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APPARENT CONSISTENCY OF RUTHERFORD'S HYPOTHESIS ON THE NEUTRON STRUCTURE VIA THE HADRONIC GENERALIZATION OF QUANTUM MECHANICS - I: NONRELATIVISTIC TREATMENT

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Abstract

Rutherford conceived the existence of the neutron back in 1920 as a "compressed hydrogen atom", i.e., as an electron compressed (say, in the core of a star) inside the proton structure. While the existence of the neutron was subsequently confirmed, Rutherford's original conception of its structure was claimed to possess several "inconsistencies", i.e. the lack of a quantitative representation of rest energy, meanlife, spin, etc. In this paper we show that these "inconsistencies" appear to be due to the excessive approximations which are inherent in the use of the underlying discipline, ordinary quantum mechanics, for the physical conditions of Rutherford's structure. In fact, quantum mechanics can only provide a point-like abstraction of particles, and thus produce a model of Rutherford's neutron as a sort of small atomic structure, while the latter physical conditions imply the total mutual penetration of the wavepackets of the electron and of the proton one inside the other. It is shown that, if a generalization of quantum mechanics specifically conceived to represent the latter conditions (under the name of hadronic mechanics), is used to treat Rutherford's hypothesis, *all* "inconsistencies" originating in the use of quantum mechanics appear to be resolved by permitting a consistent representation of all characteristics of the neutron, such as: rest energy, meanlife, size, spin, charge, space and charge parity, (anomalous) magnetic moment, as well as the neutron decay. It is therefore conjectured that Rutherford's compression of the hydrogen atom may well be the ultimate mechanism for the creation of neutrinos in Nature. A reinspection of Barut's model on the neutron structure as a bound state of one proton, one electron and an antineutrino, is suggested for possible fundamental advances in the origin of the neutrino.

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1 Introduction

In 1929 Rutherford [1] predicted the existence of the neutron n as a "compressed hydrogen atom", that is, as a bound state of one proton p^+ and one electron e^- at very small mutual distances. The hypothesis on the existence of the neutron was dramatically verified twelve years later by Chadwick [2]. Nevertheless, Rutherford's conception on the structure of the neutron, $n = (p^+, e^-)$, was claimed to be afflicted by numerous, fundamental "inconsistencies" via the use of quantum mechanics, such as the inability to achieve a consistent, quantitative, representation of the rest energy of the neutron, its mean life, its spin, etc. (see Sect. 8 for more details).

The inability to represent the neutron constituents with physical, already known particles, caused an historical change in the direction of physics, from the traditional patterns of atomic and nuclear physics (where the constituents of composite systems are ordinary particles which can be detected

in the laboratory), toward more abstract approaches. In fact, Ivanenkov [3] and Heisenberg [4] first introduced the notion of isotopic $SU(2)$ spin in which the proton and the neutron are unified in an abstract space. The $SU(3)$ symmetry and related quark models on the hadronic structure [5] then resulted to be a rather natural generalization. According to the latter theories, the neutron is a bound state of three quarks, $n = (u, d, d)$, with similar structure for the remaining elements of the family of barions.

The most visible departures from the historical patterns established previously in the study of atoms and of nuclei, is that quark models have attempted the achievement with one single model of both, the classification of all hadrons into families, as well as, jointly, the actual structure of each individual member of a given hadronic family. On the contrary, the atomic and nuclear phenomenologies were studied with two different but compatible models, one for the classification of the particles into suitable families and a different, but compatible model for the characterization of the structure of each individual member of a given family. Another relevant departure is that, while the number of constituents increases with mass in the atomic and nuclear structure, quark models imply that all elements of the same family preserve the number of quark constituents irrespective of the increase of the mass.

As of today, we can readily claim that the unitary models have indeed achieved the final classification of hadrons into families according to a simply overwhelming amount of experimental evidence, with particular reference to the capability of the models to predict new particles, in a way very similar to the historical predictions of new atoms by the Mendeleyev classification.

Regarding the separate problem of the structure of each individual element of a given family of hadrons, we have to report, for scientific accuracy, the existence for quark models of fundamental, yet unresolved problematic aspects. The most important one is the lack of achievement of a true confinement of quarks, that is, of a confinement in which, not only we have an infinite potential barrier, but also an identically null probability of tunnel effects of free quarks. According to explicit calculations [6], such probability of tunnel effect, when explicitly computed, is indeed finite at the level of conventional, nonrelativistic quantum mechanics, as it must be expected from the validity of Heisenberg's uncertainty principle in the interior structural problem. As a result, according to calculations [6], there exists a finite probability for the emission by the neutron of free quarks, which is evidently contrary to experimental evidence. Another problematic aspect of the quark models of hadronic structure is that, despite the existence of mass formulae, the models have been unable to reproduce, within a hadronic contest, the completeness of the quantitative representation achieved by the Bohr model of the hydrogen atom. In fact, it has not been possible to achieve until now one single differential equation, say for the bound state quark-antiquark, which, first of all, has only a *finite* number of admitted energy levels as requested by the complementary classification, while jointly characterizing *all* the other characteristics of the represented particles, as done by the Bohr model, such as: size, mean life, etc.

Owing to the above problematic aspects (as well as for other reasons), Barut [7] was the first to conduct a "return ad originem", i.e., a quantitative study of Rutherford's historical teaching (identification of the neutron constituents with physical particles), by assuming that the neutron is a bound state of the particle produced in its spontaneous decay, and we shall write $n = (p^+, e^-, \overline{\nu}_e)$. Equivalently, Barut's model can be interpreted also as identifying the quarks u, d, d, with the physical particles $p, e, \overline{\nu}_e$. Barut's model clearly possesses the pre-requisites for resolving the lack of true confinement, inasmuch as the finite probability of tunnel effects for free quarks [6] would imply the decay $n \rightarrow p^+ + e^- + \overline{\nu}_e$. Nevertheless, Barut's model is afflicted by other problematic aspects, such as technical difficulties in binding the elusive and massless neutrino $\overline{\nu}_e$ inside the proton structure for the very long period of time (in hadronic scale) needed to represent the neutron lifetime (of the order of 917"). As a result, the problem of the ultimate identification of the constituents of hadrons (or of quarks?) is still fundamentally open at this writing, in the personal opinion of this author.

In this paper we shall show that the "inconsistencies" of Rutherford's historical hypothesis $n = (p^+, e^-)$ are in effect due to the excessive approximations of the assumed discipline, quantum mechanics, when applied to physical conditions substantially different than the atomic ones of its original conception. In fact, a generalization of quantum mechanics under the name of *hadronic mechanics*, was submitted by this author back in 1978 [9] and subsequently developed by a number of authors (see contributions [10–25] and papers quoted therein), precisely to represent the novel physical conditions that are present in Rutherford's conception of the neutron. In this paper we shall show that the use of the generalized hadronic mechanics in lieu of quantum mechanics, permits the apparent resolution of all the "inconsistencies" in Rutherford's model.

In different terms, if a suitable generalization of quantum mechanics for the conditions considered is used, the neutron can indeed be a bound state of one proton and one electron exactly as originally conceived by Rutherford,

without any apparent need of additional particles such as the anti-neutrino $\overline{\nu}_e$. In particular, we shall submit one differential equation for the structure $n = (p^+, e^-)$ which is capable of representing all characteristics of the neutron, such as: mass, size, meanlife, spin, space and charge parity, magnetic moment, etc., in a way as similar as possible to the representational capability of Bohr's equation for the hydrogen structure.

The main contention of this article can be expressed as follows. As well known, quantum mechanics can only provide a point-like approximation of extended particles such as the proton, owing to its intrinsically local and differential structure. The use of quantum mechanics for hypothesis $n = (p^+, e^-)$ therefore implies an atom-like structure according to which the electron orbits around the point-like proton at a distance of the order of the neutron charge radius $R = 0.8 \times 10^{-13}\text{cm} = 0.8$ F.

An inspection of these physical conditions reveals serious doubts in the exact applicability of quantum mechanics. The proton is one of the densest objects detected in our laboratories. Even though its constituents are expected to be point-like, "point-like wavepackets" simply do not exist in the physical reality. Elementary calculations then show that the proton's constituents, to be physical particles, must have wavepackets with a dimension of the order of (at least) the size of the proton (about 1F), exactly as it occurs for the ordinary electrons.

As a result, physical evidence shows that the proton is far from being an empty sphere with points in it. On the contrary, its interior is constituted by a hyperdense medium (called *hadronic medium* [9]) composed by the wavepackets of all the constituents in a state of total mutual immersion-penetration. It is then evident that the electron cannot freely orbit inside such a hyperdense medium and obey exactly the same physical laws as holding for its motion in vacuum when a member of an atomic structure. Under these conditions, the exact validity of quantum mechanics for the study of the hypothesis $n = (p^+, e^-)$ is questionable and, at best, inconclusive [9].

At a deeper analysis, it emerges that, for an electron to be a member of Rutherford's compressed atom, its wavepacket must be in a state of total immersion-penetration within the hadronic medium in the interior of the proton. This implies the existence of novel interactions in the neutron structure which are: 1) of *contact* type in the sense of being caused by the actual physical contact/mutual penetration of the wavepackets without any action-at-a-distance; 2) of *nonlocal* type, in the sense that they evidently occur throughout the volume of mutual penetration and cannot be effectively reduced to interactions among a finite set of isolated points; and 3) of *non-*

Hamiltonian type, in the sense that they violate the integrability conditions for the existence of a Hamiltonian (the so-called conditions of variational selfadjointness [18]), and cannot be effectively represented via the simplistic addition of an integral potential in the Hamiltonian.

It is evident that the novel (zero range) interactions of the above type are simply outside the representational capability of quantum mechanics. The central contention of this paper is therefore that quantum mechanics is inapplicable for an effective study of Rutherford's hypothesis $n = (p^+, e^-)$ because of fundamental insufficiencies in its local-differential-Hamiltonian structure in representing the contact-nonlocal-nonhamiltonian motion of the wavepacket of the electron within the wavepackets of the proton's constituents.

Hadronic mechanics was conceived [9] precisely to represent these novel interactions in such a way to recover ordinary quantum mechanics at the limit when the constituents exit the hadronic structure and move in vacuum. The new mechanics was based on the so-called *Lie-isotopic generalization of Lie's theory* developed in the preceding memoir [8], which is essentially a realization of the theory with respect to the most general possible operator unit $\hat{I}$ (see next section). Nonlocal integral realizations of the generalized unit then allow the incorporation "ab initio" of the novel contact-nonlocal-nonhamiltonian interactions (See Sect. 4). The generalization of all the structural elements and formulations of quantum mechanics is then predictable, as we shall see, and so is the capability of the theory to recover the conventional quantum mechanics at the limit when the generalized operator unit recovers the trivial unit of quantum mechanics, $\hat{I} \to I = \text{diag}(1, 1, \ldots, 1)$. In order to avoid confusion, as well as to emphasize its arena of applicability, quantum mechanics was called *atomic mechanics* (AM) in memoir [9] and the same terminology will be used in this paper. Similarly, the name of *hadronic mechanics* (HM) was introduced in memoir [9] to stress its applicability, specifically, to the hadronic structure, that is, to bound states of particles under conditions of total mutual immersion of their wavepackets, and the same terms will be adopted in this paper.

Hadronic mechanics was suggested for the specific purpose of attempting the identification of the hadronic (or of quarks) constituents with physical, ordinary, massive particles produced in the spontaneous decays [9]. In fact, in its original rudimentary form, the new mechanics was applied for the construction of a structural model of the π^0 as a "compressed positronium"

[9] and we shall write

$$\text{Pos.} = (e^+, e^-)_{\text{AM}} \longrightarrow \pi^0 = (e^+, e^-)_{\text{HM}} \,. \tag{1.1}$$

In particular, it was shown (loc. cit. Sect. 5, pp. 819–880), that the hadronic generalization of Bohr's model for the positronium structure permits the achievement of a quantitative representation of *all* physical characteristics of the π^0, such as: rest energy, charge radius, meanlife, charge, spin, space and charge parity, magnetic and electric moments and primary decays. The reader should be aware that atomic mechanics does not allow such a quantitative representation, e.g., because of the impossibility to represent the total rest energy of 135 MeV of the π^0 as a bound state of particles with 0.5 MeV rest energy, and other reasons. In this way, the use of the hadronic generalization of atomic mechanics resulted to be *necessary* already at the level of the structure of the lightest known hadron.

The success of compression (1.1) was centrally dependent on the *suppression of the positronium spectrum of energy down to only one admissible level: the π^0 [9].* In fact, when under conditions of total mutual immersion, bound states resulted to admit only one stable state, while all other conceivable states resulted to be extremely unstable, and thus implying the constituents exiting the hadronic medium, thus reacquiring their behavior under atomic conditions. In this way, the excited states of $\pi^0 = (e^+, e^-)_{\text{HM}}$ are those of the positronium (see Fig. 1 for more details). In particular, *only the singlet state resulted to be stable in hadronic mechanics [9].* This is due to the fact that, when massive, extended, spinning particles are coupled in a singlet in condition of mutual penetration, their mutual spinning is "in phase", while the triplet state would require the spinning of one particle *against* the direction of rotation of the other, thus resulting in dissipative, highly de-stabilizing effects. It is hoped the reader begins to see the profound differences between hadronic and atomic mechanics.

Hadronic structure model (1.1) was extended in the original proposal [9] to all remaining mesons. The suppression of the energy spectrum in the π^0, as well as the need to represent the charge, implied the increase of the number of the constituents in the transition from the π^0 to the charged pions which resulted to be given by the hadronic three-body structure $\pi^\pm = (e^+, e^\pm, e^-)$, while the remaining mesons resulted to be hadronic bound states of pions, e.g., $K^0 = (\pi^+, \pi^-)$. In this way, the atomic and nuclear laws of the increase of the number of the constituents with mass was recovered in a way compatible with the established unitary models of classification [5].

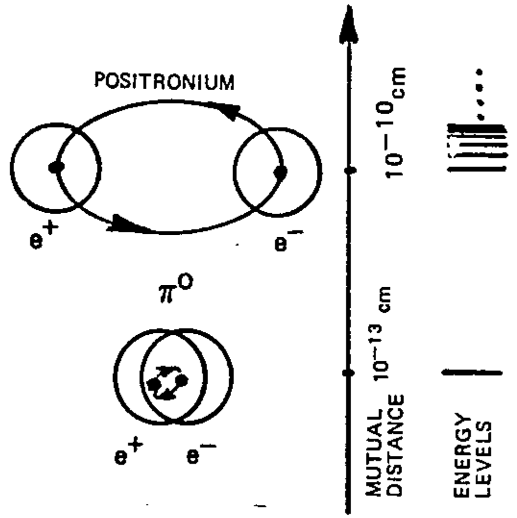

FIGURE 1: A schematic view of the first application of hadronic mechanics: the interpretation of the π^0 as a compressed positronium, i.e., as a generalized bound state of one ordinary electron and one ordinary positron, proposed by the author back in 1978 [9]. The model provided also a first concrete illustration of the apparent insufficiencies of atomic mechanics (i.e., the ordinary quantum mechanics) for an effective treatment of the structure of hadrons, and the need for a suitable generalization. In fact, when treated via atomic mechanics, the model is unable to represent the rest energy, size, and other characteristics of the π^0, while resulting in fundamental inconsistencies, such as a structure model of the atomic type with an infinite spectrum of energy outside the experimentally detected hadrons. The use instead of the covering hadronic mechanics allowed the resolution of all these inconsistencies, by providing a consistent and quantitative representation of all intrinsic characteristics of the π^0 via one single equation of structure, such as: rest energy, meanlife, size, spin, charge, magnetic and electric moments, space and charge parity, as well as primary decays. The model also produced the suppression of the infinite atomic spectrum of the positronium down to only one admissible level: the π^0. This signaled the need, within the context of the covering hadronic mechanics, of the law established for the atomic and

nuclear phenomenologies, i.e., the increase of the number of physical constituents with mass. In fact, the model was extended in proposal [9] to the remaining (light) mesons, which resulted to be interpretable as generalized bound states of massive constituents produced in the spontaneous decays, in a way fully compatible with the established unitary models of classification of hadrons into families. In order to achieve the final solution of the problem of the physical constituents of hadrons (or of quarks?), proposal [9] therefore advocated a return "ad originen", i.e., the use, as historically established for the study of atoms and nuclei, of two compatible approaches: one for the classification and a separate one for the structure of each individual element of a given family. Also, while the ordinary atomic mechanics is sufficient for the classification and exterior phenomenology of hadrons, the construction of a suitable covering mechanics resulted to be *necessary* for the identification of the hadronic constituents with physical particles. Proposal [9] ended with the indication that a central open problem for the consistency of the approach was the resolution of the known "inconsistencies" in Rutherford's conception of the neutron as a bound state of one proton and one electron (Fig. 2).

The basic mechanism for the achievement of consistency in the above structure models should be recalled already in these introductory words, because essential for the analysis of this paper. On physical grounds, memoir [9] submitted the conjecture that the ordinary electrons and positrons $e^\pm$ (as well as any other particle) experience an alteration of their intrinsic characteristics (called *mutation*) in the transition from motion in vacuum (atomic setting) to motion within a hyperdense hadronic medium (hadronic setting). According to this conjecture, the rest energy, charge, spin and other intrinsic characteristics of an ordinary electron may change in the transition from the conditions in which they have been measured until now (motion in vacuum under external, generally long range, electromagnetic interactions), to motion, say, within the core of a collapsing star (hadronic conditions).

In order to emphasize this possible mutation, the name *eletons* and the symbols $\epsilon^\pm$ were introduced in memoir [9] to denote the ordinary electrons and positrons $e^\pm$ when belonging to a hadronic bound state. The structure models $\pi = (e^+, e^-)_{\mathrm{HM}}$ and $\pi^\pm = (e^+, e^\pm, e^-)_{\mathrm{HM}}$ were then denoted

$$\pi^0 = (\epsilon^+, \epsilon^-)_{\mathrm{HM}}, \quad \pi^\pm = (\epsilon^+, \epsilon^\pm, \epsilon^-)_{\mathrm{HM}} \,. \tag{1.2}$$

The equations of motion to characterize an eleton were also introduced (loc. cit. Eq. (4.20.5), p. 800), and it resulted to be a generalization of Dirac's equation with the addition precisely of the nonpotential (variationally nonselfadjoint) terms needed to represent mutual wave penetration. Since these terms are generally dependent on the velocities, an alteration of Dirac's gamma matrices then followed. The alteration of the intrinsic characteristic, e.g., of the magnetic moment and spin, was then consequential (loc. cit. pages 801–804).

On mathematical grounds, the mutation of particles is a necessary consequence of the generalization of the unit of Lie's theory, $I \to \hat{I}$. In fact, the conjecture was submitted after the *Galilei-isotopic generalization of the Galilei symmetry* (and of the underlying relativity) worked out in the preceding memoir [8] and subsequently studied in detail in monograph [19]. In fact, the generalization of the unit implies a generally nonlinear extension of the symmetry transformations as well as a predictable alteration of the Casimir invariants. One can then readily expect a generalization of the intrinsic characteristics of particles in a covering way, i.e., in a way admitting of the conventional characteristics as a particular case.

Memoir [9] ended with the suggestion to apply hadronic mechanics to the study of the original Rutherford's hypothesis (as well as with the need to test directly the validity or invalidity of conventional atomic laws in the *interior* of hadrons). As a matter of fact, the achievement of a consistent formulation of Rutherford's hypothesis was considered a crucial test for the consistency of the new mechanics.

The objective of this paper can now be formulated in more details. In fact, we shall study Rutherford's historical hypothesis via the hadronic compression of the hydrogen atom, i.e.,

$$\text{Hydr. At.} = (p^+, e^-)_{\text{AM}} \to n = (p^+, \epsilon^-)_{\text{HM}} \,. \tag{1.3}$$

This implies the following assumptions: 1) the presence of contact-nonlocal-nonhamiltonian interactions (besides the conventional electromagnetic ones) that are absent in the atomic structure; 2) the representation of these interactions via the hadronic generalization of atomic mechanics; and 3) the alteration of the electron e^- of the atomic structure into the eleton ϵ^- when in condition of total immersion within the proton structure, while the proton p^+ is considered un-mutated in compression (1.3) owing to its much larger mass as compared to that of the eleton.

Compression (1.3) will be studied via an *isotopic lifting* of the Bohr's model of the hydrogen atom, that is, via an isotopic generalization of the underlying Schrödinger's equation (see next section). In particular, the lifting will be successful, if it achieves the suppression of the infinite spectrum of

the hydrogen atom down to only one admissible energy level: the neutron, as well as the representation of *all* the remaining characteristics of the neutron, such as charge radius, meanlife, spin, magnetic moment, etc. In particular, only the singlet state will be permitted by the covering mechanics. Again, the excited states of the hadronic structure $n = (p^+, \epsilon^-)_{\text{HM}}$ result to be precisely the infinite states of the hydrogen structure $(p^+, e^-)_{\text{AM}}$, much along the corresponding occurrences for the model $\pi^0 = (\epsilon^+, \epsilon^-)$ (see Fig. 2 for more details).

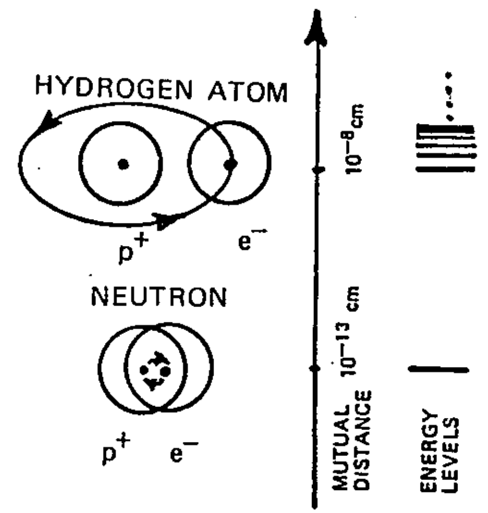

FIGURE 2: A schematic view of Rutherford's conception of the neutron as a "compressed hydrogen atom", i.e., as a bound state of one proton and one electron totally immersed in the proton structure [1]. In this paper we shall show that this historical conception is indeed consistent and capable of representin; *all* the intrinsic characteristics of the neutron, as well as its primary decay, provided that the point-like abstraction of the constituents, which is inherent in the use of atomic mechanics, is abandoned in favor of a more realistic representation of the bound state. In fact, the use of atomic mechanics leads to the representation of the proton as an imaginary sphere with points in it, and to the model $n = (p^+, e^-)_{\text{AM}}$ which is essentially a kind of small atom, with the consequential emergence of numerous "inconsistencies". Clear experimental evidence cast serious doubts on the credibility of this approach. In fact, the proton is one of the densest objects measured

in laboratory until now. Even though its constituents are expected to have a point-like charge structure, there exist no "point-like wavepackets" in Nature. On the contrary, it is well known that all massive particles (including the electron) have a wavepacket of the order of at least the size of the proton ($\sim 1F$). It then emerges from this well established experimental knowledge that the proton is a hyperdense medium, called hadronic medium [9], composed by the wavepackets of all its constituents in a state of total mutual immersion. The assumption that the electron freely orbits in such a hyperdense medium, as necessary to apply atomic mechanics, is just excessively approximated and, at best, inconclusive. A generalization of atomic mechanics under the name of hadronic mechanics was suggested for conditions which are present in Rutherford's structure model of the neutron: motion of the extended wavepacket of the electron within the hyperdense medium in the interior of the proton. In this paper we shall show that, thanks to advances made since 1978 [10–25], all "inconsistencies" claimed via the use of atomic mechanics are resolved and a quantitative representation of all the characteristics of the neutron exist. The most direct way of seeing this result is by noting that the hadronic medium inside the proton is generally inhomogeneous and anisotropic, thus demanding a suitable generalization of Galilei's relativity [8,19] and of Einstein's special relativity [22,23]. Once this aspect is seen, the construction of a consistent and quantitative representation of Rutherford's historical hypothesis is only a matter of technical details. Rather intriguingly, the biggest difficulty in the model, the achievement of a total spin $\frac{1}{2}$ from a (generalized) bound state of two particles of spin $\frac{1}{2}$, turns out to be conceptually so simple to appear trivial and merely due to constraints in the orbital motion that must be expected from the conditions considered (Figure 3).

The study of compression (1.3) along the above lines is now permitted by several advances occurred since the time of its original consideration [9]. In fact, hadronic mechanics has achieved sufficient maturity and diversification of formulation [10–25] to be suitable for practical applications. Additional advances that are crucial for this paper, have been achieved in the study of the Lie-isotopic generalization of Wigner's theorem on quantum mechanical symmetries [12]; the general formulation of Lie-isotopic symmetries on manifolds [20]; the construction (and classification) of the $\widehat{SO}(3)$-isotopic covering of the rotation group [21]; the construction (and classification) of the Lorentz-isotopic covering of the conventional Lorentz symmetry [22]; and others.

This paper is the third of a series of three, deeply inter-related papers. In the first preceding paper [23], we construct the Poincaré-isotopic symmetry $\hat{P}(3.1)$ and show its implications for the relativity laws that are applicable in the interior of a hadronic medium. In the second preceding paper [24], we then construct the *isofield equations*, i.e., the generalized form of conventional field equations which is invariant under the generalized symmetry $\hat{P}(3.1)$. A quantitative characterization of the notion of eleton was provided in the preceding two papers as a suitable representation of the Poincaré-isotopic symmetry, and realized via the isotopic generalization of Dirac's equation (*isodirac's equation*) [24]. As one can see, the original conjecture of mutation of the intrinsic characteristics of particles [9] emerges as fully justified on technical grounds, as well as possessing preliminary experimental verifications via the deformation of the charge distribution of neutrons/alteration of the magnetic moments/rotational asymmetry measured by Rauch and his associates via neutron interferometry (see paper [24] for details). Similarly, the presence of the hadronic medium, and its deviation from atomic conditions, appear to have a preliminary phenomenological verification via the numerous deviations from Einstenian laws in the behavior of the meanlife of unstable hadrons with speed appeared in the literature in the past decades (see paper [23] for details).

This paper shall present only a nonrelativistic treatment of Rutherford's hypothesis. The relativistic analysis is presented in a forthcoming paper [26] where we shall also address the problem of the compatibility of our structure models of hadrons with established unitary models and the possible identification of quarks with mutated physical particles. Nevertheless, it is important for the reader to know that such a relativistic formulation results to be possible thanks to a generalization of Dirac's equation proposed by Dirac himself in two of his last papers [27–28]. In fact, this generalized equation results to have an essential isotopic structure, and imply a *mutation of the spin from $\frac{1}{2}$ to zero*, that is, exactly the total angular momentum of the eleton which is needed to achieve consistency for structure model $n = (p^+, \epsilon^-)_{HM}$.

It is evidently impossible to provide an effective review of the body of studies that preceded this work. The true understanding of this paper therefore requires a prior in depth knowledge not only of hadronic mechanics per sé [9,14,16], but also of its classical counterpart, the so-called *Birkhoffian mechanics* [19], and of the mapping of the latter into the former, the so-called hadronization [29,30], as well as, and perhaps more importantly, of the algebraic structure underlying both classical and operator formulations, the Lie-isotopic theory [8,19] (for a recent review, one can see the forthcoming monograph [31]). The reader is warned that the conceptual, let alone technical departures from the conventional setting is such that any reading of this paper without sufficient knowledge of the field is bound to result in misinterpretations of the analysis.

Hereon, the subscripts "AM" and "HM" in models (1.2) and (1.3) shall be dropped for notational convenience whenever no confusion may arise.

2 Elements of Hadronic Mechanics

The ultimate assumption of hadronic mechanics, from which the entire theory can be derived, is the generalization of the trivial unit $I = \mathrm{diag}(1, 1, \ldots, 1)$ of atomic mechanics into an operator $\hat{I}$ which, besides being nonsingular and Hermitean, has an arbitrary, generally nonlocal-integrodifferential dependence on all possible (or otherwise admissible [23]) local quantities, such as: time t; coordinates $\vec{x}$; velocities $\dot{\vec{x}}$; accelerations $\ddot{\vec{x}}$; states ψ and their conjugates $\psi^\dagger$; density μ of the hadronic medium; temperature τ; etc., and we shall write

$$\hat{I} = \hat{I}(t, \vec{x}, \dot{\vec{x}}, \ddot{\vec{x}}, \ldots; \psi, \psi^\dagger; \mu, \tau, \cdots), \hat{I} = \hat{I}^\dagger . \tag{2.1}$$

The assumption of the above operator as the unit of the theory requires a necessary generalization of the enveloping associative algebra $\mathcal{A}$ of atomic mechanics, with elements $A, B, C, \cdots$ and trivial associative product AB into the so-called *associative-isotopic algebra* (or *isoassociative algebra* for short) originally introduced in memoir [8]

$$\hat{\mathcal{A}} : A * B \stackrel{\text{def}}{=} ATB \ , \quad T = \text{Fixed}, \quad \hat{I} = T^{-1} \ , \tag{2.2}$$

where the generalized product $A * B$ is still associative, i.e., $(A * B) * C = A * (B * C)$ (which justifies the name of "isotopy").

In fact, the generalized unit (2.1) is the consistent, right and left, unit of isoenvelope (2.2), called *isounit*

$$\hat{I} * A = A * \hat{I} \equiv A \ , \quad \forall A \in \hat{\mathcal{A}} . \tag{2.3}$$

The Heisenberg-type representation of hadronic mechanics was identified in the original proposal (ref. [9], p. 752), and constructed according to the traditional rule whereby the brackets of the time evolution are characterized by the underlying enveloping algebras, and can be written in the form called *Heisenberg-isotopic equation*

$$i\dot{A} = [A, H]_{\hat{\mathcal{A}}} = A * H - H * A = ATH - HTA \ , \tag{2.4}$$

where H is any conventional Hamiltonian. The representation of *nonhamiltonian* forces then follows from the fact that the isotopic element T multiplies H from the right and from the left.

The compatible *Schrödinger-isotopic equations* were identified subsequently and independently in papers [13,14], and can be written

$$i\frac{\partial}{\partial t}\psi = H * \psi = HT\psi \ , \tag{2.5.a}$$

$$i\psi^\dagger \frac{\overleftarrow{\partial}}{\partial t} = \psi^\dagger * \overleftarrow{H} = \psi^\dagger T \overleftarrow{H} \ , \tag{2.5.b}$$

$$T = T^\dagger . \tag{2.5.c}$$

The proof of the equivalence between isorepresentations (2.4) and (2.5) was presented in paper [14].

To avoid unnecessary restrictions, hadronic mechanics was formulated on the most general possible realization of Hilbert spaces, those along the *isohilbert* form

$$\hat{\mathcal{H}} : \langle\psi\hat{|}\psi\rangle = \int \psi^\dagger \circ \psi dv \stackrel{\text{def}}{=} \int \psi^\dagger G \psi dv \ , \tag{2.6}$$

where G is an operator generally independent of the isotopic operator T. It is easy to see that, whenever G is a positive operator, $G > 0$; composition (2.6) is inner and $\hat{\mathcal{H}}$ is a Hilbert space (the reader should however keep in mind that certain eletonic conditions may require the relaxation of the condition $G > 0$).

As an incidental note, the attentive reader may have noticed the similarity of composition (2.6) with that proposed by Gupta [32] and Bleuler [33]. It should be noted however, that generalization (2.6) is not essential for hadronic mechanics, because it can also be formulated consistently in a conventional Hilbert space. In fact, the hadronic covering of atomic mechanics is centrally dependent on the generalization of the unit, Eq. (2.1), with consequent generalization of Heisenberg's and Schrödinger's representations of the indicated isotopic type.

For mathematical consistency, the theory is formulated over the so-called *isotopic field* (or *isofield* for short), hereinafter assumed of characteristic zero)

$$\hat{C} = \{\hat{c} | \hat{c} = c\hat{I}\ , \quad c \in C\ , \quad \hat{I} \in \hat{\mathcal{A}}\}\ , \tag{2.7}$$

where the sum is the ordinary one and the product is given by

$$\hat{c}_1 * \hat{c}_2 = \widehat{c_1 c_2} = c_1 c_2 \hat{I}\ . \tag{2.8}$$

It should be stressed, however, that the eigenvalues of the theory are ordinary numbers owing to the properties

$$H * \psi = \hat{c} * \psi \equiv c\psi\ . \tag{2.9}$$

The hadronic generalization of all conventional linear operations of atomic mechanics on a Hilbert space were worked out in papers [14,16] and proved to be consistent. For instance, the conventional notion of unitary operator is lifted to that of *isounitary operator*

$$\hat{U} * \hat{U}^{\hat{\dagger}} = \hat{U}^{\hat{\dagger}} * \hat{U} = \hat{I}\ , \quad \hat{U}^{\hat{\dagger}} = \hat{U}^{-\hat{I}}\ . \tag{2.10}$$

Similarly, that of Hermitean conjugation is lifted into that of *isohermitean conjugation*

$$A^{\hat{\dagger}} = T^{-1} G A^{\dagger} T G^{-1}\ . \tag{2.11}$$

In a similar way, we have the selfexplanatory isotopic operations on $\hat{\mathcal{H}}$

$$\begin{aligned} \widehat{\mathrm{Tr}} A &= (\mathrm{Tr} A)\hat{I}\ , && (2.12.a)\\ \widehat{\mathrm{Tr}}(A \times B) &= (\widehat{\mathrm{Tr}} A) \times (\widehat{\mathrm{Tr}} B)\ , && (2.12.b)\\ \widehat{\mathrm{Det}} A &= [\mathrm{Det}(AT)]\hat{I}\ , && (2.12.c)\\ \widehat{\mathrm{Det}}(A * B) &= (\widehat{\mathrm{Det}} A) * (\widehat{\mathrm{Det}} B)\ , && (2.12.d)\\ \widehat{\mathrm{Det}} A^{-\hat{I}} &= (\widehat{\mathrm{Det}} A)^{-\hat{I}}\ . && (2.12.e) \end{aligned}$$

The expectation values of atomic mechanics now acquire generalized forms, called *isoexpectation values*

$$\langle \hat{A} \rangle = \langle \psi | o A * | \psi \rangle = \int \psi^{\dagger} o A * \psi dv = \int \psi^{\dagger} G A T \psi dv\ , \tag{2.13}$$

while the projection operators assume the form of the *isoprojection operators*

$$\hat{P} = |\psi_k\rangle\langle\psi_k| G T^{-1}\ . \tag{2.14}$$

For the isotopic generalization of the remaining aspects of hadronic mechanics we are forced to refer the interested reader to the quoted literature for brevity.

The mathematical structure of the covering mechanics can be summarized as follows. Generalization (2.1) of the unit of the Lie algebra of operators of atomic mechanics implies a predictable generalization of each and every aspect of the conventional Lie's theory and, in particular, of the structure of its main three branches: the universal enveloping associative algebras, the Lie algebras, and the topological Lie groups [8].

In fact, the isoassociative algebra $\hat{\mathcal{A}}$, Eq. (2.2), admits a consistent isotopic generalization of the main structural theorems of the universal enveloping associative algebras $\mathcal{A}(L)$ of a Lie algebra L (see, e.g., ref. [34]), as worked out in the original proposal [8]. Thus, algebras $\hat{\mathcal{A}}$ can also be *universal isoassociative enveloping algebras* $\hat{\mathcal{A}}(L)$ of a Lie algebra L. For instance, the celebrated Poincaré-Birkhoff-Witt theorem [34] admits a consistent isotopic generalization [8,19]. As a result, given a Lie algebra L of (finite) dimension n and (ordered) generators X_i , $i = 1, 2, \ldots, n$, there exist a consistent, infinite-dimensional basis in $\hat{\mathcal{A}}(L)$ given by

$$\hat{I}\ , X_i\ , \underset{i \le j}{X_i * X_j}\ , \quad \underset{i \le j \le k}{X_i * X_j * X_k}, \ldots, \tag{2.15}$$

This allows the existence and convergence into a finite sum of exponentials in $\hat{\mathcal{A}}$ which can be written for simplicity but without loss of generality for the one-dimensional case

$$e_{\hat{\mathcal{A}}}^{iXw} = \hat{I} + \frac{iXw}{1!} + \frac{(iXw) * (iXw)}{2!} + \cdots\ , \tag{2.16}$$

and can be reformulated in the conventional envelope for computational convenience

$$\begin{aligned} e_{\hat{\mathcal{A}}}^{iXw} &= \hat{I} e_{\mathcal{A}}^{iw * X} = \hat{I} e^{iwTX}\ ,\\ &= e_{\mathcal{A}}^{iX * w} \hat{I} = e^{iXTw} \hat{I}\ . \end{aligned} \tag{2.17}$$

For additional information on the universal and enveloping character of the algebra $\hat{\mathcal{A}}(L)$ we refer the interested reader to ref.s [8,19,31].

As customary, a Lie algebra is the antisymmetric algebra $\mathcal{A}^-$ attached to the envelope $\mathcal{A}$. For the case of atomic mechanics, this results in the simplest conceivable realization of the Lie product as provided by the familiar form $[A, B]_{\mathcal{A}} = AB - BA$. For hadronic mechanics, we have instead the structurally more general *Lie-isotopic algebras* as the antisymmetric algebras $\hat{\mathcal{A}}^-$ attached to $\hat{\mathcal{A}}$, introduced for the first time in memoir [8] and characterized by the product

$$[A,B]_{\hat{\mathcal{A}}} = A * B - B * A = ATB - BTA \ , \tag{2.18}$$

which verifies the Lie algebra axioms

$$\begin{aligned} [A,B]_{\hat{\mathcal{A}}} + [B,A]_{\hat{\mathcal{A}}} &= 0 \ , & (2.19.a) \\ [[A,B]_{\hat{\mathcal{A}}},C]_{\hat{\mathcal{A}}} + [[B,C]_{\hat{\mathcal{A}}},A]_{\hat{\mathcal{A}}} + [[C,A]_{\hat{\mathcal{A}}},B]_{\hat{\mathcal{A}}} &= 0 \ , & (2.19.b) \end{aligned}$$

as the reader is encouraged to verify.

The Lie-isotopic generalization of Lie's celebrated First, Second, and Third Theorems was achieved in the original proposal [8] and will not be reviewed here for brevity. This signals the existence of a fully consistent, step-by-step generalization of the theory of Lie algebras for the structural more general realization (2.18) constructed with respect to the general unit (2.1) (see locally quoted references for details).

Finally, the Lie groups are predictably generalized, for consistency with the preceding formulations, into the so-called *Lie-isotopic groups* also introduced for the first time in memoir [8] according to the *isotopic group laws*

$$\begin{aligned} \hat{U}(0) &= \hat{I} \ , & (2.20.a) \\ \hat{U}(w) * \hat{U}(-w) &= \hat{U}(-w) * \hat{U}(w) = \hat{I} & (2.20.b) \\ \hat{U}(w) * \hat{U}(w^1) &= \hat{U}(w + w^1) \ . & (2.20.c) \end{aligned}$$

It is readily seen that the *isounitary transformations*

$$\begin{aligned} \psi' &= \hat{U} * \psi \ , & (2.21.a) \\ \hat{U} &= e^{iXw}_{\hat{\mathcal{A}}} \ , \quad X^\dagger = X \ , & (2.21.b) \end{aligned}$$

first, constitute a group and, second, verify the isotopic laws (2.20), thus constituting a Lie-isotopic group. As a consequence, all conventional space-time symmetries admit, first, as consistent isotopic generalization, as worked out in general paper [20] and then in the specific contributions [21] (isorotations), [19] (isogalilei), [22] (isolorentz), and [23] (isopoincaré) (see also [35] for the $SU(3)$-isotopic symmetry and [36] for the isogauge symmetries). Second, all these symmetries admit a consistent formulation as symmetries of hadronic mechanics, that is, isounitary transformation groups on isohilbert

spaces, as established by the isotopic generalization of Winger's theorem on conventional operator symmetries [12].

For more details (such as the isotopic Baker-Campbell-Hausdorff formula, etc.), we refer the interested reader to ref.s [8,19,31].

The most interesting subcase of hadronic mechanics occurs when the isotopic element T of the operator algebra and the element G of the isohilbert space coincide and are positive definite

$$T = G > 0 \ . \tag{2.22}$$

In fact, in this case ***isohermiticity and conventional Hermiticity coincide***, as one can see from Eq. (2.11). As a consequence, under conditions (2.22), *observables of atomic mechanics remain observable of hadronic mechanics.* However, the eigenvalues of these observables are not preserved under isotopy. Thus, and contrary to popular beliefs, the *eigenvalues of Hermitean operators are not unique, because their explicit values depend on the assumed unit of the theory.* In fact, suppose that a (Hermitean) Hamiltonian H has the (spectrum of) eigenvalues E in atomic mechanics, then the *same* Hamiltonian H has a necessarily *different* set of eigenvalues E' under hadronic isotopy, and we shall write

$$H\phi = E\phi \rightarrow H * \psi = E'\psi \ . \tag{2.23}$$

We reach in this way the central technique we shall use in studying the historical hypothesis (1.3). In fact, as conceptually outlined in the Introduction, we shall study an *isotopic lifting of the conventional Hamiltonian of the hydrogen atom which is capable of suppressing the atomic spectrum of energy and reducing it down to only one admissible level: the neutron.*

The reader should be aware that the classical counterpart of hadronic mechanics, Birkhoffian mechanics, has been proved to be *directly universal*, that is, capable of representing all possible systems of the admitted topological class (*universality*) directly in the reference frame of the experimenter (*direct universality*). The reader can therefore readily predict the existence of a corresponding direct universality of hardonic mechanics, that is, the capability of representing all possible unitary and nonunitary systems (universality), directly in the frame of the experimenter (direct universality). This property has been illustrated best by Jannussis and his collaborators who have shown that even discrete systems (such as the Caldirola systems) are a particular case of hadronic mechanics [37].

This property is important to prevent the reader from attempting the study of Rutherford's hypothesis with other, often inconsistent generalizations of quantum mechanics. In fact, the Schrödinger-isotopic equation (2.5a) can be explicitly written

$$i\frac{\partial}{\partial t}\psi(t,\vec{x}) = H(t,\vec{x},\vec{p})T(b,\vec{x},\vec{p},\psi,\psi^{\dagger},\mu,\tau,\cdots)\psi \,. \tag{2.24}$$

By recalling the primitive assumptions on the isounit (2.1), the above equations are the most general known equations which are, not only *nonlinear* in all the variables, but also *nonlocal* in the same.

As a specific example, a nonlinear generalization of atomic mechanics was worked out recently by Weinberg [38]. An inspection of this theory [39] has revealed that its Schrödinger-type equation is a particular case of the Schrödinger-isotopic equation, as expected. Nevertheless, the theory is afflicted by a rather considerable number of fundamental inconsistencies because based on a *nonassociative* (Lie-admissible) enveloping algebra which, as such, does not admit a consistent unit. Weinberg's theory, therefore, does not admit consistent formulations of: the measurement theory, Planck's constant, quantization, Casimir invariants, space-time symmetries, etc. [39], not to mention basic inconsistencies identified by Okubo [40] for any nonassociative generalization of Schrödinger's equation. Hadronic mechanics evidently resolves *all* these inconsistencies because based on an operator envelope which, by central assumption, remains *associative.* As a matter of fact, and contrary to study [38], the fundamental assumption of hadronic mechanics is the existence of a consistent, right and left, unit.

The ultimate mathematical consistency of hadronic mechanics can be readily seen by noting that, under conditions (2.24), or more generally, for $T \neq G$, $T > 0, G > 0$, isounit $\hat{I}$ preserves the topological character of the original unit of the atomic mechanics (positive definiteness), isoalgebra $\hat{\mathcal{A}}$ is still associative, isohilbert space $\hat{\mathcal{H}}$ is still Hilbert, and isofield $\hat{\mathbf{C}}$ is still a field. As a result, *atomic and hadronic mechanics coincide at the abstract, realization-free level by conception* [8,9]. Atomic mechanics emerges if one assumes the simplest conceivable realizations of the axioms, such a Lie product $AB - BA$. Hadronic mechanics emerges if one assumes instead less simplistic realizations of the same axioms, e.g., via the Lie-isotopic product $A * B - B * A$ [12].

We close this outline by recalling that *hadronic mechanics is a covering of atomic mechanics* in the sense that:

A) Hadronic mechanics is based on mathematical structures (Lie-isotopic theories) more general than those of atomic mechanics (conventional Lie's theories);

B) Hadronic mechanics represents physical conditions (potential as well as contact-nonlocal-nonhamiltonian interactions) which are structurally more general than those of atomic mechanics (strictly local and potential interactions); and, last but not least,

C) Hadronic mechanics can approximate atomic mechanic as close as desired for

$$T \approx G \approx I \,, \tag{2.25}$$

and recovers atomic mechanics identically at the limits

$$T = G = I \,. \tag{2.26}$$

3 Hadronization of Jannussis' Two-Body Galilei-Isotopic Systems

We shall now initiate our study of Rutherford's hypothesis (1.3) by beginning at the primitive Newtonian level. In fact, we shall identify the primitive Newtonian systems underlying our model and show that they belong to the Birkhoffian generalization of classical Hamiltonian mechanics [8,19]. We shall then map these systems into an operator form and show that the emerging systems belong to the hadronic generalization of atomic mechanics. Once the basic differential equations have been identified in this way, we shall then pass to their explicit application to Rutherford's hypothesis.

The primitive Newtonian class of the systems underlying our study was identified in the original proposal of hadronic mechanics (ref. [9], Sect. 3.4, pages 622–633), and called *closed nonhamiltonian systems.* These are systems that are *closed* in the sense of being isolated from the rest of the universe. As such, they verify the *conventional* total conservation laws. Nevertheless, their internal forces are generally nonlocal and nohamiltonian, thus resulting in the maximization of the internal instabilities in order to maximize the internal interactions.

This *is* the general concept of hadronic structure proposed in ref. [9]. Traditionally, global stability is achieved via the stability of the orbits of the individual constituents, as it is the case in the planetary structure (classical case) and its atomic counterpart (operator case). Ref. [9] identified the existence of systems that are globally stable, but in a way in which each orbit is generally unstable. We merely have the most general possible internal exchanges of energy and other physical quantities among the constituents but in such a way that the total energy and all other conventional total quantities are conserved.

A classical example of the latter systems is given by Jupiter. Its center-of-mass trajectory verifies all conventional total conservation laws and symmetries. Nevertheless, its internal structure is highly nonconservative. Memoir [9] submitted the conjecture that *hadrons are the operator image of Jupiter.*

The classical representation of closed nonhamiltonian systems was also worked out in memoir [9] thanks to the identification of the Birkhoffian generalization of Hamiltonian mechanics of the preceding memoir [8]. In the vector field form, the equations of motion can be written as a general system of first order differential equations subject to a system of subsidiary constraints given precisely by the total conservation laws

$$\left\{\begin{array}{ll} (\dot{a}^\mu) = \begin{pmatrix} \dot{\vec{r}}_\alpha \\ \dot{\vec{p}}_\alpha \end{pmatrix} = \begin{pmatrix} \vec{p}_\alpha/\eta_\alpha \\ \vec{f}^{SA}_\alpha(\vec{r}) + \vec{F}^{\rm NSA}_\alpha(t, \vec{r}, \vec{p}, \dot{\vec{p}}, \cdots \end{pmatrix} , & (3.1a) \\ \mu=1,2,\ldots,3N, \alpha=1,2,\ldots,N, & \\ \frac{dE_{tot}}{dt} = \frac{d}{dt}(T+V) = 0, \vec{f}^{\rm SA}_\alpha = -\frac{\partial v}{\partial \vec{r}_\alpha} , & (3.1b) \\ \frac{d\vec{p}_{tot}}{dt} = \frac{d}{dt}\sum_{\alpha=1}^N \vec{p}_\alpha = 0 , & (3.1c) \\ \frac{d\vec{L}_{tot}}{dt} = \frac{d}{dt}\sum_{\alpha=1}^N \vec{r}_\alpha \times \vec{p}_\alpha = 0 , & (3.1d) \\ \frac{d\vec{G}_{tot}}{dt} = \frac{d}{dt}\sum_{\alpha=1}^N (m_\alpha \vec{r}_\alpha - t\vec{p}_\alpha) = 0 , & (3.1e) \end{array}\right.$$

where SA(NSA) represents the conditions of variational selfadjointness (nonselfadjointness), i.e., the verification (violation) of the integrability conditions for the existence of a potential (see memoir [8] for a first treatment and monograph [18] for a general presentation).

An inspection soon reveals that systems (3.1) admit *unconstrained* solutions, owing to their functional degrees of freedom. In fact, the subsidiary constraints can be reduced to the following restrictions on the NSA forces (loc. cit)

$$\sum_{\alpha=1}^N \vec{F}^{\rm NSA}_\alpha = 0 , \tag{3.2.a}$$

$$\sum_{\alpha=1}^N \vec{r}_\alpha \wedge \vec{F}^{\rm NSA}_\alpha = 0 , \tag{3.2.b}$$

$$\sum_{\alpha=1}^N \vec{p} \cdot \vec{F}^{\rm NSA}_\alpha = 0 , \tag{3.2.c}$$

which consist of *seven* independent conditions. The number of arbitrary NSA forces in systems (3.1) is 3N. Thus, unconstrained solutions exist for $N \geq 3$. The case $N = 2$ is special and will be reviewed below. The case $N = 1$ does not exist because one isolated particle is free and, as such, it experiences no force. Systems (3.1) are nonhamiltonian by conception, that is, they violate the integrability conditions for a Hamiltonian representation in the frame of the experimenter (they are the *essentially nonselfadjoint systems* of ref.s [8,19] in their general form). Nevertheless, owing to the direct universality of the covering Birkhoffian mechanics, their Birkhoffian representation *always* exists. It can be formulated via the first order, non-canonical, Birkhoffian action principle

$$\delta\hat{A} = \delta \int^{tr} [R_\mu(a)\dot{a}^\mu - B(t,a)]dt = 0 \quad , \quad \mu = 1,2,\ldots,3N , \tag{3.3}$$
$$a = (a^\mu) = (\vec{r}, \vec{p}) ,$$

where the 3N functions R_μ are (nonsingular) functions of the 3N phase space coordinates a^μ and the function B is called the *Birkhoffian* [8,19] whenever it does not represent the Hamiltonian H (although such a reduction is always possible, and it will be tacitly assumed hereon).

Birkhoff's equations are readily derived from principle (3.3) in their covariant form

$$\Omega_{\mu\nu}(a)\dot{a}^\nu = \frac{\partial B}{\partial a^\mu} , \tag{3.4}$$

where the tensor

$$\Omega_{\mu\nu}(a) = \frac{\partial R_\nu}{\partial a^\mu} - \frac{\partial R_\mu}{\partial a^\nu} , \tag{3.5}$$

is a *general symplectic tensor*, i.e., the two-form

$$\theta_2 = \frac{1}{2}\Omega_{\mu\nu}(a)da^\mu a da^\nu , \tag{3.6}$$

is a *general symplectic two-form* (nowhere degenerate and closed in the geometric sense that $d\theta_2 = 0$).

Birkhoff's equations in their contravariant form are also readily derived from Eq.s (3.4) and can be written

$$\dot{a}^\mu = \Omega^{\mu\nu}(a)\frac{\partial B}{\partial a^\nu} , \tag{3.7}$$

where the tensor

$$\Omega^{\mu\nu}(a) = \left(\|\frac{\partial R_\beta}{\partial a^\alpha} - \frac{\partial R_\alpha}{\partial a^\beta}\|^{-1}\right)^{\mu\nu} , \tag{3.8}$$

is a *general Lie tensor*, i.e., the brackets among functions A, B in phase space

$$[A \hat{,} B] = \frac{\partial A}{\partial a^\mu}\Omega^{\mu\nu}(a)\frac{\partial B}{\partial a^\nu} , \tag{3.9}$$

verify the Lie algebra axioms (loc. cit.).

The covering nature of Birkhoffian over Hamiltonian mechanics is readily shown by noting that, for the particular value of the Birkhoffian functions

$$R_\mu(a) = R^0_\mu(a) = (\vec{P}_\alpha, 0) , \tag{3.10}$$

the conventional Hamiltonian mechanics is recovered in its entirety, as the reader can verify. For these and related aspects we are forced to refer the reader to ref.s [8,19]. Particularly important are the various techniques for constructing a Birkhoffian representation from given equations of motion (3.1), as well as the study of the degrees of freedom of the theory which allow the preservation of direct physical significance of the algorithms at hand, e.g., "$\vec{p}$ " is the linear momentum "$m\dot{\vec{r}}$ ", "H" is the total energy $T + V$, etc. (note that this capability is absent in Hamiltonian mechanics for NSA forces).

Systems (3.1) and their Birkhoffian representation constitute the arena of applicability of the Galilei-isotopic relativity [8,19], which is essentially achieved by replacing the conventional canonical tensor $\omega^{\mu\nu}$ with Birkhoff's tensor (3.8), while the generators (and the parameters) remain the conventional ones. This ensures the validi y of *conventional* total conservation laws via a Birkhoffian formulation of the Noether's theorem. However, the generalized structure of the Lie tensor ensures the existence of nonhamiltonian forces, exactly as desired. For brevity, we must necessarily refer the interested reader to the locally quoted literature for details (see, in particular, ref. [19], Chap. 6).

The two-body case is particular, inasmuch as the orbits resulted to be necessarily stable [9]. In their second-order form, the systems were written

$$M\ddot{\vec{R}} = 0 , \tag{3.11.a}$$

$$m\ddot{\vec{r}} = \vec{f}^{SA}(\vec{r}) + \vec{F}^{NSA}(b, \vec{r}, \dot{\vec{r}}, \ddot{\vec{r}}, \cdots) , \tag{3.11.b}$$

$$M = m_1 + m_2 , \tag{3.11.c}$$

$$m = \frac{m_1 m_2}{m_1 m_2} , \tag{3.11.d}$$

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} , \quad \vec{r} = \vec{r}_1 - \vec{r}_2 \tag{3.11.e}$$

with a solution of conditions (3.2)

$$\vec{F}^{NSA}_1 = -\vec{F}^{NSA}_2 \stackrel{\text{def}}{=} \vec{F}^{NSA} = g(\vec{r})\left(\frac{d^{2n}\vec{r}_1}{dt^{2n}} - \frac{d^{2\mu}\vec{r}_2}{dt^{2m}}\right) , \tag{3.12}$$

under which only one orbit is possible: the circle

$$\vec{r} = \vec{r}_1 - \vec{r}_2 = \text{const} . \tag{3.13}$$

As a result, the function $g(\vec{r})$ of Eq.s (3.12) can be assumed as a constant for practical purposes.

The reader should recall that, by central condition, systems (3.1) represent extended particles under mutual *contact* interactions. The illustration of result (3.13) was therefore done in the original proposal [9] via the so-called model of *couple gears*, for which, as well known, only circular orbits are admissible for coupled conditions. This model remains conceptually effective inasmuch as it shows already at the primitive Newtonian level that only the singlet state is stable. In fact, gears can only be coupled in singlets. As one can see, these results are expected to persist at the particle level, because the notion of coupled and spinning gears is simply replaced by that of coupled, spinning and extended wavepackets (or charge distributions). The high instability of triplet states is then evident, and so is the stability of only one orbit: the circle (see the original proposal [9] for details).

The two-body generalization (3.11) of the Kepler's systems was recently studied by Jannussis and his collaborators [41] who reached the following Birkhoffian representation

$$\hat{A} = \int_{t_1}^{t_2} \left\{ \left[\left(\frac{m-g}{m}\vec{p} \cdot \dot{\vec{r}} - \left[\left(\frac{m-g}{m}\frac{\vec{p}^2}{2m} + V(\vec{r}) \right] \right\} dt , \tag{3.14.a}$$

$$R = (R_\mu) = \left(\frac{m-g}{m}\vec{p}, \vec{0}\right) , \tag{3.14.b}$$

which is defined up to a (rather vast) class of equivalence transformations in the local coordinates, called *Birkhoffian gauge transformations* [19].

Birkhoff's tensor can be readily computed from action (3.14) in its covariant (simplistic) form

$$\Omega_{\mu\nu}(a) = \rho^{-1}\omega_{\mu\nu} , \tag{3.15.a}$$
$$\rho = m/(m-g) \tag{3.15.b}$$

namely, the particular case considered is a simple scalar isotopy of the fundamental, canonical, tensor $\omega^{\mu\nu}$.

Its contravariant (Lie) form is then given by

$$\Omega^{\mu\nu} = \left(\|\Omega_{\alpha\beta}\|^{-1}\right)^{\mu\nu} = \rho\omega^{\mu\nu} , \tag{3.16}$$

and it characterizes the Galilei-isotopic symmetry of systems (3.11) [41]. Jannussis and his collaborators built and example of the generalized three-body Kepler systems, and illustrated their significance as an example of the Galilei-isotopic relativity [41]. We assume the reader is now aware of the fact that these Birkhoffian representations are important for hadronic structure models. In fact, representation (3.14) is the starting point for the two-body hadronic case characterizing the models $\pi^0 = (\epsilon^+, \epsilon^-)$ and $n = (p^+, \epsilon^-)$, while the three-body case is important for models of the type $\pi^\pm = (\epsilon^+, \epsilon^\pm, \epsilon^-)$.

We are now equipped to construct the operator image of model (3.14).

Let us recall that the Birkhoffian generalization of Hamiltonian mechanics had been suspected to be the classical image of the hadronic generalization of atomic mechanics since its original proposal [8,9], because both mechanics were constructed as more general realizations of the same underlying abstract axioms.

This expectation was first proved to be correct in paper [29] via an isotopic lifting of the conventional techniques of symplectic quantization (now called *geometric hadronization*), and then in paper [30] via the isotopic generalization of the conventional naive quantization (today called *naive hadronization*).

For the limited purposes of this paper, it is sufficient to use the latter technique. Note that action (3.14) is not canonical. As a result, it is not suitable for the conventional mapping of a canonical action functional (naive quantization)

$$\mathcal{A} \to \hbar I(-i \log \psi) , \tag{3.17}$$

under which the conventional Hamilton-Jacobi equation is mapped into their Schrödinger counterpart

$$-\frac{\partial \mathcal{A}}{\partial t} = H \to i\hbar \frac{1}{\psi}\frac{\partial}{\partial t}\psi = H , \tag{3.18}$$

with the familiar expression for the momentum

$$\frac{\partial \mathcal{A}}{\partial \vec{r}} = p \to -i\hbar \frac{1}{\psi}\vec{\nabla}\psi = p . \tag{3.19}$$

The contention of paper [30] is that Birkhoffian action (3.14) is instead suitable for the isotopic lifting of mapping (3.15) (naive hadronization)

$$\hat{\mathcal{A}} \to \hbar \hat{I}(-i \log \psi) , \tag{3.20}$$

under which the Birkhoffian form of the Hamilton-Jacobi equations [19]

$$-\frac{\partial \mathcal{A}}{\partial t} = H = \frac{1}{\rho}\frac{\vec{p}^2}{2m} - V(r) \tag{3.21.a}$$
$$\frac{\partial \mathcal{A}}{\partial \vec{r}} = \frac{1}{\rho}\vec{p} , \tag{3.21.b}$$

are mapped into the equations

$$i\hbar\frac{\partial}{\partial t}\psi = \left[H - i\left(\frac{\partial \hat{I}}{\partial t}\right)\log\psi\right] * \psi , \tag{3.22.a}$$
$$-i\hbar\vec{\nabla}\psi = \frac{1}{\rho}\left[\vec{p} + i\rho(\vec{\nabla}\hat{I})\log\psi\right] * \psi , \tag{3.22.b}$$

(where now the quantities H and $\vec{p}$ are elements of the isoassociative operator algebra $\hat{\mathcal{A}}$) which possess precisely the modular-isotopic structure of the Schrödinger-isotopic equations (2.5).

The above equations are the *fundamental (nonrelativistic), structure equations for the hadronic models $\pi^0 = (\epsilon^+, \epsilon^-)$ and $n = (p^+, \epsilon^-)$*. We shall write them in the reduced form

$$i\hbar\frac{\partial}{\partial t}\psi = \left[-\rho\frac{\hbar^2}{2m}\hat{\Delta} + \hat{V}(r) + i\left(\frac{\partial \hat{I}}{\partial t}\right)\log\psi\right] * \psi = H_{\text{eff.}} * \psi \tag{3.23.a}$$
$$-i\hbar\vec{\nabla}\psi = \frac{1}{\rho}\left[\vec{p} + i\rho(\vec{\nabla}\hat{I})\log\psi\right] * \psi = \frac{1}{\rho}p_{\text{eff.}} * \psi , \tag{3.23.b}$$

where

$$\hat{\Delta} = \Delta \hat{I}\,, \quad (3.24.a)$$
$$\hat{V} = V\hat{I}\,. \quad (3.24.b)$$

The most visible departure from the conventional two-body Schrödinger's equation is the underlying basic isotopy characterized by the yet unknown isotopic operator T (see next section), as well as the following:

1. The alteration of the conventional operator realization of the linear momentum via the factor $1/\rho$ and other terms, which will be later on crucial to compute the total angular momentum of Rutherford's eleton (Sect. 6);

2. The alteration of the conventional Laplacian via the multiplication of factor ρ which will be later on crucial to represent the total energy of the neutron (Sect. 7);

3. The natural appearance of a logarithmic term (nonlinearity in the state function), which will be crucial later on in the suppression of the atomic spectrum of energy (Sect. 8).

In performing the transition from Eq.s (3.22) to (3.23) we have assumed some familiarity with operations in $\hat{\mathcal{A}}$. For instance, the classical square of the linear momentum in Hamiltonian (3.21a) must be necessarily expressed (to preserve isolinearity [14]) as an isosquare in $\hat{\mathcal{A}}$, thus resulting in the expressions of the type

$$\frac{1}{\rho}\frac{\vec{p}^2}{2m}\Big|_{\text{clasc.}} \rightarrow \left(\frac{1}{\rho}\frac{\vec{p}*\vec{p}}{2m}\right)_{\text{oper.}} * \psi = -\hbar^2\frac{\rho}{2m}\Delta\psi = \left(-\hbar^2\frac{\rho}{2m}\hat{\Delta}\right) * \psi\,, \quad (3.25)$$

with similar forms for other terms.

4 Identification of the Generalized Unit

The identification of the isotopic element T, and related isounit (2.1) in hadronic structure equations (3.21) has been reached in ref. [42]. We shall review it below for the reader's convenience.

Consider the radial equation for the two-body Coulomb system

$$\left(-\frac{\hbar^2}{2m}\Delta_r - \frac{e^2}{r}\right)|u\rangle = E_u|u\rangle\,,$$

$$u(r) \cong \frac{b^{3/4}}{\sqrt{\pi}}e^{-br}\,, \quad b^{-1} = \frac{\hbar^2}{me^2}\,, \quad (4.1)$$

and perform, as a first step, its isotopic lifting without the logarithmic term of Eq.s (3.21)

$$\left(-\frac{\hbar^2}{2\overline{m}}\hat{\Delta}_r - \frac{e^2}{r}\right)T|u\rangle = \overline{E}|u\rangle\,, \quad \overline{m} = m/\rho\,. \quad (4.2)$$

Animalu's result is the proof that the radial equation for the structure model $\pi^0 = (\epsilon^+, \epsilon^-)$ empirically introduced in memoir [9], Eq. (5.1.14), p. 835,

$$\left(-\frac{\hbar^2}{2\overline{m}}\Delta_r - \frac{e^2}{r} - V_0\frac{e^{-br}}{1-e^{-br}}\right)\psi = E\psi\,, \quad (4.3)$$

can be derived via the Schrödinger-isotopic equation (3.21) and a suitable nonlocal realization of the unit (the reader should be aware that, at the time of ref. [9], Heisenberg-isotopic equations (2.4) were known but not their Schrödinger-isotopic counterpart).

For this purpose, consider first the isotopic operator of the idempotent type

$$T = 1 - \langle u|u\rangle\,, \quad (4.4.a)$$
$$\langle u|u\rangle = 1\,, \quad T^2 = T\,. \quad (4.4.b)$$

Then, isotopic lifting (4.2) acquires the form

$$\left(-\frac{\hbar^2}{2\overline{m}}\Delta_r + \frac{e^2}{r} - E_{\overline{u}}\langle\overline{u}|\psi\rangle\right)\psi(r) = \overline{E}\psi(r)\,, \quad (4.5)$$

namely, it becomes explicitly dependent on the *integral of wave-overlapping*

$$\langle\overline{u}|\psi\rangle = \int \overline{u}^*(\vec{r'})\psi(\vec{r'})d\vec{r'} \neq 0\,, \quad (4.6)$$

where

$$\overline{u}(r) = \overline{C}e^{-\overline{b}r}\,, \quad E_{\overline{u}} = -\overline{m}\frac{e^4}{2\hbar^2}\,, \quad (4.7.a)$$
$$\overline{C} = \frac{\overline{b}^{-3/2}}{\sqrt{\pi}}\,, \quad \overline{b}^{-1} = \frac{\hbar^2}{2\overline{m}e^2}\,. \quad (4.7.b)$$

Note that integral (4.6) has not necessarily the unit value, because in hadronic mechanics the isotopic integral (2.6) is normalized to the unit. Note also that $\overline{b}^{-1}$ is the reduced Bohr radius, i.e., Bohr's radius for the corrected mass $\overline{m}$.

In its lowest possible state, Eq. (4.5) can be written

$$\left[-\frac{\hbar^2}{2\overline{m}}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)-\frac{e^2}{r}-\overline{C}\,E_{\overline{u}}\langle\overline{u}|\psi\rangle\frac{\overline{u}(r)}{\psi(r)}\right]\psi(r)=\overline{E}\psi(r)\,, \tag{4.8}$$

where $\overline{C}$ is the normalization constant in (4.7b).

Consider now the solution of Eq. (4.3) as given in ref. [9], Eq. (5.1.29), i.e.,

$$\psi(r)=F(r)(1-e^{-br})\,. \tag{4.9}$$

The above expression is also a solution of Eq. (4.8) in the sense of Hartree's self-consistent approximation

$$F(r_0)=\overline{C}\,, \tag{4.10}$$

under which Eq. (4.8) is "linearized" to the form

$$\left[-\frac{\hbar^2}{2\overline{m}}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)-\frac{e^2}{r}+|E_{\overline{u}}|\langle\overline{u}|\psi\rangle\frac{e^{-\overline{b}r}}{1-e^{-\overline{b}r}}\right]\psi(r)=\overline{E}\psi(r)\,. \tag{4.11}$$

By comparing expressions (4.3) and (4.11), Animalu reached the following realization of the isounit

$$\hat{I}=\hat{I}(0)e^{-\frac{i}{\hbar}|E_{\overline{u}}|\langle\overline{u}|\psi\rangle t}\,, \tag{4.12}$$

under which we have the identification of the following terms of the fundamental Schrödinger-isotopic equation (3.21a)

$$i\hbar T\left(\frac{\partial T^{-1}}{\partial t}\right)=|E_{\overline{u}}|\langle\overline{u}|\psi\rangle\,, \tag{4.13}$$

and

$$\frac{1}{\psi}\log\psi\approx\frac{e^{-\overline{b}r}}{1-e^{-\overline{b}r}}\,, \tag{4.14}$$

with isotopic element

$$T=T(0)e^{\frac{i}{\hbar}|E_{\overline{u}}|(\int\overline{u}(\vec{r'})\psi(\vec{r'})d^3\vec{r'})t}\,, \tag{4.15}$$

thus proving the Schrödinger-isotopic character of the hadronic radial equation (4.3), while achieving an explicit realization of the isounit.

Animalu's isounit (4.12) is fully along the spirit of hadronic mechanics, and sufficient for the objectives of this paper. In fact, it is essentially nonlocal-integral, as needed to represent the conditions of total mutual penetration of the wavepackets of the hadronic constituents, in the nonhamiltonian way of hadronic mechanics, i.e., in a way essentially outside the representational capabilities of atomic mechanics. Also, Animalu's isounit (4.12) allows the recovering of atomic models identically, at the limit of null overlapping of the wavepackets of the hadronic constituents, i.e.,

$$\int\overline{u}(\vec{r'})\psi(\vec{r'})d^3\vec{r'}=0\,, \tag{4.16}$$

under which

$$\hat{I}=T=I=\mathrm{Diag}(1,1,\ldots,1)\,. \tag{4.17}$$

In closing this section, the reader should be aware of the central role of *Hulten's potential*

$$V_H(r)=V_H\frac{e^{-br}}{1-e^{-br}}\,, \tag{4.18}$$

in the suppression of the positronium spectrum to achieve a consistent model $\pi^0=(\epsilon^+,\epsilon^-)$ (see the review in Sect. 7). As a consequence, we can here state that the selection of Animalu's isounit for the fundamental Schrödinger-isotopic equations of structure, Eq.s (3.21), where H is the conventional Hamiltonian of the hydrogen atom, is expected to permit the suppression of the atomic spectrum of the hydrogen atom down to only one level: the neutron, along Rutherford's historical hypothesis (1.3), as we shall study in detail in Sect. 8.

We are now in a position to summarize the *rules of isotopic lifting of atomic equations*:

STEP I: Consider any given atomic equation, e.g., of the type

$$\left[-\frac{\hbar^2}{2m}\Delta+V(r)\right]U(r)=E_uU(r)\,. \tag{4.19}$$

STEP II: Lift this equation by means of an isotopic operator T, as well as: **II-a:** implement the modification of the Laplacian requested by the hadronization of the linear momentum, Eq. (3.21b); **II-b:** generalize the Laplacian and the potential via rules (3.22); and **II-c:** add the last term of Eq. (3.21a), i.e., $\frac{i}{\hbar}\left(\frac{\partial f}{\partial t}\right)\log\psi$, resulting in the liftings of the type (for model (3.14))

$$\left[-\frac{\hbar^2}{2\hat{m}}\hat{\Delta}+\hat{V}(r)+\frac{i}{\hbar}\left(\frac{\partial\hat{I}}{\partial t}\right)\log\psi\right]*\psi=E_\psi\psi\ . \tag{4.20}$$

(with evidently more general expressions for more general models).

STEP III: Assume an explicit form of the isotopic operator as suggested by the problem at end, and then compute the above equation explicitly.

For the case of Animalu's isounit, the isotopic element T does not depend on local coordinates. As a result, hadronization rule (3.21b) assumes the simpler form

$$-i\hbar\vec{\nabla}\psi=\frac{1}{\rho}\vec{p}\,,\quad \rho=\frac{m}{m-g}\ . \tag{4.21}$$

The implementation of the above rules to the conventional radial equation of the hydrogen atom then yields one hadronic equation empirically suggested in memoir [9] for the structure model $\pi^0-(\epsilon^+,\epsilon^-)$

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\psi &= \left[-\frac{\hbar^2}{2\hat{m}}\hat{\Delta}-\frac{e^2}{r}\hat{I}+\frac{i}{\hbar}\left(\frac{\partial\hat{I}}{\partial t}\right)\log\psi\right]*\psi \\ &= \left[-\frac{\rho\hbar^2}{2m}\Delta-\frac{e^2}{r}-V_H\frac{e^{-br}}{1-e^{-br}}\right]\psi=E_\psi\psi\,, \end{aligned} \tag{4.22}$$

which is here assumed as the fundamental structure equation for Rutherford's hypothesis.

5 Nonrelativistic, Hadronic, Radial Equation

The nonrelativistic radial equation of the hadronic two-body structure model was studied in detail in the original memoir [9], Sect. 5. Owing to the relevance of this study for this paper, it is important to review it here. Also, in memoir [9] the study was conducted, specifically, for the model $\pi^0=(\epsilon^+,\epsilon^-)$, while in this paper we shall conduct it for a generic system, and then apply it to both hadronic models $\pi^0=(\epsilon^+,\epsilon^-)$ and $n=(p^+,\epsilon^-)$.

Consider Eq. (4.22) in the form

$$\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)+\frac{m}{\rho\hbar^2}\left(E-\frac{e^2}{r}-V_H\frac{e^{-br}}{1-e^{-br}}\right)\right]\psi(r)=0\ . \tag{5.1}$$

First, let us recall that, for the small mutual distances under consideration here (of the order of 10^{-13} cm), Hulten's potential behaves exactly like the Coulomb potential, ref. [9], Eq. (5.1.15),

$$V_{\text{Hulten}}|_{r\approx 0}\approx\frac{V_H}{b}\frac{1}{r}\ . \tag{5.2}$$

As a consequence, the Coulomb potential can be eliminated from Eq. (5.1) with very good approximation, and replaced with a new constant of the Hulten potential

$$\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)+\frac{m}{\rho\hbar^2}\left(E-V_0\frac{e^{-br}}{1-e^{-br}}\right)\right]\psi(r)=0\ . \tag{5.3}$$

The solution of the above equation is given by the Jacobi polynomials

$$\begin{aligned} G_n(x) &= \sum_{k=1}^{n}(-1)^{k-1}\binom{n-1}{k-1}\binom{n+k+2|A|^{1/2}-1}{k}x^k & (5.4.a)\\ A &= \frac{mE}{\rho\hbar^2b^2}<0\,,\quad x=1-e^{-br}\ . & (5.4.b) \end{aligned}$$

The *hadronic binding energy* then acquires the typical spectrum of Hulten's potential

$$|E|=E^{\text{Bind}}=\frac{\rho\hbar^2b^2}{4m}\left(\frac{mV_0}{\rho\hbar^2b^2n}-n\right)^2\ . \tag{5.5}$$

Solution (5.4) can be written in the hypergeometric form, loc. cit. Eq. (5.1.21)

$$\begin{aligned} \psi(r) &= {}_2F_1(2\alpha+1+n,1-n,2\alpha+1,e^{-br})e^{-br}\frac{1-e^{-br}}{r}\,, & (5.6.a)\\ \alpha^2 &= \frac{1}{2}\frac{mE^{\text{Bind.}}}{\rho\hbar^2}=\frac{1}{2}|A|>0\,, & (5.6.b) \end{aligned}$$

with boundary conditions

$$\alpha=\frac{\beta^2-n^2}{n}\,,\quad \beta^2=\frac{mV_0}{\rho\hbar^2b^2}\ . \tag{5.7}$$

The only admissible states are therefore those for which, loc. cit., Eq. (5.1.23),

$$\beta^2>n^2\ . \tag{5.8}$$

The above condition is a crucial test for the consistency of the analysis of this paper as well as for hadronic mechanics at large. In fact, if the model $n = (p^+, \epsilon^-)$ is such that

$$\beta \cong 1 + \epsilon\,, \quad \epsilon \approx 0\,, \tag{5.9}$$

then we will succeed in suppressing the atomic spectrum down to only one energy level. If, on the contrary, the model $n = (p^+, \epsilon^-)$ is such that

$$\beta >> 1\,, \tag{5.10}$$

then we will have a finite spectrum of energy levels with a number of consequential inconsistencies. In fact, the excited states, if admitted, will be predictably of atomic type, that is, with energy levels relatively close to each other and, as such, they are expected to be fundamentally unable to represent the neutron's resonances (which must necessarily be represented, in hadronic mechanics, via an increase of the number of constituents, e.g., in order to achieve a quantitative representation of their decays).

We know from the analysis of ref. [9] that the hadronic structure model $\pi^0 = (\epsilon^+, \epsilon^-)$ does indeed verify the crucial consistency condition (5.4). The open problem to be investigated in this paper is whether the same condition is also met by Rutherford's hypothesis (1.3).

In order to have the necessary background for this study, let us continue the solution of Eq. (5.1). Assume for wavelength of the eleton the expression

$$\lambda\!\!\!^{-} = (k_1 b)^{-1}\,, \tag{5.11}$$

where k_1 is an unknown real positive number. The energy of the eleton is then given by ref. [9], Eq. (5.1.25)

$$E_\epsilon^{Kin} = k_1 \hbar bc \cong \frac{\rho \hbar^2 b^2}{2m} = \frac{p^2}{2m_\epsilon}\,, \tag{5.12}$$

and we can assume

$$\beta^2 = \frac{mV_0}{\rho\hbar^2 b^2} = k_2 = 1 + \epsilon\,, \tag{5.13}$$

where k_2 is another unknown, real, positive parameter.

The total energy of a two-body hadronic bound-state can then be written

$$\begin{aligned} E^{\rm tot} &= E^{\rm Rest}_{\rm Constituents} + E^{\rm Kin}_{\rm Constituents} - E^{\rm Bind.}\,, && (5.14.a)\\ E^{\rm Kin}_{\rm Const} - E^{\rm Bind.} &= k_1[1-(k_2-1)^2]\hbar bc = k_1(1-\epsilon^2)]\hbar bc\,. && (5.14.b)\end{aligned}$$

The above equation is in two unknowns, the k_1 and k_2 quantities. To solve it, we need one second condition, besides that of the rest energy of the particle considered, i.e., the π^0 or the n. The representation of the meanlife was selected in ref. [9] according to the expression

$$\tau^{-1} \cong 4\pi\lambda\!\!\!^{-2}|\psi(0)|\frac{\alpha E_\epsilon^{\rm Kin}}{\hbar}\,. \tag{5.15}$$

In order to compute this quantity, we need the full expression of the solution of Eq. (5.1),

$$\psi(r) = \left[\frac{\Gamma(2|A|^{1/2}+3)}{\Gamma(3)\Gamma(2)|A|^{1/2}}\right]^{1/2} e^{-|A|^{1/2}br}\frac{1-e^{-br}}{r}\,, \tag{5.16}$$

which, near the origin, becomes, ref. [9], Eq. (5.1.30)

$$\begin{aligned}\psi(0) &= \left[\frac{\frac{1}{2}(k_2-1)\Gamma\left(\frac{1}{2}(k_2-1)+2\right)}{3!\Gamma\left(\frac{1}{2}(k_2-1)\right)}\right]^{1/2} b\\ &= \left[\frac{\frac{1}{4}(k_2-1)^2\Gamma\left(\frac{1}{2}(k_2-1)+1\right)}{6\Gamma\left(\frac{1}{2}(k_2+1)\right)}\right]^{1/2} b\\ &\cong \frac{(k_2-1)^{3/2}}{(48)^{1/2}}b = \frac{\epsilon^{3/2}}{(48)^{1/2}}b\,. \end{aligned} \tag{5.17}$$

The meanlife (5.15) then becomes, loc. cit., Eq. (5.1.31),

$$\tau^{-1} = \frac{4\pi}{k_1^2 b^2}\frac{(k_2-1)^3}{48}\frac{k_1\hbar bc}{(137)^2\hbar} = \frac{4\pi}{48(137)^2}\frac{\epsilon^3}{k_1}bc\,. \tag{5.18}$$

The third condition for the representation of the structure of the hadron considered is evidently the size b^{-1} of the charge distribution. We reach in this way, the following two conditions in the two unknown quantities k_1 and k_2, loc. cit., Eq.s (5.1.32)

$$\begin{aligned} k_1[1-(k_2-1)^2] &= k_1(1-\epsilon^2) && (5.19.a)\\ &= \frac{1}{2\hbar c}(E^{\rm Tot}b^{-1})\,, &&\\ \frac{(k_2-1)^3}{k_1} = \frac{\epsilon^3}{k_1} &= \frac{48(137)^2}{4\pi c}(\tau^{-1}b^{-1})\,, && (5.19.b)\end{aligned}$$

which must be specialized to the physical characteristics $E^{\rm tot}$, τ^{-1}, and b^{-1} of the particle considered.

6 Nonrelativistic, Hadronic Angular Momentum and Spin

In this section we shall study the notions of *hadronic angular momentum, hadronic spin* and their sum. These terms stand to denote the *deviations* from conventional angular momentum and spin when the wavepacket of the particle considered is *inside* a hadronic medium, with the understanding that the latter notions are recovered identically whenever the particle considered exits hadronic matter and reacquires its atomic motion in vacuum.

The notions will be studied via the isotopic coverings of the conventional $SU(2)$-spin symmetry, hereinafter denoted with $\widehat{SU}(2)$. The mathematical *differences* between $\widehat{SU}(2)$ and $SU(2)$ shall then represent the physical differences between the hadronic and the atomic angular momentum and spin.

We shall first review the classical notion of generalized angular momentum, then study its hadronic counterpart, and finally present the general $\widehat{SU}(2)$ theory. The application to the structure models $\pi^0 = (\epsilon^+, \epsilon^-)$ and $n = (p^+, \epsilon^-)$ will be done in the next sections.

The construction and classification of the infinite family of isotopic coverings $\widehat{SO}(3)$ of $SO(3)$ was conducted in paper [21]. The first construction of the $\widehat{SU}(2)$-symmetry was done in the preceding paper [24] for the 4×4 case of the isodirac's equation. That for the 2×2 case was done in ref. [43]; an outline of its application to Rutherford's model was provided in ref. [44]; and a comprehensive representation is under preparation in volume III of [45]. For clarity, it is important to review the topic with particular emphasis on its application to Rutherford's hypothesis.

Let us recall that the conventional $SO(3)$ symmetry is the group of unimodular isometries of the sphere in Euclidean space $E(\vec{r}, \delta, \mathbf{R})$

$$SO(3) : r^t \delta r = r_i \delta_{ij} r_j = r_1 r_1 + r_2 r_2 + r_3 r_3 \,, \qquad (6.1.a)$$

$$\delta = \mathrm{Diag}(1,1,1) > 0 \,. \qquad (6.1.b)$$

The covering $\widehat{SO}(3)$ symmetries are the (infinite) groups of isounimodular isometries of all possible (infinite) ellipsoidical deformations of the sphere in isotopic space $\hat{E}(\vec{r}, g, \hat{\mathbf{R}})$ [21]

$$\widehat{SO}(3) : r^t g r = r_i g_{ij} r_j = r_1 g_{11} r_1 + r_2 g_{22} r_2 + r_3 g_{33} r_3 \,, \qquad (6.2.a)$$

$$g = \mathrm{Diag}(b_1^2, b_2^2, b_3^2) > 0 \,, \qquad (6.2.b)$$

$$b_k = b_k(t, \vec{r}, \dot{\vec{r}}, \cdots) \,, \qquad (6.2.c)$$

where $\hat{\mathbf{R}}$ is the isotopic lifting of the field of reals $\mathbf{R}$ as in structure (2.7).

All Lie-isotopic groups $\widehat{SO}(3)$ result to be isomorphic to the conventional group $SO(3)$ because of the preservation of the topology of the original metric δ (positive definiteness). If liftings (6.2) are enlarged to admit hyperbolic deformations of the sphere, then $\widehat{SO}(3)$ results to be isomorphic to the non-compact group $SO(2.1)$. These latter conditions shall be excluded hereon.

Paper [21] therefore established that, contrary to popular beliefs, the rotational symmetry remains *exact* for the ellipsoidical deformations of the sphere, because it is broken only when realized via the simplest conceivable Lie product $AB - BA$, while it has been proved to remain exact when realized via the less simplistic Lie product $A * B - B * A = AgB - BgA$, where g is precisely the isometric of Eq. (6.2). Equivalent results hold for the isotopic lifting of all Euclidean spaces $E(x, \delta, \mathsf{F})$ on the field F of real numbers, complex numbers and quaternions (see Theorem 2.1, ref. [21], p. 33 on the general treatment of isotopic liftings of space-time symmetries which are at the foundation of the analysis of this section).

The classical counterpart of $\widehat{SO}(3)$ is studied in monograph [19] as part of the formulations of Birkhoffian mechanics. The ultimate physical assumptions for both, the classical and the operator case, are embodied in the fundamental invariant (6.2) and can be expressed as follows.

As well known, the classical (canonical) and operator (unitary) notion of angular momentum and spin are centered on the following fundamental assumptions:

A) motion occurs in (empty) space which is homogeneous and isotropic;

B) particles are abstracted as being massive points; and, last but not least,

C) only perfectly rigid bodies are admitted.

The covering notions of angular momentum and spin in both Birkhoffian and hadronic mechanics are based on the following more general physical conditions:

Â) motion occurs in a physical medium which, as such, is generally *inhomogeneous* and *anisotropic*, with the understanding that the underlying space itself remains homogeneous and isotropic;

B̂) particles are not only assumed to be extended, whenever applicable, but also represented in their actual shape, e.g., of oblate spheroidal ellipsoidical form as typical for extended and spinning objects; and, last but not least, Ĉ) all possible deformations of the shape of the particles are admitted and quantitatively presented.

All the above broader physical conditions Â, B̂, and Ĉ are clearly embodied, geometrically, in metric-isotopy (6.2) which: is manifestly inhomogeneous and anisotropic because, in general, $g_{11} \neq g_{22} \neq g_{33}$ (case Â); it can represent the actual shape of the object itself [24] (case B̂); and it clearly allows the representation of all possible deformations of the shape considered owing to the admitted functional dependence of the metric elements, Eq. (6.2c) (case Ĉ). This illustrates the *deviations* from conventional, classical and operator settings that are necessary to achieve a consistent definition of generalized angular momentum and spin.

Let us begin by reviewing the classical case. Consider an arbitrary, nonhamiltonian system, e.g., of type (3.1). Under sufficient topological conditions (e.g., analyticity), a Birkhoffian representation always exists with local coordinates on phase space (cotangent bundle $T * E(\vec{r}, \delta, \mathbf{R})$), here expressed for simplicity for the one- particle case

$$\begin{aligned} a &= (a^\mu) \stackrel{\text{def}}{=} (\vec{r}, \vec{p}) = (r_i, p_i) , \\ & \mu = 1, 2, \ldots, 6 \,;\; i, j = 1, 2, 3 \,. \end{aligned} \tag{6.3}$$

This implies the lifting of the conventional, canonical realization of Lie's theory via the generalized Lie's tensor (3.8), e.g.,

$$(\Omega^{\mu\nu}) = \left(\| \frac{\partial R_\beta}{\partial a^\alpha} - \frac{\partial R_\alpha}{\partial a^\beta} \|^{-1} \right) = \begin{pmatrix} [r_i \hat{,} r_j] & [r_i \hat{,} p_j] \\ [p_i \hat{,} r_j] & [p_i \hat{,} p_j] \end{pmatrix} , \tag{6.4}$$

where the brackets are given by Eq.s (3.9).

The generators of the angular momentum are the conventional components

$$L_k = \epsilon_{kij} r_i p_j \quad \epsilon T^* E(\vec{r}, \delta, \mathbf{R}) , \tag{6.5}$$

with *isocommutations rules* which can be written in general

$$[L_i \hat{,} L_j] = \hat{C}^k_{ij}(a) L_k , \tag{6.6}$$

where the $\hat{C}$'s are the *structure functions* of $\widehat{SO}(3)$ [8,21].

For the special case of the two-body nonhamiltonian system (3.14), we have the simpler realization

$$[L_i \hat{,} L_j] = \rho \epsilon_{ijk} L_k , \tag{6.7.a}$$
$$\rho = m/(m-g) , \tag{6.7.b}$$

The above classical realization of $\widehat{SO}(3)$ is at the foundation of the Galilei-isotopic symmetry and corresponding relativity, and we refer the interested reader to monograph [19] for details. We now consider the operator version of the above setting, namely, the ***hadronic angular momentum.*** Under hadronization rule (3.20) and Animalu's isounit (4.12), the operator form of the linear momentum is given by Eq. (3.23b), i.e.,

$$\vec{p} * \psi = -i \rho \vec{\nabla} \psi , \quad \hbar = 1 . \tag{6.8}$$

The *fundamental isocommutation rules* of the hadronic, two-body structure model are then given by

$$([a^\mu \hat{,} a^\nu]) * \psi = \begin{pmatrix} [r_i \hat{,} r_j] & [r_i \hat{,} p_j] \\ [p_i \hat{,} r_j] & [p_i \hat{,} p_j] \end{pmatrix} * \psi = \begin{pmatrix} 0 & -i\rho\delta_{ij} \\ i\rho\delta_{ij} & 0 \end{pmatrix} * \psi \tag{6.9}$$

in full agreement with the classical counterpart, Eq.s (3.16), where we shall denote hereon the brackets in $\mathcal{A}^-$ with the symbol

$$[A \hat{,} B] = [A, B]_{\hat{\mathcal{A}}} = A * B - B * A . \tag{6.10}$$

The components of the *hadronic angular momentum* can then be defined via the expressions in the isoenvelope $\hat{\mathcal{A}}$ on isofield $\hat{\mathbf{C}}$

$$\hat{L}_k = \epsilon_{kij} r_i * p_j \in \hat{\mathcal{A}} , \tag{6.11}$$

and verify the isocommutation rules, readily obtained via rules (6.9),

$$\begin{aligned} \left[\hat{L}_i \hat{,} \hat{L}_j \right] &= \hat{L}_i * \hat{L}_j - \hat{L}_j * \hat{L}_i \\ &= \epsilon_{irs} \epsilon_{itu} (r_r * [p_s \hat{,} r_t] * p_u + r_t * [r_s \hat{,} p_u] * p_s) = i \rho \epsilon_{ijk} \hat{L}_k , \end{aligned} \tag{6.12}$$

where we have used the property in $\hat{\mathcal{A}}$

$$[A \hat{,} B * C] = [A \hat{,} B] * C + B * [A \hat{,} C] . \tag{6.13}$$

It is hoped in this way the reader sees the remarkable agreement between the hadronic formulation, Eq.s (6.12), and its Birkhoffian counterpart, Eq.s (6.7).

We shall here assume that the operators a^μ and $\hat{L}_k$ act on a isohilbert space with inner composition

$$\hat{\mathcal{H}}_{\hat{L}} : \langle \psi \hat{|} \psi \rangle = \int \psi^* * \psi dv = 1 , \tag{6.14}$$

that is, under the assumption that the isotopic elements of the envelope $\hat{A}$ and space $\hat{\mathcal{H}}_{\hat{L}}$ coincide. This assures the preservation of the Hermiticity of the operator $L_k = \epsilon_{kij} r_i p_j$ of atomic mechanics under isotopy, as well as the preservation of the reality of the eigenvalues (Sect. 2).

From rules (6.12) it is easy to see that, as in the case $\widehat{SO}(3)$ [21], the "square" of the operator $\hat{L}_k$ in $\hat{A}$

$$\hat{\vec{L}}^2 = \sum_{k=1}^{3} \hat{L}_k * \hat{L}_k \,, \tag{6.15}$$

is invariant

$$[\hat{\vec{L}}^2 \hat{,} L_k] = 0 \,, \quad k = 1, 2, 3 \,, \tag{6.16}$$

(for the construction of the full expression of the isocasimir in the center of $\hat{A}$, we refer the reader to ref. [21]).

We can therefore diagonalize simultaneously $\hat{\vec{L}}^2$ and $\hat{L}_3$ as in the conventional case. It is also easy to prove that the isoeigenfunctions are the conventional spherical harmonics $Y_{\ell m}(\theta, \varphi)$, although with different renormalization requested by inner product (6.14) which, for the specific case under consideration, Eq. (2.22,) can be written

$$\hat{Y}_{\ell m}(\theta, \varphi) = [T(t)]^{-1/2} Y_{\ell m}(\theta, \varphi) \,. \tag{6.17}$$

The *eigenvalues of the hadronic angular momentum* are then given by

$$\hat{L}_3 * \hat{Y}_{\ell m}(\theta, \varphi) = \rho m \hat{Y}_{\ell m}(\theta, \varphi) \,, \tag{6.18.a}$$

$$\hat{\vec{L}}^2 * \hat{Y}_{\ell m}(\theta, \varphi) = \rho\ell(\rho\ell + 1) \hat{Y}_{\ell m}(\theta, \varphi) \,, \tag{6.18.b}$$

$$\ell = 0, 1, 2, \cdots; \qquad m = \ell, \ell - 1, \ldots, -\ell, \hbar = 1 \,, \tag{6.18.c}$$

where the second equations will be justified on rigorous grounds shortly.

Owing to the assumed identity of the isotopic elements of $\hat{A}$ and of $\hat{\mathcal{H}}$ (but not in general), the isoeigenvalues coincide with the expectation values, i.e., from Eq. (2.13) and (6.14), we have

$$\langle \widehat{\hat{L}_3} \rangle = \rho m \,, \tag{6.19.a}$$

$$\langle \widehat{\hat{\vec{L}}^2} \rangle = \rho\ell(\rho\ell + 1) \,. \tag{6.19.b}$$

In conclusion, we can state that *the operator realization of the isorotation groups $\widehat{SO}(3)$ implies, for the closed two-body nonhamiltonian systems under consideration, the mutation of the atomic angular momentum*

$$\ell \to \rho\ell \,, \quad m \to \rho m \,, \quad \rho > 0 \tag{6.20}$$

and, as such, it can be assumed as a realization of the hadronic angular momentum.

The reader should recall that the quantity $\rho = m/(m - g) \neq 1$ represents precisely the deviations from the conventional, canonical-Hamiltonian action in structure (3.14), and its value $\rho \neq 1$, is then responsible for the activation of the isotopy at the operator level.

Thus, at the value $\rho = 1$, conventional formulations are recovered at *all* levels, that is, not only the hadronic angular momentum recovers the conventional one, but hadronic mechanics collapses into its atomic particula- form and, last but not least, the primitive Newtonian system reacquires its full Hamiltonian structure.

Hadronic angular momentum (6.12) is formulated, specifically, for the two-body hadronic system which is a particular case of the general, closed, nonhamiltonian systems with only one admissible orbit (the circle). As such, the orbit is stable.

Nevertheless, in the general case, total conservation laws are achieved under the most *unstable* possible conditions of the orbits of the constituents, so as to maximize the internal interactions.

Thus, the primary objective of the notion of hadronic angular momentum is that of representing the *nonconservation* (say, the decay) of the angular momentum of one constituent, or, equivalently, of one particle while moving within the hadronic medium. This setting can be illustrated also with the simple structure (6.12), e.g., for $\rho = \rho(t)$.

As an example, structure (6.12) can provide a smooth representation of the transition from a state, say, with $\ell = 2$, to one with $\ell = 1$, by simply assuming that $\rho(t)$ is a smooth function in the interval $(1, \frac{1}{2})$, under which

$$\rho\ell(\rho\ell + 1)|_{\substack{\rho=1\\ \ell=2}} = 6 \to \rho\ell(\rho\ell + 1)|_{\substack{\rho=1/2\\ \ell=2}} = 2 \,. \tag{6.21}$$

For details and more general realizations of the hadronic angular momentum, we refer the interested reader to the forthcoming work [45].

We pass now to the study of the *hadronic spin* as characterized by the isospinorial coverings $\widehat{SU}(2)$ of the $\widehat{SO}(3)$ symmetry. This study shall be conducted via the construction of the *isorepresentations* of $\widehat{SU}(2)$ [43].

Let us begin by defining $\widehat{SU}(2)$ as the infinite family of isotopes of $SU(2)$, along the general lines of ref.s [20,21]. As well known, $SU(2)$ can be interpreted as a group of unimodular isometries of the invariant in the two-dimensional complex Euclidean space $E(z, \delta\ \mathrm{C})$

$$SU(2) : z^{+}\delta z = z_i^{*}\delta_{ij}z = z_1^{*}z_1 + z_2^{*}z_2 , \tag{6.22.a}$$
$$\delta = \mathrm{Diag}(1,1) > 0 . \tag{6.22.b}$$

We shall therefore define the (infinite) family of $\widehat{SU}(2)$ coverings of $SU(2)$ as the Lie-isotopic groups of isounimodular isometries of the invariant in the isotopic space $\hat{E}(z, g, \hat{\mathrm{C}})$

$$\widehat{SU}(2) \ : \ z^{t}gz = z_i^{*}g_{ij}z_j = z_1^{*}g_{11}z_1 + z_2^{*}g_{22}z_2 , \tag{6.23.a}$$
$$g^{t} = g , \quad g = \mathrm{Diag}(g_{11}, g_{22}) > 0 , \tag{6.23.b}$$
$$g = g(t, z, z^{*}, \ldots) . \tag{6.23.c}$$

From Theorem 2.1, ref. [12], we therefore know that all isotopes $\widehat{SU}(2)$ are locally isomorphic to the conventional $SU(2)$ group, owing to the preservation by the isometric g of the positive-definiteness of the original metric δ (if this condition is relaxed, we would get also, as part of the family of isotopes $\widehat{SU}(2)$, groups locally isomorphic to $SU(1.1)$, which are not relevant for the physical objectives of this paper, even though mathematically significant).

We are interested in realizing $\widehat{SU}(2)$ with respect to the generalized unit (isounit)

$$\hat{I} = g^{-1} = \begin{pmatrix} g_{11}^{-1} & 0 \\ 0 & g_{22}^{-1} \end{pmatrix} = \frac{1}{\Delta}\begin{pmatrix} g_{22} & 0 \\ 0 & g_{11} \end{pmatrix} ,$$
$$\Delta = \mathrm{Det} g = g_{11}g_{22} > 0 \tag{6.24}$$

with universal enveloping isoassociatve algebra of generators, say, $\hat{J}_k, k = 1,2,3,$

$$\hat{\mathcal{A}}(SU(2)) : \hat{J}_i * \hat{J}_j \stackrel{\mathrm{def}}{=} \hat{J}_i g \hat{J}_j , \tag{6.25}$$

(where the preservation of the dimension 3 of the group under isotopy has been implemented from Theorem 2.1 of Ref. [12]; similar implementations will be tacitly done hereon).

Algebra $\hat{\mathcal{A}}(SU(2))$ shall act on the isohilbert space with isoinner product

$$\hat{\mathcal{H}}_{j} : \langle u \hat{|} v\rangle = \langle u| * |v\rangle = \langle u|g|v\rangle , \tag{6.26}$$

where the isotopic element has been assumed equal to that of the envelope to ensure the Hermiticity of the generators under lifting (Sect. 2). The underlying field shall be the isocomplex field (2.7).

We are interested in realizing $\widehat{SU}(2)$ as the Lie-isotopic group of isounitary transformations of type (2.10), i.e.,

$$\hat{U} * \hat{U}^{\dagger} = \hat{U}^{\dagger} * \hat{U} = \hat{I} , \tag{6.27.a}$$
$$\hat{U} = e_{\hat{\mathcal{A}}}^{i\hat{J}_k\theta_k} = \hat{I}e^{i\theta_k g\hat{J}_k} = e^{i\hat{J}_k g\theta_k}\hat{I} , \tag{6.27.b}$$

(no sum in the latter exponents), where the θ's are the conventional Euler's angles [21], under the condition of isounimodularity, which can be written from Eq. (2.12c)

$$\widehat{\mathrm{Det}}\,\hat{U} = \hat{I} , \quad k = 1,2,3 , \tag{6.28}$$

which can hold iff

$$\mathrm{Det}\ U = 1 . \tag{6.29}$$

i.e., iff

$$\mathrm{Tr}J_k g = 0 , \quad k = 1,2,3 . \tag{6.30}$$

From the local isomorphism between $\widehat{SU}(2)$ and $SU(2)$, we can write the *isocommutation relations* in the form

$$\left[\hat{J}_i \hat{,} \hat{J}_j\right] = \hat{J}_i * \hat{J}_j - \hat{J}_j * \hat{J}_i = \hat{J}_i g \hat{J}_j - \hat{J}_j g \hat{J}_i$$
$$= i\epsilon_{ijk}\hat{J}_k , \tag{6.31}$$

which are defined up to redefinitions to be identified later on in this section.

We shall now construct the *isorepresentations* of $\widehat{SU}(2)$ for the general case, and then specialize them to the hadronic angular momentum and spin (the reader should be aware that, to our knowledge, this is the first attempt at constructing isorepresentations of Lie-isotopic groups in general, which are known until now only for the regular- fundamental cases).

Isocommutation rules (6.31) imply the following consequences as for the conventional case

$$\left[\hat{\bar{J}}^{2} \hat{,} \hat{J}_k\right] = \left[\hat{\bar{J}}^{2} \hat{,} \hat{J}_{\pm}\right] = 0 , \tag{6.32.a}$$
$$\left[\hat{J}_3 \hat{,} \hat{J}_{\pm}\right] = \pm\hat{J}_{\pm} , \tag{6.32.b}$$

$$\left[\hat{J}_+ \hat{,} \hat{J}_-\right] = 2\hat{J}_3 \,, \tag{6.32.c}$$

$$\hat{\vec{J}}^2 = \sum_{k=1}^{3} \hat{J}_k * \hat{J}_k \,, \tag{6.32.d}$$

$$\hat{J}_\pm = \hat{J}_1 \pm i\hat{J}_2 \,. \tag{6.32.e}$$

Let $|b_k^d\rangle$, $k = 1, 2, \ldots, d$, be the d-dimensional *isobasis* of $\widehat{SU}(2)$ on $\hat{\mathcal{H}}_j$ with isoorthogonality properties

$$\langle b_i^d| * |b_j^d\rangle = \langle b_i^d|g|b_j^d\rangle = \delta_{ij} \,, \quad i,j = 1,2,\ldots,n \,. \tag{6.33}$$

Note that, if $|\pm\rangle$ are the conventional basis for the 2-dimensional representation of $SU(2)$, then the above isobasis are given by

$$|b_1^2\rangle = g_{11}^{-1/2}|+\rangle \,,\ |b_2^2\rangle = g_{22}^{-1/2}|-\rangle \,. \tag{6.34}$$

If $|+\rangle$, $|0\rangle$, $|-\rangle$ are the conventional basis of the three-dimensional representation of $SU(2)$, then, from Eq. (6.26) the isobasis is given by

$$|b_1^3\rangle = g_{11}^{-1/2}|+\rangle \,, |b_2^3\rangle = g_{22}^{-1/2}|0\rangle \,, |b_3^3\rangle = g_{33}^{-1/2}|-\rangle \,, \tag{6.35}$$

and similarly for the other cases. As in the conventional case, from Eq.s (6.32), we can diagonalize $\hat{\vec{J}}^2$ and $\hat{J}_3$. We can therefore assume the existence of the following *isoeigenvalues*

$$\hat{J}_3 * |b_k^d\rangle \stackrel{\text{def}}{=} \hat{J}_3 g|b_k^d\rangle = b_k^d|b_k^d\rangle \,, \tag{6.36}$$

where the b's must be necessarily real under the conditions assumed. Then, from Eq.s (6.32) we have, as in the conventional case,

$$\hat{J}_\pm * |b_k^d\rangle = |b_{k\pm1}^d\rangle \,, \tag{6.37.a}$$

$$\hat{J}_3 * |b_{k\pm1}^d\rangle = b_{k\pm1}^d|b_{k\pm1}^d\rangle \tag{6.37.b}$$

$$= \hat{J}_3 * \hat{J}_\pm * |b_k^d\rangle = (\hat{J}_\pm * \hat{J}_3 \pm \hat{J}_\pm) * |b_k^d\rangle \tag{6.37.c}$$

$$= (b_k^d \hat{J}_\pm \pm \hat{J}_\pm) * |b_k^d\rangle = (b_k^d \pm 1)|b_k^d\rangle \,.$$

Thus,

$$b_{k\pm1}^d = b_k^d \pm 1 \,. \tag{6.38}$$

Since $\hat{J}_+ * |b_d^d\rangle = 0$, we have

$$\begin{aligned}\hat{\vec{J}}^2 * |b_d^d\rangle &= (\hat{J}_- * \hat{J}_+ + \hat{J}_3 * \hat{J}_3 + \hat{J}_3) * |b_d^d\rangle \\ &= b_d^d(b_d^d + 1)|b_d^d\rangle \,,\end{aligned} \tag{6.39}$$

and similarly

$$\begin{aligned}\hat{\vec{J}}^2 * |b_1^d\rangle &= (\hat{J}_+ * \hat{J}_- + \hat{J}_3 * \hat{J}_3 - \hat{J}_3) * |b_1^d\rangle \\ &= b_1^d(b_1^d - 1) \,,\end{aligned} \tag{6.40}$$

from which

$$\begin{aligned}K &= b_d^d(b_d^d + 1) \equiv b_1^d(b_1^d - 1) \,, \\ b_1^d &= -b_d^d \,.\end{aligned} \tag{6.41}$$

It is easy to prove that, as in the conventional case, the dimension of the isorepresentation is characterized by the familiar rule

$$n = 2j + 1 \,, \quad j = 0, \frac{1}{2}, 1, \frac{3}{2}, \cdots \,. \tag{6.42}$$

The explicit form of the matrix elements can be readily computed by lifting the conventional case. In fact, we can write for $\hat{J}_-$

$$\hat{J}_- * |b_k^d\rangle = \alpha_k |b_k^d \,, \tag{6.43.a}$$

$$\alpha_k^2 = \langle b_k^d| * \hat{J}_+ * \hat{J}_- * |b_k^d\rangle \tag{6.43.b}$$

$$= \langle b_k^d| * (\hat{\vec{J}}^2 - \hat{J}_3^2 + \hat{J}_3) * |b_k^d\rangle$$

$$= k - (b_k^d)^2 + b_k^d \stackrel{\text{def}}{=} 2b_k \quad \text{for } n = 2, \quad \text{etc.} \,,$$

for $\hat{J}_+$

$$\hat{J}_+ * |b_k^d\rangle = \beta_k |b_k^d\rangle \tag{6.44.a}$$

$$\beta_k^2 = \langle b_k^d| * \hat{J}_- * \hat{J}_+ * |b_k^d\rangle = \langle b_k^d| * (\hat{\vec{J}}^2 - \hat{J}_3^2 - \hat{J}_3) * |b_k^d\rangle$$

$$= k - (b_k^d)^2 - b_k \stackrel{\text{def}}{=} -2b_k \quad \text{for } n = 2, \quad \text{etc.}\,. \tag{6.44.b}$$

The matrix elements are then given by

$$(\hat{J}_3)_{ij} = \langle b_i^d| * \hat{J}_3 * |b_j^d\rangle \,, \tag{6.45.a}$$

$$(\hat{J}_2)_{ij} = \frac{1}{2}\langle b_i^d| * (\hat{J}_+ + \hat{J}_-) * |b_j^d\rangle \,, \tag{6.45.b}$$

$$(\hat{J}_1)_{ij} = \frac{i}{2}\langle b_i^d| * (\hat{J}_- - \hat{J}_+) * |b_d^k\rangle \,, \tag{6.45.c}$$

under the subsidiary conditions (6.29) and (6.30), i.e.,

$$\det\ U = 1 \,, \quad \mathrm{Tr}(\hat{J}_k g) = 0 \,, \quad k = 1,2,3 \,, \tag{6.46}$$

which essentially imply that the identification of the b_k quantities with suitable expressions in the elements g_{kk}.

For the 2-dimensional case, the above results provide the following explicit form of the isorepresentation

$$\hat{J}_1 = \frac{1}{2\Delta^{1/2}}\begin{pmatrix} 0 & g_{11}^{1/2} \\ g_{22}^{1/2} & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & g_{22}^{-1/2} \\ g_{11}^{-1/2} & 0 \end{pmatrix}, \tag{6.47.a}$$

$$\hat{J}_2 = \frac{1}{2\Delta^{1/2}}\begin{pmatrix} 0 & -ig_{11}^{1/2} \\ ig_{22}^{1/2} & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & -ig_{22}^{-1/2} \\ ig_{11}^{-1/2} & 0 \end{pmatrix} \tag{6.47.b}$$

$$\hat{J}_3 = \frac{1}{2\Delta^{1/2}}\begin{pmatrix} g_{22} & 0 \\ 0 & -g_{11} \end{pmatrix} = \frac{\Delta^{1/2}}{2}\begin{pmatrix} g_{11}^{-1} & 0 \\ 0 & -g_{22}^{-1} \end{pmatrix}. \tag{6.47.c}$$

The isoeigenvalues of $\hat{J}^2$ and $\hat{J}_3$ are then given by

$$\hat{\vec{J}}^2 * |b_k^2\rangle = \frac{\Delta^{1/2}}{2}\left(\frac{\Delta^{1/2}}{2}+1\right)|b_k^2\rangle, \tag{6.48.a}$$

$$\hat{J}_3 * |b_k^2\rangle = \frac{\Delta^{1/2}}{2}|b_k^2\rangle, \quad k = 1,2, \tag{6.48.b}$$

and they coincide with the isoexpectation values under the conditions assumed, i.e.,

$$\langle\widehat{\hat{\vec{J}}^2}\rangle = \frac{\Delta^{1/2}}{2}\left(\frac{\Delta^{1/2}}{2}+1\right), \tag{6.49.a}$$

$$\langle\widehat{\hat{J}_3}\rangle = \frac{\Delta^{1/2}}{2}, \quad \hbar = 1. \tag{6.49.b}$$

We can therefore conclude by saying that the *Lie-isotopic liftings* $\hat{SU}(2)$ *of the* $SU(2)$*-spin symmetry imply the following mutation of the atomic spin*

$$j \to \Delta^{1/2} j, \tag{6.50}$$

$$j = 0, 1/2, 1, 3/2, \ldots; \qquad \Delta = \text{Det}.g = g_{11}g_{22}\cdots g_{nn}\ ;\ n = 2j+1.$$

As a result, *the* $\widehat{SU}(2)$ *symmetry is suitable for the characterization of the hadronic spin.*

Notice the lack of reducibility of representation (6.47) to the conventional Pauli's matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{6.51}$$

that is, the lack of existence of equivalence transformations of the type

$$\hat{J}_k = \frac{1}{2}A\sigma_k A^{-1} \tag{6.52}$$

under which the $SU(2)$ algebra of Pauli's matrices remains conventional, i.e., with the trivial Lie product $AB - BA$, and this illustrates the mathematical nontriviality of the representations of $\widehat{SU}(2)$. The nontriviality of the physical implications will be illustrated shortly via the application of the theory to Rutherford's historical hypothesis (1.3).

The degrees of freedom of the hadronic $\widehat{SU}(2)$-spin should also be illustrated for completeness, because they are considerably broader than those of the conventional theory.

The $\widehat{SU}(2)$ symmetry has been constructed until now under the specific condition of preserving the structure constants of $SU(2)$, via isocommutation rules (6.31). This has been done for the evident purpose of stressing the isomorphism of $\widehat{SU}(2)$ and $SU(2)$.

However, the general formulation of the Lie-isotopic theory, as originally proposed in memoir [8] (see monographs [19] and [31]) requires the generalization of the conventional structure constants into the so-called *structure functions*, as in Eq.s (6.6).

We can easily generalize the preceding realization of $\widehat{SU}(2)$ into the following form

$$[\hat{J}'_1 \hat{,} \hat{J}'_2] = i\hat{J}'_3, \quad [\hat{J}'_2 \hat{,} \hat{J}'_3] = i\Delta\hat{J}_1, \quad [\hat{J}'_3 \hat{,} \hat{J}'_1] = i\Delta\hat{J}'_2, \tag{6.53}$$

where the generally nonlinear and arbitrary dependence of the determinant Δ on all the local variables illustrates the structure functions of the Lie-isotopic theory.

A repetition of the representation theory previously given then yields, after tedious but simple calculations, the following matrix forms

$$\hat{J}'_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \equiv \frac{1}{2}\sigma_1, \quad \hat{J}'_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \equiv \frac{1}{2}\sigma_2, \tag{6.54.a}$$

$$\hat{J}'_3 = \frac{1}{2}\begin{pmatrix} g_{22} & 0 \\ 0 & g_{11} \end{pmatrix} = \frac{\Delta}{2}\hat{I}\sigma_3. \tag{6.54.b}$$

The isoeigenvalue equations are then given by

$$\hat{J}'_3 * |b_k^2\rangle = \frac{\Delta}{2}|b_k^2\rangle\ ,, = 1,2, \tag{6.55.a}$$

$$\hat{\vec{J}}^2 * |b_k^2\rangle = \frac{\Delta}{2}\left(\frac{\Delta}{2}+1\right)|b_k^2\rangle, \tag{6.55.b}$$

and, again, we have the preservation of the eigenvalues as the expectation values, under the assumptions considered.

One can notice the additional mutation in the transition from Eq.s (6.48) to (6.55)

$$\frac{\Delta^{1/2}}{2}j \to \frac{\Delta}{2}j\,, \tag{6.56}$$

which is nothing but an isotopic lifting of an isotopic symmetry.

To understand the occurrence from the geometric-algebraic viewpoint, one can recall the results of ref. [22,23] in the construction of the $\widehat{SO}(3.1)$ covering of the $SO(3.1)$ symmetry. As one can see, $\widehat{SO}(3.1)$ can be constructed in one single step as an isotopy of $SO(4)$ or, equivalently, in two steps, the first by constructing $SO(3.1)$ as an isotopy of $SO(4)$ and then $\widehat{SO}(3.1)$ as an isotopy of $SO(3.1)$.

Along similar lines, one should keep in mind the multiple infinite nature of all possible $\widehat{SU}(2)$ coverings of $SU(2)$, in the sense that, besides the infinite number of mutations (6.23) of the original metric (6.22), with consequential infinite numbers of $\widehat{SU}(2)$, one has in addition an infinite number of isotopies $\widehat{SU}'(2)$ of $\widehat{SU}(2)$ provided by additional mutations of the mutated metric (6.63). For illustrations regarding the different nature of the nonlinear isorotations for different isometrics, the reader is recommended to consult ref. [21].

We are now in a position to reinspect the hadronic angular momentum at a deeper level of analysis. First, the preceding general theory of $\widehat{SU}(2)$, and Eq.s (6.48) in particular, confirm the structure of isoeigenvalues (6.18) in general, and mutations (6.20) in particular. Furthermore, we gain the following interpretation of the quantity ρ of Eq.s (6.18) as an isotopy of an isotopy

$$\rho = f(\Delta) = f(\mathrm{Det}g)\,, \tag{6.57}$$

namely, the quantity ρ which appears from the primitive Newtonian action (3.14), is a functions of the determinant of the underlying metric. When the metric is conventional, $g = \delta, \rho = 1$, and conventional formulations occur. When $g \neq \delta$, then the generalized classical and hadronic formulations hold.

We pass now to the composition of hadronic angular momentum and spin. Under the assumptions that they are independent, the composition is a simple lifting of the conventional one.

Let $\hat{L}_k$ and $\hat{S}_k$ represent the hadronic angular momentum and spin operators, respectively, with corresponding determinants of the underlying, respective, metrics $\Delta_{\hat{L}}$ and $\Delta_{\hat{S}}$. Let ℓ and s represent the conventional eigenvalues of angular momentum and spin. Denote the corresponding isohilbert spaces with $\hat{\mathcal{H}}_{\hat{L}}$ and $\hat{\mathcal{H}}_{\hat{S}}$, with bases $|b_i^{d_\ell}\rangle$ and $|b_j^{d_s}\rangle$, respectively, with corresponding isoassociative envelopes $\hat{\mathcal{A}}_{\hat{L}}$ and $\hat{\mathcal{A}}_{\hat{S}}$.

Then, the enveloping associative algebra of the *total hadronic angular momentum* is given by the tensorial form

$$\hat{\mathcal{A}}_{\mathrm{Tot}} = \hat{\mathcal{A}}_{\hat{L}} \otimes \hat{\mathcal{A}}_{\hat{S}}\,, \tag{6.58}$$

and isounit

$$\hat{I}_{\mathrm{Tot}} = \hat{I}_{\hat{L}} \otimes \hat{I}_{\hat{S}}\,, \tag{6.59}$$

with selfexplanatory symbols.

The underlying isohilbert space is then given by the tensorial products and related basis

$$\hat{\mathcal{H}}_{\mathrm{Tot}} = \hat{\mathcal{H}}_{\hat{L}} \otimes \hat{\mathcal{H}}_{\hat{S}} : |b_i^{d_\ell}\rangle \otimes |b_j^{d_s}\rangle = |b_i^{d_\ell} b_j^{d_s}\rangle\,. \tag{6.60}$$

The *total hadronic angular momentum* can then be defined as the ordinary sum

$$\vec{\hat{J}}_{\mathrm{Tot}} = \vec{\hat{L}} \otimes \hat{I}_{\hat{S}} + \hat{I}_{\hat{L}} \otimes \vec{\hat{S}}\,, \tag{6.61}$$

with isoeigenvalues for realizations of type (6.48)

$$\vec{\hat{J}}^{\,2}_{\mathrm{Tot}} * |b_i^{d_\ell} b_j^{d_s}\rangle = \left[\left(\Delta_{\hat{L}}^{1/2}\ell + \Delta_{\hat{S}}^{1/2}s\right)\left(\Delta_{\hat{L}}^{1/2}\ell + \Delta_{\hat{S}}^{1/2}s + 1\right)\right] |b_i^{d_\ell} b_j^{d_s}\rangle\,, \tag{6.62.a}$$

$$\hat{J}_{\mathrm{Tot}\ 3} * |b_i^{d_\ell} b_j^{d_s}\rangle = (\Delta_{\hat{L}}^{1/2} m_\ell + \Delta_{\hat{S}}^{1/2} m_S)|b_i^{d_\ell} b_j^{d_s}\rangle\,. \tag{6.62.b}$$

Thus, given a hadronic angular momentum $\Delta_{\hat{L}}\ell$ and a hadronic spin $\Delta_{\hat{S}}S$, the total hadronic angular momentum $j^{\mathrm{hadr}}_{\mathrm{tot}}$ is given by

$$j^{\mathrm{Hadr}}_{\mathrm{Tot}} = \Delta_{\hat{L}}^{1/2}\ell + \Delta_{\hat{S}}^{1/2}s\,. \tag{6.63}$$

As we shall see in Sect. 9, the above expression will result to be fundamental in the quantitative interpretation of the total spin of the neutron.

For additional information, including the isotopic generalization of the Clebsh-Gordon and other coefficients, the interested reader may consult ref. [43].

7 Review of the Hadronic Structure Model of the π^0 Meson

It is important to review the hadronic structure model of the π^0 proposed in memoir [9] for a number of reasons. First, we can reformulate the model via the rather vast research conducted since the time of proposal [9]. Second, it is important to show that the numerical results of the original proposal are entirely unchanged by the more rigorous formulation of this section. Third, the model is clearly useful as a comparison for the hadronic structure model of the neutron of the next section.

The original hadronic structure equations of model $\pi^0 = (\epsilon^+, \epsilon^-)$, Eq.s (5.1.14), page 836, ref. [9], can now be written in the modular-isotopic language of Eq.s (4.22), originating from the primitive Newtonian model (3.14), as follow

$$
\begin{aligned}
i\hbar\frac{\partial}{\partial t}\psi &= \left[-\frac{\hbar^2}{2\overline{m}}\hat{\Delta} - \frac{\hat{e}^2}{r} + \frac{i}{\hbar}\left(\frac{\partial \hat{I}}{\partial t}\right)\log\psi\right] * \psi = E\psi \,, && (7.1.a)\\
E^{\mathrm{Tot.}}_{\pi^0} &= 2E^{\mathrm{Rest}}_{\epsilon} + 2E^{\mathrm{Kin}}_{\epsilon} - E = 135\mathrm{MeV}\,, && (7.1.b)\\
\tau^{-1}_{\pi^0} &= 4\pi\lambda\!\!\!^{-}|\psi(0)|^2\frac{\alpha E_\epsilon}{\hbar} = 10^{16}\mathrm{sec}^{-1}\,, && (7.1.c)\\
b^{-1}_{\pi^0} &= 10^{-13}\mathrm{cm}\,, \quad \overline{m} = m/\rho && (7.1.d)
\end{aligned}
$$

where the subsidiary constraints evidently express the condition to represent the rest energy of the π^0 ($\sim$ 135 MeV), its meanlife ($\sim 10^{-16}$ sec), and its size ($\sim 10^{-13}$ cm). The remaining conditions (null total angular momentum, null charge, null magnetic and electric moments, parities, etc.), could also be imposed as additional subsidiary constraints. But, as we shall see, they would be inessential because the latter total characteristics follow from Eq.s (7.1).

As reviewed in Sect. 5, Eq. (7.1a) under Animalu's isounit (4.12) reduces *exactly* to the radial equation used in ref. [9], i.e.,

$$\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right) + \frac{\overline{m}}{\hbar^2}\left(E - V_0\frac{e^{-br}}{1-e^{-br}}\right)\right]\psi = 0\,, \qquad (7.2)$$

which is consistent, if and only if Eq.s (5.20) specialized to subsidiary conditions (7.1b-7.1d), hold, i.e.,

$$
\begin{aligned}
k_1[1 = (k_2-1)^2] &= \frac{1}{2\hbar c}(135\times 10^{-13})\,, && (7.3.a)\\
\frac{(k_2-1)^3}{k_1} &= \frac{9\times 10^5}{4\pi c}(10^{16}\times 10^{-13})\,. && (7.3.b)
\end{aligned}
$$

As shown in Ref. [9], Eq. (5.1.33), the above equations are indeed consistent with solution

$$k_1 = 0.34\,, \quad k_2 = 1 + 4.27\times 10^{-2}\,. \qquad (7.4)$$

This implied the achievement of the *suppression of the positronium spectrum* because the spectrum of Hulten's potential

$$E = \frac{\hbar^2 b^2}{4\overline{m}}\left(\frac{\overline{m}V_0}{\hbar^2 b^2}\frac{1}{n} - n\right)\,, \quad n = 1\,, \qquad (7.5)$$

is reduced, under conditions (7.3), down to only one level: the π^0.

As shown in Sect. 5.1, loc. cit., the model therefore permits the representation of *all* characteristics of the π^0, such that:

1. the rest energy of 135 MeV;
2. the meanlife of 10^{-16} sec;
3. the charge radius of 10^{-13} cm;
4. the spin zero, trivially, from the singlet state (ϵ^+, ϵ^-);
5. the null total charge;
6. the null magnetic and electric dipole moments;

while the remaining characteristics, such as space and charge parity, were reduced to the selection of phases.

The model appears also to be the most direct way of representing the primary decay

$$\pi^0 \to \gamma\gamma \quad (98.85\%)\,, \qquad (7.6)$$

while the decay

$$\pi^0 \to e^+ + e^-(< 2\times 10^{-6}\%)\,, \qquad (7.7)$$

could be interpreted as resulting from the probability of tunnel effects of free eletons $\epsilon^\pm \to e^\pm$. The latter aspects related to decays were only indicated in ref. [9] without any elaboration (which we hope to present in the forthcoming Vol. III, ref. [45]).

As recalled earlier, the model was extended to the charged pions, via the hypothesis $\pi^\pm = (\epsilon^+, \epsilon^\pm, \epsilon^-)$, which resulted to be given by $\pi^\pm = (\pi^0, \epsilon^\pm)$, namely, by the model $\pi^0 = (\epsilon^+, \epsilon^-)$ with an eleton $\epsilon^\pm$ *at rest* in its center. The extension of the theory to the remaining members of the light mesons was also indicated, e.g., via the structures $K^0 = (\pi^+, \pi^-)$, and $K^\pm = (\pi^+, \pi^\pm, \pi^-)$. These latter models will be also reviewed in the forthcoming Vol. III., loc. cit.

At a first analysis, particularly from a nonrelativistic viewpoint, the eletons $\epsilon^\pm$ do not appear to have any mutation from the ordinary electrons $e^\pm$, e.g., because the spin can remain the ordinary one, and no mutation of the magnetic moment is needed. Nevertheless, a relativistic analysis indicates that the electrons must necessarily experience mutations when assumed as π^0 constituents, because the π^0 structure apparently requires a necessary modification of the Minkowski metric in order to achieve compatibility with the available phenomenology, e.g., that on the behavior of the meanlife with speed [23]. This relativistic treatment will be presented in the forthcoming paper [26].

At a deeper analysis, the use of hadronic mechanics, and the consequential mutation $e^\pm \to \epsilon^\pm$, results to be *necessary* for the consistency of model (7.1). In fact, the same equations in their conventional, atomic form with Hulten's potential (e.g., Eq. (7.2) with $\rho = 1$) are *inconsistent*, in the sense that they are unable to represent the rest energy of the π^0. As a matter of fact, this fundamental inconsistency in attempting the construction of the model $\pi^0 = (e^+, e^-)$ was a primary reason that stimulated the suggestion to construct hadronic mechanics.

The inconsistency is due to a little known property of conventional atomic mechanics according to which bound states with total energy much higher than the rest energy of the constituents are generally inconsistent. Equivalently, we can say that the typical arena of applicability of atomic mechanics is the hydrogen structure or the deuterium in which the total energy is smaller than that of the constituents, evidently because of the negative character of the binding energy (mass defect).

This occurrence was first identified in ref. [46]. Its specific application to the inconsistency of the model $\pi^0 = (e^+, e^-)$ was presented in detail in Appendix A of paper [47], which introduced the model $\pi^0 = (\epsilon^-, \epsilon^-)$ for the first time, including a first calculation of values (7.4).

It is essential for the reader to understand how hadronic mechanics resolves this inconsistency. The mechanism is essentially a sort of "renormalization" of the mass of the eleton via the expression

$$m_e = 0.5\text{MeV} \to \overline{m} = \frac{me}{\rho} \cong 67.5\text{MeV}\ . \qquad (7.8)$$

In fact, the above mechanism essentially turns the model into a conventional atomic one in which the total energy of the π^0 becomes *smaller* than the sum of the rest energies $2\overline{m}$.

In turn, mechanism (7.8) requires, as a necessary condition, the abandonment of Hamiltonian mechanics, at the classical level, and of atomic mechanics, at the operator level. In fact, it can be rigorously justified via the assumption of nonhamiltonian action functionals of type (3.14) which, in turn, imply a necessary generalization of conventional quantization, e.g., of the hadronic type (3.20). Model (7.1) then follows.

Still as of today, after reinspecting this occurrence, this author is aware of no possibility of achieving a consistent structure model of the π^0 as a bound state of one electron and one positron (that is, a structure model with a *real* rest energy) via the use of ordinary atomic mechanics.

8 Apparent Consistency of Rutherford's Historical Hypothesis on the Neutron Structure

We finally come to the crucial test of physical consistency and relevance of hadronic mechanics. In the language of the original proposal [9], it consists of the capability to provide a consistent, quantitative, representation of Rutherford's historical hypothesis here indicated with the symbol $n = (p^+, \epsilon^-)$.

The nonrelativistic, hadronic, structure equations we propose are exactly the same as those of model $\pi^0 = (\epsilon^+, \epsilon^-)$, although with the subsidiary constraints now representing the different characteristics of the neutron, i.e., rest energy of 939.6 MeV, mean life of 917″, and charge radius of 0.8×10^{-13} cm, according to the equations

$$i\hbar\frac{\partial}{\partial t}\psi = \left[-\frac{\hbar^2}{2\overline{m}}\hat{\Delta} - \frac{\hat{e}^2}{r} + \frac{i}{\hbar}\left(\frac{\partial \hat{I}}{\partial t}\right)\log\psi\right] * \psi = E\psi\ , \qquad (8.1.a)$$

$$E_n^{\text{Tot}} = E_p^{\text{Rest}} + E_\epsilon^{\text{Rest}} + E_\epsilon^{\text{Kin}} - E = 939.6\text{MeV}\ , \qquad (8.1.b)$$

$$\tau_n^{-1} = 4\pi\lambda\!\!\!^{-}|\psi(0)|^2\frac{\alpha E_\epsilon}{\hbar} = 1.09 \times 10^{-3}\text{Sec}^{-1}\ , \qquad (8.1.c)$$

$$b_n^{-1} = 0.8 \times 10^{-13}\text{cm}\ , \quad \overline{m} = m_\epsilon = m_e/\rho\ . \qquad (8.1.d)$$

As we shall see, the above model, when implemented with the notion of hadronic spin (Sect. 6), apparently resolves at this nonrelativistic level *all* "inconsistencies" of the model based on the use of atomic mechanics, i.e., for model $n = (p^+, e^-)$.

The firsts of these "inconsistencies" are given by atomic mechanics when representing precisely the rest energy, meanlife and charge radius of the neutron.

In fact, the inability to represent the total rest energy is due to the fact that, exactly as it occurred for the π^0,

$$E_n = 939.6\text{MeV} > E_p + E_e = (938.3 + 0.5)\text{MeV}. \tag{8.2}$$

The use of atomic mechanics, that is, the use of a strictly Hamiltonian model (classically and operationally), would demand a *positive* binding energy even for an electron at rest

$$E^{\text{Bind}} \cong 1.3 > 0\,, \tag{8.3}$$

which is manifestly inconsistent (see App. A of ref. [47] for details).

The second difficulty was due to the inability to reach the long life of the neutron (by particle standards), owing to the low mass of the electron.

The third difficulty was the inability to reach the charge radius, evidently because atomic mechanics would produce in model (p^+, e^-) a charge radius of an order of magnitude close to that of the atom.

It is readily seen that all the above three difficulties are resolved by hadronic mechanics. In fact, exactly as it happens for the model $\pi^0 = (\epsilon^+, \epsilon^-)$, the consistency of Eq.s (8.1) is reducible to Eq.s (5.20) for the above indicated values of the neutrons, or, equivalently, for the eleton in the external field of the proton, i.e., for

$$\begin{aligned}
E_\epsilon^{\text{Tot}} &= E_\epsilon^{\text{Rest}} + E_\epsilon^{\text{Kin}} - E^{\text{Bind}} \\
&= E_n - E_p = 1.3\text{MeV} && (8.4.a)\\
k_1[1-(k_2-1)^2] &= \frac{1}{2\hbar i}(E_\epsilon^{\text{Tot}} b_n^{-1}) \\
&= 2.5\times 10^{10}(1.3\times 0.8\times 10^{-13})\,, && (8.4.b)\\
\frac{(k_2-1)^3}{k_1} &= \frac{9.01\times 10^5}{4\pi c}(\tau_n^{-1} b_n^{-1}) \\
&= 2.38\times 10^{-6}(1.09\times 10^{-3}\times 0.8\times 10^{-13})\,. && (8.4.c)
\end{aligned}$$

Simple calculations show that the above system is consistent, with solutions

$$\begin{aligned}
k_1 &= 0.34\,, && (8.5.a)\\
k_2 &= 1 + 4.27\times 10^{-2}\,, && (8.5.b)
\end{aligned}$$

which are remarkably close, in order of magnitude, to the corresponding solutions for the π^0, Eq.s (7.4).

Also, and perhaps most importantly, solutions (8.5) imply the *suppression of the hydrogen spectrum of energy down to only one level; 939.6 MeV.* In fact, as one can see, for values (8.5) only one level of energy is admissible in Hulten's spectrum (7.5), and that value yields the neutron mass.

In conclusion, the proposed nonrelativistic hadronic model (8.1) is capable of providing a consistent, quantitative representation of the rest energy, meanlife and charge radius of the neutron.

The above results, however, could have been predicted back in 1978 from the consistency of the structure model $\pi^0 = (\epsilon^+, \epsilon^-)$. By far the biggest difficulty of model $n = (p^+, \epsilon^-)$ is to achieve a consistent, quantitative, representation of the total spin $\frac{1}{2}$ of the neutron, by recalling that the proton and the electron have both spin $\frac{1}{2}$. This was evidently the biggest "inconsistency" claimed via the use of atomic mechanics, which recommended the historical change in the direction of research toward more abstract lines recalled in Sect. 1.

Another significant technical difficulty is the representation of the anomalous value of the neutron magnetic moment

$$\mu_n = -1.9\frac{e\hbar}{2m_p c}\,. \tag{8.6}$$

The representation of the total angular momentum $j = \frac{1}{2}$ of the neutron in structure $n = (p^+, \epsilon^-)$ and its anomalous magnetic moment can be readily provided by the notion of mutation of intrinsic characteristics of particles submitted in the original proposal [9] and developed in the preceding papers [23,24], once joined to the notion of hadronic spin introduced in this paper (Sect. 6).

Recall that the proton p^+ in hadronic structure (p^+, ϵ^-) is assumed, in first approximation, to be unmutated owing to its very large rest energy as compared to that of the electron. Recall also that the proton is not an "empty sphere" with points in it, but the densest object measured in laboratory until now. As a result, the transition from empty space to the interior of the proton can be represented as a mutation of the Minkowski metric of the topology preserving class [23]

$$\begin{aligned}
\eta = \text{Diag}(1,1,1,-1) \rightarrow g &= \text{Diag}(b_1^2, b_2^2, b_3^2, -b_4^2) = T\eta\,, && (8.7.a)\\
\Delta_L &= b_1^2 b_2^2 b_3^2\,, \quad b_\mu \neq 0\,, T > 0\,. && (8.7.b)
\end{aligned}$$

Recall that, once the point-like abstraction of wavepackets is abandoned and their physical extension is duly represented, the electron can only penetrate inside the proton according to a singlet coupling, with its angular momentum parallel to the spin of the proton (see Fig. 3 for details).

The spin $s_n = \frac{1}{2}$ of the neutron is then recovered under the identities

$$\begin{aligned} s_n &= s_p + \ell_\epsilon + s_\epsilon = \frac{1}{2} , & (8.8.a) \\ j_\epsilon^{\rm Tot} &= \ell_\epsilon + s_\epsilon = 0 , & (8.8.b) \\ s_\eta &\equiv s_p = \frac{1}{2} , & (8.8.c) \end{aligned}$$

that is, (Sect. 6), for

$$\begin{aligned} j_\epsilon^{\rm Tot} = \ell_\epsilon + s_\epsilon &= \Delta_\ell m_\ell + \Delta_s m_s = \Delta_\ell - \frac{1}{2}\Delta_s = 0 , & (8.9.a) \\ m_\ell &= 1 , \quad m_s = -\frac{1}{2} , \quad \Delta_\ell = \frac{1}{2}\Delta s , & (8.9.b) \end{aligned}$$

where Δ_s is the determinant of the space isometric underlying the spin of the eleton.

The conceptual interpretation of Eq.s (8.8) is quite simple. With reference to Fig. 3, the electron can only penetrate in a stable way inside the proton with spin opposite to that of the proton (singlet state), and with orbital angular momentum parallel to the spin of the proton. Eq.s (8.8) then simply mean that, at the limit of the Rutherford's compression when the eleton reaches the center of the proton, the angular momentum must necessarily coincide, for physical consistency, with the spin of the proton, i.e.,

$$\lim_{r \to 0} \ell_\epsilon = s_p = \frac{1}{2} . \qquad (8.10)$$

Eq. (8.8) can therefore be written

$$s_n = s_p + \ell_\epsilon + s_\epsilon = \frac{1}{2} + \frac{1}{2} - \frac{1}{2} = \frac{1}{2} = s_p . \qquad (8.11)$$

Note that a number of other solutions are possible, but they require *higher* values of the hadronic angular momentum and/or of the hadronic spin. They are, therefore, excited states and, as such, extremely unstable. The *hadronic ground state* of the model $n = (p^+, \epsilon^-)$ is therefore characterized by

$$s_p = \frac{1}{2} , \quad \ell_\epsilon = \frac{1}{2} , \quad s_\epsilon = -\frac{1}{2} . \qquad (8.12)$$

Note also that, by comparison, the atomic ground state has $\ell = 0$, but this does not mean that the electron is at rest.

The anomalous value of the magnetic moment of the neutron, Eq. (8.6), can also be readily interpreted via the notion of mutation. For this purpose, we shall first compute the value of the magnetic moment of the eleton, and then pass to its interpretation.

The total magnetic moment of the model $n = (p^+, \epsilon^-)$ is evidently given by

$$\mu_n = \mu_p + \mu_\epsilon^{\rm orb} + \mu_\epsilon^{\rm Intr.} . \qquad (8.13)$$

But

$$\mu_p = +2.7 \frac{e\hbar}{2m_p c} . \qquad (8.14)$$

Therefore

$$\mu_\epsilon^{\rm Tot} = \eta_\epsilon^{\rm Orb} + \mu_\epsilon^{\rm Intr.} = -4.6 \frac{e\hbar}{2m_p c} = 2.5 \times 10^{-3} \mu_e . \qquad (8.15)$$

Recall that the plus (minus) sign in Eq. (8.13) indicates that the magnetic moment is parallel (antiparallel) to the spin of the proton. If one assumes that the intrinsic magnetic moment of the eleton is the same as that of the electron

$$\mu_\epsilon^{\rm Intr} = \mu_e = \frac{e\hbar}{2m_e c} = 1,836.13 \frac{e\hbar}{2m_p c} , \qquad (8.16)$$

the orbital magnetic moment of the eleton is then given by

$$\mu_\epsilon^{\rm Orb} = -1,840.75 \frac{e\hbar}{2m_p c} . \qquad (8.17)$$

Thus

$$|\mu_\epsilon^{\rm Orb}| > |\mu_\epsilon^{\rm Intr.}| = |\mu_e| . \qquad (8.18)$$

We should stress, however, that value (8.16) is a mere assumption made in parallel with (8.12), i.e., that the *intrinsic* spin and, therefore, magnetic moment of the electron are unmutated. Nevertheless, the reader should keep in mind that, on physical grounds, only the *total* quantities (8.8b) and (8.15) are relevant, and their mutation selfevident. Also, for these given total quantities, hadronic mechanics allows the construction of several different models for the intrinsic quantities, besides (8.8b) and (6.16) which, in the final analysis, do not have primary relevance, the eleton being at rest.

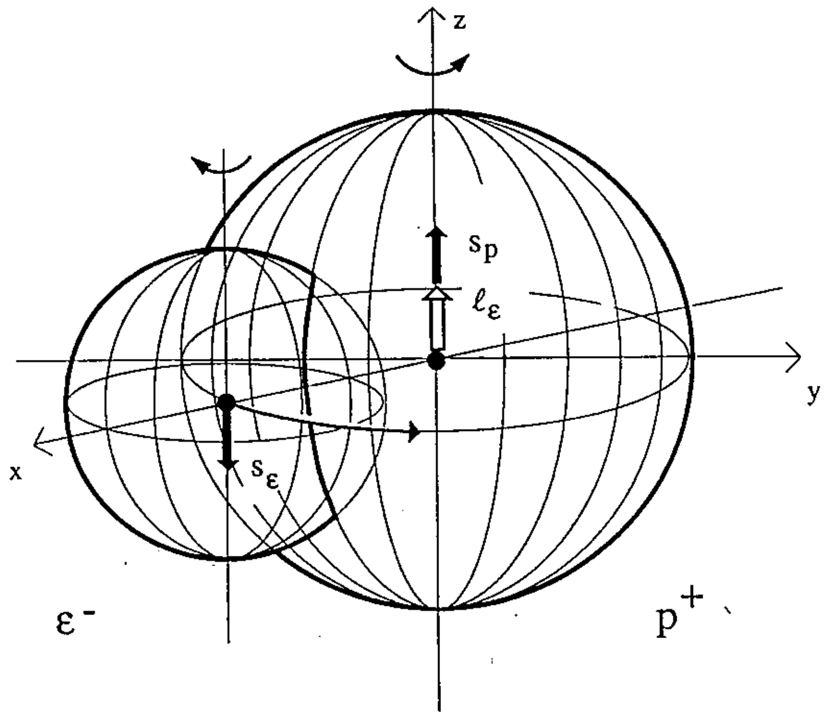

FIGURE 3. A schematic view of the structure model of the neutron $n = (p^+, \epsilon^-)$ proposed in this paper along Rutherford's historical hypothesis. The proton is represented, not as an empty sphere with points in it, but as the densest object measured in laboratory until now. As such, the proton is depicted as a sphere of radius equal to that of the charge distribution (~ 1F) filled up with the wavepackets of the constituents in conditions of total mutual overlapping (because their size is also of the order of 1F). Rutherford conceived his hypothesis on the neutron as a "compressed hydrogen atom". The figure therefore depicts the initiation of Rutherford's compression of the electron inside the proton where the electron is represented by a sphere schematizing its wavepacket, and sizes are not necessarily in scale. Once this physical setting is clearly identified, it is then easy to see that the electron can only penetrate inside the proton with the relative spinning "in phase" [9] (to avoid high dissipative effects expected from the spinning of wavepackets one against the other), and with its angular momentum parallel to the spin of the proton. These initial conditions appear to be rather plausible and well founded experimentally. The final phase of the compression is conjectural as of this writing. We argue that, when the electron is totally compressed inside the proton, the angular momentum is expected to coincide with the spin of the proton owing to their physical identity and, thus, to prevent inconsistencies in the mathematical treatment of the structure. This automatically allows to represent the spin of the neutron in model $n = (p^+, \epsilon^-)$ as coinciding with that of the proton, Eq. (8.8). The representation of all other intrinsic characteristics of the neutron is then readily allowed by the techniques of hadronic mechanics. The electron, when totally immersed in the hyperdense medium in the interior of the proton, is assumed to be mutated [9] into an eleton, and acquire intrinsic characteristics different than those of the ordinary particle. If correct, Rutherford's conception of the neutron as a compressed hydrogen atom could well result to be the mechanism at the origin of neutrinos in Nature. In fact, the eleton, when exciting the proton, must reacquire conventional characteristics which can only be achieved via the emission of an antineutrino. The process originating neutrinos would therefore be Rutherford's compression of the electron inside the proton. The understanding is that the neutrino *is not* a member of the neutron structure according to this view, and it can *only* be created when the eleton exits the proton, thus reacquiring the conventional physical characteristics.

The characterization of the above data can readily be done via Eq. (6.44), ref [24] on the mutation of the magnetic moment of the eleton

$$\mu_e \rightarrow \mu_\epsilon = \frac{b_3}{b_4}\mu_e \,. \tag{8.19}$$

Assume, for simplicity, that the medium inside the proton is homogeneous (but *not* isotropic, owing to the existence of the privileged direction due evidently to the spin). Then, from Eq. (8.7) and (8.9), we can put

$$b_1 = b_2 = b_3 \stackrel{\text{def}}{=} b = \Delta_\ell^{1/6} = \left(\frac{\Delta_s}{2}\right)^{1/6} \,. \tag{8.20}$$

Eq. (8.15) then yields the following value

$$b_4 \cong 4 \times 10^2 \left(\frac{\Delta_s}{2}\right)^{1/6} \,. \tag{8.21}$$

In different terms, the mutation of the magnetic moment of the electron can be interpreted as due to the alteration of the Minkowski metric requested by the medium itself, or, independently, by phenomenological considerations regarding the behavior of the meanlife with speed and other topics [23]. In fact, such alteration implies that of the gamma-matrices in Dirac's equations which, in turn, results into a necessary mutation of the magnetic moment and other physical quantities [24].

The best known way to acquire a deeper knowledge of the b-quantities, would be the conduction of theoretical and experimental studies on the behavior of the meanlife of the neutron with speed, as it is the case for other particles such as the pions and kaons (see next section). At this moment we have therefore to content ourselves with characterization (8.21) expressed with respect to the unknown quantity Δ_s.

Note that, to obtain a preliminary order of magnitude, one could assume

$$\Delta_s = 1 \tag{8.22}$$

in which case

$$b \cong 2^{-1/6} , \quad b_4 \cong 4 \times 10^2 \times 2^{-1/6} = 3.53 \times 10^2 . \tag{8.23}$$

Nevertheless, this is expected to be a gross approximation owing to the need to mutate the metric of the eleton, thus resulting in $\Delta_s \neq 1$.

The space and charge parities of the neutron coincide with those of the proton and are therefore automatically represented by the model $n = (p^+, \epsilon^-)$.

We can therefore conclude that *our model of the structure of the neutron along Rutherford's historical hypothesis is capable of representing with one single equation of structure, all the intrinsic characteristic of the neutron, such as:*

1. the rest mass of 939.6 MeV;
2. the mean life of 917′;
3. the charge radius of 0.8 $\times 10^{-13}$ cm;
4. the total angular momentum $\frac{1}{2}$;
5. the anomalous magnetic moment $-1.9\hbar/2m_p c$;
6. the null total charge; and
7. the space and charge parity.

Furthermore, the model $n = (p^+, \epsilon^-)$ appears to be particularly suited to represent the decay of the neutron

$$n \rightarrow p^+ + e^- + \overline{\nu}_e , \tag{8.24}$$

which can be evidently interpreted as due to the eleton ϵ^- exiting the proton structure, thus reacquiring the conventional characteristics of the electron.

This could only be possible via the emission of an (anti) neutrino, i.e., via the decay

$$\epsilon^- \rightarrow e^- + \overline{\nu}_e , \tag{8.25}$$

namely, *the neutrino originates from the constraints in the angular momentum of the eleton during Rutherford's compression*, according to Eq. (8.10). In actuality, mechanism (8.25), which is the inverse of Rutherford's compression (1.3),

$$n = (p^+, \epsilon^-) \rightarrow \text{Hydr. At.} = (p^+, e^-) + \overline{\nu}_e , \tag{8.26}$$

could well be interpreted as being at the foundation of the process of creation of neutrinos in Nature.

A relativistic treatment of the preceding analysis is presented in the forthcoming paper [26], including a reinterpretation of decay (8.25) via Barut's model $n = (p^+, e^-, \overline{\nu}_e)$.

We now close this section with the identification of the characteristics of the eleton in structure $n = (p^+, \epsilon^-)$.

In order to identify the rest energy of the eleton, we have to evaluate evidently its kinetic and binding energies in Eq. (8.4a). It is readily seen that the kinetic energy is very small, i.e.,

$$E_\epsilon^{\text{Kin}} = k_1 \hbar b c \approx 0 , \tag{8.27}$$

and, similarly, its binding energy is equally small, i.e.,

$$E = \frac{\rho^2 \hbar^2 b^2}{2m} \left(\frac{m V_0}{\rho^2 \hbar^2 b^2} \frac{1}{n} - n \right) \cong \epsilon V_0 \approx 0 , \tag{8.28}$$

as expected from he fact that contact interactions, which are primarily responsible for the model, are not derivable from a potential and, in this sense, do not carry energy, but only modify the local characteristics of particles.

As a result, the total energy of the eleton approximates its rest energy quite well, and we can write

$$E_\epsilon^{\text{Tot}} = E_\epsilon^{\text{Rest}} + E_\epsilon^{\text{Kin}} - E \cong E_\epsilon^{\text{Rest}} = 1.3\text{MeV} . \tag{8.29}$$

The remaining characteristics of the eleton have been computed earlier in this section. We can therefore conclude with the following characteristics of the eletons for Rutherford's model

$$\epsilon^{\pm}: \begin{cases} \text{mass } 1.3\text{MeV} \\ \text{meanlife } 917'; \\ \text{charge radius null;} \\ \text{total angular momentum null;} \\ \text{magnetic moment } 2.5 \times 10^{-3}\mu_e; \\ \text{space and charge parity null;} \\ \text{primary decay modes } e^-\overline{\nu}_e \text{ and } e^+\nu_e(\sim 100\%); \end{cases} \tag{8.30}$$

where the data for the eleton ϵ^+ have been based on the assumption of a fully equivalent model for the antineutron $\overline{n} = (p^-, \epsilon^+)$, where p^- is the antiproton, and the masses of particles are equal to those of antiparticles.

9 Concluding Remarks

We hope that the analysis presented in this paper illustrates the view expressed at the initiation of this line of research [8,9] according to which *physics is a discipline that will never admit terminal theories.* No matter how impressive is the experimental verification supporting the exact validity of quantum mechanics for the electromagnetic interactions at large, the construction of a covering mechanics for more general physical conditions is only a matter of time.

In fact, quantum mechanics can only provide a point-like abstraction of particles, because of its intrinsic local-differential-Hamiltonian structure. As such, it is not expected to provide a sufficient representation of effects typically related to the extended character of particles and their deformations, such as the contact-nonlocal-nonhamiltonian interactions expected in the total penetration of the wavepackets of particles within the hadronic structure.

The scientifically meaningful issue is therefore *how* to represent quantitatively these broader physical conditions. But the *need* for a suitable generalization of quantum mechanics for the novel conditions considered is simply out of the question.

The suggestion advocated in this line of research is to generalized quantum mechanics while preserving the underlying abstract axioms and mathematical structures, via more general realizations, called isotopic [8], of: enveloping associative algebras, Lie algebras, Lie groups, Hilbert spaces, space-time symmetries, etc.

The reasons for this suggestion are due to the fact that the emerging covering mechanics, called hadronic mechanics [9], does indeed provide a quantitative representation of the actual shape of the particles, as well as of all their possible deformations, while admitting the ordinary quantum mechanics as a particular case, which, as such, was called the atomic mechanics.

The primary objective of the new mechanics, for which it was conceived, is to represent the constituents of hadrons as ordinary massive particles simply produced in the spontaneous decays. In fact, the original proposal [9] presented a structure model of the light mesons essentially constituted by (generalized) bound states of ordinary electrons, which appear to represent *all* the intrinsic characteristics of the hadrons considered, as well as the primary decays. It should be stressed here that the same possibilities are strictly precluded by the use of the simpler atomic mechanics, owing to a number of technical "inconsistencies" reviewed in the text. As a matter of fact, these "inconsistencies" were the primary reasons for recommending the construction of a covering mechanics.

A central test of physical consistency, which was left open in proposal [9], is the capability by the covering mechanics to achieve a consistent, quantitative representation of the neutron as a bound state of a proton and an electron (only), along Rutherford's historical hypothesis of 1920 [1].

In this paper we have shown that, thanks to a number of advances in the study of hadronic mechanics since the time of proposal [9], all the "inconsistencies" of the model originating from the use of atomic mechanics appear to be resolvable by the covering mechanics. In particular, all the intrinsic characteristics of the neutron appear to be due to certain dynamical effects caused precisely by the condition of total immersion of the wavepackets of the electron inside the hyperdense medium in the interior of the proton. Moreover, rather than being "inconsistent", the total spin $\frac{1}{2}$ in the hadronic bound state of two particles of spin $\frac{1}{2}$ emerges not only as being possible (because of predictable constraints in the orbital motion of the electron inside the proton structure), but also as being a conceivable mechanism for the creation of neutrinos in Nature.

These preliminary results appear to be sufficient to justify a systematic study of the problem of hadrons along the lines already established in the atomic and nuclear phenomenologies, such as:

1. identification of the hadronic constituents with physical, massive, particles produced in the spontaneous decays and/or tunnel effects;

2. increase of the number of constituents with mass; and

3. use of two complementary models, the established unitary models for the classification of hadrons into families [5], and different but compatible models of structure of each individual element of a given hadronic family.

The above "return ad originem" appears to be warranted by the fact that the hadronic structure model of the octect of light mesons submitted in ref. [9] is fully compatible, at all levels, with the unitary models of classification of the particles into a hadronic family. If the same compatibility is possible for the remaining hadrons, the possibilities of guidelines 1), 2), and 3) above become real.

This scientific scene is evidently deferred to the problem of the consistency or inconsistency of the hadronic structure models considered in this paper. In this regard, it is important to recall that the mathematical consistency of hadronic mechanics has been established [14–16] and, at any rate, it can be inferred from the preservation of the underlying abstract axioms of atomic mechanics. Once the very rigid boundaries of atomic mechanics have been enlarged by our mechanism of isotopy, the consistency of the hadronic structure models under consideration is expected to be only a question of technical details.

The issue whether or not guidelines 1), 2), and 3) can indeed be implemented in hadron physics is then deferred to the fundamental test advocated in this line of research, such as:

a) The test of the validity or invalidity of Einstein's special relativity in the interior of hadrons, via the measure of the behavior of the meanlife of unstable hadrons at different energies submitted in proposal [9] and elaborated in the preceding paper [23]; and

b) The test of the validity or invalidity of the conventional $SU(2)$-spin symmetry under external nuclear-strong interactions (or, equivalently, for one hadronic constituent when considering the rest as external), also advocated in proposal [9] and elaborated in the preceding paper [24]; and

c) The test of the remaining symmetries and principles, such as the T-reversal invariance, or Pauli's exclusion principle, again, under *external* nuclear-strong interactions, also advocated in proposal [9], and elaborated in subsequent paper [11].

We therefore recommend again these so fundamental and so overdue tests, this time specialized to the neutron, that is: measure the behavior of the meanlife of the neutron at a sufficient number of different energies (Test # 1); measure the apparent $SU(2)$-spin asymmetry for thermal neutron beams under external nuclear and electromagnetic fields via neutron interferometry (Test # 2); as well as any additional basic test.

The physical relevance of these tests is evident. Test # 1 would allow the experimental verification of the existence inside the neutron of the deformation of the Minkowski metric advocated by all available phenomenological papers [23]. This, in turn, would establish the mechanism of isotopy because the (positive-definite) isotopic element T in mutation (8.7a) is the inverse of the unit of hadronic mechanics. Basic Test # 2 could confirm the alteration of the magnetic moment and other characteristics of the neutron under sufficiently intense external nuclear fields, thus providing the necessary experimental backing to the central hypothesis of mutation of particles in hadronic mechanics [9], which is at the foundation of our treatment of Rutherford's model of the neutron. The remaining basic Test # 3 would then provide all the necessary complementary aspects.

The possible alternatives are quite simple. If one conceives hadrons as ideal spheres with points in them, conventional space-time symmetries, relativities and atomic laws follow. On the contrary, if hadrons are conceived as hyperdense media composed by the wavepackets of the constituents in condition of total mutual immersion, then a suitable generalization of conventional space-time symmetries, relativities and atomic laws become necessary, trivially, because these physical media are generally inhomogeneous and anisotropic.

In this line of research we have proved that the representation of the latter conditions via our isotopies allows the preservation of the abstract symmetries, although their realizations become highly nonlinear and nonlocal, thus resulting to be irreconcilable with conventional relativities and physical laws in favor of suitable generalizations.

In conclusion, *the problem whether or not the constituents of hadrons can be interpreted as physical, massive, particles produced in the spontaneous decays, is ultimately reducible to the problem whether Einstein's special relativity is exactly or only approximately valid for the interior dynamical problems.* In fact, if the relativity is exactly valid, abstract approaches of the quark-type are unavoidable for the hadronic constituents, to our best knowledge. On the contrary, if Einstein's special relativity is only approximately valid under an exact Lorentz-isotopic symmetry and consequential covering relativity, then the guidelines 1), 2), and 3) above already established for the atomic and nuclear phenomenologies, became indeed applicable also to the hadronic phenomenology. The ultimate reduction of the physical universe to very few stable particles conjectured in the closing words of memoir [9] (p. 882), then become within theoretical reach.

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