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Mathematical, physical and chemical studies on Einstein's prediction that 'quantum mechanics is not a complete theory, " I: Basic methods

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Abstract. A. Einstein, B. Podolsky and N. Rosen expressed in 1935 their historical view that quantum mechanics could be "completed" into such a form recovering classical determinism at least under limit conditions (*EPR argument*). Following decades of preparatory studies, R. M. Santilli published in 1998 a paper showing that the historical objections against the EPR argument (such as Bell's inequality, von Neumann's theorem, et al.) are indeed valid for point-like particles in vacuum under linear, local and potential interactions (*exterior dynamical systems*), but the same objections are inapplicable (rather than being violated) for extended particles within hyperdense physical media under additional non-linear, non-local and non-potential interactions (*interior dynamical systems*) because the latter systems appear to admit an identical classical counterpart when treated with the isotopic branch of hadronic mechanics and its underlying isomathematics. In a more recent paper, Santilli has shown that quantum uncertainties of extended particles appear to progressively tend to zero when in the interior of hadrons, nuclei and stars, and appear to be identically null at the limit conditions in the interior of gravitational collapse, essentially along the EPR argument. In this first paper, we review, upgrade and specialize the basic mathematical, physical and chemical methods for the study of the EPR argument. In the subsequent second paper, we review the above results and provide specific illustrations of the important vision by Einstein, Podolsky and Rosen.

Key words: EPR argument, isomathematics, isomechanics

1. INTRODUCTION.

1.1. The EPR argument.

As it is well known, quantum mechanics does not admit classical precision in the measurement of the position or the mutual distance of particles (Figure 1) in view of Heisenberg's uncertainty principle and other physical laws.

Figure 1: *In this figure, we present a conceptual rendering of the sole representation of particles permitted by the differential calculus underlying quantum mechanics, namely, the representation as isolated points in empty space which particles, being dimensionless, can only be at a distance, with ensuing EPR argument on the need for superluminal interactions to explain quantum entanglement [1].*

Albert Einstein did not accept this uncertainty as being final for all possible conditions existing in the universe and made his famous quote *God does not play dice with the universe.*

More specifically, Einstein accepted quantum mechanics for atomic structures and other systems of point-like particles in vacuum (conditions known as *exterior dynamical problems*), but believed that quantum mechanics is an "incomplete theory," in the sense that it=could admit a "completion" into such a form to recover classical determinism at least under limit conditions.

Einstein communicated his view to the post doctoral associates, B. Podolsky and N. Rosen at the Institute for Advanced Study, Princeton, NJ, and all three together published in the 1935, May 15th issue of the Physical Review paper entitled *"Can Quantum Mechanical Description of Physical reality be Considered Complete?"* which paper became known as the *EPR argument* [1].

Soon after the appearance of paper [1], N. Bohr published paper [2] expressing a negative judgment on the possibility of "completing" quantum mechanics along the EPR argument.

Bohr's paper was followed by a variety of papers essentially supporting Bohr's rejection of the EPR argument, among which we recall *Bell's inequality* [3] establishing that the $SU(2)$ spin algebra does not admit limit values with an identical classical counterpart.

We should also recall *von Neumann theorem* [4] achieving a rejection of the EPR argument via the uniqueness of the eigenvalues of quantum mechanical Hermitean operators under unitary transforms.

The field became known as *local realism* and was centered on the rejection of the EPR argument on claims of *lack of existence of hidden variables* λ *[5] in quantum mechanics* (see the review [6] with a comprehensive literature).

Nowadays, the EPR argument is generally ignored in view of the wides-pread belief that quantum mechanics is universally valid for whatever conditions may exist in the universe without any scrutiny of the limitations and/or insufficiencies of quantum mechanics in various fields reviewed in this section.

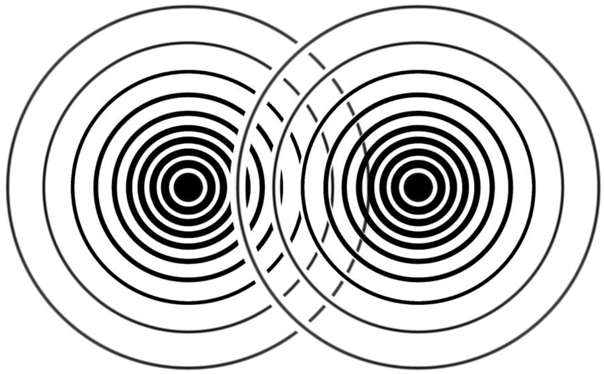

Figure 2: *A conceptual rendering of the main assumption of the apparent proofs [7] [8] of the EPR argument [1], consisting in the representation of particles as extended, deformable and hyperdense in conditions of mutual overlapping with ensuing continuous contact at a distance eliminating the need for superluminal interactions to explain quantum entanglement. Despite its simplicity, the quantitative treatment of the continuous entanglement here considered required decades of studies due to the need for a "completion" of the mathematics underlying quantum mechanics, with particular reference to the "completion" of the Newton-Leibnitz differential calculus, from its historical form solely defined at isolated points, into a covering form defined on volumes [115]. Intriguingly, the "completion" here considered turned out to be of isotopic/axiom-preserving type, thus being fully admitted by quantum mechanical axioms, merely subjected to a realization broader than that of the Copenhagen school.*

1.2. Apparent proofs of the EPR argument.

In Vol. 50, pages 177-190, 1998, of *Acta Applicandae Mathematica,* R. M. Santilli published paper [7] entitled "Isorepresentation of the Lie-isotopic $SU(2)$ Algebra with Application to Nuclear Physics and Local Realism," which paper appears to confirm Einstein's view on the existence of a "completion" of quantum mechanics into the *isotopic branch of hadronic mechanics,* or *isomechanics* for short and a "completion" of quantum chemistry into a form known as *isochemistry.* These "completions" are based on a broadening of applied mathematics known as *isomathematics* and admit progressive conditions of particles in the interior of hadrons, nuclei, stars and black holes that appear to recover classical determinism.

The proof presented in paper [7] was done via the following three main steps:

1.2.1. The proof that Bell's inequality, von Neumann's theorems and other similar objections of the EPR argument [6] are indeed correct, but under the generally tacit assumptions:

A) The point-like approximation of particles moving in vacuum (Figure 1);

B) The sole admission of Hamiltonian interactions [18];

C) The treatment of assumptions A and B via 20th century applied mathematics, including Lie's theory and the Newton-Leibnitz differential calculus;

1.2.2. The proof that the above treatments are not applicable to Einstein's vision on the existence of a "completion" of quantum mechanics based on the following assumptions:

A′) The representation of extended, therefore deformable and hyperdense particles under conditions of mutual penetration/entanglement known (Figure 2) as occurring in

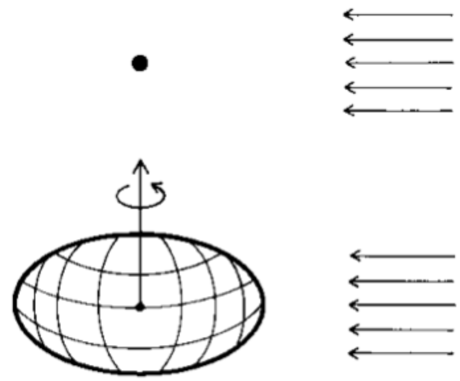

Figure 3: *An illustration of the central objective for the proof of the EPR argument consisting in the transition from the quantum mechanical representation of the Newtonian notion of massive points moving in vacuum under linear, local and potential interactions (top view), to the time invariant representation of extended particles moving within physical media under linear and non-linear, local and non-local and potential as well as non-potential interactions (bottom view).*

the structure of hadrons, nuclei, stars and black holes (systems known as *interior dynamical problems*);

B′) The emergence under condition A′ of Hamiltonian as well as contact non-Hamiltonian interactions of non-linear, non-local and non-potential character;

C′) The treatment of assumptions A′ and B′ via isomathematics that, as we shall see in Section 2, is based on:

i) The axiom-preserving isotopy $ab = ab \rightarrow a \star b = a\hat{T}b$ of the associative product ab between generic quantities a, b (numbers, functions, operators, etc.), where $\hat{T}$ is a positive-definite quantity called the *isotopic element* representing the dimension, deformability and density of particles;

ii) The ensuing axiom-preserving "completion" of Lie's theory with isotopic product $[x\hat{,}y] = x \star y - y \star x$ between Hermitean operators x, y;

iii) The reconstruction of the 20th century applied mathematics into a form compatible with isoproduct $a \star b$, including most importantly the isotopic lifting of the Newton-Leibnitz differential calculus from its centuries old definition at isolated points to its definition in the volumes of particles represented by $\hat{T}$.

1.2.3. The proof that the Lie-isotopic $\hat{SU}(2)$ algebra with isoproduct $[x\hat{,}y]$ admits limit conditions with an identical classical counterpart.

More recently, R. M. Santilli completed the above proof in paper [8] by showing that, under the above indicated conditions, the standard deviations for coordinates Δr and momenta Δp appear to progressively tend to zero for extended particles within hadrons, nuclei and stars, and appear to be identically null for extended particles within the limit conditions in the interior of gravitational collapse, essentially along Einstein's vision.

It should be noted that the above proofs of the EPR argument are centered in the *preservation* of the basic axioms of quantum mechanics, only submitted to their broadest possible *realization.*

It should be also noted that, under said broadest possible realization, quantum axioms do admit an *explicit and concrete realization of hidden variables* embedded in the structure of the Lie-isotopic product $[x\hat{,}y] = x\hat{T}y - y\hat{T}x$, for instance, via realization

Figure 4: *A first illustration of the lack of "completion" of quantum mechanics beyond scientific doubt is the time reversal invariance of the theory with equal probability for events forward and backward in time. Such a time reversibility is acceptable for atomic structures, particles in accelerators, crystals and other reversible systems, but it does not allow a consistent physical or chemical representation of energy-releasing process, such as the coal burning depicted in this figure. In fact, the time reversal image of coal burning implies that smoke must reconstruct coal with evident violation of causality.*

$\hat{T} =, Diag.(\lambda, 1/\lambda, \ \ Det\hat{T} = 1$ [7].

1.3. Insufficiencies of quantum mechanics for irreversible processes.

One of the insufficiencies of quantum mechanics well known since the 1930's is the inability to represent physical or chemical energy releasing processes, such as nuclear fusions, fuel combustion and other processes.

This is due to the fact that energy releasing processes are *irreversible over time* (namely, the time reversal image of the processes violates causality), while quantum axioms were conceived for and remain solely applicable to *systems reversible over time* (namely, the time reversal image of the systems verifies causality), such as atomic structures, particles in accelerators, crystals and other systems (Figure 4).

It is hoped that the ongoing alarming deterioration of our environment, with the consequential need for *new* clean energies, illustrate the need for a "completion" of quantum mechanics for irreversible processes outlined for completeness in Section 2.

It should be indicated that, except for the short presentation in Section 2, this and the following paper are solely devoted to the apparent proofs of the EPR argument for reversible interior systems, while the study for the broader irreversible interior systems is done elsewhere (see Refs. [9] to [83]).

This is due to the that the objections against the EPR argument were formulated for reversible exterior systems. Consequently, proofs [7] and [8] studied in these papers were formulated for reversible interior systems.

1.4. Insufficiencies of quantum mechanics in particle physics.

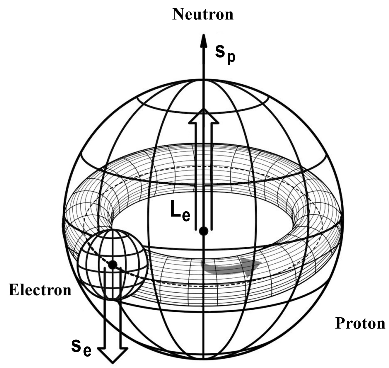

Figure 5: *A conceptual rendering of the synthesis of the neutron as a "compressed hydrogen atom" in the core of stars according to H. Rutherford [84], which synthesis cannot be represented by quantum mechanics due to a mass excess and other reasons (see Section 1.4). By contrast, all characteristics of the neutron in said synthesis have been represented at the non-relativistic and relativistic levels by the "completion" of 20th century applied mathematics and physics studied in this paper (Refs. [85] [95]).*

Quantum mechanics is justly considered to be *exactly valid* for the structure of the hydrogen atom because it achieved a numerically exact representation of *all* experimental data for the system considered.

It is hoped serious scholars will admit that quantum mechanics cannot be considered as being exactly valid for particle physics because of the known inability to achieve an exact representation of *all* experimental data of any given family of particles, despite the admission of a plethora of hypothetical neutrinos and other *ad hoc* manipulations.

Additionally, with the exception of the proton, all remaining strongly interacting particles (hadrons) are *unstable,* thus characterizing *irreversible systems* requiring Lie-admissible formulations for their correct treatment under which Einstein's lack of completion of quantum mechanics is beyond scientific doubt.

Finally, quantum mechanics is completely inapplicable (rather than violated) for the most fundamental synthesis in nature, that of the neutron from a proton and an electron as occurring in the core of stars [84], with ensuing inapplicability to the synthesis of other particles, such as that of the π^0 meson from the positronium [17].

This is due to the fact that quantum mechanical axioms have been conceived for the synthesis of particles in which the mass of the final state is *smaller* than the sum of the masses of the original constituents, resulting in the well known *mass defect* caused by *negative potentials.*

By contrast, the mass of the neutron $E_n = 939.565\ MeV$ is $0.782\ MeV$ *bigger* than the sum of the masses of the proton $E_p = 938.272\ MeV$ and of the electron $E_e = 0.511\ MeV$, resulting in a *mass excess* requiring a *positive potential* for which Schrödinger, Dirac and other quantum mechanical equations admit no physically meaningful solutions, with similar cases occurring for the synthesis of other particles [17].

The inability by quantum mechanics to represent the fundamental synthesis of the neutron in a star is ultimately due to the *point-like characterization of particles* since it is mathematically and physically impossible to fuse together two point-like particles (the proton and the electron) into a third point-like particle (the neutron).

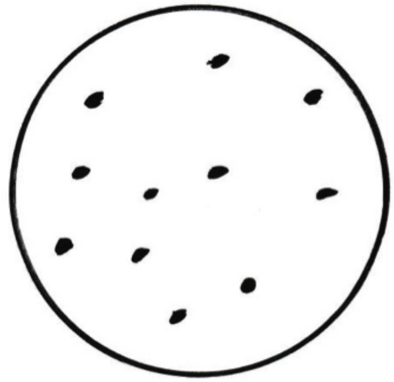

Figure 6: *The mathematics underlying quantum mechanics is local-differential, thus solely admitting a point-like approximation of particles. This figure illustrates the consequential conception of nuclei as ideal spheres with isolated points in their interior with ensuing insufficiencies beyond scientific doubt.*

In turn, this insufficiency identified the need for the representation of hadrons as extended, deformable and hyperdense, which representation is at the foundation of the EPR proof [7].

Another insufficiency of quantum mechanics is its sole capability of representing *linear interactions* (i.e., interactions linear in the wave functions). By contrast, the sole known possibility of achieving a bound state with excess mass is that via the admission of non-linear interactions for extended particles in conditions of mutual penetration/entanglement, as it is the case for the electron when totally compressed inside the proton [16] [17].

In turn, non-linear interactions are crucial for the "completion of the wavefunction" advocated by Einstein, Podolsky and Rosen [1] as we shall see in Paper II of this series.

It is nowadays known that the above insufficiencies originate from Lie's theory at the foundation of quantum mechanics because said theory solely admits linear interactions [19].

It is hoped that the above insufficiencies illustrate the significance of the "completion" of Lie's theory used for the proof of the EPR argument [7].

In fact, only following the achievement of a "completion" of 29th century mathematical and physical methods for extended, deformable and hyperdense particles in interior dynamical conditions, R. M. Santiili achieved a numerically exact representation of *all*—characteristic of the neutron in its synthesis from a proton and an electron at the non-relativistic (Refs. [85] to [87]), relativistic (Refs. [88] to [89]) and experimental (Refs. [90] to [95]) levels (see Sections 2, 3 and Paper II of this series).

1.5. Insufficiencies of quantum mechanics in nuclear physics.

There is no doubt that quantum mechanics has permitted historical achievements in nuclear physics.

However, quantum mechanics is only *approximately valid* in nuclear physics because of the inability to achieve in about one century a representation of the characteristics of

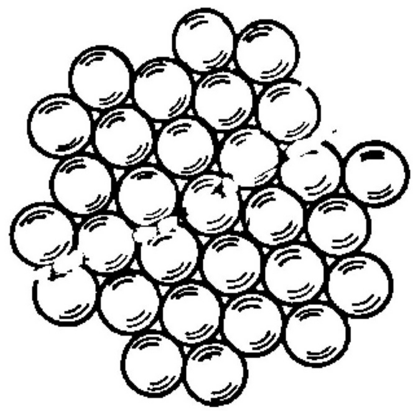

Figure 7: *A conceptual rendering of nuclei as they occur in the physical reality, i.e., a collection of extended and hyperdense protons and neutrons in conditions of partial mutual penetration, with ensuing non-linear, non-local and non-potential interactions beyond any dream of quantitative treatment via quantum mechanics.*

the simplest nucleus, the deuteron, with embarrassing deviations of the prediction of the theory from experimental data for heavier nuclei, such as the Zirconium [59].

In Santilli's view, the primary reason for the indicated insufficiency is that the *mathematics* underlying quantum mechanics, with particular reference to the *Newton-Leibnitz differential calculus,* imply the conception of nuclei as ideal spheres with isolated points in its interior (Figure 6) while in the physical reality, nuclei are composed by extended and hyperdense protons and neutrons in conditions of partial mutual penetration established by the comparison of nuclear volumes with the constituent volumes [59] (Figure 7).

The inability to represent nuclei as they are in the physical reality implies the inability to achieve an exact representation of nuclear magnetic moments. In fact, the quantum mechanical representation of the anomalous magnetic moment of the deuteron still misses $1\ \%$ despite all possible relativistic or quark-based corrections.

Additionally, quantum mechanics misses much bigger percentages of nuclear magnetic moments for heavier nuclei (as illustrated in Figure 8).

In turn, the inability to represent protons and neutrons as extended charge distributions implies the inability to represent deformations under strong nuclear forces, with related deformation of their angular momenta.

In fact, J. M. Blatt and V. F. Weisskopf state on page 31 of their treatise in nuclear physics [96]: *It is possible that the intrinsic magnetism of a nucleon is different when it is in close proximity to another nucleon* (Figure 9).

The representation of nucleons as extended, thus deformable and hyperdense charge distributions via the "completion of 20th century mathematics (Section 2) and physics (Section 3) has permitted the exact representation of the anomalous magnetic moment of the deuteron [97], as well as of heavier nuclei [98].

It should be noted that the representation of nuclear magnetic moments is presented in Ref. [7] as an illustration of the implications of the proof of the EPR arguments for extended particles in interior conditions.

A second insufficiency of quantum mechanics in nuclear physics is the lack of a consistent representation of nuclear spins despite efforts also conducted for about one century.

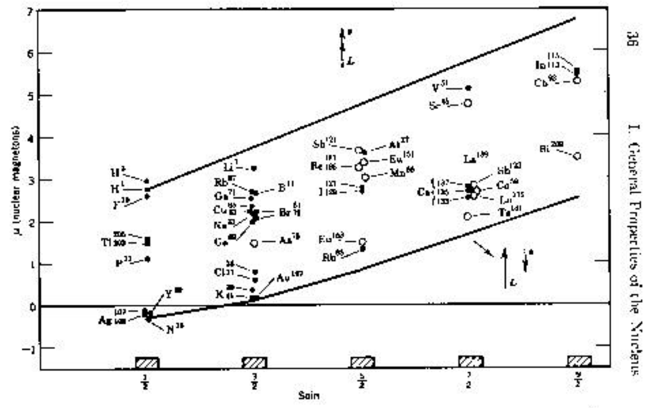

Figure 8: *A view of the experimental data of nuclear magnetic moments that cannot be exactly represented by quantum mechanics. Similar insufficiencies exist for nuclear spins.*

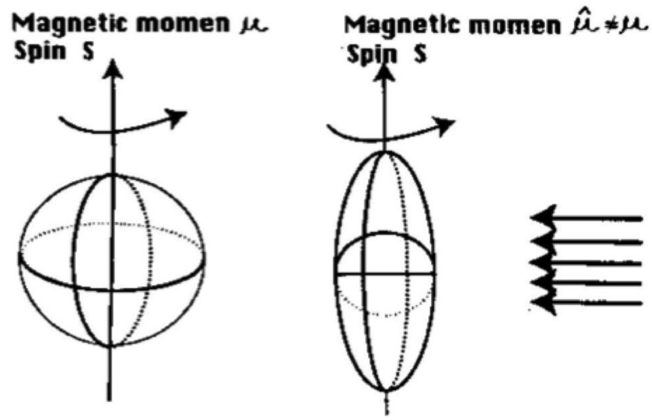

Figure 9: *A conceptual rendering of the deformability of protons and neutrons under strong nuclear interactions predicted by J. M. Blatt and V. F. Weisskopf [96] as being the origin of the inability by quantum mechanics to represent nuclear magnetic moments.*

Recall that the proton and the neutron have both spin $1/2$ and that the only stable bound state predicted by quantum mechanics between two particles with spin $1/2$ is the singlet with spin 0.

Therefore, quantum mechanics predicts that the deuteron in its ground state must have spin 0, while experimental data establish that the deuteron has spin 1.

In an attempt of salvaging quantum mechanics, the spin of the deuteron is generally represented via a combination of *excited orbital states* which, even though significant, does not represent the spin 1 of the deuteron *in its ground state*.

The achievement of the synthesis of the neutron (Refs. [85] to [95]) has permitted a resolution of the above impasse because *the deuteron emerges as being a three-body state composed by two protons and one exchange electron, with ensuing spin* 1 *in the ground state* [59].

Subsequent studies by A. A. Bhalekar and R. M. Santilli [100] based on the "completion" of 20th century mathematical and physical methods have achieved a representation of the spin of stable nuclei in their bound state.

It should be finally noted that the biggest insufficiency of quantum mechanics in nu-

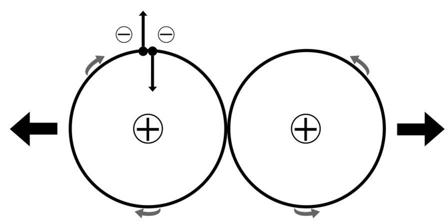

Figure 10: *This picture depicts the hydrogen molecule at absolute zero degree temperature to illustrate the fact that, despite historical achievements, quantum mechanics and chemistry have been unable to identify the force attracting identical electrons in valence bonds, because the sole admitted forces are of potential-Coulomb type, thus implying a repulsion in valence bonds with ensuing lack of a "complete" representation of molecular structures [60].*

clear physics is given by the inability to achieve a consistent representation of nuclear forces in one century of efforts.

This is due to the fact that the sole forces permitted by quantum mechanics are of *potential,* thus of action- at-a-distance type, which is solely possible, mathematical and physical conception for nuclei as ideal spheres with point-like particles in their interior (Figure 6).

One of the most important applications of the new methods studied in this work is that of representing nuclear forces as being non-linear, non-local and non-potential forces due to the mutual penetration/entanglement of the charge distribution of the hyperdense nucleons. The latter forces have emerged as being strongly attractive thus allowing the first known initiation of the understanding of the charge independence of nuclear forces [59].

1.6. Insufficiencies of quantum mechanics in chemistry.

Without doubt, quantum mechanics and chemistry have permitted chemical discoveries of historical proportions. Hence, the historical and scientific value of quantum chemistry is out of question.

Yet, it is the fate of all theories to admit, with the advancement of scientific knowledge, suitable coverings and this is the fate of quantum chemistry as well.

In fact, on strict scientific grounds quantum chemistry is only *approximately valid* in chemistry, thus admitting a suitable "completion," because of the inability in one century of efforts to achieve an exact representation of molecular experimental data from first axiomatic principles without *ad hoc* form factors and other adaptations.

Recall that quantum mechanics has achieved a numerically exact representation of all experimental data of the hydrogen atom.

By contrast, when two hydrogen atoms are bonded into the hydrogen molecule H_2, quantum mechanics and chemistry still miss the representation of $1\ \%$ of the H_2 binding energy, which is not insignificant since it corresponds to about $940\ kcal/mole$.

An in depth study of the impasse has shown that the above insufficiency is due to *the inability by quantum mechanics and chemistry to represent the attractive force between identical valence electrons in molecular bonds* (Figure 10) because according to basic axioms, *two identical valence electrons must repel each other according to quantum mechanics and chemistry due to their equal charge* [60].

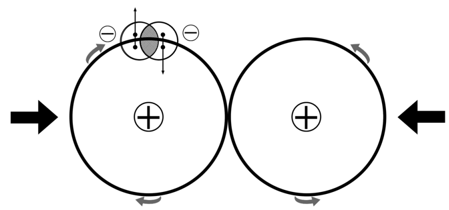

Figure 11: *This picture depicts the hydrogen molecule at absolute zero degree temperature with the strongly attractive force between identical valence electrons permitted by the representation of valence pairs as being composed by electrons with extended wavepackets in conditions of mutual penetration/entanglement with ensuing non-linear, non-local and non-potential interactions that result to be so strongly attractive [17] to overcome repulsive Coulomb forces [60]. The mutual distance d of said valence pair approaches Einstein's classical determinism and achieves it fully when it is in the interior of black holes [7].*

As it had been the case for other problems, the primary difficulty to achieve an attractive force between identical valence electrons was of *mathematical* rather than of physical or chemical character, because quantum mechanics and chemistry solely admit potential forces that, in this case, can only be of repulsive Coulomb type.

By contrast, the sole possibility of resolving the impasse was the representation of electrons as extended wavepackets, that when in conditions of mutual penetration at $1\ fm$ mutual distance, admit contact, non-linear, non-local and non-potential interactions of Hulten type.

These new interactions result to be so strong to "absorb" repulsive Coulomb forces resulting in the needed attraction [60] (Figure 11).

The new valence force was first identified in Table 5 of the 1978 paper [17] as responsible for the birth of strong interactions in the synthesis of the π^0 meson from an electron and a positron.

In 1995, A. O. E. Animalu and R. M. Santilli published paper [101] establishing that the Hulten force of paper [17] is so strong to account for the bond of the two identical electrons in the Cooper pair of superconductivity.

A generalization of superconductivity based on the new methods was then developed and it is today known as *Animalu isosuperconductivity* [102].

In 2000, R. M. Santilli and D. D. Shillady showed that valence electron pairs with a strongly attractive force, called *isoelectronia,* permit numerically exact representations from first axiomatic principles of experimental data on the hydrogen [103] and water [104] molecule, which representation had escaped quantum chemistry for about one century (see also review [105]).

The achievement of a new model of molecular structures based on the isoelectronium valence bond has permitted novel advances in larger molecules whose study has been initiated by A. A. Bhalekar and R. M. Santilli [106] with intriguing implications, e.g., possible improvements in the combustion of fossil fuels based on a more accurate representation of their molecular structure [53].

Additionally, Santilli and Shillady showed that *perturbative series of the resulting "completion" of quantum chemistry converge at least one thousand times faster than the corresponding series of quantum chemistry* (see Section 4.13).

1.7. Implications of the EPR argument.

It is hoped that the preceding sections have indicated the truly vast implications for all quantitative sciences of Einstein's view that "quantum mechanics is not a complete theory" [1], thus warranting due scientific process.

In this paper we outline the main aspects of the new mathematical and physical methods underlying proof [7], with the understanding that a technical knowledge can be solely achieved via a study of the original literature.

The reader should be aware that the literature accumulated in half a century of research in the field by numerous scientists is rather vast. Consequently, in this paper we can only quote the most important original contributions and provide comprehensive references for interested readers.

2. LIE-ADMISSIBLE "COMPLETION" OF 20TH CENTURY APPLIED MATHEMATICS.

2.1. Foreword.

R. M. Santilli never accepted quantum mechanics as a "complete" theory beginning with his graduate studies in physics at the University of Torino, Italy, in the mid 1960's because quantum axioms are invariant under time reversal, due to the invariance under anti-Hermiticity of the Lie product between Hermitean operators

$$[x, y] = -[x, y]^{\dagger}, \tag{1}$$

and other physical laws.

By recalling the fundamental character of Lie's theory, it follows that the *mathematics* (more than physical laws) underlying quantum mechanics does not allow a consistent representation of nuclear fusions and other physical or chemical energy releasing processes, due to their known irreversibility over time (Figure 4).

2.2. The historical teachings by Lagrange and Hamilton.

In view of the above lack of "completeness" of quantum mechanics, R. M. Santilli initiated his Ph. D. studies with the reading of the original works by J. L. Lagrange and studying his true analytic equations, those with external terms [9]

$$\frac{d}{dt}\frac{\partial L(r,v)}{\partial v} - \frac{\partial L(r,v)}{\partial r} = F_{ak}(t,r,v), \tag{2}$$

as well as the true Hamilton's equations, those with external terms [10]

$$\begin{aligned} \tfrac{dr}{dt} &= \tfrac{\partial H(r,p)}{\partial p}, \\ \tfrac{dp}{dt} &= -\tfrac{\partial H(r,p)}{\partial r} + F(t,r,p), \end{aligned} \tag{3}$$

where the Lagrangian L and Hamiltonian H were used to represent conservative and potential, thus notoriously reversible forces, while the irreversibility of nature was represented with their *external forces* F.

2.3. The "No Reduction Theorem."

The external terms has been truncated in the 20th century sciences on claims that irreversible systems can be decomposed into their elementary particle constituents at which level the validity of quantum mechanics is fully recovered.

However, Santilli proved the following theorem as part of his Ph. D. thesis (see Refs. [56] [25]).

THEOREM 2.3.1 (No reduction Theorem): A macroscopic time irreversible system cannot be consistently decomposed into a finite number of quantum mechanical particles and vice versa, a finite collection of quantum mechanical particles cannot reproduce a macroscopic irreversible system under the correspondence or other principle.

Consequently, a serious study of irreversible systems requires a return to the true Lagrange's and Hamilton's equations, those with external terms.

2.4. The inevitability of the Lie-admissible "completion" of 20th century science.

As it is well known, the true Lagrange and Hamilton equations cannot be assumed as a "completion" of 20th century sciences because they are not derivable from a potential.

Additionally, the brackets of the time evolution of an observable Q represented via Hamilton's equations with external terms

$$\begin{aligned} \tfrac{dA}{dt} &= (Q, H, F) = \\ &= \tfrac{\partial Q}{\partial r}\tfrac{\partial H}{\partial p} - \tfrac{\partial H}{\partial r}\tfrac{\partial Q}{\partial p} + \tfrac{\partial Q}{\partial r}F, \end{aligned} \tag{4}$$

characterizes the *triple system* (Q, H, F) that, in view of the external terms, violate the right scalar and associative axioms to characterize an algebra as currently understood in mathematics.

In the absence of a consistent algebra in the brackets of the time evolution, it was not possible to achieve a "completion" of quantum mechanics via a covering for irreversible systems.

Hence, Santilli was forced to seek the needed "completion" on algebraic grounds.

Following a year of research in the European mathematics libraries, Santilli did his Ph. D. thesis in 1965 on the "completion" of Lie algebras into A. A. Albert's *Lie-admissible and Jordan-admissible algebras* [11] with product [12]

$$(a, b) = pab - qba, \tag{5}$$

later on known as *(p, q)-deformations [13]*, where $p,\ q, p \pm q$ are non-null scalars, and time irreversibility is assured for $p \neq q$ for which irreversiblity is ensured for $p \neq q$ by the property $(x, y) \neq -(x, y)^\dagger$.

To achieve a first approximation of Hamilton's equations with external terms, Santilli introduced the following *parametric Lie-admissible generalization of Hamilton's equation* [14] [15]

$$\frac{dr}{dt} = p\frac{\partial H(r,p)}{\partial p}, \quad \frac{dp}{dt} = -q\frac{\partial H(r,p)}{\partial r}, \tag{6}$$

with corresponding *parametric Lie-admissible generalization of Heisenberg equation* for the time evolution of a Hermitean operator Q (for $\hbar = 1$)

$$i\frac{dQ}{dt} = (Q, H) = pQH - qHQ. \tag{7}$$

As one can see, all dynamical equations are manifestly irreversible over time, as desired.

2.5. Lie-admissible genomathematics and genomechanics.

In September 1977, Santilli joined the Department of Mathematics of Harvard University under DOE support during which stay he introduced the most general known realization of irreversible Lie-admissible algebras (see Refs. [16] to [23]) based on the generalization and differentiation of the ordinary product ab of arbitrary quantities (numbers, functions, operators, etc.) into the *ordered genomodular product to the right*

$$a > b = a\hat{R}b, \tag{8}$$

and that to the left

$$a < b = a\hat{S}b, \tag{9}$$

where $\hat{R}, S$ and $R \pm S$ are positive-definite operators with an unrestricted functional dependence on wavefunctions $\psi(t, r)$ and any other needed variables.

The operators R and S were called *genotopic element to the right and to the left,* respectively, where the prefix "geno" was suggested by Carla Santilli in the Greek sense of "inducing a new structure" [16].

The new genomodular products permitted the construction of new mathematics known as *genomathematics to the right and to the left,* [19] with corresponding "completion" of quantum mechanics into an irreversible covering known as *genotopic branch of hadronic mechanics* or *genomechanics for short* [19] in which irreversible energy releasing processes are represented with ordered genomodular product to the right, as it is the case for the *geno-Schrödinger equation* or *Schrödinger-Santilli genoequation* [20]

$$H > \psi = H(r, p)R(\psi, ...)\psi = E\psi, \tag{10}$$

while processes characterized by genoproducts to the left are prohibited by causality.

The time evolution of a Hermitean operator Q is given by the *Lie-admissible generalization of Heisenberg equations,* also known as *Heisenberg-Santilli genoequations* (see, e.g., Ref. [107]) first introduced in Eqs. (4.15.34), page 746 of Ref. [17] (see the 2006 general treatment [24] and the 2016 update [25]) which can be written in the infinitesimal form

$$\begin{aligned} i\tfrac{dQ}{dt} &= (Q.H) = Q < H - H > Q = \\ &= QSH - HRQ, \end{aligned} \tag{11}$$

and the finite form

$$Q(t) = e^{HRti}Q(0)e^{-itSH}. \tag{12}$$

2.6. Universality of Lie-admissible formulations.

The following simple realization of the genotopic elements

$$\begin{aligned} S = 1, \quad R &= 1 - \tfrac{1}{H}K(\psi, \partial\psi, ...), \\ i\tfrac{dQ}{dt} &= (Q.H) = [Q, H] + QK, \end{aligned} \tag{13}$$

where K is a positive-definite operator representing non-Hamiltonian interactions illustrates that the Lie-admissible generalization (11) of Heisenberg's equations constitute an operator image of Hamilton's equations with external terms (3) [56].

The double infinity of possible realizations of the genotopic elements R and S then allows Lie-admissible equations (11) to be "directly universal" for the representation of all possible (regular) non-linear, non-local and non-Hamiltonian interactions in the sense of representing all of them ("universality") directly in the frame of the experimentalism without the use of the transformation theory ("direct universality") (for details, see Ref. [24]).

It should be finally indicated that the original 1978 proposal [16] established the universality of Lie-admissible algebras because the product $(A, B) = A < B - B > A$ admit as particular case the product of all possible "algebras" as commonly understood in mathematics, including Associative, Lie, Jordan, Lie-isotopic, Jordan-isotopic, alternative, super-associative, super-Lie, super-Jordan, nilpotent, flexible and any other possible algebra.

2.7. Prediction of new clean nuclear fusions.

Scientific and industrial applications to new clean energies were initiated in the late 1990's only following the achievement of maturity in the mathematical and physical methods needed for the representation of irreversible processes.

The first application was the conception, theoretical elaboration and experimental verification of the new *Intermediate Controlled Nuclear Fusion* (ICNF) of light, natural and stable elements into light, natural and stable elements with smaller mass which occur without the emission of harmful (e.g., neutron) radiations and without the release of radioactive waste (see Ref. [26] to [32] for originating papers; Refs. [33] to [40] for independent studies and verifications; and Refs. [41] to [52] for laboratory analyses quoted in the preceding works).

2.8. Prediction of a new clean combustion of fossil fuels.

To illustrate the implications of the lack of "completion" of quantum mechanics for energy releasing processes, we should note that the current combustion of fossil fuels is essentially that at the dawn of our civilization, because we essentially strike a spark and ignite the fuel with known alarming environmental deterioration of our planet.

The achievement of the Lie-admissible representation of energy releasing processes has permitted the first known conception and initiation of tests for a new principle of combustion called *HyperCombustion* which is based on the conventional combustion of carbon and oxygen dating back to the dawn of our civilization, plus the novel synthesis of a limited number of nuclei C-12 and O-16 into Si-28 to achieve full combustion of fossil fuels as well as a significant increase of energy output [53].

2.9. Literature on hadronic mathematics and mechanics.

Due to the prediction of new clean nuclear energies, the connection between irreversible mechanics and thermo dynamics and other features, the literature on the foundations of hadronic mechanics is rather vast.

Ref. [54] provides a summary of the complete formalism of hadronic mechanics. Refs. [55] to [61] provide general presentations of hadronic mechanics. Vol. I of Refs. [61] contains a comprehensive literature up to 2008 with an upgrade to 2016 in Ref. [25].

Additional references are available in the reprint volumes [62] [63] and in the proceedings of five *Workshops on Lie-Admissible Algebras,,* twenty five *Workshops on Hadronic Mechanics,* and three international conferences on the *Lie-admissible treatment of Irreversible Processes* whose references are available from Ref. [61]. Representative independent papers are available from Refs. [64] to [74] and independent monographs are available from Refs. [75] to [83].

3. LIE-ISOTOPIC "COMPLETION" OF 20TH CENTURY APPLED MATHEMATICS.

3.1. Foreword.

As it is well known, the objections against the EPR argument (Section 1.1) were formulated for *isolated reversible systems of point particles in vacuum with linear, local and Hamiltonian interactions* called *reversible exterior dynamical problems*.

Hence, the proof of the EPR argument had to study *isolated reversible systems of extended, therefore deformable and hyperdense particles in conditions of mutual penetration/entanglement with ensuing linear and non-linear, local and non-local and Hamiltonian as well as non-Hamiltonian internal interactions.* The latter systems are called *reversible interior dynamical problems*, and they occur in the structure of hadrons, nuclei, stars and black holes.

Despite their reversible character, the latter systems could not be studied with 20th century applied mathematics, including Lie's theory, due to its strictly Hamiltonian character. Reversible interior dynamical systems could not be studied with Lie-admissible formulations due to their irreversible character. Hence, the needed new mathematics had to be built.

In this section, we review the foundations of the new mathematics for the consistent representation of reversible interior dynamical systems which was essentially constructed as a reversible particular case of universal Lie-admissible formulations.

3.2. Isoproduct.

In order to achieve a representation of the latter systems, Santilli introduced in the original proposal [16] of 1978 the axiom-preserving particular case of genomathematics called *isomathematics,* which is characterized by the genoproduct to the right being equal to that to the left, $R = S = \hat{T}$, resulting in the time-reversal invariant *isoproduct,* (first introduced in classical realization in Eqs. (3.7.10), page 352 of Ref. [16], introduced in operator form in Eq. (4.15.46), page 751, Ref. [17] and then studied in details in Ref. [19], Eq. (2), page 71 on)

$$a \star b = a\,\hat{T}\,b, \tag{14}$$

where $\hat{T}$, called the *isotopic element,* is a function, matrix or operator solely restricted to be positive-definite, but possesses otherwise an unrestricted functional dependence on all needed local variables, such as: spacetime coordinates $x = (t, r)$; linear momentum p; energy E; frequency ν; density of the medium α; temperature τ; pressure π; wavefunctions ψ; their derivatives $\partial\psi$; and any other needed variable

$$\hat{T} = \hat{T}(x, p, E, \nu, \alpha, \tau, \pi, \psi, \partial\psi, ...) > 0, \tag{15}$$

where the prefix "iso" was also suggested by Carla Santilli in its Greek meaning of preserving the axions.

The *physical* significant of isoproduct (14) is illustrated by nothing that it allows "ab initio" a *direct representation of extended, thus deformable and hyperdense particles and their non-Hamiltonian interactions* illustrated in Figure 3 (Section 1.4). This important task is achieved via simple realizations of the isotopic element of the type (needed for the neutron synthesis from the hydrogen studied in Section 2.4) [55] [56]

$$\hat{T} = Diag.(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, \frac{1}{n_4^2})e^{-\Gamma}, \tag{16}$$

with subsidiary conditions

$$\begin{gathered} n_\mu = n_\mu(x, p, E, \nu, \alpha, \tau, \pi, \psi, \partial\psi, ...) > 0, \ \mu = 1, 2, 3, 4, \\ \Gamma(x, p, E, \nu, \alpha, \tau, \pi, \psi, \partial\psi, ...) \geq 0. \end{gathered} \tag{17}$$

The isoproduct also allows a direct representation of nuclei as a collection of extended nucleons in conditions of mutual penetration/entanglement as presented in Figure 7 and Section 1.5 with broader realizations of the type (needed to represent nuclear magnetic moments and spins, or for the achievement of an attractive force between identical valence electron bonds in molecular structures) [57]

$$\begin{gathered} \hat{T} = \Pi_{k=1,...,N} Diag.(\tfrac{1}{n_{1k}^2}, \tfrac{1}{n_{2k}^2}, \tfrac{1}{n_{3k}^2}, \tfrac{1}{n_{4k}^2})e^{-\Gamma}, \\ k = 1, 2, ..., N, \quad \mu = 1, 2, 3, 4. \end{gathered} \tag{18}$$

In the above realizations of the isotopic element, n_1^2, n_2^2, n_3^2, (called *characteristic quantities*) represent the deformable semi-axes of the particle normalized to the values $n_1^2 = n_2^2, = n_3^2 = 1$, for the sphere; n_4^2 represents the *density* of the particle considered normalized to the value $n_4 = 1$ for the vacuum; and Γ represents non-linear, non-local and non-Hamiltonian interactions caused by mutual penetrations/entanglement of particles.

The *mathematical* significance of basic assumption (14) is that it requires, for consistency, a compatible "completion" of *all* aspects of 20th century applied mathematics without any known exception.

This program was initiated in the 1978 proposal [16], continued in the 1981 monograph [19] and completed in numerous works by various mathematicians (see the 1995 monograph [55] for a comprehensive presentation).

Regrettably we cannot provide a technical review of isomathematics to prevent excessive length. Nevertheless, a rudimentary outline of the main aspects of isomathematics appears to be recommendable for an understanding of proof [7] of the EPR argument.

3.3. Isonumbers.

As it is well known, physical theories are formulated over a field $F(n, \times, 1)$ of real, complex or quaternionic numbers n with product $nm = n \times m$ and multiplicative unit 1. Said field remains invariant under the unitary time evolution of quantum mechanics, thus allowing the prediction of the same numerical values under the same conditions at different times.

But the time evolutions of hadronic mechanics, such as Eqs. (11), are *non-unitary* when formulated on conventional space over a conventional field (not so for isomathematics as shown below).

This implies the loss over time of the multiplicative unit 1, and consequently, of the entire numeric field, with ensuing lack of consistent experimental verifications.

To resolve this impasse, Santilli had no other choice than that of reinspecting the historical classification of numbers, by discovering in this way that *the abstract axioms of a numeric field do not necessarily restrict the multiplicative unit to be the number* 1*, and allow for unit an arbitrary positive-definite quantity* $\hat{I}$ *provided that the multiplication is redefined for* $\hat{I}$ *to verify the unit axiom* [107].

This lead to "completion" of numeric fields $F(n, \times, 1$ into *isofields* $\hat{F}(\hat{n}, \star, \hat{I}$ of *isoreal, isocomplex, and isoquaternionic isonumbers* $\hat{n} = n\hat{I}$ with *isounit*

$$\hat{I} = 1/\hat{T} > 0, \tag{19}$$

isoproduct (14), $\hat{n} \star \hat{m} = (nm)\hat{I}$, and isounit isoaxiom

$$\hat{I} \star \hat{n} = \hat{n} \star \hat{I} = \hat{n}, \ \ \forall \hat{n} \ \in \ \hat{F}. \tag{20}$$

Isofields are completed by compatible redefinitions of all numeric operations, such as *isoquotient, isosquare, isosquareroot,* etc. [107].

It should be indicated that *isofields verify all axioms of a field. Hence, isonumbers are fully acceptable for experimental verifications* [57].

Isofields are classified into those of the *first kind* (*second kind*) depending on whether the isounit $\hat{I}$ is (is not) an element of the original field.

In this paper, we shall solely consider isofields of the second kind since the representation of extended particles and their non-Hamiltonian interactions is achieved via the isounit or equivalently, the isotopic element.

Paper [107] has stimulated various studies in number theory, among which we mention the study by C-Xu. Jiang [79], A. K. Aringazin [108], C. Corda [109] and others.

3.4. Isofunctions.

As it is well known, a necessary condition for a variable to be measurable is that it is an element of the base field, and the same holds for functions of said variable.

The implementation of the same rule under isotopic "completion" stimulated the construction of the *isofunctional isoanalysis* initiated by: J. V. Kadeisvili [110][111]; A. K. Aringazin, D. A. Kirukhin and R. M. Santilli [112]; Raul M. Falcon Ganfornina and Juan Nunez Valdes [80]; and others.

We here limit ourselves to indicate: the *isotime* $\hat{t} = t\hat{I}_t$, *isospace isocoordinates* $\hat{r} = r\hat{I}_r$, and the *isofunctions of isovariable*,

$$\hat{f}(\hat{r}) = [f(r\hat{I})]\hat{I}, \tag{21}$$

such as the *isoexponentiation*

$$\hat{e}^{\hat{X}} = [e^{\hat{X}\hat{T}}]\hat{I} = \hat{I}[e^{\hat{T}\hat{X}}]. \tag{22}$$

Similar expressions hold for virtually all conventional functions used in applications [55].

3.5. Isospaces.

The initial construction of isomathematics [19] was formulated via conventional vector or metric spaces over conventional fields.

The consistent need to formulate spaces over isofields triggered the isotopic "completion" of metric spaces into *isospaces* whose study was initiated by the mathematician Gr. Tsagas and his school [113]. In turn, these studies triggered the construction of the *isotopology* by R. M. Falcon Ganfornina and J. Nunez Valdes [114], yielding the first known topology for the characterization of extended particles, known as the *Tsagas, Sourlas, Santilli, Ganfornina and Nunez (TSSGN) isotopology.*

Let $E(r, \delta, I)$ be the conventional Euclidean space with space coordinates $r = (x, y, z)$, metric $\delta = Diag.(1, 1, 1)$, unit $I = Diag(1, 1, 1)$ and invariant

$$r^2 = (x^2 + y^2 + z^2)1, \tag{23}$$

where one should note the trivial multiplication by 1 for compatibility with the isotopic image studied below.

The representation isospace of the non-relativistic proof of the EPR argument is given by the infinite family of *iso-Euclidean isospaces,* $\hat{E}(\hat{r}, \hat{\Delta}, \hat{I})_r$, first formulated in the 1978 Ref. [16] and treated in detail in Ref. [19] , finalized in 1996 Ref. [115] and extensively treated in monograph [55].

For the simple realization of the isotopic element

$$\hat{T} = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2), \tag{24}$$

iso-Euclidean isospaces $\hat{E}(\hat{r}, \hat{\Delta}, \hat{I}_r)$ are characterized by: the *isocoordinates* $\hat{r} = r\hat{I}$, the *iso-Euclidean isometric* $\hat{\Delta} = (\hat{T}\delta\hat{I}$, and the *isospace isounit* $\hat{I}_r = 1/\hat{T} > 0$ resulting in the *iso-Euclidean isoinvariant*

$$\begin{aligned} \hat{r}^{\hat{2}} = (\hat{r}^j \star \hat{\Delta}_{jm} \star \hat{r}^m) = (r^j \hat{\delta}_{jm} r^m)\hat{I}_r = \\ = (\tfrac{r_1^2}{n_1^2} + \tfrac{r_2^2}{n_2^2} + \tfrac{r_3^2}{n_3^2})\hat{I}_r, \end{aligned} \tag{25}$$

where we should recall that, for consistency, all scalar quantities have to be elements of an isofield $\hat{F}$.

The above conditions require that: squares must be isosquares $\hat{r}^{\hat{2}} = \hat{r} \star \hat{r} = \hat{r}^2\hat{I}_r$; coordinates have to be isocoordinates $\hat{r} = r\hat{I}_r$; to be isomatrices, isometrics must have the structure $\hat{\Delta} = \hat{\delta}\hat{I}_r$; and the elaboration requires the use of the *isotrigonometric isofunctions* as well as of the *isospherical isocoordinates* (see Ref. [55] for details).

Let $M(x, \eta, I)$ be the conventional Minkowski space with spacetime coordinates $x = (x^1, x^2, x^3, x^4 = ct)$, metric $\eta = Diag.(1, 1, 1, -1)$, unit $I = Diag(1, 1, 1, 1)$ and invariant

$$\begin{aligned} x^2 = (x^\mu \eta_{\mu\nu} x^\nu)I = \\ = (x_1^2 + x_2^2 + x_3^3 - c^2t^2)I. \end{aligned} \tag{26}$$

The isospace for relativistic treatments of extended particles is given by the infinite family of *iso-Minkowski spaces* $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I}_x)$ also known as *Minkowski-Santilli isospaces,* (see e.g., Ref. [107]) first introduced in Ref. [116] and then treated in details in Ref. [56].

Figure 12: *This picture illustrates the isodifferential calculus [115] via birds flying in close formation without wing interferences, which can be best understood by assuming that birds conceive themselves as a volume encompassing their wings, rather than a mass concentrated in their center of gravity as it would be requested by the Newton-Leibnitz differential calculus.*

Iso-Minkowskian isospaces are characterized by the *isospace-time isocoordinates* $\hat{x} = x\hat{I}$; isounit $\hat{I} = 1/\hat{T}$, and *isometric* $\hat{\Omega} = \hat{\eta}\hat{I}_x = (\hat{T}\eta)\hat{I}_x$ formulated on the isoreal isonumbers $\hat{\mathcal{R}}$.

For the simple realization of the isotopic element

$$\hat{T} = Diag(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2), \tag{27}$$

we have the infinite family of realizations of the *isospace-time isoinvariant*

$$\begin{aligned} \hat{x}^{\hat{2}} = \hat{x}^\mu \star \hat{\Omega}_{\mu\nu} \star \hat{x}^\nu = (x^\mu \hat{\eta}_{\mu\nu} x^\nu)\hat{I} = \\ = (\tfrac{x_1^2}{n_1^2} + \tfrac{x_2^2}{n_2^2} + \tfrac{x_3^2}{n_3^2} - t^2 \tfrac{c^2}{n_4^2})\hat{I}, \end{aligned} \tag{28}$$

where the final multiplication by the isounit is again necessary for the invariant to be an isoscalar.

It should be noted that, in addition to the use of the isospherical isocoordinates, data elaborations in the iso-Minkowskian isospace requires the use of *isohyperbolic isofunctions* (see Ref. [56], Chapters 5 and 6).

Note also that invariant (30) is the most general possible symmetric (non-singular) invariant in $(3+1)$-dimensions, thus including as particular cases all possible Minkowskian, Riemannian, Fynslerian and all other geometries.

3.6. Isodifferential isocalculus.

Despite the above advances, numerical predictions of isomathematics lacked the crucial property of invariance over time.

In addition, isomechanics and hadronic mechanics at large, were incomplete due to the inability to formulate the isotopies and genotopies of the linear and angular momenta (see next section).

In order to resolve this impasses, Santilli had no other choice than that of reinspecting the Newton-Leibnitz differential calculus, by discovering in this way that, contrary to rather popular belief for four centuries, *the differential calculus depends on the multiplicative unit of the base field* because, when the unit depends on the variable of differentiation, said calculus has to be "completed" into the infinite family of *isodifferentials,* e.g., of the isocoordinates $\hat{r}$, first formulated in memoir [115] submitted in 1995 and published in 1996 and then treated in details in Refs. [55] [56]

$$\hat{d}\hat{r} = \hat{T}d[r\hat{I}(r,...)] = dr + r\hat{T}d\hat{I}(r,...), \tag{29}$$

with corresponding *isoderivative* [114]

$$\frac{\hat{\partial}\hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \hat{I}\frac{\partial\hat{f}(\hat{r})}{\partial\hat{r}}. \tag{30}$$

Following decades of searches, the discovery of the isodifferential isocalculus finally permitted the achievement of the invariance over time of numerical predictions, the formulation of isolinear and genolinear momenta and signaled the achievement of maturity for applications and experimental verifications (see Refs. [56] [57] for details).

All novel applications of isomathematics in physics, chemistry and other fields, including the proof of the EPR argument [7], originate from the extra term $r\hat{T}d\hat{I}(r,...)$ in isodifferential (29), which is absent in the mathematics for point particles.

The covering character of the isodifferential isocalculus over the conventional calculus is illustrated by the fact that whenever the isounit is independent from the differentiation variable or it is a constant, the conventional calculus is recovered uniquely and identically.

It should be indicated that the biggest difficulty in the use of the isodifferential isocalculus is of *conceptual,* rather than of mathematical character, because it requires the transition from the visualization of the calculus at individual points to volumes (or surfaces) represented by the isotopic element $\hat{T}$ (Figure 12).

By looking in retrospect, it appears nowadays evident that a "completion" of quantum mechanics for the representation of extended particles is fundamentally inconsistent when formulated via the conventional differential calculus, because of its sole possible characterization of particles as being point-like.

Consequently, the generalization of the differential calculus into a form defined on volumes represented by $\hat{T}$, rather than defined on coordinate points r, is essential for a consistent representation of extended particles.

Nineteen years following the discovery of the isodifferential isocalculus for extended particles, comprehensive studies in the field have been conducted by the mathematician S. Georgiev in the series of six monographs [83] which consider the broadest possible formulation of the isodifferential calculus, including the case of different isotopic elements for different isovariables.

3.7. Lie-Santilli isotheory.

In Santilli's view, the physically most important part of isomathematics is given by the isotopic "completion" of the various branches of Lie's theory, today known as the *Lie-Santilli isotheory,* which was first formulated in papers [16] [17] of 1978, systematically studied in monograph [19], finalized in Refs. [55] [56] of 1995 following the discovery of the isodifferential calculus [115] and recently studied in paper [117].

In this section, we follow the presentation of Ref. [19] of 1981 upgraded into a formulation on isospaces over isofields and elaborated via the isodifferential isocalculus.

Let L be a n-dimensional Lie algebra with Hermitean generators $X_k,\ k = 1,2,...n$ defined on a conventional space over a conventional numeric field. Then, the infinite family of isotopies $\hat{L}$ of L are characterized by the following main theorems:

THEOREM 3.7.1 [19]: (Poincaré-Birkhoff-Witt-Santilli isotheorem): The isocosets of the isounit and of the isostandard isomonomials

$$\hat{I},\ \ \hat{X}_k,\ \ \hat{X}_i \star \hat{X}_j,\ i \leq j,\ \ \hat{X}_i \star \hat{X}_j \star \hat{X}_k,\ i \leq j \leq k,\ \ldots, \tag{31}$$

form an infinite dimensional isobasis of the universal enveloping isoassociative isoalgebra $\hat{E}(\hat{L})$ (also called isoenvelope for short) of a Lie-Santilli isoalgebra $\hat{L}$.

The first illustration of the above theorem is given by isoexponential isofunction (22) whose correct derivation requires infinite basis (31).

The appearance of the non-linear, non-local and non-potential isotopic element $\hat{T}$ in the *exponent* illustrates the non-trivial character of the Lie-Santilli isotheory.

THEOREM 3.7.2 [19]: (Lie-Santilli isoalgebras) The antisymmetric isoalgebras $\hat{L}$ attached to the isoenveloping algebras $\hat{E}(\hat{L})$ verify the isocommutation rules

$$\begin{gathered}[\hat{X}_i \hat{,} \hat{X}_j] = \hat{X}_i \star \hat{X}_j - \hat{X}_j \star \hat{X}_i = \\ = \hat{C}'^k_{ij}(t, r, p, E, \mu, \tau, \psi, \partial\psi, ...) \star \hat{X}_k,\end{gathered} \tag{32}$$

where the quantities $\hat{C}$ are called the structure quantities.

The above isocommutation rules show the axiom-preserving character of the isotopies in view of their evident verification of the Lie axioms, although via a broader realization.

The Lie-Santilli isotheory is called *regular* or *irregular* depending on whether the structure quantities $\hat{C}'^k_{i,j}$ are isoscalars or isofunctions, respectively.

This classification is important because as shown in the next section, the regular Lie-Santilli isotheory can be constructed via a well defined transform of corresponding Lie theory, while irregular Lie-Santilli isotheories are truly new theories verifying Lie's axioms without a known map form the conventional formulation.

THEOREM 3.7.3 [19]: (Lie-Santilli isogroups) The isoexponentiated form $\hat{G}$ of isocommutation rules (32) defined on an isospace $\hat{S}$ with local isocoordinates $\hat{x}$ over an isofield $\hat{F}$ with isounit $\hat{I} = 1/\hat{T} > 0$ is a group mapping each element $\hat{x} \in \hat{S}$ into a new element $\hat{x}' \in \hat{S}$ via the isotransformations

$$\hat{x}' = \hat{g}(\hat{w}) \star \hat{x},\ \ \hat{x}, \hat{x}' \in \hat{S},\ \ \hat{w} \in \hat{F}, \tag{33}$$

verifying the following isomodular isoaction to the right:

1) The isomap of $\hat{g} \star \hat{S}$ into $\hat{S}$ is isodifferentiable $\forall \hat{g} \in \hat{G}$;

2) $\hat{I}$ is the left and right isounit of $\hat{G}$,

$$\hat{I} \star \hat{g} = \hat{g} \star \hat{I} \equiv \hat{g},\ \ \forall \hat{g} \in \hat{G}; \tag{34}$$

3) The isomodular isoaction is isoassociative,

$$\hat{g}_1 \star (\hat{g}_2 \star \hat{x}) = (\hat{g}_1 \star \hat{g}_2) \star \hat{x},\ \ \forall \hat{g}_1, \hat{g}_2 \in \hat{G}; \tag{35}$$

4) In correspondence with every element $\hat{g}(\hat{w}) \in \hat{G}$ with $\hat{w} \in \hat{F}$ there exists the inverse element $\hat{g}(-\hat{w})$ such that

$$\hat{g}(\hat{0}) = \hat{g}(\hat{w}) \star \hat{g}(-\hat{w}) = \hat{I}; \tag{36}$$

5) The following composition laws are verified

$$\hat{g}(\hat{w}) \star \hat{g}(\hat{w}') = \hat{g}(\hat{w}') \star \hat{g}(\hat{w}) = \hat{g}(\hat{w} + \hat{w}'), \forall \hat{g} \in \hat{G}, \ \hat{w} \in \hat{F}; \tag{37}$$

with corresponding isomodular action to the left, and general expression

$$\hat{g}(\hat{w}) = \prod_k \hat{e}^{\hat{i} \star \hat{w}_k \star \hat{X}_k} \star \hat{g}(0) \star \prod_k \hat{e}^{-\hat{i} \star \hat{w}_k \star \hat{X}_k}. \tag{38}$$

Nowadays, *isomathematics* is referred to the infinite family of isotopies of 20th century applied mathematics, with particular reference to the isotopies of Lie's theory when formulated on isospaces over isofields and elaborated via the isodifferential isocalculus. Isomathematics is then classified into *regular and irregular isomathematics* depending on whether the structure quantities $\hat{C}_{ij}$ are isoscalars or isofunctions of local isovariables, respectively.

It should be indicated that, the proof [7] of the EPR argument uses both regular and irregular Lie-Santilli isoalgebras.

Since Lie's theory is at the foundation of the axiomatic structure and applications of quantum mechanics, the covering Lie-Santilli isotheory predictably stimulated a number of independent contributions, such as the studies by the mathematicians: D. S. Sourlas and Gr. T. Tsagas [76], J. V. Kadeisvili [118], T. Vougiouklis [119] and papers quoted therein.

3.8. Simple construction of regular isomathematics.

A simple method for physicists has been identified in Ref. [120] of 1997 for the construction of regular isomathematics. The method consists of: 1) Selecting the desired representation of extended particles with non-Hamiltonian interactions via isotopic elements $\hat{T}$ of type (16) or (18); 2) Identifying a *non-unitary* transformation representing the selected isounit $\hat{I} = 1\hat{T}$

$$UU^\dagger = \hat{I}; \tag{39}$$

3) Subjecting the *totality* of conventional applied mathematics to the above nonunitary transform with no known exception, resulting in expressions of the type

$$I \to \hat{I} = UIU^\dagger = 1/\hat{T}, \tag{40}$$

$$n \to \hat{n} = UnU^\dagger = nUU^\dagger = n\hat{I}, \ n \in F, \tag{41}$$

$$f(r) \to \hat{f}(\hat{r}) = Uf(r)U^\dagger, \tag{42}$$

$$e^A \to Ue^AU^\dagger = \hat{I}e^{\hat{T}\hat{A}} = (e^{\hat{A}\hat{T}})\hat{I}, \tag{43}$$

$$\begin{gathered} AB \to U(AB)U^\dagger = \\ = (UAU^\dagger)(UU^\dagger)^{-1}(UBU^\dagger) = \hat{A} \star \hat{B}. \end{gathered} \tag{44}$$

It should be indicated that the above transformations imply the possibility of constructing the infinite family of Lie-Santilli isoalgebras via non-unitary transforms of the considered Lie algebra. This is possible in view of the transformation of commutation rules into their covering isocommutator forms,

$$[A, B] \to U[A, B]U^\dagger = [\hat{A} \hat{,} \hat{B}]. \tag{45}$$

This property is evidently important for the construction of the isorepresentation of regular isoalgebras used for physical and chemical applications.

Note that *serious inconsistencies occur, at times without their detection by non-experts, in the event only one single quantity or operation of 20th century applied mathematics is not subjected to the above non-unitary map.*

We should finally indicate that *the proof of the EPR argument used in Ref. [7] is of 'non-unitary' character,* thus implying that the physical conditions for said proof are outside the class of equivalence of quantum mechanics.

3.9. Invariance of regular isomathematics.

An additional contribution of paper [120] is the proof that *the dimension, shape and density of extended particles and their non-Hamiltonian interactions are represented by isomathematics in a form invariant over time.*

Firstly, Ref. [120] showed that, following the construction of regular isomathematics via non-unitary transformations (Section 2.2.8), isomathematics is *not* invariant under additional non-unitary transforms, e.g., because of the lack of invariance of the basic isounit

$$\hat{I} \to \hat{I}' = W\hat{I}W^\dagger \neq \hat{I}, \quad WW^\dagger \neq I, \tag{46}$$

with consequential physical inconsistencies since any structural change of the isounit implies the transition to a different physical or chemical system.

However, non-unitary transforms can always be identically rewritten as *isounitary isotransforms* according to the rule [56]

$$WW^\dagger = \hat{I}, \quad W = \hat{W}\hat{T}^{1/2}, \tag{47}$$

$$WW^\dagger = \hat{W} \star \hat{W}^\dagger = \hat{W}^\dagger \star \hat{W} = \hat{I}, \tag{48}$$

under which reformulation we have the following *invariance of the isounit of the isotopic element and of the isoproduct of regular isomathematics* [120]

$$\hat{I} \to \hat{I}' = \hat{W} \star \hat{I} \star \hat{W}^\dagger \equiv \hat{I}, \tag{49}$$

$$\hat{A} \star \hat{B} \to \hat{W} \star (\hat{A} \star \hat{B}) \star \hat{W}^\dagger =$$

$$= \hat{A}' \star \hat{B}' = \hat{A}'\hat{T}\hat{B}', \tag{50}$$

$$\hat{A}' = \hat{W} \star A \star \hat{W}^\dagger). \hat{B}' = \hat{W} \star \hat{B} \star \hat{W}^\dagger, \ \hat{T} = (W^\dagger \star \hat{W})^{-1}.$$

The invariance of the entire isomathematics follows. Note that the invariance is ensured by the *invariant numeric values of the isounit and therefore, of the isotopic element under isounitary isotransforms,*

$$\hat{I} \to \hat{I}' \equiv \hat{I}, \tag{51}$$

$$A \star B = A\hat{T}B \rightarrow A' \star' B' = A'\hat{T}'B'$$
$$= \hat{A}' \star \hat{B}' = \hat{A}'\hat{T}\hat{B}' \quad (52)$$
$$\hat{T} \rightarrow \hat{T}' \equiv \hat{T}. \quad (53)$$

By noting that, as shown in the next section, the time evolution of the isotopic "completion" of quantum mechanics is an isounitary isotransform, paper [120] established that *isomechanics is an axiom-preserving "completion" of quantum mechanics capable of representing extended particles under Hamiltonian as well as non-Hamiltonian interactions in a form invariant over time.*

In closing, we should recall other generalizations of 20th century mathematics and their applications, such as the so-called *deformations.* These generalizations are mathematically correct, but physically inconsistent because they violate causality laws (for brevity, see the *Theorems of Inconsistency of Non- Unitary Theories* in Vol. I of Refs. [61]). These inconsistencies arise from a structural generalization of Lie's algebras and other quantum laws when formulated on conventional spaces over conventional fields, thus preventing their reformulation as isounitary theories.

Note that the invariance of isomathematics reviewed in this section implies the verification of causality on isospaces over isofields in the same way as quantum mechanics verify causality laws.

4. LIE-ISOTOPIC "COMPLETION" OF QUANTUM MECHANICS

4.1. Foreword.

In this section, we review the foundation of the *isotopic branch of hadronic mechanics,* also known as *isomechanics*, which is used in the proof of the EPR argument [7].

An important difference between preceding works and the presentation in this section is that the previous works generally present isomechanics in its *projection* on conventional spaces over conventional fields.

By contrast, in this section we put the emphasis in the full formulation of isomechanics, that on isospaces over isofields because important to illustrate that, contrary to opposing view (Section 1.1), proof [7] of the EPR argument is fully compatible with quantum axioms, only subjected to a broader realization.

Isomechanics was first introduced via isoproducts defined on conventional spaces over conventional fields in the 1978 papers [16] [17] and in the 1981 monograph [19]; it first achieved mathematical maturity in the 1996 memoir [115] thanks to the discovery of the isodifferential isocalculus; isomechanics was finally presented in a systematic way in monographs [55] [56] [57] with a 2016 upgrade in Section 2 of memoir [25].

Independent studies are available in Refs. [75] to [83]. A comprehensive list of references up to 2008 is available in Vol. I of Refs. [61] while a 2016 upgrade is available in Ref. [25]. We regret the inability of reviewing all important contributions on isomechanics to prevent an excessive length and are forced to outline only the most salient structural contributions.

4.2. Iso-Newton isoequations.

As it is well known, the fundamental equations of mechanics are the historical *Newton's equations*, representing systems of point-particles with Hamiltonian (that is, *variationally selfadjoint,* SA [18]) and non-Hamiltonian (*variationally non selfadjoint,* NSA [18]) forces [18] defined on a conventional Euclidean space, $E(r, \delta, I)$ over the field of real numbers $\mathcal{R}$

$$m\frac{dv_{ak}}{dt} - F^{SA}_{ak}(r, v) - F^{NSA}_{ak}(t, r, v) = 0,$$
$$k = 1, 2, 3, \quad a = 1, 2, ..., N, \quad N \geq 2. \tag{54}$$

It is generally believed that Newton's equations with non-conservative forces can solely represent open, irreversible systems. In Ref. [19] Section 6.3, Page 236, Santilli introduced *closed non-self-adjoint systems,* which are given by systems (54) violating the integrability conditions for their representation via Lagrange's or Hamilton's equations, yet verifying all ten conservation laws of Galileo relativity under the conditions

$$\begin{aligned} &\textstyle\sum_{ak} F^{NSA}_{ak} = 0, \\ &\textstyle\sum_{ak} \mathbf{r} \cdot \mathbf{F}^{NSA} = 0, \\ &\textstyle\sum_{ak} \mathbf{r}\mathbf{F}^{NSA} = 0. \end{aligned} \tag{55}$$

which conditions are evidently applicable only for $N \geq 2$, since the case of one particle $N = 1$ is trivial.

The fundamental equations of isomechanics are given by the isotopic "completion" of Eqs. (54) known as *iso-Newton isoequations*, which were first introduced in Ref. [115] immediately following the discovery of the isodifferential isocalculus, and they are also known as *Newton-Santilli isoequations,* (see, e.g., Refs. [107] [75] [81]) defined on iso-Euclidean isospaces $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$ (Section 2.2.5) over isoreal isonumbers $\hat{\mathcal{R}}$ (Section 2.2.3)

$$\hat{m}_a \star \frac{\hat{d}\hat{v}_{ak}}{\hat{d}\hat{t}} - F^{SA}_{ak}(\hat{r}, \hat{v}) = 0. \tag{56}$$

A first important feature of Eqs. (56) is that of providing the first known consistent representation of the actual shapes and dimensions of the particles considered via the isodifferential calculus, with realization of the isotopic element of type (16).

A second important feature of Eqs. (56) is that of representing all potential-(SA) forces via conventional Newtonian forces $F^{SA}(r, v)$ while representing all non-potential-(NSA) forces via the isodifferential calculus.

This feature is treated in details in Ref. [115] and can be summarized as follows.

Note that the basic isounit of Eqs. (56) is the *isovelocity isounit,* $\hat{I} = \hat{I}_v$. Assume for simplicity that the isotime is equal to the conventional time,

$$\hat{t} = t \ \ \hat{I}_t = 1. \tag{57}$$

Consequently, from isoderivative (30), we have

$$\frac{\hat{d}\hat{v}}{\hat{d}\hat{t}} = I_t d\hat{v}/d\hat{t} = d\hat{v}/dt. \tag{58}$$

Consider then the projection of Eqs. (56) in the Euclidean space $E(v, \delta, I)$ and use the various rules of isomathematics (Section 2.2). Then Eqs. (56) can be written in the projected form (where all multiplications are conventional),

$$(m\hat{I})\hat{T}\frac{\hat{d}(v\hat{I})}{dt} - F^{SA}\hat{I} = \\ = \hat{I}[m\frac{dv}{dt} - F^{SA}] + mv\hat{T}\frac{d\hat{I}}{dt} = 0. \tag{59}$$

By dividing the above equation with $\hat{I} > o$, one obtains Newton's equations with the following realizations of the NSA forces

$$F^{NSA}(t, r, v) = mv\hat{T}\frac{d\hat{I}}{dt}. \tag{60}$$

In conclusion, *the iso-Newton isoequations (56) embed all NSA forces into the isoderivatives by therefore allowing the first known consistent operator image of non-Hamiltonian forces studied in subsequent sections.*

4.3. Iso-lagrangian and iso-Hamiltonian isomechanics.

Iso-Newton isoequations (56) admit a representation in terms of the *iso-Lagrange isoequations* first formulated in memoir [115] via the isodifferential calculus, thus being defined on an iso-Euclidean isospace over the isoreal isofield

$$\frac{\hat{d}}{\hat{d}\hat{t}}\frac{\hat{\partial}\hat{L}(\hat{r},\hat{v})}{\hat{\partial}\hat{v}_{ak}} - \frac{\hat{\partial}\hat{L}(\hat{r},\hat{v})}{\hat{\partial}\hat{r}_{ak}} = 0, \tag{61}$$

where $\hat{L} = L\hat{I}$ is an *iso-Lagrangian*, namely, a conventional Lagrangian formulated on isospaces over isofields thus being multiplied by the isounit to be an isoscalar.

Eqs. (56) also admit the isocanonically isoequivalent isorepresentation in terms of the *iso-Hamilton equations* first formulated in memoir [115] on an isophase isospaces over an isoreal isofield and also known as *Hamilton-Santilli isoequations* (see, e.g., Ref. [107])

$$\frac{\hat{d}\hat{r}_{ak}}{\hat{d}\hat{t}} = \frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{p}_{ak}}, \\ \frac{\hat{d}\hat{p}_{ak}}{\hat{d}\hat{t}} = -\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{r}_{ak}}, \tag{62}$$

where $\hat{H} = H\hat{I}$ is the iso-Hamiltonian, that is, a conventional Hamiltonian formulated on an isophase isospace over an isofield.

Note that iso-Hamiltonian isomechanics admits the following time evolution for a quantity $\hat{Q}$

$$\frac{\hat{d}\hat{Q}}{\hat{d}\hat{t}} = [\hat{Q}\hat{,}\hat{H}] = \frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{r}_{ak}}\frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{p}_{ak}} - \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{p}_{ak}}\frac{\hat{\partial}\hat{Q}}{\hat{\partial}\hat{r}_{ak}}, \tag{63}$$

where the brackets $[\hat{Q}\hat{,}\hat{H}]$ constitute a classical realization of Lie-Santilli isoalgebras.

4.4. Isovariational isoprinciple.

Another important feature of Eqs. (56) is that of permitting the first known representation of variationally non- selfadjoint/non-Hamiltonian systems via an *isovariational isoprinciple* [115],

$$\hat{\delta}\hat{A} = \hat{\delta}\hat{\int}(\hat{p}_{ak} \star \hat{d}\hat{r}_{ak} - \hat{H} \star \hat{d}\hat{t}) = 0, \tag{64}$$

which representation is notoriously impossible for NSA Newton's equations, with the consequential lack of achievement of consistent operator forms of nonconservative forces.

In view of the universality of Eqs. (56), the above isovariational isoprinciple is *directly universal,* that is, capable of representing all infinitely possible, regular, time reversal invariant Newtonian systems (54) ("universality") directly in the coordinates of the experimenter ("direct universality").

4.5. Iso-Hamilton-Jacobi isoequations.

Much along conventional analytic procedures, it is easy to prove that isovariational isoprinciple (64) implies the following *iso-Hamilton-Jacobi isoequations* also called *Hamilton-Jacobi-Santilli isoequations* [115] [25] that are at the foundation of the *isoquantization* reviewed in the next section

$$\frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{t}} + \hat{H} = 0, \tag{65}$$

$$\frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{r}_{ak}} - \hat{p}_{ak} = 0, \tag{66}$$

$$\frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{p}_{ak}} = 0. \tag{67}$$

We should recall from Section 2.2.5 that isodynamical isoequations of classical isomechanics require two different isofields, the first being the *isotime isofield* with isounits $\hat{I}_t$ and the second being the *isovelocity isofield* with isounits $\hat{I}_v$.

However, the direct universality is already achieved with the sole use of the isovelocity isounit. Hence, the isotime isounit can be assumed to be 1 without any loss of direct universality.

For non-relativistic formulations, we shall use isotime in the isodynamical equations for completeness, with the tacit understanding that, unless otherwise specified, isotime will be assumed to be equal to the conventional time.

As it will soon be evident, the Hamilton-Jacobi-Santilli isoequations (65)-(67) are truly fundamental for the construction of operator isomechanics, as well as for the proof of the EPR argument because said equations have permitted:

1) The achievement from Eqs. (65) of a unique and unambiguous map of classical into operator isomechanics;

2) The achievement from Eqs. (66) of the first known operator form of the isolinear isomomentum;

3) The achievement, from Eqs. (67) of an operator isomechanics whose *isowave Isofunctions* solely depend on local isocoordinates $\hat{\psi}(\hat{r})$. This feature is necessary for a consistent isotopic "completion" of quantum mechanics since conventional wave functions $\psi(t, r)$ do not depend on linear momenta p. Hence, any "completion" of quantum mechanics whose wavefunctions also depend on linear momenta would not be an axiom-preserving map.

It should be noted that, by comparison, the use of the Birkhoffian mechanics would imply broader Hamilton-Jacobi-Santilli isoequations (see page 205 of Ref. [19]) with 'wave functions' depending also on isomomenta, $\hat{\psi}(\hat{t}, \hat{r}, \hat{p})$, resulting in an operator mechanics beyond our current knowledge for quantitative treatments.

4.6. Naive isoquantization.

As it is well known, the conventional "naive quantization" of Hamiltonian mechanics into quantum mechanics is based on the following map generated by the conventional Hamilton-Jacobi equations

$$A = \int (p_k dx^k - H dt) \rightarrow \\ \rightarrow -i\hbar log\psi(t, r), \tag{68}$$

that identifies Planck's constant $\hbar = 1$ as the fundamental unit of the theory.

The isotopic lifting of the naive quantization, called *naive isoquantization* (first identified by A. E. O. Animalu and R. M. Santilli in Ref. [121]), characterizes the following map of (classical) iso-Hamiltonian mechanics into (operator) isomechanics via Hamilton-Jacobi-Santilli isoequation (65) (where the sum over the indices ak is omitted for simplicity)

$$\hat{A} = \hat{\int} (\hat{p} \star \hat{d}\hat{r} - \hat{H} \star \hat{d}\hat{t}) \ \rightarrow \\ \rightarrow -i\hat{I} Log\hat{\psi}, \tag{69}$$

with the following fundamental identification of the *isolinear isomomentum* from Eq. (66)

$$\hat{p} \star \psi = -\hat{i} \star \hat{\partial}_r \hat{\psi}, \tag{70}$$

and the equally fundamental, independence of the isowavefunction from the isolinear isomomentum from Eq. (67)

$$\hat{\partial}_{\hat{p}} \hat{\psi} = 0, \tag{71}$$

where we use the notion of *isolog* $\hat{log}\hat{\psi} = \hat{I} \log \psi$ (see [55]).

The above naive isoquantization identifies the central assumption of isomechanics, namely, *the map of Planck's constant $\hbar$ into an integro-differential operator, the isounit $\hat{I}$*,

$$\hbar \rightarrow \hat{I}(t, r, p, E, \mu, \tau, \psi, \partial\psi, ...) > 0. \tag{72}$$

that, from Section 2.2.8, can be achieved via a non-unitary transform of Plank's constant selected in such a way to represent the desired systems, e.g., as in model (16),

$$\hbar = 1 \ \ \rightarrow \hat{\hbar} = U \hbar U^\dagger = U U^\dagger = \hat{I}. \tag{73}$$

The above transform is then restricted by the subsidiary condition that isomechanics must recover quantum mechanics at mutual distances of particles bigger than their size d

$$Lim_{r > d/2} \hat{I} = 1. \tag{74}$$

Therefore, the studies herein reported assume that isomechanics is solely valid within the volume occupied by hadrons, nuclei or stars, while quantum mechanics is assumed to be exactly valid everywhere else (Figure 13).

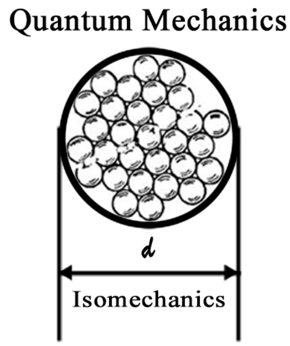

Figure 13: *An illustration of the central assumption according to which isomechanics is solely valid within the volume occupied by hadrons, nuclei, stars or black holes, while quantum mechanics is valid everywhere else thanks to the rapid convergence of isotopic elements, such as Eq. (16), to the unit value* 1.

Note that the structure of the isotopic element (16) permits a smooth transition from isomechanics to quantum mechanics.

The above condition means that, for the case of the structure of a hadron, isomechanics is solely valid within a sphere with diameter $d \approx 1fm = 10^{-15}\ cn$. For the case of the structure of the deuteron, isomechanics is solely valid within a volume with diameter $d \approx 2.50\ fm$; and the same applies for nuclei, stars and black holes.

The main objective of the "completion" of Planck's constant $\hbar$ into the integro-differential isounit $\hat{I}$ is to represent the expected, generalized, energy exchanges of particles in interior dynamical conditions (as expected for an electron in the core of a star), which exchanges cannot be the same as those occurring when particles moves in vacuum due to the surrounding pressures and other factors (see paper II for details).

4.7. Iso-Hilbert isospaces.

Another basic notion of isomechanics is its formulation on the *iso-Hilbert isospaces* $\hat{\mathcal{H}}$, also called the *Hilbert-Myung-Santilli isospace* (HMS isospace) because first introduced by H.C. Myung and R.M. Santilli in Ref. [122] of 1982 over a conventional field of complex numbers $\mathcal{C}$ and then formulated on an isocomplex isofield $\hat{\mathcal{C}}$ in Ref. [115].

HMS isospaces are characterized by (see Ref. [55] for details): *isostates* $\hat{\psi}$, with *isonormalization*

$$< \hat{\psi}| \star |\hat{\psi} >= \hat{I}, \tag{75}$$

isoexpectation isovalues of an iso-Hermitean operator $\hat{A}$,

$$\hat{<}\hat{A}\hat{>} =< \hat{\psi}| \star \hat{A} \star |\hat{\psi} >, \tag{76}$$

and basic isoidentity

$$\hat{<}\hat{I}\hat{>} = \hat{I}, \tag{77}$$

where the "hat" denotes definition on isospace over isofields.

It should be recalled from Ref. [122] that the condition of iso-Hermiticity coincides with that of Hermiticity. Therefore, *all quantities that are observable in quantum mechanics remain observable in isomechanics* (see monograph [56] for details).

4.8. Iso-Schrödinger isorepresentation.

Recall that the Schrödinger representation is crucially dependent on the realization of the linear momentum in term of the differential calculus,

$$p\psi(t,r) = -i\hbar\partial_r\psi(t,r). \tag{78}$$

Consequently, the "completion" of quantum mechanics mandated the search for the "completion" of the differential calculus [115] to achieve a consistent formulation of the linear momentum such as that of Eq. (70), that we rewrite in the more detailed form on the Hilbert-Myung-Santilli isospace $\hat{\mathcal{H}}$ over the isofield of isocomplex isonumbers $\hat{\imath calC}$ (with $\hbar = 1$)

$$\begin{aligned} \hat{p}_k \star |\hat{\psi}(\hat{t},\hat{r}) >= -\hat{i} \star \hat{\partial}_{\hat{r}^k}|\hat{\psi}(\hat{t},\hat{r}) >= \\ = -i\hat{I}\partial_{\hat{r}^k}|\hat{\psi}(\hat{t},\hat{r}) >, \end{aligned} \tag{79}$$

where: $\hat{r} = r\hat{I}$ are the isocoordinates on an iso-Euclidean isospace over an isofield, from which one can derive the *iso-Schrödinger isoequation,* also called *Schrödinger-Santilli isoequation* [17] [115]

$$\begin{aligned} \hat{i} \star \hat{\partial}_{\hat{t}}|\hat{\psi}(\hat{t},\hat{r}) >= \hat{H} \star |\hat{\psi}(\hat{t},\hat{r}) >= \\ = \hat{H}(r,p)\hat{T}(t,r,p,\psi,\partial\psi,...)|\hat{\psi}(\hat{t},\hat{r} >) = \\ = \hat{E} \star |\hat{\psi}(\hat{t},\hat{r}) >= E|\hat{\psi}(\hat{t},\hat{r}) >, \end{aligned} \tag{80}$$

where $\hat{E} = E\hat{I}$ is an isoeigenvalue defined on the isoreal isofield $\hat{\mathcal{R}}$, and E is an ordinary eigenvalue defined on the field of real numbers.

The iso-Schrödinger isorepresentation is completed by the *isocanonical isocommutation rules,* solely definable thanks to the isodifferential realization (79) of the isolinear isomomentum [115]

$$[\hat{r}_i \hat{,} \hat{p}_j]|\hat{\psi} >= \hat{i} \star \hat{\delta}_{i.j}|\hat{\psi} >= i\delta_{i.j}|\hat{\psi} >, [\hat{r}_i \hat{,} \hat{r}_j]|\hat{\psi} >= [\hat{p}_i \hat{,} \hat{p}_j]|\hat{\psi} >= 0. \tag{81}$$

Note that the characterization of extended particles at short mutual distances requires the knowledge of *two isoobservables,* the conventional Hamiltonian H for the representation of SA interactions and the isotopic element $\hat{T}$ for the representation of dimension, shape, density and NSA interactions.

On more technical grounds, Eq.s (80) are referred to as *regular iso-Schrödinger equations* to emphasize, in the sense of Theorem 3.7.2, the fact that they can be derived from the conventional Schrödinger equation via non-unitary transformations. The broader *irregular iso-Schrödinger equation* which cannot be derived via non-unitary transformations due to the addition of strong interactions, are studied in Paper II, Section 4.3., Eqs. (89).

4.9. Iso-Heisenberg isorepresentation.

Non-relativistic isomechanics is additionally based on the *iso-Heisenberg isoequations,* also called *Heisenberg-Santilli isoequations* (first formulated in Eq. (4.15.49), page 752 of the 1978 paper [17] over conventional fields and formulated via the full use of isomathematics in the 1996 memoir [115]), here written for the iso infinitesimal isotime evolution of an iso-Hermitean operator $\hat{Q}$

$$\begin{aligned} \hat{i} \star \frac{\hat{d}\hat{Q}}{\hat{d}\hat{t}} = [\hat{Q} \hat{,} \hat{H}] = \hat{Q} \star \hat{H} - \hat{H} \star \hat{Q} = \\ = \hat{Q}\hat{T}(\psi,...)\hat{H}(r,p) - \hat{H}(r,p)\hat{T}(\psi,...)\hat{Q}, \end{aligned} \tag{82}$$

and then in their isoexponentiated form

$$\hat{Q}(\hat{t}) = \hat{e}^{\hat{H}\star\hat{t}\star\hat{i}} \star \hat{Q}(0) \star \hat{e}^{-\hat{i}\star\hat{t}\star\hat{H}} = \\ = e^{\hat{H}\hat{T}ti} Q(0) e^{-it\hat{T}\hat{H}}, \tag{83}$$

where we have used isoexponentiation (22).

Note the characterization of isoinfinitesimal isoequations (82) via the Lie-Santilli isoalgebras (Theorem 3.7.2.) and the characterization of their finite form (83) via the Lie-Santilli isogroups (Theorem 3.7.3).

Note also that the Lie-isotopic equations (82) (83) are a particular case of the broader Lie-admissible equations (11) (12), respectively.

4.10. Iso-Klein-Gordon isoequation.

The relativistic isoequations of hadronic mechanics are characterized by the iso-Casimir isoinvariants of the basic symmetry of the iso-Minkowski isospace-time, the *Lorentz-Poincaré-Santilli isosymmetry* studied in paper II.

At this stage of our analysis, we merely consider the following isotopic "completion" of the second order invariant of the *Lorentz-Poincaré symmetry* formulated on iso-Minkowskian isospace $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I}$ over the isoreals $\hat{\mathcal{R}}$ (Section 2.5)

$$\hat{p}^{\hat{2}} = \hat{p}_\mu \star \hat{p}^\mu = (\hat{M} \star \hat{C})^{\hat{2}} = (mC)^2 \hat{I}, \tag{84}$$

where

$$\hat{M} = m\hat{I}, \tag{85}$$

is the *isomass,* and

$$\hat{C} = C\hat{I} = \frac{c}{n_4}\hat{I}, \tag{86}$$

is the *light isospeed* from isoinvariant (26).

By using the isolinear isomomentum (79) isoinvariant (84) characterizes the second order isoequation of isomechanics known as *iso-Klein-Gordon isoequation* [123]

$$\hat{p}_\mu \star \hat{p}^\mu |\hat{\psi}(\hat{x}) >= \hat{\Omega}^{\mu\nu} \star \hat{p}_\mu \star \hat{p}_\nu |\hat{\psi}(\hat{x}) >= \\ = \hat{\eta}^{\mu\nu}(-i\hat{I}\partial_\mu)\hat{T}(-i\hat{I}\partial_\nu)|\hat{\psi}(\hat{x}) >= \\ = -\hat{I}\hat{\eta}^{\mu\nu}\partial_\mu\partial_\nu|\hat{\psi}(\hat{x}) >= \hat{I}(mC)^2|\hat{\psi}(\hat{x}) >, \tag{87}$$

also called *Klein-Gordon-Santilli isoequation,* first introduced with the isodifferential isocalculus in Chapter 9 of Refs. [56] and in papers [123] [124], (see also review [25]).

4.11. Iso-Dirac isoequation.

The first-order relativistic isoequation of hadronic mechanics is given by the isolinearization [124] of isoinvariant (84) and it is called the *iso-Dirac isoequation,* or *Dirac-Santilli isoequation* (see Refs. [115] [56] [123] [124])

$$[\hat{\Omega}^{\mu\nu} \star \hat{\Gamma}_\mu \star \hat{\partial}_\nu + \hat{M} \star \hat{C}]|\hat{\psi}(\hat{x}) >= \\ = (-i\hat{I}\hat{\eta}^{\mu\nu}\hat{\gamma}_\mu\partial_\nu + mC)|\hat{\psi}(\hat{x}) >= 0, \tag{88}$$

where the *Dirac-Santilli isogamma isomatrices* $\hat{\Gamma} = \hat{\gamma}\hat{I}$ are given by

$$\begin{aligned} \hat{\gamma}_k &= \tfrac{1}{n_k} \begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \\ \hat{\gamma}_4 &= \tfrac{i}{n_4} \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}, \end{aligned} \tag{89}$$

where $\hat{\sigma}_k$ are the *regular iso-Pauli isomatrices* studied in Section 3.3 of paper II, with *anti-isocommutation rules*

$$\begin{aligned} \{\hat{\gamma}_\mu \hat{,} \hat{\gamma}_\nu\} = \hat{\gamma}_\mu \hat{T} \hat{\gamma}_\nu + \hat{\gamma}_\nu \hat{T} \hat{\gamma}_\mu = \\ = 2\hat{\eta}_{\mu\nu}. \end{aligned} \tag{90}$$

One should note that the anti isoanticommutators of the Dirac-Santilli isogamma isomatrices yield the isometric $\hat{\eta}_{\mu\nu}$ of the iso-Minkowski isospace-time (Section 2.5).

Recall that the iso-Minkowski isospace-time includes as particular cases all possible, non-singular, symmetric space-times, thus including the Riemannian space-time reformulated with isomathematics. In fact, the iso-Minkowskian isometric $\eta^{\mu\nu}$ admits as a particular case the Schwartzchild metric via the following simple realization of the isotopic element

$$\begin{aligned} \hat{T}_{kk} &= 1/(1 - 2M/r), \\ \hat{T}_{44} &= 1 - 2M/r. \end{aligned} \tag{91}$$

Additionally, the iso-Minkowskian isometric admits the following combination of the Schwartzchild metric for exterior gravitational problems and that for interior problems (see, Eqs. (9.5.18) page 448, Ref. [56]

$$\begin{aligned} \hat{T}_{kk} &= 1/(1 - 2M/r)n_k^2, \\ \hat{T}_{44} &= (1 - 2M/r)/n_4^2. \end{aligned} \tag{92}$$

Consequently, the Dirac-Santilli isoequation with realization (91) of the isotopic element permits the study of an electron in an *exterior gravitational field,* while realization (92) permits the study of *electrons in interior gravitational fields.*

Relativistic isoequations are far from being mere academic curiosities because, as we shall see in paper II, they have provided the first and only known relativistic representation of *all* characteristics of the neutron in its synthesis from a proton and an electron [124], *none* of which characteristics are representable via quantum mechanics. Additional advances permitted by relativistic isomechanics will be indicated in paper II.

More technically, Eq. (88) is referred to as *regular iso-Dirac equations* to emphasize, in the sense of Theorem 3.7.2 , the fact that they can be derived from the conventional Dirac equation via non-unitary transformations. The broader *irregular iso-Dirac equation* which cannot be derived via non-unitary transformations due to the addition of strong interactions, are studied in Paper II, Section 4.4., Eqs. (97).

4.12. Representation of non-linear interactions.

An important insufficiency of quantum mechanics is the inability to characterize individual constituents under non-linear internal forces in view of the inapplicability of the superposition principle.

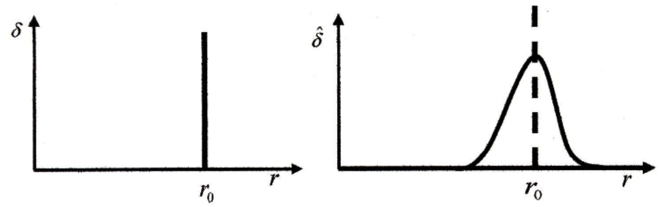

Figure 14: *An illustration on the left of the divergence of the Dirac delta distribution at its origin caused by the point-like approximation of particles, with ensuing divergencies in quantum mechanics. An illustration on the right of the removal of said divergencies by the Dirac-Myung-Santilli isodelta isofunction thanks to the representation of particles as extended, with ensuing lack of divergencies in isomechanics.*

In fact, the sole possible, quantum mechanical representation of non-linear interactions is that via the Hamiltonian

$$H(r, p, \psi, ...)|\psi(t, r) >= E|\psi(t, r) >, \tag{93}$$

under which the total state $|\psi(t, r) >$ does not admit a consistent decomposition into the individual states.

It is easy to see that this insufficiency is resolved by isomechanics thanks to the embedding of all non-linear forces in the basic invariant of the theory, the isounit (or isotopic element).

In fact, the Schrödinger-Santilli isoequation (80) can be explicitly written

$$\hat{H} \star |\hat{\psi} >= \hat{H}(\hat{r}, \hat{p})\hat{T}(\hat{\psi}, ...)|\hat{\psi} >= E|\hat{\psi} >, \tag{94}$$

and its total isostate verifies the factorization

$$\hat{\psi} = \Pi_k \hat{\psi}_k, \quad k = 1, 2, ..., N, \tag{95}$$

called *isosuperposition isoprinciple* [56].

It is evident that factorization (94) allows the characterization of individual constituents under non-linear internal interactions, thus permitting new structural models of hadrons, nuclei, stars and black holes.

4.13. Isostrong isoconvergence.

In all applications to date, the basic isotopic element (16) resulted to have a numeric value smaller than one

$$||\hat{T}|| \ll 1. \tag{96}$$

This feature has the important consequence that *perturbative and other series that are slowly convergent or divergent in quantum mechanics become strongly convergent under their isotopic "completion"* [56].

To illustrate this important feature, consider a divergent quantum mechanical series, such as the canonical series

$$A(w) = A(0) + (AH - HA)/1! + ... =\rightarrow \\ \rightarrow \infty, \quad w > 1. \tag{97}$$

But the value of the isotopic element is much smaller than the parameter w. Therefore, the isotopic "completion" of the above series

$$A(w) = A(0) + (A\hat{T}H - H\hat{T}A)/1! + ... \rightarrow \\ \rightarrow N < \infty, \tag{98}$$

is strongly convergent.

Specific examples of convergence of isoperturbative isoseries much more rapid than corresponding quantum mechanical series have been provided in Refs. [103] [104] (see Section 1.6 for additional comments).

4.14. Removal of quantum divergencies.

As it is well known, the divergencies of quantum mechanics originate from the singularity existing at the origin of the *Dirac delta distribution* (Figure 14) which divergence originates from the point-like approximation of particles.

Another important feature of isomechanics is that of avoiding these singularities as illustrated by the isotopic image of Dirac's delta "distribution", known as *Dirac-Myung-Santilli isodelta isofunction* first introduced in Ref. [122] (see also Nishioka's studies [125] to [128])

$$\hat{\delta}(\hat{r}) = \hat{\int} \hat{e}^{\hat{k} \star \hat{r}} \star \hat{d}\hat{k} = \int e^{\hat{k}\hat{T}\hat{r}} d\hat{k}, \tag{99}$$

where we have used isoexponentials and isointegrals [56].

As illustrated in Figure 14, the appearance of the isotopic element in the *exponent* of the integrant changes a sharp singularity at the origin $r = 0$ into a bell-shaped function.

In summary, *the singularities of quantum mechanics are ultimately due to the point-like abstraction of particles or equivalently, to the formulation of the differential calculus at isolated points. Whenever particles are represented with their actual extended size, and the differential calculus is extended to formulations over volumes, quantum singularities no longer hold.*

4.15. Isoscattering isotheory.

Another important application of isomechanics is the isotopic "completion" of the conventional, potential, scattering theory into the covering *isoscattering isotheory* studied by R. Mignani [129] to [131], A. K. Aringazin and D. A. Kirukhin [132], A. O. E. Animalu and R. M. Santilli [133] and others (see Chapter 12 of Ref. [56]).

Regrettably, we cannot review these studies to avoid a prohibitive length. We limit ourselves to recall that the potential scattering theory was originally conceived by Feynman and others for the electromagnetic interactions of *point-like* particles in vacuum.

Its dominant notion is the characterization of interactions via the *exchange of point-like particles.*

The historical experimental verifications of the potential scattering theory under the indicated conditions triggered its use for the scattering of *extended* hadrons all the way to their recent very high energies.

The studies herein reported on the covering isoscattering isotheory have indicated that the consistent representation of hadrons as extended, therefore deformable and hyperdense, implies necessary revisions of hadron physics beginning with a "completion" of

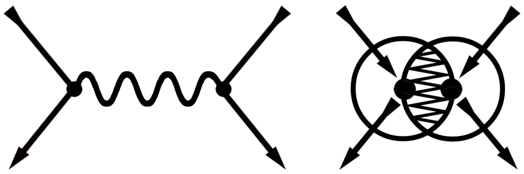

Figure 15: *The left view illustrates the scattering of point-particles, with ensuing realization of interactions via point-particle exchanges, that have been experimentally established for electromagnetic interactions. The right view illustrates the scattering of extended hadrons at high energy implying the presence of contact, non-linear, non-local and non-potential interactions in the scattering region. These conditions imply the impossibility of conventional particle exchanges evidently because of the extremely big density of the scattering regions that may approach the density of black holes, thus requiring broader scattering theories.*

its mathematical foundations, then passing to the "completion" of quantum laws. These advances then require, for consistency, the "completion" of the potential scattering theory into a covering theory in which the hyperdense character of the scattering region implies the presence of non-linear, non-local and non-potential effects preventing any consistent representation of interactions via the sole exchanges of extended hadrons in favor of covering vistas (Figure 15).

It should be stressed that the isoscattering isotheory has a Lie-Santilli algebraic structure, thus being solely applicable to time reversal invariant collisions generally given by elastic scattering. The study of inelastic scattering of extended hadrons at high energy requires the broader *genoscattering genotheory* with the covering Lie-admissible algebraic structure, which broader theory cannot be considered here for brevity (see Refs. [56] [133]).

To avoid major misconceptions, it should be indicated that *the isoscattering isotheory cannot change the numeric values of scattering angles, cross sections and other experimentally measured quantities.* However, the isoscattering isotheory does require serious revisions of the *theoretical interpretation* of measured quantities.

In closing, we should recall that rather vast studies have been conducted on *non-unitary scattering theories* (see, e.g., Ref. [134] and papers quoted therein).

As it is well known, these studies had to be abandoned because non-unitary theories violate causality laws. By contrast, *the isoscattering isotheory is isounitary on the iso-Hilbert isospace over an isofield,* thus restoring causality (Section 2.9).

This occurrence illustrates again the importance of isomathematics for the verification of the EPR argument and related applications.

4.16. Geno- and Iso-chemistry.

The Lie-admissible mathematical and physical methods of Section 2 have allowed the "completion" of quantum chemistry into *Lie-admissible hadronic chemistry,* also known as *genochemistry* [60], which is the first known chemical formulation specifically built for the consistent treatment of chemical reactions at large and energy releasing chemical processes in particular.

As it is well known but generally ignored, chemical reactions are generally *irreversible over time,* while quantum chemistry is strictly reversible. Hence, the EPR argument on the "lack of completion of quantum mechanics" does indeed apply to quantum chemistry.

In addition to a "completion" for chemical reactions, quantum chemistry needs an additional "completion," this time, for the achievement of an *attractive* force between the *identical* electrons of valence couplings in molecular structures (Section 1.6).

Since isolated molecules existing in nature are stable, thus being *reversible— over time,* and so are their valence electron bonds, there was the need of building the *Lie-isotopic particularization of the Lie-admissible hadronic chemistry* which became known as *isochemistry* because based on the isomathematics of Section 3.

Isochemistry did indeed achieve the first known attractive force between identical electrons in valence couplings [60] in a form permitting the exact representation of experimental data on the hydrogen [103] and the water [104] molecules.

As reviewed in details in Paper II, these studies essentially established that *the contact, non-potential interactions occurring in deep mutual penetration/entanglement of the wavepackets of particles (Figure 2) are strongly attractive, thus being responsible for the neutron and other hadron syntheses (Section 1.4), as well as for the attractive force between identical electrons in valence pairs (Section 1.6), thus illustrating the truly fundamental character of short range, contact, non-potential interactions in the ultimate structure of nature.*

5. CONCLUDING REMARKS

In 1935, A. Einstein, B. Podolsky and N. Rosen presented the view that *quantum mechanics is not a complete theory* [1] in the sense that the uncertainty of quantum mechanics is certainly valid for point-like particles in vacuum under sole linear, local and potential interactions (*exterior dynamical systems*), yet there could exist limit conditions in nature achieving classical determinism.

Following decades of preparatory works, R. M. Santilli published in 1998 an apparent proof of the EPR argument [7] for extended, deformable and hyperdense particles in conditions of mutual overlapping/entanglement under linear and non-linear, local and non-local and potential as well as non-potential interactions, as occurring in the structure of hadrons, nuclei, stars and black holes (*interior dynamical systems*).

In particular, Ref. [7] proved the inapplicability (rather than the violation) of Bohr's [2], Bell's [3], von Neumann's [4] and other objections against the EPR argument for extended particles in interior dynamical conditions; proved the existence of *hidden variables* [5] for the proper representation of interior systems; showed the existence of limit interior conditions admitting identical classical counterparts; and presented specific examples of interior dynamical systems progressively recovering classical determinism in the interior of hadrons, nuclei, stars and black holes.

More recently, R. M. Santilli completed the above proof in paper [8] by showing that, under the above indicated conditions, the standard deviations for coordinates Δr and momenta Δp appear to progressively tend to zero for extended particles within hadrons, nuclei and stars, and appear to be identically null for extended particles within the limit conditions in the interior of gravitational, collapse, essentially along Einstein's vision.

In this paper, we have reviewed and upgraded the mathematical, physical and chemical methods used for proof [7], with particular reference to the following aspects:

1) Review and upgrade of the axiom-preserving "completion" of 20th century applied mathematics for the representation invariant over time of extended particles in interior

conditions, which "completion" was initiated by Santilli in 1978 [16] when at Harvard University under DOE support with the proposed name of *isomathematics* and studied thereafter jointly with various mathematicians [55];

2) Review and upgrade of the axiom-preserving "completion" of quantum mechanics and chemistry permitted by isomathematics for the non-relativistic and relativistic description of extended particles in interior conditions, which "completion" was initiated by Santilli in 1978 [17] and its study continued thereafter jointly with various physicists [56];

3) Review and upgrade of the central aspect of the studies herein considered, namely, the need for a "completion" of the Newton-Leibnitz differential calculus, from its sole definition at isolated points, into a covering form defined in volumes, which "completion" was initiated by Santilli under the name of *isodifferential isocalculus* in the 1996 paper [115] and studied in details by S. Georgiev [83] and other mathematicians. In a second paper, we review the apparent proofs [7] [8] of the EPR argument and study specific cases of interior dynamical systems in particle physics, nuclear physics and chemistry that progressively approach classical determinism, and apparently achieve it under the extreme conditions in the interior of black holes.

In a possible third paper, we plan to illustrate the far reaching implications of the EPR argument for all quantitative sciences, as an illustration of Santilli's view that the *lack of completion of quantum mechanics is Einstein's most important prediction*.

Acknowledgments. The writing of this paper has been possible thanks to pioneering contributions by numerous scientists we regret not having been able to quote in this paper (see Vol. I, Refs. [61] for comprehensive literature), including:

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