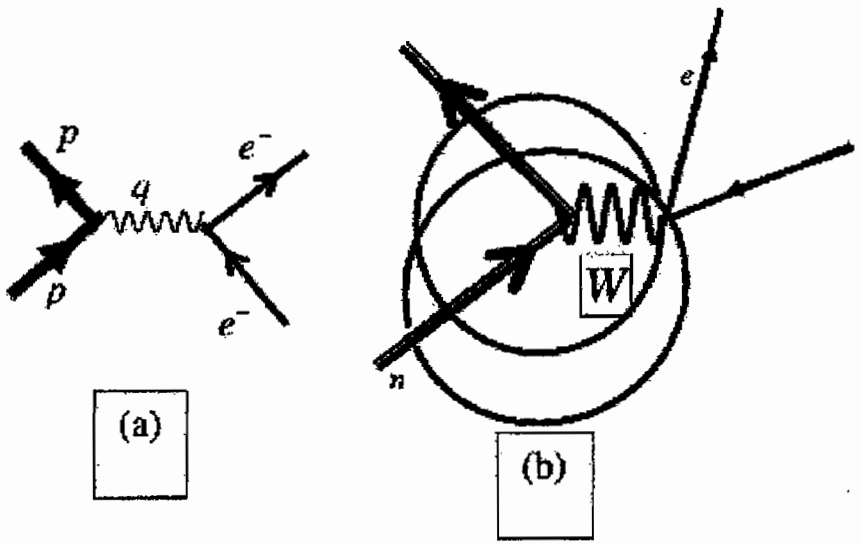



Figure 11: *Feynman graph for (a) p-e scattering and (b)neutron decay* $n \rightarrow p + e^- + \nu$

neutron in conventional QM, but build the neutron from the proton ($p$ ) and mutated electron ($\hat{e}^-$ ) in HM. Moreover, consistency of HM concept of electron mutation requires firstly that $\hat{e}^- \approx e^- \bar{a}^0$ to reach the Rutherford Santilli neutron, $n = (p, \hat{e}^-)_{HM}$ and secondly that$\hat{e}^- \sim W^- \approx e^- \bar{a}^0$, where $W^-$is the massive intermediate boson that mediates neutron weak decay as shown in Fig.6.

Consequently, a first experimental verification is expected to emerge from the lifting of the O(4,2) wave equation (2.12) for the pair $e^-, a^0$ to the following $\hat{O}(4,2)$ wave equation

$$[i\bar{\gamma}_\mu \hat{\partial}^\mu - 3\hat{m}_e - (2\hat{m}_e)^{-1}\hat{\partial}_\mu \hat{\partial}^\mu]\hat{T}\hat{\psi}_e = 0 \qquad (4.1a)$$

for $\hat{e}^-, \hat{a}^0$ . The iso-mass equation,$\hat{m}^2 + 2\hat{m}_e\hat{m} - 6\hat{m}_e^2 = 0$, has two roots

$$\hat{m}_\pm = \hat{m}_e(-1 \pm \sqrt{7}), i.e., \frac{\hat{m}_+}{\hat{m}_e} = 1.65, \frac{\hat{m}_-}{\hat{m}_e} = -3.65 \qquad (4.1b)$$

Thus, $\hat{m}_+/\hat{m}_e = 1.65 \equiv \hat{m}_{a^0}/\hat{m}_e$ as before for compatibility with negative binding energy for neutron production. However, by observing that $|\hat{m}_-|/\hat{m}_e = 3.65 \approx m_\Lambda/m_d$ , where$3m_d = m_n = 939MeV$ and $m_\Lambda \approx \frac{1}{2}(m_\Lambda + m^0_{Sigma}) = \frac{1}{2}(1116+1192) = 1142MeV$ we infer from Eq.(2.15) and (2.16) that$\hat{e}^-$ may be re-interpreted as d-isoquark. In addition, unlike the usual Dirac equation that has only positive mass, Eq.(4.1a) has both positive and negative masses, and the negative mass may be necessary for the binding of the correlated pairs of particles in O(4,2) theories.

With regards to the lifting of the electromagnetic gauge field, $A_\mu(x) \to \hat{A}_\mu(\hat{x})$ , we observe that the divergence of the Feynman graph for the $e^- - p$ Coulomb interaction in Fig. 6(a) arises basically from the factor $1/|\mathbf{k}' - \mathbf{bfk}|^2 \equiv \mathbf{1/q^2}$ associated with the Fourier transform of the long-range Coulomb potential, $V_C(r) \equiv (e/c_0)A_0(r) = -e^2/r$ which is determined by the time-component ($A_0$ ) component of the electromagnetic 4-vector potential,$A_\mu(\mu = 0, 1, 2, 3)$. For this reason, the divergence is related to the structure of the electromagnetic gauge field. However, by expressing the interparticle Coulomb force $-dV_C/dr = -e^2/r^2$ as a functional of the potential energy $V_C$ , and eliminating explicit r-dependence between $dV_C/dr$ and $V_C$, a non-linear first-order differential equation results:

$$\frac{dV_C}{dr} = \frac{V_C^2}{e^2}, or \frac{\partial A_0}{\partial r} = (1/e^2)A_0^2 \tag{4.2}$$

As this is a special case of Riccati's equation, an obvious step to achieve a progressive generalization of the Coulomb potential is to lift Eq.(4.2) to the most general iso-Riccati's equation

$$\frac{\partial A_0}{\partial r} \to \frac{\partial \hat{A}_0}{\partial \hat{r}} = (1/\hat{e}^2)\hat{A}_0^2 + \zeta \hat{A}_0^2 + \kappa \tag{4.3}$$

$\zeta, \kappa$being constants (in general, functions of $\hat{r}$ ). We note that the derivative and nonlinear parts, $[\partial A_0/\partial r - (1/e)A_0^2]$, of Eq.(4.3) may be re-interpreted as appropriate component of the SU(2) Yang-Mills gauge field,

$$F_{\mu\nu}^a \equiv \frac{\partial A_\nu^a}{\partial x_\mu} - \frac{\partial A_\mu^a}{\partial x_\nu} + g_0 \varepsilon^{abc} A_\mu^b A_\nu^c \tag{4.4}$$

where $g_0$ is a coupling constant. Moreover, an exact solution of Eq.(4.3) given in the Appendix to ref.[11] has the form of a Hulthen potential which has an approximate Yukawa form:

$$V_H(r) = \frac{-Me^{-m_0 r}}{1 - e^{-m_0 r}} \approx \frac{-Me^{-m_0 r}}{m_0 r + O(r^2)} \approx \frac{-Me^{-m_0 r}}{r} \equiv \phi(r) \tag{4.5}$$

However, by using well-known standard transformation we may convert the nonlinear first-order Riccati Eq.(4.3) into a linear second-order differential equation for $\phi$ (see, p.201 of Piaggio[18]) :

$$\hat{A}_0 = -e\frac{dlog(\phi)}{dr} \equiv -e\frac{(d\phi/dr)}{\phi} \equiv -e\frac{\phi_1}{\phi}, \tag{4.6a}$$

where $\phi_1 \equiv d\phi/dr$. Note that this transformation may be rewritten as a (Weyl-like) gauge principle in the integral form:

$$\phi = exp(-(1/e)\int_0^r \hat{A}_0 dr). \tag{4.6b}$$

From (4.6a) we find

$$\frac{d\hat{A}_0}{dr} \equiv -e\frac{\phi_2}{\phi} + e\frac{\phi_1^2}{\phi^2}, \tag{4.7}$$

so that, on substitution in Eq.(4.3), the terms in $\phi_1^2$ disappear, and hence, on multiplying the resulting equation through by $\phi/e$, we obtain a linear second-order differential equation:

$$\phi_2 - \zeta\phi_1 - (\kappa/e)\phi = 0. \tag{4.8}$$

In addition, if we select $\zeta \equiv -2/r$ and put$m_\phi^2 = \kappa/e$, this equation takes the standard form

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{\kappa}{e}\phi \equiv (\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} - m_\phi^2)\phi(r) = 0 \tag{4.9}$$

which is the static limit of the Klein-Gordon equation for a spin-0 scalar field $\phi$ of mass $m_\phi$ in units such that $h/2\pi = c_0 = 1$. The fact that the Fourier transform of $\phi(r)$ given by $-g_0^2/[q^2 + (m_\phi\hat{c})^2/\hat{h})^2]$ eliminates the divergence of the Fourier transform of $V_C$ in the limit $q \to 0$ was obtained originally from second-order Coulomb scattering S-matrix by Dalitz[19].

## 5. Concluding Remarks.

History of science has established that physical theories provide a mere approximation of nature due to its complexity generally beyond our understanding. Therefore. no matter how beautiful and correct a given theory may appear at a given time, its structural generalization is inevitable due to the advancement of scientific knowledge and the identification of conditions beyond those of the original conception. This is also the fate of the 20th century scattering theory, since the inevitability of its structural generalization, in due time, for higher and higher energies and more and more complex collisions of particles is beyond doubt.

In this and the preceding papers of this series, we have established that the 20th century scattering theory can indeed be lifted into an axiom-preserving isotopic formulation for reversible scattering processes which is universal for the class admitted, invariant over time. and admitting of the conventional theory as a simple particular case.

In particular, this and the preceding papers of this series, have established the:

1) **Conditions of exact validity of the 20th century scattering theory,** given by Coulomb and other scattering of particles under conditions admitting a valid point-like approximation (e.g., at sufficient mutual distance without collisions), as necessary for the applicability of the local-differential topology and mathematics underlying relativistic quantum mechanics.

2) **Conditions of unknown validity of the 20th century scattering theory,** given by reversible scattering events entirely representable via negative binding energies under conditions of partial or total mutual penetration of the charge distributions and/or wavepackets of particles. In this case, vast preceding studies have established the non-Lagrangian and non-Hamiltonian character of the events with expected revisions of the "experimental results" claimed from the elaboration of measured quantities vis the 20th century theory. The unsettled character of this case, clearly stated since Paper I of this series, is that the 20th century scattering and the isotopic scattering theory have exactly the same axioms, to such an extent of coinciding at the abstract realization-free level. Under this axiomatic identity, no scientific conclusion can be reached without a detailed scrutiny, whether for the validity or invalidity under isotopies of 20th century "expexperimental results" for reversible scattering events entirely representable via a negative binding energy.

3) **Conditions of inapplicability (and not violation) of the 20th century scattering theory,** given by reversible scattering events requiring "positive binding energies," as it is the case for the synthesis of the neutron from a proton and an electron, the synthesis of the $\pi^o$ meson from an electron-positron pair, and the syntheses of hadrons at large, as occurring in the core of stars, in particle accelerators or under other conditions. The inapplicability of the 20th century scattering theory for the latter class is beyond credible doubt due to its unitary character, while the events considered solely admit a quantitative representation via nonunitary theories as established by Santilli since 1978 [3].

In conclusion, this and the preceding papers of this series have indeed established the *necessity, consistency and validity* of the isoscattering theory of relativistic hadronic mechanics because it is the only known at this writing permitting quantitative representations of reversible syntheses of hadrons.

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# References


[1] R. M. Santilli and A. O. E. Animalu, Nonunitary, Lie-isotopic and Lie-admissible scattering theopries of hadronic mechanics, I, II, III, in *Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes,* C. Corda, Editor, Kathmandu University (2011) pages 423-581
http://www.santilli-foundation.org/docs/Isoscattering-I.pdf
http://www.santilli-foundation.org/docs/Isoscattering-II.pdf
http://www.santilli-foundation.org/docs/Isoscattering-III.pdf

[2] R.P. Feynman, Quantum Electrodynamics (W.A. Benjamin, Inc. N.Y.) 1962

[3] Hadronic J. **1**, 574 (1978).

[4] R. M. Santilli: Hadronic J. 13, 513 (1990) [4a]; Communication of the JINR, Dubna, Russia E4-93-352, 1993 [4b]; Chinese J. Syst. Ing. and Electr. 6, 177 (1996) [4d]; Foundations of Physics 37, 670 (20070 [4d]; *Hadronic Mathematics, Mechanics and Chemistry.* International Academic Press 2008, Internationa;l Academic Press, http://www.i-b-r.org/Hadronic-Mechanics.htm

[5] A.O. Barut, in Group Theor. Methds in Physics (Edited by W. Beiglbock, A. Bohm and T. Takasugi), SLNP Vol. 94, 490 (1979); A.O. Barut, Ann. Physik Leipzig 43 83 (1986); see also, A.O.E. Animalu, Lepton and Hadron Currents in O(4,2) Current Algebra Letters Nuovo Cim. 3, 729 (1972).

[6] A. O. Barut, P. Cordero and G.C. Ghirardi, Phys. Rev. 182, 1844 (1969)and Nuov. Cim. 66A, 36 (1970); A.O.E. Animalu, Mass Ratio of Quarks, Phys. Rev. D3, 1106 (1971); Charge Spectrum of 4-Component Fields with O(4,2) Dynamical Symmetry, Phys. Rev. 4D, 1992-1925 (1971).

[7] J. Bernstein, in Elementary Particles and their Currents (Freeman, San Francisco, 1947) p. 41.

[8] A.O.E. Animalu, Scale Symmetry, Physics Today p.76, June (1972)

[9] M. Gell-Mann, R.J. Oakes and B. Renner, Phys. Rev. 175,2195 (1968); M. Gell-Man, Phys. Letters 8, 214 (1964); G. Zweig, CERN Report 8419TH 412 (1964).

[10] H.C. Myung and R.M. Santilli, Foundations of the Hadronic Generalization of the Atomic Mechanics, II: Modular-Isotopic Hilbert Space Formulation of the Exterior Strong Problem, Hadronic J. 5, 1277-1366 (1982); M. Nishioka,

Extension of the Dirac-Myung-Santilli Delta function to Field Theory, Lett. Nuov. Cim Ser. 2, Vol. 39, pp.369-372 (1984).

[11] A.O.E. Animalu, Gauge-Invariant Relativistic theory of the Rutherford-Santilli Neutron, Hadronic J 27,599-624 (2004).

[12] John Evans, Mind, Body and Electromagnetism (Element Books 1986), p.64-66.

[13] J. Todd, Projective and Analytical Geometry (Pitman, London 1958) p.62.

[14] P.A.M. Dirac P.A.M. Dirac, Quantum theory of the Electron, Proc. Roy. Soc. (Lond.) A117, 610 (1928), Part II, A118 351 (1928); A Positive-Energy relativistic Wave equation, Proc. Roy. Soc. (Lond.) A322, 435 (1971); A328, 438 (1972).

[15] J. D. Bjorken and S. Drell Advanced Quantum Mechanics Vols. 1 and 2 (1966).

[16] A.O.E. Animalu and C.E. Ekuma, Review of deep-Inelastic e-p Scattering: A Hadronic Mechanics Viewpoint, African J. Phys. Vol. 1, p.133-153 (2008).

[17] S. T. Weinberg, Phys. Rev. Lett 18, 1264 (1967).

[18] H. T. Piaggio. An Elementary Treatise on Differential Equations and their Applications London) (1961).

[19] R.H Dalitz Proc Roy Soc 206A,509(1951)

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
Kathmandu University, Nepal, April (2011) pages 581-638

# NONUNITARY LIE-ISOTOPIC AND LIE-ADMISSIBLE SCATTERING THEORIES OF HADRONIC MECHANICS, V: Foundations of the Genoscattering Theory for Irreversible Processes

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## Abstract

In four previous papers of this series, we presented the foundations of a nonunitary-isounitary generalization of the unitary relativistic scattering theory as characterized by Santilli's Lie-isotopic theory. The generalized theory was called *isoscattering theory* due to the use of the underlying isomathematics. Even too time-reflection noninvariance can be accommodated via a time-dependence of the isounit, the axioms of the isoscattering theory have no "arrow of time" and, therefore, are essentially applicable to reversible scattering events, such as Coulomb scattering without collisions. In view of these limitations, in this paper we present, apparently for the first time, an irreversible covering of the isoscattering their as characterized by Santilli's Lie-admissible covering of the Lie-isotopic theory. The latter theory is presented under the name of *genoscattering theory* due to the use of the underlying genomathematics, and it is specifically intended for irreversible scattering processes, such as deep inelastic scattering. Besides a number of divergences between the data interpretation via the genoscattering and the conventional scattering theory, a significant result identified in this paper is that the irreversible treatment of inelastic processes among extended particles or wavepackets implies numerical values of the masses of intermediate states, such as that of the Higgs boson, largely different than those predicted by the conventional reversible scattering theory among point-like particles.

Key words scattering theories, nonunitary theories, isounitary theories
PACS 03.65.Nk, 11.30.Cp, 03.65.-w

TABLE OF CONTENT

1. Introduction
   1.1. Brief outline of Preceding Papers
   1.2. The Role of thge Isodual Theory for Antimatter
   1.3. Main Objectives of this paper

2. Santilli Lie-Admissible Genomathematics
   2.1 Genounits, Genoproducts and their Isoduals
   2.2. Genonumbers, Genofunctional Analysis and their Isoduals
   2.3. Genogeometries and Their Isoduals
   2.4. Santilli Lie-admissible Theory and its Isodual

3. Elements of Lie-Admissible Hadronic Mechanics.
   3.1. Basic Dynamical Equations
   3.2. Simple Construction of Lie-Admissible Theories
   3.3. Invariance of Lie-Admissible Theories
   3.4. Genotopy of Pauli-Santilli Isomatrices and
   3.5. Genotopy of Dirac-Santilli isoequation

4. Lie-Admissible Invariant Genoscattering Theory
   4.1. The Fundamental Lie-Admissible Scattering Matrix
   4.2. Review and Genotopy of the Isoscattering Formalism
   4.3. $\hat{O}^{>}(4,2) \times \hat{SU}^{>}(3)$ Dynamical Symmetries of the Scattering Region.
   4.4. Genotopy of Lepton and Hadron Currents

5. Initial Applications
   5.1 Genotopy of S-matrix and Feynman Graphs/Rules.
   5.2 Deep-Inelastic e-p Scattering Experimental Results
   5.3 Genotopy of Bjorken Variable and Structure Functions
   5.4. Mass Genorenormalization
   5.5. Concluding Remarks

Acknowledgments

References

# 1. INTRODUCTION

## 1.1. Brief Outline of Preceding Papers

An important property of quantum mechanics, which is at the foundation of its physical relevance when applicable, is its *invariance over time,* namely, the capability of predicting the same numerical values under the same conditions at different times. As it is well known, this property originates from the fact that the time evolution of quantum mechanics characterizes a *unitary transformation* on a Hilbert space over a field.

However, quantum mechanics was conceived and verified for closed-isolated systems of point particles in vacuum (exterior dynamical problems), such as the atomic structures, that are *reversible over time,* namely, their time reversal images verify causality and conservation laws. This feature is reflected in the fact that the basic mathematical and physical axioms of quantum mechanics have no "time arrow," namely, they are as reversible as the systems intended for representation.

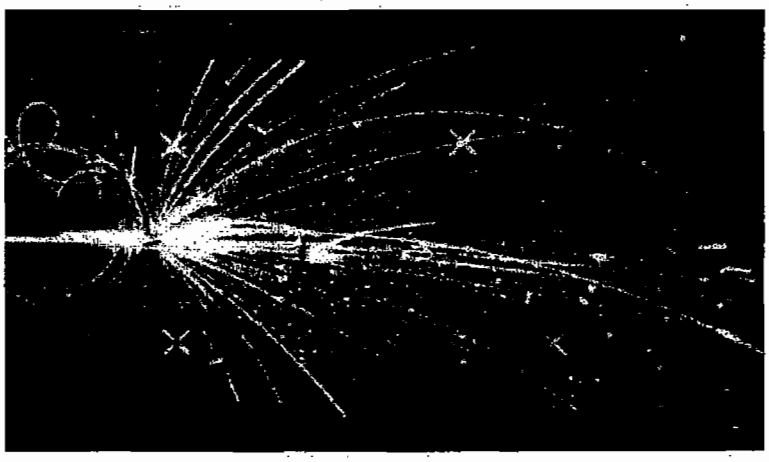


Figure 1: *A view of inelastic scattering events studied in this paper to illustrate their irreversible character.*

In the first paper of this series [1], we recalled the historical legacy by Lagrange, Hamilton and Jacobi according to which the irreversibility of natural processes originates from contact nonpotential interactions represented with the external terms in the analytic equations (interior dynamical systems); we recalled the *No Reduction Theorems* prohibiting the reduction of macroscopic irreversible systems to a finite number of elementary particles all in reversible conditions, thus establishing the origin of nonpotential/non-Hamiltonian forces at the most elementary level of na-

ture; and we recalled that the covering *hadronic mechanics* was built for the specific purpose of achieving an invariant representation of systems with conventional potential Hamiltonian as well as contact non-Hamiltonian interactions, the latter demanding a time evolution which is necessarily *nonunitary* from the non-Hamiltonian character of the forces.

In the preceding Papers I, II and III of this series [1], w reviewed the *Theorems of Catastrophic Mathematical and Physical Inconsistencies of Noncanonical and Nonunitary Theories* when treated with the mathematics of canonical and unitary theories, respectively. Along these lines, we pointed out that nonunitary time evolutions violate causality when formulated on a conventional Hilbert space over a conventional field. We then reviewed *Santilli Lie-isotopic mathematics.* or *isomathematics* for short, which provides the only known method capable of resolving said catastrophic inconsistencies, by regaining invariance over time and causality. In the same Papers I, II, III, we then reviewed the *Lie-isotopic branch of hadronic mechanics,* or *isomechanics* for short, and specialized it to the scattering problem.

It may be useful to recall that Santilli's Lie-isotopic formulations are achieved at both, the mathematical and the physical levels, by using a nonunitary transformation of a generic quantum mechanical quantity Q

$$UU^\dagger \neq I, \tag{1,1a}$$

$$Q \to \hat{Q} = UQU^\dagger, \tag{1.1b}$$

applied to the *totality of quantum mechanical, mathematical and physical quantities and their operations* with no known exception, thus including the isotopic lifting of basic units, numbers, functions, differentials, etc. Invariance over time and causality are achieved indeed, but under the condition of elaborating isotopic theories with isomathematics, since elaborations of isotopic theories with conventional mathematics, or elaborations of conventional theories with isomathematics, are evidently inconsistent.

In Paper IV of this series [1], we then presented in operational details the *Lie-isotopic scattering theory,* or *isoscattering theory* for short, including all necessary foundations, such as the Dirac-Santilli equation, the Feynman-Animalu diagrams, and related procedure. The resulting theory emerges as a significant covering of the conventional unitary scattering theory since it possesses an essential nonunitary structure, yet it is as invariant and causal as the conventional theory. The nonunitary character permits significant advances in the representation of scattering processes, such as the representation of particles as being extended, resulting in a scattering region no longer constituted by ideal points, but consisting of a hyperdense

medium with Hamiltonian and non-Hamiltonian internal interactions verifying the laws of hadronic mechanics, while quantum mechanics is recovered uniquely identically in the exterior of the scattering region under the limit $LimUU^{\dagger} = I$.

Despite these advances, *the Lie-isotopic scattering theory remains as reversible over time as the conventional scattering theory.* This feature was expected *ab initio* since the formulation was constructed via the use of *Santilli isotopies* that are known to be *axiom-preserving* by conception and technical realization.

Despite this limitation, in Paper IV we showed that the isoscattering theory is non-trivial because it allows the inclusion of scattering processes that are prohibited by the conventional theory due to its unitarity. As an illustration, we provided the representation via the isoscattering theory of the following events

$$e^- + e^+ \to \pi^0 \to e^- + e^+ \tag{1.2a}$$

$$e^- + p \to n + \nu \ \to p + e^- + \nu + \bar{\nu}, \tag{1.2b}$$

which are manifestly reversible over time, yet requiring a nonunitary scattering theory because: the rest energy of the synthesized hadron is bigger than the sum of the rest energies of the original particles; the missing energy cannot be provided by the relative kinetic energy due to the related excessively small cross section and ensuing inability to achieve the indicated syntheses of hadrons; and the only known consistent dynamical equations is the *Schrödinger-Santilli equation,* namely, a nonunitary image of the Schrödinger equation proposed by Santilli since his original memoir of 1978 to build hadronic mechanics [2].

### 1.2. The Role of the Isodual Theory for Antimatter

As it is well known, during the 20th century matter was treated at all level of study, from Newtonian Mechanics to second quantization, while antimatter was solely treated at the level of second quantization, due to the lack of technical means in Einstein's special and general relativities to provide any distinction between *neutral* matter and antimatter. This resulted in the lack of scientific democracy in the treatment of matter and antimatter with deep implications at all levels of study.

Santilli (see general review [5] and original papers quoted therein) resolved the above imbalance via the construction of a new mathematics, today known as *Santilli isodual mathematics,* the related *isodual mechanics and relativity* and the resulting *isodual theory of antimatter.* The main idea of these studies can be outlined as follows. Recall that the conventional *charge conjugation* is defined on a Hilbert space $\mathcal{H}$ with states $\psi(x)$ over the field of

complex numbers $C$ and can be characterized by expressions of the type

$$C\ \psi(x) = -\ \psi^{\dagger}(x), \tag{1.3}$$

where $x$ is the coordinate of the representation space, such as the Minkowski spacetine.

Santilli [28] constructed the isodual mathematics, mechanics and relativity are via an anti-Hermitean conjugation, called *isoduality* and denoted with the upper index $d$, applied to the totality of the mathematics and physics used for matter with no known exception to avoid catastrophic inconsistencies when mixing conventional and isodual formulations. Therefore, the isodual conjugation of an arbitrary classical or operator quantity $A(x, p, ....)$ depending on coordinates $x$, momenta $p$, and any other needed variable is given by

$$A(x, p, ....) \ \rightarrow\ A^d(x^d, p^d, ....) = A(-x^{\dagger}, -p^{\dagger}, ....). \tag{1.4}$$

This conjugation characterizes the novel *isodual unit* $1^d = -1^{\dagger}$, *isodual real, complex or quaternionic numbers* $n^d = -n^{\dagger}$, *isodual product* $n^d \times^d m^d = n^s \times (1^d)^{-1} \times m^d$, *isodual functional analysis*, isodual differential calculus, etc. (see Ref. [2] for brevity). In particular, the reader should keep in mind that isoduality is the only known consistent procedure for the differentiation between *neutral* as well as charged matter and antimatter at all levels of treatment.

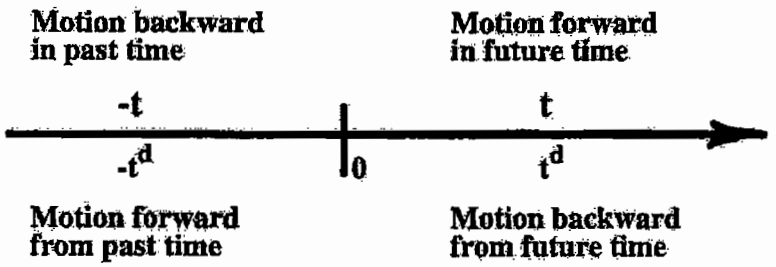



Figure 2: *An illustration of the four different directions of time and the consequential need of the isodual conjugation in addition to the conventional time inversion for the representation of all four directions.*

Even though charge and isodual conjugations are both anti-Hermitean, their differences are not trivial. From a physical viewpoint, charge conjugation conjugates states in a Hilbert space, but does not conjugate the

local coordinates $x$. This implies that, for 20th century theories, antimatter exists in the same spacetime of matter. At any rate, the relegation of antimatter at the level of second quantization, e.g., via Dirac's "hole theory," leaves the Minkowski spacetime unique, thus entirely characterized by the fundamental Poincaré symmetry and special relativity.

By contrast, the isodual conjugation additionally maps spacetime coordinates $x$ into the novel *isodual coordinates* $x^d = -x^\dagger$ that are defined on the *Minkowski-Santilli isodual spacetime* $M^d(x^d, \eta^d, 1^d)$, where $\eta$ is the usual Minkowski metric. Therefore, under isoduality, the *Poincaré-Santilli isodual symmetry,* and the *isodual special relativity,* antimatter is predicted to exist in a new spacetime which is distinct from, yet coexisting with our spacetime. In particular, the differences of conventional and isodual spacetimes are not trivial. e.g., because the isodual conjugation of coordinates is different than inversions [2].

It should be stressed to prevent possible scientific misrepresentations that *the isodual theory verifies all available experimental data on antimatter at both the classical and operator levels.* In fact, the *Newton-Santilli isodual equations* for antiparticles verifies all available data for charged particles and antiparticles, while isoduality is equivalent to charge conjugation at the operator level by conception and construction, as recalled via Eqs. (1) and (2) (see Ref. [2] for details).

As expected, *the isodual theory of antimatter suggested a re-interpretation of Dirac's equation with deep implications for the scattering theory.* Recall that Dirac was forced to voice the "hole theory" for the consistent representation of antiparticles due to the unphysical character of negative energy solutions.

The isodual theory of antimatter resolved the latter issue since negative energies are referred to negative units, thus being as causal as positive energies referred to positive units. In any case, the isodual theory of antimatter achieves a consistent representation of antiparticles at the Newtonian level, let alone that in first quantization. Consequently, the conventional Dirac equation

$$[\gamma^\mu \times (p_\mu - e \times A_\mu/c) + i \times m] \times \Psi(x) = 0, \tag{1.6a}$$

$$\gamma_k = \begin{pmatrix} 0 & -\sigma_k \\ \sigma_k & 0 \end{pmatrix}, \ \gamma^4 = i \times \begin{pmatrix} I_{2\times 2} & 0, \\ 0 & -I_{2\times 2} \end{pmatrix}, \tag{1.6bb}$$

$$\{\gamma_\mu, \tilde{\gamma}_\nu\} = 2\times\eta_{\mu\nu}, \ \Psi = i \times \begin{pmatrix} \Phi \\ -\Phi^\dagger \end{pmatrix} \tag{1.6c}$$

has been subjected to the following re-interpretation solely permitted by

the isodual theory [5]

$$[\tilde{\gamma}^{\mu} \times (p_{\mu} - e \times A_{\mu}/c) + i \times m] \times \tilde{\Psi}(x) = 0, \tag{1.7a}$$

$$\tilde{\gamma}_k = \begin{pmatrix} 0 & \sigma_k^d \\ \sigma_k & 0 \end{pmatrix}, \quad \tilde{\gamma}^4 = i \begin{pmatrix} I_{2\times 2} & 0, \\ 0 & I_{2\times 2}^d \end{pmatrix}, \tag{1.7b}$$

$$\{\tilde{\gamma}_{\mu}, \tilde{\gamma}_{\nu}\} = 2^d \times^d \eta_{\mu\nu}^d, \quad \tilde{\Psi} = -\tilde{\gamma}_4 \times \Psi = i \times \begin{pmatrix} \Phi \\ \Phi^d \end{pmatrix} \tag{1.7c}$$

in which the electron and the positron are both treated in first quantization without any need for the "hole theory."

The main conclusion of re-interpretation (1.xxx) is that *the Dirac equation directly represents the Kronecker product of an electron and its antiparticle.* This conclusion is equally reached, rather forcefully, on algebraic grounds. Santilli [28] first noted that there exists no *irreducible* four-dimensional representation of the SU(2) symmetry for spin 1/2, and there exists no *reducible* four-dimensional representation of SU(2) with the structure of Dirac's gamma matrices. Therefore, the sole known algebraically consistent meaning of the gamma matrices is that they characterize an *irreducible* representation for spin 1/2 of the Kronecker product $SU(2) \times SU(2)^d$, thus representing a Kronecker product of an electron and its antiparticle as indicated above.

Since Feynman's diagrams for electrons and positrons are centrally dependent on Dirac's equation, it is evident that the above reformulation of the latter equation requires a necessary re-inspection of the former. In fact, the annihilation process in Feynman's diagrams

$$e^- + e^+ \rightarrow 2\,\gamma, \tag{1.8}$$

exhibits a number of asymmetries, such as: the l.h.s. is isoselfdual (invariant under isoduality), but the r.h.s is not; the annihilation process is assumed to occur via the exchange of a particle (an electron or a photon)which is not isoselfdual; and others.

One of the major implications of the isodual conjugation which is not possible for charge conjugation is the prediction that *antimatter emits a new light with experimentally verifiable physical differences with the ordinary light emitted by matter.* In fact, charge conjugation is evidently inapplicable to the photon, while isoduality predicts the isodual photon $\gamma^d$ with energy $E^d = \hbar^d \times^d \nu^d = -E$ referred to the unit $MeV^d = -MeV$ which is predicted as being repelled by the gravitational field of matter, thus being physically distinguishable from the ordinary photon $\gamma$ with g energy $E = \hbar \times \nu$ referred

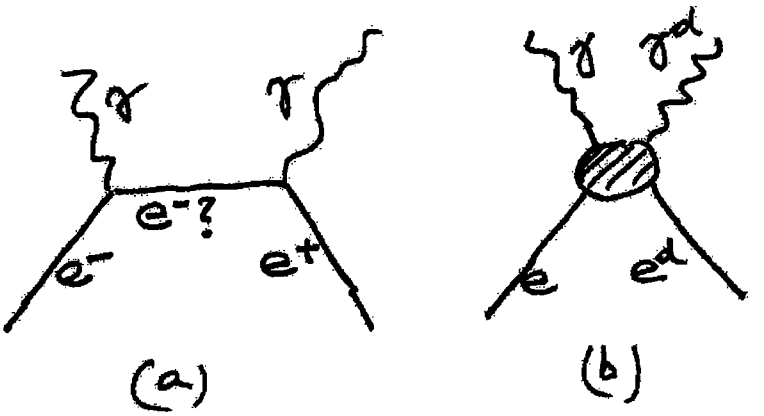



Figure 3: *A view of the electron-positron annihilation according to Dirac-Feynman theories (l.h.s) and the same annihilation as predicted by Santilli's isodual theories (r.h.s). Note the verification for the latter of all isodual laws, as well as the absence of the isoselfduality violating exchange of the former, since annihilation requires actual physical contact of particles antiparticles that cannot be triggered by particle exchanges at a distance as represented by the Feynman-Animalu diagrams.*

**to the unit $MeV$. Consequently, the isodual theory requires the following re-interpretation of Feynman's diagram for annihilation ()1.8) [5]**

$$e + e^d \equiv (e + e^d)^d \ \rightarrow \ \gamma + \gamma^d \equiv (\gamma + \gamma^d)^d, \quad e = e^-, \quad e^d = e^+, \tag{1.9}$$

**that provides an evident resolution of all ambiguities and asymmetries of annihilation (1.8). Moreover, in the latter case, there is no exchange of particles, since annihilation is predicted to occur under actual physical contact or mutual penetration of the wavepackets of particles and antiparticles in accordance with *Feynman-Animalu diagrams* (see Fig. 3).**

**The insidious character of the lack of full democracy in the treatment of matter and antimatter is illustrated by comparing reactions (1.8) and (1.9). Reaction (1.8) is rather universally treated in first quantization, resulting in clear inconsistencies since, at that level, the electron and the photons can indeed be fully treated, yet the positron has negative energy in first quantization, thus prohibiting such a treatment for the sole consistent treatment in second quantization. by comparison, Reaction (1.9) can be consistently treated at the level of first quantization, its treatment at the level of second quantization being under study by V. de Haan (private communication).**

Needless to say, there exists a large number of experiments in electron-positron annihilation and the emitted two photons. However, a deep inspection reveals that available experiments have provided no consideration on the possible differences between the two emitted photons, trivially, because no such difference was provided by the uses basic theories. Specific experiments for the resolution whether the two photons of reaction (1.9) are identical or physically different has been proposed in Ref. [29].

All in all, it is hoped the reader can see that *a reinspection of the 20th century scattering theory is necessary for the sole advances in antimatter, let alone because of its reversible structure compared to the general irreversibility of scattering events.*

### 1.3. Main Objectives of this Paper

In this paper, we present, apparently for the first time, the foundation of the *Lie-admissible scattering theory* as a covering of the conventional or the isoscattering theory for the specific purpose of representing *deep inelastic scattering processes involving particles and antiparticles* that are notoriously irreversible over time (see Fig. 1). In fact, in the latter processes, we have the lack of rigorous applicability of the conventional scattering theory due to its reversibility since the selection of the appropriate scattering theory for irreversible scattering events is indeed open to scientific debates, but not its need.

The main difficulties of this paper are numerous. First, it is essential to achieve a formulation which is truly irreversible in both its mathematical and physical formulations. This objective is implemented via *Santilli's forward and backward maps* characterized by *two* nonunitary transformations of each quantum mechanical quantity Q

$$ZZ^\dagger \neq I \quad WW^\dagger \neq I, \quad ZW^\dagger \neq I, \tag{1.10a}$$

$$Q \to \hat{Q}^> = ZQW^\dagger, \quad Q \to^< \hat{Q} = WQZ^\dagger. \tag{1.10b}$$

The above liftings essentially set the "time arrow" in all mathematical and physical structures and their operations. Irreversibility is then assured by the inequivalence of the forward and backward processes.

The second major difficulty is in the achievement of invariance over time not only for nonunitary theories, but also of their realization in an irreversible form. This task cannot any longer be achieved via the isomathematics, thus mandating the use of a yet broader *Santilli Lie-admissible mathematics,* also known as*genomathematics,* the resulting scattering theory being also proposed under the shorter name of *genoscattering theory.*

The final difficulty is the verification that the ensuing Lie-admissible scattering theory is indeed physically valid and carries nontrivial implications in the data elaboration of deep inelastic scattering events. By remembering that the conventional scattering theory was established only following about half a century of research, the authors hope that this initiating paper will essentially set up the foundation of the new scattering theory for irreversible scattering process for a collegial future finalization.

The results of this paper are largely dependent on Santilli's lifelong studies on Lie-admissible algebras with particular reference to the latest memoir [4], as well as on Animalu's first isotopies of Feynman's diagrams [30].

## 2. SANTILLI LIE-ADMISSIBLE GENOMATHEMATICS

### 2.1 Genounits, Genoproducts and their Isoduals

The most fundamental notion of Santilli's genomathematics from which the entire formulation is build via compatibility arguments, is a dual generalization of the basic unit of quantum mechanics $\hbar = 1$ into two non-Hemitean (nonsingular) generalized units, called *genounits,* one used to represent motion forward in time and the other for motion backward in time, but having a nonsingular, but otherwise arbitrary dependence on time $t$, coordinates $r$, the density $\mu$ of the region considered (e.g. the scattering region), wavefunctions $\psi$, their derivatives $\partial\psi$, etc. [3]

$$\hat{I}^{>}(t,r,\mu,\psi,\partial\psi,...) = 1/\hat{T}^{>}, \quad {}^{<}\hat{I}(t,r,\mu,\psi,\partial\psi,...) = 1/{}^{<}\hat{T}, \tag{2.1a}$$

$$\hat{I}^{>} \neq {}^{<}\hat{I}, \quad \hat{I}^{>}(t,...) \neq^{>} (-t,...), \quad {}^{<}\hat{I}(t,...) \neq^{<} \hat{I}(-t,...(, \quad \hat{I}^{>} = ({}^{<}\hat{I})^{\dagger}, \tag{2.1b}$$

with two additional *isodual genounits* for the description of antimatter

$$(\hat{I}^{>})^{d} = -(\hat{I}^{>})^{\dagger} = -{}^{<}\hat{I} = -1/{}^{<}\hat{T}, \quad ({}^{<}\hat{I})^{d} = -\hat{I}^{>} = -1/\hat{T}^{>}. \tag{2.2}$$

Santilli selected since the original memoirs [2] of 1978 the "genotopic" from the Greek meaning of "inducing new structures and to have a differentiation with the word "isotopic" used in the preceding papers that stands for preserving the original; structures.

Jointly, all conventional and/or isotopic products $A\hat{\times}B$ among generic quantities (numbers, vector fields, operators, etc.) are lifted in such a form to admit the genounits as the correct left and right units at all levels, i.e.,

$$A > B = A \times \hat{T}^{>} \times B, \quad A > \hat{I}^{>} = \hat{I}^{>} > A = A, \tag{2.3a}$$

$$A < B = A \times^{<} \hat{T} \times B, \quad A <^{<} \hat{I} =^{<} \hat{I} < A = A, \tag{2.3b}$$

$$A >^{d} B = A \times \hat{T}^{>^{d}} \times B, \quad A >^{d} \hat{I}^{>^{d}} = \hat{I}^{>^{d}} >^{d} A = A, \tag{2.3c}$$

$$A <^{d} B = A \times^{<} \hat{T}^{d} \times B, \quad A <^{d} {}^{<}\hat{I}^{d} = {}^{<}\hat{I}^{d} <^{d} A = A, \tag{2.3d}$$

for all elements $A$, $B$ of the set considered.

In different words, the central idea in the Lie-admissible representation of irreversible processes is to lift the conventional associative product of quantum mechanics $A \times B$ into *two* products, one for the product to the right $A > B$ and one for the product to the left $A < B$. Irreversibility is then solely guaranteed under the condition of their nonequivalent, that is, $A > B \neq A < B$. In turn, this inequivalence is impossible under the conventional associative product $A \times B$, thus mandating inequivalent genotopies $A > B = A \times \hat{T}^{>} \times B$ and $A < B = A \times^{<} \hat{T} \times B$. In the next section we shall then show that the quantity $\hat{T}^{>}$ precisely represents the external forces of Lagrange and Hamilton equations for motion forward in time and ${}^{<}\hat{T}$ represents the inequivalent time reversal image.

The assumption of all *ordered product to the right* $>$ permits the representation of matter systems moving forward in time, the assumption of all *ordered products to the left* $<$ can represent matter systems in the scattering region moving backward in time, with corresponding antimatter systems represented by the respective isodual ordered products $>^{d} = - >^{\dagger}$ and $<^{d} = - <^{\dagger}$. Irreversibility is represented *ab initio* by the inequality $A > B \neq A < B$ for matter and $>^{d} \neq <^{d}$ for antimatter.

We recall here that the simpler isotopic subclass are given by $\hat{I}^{>} =^{<} \hat{I} = \hat{I} = \hat{I}^{\dagger} > 0$ for matter and $\hat{I}^{>^{d}} =^{<} \hat{I}^{d} = \hat{I}^{d} = \hat{I}^{d\dagger} < 0$ for antimatter.

The reader should be aware (by looking at Fig. 2) that *Santilli's genomathematics consists of four branches, namely the forward and backward genomathematics for matter and their isoduals for antimatter,* each pair being interconnected by time reversal, and the two pairs being interconnected by isodual map

$$\begin{aligned} Q(t, r, \psi, \partial\psi, ...) &\to Q^{d}(t^{d}, r^{d}, \psi^{d}, \partial^{d}\psi^{d}, ...) \\ &= -Q^{\dagger}(-t^{\dagger}, -r^{\dagger}, -\psi^{\dagger}, -\partial^{\dagger}(-\psi^{\dagger}), ...) \end{aligned} \tag{2.4}$$

that, as it is well known [5], is equivalent to charge conjugation.

### 2.2. Genonumbers, Genofunctional Analysis and their Isoduals

Genomathematics began with Santilli's discovery in paper [3] of 1993, that *the axioms of a field still hold under the ordering of all products to the right or, independently, to the left.* This property permitted the formulation of *new*

*numbers* **that can be best introduced as a generalization of the isonumbers, although they can also be independently presented as follows:**

*DEFINITION : Let* $F = F(a, +, \times)$ *be a field of characteristic zero. Santilli's forward genofields are rings* $\hat{F}^{>} = \hat{F}(\hat{a}^{>}, \hat{+}^{>}, \hat{\times}^{>})$ *with: elements*

$$\hat{a}^{>} = a \times \hat{I}^{>}, \tag{2.5}$$

*where* $a \in F$, $\hat{I}^{>} = 1/\hat{T}^{>}$ *is a non singular non-Hermitean quantity (number, matrix or operator) generally outside* $F$ *and* $\times$ *is the ordinary product of* $F$*; the genosum* $\hat{+}^{>}$ *coincides with the ordinary sum* $+$*,*

$$\hat{a}^{>}\hat{+}^{>}\hat{b}^{>} \equiv \hat{a}^{>} + \hat{b}^{>}, \ \forall \hat{a}^{>}, \ \hat{b}^{>} \in \hat{F}^{>}, \tag{2.6}$$

*consequently, the additive forward genounit* $\hat{0}^{>} \in \hat{F}$ *coincides with the ordinary* $0 \in F$*; and the forward genoproduct* $>$ *is such that* $\hat{I}^{>}$ *is the right and left isounit of* $\hat{F}^{>}$*,*

$$\hat{I}^{>}\hat{\times}\hat{a}^{>} = \hat{a}^{>} > \hat{I}^{>} \equiv \hat{a}^{>}, \ \forall \hat{a}^{>} \in \hat{F}^{>}. \tag{2.7}$$

*Santilli's forward genofields verify the following properties:*

*1) For each element* $\hat{a}^{>} \in \hat{F}^{>}$ *there is an element* $\hat{a}^{>-\hat{I}^{>}}$*, called forward genoinverse, for which*

$$\hat{a}^{>} > \hat{a}^{>-\hat{I}^{>}} = \hat{I}^{>}, \ \forall \hat{a}^{>} \in \hat{F}^{>}; \tag{2.8}$$

*2) The genosum is commutative*

$$\hat{a}^{>}\hat{+}^{>}\hat{b}^{>} = \hat{b}^{>}\hat{+}^{>}\hat{a}^{>}, \tag{2.9}$$

*and associative*

$$(\hat{a}^{>}\hat{+}^{>}\hat{b}^{>}) +^{>} \hat{c}^{>} = \hat{a}^{>}\hat{+}^{>}(\hat{b}^{>}\hat{+}^{>}\hat{c}^{>}), \ \forall \hat{a}, \hat{b}, \hat{c} \in \hat{F}; \tag{2.10}$$

*3) The forward genoproduct is associative*

$$\hat{a}^{>} > (\hat{b}^{>} > \hat{c}^{>}) = (\hat{a}^{>} > \hat{b}^{>}) > \hat{c}^{>}, \ \ \forall \hat{a}^{>}, \hat{b}^{>}, \hat{c}^{>} \in \hat{F}^{>}; \tag{2.11}$$

*but not necessarily commutative*

$$\hat{a}^{>} > \hat{b}^{>} \neq \hat{b}^{>} > \hat{a}^{>}, \tag{2.12}$$

*4) The set* $\hat{F}^{>}$ *is closed under the genosum,*

$$\hat{a}^{>}\hat{+}^{>}\hat{b}^{>} = \hat{c}^{>} \in \hat{F}^{>}, \tag{2.13}$$

*the forward genoproduct,*

$$\hat{a}^{>} > \hat{b}^{>} = \hat{c}^{>} \in \hat{F}^{>}, \tag{2.14}$$

*and right and left genodistributive compositions,*

$$\hat{a}^{>} > (\hat{b}^{>} \hat{+}^{>} \hat{c}^{>}) = \hat{d}^{>} \in \hat{F}^{>}, \tag{2.15a}$$

$$(\hat{a}^{>} \hat{+}^{>} \hat{b}^{>}) > \hat{c}^{>} = \hat{d}^{>} \in \hat{F}^{>} \quad \forall \hat{a}^{>}, \hat{b}^{>}, \hat{c}^{>}, \hat{d}^{>} \in \hat{F}^{>}; \tag{2.15b}$$

*5) The set $\hat{F}^{>}$ verifies the right and left genodistributive law*

$$\hat{a}^{>} > (\hat{b}^{>} \hat{+}^{>} \hat{c}^{>}) = (\hat{a}^{>} \hat{+}^{>} \hat{b}^{>}) > \hat{c}^{>} = \hat{d}^{>}, \quad \forall \hat{a}^{>}, \hat{b}^{>}, \hat{c}^{>}, \in \hat{F}^{>}. \tag{2.16}$$

*In this way we have the forward genoreal numbers $\hat{R}^{>}$, the forward genocomplex numbers $\hat{C}^{>}$ and the forward genoquaternionic numbers $\hat{Q}C^{>}$ while the forward genooctonions $\hat{O}^{>}$ can indeed be formulated but they do not constitute genofields [6].*

*The backward genofields and the isodual forward and backward genofields are defined accordingly. Santilli's genofields are called of the first (second) kind when the genounit is (is not) an element of F.*

**The basic axiom-preserving character of genofields is illustrated by the following:**

*LEMMA: Genofields of first and second kind are fields (namely, they verify all axioms of a field).*

**Note that the conventional product "2 multiplied by 3" is not necessarily equal to 6 because, for isodual numbers with unit $-1$ it is given by $-6$. The same product "2 multiplied by 3" is not necessarily equal to $+6$ or $-6$ because, for the case of isonumbers, it can also be equal to an arbitrary number, or a matrix or an integrodifferential operator depending on the assumed isounit [3].**

**In this section we point out that "2 multiplied by 3" can be ordered to the right or to the left, and the result is not only arbitrary, but yielding different numerical results for different orderings, $2 > 3 \neq 2 < 3$, all this by continuing to verify the axioms of a field per each order [3].**

**Once the forward and backward genofields have been identified, the various branches of genomathematics can be constructed via simple compatibility arguments.**

**For specific applications to irreversible processes there is first the need to construct the *genofunctional analysis,* studied in Refs. [6,7] that we shall not review here for brevity. It should, however, be clear to the reader**

**that any elaboration of irreversible processes via Lie-admissible formulations based on conventional or isotopic functional analysis leads to catastrophic inconsistencies because it would be the same as elaborating quantum mechanical calculations with genomathematics. Recall the theorem of catastrophic inconsistencies[8] which states that:**

*All theories with a non-unitary time evolution, $W(t)W^+(t) \neq I$ when formulated with mathematical methods of unitary theories (conventional fields, spaces, functional analysis, differential calculus, etc) do not preserve the said mathematical methods over time thus being afflicted by catastrophic mathematical inconsistencies and do not preserve over time the basic units of measurements, Hermiticity-observability, numerical predictions and causality, thus suffering catastrophic physical inconsistencies.*

**And observe that this theorem is activated unless one uses the ordinary differential calculus is lifted, for ordinary motion in time of matter, into the following** *forward genodifferentials and genoderivatives*

$$\hat{d}^> x = \hat{T}_x^> \times dx, \quad \frac{\hat{\partial}^>}{\hat{\partial}^> x} = \hat{I}_x^> \times \frac{\partial}{\partial x}, \text{ etc.} \tag{2.16}$$

**with corresponding backward and isodual expressions here ignored,**

**Similarly, all conventional functions and isofunctions, such as isosinus, isocosinus, isolog, etc., have to be lifted in the genoform**

$$\hat{f}^>(x^>) = f(\hat{x}^>) \times \hat{I}^>, \tag{2.17}$$

**where one should note the necessity of the multiplication by the genounit as a condition for the result to be in $\hat{R}^>$, $\hat{C}^>$, or $\hat{O}^>$.**

### 2.3. Genogeometries and Their Isoduals

**Particularly intriguing are** *Santilli's genogeometries* **which are characterized by a step-by-step genotopy of isogeometries, Consider the Minkowski isospacetime (see Paper III [1])**

$$\hat{M}(\hat{c}, \hat{\eta}, \hat{I}): \quad \hat{x} = x\hat{I}, \quad \hat{\eta} = \hat{T}(x, ...) \times \eta, \quad \hat{I}(x, ...) = \hat{I}^\dagger(x, ...) = 1/\hat{T} > 0, \tag{2.18}$$

**and introduce two nonunitary four-dimensional matrices $C, D$. Then the** *Minkowski-Santilli genospacetime* **is given by [4]**

$$\hat{M}^>(\hat{x}^>, \hat{\eta}^>, \hat{I}^>): \quad \hat{x}^> = C \times \hat{x} \times D^\dagger = x \times \hat{I} >, \tag{2.19a}$$

$$\hat{\eta}^> = C \times \hat{\eta} \times D^\dagger, = \hat{T}^> \times \eta, \quad \eta = Diag.(1, 1, 1, -1), \tag{19b}$$

$$I^> = CD^\dagger = 1/T^>, \qquad CC^\dagger \neq I, \quad DD^\dagger \neq I, \quad CD^\dagger \neq I. \tag{2.19c}$$

Genospaces and related geogeometries can also be independently defined, based on one of the fundamental axiomatic principles of hadronic mechanics, namely, that *irreversibility is directly represented with the background geometry and, more specifically, with its nonsymmetric metric.*

In fact, a central feature of genospacetime p2.19) is that its geometric $\hat{\eta}^{>}$ is *nonsymmetric* by conception and construction. Alternatively, it is easy to prove that a geometry with a symmetric metric cannot possible characterize irreversible processes. In this way, Santilli has initiated a new chapter in geometry, the first known to the authors with a realistic capability of achieving the much needed compatibility of geometries and thermodynamical laws, the latter being strictly irreversible over time.

Since the Minkowski-Santilli genospacetime is the ultimate and fundamental method for our relativistic representation of high energy inelastic scattering events, a simple illustration appears recommendable. Consider the following realization of the $C, D$ matrices

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ p & 0 & 0 & 1 \end{pmatrix} ; \; D = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ q & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{2.20}$$

where $p \neq q$ are non-null real numbers, under which we have the following forward and backward genotopy of the Minkowskian line element

$$x^2 \to x^{>2>} = Cx^2D^\dagger = C(x^t\eta x)D^\dagger =$$

$$= (C^t x^t D^{t\dagger})(CD^\dagger)^{-1}(C\eta D^\dagger)(CD^\dagger)^{-1}(CxD^\dagger) =$$

$$= (x^t I^{>})T^{>}\eta^{>}T^{>}(I^{>}x) = x^\mu \eta^{>}_{\mu\nu} x^\nu =$$

$$= (x^1x^1 + x^1qx^3 + x^2x^2 + x^3x^3 + x^1px^4 - x^4x^4), \tag{2.21a}$$

$$Dx^2C^\dagger = D(x^t\eta x)C^\dagger =$$

$$= (x^{t<} I)^{<}T^{<}\eta^{<}T(^{<}Ix) = x^{\mu <}\eta_{\mu\nu} x^\nu =$$

$$= (x^1x^1 + x^1px^3 + x^2x^2 + x^3x^3 + x^1qx^4 - x^4x^4), \tag{2.21b}$$

resulting in the forward and backward nonsymmetric genometrics

$$\eta^{>} = T^{>}\eta T^{>} = \begin{pmatrix} 1 & 0 & q & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ p & 0 & 0 & -1 \end{pmatrix}, \quad {}^{<}\eta = {}^{<}T\eta {}^{<}T = \begin{pmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ q & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{2.22}$$

exactly as desired. That is, the first expression of the genoinvariant is on genospaces while the second is its projection in our spacetime.

Note that irreversibility selects a mutation of the line elements along a pre-selected direction of space and time.

Note also that the quantities $p$ and $q$ can be functions of the local spacetime variables, in which case the resulting *Minkowskian genogeometry* can be equipped by a suitable lifting of the machinery of the Riemannian geometry (see Ref. [7] for the isotopic case). Moreover, because Minkowski-Santilli genospace has such an explicit dependence on spacetime coordinates, it is equipped with the entire formalism of the conventional Riemannian spaces covariant derivative, Christoffel's symbols, Bianchi identity, etc. only lifted from the isotopic form into the genotopic form.

The central property of genospaces, their lack of symmetric character, is evidently expressed by

$$\hat{\eta}^{>}_{\mu\nu} \neq \hat{\eta}^{>}_{\nu\mu}. \tag{2.23}$$

Consequently, *genotopies permit the lifting of conventional symmetric metrics into nonsymmetric forms,*

$$\eta^{Minkow.}_{Symm} \rightarrow \hat{\eta}^{>Minkow.-Sant.}_{NonSymm} \tag{2.24}$$

We note in particular the following *invariance under genotopy*

$$(x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \equiv [x^\mu \times (\hat{T}^{>} \times \eta_{\mu\nu}) \times x^\nu] \times T^{>-1} \equiv$$
$$\equiv (x^\mu \times \hat{\eta}^{>}_{\mu\nu} \times x^\nu) \times \hat{I}^{>}, \tag{2.25}$$

that evidently occurs for the particular case in which $\hat{T}^{>}$ is a complex number, with the understanding that such an invariance does not hold in general.

### 2.4. Santilli Lie-Admissible Theory and its Isodual

As it is well known, the methodological pillar of the entire 20th century physics is *Lie's theory.* In full awareness of this feature, Santilli first introduced in the original memoirs [2] of 1978 in rather large details the *Lie-isotopic theory* which is at the foundation of the isoscattering theory of the preceding Paper IV [1]. In the same original memoirs [2], Santilli introduced the yet broader *Lie-admissible theory* for the specific intent of characterizing open irreversible systems, and the latter theory is at the foundation of the genoscattering theory of this paper.

It should be noted that, except for the achievement of invariance, that was achieved only in the late 2000 (and studied in the next section), the

Santilli's Lie-admissible theory, also called *Lie-Santilli genotheory,* has remained essentially that of the original formulation [2]. The main advances has been the formulation of the theory on genospaces over genofields.

In accordance with the original proposal of 1978, the lie-admissible theory is characterized by the structural broadening of the following main branches of Lie's theory widely used in physics (See monographs [7] for the most general formulation to date):

*GENOTOPIES OF ENVELOPING ALGEBRAS:* They are characterized by the *forward and backward universal enveloping genoassociative algebra* $\hat{\xi}^{>}$, $^{<}\hat{\xi}$, with infinite-dimensional basis characterizing the *Poincaré-Birkhoff-Witt-Santilli genotheorem*

$$\hat{\xi}^{>} : \hat{I}^{>},\ \hat{X}_i,\ \hat{X}_i > \hat{X}_j,\ \hat{X}_i > \hat{X}_j > \hat{X}_k,\ \ldots,\ i \leq j \leq k, \tag{2.26a}$$

$$^{<}\hat{\xi} : \hat{I},\ ^{<}\hat{X}_i,\ \hat{X}_i < \hat{X}_j,\ \hat{X}_i < \hat{X}_j < \hat{X}_k,\ \ldots,\ i \leq j \leq k; \tag{2.26b}$$

where the "hat" on the generators denotes their formulation on genospaces over genofields and their Hermiticity implies that $\hat{X}^{>} =^{<} \hat{X} = \hat{X}$;

*GENOTOPIES OF LIE ALGEBRAS:* They are characterized by the *Lie-Santilli genoalgebras* characterized by the universal, jointly Lie- and Jordan-admissible brackets,

$$^{<}\hat{L}^{>} : (\hat{X}_i, \hat{X}_j) = \hat{X}_i < \hat{X}_j - \hat{X}_j > \hat{X}_i = C_{ij}^{k} \times \hat{X}_k, \tag{2.27}$$

here formulated formulated in an invariant form (see below);

*GENOTOPIES OF LIE GROUPS:* They are characterized by the *Lie-Santilli genotransformation groups*

$$^{<}\hat{G}^{>} : \hat{A}(\hat{w}) = (\hat{e}^{\hat{i}\hat{x}\hat{X}\hat{x}\hat{w}})^{>} > \hat{A}(\hat{0}) <^{<} (\hat{e}^{-\hat{i}\hat{x}\hat{w}\hat{x}\hat{X}}) =$$

$$= (e^{i\times\hat{X}\times\hat{T}^{>}\times w}) \times A(0) \times (e^{-i\times w\times^{<}\hat{T}\times\hat{X}}), \tag{2.28}$$

where $\hat{w}^{>} \in \hat{R}^{>}$ are the *genoparameters;* the *genorepresentation theory,* etc.

the most salient mathematical aspect of Santilli's Lie-admissible theory is that its representation requires the necessary use of a *genobymodules,* referred to conventional modules whose action to the right and that to the left remain indeed associative, but in order to be different they have to be genoassociatives, e.g.

$$H > |\hat{a}^{>} >= H \times \hat{T}^{>} \times |\hat{a}^{>} >, \quad <^{<} \hat{b}| < H =<^{<} \hat{b}| \times^{<} \hat{T} \times H. \tag{2.29}$$

Consequently, the representation theory of Lie algebras for the conventional scattering theory is done on a conventional module with the conventional associative composition law. The representation theory of the

isoscattering theory is done on an isomodule, that is a module with isoassociative composition law. Finally, the representation theory of the genoscattering theory of this paper is done on a genobymodules, as indicated above, consisting of the necessary use of a module for the product to the right and one for the product to the left whose composition law is still associative, but it is characterized by two different isotopic elements as a necessary condition to represent irreversibility./

## 3. ELEMENTS OF LIE-ADMISSIBLE HADRONIC MECHANICS

### 3.1. Basic Dynamical Equations

The *Lie-admissible branch of hadronic mechanics* comprises four different formulations, the *forward and backward genomechanics for matter and their isoduals for antimatter* [4.7].The forward genomechanics for matter is characterized by the following main structures:

1) The nowhere singular (thus everywhere invertible) non-Hermitean *forward genounit* for the representation of all effects causing irreversibility, such as contact nonpotential interactions among extended particles,

$$\hat{I}^{>} = 1/\hat{T}^{>} \neq (\hat{I}^{>})^{\dagger}, \tag{3.1}$$

with all corresponding ordered product to the right, forward genoreal $\hat{R}^{>}$ and forward genocomplex $\hat{C}^{>}$ genofields;

2) *Hilbert-Santilli forward genospace* $\hat{\mathcal{H}}^{>}$ with *forward genostates* $|\hat{\psi}^{>}>$, *forward genoinner product*

$$<^{<} \hat{\psi}| > |\hat{\psi}^{>} > \times \hat{I}^{>} =<^{<} \hat{\psi}| \times \hat{T}^{>} \times |\hat{\psi}^{>} > \times \hat{I}^{>} \in \hat{C}^{>}, \tag{3.2}$$

and fundamental property

$$\hat{I}^{>} > |\hat{\psi}^{>} >= |\hat{\psi}^{>} >, \tag{3.3}$$

establishing that $\hat{I}^{>}$ is indeed the correct unit for motion forward in time, and *forward genounitary transforms*

$$\hat{U}^{>} > (\hat{U}^{>})^{\dagger} = (\hat{U}^{>})^{\dagger} > \hat{U}^{>} = \hat{I}^{>}; \tag{3.4}$$

3) Santilli's Lie-admissible equations, first proposed in the original proposal [2] of 1978, formulated on genospaces and genodifferential calculus on genofields, today known as *Heisenberg-Santilli genoequations,* which can be written in the finite form

$$\hat{A}(\hat{t}) = \hat{U}^{>} > \hat{A}(0) <^{<} \hat{U} = (\hat{e}_{>}^{\hat{i}\hat{\times}\hat{H}\hat{\times}\hat{t}}) > \hat{A}(\hat{0}) < (_{<}\hat{e}^{-\hat{i}\hat{\times}\hat{t}\hat{\times}\hat{H}}) =$$

$$= (e^{i\times\hat{H}\times\hat{T}^{>}\times t}) \times A(0) \times (e^{-i\times t\times^{<}\hat{T}\times\hat{H}}), \tag{3.5}$$

with corresponding infinitesimal version

$$\hat{i}\hat{\times}\frac{\hat{d}\hat{A}}{\hat{dt}} = (\hat{A}\hat{;}\hat{H}) = \hat{A} < \hat{H} - \hat{H} > \hat{A} =$$

$$= \hat{A} \times^{<} \hat{T}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \ldots.) \times \hat{H} - \hat{H} \times \hat{T}^{>}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \ldots) \times \hat{A}, \tag{3.6}$$

where there is no time arrow, since Heisenberg's equations are computed at a fixed time.

4) The equivalent *Schrödinger-Santilli genoequations*, can be written as

$$\hat{i}^{>} > \frac{\hat{\partial}^{>}}{\hat{\partial}^{>}\hat{t}^{>}}|\hat{\psi}^{>}>= \hat{H}^{>} > |\hat{\psi}^{>}>=$$

$$= \hat{H}(\hat{r}, \hat{v}) \times \hat{T}^{>}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \hat{\partial}\hat{\psi}\ldots) \times |\hat{\psi}^{>}>= E^{>} > |\psi^{>}>, \tag{3.7}$$

where the time orderings in the second term are ignored for simplicity of notation;

5) The *forward genomomentum*

$$\hat{p}_k^{>} > |\hat{\psi}^{>}>= -\hat{i}^{>} > \hat{\partial}_k^{>}|\hat{\psi}^{>}>= -i \times \hat{I}_k^{>i} \times \partial_i|\hat{\psi}^{>}>, \tag{3.8}$$

6) The *fundamental genocommutation rules*

$$(\hat{r}^i\hat{;}\hat{p}_j) = i \times \delta_j^i \times \hat{I}^{>}, \quad (\hat{r}^i\hat{;}\hat{r}^j) = (\hat{p}_i\hat{;}\hat{p}_j) = 0, \tag{3.9}$$

7) The *genoexpectation values* of an observable for the forward motion $\hat{A}^{>}$

$$\frac{<^{<}\hat{\psi}| > \hat{A}^{>} > |\hat{\psi}^{>}>}{<^{<}\hat{\psi}| > |\hat{\psi}^{>}>} \times \hat{I}^{>} \in \hat{C}^{>}, \tag{3.10}$$

under which the genoexpectation values of the genounit recovers the conventional Planck's unit as in the isotopic case,

$$\frac{<\hat{\psi}| > \hat{I}^{>} > |\hat{\psi}>}{<\hat{\psi}| > |\hat{\psi}>} = I. \tag{3.11}$$

Note that, unlike conventional quantum mechanics, physical quantities are generally *nonconserved*, as it must be the case for the energy,

$$\hat{i}^{>} > \frac{\hat{d}^{>}\hat{H}^{>}}{\hat{d}^{>}\hat{t}^{>}} = \hat{H} \times (^{<}\hat{T} - \hat{T}^{>}) \times \hat{H} \neq 0. \tag{3.12}$$

**Therefore,** *the genotopic branch of hadronic mechanics is the only known operator formulation permitting nonconserved quantities to be Hermitean as a necessary condition to be observability.* **Other formulations attempt to represent nonconservation, e.g., by adding an "imaginary potential" to the Hamiltonian, as it is often done in nuclear physics. In this case the Hamiltonian is non-Hermitean and, consequently, the nonconservation of the energy cannot be an observable. Moreover, since the said "nonconservative models" with non-Hermitean Hamiltonians are nonunitary and are formulated on conventional spaces over conventional fields, they are plagued by all the catastrophic inconsistencies cited earlier. However, we should stress that the representation of irreversibility and nonconservation beginning with the most primitive quantity, the unit and related product.** *Closed irreversible systems* **are characterized by the Lie-isotopic subcase in which**

$$\hat{i}\hat{\times}\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = [\hat{A},\hat{H}] = \hat{A}\times\hat{T}(t,\ldots)\times\hat{H} - \hat{H}\times\hat{T}(t,\ldots)\times\hat{A}, \tag{3.13a}$$

$$^{<}\hat{T}(t,\ldots) = \hat{T}^{>}(t,\ldots) = \hat{T}(t,\ldots) = \hat{T}^{\dagger}(t,\ldots) \neq \hat{T}(-t,\ldots), \tag{3.13b}$$

**for which the Hamiltonian is manifestly conserved. Nevertheless the system is manifestly irreversible. Note also the first and only known observability of the Hamiltonian (due to its iso-Hermiticity) under irreversibility.**

**The above formulation must be completed with three additional Lie-admissible formulations, the backward formulation for matter under time reversal and the two additional isodual formulations for antimatter. For brevity, their study is left to the interested reader.**

### 3.2. Simple Construction of Lie-Admissible Theories

**As it was the case for the isotopies, a simple method for the construction of Lie-admissible (geno-) theories from any given conventional, classical or quantum formulation consists in** *identifying the genounits as the product of two different nonunitary transforms,*

$$\hat{I}^{>} = (^{<}\hat{I})^{\dagger} = U\times W^{\dagger}, \quad ^{<}\hat{I} = W\times U^{\dagger}, \tag{3.15a}$$

$$U\times U^{\dagger}\neq 1, \quad W\times W^{\dagger}\neq 1, \quad U\times W^{\dagger} = \hat{I}^{>}, \tag{3.15b}$$

**and subjecting the totality of quantities and their operations of conventional models to said dual transforms,**

$$I\to\hat{I}^{>} = U\times I\times W^{\dagger}, \quad I\to^{<}\hat{I} = W\times I\times U^{\dagger}, \tag{3.16a}$$

$$a\to\hat{a}^{>} = U\times a\times W^{\dagger} = a\times\hat{I}^{>}, \tag{3.16b}$$

$$a \to^{<} \hat{a} = W \times a \times U^{\dagger} =^{<} \hat{I} \times a, \tag{3.16c}$$

$$a \times b \to \hat{a}^{>} > \hat{b}^{>} = U \times (a \times b) \times W^{>} =$$

$$= (U \times a \times W^{\dagger}) \times (U \times W^{\dagger})^{-1} \times (U \times b \times W^{\dagger}), \tag{3.16d}$$

$$\partial/\partial x \to \hat{\partial}^{>}/\hat{\partial}^{>}\hat{x}^{>} = U \times (\partial/\partial x) \times W^{\dagger} = \hat{I}^{>} \times (\partial/\partial x), \tag{3.16e}$$

$$< \psi| \times |\psi > \to <^{<} \psi| > |\psi^{>} >= U \times (< \psi| \times |\psi >) \times W^{\dagger}, \tag{3.16f}$$

$$H \times |\psi > \to \hat{H}^{>} > |\psi^{>} >=$$

$$= (U \times H \times W^{\dagger}) \times (U \times W^{\dagger})^{-1} \times (U \times \psi > W^{\dagger}), \text{ etc.} \tag{3.16g}$$

**As a result, any given conventional, classical or quantum model can be easily lifted into the genotopic form. Note that the above construction implies that** *all conventional physical quantities acquire a well defined direction of time.* **For instance, the correct genotopic formulation of energy, linear momentum, etc., is given by**

$$\hat{H}^{>} = U \times H \times W^{\dagger}, \ \hat{p}^{>} = U \times p \times W^{>}, \text{ etc.} \tag{3.17}$$

**In fact, under irreversibility, the value of a nonconserved energy at a given time $t$ for motion forward in time is generally different from the corresponding value of the energy for $-t$ for motion backward in past times. This explains the reason for having represented in this section energy, momentum and other quantities with their arrow of time $>$. Such an arrow can indeed be omitted for notational simplicity, but only after the understanding of its existence.**

**Note finally that a conventional, one dimensional, unitary Lie transformation group with Hermitean generator $X$ and parameter $w$ can be transformed into a covering Lie-admissible group via the following nonunitary transform [4]**

$$Q(w) \times Q^{\dagger}(w) = Q^{\dagger}(w) \times Q(w) = I, \ w \in R, \tag{3.18a}$$

$$U \times U^{\dagger} \neq I, \quad W \times W^{\dagger} \neq 1, \tag{3.18b}$$

$$A(w) = Q(w) \times A(0) \times Q^{\dagger}(w) = e^{X \times w \times i} \times A(0) \times e^{-i \times w \times X} \to$$

$$\to U \times (e^{X \times w \times i} \times A(0) \times e^{-i \times w \times X}) \times U^{\dagger} =$$

$$\equiv [U \times (e^{X \times w \times i}) \times W^{\dagger} \times (U \times W^{\dagger})^{-1} \times A \times A(0) \times$$

$$\times U^{\dagger} \times (W \times U^{\dagger})^{-1} \times [W \times (e^{-i \times w \times X}) \times U^{\dagger}] =$$

$$= (e^{i \times X \times X})^{>} > A(0) <^{<} (e^{-1 \times w \times X}) = \hat{U}^{>} > A(0) <^{<} \hat{U}, \tag{3.18c}$$

which equations confirm the property that under the necessary mathematics *the Lie-admissible theory is indeed admitted by the abstract Lie axioms, and it is a realization of the latter being broader than the isotopic form.*

### 3.3. Invariance of Lie-Admissible Theories

It is easy to see that Lie-admissible formulations are not invariant under noniunitary transformations, in which case they verify the Theorems of catastrophic Inconsistencies of Nonunitary. Theores reviewed in Paper I [1]. The crucial invariance permitting the prediction of the same numbers under the same conditions at different times was first achieved by Santilli in Ref. [28] of 1997 and can be reviewed as follows.

Invariance of Lie-admissible formualtions is provided by reformulating any given nonunitary transform in the *genounitary form*

$$U = \hat{U} \times \hat{T}^{>1/2}, W = \hat{W} \times \hat{T}^{>1/2}, \tag{3.19a}$$

$$U \times W^{\dagger} = \hat{U} > \hat{W}^{\dagger} = \hat{W}^{\dagger} > \hat{U} = \hat{I}^{>} = 1/\hat{T}^{>}, \tag{3.19b}$$

and then showing that genounits, genoproducts, genoexponentiation, etc., are indeed invariant under the above genounitary transform in exactly the same way as conventional units, products, exponentiations, etc., are invariant under unitary transforms,

$$\hat{I}^{>} \to \hat{I}^{>'} = \hat{U} > \hat{I}^{>} > \hat{W}^{\dagger} = \hat{I}^{>}, \tag{3.20a}$$

$$\begin{aligned} \hat{A} > \hat{B} \to \hat{U} > (A > B) > \hat{W}^{\dagger} = \\ = (\hat{U} \times \hat{T}^{>} \times A \times T^{>} \times \hat{W}^{\dagger}) \times (\hat{T}^{>} \times W^{\dagger})^{-1} \times \hat{T}^{>} \times \\ \times (\hat{U} \times \hat{T}^{>})^{-1} \times (\hat{U} \times T^{>} \times \hat{A} \times T^{>} \times \hat{W}^{>}) = \\ = \hat{A}' \times (\hat{U} \times \hat{W}^{\dagger})^{-1} \times \hat{B} = \hat{A}' \times \hat{T}^{>} \times B' = \hat{A}' > \hat{B}', \text{ etc.} \end{aligned} \tag{3.20b}$$

from which all remaining invariances follow, thus resolving the catastrophic inconsistencies.

Note that the *numerical invariances of the genounit* $\hat{I}^{>} \to \hat{I}^{>'} \equiv \hat{I}^{>}$, *of the genotopic element* $\hat{T}^{>} \to \hat{T}^{>'} \equiv \hat{T}^{>}$, *and of the genoproduct* $>\to>'\equiv>$ are necessary to have invariant numerical predictions.

### 3.4. Genotopy of Pauli-Santilli Isomatrices

We now proceed to define the genotopy of the Pauli spin matrices and of the Dirac equation, which will be required for constructing genoscattering

theory in Sec. 4. We begin by defining the Pauli-Santilli iso-spin matrices, without spin mutation (see Papers II and III):

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1 \times n_2 \\ n_1 \times n_2 & 0 \end{pmatrix}, \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times n_1 \times n_2 \\ i \times n_1 \times n_2 & 0 \end{pmatrix},$$

$$\hat{\sigma}_3 = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, \hat{T} = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix} \tag{3.21a}$$

where $n_k^{-2} = b_k^2 (k = 1, 2, 3), (b_1^2 \times b_2^2 \times b_3^2) = 1$ for an ellipsoidal deformation of a spherical scattering region; and, with spin mutation:

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1^2 \\ n_2^2 & 0 \end{pmatrix}, \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times n_1^2 \\ i \times n_2^2 & 0 \end{pmatrix}, \hat{\sigma}_3 = \begin{pmatrix} w \times n_1^2 & 0 \\ 0 & w \times n_2^2 \end{pmatrix} \tag{3.21b}$$

Consequently, the isotopies provide five additional quantities [the four ( $b_k, k = 1, ...4$) for spacetime mutation and one ($w$ ) for the spin] for the representation of experimentally measureable features of the scattering region in isoscattering theory, such as shape, deformation, scaling, density, anisotropy, etc . The construction of the genotopies is most conveniently done by subjecting the conventional Pauli's matrices to two different nonunitary transforms. To avoid un-necessary complexity, one may select the following two matrices

$$A = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}, \quad AA^\dagger \neq I, \ ] \ BB^\dagger \neq I, \tag{3.22}$$

where $a$ and $b$ are non-null real numbers, and observe that if $Q, P$ are idempotent (creation and annihilation) matrices, then one may write $A = I + aQ \equiv exp(a \times Q), B = I + bP \equiv exp(b \times P)$. Accordingly, one has the following forward and backward genounits and related genotopic elements

$$I^> = AB^\dagger = \begin{pmatrix} 1 & b \\ a & 1 \end{pmatrix}, \quad T^> = \frac{1}{(1-ab)} \begin{pmatrix} 1 & -b \\ -a & 1 \end{pmatrix}, \tag{3.23a}$$

$${}^<I = BA^\dagger = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}, \quad {}^<T = \frac{1}{(1-ab} \begin{pmatrix} 1 & -a \\ -b & 1 \end{pmatrix}, \tag{3.23b}$$

The *forward and backward Pauli-Santilli genomatrices* are then given respectively by

$$\sigma_1^> = A\sigma_1 B^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & (a+b) \end{pmatrix}, \quad \sigma_2^> = A\sigma_2 B^\dagger = \begin{pmatrix} 0 & -i \\ i & (a+b) \end{pmatrix}, \tag{3.24a}$$

$$\sigma_3^> = A\sigma_3 B^\dagger = \begin{pmatrix} 1 & b \\ a & -1 \end{pmatrix}, \quad {}^<\sigma_1 = B\sigma_1 A^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & (a+b) \end{pmatrix},, \tag{3.24b}$$

$${}^<\sigma_2 = B\sigma_2 A^\dagger = \begin{pmatrix} 0 & -i \\ i & (a+b) \end{pmatrix}, \quad {}^<\sigma_3 = A\sigma_3 B^\dagger = \begin{pmatrix} 1 & a \\ b & -1 \end{pmatrix}, \tag{3.24c}$$

**in which the direction of time is embedded in the structure of the matrices.**

**It is an instructive exercise for any interested reader to verify that** *conventional commutation rules and eigenvalues of Pauli's matrices are preserved under forward and backward genotopies,*

$$\sigma_i^> > \sigma_j^> - \sigma_j^> > \sigma_i^> = 2i\epsilon_{ijk}\sigma_k^> \tag{3.25a}$$

$$\sigma_3^> > |> = \pm 1|>, \quad \sigma^{>2>} > |> = 2(2+1)|>, \tag{3.25b}$$

$${}^<\sigma_i >^< \sigma_j - {}^<\sigma_j >^< \sigma_i = 2i\epsilon_{ijk}^< \sigma_k \tag{3.25c}$$

$$< | <^< \sigma_3 =< | \pm 1, \ ; < | <^< \sigma^{2>} =< |(2(2+1). \tag{3.25d}$$

**We can, therefore, conclude by stating that** *Pauli's matrices can indeed be lifted in such an irreversible form to represent the direction of time in their very structure.*

### 3.5. Genotopy of Dirac-Santilli isoequation

**The Dirac-Santilli isomatrices ($\hat{\gamma}_\mu$) are defined as follows (see Papers II and III):**

$$\hat{\gamma}_k = b_k \times \begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \hat{\gamma}_4 = i \times b_k \times \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix} \tag{3.26a}$$

$$[\hat{\gamma}_\mu, \hat{\gamma}_\nu] \equiv \hat{\gamma}_\mu \times T \times \hat{\gamma}_\nu + \hat{\gamma}_\nu \times T \times \hat{\gamma}_\mu = 2 \times \hat{\eta}_{\mu\nu}.$$

**To construct the simplest possible genotopy of Dirac's equation via the genotopies of the Pauli-spin matrices and space-time structure, we shall use Dirac's equation in its isodual re-interpretation representing a direct product of one electron and one positron, the latter without any need of second quantization. We note, however, that the latter re-interpretation requires the use of the** *isodual transform* $A \to A^d = -A^\dagger$) **as being distinct from Hermitean conjugation. Under this clarifications, the** *forward Dirac genoequation* **can be written**

$$(\eta^{>\mu\nu}\gamma_\mu^> T^> p_\nu^> - im)T^> |\psi^> >= 0 \tag{3.27a}$$

$$p_\nu^> T^> |\psi^> >= -i\frac{\partial^>}{\partial^> x^{>\nu}}|\psi^> >= -iI^> \frac{\partial}{\partial x^>}|\psi^> >, \tag{3.27b}$$

with *forward genogamma matrices*

$$\gamma_4^> = \begin{pmatrix} A & 0 \\ 0 & B^d \end{pmatrix} \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix} \begin{pmatrix} A^d & 0 \\ 0 & B \end{pmatrix} = \begin{pmatrix} AA^d & 0 \\ 0 & -B^dB \end{pmatrix} \tag{3.28a}$$

$$\gamma_k^> = \begin{pmatrix} A & 0 \\ 0 & B^d \end{pmatrix} \begin{pmatrix} 0 & \sigma_k \\ \sigma_k^d & 0 \end{pmatrix} \begin{pmatrix} A^d & 0 \\ 0 & B \end{pmatrix} = \tag{3.28b}$$

$$= \begin{pmatrix} 0 & A\sigma_k B^\dagger \\ B\sigma_k^d A^d & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma_k \\ \sigma_k^d & 0 \end{pmatrix} = \begin{pmatrix} 0 & \sigma_k^> \\ <\sigma_k^d & 0 \end{pmatrix}. \tag{3.28c}$$

$$\{\gamma_\mu^>, \gamma_\nu^>\} = \gamma_\mu^> T^> \gamma_\nu^> + \gamma_\nu^> T^> \gamma_\mu^> = 2\eta_{\mu\nu}^>, \tag{3.28d}$$

where $\eta_{\mu\nu}^>$ is given by the same genotopy of Eqs. (3.26a). The backward genoequation may be constructed similarly.

For additional insight into the mathematical structure of the genotopy of the spin matrices, it should be noted, at this juncture, that in terms of the larger group O(4,2), considered as a set of linear transformations in a six-dimensional linear vector space which leave the quadratic form, $g_{IJ}X^IX^J$ with $I, J = 1, 2, 3, 4, 0 = 5, 6; g_{IJ} = (- - - - + +)$ invariant, it is well known that the O(4,2) group generators can be explicitly written in terms of creation and annihilation operators of a spin-$\frac{1}{2}$ field as follows:

$$L_{ij} = L_k = \frac{1}{2}[a^+\sigma_k a + b^+\sigma_k b]; L_{44} = A_i = -\frac{1}{2}[a^+\sigma_i a - b^+\sigma_i b];$$

$$L_{i5} = M_i = -\frac{1}{2}[a^+\sigma_i C b^+ - aC\sigma_i b]; L_{i6} = \Gamma_i = -\frac{i}{2}[a^+ C b^+ - aCb];$$

$$L_{45} = T = -\frac{i}{2}[a^+ C b^+ - aCb]; L_{46} = S = \frac{1}{2}[a^+ C b^+ + aCb];$$

$$L_{56} = \Gamma_5 = \frac{1}{2}[a^+ a + b^+ b + 2] \tag{3.29}$$

where $i, j, k = 1, 2, 3$

$$a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}; b = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix};$$

and $C = i\sigma_2$, where the matrices $\sigma_1, \sigma_2, \sigma_3$ are the Pauli spin matrices. The group operators given in Eq.(2.29) are readily shown to satisfy the commutation relation for the 15 generators of the group considered as an antisymmetric tensor $L_{IJ} = -L_{JI}$

$$[L_{IJ}, L_{KL}] = -i[g_{IK}L_{JL} - g_{JL}L_{IK} - g_{JK}L_{IL} - g_{IL}L_{JK}] \tag{3.30}$$

One can then see in this way the implicvations of the transition from the conventional to the genotopic scattering theory.

## 4. LIE-ADMISSIBLE INVARIANT GENOSCATTERING THEORY

### 4.1. The Fundamental Lie-Admissible Scattering Matrix

As it is well known, the conventional (relativistic scattering theory is based on the *scattering matrix* $S$ for the connection of initial $(i)$ and final $(f)$states

$$S = (S_{if}), \tag{4.1}$$

whose central feature is that of being *unitary* on the base Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$,

$$S \times S^\dagger = S^\dagger \times S = I. \tag{4.2}$$

The latter feature then assures the verification of *causality,* the predictions of the same numerical values under the same conditions at different times (*invariance*), and the remaining axiomatic features of relativistic quantum mechanics, including the characterization of symmetries via the fundamental *Lie theory.*

Despite historical advances, the above conception of the scattering matrix has the same insufficiencies as those of the underlying disciplines, namely, the theory is based on the necessary reduction of scattering processes to dimensionless points. As indicated in the preceding Papers I-IV [1], this abstraction of reality is effective for a number of scattering events, such as Coulomb scattering without collisions, but it is manifestly insufficient for the characterization of high energy scattering processes, e.g., because of the inability to characterize the hyperdense scattering region and the consequential expected non-hamiltonian internal effects.

To initiate the process toward a more accurate description of high energy scattering processes, in Paper IV [1] we have reviewed and expanded the notion of the *isoscattering matrix*

$$\hat{S} = (\hat{S}_{if}), \tag{4.3}$$

whose primary feature is that of being *nonunitary* when formulated on $\mathcal{H}$ over $\mathcal{C}$,

$$\hat{S} \times \hat{S}^\dagger \neq I, \tag{4.4}$$

but of being *unitary,* namely, of verifying the conditions of unitarity on the Hilbert-Myung-Santilli isospace $\hat{\mathcal{H}}$ over Santilli isofield $\hat{\mathcal{C}}$

$$\hat{S}\hat{\times}\hat{S}^\dagger = \hat{S} \times \hat{T} \times \hat{S}^\dagger = \hat{S}^\dagger\hat{\times}\hat{S} = \hat{S}^\dagger \times \hat{T} \times \hat{S} \equiv \hat{I}1/\hat{T} > 0, \tag{4.5}$$

which property resolves the historical inconsistencies of nonunitary scattering theories by achieving full causality and invariance when properly elaborated, that is, treated over $\hat{\mathcal{H}}$ over $\hat{C}$, by allowing the use of the axiomatically consistent formulations of the isotopic branch of hadronic mechanics, including the fundamental use of the *Lie-Santilli isotheory* for all symmetry needs.

The main advantage of the transition from the scattering to the isoscattering theory is that of achieving a more realistic representation of the scattering region at distances of 1 $fm$, while recovering the conventional scattering matrix at bigger distances. This is possible because the isounit $\hat{I}$ and, therefore, the isoscattering matrix $\hat{S}$, have a completely unrestricted functional dependence on all needed variables and quantities, including local coordinates $x$, momenta $p$, density $d$, wavefunctions $\psi$, their derivatives $\partial\psi$, etc.

$$\hat{S} = \hat{S}(x, p, d, \psi, \partial\psi, ...). \tag{4.6a}$$

$$Lim_{r\ >\ 1\ fm} \hat{S} \equiv S. \tag{4.6b}$$

The above feature permits, for the first time in the field, a quantitative, causal and invariant representation of nonlinear, nonlocal and nonpotential effects that are inevitable in the scattering of particles at high energy.

Despite the above advances, the isoscattering theory has "no arrow of time" in its axioms and technical realization, thus being solely applicable to *high energy reversible scattering processes requiring a nonunitary-isounitary structure* as indicated in Section 1.

The latter insufficiency has mandated the studies presented in this paper, that are centered in the notion of *forward and backward genoscattering matrices*

$$\hat{S}^{>} = (\hat{S}^{>}_{if}), \tag{4.7a}$$

$$^{<}\hat{S} = (^{<}\hat{S}_{if}), \tag{4.7b}$$

whose primary feature is that of being *nonunitary* as well as *non-isounitary,* yet being *genounitary,* namely, verifying the conditions of unitarity on the forward and backward genospaces over genofields, respectively,

$$\hat{S}^{>}\hat{\times}\hat{S}^{>\dagger} = \hat{S}^{>} \times \hat{T}^{>} \times \hat{S}^{>\dagger} =$$

$$= \hat{S}^{>\dagger}\hat{\times}^{>}\hat{S}^{>} = \hat{S}^{>\dagger} \times \hat{T}^{>} \times \hat{S}^{>} \equiv \hat{I}^{>} 1/\hat{T}^{>} > 0, \tag{4.8a}$$

$$^{<}\hat{S}^{<}\hat{\times}^{<\dagger}\hat{S} =^{<} \hat{S} \times^{<} \hat{T} \times^{<\dagger} \hat{S} =^{<\dagger} \hat{S}^{<}\hat{\times}^{<}\hat{S} =$$

$$=^{<} \hat{S}^{\dagger} \times^{<} \hat{T} \times^{<} \hat{S} \equiv {}^{<}\hat{I} 1/^{<}\hat{T} > 0, \tag{4.8b}$$

which can be constructed from the isoscattering or conventional scattering theory via the rule (1.10) formulated on genospaces over genofields under the crucial condition of preserving limit (4.6b)

$$ZZ^{\dagger} \neq I \quad WW^{\dagger} \neq I, \quad ZW^{\dagger} \neq I, \tag{1.9a}$$

$$\hat{S} \to \hat{S}^{>} = Z \times Q \times W^{\dagger}, \tag{4.9b}$$

$$Lim_{r>1\ fm} \hat{S}^{>} \equiv S. \tag{4.9c}$$

$$\hat{S} \to^{<} \hat{S} = W\hat{S}Z^{\dagger}. \tag{4.9d}$$

$$Lim^{<}_{r>1\ fm} \hat{S} \equiv S. \tag{4.9e}$$

Under the latter conditions the forward and backward genoscattering matrices recover, individually, unitarity, invariance and the other features of the genotopic branch of hadronic mechanics, including its fundamental treatment via *Santilli Lie-admissible theory.*

Note that *the forward and backward genoscattering matrices are individually causal,* because they are necessary for the consistency of the theory as well as for the treatment of antimatter requiring a backward treatment. Nevertheless, *the Lie-admissible scattering theory is indeed irreversible over time because of the strict inequivalence between the forward and backward scattering matrix.*

We shall now pass to a more detailed the formulation of the Lie-admissible scattering theory, by restricting our attention for simplicity to the forward scattering matrix.

4.2. Genotopy of the Isoscattering Formalism

As is well known[2], the usual Feynman propagator in conventional QED of spin-$\frac{1}{2}$ particles can be characterized as follows in the O(3,1) carrier space of a relativistic quantum mechanics:

$$S_F(x) = (\gamma^{\mu} p_{\mu} + im)\Delta_F(x), \Delta_F(x) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ipx}}{p^2 - m^2 + i\epsilon} \tag{4.10}$$

with corresponding expression in momentum 4-vector space:

$$S_F(p) = (\gamma^{\mu} p_{\mu} + im)\Delta_F(p), \Delta_F(p) = \frac{\gamma^{\mu} p_{\mu} + im}{p^2 - m^2 + i\epsilon} \tag{4.11}$$

In terms of the "isounit" $(\hat{I})$and isotopic element $(\hat{T} = \hat{I}^{-1})$defined in Sec.2.1, and represented as $\hat{I}_{st}$ and $T_{st}$,the generalized Feynman (which may be called iso-Feynman) propagator in the $\hat{O}(3,1)$ carrier space of hadronic mechanics is given by the corresponding expressions as follows

$$\hat{S}_F(\hat{x}) = (\hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}^{\mu} \times \hat{p}_{\mu} + i \times \hat{m}) \times \hat{T}_{st} \times \hat{\Delta}_F(\hat{x}),$$

$$\hat{\Delta}_F(\hat{x}) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip\times\hat{T}_{st}\times x}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i\times\hat{\epsilon}} \tag{4.12}$$

**with corresponding expression in iso-momentum 4-vector space**

$$\hat{S}_F(\hat{p}) = (\hat{\eta}_{st}^{\mu\nu} \times \hat{\gamma}^{\mu} \times \hat{p}_{\mu} + i \times \hat{m}) \times \hat{T}_{st} \times \hat{\Delta}_F(\hat{p}),$$

$$\hat{\Delta}_F(\hat{p} = \frac{(\hat{\eta}_{st}^{\mu\nu} \times \hat{\gamma}^{\mu} \times \hat{p}_{\mu} + i \times \hat{m}) \times \hat{T}_{st}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i \times \hat{\epsilon}} \tag{4.13}$$

**In terms of the "genounits" $(\hat{I}^{>}, {}^{<}\hat{I})$ genotopic elements $(\hat{T}^{>} = 1/\hat{I}^{>}$, ${}^{<}\hat{T} = 1/{}^{<}\hat{I})$ defined in Sec. 3.4 and represented as $(\hat{I}^{>}_{st}, {}^{<}\hat{I}_{st})$ and $(T^{>}_{st}\ {}^{<}T_{st})$, the generalized Feynman (which may be called geno-Feynman) propagator in the $(\hat{O}^{>}(3,1)$, ${}^{<}\hat{O}(3,1)$ carrier genospaces of the Lie-admissible branch of hadronic mechanics is given by the corresponding expressions as follows**

$$\hat{S}^{>}_F(\hat{x}) = ((\hat{\eta}^{>}_{st})^{\mu\nu} \times \hat{\gamma}^{>}_{\mu} \times \hat{p}_{\mu} + i \times \hat{m}) \times \hat{T}^{>}_{st} \times \hat{\Delta}^{>}_F(\hat{x}),$$

$$\hat{\Delta}^{>}_F(\hat{x}) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip\times\hat{T}^{>}_{st}\times x}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i \times \hat{\epsilon}} \tag{4.14a}$$

$${}^{<}\hat{S}_F(\hat{x}) = ({}^{<}\hat{\eta}_{st}^{\mu\nu} \times^{<} \hat{\gamma}_{\mu} \times \hat{p}_{\mu} + i \times \hat{m}) \times^{<} \hat{T}_{st} \times^{<} \hat{\Delta}_F(\hat{x}),$$

$${}^{<}\hat{\Delta}_F(\hat{x}) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip\times^{<}\hat{T}_{st}\times x}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i \times \hat{\epsilon}} \tag{4.14b}$$

**with corresponding expression in geno-momentum 4-vector space**

$$\hat{S}^{>}_F(\hat{p}) = ((\hat{\eta}^{>}_{st})^{\mu\nu} \times \hat{\gamma}^{>}_{\mu} \times \hat{p}_{\mu} + i \times \hat{m}) \times \hat{T}^{>}_{st} \times \hat{\Delta}^{>}_F(\hat{p}),$$

$$\hat{\Delta}^{>}_F(\hat{p}^2) = \frac{((\hat{\eta}^{>}_{st})^{\mu\nu} \times \hat{\gamma}^{>}_{\mu} \times \hat{p}_{\mu} + i \times \hat{m}) \times \hat{T}^{>}_{st}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i \times \hat{\epsilon}} \tag{4.14c}$$

$${}^{<}\hat{S}_F(\hat{p}) = ({}^{<}\hat{\eta}_{st}^{\mu\nu} \times^{<} \hat{\gamma}_{\mu} \times \hat{p}_{\mu} + i \times \hat{m}) \times^{<} \hat{T}_{st} \times^{<} \hat{\Delta}_F(\hat{p}),$$

$${}^{<}\hat{\Delta}_F(\hat{p}^2) = \frac{({}^{<}\hat{\eta}_{st}^{\mu\nu} \times^{<} \hat{\gamma}^{\mu} \times \hat{p}_{\mu} + i \times \hat{m}) \times^{<} \hat{T}_{st}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i \times \hat{\epsilon}} \tag{4.14d}$$

**In the presence of an external electromagnetic field, the solution of the (regular) Dirac-Santilli isoequation takes the form**

$$\hat{\Psi} = \hat{\psi}(\hat{x}) + \hat{e}\hat{\times}\hat{\int}\hat{d}^4\hat{x}'\hat{\times}\hat{S}_f(\hat{x} - \hat{x}')\hat{\times}\hat{\gamma}.\hat{A}(\hat{x}')\hat{\times}\Psi(\hat{x}$$

$$= \hat{\psi}(\hat{x}) + \hat{e}\hat{\times}\hat{\int}\hat{d}^4\hat{x}'\hat{\times}\hat{S}_f(\hat{x}-\hat{x}')\hat{\times}\hat{\gamma}\hat{\cdot}\hat{A}(\hat{x}')\hat{\times}\hat{\psi}(\hat{x})$$

$$+\hat{e}^{\hat{2}}\hat{\times}\hat{\int}\hat{d}^4\hat{x}'\hat{\int}\hat{d}^4\hat{x}"\hat{\times}\hat{S}_f(\hat{x}-\hat{x}')\hat{\times}\hat{\gamma}\hat{\cdot}\hat{A}(\hat{x}')\hat{\times}\hat{S}_f(\hat{x}'-\hat{x}")\hat{\times}\hat{\gamma}\hat{\cdot}\hat{A}(\hat{x}")\hat{\times}\hat{\psi}(\hat{x}")+... \quad (4.15)$$

**This leads to a formal definition of the iso-Feynman propagator either as a series**

$$\hat{S'}_f(\hat{x},\hat{x}') = \hat{S}_f(\hat{x}-\hat{x}') + \hat{e}\hat{\times}\hat{\int}\hat{d}^4\hat{x}"\hat{\times}\hat{S}_f(\hat{x}-\hat{x}")\hat{\times}\hat{\gamma}\hat{\cdot}\hat{A}(\hat{x}")\hat{\times}\hat{S}_f(\hat{x}'-\hat{x}")+... \quad (4.16)$$

**or as an integral equation**

$$\hat{S'}_f(\hat{x},\hat{x}') = \hat{S}_f(\hat{x}-\hat{x}') + \hat{e}\hat{\times}\hat{\int}\hat{d}^4\hat{x}"\hat{\times}\hat{S}_f(\hat{x}-\hat{x}")\hat{\times}\hat{\gamma}\hat{\cdot}\hat{A}(\hat{x}")\hat{\times}\hat{S'}_f(\hat{x}'\hat{.}\hat{x}") \quad (4.17)$$

**where $\hat{\gamma}\hat{\cdot}\hat{A}(\hat{x}') \equiv \hat{\eta}_{st}^{\mu\nu} \times \hat{\gamma}_\mu \times \hat{A}_\nu(\hat{x}')$ and $\hat{A}_\nu(\hat{x}')$ is the iso-electromagnetic four-vector potential given by the corresponding iso-gauge principle[xx]. This leads, in turn, to a formal definition of geno-Feynman propagators either as series:**

$$\hat{S'}^{>}_f(\hat{x},\hat{x}') = \hat{S}^{>}_f(\hat{x}-\hat{x}') + \hat{e} > \hat{\int}\hat{d}^4\hat{x}" > \hat{S}^{>}_f(\hat{x}-\hat{x}") > \hat{\gamma}^{>}\hat{\cdot}\hat{A}^{>}(\hat{x}") >$$

$$\hat{S}^{>}_f(\hat{x}'-\hat{x}")+...$$

$$^{<}\hat{S'}_f(\hat{x},\hat{x}') =^{<} \hat{S}_f(\hat{x}-\hat{x}') + \hat{e} < \hat{\int}\hat{d}^4\hat{x}" <^{<} \hat{S}_f(\hat{x}-\hat{x}") <^{<} \hat{\gamma}\hat{\cdot}^{<}\hat{A}(\hat{x}") <$$

$$^{<}\hat{S}_f(\hat{x}'-\hat{x}")+... \quad (4.18)$$

**or as an integral equation**

$$\hat{S'}^{>}_f(\hat{x},\hat{x}') = \hat{S}^{>}_f(\hat{x}-\hat{x}') + \hat{e} > \hat{\int}\hat{d}^4\hat{x}" > \hat{S}^{>}_f(\hat{x}-\hat{x}") >$$

$$\hat{\gamma}^{>}\hat{\cdot}\hat{A}^{>}(\hat{x}") > \hat{S'}^{>}_f(\hat{x}'\hat{.}\hat{x}")$$

$$^{<}\hat{S'}_f(\hat{x},\hat{x}') =^{<} \hat{S}_f(\hat{x}-\hat{x}') + \hat{e} < \hat{\int}\hat{d}^4\hat{x}" < \hat{S}^{>}_f(\hat{x}-\hat{x}") <$$

$$^{<}\hat{\gamma}\hat{\cdot}^{<}\hat{A}(\hat{x}") <^{<} \hat{S'}_f(\hat{x}'\hat{.}\hat{x}") \quad (4.19)$$

**where $\hat{\gamma}^{>}\hat{\cdot}\hat{A}^{>}(\hat{x}') \equiv (\hat{\eta}^{>}_{st})^{\mu\nu} \times \hat{\gamma}^{>}_\mu \times \hat{A}^{>}_\nu(\hat{x}')$ and $\hat{A}^{>}_\nu(\hat{x}')$, and similarly for $^{<}\hat{\gamma}\hat{\cdot}^{<}\hat{A}(\hat{x}")$**

Note that, in the limit of unitary transformation, we recover exactly the conventional expressions. For this reason, the primary interest of isoscattering and genoscattering theories lies in the formal relationship/differentiation of the isoscattering and genoscattering profiles (1.1) and (1.2) for interpreting the existing and future scattering experimental data. As our interest is to elaborate the basic physical concepts in terms of Feynman diagrams for electron scattering with an electromagnetic field and electroweak neutron decay, as well as remove divergences from the theory, it is useful to characterize the differences by noting that, in 1st quantization scheme, the generalized S-matrix for isoscattering theory is given by

$$\hat{S}_{f,i} = lim_{t\to \inf} \hat{\int} \hat{d}^3\hat{x} \hat{\times} \hat{\psi}^{+\hat{s}'}_{\hat{p}} \hat{\times} \hat{\Psi}^{\hat{s}}_{\hat{p}} \tag{4.20}$$

where $\hat{\Psi}^{\hat{s}}_{\hat{p}}$ is the exact solution given by

$$\hat{\Psi}^{\hat{s}}_{\hat{p}}(\hat{x}) = \hat{\psi}^{\hat{s}}_{\hat{p}}(\hat{x}) + \hat{e}\hat{\times} \hat{\int} \hat{d}^4\hat{x}' \hat{\times} \hat{S}_f(\hat{x} - \hat{x}') \hat{\times} \hat{\gamma} \hat{.} \hat{A}(\hat{x}') \times \hat{\Psi}^{\hat{s}}_{\hat{p}}(\hat{x}') \tag{4.21}$$

with the normalization

$$\hat{\int} \hat{d}^3\hat{x} \hat{\times} \hat{\psi}^{+\hat{s}}_{\hat{p}}(\hat{x}) \hat{\times} \hat{\psi}^{+\hat{s}'}_{\hat{p}'}(\hat{x}) = \hat{\delta}_{\hat{s}\hat{s}'} \hat{\times} \hat{\delta}^3(\hat{p} - \hat{p}') \tag{4.22}$$

and similarly for the geno case. Consequently, as indicated in Fig. 2, the correspondence principle in 1st quantization scheme involves a lifting of the Coulomb vertex in QED into the approximate Yukawa vertex in hadronic mechanics. In 2nd quantization scheme, one has additionally the lifting from Bose-Einstein to Fermi-Dirac statistics, i.e., mutation of spin under sufficiently high energies and further differences indicated in Sec 3.4 above.

The correspondence between Feynman graphs/rules and their isotopic images for computation of contributions to the S-matrix in QED of spin-$\frac{1}{2}$ particles have been summarized in table 2 of Ref.[1, paper IV] and need not be repeated here for brevity.

4.3. $\hat{O}^{>}(4,2) \times \hat{SU}^{>}(3) \times \hat{U}^{>}(1)$ Dynamical Symmetries of the Scattering Region.

We now turn to a more detailed specification of the structure of the scattering region. While the isotopy of Dirac matrices characterizes the lifting of the Lorentz group $O(3,1) \to \hat{O}(3,1)$ , in terms of five additional quantities, namely the four $b_k (k = 1, ..., 4)$ for spacetime mutation and one ($w$ )

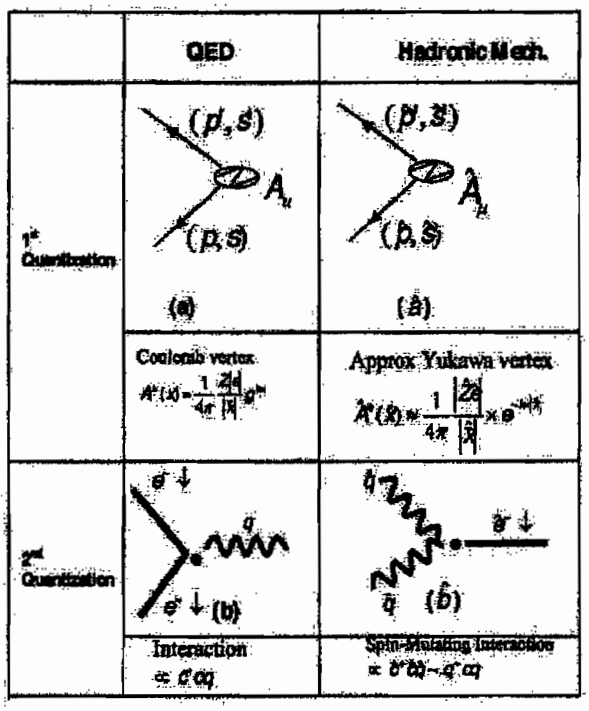



Figure 4: *Expected Modification of QED in HM*

for the spin, in regard to analyzing experimentally measureable features of the scattering region, such as its shape, deformation, anisotropy, etc, we expect the genotopy to characterize, in addition, time-irreversibility and the breaking of the associated discrete symmetry of parity by electroweak forces responsible for neutron decay in the standard (V-A, i.e. vector-axialvector) current-current interaction model.

We wish the genotopy to accommodate this feature in such a way that correlated pairs of spin-$\frac{1}{2}$ particles,$e^-, \nu$ and $e^-, a^0$ , can be subsumed and long-range 1/r-potential between pairs of particles $(p, e^-)$ eliminated simultaneously in the representation of the conventional O(4,2) dynamical symmetry group, in terms of the most general conserved current in the O(4,2) algebra of Dirac matrices[10] which includes not only (parity-conserving) vector current but also axial-vector (parity non-conserving) current as well as certain "convective" currents proportional to the total momentum of the particle-antiparticle system. The most important distinctive features of the scattering region for the three profiles of $e^- - p$ (Coulomb) action-at-a-distance between point-particles, penetration of point-like particle into an extended wave-packet, and overlap of two extended wavepackets as well as current-current interaction processes indicated in Fig. 3 are realized as indicated in Fig. 4 in terms of the progressive lifting of the larger dynamical group:

$$O(4,2) \to \hat{O}(4,2) \to \left\{ \begin{array}{c} \hat{O}^{>}(4,2) \\ . \\ {}^{>}\hat{O}(4,2) \end{array} \right\} \tag{4.23}$$

To elaborate the characteristics of the isoscattering region in Figs. 3 and 4, let us examine the generalized iso-current given by the isotopic lifting of the O(3,1) (Dirac) vector current into $\hat{O}(4,2)$ vector and convective currents:

$$J_\mu \equiv \bar{\psi}\gamma_\mu\psi \to \hat{\bar{\psi}} \times \hat{T} \times (\hat{\gamma}_\mu - i \times \kappa_0 \times \hat{\bar{\partial}}_\mu) \times \hat{T} \times \hat{\psi} = \hat{J}_\mu \tag{4.24}$$

The generalized wave equation that conserves $\hat{J}_\mu$ is given by the iso-Lagrangian density

$$\hat{L} = -\frac{1}{2}\bar{\psi}(\hat{x}) \times \hat{T} \times (-i \times \hat{\gamma}^\mu \times \hat{\bar{\partial}}_\mu + \kappa_1) \times \hat{T} \times \hat{\psi}(\hat{x}) -$$

$$\hat{\bar{\psi}}(\hat{x}) \times \hat{T} \times \kappa_0 \times \hat{\bar{\partial}}^\mu \hat{\bar{\partial}}^\mu \times \hat{T} \times \hat{\psi}(\hat{x}) \tag{4.25}$$

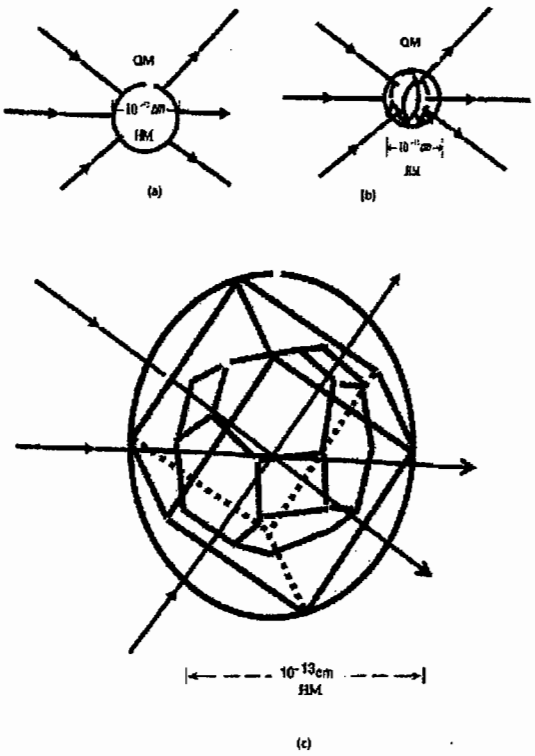



Figure 5: *A schematic view of the main assumption of (a) Lie-isotopic isoscattering theory[1] and its generalizations (b) and (c) in Lie-admissible genoscattering theory, namely, the exact validity of quantum mechanics (QM) everywhere in exterior conditions, and the validity of hadronic mechanics (HM) for the interior conditions of the scattering region of generally arbitrary shape, but represented as (a) spherical "extended" particle with radius of about* $10^{-13}cm$ *in* $\hat{O}(3,1)$ *and (b) as a deformed sphere in* $\hat{O}(4,2)$ *isoscattering theories, and as (c) overlap/penetration of a cube and its (dual) polyhedron (Wigner-Seitz unit cell) defined by the plane-coordinates representing fermion current-current interaction in genoscattering theory.*

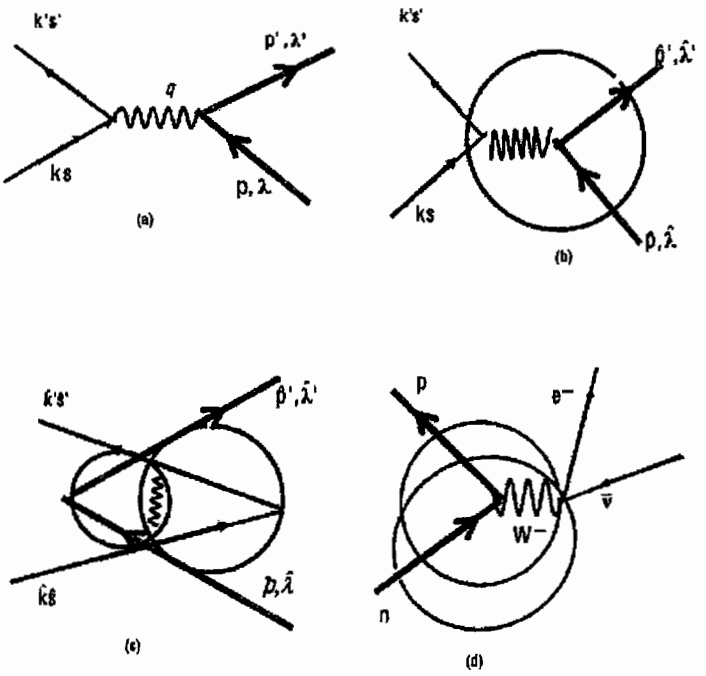



Figure 6: *Feynman diagrams for (a) conventional $e^- - p$ long-range Coulomb interaction via "virtual" photon exchange; (b) point-electron contact/penetration into an extended proton wave packet,$e^- + p \rightarrow n + \nu$ which implies $n \sim (pe^-\bar{\nu})_{QM} \rightarrow (pe^-a^0)_{QM}$; (c) mutual overlap of extended electron and extended proton wave packets, $e^- + p \rightarrow n + \nu + \bar{\nu}$ which implies $n \equiv (\hat{e}^-, \hat{p})_{HM}$ involving mutation of spin and in the similar scattering process $e^+ + e^- \rightarrow \pi^0 \rightarrow e^+ + e^-$ which implies $\pi^0 \equiv (\hat{e}^-, \hat{e}^+)_{HM}$;* and (d) electroweak decay of the neutron, $n \rightarrow p + W^- \rightarrow p + e^- + \bar{\nu}$ usually characterized by vector and axial-vector currents in unified strong and electroweak current-current interaction models.

as (cf Eq.(3.2) of Barut, Cordero and Ghirardi[10])

$$(i \times \hat{\gamma}^{\mu} \times \hat{\vec{\partial}}_{\mu} + \kappa_0 \times \hat{\vec{\partial}}^{\mu}\hat{\vec{\partial}}_{\mu} - \kappa_1) \times T \times \hat{\psi}(\hat{x}) = 0 \tag{4.26}$$

where $\kappa_0, \kappa_1$ are constants. It is of interest to note that the last term in Eq.(4.10) (due to convective currents) gives rise to the Pauli magnetic transitions, inasmuch as for any Dirac spinor $\psi$, it is easy to establish from the relation, $(\partial^{\mu}\bar{\psi})(\partial^{\mu}\psi) = (\partial^{\mu}\bar{\psi})\gamma^{\mu}\gamma^{\nu}(\partial^{\nu}\psi)$, a connection with the intrinsic Pauli-moment coupling which is tantamount to inclusion of a non-potential term,$-i\partial^{\mu}(\bar{\psi}\sigma_{\mu\nu}\partial^{\nu})\psi$ , in the free Dirac Lagrangian density[11]. Thus even the ($\hat{T} \to 1$ ) limit of $\hat{O}(4,2)$ corresponding to the conventional $O(4,2)$ provides a simple non-trivial profile of neutron production in ($e^{-}, p$ ) scattering, summarized in Fig.5(Table 1), as follows.

If one puts the leptons into a triplet $l = (\nu, e^{-}, \mu^{-})$ or $(a^0, e^{-}, \mu^{-})$ with integral lepton number $L = Diag(1,1,1)$ and charges $Q_L = Diag(0,-1,-1)$ (in units of the proton charge) and compares with the quark tripet $(u,d,s)$ with fractional baryon number $B = \frac{1}{3}Diag(1,1,1)$ and fractional electric charges $Q_B = \frac{1}{3}Diag(2,-1,-1)$, then one finds[12] that $L + Q_L = B + Q_B = Diag(1,0,0) \equiv F + Q_F = P$ is idempotent (i.e.,$P^2 = P$) and, therefore[13], that quarks could be obtained from the leptons by shifting 2/3 of the lepton number to the leptonic electric charge, i.e., $B \equiv L - \frac{2}{3}L, Q_B \equiv Q_L + \frac{2}{3}L$. This motivated Barut's[13] model of the neutron, and subsequently, its variant as Santilli's [14] "etherino" model.

Indeed, according to Figure 5(table 1), if in the scattering process, $e^{-} + p \to n + \nu$ , the proton is treated as pointlike particle described by the conventional Dirac equation with $O(3,1)$ symmetry, then the electron with an associated massless neutrino may be described by the simplest (scale-invariant[15]) equation with $O(4,2)$ dynamical symmetry,

$$(i\gamma_{\mu}\partial_{\mu} - m_e^{-1}\partial_{\mu}\partial^{\mu})\psi_e = 0 \tag{4.27}$$

whose mass equation has two roots,$m = 0, m_e$, and therefore leads to Barut's model[13] of neutron production, $n \sim (pe^{-}\bar{\nu})_{QM}$ (which is not compatible with negative binding energy). Alternatively, if one adopts Santilli's "etherino hypothesis"[14] (for compatibility with neutron decay and negative binding energy for $n = (pe^{-}a^0)_{QM}$ ), the electron with an associated massive "etherino" may be described by the more general equation

$$[i\gamma_{\mu}\partial^{\mu} - 3m_e - (2m_e)^{-1}\partial_{\mu}\partial^{\mu}]\psi = 0, \tag{4.28}$$

whose mass equation and its two non-zero roots are respectively given by:

$$m^2 + 2m_e m - 6m_e^2 = 0, \tag{4.29}$$

| Table 1: O(4,2) profile of $e^- - p$ scattering & neutron production/decay | | | Wave Equations | Symm-etry Group |
|---|---|---|---|---|
| $e^- + p$ $\rightarrow n + \nu \Rightarrow$ | $n \sim (p e^- \nu_e)_{QM}$ (Barut's model[13]) | $p$ | $(i\gamma_\mu \partial_\mu - m_p)\psi_p = 0$ | O(3,1) |
| | | $(e^-, \nu_e)$ | $(i\gamma_\mu \partial_\mu - m_e^{-1}\partial_\mu \partial^\mu)\psi_e = 0$ | O(4,2) |
| $e^- + p + a^0$ $\rightarrow n + \nu \rightarrow$ $e^- + p + \bar{\nu} \Rightarrow$ | $n = (p, e^- a^0)_{QM}$ (Santilli's "etherino" model [14] | $(e^-, a^0)$ | $(i\gamma_\mu \partial^\mu - 3m_e$ $-(2m_e)^{-1}\partial_\mu \partial^\mu)\psi = 0$ | O(4,2) |
| $n \xrightarrow{W} p + W$ $\rightarrow p + e^- + \bar{\nu}$ | $n \sim (p, W^-)_{QM}$ $\Rightarrow W^- \cong \hat{e}^- \approx e^- a^0$ | $(\hat{e}^-, \hat{a}^0)$ | $(i\hat{\gamma}_\mu \hat{\partial}^\mu - 3\hat{m}_e$ $-(2\hat{m}_e)^{-1}\hat{\partial}_\mu \hat{\partial}^\mu)\hat{T}\hat{\psi} = 0$ | $\hat{O}(4,2)$ |

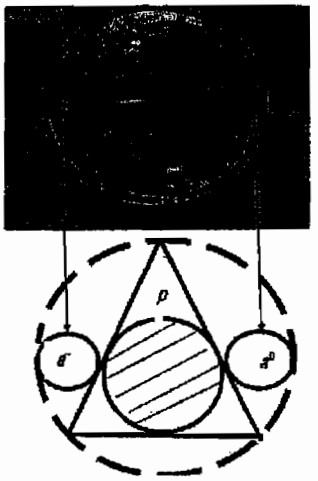



Figure 7: *Table 1: O(4,2) Profile of e-p scattering and neutron production; and 2-dimensional projection of the Macdonough representation of the Rutherford-Santilli neutron showing its relationship to Santilli's "etherino" model of the neutron.*

$$m_{\pm} = m_e(-1 \pm \sqrt{7}), i.e., \frac{m_+}{m_e} = 1,65; \frac{|m_-|}{m_e} = 3.6 \tag{4.30}$$

**Consequently, since $0.78MeV = 1.53!m_e$ , it follows by setting $m_{a^0} \equiv m_+ = 1.65m_e$ that one may validly characterize a quantum mechanical bound state of $n = (p, e^-, a^0)$ system with negative binding energy: $m_n - (m_p + m_e + m_{a^0}) \equiv -0.18m_e$ .**

**Intriguingly, the numerical coefficients in the wave equations (4.18) and (4.19) are uniquely related in terms of Gell-Mann SU(3) $\lambda$ - generators,**

$$\lambda_0 = \sqrt{\frac{2}{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \lambda_8 = \sqrt{\frac{1}{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \lambda_8^{-1} = \sqrt{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}, \tag{4.16}$$

**and a triplet field**

$$\Psi = \begin{pmatrix} \psi_\nu \\ \psi_e \\ \psi_{a^0} \end{pmatrix},$$

**as the components of the wave equation**

$$(i\gamma_\mu \partial^\mu - m_e\sqrt{\frac{3}{2}}(\lambda_0 - \sqrt{2}\lambda_8) + (\frac{1}{m_e\sqrt{3}})\lambda_9^{-1}\partial_\mu\partial^\mu)\Psi = 0 \tag{4.31a}$$

**where**

$$m_e\sqrt{\frac{1}{2}}(\lambda_0 - \sqrt{2}\lambda_8) = (3m_e)\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

$$(\frac{1}{m_e\sqrt{3}})\lambda_8^{-1} \equiv (\frac{1}{3m_e})\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}. \tag{4.31b}$$

**As the three equations in this system are uncoupled except insofar as there is only one characteristic mass,$m_e$ , for the whole triplet, Eq.(4.17a) implies that in the absence of convective currents, the apparent chiral $SU(3) \times SU(3)$ symmetry of the leptonic triplet ($\Psi$ ) is broken in the manner prescribed by Gell-Mann, Oakes and Renner[16]. This observation provides, therefore, a heuristic motivation to include both vector currents and (parity-violating) axial-vector currents in the genotopy of lepton and hadron currents in $\hat{O}^{>}(4,2) \times \hat{S}^{>}(3) \times \hat{U}^{>}(1)$ genospace suitable for a sufficiently broader genoscattering model of electroweak current-current interaction indicated in Fig.**

3 which we now proceed to elucidated further in Sec.4.3.

### 4.3 Genotopy of Lepton and Hadron Currents

In order for formal genoscattering theory involving strong and electroweak forces to be expressible as current-current interaction, it is necessary to construct the genotopy of the iso-current defined in Eq.(4.9) in the following broader form:

$$J_\mu \equiv \bar{\psi}\gamma_\mu\psi \to \hat{\bar{\psi}}\hat{T}(\hat{\gamma}_\mu - i\kappa_0\hat{\bar{\partial}}_\mu)\hat{T}\hat{\psi} \equiv \hat{J}_\mu$$

$$\to \left\{ \begin{array}{c} \hat{\bar{\Psi}}_F^{>}\hat{T}^{>}(\hat{\gamma}_\mu^{>} - i^{>}\kappa_0^{>}\hat{\bar{\partial}}_\mu^{>})(\hat{I}^{>} + \hat{\gamma}_5^{>})\hat{Q}_F^{>}\hat{T}^{>}\hat{\Psi}_F^{>} \equiv \hat{J}_\mu^{>} \\ or \\ {}^{<}\hat{\bar{\Psi}}_F{}^{<}\hat{T}({}^{<}\hat{\gamma}_\mu - {}^{<}i{}^{<}\kappa_0{}^{<}\hat{\bar{\partial}}_\mu)({}^{<}\hat{I} + {}^{<}\hat{\gamma}_5){}^{<}\hat{Q}_F{}^{<}\hat{T}{}^{<}\hat{\Psi}_F \equiv {}^{<}\hat{J}_\mu \end{array} \right\} \tag{4.32}$$

where $\hat{Q}_F^{>}$ is the Fermion (i.e. lepton ($L$) or baryon ($B$)) charge, and $\hat{\Psi}_F^{>}$ is the corresponding Fermion $\hat{O}^{>}(4,2)\times\hat{SU}^{>}(3)\times\hat{U}^{>}(1)$ multiplet, and similarly for ${}^{<}\hat{Q}_F$ etc.

In order to highlight the physical content of the above generalization, it is instructive to discuss the most familiar (baryon) limit of this expression provided by Cabibbo's[17] representation of the relative strengths of the vector and axial-vector currents given in the quark triplet $q_h = (u,d,s)$ model by,

$$J_{h\mu}^{wk} = \bar{q}_h\gamma_\mu Q_h^{wk} q_h \tag{4.33}$$

where

$$Q_h^{wk} = \frac{1}{2}cos\theta(\lambda_1 + i\lambda_2) + \frac{1}{2}sin\theta(\lambda_4 + i\lambda_5) = \begin{pmatrix} 0 & cos\theta & sin\theta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{4.34}$$

and $\theta \sim 15^0$ (empirically) is the mixing (*Cabibbo*) angle. The fact that the length of the vector $(0, cos\theta, sin\theta)$ is unity expresses the so-called *lepton-hadron universality* in weak interactions. Similarly, for the lepton triplet $(\nu, e^-, \mu^+)$ first introduced in 1968 by Salam[18] where $\nu$ is a 4-component neutrino $(\nu_e, \bar{\nu}_\mu)$, or more appropriately, the "Santilli" triplet, $q_l = (a^0, e^-, \mu^+)$ we have,

$$J_{l\mu}^{wk} = \bar{q}_l\gamma_\mu Q_l^{wk} q_l \tag{4.35}$$

where

$$Q_l^{wk} = \frac{1}{\sqrt{2}}(\lambda_1 - i\lambda_2) + \frac{1}{\sqrt{2}}(\lambda_4 - i\lambda_5) = \begin{pmatrix} 0 & 0 & 0 \\ cos45^0 & 0 & 0 \\ sin45^0 & 0 & 0 \end{pmatrix} \tag{4.36}$$

Thus, $Q_l^{wk} \equiv Q_h^{wk\dagger}$ apart from the difference in the numerical values of the leptonic and hadronic Cabibbo angles, in which case we may regard the hadronic Cabibbo angle $\theta$ as a "mutation" of the leptonic Cabibbo angle $\phi$.

In like manner, we observe that the electric charges may be expressed in the forms

$$Q_h^\gamma \equiv \frac{1}{2}(\lambda_3 - \lambda_8/\sqrt{3})$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{pmatrix} = U\tilde{Q}_h^\gamma U^{-1} \tag{4.37a}$$

where

$$\tilde{Q}_h^\gamma = \frac{1}{3}(\lambda_1 + \lambda_4 + \lambda_6) = \frac{1}{3}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix};$$

$$U = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{3} & 1/\sqrt{2} & -1/\sqrt{2} \\ 2/\sqrt{6} & -1/\sqrt{6} & -1/\sqrt{6} \end{pmatrix}; U^{-1} = U^\dagger.$$

$$Trace(U) = 1/\sqrt{3} + 1/\sqrt{2} - 1/\sqrt{6} \equiv 2cos\theta - 1 \tag{4.37b}$$

and for the leptonic charge,

$$Q_l^\gamma \equiv \frac{1}{2}(\lambda_3 - \sqrt{3}\lambda_8) = V\tilde{Q}_l^\gamma V^{-1}$$

$$\tilde{Q}_l^\gamma = \frac{1}{3\sqrt{3}}(\lambda_2 + \lambda_5 + \lambda_7) = \frac{1}{3\sqrt{3}}\begin{pmatrix} 0 & -i & -i \\ i & 0 & -i \\ i & i & 0 \end{pmatrix} \tag{4.2238a}$$

where

$$V = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & e^{-i\phi} \\ -1 & e^{-i\phi} & 1 \\ 1 & -e^{i\phi} & e^{i\phi} \end{pmatrix}; V^{-1} = V^\dagger, \phi = 60^0. \tag{4.38b}$$

As a result, the lepton and hadron electromagnetic currents can be added (like Cabibbo) in terms of a single triplet,$q_F$ such that

$$J_\mu^\gamma = \bar{q}_F \gamma_\mu \tilde{Q}_F^\gamma q_F \tag{4.39}$$

where

$$\tilde{Q}_F^\gamma = \frac{2}{3\sqrt{3}}[cos\phi(\lambda_2 + \lambda_5 + \lambda_7] + sin\phi[\lambda_1 + \lambda_4 + \lambda_6]$$

$$= \frac{2}{3\sqrt{3}} \begin{pmatrix} 0 & e^{-i\phi} & e^{-i\phi} \\ e^{i\phi} & 0 & e^{-i\phi} \\ e^{i\phi} & e^{i\phi} & 0 \end{pmatrix} \tag{4.40}$$

Consequently, the eigenvalues of the symmetric part of $\tilde{Q}_F^\gamma$ give the eigenvalues of $Q_l^\gamma$ while the eigenvalues of its antisymmetric part give the quark charges $Q_h^\gamma$. This is the unifying feature of genoscattering theory that we are after, which we now proceed to use as a framework for initiating applications of the theory to inelastic scattering involving spin-$\frac{1}{2}$ fermion (baryon+lepton) systems.

## 5. INITIAL APPLICATIONS

### 5.1 Genotopy of the S-matrix and Feynman Graphs/Rules.

In order to familiarize the reader with the use of generalized Feynman graphs/rules for computation of the S-matrix we recapitulate the Lie-isotopic elaboration of Feynman graph for reversible electron-proton Coulomb scattering shown in Fig. 6(a) which will lead to an isotopic generalization of the familiar Mott scattering cross-section, and will enable us to identify the characteristic features of inelastic scattering processes in Fig. 6(b) requiring an application of the genoscattering theory.

To write down the S-matrix for the reversible scattering process shown in Fig.6(a) using the Feynman rules, one starts in the direction of the top left arrow to right and, at each vertex, inserts all other factors between the incoming and outgoing arrows. If loop closes, one takes trace to get:

$$\hat{S}_{fi} = -4i\pi \int \frac{d^4\hat{q}}{(2\pi)^4} \times [\frac{1}{\sqrt{(2\pi)^3}} \sqrt{\frac{\hat{m}\hat{c}}{\hat{k}'_0}} \times \hat{\hat{u}}^{\hat{S}'}(\hat{k}')] \times$$

$$[-i(4\pi e)\hat{\gamma}^{\mu'}(2\pi)^4 \times \hat{\delta}^4(\hat{k}' - \hat{k} + \hat{q})] \times [\frac{1}{\sqrt{(2\pi)^3}} \sqrt{\frac{\hat{m}\hat{c}}{\hat{k}_0}} \times \hat{u}^{\hat{S}}(\hat{k})] \times$$

$$[-i\hat{g}^{\mu\nu}\hat{D}_F(\hat{q}^2)] \times [\frac{1}{\sqrt{(2\pi)^3}} \sqrt{\frac{\hat{M}\hat{c}}{\hat{P}'_0}} \times \hat{\hat{U}}^{\hat{\lambda}'}(\hat{P}')] \times$$

$$[-i(4\pi e)\hat{\gamma}^{\mu'}(2\pi)^4 \times \hat{\delta}^{(4)}(\hat{P}' + \hat{P} - \hat{q})] \times [\frac{1}{\sqrt{(2\pi)^3}} \sqrt{\frac{\hat{M}\hat{c}}{\hat{P}_0}} \times \hat{U}^{\hat{\lambda}}(\hat{P})]$$

$$= -4\pi i \int \frac{d^4\hat{q}}{(2\pi)^4} \sqrt{\frac{(\hat{m}\hat{c})^2(\hat{M}\hat{c})^2}{\hat{k}'_0\hat{k}_0\hat{P}'_0P_0}} \times \hat{\hat{u}}^{\hat{S}'}(\hat{k}')\hat{\gamma}^{\mu}\hat{u}^{\hat{S}}(\hat{k}) \times$$

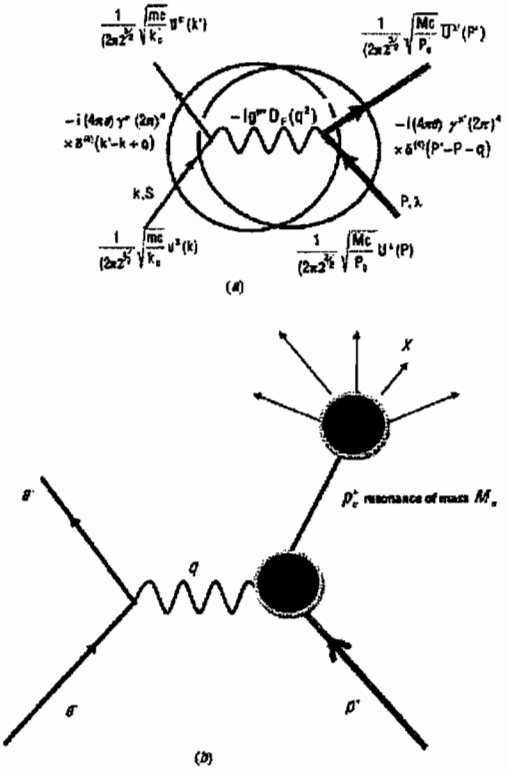



Figure 8: *generalized Feynman graphs for (a) electron-proton isoscattering and (b) resonance model of deep-inelastic* $e^- - p^+$ *genoscattering.*

$$\frac{-4\pi i\alpha}{(4\pi)^4}(2\pi)^4\hat{\delta}^{(4)}(\hat{k}'+\hat{P}'-\hat{k}-\hat{P})\times$$

$$[(\hat{k}'-\hat{k})^2+(m_\phi\hat{c})^2+i\hat{\varepsilon}]^{-2}\times\hat{\bar{U}}^{\hat{\lambda}'}(\hat{P}')\hat{\gamma}_\mu\hat{U}^{\hat{\lambda}}(\hat{P}). \tag{5.1}$$

**This may be rewritten in the form of** *generalized current-current interaction*:

$$\hat{S}_{fi}=-4i\pi\int\frac{d^4\hat{q}}{(2\pi)^4}\times\hat{j}^\mu(\hat{k}'\hat{k},-\hat{q})\times\frac{1}{[\hat{q}^2+(m_\phi\hat{c})^2]^2}\times\hat{J}_\mu(\hat{p}'\hat{p};\hat{q}) \tag{5.2}$$

**where**

$$\hat{j}^\mu(\hat{k}'\hat{k},-\hat{q})=(2\pi)^4\times\hat{\delta}(\hat{k}'-\hat{k}+\hat{q})]\times\frac{e}{(2\hat{m}\hat{c})^3}\times\sqrt{\frac{(\hat{m}\hat{c})^2}{\hat{k}_0'\hat{k}_0}}\hat{\bar{u}}^{S'}(\hat{k}')\hat{\gamma}^\mu\hat{u}^S(\hat{k}) \tag{5.3a}$$

$$\hat{A}_\mu(\hat{k}'-\hat{k})=\frac{4\pi}{(\hat{k}'-\hat{k})^2+(m_\phi\hat{c})^2+i\hat{\varepsilon}}\hat{J}_\mu(\hat{p}',\hat{p};\hat{k}'-\hat{k}) \tag{5.3b}$$

**are, respectively, the generalized electromagnetic current**$\hat{j}^\mu$ **and generalized Moller current**$\hat{J}_\mu$ **associated with the generalized electromagnetic vector potential,** $\hat{A}_\mu$ **.**

**The differential cross section with no polarization for initial particles in the laboratory frame,**$\hat{p}=(M\hat{c};0)$ **is given by**

$$d\hat{\sigma}=\frac{1}{2}\sum_{ij}\frac{1}{|\frac{\hat{k}}{\hat{k}_0}|\frac{1}{(2\pi)^3}\frac{V}{(2\pi)^3}}\frac{|\hat{S}_{if}|^2}{T}d^3\hat{p}'d^3\hat{k}' \tag{5.4}$$

**We observe that, in the limit** $\hat{T}\to 1$ **this leads to the standard expression of "potential scattering theory" for electron-point-proton scattering (with unpolarized initial state and no observation of final spin):**

$$\frac{d\sigma_{e-p}}{d\Omega}=\frac{\alpha^2E^2(1-\beta sin^2(\theta/2))}{4P^4sin^4(\theta/2)}=\frac{\alpha^2cos^2(\theta/2)}{E^2sin^4(\theta/2)}\equiv\frac{d\sigma_{Mott}}{d\Omega} \tag{5.5a}$$

$$\beta=\frac{|\mathbf{P}|}{E},\frac{1}{2}(\mathbf{P}'-\mathbf{P})^2=(\mathbf{q})^2=\mathbf{2P^2(1-\cos(\theta))=4P^2\sin^2(\theta/2)} \tag{5.5b}$$

**In the isoscattering theory of reversible processes, three novel features arise: firstly, from the generalized internal photon line** $\hat{D}_F(\hat{q}^2)$**, which is no longer divergent in the limit** $\hat{q}\to 0$**; secondly, from the generalized Dirac matrices** $\hat{\gamma}^\mu$**; and thirdly, from the generalized currents,**$\hat{j}^\mu$ **and** $\hat{J}_\mu$**. These features persist in genoscattering theory of irreversible processes, which**

we now proceed to explore, beginning with a review of the results of deep-inelastic electron-positron and electron-proton scattering experiments.

5.2 Deep Inelastic e-p Scattering Experimental Results

In the resonance model [19] of the 1967 SLAC-MIT experiments [20,21] on a program of inelastic electron-proton ($e^-p^+$ ) scattering on the 20GeV Stanford linear accelerator to study electro-production of resonances as a function of momentum transfer, the process is viewed as a generalization of the Feynman graph for electron-point-proton scattering in Fig.6(a) in which virtual space-like photon emitted by the in-coming electron violently collides with the interacting target proton and under the kinematical condition

$$M_n^2 = M^2 + 2M\nu + q^2, (M^2 \equiv P^2, \nu \equiv q.P), \tag{5.6}$$

a resonance of mass $M_n$ is produced. Subsequently, after a short flight this resonance decays into a multitude of stable hadrons as shown in Fig.6(b).

In their rest frame, these resonances are classified under the product group, $SU(3) \times G$ where the unitary unimodular symmetry group $SU(3)$ provides the internal symmetry through quantum numbers, such as isotopic spin(I), hypercharge (Y), etc, and $G$ represents a dynamical group, frequently chosen to be SL(2,C) and $SL(2) \times d$ or O(4,2). And in order to compare with the SLAC-MIT experimental data [19] one generalizes Eq.(5.5) to [21]:

$$\frac{d^2\sigma}{d\Omega dE} = \frac{d\sigma_{Mott}}{d\Omega}[W_2(\nu, q^2) + 2W_1(\nu, q^2)tan^2\frac{\theta}{2}] \tag{5.7a}$$

or in the form

$$\frac{d^2\sigma}{d\Omega dE} = \frac{d\hat{\sigma}_{Mott}}{d\Omega}[W_2(\nu, q^2)cos^2\frac{\theta}{2} + 2W_1(\nu, q^2)sin^2\frac{\theta}{2}]; \tag{5.7b}$$

where the structure functions, $W_1(\nu, q^2)$ and $W_2(\nu, q^2)$ , depend on the properties of the target system, and $\hat{\sigma}_{Mott} \equiv \sigma_{Mott}sec^2\frac{\theta}{2}$. The fact that two such functions are required because there are two (transverse and longitudinal) polarization states of the virtual photon also lends itself to an interpretation of the scattering angle $\frac{\theta}{2}$ as a Cabibbo-like "mixing" angle envisaged in Sec.4.3.

In models that satisfy $SU(3)\times SU(3)\times O(3,1)$ current algebra, Bjorken[22] conjectured that, in the limit, $M^2 = -q^2 \to \inf, \mu = q.P \to \inf$ such that $2M\nu/q^2 = \omega$ is fixed, one should expect the following dependence on only:

$$2MW_2(\nu, q^2) = F_1(\omega), \nu W_2(\nu, q^2) = F_2(\omega) \tag{5.8a}$$

He also derived a sum-rule for inelastic electron scattering

$$\int_{q^2/2M}^{\inf} d\nu[W_2^p(\nu,q^2)+W_2^n(\nu,q^2)] \geq \frac{1}{2} \tag{5.8b}$$

where$W_2^p(\nu,q^2)$ and $W_2^n(\nu,q^2)$ are the structure functions for the proton and neutron respectively. This scaling behavior in Eq.(3.4a) was subsequently found experimentally [19]; and a value over the range of MIT-SLAC data [19,20] was found for the weighted sum

$$\int^{20} \frac{d\omega}{\omega}\nu W_2^p = 0.78 \pm 0.04 \tag{5.9}$$

It is intriguing that this is comparable to the dimensionless mass defect, $[m_n-(m_p+m_{e^-})]/(MeV)=0.78$.

However, from the 1990 Nobel Lecture by J. Friedman[23] entitled Deep Inelastic Scattering: Comparisons with the Quark Model, it is apparent that theoretical and practical difficulties have arisen in the analysis of the unavoidably limited experimental data from the infinities characterizing the definition of the Bjorken variable, $x$ (or $\omega=1/x$) and its physical interpretation in Feynman's quark-parton model[24] as the fraction of proton momentum carried by point-like constituents of the proton (usually identified with free quarks) in the so-called infinite momentum frame ($p\rightarrow$ inf) . Also comparison of theory with limited experimental data has been largely limited to sum rule predictions (requiring integration over all $x$ or $\omega$ ). Moreover, the replacement of the electron altogether by a presumed space-like photon (traveling at superluminal speed $V > c_0$ ) interacting with the proton (travelling at subluminal speed $v < c_0$ ) involves the singularity in the special relativity theorem of addition of velocities, V and v, when $vV = c_(0)^2$ . It is not surprising, therefore, that, none of the existing models has satisfactorily explained the data, especially why scaling behavior should set in at energies as low as observed.

It was argued by Jackiw[25] in his 1972 Physics Today review article on Scale Symmetry, that the experimental results point towards approximate O(3,1) scale symmetry (broken by non-vanishing divergence of the dilatation current density due to non-zero masses in the O(3,1) kinematical group sector of the $SU(3)\times O(3,1)$ theory). However, as subsequently pointed out by Animalu[26] in a letter to Physics Today on Jackiw's article, scale symmetry breaking can be remedied by "lifting" the $SU(3)\times O(3,1)$ to $SU(3)\times O(4,2)$ current algebra involving fundamental length or mass scale. It was from this point of view that we have characterized the scattering region in Sec.4.2 while the basically non-unitary character of scale symmetry

and irreversibility provide the justification for the application of genoscattering theory to the understanding of deep-inelastic electron-positron and electron-proton scattering data.

5.3 Genotopy of Bjorken Variable and Structure Functions.
In order to avoid infinites in the characterization of the Bjorken limit $M^2 = -q^2 \to \inf, \mu = q.P \to \inf$ such that $2M\nu/q^2 = \omega$, we may define geno-Bjorken variables from the left genotopy of the "point" sphere, $P^2 + q^2 = 0$ into a torus in terms of the two Lorentz scalars, $\hat{P}^2 - \hat{q}^2$ and , $\hat{q}.\hat{P}$ given by:

$$(q,p)\begin{pmatrix} 1 & -x \\ x & 1 \end{pmatrix}\begin{pmatrix} q \\ p \end{pmatrix} \to (\hat{q},\hat{P})\begin{pmatrix} -1 & \hat{x} \\ \hat{x} & 1 \end{pmatrix}\begin{pmatrix} \hat{q} \\ \hat{P} \end{pmatrix} = 0$$

$$i.e., q^2 + P^2 \to -(\hat{q}^2 - \hat{P}^2) + 2\hat{x}\hat{q}.\hat{P} = 0 \tag{5.10a}$$

where the underlying metrics are related as follows,

$$\eta \equiv \begin{pmatrix} 1 & -x \\ x & 1 \end{pmatrix} \to \begin{pmatrix} -1 & x \\ x & 1 \end{pmatrix} \equiv TC\eta; \tag{5.10b}$$

and right genotopy given by:

$$(\hat{q},\hat{P})\begin{pmatrix} 1 & -\omega \\ \omega & 1 \end{pmatrix}\begin{pmatrix} \hat{q} \\ \hat{P} \end{pmatrix} \to (\hat{q},\hat{P})\begin{pmatrix} -\hat{\omega} & 1 \\ 1 & \hat{\omega} \end{pmatrix}\begin{pmatrix} \hat{q} \\ \hat{P} \end{pmatrix}$$

$$i.e., q^2 + P^2 \to -\hat{\omega}(\hat{q}^2 - \hat{P}^2) + 2\hat{q}.\hat{P} = 0 \tag{5.10c}$$

where the underlying metrics are related as follows,

$$\eta \equiv \begin{pmatrix} 1 & -x \\ x & 1 \end{pmatrix} \to \begin{pmatrix} -x & 1 \\ 1 & x \end{pmatrix} \equiv \eta DT; \tag{5.10d}$$

and

$$C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \equiv \sigma_1; T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \equiv -i\sigma_2; D = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \equiv -\sigma_3.$$

We observe that if $x = 1$, then $TC\eta - \eta DT = 0$, with $C \neq D^\dagger \neq I$ is a Lie-admissible relation. From the relations on the right of Eqs.(5.10a) and (5.10c) we obtain the geno-Bjorken variables $(\hat{x}, \hat{\omega})$:

$$\hat{x} = \hat{\omega}^{-1} = (\hat{P}^2 - \hat{q}^2)/2\hat{q}.\hat{P} \tag{5.11}$$

Consequently, following Animalu and Ekuma[27], the genotopy of the deep-inelastic structure functions, $\hat{F}_1$ and $\hat{F}_2$ , can be represented (cf Myung[28]) by a pair of Riccati's equations:

$$d\hat{F}_1/d\hat{\omega} = \hat{x}_0\hat{F}_1 - \gamma\hat{F}_1^2$$
$$d\hat{F}_2/d\hat{\omega} = -\hat{x}_0\hat{F}_2 + \gamma\hat{F}_2^2 \qquad (5.9a)$$

subject to the boundary conditions, $\hat{F}(0)_1 = \hat{F}_{10}, \hat{F}(0)_2 = \hat{F}_{20}$ . The solutions[23]

$$\hat{F}(\hat{\omega})_1 = \frac{\hat{x}_0/\gamma}{1 - (\hat{x}_0/\gamma\hat{F}_{10} - 1)e^{+\hat{x}_0\hat{\omega}}}$$
$$\hat{F}(\hat{\omega})_2 = \frac{\hat{x}_0/\gamma}{1 - (\hat{x}_0/\gamma\hat{F}_{20} - 1)e^{-\hat{x}_0\hat{\omega}}} \qquad (5.9b)$$

have the features shown for $\hat{F}_1(0) > \hat{x}_0/\gamma$ and $\hat{F}_2(0) < \hat{x}_0/\gamma$ in Fig.7 where they are also compared with experimental data. The agreement between theory and experiment is quite good. In terms of the reciprocal Bjorken variable $\hat{\omega}$ , the corresponding curve for $\hat{F}_2(\hat{x}_0)$ turns out to be an image of $\hat{F}_2(\hat{\omega})$ and has the form

$$\hat{F}_2(\hat{x}) = \frac{\hat{\omega}_0/\xi}{[1 + (\hat{\omega}_0/\xi\hat{F}_{20} - 1)e^{+\hat{x}\hat{\omega}_0}]} \qquad (5.10)$$

whose feature is as shown for $\hat{F}_{20} > \hat{\omega}_0/\xi$ in Figs. 8(a) and 8(b), where it is also compared with experimental data for $\nu W_2 \equiv F_2(x)$. The agreement between theory and experiment is again quite good.

5.4. Mass Genorenormalization

In the preceding sections of this paper, we have established that an irreversible scattering theory implies rather serious differences in the data elaboration of the same inelastic scattering compared to the elaboration done with the conventional reversible scattering theory.

To complete the understanding of the implications, it is important to out;line the additional implication according to which *irreversible nonlinear, nonlocal and nonpotential effects in the interior of the scattering region cause an alteration of the numerical value of internal masses.*

This new occurrence can be seen by recalling from Paper II that nonlinear, nonlocal and nonpotential effects are representable with a general symmetric metric here expressed for simplicity in (1 + 1)-dimensions with the *light genocone* [7b]

$$\frac{r^2}{n_r^{2>}} - t^2\frac{c^2}{n_4^{2>}} = 0, \qquad (5.11)$$

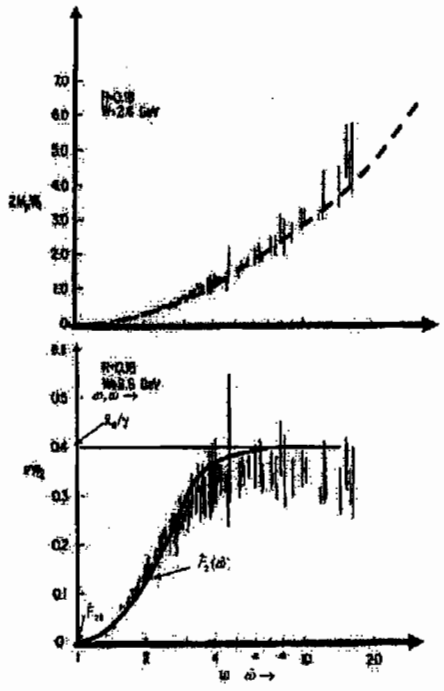


Figure 9: *Comparison of* $\hat{F}_1(\hat{\omega})$ $\hat{F}_2(\hat{\omega})$ *for* $\hat{F}_{20} < \hat{x}_0/\gamma$ *with* $2MW_1 = F_1(\omega)$ *and* $\nu W_2 = F_2(\omega)$ *where* $\omega = 2M\nu/q^2$, *for proton;* $W > 2.6GeV, q^2 > 1(GeV/c_0^2$ *and R=0.18. Data from G. Miller et al Phys. Rev.D5, 528 (1972)[19].*

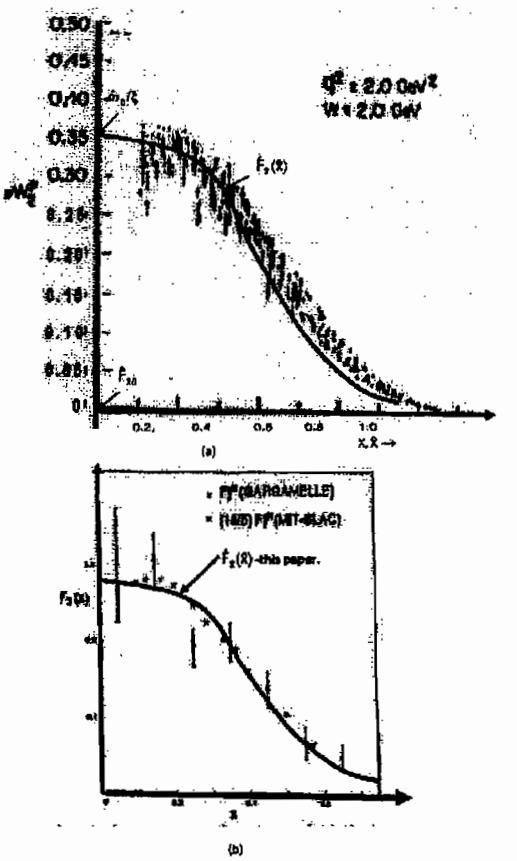


Figure 10: *(a) Comparison of* $\hat{F}_1(\hat{\omega})$ *for* $\hat{F}_{20} < \hat{\omega}_0/xi$ *(solid line) with* $W_2$ *versus* $x = q^2/2M\nu$ *for the proton for* $W > 2.0GeV, q^2 > 2(GeV/c_0^2$ *Data from ref.[20] : A.Bodek et al Phys. Rev .Lett. 30,1087 (1973); Phys. Lett. 51B, 417(1974); Phys. Rev. D20, 1471 (1979) . (b) Early Gargamelle measurements of* $F_2^{\nu N}$ *compared with*$(18/5)F_2^{eN}$ *calculated from the MIT-SLAC results ) and with HM model* $\hat{F}_2^{\nu N}(\hat{x})$ *. (Source of data J.I. Friedman Nobel Lecture 1990, Physics 1990 p. 715 ref.[23].*

where the arrow in the characteristic quantities oi the interior medium indicates their lack of time-reversal invariance and the selection of their value for the forward motion. The preceding light genocone then characterizes the following *maximal causal genospeed*

$$V^{>}_{max} = c\frac{n^{>}_{r}}{n^{>}_{4}}. \tag{5.12}$$

The second order Casimir invariant of the Lorentz-Poincaré symmetry is then lifted from the conventional expression $p^2 - m^2c^2 = 0$ characterizing the mass $m$ into the genotopic form

$$p^2 - m^2V^{2>}_{max} = 0 \tag{5.13}$$

The identification from the outside of the numerical value of a mass in the interior of the scattering region, such as the mass of an exchange particle, requires the projection of the above expression in our spacetime, resulting in the *mass renormalization*

$$m \ \rightarrow \ m^{>} = m\frac{n^{>}_{r}}{n^{>}_{4}} \tag{5.14}$$

As a concrete illustration, the mass of the hypothetical;l *Higgs boson* is estimated as being between 115 and 185 GeV/c2 although it is admitted as being a model-dependent upper bound. Our genoscattering theory establishes that, for highly irreversible processes such as those of Figure 1, such a numerical prediction is additionally dependent on the energy of the scattering particles, the geometry of their collision and numerous additional features, to such a extent that the "search for the Higgs boson" is experimentally meaningless as current stated.

As illustrated in Ref., [31], the above potentially large variation of the value of the masses of extended particles is rather general and essentially due to the fact that the value $c$ can be safely assumed as being the maximal causal speed solely for *point-like* particles. When considering *extended* particles, the value of the mass depends on the maximal causal speed in its interior that is expected to vary dependent on the density and other features, thus increasing with the mass (since hadrons have essentially the same charge distribution).

The necessity for a new renormalization of the masses for interior problems, caused by nonlinear, nonlocal and non-Lagrangian/non-Hamiltonian internal effects, was first established by Santilli in Ref. [2b] as being necessary for a quantitative representation of the synthesis of the $\pi^o$ meson

from an electron and a positron, $e^+ + e^- \rightarrow \pi^o$. In this case, the total rest energy of the final state (134 $MeV$) is much bigger than the sum of the rest energies of the two original states (1 $MeV$), under which conditions the Schrödinger equation became inconsistent (due to the need of a "positive" binding energy which is anathema in quantum, mechanics).

Santilli (see Section 5 of paper [2b]) then established the sole known methods to achieve a quantitative representation of the considered synthesis is that of subjecting the inconsistent Schrödinger equation to a nonunitary transform. In this case, consistency of the equation is achieved via a novel renormalization of the masses of the original constituents of unitary, thus of non-Lagrangian and non-Ham,Hamiltonian type with numerical value

$$m_e = 0.5\ MeV \rightarrow m_{\hat{e}} \approx 75\ MeV, \tag{5.15}$$

which yields indeed the final value $m_{\pi^o} = 134! MeV$ in view of the mass defect caused by the Coulomb attraction between the electron and the positron.

The above novel renormalization was subsequently confirmed, also by Santilli [32], via the representation of *all* characteristics of the neutron in its synthesis inside stars from a proton and an electron inside a star, $p^+ + e^- \rightarrow !n+?$, that also required a nonunitary lifting of the Schrödinger equation (since the rest energy of the neutron is bigger than the sum of the rest energies of the proton and the electron). This representation was first achieved in 1990 [32a] at the nonrelativistic level and then in 1993 [32b] at the relativistic level (see Kadeisvili [33] for a comprehensive review).

Numerous additional data have independently confirmed the need for they isorenormalization of internal masses for the case of reversible interior problems and of the broader renormalization for the case of irreversible processes (see the experimental lectures in www.world-lecture-series.org).

### 5.5. Concluding Remarks

In this series of papers, we have shown that the so-called "experimental results" obtained in the data elaboration of inelastic scattering experiments via the conventional, quantum mechanical, scattering theory, are mere personal opinions by the issuing experimentalists, rather than incontrovertible experimental truth.

On mathematical ground, the above conclusion can be seen from the very axiomatic structure of quantum mechanics which is notoriously *local-differential,* thus solely capable of characterizing *a finite number of point-like particles without collisions* (in any case, collisions are meaningless for point particles). Consequently, we can expect that the relativistic scattering

theory can indeed provide an exact representation of scattering events without collisions, as it is the case for the Coulomb scattering. However, the local-differential mathematical foundations of the relativistic scattering theory prevents even a consistent definition of collision, let alone its quantitative treatment, under which conditions any expectation of exact results is ascientific.

On physical grounds, the very notion of collision requires the representation of particles as *extended.* But then, the scattering/collision of extended particles implies the presence of conventionalize Hamiltonian as well as contact, zero-range, nonlinear, nonlocal-integral and nonpotential/non-Hamiltonian interactions. The insufficiency of the conventional linear, local-differential, and potential/Hamiltonian scattering theory is then beyond scientific or otherwise credible doubts.

To state it in a nutshell, the papers of this series confirm the expectation expressed in the first lines of the first paper, namely, that time-reversal invariant theories, such as Einstein's special relativity and relativistic quantum mechanics, cannot possibly or otherwise credibly be assumed as being exactly valid for irreversible scattering events.

By looking in retrospect, the above conclusions have been known to the authors for decades. The resolution of the technical difficulties for their quantitative treatment has requested decades of research by Santilli because of the prior need to develop the new Lie-isotopic and Lie-admissible formulations for the invariant representation of the extended character of particles and/or of their wavepackets, and their most general known interactions, after which Animalu's broadening of Feynman's diagrams could be subjected to proper treatment and development.

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# References


[1] R. M. Santilli and A. O. E. Animalu, Nonunitary Lie-isotopic and Lie-admissible scattering theories of hadronic mechanics,", Papers I, II, III, IV, contributed papers in the *Proceedings of the Third Lie-Admissible Treatment of Irreversible Processes,* C. Corda, Editor, Kathmandu University. Nepal (2011), a;spavailable in pdf download from
http://www.santilli-foundation.org/docs/Isoscattering-I.pdf
http://www.santilli-foundation.org/docs/Isoscattering-II.pdf
http://www.santilli-foundation.org/docs/Isoscattering-III.pdf
http://www.santilli-foundation.org/docs/Isoscattering-IV.pdf

[2] R.M. Santilli, Hadronic Journal 1., 224 (1978) [2a] and, 1,574 (1978) [2b] and 1,1267 (1978).[2c].

[3] R. M. Santilli, Algebras, Groups and Geometries 10, 273 (1993).

[4] R. M. Santilli, "Lie-admissible invariant representation of irreversibility for matter and antimatter at the classical and operator levels," Nuovo Cimento B bf 121, 443 (2006), available in free pdf download from the link
http://www.santilli-foundation.org/docs/Lie-admiss-NCB-I.pdf

[5] R. M. Santilli, *Isodual Theory of Antimatter, with Application to Antigravity, Grand Unification and Cosmology,* Springer (2006), available in pdf download from http://www.santilli-foundation.org/docs/santilli-79.pdf

[6] R. M. Santilli, Rendiconti Circolo Matematico di Palermo, Supplemento 42, 7 (1996).

[7] R. M. Santilli, *Elements of Hadronic Mechanics*, Vol. I: *Mathematical Foundations* (1995), Vol. II: *Theoretical Foundations* (1995), Vol. III: *Recent Developments, Experimental verifications and Industrial Applications* (available in pdf format at http://www.i-b-r.org) Ukrainian Academy of Sciences, Kiev.

[8] R.M. Santilli, New Problematic Aspects of Current String Theories and Their Invariant Resolution, Found. Phys. Vol. 37. No. 7 (2002).

[9] R. M. Santilli, Intern. J. Modern Phys. D 7, 351 (1998).

[10] A. O. Barut, P. Cordero and G.C. Ghirardi, Phys. Rev. 182, 1844 (1969)and Nuov. Cim. 66A, 36 (1970); A.O.E. Animalu, Mass Ratio of Quarks, Phys. Rev. D3, 1106 (1971); Charge Spectrum of 4-Component Fields with O(4,2) Dynamical Symmetry, Phys. Rev. 4D, 1992-1925 (1971).

[11] J. Bernstein, in Elementary Particles and their Currents (Freeman, San Francisco, 1947) p. 41.

[12] A.O.E. Animalu, Lepton and Hadron Currents in O(4,2) Current Algebra Letters Nuovo Cim. 3, 729 (1972).

[13] A.O. Barut, in Group Theor. Methods in Physics (Edited by W. Beiglbock, A. Bohm and T. Takasugi), SLNP Vol. 94, 490 (1979); A.O. Barut, Ann. Physik Leipzig 43 83 (1986).

[14] R.M. Santilli, The Etherino and/or the Nuetrino Hypothesis, Foundations of Physics 37, 670 (2007).

[15] A.O.E. Animalu, Scale Symmetry, Physics Today p.76, June (1972)

[16] M. Gell-Mann, R. J. Oakes and B. Renner, Phys. Rev. 175,2195 (1968); M. Gell-Man, Phys. Letters 8, 214 (1964); G. Zweig, CERN Report 8419TH 412 (1964).

[17] N. Cabibbo, Phys. Rev. Letters 10, 513 (1963).

[18] A. Salam in Elementary Particle theory (Nobel Symposium 8, ed. N. Svartholm, Wiley and Sons Inc) p. 367 (1968)

[19] G. Miller et al Phys. Rev.D5, 528 (1972)

[20] A. Bodek et al Phys. Rev .Lett. 30,1087 (1973); Phys. Lett. 51B, 417(1974); Phys. Rev. D20, 1471 (1979)

[21] S.D. Drell and J. D. Walecka, Ann. Phys. (NY) 28, 18 (1964).

[22] J.D. Bjorken, Phys. Rev. 179, 1547 (1969)

[23] J.I. Friedman, Deep Inelastic Scattering: Comparisons with the Quark Model, Dec. 8 1990 Nobel Lecture, Physics p.715-738.

[24] R.P. Feynman, Phys. Rev. Lett. 23, 1415 (1969).

[25] R. Jackiw, Introducing Scale Symmetry Physics Today p.23, Jan 1972.

[26] A.O.E. Animalu, Scale Symmetry Physics Today p.76 June 1972.

[27] A.O.E. Animalu and E. C. Ekuma, Review of Deep-Inelastic e-p Scattering: A Hadronic Mechanics Viewpoint, African J. Phys. Vol. 1, p.133-153 (2008)

[28] R. M. Santilli, Found. Phys., 27 (1997,) 1159.

[29] R. M. Santilli, "Can antimatter asteroids, stars and galaxies be detected with current means?"in press in the Proceedings of the *Third International Conference on the lie-admissible Treatment of Irreversible Systems,* University of Kathmandu, Nepal, (2011) pages 25-36
http://www.santilli-foundation.org/docs/Antimatter-Asteroid.pdf

[30] A. O. E. Animalu, Hadronic J. bf 31, 248 (2008).

[31] R. M. Santilli, "Are masses of elementary particles and of the Solar system truly known?" in *Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes,* C. Corda, Editor, Kathmandu University (2011), pages 37-44
http://www.santilli-foundation.org/docs/Masses-Solar-Syst.pdf

[32] R. M. Santilli, Hadronic J. 13, 513 (1990) [32a]' Communication of the Joint Institute for Nuclear Research, Dubna, Russia, number JINR-E4-93-352 (1993) [32b]; and Chinese J. Syst. Eng. and Electr. and Electronics 6, 177 (1996) [32c].

[33] J. V. Kadeisvili, "The Rutherford-Santilli neutron," Hadronic J. 31, 1 2008
http://www.i-b-r.org/Rutherford-Santilli-II.pdf

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
Kathmandu University, Nepal, April (2011) pages 639-656

# Iso-superconductivity model: generalization of the Cooper pair formation model of superconductors

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## Abstract

The successful application of the electron-phonon interaction (EPI) mechanism in formulating the Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity is among the most outstanding intellectual achievements in theoretical physics because of the successful application of the theory to the conventional superconducting materials. Therefore, its unsuccessful application to the nonconventional superconducting materials has led to the search for new theories which generalized it, include an interplay with other mechanisms or are formulated from non – EPI mechanisms. We observe in this current study that to achieve a generalized theory of superconductivity, there is need to first developed a quantitative structure model of the Cooper pair formation (CPF) in line with the formulation of the molecules in nature which has given birth to hadronic mechanics. This generalized formulation is the iso-superonductivity model which is based on the observation by Animalu that the Cooper pair of the standard BCS model may have a nonlocal-nonhamiltonian structure equivalent to the strong interaction ("hadronic" mechanics (HM)) structure of the neutral pion, as compressed positronium atom at short distances ($< 1$ F $\sim 10^{-13}$ cm) which has been proposed by Santilli. The equivalent but approximate description at large distances ($> 1$ F) is by the quantum mechanical superexchange interaction. This generalized CPF has been used to successfully explain the high-$T_c$ superconductivity in the cuprates as well as to account for the transition temperatures of these materials. It is also used to account for the high-$T_c$ iron based superconducting materials.

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## Introduction

This year, 2011, has been declared by the international scientific community as the centenary celebration of the discovery of superconductivity (SC) in Hg at about 4 K by the Dutch physicist, Heike kamerlingh Onnes on August 8, 1911. The defining electromagnetic property of superconductivity is the complete disappearance of the dc electrical resistivity at and below a certain temperature called the critical or transition temperature $T_c$. In other words, if a superconducting material with $T \leq T_c$ is an element in a dc network, the voltage drop across it will be zero. Today, there is metallic/non-metallic, conventional/nonconventional, doped/undoped and high/low $T_c$ superconducting materials [1]. There have been quite a number of failed theories postulated since 1911 to account for superconductivity and these include those of some of the respected 20th century physics intellectuals such as Einstein, Heisenberg, Feynman, etc (for a recent review of the failed theories, see Ref [2]). The only generally accepted theory is that by Bardeen-Cooper-Schrieffer (BCS) in 1957 and its salient feature is the electron-phonon interaction (EPI) leading to the formation of an ensemble of Cooper pairs which can propagate coherently as the superconducting state of the parent material. The BCS theory has been used to account for a number of metallic and intermetallic superconductivity and all such materials are known as conventional superconductors and their transition temperatures are low [3,4]. This BCS theory has led to a number of important applications like superconducting magnets for laboratory use (spanning from small-scale laboratory experiments to the LHC's bending magnets), and importantly, for MRI systems as well as explanation of puzzling experimental data such as nuclear magnetic resonance (NMR) relaxation rate and Josephson tunneling [5]. However, the BCS failed to explain the Meissner effect which is a fundamental property of superconductors [6] and has been proven incapable of predicting high – temperature superconductors [7] and providing the guidelines to search for new materials [8]. Therefore the BCS theory did not only fail to predict the relatively high $T_c$ of the superconducting copper oxide compounds commonly known as cuprates discovered in 1986 [9] but also failed to account for this class of superconductors [10 - 13]. In general, the BCS has failed in its application to a number of new classes of superconductors discovered since 1970 [6]. To account for these superconductors now collectively known as non-conventional superconducting materials, there has been a deluge of proposals of new theories which generalized the BCS theory by replacing the phonon with other bosons [14 – 16], introduce an interplay of the EPI and other mechanisms [17 - 19] or are formulated from non – EPI mechanisms [20 - 25] as well as those that even question the validity of the BCS theory [6, 26]. However, the partial success of these theories for the superconducting materials they are developed for, means the main properties of the superconducting phenomena are still poorly understood and new concepts are needed [27 - 29]. The basic requirement of any theory of superconductivity is to first provide a mechanism of the Cooper pair formation (CPF) and its coherent propagation. Therefore the basic

question man has been struggling with is how does two electrons which should repel as provided by quantum mechanics come to bind to form the Cooper pair? A general answer to this question can be achieved by providing a quantitative and qualitative structure model of the CPF possible in all superconducting materials. In this current study, we have shown that at short distances, the CPF formation is by a nonlinear, nonlocal and nonhamiltonian strong hadronic-type interactions due to deep wave-overlapping of spinning particles leading to Hulthen potential that is attractive between two electrons in singlet couplings [20, 21, 29] while at large distance the CPF is by superexchange interaction which is purely a quantum mechanical affairs [23, 26, 30]. We observe that for both distances the control parameter responsible for the superconducting state is the effective valence z. Therefore $T_c$ expressions depending on the effective valence were obtained for both the superconducting cuprates and the iron based compounds to verify experimental results.

## Brief review of the EPI mechanism of CPF

The mechanism of electron-electron interaction mediated by phonon emanates from the discovery of the isotopic effect in superconductivity [31] which suggested that the vibration of the lattice could be involved in the interaction. Usually, electrons are scattered from one state k to another state K' leading to electrical resistivity. The lattice vibrations which distort the local crystal structure and consequently the local band structure can be absorbed and emitted in the scattering process. The first electron, which absorbed the phonon, has its effective mass increased and consequently will attract it to a second electron thereby leading to the CPF. Together, all the paired electrons form a condensate that moves as a single entity resulting in the zero resistance. This proposal was boosted by the earlier observation that the $T_c$ of superconductors depends on the isotopic mass of the lattice, in the independent studies of Reynolds et al. [32] and Maxwel [33]. Therefore one of the key features of the emanating BCS theory is its $T_c$ expression,

$$T_c \approx \frac{\hbar\omega}{k_\beta} e^{-1/g}, \tag{1a}$$

where $\omega$ is the relevant or characteristic phonon frequency, $\hbar$ is the Plank's constant and $k_\beta$ is the Boltzman's constant while the coupling factor g is define by

$$g = \lambda + \mu^* \approx N(E_F)V = \lambda, \tag{1b}$$

where $\lambda$ is the EPI coupling constant, $N(E_F)$ is the density of state at the Fermi level and V is the EPI coupling strength. Note that the effect of the Coulomb repulsive parameter $\mu^*$ is considered negligible, that is, the V is completely dominant.

It is easy to observe in Eq. (1a) that when the coupling constant $\lambda$ is small and the value of $k_\beta T_c$ is of the same order of magnitude as the binding energy of a Cooper pair, at $T > T_c$, the Cooper pairs break apart and the material returns to normal state.

As expected from a successful theory, the BCS $T_c$ formula has been used to obtain the $T_c$ of many conventional superconductors that agree fairly with

experiments. The $T_c$ expression however, has some shortcomings such as its inability to account for the deviation of some superconductors from the isotopic shift predicted in the study of Ref. [38]: the property of isotopic effect in superconductivity is that the $T_c$ is proportional to the isotopic mass, that is,

$$T_c \propto M^{-\beta}. \quad (2)$$

where M is the isotopic mass and β is the isotopic shift. It is obvious in Eq. (1a) that the $T_c$ is proportional to ω. The ω however depends on the ionic mass M, that is

$$\omega = (k/M)^{1/2} \quad (3)$$

where k is a constant.

The implication is that two isotopes of the same superconducting materials have different critical temperatures because as shown by Eqs (2) and (3), they have different bandwidths or frequencies. The β observed for most non-transition metals is about 0.5 while for transition metals, the values are much smaller than the BCS predicted values and nearly absent in some metals like Zr and Ru (β = 0.00 + 0.05). Similarly, the BCS $T_c$ expression could not account for why the gap ratio of some metals deviate from the constant value, i.e. $\Delta_0(0)/K_\beta T_c = 3.52$, predicted for them.

In Eq. (1a), the $T_c$ obviously depends on two factors: $\omega_{ph}$ and λ. An increase in any of these parameters will increase the $T_c$ value. However, the basic assumption as stated above, requires that the λ should be small. This implies that the $T_c$ cannot be increased by arbitrarily increasing the λ. It is this restriction of the value of the λ to small values that makes the BCS theory to be applicable only to weak-coupling superconductivity and hence the limitation of its predicting power to $T_c \leq 25\ K$ [7].

As stated above, to generalize the BCS theory, there have been various attempts in the literature such extension of the EPI to strong coupling regime resulting in the Migdal- Eliashberg Theory [34, 25], polaron formation believed to occur beyond the maximum coupling limit of a normal EPI [36], replacement of the phonons with higher energy bosons (like excitron or charge fluctuations [14], plasmons [15], magnons [16], etc), including the effect of Coulomb repulsion to the original BCS interactive potential [37], including other bands [38], etc. All these attempts have not recorded generally accepted success as expected in their application beyond the conventional superconductors.

### A possible structure model of the CPF in all superconducting materials

A simplistic overview of the emergence of superconducting materials since 1970 for which there is a consensus that they cannot be described by the EPI mechanism of the BCS theory or at least there are serious doubts whether they can, identify at least ten distinct materials or families of materials [6]: (i) the cuprates, hole-doped ($YBa_2Cu_3O_7$) and electron-doped ($Nd_{1-x}Ce0_xCuO_{4-y}$); (ii) heavy fermion materials ($CeCu_2Si_2$, $UBe_{13}$, $UPt_3$); (iii) organics ($TMTSF_2PF_6$); (iv) strontium–ruthenate ($Sr_2RuO_4$); (v) fullerenes ($K_3C_{60}$, $Cs_3C_{60}$); (vi) borocarbides ($LuNi_2B_2C$, $YPd_2B_2C$); (vii) bismuthates ($Ba_{1-x}K_xBiO_3$, $BaPb_{1-x}Bi_xO_3$); (viii) 'almost' heavy fermions ($U_6Fe$, $URu_2Si_2$, $UPd_2Al_3$); (ix) iron arsenide compounds ($LaFeAsO_{1-x}F_x$ ,

$La_{1-x}Sr_xFeAs$); (x) ferromagnetic superconductors ($UGe_2$, $URhGe_2$). It is pertinent to mention the recent prediction of superconductivity in neutron stars [38, 39], alcoholic beverages induce superconductivity in $FeTe_{1-x}S_x$ [40] as well as the possibility of superconductivity in a vacuum [41]. The theoretical proposals to account for the superconductivity in these materials are based on their possible structures and the possible means of naturally achieving the CPF from the appropriate bonding of the elements yielding these structures. The first quantitative representation in line with this thinking emanates from the Santilli's proposal in 1978 [29, 42] to build the foundation of hadronic mechanics wherein a bound state of one electron and one positron at a short distance (< 1 F ~ $10^{-13}$ cm) with non-local, non-linear and non-potential is due to deep overlapping of their wavepackets. Animalu observed that at such distances, the magnetically induced Hulthen potential which is an attractive force will dominate the Coulomb repulsion between two electrons to allow them to bond into singlet coupling as in the CPF in the cuprates [20, 21, 25]. At large distances (> 1 F ), the Hulthen potential no longer dominates and it has been suggested by Akpojotor [25] that the CPF is by superexchange interaction which naturally affects electrons that are close enough to have (no deep) overlapping wavefunctionis and this is purely a quantum mechanical affairs [29]. This dichotomy emanates from the regimes of validity of hadronic mechanic and quantum mechanics. For sufficiently large distances, particles can be described effectively in point-like approximation so that there is the sole presence of action-at-a distance potential interaction which can be represented by a Hamiltonian. This approximate point-like description is no longer valid at sufficiently small distances so that their interaction is dominated by a contact non-potential character. As explained in Ref. [29], the quantum mechanical point-like description is only not applicable in this regime since it is not violated because the condition in the regime was beyond what quantum mechanics was conceived for. Thus the possible structure model of the CPF emanates from the strong valence at short distances of hadronic mechanics while at large distances the equivalent mechanism is by the quantum mechanical superexchange interaction. This is the foundation of the isosuperconductivity model of superconductivity.

It has been suggested in Refs. [20, 21, 25, 43] that the control parameter to drive these interactions which is known in hadronic mechanics as the 'trigger' is the effective valence. Here the 'trigger' assumes the same meaning as the often ambiguous 'under favourable condition' that is required for the EPI to lead to the CPF in the BCS theory. The achievement of the effective valence as the natural favourable condition for the CPF by either the hadronic mechanical strong valence or quantum mechanical superexchange interaction is in line with the common knowledge that in material design, the valence electrons are known to govern the crystal structure [44].

## Application of the iso-superconductivity model

The two classes of superconducting materials with relatively high transition temperatures than the others are the superconducting cuprates and iron based

compounds. Consequently these two classes of materials have received the highest interest of researchers in the field of superconductivity which can be grouped into those seeking for a theory to account for these materials and those seeking for how to increase their $T_c$ to room temperature. Thus our application here of the isosuperconductivity model will be restricted to these two classes of materials. In particular, it is straightforward to apply the model to the superconducting cuprates since they have the $CuO_2$ planes common to all members of the family as the key to understanding them. Further, the superconductivity is driven by the effective valence of the Cu so that a $T_c$ that depends on the z is obtained. The iron based compound, however, do not have a common plane: the consensus feature of members of this family is that the Fe is responsible for their high $T_c$ [43, 45]. Thus a $T_c$ that depends on the z will also be obtained for the superconducting iron based compounds.

### *The superexchange interaction in the $CuO_2$ planes of the superconducting cuprates*

In addition to the centenary celebration of the superconducting phenomenon this year, the international scientific community is also celebrating the silver jubilee of the discovery of superconductivity in the cuprates by Bednorz and Muller in September, 1986. The initial material was the ternary $(La_{1-x}Ba_x)_2CuO_4$ in the 30 K range which is higher than the hitherto highest $T_c$ of 23.2 K of $Nb_3Ge$ discovered in 1973 [46]. Within the first year of the discovery of superconductivity in cuprates, a number of laboratories across the globe had reproduced and even extended not only the $T_c$ but the superconductivity to other copper oxides. The current highest $T_c$ at normal condition is 134 K in $HgBa_2Ca_2Cu_4O_{8+\delta}$ [47].

One early consensus after the discovery of the high $T_c$ superconducting cuprates, is that the key to understanding these materials is the $CuO_2$ planes common to all of them. Another general consensus is the Anderson observation that the superconductivity phenomenon in the cuprates involving the Cu 3d and O 2p bands of the $CuO_2$ plane can be reduced to an effective single band pairing problem via his doped resonant valence bond (RVB) model [48]. Zhang and Rice generalized the Anderson approach by showing that it is possible to obtain the Cooper pair as a bound singlet state of the $CuO_2$ plane within a t-J model [49]. This singlet state now commonly known as the Zhang-Rice singlet (ZRS) which is the Cooper pair of the superconducting cuprates has been achieved [23] by the quantum mechanical superexchange interaction using the first electron removal (FER) approach [50] and a highly simplified correlated variational approach [51] to obtain a t-J model with the interaction part, J being the XY limit of the anisotropic Heisenberg exchange interaction in second quantization language:

$$H_{t_{pd}} = -t_{pd}\left[\sum_{\{i\}}\sum_{<j,k>\in\{i\}} d^{+}_{i_r,\sigma} p^{+}_{j_{r'},\sigma} d_{i_r\sigma} p_{k_r\sigma} + H.C.\right] + J_{dp}\left[\sum_{\{i\}}\sum_{<j,k>\in\{i\}} d^{+}_{i_r,\sigma} p^{+}_{j_{r'},\bar{\sigma}} d_{i_r\bar{\sigma}} p_{k_r\sigma}\right] \quad (4)$$

where $d^{+}(d)$ is the creation (annihilation) of the carrier at the Cu $3dx^2 - y^2$ orbital and $p^{+}(p)$ is the creation (annihilation) of the carrier at the O $2p_x$ and O $2p_y$ orbitals and the $t_{pd}$ denotes a hopping between a Cu and O in the same plane.

This model gives the experimental bandwidth of W ≈ 1 eV [52] for the common value in the literature ($J_{dp}/t_{pd}$ = 0.3) [53]. One important evidence supporting our mechanism of Cooper pairs formation and their propagation is the observation in [54] that the critical $T_c$ of the superconducting cuprates depends on the number of $CuO_2$ planes within a short distance of each other in the structure. The implication is that the smaller the $CuO_2$ planes, the more the enhancement of the hybridization of the carriers in the Cu and O sites into the ZRS due to decreasing distances of their overlapping wavefunctions as expected from the structure mode of the CPF in this current study. Finally we re-emphasize that the description in this section is valid as the quantum mechanical approximation of the hadronic mechanical reality, to which we now turn to.

### *The strong valence CPF and* the $T_c$ expression for *the superconducting cuprates*

The strong valence hadronic mechanical CPF began with the observation by Animalu [20 - 21] that for the strongly correlated high-$T_c$ cuprate materials, $(...)Cu_mO_{n-x}$, the Santilli representation of the neutral pion as a compressed positronium system ($e^+$ $e^-$ ) system [42] is equivalent to a state of mutual overlap/non-orthogonality of the paired Cu 3d and O 2p electrons wavefunctions, $\psi_\uparrow$ and $\psi_\downarrow$ such that $<\psi_\uparrow / \psi_\downarrow > \neq 0$.

By iso-unitary transformation in line with the Santilli Lie-isotopic/Lie admissible approach (29), Animalu was able to transform the BCS theory into a t-J model for the superconducting cuprates [20, 21, 25]. Starting with the Lurie-Cremer [55] quasiparticle wave equation,

$$i\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = H\Psi(\mathbf{r},t), \qquad H \equiv \frac{1}{2m}\mathbf{p}^2\tau_3 + \Delta\tau_1 \quad (5)$$

via the non-unitary ("isotopic lifting") transformation of the underlying "metric" ($g$),

$$g \equiv \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \rightarrow \hat{\tau}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -T \end{pmatrix} \equiv g \qquad (6)$$

which is characterized by a nonlocal integral (pseudopotential) opertar defined by

$$T\psi_{\downarrow}^{*}(\mathrm{r}) = \int d^3 r' \left[\delta(r-r') - \psi_{\uparrow}^{*}(\mathrm{r})\psi_{\uparrow}(\mathrm{r}')\right]\psi_{\downarrow}^{*}(\mathrm{r}') \qquad (7)$$

where $\psi_{\uparrow}^{*}(\mathrm{r})$ and $\psi_{\downarrow}(\mathrm{r})$ are the two spinor components of the quasi-particle wavefunction $\Psi(\mathrm{r},0)$ in the Nambu representation, $p^2/2m$ being the kinetic energy operator (measured from the Fermi level) and $\Delta$ is the pair potential energy. It is apparent from Eq. (7) that when the overlap integrals or "orthogonalization term"

$$Z^{\frac{1}{2}} = \int d^3 r' \psi_{\downarrow}^{*}(\mathrm{r}')\psi_{\uparrow}(\mathrm{r}') \equiv \left\langle \psi_{\downarrow}^{*} \middle| \psi_{\uparrow} \right\rangle \qquad (8)$$

is zero, $T$ reduces to unity and we recover the standard (BCS) model exactly. Since we may rewrite $T$ in the form

$$T = 1 - \left|\psi_{\downarrow}\right\rangle\left\langle\psi_{\uparrow}^{*}\right| \qquad (9a)$$

so that $T^2 = T$ if $\left\langle \psi_{\downarrow}^{*} \middle| \psi_{\uparrow} \right\rangle \neq 0$, the physical effect of $T$ is that the charge on the $e^{-}\uparrow$ representated by the expectation value of T, i.e.,

$$\left\langle \psi_{\downarrow}^{*} \middle| T \middle| \psi_{\downarrow} \right\rangle = 1 - Z \qquad (9b)$$

is "depleted" by an amount Z (called the "orthogonalization charge") whereas the charge on $e^{-}\downarrow$ appears to vanish, i.e.,

$$\left\langle \psi_{\uparrow}^{*} \middle| T \middle| \psi_{\uparrow} \right\rangle = 0 \qquad (9c)$$

In other words, $e^{-}\uparrow$ behaves like a neutral spin-$\frac{1}{2}$ quasiparticle (spinion) while $e^{-}\downarrow$ behaves like a fractionally-charged quasiparticle ("anyon"). Consequently, in the solid state where the wavefunction $\psi_{\sigma}(r,t)$ to which the nonlocal transformation in Eq.(6) is to be applied is related to the $\phi_i(r)$ and $c_{k\sigma}(t)$ of the second quantized formulation by

$$\psi_{\sigma}(r,t) = \sum c_{k\sigma}(t)\phi_i(r) \qquad (10)$$

the corresponding transformation of the corresponding creation and annihilation operators, $c_{k\sigma}^{+}$ and $c_{k\sigma}$, into iso-creation and iso-annihilation operators, is defined by

$$\hat{c}_{ik\sigma}^{+} = T_{ik\sigma} c_{ik\sigma}^{+} \equiv (1 - n_{ik\sigma}), \quad n_{ik\sigma} = c_{ik\sigma}^{+} c_{ik\sigma} \qquad (11)$$

and similarly for $\hat{c}_{i\bar{\sigma}}$ where $\sigma = \uparrow$ for $\bar{\sigma} = \downarrow$ and vice versa. This has the effect of transforming the hopping (kinetic energy) term *exactly* into

$$-t\sum_{\langle ij\rangle,\sigma}\hat{c}^{+}_{i\sigma}\hat{c}_{j\sigma} = -t\sum_{\langle ij\rangle,\sigma}(1-n_{i\sigma})c^{+}_{i\sigma}c_{j\sigma}(1-n_{j\bar{\sigma}}). \tag{12}$$

as in Eq.(4) characterizing the $t-J$ model. It follows that the difference between the $t-J$ model and the isosuperconductivity model lies in the replacement of the U-term in the Hubbard model by the J–term in the t – J model (with the antiferromagnetic exchange constant $J = t^2/U$ via second-order perturbation theory). Typically, *i(j)* = *d, p* label electrons (bands) of Cu 3d and/or O 2p characters whose wavefunctions may overlap and/or bands hybridize; and (i(j) = 1, 2, ...,N) in the nearest-neighbour electron transfer (hopping) integral.

By virtue of the transformation defined by Eq.(11), only single occupancy per spin site is permitted but double occupancy of an orbital site is not forbidden. Another feature of the second-quantized theory form of the iso-creation and iso-annihilation operators is that the waveoverlapping is associated with the coexistence of a non-zero antiferromagnetic spin wave state, $\langle c^{+}_{i\downarrow}c_{i\uparrow}\rangle \neq 0$ and Cooper pair state $\langle c_{i\uparrow}c_{i\downarrow}\rangle \neq 0$ under Gor'kov's factorization of the products of three fermion creation and annihilation operators involved in the transformation

$$\begin{aligned} T_{ij}c_{i\downarrow} &= (1-c^{+}_{i\uparrow}c_{i\uparrow})c_{i\downarrow} \\ &\equiv c_{i\uparrow}c^{+}_{i\uparrow}c_{i\downarrow} \rightarrow \langle c_{i\uparrow}c^{+}_{i\uparrow}\rangle c_{i\downarrow} - \langle c_{i\uparrow}c_{i\downarrow}\rangle c^{+}_{i\downarrow} + \langle c^{+}_{i\uparrow}c_{i\downarrow}\rangle c_{i\downarrow} \end{aligned} \tag{13}$$

In this (mean field) sense, one can derive from the isosuperconductivity model one of the primary objectives of the $t-J$ model which is to describe the coexistence of superconductivity and antiferromagnetismin high-$T_C$ materials as a function of band filling. Thus we have shown here the CPF by either the hadronic mechanical strong valence or quantum mechanical superexchange interaction can be used to formulate a t-J model that can be used to account for the formation and propagation of the ZRS in the superconducting cuprates. The most important difference between the standard t–J model and the iso-superconductivity model lies in the ability of the latter to predict $T_c$ from an exact solution of the model. Another important feature of the isosuperconductivity model is that instead of solving an integral equation for the energy gap as done in the conventional BCS model, the desired result comes from the self-consistent solution of the conventional Schrodinger equation for one spin state,( $\psi_{\downarrow}$ ) say,

$$H\psi_{\downarrow} \equiv (p^2/2m + V_c)\psi_{\downarrow} = E_{\downarrow}\psi_{\downarrow}, \tag{14a}$$

in the Coulomb field $V_C$ of the $Cu^{z+}$ ion "trigger" in Figure. 1b, and an iso-Schrodinger equation

$$HT\psi_{\uparrow} \equiv \left(p^2/2m + V_H\right)\psi_{\uparrow} = E_{\uparrow}\psi_{\uparrow}, \qquad (14b)$$

for the opposite spin state($\psi_{\uparrow}$), where T is the non-local (psuedopotential) integral operator defined by Eq.(7). This has the effect of replacing the Coulomb potential, $V_C$, by an effective Hulthen potential, $V_H$ in Eq.(14b) for the Zhang-Rice singlet ($e-\downarrow, e-\uparrow$):

$$V_C \rightarrow V_C - \frac{E_{\downarrow}\langle\psi_{\downarrow}|\psi_{\uparrow}\rangle\psi_{\downarrow}}{\psi_{\uparrow}} = -V_0\frac{1}{e^{kr}-1} \equiv V_H \qquad (15)$$

where $V_0$ is proportional to $\langle\psi_{\downarrow}|\psi_{\uparrow}\rangle$. From the exact solution of Eq.(14b), Animalu [20, 21] derived the following formula for the critical temperature having the general form:

$$T_C = \frac{\Theta_J}{\left[\exp(\frac{1}{NV})-1\right]} \qquad (16a)$$

where $NV$ reprsents the dimensionless coupling constant while

$$\Theta_J = \frac{\hbar\omega_p}{k_B\sqrt{d\varepsilon(q_D)}} \qquad (16b)$$

is the "jellium" temperature, $d = 1, 2, 3$ being the effective dimensionality of the system and $\varepsilon(q_D)$ the Hatree dielectric function evaluated at the Debye wavenumber $q_D$. We observe that in the weak coupling limit $NV < 1$, we may express the result in the BCS form:

$$T_C = \Theta_J \exp(-1/NV) \qquad (17a)$$

But in the strong coupling limit, i.e. if $NV > 1$, we may expand the exponential in the denominator of Eq.(2.16a) to first order in $1/NV$ to get

$$T_C = \Theta_J NV \qquad (17b)$$

Solving the explicit form of Eq.(17a) used in Ref. 21 for the verification with experimental data in the cuprates with structural formula $(...)Cu_mO_{n-x}$ yields:

$$T_C = \Theta_J \exp(-\frac{13.6}{z}) = \left(\frac{367.3z}{\sqrt{d\varepsilon}}\right)\exp\left(-\frac{13.6}{z}\right) \ (^0K) \qquad (18)$$

where the effective valence z of the $Cu^{z+}$ ion is given by $z = \frac{2(n-x)}{m}$.

Eq. (18) emphasizes the foundation of the iso-superconductivity model for the superconducting cuprate materials, $(...)Cu_mO_{n-x}$, that a $Cu^{z+}$ ion of effective valence, $z \equiv 2(n - x)/m$ provides a "trigger" for the overlapping (i.e., "covalent" mixing) of electron wavefunctions to form a singlet pair, $(e{-}\downarrow, e{-}\uparrow)_{HM}$. (see Figure 1). The results of Eq. (18) which have been published in earlier works [20, 21, 25] and compared with experimental results are shown in Table 1 and Figure 2.

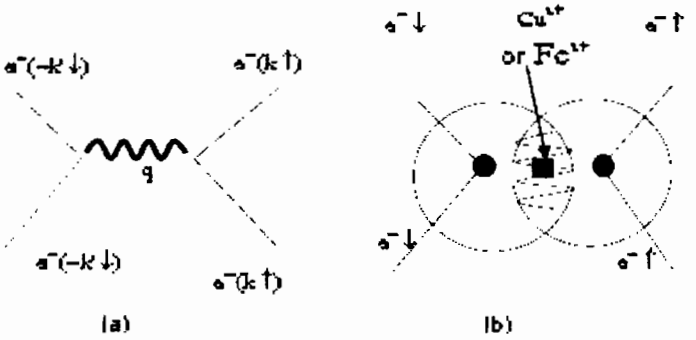



FIGURE 1. (a) Attractive electron-electron interaction mediated by virtual phonon exchange in the conventional BCS model; (b) attractive electron-electron pairing due to deep overlapping of electron wavefunctions around $Cu^{z+}$ or $Fe^{z+}$ ion "trigger" in orbital s(↑,↓) state envisaged in iso-superconductivity model.

### *The Cooper pair formation and $T_c$ prediction in the iron based superconducting materials*

In January 2008, the group of Hideo Hosono in Japan reported that a layered iron arsenide material (LaOFeAs) is superconducting with a transition temperature ($T_c$) of 26 K [56]. The two surprising issues here are [43]: (1) it was believed before then that iron ions in many compounds have magnetic moments and consequently form an ordered magnetic state [57] rather than a superconducting state though the Hosono group had earlier obtained a $T_c$ of 5 K in LaOFeP [58]; (2) The $T_c$ of 26 K is higher than any intermetallic compounds except that of 35 K in $MgB_2$ [59]. Thus it has attracted a deluge of experimental and theoretical studies of the iron based compounds commonly called the iron pnictides (pnictides means compounds of the nitrogen group). Today, a $T_c$ of 55 K has been achieved [60] and there are four major classes of this family of superconducting materials: iron oxypnictides/single layered LnOMPn (Ln = La, Ce, Pr, Nd, Sm, Gd, Tb, Dy, Ho and Y: M = Mn, Fe, Co and Ni: Pn = P and As) ; oxyfree-pnictides/single layered AMPn (A = LnO = Li and Na: M = Mn, Fe, Co and Ni: Pn = P and As); oxyfree-pnictides/double layered ALM2Pn2 (AL = Ba, Sr, Ca: M = Mn, Fe, Co and Ni: Pn = P and As) and chalcogen/non-layered MCn (M = Mn, Fe, Co and Ni: Cn = S, Se and Te).

It has been observed that the superconducting $T_c$ of the iron based compound is critically dependent on extremely small changes in the iron stoichiometry [45]. For example, the non-iron based superconductor, LaONiP is isostructural to LaOFeAs, yet it can be accounted for by electron-phonon coupling [61] while the electron-phonon interaction (EPI) of the iron-based compounds has been shown to be too weak to produce their remarkably high $T_c$ [62 - 63]. This observation led to the speculation that the superconductivity of the iron-based compounds might be related to that of the cuprates. The speculation was boosted by early observation that like the cuprates, the parent compounds of the iron pnictides are antiferromagnetic (AF) and only become superconducting when doped [64 - 65]. Further, just as the three bands problem of the $CuO_2$ planes common to the cuprates can be reduced to an effective single band pairing problem via the Anderson doped resonant valence bond model and its generalization by Zhang and Rice into the t – J model, so also the high- $T_c$ superconductivity in the iron-based compounds known to involve multi-orbital effects of the Fe-3d with filling of approximately six electrons per Fe-site in the pnictides has been shown in the 2009 selfconsistent fluctuation exchange (FLEX) model by Zhang et al [66] to be reducible to an orbital s(↑, ↓) coupling affair also known as the $s_{\pm}$ state [67]. It has been shown that such an $s_{\pm}$ state can be achieved as the CPF from superexchange interaction within a quantum mechanical treatment [43, 68 - 69]. The analogy for the natural description of the CPF within hadronic mechanics is similar to the situation in the foundation of the iso-superconductivity model for the high-$T_c$ cuprate materials: here an $Fe^{z+}$ ion of effective valence of appropriate value provides a "trigger" for the deep overlapping of electron wavefunctions to form the $s_{\pm}$ state $(e-\downarrow, e-\uparrow)_{HM}$ (see Figure 1). A common evidence to support this mechanism is the observation that for FeSe which is the simplest member of the iron-based compounds, its structure and magnetic properties depend sensitively on the ratio of Se:Fe [57]. For example its $T_c$ of 8 k can be pushed up to 14 K simply by replacing Se with Te. Since $Se^{2-}$ and $Te^{2-}$ have the same valence but different ionic radii, the substitution does not directly lead to charge carrier doping but to a new effective valence of the $3d^{z+}$.

In order to compare with experimental data for the iron based compounds as done for the cuprates, we now turn to a realization of the formula in Eq.(17b) in a similar form [25]:

$$T_C = 467.0 z \exp\left(-\frac{13.6}{z}\right)(^0 K) \tag{19}$$

where 467.0 is the experimental Debye temperature of iron (see Table 1). It is also plotted alongside the result for the cuprates in Table 1. There is good agreement with the experimental data in the iron based compounds as shown in Table 1 and Figure 2. See also the fair comparison with the result from neutron scattering (see Figure 3) [70].

TABLE 1. Dependence of $T_c$ on the effective valence z of $Cu^{z+}$ in the cuprates and $Fe^{z+}$ in the iron pnictides.

| z | $T_c(Cu^{z+})$ | $T_c(Fe^{z+})$ | | | |
|---|---|---|---|---|---|
| 1.0 | 0.0005 | 0.0006 | 4.0 | 49.0320 | 62.2413 |
| 1.5 | 0.0636 | 0.0809 | 4.1 | 54.6032 | 69.4247 |
| 2.0 | 0.8182 | 1.0403 | 4.2 | 60.5318 | 76.9625 |
| 2.5 | 3.9847 | 5.0663 | 4.3 | 66.8201 | 84.9477 |
| 3.0 | 11.8397 | 15.0535 | 4.4 | 73.4698 | 93.4125 |
| 3.2 | 16.7656 | 21.3165 | 4.5 | 80.4820 | 102.328 |
| 3.4 | 22.8729 | 29.0816 | 5.0 | 120.9790 | 153.817 |
| 3.5 | 26.3964 | 33.5614 | 5.5 | 170.4087 | 216.664 |
| 3.6 | 30.2451 | 38.4549 | 6.0 | 228.4397 | 290.447 |
| 3.8 | 38.9482 | 49.5203 | | | |

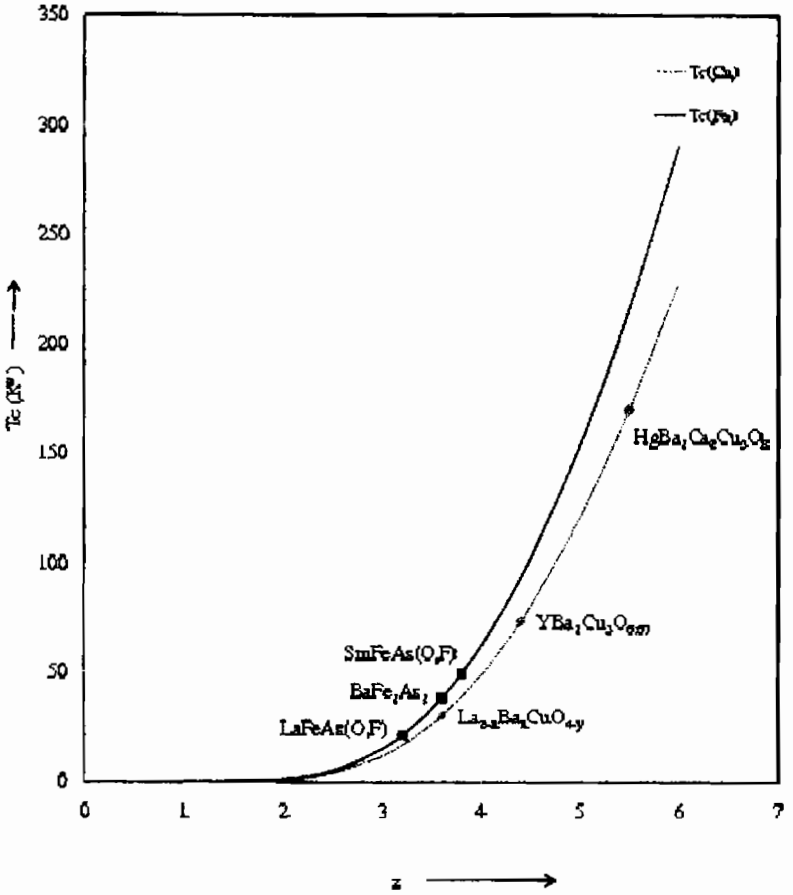



FIGURE 2. (Colour online) Predicted dependence of the transition temperature $T_c$ on the effective valence z for the cuprates in Eq.(18) and the iron based superconductors in Eq.(19)

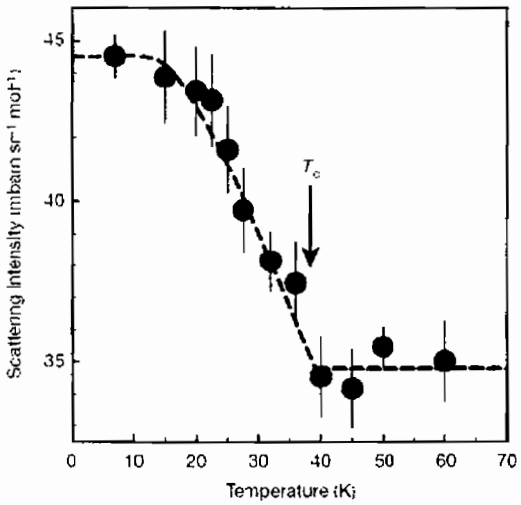



FIGURE 3. (Colour online) Experimentally predicted $T_c$ for $Ba_{0.6}K_{0.4}Fe_2As_2$ [70]

## Summary and Conclusion

The absence of the electron-phonon interaction mechanism of the BCS theory in other areas of particle physics [29], its limitation to conventional superconductors and predictive power to 25 K [7] and a number of other flaws (see Ref. [6]) instigate the need to seek a more natural mechanism for the formation of the Cooper pair. The natural description is the foundation of the iso-superconductivity model , wherein the $Cu^{z+}$ ion of effective valence, $z \equiv 2(n - x)/m$ or an $Fe^{z+}$ ion of effective valence of appropriate value provides a "trigger" for the deep overlapping of electron wavefunctions to form the Cooper pair ($e-\downarrow$, $e-\uparrow$) for both the cuprates and iron based compounds respectively. An obvious open problem now is how to design real materials from the predictive values of the effective valence. This requires further investigation on how parameters from the periodic table database, electronegativity spectrum and material database can be used to obtain the effective valence of Cu and Fe as the control parameter for both the superconducting cuprates and iron based compounds [43]. It will also be interesting to investigate the effect of doping on the iso-superconductivity model for non-conventional superconductors. Speculatively, if the approach of assuming the Coulomb interaction as a screened interaction mediated by a Bosefield is to merely allow one to define different approximations as pointed out in Ref [26], such that the electron-phonon interaction of the BCS theory is a mathematical artifact as being canvassed by some workers [6, 8, 29], then the electron-phonon coupling constant will also be a mathematical artifact that has been simulating some material specific parameters of the effective valence. These are important investigations to be made on the foundation of iso-superconductivity in analogy between the Santilli's model of the

neutral pion as a compressed positronium atom which is the foundation of hadronic mechanics. It is hoped that such studies under the subject-matter of the isotopic branch of "hadronic" mechanics will have far-reaching implications not only for superconductivity physics beyond its first 100 years but also general physics in the 21st Century and with the ultimate goal of remarkable technological harvest.

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## REFERENCES

1. R. Hott, R. Kleiner, T. Wolf and G. Zwicknagl in *Frontiers of superconducting materials*, edited by A. V. Narlikar, Berlin: Springer, 2005.
2. J. Schmalian, *in Bardeen Cooper and Schrieffer: 50 YEARS,* edited by L. N. Cooper and D. Feldman, Singapore: World Scientific Publishing Co Pte Ltd, pp 41 - 55, 2011.
3. J. C. Carbotte, *Rev. Mod. Phys. 62, 1027* (1990).
4. K. H. Bennemann and J. B. Ketterson, in *Superconductivity Vol 1: Conventional and Unconventional Superconductors,* edited by K. H. Bennemann and J. B. Ketterson, Berlin: Springer, 2008, pp. 3 - 25.
5. W. Buckel and R. Kleiner, *Superconductivity: Fundamentals and Applications (Physics)* (2nd, Revised and Enlarged Edition), Berlin: Wiley-VCH, 2004.
6. J E Hirsch, Phys. Scr. 80 0357021 (2009)
7. V.L. Ginzburg and D.A. Kirzhnits, *in Superconductivity, Supermagmetism, Superfluidity,* Moscow, Mir Publishers, 1987 pp11-17.
8. T Matthias Physica 69 54 (1973)
9. J. G. Bednorz and K. A. Mueller, Z. Phys. B 64(2), 189 (1986)
10. W. Weber and L.F. Mattheiss, Phys. Rev. B 37, 599 (1988)
11. R. Lal and S. K. Joshi, Phys. Rev. B 45, 361 (1992).
12. J P Hague J. Phys.: Conf. Ser. 92 012119 (2007).
13. R. Heid, K. P. Bohnen, R. Zeyher and D. Manske, Phys. Rev. Lett 100, 137001 (2008).
14. A. Pashkin, M. Porer, M. Beyer, K. W. Kim, A. Dubroka, C. Bernhard, X. Yao, Y. Dagan, R. Hackl, A. Erb, J. Demsar, R. Huber and A. Leitenstorfer, Phys. Rev. Lett. 105, 067001 (2010)
15. V.S. Kresin and H. Moranwitz, Physica C 153, 1327 (1988).
16. J.R. Schrieffer, X.G. Wen and S.C. Zhang, Phys. Rev. B 39, 11663 (1989).
17. J. D. Lee, K. Kang and B. I. Min, Phys. Rev. B 51, 3850 (1995).
18. M. L. Kulic, Physics Reports 338 1 (2000).

19. O. Gunnarsson and O. Rosch, Phys. Rev. Lett. 92, 146031 (2004): O. Gunnarsson and O. Rosch, J. Phys. Condens. Matter 20, 043201 (2008).
20. A.O.E. Animalu, Hadronic J. 14, 459 (1991); A.O.E. Animalu, Hadronic J. 17, 349 (1994).
21. A.O.E. Animalu and R.M. Santilli, Int. J. Quantum Chemistry 29, 175 (1995).
22. P. W. Anderson, *The Theory of Superconductivity in the High-Tc Cuprates,* Princeton: Princeton University Press, 1997.
23. G. E. Akpojotor, Physics Letters A 372, 6992 (2008).
24. P. A. Lee, Rep. Prog. Phys. 71, 012501 (2008)
25. A.O.E. Animalu and G. E. Akpojotor, AJP 2, 46 (2009)
26. M. Ogata and H. Fukuyama, Rep. Prog. Phys. 71, 036501 (2008)
27. E.W. Carlson, V. J. Emery, S.A. Kivelson, and D. Orgad in *Superconductivity Vol. 2: Novel Superconductors,* edited by K. H. Bennemann and J. B. Ketterson, Berlin: Springer, 2008, pp. 1225 - 1327.
28. O. Stockert, J. Arndt, E. Faulhaber, C. Geibel, H. S. Jeevan, S. Kirchner, M. Loewenhaupt, K. Schmalzl, W. Schmidt, Q. Si and F. Steglich, Nature Physics 7 119 (2011).
29. I. Gandzha. and J. Kadeisvily, *New sciences for a new era: Mathematical, Physical and chemical Discovery of Ruggero Maria Santilli,* Florida: The R. M. Santilli Foundation, 2010, pp. 262 – 272.
30. Z. Tesanovic, Nature Phys. 4, 408, (2008)
31. J. Bardeen, Theoretical Part. Handbuch der Physik 15, 274 (1956).
32. C.A.Reynolds, B. Serin, W.H Wright,. and L.B.Nesbitt, Phys. Rev. 78, 487 (1950).
33. E. Maxwell, Phys. Rev. 78, 477 (1950)
34. A. B.Migdal, Sov. Phys. JETP 7, 996 (1958).
35. G.M. Eliashberg, Sov. Phys. JETP 11, 696 (1960).
36. A.S. Alexandrov and V.V. Kabanov, Phys. Rev. B 46, 14932 (1992).
37. G. C.. Asomba, Physica C 245 355 (1995).
38. P. S. Shternin, D. G. Yakovlev, C. O.Heinke, W. C. G Ho and D. J. Patnaude Mon. Not. R. Astron. Soc. 000, 1 (2010)
39. D. Page, M. Prakash, J. M. Lattimer and A. W. Steiner, Phys. Rev. Lett. 106, 081101 (2011).
40. K. Deguchi, Y. Mizuguchi,, Y. Kawasaki, T Ozaki,, S. Tsuda, T. Yamaguchi and Y. Takano, Supercond. Sci. Technol. 24 055008 (2011)
41. M. N. Chernodub, Phys. Rev. Lett. 106, 142003 (2011).
42. R.M. Santilli, Hadronic J. 1, 574 (1978). See also Foundations of Hadronic Chemistry with Applications to new Clean Energies and Fuels (Kluwer Academic Publishers, Dordrecht, Boston, London, (2001).
43. G. E. Akpojotor, J. of NAM P 15, 297 (2009).
44. K. Shiraishi, H. Tamura and H. Takayanagi, Appl. Phys. Lett, 78 3702 (2001).
45. T. Yildirim, Phys. Rev. Lett. 102, 037003 (2008).
46. J. R. Gavaler, Appl. Phys. Lett. 23, 480 (1973).

47. A. Schilling, M. Cantani, J. D. Guo and H. R. Oti, Nature 362 56 (1993).
48. P. W. Anderson, Science 235 1196 (1987).
49. F.C. Zhang and T.M. Rice, Phys. Rev. B 37 3759 (1988).
50. C. Durr, S. Legner, R. Hayn, S.V. Borisenko, Z. Hu, A. Theresiak, M. Knupfer, M.S. Golden, J. Fink, F. Ronning, Z.-X. Shen, H. Eisaki, S. Uchida, C. Janowitz, R. Muller, R.L. Johnson, K. Rossnagel, L. Kipp, R. Reichardt, Phys. Rev. B 63, 014505 (2000).
51. G.E. Akpojotor, in *Lectures on the Physics of Highly Correlated Electron Systems XII* (edited by A. Avella, F. Mancini), AIP Conf. Proc. 1014, American Institute of Physics, Melville, NY, 2008, pp. 251 - 259.
52. A. Ino, C. Kim, M. Nakamura, T. Yoshida, T. Mizokawa, Z.-X. Shen, A. Fajimori, T. Kakeshita, H. Eisaki, Uchida, Phys. Rev. B 62 4137 (2000).
53 B. Edegger, V.N. Muthukumar, C. Gros, P.W. Anderson, Phys. Rev. Lett. 96 207002 (2006)
54. R. Thomale and M. Greiter, Phys. Rev. B 77 094511 (2008).
54. E. Dagotto, Rev. Mod. Phys. 66 763 (1994).
55. D. Lurie and S. Cremer, Progr. Theor. Phys. 44, 300 (1970).
56. Y. Kamihara, T. Watanabe, M. Hirano, and H. Hosono, J. Am. Chem. Soc. 130, 3296 (2008).
57. M. H. Fang, H. M. Pham, B. Qian, T. J. Liu, E. K. Vehstedt, Y. Liu, L. Spinu and Z. Q. Mao, Phys. Rev. B 78, 224503 (2008).
58. Y. Kamihara, H. Hiramatsu, M. Hirano, R. Kawamura, H. Yanagi, T. Kamiya, and H. Hosono, J. Am. Chem. Soc. 128, 10012 (2008).
59. J. Nagamatsu, N. Nakagawa, T. Muranaka, Y. Zenitani and J. Akimitsu, Nature 410, 63 (2001).
60. Z A Ren, W Lu, J Yang, W Yi, X L Shen, Z C Li, G C Che, X L Dong, L L Sun, F Zhou, and Z X Zhao, Chin. Phys. Lett. 25, 2215 (2008).
61. A. Subedi, L. Zhang, D. J. Singh and M. H. Du, Phys. Rev. B 78, 060506 (2008).
62. L. Boeri, O. V. Dolgov, and A. A. Golubov, Phys. Rev. Lett. 101, 026403 (2008).
63. K. Haule, J. H. Shim, and G. Kotliar, Phys. Rev. Lett. 101, 097010 (2008).
64. M. Norman, Physics 1, 2 (2008)
65. I. I. Mazin, M. D. Johannes, L. Boeri, K. Koepernik and D. J. Singh, Phys. Rev. B 78, 085104 (2008).
66. J. Zhang, R. Sknepnek, R. M. Fernandes and J. Schmalian, arXiv:0903.4473
67. I. I. Mazin, D.J. Singh, M.D. Johannes, M.H. Du, Phys. Rev. Lett. 101 057003 (2008)
68. T. Yildirim, Phys. Rev. Lett. 101, 057010 (2008); Physica C 469 425 (2009)
69. J. Zhao, D. T. Adroja, Dao-Xin Yao, R. Bewley, S. Li, X. F. Wang, G. Wu, X. H. Chen, J. Hu and P. Dai, Nature Physics 5 555 (2009)
70. A. D. Christianson, E. A. Goremychkin, R. Osborn, S. Rosenkranz, M. D. Lumsden, C. D. Malliakas, I. S. Todorov, H. Claus, D. Y. Chung, M. G. Kanatzidis, R. I. Bewley and T. Guidi, Nature 456, 930 (2008).

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# It's About Time

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**Abstract.** Some simple, fundamental questions in basic physics remain unanswered. Many of these relate to the nature of time. This article argues that those questions can only be answered if we abandon the unique space-time manifold of relativity and consider instead a connected, multi-metrical structure. Mass, length and time are introduced as the primary dimensions of three, distinct metrical spaces, which are inter-connected by particle wave functions. Then the laws of quantum mechanics and the symmetry of relativity both arise as consequences. But there are other consequences, which describe the quantisation of collective, periodic motion and the free rotation of solid objects. The gaps between classical and quantum mechanics are bridged, providing logically complete descriptions of such processes as heat flow and measurement in quantum mechanics – and resolving the fate of Schrödinger's cat. The basic mechanisms of causal time and thermodynamic time, within the structure of matter, are made apparent.

**Keywords:** Historical time, multi-metric, inter-metrical connections, Tri-space
**PACS:** <Missing classification>

**PACS:** 03.30.+p , 03.65.Fd , 03.65.Pm , 03.65.Ta , 03.65.Ud , 03.65.Yz

## INTRODUCTION

In spite of our advanced state of knowledge in particle physics and the physics of materials, some very simple questions in fundamental physics have never been successfully answered. Many of these relate, in some way, to the nature of time. In this article, I develop the suggestion that we should go beyond relativity and think in terms of inter-connected metrical spaces, in order to answer these.

*The local arrows of time, within matter*

- How does time manage to elapse, always and everywhere in the same direction, if it is just a dimension on a metric?

I am thinking here about causal time and thermodynamic time. These both seem to be associated with particulate matter. Only massive particles that bind together by attractive forces can generate elapsed (historical) time, or have a thermodynamic temperature. Yet relativity assigns measurable space and time co-ordinates to all frames of reference, whether or not they contain matter.

*The Schrödinger equation and quantum mechanics*

- How is time in the Hamiltonian of quantum mechanics (the frequency of oscillation of a particle wave function) related to time on a metric?

- Why do physicists (who believe in relativity) still use the Schrödinger / Pauli equations to describe multi-particle systems (when these equations are inconsistent with relativity)?

*Quantum field theory* is not based on actual particle wave equations, but on a set of arbitrary rules applied to waves propagating on (multiple copies of) the local metric.

- But what causes the particles to obey those rules?
- What is a measurement, in quantum mechanics?
- How are observers "outside the system" and how should such observers be described?
- How and when was the fate of Schrödinger's cat sealed?

In this famous paradox, a live cat is put into a closed box, together with a quantum trigger that spills a phial of poison, so that the cat will die if a single particle decays. Thereafter, the system is described in quantum mechanics as a superposition of states in which the cat is both alive and dead at the same time.

*Quantisation and heat*

- How and why does the quantisation of periodic motion occur, when the classical equations of motion are quite perfect (and not quantised)?
- Where does quantisation set in, as systems become smaller / simpler?
- How is thermal motion distinct from kinematical motion (ie how is heat distinct from free energy, at the fundamental level)?

I show here that these questions can all be fully resolved, but that these issues cannot be fixed after the metrical structure of space and time has been over-simplified.

## THE ORIGIN OF QUANTISATION - A WORKING METRIC FOR SPACE AND TIME

I now ask you to suppose that **mass, length and time arise as the primary dimensions of three, different metrical spaces, and that these spaces connect together to form the elementary particles** (depicted in Figure 1). Then measurable events in space and time must coincide with discrete changes involving massive particles.

A massive particle has a wave function that spreads out in space, over time. It can interact at any point where it has a finite wave amplitude, and that interaction changes the spatial wave function everywhere, instantaneously. No information can be passed by this collapse of the wave function, so the speed of light is not exceeded by any signal. However, it does spread the particle irreversibly outwards into the space, leading to the causal arrow of time. The collapse can never be undone. This phenomenon can be understood in terms of two, "complementary metrics" working together.Time is the principal dimension of the global, driving metric "tempospace", which connects to the other metrical spaces when a particle is created. Length is the principal dimension in the local space of 3-dimensional projections - "real space". Mass arises as the frequency of oscillation of an unseen, oscillation space - "modespace", giving the same mass values to all particles that connect in the same way, throughout all of space and time. Inertial mass is equivalent to the total driven frequency, times the Planck constant divided by the square of the speed of light, giving about $10^{50}$ Hz/Kg. The resulting, entire, multi-metrical structure is called "tri-space".

### Complementary temporal and spatial metrics

The global, driving metric involves time and distance intervals in empty space $(dt;dr_t)$ (denoting small intervals of time and distance corresponding to the same driven frame of reference). The local space of projection involves physical length intervals and an empty time dimension $(dr_s;dr_0)$ (denoting small intervals of length and relative time corresponding to the same mapping), with a separate mapping for every separate particle. These give two, distinct, positive intervals, both involving the speed of light (c):-

$$c^2d\tau^2 = c^2dt^2 - dr_t^2 \text{ locally } (>0 \text{ for physical intervals})$$

$$ds^2 = dr_s^2 - c^2dr_0^2 \text{ locally } (>0 \text{ for physical intervals})$$

$$\text{and } dr_s.dr_t = c^2dtdr_0 - dr_t.dr_s = 0 \text{ locally.}$$

The last equation shows that these two metrics are fully complementary and locally orthogonal, in all physical frames of reference. There are 8 space and time dimensions involved, but only 4 are real (having the same sign as the corresponding interval) – time $t$ and local space $r_s$. These define the local rest frame for the particular particle or

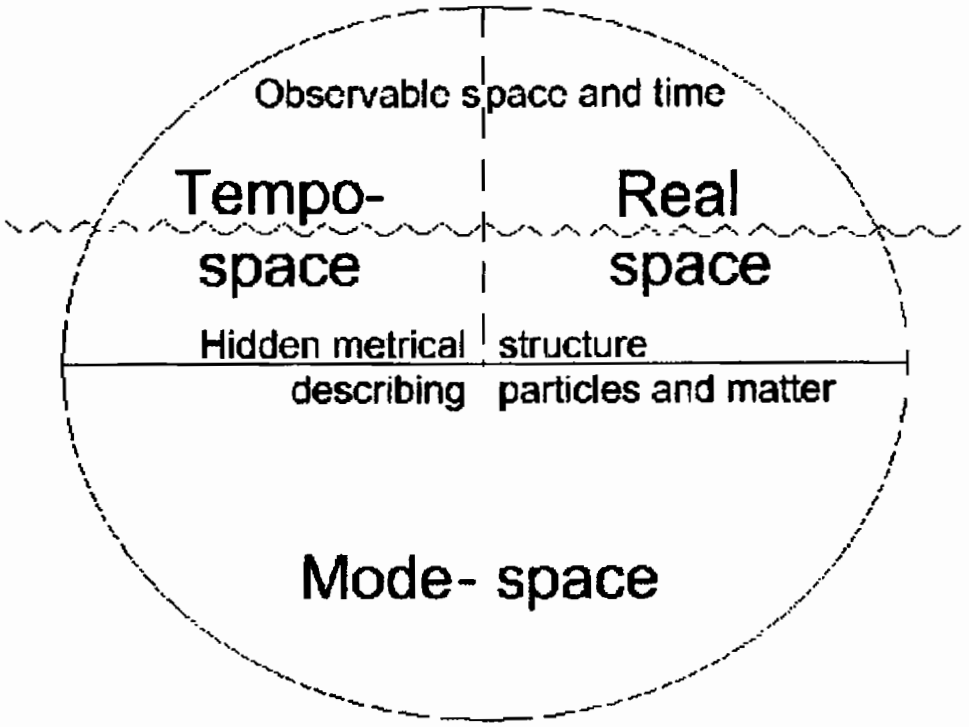



**FIGURE 1.** The Iceberg Diagram

object. Dimensions that make a negative contribution to their interval are "virtual" – like empty space $r_t$. (Metrics are physically real and cannot be related by the square root of -1.)

In a co-moving frame, both metrics make the same Lorentz transformation, so they remain orthogonal and they are not mixed with each other. Separate vector and scalar operators can be constructed for the two metrics, eg the gradients $\partial_t = \left(\frac{\delta}{c\delta t}, \frac{\delta}{\delta r_t}\right)$ and $\partial_s = \left(\frac{\delta}{\delta r_s}, \frac{\delta}{c\delta r_0}\right)$, giving distinct scalar contractions:-

$$\partial_t^2 = \left(\frac{1}{c^2}\frac{\delta^2}{\delta t^2} - \frac{\delta^2}{\delta r_t^2}\right) \qquad \partial_s^2 = \left(\frac{\delta^2}{\delta r_s^2} - \frac{1}{c^2}\frac{\delta^2}{\delta r_0^2}\right)$$

The first describes the square of the temporal frequency operator; the second is the spatial Laplacian operator; both are made covariant, without mixing.

## Particle wave functions and the role of energy

Energy on the driving metric is just the Planck constant times the driving frequency. A particle is a discrete connection of the driving metric to the other metrical spaces. Elementary particles must connect all of the metrics, but due to symmetries in the topology of connection, not all particles reveal all of the metrical structure. Particles with no mass cannot reveal structure in modal space, for example.

The particle "wave function" emerges as the amplitude of inter-metrical connection – not a physically tangible wave. The amplitude of a wave function has no physical meaning – just the topology of the inter-metrical connection that it represents. Two connections with the same topology must be identical particles, so where these are driven from the same rest frame, they can interfere and the wave functions must have a definite, spatial exchange symmetry. In theory, they must have separate exchange symmetries in each connected metrical space: then the product of these separate exchange symmetries gives the overall exchange symmetry – odd for fermions and even for bosons. (By implication, complete particles must be even in modespace.)

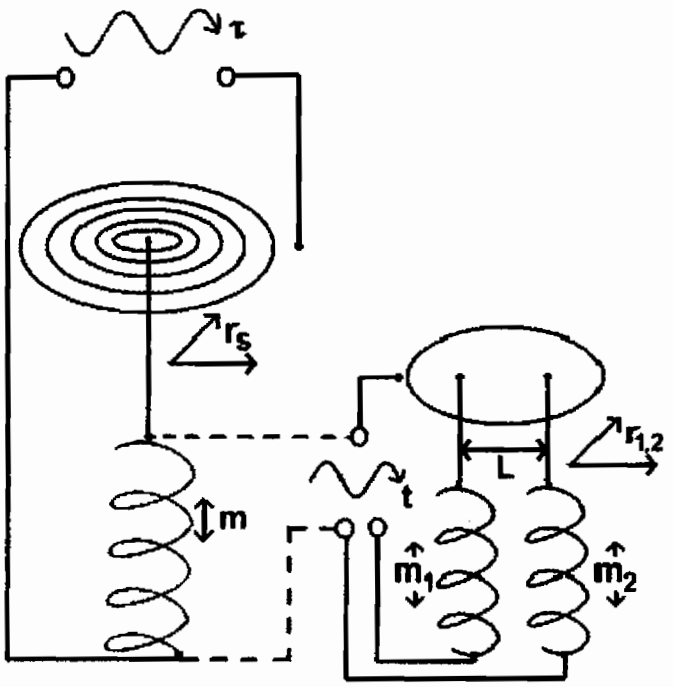



**FIGURE 2.** Depicting the General Connection. The left side shows an isolated mass $m$ (of frequency $mc^2/\hbar$) driven externally ($\tau$) and projected into its real space $r_s$. The alternative connection on the right shows two point masses driven as a bound state (of total mass $m$), with internal time $t$ and size $L$, in the relative space $r_{1,2}$. Then $r_{1,2}$ and $t$ define the local space-time metric.

But what is the role of energy, in the different metrical spaces? When connected, each space represents a distinct form of internal energy (not potential or interaction energy), so that the driving metric provides the total energy supplied to the other metrics. Point mass is the frequency of oscillation in modespace and overall inertial mass corresponds to the driven frequency, in the rest frame of a massive object. The space of projection has its own energy, corresponding to the motion of the projected wave function. This is the kinetic energy associated with directionless, wave-like motion. It includes orbital and thermal motion, which cannot be measured directly. This connection is depicted on the left in Figure 2.

## Particle wave equations

The distribution of energy is expressed by the "General Connection Equation" (GCE for short), which must describe all elementary particles and involves a wave function $\psi(r_s;r_t)$. This resembles the known particle wave equations, but it connects multiple metrical spaces:-

$$\begin{aligned}\text{tempospace energy} &= \text{constant driving energy } (i\hbar\tfrac{\delta}{\delta\tau}\psi)\\ &= \text{real space energy} + \text{modespace energy}\\ \hbar c\sqrt{-\partial_t^2}\,\psi(r_s;r_t) &= E_0\psi(r_s;r_t) = (c\sqrt{-\hbar^2\partial_s^2+m^2c^2}+V_0)\psi(r_s;r_t)\end{aligned}$$

where i denotes $\sqrt{-1}$ and $\hbar$ is the Planck constant divided by $2\pi$.

The corresponding, free particle dispersion relation (with the potential $V_0 = 0$) involves the temporal momentum $\left(p = -i\hbar\frac{\delta}{\delta r_t}\psi\right)$ and the spatial momentum $\left(k = -i\hbar\frac{\delta}{\delta r_s}\psi\right)$ (whose distinct roles are indicated in Figure 3):-

$$E^2 - p^2c^2 = E_0^2 = k^2c^2 - k_0^2 + m^2c^4$$

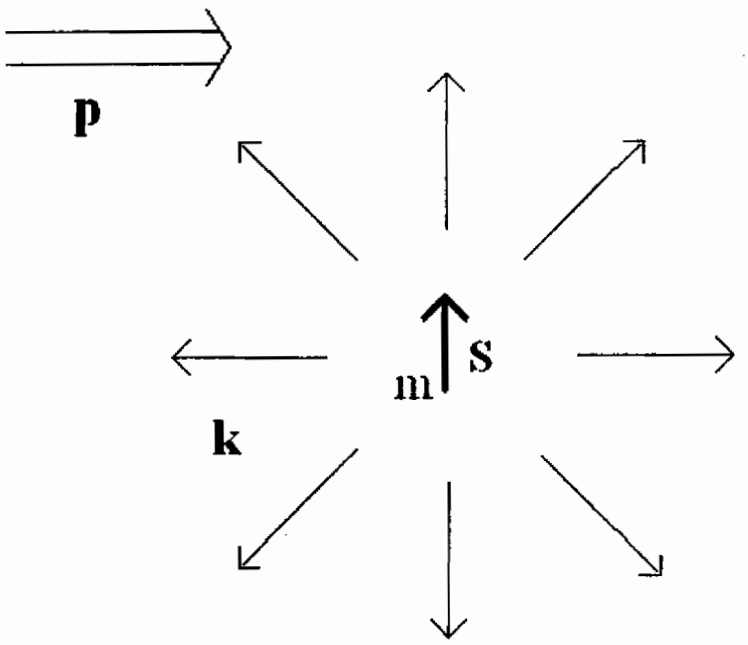



**FIGURE 3.** Free particle wave function. A point particle of mass $m$ and spin S spreads outwards with spatial momentum k

The former is directed, according to the relative velocity. The latter is directionless, or rotational, but always with zero average. There can be no rotational vectors on the driving metric and no translational vectors on the local metric. Thus physical vectors like particle currents have 8 components, with 4 in each metric, but only 6 are independent (making "full-vectors").

In the co-moving frame of massive particles, the GCE simplifies to:-

$$i\hbar\frac{\delta}{\delta t}\psi(r_s;t) = E_0\psi(r_s;t) = \left(-\frac{\hbar^2}{2\mu}\frac{\delta^2}{\delta r_s^2} + mc^2 + V(r_s)\right)\psi(r_s;t)$$

where $2\mu = 2m + \frac{E_s}{c^2}$ and $E_s$ is the real space energy ($E_s = E_0 - V - mc^2$) and $V(r_s)$ is the interaction potential between the particles.

This is equivalent to the Klein-Gordon equation, in time and the local spatial dimensions. It becomes the Schrödinger equation in the non-relativistic approximation (where $2\mu \to 2m$ and ignoring the $mc^2$ term), linear in time but quadratic in the local spatial dimensions. These are not the dimensions that are mixed together in special relativity.

For massive, point fermions, replace $\partial_s^2$ with $(\sigma.\partial_s)^2$ in the GCE, where $\sigma$ are the Pauli spin matrices, describing intrinsic spin $\frac{1}{2}\hbar$. The result is equivalent to the Dirac equation, or to the Pauli equation in the non-relativistic approximation. Intrinsic spin is a polar vector associated with tempospace.

The free solution for $\psi(r_s;r_t)$ in the GCE is just a plane wave in the driving metric, corresponding to definite energy and linear momentum $(E,p)$, multiplied by a spherical, outgoing wave in the local metric, corresponding to a definite radial momentum (of the form $\frac{1}{r_s}\exp(\frac{-i}{\hbar}\int(p.dr_t - k.dr_s))$ depicted by Figure 3). Only the outgoing spherical wave solutions are physical, because any incoming wave encounters a singularity at the origin $r_s = 0$. **Thus free object wave functions always expand in their local metrics, defining the forward sense of time.** Momentum is conserved in both spaces, but when objects collide or particles scatter, kinetic energy flows from the linear momentum ($p$) to the spatial momentum ($k$).

The wave function for multiple particles is the product of the single particle wave functions, so that energy and momentum are added on the driving metric, but the local metrics are distinct. Where identical particles are driven with the same temporal co-ordinates, the spatial wave functions must be constructed with the required exchange symmetry. These are well-known features of quantum mechanics which are also used in quantum field theory.

For zero mass fermions, the GCE corresponds to the Weyl equations for neutrinos, or simply $(\partial_t^2 - \partial_s^2)\psi = 0$ for bosons. Particles with no mass cannot distinguish tempospace from real space – they appear as one.

The Maxwell equations for electromagnetism are also consistent with this multi-metrical structure, with temporal electric potentials and real spatial magnetic vector fields, making a full-vector electromagnetic potential in the rest frame of any massive object. The electromagnetic interaction is just the full-vector contraction of this with the vector currents of charged particles.

So the general connection gives rise to the known equations of single particle quantum mechanics, and causal reality can be effected by two, complementary metrics working together. But how can this be consistent with special relativity, where space and time intervals are mixed together (by a Lorentz transformation) for observers in relative motion?

## COMPOSITE MATTER AND THE SEPARATION OF SPATIAL AND TEMPORAL METRICS

So far, I have just described individual, point particles, which move about and spread out independently. But when particles attract one another, and their wave functions overlap, there is a chance that they will bind into a composite object. When two or more particles bind together, a "separation of metrics" occurs. Then their relative positions and their collective centre-of-mass position are projected independently, giving separately-driven, real space wave functions.

The first determines the structure and mass of the composite object and the second determines its collective motion, so that each component mass is driven once at each level of structure. That doesn't mean double the point mass! It means that the two tempospace connections ("tempodrivers") are nested (with one driving the other – see Figure 2), where before they were independent connections to tempospace. Such a separation can only occur between complementary metrical spaces and for particles with mass.

### Separated particle wave equations

For component objects, the general connection becomes:-

$$i\hbar\frac{\delta}{\delta t}\phi(r_{1,2};t) = Mc^2\phi(r_{1,2};t) = \left(-\frac{\hbar^2}{2\mu_{1,2}}\nabla_{1,2}^2 + V(r_{1,2}) + m_1c^2 + m_2c^2\right)\phi(r_{1,2};t)$$

here $\nabla_{1,2}^2 = \frac{\delta^2}{\delta r_{1,2}^2}$ for the relative real spatial coordinates, and the reduced mass $\mu_{1,2}$ is derived (for the case of 2 bodies):-

$$\frac{1}{2\mu_{1,2}} = \frac{1}{2m_1 + E_{s1}/c^2} + \frac{1}{2m_2 + E_{s2}/c^2}$$

The internal tempodriver has no separate freedom to move around. It is just a constant frequency associated with the bound object (of magnitude $Mc^2/\hbar$).

But the external tempodriver is free, so that the composite behaves as a single object, of net mass $M < (m_1 + m_2)$. The missing mass corresponds to the net binding energy of the particles, by Einstein's equivalence formula. (The equivalence of mass and energy is explicit in all forms of multi-metrical connection, arising from the GCE.) Then the scope of the GCE is extended so that "modespace energy" becomes "internally driven energy". Tempodrivers for composites have no direct connections to modespace. Developing this idea, we see that any internal mode of harmonic oscillation within matter can be independently tempo-driven.

### Special relativity

But the composite object has a finite size which projects into tempospace as a proper length interval (actually just a spherical atom at this level of structure). Thus objects combined by the separation of metrics can acquire fundamental properties that did not exist in the separate components. This cannot occur in a theory based on a single metric.

Two-particle composites are just the simplest case – any number n of particles can combine by a single separation of metrics, in which case there are $(n-1)$ relative real spaces. These are each 3-dimensional, and each has a separate, internal connection to tempospace, but the composite still has a single external wave function. This idea is already established in physics, as the quantum mechanics of Hilbert spaces. However, this method has never before been reconciled, either mathematically or philosophically, with the theory of relativity.

By binding many copies of the simplest composites (atoms) into an extended structure, macroscopic, solid objects are made – including the clocks and measuring rods that we can use to assert the validity of special relativity. We can ignore the wave functions of macroscopic objects (which usually are very small), and make discrete, point measurements separated by measured, proper length and time intervals. Observers in relative rectilinear motion will judge such space and time intervals to be mixed by a Lorentz transformation – **because both kinds of interval are now embedded into tempospace.**

## Matter with multiple levels of structure

The separation of real space metrics cannot be described by a continuous transformation of the space and time coordinates, but at non-relativistic energies, the continuous approximation (as described by the Schrödinger equation in quantum mechanics) is a very good one. To describe multiple levels of structure, we get a nest of separated metrics, giving a distinct wave equation for each level.

So, when I look at the palm of my hand, what type of length am I seeing? The answer must be:- both types, superimposed together. The fixed, inert length corresponds to tempospace, but all of the dynamical processes occurring in the same region are in separate mappings of real space. **There are no unique space-time coordinates with which to describe the fundamental processes of physics**, not even locally.

Causal time elapses when metrics separate or re-connect, making external wave function propagation between re-connection events (which always involves expansion in real space) irreversible. The point particle masses determine the frequency of oscillation, and thus the universality of the rate at which time passes, in any material rest frame. These point masses must be determined by the structure of modespace. Thus mass, length and time correspond to the primary dimensions of three, distinct metrics. Historical time cannot elapse without including the effects of all three metrics. Special relativity and quantum mechanics are both consequences of this underlying structure – but there are many other interesting consequences.

## COLLECTIVE MOTION AND GLOBAL QUANTISATION

Spatial angular momentum is described by a polar operator acting on the wave function in the GCE:-

$$L = -i\hbar r_s \times (\partial_s + \partial_t)\psi = r_s \times (k + p)$$

This is the sum of internal and external parts from the two metrics. $r_s$ does not commute with $\partial_s$ but it does commute with $\partial_t$, so that only local, internal angular momentum is quantised (in units of the Planck constant $\hbar$). Total vector angular momentum is conserved in space and time, but the z-axis component is quantised and separately conserved in the local metrics only.

For particles and for component structures within objects, the driving phase of tempospace is the inner product of the corresponding position and momentum full-vectors ($\int(p.dr_t - k.dr_s)$) – this is a general property of solutions of the GCE. This leaves considerable freedom to introduce local "gauge transformations" into the collective wave function (where a local, scalar function of the coordinates is added to the driving phase).

The rotation of a condensed matter object can be considered as that of the outermost real space projection only, within tempospace. This is described mathematically by a gauge transformation, which is applied to every bound tempodriver, involving the contraction of the full-vector displacement with the "rotational velocity full-vector":-

$$R(\omega) = \beta[c, 0; 0, (\omega \times r_s)] \text{ where } \beta = \left(1 - \frac{(\omega \times r_s)^2}{c^2}\right)^{-\frac{1}{2}}$$

Here $r_s$ is the local displacement from the centre of mass and $\omega$ is the collective angular velocity. $\beta$ is the local, relativistic factor for the rotating motion (which prevents matter from exceeding the speed of light $c$).

The separate spatial and temporal metrics allow the independent description of rotation and translation, even for macroscopic objects. The disintegration of a rotating object involves a re-joining of the metrics (described later).

One essential outcome is that the total, rotational angular momentum of any object is quantised (in integer units of $\hbar$). The energy of any collective, harmonic oscillation is also quantised, in both cases to keep the total driving phase of tempospace unique. (Any driven system must have a unique phase angle at all points in tempospace – but you can add any integer times $2\pi$, for each completed cycle.) This nicely bridges the gap between the classical and quantum mechanical descriptions of rotation and oscillation.

## HEAT AND THERMODYNAMICS IN A MULTI-METRIC

When particles are contained in prolonged, spatial contact, if they do not bind into greater composites, they still develop a common rest frame and share their spatial kinetic energy, defining a temperature (as in the co-moving form of the GCE). Initially, the collective wave function describes particles with different velocities $\Psi_{1,2...n} = \psi_1(r_{s1};r_{t1})\psi_2(r_{s2};r_{t2})...\psi_n(r_{sn};r_{tn})$ Then the particles lose any relative vector momentum, so that their kinetic energies become purely spatial, developing a thermal distribution in the common rest frame. This is the gaseous state, and the multi-particle wave function has the form $\Psi_{1,2...n} = \psi_1(\mathbf{r}_1;t)\psi_2(\mathbf{r}_2;t)...\psi_n(\mathbf{r}_n;t)$ Here, the relative particle positions also describe the size in tempospace.

Systems not in thermodynamic equilibrium contain multiple, distinct, top-level connections to tempospace – the parts are separately driven, so **there is no single wave function description.** This must apply to working engines and all living things, for example. Free energy is that available on the driving metric, such as that due to relative, kinematical motion. It includes any periodic motion with a measurable phase angle.

Heat is unobservable motion on the spatial metric, which has zero net momentum and is locally directionless (see Figure 3). Elastic scattering between colliding particles transfers energy from the temporal metric (work) to the spatial metric (heat) – so it is time-irreversible, even though the wave function may not collapse. The outgoing, scattered wave function carries momentum in many directions simultaneously (so $E_0$ has increased).

To define a temperature, the energy must be distributed equally between all accessible spatial modes. But if two particles / modes have different energy, this is equivalent to the same particles at the lesser spatial energy but with a relative temporal motion. This temporal kinetic energy is partly available to do external work, but it is soon dissipated (by random scattering) into heat that is shared between the particles / modes. This describes the thermodynamic arrow of time, and how confined systems develop a common rest frame, always moving towards thermodynamic equilibrium. Stable objects which are in thermodynamic equilibrium cannot record time.

Then Brownian motion must be due to thermal fluctuations, where spatial, heat energy is randomly inter-converted with temporal, observable motion. These are essential to the multi-metrical hypothesis, including the possibility of particle pair creation at high temperatures. No useful work can be done by this motion, in the absence of a temperature gradient.

## THE RE-JOINING OF SEPARATED METRICS

But what happens when energy is added to a composite object, causing it to disintegrate? How does this affect the metrical structure? This could be heat put into an atom or molecule, or kinetic energy put into a spinning wheel – the descriptions correspond. Alternatively, the energy excess could come from within, as in particle decay or combustion events.

If a composite disintegrates, the metrics at first remain separated, giving "entangled" wave functions in real space, all driven by the same connection to tempospace. Many possible outcomes can be simultaneously driven, and their amplitudes can interfere in real space, or scatter elastically with spectator objects (as in a double slit interference pattern). This is depicted on the left side in Figure 4.

When any one part interacts inelastically with some external object (changing its internal energy), then the metrics become re-joined as tempospace re-connects. After this, tempospace drives each part separately again, acquiring coordinates in the virtual dimensions $r_0$ and $r_t$, according to the probability distribution given by the intensity of the entangled spatial wave function – which then vanishes. The spatial symmetry is spontaneously broken and the particular outcome is selected by this process.

For example, the probability of detecting momentum $p$ in Figure 4:-

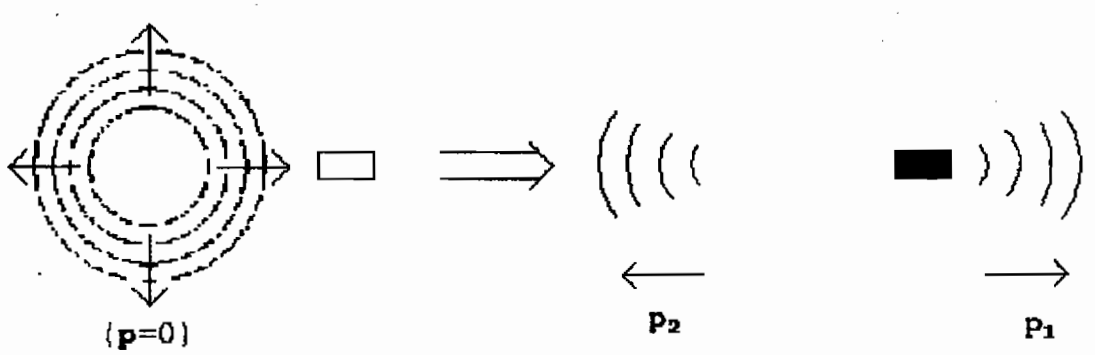



**FIGURE 4.** Resolution of a 2-body entangled state by a particle detector. An object decays into two products with outward spatial momentum **k**, one of which triggers a detector, measuring its linear momentum to be $\mathbf{p}_1$ which causes the other to have linear momentum $\mathbf{p}_2 = |\mathbf{k}|$ in direction $-\mathbf{p}_1$

$$\rho_2(p) = \phi^*_{1,2}(k=p)D_1(p)\phi_{1,2}(k=p)$$

Here $\phi^*_{1,2}$ is the complex conjugate of the entangled wave function and $D_1(p)$ is the detector acceptance function for particle 1. The phase of particle 2 is indeterminate, after the detection of particle 1.

The (outermost) spatial wave function collapses at any re-connection event - only its quantum numbers remain, but any internal structure of these objects remains separately driven. For elementary particles, modespace is also resolved into a definite state. Thus particle decay or combustion events destroy the nest of separated metrics that leads to a single wave function description at the highest level.

So history is made by the sequence of real space expansions and external re-connections of object wave functions, all connected to the common driving metric of tempospace. We can infer the existence of a globally-conserved, positive quantity - energy. Also, that the action of a closed system is just the total tempospace drive phase, added over all of the connected particles. Then observers must be described by separately-driven frames of reference, having independent real space functions until an observation event occurs, like the particle detector in Figure 4. No human intervention is required.

This mechanism for resolving the wave function collapse accords with the Copenhagen interpretation of quantum mechanics, as verified by numerous experiments over the last 80 years. The re-connection occurs at a unique value of the driving phase, simultaneous for all of the resolved objects.

But when such an entangled, spatially-distributed system re-connects, observers in relative motion cannot agree that all of the parts have re-connected simultaneously. This time difference is an interval of $r_0$, but it can never be measured! There are no hidden variables in this theory – just one hidden time dimension. The result appears just like a single, space-time metric, where history would be inexplicably determined by matter and with spooky action at a distance.

## Live and dead cats

Applying the theory described here, Schrödinger's cat is either alive or dead, at any time after the box is closed, for two separate reasons.

The first concerns the state of living things, which can never be in thermodynamic equilibrium. (For this argument, the cat could be replaced by a heat engine which is either running or stopped.) They can only be described by wave functions with multiple, independent connections to the driving metric. The box may be closed around the cat, but the metrics cannot be separated, so that wave functions describing the cat are continually collapsing as the life processes go on inside the animal. There is no single wave function description.

The second concerns the phial of poison, which is spilled by a single quantum decay event, leading to the death of the cat. Decay or combustion events upset the thermal equilibrium of the system, destroying the nest of separated metrics that leads to a single wave function description. Thus the wave function also collapses and metrics re-connect,

when the particle decay has occurred and when the poison is spilled. No human observer is required, but there should be adequate time to open the box and rescue the cat, after the quantum trigger has decayed.

## CONCLUSIONS

The basic laws of physics (quantum mechanics, relativity and thermo-dynamics) can be described by the inter-connections of multiple metrical spaces. Then mass, length and time are the primary dimensions of three distinct metrics, but the metrics corresponding to length and time are fully complementary (and therefore locally orthogonal). Then composite objects appear to move on a single space-time manifold, by discrete observations, but there are no unique space-time coordinates with which to describe the fundamental processes of physics.

The quantisation of classical motion, particle wave equations and other laws of quantum mechanics and the symmetry of relativity all follow from this suggestion. (The structure of modal space is not discussed here.) The process of quantum measurement and the inherent unpredictability of causal history are completely explained within this theory. The wave functions of massive objects always expand into real space between external, inelastic events, at which the spatial wave function must collapse. **Historical time can only elapse in the rest frame of massive objects.** Time in empty space has no meaning. Particles with no mass cannot make history.

The nature of heat (as distinct from directed, kinetic energy) is also apparent and the basic, thermodynamic laws can be deduced, providing a proper basis for describing the thermodynamics of quantum systems. Heat flow provides a second arrow of time, within composite matter, but this also involves expansion in real space, so it must always be parallel to the causal arrow. Cosmological time could also be determined from causal time within matter.

The pill you have to swallow:-

> *Special relativity, though mathematically correct, gives a highly misleading and incomplete description of the relationship between time and space-filling matter. The success of the Schrödinger equation, in describing the structure of matter, should have told us this.*

I believe it has been the underlying assumptions of fundamental physics that have impeded progress in understanding the true nature of time, for the last 100 years. Time is not just another dimension of space, it is a very different kind of dimension. All such differences must arise from the underlying metrical structure. For more on this thinking, please visit our web-site : *www.tri-space-lab.com.*

*PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)*
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# Group Representation of Johansen Revolving Method to Generate the Prime Numbers with Implications to Genonumbers

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## Abstract

**Stein E. Johansen (2010) has published a complete deduction of the formulas generating all prime numbers exactly in a strict and unique pattern, coined Johansen Revolving Prime Number Code (JR). This result was achieved by applying a geometrical interpretation of the natural numbers composed as joint products of 5- and 3-multiples and located at specified positions in a certain rotational structure. The present paper offers a suggestion for a group representation of JR, relating the eight departing positions to the eight arriving positions in an exhaustive formulation, whatever the amount of chamber rotations, by a transformation to a certain 8x8-matrix of finite, cycling groups of eight elements. Further, the paper performs a Santilli genonumber representation of this group matrix. Thereafter, it is showed that any of these genonumbers can be represented as isonumber combinations.**

## REPRESENTATION THEORY

Abstract algebra is the subject area of mathematics that studies algebraic structures, such as groups, rings, fields, modules, vector spaces, and algebras. The term abstract algebra arose at the turn of the $20^{th}$ century to *distinguish* this area from what was usally referred to as algebra, the study of the rules for manipulating formulas and algebraic expressions involving unknowns and real or complex numbers, often now called elementary algebra. The distinction is rarely made in more recent writings.

Contemporary mathematics and mathematical physics are making extensive use of abstract algebra; for example, the study of X-ray diffractograms depicting crystal structures using symmetry groups. Subject areas such as algebraic number theory, algebraic topology, and algebraic geometry apply algebraic methods to other areas of mathematics. Representation theory is a branch of mathematics that studies abstract algebraic structures by representing their *elements* as linear transformations of vector spaces. Representation theory, roughly speaking, takes the 'abstract' out of 'abstract algebra', studying the *concrete* side of a given structure. In essence, a representation makes an abstract algebraic object more concrete by describing its elements by matrices and the algebraic operations in terms of matrix addition and matrix multiplication. The algebraic objects amenable to such a description include groups, associative algebras and Lie algebras. The most prominent of these, and historically the first, is the representation theory of groups, in which elements of a group are represented by invertible matrices in such a way that the group operation *is* matrix multiplication.

Representation theory is useful because it reduces problems in abstract algebra to problems in linear algebra, a subject which is well understood. Representation theory is also important in physics because it describes *how* the symmetry group of a physical system affects the solutions of equations describing that system.

# GROUP REPRESENTATION OF JOHANSEN 8X8

# ROTATING APPROACH

Given two groups (G, $*$) and (H, $\odot$), a group isomorphism from (G, $*$) to (H, $\odot$) is a bijective group homomorphism from G to H. This means that a group isomorphism is a bijective function $f : G \rightarrow H$ such that for all u and v in G it holds that

$$f(u * v) = f(u) \odot f(v) \tag{0.1}$$

The two groups $(G, *)$ and $(H, \odot)$ are isomorphic *if an isomorphism exists.*

Consider the 8x8 boxes underlying the Johansen approach. Let $P = \{11, 13, 17, 19, 23, 27, 29, 31\}$ be the set of all position numbers. We know that there are 8 boxes for each of the 8 position numbers, that is, *there are 8 boxes producing rotations that use a given position number.* We also know that from each position number, we can produce 8 boxes (using the given position number as a starting multiple), that use each position number once and only once. There is therefore a degree of symmetry present, which allows the mechanisms to be described precisely by group theory.

The information needed to uniquely identify a box, is either the starting position number and its multiple, or its starting position number and position number used with the rotations produced. For example, the box (11, 17), having a basis in the product 11x17, can be identified by "11" and "17", or equivalently, by "11" and "31" (11x17 = 181 = 31 + 5x30). Mathematically speaking, letting $B(P_S, P_E)$ denote a given box, where $P_S$ is the starting position number and $P_E$ $(S, E \epsilon \{0, 1, 2, 3, 4, 5, 6, 7\}$ is the ending position number (that is, the position number used together with the rotations produced) this means that:

$$B(P_S, P_E) \rightarrow P \times P \tag{0.2}$$

Let $\mathbb{Z}_8$ be the finite, cyclic group of 8 elements. Since $\mid P \mid = 8$, and we can identify a linear transformation $\varphi : P \rightarrow \mathbb{Z}_8$ by $\varphi(P_S) = s$, $s \epsilon \mathbb{Z}_8$ , then we have an isomorphism $P \backsimeq \mathbb{Z}_8$. It then follows that:

$$B(P_S, P_E) \rightarrow \mathbb{Z}_8 \times \mathbb{Z}_8 \tag{0.3}$$

Note that $8 = 2^3$, and since we *therefore cannot find two factors that are coprime* (only 2, 4 and 8), the above representation cannot be isomorphic to a direct sum of smaller cyclic groups. This algebraic structure is therefore *uniquely represented* by the above transformation. The matrix representation of the underlying structure is well described by 8x8 matrices. For example, for the basis number we have:

$$B_{64} = \begin{pmatrix} 2 & 2 & 2 & 20 & 4 & 10 & 2 & 2 \\ 12 & 6 & 4 & 4 & 2 & 2 & 6 & 18 \\ 2 & 2 & 2 & 4 & 4 & 4 & 26 & 2 \\ 2 & 6 & 12 & 4 & 18 & 4 & 6 & 2 \\ 2 & 2 & 4 & 2 & 2 & 4 & 2 & 2 \\ 8 & 2 & 2 & 4 & 2 & 2 & 4 & 2 \\ 0 & 8 & 18 & 20 & 6 & 2 & 2 & 12 \\ 6 & 2 & 6 & 2 & 6 & 8 & 6 & 4 \end{pmatrix} \tag{0.4}$$

Note that there exists a linear 1-1 correspondence between each indice and each of the 64 boxes, describing the system in its simplest form.

# SANTILLI GENONUMBERS REPRESENTATION

Let $a^>$ be a genonumber in any genofield $\mathbb{F}^>$. Using Santilli references, we know that $a^>$ can be written uniquely as $a^> = a \times I^>$ , where a is an element of the conventional field $\mathbb{F}$.

If we consider the lifting $\mathbb{F} \to {}^<\mathbb{F}$, $\mathbb{F} \to \mathbb{F}^>$, and that the necessairy symmetry is present in the *bisection embedded* in the definition of the bigenofield $\{{}^<\mathbb{F}, \mathbb{F}^>\}$, we can identify a linear transformation $\varphi : \mathbb{F}^> \to \mathbb{Z}_{dim({}^<\mathbb{F})} \times \mathbb{Z}_{dim(\mathbb{F}^>)}$ by $\varphi(a^>) = \varphi(a \times I^>) = z_i \times z_j$, $z_i, z_j \epsilon \mathbb{Z}_{dim(\mathbb{F}^>)}$, such that each element of $\mathbb{F}^>$ can be expressed as a position in $\mathbb{Z}_{dim({}^<\mathbb{F})} \times \mathbb{Z}_{dim(\mathbb{F}^>)}$. Note that taking $\varphi(I^>) = \hat{I}$ does not violate the uniqueness condition. It then follows that:

$$\mathbb{F}^> \to \mathbb{Z}_{dim({}^<\mathbb{F})} \times \mathbb{Z}_{dim(\mathbb{F}^>)} \tag{0.5}$$

Note that $dim({}^<\mathbb{F}) = dim(\mathbb{F}^>)$, so the two direct products are equivalent groups.

How then, are the elements within this matrix organized? If we consider that the definitions of the genofield carries certain axioms, and that the amount of elements is pre-determined, together with the fact that carrying out elemental operations in the genofield must produce elements still within the genofield, we get that the only possible organizing of the elements are the following (taking $dim(\mathbb{F}) = 8$ as an example):

$$a^> \to \begin{pmatrix} \mathbb{Z}_8 \\ 1+\mathbb{Z}_8 \\ 2+\mathbb{Z}_8 \\ 3+\mathbb{Z}_8 \\ 4+\mathbb{Z}_8 \\ 5+\mathbb{Z}_8 \\ 6+\mathbb{Z}_8 \\ 7+\mathbb{Z}_8 \end{pmatrix} \to \begin{pmatrix} a_1 x \hat{I} & a_2 x \hat{I} & a_3 x \hat{I} & a_4 x \hat{I} & a_5 x \hat{I} & a_6 x \hat{I} & a_7 x \hat{I} & a_8 x \hat{I} \\ a_2 x \hat{I} & a_3 x \hat{I} & a_4 x \hat{I} & a_5 x \hat{I} & a_6 x \hat{I} & a_7 x \hat{I} & a_8 x \hat{I} & a_1 x \hat{I} \\ a_3 x \hat{I} & a_4 x \hat{I} & a_5 x \hat{I} & a_6 x \hat{I} & a_7 x \hat{I} & a_8 x \hat{I} & a_1 x \hat{I} & a_2 x \hat{I} \\ a_4 x \hat{I} & a_5 x \hat{I} & a_6 x \hat{I} & a_7 x \hat{I} & a_8 x \hat{I} & a_1 x \hat{I} & a_2 x \hat{I} & a_3 x \hat{I} \\ a_5 x \hat{I} & a_6 x \hat{I} & a_7 x \hat{I} & a_8 x \hat{I} & a_1 x \hat{I} & a_2 x \hat{I} & a_3 x \hat{I} & a_4 x \hat{I} \\ a_6 x \hat{I} & a_7 x \hat{I} & a_8 x \hat{I} & a_1 x \hat{I} & a_2 x \hat{I} & a_3 x \hat{I} & a_4 x \hat{I} & a_5 x \hat{I} \\ a_7 x \hat{I} & a_8 x \hat{I} & a_1 x \hat{I} & a_2 x \hat{I} & a_3 x \hat{I} & a_4 x \hat{I} & a_5 x \hat{I} & a_6 x \hat{I} \\ a_8 x \hat{I} & a_1 x \hat{I} & a_2 x \hat{I} & a_3 x \hat{I} & a_4 x \hat{I} & a_5 x \hat{I} & a_6 x \hat{I} & a_7 x \hat{I} \end{pmatrix}, \; a_i \epsilon \mathbb{F} \tag{0.6}$$

where we recognize the elements $a_i x \hat{I}$, $i \epsilon dim(\mathbb{F})$, as *isonumbers*. *Any* genonumber can therefore be uniquely written as a combination of isonumbers, with increasing complexity as the dimension of the overlying field increases. The difference between these two numbers diminishes as the dimension is reduced, a similarity embedded in their very definitions. In computations where the nature of the problem *simplifies* using isonumbers instead of genonumbers (up to the dimension of the overlying

algebraic field), it could therefore be useful to transform the genonumbers to their respective representations using isonumbers. One could say that we effectively transforms the *genoproblem into an isoproblem.*

# References


[1] Johansen, Stein E. 2010 Complete Exposition of Non-Primes Generated from a Geometric Revolving Approach by 8x8 Sets of Related Series, and thereby ad negativo Exposition of a Systematic Pattern for the Totality of Prime Numbers. Journal of Dynamical Systems and Geometric Theories 8 (2), 101-171.

[2] Santilli, R.M. 2001 Foundations of Hadronic Chemistry with Applications to New Clean Energies and Fuels. Boston-Dordrecht-London: Kluwer Academic Publishers.

[3] Santilli, R.M. 2003 Elements of Iso-, Geno-, Hyper-Mathematics for Matter, Their Isoduals for Antimatter, and Their Applications in Physics, Chemistry, and Biology. Foundations of Physics 33, 1373-1416.

[4] Santilli, R.M. 2006 Isodual Theory of Antimatter with applications to Antigravity, Grand Unification and Cosmology. Dordrecht, Netherlands: Springer.

[5] Santilli, R.M. 2008 Hadronic Mathematics, Mechanics and Chemistry. Vol. I-V, US-Europe-Asia: International Academic Press. http://www.i-b-r.org/Hadronic-Mechanics.htm

# Third International Conference on Lie-Admissible Treatment of Irreversible Processes (ICLATIP-3)

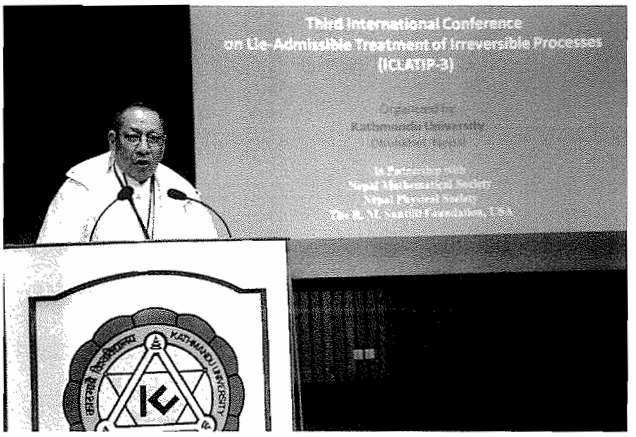
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**January 4 to 7, 2011**

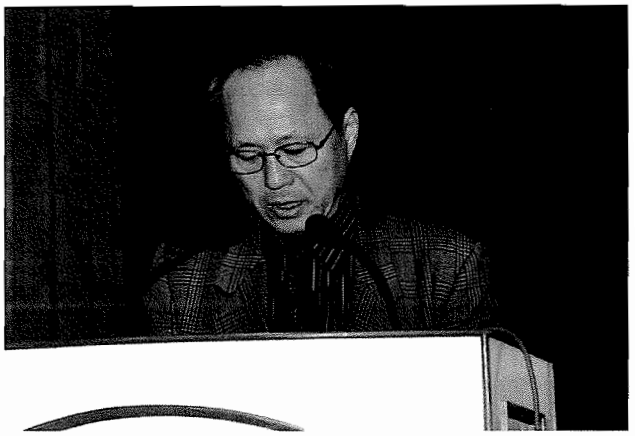
## Opening Ceremony



Inauguration jointly by: Prof. Suresh Raj Sharma, Vice Chancellor, Kathmandu University
His Excellency Hong Sungmog, Ambassador, Republic of Korea to Nepal



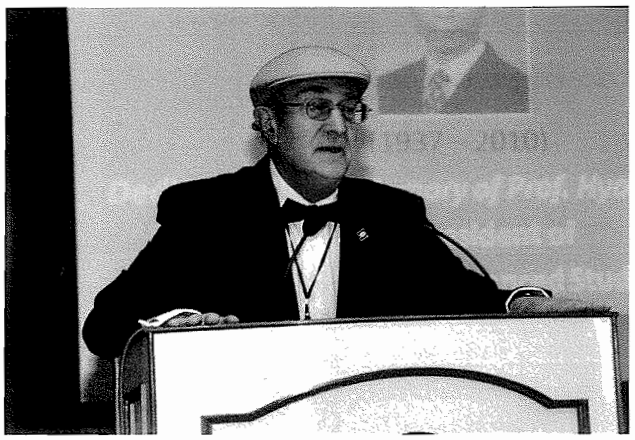

Welcome address by Prof. Bhadra Man Tuladhar,
Chair, ICLATIP-3 Nepal



Memory of Prof. Hyo Chul Myung Prof. Jongmann Yang,
President, The Korean space Science Society



Greeting by His Excellency Hong Sungmog,
Ambassador, Republic of Korea to Nepal

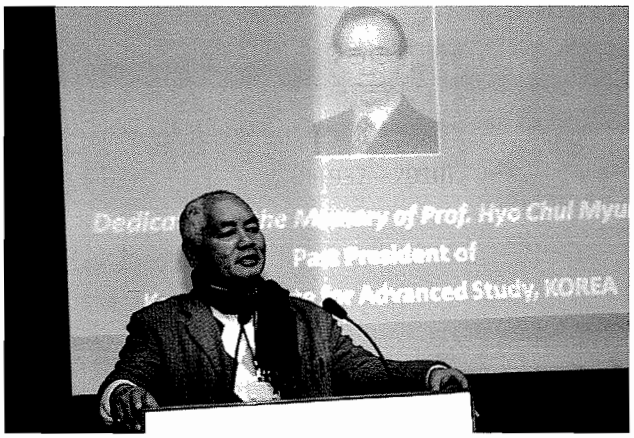


High-light of the Conference - Prof. Ruggero Maria Santilli,
Honorary Chairman, ICLATIP-3 Nepal &
President, The Institute for Basic Research, USA

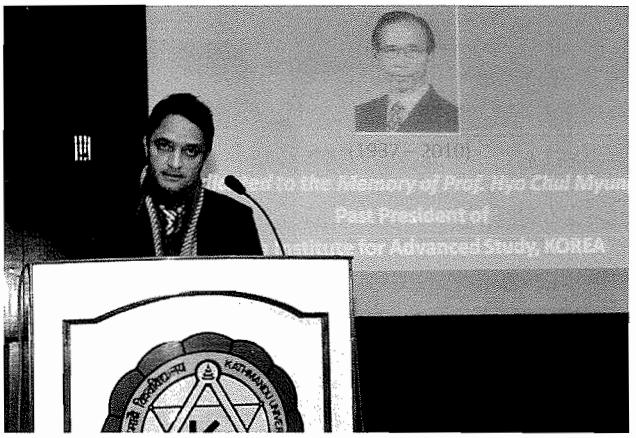
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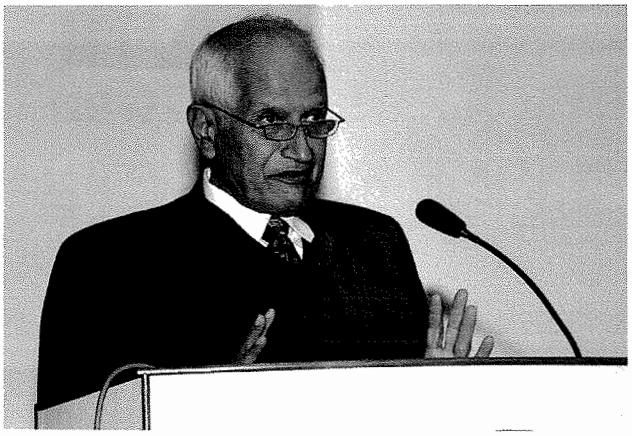
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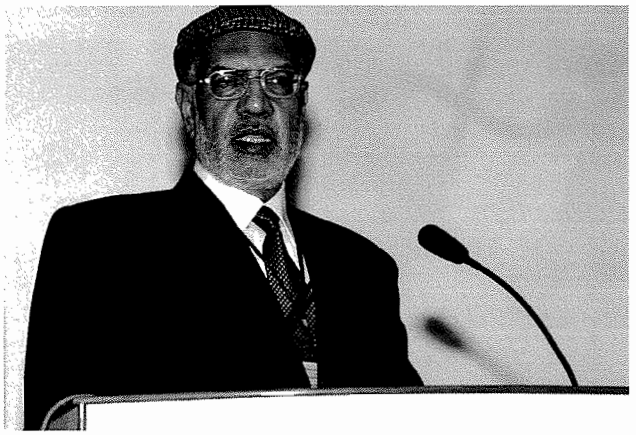

Greetings by Prof. Shekhar Gurung
President, Nepal Physical Society




Greetings by Dr. Chet Raj Bhatta
Secretary, Nepal Mathematical Society



Remarks by Prof. Suresh Raj Sharma,
Vice Chancellor, Kathmandu University



Vote of Thanks by Prof, Pushpa Raj Adhikary,
Chair, LOC, ICLATIP-3, Nepal



Participants and Guests

# Third International Conference on
# Lie-Admissible Treatment of Irreversible Processes (ICLATIP-3)

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## Scientific Program













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