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Summary. – We wonsider the hadronic isotopic generalization of the Dirac delta-function proposed by MYUNG and SANTILLI, and we introduce its extension to field theory. Some of the first implications are identified, such as the capability of removing the singularities of the propagation function on the light-cone.

As is well known, the Dirac delta-function is at the ultimate foundations of quantum mechanics, in as much as it is the central representative of the pointlike description of particles which is inherent in the mechanics itself.

In a recent paper (1), MYUNG and SANTILLI proposed the following hadronic-isotopic generalization of the Dirac delta-function

$$\delta^*(x) = \frac{1}{2\pi}\int \exp\left[iz * x\right] dz = \frac{1}{2\pi}\int \exp\left[izTx\right] dz\,, \tag{1}$$

where T is an operator generally restricted to be nonsingular and Hermitian. Generalization (1), hereon referred to as the Dirac-Myung-Santilli delta-function, was proposed to achieve a representation of hadrons as extended-deformable charge distributions.

In fact, eq. (1) can be formally written as

$$\delta^*(x) = \delta(Tx)\,, \tag{2}$$

where T can be, in particular, also a function of x, thus illustrating the possibility of « smoothing » the distribution character of the Dirac delta, and of spreading it over a finite region of space, which is exactly the original proposal (1).

MYUNG and SANTILLI continued their analysis for the particular case when T is a constant, for which they wrote $\delta(Tx) = (1/T)\,\delta(x)$. The understanding is that this simplified form does not hold for the general case when T is an operator.

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(1) H. C. MYUNG and R. M. SANTILLI: *Hadronic J.*, **5**, 1277 (1982).

In this paper we shall initiate the studies on the extension of the Dirac-Myung-Santilli delta-function to field theory. For this purpose, let us recall the definition of the Dirac delta-function in a Minkowski space-time with local co-ordinates x^μ, $\mu = 0, 1, 2, 3$,

$$\delta(x) = \frac{1}{(2\pi)^4}\int \exp\,[ik_\mu x^\mu]\,\mathrm{d}k\,, \tag{3}$$

where $\mathrm{d}k = \mathrm{d}k_0\,\mathrm{d}k_1\,\mathrm{d}k_2\,\mathrm{d}k_3$ and $k_\mu x^\mu$ is Lorentz invariant.

Following (1), we shall, therefore, introduce the following hadronic-isotopic lifting of structure (3):

$$\delta^*(x) = \frac{1}{(2\pi)^4}\int \exp\,[ik_\mu * x^\mu]\,\mathrm{d}k = \frac{1}{(2\pi)^4}\int \exp\,[ik_\mu Tx^\mu]\,\mathrm{d}k\,, \tag{4}$$

where T is assumed to be, for simplicity, a (scalar) function of Lorentz-invariant quantities.

We shall show that generalization (4) offers a number of intriguing possibilities in field theory, such as that of removing light-cone singularities of propagators, or at least rendering them more manageable.

Consider the invariant distance with the Minkowski metric

$$\tau^2 = x_0^2 - x_1^2 - x_2^2 - x_3^2\,. \tag{5}$$

T is assumed to be a function of τ, that is to say

$$T = f(\tau)\,. \tag{6}$$

If we set

$$x'^\mu = Tx^\mu = f(\tau)\,x^\mu\,, \tag{7}$$

we have for the invariant square

$$x'_\mu x'^\mu = f^2(\tau)\,x_\mu x^\mu = f^2(\tau)\,\tau^2\,. \tag{8}$$

Then it follows from (8) that the space-time of x'^μ differs from the Minkowski space-time.

A Lie isotopic generalization of Einstein's special relativity has been recently proposed by SANTILLI [2] for the generalized principle according to the invariance of the maximal speed of propagation of causal signals, *whose value depends on the local physical conditions in which the signal propagates.*

The generalization is based on the construction of the Lorentz group on the identity $\hat{I} = g(x, \ldots)^{-1}$, where g is a nonsingular and Hermitian metric characterizing the space-time separation $x^2 = x^t g x$ which is left invariant by the Lorentz-isotopic transformations identified in ref. [2]. It is evident that (8) is a particular case of the relativity of ref. [2] for the case when $g = f^2(\tau)\,\eta I$, η is the Minkowski metric, $I = \mathrm{diag}\,[+1, +1, +1, +1]$.

It should be recalled that one of the implications of the relativity proposed by SANTILLI is the *variation* of the cone of light depending on local physical conditions

[2] R. M. SANTILLI: *Lett. Nuovo Cimento*, **37**, 545 (1983).

(density of medium, temperature, etc.), which is experimentally established for light propagating in transparent, material media (such as for the Čerenkov light), and conjectured for the interior of hadrons ([2]). Evidently, the same modification exists for our particular case of metric (8). This deformation of the light-cone is important to understand the physical origin of the removal of the singularities on the conventional light-cone in vacuum (empty space), as we shall see.

If we consider the D'Alembert equation of scalar fields $\psi(x'^\mu)$ in x'^μ space-time,

$$\Box\,\psi(x'^\mu) = 0\,, \tag{9}$$

the plane-wave solution is of course given by

$$\psi = \exp\,[ik_\mu x'^\mu]\,. \tag{10}$$

Equation (9) written in terms of x^μ will be slightly complicated, but the plane-wave solution will be

$$\psi = \exp\,[ik_\mu Tx^\mu] = \exp\,[ik_\mu f x^\mu]\,, \tag{11}$$

where we used (7). We evidently assume that the wave propagation occurs, not in vacuum, but within a material medium, as in ref. ([1,2]). Dirac's delta-function with argument $x_\mu x^\mu$ is often used in field theory ([3]), so we consider it with argument $x'_\mu x'^\mu$

$$\delta(x'_\mu x'^\mu) = 4\pi D(x'^\mu)\,, \tag{12}$$

where $D(x'^\mu)$ is a propagation function in field theory. From (8) and (12) $D(x'^\mu)$ is expressed by

$$D(x'^\mu) = \frac{1}{4\pi}\,\delta(f^2\tau^2)\,. \tag{13}$$

If we assume that $f(\tau)$ has the following form:

$$f^2(\tau) = \frac{h(\tau)}{\tau^2}\,, \tag{14}$$

and that $h(\tau)$ obeys the following conditions: the equation $h(\tau) = 0$ has the roots s_i and $h'(s_i)$ do not vanish, where $h'(z) = \mathrm{d}h/\mathrm{d}z$, then D of (13) is expressed as

$$D(x'^\mu) = \frac{1}{4\pi}\sum_i \frac{1}{|h'(s_i)|}\,\delta(\tau - s_i)\,. \tag{15}$$

If s_i do not vanish, from (15) the singularities of the propagation function D do not lie on the light-cone. So the field theory with (13) is not the ordinary field theory, examples of this type were given by several authors ([4-6]).

([3]) C. ITZYKSON and J.-B. ZUBER: *Quantum Field Theory* (McGraw-Hill Co., New York, N. Y., 1980).
([4]) M. A. MARKOV: *Nucl. Phys.*, **10**, 140 (1959).
([5]) A. A. KOMAR and M. A. MARKOV: *Nucl. Phys.*, **12**, 190 (1959).
([6]) F. BOPP: *Ann. Phys.* (Leipzig), **38**, 345 (1940).

From (4) and (7) we obtain

$$\delta^*(x) = \frac{1}{(2\pi)^4}\int \exp\,[ik_\mu x'^\mu]\,\mathrm{d}k = \delta(x')\,. \tag{16}$$

It follows from (16) that $\delta^*(x)$ corresponds to the four-dimensional Dirac's delta-function in the x'^μ space-time the relation of which to the Minkowski space-time is given by (7). As seen above, $\delta^*(x)$ plays an important role to modify the ordinary field theory, especially for escaping out from the singularity on the light-cone of the propagation function. Additional studies will be presented elsewhere.

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