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FOR THE

NOBEL PRIZE IN PHYSICS FOR I992

submitted by

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PART I:

SANTILLI'S LIE-ISOTOPIC AND LIE-ADMISSIBLE GENERALIZATIONS OF GALILEI'S RELATIVITY FOR CLASSICAL DYNAMICAL SYSTEMS WITHIN PHYSICAL MEDIA

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THE NOMINATION

Physics is a discipline that will never admit *final theories* . No matter how authoritative current theories are, their generalization is only a matter of time.

Physics is also a discipline centrally dependent on mathematical elaborations of the physical reality in a quantitative form suitable for experimental verification.

The contemporary relativities, *Galilei's, Einstein's special and Einstein's general relativities*, and related physical theories, are based on an articulated body of mathematical methods comprising:

I) *Algebras* , e.g., Lie's theory in its various branches such as enveloping algebras, Lie algebras, Lie groups, representation theory, etc.;

II) *Geometries* , e.g., the Euclidean, symplectic, and Riemannian geometries; and

III) *Mechanics* , e.g., conventional nonrelativistic and relativistic Lagrangian and Hamiltonian mechanics;

and others.

Professor **Ruggero Maria Santilli**, while working first at the *Istituto di Fisica Teorica* of the *Universitá degli Studi* , Torino, Italy (where he obtained his PhD in Physics), then at the *Department of Mathematics* of *Harvard University* , Cambridge, MA, USA, and more recently at the *International Centre for Theoretical Physics* of Trieste, Italy, has achieved an unprecedented series of discoveries consisting, first, of the identification of new mathematical methods, including:

I') *Certain generalizations of Lie algebras called of isotopic type*;

II') *Isotopic generalizations of the Euclidean, symplectic and Riemannian geometries*; and

III') *Isotopic generalizations of conventional nonrelativistic and relativistic, classical and quantum mechanics* .

Then, via the use of these broader mathematical tools, Santilli succeeded in constructing certain *generalizations-coverings of Galilei's relativity, Einstein's special relativity, and Einstein's general theory of relativity* for novel physical conditions in which the conventional relativities are inapplicable.

As well known, the conventional relativities describe particles which can be approximated as being point-like while moving in vacuum under action-at-a-distance, potential forces (historically called *exterior dynamical problem* , see Sect. I).

Santilli's new relativities describe instead the most general known physical systems, namely, extended and therefore deformable particles while moving within generally inhomogeneous and anisotropic physical media, resulting in equations of motion that are nonlinear, nonlocal, as well as not representable via the usual Lagrangian or Hamiltonian (historically referred to as *interior dynamical problem*, see Sect. I).

Also, ***Santilli's new relativities are a covering of the conventional ones*** in the sense that: 1) they are based on more general mathematical methods; 2) they represent structurally more general physical conditions; and 3) they admit the conventional relativities as particular cases.

There is little doubt that ***Santilli's discoveries are among the most important ones which can be brought to the attention of the NOBEL COMMITTEE***. In actuality, it appears that Santilli's discoveries are unprecedented in physics as an achievement by one single individual. In fact, virtually all discoveries made by physicists until now were based on mathematical tools previously established by mathematicians. The unprecedented aspect of Santilli's discoveries is that, before being in a position to generalize conventional relativities, he had to discover all needed new mathematical methods because unavailable in the mathematical literature for the needed application: the treatment of nonlinear, nonlocal-integral, nonlagrangian and nonhamiltonian systems of the interior dynamical problem.

The purpose of this presentation, specifically written for the NOBEL COMMITTEE, is multifold. First, we would like to indicate the fundamental novelty, comprehensive character, and historical dimension of the discoveries.

Second, Santilli has written in the topic some seven monographs and over 100 articles in numerous international Journals. By adding the contributions of other independent scientists, we are dealing with a field that has surpassed the mark of ten thousand pages of published research. The second objective of this presentation is, therefore, that of identifying for the NOBEL COMMITTEE the most salient aspects of the discoveries and their original reference among such a disparate literature.

Third, the novel scientific edifice emerging from Santilli's discoveries implies a generalization of the entirety of contemporary physics, including generalizations of: classical nonrelativistic and relativistic mechanics, nonrelativistic and relativistic quantum mechanics, quantum field theory, gravitation, classical and quantum statistics, etc. It is easily predictable that in a scientific scene of this dimension, we have a spectrum of conditions, including: discoveries which can be safely considered as established at this writing; discoveries in need of additional theoretical and experimental studies; and others only at their initiation.

> ***This Nomination of Professor RUGGERO MARIA SANTILLI for the NOBEL PRIZE IN PHYSICS FOR 1992 is solely based on those discoveries which are fully established at this writing on both grounds of mathematical consistency and physical validity, and which, as outlined in this Part I, consists of Santilli's classical generalization of Galilei's relativity for nonlinear, nonlocal and nonhamiltonian dynamical systems of the interior problem.***

according to the following primary publications:

1) The discovery originally appeared in the memoir (Santilli (1978a); see Document A for its front page); and was then subjected to a variety of specialized studies in a number of papers identified in this Part I;

2) The discovery was then presented in all the necessary details in the four monographs:

R. M. Santilli, ***Foundations of Theoretical Mechanics,***
Volume I: ***The Inverse Problem in Newtonian Mechanics (1978b)*** ,
Volume II: ***Birkhoffian Generalization of Hamiltonian Mechanics (1982a)*** , and
printed by *Springer-Verlag* of Heidelberg, Germany,

R. M. Santilli, ***Lie-admissible Approach to the Hadronic Structure***
Volume I: ***Nonapplicability of Galilei's and Einstein's Relativities ? (1978c)*** ,
Volume II: ***Generalizations of Galilei's and Einstein's Relativities ? 1981a)***
published by *Hadronic Press Inc.,* Palm Harbor, FL 34682-1577 USA, and

3) The discovery was then finalized in the memoir (Santilli (1988a)). Its novel mathematical structures were studied in detail, first, in the memoir (Santilli (1988b)), and then in the two memoirs appeared in a mathematical Journal (Santilli (1991a, b). The discovery was then finalized in its physical contents in the two additional monographs:

R.M.Santilli, ***Isotopic Generalizations of Galilei's and Einstein's Relativities***
Volume I: ***Mathematical Foundations (1991c)***
Volume II: ***Classical Isotopies (1991d)***

In our opinion, the content of the two monographs published by Springer-Verlag in 1978 and 1982 is sufficient to warrant, alone, a NOBEL PRIZE IN PHYSICS. In fact, the title of Volume II reads ***"Birkhoffian Generalization of Hamiltonian Mechanics"*** and does indeed present a completely new physical discipline (see Section IV for a brief outline). Similarly, the title of Chapter 6, p. 199, Volume II, reads ***"Generalization of Galilei's Relativity"*** and presents a generalization that appears in print for the first time after about four centuries from the original Galilean conception.

As a result of this occurrence, ***copies of Prof. Santilli's two monographs published by Springer-Verlag are enclosed as an integral part of this Nomination*** . The more recent monographs 3) will be separately mailed to the NOBEL COMMITTEE as addenda.

Part II, which is under preparation for the NOBEL COMMITTEE, outlines *Santilli's isotopic generalization of Einstein's special relativity for light and/or relativistic systems of extended-deformable particles moving within inhomogeneous and anisotropic physical media* , and it is scheduled for submission sometime in 1992. This latter new relativity is mathematically consistent but, unlike the Galilean case, its novel predictions (e.g., that for a redshift of light propagating within inhomogeneous and anisotropic transparent media, and others) needs specific experimnental verifications.

Part III, also in preparation for the NOBEL COMMITTEE for delivery sometime in 1992, outlines *Santilli's isotopic generalization of Einstein's gravitation for the most general known nonlinear, nonlocal and nonlagrangian interior gravitational conditions, as expected, say, for a star undergoing gravitational collapse or for any interior gravitation at large.* This third new relativity is also mathematically consistent at this writing but its physical consistency, in addition to the experimental verifications for the local relativistic interior behavior indicated above, requires additional studies connected to the numerous and now vexing problematic aspects of Einstein's gravitation.

Part IV, also in preparation for the NOBEL COMMITTEE, outlines *the operator formulation of Santilli's coverings of Galilei's and Einstein's special relativities for elementary particles with extended wavepackets when in conditions of total mutual penetration, as conceivable for the hadronic structure, which result in expected, short range, nonlinear, nonlocal and nonhamiltonian internal effects without any visible effect in the exterior dynamics* . These studies, which have resulted in a generalization of quantum mechanics called *hadronic mechanics* , are also mathematically consistent at this writing, but in need of a number of additional theoretical elaborations and experimental verifications.

A final Part V may be prepared for the NOBEL COMMITTEE at some future time on certain ongoing efforts to achieve an isotopic generalization of unified gauge theories for the possible inclusion of gravitational and strong interactions, known under the name of *iso-grand-unification* .

This Nomination for the NOBEL PRIZE IN PHYSICS OF 1992 is solely referred to this Part I. Nevertheless, the NOBEL COMMITTEE should be aware of the additional discoveries outlined in the remaining parts, because important to reach a mature judgment on their dimension, depth, implications and interrelations.

It may be appropriate here to recall that Santilli's discoveries were called

"Truly epoc making", by Prof. H.P.Leipholz, Univ. of Waterloo, Canada
(official reviewer for Springer-Verlag, see enclosed Document B).

In recognition of his discoveries, Santilli received ***two Gold Medals*** , one from the City of Orléans, France, and another from the city of Campobasso, Italy, in conjunctions with international Conferences in which he presented his discoveries (see Document C).

But the biggest honor was granted until now by the Estonian Academy of Science in Tartu which, in the occasion of an International Conference in algebras of 1989, prepared two official charts on the most historical contributions in physics and mathematics from 1800 till today (Document D) which were presented at the Conference during the opening talk of the organizers and subsequently printed in 1990 (ISSN0134-627X). As the NOBEL COMMITTEE can see, the name RUGGERO SANTILLI is listed with the year 1967 of initiation of the discoveries that lead to this Nomination, jointly with the best names in the history of physics and mathematics, such as GAUSS (1820), CAUCHY (1847), HAMILTON (1843), CAYLEY (1854), LIE (1880), POINCARE' (1884), CARTAN (1894), NOETHER (1929), EDDINGTON (1928), WEYL (1926), DIRAC (1928), JORDAN (1932), VON NEUMANN (1934), WIGNER (1934), ALBERT (1948), and others.

SECTION I: INTRODUCTION.

I.1: THE FIELD OF UNEQUIVOCAL VALIDITY OF CONVENTIONAL RELATIVITIES.

Galilei's relativity (see Galilei (1638), Newton (1687) and, for a contemporary account, Sudarshan and Mukunda (1974)), *Einstein's special relativity* (see Lorentz (1904), Poincaré (1905), Einstein (1905), Minkowski (1913) and, for a historical review, Pauli (1921)) and *Einstein's general relativity* (see Riemann (1868), Einstein (1916) and, for a historical account, Pauli (loc. cit.)) were conceived for physical conditions referred to by Lagrange (1788), Hamilton (1834), Jacobi (1837) and other Founders of contemponary physics, as those of the *exterior dynamical problem*, that is, the study of particles which can be well approximated as being point-like, while moving within the (homogeneous and isotropic) vacuum, under action-at-a-distance interactions derivable from a potential energy.

The point-like character of the particles implies the exact validity of conventional local-differential geometries, such as the symplectic, affine or Riemannian geometries. The action-at-a-distance, potential nature of the interactions then implies the exact validity of all conventional Lagrangian-Hamiltonian disciplines, such as the conventional nonrelativistic and relativistic, discrete or continuous, classical or quantum mechanics.

An overwhelming amount of experimental evidence has nowadays established the validity of the conventional relativities in the arena considered beyond any possible doubt. It is here appropriate to recall, as a classical illustration, the majestic successes of the NASA missions throughout our Solar system and, as a quantum mechanical illustration, the equally majestic successes in the description of the atomic structure.

The exact validity of conventional relativities, within the above identified conditions, is assumed by Santilli's as the sound foundations of his research. The NOBEL COMMITTEE should therefore expect *no conflict whatever* between conventional relativities and Santilli's generalizations, but only a continuity of mathematical and physical thought, as we shall see.

I.2: THE DIFFERENT FIELD OF VALIDITY OF SANTILLI'S COVERING RELATIVITIES.

Santilli has devoted his research life to the study of physical conditions fundamentally different and substantially more complex than the above. In particular, he has studied conditions which were historically referred to by Lagrange (loc. cit.), Hamilton (loc. cit.), Jacobi (loc. cit.) and other Founders, as those of the *interior dynamical problem*, that is, the study of extended particles which cannot be approximated as being point-like, while moving within generally *inhomogeneous* and *anisotropic* physical media under action-at-a-distance, potential forces, as well the additional contact forces with the physical media for which the notion of potential has no meaning.

Lagrange and Hamilton knew well that the contact forces between extended objects and the physical media in which they move are outside the representational capabilities of their functions and, for this reason, they formulated their historical equations with external terms. In fact, for a system of N particles represented by the index a = 1, 2, ..., N, in three-dimensional Euclidean space with local coordinates $r = (r_{ka})$, k = 1, 2, 3 (= x, y, z)[1], the historical Lagrangian and Hamilton's equations *are not* those given in the contemporary texbooks of physics and mathematics, but instead by the forms

$$\frac{d}{dt}\frac{\partial L(t.\ r, \dot{r})}{\partial \dot{r}_{ka}} - \frac{\partial L(t, r, \dot{r})}{\partial r_{ka}} = F_{ka}(t, r, \ddot{r},...), \qquad \text{(I.1a)}$$

$$\dot{r}_{ka} = \frac{\partial H(t, r, p)}{\partial p_{ka}}, \qquad \dot{p}_{ka} = -\frac{\partial H(t, r, p)}{\partial r_{ka}} + F_{ka}(t, r, \dot{r}, ...) \qquad \text{(I.1b)}$$

$$k = 1, 2, 3\ (= x, y, z), \qquad a = 1, 2, ..., N,$$

where the external terms F_{ka} represent precisely the contact, nonlagrangian/nonhamiltonian forces of our physical reality. Similarly, Jacobi formulated his historical theorem, not for the "contemporary Lagrange's and Hamilton's equations", those *without* external terms, but for the original ones *with* external terms.

With the passing of time, the external terms were removed as a result of a historical process still ignored by historians until now, such as: the advent of Lie's theory (1983), the classical and quantum mechanical successes for the description of exterior planetary and atomic systems, respectively, and other reasons. In this way, Lagrange's and Hamilton's equation acquired the contemporary "truncated form" without external terms.

As a result of this process, the original, historical distinction between the exterior and interior dynamical problem was progressively lost, up to the contemporary scientific scene which is virtually without any remnant of the historical distinction.

Santilli essentially dedicated his research life to a comprehensive classical and quantum mechanical study of historical equations (I.1) *with* external terms. In fact, he first identified their algebraic character as being that of a generalization of Lie algebra called *Lie-admissible algebras* (Sect. III.3) and discovered their underlying new geometry, which he called *symplectic-admissible geometry* (Appendix A),. Santilli then succeeded in identifying their operator image (see the forthcoming Part IV). This first group of methods, now known as *Santilli's Lie-admissible formulations* (Sect. III.3 and Appendix A), is particularly suited for the direct study of Lagrange's and Hamilton's interior problem in its original conception, that is, under open-nonconservative conditions.

Santilli then identified a second group of methods, now known as *Santilli's Lie-*

[1] For simplicity, we shall ignore any disctinction between covariant and contravariant indeces for coordinates $r = (r_{ka})$ and momenta $p = (p_{ka})$, but introduce the dictinction later on for the unified notation in phase space $a = (a^\mu) = (r, p)$, $\mu = 1, 2, ..., 6N$

isotopic formulations , which essentially consist of an alternative approach in which the external terms are removed, and replaced by a generalized unit of the theory, by resulting in a structural generalization of Lie algebras, symplectic geometry and Hamiltonian mechanics, called by Santilli *Lie-isotopic algebras, symplectic-isotopic geometry and Birkhoff mechanics* , as outlined in Sect. IV. Santilli also succeeded in identifying the operator countepart of these alternative formulations, which is outlined in the forthcoming Part IV. These latter formulations are particularly suited for "closing" Eq.s (I.1) via the addition of the external media, thus resulting in isolated systems verifying all conventional total conservation laws, while the internal forces are nonlinear, nonlocal and nonhamiltonian (Sect. II).

The technical foundations of all these studies are provided by the so-called *conditions of variational selfadjointness* , which are presented in details in the enclosed first monograph by Santilli under the title ***Foundations of Theoretical Mechanics, Vol. I: The Inverse Problem in Newtonian Mechanics***, published by *Springer-Verlag* , Heidelberg (1978b). As the NOBEL COMMITTEE can see, this is a very scholarly work providing the first comprehensive presentation of the *necessary and sufficient conditions for given forces to admit a potential or, more generally, for given equations of motion to admit a Lagrangian or a Hamiltonian*.

The NOBEL COMMITTEE should also be aware of the historical search conducted by Santilli in the scientific libraries of Cambridge, Massachusetts as an essential part of this monograph. In fact, the paternity of the integrability conditions for the existence of a Lagrangian were essentially unknown in the 70's, with contrasting quotations generally existing in advanced mathematical papers. In his comprehensive library search, which lasted from 1975 to 1978, Santilli succeeded in establishing that Helmholtz (1887) had been the originator of the conditions of variational selfadjointness, and then identified all subsequent contributions (see Vol. I of the enclosed monographs, pages 12, 13).

The conditions of variational selfadjointness are the true technical foundations for both the Lie-admissible and the Lie-isotopic formulations, inasmuch as they provide all the necessary quantitative means for studying the structure of any given force, the conditions when it is reducible to the Lie-isotopic formulations, and the conditions under which the more general Lie-admissible methods are requested.

The NOBEL COMMITTEE can find in Document E the outline of a post-graduate course Santilli taught in 1978 in the field at the *Lyman Laboratory of Physics* of *Harvard University* .

I.3: INEQUIVALENCE OF THE INTERIOR AND EXTERIOR PROBLEMS.

One of the first introductory points the NOBEL COMMITTEE can find in Santilli's writings is the proof of the inequivalence of the interior and exterior problems (Santilli (1978a, c), (1982a), (1985c), (1988a), (1991c)).

In fact, the exterior problem is based on the point-like abstraction of particles under interactions derivable from a potential V and are representable in their first-order form via the familiar Hamiltonian vector-fields

$$a = (\dot{a}^{\mu}) = \begin{pmatrix} \dot{r}_{ka} \\ \\ \dot{p}_{ka} \end{pmatrix} = \Phi = (\Phi^{\mu}(a)) = \begin{pmatrix} p_{ka}/m_a \\ \\ -\dfrac{\partial V}{\partial r_{ka}} \end{pmatrix} \qquad (I.2)$$

$$a = (a^{\mu}) = (r, p), (r_{ka}, p_{ka}), \quad \mu = 1, 2, ..., 3N, \quad k = 1, 2, 3, (= x, y, z)$$

and, as such, it implies the exact validity of conventional local-differential geometries, such as the symplectic, affine and Riemannian geometries.

On the contrary, interior dynamical problems describe, say, a satellite during re-entry in Earth's atmosphere, where, in addition to the conventional local-differential forces derivable from a potential V, experimental evidence establishes the existence of the contact interaction with the medium which can be reduced to local-differential nonlinear and nonhamiltonian forces $F_k(t, r, p, ...)$, plus additional nonlocal-integral forces also evidently not derivable from a Hamiltonian (see, e.g., Hofstadter et al. (1970), Fijimura et al. (1971), and quoted papers)

$$a = (\dot{a}^{\mu}) = \begin{pmatrix} \dot{r}_{ka} \\ \\ \dot{p}_{ka} \end{pmatrix} = \Gamma = (\Gamma^{\mu}(t, a, ..)) = \begin{pmatrix} p_{ka}/m_a \\ \\ -\dfrac{\partial V}{\partial r} + F_{ka}(t, r, p, ...) + \int_{\sigma} d\sigma \mathcal{F}_{ka}(t, r, p, ...) \end{pmatrix} \qquad (I.3)$$

Thus, interior dynamical systems are characterized by the most general known systems of differential equations which are: 1) nonlinear and nonlocal in all variables; 2) nonlagrangian and nonhamiltonian, in the sense that the conventional functions $L(t, r, \dot{r})$ or $H(t, r, p)$ are insufficient to represent the system; and 3) nonnewtonian in the sense that the acting forces generally dependent on the accelerations.

As the NOBEL COMMITTEE can see, systems (I.3) can be readily represented by the original equations (I.1), where L or H represent the kinetic energy T as well as the potential energy V, $L = T - V$ or $H = T + V$, while the external terms F_{ka} represent all the nonpotential forces.

The inequivalence of the interior and exterior problems is then established at all levels of studies. In fact:

A) *Topologically*, the nonlocal character of systems (3) imply the inapplicability of all basic geometries of contemporary physics, the symplectic, affine and Riemannian geometries because of their strict local-differential nature;

B) *Algebraically*, the nonlinear, nonlocal and nonhamiltonian character of the systems imply the inapplicability of the conventional canonical formulation of Lie's theory;

C) *Analytically*, the nonlagrangian and nonhamiltonian character implies the inapplicability of current analytic mechanics;

etc.

Even by approximating the nonlocal-integral forces with local-differential expressions (which are usually done via power series expansions in the velocities truncated to a sufficiently high power), the nonlagrangian and nonhamiltonian character of the systems persists. In fact, it is well known in engineering circles (but not yet in physical circles) that the computerized guidance systems of missiles in atmosphere may require contact forces *up to the tenth power in the velocity and more* , thus being strictly nonlagrangian and nonhamiltonian.

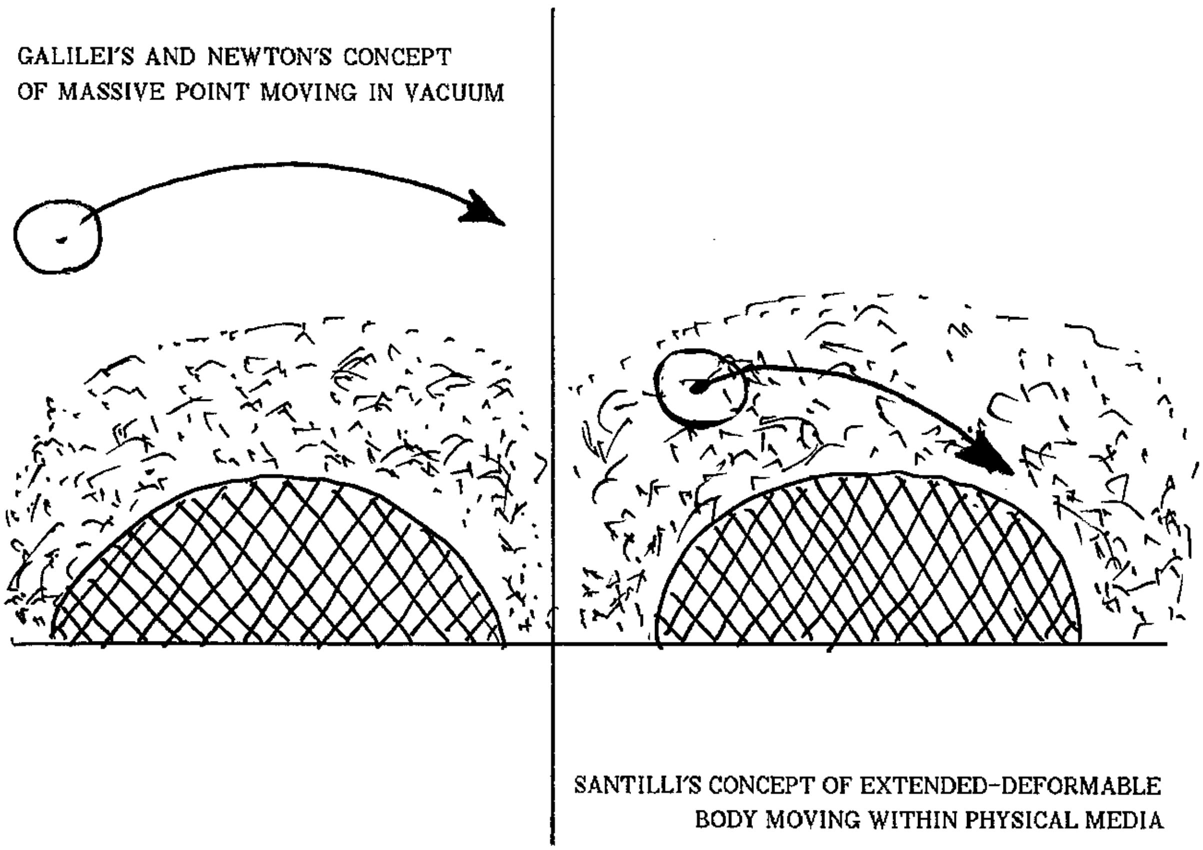

FIGURE I.1: A schematic view of the funndamental concepts in conventional and Santilli's relativities. Consider an object moving in empty space, such as a satellite in a stationary orbit around our Earth. Since motion occurs in vacuum, the extended character of the object and its actual shape do not affect its dynamical evolution. One recovers in this way ***the historical notion of "massive point" by Galilei (1638) and Newton (1687)***, namely, the satellite can be assumed to be a massive point concentrated in its center of gravity without any consequential approximation in the dynamics. This centuries hold concept has profound, contemporary, topological implications. In fact, it implies the exact validity of the local-differential geometries of contemporary mathematics, such as the differential, affine and Riemannian geometries. The Newtonian equations of motion are then given by Eq.s (I.2). The exact validity of Galilei's relativity, Einstein's special relativity and Einstein's gravitation for the satellite in exterior conditions is then consequential.

Consider now the satellite during re-entry in Earth's atmosphere. The dynamical conditions are then profoundly altered. In fact, the actual shape of the satellite now directly affects its dynamical evolution (e.g., spherical and nonspherical satellites of equal mass have essentially different trajectories in atmosphere). As a result, the extended character of the satellite must be represented in the equations of motion. Moreover, perfectly rigid objects do not exist in Nature. One therefore has deformations of the shape of the satellite which must also be taken into consideration. We reach in this way ***Santilli's concept of extended and therefore deformable object moving within generally inhomogeneous and anisotropic physical media*** of equally historical character which is at the foundation of this Nomination. The mathematical implications of the latter concept are far reaching. In fact, the representation of the shape in the dynamical evolution requires forces of integral type as in Eq.s (I.3) where σ now represents the surface of the satellite. In turn, this implies the irreconcilable inapplicability of conventional geometries, such as the symplectic, affine and Riemannian geometries, because of their strictly local-differential topology. The inapplicability of conventional relativities is then consequential, as outlined in the text.

While a member of the *Department of Mathematics* of *Harvard University* in the late 70's, Prof. Santilli studied all available efforts by pure mathematicians in the construction of the so-called "integral topologies" and "integral geometries" for the purpose of ascertaining their effectiveness in the treatment of systems (I.3). He found none available in the pure mathematical literature which would verify all his requirements, including the conditions of: a) admitting nonlocal-integral forces of nonlagrangian and nonhamiltonian character; b) being simple in use and effective in physical applications; and, last but not least, c) permitting the constructions of covering relativities. He therefore constructed novel geometries and mathematical tools for the needed quantitative treatment of *"extended"* and *"deformable"* objects moving within *"inhomogeneous"* and *"anisotropic"* physical media (see Sect.s III and IV). The construction of generalized relativities was then consequential (Sect. V).

These experimental facts establish that interior trajectories are structurally beyond the representational capabilities of the symplectic and Riemannian geometries (a property also known as the *Cartan's legacy*) , thus establishing the need for suitable generalizations.

1.4: IRREDUCIBILITY OF THE INTERIOR TO THE EXTERIOR DYNAMICAL PROBLEM.

When exposed to interior systems (I.3), contemporary physicists generally provide all conceivable efforts in reducing them to the simpler form (I.2). Santilli (1985c) has proved that such a reduction is inconsistent and not realizable in technical terms.

First, when exposed to systems of type (I.3), physicists tend to perform their transformation, from the original coordinate system r of their experimental detection, into an imaginary frame r′ in which the systems become Hamiltonian. Under a number of approximations and conditions (locality, regularity and analyticity in a star-shaped region), the existence of such a transformation is ensured by the *Lie-Koening Theorem* (Santilli (1982a)). However, it is a mere mathematical curiosity, because the original system is nonhamiltonian as well as nonlinear. Thus, the transformation $r \Rightarrow r'$ must necessarily be *noncanonical* , as well as highly *nonlinear* . This renders inapplicable all conventional relativities to the hypothetical, transformed system, evidently because the transformed frame r′ is highly noninertial, as well as nonrealizable in experiments (e.g., r′ = a exp{b sinh (c r) }, a, b, c ∈ ℜ).

Santilli (loc. cit.) therefore insists that ***systems (I.3) must be represented in the physical coordinates r of their experimental detection*** (which he calls "direct representation"). Besides, systems (I.3) are nonlocal-integral, in which case the Lie-Koening Theorem is known to be *inapplicable* and the reduction to a Hamiltonian form impossible.

After recognizing the impossibility of effectively reducing systems (I.3) to the simpler form (I.2) treatable via current relativities, contemporary physicists claim that their differences are *"illusory"* (sic) because, when a macroscopic body of the interior dynamical problem, such as a satellite during re-entry, is reduced to its elementary particle constituents, one recovers point-like particles in stable orbits under potential interactions, with the consequential validity of conventional geometries, disciplines and relativities.

In an invited talk at an International Conference held in Calcutta in 1985, Santilli (1985c) presented a series of *"No Reduction Theorems"* which establish the impossibility of any consistent reduction of a classical, nonconservative and nonlagrangian-nonhamiltonian system to a collection of conservative, Lagrangian-Hamiltonian particles. Viceversa, he proved that a (finite) collection of elementary particles in stable orbits and in unitary time evolution simply cannot reproduce a mascroscopic system which is in highly nonconservative conditions and not representable by a Hamiltonian.

When the impossibility of a consistent elimination of the interior nonlocality is finally acknowledged, contemporary physicists still attempt other mechanisms in the hope of salvaging established doctrines for interior conditions.

One of them is the addition of an "integral potential" to conventional Lagrangians and Hamiltonians. The simplistic argument is that the salvaging of the canonical formalism implies the preservation of conventional relativity. Santilli (1978c) has proved the inconsistency of these latter attempts on numerous mathematical and physical grounds reviewed in Sect. II.2 (see Footnote[2] in particular), such as the invalidation of the conventional local-differential topology with consequential loss of topological symmetries, as well as the necessary impact on the *exterior* trajectory caused by the *internal* effects (because of its "potential" interpretation), which is against clear physical evidence.

The mathematical roots of Santilli's "No Reduction Theorems" is the evidence that the unstable orbit, say, of a satellite during re-entry with monotonically decaying angular momentum, simply cannot be decomposed into a collection of stable orbits, each one with conserved angular momentum. Viceversa, a collection of stable orbits each with conserved angular momentum simply cannot reproduce a macroscopic body with monotonically decaying angular momentum.

The physical roots are given by the ***legacy of Fermi (1949), Bogoliubov (1960), and other Founders of particle physics on the ultimate non-locality of the structure of strongly interacting particles*** . In fact, while the atomic constituents are at large mutual distances when compared to their wavelength, the hadronic constituents must necessarily be in conditions of total mutual penetration and overlapping because their wavelength is precisely of the order of magnitude of the size of all hadrons (about $1\ F = 10^{-13}$ cm). Thus the atomic structure is a typical example of *exterior quantum mechanical problems* , while the hadronic structure is expected to be a typical case of *interior quantum mechanical problems*.

As well known, current theories on the structure of hadrons are dominated by the hypothesis that the constituents of hadrons are the *quarks* (see the reprints of the original contributions edited by Lichtenberg et al. (1980)). Now, even though there is experimental evidence (Bloom et al (1969)) that the hadronic constituents have a point-like charge structure (for which NOBEL PRIZES were recently granted), "point-like wave packets" do not exist in Nature. Quarks, to be physical particles, must therefore have extended wavepackets with the dimension of the entire hadron. The historical legacy on the ultimate nonlocality of strong interactions then follows.

One may argue from the clear successes of the quark theories that such nonlocal effects could be small. Nevertheless, if one passes to more limiting conditions, they simply are not ignorable. The NOBEL COMMITTEE can consider in this respect the core of a collapsing star, in which we have not only total mutual penetration of the wavepackets of the particles constituents, but also their compression in very large numbers in an extremely small region of space. Under these conditions, the validity of the historical legacy on the ultimate nonlocality of the structure of matter becomes beyond any credible scientific doubt.

This illustrates the ***necessity of studying the interior dynamical problem at all its levels, nonrelativistic, relativistic, gravitational, classical and quantum mechanically***, precisely as done by Santilli.

Moreover, despite their successes, quark theories are still afflicted by fundamental, now vexing, open problems. As an example, all nonrelativistic quark theories have a finite nonnull probability of tunnel effects for free quarks when near the infinite potential barrier (Chattarjee et al. (1986)), which is contrary to experimental evidence. This occurrence is a necessary consequence of the assumption of quantum mechanics in general, and Heisennberg's uncertainty principle in particular, in the interior and in the exterior problems of hadrons. The same nonnull probability of tunnel effects is expected to persist at all subsequent levels of treatment, such as that of QCD, because inherent in Heisenberg's uncertainty principle.

Explicitly stated, Heisenberg's uncertainty principle implies the consequence, beyond any reasonable doubt, that ***current quark theories have a finite nonnull probability that the ordinary protons and neutrons should spontaneously emit free quarks, which is evidently contrary to experimental evidence***.

In Part IV of this Nomination we shall review for the NOBEL COMMITTEE the fact that this (and other) vexing open problems of current quark theories may be due precisely to their lack of treatment of the historical legacy on the ultimate nonlocality of the hadronic structure. In fact, short range, nonlocal effects can be admitted only in the interior problem, and are definitely null in the large mutual distances of the exterior problem.

This results into a structural difference between the interior and the exterior problem under which the probability of tunnel effects of free quarks can indeed be made identically null, e.g., by rendering incoherent the interior and exterior Hilbert spaces and other means.

Besides, the isotopic generalization of the SU(3) symmetry is locally isomorphic to the conventional symmetry (Mignani (1984)). As a result, the representation of the historical legagy of the ultimate nonlocality of the strong interactions via Santilli's isotopic techniques offers the possibility of genuine advances in quark theories, while leaving the unitary symmetries essentially unchanged.

The NOBEL COMMITTEE is warned against high ranking "false experts", and encouraged to dismiss superficial opinions expressed by individuals without an established record of expertise on the methods necessary for an ethically and scientifically sound judgment: the conditions of variational selfadjointness (see the encloseed Vol. I of Santilli's *Foundations of Theoretical Mechanics*).

I.5: INAPPLICABILITY OF CONVENTIONAL RELATIVITIES FOR THE INTERIOR DYNAMICAL PROBLEM.

The conventional relativities of contemporary physics are *inapplicable* (rather than "violated") for an effective characterization of interior dynamical systems (I.3) beyond any scientific doubt, for a variety of independent mathematical and physical reasons, such as:

1) The fundamental transformations of contemporary relativities, Galilei's, Lorentz's and Poincaré's transformations, are *linear* and *local*, as well known, while systems (I.3) are strictly *nonlinear* and *nonlocal*;

2) Contemporary relativities are centered on the *canonical-Hamiltonian* formalism, while systems (I.3) are strictly *nonhamiltonian* in the frame of their experimental detection;

3) Contemporary relativity are based on *Lie's symmetries* in their canonical realization, while the *conventional Lie's theory is fundamentally inapplicable for systems* (I.3);

4) Conventional relativities are centered on a *local-differential topology* (e.g., the Zeeman topology), while systems (I.3) require an essential *nonlocal-integral topology*;

5) Conventional relativities are centrally dependent on the *homogenuity and isotropy of space*, while interior physical media are manifestly *inhomogeneous and anisotropic*;

and numerous other independent technical reasons worked out by Santilli in all necessary details. In particular, the breakings of conventional symmetries for interior systems (I.3) were classified by Santilli (1978e), Sect. A.12, pp. 344-348) into: *isotopic, selfadjoint, semicanonical, canonical and essentially nonselfadjoint breakings.*

> *These studies establish beyond any credible scientific doubt the inapplicability for interior systems (I.3) of the mathematical foundations of Galilei's relativity, Einstein's special relativity and Einstein's general relativity, let alone the inapplicability of the relativities themselves, by therefore establishing the need to identify new mathematical methods and construct new coverinfg relativities.*

As an illustration, ***the insistence in the exact validity for interior***

dynamical problems of the relativities for the exterior problem literally implies the acceptance of the perpetual motion in a physical environment, trivially, from the necessarily exact validity of their local rotational symmetry, with consequential necessary conservation of the angular momentum, without any possibility of escaping from these nonscientific conclusions because of the "No Reduction Theorems" recalled earlier.

SECTION II: CONCEPTUAL OVERVIEW OF SANTILLI'S DISCOVERIES

II.1: COMPREHENSIVE CHARACTER OF SANTILLI'S RESEARCH.

Santilli has conducted a truly vast study of the interior dynamical problem at the nonrelativistic, relativistic and gravitational levels, from a discrete and continuous viewpoint, as well as for classical and quantum mechanical treatments, all this repeated twice, one for the study of interior systems as closed-isolated (thus verifying conventional total conservation laws), and one for their open-nonconservative treatment (as conceived by Lagrange and Hamilton).

Moreover, in each of the above two main lines, Santilli constructed suitable generalizations of conventional mechanics, algebras and geometries, by resulting in this way in a novel scientific edifice of truly unique dimension, diversifications and interrelations, which is rather remarkable as the achievement by one single individual.

In a scientific edifice of this type, it appears recommendable to point out first the main conceptual lines, and then pass to a technical review.

II.2: SANTILLI'S CLOSED NONHAMILTONIAN SYSTEMS.

It is generally believed that the global stability of a system is due to the stability of the orbits of each constituent, as it is the case for the planetary and atomic structures.

Santilli (1978d) proved that, by no means, these systems exhaust all possible systems of the Universe. In fact, he identified a class of systems, at both classical and quantum mechanical levels, which he called *"closed nonhamiltonian" systems"*. These are systems whose total physical quantities are conventionally conserved (closure), but the internal orbits of the constituents are generally unstable because of contact interactions with the physical medium (nonhamiltonian character). In these broader systems we merely have *internal* exchanges of energy, angular momentum and other physical quantities, but in a way compatible with total conservations.

At the classical level, Santilli represented these novel systems with the equations

$$(\dot{a}^{\mu}) = \begin{pmatrix} \dot{r}_{ka} \\ \\ \dot{p}_{ka} \end{pmatrix} = \Gamma = (\Gamma^{\mu}(t, a, ..)) = \begin{pmatrix} p_{ka}/m_a \\ \\ -\dfrac{\partial V}{\partial r} + F_{ka}{}^{NSA}(t, r, p, ...) + \int_{\sigma} d\sigma \mathcal{F}_{ka}{}^{NSA}(t, r, p, ...) \end{pmatrix} \tag{II.1a}$$

$$\dot{X}_k = (\partial X_k / \partial a^{\mu})\, \dot{a}^{\mu} + \partial X_k / \partial t \equiv 0, \qquad k = 1, 2, ..., 10. \tag{II.1b}$$

where: the (ordered set of) ten conserved quantities X_k represent the conventional Galilean conservation laws of the energy, $\dot{H} = 0$, total linear momentum, $\dot{P} = 0$, total

JUPITER's STRUCTURE

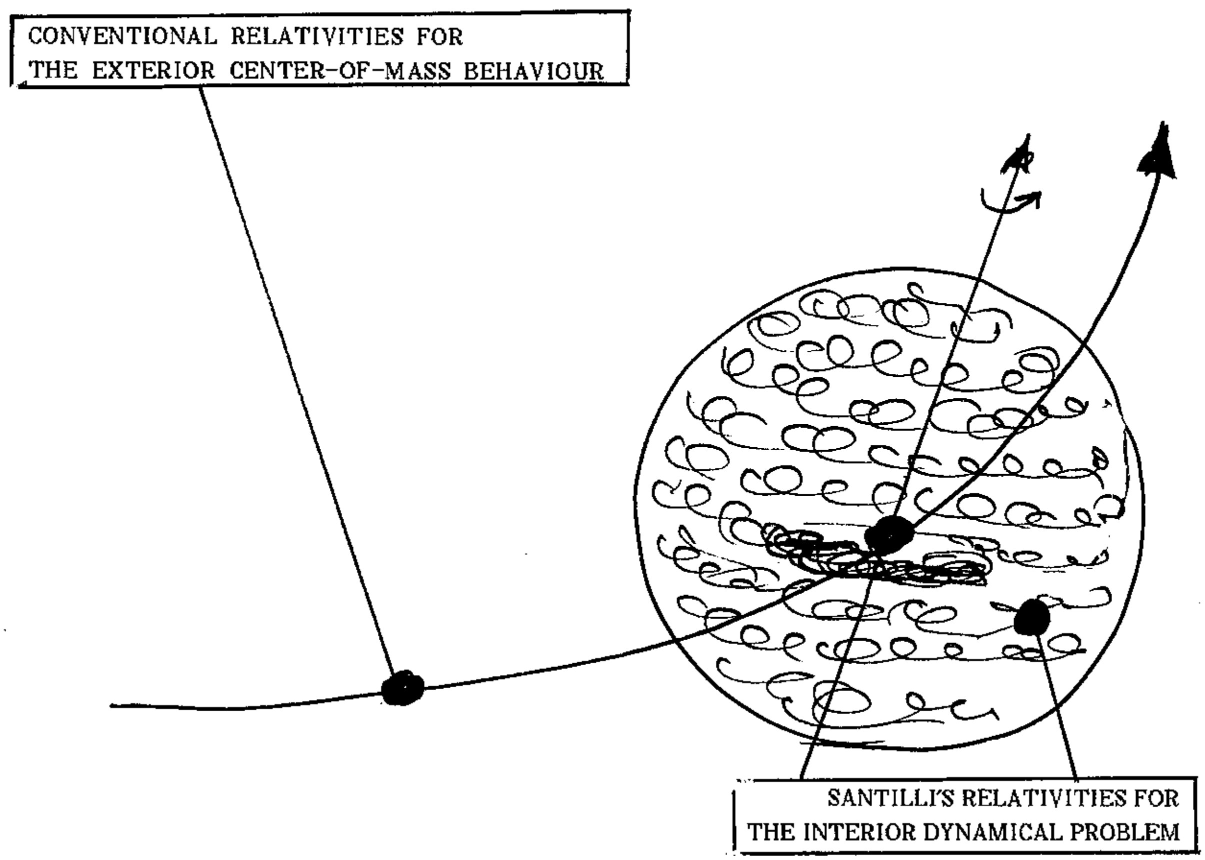

FIGURE II.1: A conceptual view often presented in Santilli's publications (1978c, d), (1981a), (1982a), etc.). The origin of all current relativities can be identified with the first visual observation of the Jovian system by Galileo Galilei back in 1609. Santilli's generalizations of Galilei's and Einstein's relativities can also be identified with a direct visual inspection of the Jovian systrem. The recent NASA missions to Jupiter clearly reveal a dichotomy of historical character: the exact *validity* of the conventional relativities for Jupiter's center-of-mass, EXTERIOR dynamics in the Solar system, jointly with the manifest *inapplicability* (and not "violation") of the same relativities for Jupiter's INTERIOR structural problem, as established by vortices with continuously varying angular momenta, etc. Also, when considered as isolated from the rest of the universe, Jupiter's is a majestic illustration of Santilli's "closed nonhamiltonian systems", because of its clear global stability and verification of conventional total conservation laws, while its interior dynamics is structurally nonconservative, nonlinear, nonlocal and nonhamiltonian. The above view has particularly conceptual value, because Santilli's generalized relativities provide a form-invariant description of Jupiter's structure conceived as aclosed nonhamiltonian system at the various Newtonian (Part I), relativistic (Part II) and gravitational (Part III) levels. As we shall see in Part IV, the operator counterpart of Jupiter's leads to a conceptually similar structure of hadrons, namely, an operator bound system whose center-of-mass

EXTERIOR dynamics (e.g., in a particle accelerator), verifies all conventional relativities, nevertheless, the INTERIOR structural problem is nonhamiltonian to acccount for the historical nonlocality of matter. This is the reason why Santilli repeats in his writings that the structure of hadrons may well be analytically equivalent to the structure of Jupiter.

angular momentum $\dot{J} = 0$, and uniform, motion of the center of mass, $\dot{G} = 0$, and NSA stands for variational nonselfadjointness, namely, for the violation of the integrability conditions for the existence of a Hamiltonian (see the enclosed Vol. I of Santilli's *Foundations of Theoretical Mechanics*).

Systems (II.1) constitute underdetermined systems of 6N differential equations with ten subsidiary constraints given by the total conservation laws, which are reducible for certain technical reasons to seven independent constraints. As such, the systems are consistent under sufficient continuity conditions generally verified in the physical reality.

In particular, Santilli proved that *unconstrained* solutions in the nonselfadjoint forces always exist for given potential forces.

In essence, Santilli's systems (II.1) establish that, in "closing" the satellite of Fig. I.1 with the surrounding atmosphere (or, equivalently, by "closing"Lagrange's and Hamilton's historical equations (I.1) via the inclusion of the medium in which motion occur), by no means, the contact, nonlocal, nonlagrangian or hamiltonian forces F_{ka} "disappear" according to a rather widespread but erroneous belief. On the contrary, Lagrange's and Hamilton's exterior forces F_{ka} persist in their entirety because variationally nonselfadjoint.

Closed nonhamiltonian systems for N = 2 were studied in detail in the original proposal (Santilli (1978d)). Their general theory was then studied in the enclosed Vol. II of Santilli's *Foundations of Theoretical Mechanics* (1982a). Additional basic advances were made in the memoir (Santilli (1988a)). Examples for N = 2 and 3 were worked out in details by Jannussis, Mijatovic and Veljanoski (1991) as the first examples of Santilli's generalization of Galilei's relativity (see Sect. V.7). Systems (II.1) were also studied from a statistical viewpoint by Fronteau et al. (1982), Tellez-arenas et al. (1982), and others.

Systems (II.1) are expected to have an intriguing connection with *Prigogine statistics* (see Prigogine (1962), (1968), (1990) and quoted references), which is currently under study by Jannussis et al. In fact, the systems recover the conventional time reversal symmetry for the exterior center-of-mass behavior, while admitting an intrinsically irreversible interior dynamics. The operator counterpart of the above results is presented in Part IV.

In summary, the central requirements for the closed-isolated approach to interior dynamical systems are the following:

I) All total, conventional, Galilean, Lorentzian or Riemannian conservations are verified.

II) The particles considered are extended-deformable while moving within a generally inhomogeneous and anisotropic medium; and

III) The forces are a combination of conventional local-potential, as well as nonlinerar, nonlocal and nonhamiltonian forces.

Note that, by conception and practical realization, no generalized interior dynamics can be detected from the outside, trivially, because of the exact validity of conventionbal relativities (Figure II.1).

This is nothing but a consequence of the fact that internal contact interactions for which the notion of potential energy has no meaning, cannot possibly have an impact on the exterior dynamical behaviour, as majestically established at the nonrelativistic level by Jupiter (Fig. II.1)

In Part II the NOBEL COMMITTEE will see the relativistic counterpart of the above setting. In fact, the historical legacy on the nonlocality of the hadronic structure can at best deal with *internal short range effects* also of nonpotential type[2] which, as such, cannot possibly affect the *exterior dynamical behaviour.* This leads to a dichotomy fully analogous of that of Jupiter, whereby the center-of-mass trajectory of a hadron in a particle accelerator strictly obeys Einstein's special relativity in a way fully compatible with the possible validity of Santilli's covering relativity for its interior structural problem.

As well known, *"closed Hamiltonian systems"* (i.e., isolated systems of point-like particles with only potential internal forces) constitute the physical foundations of contemporary relativities. Santilli's more general closed nonhamiltonian systems then constitute the physical foundations of his covering relativities. The main sections of this Nomination are therefore devoted to a review of the mathematical methods and relativities for closed nonhamiltonian systems

II.3: LAGRANGE'S AND HAMILTON'S OPEN SYSTEMS.

The second complementary approach identified by Santilli is the study of systems (I.3) as originally conceived by Lagrange's and Hamilton's, namely, with a total energy H = T + V which is NONCONSERVED because of exchanges with the physical medium which is considered as external.

This alternative conception is a necesary complement of the preceding one on numerous counts indicated later on in this presentation, including the proper definition of ONE individual "particle" in interior conditions.

[2] A fundamental point for the consistency of this dichotomy is the *lack* of potential character of the contact internal forces, as conceived by Santilli and quantitatively treated via the *conditions of nonselfadjointness* . In fact, ***internal forces with a potential would directly affect the exterior dynamical behaviour, contrary to experimental evidence*** (Fig. II.1). This point is important for the NOBEL COMMITTEE to separate "true experts" from "false experts" (Sect.4). When exposed to interior nonlocal dynamical problems, the formers use the conditions of selfadjointness or other means to treat forces are they are in the physical reality, and proceeds to the consequential necessary *generalization* of current doctrines,. By contrast, the latters merely add "nonlocal potentials" to their trivial Lagranmgian or Hamiltonians, for the intent of *preserving* conventional relativities for the interior dynamics. This latter approach is however inconsistent on numerous mathematical and physical counts identified by Santilli (1981c) in details. In fact, the latter approach implies the invalidation of the Zeeman topology (evidently because strictly local-differential and therefore incompatible with "nonlocal potentials', with the consequential loss of the integrability to the finite Galilei's or Lorentz's symmetries. Physically, the addition of a "nonlocal potential" to a Lagrangian or a Hamiltonian implies the necessary alteration of the exterior trajectory (e.g., Jupiter's center-of-mass motion would be in part dependent on internal nonconservative effects), which is manifestly against clear experimental evidence.

Therefore, the contemporary study of the historical interior problem can be reinterpreted today as the study of *one particle in the most complex possible physical conditions* , e.g., a proton in the core of a star undergoing gravitational collapse. The complementarity of this open-nonconservative notion with that of the closed-isolated system as a whole is then evident, and equally evident is the need to identify the most effective means to treat each approach.

To clarify the above setting, recall that only one approach is needed for the characterization of a closed Hamiltonian system as a whole as well as for the characterization of one of its constituent. This is due to the fact that *both, the system as a whole and its individual constituents are in stable orbits*. A theory characterizing conservation laws is therefore sufficient for both profiles (see next section).

In the transition to closed nonhamiltonian systems the situation is, again, fundamentally different. In fact, in this case *the system as a whole is stable, thus requiring theories with the emphasis on conservation laws, while individual constituents are in generally unstable conditions, thus requiring more general theories which put the emphasis on time-rate-of-variations of physical quantities (see next section)*

This complementary open-nonconservative approach is outlined in Appendix A of this Nomination.

SECTION III: MATHEMATICAL OVERVIEW OF SANTILLI'S DISCOVERIES

III.1: STATEMENT OF THE PROBLEM.

As recalled in the foreword, the mathematical foundations of contemporary theoretical physics at large and, in particular, of the conventional Galilei's and Einstein's spacial relativities for the exterior problem, are given by *Lie algebras*, the *symplectic geometry*, and conventional *Hamiltonian mechanics*

The central equations for an exterior system of N particles in phase space are Hamilton's equations in their contemporary "truncated" form without external terms, which can be written in the local coordinates $a = (a^{\mu})$ of Eq.s (II.1)

$$\dot{a}^{\mu} = \omega^{\mu\nu} \frac{\partial H(a)}{\partial a^{\nu}}, \qquad H(a) = H(r,p) = T(p) + V(r), \ \mu = 1, 2, ..., 6N \qquad \text{(III.1)}$$

where the tensor $\omega^{\mu\nu}$, called the *canonical Lie tensor*, is given by

$$(\omega^{\mu\nu}) = \begin{pmatrix} 0_{3Nx3N} & I_{3Nx3N} \\ -I_{3Nx3N} & 0_{3Nx3N} \end{pmatrix} \qquad \text{(III.2)}$$

The underlying brackets are the familiar *Poisson brackets*

$$[A,B] = \frac{\partial A}{\partial a^{\mu}} \omega^{\mu\nu} \frac{\partial B}{\partial a^{\nu}} = \frac{\partial A}{\partial r_{ka}} \frac{\partial B}{\partial p_{ka}} - \frac{\partial B}{\partial r_{ka}} \frac{\partial A}{\partial p_{ka}}. \qquad \text{(III.3)}$$

$$k = 1, 2, 3, \qquad a = 1, 2, ..., N$$

Their most salient feature is that they characterize a Lie algebra, i.e.,. they verify the axioms

$$[A,B] + [B,A] = 0, \quad [A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0. \qquad \text{(III.4)}$$

from which the symplectic geometry and Hamiltonian mechanics follow, as well known (see, e.g., Abraham and Marsden (1967)).

But, the systems represented by these mathematical methods are local-differential and potential-Hamiltonian, while no effective mathematical method existed, at the time of initiation of Santilli's studies, for the treatment of interior, nonlinear, nonlocal and nonhamiltonian systems.

Thus, in order to be able to study of the generalization of current relativities, Santilli was forced to construct, first, suitable generalizations of their mathematical foundations, and then construct the covering relativities themselves.

Moreover, the novel mathematical methods had to be of dual character, a first class for the characterization of interior systems as closed-conservative (e.g., Jupiter's structure of Fig. II.1), and a second class for their open-nonconservative version (e.g., the satellite in Jupiter's atmosphere considered as external of Fig. I.1).

III.2: SANTILLI'S LIE-ISOTOPIC FORMULATIONS.

The formulations needed for the characterization of closed nonhamiltonian systems (II.1) must verify the following conditions:

1) Their algebra must possess a totally antisymmetric product, say $[A \hat{,} B] = -[B \hat{,} A]$ (or, equivalently, the underlying exterior calculus must be totally antisymmetric) as a necessary condition to represent the conservation of the total energy, $\dot{H} = [H \hat{,} H] \equiv 0$.

2) The formulations must be able to represent consistently nonlinear, nonlocal and nonhamiltonian forces, and must therefore have in particular a suitable nonlocal topology; and

3) All conventional formulations and disciplines must be contained as a particular case, and recovered identically when the nonhamiltonian forces are null.

The first identification of the above generalized methods was made by Santilli in his memoir (1978a) written at the *Lyman Laboratory of Physics* of *Harvard University* under contract from the U. S. Department of Energy No. ER-78-S-02-4742.A000 (see Docunment A), and then subsequently presented in the enclosed Vol. II of *Foundations of Theoretical Mechanics* (1982a). It consists of the discovery of a generalization of Lie's theory verifying conditions 1), 2) and 3) above, which he called *"Lie-isotopic theory"*, and which has been called in the literature the *Lie-Santilli theory* (see, .Kadeisvili (1991), Aringazin et al. (1992), and others).

In the subsequent years, he worked out the necessary details for the construction of the foundations of the various branches of his generalized theory, including the compatible formulation of fields, vector spaces, transformations, representations, analytic mechanics, symplectic and Riemannian geometries, ect. , as reviewed in the memoirs Santilli (1988a, b) and (1991a, b) (see the outline of Sect. IV below).

The central analytic equations are a generalization of the "truncated" Hamilton's equations (III.1) of the form, now called *Hamilton-Santilli equations*

$$\dot{a}^{\mu} = \omega^{\mu\alpha}\, \hat{1}_{\alpha}{}^{\nu}(a) \frac{\partial H(a)}{\partial a^{\nu}}, \quad H(a) = H(r,p) = T + V, \quad \mu, \nu, =1, 2, ..., N, \quad (III.5)$$

where $\hat{1}$ is the generalized unit of the theory (see next section), with generalized brackets

$$[A \hat{,} B] = \frac{\partial A}{\partial a^{\mu}} \omega^{\mu\alpha} I_{\alpha}{}^{\nu} \frac{\partial B}{\partial a^{\nu}} =$$

$$= \frac{\partial A}{\partial r_{ia}} I_{ij} \frac{\partial B}{\partial p_{ja}} - \frac{\partial B}{\partial r_{ia}} I_{ij} \frac{\partial A}{\partial p_{ja}}, \qquad (III.6)$$

verifying the axioms

$$[A \hat{,} B] + [B \hat{,} A] = 0, \quad [A \hat{,} [B \hat{,} C]] + [B \hat{,} [C \hat{,} A]] + [C \hat{,} [A \hat{,} B]] = 0, \qquad (III.7)$$

The terms *"Lie-Santilli formulations"* today encompass a generalization of the virtual entirety of the mathematical foundations of contemporary classical (and quantum) mechanics, as we shall outline in the next section.

One of the objectives of this Nomination is to indicate that the generalization of contemporary relativities were achieved by Santilli only AFTER discovering these generalized new mathematical tools.

III.3: SANTILLI'S LIE-ADMISSIBLE FORMULATIONS.

Quite significantly, the listing by the Estonian Academy of Science of Santilli's name among the most illustrious names in the history of physics and mathematics (Document D) was *not* done for the Lie-Santilli formulations, but for yet more general formulations which include the preceding ones as particular case.

They treat the more general class of open nonconservative systems as originally conceived by Lagrange (1788) and Hamilton (1834), those via the original equations (I.1). At the time of Santilli's PhD studies at the *Istituto di Fisica* of the *Universita' di Torino* , Italy, the algebraic character of Eq.s (I.1) was entirely unknown.

In three historical papers (1967), (1968) and (1969) (see also the enclosed monograph of (1982a), p. 90 and ff.) Santilli first proved that the brackets of the original Hamilton's equations,

$$AxB = [A,B] + \frac{\partial A}{\partial p_{pa}} F_{ka}, \qquad (III.8)$$

not only violate the Lie algebra axioms (III.4) or (III.7), but *they violate the conditions for the validity of any algebra as commonly understood in contemporary mathematics* (see, e.g., Schafer (1966)), because they verify the left scalar and distributive laws, but violate the corresponding right laws

$$\alpha x(BxC) = Ax(\alpha xB) = (\alpha xA)xB, \qquad (A + B)xC = AxC + BxC, \qquad (III.9a)$$

$$(AxB)x\alpha \neq Ax(Bx\alpha) \neq (Ax\alpha)xB, \qquad Ax(B + C) \neq AxB + AxC, \qquad (III.9b)$$

Santilli then reformulated Hamilton's equations (I.1) in the form

$$\dot{a}^{\mu} = S^{\mu\nu}(t, a) \frac{\partial H(a)}{\partial a^{\nu}}, \quad (S^{\mu\nu}) = (\omega^{\mu\nu}) + diag(0, s), \quad s = F/(\partial H/\partial p) \qquad (III.10)$$

where the F's are the external forces, whose brackets are now given by

$$(A,B) = \frac{\partial A}{\partial a^{\mu}} S^{\mu\nu}(t, a) \frac{\partial B}{\partial a^{\nu}}. \qquad (III.11)$$

He then showed that the brackets (A,B) of modified Equations (III.10)) do indeed characterize a consistent algebra, because they do verify the *left* and right scalar and distributive laws. Santilli also showed that the consistent algebras characterized by brackets (A,B) admit the Lie algebras as a particular case evidently when all external forces are null and, in this sense, they are "coverings" of Lie algebras.

Prior to publishing his first paper, Santilli (1967) then conducted an extensive search in the mathematical libraries of the Universita' di Torino, Italy, which lasted well in excess of one full year, and finally succeeeded in identifying the algebras with brackets (A,B) as being the (nonassociative) algebras introduced by Albert (1948) under the name of *Lie-admissible algebras* .

The NOBEL COMMITTEE should note that, as today known (see the extensive mathematical bibliography by Balzer et al. (1984) in nonassociative algebras), prior to the paper (Santilli (1967)), only two other papers had appeared in the specialized mathematical literature besides the original paper by Albert, and given by a paper by Weiner (1957) published in a little known Journal (*Rev. Univ. Tucuman*) and a paper by Laufer et al. (1962) appeared in *Can. J. Math.* Despite that, Santilli did succeed in identifying prior contributions, as a necessary pre-requisite for truly scolarly work.

In fact, the NOBEL COMMITTEE can see the quotation of the mathematical article by Albert (1948) published by *Trans. Amer. Math. Soc.* , in the physics article by Santilli (1967) published by *Il Nuovo Cimento* , at the bottom of the first page 570. Santilli then continued his laborious library search in 1967-1968, this time, at the *University of Miami* , Coral Gables, Florida, and did indeed succeed in identifying the remaining two articles by Weiner (1957) and Laufer et al. (1962) which he quoted in the subsequent paper (Santilli (1968), p. 1243 and 1244, respectively.

Albert had introduced the abstract definition of Lie-admissible algebras as nonassociative algebras U with elements a, b, ... and abstract product ab over a field F which are such that the attached algebra U^-, which is the same vector space U (namely, the elements of U and U^- coincide) is Lie, that is, the product $[a,b]_U = ab - ba$ is Lie. Weiner (loc. cit.) and Laufer et al. (loc. cit.) had studied some preliminary properties of these algebras, but always at the abstract mathematical level, while none of these authors had identified specific cases of Lie-admissible algebras.

Santilli was unequivocally the first to identify a rather fundamental realization of Lie-admissible algebras and their application as providing the algebraic characterization of the historical Hamilton's equations with external terms, when written in the modified form (III.10). In fact, the algrebra characterized by brackets (A,B) is Lie-admissible because that characterized by the attached brackets (A,B) - (B,A) is Lie,

$$(A,B) \text{ is LIE-ADMISSIBLE because } \quad (A,B) - (B,A) = 2[A,B] \text{ is LIE.} \qquad (III.12)$$

Subsequently, Santilli extended his studies of Lie-admissible algebras in memoir (1978a) at the abstract level as well as in their classical realization. In the subsequent memoir (1978d) also written at Harvard University under contract with the U.S. Department of Energy, Santilli succeeded in introducing an ***operator realization of the Lie-admissible algebras, that is, an operator counterpart of the historical Hamilton's equations with external terms,*** which signaled the birth of a generalization of quantum mechanics as outlined for the NOBEL COMMITTEE in the forthcoming Part IV. Finally he expanded his Lie-admissible studies in the monographs Santilli (1978c) and (1981a).

By recalling that Lie algebras constituite the ultimate mathematical structure of both classical and quantum mechanics, ***the listing by the Estonian Academy of Science of Ruggero Santilli in their chart of famed mathematicians and physicists since 1800 (Document D) for his first article of 1967 was very appropriate indeed, because that particular article can be historically referred to as signaling the birth of the Lie-admissible generalizations of both classical and quantum mechanics for open interior trajectories, the complementary classical (1978a) and quantum mechanics (1978d) Lie-isotopic generalization for closed nonhamiltonian systems being a mere particular case.***

Nowadays, the terms *"Santilli's Lie-admissible formulations"* are referred to a rather vast body of methodological tools encompassing Lie-admissible generalizations of classical and operator Hamiltonian mechanics, as well as symplectic and Riemannian geometries, which we cannot possibly review in this Nomination in detail, but merely indicate in the appendices of: this Part I for the classical Galilean profile, of Part II for the relativistic profile, of Part III for the gravitational profile, of Part IV for the operator profile, and of Part V for the unified theories.

When inspecting the rather vast body of novel mathematical and physical knowledge of this Nomination, the NOBEL COMMITTEE should therefore keep in mind the following chain of mathematical formulations of increasing complexity and methodological need

$$\begin{pmatrix}\text{LIE}\\ \text{FORMULATIONS}\end{pmatrix} \subset \begin{pmatrix}\text{LIE-ISOTOPIC}\\ \text{FORMULATIONS}\end{pmatrix} \subset \begin{pmatrix}\text{LIE-ADMISSIBLE}\\ \text{FORMULATIONS}\end{pmatrix} \quad \text{(III.13)}$$

which represent systems of corresponding, progressively more complex physical conditions

$$\begin{pmatrix}\text{CLOSED}\\ \text{LOCAL}\\ \text{HAMILTONIAN}\end{pmatrix} \subset \begin{pmatrix}\text{CLOSED}\\ \text{NONLOCAL}\\ \text{NONHAMILTONIAN}\end{pmatrix} \subset \begin{pmatrix}\text{OPEN}\\ \text{NONLOCAL}\\ \text{NONHAMILTONIAN}\end{pmatrix} \quad \text{(III.14)}$$

with related, progressively more general relativities

$$\begin{pmatrix}\text{CONVENTIONAL}\\ \text{LIE}\\ \text{RELATVITIES}\end{pmatrix} \subset \begin{pmatrix}\text{SANTILLI'S}\\ \text{LIE-ISOTOPIC}\\ \text{RELATIVITIES}\end{pmatrix} \subset \begin{pmatrix}\text{SANTILLI'S}\\ \text{LIE-ADMISSIBLE}\\ \text{RELATIVITIES}\end{pmatrix} \quad \text{(III.15)}$$

SECTION IV: OUTLINE OF SANTILLI'S LIE-ISOTOPIC FORMULATIONS

IV:1: THE CENTRAL MATHEMATICAL IDEA.

The fundamental mathematical notion introduced by Santilli in his memoir (1978a) is the generalization of the conventional trivial unit I of contemporary Lie's formulations into a quantity $\hat{1}$, called *isotopic-unit* (or *isounit* for short), which, besides being nonsingular and Hermitean, has an arbitrary, nonlinear and nonlocal dependence on all possible quantities and their derivatives (see, later on, the examples of Sect. V.8),

$$\hat{1} = \hat{1}(t, r, p, \dot{p}, \ldots). \tag{IV.1}$$

Since Lie's theory is insensitive to the topology of its unit, the above generalization ensures the possibility of representing nonlocal interactions, while achieving the desired mathematically simple and physically effective nonlocal topology.

The generalization of the unit evidently requires a corresponding, compatible generalization of the entire Lie's theory, including: the universal enveloping associative algebras; the theory of Lie algebras; the theory of Lie groups; the representation theory; etc.. In turn, such a generalized theory requires a corresponding generalization of virtually all mathematical structures of current use in physics, such as fields, vector spaces, metric spaces, transformations, representations, geometries, etc., as established in the recent comprehensive presentation for mathematicians (Santilli (1991a,. b). No appraisal of Santilli's physical discoveries is possible, without a knowledge of these novel mathematical structures outlined below in this section.

The mathematical method underlying the generalization $I \Rightarrow \hat{1}$ is an *isotopy* in the Greek meaning of "preserving the configuration", namely, a generalization of a given mathematical structure which preserves the basic axioms. In fact, the axioms of the trivial unit I of Lie's theory are nonsingularity, Hermiticity and positive-definiteness, which are preserved by the generalized unit $\hat{1}$.

Prior to releasing the memoir (1978a), Santilli therefore embarked in his third[3] and perhaps most time consuming search in the Cantabridgean libraries

[3]The first extensive library search was that recalled in Sect.III.3 from 1966 to 1968 to identify prior references on the Lie-admissible algebras, and lead to the establishment of Albert (1948) as the first contribution in the field (Santilli (1967), (1968)). The second extensive library search was that recalled in Sect. I.2 from 1975 to 1978 to identify all the historical contributions on the integrability conditions for the existence of a Lagrangian or a Hamiltonian, and lead to the establishing of Helmholtz (1887) as the originator of the studies. We are here referring to the third extensive library search on prior contributions on Santilli's isotopic generalization of Lie's theory wich resulted to be fruitless because, as today well known (see the comprehensive bibliography on nonassociative algebras by Balzer et al. (1984)), Santilli was the originator of this new mathematical theory. Numerous, additional, extensive

for prior references on the isotopic generalization of Lie's theory. This search lasted from 1975 to 1978 and essentially resulted to be fruitless.

The *only* mathematical book that Santilli could identify at that time with a mention of the notion of isotopy, was Bruck (1958), who points out that the notion of isotopy dates back to the early stages of *set theory*. Still as of today, we are aware of no mathematical texbook (let alone physics textbook) that reviews the notion of isotopy.

In regards to articles, the *sole* use of the isotopy Santilli could identify was in regard to certain nonassociative algebras (the so-called *commutative and noncommutative Jordan algebras*), as subsequently confirmed by the comprehensive bibliography in nonassociative algebras by Baltzer et al. (1984).

These historical notes are here presented to the NOBEL COMMITTEE to illustrate the fact that the name *"Lie-Santilli theory"* in Kadeisvili (1991), Aringazin et al (1992) and othersis fully sound indeed.

IV.2: SANTILLI'S ISOFIELDS.

Let F be a field of current use in physics, such as the field of real numbers $\Re$ or of complex numbers C, with elements α, β, ..., conventional sum, $\alpha + \beta$, product $\alpha\beta$, and multiplicative unit given by the trivial element 1, $1\alpha = \alpha 1 \equiv \alpha$ for $\forall\ \alpha \in F$.

At his invited speech at the 1980 *Conference on Differential Geometric Methods in Mathematical Physics*, held in Clhaustal, Germany, Santilli ((1980, (1981), (1983a), (1985a)) introduced the notion of *isofield* $\hat{F}$ as the structure

$$\hat{F} = \{\hat{\alpha} \,|\, \hat{\alpha} = \alpha \hat{1},\ \ \alpha \in F,\ \ \hat{1} = T^{-1}\}, \tag{IV.2}$$

equipped with the conventional sum $\hat{\alpha} + \hat{\beta} = (\alpha + \beta)\,\hat{1}$ and the isotopic product $\hat{\alpha}*\hat{\beta} = \hat{\alpha}T\hat{\beta} = (\alpha\beta)\,\hat{1}$, where T is a fixed quantity (generally outside the original field), called the *isotopic element*. Then $\hat{1}$ is the correct right and left multiplicative unit of the isofield $\hat{F}$ because

$$\hat{\alpha}*\hat{1} = \hat{1}*\hat{\alpha} \equiv \hat{\alpha}, \qquad \forall\ \hat{\alpha} \in \hat{F} \tag{IV.3}$$

Santilli then proved that the abstract notion of *isoreals* (that is, the isotope $\hat{\Re} = \Re\hat{1}$ of the field of real number $\Re$) includes as particular cases all fields used in physics, such as the reals, complex numbers and quaternions. In fact, the field of complex numbers C with structure $c = \alpha 1 + \beta 1_1$ ($\alpha, \beta \in \Re$ and $1_1 = i$), and the field of quaternions with structure $Q = A1 + \beta 1_1 + \gamma 1_2 + \delta 1_3$ (where α, β, γ, $\delta \in \Re$ and the 1_k, k = 1, 2, 3 are Pauli's matrices), are isotopes of the field of real

library searches, that individually lasted for years, were also conducted by Santilli, such as: the fourth extensive library search that lead to the establishment of Birkhoff (1927) as the discoverer of the analytic equations underlying generalized Poisson brackets (Sect. IV.8); the fifth extensive library search to identify any preceding true generalization of Einstein's special relativity indicated in the forthcoming Part II; and a number of additional extensive searches outlined in the subsequent parts of this Nonimation. It is a pleasure for these authors to bring to the attention of the NOBEL COMMITTEE the beautiful historical value of these very time consuming searches conceived and conducted by Santilli alone.

numbers with isounits $\hat{1} = 1 \times 1_1$ and $\hat{1} = 1 \times 1_1 \times 1_2 \times 1_3$, respectively.

To begin the illustration of the unifying power of Santilli's isofields, the NOBEL COMMITTEE should be aware that *the known quaternionic generalization of quantum mechanics is an isotopy of quantum mechanics* (see Part IV).

It should be indicated from the outset that *the liftings $\Re \Rightarrow \hat{\Re}$, $C \Rightarrow \hat{C}$, etc., imply no alteration of the physical numbers of isotopic theories, both classically and quantum mechanically*. This is due to the fact that the isomultiplication of an isonumber $\hat{\alpha}$ by any physical quantity Q coincides with the conventional multiplication by an ordinary number, i.e., $\hat{\alpha} * Q \equiv \alpha Q$. As a result classical and quantum "isoeigenvalues" coincide with ordinary eigenvalues.

IV.3: SANTILLI'S ISOSPACES.

Let M(x,g,F) be an n-dimensional metric or pseudo-metric space of current use in physics with local coordinates x, metric g, and composition $x^2 = x^i g_{ij} x^j$, i, j = 1, 2, ..., n, over the field F . Well known examples of spaces used in physics are:

1) The 3-dimensional *Euclidean space* $E(r,\delta,\Re)$ with metric δ = diag. (1,1,1) over the reals $\Re$;

2) The (3+1)-dimensional *Minkowski space* $M(x,\eta,\Re)$ with the metric η = diag. (1,1,1,-1); and

3) The (3+1)-dimensional *Riemannian space* $R(x,g,\Re)$ with symmetric metric $g = g(x)$;

which are the fundamental carrier spaces of Galilei's relativity, Einstein's special relativity and Einstein's general relativity, respectively.

In one of his most important papers written on the isotopic generalization of Einstein's special relativity (see Part II), Santilli (1983a) discovered the notion of *isospaces* as the generalized spaces $\hat{M}(x,\hat{g},\hat{F})$ which have the same coordinates x of the original space M(x,g,F), but are equipped with the generalized metric $\hat{g} = Tg$ called *isometric*, and are defined over the isofields $\hat{F} = F\hat{1}$, $\hat{1} = T^{-1}$, with composition

$$x^{\hat{2}} = (x^i \hat{g}_{ij}(t, x, \dot{x}, ...) x^j)\, \hat{1} \in \hat{F}, \qquad \text{(IV.4)}$$

called *isocomposition*.

The fundamental meaning of isospaces is to provide a *geometrization of interior physical media*, via the direct representation of their inhomogenenuity, anisotropy and other physical characteristics. In fact, as we shall see in the physical review of Sect. V, *the transition from a conventional space to its isotopic extension is a direct representative of the transition from motion in vaucuum to motion within physical media*.

Santilli ((1983a), (1985a), (1991a)) therefore studied in details the isotopes of spaces $E(r,\delta,\Re)$, $M(x,\eta,\Re)$ and $R(x,g,\Re)$ given by:

$\hat{1}$) The *isoeuclidean spaces* $\hat{E}(r,\hat{\delta},\hat{\Re})$, $\hat{R} = \Re\hat{1}$, $\hat{1} = T^{-1}$, $\hat{\delta} = T(r, \dot{r}, \ddot{r}, ...)\delta$, $T^{\dagger} = T > 0$, which are the carrier spaces of *Santilli's isogalilean relativities* (Sect.V);

$\hat{2}$) The *isominkowski spaces* $\hat{M}(x,\hat{\eta},\hat{R})$, $\hat{\Re} = \Re\hat{1}$, $\hat{1} = T^{-1}$, $\hat{\eta} = T(x, \dot{x}, \ddot{x}, ...)\eta$, $T^{\dagger} = T > 0$, which are the carrier spaces of *Santilli's special relativities* (Part II); and

$\hat{3}$) The *isoriemannian spaces* $\hat{R}(x,\hat{g},\hat{\Re})$, $\hat{\Re} = \Re\hat{1}$, $\hat{1} = T^{-1}$, $\hat{g} = T(x, \dot{x}, \ddot{x}, ...)g(x)$, $T^{\dagger} = T > 0$, which are the carrier spaces of *Santilli's isogravitation* (see Part III).

The NOBEL COMMITTEE should be aware of the unifying power of Santilli's isospaces. In fact, by keeping the nonsingularity and Hermiticity of the isounit, and by relaxing the condition $T > 0$, the isoriemannian and isominkowski spaces are particular cases of the isoeuclidean spaces of the same dimension.

This permitted Santilli to achieve a remarkable *geometric unification of all relativities for the interior and the exterior problem* which will be outlined in Part III via the reduction of all conceivable interior and exterior, linear and nonlinear, local and nonlocal, Lagrangian and nonlagrangian, relativistic and gravitational systems to only one abstract symmetry: *Santilli's Poincaré-isotopic symmetry*. In this Part I we can only outline the first step of this chain of unified relativities.

In the same paper (1983a) Santilli showed that the isospaces are useful also for conventional theories. In fact, the Minkowski space $M(x,\eta,\Re)$ can be interpreted as the isotope of the 4-dimensional Euclidean space $M(x,\eta,\hat{\Re}) \approx \hat{E}(x,\hat{\delta},\hat{\Re})$, $\hat{\Re} = \Re\hat{1}$, $\hat{1} = \eta^{-1}$, $\hat{\delta} = \eta\delta$. Similarly, the Riemannian space $R(x,g,\Re)$ can be interpreted as the isotope of the Euclidean space $\hat{E}(x,\hat{\delta},\hat{\Re})$, $\hat{\Re} = \Re\hat{1}$, $\hat{1} = g^{-1}$, $\hat{\delta} = g(x)\delta$, or as the isotope of the Minkowski space $\hat{M}(x,\hat{\eta},\hat{\Re})$, $g(x) = T(x)\eta$, $T > 0$.

These novel mathematical methods permitted *the interpretation of Einstein's exterior gravitation as an isotope of the special relativity* as well as the discovery of the global symmetry of gravitational theories which was only locally known prior to Santilli's research (see Part III).

Finally, the NOBEL COMMITTEE should keep in mind the fundamental role of the isofields in the very definition of isospaces.

IV.4: SANTILLI'S ISOTRASFORMATIONS.

Conventional relativities are fundamentally dependent on the notion of *linear and local* transformations $x' = A(w)x$ on a manifold $M(x,F)$. In fact, Galilei's, Lorentz's and Poincaré's transformations are strictly linear and local transformations, as well known.

Another fundamental discovery by Santilli in the original, memoir (1978a) is that of *isotransformations*

$$x' = A(w)*x = A(w)\,T\,x, \quad T = \text{fixed}, \qquad \text{(IV.5)}$$

subsequently formulated on an isomanifold $\hat{M}(x,\hat{F})$, $\hat{F} = F\hat{1}$, $\hat{1} = T^{-1}$(Santilli (1980), (1983a), (1985a), (1991a)). The relevance of the above transformations is that they verify all the conditions of linearity and locality in $\hat{M}(x,\hat{F})$, called *isolinearity* and *isolocality*. Nevertheless, when projected in the original manifold M(x,F), they are intrinsically nonlinear and nonlocal, because of the nonlinear and nonlocal dependence of the isotopic element T on all variables and their derivatives

$$x' = A(w)\,T(t, x, \dot{x}, ...)\,x, \qquad \text{(IV.6)}$$

In turn, this notion is at the foundations of Santilli's construction of nonlinear and nonlocal generalizations of conventional relativities for the characterization of the most general known interior systems (II.1).

The discovery of isolinearity and isolocality has far reaching mathematical and physical implications which will predictably require considerable time for their full development. Mathematically, the discovery implies that

All nonlinear and nonlocal systems can always be written in an equivalent isolinear and isolocal form.

In fact, given a nonlinear transformation $x' = A(w, x)\,x$, there always exists an isotopic element T(x) and a linear transformation B, such that

$$x' = A(w, x)\,x = \text{NONLINEAR} \equiv B(w)*x = B(w)T(x)\,x = \text{ISOLINEAR} \qquad \text{(IV.7)}$$

and the same evidently occurs for isolocality.

But at the abstract level, isotransformations A*x and Ax coincide. Thus, the notions of isolinearity and isolocality have permitted a new horizon for the mathematical treatment of nonlinear and nonlocal systems via an isotopy of conventional methods for linear and local systems.

Physically, the implications are equally far reaching in all branches of physics, classically and quantum mechanically. In fact, classically, the discovery implies that

The isotopies of a given linear and local theory characterize all its possible nonlinear and nonlocal generalizations.

To see the physical implications, we can note that *the historical open legacy of the ultimate (nonlinearity and) nonlocality of the strong interactions (Sect. I) becomes quantitatively treatable via Santilli's isotopies of quantum mechanics*. In fact, the classification of such isotopies implies the achievement of all possible nonlinear and nonlocal generalizations of quantum mechanics without altering its basic axioms (Part IV).

In this Nomination we deal with classical systems. We can therefore say that *the transition from the treatment of closed, local-differential, Hamiltonian systems to their closed nonlocal and nonhamiltonian generalization (II.1) can be quantitatively studied via Santilli's isotopies of classical Hamiltonian mechanics and of its underlying methodologies*, as outlined in the remaining parts of this section.

Almost needless to say, the generalization of the notion of transformation implies that of representations, which we regrettably cannot review here for brevity. We merely point out that in conventional Lie's theory the underlying vector space of a Lie algebra is a *one-sided module* of the algebra, with the action to the right being equivalent to that to the left from the antisymmetry of the Lie product (see, e.g., Schafer (1966)).

In the transition to the covering Lie-isotopic theory, the above structure is axiomatically preserved as it is the case for all isotopies, but it is nevertheless realized in its most general possible, isolinear and isolocal form. This lead Santilli's (1979) to the further discovery of the *isorepresentations* as characterized by the most general possible *modular-isotopic action* to the right or to the left, further developed and applied in Santilli (1982a), (1989c), and in the ICTP preprint (1991g).

The physical implications of Santilli's isotopies then become visible. In fact, the transition from conventional, linear and local representations of Lie algebras to Santilli's isolinear and isolocal isorepresentations of Lie-isotopic algebras implies *a necessary generalization of the notion of particle into that of isoparticle* (Sect. V.8) [4].

IV.5: LIE-SANTILLI THEORY IN ABSTRACT REALIZATION.

As well known, Lie's theory characterizes all contemporary, classical and quantum mechanical structures. In particular, Lie's theory provides a direct characterization of quantum mechanics when realized via matrices, and a direct characterization of classical Hamiltonian mechanics when realized via functions in phase space.

Santilli's original and most basic intuition during his graduate studies at the *Universita' di Torino*, Italy, was that

Any generalization of Lie algebras implies a consequential

[4] The non triviality of the covering Lie-admissible algebras over the Lie-isotopic and conventional Lie algebras can be seen at this point by noting that the Lie-admissible product is no longer totally antisymmetric. This implies that the modular action to the right is no longer equivalent to that to the left. This lead Santilli (1979) to the further discovery of the *isobirepresentations*, that is, the two-sided, left and right, modular-isotopic representations of a Lie-admissible algebras, which is one of the most complex concepts of contemporary mathematics whose technical development will require mathematical studies well into the next century. Santilli (1989a), (1991a) conceived one open-nonconservative particle of the interior system as an *isobiparticle*, that is, as an isoparticle in irreversible conditions with inequivalent motions forward (modular action to the right) and backward in time (modular action to the left). In this way, Santilli represented a proton in the core of a star undergoing gravitational collapse as an isobiparticle characterized by an isobirepresentation of a Lie-admissible algebras, as one way to express the view that we are dealing with one of the most complex physical conditions that can be conceived by our current knowledge.

generalization of classical and quantum mechanical disciplines .

He therefore identified two sequential generalizations of Lie algebras, the Lie-isotopic and Lie-admissible algebras, and showed that each of them admits consistent generalizations of conventional classical and quantum mechanics theories.

In this section we shall outline the idea of the abstract (e.g., matrix) formulation of the Lie-isotopic theory, today called *Lie-Santilli theory* (see, e.g., Aringazin et al (1992)), which constitutes the algebraic foundation of the isotopic generalization of quantum mechanics reviewed in Part IV. The classical realization of the Lie-Santilli theory will be outlined in the next section. The more general classical realizations of the Lie-admissible theory will be outlined later on in Appendix A.

The current formulation of Lie's theory is centered in the trivial unit $I = \mathrm{diag.}(1,1,..,1)$. The generalization of the unit I into the nontrivial isounit $\hat{1}$ (Sect. IV.1) then implies a necessary, consequential generalization of the entire theory, as indicated earlier.

Let $\xi(L)$ be the conventional *universal enveloping associative algebra* of a Lie algebra L (see, e.g., Jacobson (1962)) with elements A, B, ..., trivial associative product AB, and left and right unit I, $IA = AI \equiv A$ for $\forall A \in \xi(L)$. Let L be an n-dimensional Lie algebra with the (ordered) basis $X = X^{\dagger} = (X_k)$, $k = 1, 2,.., n$. In memoir (1978a) Santilli discovered a generalization of the envelope $\xi(L]$ into a form $\hat{\xi}(L)$ admitting $\hat{1}$ as the generalized unit, which he called the *universal enveloping associative-isotopic (or isoassociative) algebras*, characterized by the isotopic product

$$\hat{\xi}: \quad A*B = ATB, \quad T = \text{fixed} \tag{IV.8}$$

under which $\hat{1}$ is indeed the correct right and left unit of the new theory, $\hat{1}*A = A*\hat{1} \equiv A$, $\forall A \in \hat{\xi}(L)$.

Moreover, Santilli proved, in the same memoir (loc. cit.), that the celebrated *Poincaré-Birkhoff-Witt Theorem* (see, e.g., Jacobson (loc. cit.)) admits a consistent isotopic generalization for $\hat{\xi}(L)$, resulting in the new infinite-dimensional basis

$$\hat{\xi}: \; \hat{1}, \; X_k, \quad X_i*X_j \; (i \leq j), \quad X_i*X_j*X_k \; (i \leq j \leq k), ... \tag{IV.9}$$

More recently, Santilli (1991a) proved that the notion of abstract isoenvelope $\hat{\xi}(L)$ is capable of unifying the envelopes of all simple (nonexceptional) Lie algebras of the same dimension in Cartan's classification (see, e.g., Gilmore (1974)). In essence, the conventional envelope $\xi(L)$ characterizes only one Lie algebra L. On the contrary, the isoenvelope $\hat{\xi}(L)$ in the same original basis X but with infinitely many possible isotopic elements T, can recover via the attached algebras $[\hat{\xi}(L)]^{-}$all possible Lie algebras of the same dimensional of L.

Note that $[\xi(L)]^{-} \approx L$, but that, in general, $[\hat{\xi}(L)]^{-} \approx \hat{L} \not\approx L$, namaly, the Lie-isotopic algebra $\hat{L}$ attached to the isoenvelope $\hat{\xi}(L)$ is not necessarily isomorphic to the original Lie algebra L.

The preservation of the original symbol L in the isoenvelope $\hat{\xi}(L)$ was submitted since the original proposal (Santilli (1978a)) to stress the fact that the original basis and parameters are preserved by the isotopies. This is the technical foundation of the preservation of the conventional total conservation laws for closed nonhamiltonian systems (II.1).

Note also that the isotopy $\xi(L) \Rightarrow \hat{\xi}(L)$ implies a nonlinear and nonlocal generalization of the original structure.

Contemporary mathematics and physics are based on the simplest conceivable Lie product, the familiar Lie product $[A,B] = [A,B]_{\xi} = AB - BA$ attached to the envelope $\xi(L)$. Another historical discovery made by Santilli in the same memoir (loc. cit.) was the *isotopic generalization of Lie algebras* as the algebras $\hat{L} \approx [\hat{\xi}(L)]^{-}$ now attached to the isoenvelopes $\hat{\xi}(L)$ which are characterized by the new product

$$\hat{L}: \quad [A,B]_{\hat{\xi}} = A*B - B*A = ATB - BTA, \qquad \text{(IV.10)}$$

originally called *Lie-isotopic product* , and today called *Lie-Santilli product* (see, e.g., Aringazin et al (loc. cit.).

It is easy to see that brackets (IV.10) do verify the Lie algebra axioms, although in the generalized form

$$[A,B]_{\hat{\xi}} + [B,A]_{\hat{\xi}} = 0, \quad [A,[B,C]_{\hat{\xi}}]_{\hat{\xi}} + [B,[C,A]_{\hat{\xi}}]_{\hat{\xi}} + [C,[A,B]_{\hat{\xi}}]_{\hat{\xi}} = 0 \qquad \text{(IV.11)}$$

In more recent studies, the notion of Lie-Santilli algebras was brought to its full mathematical maturity, via its formulation on isofields $\hat{F}$ (Santilli (1989a), (1991a)).

In the original memoir (loc. cit.), Santilli then proved the isotopic generalizations of a number of conventional structural theorems of Lie's theory, including the *isotopic generalization of Lie's First, Second and Third Theorems*.

One of the most intriguing applications of the Lie-isotopic algebra is the unification of all possible compact and noncompact simple (nonexceptional) Lie algebras of the same dimension in Cartan's classification into one single, abstract, Lie-Santilli algebra.

This unification was first illustrated in Santilli (1978a) where one can see that the abstract rotational-isotopic algebra $\hat{O}(3)$ (see Sect. V.3) unifies all simple three-dimensional Lie algebras, the algebras O(3) and O(2.1). In Santilli (1983a) and (1989b), (1991d) one can see the unification of all simple six-dimensional Lie algebras into the abstract isotope $\hat{O}(4)$, which includes as particular cases O(4), the Lorentz algebra O(3.1), and O(2.2), as well as all their infinitely possible nonlinear and nonlocal realizations.

The underlying idea is so simple to appear trivial. In conventional Lie's theory it is generally believed (in both mathematics and physics) that one given basis uniquely identifies a Lie algebra (up to local isomorphisms). Thus, the transition from the compact O(3) algebra to the noncompact O(2.1) algebra requires a change of the basis. Santilli proved this belief to be erroneous because based on the *assumption* of the simplest conceivable Lie product "AB – BA". If, on the contrary, one assumes the more general product "ATB – BTA", then the same basis can evidently characterize nonisomorphic algebras.

In fact, the unification of O(3) and O(2.1) was illustrated in Santilli (1978a) (see also (1985b) for technical details) via the use of the generators of O(3), the components of the conventional angular momentum, with two essential isounits $\hat{1}$ = diag. (1,1,1), and $\hat{1}$ = diag. (1,1,−1), the former leading to O(3) and the latter leading to O(2.1).

Moreover, Santilli introduced, also for the first time, the generalized notion of *Lie-isotopic groups* $\hat{G}$, today called *Lie-Santilli groups* .(Aringazin et al. (1992)) Let G be a connected, linear, Lie group with Lie algebra L and envelope $\xi(L)$. As well known, G is characterized by exponentials in $\xi(L)$, namely, by power series expansions in the conventional associative envelope ξ, $G = \exp(iwX)$, $w \in F$, $X^{\dagger} = X$.

Consider now a Lie-Santilli algebra $\hat{L}$ over an isofield $\hat{F}$. The use of the same exponential in $\xi(L)$ is now prohibited because it would violate the isolinearity condition. Santilli therefore introduced the realization of the connected isotopic group $\hat{G}$ as the power series expansion, this time, in the isoenvelope $\hat{\xi}(L)$, i.e.,

$$\hat{G}: \quad \hat{A}(w) = \hat{1} + (iwX)/1! + (iwX)*(iwX)/2! + \ldots$$

$$= e_{\hat{\xi}}^{iwX} = (e_{\xi}^{XTwi})\,\hat{1} = \hat{1}\,(e_{\xi}^{iwTX}) \qquad \text{(IV.12)}$$

The conventional group laws were then generalized into the *Lie-Santilli group laws*

$$\hat{A}(0) = \hat{1}, \quad \hat{A}(w)*\hat{A}(w') = \hat{A}(w')*\hat{A}(w) = \hat{A}(w + w'), \quad \hat{A}(w)*\hat{A}(-w) = \hat{1}. \qquad \text{(IV.13)}$$

Always in the same memoir (loc. cit.), Santilli proved that the basic properties of Lie groups admit a consistent isotopic extension. As an illustration, he proved that the celebrated *Baker-Campbell-Hausforff Theorem* (see, e.g., Gilmore (loc. cit.)) admits a consistent isotopic image given by

$$(e_{\hat{\xi}}^{X_1})*(e_{\hat{\xi}}^{X_2}) = e_{\hat{\xi}}^{X}$$

$$X = X_1 + X_2 + [X_1,X_2]_{\hat{\xi}}/2 + [(X_1 - X_2),[X_1,X_2]_{\hat{\xi}}]_{\hat{\xi}}/12 + \ldots \qquad \text{(IV.}$$

In this way Santilli identified the foundations of a structural generalization of every branch of Lie's theory of fundamental mathematical and physical relevance. In fact, all the generalized mathematical structures recalled earlier now emerge as one single mathematical edifice (Santilli (1988b), (1991a)). As an example, Lie-Santilli transformation groups can only be consistently defined on an isospace over an isofield.

Needless to say, the contemporary Lie's theory has reached an outstanding maturity and diversification, as a results of hundred of thousands of contributions by mathematicians and physicists for over one century. By comparison, the Lie-Santilli theory, being the result of only one individual[5], is at its fist infancy at this writing. Despite that, the NOBEL COMMITTEE should be reassured that all the most essential parts of the new theory needed for physical applications are fully available.

Following the original proposal Santilli (1978a), the generalized theory was reviewed and expanded in the enclosed Vol. II of Santilli's *Foundations of Theoretical Mechanics* (1982a), see page 148 and ff., and then further elaborated in the recent mathematical presentation (Santilli (1991a) and monograph (1991c).

Note that the Lie-Santilli groups are formally linear and local, but structurally nonlinear and nonlocal as it is the case for the isotransformations (IV.5), and this illustrates the mathematical and physical nontriviality of the generalized theory.

The above theory was constructed by Santilli as the foundations of a generalization of quantum mechanics reviewed in Part IV.

IV.6: LIE-SANTILLI THEORY IN CLASSICAL REALIZATION.

Let $E(r,\delta,\Re)$ be a conventional Euclidean space and $T^*E(r,\delta,\Re)$ the corresponding cotangent bundle (phase space) with local coordinates $a = (a^\mu) = (r, p) = (r_{ka}, p_{ka})$, $\mu = 1, 2, ...,6N$, $k = 1, 2, 3$ (= x, y, z), $a = 1, 2, ..., N$. The central algebraic tool for the characterization of potential-Hamiltonian systems (I.2) is then given by the the classical realization of the Lie algebras via the familiar Poisson brackets (III.3) among generic functions A(a) and B(a) in phase space, i.e.,

$$[A\ B] = \frac{\partial A}{\partial a^\mu}\,\omega^{\mu\nu}\,\frac{\partial B}{\partial a^\nu} = \frac{\partial A}{\partial r_{ka}}\,\frac{\partial B}{\partial p_{ka}} - \frac{\partial B}{\partial r_{ka}}\,\frac{\partial A}{\partial p_{ka}}, \qquad \text{(IV.14)}$$

where $\omega^{\mu\nu}$ is the canonical Lie tensor (III.2) The exponentiated form of brackets (IV.14), say, for the case of n generators X_i and parameters w_i, characterizes the classical realizations of connected Lie groups

$$A'(a') = \{e^{w_i \omega^{\mu\nu}(\partial_\nu \partial X_i)\,(\partial_\mu)}\}\,A(a). \qquad \text{(IV.15)}$$

with the discrete part being characterized by *inversions* of the type $Pr = -r$.

In particular, the celebrated Lie's (1893) First, Second and Third Theorems provide a direct characterization of Lie algebras in their classical realization (IV.14).

The NOBEL COMMITTEE should keep in mind that all fundamental space-time symmetries of contemporary physics, such as Galilei's symmetry G(3.1) and Poincare' symmetries P(3.1), have precisely a structure of type (IV.14) and (IV.15).

In his memoir (1978a), Santilli discovered the following classical realization

[5] No contribution by mathematicians or physicists has appeared in print to this day in the specific study of the Lie-isotopic algebras, to our best knowledge.

of the Lie-isotopic theory, that via the most general possible, classical and regular, Lie product in phase space

$$[A \hat{,} B] = \frac{\partial A}{\partial a^{\mu}} \Omega^{\mu\nu}(a) \frac{\partial B}{\partial a^{\nu}}, \qquad (IV.16)$$

where $\Omega^{\mu\nu}$ is the *Lie-isotopic tensor*, expressible, for certain analytic and geometric reasons recalled below, via the form

$$\Omega^{\mu\nu} = \{ \|(\partial_{\alpha}R_{\beta} - \partial_{\beta}R_{\alpha})\|^{-1}\}^{\mu\nu}. \qquad (IV.17)$$

Santilli then introduced the following classical realization of the connected isotopic groups

$$A'(a') = \{ e^{w_i \Omega^{\mu\nu}(a) (\partial_{\nu}X_i) (\partial_{\mu})} \} A(a). \qquad (IV.18)$$

with discrete components characterized by the *isoinversions* $\hat{P}*r \equiv Pr = -r$, $\hat{P} = P\hat{1}$.

Another fundamental result of the same memoir is the discovery of the *isotopic generalization of Lie's First, Second and Third Theorems*, and the proof that they provide a direct characterization of the isotopic algebras in realization (IV.16), and not of the conventional canonical form (IV.14).

The covering character of the Lie-Santilli theory over the canonical formulation of Lie's theory is evident. In fact, under the particular value $R(a) \Rightarrow R^{\circ} = (p_{ka}, 0_{ka})$), generalized Lie's tensor (IV.17) reduces to the canonical tensor (III.2), and all conventional canonical formulations are recovered identically.

The above generalized classical theory was then used by Santilli, always in his memoir (1978a), to outline the foundations of a generalization of Hamiltonian mechanics which he called, for certain historical reasons, *Birkhoffian mechanics* (see Sect. IV.8).

In 1979 Santilli became a member of the *Department of Mathematics* of *Harvard University* as Coprincipal Investigator of the DOE contract ER-78-S-02-4742.A000 with the known mathematician S. Sternberg as Principal Investigator.

In such a position, he studied his Birkhoffian mechanics in all the necessary details and released his results in the enclosed monograph (1982a).

Significantly, Birkhoffian mechanics with its underlying Lie-Santilli theory resulted to be "directly universal" for all possible nonlinear and nonhamiltonian Newtonian systems, but *only in local-differebntial approximation*, owing to the use of the conventional symplectic geometry as the background geometry (see next section).

As a result of this limitation, and following the release of the monograph (1982a), Santilli continued at the *Department of Mathematics* of *Harvard University* the search for a suitable generalization permitting the direct treatment of nonlocal-integral systems. The studies were then continued at the

International Centre for Theoretical Physics of Trieste, Italy.

These studies were completed in the late 80's, and first published in the memoirs Santilli (1988a, b, c, d) resulting in a new formulation of the Lie-Santilli theory as the true classical counterpart of the abstract formulation. The studies were also release in a series of ICTP preprints (1991), and were finally presented in a comprehensive way in the recent mathematical papers (1991a, b) and monograph (1991c).

The latter additional discovery was achieved via the formulation of the theory on the isospace $T^*\hat{E}(r,\hat{\delta},\hat{\mathfrak{R}})$, with 6N×6N isounit $\hat{1} = \hat{1}(a, ...) = (\hat{1}_{\alpha}{}^{\beta}) = (\hat{1}^{\beta}{}_{\alpha})$, and classical Hamilton-Santilli brackets (III.6) here expressed in the more general Birkhoff-Santilli form (see Sect. IV.8 for details)

$$[A \hat{,} B] = \frac{\partial A}{\partial a^{\mu}} \, \Omega^{\mu\alpha}(a) \, \hat{1}_{\alpha}{}^{\nu}(a) \, \frac{\partial B}{\partial a^{\nu}}, \tag{IV.19}$$

under certain integrability conditions indicated in the next section.

The exponentiation of structure (IV.19) follows the same pattern as above, but it is now defined on an isospace and, as such, can only characterize isotransformations, resulting in the classical realization of Lie-Santilli groups

$$A'(a') = \{ [e^{w_i \Omega^{\mu\alpha}(a) \, \hat{1}_{\alpha}{}^{\nu}(a) \, (\partial_{\nu} X_i) (\partial_{\mu})}] \, \hat{1} \} * A(a). \tag{IV.20}$$

whose transformations can be explicitly computed in their finite form from the sole knowkledge of the old generators and parameters, and of the new quantities $\Omega^{\mu\nu}$ and $\hat{1}$.

The important point is the appearance, for the first time in mathematics and physics, of the isounit $\hat{1}$ directly in the classical brackets of the theory. This allowed the direct representation of *nonlocal* , as well as nonlinear and nonhamiltonian forces via their embeding precisely in the isounit, without requesting a new complicated topology (see Sect. IV.8 for the computation of the equations of motion).

It is evident that the Lie-isotopic groups are at the foundations of Santilli's covering relativities in classical and quantum mechannics, of course, after their formulation as symmetries of closed nonhamiltonian systems. But, before reaching that stage, Santilli had to identify the underlying geometry and analytic mechanics.

IV.7: SANTILLI'S SYMPLECTIC-ISOTOPIC GEOMETRY.

The symplectic geometry in canonical realization is the geometric structure of Lie algebra in their classical realization. Consider again the cotangent bundle $T^*E(r,\delta,\mathfrak{R})$ with local coordinates in the unified notation $a = (r, p)$ and the canonical one-form

$$\theta = R^{\circ}_{\mu}\, da^{\mu} = p_{ka}\, dr_{ka}, \quad R^{\circ} = (p, 0), \tag{IV.21}$$

Then, the exact, symplectic, canonical two-form can be written

$$\omega = d\theta = \tfrac{1}{2}\,\omega_{\mu\nu}\, da^{\mu} \wedge da^{\nu} \equiv dr_{ka} \wedge dp_{ka}. \tag{IV.22}$$

where $\omega_{\mu\nu} = \partial_{\mu}R^{\circ}_{\nu} - \partial_{\nu}R^{\circ}_{\mu}$ is the covariant, canonical, symplectic tensor related to the contravariant Lie tensor $\omega^{\mu\nu}$ of Eq. (III.2) by the rules

$$\omega_{\mu\nu} = (|\omega^{\alpha\beta}|^{-1})_{\mu\nu} \tag{IV.23}$$

As is well known, canonical two-form (IV.22) is exact because $\omega = d\theta$, and symplectic because $d\omega = d(d\theta) \equiv 0$. In turn, the exact symplectic character of two-form (IV.22) provides the necessary and sufficient conditions for the corresponding Poisson brackets (IV.14) to be Lie (see, e.g., Abraham and Marsden (1967)).

Moreover, under sufficient topological conditions herein ignored (regularity and analyticity or at least class C^{∞}), the local-differential, potential system Φ, Eq.s (I.2) is a *Hamiltonian vector-field* , namely, there exists a function H(a), the *Hamiltonian*, such that

$$\Phi \rfloor \omega = -dH, \tag{IV.24}$$

(see next section for the explicit form).

In his memoir (1978a), Santilli proved that the conventional symplectic geometry in its most general possible exact form, is the geometry underlying the classical, local-differential realizations of Lie-Santilli algebras. Consider the most general possible one-form on $T^{*}E(r,\delta,\Re)$

$$\Theta = R_{\mu}(a)\, da^{\mu}, \tag{IV.25}$$

and introduce its exterior derivative

$$\Omega = d\Theta = \tfrac{1}{2}\,\Omega_{\mu\nu}(a)\, da^{\mu} \wedge da^{\nu}, \tag{IV.26}$$

which is therefore an exact two-form, where the tensor $\Omega_{\mu\nu}$ is assumed to be nowhere degenerate with explicit form

$$\Omega_{\mu\nu} = \partial_{\mu}R_{\nu} - \partial_{\nu}R_{\mu}. \tag{IV.27}$$

Then, from the Poincaré Lemma, $d\Omega = d(d\Theta) \equiv 0$, two-form (IV.25) becomes the most general possible exact, symplectic two-form in local coordinates.

The symplectic character of two-form (IV.26) then provided the necessary and sufficient conditions for the corresponding isotopic brackets (IV.16) to be

Lie-Santilli (see the analytic, algebraic and geometric proofs in the enclosed Vol. II of Santilli's *Foundations ofTheoretical Mechanics* (1982a).

It is significant to recall that, in the same memoir (1978a), Santilli called two-form (IV.26) an *isotope* of the canonical two-form (IV.22), in the sense that the former preserves the basic axioms of the latter (nowhere degeneracy, exact and symplectic characters), although expressed in their most general possible local form.

This terminology was introduced to parallel the corresponding terminology for the algebraic brackets (IV.16) and (IV.14), the former being called an *isotope* of the latter, again, because the former preserves the basic axioms of the latter (nondegeneracy and Lie character).

The topology of the conventional symplectic geometry is however local in character. Santilli therefore considered the most general possible nonlinear and nonhamiltonian systems Γ, Eq.s (I.3), in their local-differential approximation, and achieved in the enclosed memoir (1982a) another remarkable result, *the proof of thedirect universality of the conventional symplectic geometry for the most general possible local-differential, but nonlinear and nonhamiltonian systems (I.3)* .

In different terms, Santilli proved (under sufficient topological conditions) that, whenever Hamiltonian condition (IV.24) is violated, there exists a general exact two-form Ω (or, equivalently, 6N functions $R_\mu(a)$) and a function B(t,a), he called the *Birkhoffian* (see next section), for any given nonhamiltonian system Γ of the class admitted, such that

$$\Gamma \rfloor \Omega = -dB \qquad \text{(IV.28)}$$

("universality"), directly in the local a-frame of the experimenter ("direct universality"). The nonhamiltonian vector-field $\hat{h}$ was then called by Santilli a *Birkhoffian vector-field*.

The above direct universality of the conventional symplectic geometry in Newtonian mechanics is an evident basic result of the enclosed monograph (1982a), and illustrates the power of Santilli's isotopic techniques for specific physical applications.

In order to achieve the "direct universality" for all possible systems (I.3) of nonlocal-integral as well as nonhamiltonian type, Santilli discovered in memoir (1988a) (see also (1988b) and (1991a, b)) a new geometry which he called the *symplectic-isotopic geoometry* , and which is the full geometry underlying the Lie-Santilli algebras (recall that the latter are naturally set for the representation of nonlocal forces via their embedding in the isounit).

Evidently we cannot possibly review the new geometry here in the necessary details. We merely indicate that it is formulated in the *isocotangent bundles* $T^*\hat{E}(r,\hat{\delta},\hat{\mathfrak{R}})$, $\hat{\mathfrak{R}} = \mathfrak{R}\hat{1}$, $\hat{1} = T^{-1}$, with *isotransformations* $x' = A{*}x = ATx$, which require the necessary lifting of the conventional differentials da′ and da (those connected by the linear and local transformations da′ = Ada) into the *isodifferentials* $\hat{d}a'$ and $\hat{d}a$ connected by the isolinear and isolocal transformations

$$\hat{d}a' = A{*}\hat{d}a = AT(t, a, \dot{a}, ...)\, \hat{d}a. \qquad \text{(IV.29)}$$

From the above notions he constructed the one–, two–, and p–isoforms,

$$\hat{\Phi}_1 = A * \hat{\partial} a = A_\mu T^\mu{}_\nu \hat{\partial} a^\nu, \quad \hat{\Phi}_2 = A_{\mu\nu} T^\mu{}_\alpha T^\nu{}_\beta \hat{\partial} a^\alpha \hat{\wedge} \hat{\partial} a^\beta, \quad \text{etc.} \tag{IV.30}$$

and proved that the new geometry verifies a generalized version of the celebrated *Poincaré Lemma*, namely, that a two–isoform $\hat{\Phi}_2$ which is isoexact, i.e., admitting of a one–isoform $\hat{\Phi}_1$ such that $\hat{\Phi}_2 = \hat{\partial}\hat{\Phi}_1$, is isoclosed, $\hat{\partial}\hat{\Phi}_2 \equiv 0$.

He then proved in all the necessary technical details that the symplectic–isotopic geometry is indeed the proper geometry underlying the classical Lie–Santilli brackets (IV.20). We indicate here the main lines for the simpler case of the Hamilton–Santilli brackets (III.6).

The starting point is the restriction of a general one–form to the structure

$$\hat{\Theta}^\circ = R^\circ{}_\alpha \hat{T}_1{}^\alpha{}_\mu \hat{\partial} a^\mu = p_{ia} \hat{\delta}^{ij} dr_{ja}, \quad R^\circ = (p, 0), \quad \hat{T}_1 = \text{diag.}(\hat{\delta}, \hat{\delta}) > 0, \tag{IV.31}$$

where $\hat{\delta}$ is precisely the isometric of the isospace $T^*\hat{E}(r,\hat{\delta},\hat{\mathfrak{R}})$ for one–isoforms, often denoted with $T^*\hat{E}_1(r,\hat{\delta},\hat{\mathfrak{R}})$.

Santilli then restricted the two–isoform $\hat{\Theta}^\circ = \hat{\partial}\hat{\Theta}^\circ$ characterized by one–isoform (IV.31) to possess a structure in which the canonical two–form ω is factorizable, i.e., a structure of the type

$$\hat{\Omega}^\circ = \hat{\partial}\hat{\Theta} = \tfrac{1}{2} \hat{T}_{2\mu}{}^\alpha(a)\, \omega_{\alpha\nu}\, \hat{\partial} a^\mu \hat{\wedge} \hat{\partial} a^\nu, \tag{IV.32}$$

where

$$(\Omega^\circ{}_{\mu\nu}) = (\hat{T}_{2\mu}{}^\alpha \omega_{\alpha\nu}) = \begin{pmatrix} 0_{3N\times3N} & \hat{G}_{3N\times3N} \\ -\hat{G}_{3N\times3N} & 0_{3N\times3N} \end{pmatrix} \tag{IV.33}$$

and

$$\hat{G} = \left(\hat{\delta}_{ij} + p_{ka} \frac{\partial \hat{\delta}^k{}_i}{\partial p_{ja}} \right)_{3N\times3N} \tag{IV.34}$$

Then, the following major features result:

I) *The simplectic–isotopic character of two–form (IV.32) provides the necessary and sufficient conditions for the Hamilton–Santilli brackets (III.3) to be Lie–isotopic*, i.e., for brackets

$$[A \hat{,} B] = \frac{\partial A}{\partial a^\mu} \omega^{\mu\alpha} 1_\alpha{}^\nu \frac{\partial B}{\partial a^\nu} =$$

$$= \frac{\partial A}{\partial r_{ia}} I_{ij} \frac{\partial B}{\partial p_{ja}} - \frac{\partial B}{\partial r_{ia}} I_{ij} \frac{\partial A}{\partial p_{ja}}, \qquad I = \hat{G}^{-1} \tag{IV.35}$$

to verify the axioms

$$[A \hat{,} B] + [B \hat{,} A] = 0, \quad [A \hat{,} [B \hat{,} C]] + [B \hat{,} [C \hat{,} A]] + [C \hat{,} [A \hat{,} B]] = 0, \tag{IV.36}$$

II) *The symplectic-isotopic geometry can fully represent nonlocal-integral forces, provided that they are totally embedded in the isotopic element T_1 or T_2.* In fact, the conventional local-differential topology of the symplectic geometry remains fully valid, because of the factorization of the canonical two-form ω, while the geometry is insensitive to the topology of the isotopic element because at the abstract, coordinate-free level all distinctions between the conventional p-forms and p-isoforms (conventional symplectic and symplectic-isotopic geometry) cease to exist by construction.

III) *The isometrics of two-isoforms (hat is, of the algebraic brackets of the theory) are different than the isometrics of one-isoforms (that is, of the integrand of variational principles, see also next section).* In the conventional case the metric is constant, δ = diag. (1,1,1) and therefore applies for both one- and two-forms. As a result, there is no change of metric in the transition from a variational principle to the analytic equations and underlying algebraic brackets.

In the Santilli's covering geometry the situation is different. In fact, in the transition from one-isoform (IV.31) to two-isoforms (IV.32), we have the transition from the original isometric $\hat{\delta}$ of $T^*\hat{E}_1(r,\hat{\delta},\hat{\mathfrak{R}})$, to the new isometric $\hat{G}$, Eq.s (IV.34). The isocotangent bundle for two-isoforms is therefore different than that of one iso-forms and denoted with $T^*\hat{E}_2(r,\hat{\delta},\hat{\mathfrak{R}}) = T^*\hat{E}(r,\hat{G},\hat{\mathfrak{R}})$.

This implies that the isospaces for Lie-isotopic analytic equations and symmetries are $T^*\hat{E}(r,\hat{G},\hat{\mathfrak{R}})$ and not $T^*\hat{E}(r,\hat{\delta},\hat{\mathfrak{R}})$. Needless to say, when the iosometrics $\hat{\delta}$ are constants, $\hat{\delta} \equiv \hat{G}$ and $T^*\hat{E}(r,\hat{\delta},\hat{\mathfrak{R}}) \equiv T^*E(r,\hat{G},\hat{\mathfrak{R}})$.

The knowledge of the transition from the isometric of one-isoforms to that of two-isoforms is a fundamental point for the understanding of Santilli's generalized symmetries for nonlinear, nonlocal and nonhamiltonian systems (I.3) or (II.1).

Santilli also worked out the most general possible case of the symplectic-isotopic geometry which is that characterized by the exact two-isoform[6]

$$\hat{\Omega} = \hat{d}\hat{\Theta} = \tfrac{1}{2} \hat{T}_{2\mu}{}^{\alpha}(a)\, \Omega_{\alpha\nu}(a)\, \hat{d}a^{\mu} \hat{\wedge} \hat{d}a^{\nu}, \tag{IV.37}$$

namely, by two-isoform with the factorization of the most general possible nowhere degenerate and exact symplectic tensor $\Omega_{\alpha\mu}(a)$.

[6] From now on, the symbols $\hat{\Theta}°$ and $\hat{\Omega}°$ shall denote one-and two-isoforms with the factorization of the canonical forms, while the symbols $\hat{\Theta}$ and $\hat{\Omega}$ shall denote the most general possible one- and two-isoforms, respectively.

To complete his geometrical studies, Santilli proved (1989a) that *the symplectic-isotopic geometry in the above general form, achieves "direct universality" for all, most general possible, nonlinear, nonlocal and nonhamiltonian systems (I.3)*. In fact, he proved that, under the necessary topological conditions indicated earlier, for any given nonhamiltonian and nonlocal vector-field Γ of Eq.s (I.3) on $T^*\hat{E}(r,\hat{\delta},\hat{\Re})$, there always exist: 1) 6N functions $R_\mu(a)$ (or a general symplectic tensor $\Omega_{\mu\nu}(a)$), 2) a isounit $\hat{I} = \text{diag.}(\hat{G}^{-1}, \hat{G}^{-1})$, and 3) a Birkhoffian function B(t, a), such that

$$\Gamma \hat{\rfloor} \hat{\Omega} = - dB, \tag{IV.38}$$

(see next section for explicit expressions).

These far reaching discoveries should constitute, per sé, sufficient grounds for the Nomination of Prof. Santilli for the NOBEL PRIZE IN PHYSICS OF 1992. Nevertheless, they constitute the mere geometric background of the true reason for this Nomination, the classical generalizations of Galilei's relativity (Sect. V).

IV.8: BIRKHOFF-SANTILLI AND HAMILTON-SANTILLI MECHANICS.

As clearly expressed in the original presentations (1978a) and (1988a), Santilli addressed the physical issue of the analytic representation of local nonhamiltonian, and nonlocal nonhamiltonian systems, respectively, only *after* having discovered the underlying new algebras and geometries.

The same original approach to analytic mechanics, which is the sole capable of ensuring mathematical consistency, has been followed in this presentation as close as possible.

Recall that conventional Hamiltonian mechanics is based on the canonical variational principle in phase space $T^*E(r,\delta,\Re)$ with local coordinates $a = (a^\mu) = (r, p) = (r_{ka}, p_{ka})$ (see, e.g., Sudarshan and Mukunda (1974)),

$$\delta A = \delta\int_{t_1}^{t_2}[\, p_{ka}\dot{r}_{ka} - H(t, r, p)]\, dt = \delta\int_{t_1}^{t_2}[\, R^\circ_\mu \dot{a}^\mu - H(t,a)]\, dt \equiv 0, \tag{IV.39}$$

where $R^\circ = (p, 0)$, which characterizes the *conventional, truncated Hamilton's equations*, without external terms in their covariant form

$$\omega_{\mu\nu}\dot{a}^\nu = \partial H / \partial a^\mu, \tag{IV.40}$$

from which the contravariant form follow

$$\dot{a}^\mu = \omega^{\mu\nu}\, \partial H / \partial a^\nu \tag{IV.41}$$

with *Hamilton-Jacobi equations*

$$\partial_t A + H = 0, \qquad \partial_\mu A = R^\circ_\mu. \qquad \text{(IV.42)}$$

Hamiltonian vector-field (IV.24) can then be written explicitly (see, e.g., Abraham and Marsden (1967))

$$\omega_{\mu\nu}\,\Phi^\mu\, da^\nu = -\,dH, \quad \text{or} \quad \Phi \equiv \Phi^\mu \frac{\partial}{\partial a^\mu} = \omega^{\mu\nu} \frac{\partial H}{\partial a^\nu}\frac{\partial}{\partial a^\mu}, \qquad \text{(IV.43)}$$

Another discovery made by Santilli in his memoir (1978a) is the construction of a step-by-step generalization of Hamiltonian mechanics into a form which he called *Birkhoffian mechanics* for historical reasons indicated below. As a matter of fact, he introduced the generalized, algebraic and geometric theories outlined in the preceding sections precisely for the purpose of constructing the generalized mechanics.

The mechanics was further studied in all the necessary details while at the *Department of Mathematics* of *Harvard University*, and published in Volume II of Santilli's *Foundations of Theoretical Mechanics* with the subtitle *Birkhoffian Generalization of Hamiltonian Mechanics*.

Again, as it had been the case for all other discoveries (such as those of the Lie-admissible algebras, the conditions of variational selfadjointness, and the Lie-isotopic algebras), Santilli first identified the generalization of Hamiltonian mechanics needed for the interior problem, and then conducted his *fourth* extensive library search (see Footnote[3], page 28) to identify preceding contributions. In this way, he discovered that G.D.Birkhoff (1927)[7] had previously identified the basic equations of the mechanics, although without the identification of their algebraic and geometric structures. Also, Birkhoff had applied his equations to typical aspects of the exterior problem, such as the stability of the planetary orbits.

Birkhoff's (1927) studies went essentially un-noticed for over half a century, until rediscovered by Santilli (1978a) who:

A) proved the "direct universality" of Birkhoff's equations for all nonlinear and nonhamiltonian interior systems in local approximation;

B) identified the algebraic and geometric structures of the equations as being those of his Lie-isotopic algebras and of the conventional symplectic geometry in its most general possible exact formulation; and

C) used the equations for a step-by-step generalization of Hamiltonian mechanics which had not beed considered by Birkhoff.

To outline the enclosed monograph in a few words, Birkhoffian mechanics can be constructed from the most general possible, first-order, Pfaffian variational principle

[7] The initiator of "Birkhoffian mechanics", G.D.Birkhoff, was the father of G.Birkhoff, the former colleague of Santilli at Harvard and co-author of the Poincaré-Birkhoff-Witt Theorem of Sect. IV.5.

$$\delta \hat{A} = \delta \int_{t_1}^{t_2} [R_\mu(a)\, \dot{a}^\mu - B(t, a)]\, dt = 0, \qquad \text{(IV.44)}$$

(here presented for simplicity in the so-called *semiautonomous* form with R = R(a) and B = B(t,a)), where the function B was called by Santilli the *Birtkhoffian* because it does not necessarily represent the total energy H = T + V.

Principle (IV.44) characterizes the *contravariant Birkhoff's equations*

$$\Omega_{\mu\nu}(a)\, \dot{a}^\nu = \partial B(t, a) / \partial a^\mu \qquad \Omega_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu, \qquad \text{(IV.45)}$$

with contravariant form

$$\dot{a}^\mu = \Omega^{\mu\nu}(a)\, \partial B(t, a) / \partial a^\nu \qquad \Omega^{\mu\nu} = \{ |(\Omega_{\alpha\beta})|^{-1} \}^{\mu\nu}, \qquad \text{(IV.46)}$$

while Hamilton-Jacobi equations (IV.42) assume the generalized expression

$$\partial_t \hat{A} + B(t,a) = 0, \qquad \partial_\mu \hat{A} = R_\mu(a). \qquad \text{(IV.47)}$$

The connection with the preceding algebraic and geometric studies is given by the fact that the brackets of the time evolution characterized by Birkhoff's equations (IV.46) are given precisely by the Lie-isotopic brackets (IV.16), i.e.,

$$[A \hat{,} B] = \frac{\partial A}{\partial a^\mu}\, \Omega^{\mu\nu}(a)\, \frac{\partial B}{\partial a^\nu}, \qquad \text{(IV.48)}$$

while the underlying geometric structure is precisely the general, exact two form (IV.26), i.e.,

$$\Omega = d\Theta = \tfrac{1}{2}\, \Omega_{\mu\nu}(a)\, da^\mu \wedge da^\nu, \qquad \text{(IV.49)}$$

which illustrates the algebraic and geometric characters of Birkhoff's equations recalled earlier.

By using these techniques, he then proved the analytic counterpart of the "direct universality of the symplectic geometry for local nonhamiltonian system,s indicated earlier. in fact, he proved that, under sufficient topological conditions, for any nonhamiltonian vector-field Γ, there always exist 6N functions $R_\mu(a)$ and a Birkhoffian B(t,a), such that the following equations

$$(\partial_\mu R_\nu - \partial_\nu R_\mu)\ \Gamma^\nu(t,a) = \partial B(t, a) / \partial a^\mu, \qquad \text{or} \qquad \Omega_{\mu\nu}\Gamma^\mu da^\nu = - dB \qquad \text{(IV.50)}$$

hold identically.

Santilli then identified numerous methods for the construction of the above Birkhoffian representation from given equations of motion (I.3), and studied in detail their degrees of freedom, e.g., the so-called Birkhoffian gauge transformations

$$R_\mu(a) \Rightarrow R'_\mu(a) = R_\mu(a) + \partial G / \partial a^\mu, \quad B(t, a) \Rightarrow B'(t, a) = B(t, a) = \partial G / \partial t, \ \text{(IV.51)}$$

which evidently leave invariant principle (IV.44).

A step-by-step generalization of Hamiltonian mechanics was then presented, including a generalized transformation theory, generalized symmetries and conservation laws, etc.

Note that the new mechanics is a "covering" of the Hamiltonian mechanics, in the sense that:

1) The new mechanics is constructed via mathematical tools (Lie-Santilli theory) structurally more general than those of the conventional mechanics (Lie's theory in the simplest possible, canonical formulation);

2) The new mechanics represents physical systems (the local, but most general possible nonlinear and nonhamiltonian systems) structurally more general than those of the conventional ones (conservative-potential systems); and

3) The new mechanics admits the old mechanics as a particular case. In fact, for $R = R^\circ = (p, 0)$ all conventional Hamiltonian formulations are recovered identically.

Despite these achievements, Santilli remained unsatisfied and continued his studies to reach a still more general formulation of Birkhoffian mechanics capable of achieving "direct universality" for the most general systems known to mankind today, the nonlocal, as well as nonlinear and nonhamiltonian interior systems.

This further advancement was reached for the first time in the memoir Santilli (1989a); it was then reviewed and expanded for mathematicians in Santilli (1991a); and finally presented in a comprehensive way in the monographs Santilli (1991c, d).

The central starting point is the reformulation of Birkhoffian mechanics in the isophase space $T^*\hat{E}(r,\hat{\delta},\hat{\mathfrak{R}})$ with the local coordinates $a = (r, p)$. This implies the generalization of the trivial six-dimensional unit of the conventional phase space, $I = \text{diag.}\ (1, 1, ..., 1)$, into Santilli's isounit $\hat{1} = \hat{1}(t, a, \dot{a}, ...)$ which, in turn, implies the necessary lifting of the conventional linear and local transformation theory into the isolinear and isolocal form of Sect. IV.4.

The emerging generalization of the Birkhoffian mechanics was called by Santilli the *Birkhoffian-isotopic mechanics* , and it is now known as *Birkhoff-Santilli mechanics* (Aringazin et al (1992)) Its general form is rather complex for the rudimentary nature of this review. We shall therefore outline here its simplest possible realization, that called *Hamilton-Santilli mechanics.*

The latter mechanics is based on the following form of Pfaffian variational principle

$$\delta \hat{A}^\circ = \delta \int_{t_1}^{t_2} [\, R^\circ_\mu T_1{}^\mu{}_\nu \, \dot{a}^\nu \; - \; H(t, a) \,] \, \hat{d}t \; \equiv \; 0, \qquad \text{(IV.52)}$$

where the integrand characterizes a one-isoform (IV.31), i.e.,

$$\hat{\Phi}_1 \; = \; R^\circ {*} \hat{d}a^\mu \; = \; R^\circ_\mu T_1{}^\mu{}_\nu \, \hat{d}a^\nu. \qquad \text{(IV.53)}$$

The above variational principle characterizes the following equations, called the *convariant Hamilton-Santilli equations*

$$\hat{T}_{2\mu}{}^{\alpha}\, \omega_{\alpha\nu}\, \dot{a}^{\nu} = \partial H(t, a) / \partial a^{\mu}, \qquad \text{(IV.54)}$$

with *contravariant Hamilton-Santilli form*

$$\dot{a}^{\mu} = \omega^{\mu\alpha}\, \hat{I}_{2\alpha}{}^{\nu}\, \partial H(t, a) / \partial a^{\nu}, \qquad \text{(IV.55)}$$

where, from Eq.s (IV.33) and (IV.34)

$$(\hat{\Omega}^{\circ\mu\nu}) = \omega \times \hat{I}_2 = \hat{I}_2 \times (\omega^{\mu\nu}) = (\omega^{\mu\alpha}\, \hat{I}_{2\alpha}{}^{\nu})$$

$$= \begin{pmatrix} 0_{6N\times 6N} & (I_2)_{6N\times 6N} \\ -(I_2)_{6N\times 6N} & 0_{6N\times 6N} \end{pmatrix} \qquad \text{(IV.56)}$$

and the quantities $I_2 = \hat{G}^{-1}$ are computed from the integrand of the variational principle via rukles (IV.34).

Eq.s (IV.55) can be written in the disjoint r- and p-coordinates

$$\dot{r}_i = I_{2\,ij}(r, p, ...) \frac{\partial H(t, r, p)}{\partial p_j}, \qquad \text{(IV.57a)}$$

$$\dot{p}_i = -\, I_{2\,ij}(r, p, ...) \frac{\partial H(t, r, p)}{\partial r_j}, \qquad \text{(IV.57b)}$$

The NOBEL COMMITTEE can see in this way that the Hamilton-Santilli mechanics has indeed the structure of the symplectic-isotopic geometry.

The algebraic structure is evidently that of the Lie-isotopic brackets (III.3), i.e.,

$$[A \hat{,} B] = \frac{\partial A}{\partial a^{\mu}}\, \omega^{\mu\alpha}\, I_{2\alpha}{}^{\nu}(a, ..)\, \frac{\partial B}{\partial a^{\nu}} \qquad \text{(IV.58)}$$

which can also be written in the disjoint r- and p-form

$$[A \hat{,} B] = \frac{\partial A}{\partial r_i}\, I_{2ij}(r, p, ..)\, \frac{\partial B}{\partial p_j} - \frac{\partial B}{\partial r_i}\, I_{2ij}(r, p, ...)\, \frac{\partial A}{\partial p_j}, \qquad \text{(IV.59)}$$

which exhibit rather clearly the generalized structure over the conventional Poisson brackets.

The analytic representation of nonlocal nonhamiltonian vectort-fields Γ (I.3] then follows according to the rules

$$\hat{T}_{2\mu}{}^{\alpha}\ \omega_{\alpha\nu}\,\Gamma^{\nu} = \partial H / \partial a^{\mu}, \quad \text{or} \quad \Gamma^{\mu} = \omega^{\mu\alpha}\,\hat{1}_{\alpha}{}^{\nu}\,\partial H / \partial a^{\nu}. \qquad \text{(IV.60)}$$

One can now see the importance of the symplectic-isotopic geometry in the representation of nonlocal systems (I.3). In fact, the Hamiltonian $H = H(t, a) = H(t, r, p) = T(p) + V(r)$ represent all conventional potential forces, while the isounit $\hat{1}$ represents all nonlocal and nonhamiltonian forces (see Sect. V for examples).

In conclusion, the conventional Hamiltonian mechanics is based on the knowledge of only one function, the Hamiltonian $H = T + V$, and can therefore represent only local-potential forces (as well as a very limited class of local nonpotential forces, see Santilli (1978b)).

The Hamilton-Santili mechanics is based instead on two quantities, the conventional Hamiltonian $H = T(p) + V(r)$, plus the isounit $\hat{1}(t, r, p, ...) = \text{diag.}\ (I_2, I_2)$. As such, it can represent not only all conventional potential forces, but also a large class of nonlinear, nonlocal and nonhamiltonian forces.

The direct universality is achieved for the more general Birkhoff-Santilli case in which, besides the Birkhoffian $B(t, a)$ and the isounit $\hat{1}(t, a, \dot{a}, ...)$, one has available the 6N additional functions $R_{\mu}(a)$.

A few comments on the operator counterpart of Santilli's Lie-isotopic formulations are presented in Fig IV.1, as an advance outline of the more detailed treatment scheduled for Part IV.

IV.9: SANTILLI'S ISOSYMMETRIES AND CONSERVATION LAWS.

In our outline of Santilli's journey of discoveries, we have reviewed until now the background new tools, such as the notions of isofields, isospaces and isotransformations, and then the isotopies of Lie's theory, symplectic geometry and analytic mechanics.

All these discoveries were used by Santilli for a final central notion needed for the construction of the generalized relativities, *the symmetries of interior dynamical systems at large, and of closed nonhamiltonian systems, in particular* (Sect. II.2), which he called *isosymmetries* to stress the point that they must be defined on an isospace and possess an isotopic algebraic structure.

Santilli's contributions in this field are rather numerous (see his curriculum). We can therefore indicate here only the most salient ones.

The generalization of the conventional Noether's theorem was first achieved in the enclosed monograph (Santiilli (1982a)) for the case of the most general possible nonlinear and nonhamiltonian interior systems, although in local approximation.

The generalized symmetries for nonlinear, nonlocal and nonhamiltonian systems were achieved in the subsequent paper Santilli (1985a), then applied to the construction of the generalized rotational symmetry (1985b), the generalized Lorentz

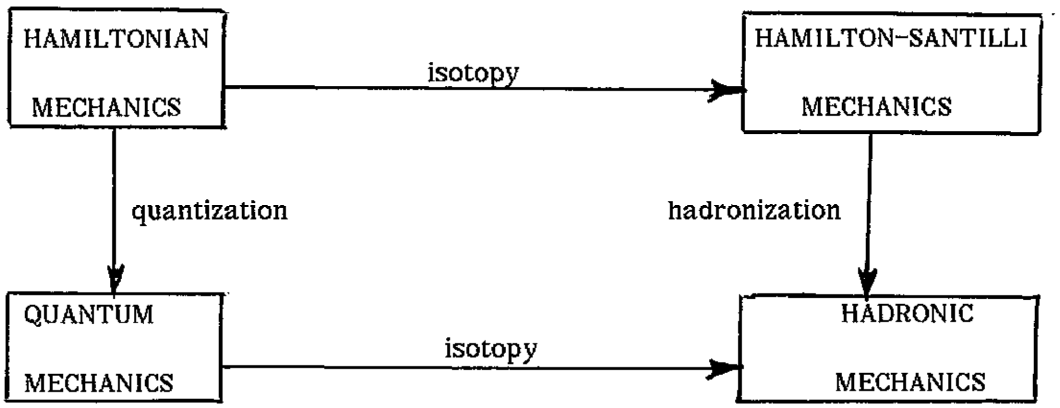

FIGURE IV.1: A schematic view of the deep inter-relation between Santilli's classical and operator formulations, the latter being outlined in more details in the forthcoming Part IV. The advance information that may be useful to the NOBEL COMMITTEE is that *the very existence of the Hamilton-Santilli generalization of Hamiltonian mechanics implies the existence of a corresponding operator generalization of quantum mechanics* of which it is the classical limit. Also, *the methods used for the construction of the classical generalization can also be used for the construction of the operator image*. In essence, the Hamilton-Santilli mechanics is constructed via the isotopies of the conventional analytic, algebraic and geometric structures. In particular, the isotopies are such that all distinctions between Hamiltonian and Hamilton-Santilli mechanics cease to exist at the abstract realization-free level. The first aspect the NOBEL COMMITTEE should know for an appraisal of the classical profile is that a fully similar occurrence emerges at the operator level. In fact, in another pioneering memoir (1978d) (see also the comprehensive presentations (1989a, b, c, d))) Santilli proposed the construction of the *"hadronic generalization of quantum mechanics"*, or *"hadronic mechanics"* for short, as an isotope of quantum mechanics, and provided the fundamental Lie-isotopic generalization of Heisenberg's equations (see below in this footnote).

In essence, quantum mechanics is characterized by
1) The enveloping associative operator algebra ξ with elements A, B, ...and trivial associative product AB (say, of matrices) and unit I ;
2) The field F of real numbers $\Re$ or of complex numbers C; and
3) The Hilbert space $\mathcal{H}$ with states $|\psi>$ and inner product $<\phi|\psi> \in F$,
from which the entire theory follows, such as Heisenberg's equations $i\dot{A} = [A,H]_\xi = AH - HA$, $\hbar = 1$, Schrödinger's equation $i\partial_t|\psi> = H|\psi> = K|\psi>$, $K \in \hat{F}$, the linear (and local) operations on $\mathcal{H}$, the unitary transformatio theory, etc.

Thanks to the contributions of several independent scientists besides Santilli's origination (see Part IV for their full identification and bibliography), in today's language we can say that hadronic mechanics is based on the isotopy of the above structures, that is:
$\hat{1}$) The isoassociative operator algebra $\hat{\xi}$ of Sect. IV.5 with isounit $\hat{I}$ and isotopic product $A*B = ATB$, $\hat{I} = I^\dagger = T^{-1} > 0$;
$\hat{2}$) The isofield $\hat{F}$ of isoreals numbers $\hat{\Re}$ or of isocomplex numbers $\hat{C}$; and

$\hat{3}$) The isohilbert space $\hat{\mathcal{H}}$ with isoinner product $<\phi \hat{|} \psi> = <\phi |T| \psi>\hat{I} \in \hat{F}$;
from which the entire new operator mechanics can be built, including Santilli's isoheisenberg's equations $i\dot{A} = [A,H]_{\hat{\xi}} = A*H - H*A = ATH - HTA$, the isoschrödinger's equation $i\partial_t| \psi> = H*| \psi> = \hat{K}*| \psi> \equiv K| \psi>$, $\hat{K} \in \hat{F}$, $K \in F$, (derived by Mignani, Myung and Santilli) the isolinear and isolocal operations on $\hat{\mathcal{H}}$, the isounitary transformation theory, etc. (see Part IV for details). In particular, the isohermiciticy coincides with the conventional Hermicicity, as a result of which observables $H = H^\dagger$ of quantum mechanics remain observable for hadronic mechanics. Note that hadronic and quantum mechanics coincide at the abstract, realization-free level by construction, exactly as it occurs at the classical level. In fact, under the assumed positive definiteness $T > 0$, the isotopes $\hat{\xi}$ remain associative, the isotopes $\hat{F}$ remain fields and the isotopes $\hat{\mathcal{H}}$ remain Hilbert spaces.

Moreover, hadronic mechanics can be obtained via an isotopy of the conventional methods of quantization called ***hadronization*** . In fact, under the (naive) quantization of the canonical action $A \Rightarrow i\ I \log\psi$, where I is the trivial unit, the Hamilton-Jacobi equations $\partial_t A + H = 0$ becomes $i\ \partial_t\ \psi + H\psi = 0$, which is precisely Schrödinger's equation. Along exactly the same lines, under the hadronization of Santilli's isotopic action $\hat{A} \Rightarrow i\ \hat{I} \log\psi$, where $\hat{I} = T^{-1}$ is now the isounit, the equation of the Hamilton-Santilli mechanics $\partial_t\hat{A} + H = 0$ become $i\ \partial_t\psi + H*\psi = i\partial_t\psi + HT\psi = 0$, which is precisely the isoschrödinger's equation of hadronic mechanics. Thus, hadronic mechanics is indeed the operator image of the Hamilton-Santilli classical mechanics and, viceversa, the latter is the classical limit of the former.

IN summary:
a) Quantum mechanics can only represent particles as being point-like with action-at-a-distance, potential forces, as well known from its mathematical structure. On the contrary, hadronic mechanics can directly represent all conventional potential forces via the Hamiltonian H plus nonlinerar and nonlocal forces in all variables as well weavefunctions AND their derivatives via the appropriate selection of the isotopic element, $T = T^\dagger = T(r, p, \psi, \psi^\dagger, \partial\psi, \partial\psi^\dagger, ...)$;
b) Hadronic mechanics is fully consistent on mathematical grounds at this writing, although in need of experimental verification;
c) The additionnal nonlinear, nonlocal and nonhamiltonian interactions represented by hadronic mechnics are strictly short range and internal, as expected fronm the overlapping of their wavepackets (Sect. I) and, as such, not detectable from the outside. Thus, to our best knowledge, no experimental information is currently available to disprove hadronic mechanics, and the issue of its validity or invalidity must be resolved via direct experimental verificvations of its novel predictions in particle physics, such as that of the deformability of the extended charge distributions of hadrons, with their consequential alteration of their intrinsic magnetic moment, and others intriguing predictions (see again Part IV).
In conclusion, the operator image of the entire content of this Part I has already been identified in the literature, and will be outlined in Part IV, including the operator image of: closed nonhamiltonian systems, isosymmetries and conservation laws, isogalilean symmetries, Santilli's isogalilean relativity, etc.

symmetry (1983a) [8]. A comprehensive study of the nonlocal case first appeared in

[8] The papers Santilli (1985a, b) on the general methodology for isosymmetries and its application to isorotations were evidently written prior to paper (1983a) on the isotopic generalization of the Lorentz symmetry, but they appeared in print two years later because of truly increedible difficultiers in their

the memoirs (1989a); it was rewritten for mathematicians in the recent paper Santilli (1991a); and finally presented for the physical audience in monograph (1991c).

Again, we cannot possibly review this extensive reseach, but only indicate a few salient aspects. The first result to bring to the attention of the NOBEL COMMITTEE is the following

THEOREM IV.9.1 (SANTILLI'S FUNDAMENTAL THEOREM ON ISOSYMMETRIES (1985a)): Let G_r be an r-dimensional Lie group of isometries of an n-dimensional metric or pseudometric space $M(x,g,F)$ over the field of real numbers $\Re$, complex numbers C or quaternions Q,

$$G_r : x'^{\dagger} = x^{\dagger} A^{\dagger}(w), \quad x' = A(w)\, x, \tag{IV.61a}$$

$$[(x-y)^{\dagger} A^{\dagger}(w)]\, g\, [A(w)\,(x-y)] \equiv (x-y)^{\dagger}\, g\, (x-y). \tag{IV.61b}$$

$$A^{\dagger} g\, A = A\, g\, A^{\dagger} = gI, \tag{IV.61c}$$

$$\det A = \pm 1. \tag{IV.61d}$$

Then, the infinitely possible isotopes $\hat{G}_r$ of G_r characterized by the same parameters and generators of G(m), and the infinitely possible, nowhere singular, Hermitean and sufficiently smooth isounits $\hat{I} = T^{-1}$, leave invariant the isocomposition $(x^{t}Tgx)\,\hat{I}$ of the isotopic spaces $\hat{M}(x,\hat{g},\hat{F})$, $\hat{g} = Tg$, $\hat{F} = F\hat{I}$, $\hat{I} = T^{-1}$,

$$\hat{G}_r = x'^{\dagger} = x^{\dagger} * \hat{A}(w) = x^{\dagger} T\, A^{\dagger}(w), \quad x' = A(w) * x = A(w) T x, \tag{IV.62a}$$

$$[\,(x-y)^{\dagger} * \hat{A}^{\dagger}]\ \hat{g}\ [\hat{A} * (x-y)\,] \equiv (x-y)^{\dagger}\, \hat{g}\, (x-y), \tag{IV.62b}$$

$$\hat{A}^{\dagger}\, \hat{g}\, A = \hat{A}\, \hat{g}\, \hat{A}^{\dagger} = \hat{g}\hat{1}, \tag{IV.62c}$$

$$\mathrm{Det}\,(\hat{A}\hat{g}) = \det\hat{B} = \pm 1. \tag{IV.62d}$$

The following comments are in order:

1) Each given isometry Gr admits an infinite number of different isotopes $\hat{G}_r$ characterized by infinitely possible, different isounits which, from a physical viewpoint, represent infinitely possible interior physical media.

2) Each of the infinite isotopes can be explicitly computed, from expansions (IV.20), via the knowledge of the old isometry G(m) and of the isotopic element T.

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publication, Santilli felt obliged to formally report in paper (1985a), p. 26.

3) Even though the mathematical formulation can be unified for all possible isotopes $\hat{G}(m)$, the explicit form of the isotransformations is different for different isounits $\hat{1}$ (see later on).

4) As indicated earlier, the isotransformations are generally nonlinear and nonlocal, because of the dependence of T.

6) All isotopes $\hat{G}_r$ are *coverings groups* of the original isometry G_r under the sole condition that the old metric g is admitted as a particular case (or the isotopic element T admits the trivial unit I as a particular case).

7) All Lie algebras admit the following *trivial isotopy* $X_k \Rightarrow \hat{X}_k = X_k\hat{1}$, under which

$$\hat{G}^\circ_r: \quad [\hat{X}_r, \hat{X}_s]_{|\xi} = \hat{X}_r * \hat{X}_s - \hat{X}_s * \hat{X}_r = [X_r, X_s]_{|\xi}\, \hat{1} = (C_{rs}{}^t X_t)\, \hat{1} = C_{rs}{}^t \hat{X}_t. \tag{IV.63}$$

The above isotopies are *excluded* from the above theorem because they do not produce the invariance of the new isoseparation;

8) The dimension m of the original isometries G_r is preserved by all infinitely possible isotopic isometries $\hat{G}_r$. In particular, the condition for closure of $\hat{G}_r$ are reducible to those for G_r.

9) The isotopic isometries $\hat{G}r$ are generally nonisomorphic to the original symmetry G_r. However, as we shall see in the subsequent section, all infinitely possible isotopes $\hat{G}_r$ can be restricted to be locally isomorphic to the original isometry G(m) under the sole condition of positive– (or negative–) definiteness of the isotopic element T.

To understand the physical relevance of Theorem IV.9.1, one should be aware that all isotopic generalizations of Galilei's relativity, of Einstein's special relativity and of Einstein's general relativity are particular applications of Theorem IV.9.1

Next, we indicate the following

THEOREM IV.9.2 (BIRKHOFFIAN NOETHER'S THEOREM (Santilli (1982a)): If Birkhoff's equations admit a symmetry under an r–dimensional, connected Lie group G_r of infinitesimal transformations

$$G_r: \quad b \Rightarrow b' = b + \delta b = (b^\mu + w^i \alpha^\mu{}_i(b)) = \begin{pmatrix} t + w^i \rho_i(t, a) \\ a^\mu + w^i \eta^\mu{}_i(t, a) \end{pmatrix} \tag{IV.64}$$

then there exist r first integrals $\mathcal{F}_i$ (b) of the equations of motion which are conserved along an actual path $\tilde{E}$

$$\frac{d}{dt} \mathcal{F}_i(b)|_{\tilde{E}} \equiv 0, \tag{IV.65}$$

namely, there exist r linear combinations of Birkhoff's equations which are exact differentials along $\tilde{E}$, i.e.,

$$\frac{d}{dt} \mathcal{F}_i(b) = \tilde{\Omega}_{\mu\nu}(b)\, b^{\nu}\, \alpha^{\mu}{}_i , \tag{IV.66}$$

given explicitly by

$$\mathcal{F}_i(b) = \tilde{R}_{\mu} a^{\mu}{}_i$$

$$= R_{\mu}(t, a)\, \eta^{\mu}{}_i(t, a) - B(t, a)\, \rho(t, a) + G(_i(t, a). \tag{IV.67}$$

Note that the "new time" t' in Birkhoffian mechanics is a function of the old time t as well as of the coordinates r and momenta p, t' = t' (t, r, p). This property is important to understand the isotranslations in time of the new relativities presented in the next csection.

Intriguingly, this property is typical of relativistic formulations. The Birkhoffian mechanics then achieves a form of symmetric behaviour of time for both nonrelativistic and relativistic formulations.

Note that the symmetry G_r of Theorem IV.9.2 is a *conventional Lie symmetry* defined on a *conventional space.*

Considers the Birkhoff-Santilli representation of systems (I.3) on a on isospace $T^*\hat{E}(r,\hat{G},\hat{\mathfrak{R}})$ from the preceding section, i.e.,

$$\hat{T}_{2\mu}{}^{\alpha}(a)\, \Omega_{\alpha\nu}(a)\, \Gamma^{\nu}(t,a) = \frac{\partial B(t, a)}{\partial a^{\mu}}, \tag{IV.68}$$

where $\Omega_{\mu\nu}$ is the symplectic tensor and $\hat{T}_2$ = diag. $(\hat{G}, \hat{G})$ isthe isotopic element of $T^*\hat{E}(r,\hat{G},\hat{\mathfrak{R}})$.

An r-dimensional symmetry of Birkhoff-isotopic equations (IV.68) is an *"isosymmetry"* $\hat{G}_r$ when it is defined on isospaces $T^*\hat{E}(r,\hat{G},\hat{\mathfrak{R}})$ and admits infinitesimal transformations of the Lie-isotopic type

$$a'^{\mu} = a^{\mu} + w^i\, \Omega^{\mu\alpha}(a)\, \hat{1}_{2\alpha}{}^{\nu} \frac{\partial X_i}{\partial a^{\nu}} , \tag{IV.69}$$

where $\hat{1}_2 = \hat{T}_2^{-1}$ is the isounit of the isospace, the w's are the parameter and the X's are the generators of $\hat{G}_r$, with *isocommutation rules*

$$[X_r \hat{,} X_s] = \frac{\partial X_r}{\partial a^\mu} \Omega^{\mu\alpha} \hat{1}_{2\alpha}{}^{\nu} \frac{\partial X_s}{\partial a^\nu} = \hat{C}_{rs}(a)\, X_s, \qquad (IV.70)$$

where the $\hat{C}$'s are the so-called structure functions of the Lie-Santilli algebra (Santilli 1978a), (1982a))

It is easy to see that a *necessary* condition for transformations $a \Rightarrow a'$ to be a symmetry of Birkhoff-Santilli equations is that they have a isotopic structure. This renders *necessary* the use of the Lie-Santilli theory for the study of isosymmetries and their first integrals.

THEOREM IV.9.3 (INTEGRABILITY CONDITIONMS FOR THE EXISTENCE OF AN ISOSYMMETRY (Santilli (1982a), (1988b), (1991a)): Necessary and sufficient conditions for a smoothness and regularity preserving transformation (IV.69) to be an isosymmetry of the Birkhoff-isotopic equations (IV.68) is that they leave the Birkoffian invariant, i.e.,

$$B'(a') = B(a) + w_i\,[X_i \hat{,} B] \equiv B(a), \qquad (8.86)$$

which can hold iff the Birkhoffian B isocommutes with all generators X_i, i.e.,

$$[X_i \hat{,} B] \equiv 0, \quad i = 1, 2, ..., r. \qquad (8.87)$$

The Hamilton-Santilli subcase is simpler and readily usable in practical cases. Consider a closed Hamiltonian system of N particles and let G_r be its symmetry with generato.rs X_k and parameters w_k

$$G_r: \quad A'(a') = \{ e^{w_k\, \omega^{\mu\nu}\, (\partial_\nu X_k)\, (\partial_\mu)} \}\, A(a). \qquad (IV.88)$$

Suppose now that the system is generalized into a closed nonhamiltonian form (II.2). The construction of the isotope $\hat{G}_r$ of G_r leaving invariant the new system can be achieved via the following steps (Santilli (1988a), (1991d)):

1) Construct an analytic representation of the system in terms of the Pfaffian principle (IV.52) on $T^*\hat{E}(r,\hat{\delta},\hat{\mathfrak{R}})$ via the use of any of the techniques of the enclosed Vol. II of Santilli's *Foundations of Theoretical Mechanics* (1982a);

2) Construct the contravariant Hamilton-Santilli equations (IV.55) from principle (IV.52), by making sure that the isounit $\hat{1}_2$ (or isotopic element $\hat{T}_2$) verifies the conditions for the underlying brackets to be Lie-isotopic (the underlying two-isoform to be simpletic-isotopic). This essentially implies that the symmetries of the system are now studied in the new isospace $T^*\hat{E}_2(r,\hat{\delta},\hat{\mathfrak{R}}) = T^*(r,\hat{G},\hat{\mathfrak{R}})$, where $\hat{G}$ is given by Eq.s (IV.34);

3) The isotope $\hat{G}_r$ of G_r is given by structure

$$\hat{G}_r: \quad A'(a') = \{ [e^{w_k \omega^{\mu\alpha} \hat{I}_{2\alpha}{}^{\mu}(a) (\partial_\nu X_k) (\partial_\mu)}] \hat{I} \} * A(a)$$

$$= \{ e^{w_k \omega^{\mu\alpha} \hat{I}_{2\alpha}{}^{\mu}(a) (\partial_\nu X_k) (\partial_\mu)} \} A(a). \qquad \text{(IV.89)}$$

and it is an invariant of system (II.2) when H is an isoscalar on $T^*\hat{E}(r,\hat{G},\hat{\mathfrak{R}})$, i.e., has the form

$$H = p_{ia} \hat{G}^{ij}(r, p, \dot{p}, \ldots) \, p_{ja} \, / \, 2m_a + \hat{V}(r_{ab}), \qquad \text{(IV.90a)}$$

$$r_{ab} = | (r_{ia} - r_{ib}) \, \hat{G}^{ij} \, (r_{ja} = r_{jb}) |^{\frac{1}{2}}, \qquad \text{(IV.90b)}$$

(see next section for examples).

We have completed in this way our review of the network of Santilli's discoveries which do indeed provide the methodological tools for constructing the most general possible nonlinear and nonlocal space-time symmetries for the interior dynamical problem and, therefore, for constructing new relativities.

SECTION V: SANTILLI'S LIE-ISOTOPIC GENERALIZATIONS OF GALILEI'S RELATIVITY IN CLASSICAL MECHANICS

V.1: STATEMENT OF THE PROBLEM.

The exact validity of the conventional Galilei's relativity for the exterior dynamical problem, such as for the form-invariant characterization of the stability of the planetary orbits, has been recalled in the introductory words as being assumed at the foundations of Santilli's studies.

However, the insistence in the exact validity of the same relativity for the interior dynamical problem implies a number of inconsistencies of mathematical, physical and epistemological nature also recalled in Sect. I, including the direct acceptance of the perpetual motion in a physical environment, without any possibility of compromise because of the Theorems preventing the reduction of actual interior systems to idealistic exterior conditions (Sect. I).

Santilli therefore embarked in his most demanding and successful task: use the generalized mathematical structures outlined in the preceding section for the construction of suitable generalizations of Galilei's relativity for the interior dynamical problem which:

a) are "directly universal" for closed, nonlinear, nonlocal and nonhamiltonian interior systems (II.1), that is, capable of providing the form-invariant description of all systems considered (universality), directly in the frame of the experimenter (direct universality);

b) the generalized isotransformations and isosymmetries can be explicitly constructed from each given equations of motion, resulting in this way in an infinite number of generalized relativities (infinite number of different isounits), one per each given closed nonhamiltonian system; and, last but not least,

c) all possible generalized relativities admit the conventional Galilei's relativity identically when all nonhamiltonian forces are null.

Santilli's successful achievement of the above generalizations is a historical event, because it deals with the very first comprehensive generalizations of Galilei's relativity in all its mathematical foundations, physical applications and epistemological profiles, after some four centuries from its inception by Galileo Galilei in the early 1600's.

V.2: PHYSICAL MEANING OF SANTILLI'S ISOSPACES.

We now outline the physical applications of the novel mathematical tools of the preceding section, beginning with the physical meaning of the isospaces of Sect. IV.3.

The conventional Euclidean space $E(r,\delta,\mathfrak{R})$ essentially provides a geometrization of the homogeneous and isotropic vacuum (empty space).

The first meaning of Santilli's isospaces $\hat{E}(r,\hat{G},\hat{\mathfrak{R}})$ [9] is that of providing a geometrization of the interior physical media. Thus, the differences between $E(r,\delta,\mathfrak{R})$ and $\hat{E}(r,\hat{G},\hat{\mathfrak{R}})$ are representative of the transition from motion of an extended object in vacuum to motion of the same object within a physical medium.

Suppose that the interior problem is characterized by a surface S_0 of local radius R_0 encompassing all matter of the body considered, including its atmosphere. Then, the first fundamental condition of Santilli's relativities is that all isometrics $\hat{G} = T\delta$, recover the conventional metric δ, or, equivalently, all isotopic elements T acquire their conventional value I at distances $r > R_0$, i.e.,

$$\hat{I}\big|_{r>R_0} = I, \qquad \hat{T}\big|_{r>R_0} = I. \tag{IV.1}$$

This ensures the full compatibility of the generalized relativities with the conventional one, as well as the capability of the former to admit the latter as a particular case (Condition c above). Conditions a) and b) are ensured by the Lie-Santilli formulations.

Because of the assumed nonsingularity and Hermiticity, all isometrics $\hat{G}$ can always be diagonalized into the forms

$$\hat{G} = \text{diag.}\,(B_1^2, B_2^2, B_3^2). \tag{V.2}$$

where the B's are called *characteristic functions of the interior physical medium*, and depend on all possible variables and their derivatives, such as: coordinates r, their derivatives $\dot{r}$, $\ddot{r}$, ..., the local density $\mu(r)$, the local temperature $\tau(r)$, the local index of refraction n(r) (if any), etc.,

$$B_k = B_k(t, r, \dot{r}, \ddot{r}, \mu, \tau, n, ...) > 0. \tag{V.3}$$

where the positive-definite value is assumed to ensure the isomorphism of the generalized and conventional symmetries (see bellow).

Next, a central physical difference between empty space and physical media is that the latter are generally *inhomogeneous* (e.g., because of the local variation of the density) as well as *anisotropic* (e.g., because of an intrinsic angular momentum which creates a preferred direction in the medium). A typical physical example is given by our Earth's atmosphere, which is exactly inhomogeneous and anisotropic.

Another important application of the isotopic element T is therefore that of

[9] Santilli (1988a), (1991a, d) considers the isospace $\hat{E}(r,\hat{\delta},\hat{\mathfrak{R}})$ when dealing with variational principles (IV.52) and the more general isospaces $\hat{E}(r,\hat{G},\hat{\mathfrak{R}})$ when dealing with Hamilton-Santilli equations (IV.55) or, equivalently, with the isosymmetries of the equations of motion. In fact, the latter are characterized by the brackets of the analytic equations and not by the metric of the variational principle. When $\hat{G}$ is constant, then the two spaces coincide, $\hat{E}(r,\hat{\delta},\hat{\mathfrak{R}}) \equiv \hat{E}(r,\hat{G},\hat{\mathfrak{R}})$.

representing the inhomogenuity of the physical media, e.g., via the values $B_1 \neq B_2 \neq B_3$, as well as its possible anisotropy, e.g., via the factorization $\hat{G} = F(r)\,\hat{\delta}$, where r represents a preferred direction.

The NOBEL COMMITTEE should be aware that, in Santilli's writings, *the above inhomogenuity and anisotropy is solely referred to the experimental evidence pertaining to interior physical media, while the underlying space itself remains exactly homogeneous and isotropic*.

This is the reason for the factorization of the isometrics used throughout Santilli's writings, $\hat{G} = T\delta$, where the conventional metric δ represents the homogenuity and isotropy of (empty) space, and the isotopic element T represents the *"mutations"* caused by physical media.

A further important distinction is the global effect of interior media (e.g., for light propagating through the entire Earth's atmosphere), as compared to the local interior behavior (say, the motion of a satellite during re-entry in Earth's atmosphere). The explicit functional dependence (V.3) is essential for the latter case, but for the former case, the characteristic B-functions can be averaged into constants via any averaging process,

$$<|B_k|> = b_k = \text{constants} > 0. \qquad \text{(V.4)}$$

In turn, the regaining of the costant value of the isometrics has important epistemological implications. For instance, it implies the return to a linear and local transformation theory with $r' = A{*}r = ATr$, $T = \text{constant}$. Still in turn, this implies the possibility of preserving the *inertial* approximation of the observer as currently used in physics.[10]

Another meaning of the conventional metric $\delta = \text{diag.}\,(1,1,1)$ is that of representing the perfect sphere,

$$r^2 = r_i\,\delta^{ij}\,r_j = r_1 r_1 + r_1 r_2 + r_3 r_3 = \text{const.} \qquad \text{(IV.5)}$$

A further meaning of Santilli's isometric $\hat{\delta}$ is, first, that of representing the actual shape desired, say an oblate spheroidal ellipsoid

$$r^{\hat{2}} = r_i \hat{G}^{ij}\,r_j = r_1 B_1{}^2 r_1 + r_2 B_2{}^2 r_2 + r_3 B_3{}^2 r_3 = \text{const.} \qquad \text{(V.6)}$$

and then all possible deformations of the original shape, e.g., via a dependence of the characteristic B-functions on the external pressure or other physical causes.

A number of additional physical interpretations and applications of Santilli's isospaces have been identified in the specialized literature and they will be pointed out later on (the most intriguing ones are of operator nature and, as such, are indicated in Part III).

Finally, the NOBEL COMMITTEE should note that the above applications are a mere classical limit of the applications in particle physics to represent the historical legacy on interior, short range, nonlocal and nonhamiltonian effects (Part IV.).

[10] Recall from Sect. IV.3 that the isotransformations are nonlinear and nonlocal only for a nontrivial functional dependence of the isometric, while they return to be linear and local for constant isometrics (or constant isotopic elemts).

V. 3: SANTILLI'S ISOROTATIONAL SYMMETRIES.

The pillar of contemporary theoretical physics is the rotational symmetry O(3), classically and quantum mechanically.

The first generalized symmetry discovered by Santilli for nonlinear, nonlocal and nonhamiltonian systems was the generalization of the rotational symmetry into a form he called *rotational-isotopic symmetry*, or *isorotational symmetry* $\hat{O}(3)$.

The notion of "isorotational group" first appeared with examples in memoir Santilli (1978a), and was then expanded in the enclosed Vol. II of Santilli's *Foundations of Theoretical Mechanics* (1982a) for the nonlinear and nonhamiltonian but local case. The theory was then formulated for the most general possible abstract nonlocal case in Santilli (1985a). The first classical, nonlinear, nonlocal and nonhamiltonian realizations of $\hat{O}(3)$ appeared in Santilli (1988a), and reached their final form in the monograph Santilli (1991d).

The NOBEL COMMITTEE should be aware that *Santilli is, not only the originator, but also the sole contributor in the field until now*, that is, we are aware of no independent study by other mathematicians or physicists on the realization of the rotational symmetry with *a structurally generalized Lie product* other than the Poisson brackets (III.3) or the simplest possible Lie product $[A,B]_\xi = AB - BA$.

In essence, the conventional theory of rotations provides the symmetry only of *perfect spheres* (V.5), and can only characterize *rigid bodies*, as well known. But perfectly spherical and/or perfectly rigid bodies do not exist in the physical reality. Santilli's theory of isotopic rotations $\hat{O}(3)$ provide the symmetries, first, of any given nonspherical shape (V.6) and, second, of the infinitely-possible deformations of that original shape.

Santilli therefore discovered a generalized theory of rotations which is a *theory of extended and therefore deformable bodies*.

The *"rotational isotopic groups"* $\hat{O}(3)$, or *"isorotational groups"*, are the largest possible isolinear and isolocal groups of isometries of the infinitely possible isotopes $\hat{E}(r,\hat{G},\hat{\mathfrak{R}})$ of the three-dimensional Euclidean space $E(r,\delta,\mathfrak{R})$, i.e.,

$$E(r,\delta,\mathfrak{R}) \Rightarrow \hat{E}(r,\hat{G},\hat{\mathfrak{R}}], \ \delta = I_{3\times 3} \Rightarrow \hat{G} = T(r,\dot{r},\ddot{r},...)\,\delta, \tag{V.7a}$$

$$\det. T \neq 0, \quad T = T^{\dagger}, \quad \mathfrak{R} \Rightarrow \hat{\mathfrak{R}} = \mathfrak{R}\hat{1}, \quad \hat{1} = T^{-1} = \hat{G}^{-1} \tag{V.7b}$$

$$(r,r) = r_i\,\delta^{ij}\,r_j \Rightarrow (r\,\hat{,}\,r) = (r,\hat{G}r)\,\hat{1} = [r_i\hat{G}^{ij}(r,\dot{r},\ddot{r},..)\,r_j]\,\hat{1}, \tag{V.7c}$$

characterized by: the right, modular-isotopic transformations

$$r' = \hat{R}(\theta)*r = \hat{R}(\theta)\,\hat{G}\,r, \qquad \hat{G} = \text{fixed}, \tag{V.8}$$

where the θ's are the conventional Euler's angles, whose elements $\hat{R}(\theta)$ verify the properties

$$\hat{R}*\hat{R}^t = \hat{R}^t*\hat{R} = \hat{1} = \hat{G}^{-1}, \qquad (V.9)$$

or, equivalently, $\hat{R}^t = \hat{R}^{-\hat{1}}$, and form Lie-Santilli groups, i.e., verify the group-isotopic rules

$$\hat{R}(0) = \hat{1}, \qquad \hat{R}(\theta)*\hat{R}(\theta') = \hat{R}(\theta')*\hat{R}(\theta) = \hat{R}(\theta+\theta'), \qquad \hat{R}(\theta)*\hat{R}(-\theta) = \hat{1}. \qquad (V.10)$$

Finally the isogroups $\hat{O}(3)$, like the conventional group O(3), are simple, three-dimensional, and admit a decomposition into connected isosubgroups $S\hat{O}(3)$ occurring, from Eq.s (V.9), for

$$\det(\hat{R}\hat{G}) = +1, \qquad (V.11)$$

and a discrete component $\hat{\phi}(3)$ characterized by the *isoinversions*

$$\hat{P}*r = Pr = -r, \qquad \hat{P} = P\hat{1}. \qquad (V.12)$$

Let us briefly outline the classical realization of Santilli's ((1985a), (1988a), (1991d)) isorotations, owing to its truly fundamental character. For simplicity, but without lack of generality, we shall assume diagonalized isometrics with elements b_k independent from the local coordinates and momenta (but depending on all other quantities of interior media)

$$\hat{E}(r,\hat{G},\hat{\mathfrak{R}}) \equiv \hat{E}(r,\hat{\delta},\hat{\mathfrak{R}}), \qquad \hat{\delta} = \text{diag.}\,(b_1^2, b_2^2, b_3^2), \qquad b_k > 0, \qquad (V.13a)$$

$$r^{\hat{2}} = r_i\,\hat{\delta}^{ij}\,r_j = r_1 b_1^2 r_1 + r_2 b_2^2 r_2 + r_3 b_3^2 r_3 = \text{isoinv.} \qquad (V.13b)$$

The Lie-Santilli brackets (IV.35) then assume the simpler form

$$[A \,\hat{,}\, B] = \frac{\partial A}{\partial r_{ka}}\, b_k^{-2}\, \frac{\partial B}{\partial p_{ka}} - \frac{\partial B}{\partial r_{ka}}\, b_k^{-2}\, \frac{\partial A}{\partial p_{ka}}, \qquad (V.14)$$

By central condition of the Lie-Santilli theory, the original generators and parameters do not change[11]. The generators of $S\hat{O}(3)$ are then given by the *conventional generators of O(3)* , $J_k = \epsilon_{kij}\, r_i\, p_j$. The isocommutation rules of the (compact, connected) isorotational algebra **$S\hat{O}(3)$** are then given by (loc. cit.)

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[11] Technically, this originates from the PROPOSITION: *The basis of a vector space is left unchanged by isotopies* (Santilli 1991a), which then propagates to Lie algebras, Lie groups, etc. This so simple a mathematical property has funbdamental physical implications. The basis of a Lie algebra in physics, say, that of the Galilei algebra G(3.1), is given by the conventional total conserved quantities. The above Proposition then provides *a geometrization of closed nonhamiltonian systems* , that is, the conservation of conventional total physical quantities (the basis of the old symmetry) under additional nonhamiltonian forces. Equivalently, the Proposition confirm the *purely interior character of Santilli's nonlocal effects*, not only at this introductory classical level, but much so at the level of the structure of hadrons (Part IV).

$$\hat{SO}(3): \qquad [J_i \hat{,} J_j] = C_{ij}{}^k J_k = \epsilon_{ijk} b_k{}^{-2} J_k; \tag{V.15}$$

The proof of the local isomorphisms $\hat{SO}(3) \approx \hat{S\hat{O}}(3)$ was then done via the redefinition

$$\hat{J}_k = \epsilon_{kij} \; b_i r_i \; b_j p_j, \tag{V.16}$$

for which

$$[\hat{J}_i \hat{,} \hat{J}_j] = \epsilon_{ijk} \hat{J}_k, \tag{V.17}$$

which recovers the structure constants ϵ_{ijk} of the original symmetry. However, the physical generators remain J_k and the physical isocommutation rules remain rules (V.15).

The *isocasimir* (the invariant quantity) is given by the "square" of J, but properly computed in the isospace $\hat{E}(r,\hat{\delta},\Re)$, i.e., the *isosquare*

$$J^{\hat{2}} = J{*}J = J_i \, \hat{\delta}^{ij} J_j = J_k \, b_k{}^2 \, J_k. \tag{V.18)}$$

while the conventional Hamiltonian quantity

$$J^2 = J_i \, \delta^{ij} J_j = J_k J_k, \tag{V.19}$$

is no longer invariant as one can easily prove via rules (V.15).

The classical realization of the Lie-Santilli group $\hat{SO}(3)$ is then given by ((loc. cit.)

$$\hat{SO}(3): \quad \hat{R}(\theta) = [\, \prod_{k=1,2,3} e_{|\xi}^{\theta_k \omega^{\mu\alpha} \hat{1}_{2\alpha}{}^{\nu}(a) \, (\partial J_k/\partial a^{\nu}) \, (\partial/a^{\mu})} \} \, \hat{1} = S_{\hat{\delta}}(\theta) \, \hat{1}, \tag{V.20}$$

where the exponentials are expanded in the conventional associative envelope ξ.

Note the true realization of Santilli's notion of isotopic lifting of a Lie symmetry, i.e., the preservation of the original generators and parameters of the symmetry, and the isotopic generalization of the *structure* of the Lie group via the liftings $I \Rightarrow \hat{1}_2$. Note also the existence of infinitely many different isorotational symmetries $\hat{O}(3)$, which are however all locally isomorphic among themselves and to the conventional symmetry O(3).

The isorotational transformations can be explicitly computed from expansion (V.20) via the sole knowledge of the new metric $\hat{\delta}$, and their convergence into a finite form is ensured by the original convergence (under the assumed topological restrictions of the isounit). For instance a (classical) *isorotation around the third axis* is given by (loc. cit.)

$$r' = \hat{R}(\theta_3){*}r = S_{\hat{\delta}}(\theta_3) \, r = \tag{V.21}$$

$$= \begin{pmatrix} r'_1 \\ r'_2 \\ r'_3 \end{pmatrix} = \begin{pmatrix} r_1\cos(\theta_3 b_1 b_2) - r_2 \frac{b_2}{b_1}\sin(\theta_3 b_1 b_2) \\ r_1 \frac{b_1}{b_2}\sin(\theta_3 b_1 b_2) + r_2\cos(\theta_3 b_1 b_2) \\ r_3 \end{pmatrix}$$

The invariance of isoseparation (V.13b) under the above transformation then follows.

Santilli ((1982a), (1985b), (1988a)) therefore introduced the notion of *isorotational symmetry* (at times also called *rotational isosymmetry*, depending on the desired emphasis) which is verified by systems (II.1) when they are: 1) properly written in $T^*\hat{E}(r,\hat{\delta},\hat{\mathfrak{R}})$; 2) represented by Hamilton-Santilli equations (IV.55) via the identification of the Hamiltonian H and of the isounit $\hat{1}$; and 3) the Hamiltonian H is isorotationally invariant, i.e.,

$$H(r, p) = H(r + \delta\theta\, n\hat{\wedge}J, p + \delta\theta\, n\hat{\wedge}J) = \{ e_{|\xi}^{-\delta\theta\, n\hat{\wedge}J} \} B(r, p), \qquad (V.22)$$

which can hold iff $[J_k \hat{,} B] = 0$, $k = 1, 2, 3$, as expected. The above conditions were proved to be verified when the Hamiltonian has the structure in $\hat{E}(r,\hat{\delta},\hat{\mathfrak{R}})$

$$H = T(p) + V(r) = \frac{p_{ia}\,\delta^{ij}\,p_{ja}}{2m_a} + V(r), \quad r = | r_{ia}\,\delta^{ij}\,r_{ja} |^{\frac{1}{2}}. \qquad (V.23)$$

Moreover, Santilli (loc. cit.) worked out numerours additional developments and classical applications also of fundamental character, such as:

1) the most general poassible nonlinear, nonlocal and nonhamiltonian isorotational symmetries $\hat{O}(3)$, including isoinversions, whuich are essentially given by structures (V.15), (V.20) and (V.21) with the b-quantities replaced by the characteristic B-functions, while isocasimir (V.18) assumes a local meaning;

2) the proof of the local isomorphisms $\hat{O}(3) \approx O(3)$ for the above most general possible case under the conditions of positive-definiteness of the B-functions;

3) the proof of the $\hat{O}(3)$ invariance for all possible extended, nonspherical shapes;

4) the application for the direct representation of all infinitely possible deformations of a given shape;

5) the establishing of the isorotational symmetries $\hat{O}(3)$ as the fundamental isosymmetries of the theory of elasticity;

6) the isotopic generalization of the historical Euler's theorem, according to which the general displacement of an elastic body with one point fixed is a (compact) isorotation around an axis through that point;

7) the construction of the isorepresentation theory of the covering, isotopic SU(2) symmetries, including the constructioon of the iso-Clebsh-Gordon coefficients, etc.;

and numerous other advances we cannot possibly review here.

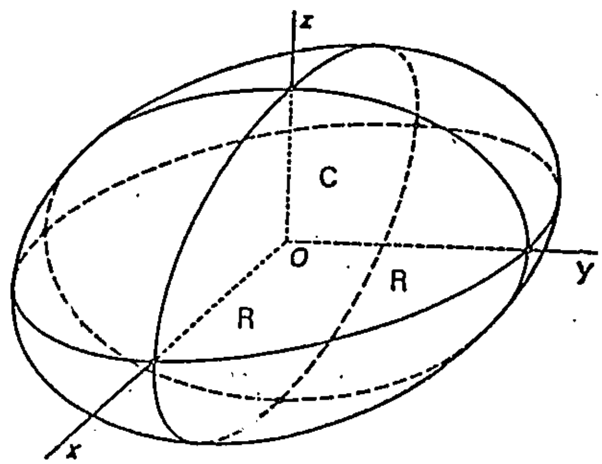

FIGURE III.3.2: An illustration of the possibilities of Santilli's (1978a), (1982a), (1985b), (1988a), (1991d) discovery of the classical isorotations: the direct representation, first, of the actual shape of a given body via the isometric $\hat{\delta}$ (in the figure, an oblate spheroidal ellispoid), and then the direct representation of all its infinitely possible deformations under sufficiently intense external forces, collisions or other causes, all this already at a semiclassical level. By comparison, contemporary theories can indeed reach a representation of the extended character of particles via form factors, but, as well known: 1) the representation occurs only after the rather complex second quantization; 2) the form factors provide only a remnant of the actual shape and cannot characterize the actual shape itself (at any rate, the representation of non-spherical shapes would imply the breaking of the conventional rotational symmetry); and 3) the representation cannot possibly characterize the deformations of said original shape (because, again, the conventional rotational symmetry refers strictly to rigid bodies and it is broken under the deformations of the physical reality). The advantages of Santilli's covering isorotational symmetries over the conventional rotational symmetry is then multiple, diversified, and beyond any scientific doubt. But, the most far reaching implications of Santilli's isorotations occurs in particle physics (Part III), where they directly imply a generalization of the very notion of particle.

A central discovery the NOBEL COMMITTEE should keep in mind is the following. *It is generally believed, in both the mathematical and physical literatures, that all deformations (V.13b) of the perfect sphere (V.5) violate the rotatioinal symmetry. Santilli proved this belief to be erroneous. In fact, since his isorotational symmetries $\hat{O}(3)$ are locally isomorphic to the conventional rotational symmetry O(3), Santilli reconstructed as exact the rotational symmetry for all possible ellipsoidical deformations of the sphere, of course, at the covering isotopic level.*

The same reconstruction of the exact character at the isotopic level also exists for all remaining space-time symmetries when believed to be broken in conventional formulations, including the Galilei symmetry, the Lorentz symmetry, and the Poincaré symmetry.

As a matter of fact, one of the reasons for which Santilli constructed the isotopies of conventional space-time symmetries is precisely to prove that the claim of their being violated is erroneous and due to insufficient mathematical knowlede because, when the same symmetry is realized in a sufficiently more general way, it returns to be exact.[12]

The NOBEL COMMITTEE should be aware that, *by far the most fundamental applications of Santilli's isorotational symmetries exist in particle physics*. As an illustration, let us recall that, after over half a century, nuclear physics has been unable to explain the total magnetic moment of nuclei. Santilli's isorotational symmetry predicts that, under the "contact" conditions of the nuclear structure, the extended charge distribution of protons and neutrons experiences a small deformation of shape, e.g., into the oblate form of Fig. V.1. Such a deformation of the charge distribution then implies a necessary alteration of the intrinsic magnetic moment of the necleons as requested by (classical and quantum) Maxwell's electrodynamics, which is finally capable of representing the total magnetic monment of nuclei (for details, see Part IV).

An important aspect of these latter advances in particle physics is that the deformation of the charge distribution of protons and neutrons and related phenomenology are represented by preserving the exact character of the rotational symmetry, of course, at the isotopic level.

V.4: SANTILLI'S ISOEUCLIDEAN SYMMETRIES.

The conventional Euclidean symmetry for the exterior problem on $T^*E(r,\delta,\Re)$ is given by $E(3) = O(3) \times T(3)$, where $O(3)$ is the group of rotations and $T(3)$ represents the translations. The Euclidean transformations are given by the familiar expressions

$$r' = R(\theta)r, \qquad r' = r + r^{\circ}, \tag{V.24}$$

and leave invariant the composition in $E(r,\delta,\Re)$ for the relative distances among any pair of particles

$$(r_a - r_b)^2 = (r_{ia} - r_{ib})\,\delta^{ij}\,(r_{ja} - r_{jb}). \tag{V.25}$$

The next space-time isosymmetries discovered by Santilli was the generalization of the above Euclidean symmetry for the most general possible, nonlinear, nonlocal and nonhamiltonian conditions, called *Euclidean-isotopic symmetries*, or *isoeuclidean symmetries* $\hat{E}(3)$.

The discovery was first made as implicitly contained in the more general Galilei

[12] This reconstruction of the exact space-time symmetries when believed to be broken at the simplest possible level of Lie's theory is fully established at the classical level, but not yet at the operator level. In fact, in Part III we indicate Santilli (1984a) conjecture on the possible regaining of the *exact character of parity for weak interactions*, of course, when realized at the covering isotopic level. These studies are under way at this writing and, if confirmed, would be of such a magnitude to deserve, alone, a NOBEL PRIZE IN PHYSICS.

isosymmetries in Santilli (1978a) and in the enclosed monograph Santilli (1982a). The explicit treatment of the isoeuclidean symmetry first appeared in Santilli (1988a) and was then treated in full details in the monograph Santilli (1991d).

The NOBEL COMMITTEE should be aware that, again, Santilli is the originator and the sole contributor on the isoeuclidean symmetry to this day.

The discovery can be essentially presented via the following

THEOREM V.4.1: The Euclidean-isotopic symmetries $\hat{E}(3)$ are defined on the isotopes $\hat{E}(r,\hat{G},\hat{\Re})$ of the Euclidean spaces $E(r,\delta,\Re)$, leave invariant the relative isoseparation

$$(r_a - r_b)^{\hat{2}} = (r_{ia} - r_{ib})\, \hat{G}^{ij}\, (t, r, \dot{r}, ...)\, (r_{ja} - r_{jb}), \qquad (V.26)$$

possess the structure

$$\hat{E}(3) \ : \ \hat{O}(3) \otimes \hat{T}(3), \qquad (V.27)$$

where $\hat{O}(3)$ is given by the isorotational symmetries of Sect. V.3

$$\hat{O}(3): \quad r'_a = \hat{R}(\theta) * r_a = \hat{R}(\theta)\, \hat{G}(r, \dot{r}, ...)\, r_a, \qquad (V.28a)$$

$$\hat{R}^{\hat{t}} * \hat{R} = \hat{R} * \hat{R}^{\hat{t}} = \hat{1} = \hat{G}^{-1}, \qquad (V.28b)$$

$$r_a = (r_{ia}), \quad i = 1, 2, 3 \ (=x, y, z), \quad a = 1, 2, ..., N,$$

and $\hat{T}(3)$ is the largest possible group of isolinear and isolocal translations

$$\hat{T}(3): \quad r'_{ia} = r_{ia} + r_i^{\circ}\, \hat{B}_i^{-2}(t, r, \dot{r}, \ddot{r}, \mu, \tau, n....), \qquad (V.29a)$$

$$p'_{ia} = p_{ia}, \qquad (V.29b)$$

with realization in $T{*}\hat{E}(r,\hat{G},\hat{\Re})$ characterized by:

1) The same set of parameters $(w_k) = (\theta_i, r^{\circ}_i)$ of the conventional symmetry $E(3)$;
2) the same set of generators $(X_k) = (J, P)$, $J = \sum_a J_a$, $P = \sum_a p_a$, of the conventional symmetry $E(3)$,
3) the Lie-Santilli algebras

$$\hat{E}(3): \quad [J_i \hat{,} J_j] = \epsilon_{ijk}\, B_k^{-2}\, J_k, \qquad (V.30a)$$

$$[J_i \hat{,} P_j] = \epsilon_{ijk} B_j^{-2} P_k, \quad \text{(V.30b)}$$

$$[P_i \hat{,} P_j] = 0. \quad \text{(4.30c)}$$

4) the Lie-Santilli groups

$$\hat{E}(3): \ \hat{e}(w)*a = \{\hat{R}(\theta), \hat{T}(r^\circ)\}*a$$

$$= \{ [\prod_k e_{|\xi}^{w_k \omega^{\mu\sigma} \hat{1}_2^{\sigma\nu} (\partial_\nu X_k)(\partial_\mu)}] \hat{1}_2\}*a, \quad \text{(V.31)}$$

5) the realization of the $\tilde{B}$-functions

$$\tilde{B}_i^{-2}(t,r,p,.....) = B_i^{-2} + r^\circ_j [B_i^{-2} \hat{,} P_j] /2! + r^\circ_j r^\circ_l [[B_i^{-2} \hat{,} P_j] \hat{,} P_l] / 3! + ... \quad \text{(V.32)}$$

and the local isocasimir invariants given by the following multiples of the isounits[13]

$$\hat{C}^{(o)} = \hat{1}_2, \quad \hat{C}^{(1)} = (P*P)\hat{1}_2, \quad \hat{C}^{(2)} = (J*P)\hat{1}_2, \quad \text{(V.33)}$$

Finally, all the above infinitely possible Euclidean-isotopic symmetries $\hat{E}(3)$ result to be locally isomorphic to the conventional Euclidean symmetry E(3) under the sole conditions of sufficient smoothness, nonsingularity and positive-definiteness of the isometrics $\hat{G}$.

The NOBEL COMMITTEE should note the achievement of the most general possible nonlinear and nonlocal realizations, not only of the group of rotations, but also of the translations, as clearly expressed in Eq.s (V.29) via the arbitraryness of the B-functions.

Also, the above theorem provides the foundations of the isotopic generalization of the Poincare' symmetry in (2+1)-space-time dimension (see Part II), via tits mere reformulation for the isonmetric $\hat{G} = \text{diag.}(+B^2, +B_2^2, -B_3^2)$.

V.5: SANTILLI'S ISOGALILEAN SYMMETRIES

Recall that the conventional *Galilei's symmetry* G(3.1) (see, e.g., Levy-Leblond (1971) or Sudarshan and Mukunda (1974)) is the largest Lie group of *linear and local* transformations leaving invariant the separations

$$t_a - t_b = \text{inv.},$$

[13] It is appropriate to recall here that the realization of the isoeuclidean symmetry under consideration here is classical, that is, via *functions* in $T^*\hat{E}(r,\hat{G},\hat{\Re})$. As a result, the generators and, therefore, the isocasimir invariants, must necessarily be functions, while the isounit $\hat{1}_2$ of Eq.s (4.18) is a matrix of functions. When passing to the operator realization of the isorotational symmetry of Part III, then the entire expressions of rthe isocasimirs must be used.

$$(r_{ia} - r_{ib})\, \delta^{ij}\, (r_{ja} - r_{jb}) = \text{inv.} \;\; \text{at} \;\; t_a = t_b, \tag{V.34}$$

$$i,j = 1, 2, 3 \;\; (= x, y, z), \qquad a = 1, 2, \ldots, N$$

in $\Re_t \times T^*E(r,\delta,\Re)$, where $\Re_t$ represents time, $E(r,\delta,\Re)$ is the conventional Euclidean space, and T^*E its cotangent bundle (phase space), with metric $\delta = \text{diag.}\ (1,1,1)$ over the reals $\Re$.

The explicit form of the celebrated *Galilei transformations* is given by the familiar expressions

$$t' = t + t^\circ, \quad \text{translations in time} \tag{V.35a}$$

$$r'_{ia} = r_{ia} + r^\circ_i, \quad \text{translations in space} \tag{V.35b}$$

$$r'_{ia} = r_{ia} + t^\circ v^\circ_i, \quad \text{Galilei bosts} \tag{V.35c}$$

$$r'_a = R(\theta)\, r_a, \quad \text{rotations.} \tag{V.35d}$$

The use of the totality of the preceding mathematical and physical studies, including the Lie-isotopic theory, the Birkhoffian-Santilli mechanics, the symplectic-isotopic geometry, etc., then permitted Santilli to achieve the most general possible nonlinear, nonlocal and nonhamiltonian generalizations $\hat{G}(3.1)$ of the conventional Galilei symmetry $G(3.1)$ for the interior problems, which he called *Galilei-isotopic symmetries* , or *isogalilean symmetries* , but which are now called *Santilli's isogalilean symmetries* (see, Kadeisvili (1991), Aringazin et al (1992) and others).

The generalizations were first achieved in the memoir Santilli (1978a) as a particular case of their broader Lie-admissible forms (see Appendix A); they were formalized for the local case in the enclosed monograph Santilli (1982a); the isosymmetries were then formulated for the most general possible nonlocal case in the memoir Santilli (1988a); and finally treated in details in the recent monograph Santilli (1991d).

The NOBEL COMMITTEE should note that, again, Santilli is the originator and the sole contributor in the study of the isogalilean symmetries [14]. In fact, despite the appearance of the enclosed monograph printed by *Springer-Verlag* (and advertized rather widely), no independent mathematician or physicist elected to conduct research in the field. Santilli therefore continued his studies alone.

THEOREM V.5.1: The general nonlinear and nonlocal, classical realization of Santilli's isogalilean symmetries $\hat{G}(3.1)$ are the isotopic groups of the most general possible isotransformations on $\hat{\Re}_t \times T^*\hat{E}(r,\hat{G},\hat{\Re})$ leaving invariant the isoseparations

$$t_a - t_b = \text{inv.}, \tag{V.36a}$$

$$(r_{ka} - r_{kb})\, B_k^2(t, r, p, \ldots)\, (r_{ka} - r_{kb}) = \text{inv.} \;\; \text{at} \;\; t_a = t_b\, , \tag{V.36b}$$

[14] The only independent article in Santilli's isotopic generalizations of Galilei's symmetry known to these authors is that by Jannussis, Mijatovic and Veljanoski (1991) reviewed later on in Sect. V.7 which however studies *examples* of the isogalilean symmetries.

$$t_a, t_b \in \hat{\Re}_t, \qquad r_a, r_b \in T^*\hat{E}(r,\hat{G},\hat{\Re}) \qquad \text{(V.36c)}$$

where $\hat{\Re}_t$ is an isotopic lifting of the conventional field $\Re_t$, called isotime field, with explicit structure

$$\hat{\Re}_t = \Re\,\hat{I}_t, \quad \hat{I}_t = B_4^{-2}(t, r, p, ...), \qquad B_4 > 0, \qquad \text{(V.37)}$$

$T^*\hat{E}(r,\hat{G},\hat{\Re})$ is the isocotangent bundle for symplectic-isotopic two-isoforms with isometrics $\hat{G}$ diagonalizable to form (V.2), and the four functions B_1, B_2, B_3 and B_4 besides being independent and positive-definite, are arbitrary nonlinear and nonlocal (e.g., integral) functions on all possible, or otherwise needed local variables and quantities. Santilli's isogalilean transformations can be explicitly written

$$t' = t + t^\circ \tilde{B}_4^{-2}, \qquad \text{iso-time translations} \qquad \text{(V.38a)}$$

$$r_i' = r_i + r^\circ_i \tilde{B}_i^{-2}, \qquad \text{iso-space translations} \qquad \text{(V.38b)}$$

$$r_i' = r_i + t^\circ v^\circ_i \tilde{B}_i^{-2}, \quad \text{iso-Galilei boosts} \qquad \text{(V.38c)}$$

$$r' = \hat{R}(\theta) * r, \qquad \text{isorotations,} \qquad \text{(V.38d)}$$

where the $\tilde{B}$-functions are generally nonlinear and nonlocal in all possible local variables and quantities to be identified shortly. Moreover, the symmetries $\hat{G}(3.1)$ are characterized by the Lie-Santilli brackets underlying the exact symplectic-isotopic two-forms, with explicit expression

$$[A\hat{,} B] = \frac{\partial A}{\partial r_{ka}} B_k^{-2} \frac{\partial B}{\partial p_{ka}} - \frac{\partial A}{\partial p_{ka}} B_k^{-2} \frac{\partial B}{\partial r_{ka}}, \qquad \text{(V.39)}$$

and possess the following structure:
1) the conventional parameters of G(3.1),

$$w = (w_k) = (\theta_i, r^\circ_i, v^\circ_i, t^\circ), \quad k = 1, 2, ..., 10, \qquad \text{(V.40)}$$

and the conventional generators of G(3.1), but now defined on isospace $\hat{\Re}_t \times T^*\hat{E}_2(r,G,\hat{\Re})$, i.e.,

$$J_i = \sum_a \epsilon_{ijk}\, r_{ia} p_{ia}, \qquad P_i = \sum_a p_{ia}, \qquad \text{(V.41a)}$$

$$G_i = \sum_a (m_a\, r_{ia} - t\, p_{ia}), \qquad \text{(V.41b)}$$

$$H = p_{ka} B_k^2 p_{ka} / 2m_a + V(r_{ab}), \tag{V.41c}$$

$$r_{ab} = |r_a - r_b|^{\hat{2}} = \{(r_{ka} - r_{kb}) B_k^2 (r_{ka} - r_{kb})\}^{\frac{1}{2}}, \tag{V.41d}$$

2) the Lie-Santilli algebras

$$\hat{G}(3.1) \quad [J_i \hat{,} J_j] = \epsilon_{ijk} B_k^{-2} J_k, \qquad [J_i \hat{,} P_j] = \epsilon_{ijk} B_j^{-2} P_k, \tag{V.42a}$$

$$[J_i \hat{,} G_j] = \epsilon_{ijk} B_j^{-2} G_k, \qquad [J_i \hat{,} B] = 0, \tag{V.42b}$$

$$[G_i \hat{,} P_j] = \delta_{ij} M B_j^{-2}, \qquad [G_i \hat{,} B] = 0, \tag{V.42c}$$

$$[P_i \hat{,} P_j] = [G_i \hat{,} G_j] = [P_i \hat{,} B] = 0, \tag{V.42d}$$

3) the (connected component of the) Lie-Santili groups

$$\hat{G}(3.1: \qquad r' = \{ [\prod_k e_{|\xi}^{w_k \omega^{\mu\sigma} \times \hat{1}_2^{\sigma\nu} (\partial_\nu X_k)(\partial_\mu)}] \hat{1}_2 \} * r, \tag{V.43}$$

4) the local isocasimir invariants

$$\hat{C}^{(o)} = \hat{1}_2, \qquad \hat{C}^{(1)} = (P \hat{G} P - MH) \hat{1}_2, \tag{V.44a}$$

$$\hat{C}^{(2)} = (MJ - G \wedge P)^{\hat{2}} = \{ (MJ - G \wedge P) \hat{G} (MJ - G \wedge P) \} \hat{1}_2, \tag{V.44b}$$

5) the explicit expressions of the $\tilde{B}_j$ functions

$$\tilde{B}_i^{-2}(r^\circ) = B_i^{-2} + r^\circ_j [B_i^{-2} \hat{,} P_j] / 2! + r^\circ_m r^\circ_n [[B_i^{-2} \hat{,} P_m] \hat{,} P_n] / 3! + \ldots \tag{V.45a}$$

$$\tilde{B}_i^{-2}(v^\circ) = B_i^{-2} + v^\circ_j [B_i^{-2} \hat{,} G_j] / 2! + v^\circ_m v^\circ_n [[B_i^{-2} \hat{,} G_m] \hat{,} G_n] + \ldots \tag{V.45b}$$

while $\tilde{B}_4^{-2}(t^\circ)$ is the solution of the algebraic equation

$$r(t + t^\circ \tilde{B}_4^{-2}) = \{ e_{|\xi}^{t^\circ \omega^{\mu\sigma} \times \hat{1}_2^{\sigma\nu} (\partial_\mu H)(\partial_\nu)} \} r. \tag{V.46}$$

The infinite possible isogalilean symmetries $\hat{G}(3.1)$ so constructed result to be all locally isomorphic to the conventional Galilei symmetry G(3.1) under the sole condition of (sufficient smoothness, nonsingularity and) positive-definitness of the isounits. Finally, all isosymmetries $\hat{G}(3.1)$ can approximate the conventional symmetry G(3.1) as close as desired whenever the isounits approach the conventional unit, and they all admit the conventional symmetry as a particular case by construction.

The follwoving particular case is sufficiently improtant to warrant a mention for the NOBEL COMMITTEE.

COROLLARY V.5.1A: In the particular case when the characteristic B-functions of the interior physical media are averaged to constant b-quantities, we have

$$\hat{\mathfrak{R}}_t \times T^*\hat{E}(r,\hat{\delta},\hat{\mathfrak{R}}) \equiv \hat{R}_t \times T^*\hat{R}(r,G,\hat{\mathfrak{R}}) \qquad (V.47a)$$

$$g = G = \mathrm{diag.}\ (b_1^2, b_2^2, b_3^2), \quad b_k = \mathrm{cost.} > 0; \qquad (V.47b)$$

the $\hat{B}$-quantities coincide with the diagonal elements of the isounits,

$$\hat{B}_i^{-2}(r^\circ) \equiv \hat{B}_i^{-2}(v^\circ) \equiv B_i^{-2} \equiv b_i^{-2}, \qquad \hat{B}_4^{-2}(t^\circ) \equiv b_4^{-2}. \qquad (V.48)$$

and the general isogalilean transformations (V.38) become linear and local, i.e., they assume the simplified form called "restricted isogalilean transformations"

$$t' = t + t^\circ b_4^{-2}, \qquad (V.49a)$$

$$r'_i = r_i + r^\circ_i\, b_i^{-2}, \qquad (V.49b)$$

$$r'_i = r_i + t^\circ\, v^\circ_i\, b_i^{-2}, \qquad (V.49c)$$

$$r' = \hat{R}(\theta) * r. \qquad (V.49d)$$

The above properties imply that the Santilli's isogalilean symmetries can indeed preserve inertial frames, but, of course, in their linear particularization, e.g., following averaging of the characteristic B-functions of the interior medium of type (V.4).

Thanks to the body of formulations outlined in the preceding section, the application of the symmetries $\hat{G}(3.1)$ to the characterization of closed nonselfadjoint systems was readily achieved by Santilli. In fact, Theorems IV.9.2 and IV.9.3 readily yield the following

LEMMA V.5.1 (loc. cit.): Necessary and sufficient conditions for the isoinvariance of closed nonselfadjoint systems (II.1) under Santilli's isogalilean symmetries $\hat{G}(3.1)$ are that tthe systems can be consistently written in isospaces $\hat{R}_t \times T^*\hat{E}(r,\hat{G},\hat{\mathfrak{R}})$ and admit the representation in terms of the Hamilton-Santilli equations

$$\omega_{\mu\sigma}\, T_2{}^{\sigma}{}_{\nu}(a)\, \dot a^{\nu} = \frac{\partial H(t,a)}{\partial a^{\mu}}, \qquad \dot a^{\mu} = \omega^{\mu\sigma}\, \hat 1_{2\sigma}{}^{\nu}(a)\, \frac{\partial H(t,a)}{\partial a^{\nu}}, \tag{V.50a}$$

$$H = p_{ia}\, \hat G^{ij}(t, r, p, ...)\, p_{ja}/2m_a + V(r_{ab}), \tag{V.50b}$$

$$r_{ab} = \{(r_{ia} - r_{ib})\, \hat G^{ij}(r, p, ...)\, (r_{ja} - r_{jb})\} \tag{V.50c}$$

in which case all total physical quantities are not subsidiary constraints, but first integrals of the equations of motion.

In summary, Santilli not only discovered the class of closed nonhamiltonian systems (II.1) with subsidiary constraints ensuring the conventional total conservation laws, but also the technical means for eliminating the subsidiary constraints, and reduce the conservation laws to generalized symmetries. In fact, the imposition of the invariance under the isogalilean symmetries $\hat G(3.1)$ ensures the ten conventional total conservation laws.

Remarkably, the same results hold all subsequent levels of study, such as the relativistic (Part II), gravitational (Part III), quantum mechanical (Part IV), gauge, etc.

V.6: SANTILLI'S CLASSICAL ISOGALILEAN RELATIVITY.

As well known, *Galilei's relativity* (see, e.g., Levy-Leblond (1971) or Sudarshan and Mukunda (1974)) is a description of physical systems via their form-invariance under the Galilei's symmetry $G(3.1) = [O_\theta(3) \otimes T_{r^\circ}(3)] \times [T_{v^\circ}(3) \times T_{t^\circ}(1)]$, or, equivalently, under the celebrated *Galilei's transformations*

$$t' = t + t^\circ, \quad \text{translations in time,} \tag{V.51a}$$

$$r'_i = r_i + r^\circ_i, \quad \text{translations in space,} \tag{V.51b}$$

$$r'_i = r_i + t^\circ v^\circ_i, \quad \text{Galilei boosts,} \tag{V.51c}$$

$$r' = R(\theta)\, r\,, \quad \text{rotations.} \tag{V.51d}$$

The relativity is verified in our physical reality only for a rather small class of Newtonian systems, called *closed selfadjoint systems* . These are systems (such as our planetary system) which verify the conventional total Galilean conservation laws when isolated from the rest of the universe, and admit internal forces which are local (differential), potential and (variationally) selfadjoint (Helmholtz (1887), Santilli (1978b)).

For all remaining Newtonian systems, Galilei's relativity is inapplicable because of the several; reasons outlined in Sect. III.1. In the final analysis, the limitations of Galilei's relativity are inherent in its mathematical structure, because:

1) The *linear* character of Galilei's transformations is at variance with the

generally *nonlinear* structure of the systems of the physical reality of the interior dynamical problem, as established by incontrovertible evidence;

2) The *local* (differential) character of Galilei's relativity is at variance with the generally *nonlocal* (integral) nature of our Earthly environment;

3) The *Hamiltonian* (canonical) structure of Galilei's relativity is at variance with the generally *nonhamiltonian* character of physical systems of our reality,

and so on.

An infinite family of isotopic generalizations of the Galilei symmetry, under the name of *Santilli's isogalilean symmetries* $\hat{G}(3.1)$ has been reviewed in the preceding section. In particular, we have indicated that:

A) The symmetries $\hat{G}(3.1)$ characterize *closed non-selfadjoint systems* (II.1).These are systems (such as Jupiter) which verify the conventional, total, Galilean conservation laws when isolated, while admitting the additional class of nonlinear, nonlocal and nonhamiltonian internal forces.

B) The Galilei-isotopic symmetries possess the structure

$$\hat{G}(3.1) = [\hat{O}_{\theta}(3) \otimes \hat{T}_{r}{}^{\circ}(3)] \times [\hat{T}_{V}{}^{\circ}(3) \times T_{t}{}^{\circ}(1)] , \qquad (V.52)$$

which results to be locally isomorphic to the conventional symmetry G(3.1) under the positive-definiteness of the isounits, by admitting the latter as a particular case. In this sense, $\hat{G}(3.1)$ provides an infinite family of *isotopic coverings* of G(3.1)

.

C) All symmetries $\hat{G}(3.1)$ can be explicitly constructed via the Lie-Santilli theory, that is, via the use of the same parameters and generators (conserved quantities) of the conventional symmetry, and the most general possible, axiom-preserving realization of Lie algebras and Lie groups. In this way, an infinite number of symmetries $\hat{G}(3.1)$ can be constructed for each given Hamiltonian H = T + V (i.e., for each given potential-selfadjoint forces), as characterized by an infinite number of posssible interior physical media.

By using the above discoveries, Santilli finally constructed his generalizations of the conventional Galilei's relativity for the most general known interior dynamical systems, hereinafter referred to as *Santilli's general isogalilean relativities* (Theorem V.5.1), with the *special isogalilean relativities* holding for the case of linear and local isotransformation (Corollary V.5.1A).The relativities were first submitted as a particular case of the more general Lie-admissible relativities in the memoir (Santilli (1978a)). The new relativities were then submitted in the enclosed monograph Santilli (1982a) for the most general possible local case. Finally, the extension to the most general possible nonlinear, nonlocal and nonhamiltonian systems was achieved in Santilli (1988a) and then presented in details in the recent monograph (Santilli (1991d)).

DEFINITION V.6.1: The "general, nonlinear and nonlocal, Santilli's isogalilean relativities" are a form-invariant description of physical systems characterized by the infinite family of isogalilean symmetries $\hat{G}(3.1)$ on

$\hat{\Re}_t \times T^*\hat{E}(r,\hat{G},\hat{\Re})$, $\hat{\Re} = \Re \, \hat{I}_2$, $\hat{I}_2 = \text{diag.}\ (\hat{G}^{-1}, \hat{G}^{-1})$,, $\hat{\Re}_t = \Re \, \hat{I}_t$, $\hat{I}_t > 0$, $\hat{I}_2 > 0$, *with corresponding, infinite family of general isotransformations (V.38).*

Note that the conventional Galilei transformations are unique, while Santilli's isogalilean transformations admit an infinite number of different forms. Also, within the context of the Galilei's relativity, one first selects the symmetry transformations (), and then restricts the physical reality to be invariant under them. On the contrary, within the context of Santilli's relativities, one first identifies the physical systems as they occur in Nature, and then constructs the corresponding isosymmetries.

Note also that the above definition implies the restriction $\hat{G}(3.1) \approx G(3.1)$. This is due to the fact that, if such restriction is lifted (i.e., if the isounits are not necessarily positive-definite), isosymmetries $\hat{G}(3.1)$ still formally exist, but they do not qualify for the characterization of covering relativities. See in this respect the classification of all possible compact and noncompact isotopes $\hat{O}(3)$ of $O(3)$ of Santilli (1985b).

Finally, the NOBEL COMMITTEE should note that Santilli's isotopic formulations have been constructed in such a way to coincide with the original formulations at the abstract, realization-free level. We should therefore expect that the Galilei-isotopic relativities too coincide, by construction, with the conventional relativity at the coordinate-free level.

In the following we briefly illustrate the generalized structure of Santilli's relativities. Consider the historical *Galilei's boosts*

$$r'_i = r_i + t^\circ v^\circ_i, \qquad p'_i = p_i + m v^\circ_i, \qquad \text{(V.53)}$$

which, as well known, characterize a particle with constant speed, under the (often tacit) assumption that motion occurs in vacuum.

Suppose now that the particle considered is extended and penetrates within a physical medium at a given instant of time t. Then, Galilei's transfortmations are evidently inapplicable, e.g., because of their linearity, locality and Hamiltonian character, while the particle experiences a drag force that is nonlinear, nonlocal and nonhamiltonian.

Santilli's generalized tranformations

$$r'_i = r_i + t^\circ v^\circ_i \tilde{B}_i^{-2}(t, r, p,...), \qquad p'_i = p_i + m v^\circ_i \tilde{B}_i^{-2}(t, r, p,...), \qquad \text{(V.54)}$$

are then applicable to represent the *deviations* from the original uniform motion. In particular, Eq.s (V.54) can represent a (monotonic) increase or decrease of speed depending on the sign of the v°-parameter (since the $\tilde{B}^{-2}$-terms are always positive definite). In the former case we have the usual drag force caused by motion within the physical medium. In the latter case we have instead an extended particle penetrating a medium in such highly dynamical conditions to cause an increase of its speed.

The important point is that, in the transition from the linear, local and Hamiltonian transformations (V.34) to their nonlinear, nonlocal and nonhamiltonian generalizations (V.38) Santilli's relativities preserve the underlying Galilean axioms for the uniform motion, as ensured by the local isomorphisms $\hat{G}(3.1) \approx G(3.1)$.

To elaborate better these aspects, note that law (V.53) of the uniform motion in vacuum is geometrically expressed by the Lie group

$$T(v^\circ)\, r_i = r_i - t^\circ v^\circ_i, \qquad T(v^\circ)\, p_i = p_i - m v^\circ_i, \tag{V.55a}$$

$$T(v^\circ) = e_{|\xi}^{\, v^\circ_j\, \omega^{\mu\nu}\, (\partial_\nu G_j)\, (\partial_\mu)}, \tag{V.55b}$$

namely, *the structure of the the Galilean law of uniform motion is provided by the right modular (associative) action of the finite Galilei boosts $T(v^\circ)$ on the coordinates and momenta.*

But isotopic laws (V.54) are geometrically expressed by the Lie–Santilli group

$$\hat{T}(v^\circ) * r_i = r_i - t^\circ v^\circ_i\, \hat{B}_i^{-2}, \qquad \hat{T}(v^\circ) * p_i = p_i - m v^\circ_i\, B_i^{-2}, \tag{V.56a}$$

$$\hat{T}(v^\circ) = \{ [\, e_{|\xi}^{\, v^\circ_j\, \omega^{\mu\sigma}\, \hat{1}_2^{\sigma\nu}\, (\partial_\nu G_j)\, (\partial_\mu)}]\, \hat{1}_2 \}. \tag{V.56b}$$

Thus, *the structure of the variable motion within a physical medium is characterized by the modular-isotopic (associative-isotopic) action $\hat{T}(v^\circ)*r$ of Santilli's isoboosts on coordinates and momenta.*

But the conventional action $T(v^\circ)r$ coincides with the modular-isotopic action $\hat{T}(v^\circ)*r$ at the abstract, realization-free level by construction. This shows that *the abstract axioms underlying the Galilean uniform motion are preserved by Santilli's covering relativities*.

Note the unity of physical and mathematical thought in Santilli's covering relativities. In fact we can introduce only one abstract law of rectilinear motion, say, $\mathbf{T(v^o)r}$, with infinitely many different, but locally isomorphic, realizations $\hat{T}(v^\circ)*r$ representing the infinitely many nonuniform motions within different physical media, and only one canonical realization $T(v^\circ)r$, representing uniform motion in vacuum.

The extension of the above results to other physical laws is straightforward and not reviewed here for brevity.

We reached in this way another most important physical result achieved by Santilli (1989a), (1991d), which can be expressed as follows:

THEOREM V.6.1: All infinitely possible Santilli's isogalilean relativities on $\hat{\mathfrak{R}}_t \times T^*\hat{E}(r,\hat{G},\hat{\mathfrak{R}})$ coincide with the conventional Galilei relativity on $\mathfrak{R}_t \times T^*E(r,\delta,\mathfrak{R})$ at the abstract, realization-free level, that is, not only all infinitely possible isosymmetries $\hat{G}(3.1)$ coincide with the conventional symmetry $G(3.1)$, but also all the infinite class of isogalilean transformations coincide with the conventional Galilei transformations, and the same holds for the related physical laws .

The NOBEL COMMITTEE should keep in mind that the above unity will appear in its full light only at the gravitational level of Part III where we shall see that the axiomatic unity between Galilei's boosts and their isotopic extensions is a particular case of a much broader geometric unity within the context of Santilli's Riemannian–

isotopic geometry.

Systems:		Systems:		Systems:
closed.		closed.		closed.
Forces:		Forces:		Forces:
local, SA.		local NSA.		nonloc. NSA.
Space:		Space:		Space:
$\Re_t \times T^*E(r,\delta,\Re)$		$\Re_t \times T^*\hat{E}(r,g,\hat{\Re})$		$\hat{\Re}_t \times T^*\hat{E}_2(r,G,\hat{\Re})$
$\delta = 1.$		$\hat{\delta} = const>0.$		$\hat{G} = \hat{G}(r,p,..) > 0.$
Frames:	$\subset$	Frames:	$\subset$	Frames:
Inertial.		Inertial.		Noninertial.
Methods:		Methods:		Methods:
Lie's theory.		Lie-Santilli theory.		Lie-Santilli theory.
Relativity:		Relativity:		Relativity:
Convent. Galilei's relativity.		Restricted isogalil, relativity.		General isogalil. relativity.

FIGURE V.1: A classification of physical systems, with their carrier spaces, observer's frames (assumed at rest with respect to the center-of-mass of the system), and related methodology. The first column depicts the conventional linear-local-inertial-Hamiltonian setting; the second column depicts the first nontrivial isotopic generalization, that of linear-local-inertial but of isotopic type; and the third column depicts the most general possible nonlinear, nonlocal, nonhamiltonian *and noninertial* setting. The first two columns have equivalent inertial characterizations, because they are both defined on inertial frames. Also, the first column treats rigid bodies, while the second represents deformable bodies. The third column represents the most general possible conditions of extended-deformable bodies in regard to both acting forces and observer frames.

In particular, in Part II we shall see that *the transition from the Galilean, exterior, uniform motion in vacuum to Santilli's extensions within physical media does not imply a change in geodesic motion. but only the transition from geodesics within conventional spaces, to geodesics within isospaces.*

Despite such ultimate mathematical and physical unity, the physical differences between the Galilei's and Santilli's relativities are nontrivial. To illustrate of this point, let us recall that *Galilei's relativity establishes the equivalence of all inertial frames* , as well known.

But inertial frames are a philosophical abstractions inasmuch as they do not exist in our earthly environment, nor are they attainable in our Solar or Galactic systems.

Santilli's relativities were therefore conceived for the actual, physical frames of the experimenter, that is, for the actual noninertial frames of our Earthly laboratories. In fact, *Santilli's relativities establish equivalence subclasses of noninertial frames, those with respect to the center-of-mass frame of the interior medium* , each class being characterized by each relativity (i.e., by each physical medium). The understanding is that different systems imply different subclasses of isotopically equivalent frames.

But Santilli's relativities are coverings of the conventional one. This means that the conventional inertial aspects are not lost, but fully included and actually generalized in the broader isotopic setting. This concept can be made more clear by nothing that, when the isometrics $\hat{\delta}$ are constants, Santilli's isogalilean transform,ations are given by the linear and local form (V.49). In this case the covering relativity is called *restricted isogalilean relativities*.

The generalized nature of the relativities is soon illustrated by the fact that the isotopic relativities characterize *deformable bodies*, while the conventional relativity characterizes *rigid bodies* (see the examples of Sect.s V.7 andV.8).

The NOBEL COMMITTEE should keep in mind that the network of classical nonrelativistic discoveries outlined until now were merely preparatory for *Santilli's (1989) isotopic generalization of Galilei's relativity in quantum mechanics* (Part IV), which he proved in the most rigorous possible way (via an isotopic lifting of Wigner's theorem on unitary symmetries and Mckay imprimitivity theorem), and which does establish the mathematical consistency of the *"hadronic generalization of quantum mechanics"* (see Fig. IV.1). The discoveries were also conceived as a basis for the *isotopic generalizations of Einstein's special relativity* for interior dynamical systems (Part II). However, the true geometric understanding and ultimate unity of mathematical and physical thought can be seen only at the level of the *isotopic generalizations of Einstein's gravitation* on the Santilli's novel isoriemannian geometry for the most general nonlinear, nonlocal and nonlagrangian interior gravitational conditions known to mankind today (Part III).

V.7: EXAMPLES OF COMPOSITE ISOGALILEAN SYSTEMS.

All the classical nonrelativistic discoveries reviewed so far have been made by Santilli alone. The sole contribution in the field we are aware of is the paper by Jannussis, Mijatovic and Veljanoski (1991) on *examples of the classical Santilli's isogalilean relativities* consisting of two-body and three-body, closed nonhamiltonian bound systems (II.1).

The two-body case is the simplest possible one and is geometrically expressed by the so-called scalar isotopy of the Lie tensor

$$\omega^{\mu\nu} \Rightarrow \Omega^{\mu\nu} = b^2\, \omega^{\mu\nu}, \quad b = \text{constant} > 0. \qquad \text{(V.57)}$$

Despite this simplicity, the system is analytically nontrivial. In fact, contrary to the conventional Kepler's system where both circular and elliptic orbits are possible, *a two-body closed system with contact internal forces admits only the circle as a stable orbit* . Moreover, *scalar isotopy (V.57) implies a renormalization of the massses of the two bodies already at this classical level* (Santilli (1988a), (1991d), with truly intriguing implications at the operator level (see Part III).[15]

[15] As an illustration of advances reviewed in Part IV, recall that *a structure model of the π° pasrticle as a bound state of an ordinary electron and a positron is inconsistent within the context of quantum mechabics* . This is so on numeorus counts, such as the inability to represent the total energy of 140 Mev as a bound state of two very light constituents with 0.5 MeV rest energy, the inability to represent the relatively long mean life, and others. In his very first proposal of the *"hadronic generalization of quantum mechanics"* (1978d, Sect. V), Santilli proved that *a structure model of the π° particle as a*

Three-body closed nonhamiltonian systems are less trivial then the two-body ones and much richer in structure, possible configurations and implications. The simplest stable configurations is that for the three bodies on a straight line, the system rotating rigidly as a whole on its center-of-mass. The next stable configuration is *Lagrange's historical triangle* with contact internal forces. These configurations occur without subsidiary constraints and they all imply circular orbits for the individual constituents.

The most general configuration was studied by Jannussis et al. (loc. cit.) and occurs for nontrivial subsidiary constrtaints (II.1b). This configuration is analytically intriguing because it permits the constituents to have individually unstable-nonconservative orbits, while all total quantities are conserved (this possibility is precluded for the two-body case).

Santilli (1978a), (1982a) (1988a), (1991d) also studied examples of his classical relativities and made another discovery. In essence, the *nucleus* for the conventional Kepler's systems must have a mass much bigger than that of the peripheral constituents, as verified by our Solar system. However, when the constituents are under mutual contact interactions, the particle at the center, called by Santilli *isonucleus*, can instead have an arbitrary mass, including a mass much smaller than that of the peripheral constituents.

These features have predictable far reaching, quantum mechanical implications, as we shall indicate in Part IV. In fact, they imply profound departures from conventional quantum mechanical bound states, e.g., of atomic type, into fundamentally novel composite systems with short range, nonlocal and nonhamiltonian internal effects. These systems possess basically novel features, such as "isocenters" with mass much smaller than those of the peripheral particles, spontaneous renormalization of the masses and other characteristics. In turn, these novel features permit fundamentally new approaches to all conditions in which short range nonlocal effects are expected, such as: the structure of hadrons; the structure of the Cooper's pairs in superconductivity, the origin of the Bose-Einstein correlations; etc.

In conclusion, the studies by Jannussis et al. (loc. cit.) via specific cases of generalized two-body and three-body systems have confirmed the existence and consistency of Santilli's isogalilean relativities. Moreover, the emerging generalized bound states possess truly novel features at the classical level, with far reaching implications at the quantum mechanical level.

"compressed positronium"; i.e., as a bound state of one electron and one positron at distances maller than their wavelength, is fully consistent within the context of the covering hadronic mechanics, and in fact capable of representing with one single equation of structure ALL the characteristics of the π^0, such as: rest energy, mean life, charge radius, charge, spin, space and charge parity, and the (null) electric and magnetic moments. The consisency is given precisely by the short range internal nonlocal effects caused by the total mutual penetration of the electron and positron. In fact, when such internal force is represented by an isotopy of quantum mechanics (Fig. IV.1), it implies the renormalization of the masses already existing at the classical level (Santilli (loc. cit), Jannussis et al. (loc. cit.)) which turns inconsistent quantum mechanical equations into consistent hadronic equations. We reach in this way what appears to be one of most important aspects of Santilli's research: the possibility that the imposition of established relativities for the characterization of the hadronic constituents PREVENTS their identification with physical particles freely emitted in their spontaneous decays. On the contrary, if generalized relativities and discipllies are build for the interior structural problem of hadrons following the historical legancy on its lonlocality, the identification of the hadronic constituents with ordinary particles appears within technical reach. As clearly stated in his contributions, see (1978a, c, d), (1981a), and others, Santilli constructed his scientific edifice for one ultimate purpose: achieve, in due time, the identification of the hadronic constituents with physical particles freely produced in the spontaneous decays, as established for the preceding nuclear and atomic structures.

V.8: EXAMPLES OF ISOGALILEAN PARTICLES

Another most effective way of appraising the physical relevance of any generalized relativity is by identifying its impact in the characterization of the notion of particle. It is at this point where Santilli's relativities acquire their full light, inasmuch as they imply a profound revision of the very notion of particle, into a generalized notion he called *isoparticle* (Santilli (1988a), (1991b)).

As well known, the conventional notion of *classical nonrelativistic particle* is a representation of the Galilei group $G(3.1) = [O_\theta(3) \otimes T_r{\circ}(3)) \times (T_v{\circ}(3) \times T_t{\circ}(1)]$ on $\Re_t \times T^*E(r,\delta,\Re)$ and, as such, it is characterized by conventional units, the scalar unit 1 for the time field $\Re_t$, and the six-dimensional unit matrix I for the cotangent bundle.

By recalling that the Galilei symmetry holds only for interactions which are of local and potential type, the notion essentially characterizes the historical Galilei's concept of *"massive point"* moving in vacuum under action-at-a-distance interactions (Fig. I.1).

In particular, the intrinsic characteristics of the particle (mass, spin, charge, etc.) are immutable, classically and quantum mechanically, because points are immutable geometrical objects.

This perennial character of the intrinsic characteristics of particles is lost under Santilli's covering relativities. In fact, as we shall illustrate shortly, the new relativities represent the actual shape of the particle, as well as all its possible deformations, called *mutations* in Santilli (1978c).

The NOBEL COMMITTEE can now see better why the Galilei symmetry G(3.1) is unique, while there are infinitely many isotopic coverings Ĝ(3.1). In fact, the latter essentially represent the physical reality according to which one extended particle can assume an infinite variety of different configurations, evidently depending on the local physical conditions.

Consider a spherical particle, say, in vacuum. When the same particle penetrates within an inhomogeneous medium, it can assume an infinite number of different shapes depending on the local density, pressure, etc. All these infinitely different configurations, including the simpler isogalilean one, are represented with the following

DEFINITION III.7.1 (loc. cit): A nonrelativistic isoparticle is a representation of one of the infinitely possible Santilli's isogalilean symmetries Ĝ(3.1) on isospace $\hat{\Re}_t \times T^*\hat{E}(r,G,\hat{\Re})$, Theorem V.5.1,

$$\hat{G}(3.1): \; a' = \hat{g}(w) * a = \hat{g}(w)\,\hat{T}_2 a = \{[e^{w_k\, \omega^{\mu\sigma}\, \hat{I}_2^{\sigma\nu}\, (\partial_\nu X_k)\, (\partial_\mu)}\,]_{|\xi}\, \hat{I}_2\} * a, \qquad (V.58))$$

Equivalently, Santilli's nonrelativistic isoparticle can be defined as the generalization of the conventional particle induced by the isotopic liftings of the unit

$$I_t = 1 \in \Re_t \Rightarrow \hat{I}_t \in \hat{\Re}_t, \qquad I \in T^*E(r,\delta,\Re) \Rightarrow \hat{I}_2 = \mathrm{diag}\,(\hat{G}^{-1}, \hat{G}^{-1}) \in T^*\hat{E}(r,\hat{G},\hat{\Re}). \qquad (V.59b)$$

$$\delta = \text{diag.}\,(1,1,1) \Rightarrow \hat{G} = \text{diag.}\,(B_1^2, B_2^2, B_3^2) > 0. \qquad (V.59c)$$

We shall now present a few classical nonrelativistic examples of isoparticles. A technicalk knowledge of the preceding analysis is assumed.

"FREE" ISOPARTICLE. In this case, N = 1, all selfadjoint and nonselfadjoint forces are null, and the isometrics $\hat{\delta} = \hat{G}$ must evidently be constants, and we have (Santilli (1988a), (1991d))

$$\hat{\delta} = \hat{G} = \text{diag.}\,(b_1^2, b_2^2, b_3^2), \qquad b_i = \text{ constants } > 0. \qquad (V.60)$$

Hamilton–Santilli equations (IV.55) then describe, as expected, the free particle

$$\dot{r}_i = b_i^{-2}\, \partial H / \partial p_i = p_i/m = v_i, \qquad (V.61a)$$

$$\dot{p}_i = -b_i^{-2}\, \partial H / \partial r_i = 0. \qquad (V.61b)$$

namely, the isoequations of motion are identical to those of the conventional Galilei's relativity.

Despite that, the use of the Santilli's relativities is not trivial, because it permits the direct and immediate representation of:

1) the extended character of the particle;

2) the actual shape of the particle considered; and

3) an infinite class of possible deformations of the shape itself;

all the above already at this primitive, classical, nonrelativistic level.

By comparison, if one insists in preserving the conventional Galilei relativity:

1') the extended character of the particle can be represented only after the rather complex process of *second quantization;*

2') the second quantization does not represent the actual shape of a particle, say, an oblate spheroidal ellipsoid, but provides only the remnants of the actual shape; and, last but not least,

3') possible deformations of extended particles are strictly excluded, as well known, for numerous reasons, e.g., because they imply the breaking of the conventional rotational symmetry.

As an illustration, there are reasons to suspect that the charge distribution of the proton is not perfectly spherical, but characterized instead by a deformation of the sphere of the oblate type computed by Nishioka and Santilli (1992)

$$\delta = \text{diag.}\,(1,1,1) \quad \Rightarrow \quad g = G = (\text{diag.}\,(1, 1, 0.6), \qquad (V.62)$$

(where the third axis is assumed to be that of the intrinsic angular momentum), which permits an interpretation of the anomalous magnetic moment of the particle.

Oblate spheroidal ellipsoid (V.62) can be directly and exactly represented by Santilli's relativities, already at the classical nonrelativistic level of this treatment via the value of the isometric

$$b_1^2 = 1, \quad b_2^2 = 1, \quad b_3^2 = 0.6. \tag{V.63}$$

It is evident that such an actual, direct and immediate representation of the shape of the proton is impossible within the context of the conventional Galilei's relativity, classically and quantum mechanically.

We shall indicate in subsequent strudies that, in the transition to the operator version of the theory, the representational capabilities are enhanced because of the appearance of additional degrees of freedom besides that offered by the isounit of the enveloping algebra.

The above case illustrates the simplest conceivable (and perhaps most fundamental) mutation of a Galilean particle. In fact, the original particle has the perfectly spherical shape expressed by the underlying metric δ = diag. (1,1,1), while Santilli's isoparticle can acquire any one of the infinitely many ellipsoidical deformations of the original sphere expressed by the isometrics $\hat{\delta}$.

The case also illustrates a first use of the isoeuclidean spaces $\hat{E}(r,\hat{\delta},\hat{\mathfrak{R}})$, the geometric one for the characterization of shape only without any force.

In particular, it should be indicated that this is a sort of limiting case because *the notion of isoparticle generally requires nontrivial interactions*. With the terms "free" isoparticles we therefore refer to a conventionally free particles which however have nongalilean characteristics.

In conclusion, the mutation of shape under consideration at this primitive Newtonian level is intrinsically contained in the lifting of the underlying metric $\delta \Rightarrow \hat{\delta}$, with consequential liftings of fields, metric spaces, space-time symmetries, etc. Equivalently, it can be geometrically expressed by the symplectic isotopy $\omega_2 \Rightarrow \hat{\Omega}^\circ_2 = \omega_2 \times T_2$ and it is algebraically/group theoretically characterized by the Lie-Santilli isotopy $\omega^{\mu\nu} \Rightarrow \Omega^{\circ\mu\nu} = \omega^{\mu\sigma}\, \hat{1}_2{}^{\sigma\nu}$, $\hat{1}_2 = T_2^{-1} > 0$.

ISOPARTICLE UNDER EXTERNAL POTENTIAL INTERACTIONS. The simplest possible generalization of the preceding case is the extended-deformable particle under conventional, external, *potential* interactions. In this case, N = 1, $V \neq 0$, $F^{SA} \neq 0$, all nonselfadjoint forces are null, and the b-quantities can still be assumed to be independent of the local coordinates in first approximation, although they can be dependent on the local strength of the potential and other local quantities. The equations of motion are then given by (loc. cit.)

$$\dot{r}_i = b'^{-2}_i\, \partial H / \partial p_i = p_i / m = v_i, \tag{V.64a}$$

$$\dot{p}_i = - b'^{-2}_i\, \partial H / \partial r_i = -(\partial V/\partial r)\, r_i/r, \tag{V.64b}$$

where one should assume that the deformation of shape $\hat{\delta} \Rightarrow \hat{\delta}'$ is volume preserving

$$\hat{\delta} \Rightarrow \hat{\delta}, \quad \det \hat{\delta} \equiv \det. \hat{\delta}'. \tag{V.65}$$

Equations of motion (7.7) also coincide with the conventional Galilean equations when

$$r = (r_i b_i^2 r_i)^{\frac{1}{2}} = \overline{r} = (\hat{r}_i \delta^{ij} \hat{r}_j)^{\frac{1}{2}}, \qquad \hat{r}_i = r_i b_i, \tag{V.66}$$

namely, when the coordinates r_i in the geometrical space $\hat{E}(r,\hat{\delta},\hat{\Re})$ coincide with the distance $\hat{r}_i = r_i b_i$ in our physical space $E(\overline{r},\delta,\Re)$.

Again, the transition from the Galilei to Santilli's covering relativities is not trivial. In fact, it first allows the direct representation of the actual shape of the particle, as in the free case. In addition, *the new relativities can represent the deformations of the original shape caused by the external force .*

In fact, starting from an extended particle with the shape represented by the isometric $\hat{\delta}$, we have to expect from simple mechanical considerations that the application of the external potential force F^{SA} will cause a deformation of the shape into the isometric $\hat{\delta}'$.

Needless to say, one may argue that such deformation could be small for given conditions. The point is that perfectly rigid bodies do not exist in the physical reality. The *amount* of deformations for given conditions is evidently an open scientific debate, but its *existence* is out of any scientific doubt.[16]

In conclusion, the rotational and Galilei symmetries characterize a *theory of rigid bodies ,* as well known. The above examples confirm that Santilli's isorotational and isogalilean symmetries characterize instead a *theory of deformable bodies* (Figure IV.1) without violating the abstract O(3) and G(3.1] symmetries, but by realizing them instead in their most general possible form.

ISOPARTICLE UNDER EXTERNAL NONHAMILTONIAN INTERACTIONS: The next examples are those of an extended-deformable isoparticle under, this time, *nonpotential* external fields caused by motion within a physical medium. *Note that this class of interactions is strictly excluded by the conventional Galilei relativity, but it is rather natural for Santilli's covering relativities.*

In this case, N = 1, the selfadjoint interactions can be assumed to be null (V = 0) for simplicity, but we have nontrivial nonselfadjoint interactions represented via Lie-Santilli formulations.

A first simple case in one space-dimension is given by a particle moving within a resistive medium under a quadratic damping force (loc. cit.)

$$m\ddot{r} + \gamma \dot{r}^2 = 0, \tag{V.67}$$

with the Birkhoffian representation (Santilli (1982a))

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[16] In particle physics, this is precisely the case of protons and neutrons (see Part IV). Their extended charge distribution (of about 1F in radius) is in fact expected to experience deformations under sufficiently intense external forces or collisions, resulting in a consequential alteration of their intrinsic magnetic monent, as indicated at the end of Sect. V.3. These novel conditions are predicted and directly represented by Santilli's isogalilean relativities, classically (1989a) and quantum mechanicaly (1991f). Again, the amount of deformation of the charge distribution/intrinsic magnetic moment of protons and neutrons under given external forces is debatable, but the idea that these partecles have a perfectly rigid charge distribution and a consequently perennial magnetic moment under whatever conditions exist in the Universe, has no physical value.

$$\hat{R}^\circ = (p\hat{\delta}, 0), \qquad B = p\hat{\delta}p/2m, \quad \hat{\delta} = e^{\gamma r}, \tag{V.68}$$

which provides a first approximation of systems such as a satellite penetrating Jupiter's atmosphere or, along similar conceptual grounds, a proton moving within the core of a star.

The above case illustrates a second use of the isospaces, namely, that for the characterization of nonpotential forces of the interior dynamics. In fact, the lifting $E(r,\delta,\mathfrak{R}) \Rightarrow \hat{E}(r,\hat{\delta},\hat{\mathfrak{R}})$, $\delta = I \Rightarrow \hat{\delta} = \exp \gamma r > 0$, essentially represents the local, but nonlinear and nonselfadjoint resistive force $F^{NSA} = -\gamma\dot{r}^2$.

Note that the resistive forces imply an evident breaking of the conventional Galilei symmetry, while Santilli's techniques permit the restoration of the exact symmetries at the broader isotopic level. Note also that the case considered is a direct illustration of physical validity of Santilli's isoboosts.

The above example can be readily extended to three dimensions, e.g., for motion along the third axis

$$\hat{R}^\circ = (p_i \hat{\delta}_{ij}, 0_j), \quad \hat{\delta} = \text{diag.}\,(b_1^2, b_2^2, b_3^2\, e^{\gamma r}), \qquad B = p_i\, g_{ij}\, p_j\, /\, 2m, \tag{V.69}$$

with a deformation of shape, this time, due to contact interactions[17].

Along similar lines, one can have an isoparticle subject to selfadjoint ($V \neq 0$) and nonselfadjoint ($\hat{\delta} = \hat{\delta}(r)$) interactions. In this case, a simple example is given by the quadratically damped oscillator

$$\ddot{r} + r + \tfrac{1}{2}\gamma\dot{r}^2 + \tfrac{1}{2}\gamma r^2 = 0, \qquad m = k = 1, \tag{V.70}$$

with Birkhoffian representation (loc. cit.)

$$\hat{R}^\circ = (pe^{\gamma r}, 0), \qquad B = \tfrac{1}{2}pe^{\gamma r}p + \tfrac{1}{2}re^{\gamma r}r. \tag{V.71}$$

Similarly, an illustration of a nonlocal and nonselfadjoint internal forces is provided by the isometric

$$\hat{\delta} = \exp\,[\,\gamma r + k\int_\sigma d\sigma\, \mathcal{F}(t, r, p, ..)], \tag{V.72}$$

where the first term characterizes the damping of the center-of-mass trajectory due to the quadratic forces in the velocity, and the second term provides the nonlocal correction due to the shape σ of the particle considered.

Numerous additional examples can be worked out in any desired combination of selfadjoint and nonselfadjoint forces, the latter being local-differential or nonlocal-integral, as desired. A number of explicit examples of nonlocal interactions can be found in, e.g., Hostandter et al (1971) or Fujimura et al (1971) and in the references quoted herein.

In the transition to elementary particles, we shall encouter nonlocal interactions directly expressing the overlapping of the wavepackets ψ and ϕ of two particles

[17] This case is particularly relevant in elementary particle physics to represent the deformation of the charge distribution of hadrons under sufficiently intense external fields and/or collisions, as we shall see in Part IV.

represented by *Animalu's (1991) isounit*

$$\hat{1} = I \exp tC\int dr' \psi(r') \phi(r') \qquad (V.73)$$

where one can see that, for nonoverlapping waves, the isounit $\hat{1}$ recovers the trivial unit I of quantum mechanics. Isounit $\hat{1}$ illustrates that the systems outlined in this section are intended to be a mere classical image of the systems expected in Santilli's nonlocal realization of the strong interactions (Part III).

In all the above cases, Santilli's covering relativities permit the explicit construction of the generalized invariance $\hat{G}(3.1)$ via the computation of the isounit directly from the equations of motion, and its use in the exponential characterizing $\hat{G}(3.1)$ as reviewed in Sect. V.5, all in a way which reconstructs the exact Galilei symmetry in isospaces $\hat{\mathfrak{R}}_t \times T^*\hat{E}(r,\hat{G},\hat{\mathfrak{R}})$, while the conventional symmetry is manifestly broken in $R_t \times T^*E(r,\delta,\mathfrak{R})$.

V. CONCLUDING REMARKS.

The most important conlusion of this Part I which is here submitted for consideration by the NOBEL COMMITTEE is that

Santilli's classical isogalilean relativities are fully consistent on mathematical grounds, and amply verified in the interior physical reality of our environment.

Stated differently, and contrary to the relativistic case of Part II which dooes need experimental verificastion, Santillis classical isogalilean relativities need no further experimental verification.

The grounds for the above statements are the following:

The existence theorems of Santilli's isotopic formulations[18] permit the construction of the isogalilean symmetry for all closed, linear and nonlinear, local and nonlocal and Hamiltonian or nonhamiltonian systems of the exterior and interior physical reality. Santilli's isogalilean relativities are then "directly universal" in classical Newtonian mechanics, that is, they hold for all possible interior and exterior[19] systems (universality), directly in the frame of the experimenter (direct universality).

Also, we would like to take the liberty to point out to the NOBEL COMMITTEE that the possible granting of the NOBEL PRIZE to Santilli would be different than any of the recent NOBEL PRIZES, essentially consisting of awards for clearly outstanding but *past* accomplishments without significant implications for the *future* of physics. We

[18] See the *Theorem of Direct Universality of Birkhoffian Mechanics* for all local Newtonian system of the enclosed monograph Santilli (1982a), p. 54, and the corresponding *Theorem of Direct Universality of the Birkhoffian-isotopic Mechanic* for all nonlocal systems of Santilli (1991c).

[19] The conventional Galilei's relativity is a simple particular case of Santilli's covering relativities.

therefore closed this Nomination with the remarks that

> ***The possible granting of a NOBEL PRIZE IN PHYSICS to Professor RUGGERO MARIA SANTILLI would have a profound influence in the FUTURE of physics, because it would provide the necessary authority to terminate a rather widespread scientific inertia against truly fundamental advances, it would launch a fundamentally new generation of relativities with expected implications in all branches of physics and, in the words of Prof. Leipholtz of the University of Waterloo, Canada, it would confirm the birth of a "new epoc" in physics , fundamentally beyond Galilei, Newton and Einstein.***

APPENDIX A:

SANTILLI'S CLASSICAL LIE-ADMISSIBLE FORMULATIONS

The Lie-isotopic formulations outlined in the main text were achieved by Santilli as a particular case of the still more general formulations of Lie-admissible type, as indicated in the very title of a main reference of this Nomination, the memoir Santilli (1978a), see the title of the memoir in Exhibit D.

No in depth appraisal of Santilli's research can be achieved without some knowledge of at least the analytic, algebraic and geometric structures underlying these broader formulations. One can then see in this way the birth of a second generation of covering theories for yet more general physical conditions..

The study of the classical Lie-admissible formulations was initiated in papers (Santilli (1967), (1968), and (1969); then developed in the memoir (1978a); and finalized in monographs (Santilli (1978c) and (1981a) (the operator Lie-admissible formulations originating from the second memoir off the same year, Santilli (1978c) will be outlined in Part III). As it was the case for the Lie-isotopic formulations, Santilli is the originator and remains the sole author for fundamental reseaerch on *classical* Lie-admissible formulations. However, unlike the Lie-Santilli algebras which have seen no contribution by mathematicians to this day, Lie-admissible algebras have been the subject of considerable mathematical research identified in the bibliography by Baltzer et al. (1984). Also, numerous physicists conducted independent research in the Lie-admissible formulations, although mostly in the operator formulation of the theory and, for this reason, thety will be identified in Part III.

A.1: ANALYTIC PROFILE.

As recalled in Sect. III, the central physical objective of the classical Lie-admissible formulation is the representation of the most general known nonlinear, nonlopcal and nonhamiltonian systems (I.3) in open-nonconservative conditions.

This physical arena can be best visualized by considering *one* extended-deformable object moving within a physical medium which is considered as *external* , such as a satellite during re-entry in Earth's atmosphere.

As a result, while conventional Lie and Lie-isotopic formulations use symmetries for the characterization of conservation laws, *the more general Lie-admissible formulations use symmetries for the characterization of time-rate-of-variations of physical quantities* , of course, in a covering way, that is, in such a way to recover the conventional Lie-isotopic or the simpler Lie formulation whenever the physical quantities considered are conserved.

As recalled in Sect. III, the analytic foundations of the approach are given by the equations originally conceived by Hamilton (1834) for interior dynamical systems, those with external terms

$$\dot{r}_{ka} = \frac{\partial Ht, r, p)}{\partial p_{ka}} = p_{ka}/m_a, \tag{A.1a}$$

$$\dot{p}_{ka} = - \frac{\partial H(t, r, p)}{\partial r_{ka}} + F_{ka}, \tag{A.1b}$$

$$H = p_{ka}\, p_{ka} / 2m_a + V(r), \tag{A.1c}$$

$$F_{ka} = F^{NSA}{}_{ka}(t, r, p, \dot{p},.) + \int_\sigma d\sigma \mathfrak{F}^{NSA}{}_{ka} t, r, p, \dot{p},..), \tag{A.1d}$$

$$k = 1, 2, 3\ (= x, y, z),\ a = 1, 2, ..., N.$$

As one can see, the "direct universality" of the equations for the representation of all possible systems (I.3) in the coordinates of the exterimenter is immediate, because the Hamiltonian H = T + V represents all local and potential forces, while the external terms F_{ka} represents all remaining nonlinear, nonlocal and nonhamiltonian forces. However, in so doing, the Hamiltonian H is necessarily nonconserved and, for this reason, the equations characterize an open nonconservative system.

But, as recalled in Sect. III, the brackets of the time evolution characterized by Eq,.s (A.1)

$$AxB = [A,B] + \frac{\partial A}{\partial p_{pa}} F_{ka}. \tag{A.2}$$

(where [A,B] are the conventional Poisson brackets) violate the left scalar and distributive laws (Santilli (1978a)). For this reason they were rewritten by Santilli in the unified form in phase space $T^*E(r,\delta,\Re)$ with the now familar local coordinates a = (r, p),

$$\dot{a}^\mu = S^{\mu\nu}(t, a, \dot{a}, ...) \frac{\partial H(t, a)}{\partial a^\nu} =$$

$$= \begin{pmatrix} \dot{r}_{ka} \\ \dot{p}_{ka} \end{pmatrix} = \begin{pmatrix} \partial H/\partial p_{ka} \\ - \partial H/\partial r_{ka} + s_{kajb}\, \partial H/\partial p_{jb} \end{pmatrix} = \begin{pmatrix} \partial H / \partial p_{ka} \\ - \partial H/\partial r_{ka} + F_{ka}. \end{pmatrix} \tag{A.3}$$

called *Hamilton-admissible equations* , where

$$S^{\mu\nu}(t,a) = \omega^{\mu\nu} + s^{\mu\nu}(t,a), \tag{A.4}$$

$\omega^{\mu\nu}$ is the (totally antisymmetric) canonical Lie tensor (III.2), and $s^{\mu\nu}$ is the totally symmetric tensor

$$s = (s^{\mu\nu}) = \text{diag.}(0, s), \quad s = F/(\partial H/\partial p) \tag{A.5}$$

In the disjoint r– and p–variables, Eq.s (A.2) can be written

$$\dot{r}_{ka} = \frac{\partial H(t, r, p)}{\partial p_{ka}}, \tag{A.6a}$$

$$\dot{p}_{ka} = -\frac{\partial H(t, r, p)}{\partial r_{ka}} + s_{ka\,ia}(t, r, p, \dot{p}, ...)\frac{\partial H(t, r, p)}{\partial p_{ia}}, \tag{A.6b}$$

The brackets of the theory are now given by

$$(A,B) = \frac{\partial A}{\partial a^{\mu}} S^{\mu\nu}(t,a, ...) \frac{\partial B}{\partial a^{\nu}} = [A,B] + \frac{\partial A}{\partial p_{ia}} s_{iaja} \frac{\partial B}{\partial p_{ja}}, \tag{A.7}$$

The consistency of Santilli's reformulation (A.3) is soon established by the fact that brackets (A.8) correctly represent the time-rate-of-variation of the energy

$$\dot{H} = (H,H) = \frac{\partial H}{\partial p_{ka}} F_{ka} = v_{ka}F_{ka}, \tag{A.8}$$

Santilli (1978a) (1981a) discovered that the most general possible nonautonomous Birkhoff's equation in $T^*E(r,\delta,\Re)$

$$\dot{a}^{\mu} = \Omega^{\mu\nu}(t,a)\left[\frac{\partial B(t, a)}{\partial a^{\nu}} + \frac{\partial R_{\mu}(t, a)}{\partial t}\right], \quad \mu, \nu = 1, 2, ..., N, \tag{A.9a}$$

$$\Omega^{\mu\nu} = (|\Omega_{\alpha\beta}|^{-1})^{\mu\nu}, \tag{A.9b}$$

$$\Omega_{\mu\nu} = \frac{\partial R_{\nu}(t, a)}{\partial a^{\mu}} - \frac{\partial R_{\mu}(t,a)}{\partial a^{\nu}}, \tag{A.9c}$$

also do not characterize a consistent algebra, in the sense that the brackets of their time evolution also violate the right scalar and distributive laws.

For this reason, he reformulated Eq.s (A.9) in the identical form

$$\dot{a}^{\mu} = \hat{S}^{\mu\nu}(t, a, ...)\frac{\partial B(t, a)}{\partial a^{\nu}} \equiv \Omega^{\mu\nu}(t, a)\frac{\partial B(t, a)}{\partial a^{\nu}} + \frac{\partial R_{\mu}(t, a)}{\partial t}, \tag{A.10}$$

called *Birkhoff-admissible equations*, where

$$\hat{S}^{\mu\nu} = \Omega^{\mu\nu} + \hat{s}^{\mu\nu}, \tag{A.11}$$

namely, where the antisymmetric part attached to the S-tensor is the most general possible Birkhoff's (rather than the canonical) tensor.

The brackets of reformulation (A.10) are now given by (Santilli (1981a))

$$(A \,\hat{,}\, B) = \frac{\partial A}{\partial a^{\mu}} \hat{S}^{\mu\nu}(t, a, ...) \frac{\partial B}{\partial a^{\nu}} \tag{A.12}$$

and they are now algebraically consistent as well as characterizing a Lie-admissible algebra. Moreover, note that the mapping of brackets (A.7) into brackets (A.12)

$$(A,B) \quad \Rightarrow \quad (A \,\hat{,}\, B) \tag{A.13}$$

constitutes a Lie-admissible isotopy, in same way as the transition from the Poisson brackets [A,B] to the Birkhhoffian brackets $[A \,\hat{,}\, B]$ constitutesd a Lie isotopy (loc. cit.).

Even though Eq.s (A.10) might be considered physically redundant over Eq.s (A.3), (because the latter are already directly universal for all possible systems (I.3)), the generalization is analytically nontrivial. As an example, it allows the derivation of the analytic equations from the most general possible nonautonomous Pfaffian variational principle

$$\delta\hat{A} = \delta\int_{t_1}^{t_2} [R_{\mu}(t, a)\, \dot{a}^{\mu} - B(t, a)] \equiv 0, \tag{A.14}$$

from which Eq.s (A.9) and their identical reformulation (A.10) follow.

Regrettably, we are forced for brevity to refer the interested Reviewer to Santilli (1981a) for further analytic details.

A.2: ALGEBRAIC PROFILE

By no means, the loss of a consistent algebra in the brackets of the time evolution laws is a mere "mathematical occurrence", because it carries serious physical impocations.

As an example the angular momentum can be consistently formulated in classical Hamiltonian mechanics because the underlying Poisson bracklets, first of all, verify both the right and left scalar and distributive laws and, secondly, characterize a Lie algebra.

When the same system becomes open-nonconservative and it is represented via Hamilton's historical equations (A.1), the entire theory of the angular momentum cannot be consistently formulated any more, let alone applied to specific cases. In particular, the magnitude $J^2 = J_k J_k$ of the angular momentum, while having a fully consistent meaning in conventional Hamiltonian mechanics (as the Casimir invariant of O(3)), it loses any consistent formulation when referred to Eq.s (A.1).[20]

[20] As it is the case for most of Santilli's studies, these classical occurrences are a mere basis for the identification of corresponding implications in quantum mechanics. In this case, the operator equations corresponding to Eq.s (A.1) are given by Schrödinger's equations with nonhermitean Hamiltonians $H \neq H^{\dagger}$, as widely used, e.g., in nuclear physics. The brackets of the theory are then given by $A \otimes H = AH^{\dagger} - HA$

Santilli arrived at the consistent reformulation (A.3) by imposing the following conditions on the new brackets (A.B):

I) They must be algebraically consistent, that is, verify the right and left scalar and distributive laws);

II) They must *not* be totally antisymmetric as a necessary condition to represent the time-rate-of-variation of the energy $\dot{H} = (H,H) \neq 0$; and

III) They must admit the convenmtional (Lie) Poisson brackets as a particular case when all external forcees are null, i.e.,

$$(A,B)_{|F_{ka}=0} \equiv [A,B]. \qquad (A.15)$$

Santilli (loc. cit.) then proved that the above conditions identify uniquely the so-called *general Lie-admissible algebras* .

According to Albert (1948), an algebra **U** with (abstract) elements a, b, c,...and (generally nonassociative, abstract) product ab over a field F is called a *Lie-admissible algebra,* when the attached algebra $\mathbf{U}^-$, which is the same vector space as **U**, but equipped with the product

$$\mathbf{U}^- : \ [a,b]_{\mathbf{U}} = ab - ba, \qquad (A.16)$$

is Lie.

The simplest Lie-admissible algebras are the associative algebra ξ with elements A, B, ... (e.g., matrices) and product AB. Then

$$\xi^- : \quad [A,B]_\xi = AB - BA, \qquad (A.17)$$

is the familiar Lie produce of current use in mathematics and physics.

The next Lie-admissible algebras are the Lie algebras L themselves because, fo4r $ab = [A,B]_\xi$, we have

$$L^- : \quad [A,B]_L = 2\,[A,B]_\xi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (A.18)$$

Santilli (1978a) called the most general possible algebras of the type considered the *general Lie-admissible algebras U* when they verify no condition other than the general Lie-admissibility law

$$(A, B, C) + [B, C, A) + (C, A, B) = (C, B, A) + (B, A, C) + (A, C, B), \qquad (A.19)$$

where

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and they also do not characterize a consistent algebra any more, let alone they violate the lie algebra axioms. Under these conditions, and contrary to a rather popular but erroneous use, quantities centrally dependent on a consistent algebra, such as the notion of spin,. cannot be consistently *defined* , let alone consistently applied (see Santilli (1978d) for details). This illustrates rather forcefully the serious physical implications originating from the mathematical loss of a consistent algebra in the brackets of the time evolution.

$$(A, B, C) = (A, (A, B) - ((A, B), C) \qquad (A.20)$$

is called the associator, and (A, B) is the Lie-admissible product (A.7)

Simpler Lie-admissible algebras are those of the so-called *flexible* type, and they occur when, in addition to axiom (A<), verify the law. Yet simpler Lie-admissible algebras are the Lie algebras themselves. The simplest possible Lie-admissible algebras are the associative algebras.

It is easy to see that brackets (A.7) verify all conditions I, II and III above. Moreover, the brackets are Lie-admissible because the attached brackets are Lie (Santilli (loc. cit.)),

$$\mathbf{U}^- : \quad [A,B]_{\mathbf{U}} = (A,B) - (B,A) = 2[A,B]_{Poisson}. \qquad (A.21)$$

or, equivalently, because the attached antisymmetric tensor

$$S^{\mu\nu} - S^{\nu\mu} = 2\omega^{\mu\nu} \qquad (A.22)$$

is canonical, and the same occurs for the more general brackets (A.12).

The regaining of a consistent algebraic structure carries rather important physical implications.

As an example, *Eq.s (A.1) do not admit a consistent exponentiation into a finite group or semigroup*. On the contrary, when written in their equivalent Lie-admissible form (A.3), they can be easily exponentiated into the form (Santilli (1978a), (1981a))

$$a' = \{ e|_{\mathbf{A}}^{t \circ S^{\mu\nu} \partial_\nu H) (\partial_\mu)} \} a, \qquad (A.23)$$

In particular, *the above structure leaves invariant the equations of motion*. In fact, from a general property of vector-fields on manifolds we have (loc. cit.)

$$\Gamma'(t,a') =$$

$$\{ e|_{\mathbf{A}}^{t \circ S^{\mu\nu} (\partial_\nu H) (\partial_\mu)} \} \Gamma(t,a) = \Gamma(t,a'), \qquad (A.24)$$

For this reason, structures of type (A.23) constitutes an intriguing generalization of the notion of Lie-isotopic symmetry (Sect. IV.9) called *Lie-admissible symmetry* (loc. cit.).

Also, *the broader Lie-admissible symmetry (A.23) represents the time-rate-of-variation of the energy, exactly as desired,*

$$\dot{H} = H(t,a) - \{ e|_{\mathbf{A}}^{t \circ S^{\mu\nu} (\partial_\nu H) (\partial_\mu)} \} H(t,a) = v_{ka} F_{ka}. \qquad (A.25)$$

Moreover, exponentiation (A.23) admits the following explicit form

$$\{ e_{|A}^{t^o S^{\mu\nu} (\partial_\nu H)(\partial_\mu)} \} A =$$

$$A + t^o (A, H)/1! + t^{o2}((A, H), H)/2! + \ldots.. \tag{A.26}$$

namely, *symmetries (A.24) admit non-Lie, Lie-admissible algebras in the neighborhood of the identity.*

The above properties signal the birth of a generalization of the entire Lie's and Lie-isotopic theories in a yet more general Lie-admissible form. We are here referring to a Lie-admissible generalization of the entire Lie's theory, such as enveloping algebras, Lie algebras, Lie groups, representation theory, etc.

These studies were initiated by Santilli (1978a), (1981a), (1982a) via a (flexible) Lie-admissible generalization of the Popincaré-Birkhoff-Witt Theorem (see also Ktorides, Myung and Santilli (1980)), the Lie-admissible generalization of Lie's First, Second and Third Theorem, the notion of Lie-admissible group, the bimodular representation theory (see also Santilli (1979)), and other aspects, with the understanding that so much remains to be done. For additional mathematical studies of abstract Lie-admissible algebras, one can consult the bibliography by Balzer (1984).

A.3: GEOMETRICAL PROFILE

As stressed in Santilli's work, *physical theories in general, and relativities in particular, are a simbiotic expression of deeply inter-related, analytic, algebraic and geometric formulations*. No truly novel relativity can be achieved without prior achievement of a generalization of each of these three basic structures.

After identifying the appropriate analytic structure of the equation and their Lie-admissible character, Santilli proceeded with the discovering a *yet new geometry* (Santilli (1978a), (1981a)), he called *symplectic-admissible geometry*, which is the geometric counterpart of the Lie-admissible algebras, in exactly the same way as the symplectic geometry is the geometric counterpart of the Lie algebras, and the symplectic-isotopic geometry is the geometric counterpart of the Lie-isotopic algebras.

Recall that the direct geometric structure underlying the conventional Hamilton's equations is the exact, canonical, symplectic two-form (A.

$$\omega = \tfrac{1}{2}\, \omega_{\mu\nu}\, da^\mu \wedge da^\nu; \tag{A.27}$$

the geometric structure underlying Birkhoff's equations is the exact, symplectic, Birkhoffian two-forms

$$\Omega = \tfrac{1}{2}\, \Omega_{\mu\nu}(a)\, da^\mu \wedge da^\nu; \tag{A.28}$$

and the geometric structure underlying the Hamilton-Santilli equations is the isoexact, symplectic-isotopic two-form on isospace $T^*\hat{E}(r,\hat{\delta},\hat{\mathfrak{R}})$

$$\Omega^{\circ} = \tfrac{1}{2}\,\hat{1}_{\mu}{}^{\alpha}(a)\,\omega_{\alpha\nu}\,\hat{d}a^{\mu} \,\hat{\wedge}\, \hat{d}a^{\nu} \quad , \tag{A.29}$$

Finally, recall that the covariant (geometric) tensor and the contravariant (algebraic) tensor are inverse of each other, and that the symplectic (or symplectic-isotopic) character of the two-form provides the necessary and sufficient conditions for the cortresponding brackets to be Lie (or Lie-isotopic).

The first point identified by Santilli (loc. cit.) in the study of the geometry of Eq.s (A.3) is that *the symplectic geometry and related exterior calculus, whether in their conventional or isotopic formulation, are intrinsically unable to characterize the Lie-admissible algebras.*

This is due to the fact that the calculus of exterior forms is essentially *antisymmetric* in the indeces, while the Lie-admissible tensors $S^{\mu\nu}$ are not, and the same occurs for the covariant counterpart

$$S_{\mu\nu}(t, a) \;=\; (\| S^{\alpha\beta}\|^{-1})_{\mu\nu} \neq \pm\, \hat{S}_{\nu\mu} \tag{A.30}$$

. In fact, the construction of a conventional exterior two-form with the above tensor implies the reduction

$$S_{\mu\nu}\, da^{\mu} \wedge da^{\nu} \;\equiv\; \tfrac{1}{2}\,\omega_{\mu\nu}\, da^{\mu} \wedge da^{\nu}, \tag{A.31}$$

namely the symplectic geometry automatically eliminates the symmetric component of the S-tensor, thus characterizing only its Lie content.

The main idea of Santilli's symplectic-admissible geometry is that of generalizing the conventional exterion calculus, say, of two differentials da^{μ} and da^{ν} into a more general calculus of differentials called *exterior-admissible calculus*. It is based on a product, say $\odot$, which is neither totally symmetric nor totally antisymmetric, but such that its antisymmetric component is the conventional exterior one,

$$da^{\mu} \odot da^{\nu} \;=\; da^{\mu} \wedge da^{\nu} \;+\; da^{\nu} \times da^{\nu}, \tag{A.32a}$$

$$da^{\mu} \wedge da^{\nu} \;=\; -\; da^{\nu} \wedge da^{\mu}, \qquad \hat{d}a^{\mu} \times \hat{d}a^{\nu} \;=\; \hat{d}a^{\nu} \times \hat{d}a^{\mu}, \tag{A.32c}$$

This allows the introduction of the *exterior-admissible forms*

$$\hat{S}_{o} = \phi(a), \qquad \hat{S}_{1} = \hat{S}_{\mu}\, da^{\mu}, \qquad \hat{S}_{2} = \hat{S}_{\mu\nu}\, da^{\mu} \odot da^{\nu}, \ldots\ldots\ldots \tag{A.33}$$

The *exact exterior-admissible forms* are then given by

$$\hat{S}_{1} = d\hat{S}_{o} = \frac{\partial\phi}{\partial a^{\mu}}\, da^{\mu}, \tag{A.34a}$$

$$\hat{S}_2 = d\hat{S}_1 = \frac{\partial A_\nu}{\partial a^\mu} \, da^\mu \odot da^\nu, \tag{A.34b}$$

....

The calculus of exterior-admissible forms can indeed characterize the Lie-admissible algebras, because they characterize, not only the antisymmetric component of the Lie-admissible algebras, but also their symmetric part, via the two-forms

$$S_{\mu\nu}(t,a) \, da^\mu \odot da^\nu = \omega_{\mu\nu}(a) \, da^\mu \wedge da^\nu + s_{\mu\nu}(t, a) \, da^\mu \times da^\nu, \tag{A.35}$$

Structures (A.34b) were called by Santilli (loc. cit.) *symplectic-admissible two-forms* because their antisymmeyric component is symplectic, in a way fully parallel to the property whereby the antisymmetric part of the Lie-admissible algebras is Lie. Spaces $T^*E(r,\delta,\Re)$, when equipped with two-form (A.34b) were called *symplectic-admissible manifolds* and the related geometry *symplectic-admissible geometry*.

The most salient departure of the exterior-admissible calculus from the exterior calculus in its conventional or isotopic formulation is that the Poincare′ Lemma no longer holds, i.e., for exact symplectic-admissible two-forms we have in general

$$\hat{S}_2 = d\hat{S}_1, \qquad d\hat{S}_2 = d(d\hat{S}_1) \neq 0. \tag{A.36}$$

In actuality, within the contest of the exterior-admissible calculus, the Poincare′ Lemma is generalized into a rather intriguing geometric struture which evidently admits the conventional Lemma as a particular case when all symmetric components are null.

The geometric understanding of the Lie-isotopic algebras requires the understanding that *the validity of the Poincaré Lemma within the context of the symplectic-isotopic geometry is a necessary condition for the representation of the conservation of the total energy under nonhamiltonian internal forces,* as putlined in the main text

By the same token, the geometric understanding of the Lie-admissible algebras requires the understanding that *the lack of validity of the Poincaré Lemma within the context of the more general symplectic-admissible geometry is a necessary condition for the representation of the time-rate-of-variation of the energy of an interior dynamical system.*

By using the above generalized, Lie-admissible, analytic, algebraic and geometric structures, Santilli then formulated his *Lie-admissible generalizations of Galilei's relativity for* open nonconservative systems, as stated in the title of the memoir (1978a), and then elaborated in the monograph Santilli (1981a).

The NOBEL COMMITTEE should know that such generalized relativity has a *bimodular structure* (Santilli (1979), namely, a structure which, in its most general possible form, is composed of two inequivalent modular-isotopic actions, one to the right (forward in time) and one to the left (backward in time), which is one of the most complex notions of contemporary mathematics, of which we possess today rather limited knowledge beginning at the pure matheatical level. While the Lie-isotopic formulations are at an advanced stage, the study of the still more general Lie-admissible formulations will certainly continue well into next century.

To conclude this Nomination, in the main test we have reviewed the series of discoveries that lead Santilli to a Lie-isotopic generalization of Galilei's relativity for closed systems with nonlinear, nonlocal and nonhamiltonian interior forces. In this case the abstract Lie character is preserved, but realized in its most general possible form preserving the original generators. This guarantees the validity of the conventional total conservation laws, while permitting generalized interior forces.

In this Appendix we have outlined the foundations of a further generalization of Galilei's relativity of Lie-admissible character. In this case, the Lie structure itself is abandoned in favor of a covering algebraic structure, while the emerging generalized symmetries directly represent the covering notion of time-rate-of-variation of physical quantities.

The sequential quantum leaps in physical and mathematical knowledge are then rather evident.

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On a possible Lie-admissible covering of the Galilei relativity in Newtonian Mechanics for nonconservative and Galilei form-noninvariant systems

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Abstract

In order to study the problem of the relativity laws of nonconservative and Galilei form-noninvariant systems, two complementary methodological frameworks are presented. The first belongs to the so-called Inverse Problem of Classical Mechanics and consists of the conventional analytic, algebraic and geometrical formulations which underlie the integrability conditions for the existence of a Lagrangian or, independently, of a Hamiltonian. These methods emerge as possessing considerable effectiveness in the identification of the mechanism of Galilei relativity breaking in Newtonian Mechanics by forces not derivable from a potential. Nevertheless, they do not exhibit a clear constructive capability for a possible covering relativity. For this reason, the second methodological framework is presented. It belongs to the so-called Lie-Admissible Problem in Classical Mechanics and consists of the covering analytic, algebraic and geometrical formulations which are needed for the equations originally conceived by Lagrange and Hamilton, those with external terms. These formulations are characterized by the Lie-admissible algebras which are known to be genuine algebraic covering of Lie algebras, and which in this paper are identified as possessing (a) a direct applicability in Newtonian Mechanics for the case of forces not derivable from a potential, (b) an analytic origin fully parallel to that of Lie algebras, i.e., via the brackets of the time evolution law, (c) a covering of the conventional canonical formulations as classical realizations, (d) an implementation at a number of levels of Lie's theory, including a fundamental realization as enveloping nonassociative algebras, (e) a generalization of symplectic and contact geometry as geometrical backing and (f) the capability of recovering conventional formulations identically at the limit of null external forces, here interpreted as relativity breaking forces. A covering of the Galilei relativity, called Galilei-admissible relativity, is then conjectured for independent scrutiny by interested researchers. A number of potential implications, particularly for hadron physics, are then briefly considered for future detailed treatment.

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Quasigroups and nonassociative algebras in physics

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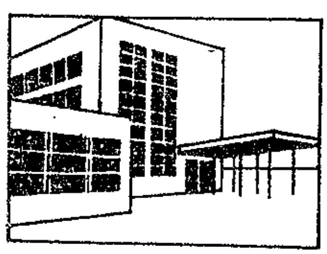

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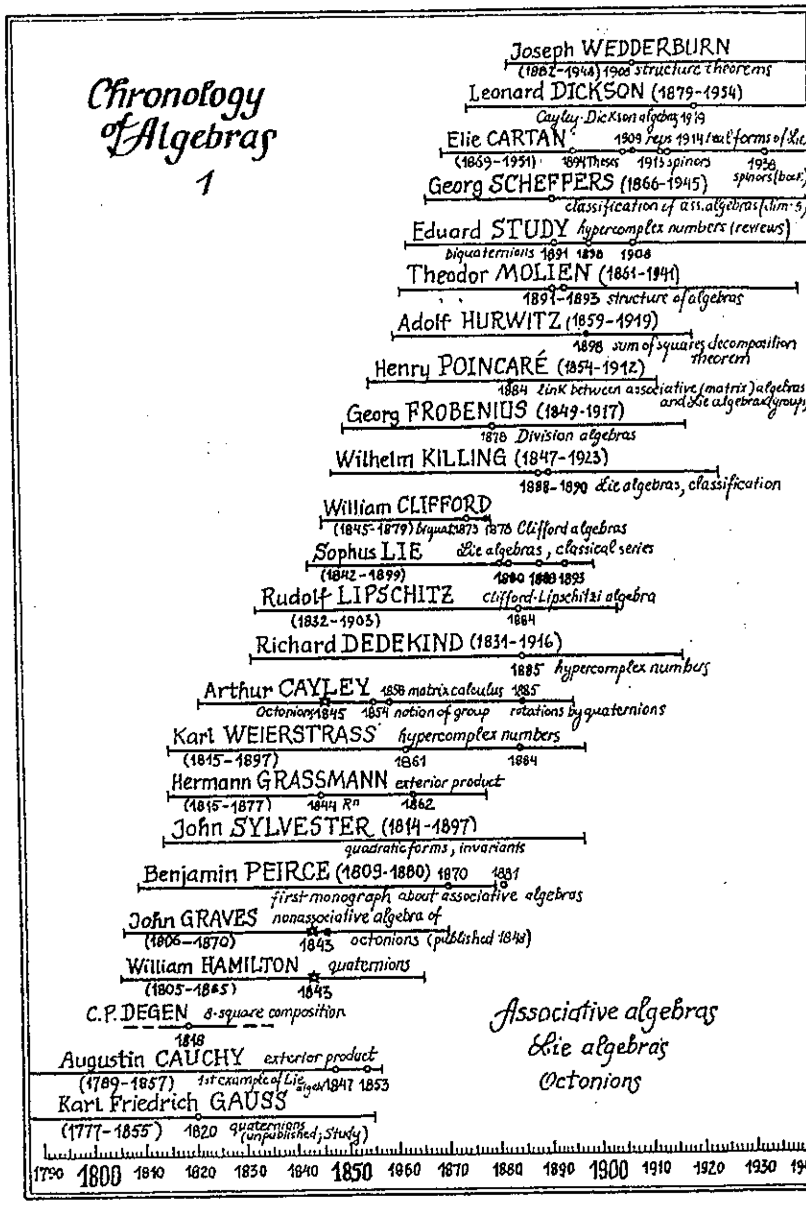
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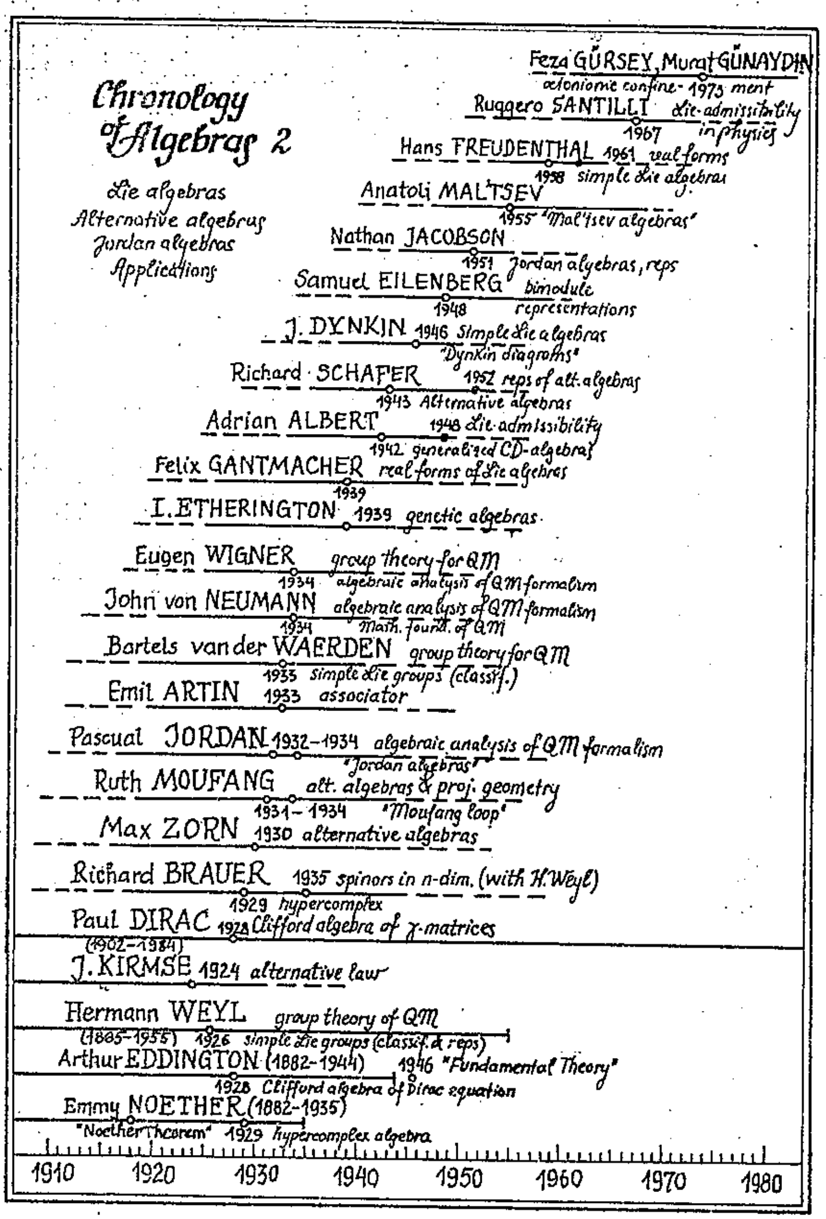
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"It is...a hope and a prophecy that this book will soon become essential knowledge for all those engaged in the development of science...Professor Santilli's discussion of the subject distinguishes itself by completeness, mathematical rigor, and clarity of physical applications."

R. Mignani
University of Rome

Foundations of Theoretical Mechanics I and II

Ruggero M. Santilli, Basic Institute for Research, Cambridge, Massachusetts

The Importance...

The contemporary conception of Galilei's relativity is based on systems of particles that are both closed—in the sense that they verify conservation laws—and are Hamiltonian-type.—or have constituents that are approximated as being point-like. Because most real systems are non-Hamiltonian, its applications have, for the most part, been limited to theoretical exercises.

Professor Santilli, drawing from mechanics, geometry, and algebra, develops a generalization of Galilei's relativity for closed systems that considers both Hamiltonian and non-Hamiltonian internal forces. Based on original research, Santilli's discovery essentially establishes a new branch of physics, with important applications in statistical and space mechanics, theoretical biophysics, particle physics, and engineering.

The Books...

Volume I, **The Inverse Problem in Newtonian Mechanics** develops the conditions in which an arbitrary Newtonian system can be integrated to admit a Lagrangian or, independently, a Hamiltonian representation. In addition, it surveys results in the field going back three centuries.

Building on the findings of the first volume, **Birkhoffian Generalization of Hamiltonian Mechanics** constructs a step-by-step generalization of Hamiltonian mechanics—a generalization that on one hand, preserves the analytic, Lie, and symplectic characters of Hamilton's equations, yet, on the other, it expresses the principle in the most general form possible. This constitutes the foundations of generalized relativity. The numerous examples and applications which are cited throughout the text emphasize the direct universality of the new mechanics for local non-Hamiltonian systems.

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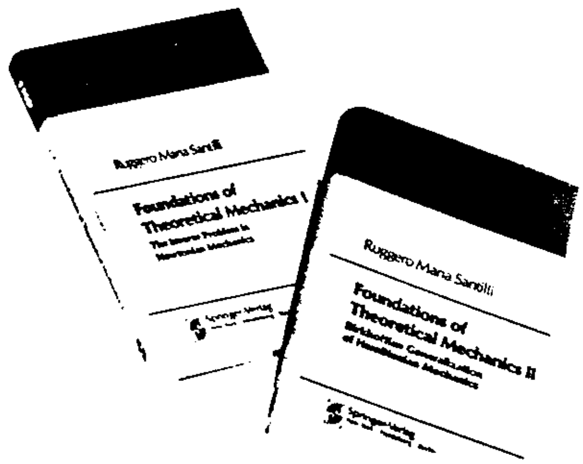

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"Truly epoche-making...It can be said that this is a remarkable book of highest quality and greatest importance to anyone who is interested in the progress of theoretical mechanics."
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Volume II:

"It's all to Professor Santilli's credit that in the second volume of his treatise he proves the existence of a mathematical structure of Lie type capable of these (non-potential) interactions."

"The monograph is an excellent completion of the existing literature. It deserves the attention of all students as well as researchers, who deal with classical mechanics and/or field theory."
—J. Kobussen, Swiss Federal Institute for Reactor Research

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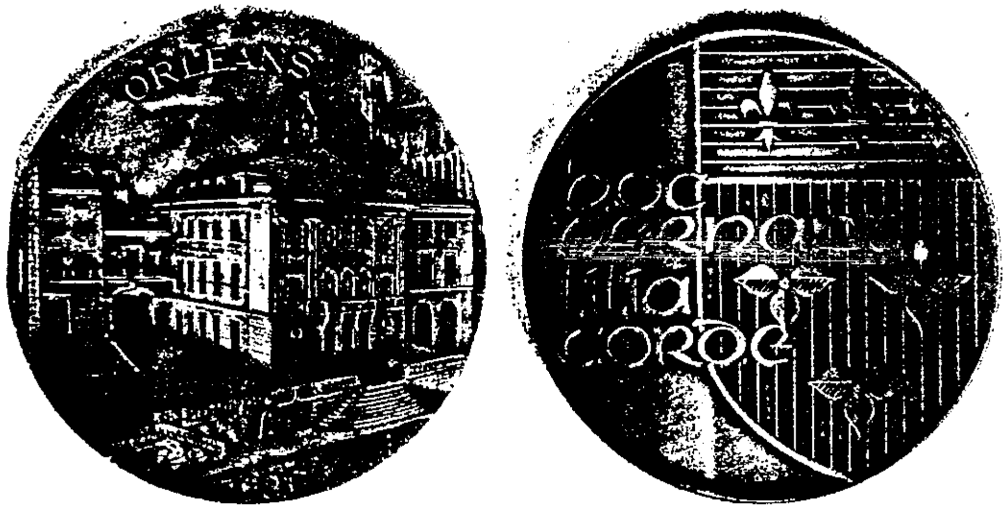
OFFICIAL ANNOUNCEMENT OF PROF. SANTILLI'S DISCOVERY APPEARED IN PHYSICS TODAY, ISSUE OF JUNE 1983



Images of the MEDAL OF HONOR received by

Professor RUGGERO MARIA SANTILLI

from the CITY OF ORLÉANS, FRANCE, in the occasion of the First International Conference on nonpotential interactions and their Lie–admissible Treatment.

Orléans, le Mercredi 6 Janvier 1982, à 18 heures.

Harvard University, Lyman Laboratory
Informal Seminar Course

September 1977

by Ruggero Maria Santilli

Title	**The Inverse Problem in Newtonian Mechanics and Field Theory**
Objective	a study of Lagrange's and Hamilton's equations
Means	1. integrability conditions for analytic representations of systems with couplings not necessarily derivable from a potential, 2. methods for the computation of a Lagrangian or, independently, of a Hamiltonian, when they exist, from given arbitrary systems, 3. outline of the applications to Newtonian mechanics, space mechanics, optimal control theory, plasma physics and classical field theory.
Prerequisites	knowledge of mechanics and differential equations
Duration	25 lectures of 1.5h each
Organization	(a) 50% of time dedicated to the methodology of the Inverse Problem for ordinary differential equations (Newtonian mechanics, etc.), (b) 30% of time dedicated to the extension of the methodology to partial differential equations (field theory), (c) 20% of time dedicated to applications and open problems, (d) several illustrative examples will be either worked out in class or distributed, (e) a number of homeworks will be assigned.
Reference	a limited number of copies of two forthcoming monographs on the Inverse Problem by R.M. Santilli (to be published by Springer-Verlag) will be made available.
Outline	enclosed in a tentative form
Organizational Meetings	Time: 4:30 p.m., October 3, 1977. Place: Room 267 (alternatively, the interested persons can contact R.M. Santilli at Room 437, Lyman Lab, Harvard University, office 495-3212, home 969-3465).

The Inverse Problem in Newtonian Mechanics and Field Theory

Course Outline

1. Introduction. Significance of systems with arbitrary couplings in Newtonian Mechanics, Space Mechanics, Optimal Control Theory and other disciplines. Need for the applicability of known analytic methods to the broader systems considered. Identification of the Inverse Problem.

2. Elemental Mathematics. Review of the existence theory for implicit functions, solutions and derivatives in the parameters. Review of the calculus of differential forms, the Poincaré-Lemma and its converse. Review of the calculus of variations.

3. Variational Approach to Self-Adjointness. Equations of variation, adjoint system, conditions of self-adjointness for quasi-linear second order systems. Reduction to first order systems and extension to arbitrary order.

4. Lagrange's and Hamilton's Equations. Hilbert Differentiability Theorem and basic properties. Equations of variation of Lagrange's and Hamilton's equations and their properties. Self-adjointness of Lagrange's and Hamilton's equations. Characterization of the ordered analytic representations of arbitrary systems.

5. Fundamental Theorems. Theorems on the necessary and sufficient conditions for the existence of a Lagrangian or, independently, of a Hamiltonian. Methods for the computation of these functions from the given equations of motion. Analysis of their structure from the viewpoint of the interactions and classification of the admissible couplings.

6. Application to the Newtonian Transformation Theory. Algebraic and geometrical significance of the conditions of self-adjointness. The new class of equivalence transformations of a Lagrangian characterized by its integrability conditions (isotopic transformations).

7. Application to Symmetries and First Integrals. Noether's theorem, its inverse and its generalization to higher orders. Use of the isotopic transformations for the computation of new first integrals. The concept of isotopically related symmetry groups, algebras and brackets.

8. Extension of the Inverse Problem to Field Theory. Conditions of variational self-adjointness for quasi-linear partial differential equations and relation to the conventional concept of self-adjointness in linear spaces. Fundamental theorems on the necessary and sufficient conditions for the existence of a Lagrangian density and its construction. Analysis of its structure and the relation with chiral Lagrangians. Application of the methodology to the transformation theory.

9. Outline of Applications. Electric circuits inclusive of losses. Nonlinear nonconservative plasma equations. Spinning top with drag torques. Space mechanics with drag forces for interplanetary dust. A missile trajectory problem. Nonconservative equations in continuum mechanics. Variational analysis of the Weinberg-Salam model of unified gauge theories of weak and electromagnetic interactions and the problem of its extension for the inclusion of strong interactions.