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The novel Einstein-Podolsky-Rosen entanglement and its representation of the anomalous magnetic moment of the muons

Ruggero M. Santilli
The Institute for Basic Research
35246 US 19 North No. 215, Palm Harbor, FL 34684, U.S.A.
Email: research@i-b-r.org

Abstract. In this paper, we review recent experiments supporting the 1935 view by Einstein-Podolsky-Rosen that *"quantum mechanics is not a complete theory"* (EPR argument), as well as the axiom-preserving completion of quantum mechanics via isomathematics and isomechanics. Following a review of the background methods, we present the completion of quantum entanglement for point particles into a covering notion, here submitted under the name of *Einstein-Podolsky-Rosen entanglement,* for the representation of the instantaneous and continuous communications between extended particles via the overlapping of their wavepackets, with ensuing non-linear, non-local and non-potential interactions. We then show that the EPR entanglement allows a three-body structure model of the muons composed by electrons and positrons produced free in the spontaneous decay with the lowest mode, while apparently representing all characteristics of the muons, including their recently measured, anomalous magnetic moment.

Keywords: Einstein-Podolsky-Rosen argument, isomathematics, isomechanics

1. Introduction.

Recent, very accurate measurements [1] have established the following difference between the measured *muon* g-factor g_μ^{EXP} and its prediction via quantum electrodynamics g_μ^{QED}

$$\begin{aligned} & g_\mu^{EXP} - g_\mu^{QED} = \\ & = 2.00233184122 - 2.00233183620 = \\ & = 0.00000000502 > 0. \end{aligned} \tag{1}$$

Additional accurate measurements [2] have shown deviations from quantum mechanical predictions for *atoms* in condensed matter, and measurements [3] have indicated bigger deviations from the predictions of quantum mechanics for *heavy ion.*

The above experiments support:

1) The validity of the historical 1935 argument by A. Einstein, B. Podolsky and N. Rosen that *"quantum mechanics is not a complete theory"* (EPR argument) [4];

2) The significance of historical completions of quantum mechanics, such as the non-linear completion by W. Heisenberg [5], the non-local completion by L. de Broglie and D. Bohm [6], and the completion via *hidden variables* by D. Bohm [7];

3) The validity of the recent verifications of the EPR argument by R. M. Santilli [8] [9] [10] based on the axiom-preserving (called *isotopic*) completion of 20th century ap-

plied mathematics into the novel *isomathematics,* with ensuing completion of quantum mechanics into the branch of hadronic mechanics known as *isomechanics,* for the time-invariant representation of extended particles/wavepackets under non-linear, non-local and non-potential interactions due to deep wave-overlapping (see Refs. [11] [12] [13] for an outline of the basic methods, Refs. [14] [15] for recent overviews and Refs. [16] [17] [18] for detailed presentations).

2. Isomathematics and isomechanics.

As it is well known, 20th century applied mathematics is characterized by a universal enveloping associative algebra ξ with conventional associative product $AB = A \times B$ between arbitrary quantities A, B, such as numbers, functions, operators, etc.

The axiom-preserving, thus isotopic image of 20th century applied mathematics, called *isomathematics* and used in the indicated verifications of the EPR argument, is based on the *universal enveloping isoassociative isoalgebra* $\hat{\xi}$ characterized by the completion of the conventional associative product AB into the associativity-preserving *isoproduct* (first introduced in Eq. (5), page 71 of Ref. [17] and treated in detail in Refs. [18])

$$A \star B = A\hat{T}B, \ \hat{T} > 0, \tag{2}$$

where the quantity $\hat{T}$, called the *isotopic element,* is positive-definite but possesses otherwise an unrestricted dependence on all needed local variables (herein tacitly assumed).

The branch of hadronic mechanics [18] characterized by isomathematics, and called *isomechanics,* is a step-by-step axiom-preserving image of quantum mechanics characterized by the completion of all associative products AB into isoproduct (2).

An axiomatic meaning of isoproduct (2) is to provide an explicit and concrete realization of *hidden variables* [7] via the isotopic element $\hat{T}$, by therefore supporting D. Bohm's view that quantum mechanics does indeed admit hidden degrees of freedom, since the hidden variables become evident when the basic quantum axions are realized in a way more general than that of the Copenhagen school.

A physical meaning of isoproduct (2) is to provide a quantitative representation of extended particles in condition of mutual penetration, as occurring in a nuclear structure, with expected interactions of non-linear (in the wavefunctions), non-local (due to integrals on volumes) and non-derivable from a potential. Hence these interactions are not representable with a Hamiltonian (hereon referred to as *non-Hamiltonian interactions*). Said interactions can be represented by realizations of the isotopic element of the type [13]

$$\begin{gathered} \hat{T} = \Pi_{\alpha=1,2} Diag.(\tfrac{1}{n_{1,\alpha}^2}, \tfrac{1}{n_{2,\alpha}^2}, \tfrac{1}{n_{3,\alpha}^2}, \tfrac{1}{n_{4,\alpha}^2}) e^{-\Gamma}, \\ n_{\mu,\alpha} > 0, \quad \Gamma > 0, , \ \ \mu = 1, 2, 3, 4, \ \ \alpha = 1, 2, ..., N, \end{gathered} \tag{3}$$

where: $n_{k,\alpha}^2$, $k = 1, 2, 3$, represent the extended, thus deformable shape of particles with n_3 parallel to the spin; $n_{4,\alpha}^2$ represents the density of particles; all n-characteristic quantities are normalized to the value $n_{\mu,\alpha}^2 = 1$ for the vacuum; and the term e^{Γ}, where Γ is positive-definite, represents all non-Hamiltonian interactions.

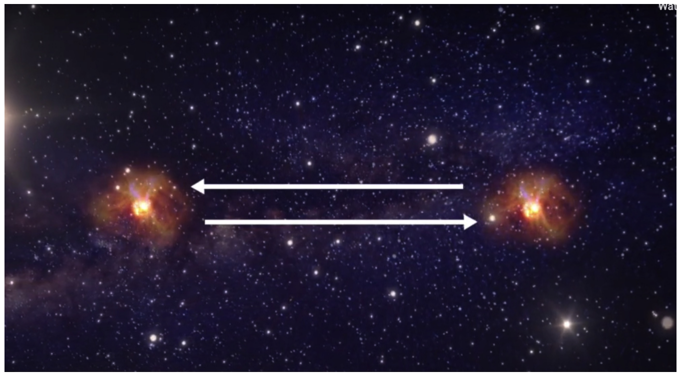

Figure 1: *We reproduce a view of the quantum entanglement of particles from the video world-lecture-series.org/legacy-of-einstein-for-new-clean-energies illustrating Einstein's argument on its need for superluminal communications due to the local character of differentials, potentials and wavefunctions of the Schrödinger equation solely allowing a point-like characterization of particles [4].*

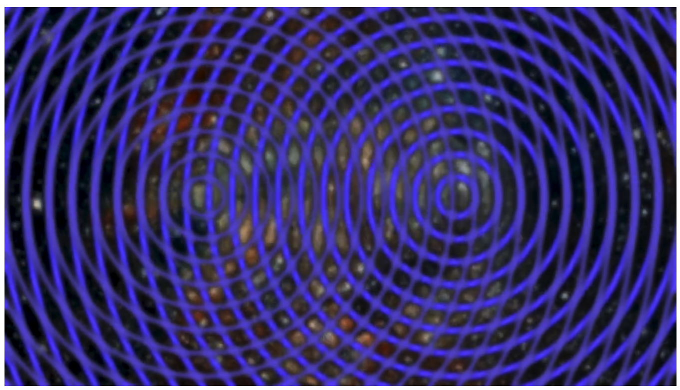

Figure 2: *We reproduce a view from the video world-lecture-series.org/legacy-of-einstein-for-new-clean-energies of the new entanglement of particles treated in detail in the forthcoming Ref. [14] and presented in this paper under the name of EPR entanglement, which is characterized by non-linear, non-local and non-potential interactions expected in the overlapping of the wavepackets of particles when represented via isotopic elements of type (3). As such, the EPR entanglement prevents the applicability of Bell's inequality [19], allows an explicit and concrete realization of Bohm's hidden variables [7], and permits a progressive recovering of Einstein's determinism in the interior of hyperdense particles, with its full recovering at the limit of gravitational collapse [8] [9] [10].*

It should be noted that isoproduct (2) with realizations (3) of the isotopic element provide a quantitative representation of the completion of the conventional *quantum entanglement* for systems of *point-like* particles under Hamiltonian interactions, into a covering notion, here called *Einstein-Podolsky-Rosen entanglement,* which is applicable to systems of *extended* particles with non-Hamiltonian interactions due to deep wave-overlapping (for details, see Figures 1 and 2 and references quoted therein).

Despite its simplicity, isoproduct (2) requires, for consistency, a compatible isotopy of the entire 20th century applied mathematics, with no exception known to the author [11]. In fact, isoproduct (2) requires the following completions (see Vol. I of Refs. [18] for a general treatment):

A) The compatible, completion of the basic unit $\hbar = 1$ of quantum mechanics into the integro-differential *isounit*

$$\hat{I} = 1/\hat{T} > 0, \quad \hat{I} \star A = A \star \hat{I} = A, \tag{4}$$

with ensuing completion of the conventional numeric field $F(n, \times, 1)$ of real $\mathcal{R}$, complex $\mathcal{C}$ and quaternionic $\mathcal{Q}$ numbers n into the *isofields* $\hat{F}(\hat{n}, \star, \hat{I})$ of *isoreal* $\hat{\mathcal{R}}$, isocomplex $\hat{\mathcal{C}}$ and *isoqquaternionic* $\hat{\mathcal{Q}}$ *isonumbers* $\hat{n} = n\hat{I}$ and related isotopic operations [20] (see Ref. [21] for an independent study).

B) The completion of conventional functions $f(r)$ of a local variable r into *isofunctions* that, to have value on an isofield, must have the structure [22] (see Ref. [23] for an independent study)

$$\hat{f}(\hat{r}) = [f(r\hat{I})]\hat{I}, \tag{5}$$

and related isodifferential isocalculus [22] (see Refs. [24] for independent studies).

C) The completion of conventional spaces S over F into *isospaces* $\hat{S}$ over isofields $\hat{F}$ [22]. In particular, the conventional Minkowski space $M(x, \eta, I)$ over $\mathcal{R}$ with spacetime coordinates $x \in \mathcal{R},\ x^4 = ct$, metric $\eta = Diag(1,1,1,-1)$ and unit $I = Diag(1,1,1,1)$, is mapped into the *iso-Minkowski isospace* $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I})$ over the isoreal isonumbers $\hat{\mathcal{R}}$ [25] (see Ref. [26] for an independent study) with spacetime isocoordinates $\hat{x} = x\hat{I} \in \hat{\mathcal{R}}$, isometric $\hat{\Omega} = (\hat{\eta})\hat{I} = (\hat{T}\eta)\hat{I}$, and isointerval

$$\begin{aligned}(\hat{x}^\rho - \hat{y}^\rho)^{\hat{2}} &= (\hat{x}^\rho - \hat{y}^\rho) \star \hat{\Omega}_{\rho\nu} \star (\hat{x}^\nu - \hat{y}^\nu) = \\ &= [\tfrac{(x_1-y_1)^2}{n_1^2} + \tfrac{(x_2-y_2)^2}{n_2^2} + \tfrac{(x_3-y_3)^2}{n_3^2} - \tfrac{(t_x-t_y)^2c^2}{n_4^2}]\hat{I},\end{aligned} \tag{6}$$

where the exponential term $exp\{-\Gamma\}$ is imbedded into the n-characteristic quantities.

D) The compatible completion of all branches of Lie's theory first studied in Ref. [17] (see Vol. II of Refs. [18] for a general treatment and Ref. [27] for an independent study). For instance, a N-dimensional Lie algebra L with Hermitean generators $X_k,\ k = 1, 2, ...N$ is completed into the infinite family of Lie-Santilli isoalgebras $\hat{L}$ with isocommutation rules

$$[X_i \hat{,} X_j] = X_i \star X_j - X_j \star X_i = C_{ij}^k X_k, \tag{7}$$

which isoalgebras are called *regular* or *irregular* depending on whether the structure quantities C_{ij}^k are constant or functions of local variables, respectively.

E) The completion of well known space-time symmetries into the *isosymmetries* of iso-space-time (6), including the completion of the Lorentz symmetry $SO(3.1)$ into the *Lorentz-Santilli isosymmetry* $\hat{SO}(3.1)$ [25] with isotransformations

$$\begin{aligned}&x^{1'} = x^1, \ \ , x^{2'} = x^2, \\ &x^{3'} = \hat{\gamma}(x^3 - \hat{\beta}\tfrac{n_3}{n_4}x^4), \quad x^{4'} = \hat{\gamma}(x^4 - \hat{\beta}\tfrac{n_4}{n_3}x^3),\end{aligned} \tag{8}$$

where

$$\hat{\beta}_k = \frac{v_k/n_k}{c_o/n_4}, \quad \hat{\gamma}_k = \frac{1}{\sqrt{1-\hat{\beta}_k^2}}, \tag{9}$$

which provide the invariance of the local speed of light

$$C = \frac{c}{n_4}, \tag{10}$$

with consequential *isorenormalization* of the energy (namely, renormalization caused by non-Hamiltonian interactions)

$$E = mc^2 \ \rightarrow \ \bar{E} = mC^2 = \frac{E}{n_4^2}. \tag{11}$$

Additionally, the isotopic representation of the anomalous magnetic moment of the muons requires the completion of the Lorentz-Poincaré symmetry $P(3.1)$ into the *Lorentz-Poincaré-Santilli isosymmetry* $\hat{P}(3.1)$ [28], as well as, more particularly for the muons due to their spin $1/2$, the completion of the spinorial covering of the Lorentz-Poincaré symmetry into the *isospinorial covering of the Lorentz-Poincaré-Santilli isosymmetry* $\hat{\mathcal{P}}(3.1)$ [29] (see Ref. [12] for a recent review and Ref. [26] for independent studies).

We should also recall that all aspects of regular isomathematics and isomechanics can be constructed via the simple *non-unitary transform*

$$UU^\dagger = \hat{I} \neq I, \tag{12}$$

of *all* conventional mathematical or physical aspects, under which the unit of quantum mechanics is mapped into the isounit of isomechanics

$$\hbar = 1 \rightarrow U1U^\dagger = \hat{I}, \tag{13}$$

the conventional associative product AB is mapped into the isoproduct

$$AB \rightarrow U(AB)U^\dagger = (UAU^\dagger)(UU^\dagger)^{-1}(UBU^\dagger) = \hat{A}\hat{T}\hat{B}, \tag{14}$$

and the same holds for the construction of all remaining regular isotheories.

Finally, we recall that the isotopic element $\hat{T}$ represents physical characteristics of particles. Hence, the invariance of its numeric value is important for the consistency and exerimental verification of any isotheory. Such an invariance does indeed occur under the *infinite class of isoequivalence* of isotopic methods which is given by the isotopic reformulation of non-unitary transforms called *isounitary isotransforms*

$$UU^\dagger = \hat{I} \neq I, \quad U = \hat{U}\hat{T}^{1/2}, \quad \hat{U} \star \hat{U}^\dagger = \hat{U}^\dagger \star \hat{U} = \hat{I}, \tag{15}$$

under which we have the numeric invariance of the isounit [30]

$$\hat{I} \rightarrow \hat{U} \star \hat{I} \star \hat{U}^\dagger = \hat{I}' \equiv \hat{I}, \tag{16}$$

and of the isotopic element

$$\hat{A} \star \hat{B} \to \hat{U} \star (\hat{A} \star \hat{B}) \star \hat{U}^{\dagger} = \hat{A}'\hat{T}'\hat{B}', \quad \hat{T}' \equiv \hat{T}. \tag{17}$$

3. The structure of muons.

As it is well known, muons $\mu^{\pm}$ are assumed to be elementary particles by the standard model, under which assumption, the sole known possibility of representing deviation (1) is the search for new particles and/or new interactions.

In this paper, we study the view presented in Table 5.2, page 849 of the 1978 paper [31], (see also Section 2.5.5, page 163 of Ref. [13] for a recent update) according to which muons need a structure for a quantitative representation of their spontaneous decay.

Along these lines, muons were represented as a hadronic bound state of particles produced free in the spontaneous decays with the lowest mode (tunnel effect of the physical constituents), resulting in the three-body structure model of muons according to hadronic mechanics (hm)

$$\mu^{\pm} = (\hat{e}^{-}_{\uparrow}, \hat{e}^{\pm}_{\downarrow}, \hat{e}^{+}_{\uparrow})_{hm}, \tag{18}$$

where the presence of positrons was important for the representation of the muon instability, and the "hat" in the symbol $\hat{e}^{\pm}$ denotes the presence of contact, non-Hamiltonian interactions due to EPR entanglement of the constituents (see Figure 3 for details).

A non-relativistic representation of the mass, charge radius, mean life, spin, and parity of the muons was then provided via the iso-Schrödinger equation of isomechanics, Eqs. (5.2.4), Ref. [31], here omitted for brevity (see also Eqs. (54), page 165, Ref. [13]).

The 1978 paper [31] provided a representation of the magnetic moment of the muons known at that time, that is, coinciding with the magnetic moment of the central electrons. This is due to the lack of contribution to the total magnetic moment from the peripheral electron-positron pair due to their opposite charges under the assumption of a circular orbit (which appears to be the only possible at short distances under deep mutual penetration of particles).

4. Anomalous magnetic moment of the muons.

By keeping in mind that electrons have a point-like *charge* structure, it appears that the EPR entanglement of the constituents of model (18) (Figure 1) causes a very small deformation (called *mutation* [31]) of the wavepackets of the constituents such to produce deviation (1), whose quantitative representation can be preliminarily derived via the following isotopic procedure.

The relationship under isotopies between the magnetic moment and the spin of charged particles has been identified in Eq. (6.5), page 190 Ref. [29] with ensuing relation for the g-factors (here reproduced for brevity without its derivation)

$$\hat{g}^{EXP}_{\mu} = \frac{n_4}{n_3} g^{QED}_{\mu}. \tag{19}$$

From value (1), we can then write

$$\frac{n_4}{n_3} = 1.00000000502. \tag{20}$$

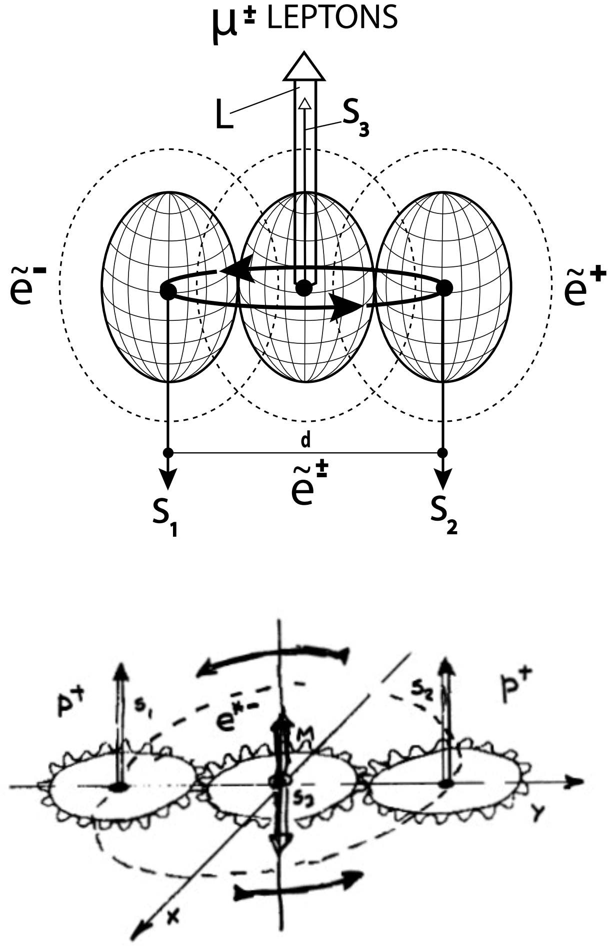

Figure 3: *In the top, we show an illustration of the structure model of muons $\mu^{\pm}$ presented in Table 5.2, page 849 of the 1978 paper [31], (see also Section 2.5.5, page 163 of Ref. [13] for an update) as three-body hadronic bound states $\mu^{\pm} = (\hat{e}^{-}_{\uparrow}, \hat{e}^{\pm}_{\downarrow}, \hat{e}^{+}_{\uparrow})_{hm}$ of elementary electrons identified in the decays with the lowest mode. The lower view reproduces the "gear-model" used in Figure 2, page 852, Ref. [31], to illustrate the stability of the model in order to represent the relatively long mean life of the muons (for particle standard), as well as to illustrate the presence of non-Hamiltonian interactions due to the mutual penetration of the wavepackets of the constituents. Note that said constituents are isoelectrons $\hat{e}^{\pm}$, rather than ordinary electrons $e^{\pm}$, due to their non-Hamiltonian interactions requiring their characterization via an isoirreducible isounitary isorepresentation of* $\hat{\mathcal{P}}(3.1)$ *[29]. Note also that, since all constituents have a point-like charge, the charge radius of the model is given by the radius of the orbit of the peripheral isoelectrons.*

The identification of the individual values of the characteristic quantities n_μ requires a structure model of the muons as a composite system, for which muons are extended particles with density n_4 different than that of the vacuum $n_4 = 1$.

Under the assumption of model (18) (Figure 3), the total rest energy of the constituents is $E_{cons} = 3E_e = 1.533\ MeV$, while the muon rest energy is given by the familiar value $E_\mu = 105.7\ MeV$. This implies the excess energy

$$\Delta E = 105.7\ MeV - 1.533\ MeV = 104.167\ MeV, \tag{21}$$

under which the Schrödinger equation no longer admits physically meaningful solutions [13] [31].

For this reason, isotopic methods provide a mathematical representation of excess energy (21) via isorenormalization (11) with numeric value of the density

$$n_4^2 = \frac{3E_e}{E_\mu} = \frac{1.533}{105.7} = 0.0149, \tag{22}$$

under which the consistency of the Schrödinger equation is restored at the isotopic level [13], while excess energy (21) can ne physically interpreted, e.g., via the kinetic energy of the peripheral muon constituents.

The use of normalization

$$n_1^2 + n_2^2 + n_3^2 = 1, \tag{23}$$

then provides the desired first approximation of the charge distribution and shape of muons

$$n_1^2 = n_2^2 \approx 0.4926, \ \ n_3^2 \approx 0.0149. \tag{24}$$

The above data confirm the expected very prolate character of the assumed structure model of the muon shown in Figure 3 due to the point-like character of the constituents.

5. Concluding remarks.

In the author's view, the most important notion emerging from the preceding study is that of the Einstein-Podolsky-Rosen entanglement representing the instantaneous and continuous communications between extended particles due to the overlapping of their wavepackets, with ensuing non-Hamiltonain interactions according to Eqs. (2) and (3), whose treatment required the construction of isomathematics and isomechanics [18].

In fact, the EPR entanglement: prevents the applicability of Bell's inequality [19] due to the presence of non-Hamiltonian interactions first studied in Ref. [9]; provides an explicit and concrete realization of Bohm's hidden variables [7] in terms of the isotopic element first studied in Ref. [10]; and permits a preliminary, yet numeric and time invariant representation of all characteristics of muons, including their anomalous magnetic moment [1].

Therefore, it may be of some interest to indicate the expected completion of other branches of physics, such as the completion of quantum computers into a covering notion, here suggested under the name of *EPR computers,* essentially consisting of electrons and other constituents in continuous and instantaneous communications through the overlapping of their wavepackets, by therefore approaching the new notion of living organisms attempted in Ref. [32], with the following expected advances:

1) Faster computations, due to fact that all possible values of the isotopic element (3) are very small according to all available fits of experimental data [14], with ensuing rapid convergence of isoperturbative series (see also Corollary 3.7.1, page 128 of Ref. [12]). As a confirmation of this expectation, we recall the achievement via isomathematics and isochemistry of an *attractive* force between the *identical* electrons of valence coupling (see Chapter 4 of Ref. [33]) resulting in a *strong valence bond* that allowed the exact representation of the experimental data for the hydrogen [34] and water [35] molecules with isoperturbative calculations at least one thousand times faster than their conventional chemical counterparts.

2) Better cybersecurity, due to the formulation over isonumeric isofields, with consequential natural availability of isocryptograms equipped with an algorithm changing the numeric value of the isounit with such a frequency to prevent the achievement of a solution within a finite period of time (see Appendix 2C, Vol. I , page 84 of Ref. [18]).

3) Increased efficiency, due to the fact that EPR entanglements are caused by *interactions without potential energy,* thus being more energy efficient than the potential energy dependent quantum computers.

In closing, there seems to be grounds for a new physics, with expected corresponding advances in chemistry and biology, via the axiom-preserving completion of the Copenhagen simplest possible realization of quantum axioms into their broadest possible realization suggested by hadronic mechanics [14].

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