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Abstract

This paper is devoted to the origin and methodology of the several phenomenological predictions of deviations from Einstein's Special Relativity and related Lorentz symmetry in the behaviour of the lifetime of unstable hadrons at different speeds, that exist in the literature since the early '60's. After reviewing the background phenomenological literature, we outline the Lie-isotopic symmetry of the emerging deformations of the Minkowski metric introduced in a preceding paper, and extend the results to the construction of the full Poincaré-isotopic symmetry. The local isomorphism of the Poincaré-isotopic symmetry with the conventional symmetry is proved for all possible topology-preserving deformations of the Minkowski metric. In this way we establish that the phenomenological predictions of deviations recalled earlier must be specifically referred to Einstein's Special Relativity, but they cannot be referred to the Lorentz (or to the Poincaré) symmetry which remains exact. Particular attention is devoted to the proof of the compatibility of the exact validity of the Special Relativity for the center-of-mass trajectory of a hadron in a particle accelerator, with conceivable deviations from the same relativity in the interior structural problem. Stimulated by shear scientific curiosity, we identify the generalization of the basic laws of the Special Relativity that are characterized by the Lorentz-isotopic relativity, in order to achieve compatibility with available phenomenological information in the interior of hadrons. For completeness, the analysis is complemented with a few remarks on the gravitational profile. First, we review the pioneering Lie-isotopic generalization of Einstein's Gravitation worked out by Gasperini, which possesses precisely a locally Lorentz-isotopic (Poincaré-isotopic) structure. We then restrict this theory to the interior gravitational problem in order to achieve compatibility with the particle setting. In this way, we are in a position to show that Gasperini's Gravitation is capable of resolving at least some of the rather numerous problematic aspects of Einstein's Gravitation. The paper concludes with a review of the need to finally conduct direct experimental measures of the lifetime of unstable hadrons at different speeds (which continue to be ignored decade after decade), in order to finally resolve whether Einstein's Special and General Relativities are locally valid in the interior of hadrons, or structurally more general relativities must be worked out.

1. Introduction

One of the basic open legacies of the Founding Fathers of the theory of strong interactions is their ultimate *nonlocal* nature (see, e.g, refs. [1] and quoted papers).

The legacy is based on the well established experimental information that all known hadrons have a wavepacket (as well as a charge distribution) of the order of the range of the strong interactions themselves ($1\ F = 10^{-13}$cm). A necessary condition to activate the strong interactions is therefore that hadrons must enter into a state of mutual penetration and overlapping of their wavepackets (as well as of their charge distributions). This physical condition evidently results in nonlocal interactions, in the sense that they occur throughout the volume of overlapping, and cannot be reduced to a finite number of isolated points.

The theories on strong interactions currently receiving the majority of consensus, the quark theories and QCD (see, e.g., ref. [2]), have attempted to bypass the above historical legacy by assuming that the constituents of hadrons, the quarks, are point-like.

Rather than resolving the historical legacy, this has created an unreassuring condition of this sector of research, which is growing in time, as illustrated below. In fact, under a number of approximations (e.g., the assumption of truly elementary character), quarks can eventually have a point-like charge distribution, but "point-like wavepackets" simply do not exist in the physical reality, nor can be admitted under any meaningful approximation. Elementary quantum mechanical calculations then lead to the expectation that the size of the wavepacket of quarks is of the order of magnitude of the size of all other hadrons (~ 1 F). In order for a quark to be the true constituent of hadrons, its wavepacket must therefore be in a state of total immersion within the wavepackets of all other quarks. This evidently results again in the historical legacy, this time referred specifically to the ultimate nonlocality of the hadronic structure.

An apparent reason for the virtual complete silence on the historical legacy by the technical literature on quarks and QCD [2], is that, as well known, nonlocal interactions imply the invalidation of Einstein's Special Relativity in an irreconcilable way.

This occurrence was originally identified via the breakdown of the local commutation rules in field theory (see, e.g., [1]). Subsequently, various independent studies have established that nonlocal interactions imply the breakdown of the Special Relativity at *all* its levels, mathematical, physical, and epistemological. In fact, one has first the breakdown of the conventional topology in Minkowski space (e.g., the Zeeman's topology) owing to its strict local-differential character. The breakdown of the topology then implies the violation of the integrability conditions to reach the Lorentz group (e.g., Nelson's integrability conditions). In turn, this implies the breakdown of the physical laws, with consequential inapplicability of known epistemological arguments. Needless to say, the contribution from nonlocal terms is expected to be, in general, small as compared to those from conventional potential terms.

Stated in a nutshell, nonlocal interactions are known in classical mechanics to be of *contact* type, that is, resulting from the actual, physical, contact of an extended object within a physical medium, such as a satellite during re-entry in Earth's atmosphere. These interactions are known to be: a) of *nonlocal* type precisely in the sense indicated earlier b) of *zero range*, or *instantaneous* nature, and, last but not least, c) of *non-Hamiltonian* type, in the sense that, in general, not only they are of non-potential type, but they actually violate the integrability conditions for the existence of a Hamiltonian, the so-called conditions of variational selfadjointness (that is, they cannot be generally represented via the simplistic addition of a nonlocal potential to the kinetic energy). Interactions of this type are manifestly outside the representational capability of Einstein's Special Relativity without any hope for a serious reconciliation.

The scientific scene emerging from these studies is essentially the following:

A) *Einstein's Special Relativity is exactly valid for the arena of its original conception, i.e., point-like particles moving in vacuum (empty space) under action-at-a-distance interactions*;

B) *The historical legacy of the nonlocality of the strong interactions implies the lack of exact character of the Special Relativity for all strong interactions at large, and for the hadronic structure in particular*; and

C) *Even when violated, the Special Relativity is still expected to remain approximately valid, because of the expected small contribution of nonlocal terms as compared to conventional potential terms.*

A courageous effort was initiated in the 60's to achieve a phenomenological characterization of the above setting, via studies on the lifetime of unstable hadrons at different speeds. Consider, say, a kaon in a particle accelerator. The center-of-mass trajectory of the particle must strictly obey Einstein's Special Relativity because all conditions originally conceived by Einstein are verified. Conceivable departures from the Special Relativity can therefore occur only in the interior dynamics of the particle. In turn, the only known way according to which these expected internal deviations can manifest themselves in the outside is via deviations of the behavior of the lifetime with speed from the Einsteinian law

$$\tau = \tau_0\gamma, \qquad \gamma = (1-\beta^2)^{-1/2}, \quad \beta = v/c_0. \tag{1.1}$$

Intriguingly, *the totality of the available phenomenological studies on the behaviour of the lifetime of unstable hadrons with speed show clear deviations from Einstein's Special Relativity, without one single exception known to this author.*

We are referring to the research by Blokhintsev [3], Redei [4], Kim [5], Nielsen and Picek [6], Huerta-Quintanilla and Lucio [7], Aronson, Block, Cheng and Fishback [8] and others.

As an example, Nielsen and Picek [6] have studied the Higgs sector of the spontaneous symmetry breakdown for the (interior of) pions and kaons and

identified, via the use of available phenomenological information, the apparent generalization of the conventional Minkowski metric

$$\eta = (\eta_{\mu\nu}) = (1,1,1,-1), \qquad \mu,\nu = 1,2,3,4, \tag{1.2}$$

into the form

$$g = (g_{\mu\nu}) = \left((1-\tfrac{1}{3}\alpha),(1-\tfrac{1}{3}\alpha),(1-\tfrac{1}{3}\alpha),-(1+\alpha)\right), \tag{1.3}$$

where the α-parameter assumes the following value for pions

$$\alpha = (-3.79 \pm 1.37) \times 10^{-3}, \tag{1.4}$$

and the different one for kaons

$$\alpha = (0.61 \pm 0.17) \times 10^{-3}. \tag{1.5}$$

In particular, Einsteinian law (1.1) is modified by metric (1.3) into the form

$$\tau = \tau_0\gamma\left(1 + \frac{4\alpha\gamma^2}{3}\right). \tag{1.6}$$

The breakdown of the exact validity of Einstein's Special Relativity for generalized metric (1.3) is evident (Occurrence B above). Equally evident is the approximate validity of the relativity (Occurrence C above) because of the small value of the "Lorentz-asymmetry parameter" α.

Most importantly, generalized lifetime (1.6) can indeed be tested experimentally via direct measures of the lifetime of the particles at a sufficient number of different speeds (see the final remarks). Similar situations occur for all other investigations [3–8].

In a preceding paper [9] we have investigated generalized metrics of type (1.3) and shown, via the use of the Lie-isotopic generalization of Lie's theory [10, 11], that *the Lorentz symmetry remains exact for all phenomenological studies* [3–8] *when realized at the covering Lie-isotopic level. As a result, the deviations under consideration here must be specifically restricted to Einstein's Special Relativity and cannot be extended to the Lorentz symmetry.*

The proposal by this author to construct the Lie-isotopic generalization of conventional Lie's theories [10] was centered on the generalization of the unit of the theory, from the trivial form generally assumed in contemporary formulations, e.g., $I = \mathrm{diag}\,(1,1,\ldots,1)$, into the most general possible unit $\hat{I}$ verifying certain topological restrictions (e.g., invertibility and Hermiticity) but with an otherwise arbitrary dependence on the local variables such as coordinates x, velocities $\dot{x}$, density μ, temperature t, etc., $\hat{I} = \hat{I}(x, \dot{x}, \mu, t, \dots)$.

The underlying universal enveloping associative algebra is then generalized from the conventional form ξ with the trivial associative product AB,

$$\xi \colon AB, \qquad IA = AI = A, \qquad A, B \; \forall \xi, \tag{1.7}$$

into an isotopic form $\hat{\xi}$ with a product that is still associative (as a necessary condition for isotopy), but of the less trivial type

$$\begin{aligned} &\hat{\xi} \colon A * B \overset{\text{def.}}{=} ATB, \qquad T = \text{fixed and invertible}, \\ &\hat{I} * A = A * \hat{I} = A, \qquad A, B \; \forall \hat{\xi}. \qquad \hat{I} = T^{-1} \end{aligned} \tag{1.8}$$

The Lie algebra $\underline{\mathrm{G}} \approx \xi^-$ attached to ξ is consequently generalized from the familiar realization in term of the simplest conceivable product $AB - BA$, into the lesser trivial Lie-isotopic form

$$\hat{\underline{\mathrm{G}}} \colon [A, B]_{\hat{\xi}} = A * B - B * A = ATB - BTA. \tag{1.9}$$

Since all the basic theorems on enveloping algebras admit a consistent lifting under isotopy [10], the Lie-isotopic algebra $\hat{\underline{\mathrm{G}}}$ can be exponentiated into a Lie-isotopic group with power-series expansions evidently in $\hat{\xi}$

$$\hat{G}(\theta) \colon \hat{R}(\theta) = e^{i\theta A}\Big|_{\hat{\xi}} = e^{i\theta TA}\Big|_{\xi} \hat{I}, \tag{1.10}$$

which verify the isotopic group laws

$$\begin{aligned} &\hat{R}(0) = \hat{I}, \\ &\hat{R}(\theta_1) * \hat{R}(\theta_2) = \hat{R}(\theta_1 + \theta_2), \\ &\hat{R}(\theta) * \hat{R}(-\theta) = \hat{I}. \end{aligned} \tag{1.11}$$

The above theory was submitted by the author precisely to attempt a quantitative treatment of the historical open legacy on the nonlocal nature of the strong interactions. In fact, the Lie-isotopic theory implies the following generalization of Heisenberg's equation [12]

$$i\dot{A} = [A, H]_{\hat{\xi}} = ATH - HTA, \qquad \hbar = 1. \tag{1.12}$$

The use of conventional Hamiltonians H then allows the preservation of conventional local models of strong interactions, while the use of suitable integral forms of the isotopic element T allows the representation of the expected nonlocal component. The assumption $\hat{I} \approx I$ then allows the preservation of conventional relativities in first approximation.

For additional studies along these operator lines, we refer the interested reader to Myung and Santilli [13] and Mignani, Myung and Santilli [14].

The main results of paper [9] were the following. First, the isotope $\hat{O}(3.1)$ of the Lorentz group $O(3.1)$ was explicitly constructed (with illustrative examples), and tentatively called the *Lorentz-isotopic group*. Second, it was proved that the group $\hat{O}(3.1)$ is locally isomorphic to $O(3.1)$ for all modifications of the Minkowski metric of refs. [3–8], because they all preserve the topological structure of the conventional metric (1.2). Third, the generalization of the main laws of Einstein's Special Relativity implied by symmetry $\hat{O}(3.1)$ was initiated for the future achievement of a relativity that is exactly valid for generalized metrics of type (1.3). The emerging formulation was tentatively called the *Lorentz-isotopic relativity*.

As a consequence of these results, the statements of "Lorentz noninvariance" for generalized metrics of the topological type (1.3) are erroneous, unless specifically referred to the "simplest possible realization" of the Lorentz symmetry, because the symmetry remains exact when realized in lesser trivial forms.

Nevertheless, the Lorentz-isotopic symmetry characterizes irreconcilable deviations from Einsteinian laws, even though the latters are admitted as a particular case when the isotopic unit recovers the conventional unit. Also, the generalized relativity results to be "directly universal", in the sense of being able to represent all possible topology-preserving modifications of the Minkowski metric (universality) in the frame of the experimenter (direct universality).

The Galilean-isotopic counterpart had been previously worked out in monograph [11] with structurally similar lines, including the proof of the "direct universality" for the representation of all possible Newtonian systems verifying certain topological conditions, directly in the frame of the experimenter.

The operator formulation of the Poincaré-isotopic symmetry, based on the operator mechanics of papers [12, 13, 14], is under preparation [15].

The objectives of this paper are the following. First, for the reader's convenience, we shall review the Lorentz-isotopic symmetry. Then, we shall extend the results to the full Poincaré-isotopic symmetry. We shall then review and somewhat expand the central laws of the Lorentz-isotopic relativity. Particular attention will be devoted to the proof of compatibility of conventional relativity laws for the center-of-mass motion with generalized laws for the interior dynamics. This compatibility has been proved at the classical Galilei-isotopic level via the notion of closed non-Hamiltonian systems [11], and the consistency of the corresponding operator image has been studied in ref. [14]. The compatibility under consideration here will therefore be a mere relativistic extension of known results.

We shall then pass to a few remarks of gravitational nature. A possible invalidation of the Special Relativity for the interior problem of hadrons will evidently imply a corresponding invalidation of the General Relativity (also for the interior problem only), with the loss of the Riemannian geometry (evidently because of its strictly local character), as well as the loss of the locally Lorentz character of the theory.

In a series of pioneering papers, Gasperini [16–18] has worked out in considerable details the Lie-isotopic generalization of Einstein's Gravitation which is precisely locally Lorentz-isotopic in character. In this paper we shall reinspect these results to restrict the applicability of the gravitational isotopy to the interior problem only, so as to achieve compatibility with experimental data not only in gravitation but also in particle physics.

This paper ends with further calls for the conduction of the much overdue experiments on the measure of the lifetime of unstable hadrons at different speeds, which continue to be ignored decade after decade, despite the existence of authoritative predictions of violations [3–8]. The fundamental character of these experiments over others under consideration or conduction is evident. In fact, if phenomenological predictions of violations [3–8] can be experimentally confirmed in a direct way, they will evidently establish the need for suitable generalizations of both Einstein's Special *and* General Relativity for the interior problem. The understanding is that the lines studied in this paper should be considered as only initial and tentative steps towards the possible, future, construction of these generalizations via the usual scientific process of trial and error.

In the adjoining paper [19], we shall review additional experimental information favoring the Lie-isotopic space-time symmetries over the conventional ones for the interior problem. In particular, we shall consider certain experiments on the apparent deformation of the charge distributions of hadrons under sufficiently intense external fields, with consequential alteration of the intrinsic magnetic moment and other characteristics. Paper [19] then presents the notion of particle characterized by the Poincaré-isotopic symmetry which results to be an altered (mutated) form of conventional Poincaré particles. This was expected from proposal [12] to represent the transition from motion in vacuum (Einsteinian conditions) to motion under full immersion within an hyperdense hadronic medium (isotopic conditions).

In the final analysis, the mutation of the physical characteristics of particles is only one of the (rather intriguing) implications in the transition from conventional to isotopic spacetime symmetries.

As a final introductory comment, the reader should be aware that the Lie-isotopic theory, rather than disproving current quark theories and QCD, can provide genuine new possibilities for the resolution of at least some of their problematic aspects, such as: achievement of a true confinement of quarks (via the incoherence of the isotopic-interior and conventional-exterior Hilbert space): convergence of conventionally divergent QCD expansions (evidently possible in our formalism via suitable selections of the isotopic element T); identification of the quark constituents with massive, physical, already known particles (thanks to the mutation of their physical characteristics, for otherwise a new generation of hypothetical particles must be invented again), and other.

2. Reconstruction of the Exact Lorentz-Isotopic Symmetry when Conventionally Broken

A primary objective of ref. [9] was to show that the Lorentz symmetry is indeed exact for deformations of the Minkowski metric of type (1.3), provided that the symmetry is realized, not with the simplest conceivable Lie product $AB - BA$, but with the more general Lie-isotopic product.

Consider the following Minkowski-isotopic spaces (introduced in ref. [9])

$$\hat{M}(x, g, \hat{\mathbb{R}}): x^{\hat{2}} = x^{\mu} g_{\mu\nu} x^{\nu} \hat{I}, \tag{2.1a}$$

$$\hat{R}: \{\hat{N} | \hat{N} = N\hat{I},\ N \in \mathbb{R}\}, \tag{2.1b}$$

$$\hat{I} = T^{-1}, \qquad g = \mathrm{diag}(g_{11}, g_{22}, g_{33}, g_{44}),$$

$$= T\eta, \eta = \mathrm{diag}(1,1,1,-1), \tag{2.1c}$$

$$g_{\mu\mu} = g_{\mu\mu}(x, \dot{x}, \ddot{x}, \ldots) \neq 0, \qquad \mu, \nu = 1,2,3,4, \tag{2.1d}$$

(which include all available phenomenological investigations of refs. [3–8]). The related, intrinsically nonlinear, but formally isolinear transformation theory is characterized by the isotopic law

$$x' = A * x \stackrel{\text{def}}{=} ATx. \tag{2.2}$$

The *Lorentz-isotopic symmetry* introduced in ref. [9] is the Lie-isotopic transformation group

$$\begin{aligned} \hat{\Lambda}(u) * \hat{\Lambda}(-u) &= \hat{I}, \\ \hat{\Lambda}(u) * \hat{\Lambda}(u') &= \hat{\Lambda}(u + u'), \\ \hat{\Lambda}(0) &= \hat{I}, \end{aligned} \tag{2.3}$$

verifying the conditions

$$\hat{O}(3.1): \begin{array}{l} \hat{\Lambda}^t g \hat{\Lambda} = \hat{\Lambda} g \hat{\Lambda}^t = g^{-1}, \\ \det(\hat{\Lambda}) = \pm\det(\hat{I}). \end{array} \tag{2.4}$$

The explicit realization of $\hat{O}(3.1)$ is the following. Let $\xi(\mathbf{O}(3.1))$ be the conventional, enveloping, associative algebra of the Lie algebra $\mathbf{O}(3.1)$. Then the enveloping algebra of the isotope $\hat{\mathbf{O}}(3.1)$ is characterized by the basis

$$\hat{\xi}: \hat{I}, X_i, X_i * X_j, X_i * X_j * X_k, \ldots, \qquad i \leq j, \qquad i \leq j \leq k, \tag{2.5}$$

where the X's are any ordered set of the *conventional* six generators of $\mathbf{O}(3.1)$

$$\{X_k\} = \{J_1 = M_{23}, J_2 = M_{31}, J_3 = M_{12}, M_1 = M_{14}, M_2 = M_{24}, M_3 = M_{34}\},$$
$$= \{M_{\mu\nu}\}, \tag{2.6}$$

in their fundamental 4×4 representation (see, e.g., ref. [20], p. 40)

$$J_1 = M_{23} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad J_2 = M_{31} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$
$$J_3 = M_{12} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{2.7}$$

$$M_1 = M_{14} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \quad M_2 = M_{24} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix},$$

$$M_3 = M_{34} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}.$$

The Lie algebra attached to $\hat{\xi}$ is evidently of the broader Lie-isotopic type with basic product (1.9), i.e.,

$$[A \hat{;} B] \overset{\text{def}}{=} A * B - B * A = ATB - BTA. \tag{2.8}$$

It is then easy to verify the following isocommutation rules

$$\begin{aligned} [J_i \hat{;} J_j] &= -\varepsilon_{ijk} g_{kk} J_k, \\ [M_i \hat{;} M_j] &= -g_{44} \varepsilon_{ijk} J_k, \\ [J_i \hat{;} M_j] &= -g_{jj} \varepsilon_{ijk} M_k, \end{aligned} \tag{2.9}$$

which can be written in the unified notation

$$[M_{\alpha\beta} \hat{;} M_{\gamma\delta}] = -g_{\alpha\gamma} M_{\beta\delta} + g_{\alpha\delta} M_{\beta\gamma} + g_{\beta\gamma} M_{\alpha\delta} - g_{\beta\delta} M_{\alpha\gamma}, \tag{2.10}$$

to exhibit more clearly the isotopy with respect to the conventional commutation rules.

The explicit structure of the isotopic group was also worked out in ref. [9], thanks to the existence of a consistent isotopic lifting of the conventional Poincaré-Birkhoff-Witt theorem and related methodology [10, 11] which resulted in the forms

$$\begin{aligned} \hat{SO}^{\uparrow}_{+}(3.1): \hat{\Lambda}(\boldsymbol{\theta}, \mathbf{w}) &= \sum_{k=1}^{3}{}^{*} \exp(J_k \theta_k) \Big|_{\hat{\xi}}^{*} \sum_{k=1}^{3}{}^{*} \exp(M_k w_k) \Big|_{\hat{\xi}} \\ &= \left(\sum_{k=1}^{3} \exp(J_k T \theta_k) \Big|_{\hat{\xi}} \sum_{k=1}^{3} \exp(M_k T w_k) \Big|_{\hat{\xi}} \right) \hat{I}, \end{aligned} \tag{2.11}$$

where the θ's are the parameters of the conventional group of rotations and the w's are the parameters of the conventional Lorentz boosts.

The discrete components of $\hat{O}(3.1)$ resulted to be given by the *isoinversions* [9]

$$\begin{aligned} \hat{P} * x &= P(\mathbf{r}, t) = (-\mathbf{r}, t), \\ \hat{T} * x &= T(\mathbf{r}, t) = (\mathbf{r}, -t), \\ \widehat{PT} = \hat{P} * \hat{T} &= PT(\mathbf{r}, t) = (-\mathbf{r}, -t), \end{aligned} \tag{2.12}$$

where P and T are the conventional inversions.

In order to identify the *isocasimir invariants*, the following redefinition of the basis was introduced

$$\begin{aligned} \hat{J}_1 &= g_{22}^{-1/2} g_{33}^{-1/2} J_1, \qquad \hat{J}_2 = g_{11}^{-1/2} g_{33}^{-1/2} J_2, \\ \hat{J}_3 &= g_{11}^{-1/2} g_{22}^{-1/2} J_3 \qquad \hat{M}_k = g_{kk}^{-1/2} M_k, \end{aligned} \tag{2.13}$$

with redefinition of the isocommutators

$$\begin{aligned} [\hat{J}_i \hat{;} \hat{J}_j] &= -\varepsilon_{ijk} \hat{J}_k, \\ [\hat{M}_i \hat{;} \hat{M}_j] &= -g_{44} \varepsilon_{ijk} \hat{J}_k, \\ [\hat{J}_i \hat{;} \hat{M}_j] &= -g_{jj}^{-1/2} \varepsilon_{ijk} \hat{M}_k. \end{aligned} \tag{2.14}$$

The isocasimirs then resulted to be the isounit $\hat{I}$ and the two additional expressions [9]

$$\begin{aligned} \hat{C}_1 &= \hat{\mathbf{J}}^2 + \frac{1}{g_{44}}\hat{\mathbf{M}}^2 = \sum_{k=1}^{3}\left(\hat{J}_k T\hat{J}_k + \frac{1}{g_{44}}\hat{M}_k T\hat{M}_k\right) = -3\hat{I}, \\ \hat{C}_2 &= \hat{\mathbf{J}} * \hat{\mathbf{M}} = \sum_{k=1}^{3}\left(\hat{J}_k T\hat{M}_k\right) = 0. \end{aligned} \tag{2.15}$$

Note that *all isocasimirs are a numerical multiples of the isounit and not of the conventional unit*, as it must be for mathematical consistency.

The invariance of the isotopic separation (2.1a) under the continuous Lorentz-isotopic group followed from the properties identified in [21, 22]

$$e^{-\theta TJ}T\,e^{JT\theta} \equiv T, \tag{2.16}$$

with trivial corresponding properties for the isoinversions, as the reader is encouraged to verify.

The main result of ref. [9] can therefore be expressed via the following

THEOREM 2.1. The Lie-isotopic group $\hat{O}(3.1)$ with invariant separation $x^2 = x^\mu g_{\mu\nu} x^\nu$ in the Minkowski-isotopic space $\hat{M}(x, g, \hat{\mathbb{R}})$ is locally isomorphic to the conventional Lorentz group $O(3.1)$ whenever the generalized metric g possesses the same topological structure of the conventional Minkowski metric, e.g., for

$$\begin{aligned} g &= T\eta, \\ T &= \mathrm{diag}\left(b_1^2, b_2^2, b_3^2, b_4^2\right), \\ b_\mu &\neq 0, \end{aligned} \tag{2.17}$$

otherwise the Lorentz isotopic group $\hat{O}(3.1)$ is locally isomorphic to any other simple six-parameters group of Cartan's classification, such as $O(2.2)$ or $O(4)$.

The proof of the last statement is based on a simple application of the techniques of refs. [21, 22] to the case at hand.

As an illustration, the following explicit form of Lorentz-isotopic transformations was computed in ref. [9]

$$\begin{aligned} x^{1'} &= x^1, \\ x^{2'} &= x^2, \\ x^{3'} &= \hat{\gamma}(x^3 - \hat{\beta}x^4), \\ x^{4'} &= \hat{\gamma}(x^4 - \hat{\beta}\cdot x^3), \end{aligned} \tag{2.18}$$

$$\hat{\gamma} = \left(1 - \hat{\beta}^2\right)^{-1/2}, \hat{\beta}^2 = vb_3^2v/c_0b_4^2c_0.$$

In summary, the isotopic lifting of the Lorentz group preserves, by construction, the generators and parameters of the original group, and generalizes instead the structure of the Lie product itself. This is due to the fact that generators and parameters generally have a direct physical meaning which is independent from the class of interactions considered and, as such, must be preserved unchanged. The generalization of the structure of the Lie product represents a class of new interactions which, by conception, are of non-Hamiltonian type. They are the contact, zero range, instantaneous interactions experienced by extended bodies when moving within a material medium. Such generalized physical conditions are represented via deviations (2.1) from the Minkowski metric much along the lines of paper [6]. The emerging Lorentz-isotopic group (2.11) then results to be automatically a symmetry of the generalized separation.

Equivalently, the entire content of paper [9] can be expressed via the following single concept: generalizations $g = T\eta$ of the Minkowski metric η representing motions within a physical medium (or other non-Einsteinian conditions), when represented via a generalization $\hat{I} = T^{-1}$ of the unit of Lie's theory, produce automatically the symmetries of g, trivially, because Lie's theory leaves invariant its unit.

An important difference between the realization of the $\hat{O}$ (3.1) symmetry of this section and that of ref. [9] should be pointed out. In ref. [9] the full metric g was used for isotopic element with isounit $\hat{I} = g^{-1}$, while in this section we have used the decomposition $g = T\eta$ of Eqs. (2.1) and the isotopic element T with isounit $\hat{I} = T^{-1}$. The difference is due to the fact that the generators used in ref. [9] are those of the *compact* $O(4)$ (see the generators used in ref. [28] quoted in page 550), while in this section we have used the generators of the *noncompact* $O(3.1)$.

This comment also serves to illustrate the fact that the $O(3.1)$ symmetry can be obtained via an isotopic lifting of $O(4)$ with isounit $\hat{I} = \eta^{-1}$, as pointed out in refs. [9, 22]. $\hat{O}(3.1)$ is then generated by a second lifting of $O(4)$ via isounit $\hat{I} = T^{-1}$, $g = T\eta$ (as done in this section), or, directly, via a single isotopic lifting of $O(4)$ with isounit $\hat{I} = g^{-1}$ (as done in ref. [9]).

3. Construction of the Poincaré-Isotopic Symmetry

We now pass to the extension of the results of ref. [9] to the *inhomogeneous Lorentz-isotopic group (Poincaré-isotopic group)* $\hat{P}(3.1)$. For this purpose, we introduce the conventional generators P_μ of translations in space-time, and restrict hereon all generalized metrics g to be independent from local coordinates (but preserve their dependence on velocities as well as any additional physical characteristics of the medium considered, such as density, temperature, etc.).

We shall now define as the Poincaré-isotopic group the set of isotransformations

$$\hat{P}(3.1): x' = \{\hat{\Lambda}, \hat{T}\} * x = \hat{\Lambda} * x + a, \qquad \hat{\Lambda} \in \hat{O}(3.1), \qquad a = \text{const.}, \tag{3.1}$$

which leave invariant the separation in Minkowski-isotopic space

$$(x' - y')^{\mu} g_{\mu\nu} (x' - y')^{\nu} \equiv (x - y)^{\mu} g_{\mu\nu} (x - y)^{\nu}, \tag{3.2}$$

with evident isocomposition rule

$$\{\hat{\Lambda}_1, \hat{T}_1\} * \{\hat{\Lambda}_2, \hat{T}_2\} = \{\hat{\Lambda}_1 * \hat{\Lambda}_2, \hat{T}_1 + \hat{\Lambda}_2 * T_2\}. \tag{3.3}$$

The explicit construction of $\hat{P}(3.1)$ is then straightforward from the results reviewed in the preceding section. In fact, it is easy to prove that the enveloping algebra $\hat{\xi}(\mathbf{P}(3.1))$ of $\hat{P}(3.1)$ is structure (2.5) where the X's now refer to an (ordered) basis

$$\{X_k\} = \{M_{\alpha\beta}, P_{\gamma}\}. \tag{3.4}$$

By recalling that the generators P_{μ} are diagonal, it is easy to prove the following Lie-isotopic algebra $\hat{\mathbf{P}}(3.1)$

$$\begin{aligned} [M_{\alpha\beta} \hat{,} M_{\gamma\delta}] &= -g_{\alpha\gamma} M_{\beta\delta} + g_{\alpha\delta} M_{\beta\gamma} + g_{\beta\gamma} M_{\alpha\delta} - g_{\beta\delta} M_{\alpha\gamma}, \\ [M_{\alpha\beta} \hat{,} P_{\gamma}] &= g_{\beta\gamma} P_{\alpha} - g_{\alpha\gamma} P_{\beta}, \\ [P_{\alpha} \hat{,} P_{\beta}] &= 0. \end{aligned} \tag{3.5}$$

The structure of the Lie-isotopic group $\hat{P}(3.1)$ is then given by exponentials (2.11) multiplied by the *isotranslation group*

$$\hat{T}(3.1): \hat{T}(a) = \exp(P^{\mu} \eta_{\mu\nu} a^{\nu})|_{\hat{\xi}} = |[\exp(P^{\mu} g_{\mu\nu} a^{\nu})|_{\hat{\xi}}^{\hat{I}}, \tag{3.6}$$

plus the inversions (2.12), resulting in the generalized structure

$$\hat{P}(3.1) = \hat{O}(3.1) \otimes \hat{T}(3.1). \tag{3.7}$$

The reader should be aware that, despite the similarities with the conventional commutation rules, algebras $\hat{\mathbf{P}}(3.1)$ and $\mathbf{P}(3.1)$ are *not*, in general, isomorphic. Nevertheless, they are isomorphic when the generalized metric g possesses the same topological structure of the conventional Minkowski metric, as in Theorem 2.1.

In order to identify the isocasimir invariants of $\hat{\mathbf{P}}(3.1)$, we now introduce the contravariant metric tensor via conventional geometric (e.g. affine) approaches

$$g^{\mu\nu} = (g_{\mu\nu})^{-1} = (g_{\mu\nu}^{-1}), \qquad g^{\mu\alpha} g_{\alpha\nu} = \delta^{\mu}_{\nu}. \tag{3.8}$$

The contraction from contravariant to covariant forms and vice versa is then given by expressions of the type

$$\begin{aligned} x_{\mu} &= g_{\mu\nu} x^{\nu}, \qquad x^{\mu} = g^{\mu\nu} x_{\nu}, \\ P_{\mu} &= g_{\mu\nu} P^{\nu}, \qquad P^{\mu} = g^{\mu\nu} P_{\nu}, \end{aligned} \tag{3.9}$$

under which the basic invariant can be written in any of the following equivalent forms

$$x^{\mu} g_{\mu\nu} x^{\nu} = x_{\mu} g^{\mu\nu} x_{\nu} = x^{\mu} x_{\mu} = x_{\mu} x^{\mu}. \tag{3.10}$$

We now introduce the iso-Pauli-Lubanski four vector

$$W_{\mu} = \tfrac{1}{2} \varepsilon_{\mu\alpha\beta\gamma} M^{\alpha\beta} * P^{\gamma}, \tag{3.11}$$

with properties

$$[M_{\alpha\beta} \hat{,} W_{\gamma}] = g_{\beta\gamma} W_{\alpha} - g_{\alpha\gamma} W_{\beta}, \qquad [P_{\alpha} \hat{,} W_{\beta}] = 0, \tag{3.12}$$

where use has been made of the property

$$[A * B \hat{,} C] = A * [B \hat{,} C] + [A \hat{,} C] * B. \tag{3.13}$$

The *isocamimir invariants* of $\underline{\hat{\mathbf{P}}}(3.1)$ are then given by the isounit $\hat{I}$ and the two additional forms

$$P^{\hat{2}} = (P^{\mu} g_{\mu\nu} P^{\nu}) \hat{I}, \qquad W^{\hat{2}} = (W^{\mu} g_{\mu\nu} W^{\nu}) \hat{I}. \tag{3.14}$$

Despite the similarities with the conventional setting, the lifting of the conventional Poincaré group into isostructure (3.7) implies an alteration of the physical characteristics of particles exactly along the notion of "mutation" proposed by this author back in 1978 [12]. We are here referring to the hypothesis that conventional elementary (or composite) particles experience a mutation of (at least some of) their physical characteristics (rest energy, charge, spin, magnetic and electric moments, etc.) in the transition from motion in vacuum (Einstenian conditions) to motion inside dense hadronic matter, with consequential full immersion of their wavepackets within the generally inhomogeneous and anisotropic medium composed by the wavepackets of the hadronic constituents. This hypothesis was introduced following the construction of the Galilei-isotopic symmetry of ref. [10]. The transition to the Poincaré setting is a mere relativistic extension of the above Newtonian setting.

A study of the hypothesis of mutations of elementary particles is provided in the adjoining paper [19] on the *isofield equations*, that is, field equations that are invariant under the Poincaré-isotopic group.

4. A Generalization of Einstein's Special Relativity?

A second objective of paper [9] was to show that the reconstruction of the exact Lorentz symmetry for deformation of the Minkowski metric of the Nielsen-Picek type (1.3) implies an irreconcilable departure from Einstein's Special Relativity. This result is expected from a number of independent aspects, all converging toward the same conclusion, and all centered on the profound differences between the physical conditions conceived by Einstein and those under consideration here. In particular, the generally inhomogeneous and anisotropic nature of the physical media in which the Lorentz-isotopic symmetry holds is sufficient, per se, to require a suitable generalization of the Special Relativity.

In view and consideration of the above, and out of shear curiosity, the rudiments of a conceivable generalization of the Special Relativity were studied in ref. [9], as a necessary prerequisite to assess the possible physical relevance of the isotopic space-time symmetries. In the following, we shall review and somewhat expand the generalization of Special Relativity that is required by motion of extended-deformable particles within physical media.

In this section we shall restrict our attention to the Minkowski-isotopic spaces $\hat{M}(x, g, \hat{\mathbb{R}})$ characterized by

$$\begin{aligned} &g = T\eta, \\ &\eta = \operatorname{diag}(1, 1, 1, -1), \\ &T = \operatorname{diag}(b_1^2, b_2^2, b_3^2, b_4^2), \\ &b_\mu > 0, \quad c = c_0 b_4, \quad b_1 = b_2 = b_3 = b, \end{aligned} \tag{4.1}$$

for which the Lorentz-isotopic group is locally isomorphic to the conventional group. The generalization of the Special Relativity that apparently follows was tentatively called in ref. [9] the *Lorentz-isotopic relativity* and the same terminology will be kept in this work.

The first, most visible, departure from the Special Relativity is provided by the following

POSTULATE 1. The maximal possible speed of massive, physical, particles (or causal signals) predicted by the Lorentz-isotopic relativity can be smaller, equal or higher than the speed of light in vacuum c_0

$$V_{\text{Max}} = \left|\frac{d\mathbf{r}}{dt}\right|_{\text{Max}} \stackrel{\text{def}}{=} C \gtreqless c_0, \tag{4.2}$$

depending on the local characteristics of the medium in which motion occurs.

In fact, separation (4.1) for null values can be written

$$\begin{aligned} &\sum_{1^k}^{3} dr^k\, b^2\, dr^k - dt\, c^2\, dt = 0, \\ &\left(\frac{d\mathbf{r}}{dt}\right) = \frac{c}{b} \gtrless c_0, \end{aligned} \tag{4.3}$$

yielding precisely maximal value (4.2).

A direct illustration of Postulate 1 is provided by Nielsen-Picek metric (1.3) [6]. In fact, for pions we have

$$\begin{aligned} &V_{\text{Max}} = c_0 \frac{1+\alpha}{1-\frac{1}{3}\alpha} < c_0, \\ &\alpha = (-3.79 \pm 1.37) \times 10^{-3}, \end{aligned} \tag{4.4}$$

while for kaons we have

$$\begin{aligned} &V_{\text{Max}} = c_0 \frac{1+\alpha}{1-\frac{1}{3}\alpha} > c_0, \\ &\alpha = (0.61 \pm 0.17) \times 10^{-3}, \end{aligned} \tag{4.5}$$

thus indicating the possibility of an increase of the maximal causal speed with the density beyond c_0.

Following the proposal of Postulate 1, de Sabbata and Gasperini [23] conducted explicit calculations via the use of gauge theories and concluded with the conceivable value for causal signals in the interior of (heavy) hadrons

$$V_{\text{Max}} \approx 75 c_0. \tag{4.6}$$

In general, we can say that *any modification of the Minkowski metric implies an alteration of the maximal causal speed, which can evidently result to be higher, lower or equal to c_0 depending on the specific condition at hand.*

The reader should be aware that, while c_0 is a universal constant for Einstein's Special Relativity, the value C of Postulate 1 is a local invariant, in the sense that it should be conceived as being applicable in the neighborhood of a point (e.g., in the vicinity of the center of a pion or a kaon).

Also tachyons should not be confused with the physical particles (with ordinary, positive, rest energy) of Postulate 1 when traveling at speeds higher than c_0.

The emergence of Postulate 1 becames rather natural when the type of the forces considered is duly taken into account. Within the context of the conventional Special Relativity, the only applicable forces are those of potential type. Under these conditions, it takes infinite energy to accelerate an ordinary massive particle to c_0, as well known.

The physical arena under consideration is different, inasmuch as, besides potential forces, the particle experiences *contact*, *zero range*, *instantaneous*, and *non-hamiltonian forces* (as it is the case, say, for a proton in the core of a collapsing star). These latter forces are substantially outside the technical capabilities of the Special Relativity. As such, they characterize a different value of the maximal speed, e.g., because of the lack of potential character for which the energy considerations of the Special Relativity are evidently inapplicable.

Finally, the quantity $c^2 = -g_{44}$ has, in general, a purely geometrical meaning and does not necessarily represent a physical speed. This can be illustrated, again, with Nielsen-Picek metric (1.3) which implies the value $c^2 = c_0^2(1 + \alpha) > c_0^2$ for the interior of the particle. The point is that such a "speed" c does not represent the speed of any causal signal. Also, the value $c > c_0$ is not sufficient, per se, to warrant the existence of actual causal speeds higher than c_0, which can be established only via the calculation of the maximal causal speed admitted by the underlying relativity, i.e., Eq. (4.5).

As an incidental note, we *do not* believe that certain reported speeds of giant quasars exceeding the speed of light in vacuum, as predicted from the Einstenian redshift, are plausible, and, therefore, we *do not* recommend them as an illustration of Postulate 1. In fact, the motion of the quasars in our Universe is Einstenian, being essentially in vacuum, in which case Einstein's Special Relativity must apply for their center-of-mass trajectories, including c_0 as the maximal possible causal speed.

On the contrary, certain reported expulsions of jets of matter from the core of certain quasars at speeds exceeding that of light in vacuum *do* belong to Postulate 1. In fact, these jets are primarily subjected to *non-Hamiltonian* forces, the origin of the ejection being internal explosions and the corresponding *contact* effects on the jet's matter. As a result, and unlike the preceding case, these latter physical conditions are beyond the representational capabilities of Einstein's Special Relativity, and do indeed belong to the physical arena of Postulate 1.

The above two astrophysical cases provide a rather clear illustration of the physical conditions in which Einstein's Special Relativity must be applied, and the different physical conditions in which the covering relativity proposed in this paper may have physical relevance.

The regions of space characterized by the deformed (mutated) light cone under Postulate 1 shall be called

$$\begin{aligned} &\textit{Isotimelike}, && \text{when} \quad x^{\hat{2}} < 0, \\ &\textit{Isonul}, && \text{when} \quad x^{\hat{2}} = 0, \\ &\textit{Isospacelike}, && \text{when} \quad x^{\hat{2}} > 0. \end{aligned} \tag{4.7}$$

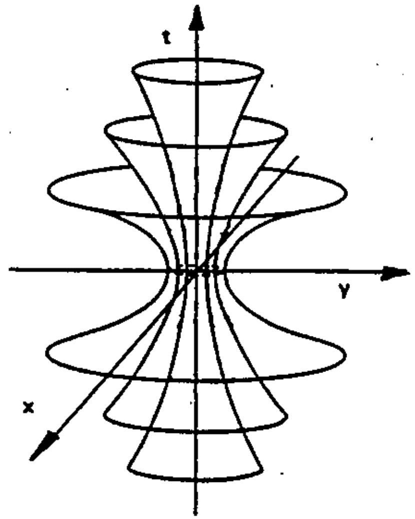

FIG. 1. The deformation (mutation) of the light cone caused by contact, zero range, instantaneous, and nonhamiltonian interactions experienced by an extended particle while moving within an inhomogeneous and anisotropic material medium, as identified in ref. [9]. The figure above essentially represents the conditions of Postulate 1 of Section 4. The deformed cones characterize the maximal speed for massive physical particles or causal signals, one for maximal speeds smaller than c_0 (e.g., as per Eq. (4.4)), and one for maximal speeds higher than c_0 (e.g., as per Eqs. (4.5)). The following four possible cases were identified [9].

1) $V_{Max} = c_0$, $c < c_0$. This is typically the case of the Cherenkov light in water for which the speed of light is $c = c_0/n < c_0$ (where n is the index of refraction) and $V_{Max} = c_0$ (e.g. the speed of electrons). This first case can be represented via the scalar isotopy

$$x^{\hat{2}} = x^\mu g_{\mu\nu} x^\nu = \frac{1}{n^2}\mathbf{r}\cdot\mathbf{r} - \frac{1}{n^2}tc_0^2t = \frac{1}{n^2}x^2, \qquad V_{Max} = \frac{c}{b} = c_0.$$

2) $V_{Max} < c_0$, $v < c_0$. This case is conjectured to represent motion within physical media in which causal signals (whether light or massive particles) can propagate at speeds smaller than c_0 (recall that this is a first classical approximation and, as such, neutrinos are excluded). The isotopy is no longer trivial as in Case 1, but of the type

$$x^{\hat{2}} = x^\mu g_{\mu\nu} x^\nu = \frac{1}{n_1^2}\mathbf{r}\cdot\mathbf{r} - \frac{1}{n_2^2}tc_0^2t,$$

$$V_{Max} = \frac{c}{b} = \frac{n_1}{n_2}c_0 < c_0, \qquad 1 < n_1 < n_2.$$

A conceivable physical arena for this case was identified as the dynamics of electrons in metals. Note also that the dynamics in the interior of pions according to Nielsel-Picek case (4.4) is precisely a realization of Case 2.

3) $V_{Max} < c_0$, $c > c_0$. This case is represented by the isotopy

$$x^{\hat{2}} = x^\mu g_{\mu\nu} x^\nu = \frac{r}{n_1^2}\mathbf{r}\cdot\mathbf{r} - \frac{1}{n_2^2} t c_0^2 t,$$

$$V_{Max} = \frac{c}{b} = \frac{n_1}{n_2} c_0 < c_0, \qquad n_1 < n_2 < 1,$$

and it has been conjectured in ref. [9] as representing the dynamical conditions in the nuclear structure, in which the quantity $c > c_0$ assumes a purely geometrical meaning (as in the Nielsen-Picek metric for the pions), while V_{max} is inferred as being smaller than c_0 from the excellent approximation providing by nonrelativistic treatments. The classical nature of the study must be stressed here again, to prevent the inclusion of effects, such as photon exchanges, that are quantum field theoretical.

4) $V_{Max} > c_0$, $c > c_0$. This is the case conjectured in ref. [9] for the interior of hadrons (beginning with kaons) up to the limiting conditions in the core of stars undergoing gravitational collapse. The dynamical treatment by Nielsen-Picek in the interior of kaons according to Eq. (4.5) is an illustration of the case under consideration. Different but higher values of V_{Max} are expected for the remaining hadrons owing to the increase of the density with mass (remember that all known hadrons have approximately the same size). This case is represented by the isotopy

$$x^{\hat{2}} = x^\mu g_{\mu\nu} x^\nu = \frac{1}{n_1^2}\mathbf{r}\cdot\mathbf{r} - \frac{1}{n_2^2} t^2, \qquad V_{Max} = \frac{n_1}{n_2} c_0 > 0, \qquad 1 > n_2 > n_1.$$

Note that in all cases considered here, the three-dimensional space has been assumed as being homogeneous and isotropic for simplicity (but without loss of generality). Note also that, for extreme limiting conditions conceivable under gravitational collapse, V_{Max} can indeed be infinite.

POSTULATE 2. The invariant speed of the Lorentz-isotopic relativity is not, in general, the speed of light, but the maximal speed of propagation of massive particles (or of causal signals) $V_{Max} \gtrless c_0$.

This case can be readily seen by noting that if one puts $v_1 = v_2 = c$ in the composition of speeds, the total speed is given by

$$V_{Tot} = \frac{2c}{1+b^2} \neq c. \tag{4.8}$$

On the contrary, if $v_1 = v_2 = V_{Max}$, we have

$$V_{Tot} = \frac{2c/b}{2} = \frac{c}{b} = V_{Max}. \tag{4.9}$$

Einstein's Special Relativity is a trivial particular case of Postulate 2, owing to the fact that, in this case, $V_{Max} = c = c_0$. Nevertheless it is remarkable to note that the invariant quantity is V_{Max} rather than c_0.

Postulate 2 is quite plausible. Consider in fact the case of the Cherenkov light in which $c = c_0/n$. Since $c < c_0$ (and also c changes from transparent medium to transparent medium), the speed of light c cannot be an invariant of any relativity, whether the Special or any of its conceivable generalizations. Instead, for physical consistency, it is essential that the maximal causal speed is the invariant of the theory, as it is the case for the Lorentz-isotopic relativity.

POSTULATE 3. The dependence of time intervals with speed in the Lorentz-isotopic relativity follows the law of isotopic time dilation

$$\Delta t = \hat{\gamma}\,\Delta t_0, \qquad \hat{\gamma} = (1-\hat{\beta}^2)^{-1/2}, \qquad \hat{\beta}^2 = \frac{v b^2 v}{c^2}, \tag{4.10}$$

while space intervals follow the law of isotopic space contraction

$$\Delta l = \hat{\gamma}^{-1}\Delta l_0 = \Delta l_0 (1+\hat{\beta}^2)^{1/2}. \tag{4.11}$$

Postulate 3 may have a crucial role for conceivable experimental tests aimed at the resolution whether the isotopic or conventional space-time symmetries hold for the interior of hadrons. In fact, law (4.10) is that expected for the mean life of unstable hadrons at different speed

$$\tau = \frac{\tau_0}{\sqrt{1 - \dfrac{v b^2 v}{c^2}}}. \tag{4.12}$$

It is an instructive exercise for the interested reader to prove that Nielsen-Picek lifetime (1.3) is a particular case of law (4.12). This situation is expected for all available studies on the behaviour of the mean life (and other parameters) with speed, owing to the arbitrariness of the underlying metric g.

Consider now an *iso-plane-wave* from Eq. (3.6)

$$\hat{\psi}(x) = \hat{A} e^{ik^\mu g_{\mu\nu} x_\nu}, \qquad \hat{A} = A\hat{I}, \tag{4.13}$$

where k is an isonull vector

$$k^\mu g_{\mu\nu} k^\nu = \mathbf{k} b^2 \mathbf{k} - w^2 = 0,$$

$$k = \left(\mathbf{k}, \frac{w}{c}\right), \qquad \frac{w}{c} = \frac{2\pi}{\lambda}. \tag{4.14}$$

Suppose that such a wave (again propagating within a physical medium such as a dense atmosphere) is detected by two observers S and S', one at rest with respect to the source of the iso-wave, and the other in motion with respect to it at a relative speed v along the x^3-axis. Let α be the angle between k and x^3. Let ω', k', α' be the corresponding quantities in S'. For the form-invariance of the iso-wave under the Lorentz-isotopic group $\hat{O}(3.1)$, it is then easy to see that

$$k^{\mu\prime} g_{\mu\nu} k^{\nu\prime} = k^{\mu} g_{\mu\nu} k^{\nu},$$

$$\begin{aligned} k'^1 &= k^1, \qquad k'^2 = k^2, \\ k'^3 &= \hat{\gamma}(k^3 - \hat{\beta} k^4) = |k'| \cos \alpha', \\ k'^4 &= \hat{\gamma}(k^4 - \hat{\beta} k^3) = \frac{w'}{c}. \end{aligned} \tag{4.15}$$

This leads to the following

POSTULATE 4. The Doppler frequency shift for electromagnetic waves propagating within a physical medium (iso-plane waves) follow the isotopic law

$$\begin{aligned} \omega' &= \omega \hat{\gamma}(1 - \hat{\beta} \cos \alpha), \\ \hat{\gamma} &= (1 - \hat{\beta}^2)^{-1/2}, \qquad \hat{\beta}^2 = \frac{v b^2 v}{c^2}, \end{aligned} \tag{4.16}$$

with isotopic aberration rule

$$\cos \alpha' = (\cos \alpha - \hat{\beta})/(1 - \hat{\beta} \cos \alpha). \tag{4.17}$$

Postulate 4 has potential experimental relevance for astrophysics. In fact, space can be considered empty only in a first approximation, being in actuality filled up with radiation, dark matter and other entities (e.g., elementary particles). In this sense, space is Einstenian only locally, say, in the neighborhood of our Solar systems (in which the presence of light, dark matter and particles can be ignored), but it becames an isotopic space at Galactic distances or whenever a more exact treatment is desired.

A number of astrophysical studies can be conceived to ascertain whether current data can select the Einstenian prediction of the Doppler effect over those of Postulate 4.

Note that the Doppler's shift in water follows Einsteinian laws, as well known. To have a significant departure from Einstenian predictions, one must have a transparent physical medium characterized by an isotopy other than the scaler one (Case 1 of Figure 1).

Particularly intriguing is the application of Postulate 4 to the vexing problem of astrophysical redshifts, e.g., those of giant quasars. We have indicated earlier that we consider unplausible the current explanation because, while on one side it is fundamentally dependent on the use of Einstein's Special Relativity for the computation of the Doppler's frequency shift, it leads to the evident contradiction that certain quasars move in our Universe with speeds much higher than that of light in vacuum, the contradiction lying on the fact that the quasar motion is fully Einstenian and without any conceivable non-Hamiltonian effect, in which case c_0 must be the maximal possible causal speed.

The covering of Einstein's Special Relativity submitted in this paper appears to offer genuine hopes for resolving such an unplausible and manifestly inconsistent scientific case. In fact, light is emitted by the quasars from their interior. As a consequence, before reaching us, light must first pass through the hyperdense transparent media in the structure of the quasars themselves, thus leading to a first sizable use of Eq. (4.16) without any need of reaching unplausible speeds for such large objects. A second, quantitatively much smaller use of generalized Doppler's shift (4.16) is due to the fact that, after leaving the hyperdense media of the quasars, light does not propagate in vacuum because intergalactic space is not empty, as noted before, but filled up with energy, radiations, dust and elementary particles. The study of this intriguing case of the generalized relativity under study is here recommended to interested researchers.

We now introduce the rudiments of *isotopic kinematics*, i.e. the generalization of the relativistic kinematics required by motion in isotopic space $\hat{M}(x, g, \hat{\mathbb{R}})$. From the basic invariant

$$\begin{aligned} ds^2 &= -dx^{\mu}\, g_{\mu\nu}\, dx^{\nu} \\ &= dx^4\, c^2\, dx^4 - dx^k\, b_k^2\, dx^k, \end{aligned} \tag{4.18}$$

we can write

$$\frac{dx^{\mu}}{ds} g_{\mu\nu} \frac{dx^{\nu}}{ds} = -1. \tag{4.19}$$

We now define as *iso-four-velocity* in $\hat{M}$ the expression:

$$u^{\mu} = \frac{dx^{\mu}}{ds}. \tag{4.20}$$

But

$$\left(\frac{dx^4}{ds}\right)^2 \left(c^2 - \frac{dx^k}{dx^4} b_k^2 \frac{dx^k}{dx^4}\right) = 1. \tag{4.21}$$

Thus, the components of u are given by

$$u^4 = \frac{dx^4}{ds} = \frac{dt}{ds} = \hat{\gamma} c,$$
$$u^k = \frac{dx^k}{ds} = \frac{dx^4}{ds}\frac{dx^k}{dx^4} = \hat{\gamma} c v^k. \tag{4.22}$$

We now define as *iso-four-momentum* in $\hat{M}$ the quantity

$$p^\mu = mu^\mu, \qquad p = (m_0 \hat{\gamma} c \mathbf{v}, m_0 \hat{\gamma} c). \tag{4.23}$$

From Eqs. (3.14) we therefore reach the *fundamental iso-invariant* of the Lorentz-isotopic relativity

$$\begin{aligned} p^{\hat{2}} &= p^\mu g_{\mu\nu} p^\nu = p^k b_k^2 p^k - p^4 c^2 p^4 \\ &= m_0^2 \hat{\gamma}^2 c^2 v^k b_k v^k - m_0^2 \hat{\gamma}^2 c^4 \\ &= -m_0^2 c^4 \hat{\gamma}^2 \left(1 - \frac{v^k b^2 v^k}{c^2}\right) \\ &= -m_0^2 c^4, \end{aligned} \tag{4.24}$$

which, via the familiar expression $p^4 = \mathrm{E}/c$ where E is the physical energy of the particle, can be written

$$\begin{aligned} \hat{p}^2 &= p^k b_k^2 p^k - p^4 c^2 p^4 \\ &= p^k b_k^2 p^k - \mathrm{E}^2 = -m_0^2 c^4. \end{aligned} \tag{4.25}$$

POSTULATE 5. The mass m_0 of a particle moving within a physical medium characterized by the Minkowski-isotopic space $\hat{M}(x, g, \hat{\mathbb{R}})$ varies with speed according to the isotopic law

$$m = m_0 \hat{\gamma}, \qquad \hat{\gamma} = (1 - \hat{\beta}^2)^{-1/2}, \qquad \hat{\beta}^2 = \frac{v^k b_k^2 v^k}{c^2}, \tag{4.26}$$

and the equivalent value of the energy for at rest conditions is given by

$$\mathrm{E} = m_0 c^2 = m_0 c_0^2 b_4^2, \tag{4.27}$$

where c is a geometrical quantity characterized by the medium itself.

Note that, again, for the case of water, $\hat{\beta} = \beta$, $\hat{\gamma} = \gamma$ and the isotopic law (4.26) coincides with the conventional law. Also, in this case, $p^{\hat{2}} = (1/n^2)p^2$ and the conventional expression $\mathrm{E} = m_0 c_0^2$ is recovered. Again in order to have a departure from Einstenian predictions we must have an isotopy other than the scalar one.

For the case of a particle moving inside a kaon described via Nielsen-Picek metric (1.3), we have

$$m = m_0 \left(1 - \frac{v^2(1 - \frac{1}{3}\alpha)}{c_0^2(1 + \alpha)}\right)^{-1/2},$$
$$\mathrm{E} = m_0 c_0^2 (1 + \alpha), \qquad \alpha > 0, \qquad \alpha \approx 10^{-3} \tag{4.28}$$

that is, *the mass of the particle is not infinite at the speed of light in vacuum c_0, and the energy equivalent is higher than the Einstenian one.*

Conceivably, the above deviations from the predictions of the Special Relativity should be suitable for future experimental resolutions. It should be stressed here that predictions (4.26) and (4.27) refer, by conception, to one particle *inside* hadronic matter, e.g., for one constituent of a kaon when considering the rest as external. As a result, the experimental resolutions of isotopic laws will require a new generation of experiments, those capable of achieving measures under actual, external, *strong* interactions (the only experimental capability currently available is that of measures under external electromagnetic interactions).

Note that the fundamental isoinvariant (4.24) is the starting point of the isofield theory of the adjoining paper [19].

One of the most intriguing applications of Postulate 5 is, again, in astrophysics. In fact, the existence in the Universe of energy sources beyond the interpretational capabilities of the current law $E = mc_0^2$ (e.g., in the supernova explosion) is today well established. But, these energy effects occur in the core of the astrophysical objects, that is, within physical media which are much denser than those in the interior of pions or of kaons as per modified Minkowski metrics of type (1.3). A mutuation of the local metrics in the core of the astrophysical objects considered is then highly plausible, independently from the predictions of refs. [3–8], trivially, because motion in these regions cannot possibly be in vacuum. The emergence of the Minkowski-isotopic spaces $\hat{M}(x, g, \hat{\mathbb{R}})$ is then unavoidable, to our best knowledge at this time. Covering energy law (4.27) is then consequential. The achievement of energy effects higher than those predicted by the Einstenian law in vacuum is illustrated in Eqs. (4.28).

A perspective view of the above considerations Postulates 1–5 appears to be in order. In the preceding analysis we have stressed the necessary applicability of Einstein's Special Relativity for the original physical conditions conceived by Einstein and, in particular, for the center-of-mass trajectories of material objects moving in vacuum. Thus, the center-of-mass of a hadron in a particle accelerator must strictly obey Einstein's Special Relativity. Along the same lines, the center-of-mass trajectory of a quasar in our University must also obey, locally, the same laws and, for this reason, we have dismissed as unplausible recent claims from large Doppler's shifts that extremely massive objects such as giant quasars could possibly accelerate in vacuum to speed beyond c_0.

The only physical conditions in which the generalization of Einstein's Special Relativity submitted in ref. [9] should be taken into consideration are those for motion within a physical medium at large, and for all *interior* dynamical problems, whether in the interior of a hadron, or in the interior of a quasar. In this latter case, we have the incontrovertible presence of contact interactions which are beyond the representational capabilities of the Hamiltonian, that is, beyond the predictions of the conventional relativity. At this point, the selection of the final, quantitative, treatment is still unresolved and will likely emerge from the traditional scientific process of trial and error. But the existence of non-Einstenian conditions in the interior of hadrons or of quasars should be beyond any credible doubt (e.g., because of the nonlocal internal effects due to mutual wave penetration of particles, with consequential breakdown of Einstein's Special Relativity in each of its various aspects, from the Zeeman topology of the Minkowski space, to Nelson's integrability conditions to reach the Lorentz group, etc., see Section 1).

The quantitative treatment of the interior dynamics advocated in ref. [9] and in this paper is the representation of the contact, non-Hamiltonian, and nonlocal internal forces via a mutation of the Minkowski metric of type (2.1). Phenomenological studies then restrict these mutation to those of class (4.1), i.e., $g = T\eta$, with T a positive-definite diagonal matrix. The mutation T of the original Minkowski metric η is then assumed as the inverse of the unit of the theory. In this way, all nonlocal and non-Hamiltonian effects are embedded in the generalized unit of Lie's theory. This quantitative treatment was selected since memoir [10] because Lie's theory is known to be insensitive to the topology of its unit. The embedding of all non-local and non-Hamiltonian effects in the unit then avoid truly serious problems that are today unresolved at the level of pure mathematics, such as the need for a new, integrodifferential topology.

The notion of mutation of the Minkowski space into its isotope (2.1), and its treatment via the Lie-isotopic theory imply a necessary generalization of Einstein's Special Relativity which is embodied in Postulates 1–5 of this section. The central issue is then whether experimental evidence confirms or denies the predictions of Postulates 1–5. This issue is evidently complex, inasmuch as it implies a classical and an operator profile, as well as specific sector of physics, such as elementary particles or astrophysics. Evidently, we can provide only preliminary and predictably inconclusive comments on it. Intriguingly, we are aware of no direct or indirect experimental or phenomenological information that could disprove at this writing the predictions of Postulates 1–5, while all available information appears to support these prediction in a rather encouraging way, with the clear understanding that the resolution of the issue will require a predictably long and diversified scientific process (Section 8).

Within the context of elementary particle physics the available information is encouraging indeed. In fact, *all* available phenomenological information on the behaviour of the meanlife with speed (e.g. refs. [3–8]) violate Einstein's Special Relativity quite clearly in favor of its Lorentz-isotopic covering. The same is true for all possible phenomenological studies dealing with the interior dynamics which result into a modification of the Minkowski metric of type (4.1). Postulates 1–5 then follow. Direct phenomenological calculations on the maximal possible causal speed in the interior of hadrons [23] independently favor Postulate 1. Additional, preliminary, but direct experimental support is submitted in the second paper of this series, ref. [19], in which we show that the isotopic lifting of the field equations provides an intriguing representation of Rauch's interferometric measures on the apparent deformation of the magnetic moment of neutrons under external electromagnetic *and* nuclear interactions. These results provide direct support for Postulates 1–5 because they are based on the fundamental isoinvariant underlying the postulates themselves, Eq. (4.25), while the isofield equations constitute the ultimate characterization of the Lorentz-isotopic relativity. Additional experimental data, e.g., related to the so-called Berry phase and other aspects, will be presented at some subsequent time.

But, by far, the most encouraging information supporting the generalized relativity originates from astrophysics, and ranges from the possibility of interpreting extremely high speeds of jets of matter, to the possibility of resolving the unplausible current interpretation of large redshifts of giant quasars, to energy phenomena simply beyond the capabilities of Einstenian laws. The understanding is that each of these astrophysical aspects deserves a specific study we could not possibly present here.

5. Closed-Isolated Non-Hamiltonian Systems and the No-No-Interactions Theorem

As stressed since the introduction, unstable hadrons in particle accelerators must obey Einstein's Special Relativity exactly. Any conceivable deviation in the interior dynamics, to be plausible, must therefore be compatible with such experimental evidence.

The compatibility of conventional relativities for the center-of-mass motion with generalized relativities for the interior dynamics has been proved for the classical Galilean case in ref. [11] and for the operator counterpart in ref. [14], via the notion of closed (isolated) non-Hamiltonian systems. In this section we shall point out the existence of a compatible and consistent, relativistic extension that is invariant precisely under the Lorentz-isotopic (or Poincaré-isotopic) group. In this way, no available experimental information on the behaviour of a particle as a whole can be used to rule out possible generalized laws for the interior dynamics, and a new generation of experiments, specifically tailored for the new conditions considered, must be conducted.

Let us begin by introducing the *iso-four-force* in the Minkowski-isotopic space (4.1)

$$K = (K^\mu) = \left(\mathbf{K}, \frac{K^i b_i K^i}{c^2 u^4}\right) = K_{\mathrm{SA}} + K_{\mathrm{NSA}}, \tag{5.1}$$

where SA (NSA) stands for variational self-adjointness (nonself-adjointness) [11]. Note that K is orthogonal to u, not on M, but on $\hat{M}$, i.e.

$$K^\mu g_{\mu\nu} u^\nu = K^k b_k^2 u^k - K^4 c^2 u^4 = 0. \tag{5.2}$$

The dynamical equations can then be written

$$m_0 \frac{du^\mu}{ds} = K^\mu_{\mathrm{SA}} + K^\mu_{\mathrm{MSA}}, \tag{5.3}$$

and they imply, by conception, the violation of the integrability conditions for the existence of a Hamiltonian representation in the x-coordinates of the experimenter.

For the space component we have

$$m_0 \frac{d\mathbf{u}}{ds} = m_0 \hat{\gamma} c \frac{d\mathbf{u}}{dt} = \mathbf{K}, \tag{5.4}$$

thus yielding

$$\mathbf{K}_{\mathrm{Newton}} = \frac{1}{\hat{\gamma} c} \mathbf{K}_{\mathrm{Isotopic}}. \tag{5.6}$$

With these simple preliminaries, it is easy to submit the following *relativistic extension of the closed non-Hamiltonian systems* of ref. [11]

$$m_{0a} \frac{du^\mu_a}{ds} = K^\mu_{a\,\mathrm{SA}} + K^\mu_{a\,\mathrm{MSA}}, \qquad a = 1, 2, 3, \ldots, N \tag{5.7a}$$

$$\frac{d}{ds} P^\mu_{\mathrm{tot}} = \frac{d}{ds} \sum_{a=1}^{N} p^\mu_a = 0, \tag{5.7b}$$

$$\frac{d}{ds} M^{\mu\nu}_{\mathrm{tot}} = \frac{d}{ds} \sum_{a=1}^{N} M^{\mu\nu}_a = 0, \tag{5.7c}$$

where the reader will recognize the conventional *Einstenian* total conservation laws imposed as subsidiary constraints.

It should be indicated that, exactly as it happens at the Galilean level, systems (5.7) admit consistent unconstrained solutions. In fact, the conservation laws consist of a total of 10 conditions, while the equations of motion admit $4N$ functional degrees of freedom, the components of the NSA forces. Algebraic solutions therefore exist for $N \geq 3$. The case $N = 2$ can also be proved to be consistent because the motion occurs necessarily on a plane. The case $N = 1$ does not exist because one isolated particle is free and therefore cannot admit non-Hamiltonian forces.

Systems (5.7), when inspected from the outside, show no deviations from the Einstenian description, evidently because an outside experimenter observes total quantities. Nevertheless, the interior dynamics is structurally outside the representational capabilities of Einstein's Special Relativity.

This occurrence can be realized even in stricter terms, such as via the systems

$$m_{0a} \frac{du^\mu}{ds} = K^\mu_{a\,\mathrm{SA}} + K^\mu_{a\,\mathrm{NSA}}, \qquad a = 1, 2, 3, \ldots, N, \tag{5.8a}$$

$$\frac{d}{ds} P^\mu_{\mathrm{tot}} = \frac{d}{ds} \sum_{a=1}^{N} P^\mu_a = 0, \qquad \frac{d}{ds} M^{\mu\nu}_{\mathrm{tot}} = \frac{d}{ds} \sum_{a=1}^{N} M^{\mu\nu}_a = 0, \tag{5.8b}$$

$$\frac{d}{ds}\left(P^\mu_{\mathrm{tot}} \eta_{\mu\nu} P^\nu_{\mathrm{tot}}\right) = 0, \tag{5.8c}$$

$$\frac{d}{ds}(W_\nu W^\nu) = 0, \qquad W_\alpha = \tfrac{1}{2} \varepsilon_{\alpha\mu\nu\lambda} M^{\mu\nu}_{\mathrm{tot}} P^\lambda_{\mathrm{tot}}, \tag{5.8d}$$

$$\dot{X}^\mu_{\mathrm{tot}} \eta_{\mu\nu} \dot{X}^\nu = -1, \qquad \dot{X}^\mu_{\mathrm{tot}} = \sum_{a=1}^{N} x^\mu_a, \tag{5.8e}$$

where the reader recognizes, not only the conventional total conservation laws, but also the conditions for such laws to act on a conventional Minkowski space. Under these additional requests, no departure from Einstenian settings can be identified from the outside. The experimental resolution of possible generalization of the Special Relativity in the interior dynamics must be achieved via other means, such as the nature of the lifetime of unstable hadrons at different speeds, or (at some future time) via direct experimental measures under *external strong* interactions.

Note that systems (5.8) admit functional, unconstrained solutions. In fact, the total number of subsidiary constraints is 13, with $4N + 4$ functional degrees of freedom, the $4N$ components of the nonself-adjoint forces, plus the 4 elements of the deformation g of the Minkowski metric. Solutions, again, exist for $N \geq 3$, with the case $N = 2$ resulting to be consistent from the reduction of the motion to a plane.

By no means systems (5.7) and (5.8) exhaust all possibilities of constructing relativistic, closed, non-Hamiltonian systems. In fact, one can add the N constraints to system (5.7)

$$\dot{x}^{\mu}_{a} g_{\mu\nu} \dot{x}^{\nu}_{a} = -1, \qquad a = 1, 2, \ldots, N, \tag{5.9}$$

and still preserve consistency. In fact, in this case we have $N + 10$ subsidiary conditions on $4N + 4$ functional degrees of freedom. An algebraic solution then exists for $N \geq 2$ when g is arbitrary, and for $N \geq 3$ when g is assigned. A similar extension to include conventional relativistic constraints can be done to both systems (5.7) and (5.8) with similar results.

A further extension of particular relevance for this work is that when the Minkowski-isotopic space $\hat{M}(x, g, \hat{\mathbb{R}})$ is curved. In this case, Gasperini [16] has constructed the equations of motion which result to be of the *non-geodesic* type

$$m_0 \frac{du^{\mu}}{ds} + \left\{ {\mu \atop \nu\alpha} \right\} p^{\alpha} p^{\nu} = K^{\mu}_{\mathrm{SA}} + K^{\mu}_{\mathrm{NSA}}, \tag{5.10}$$

where $\left\{ {\mu \atop \nu\alpha} \right\}$ are the Christoffel's symbols of the second kind.

It is interesting to note that the systems considered exist also at this latter level, and consistent algebraic solutions are admitted for a sufficient number of particle constituents. In this way the existence and consistency of the closed non-Hamiltonian systems has been proved at the Galilean, relativistic, gravitational and operator levels.

We now provide an alternative approach in which the conventional total conservation laws are guaranteed by Noether's theorem (of course, in a form suitably generalized for Birkhoffian formulations [11]), while the interior dynamics is essentially non-Hamiltonian.

Consider a conventional relativistic Hamiltonian (see, e.g., ref. [24])

$$H = \sum_{a=1}^{N} \left[p^{\mu}_{a} \eta_{\mu\nu} p^{\nu}_{a} - m_a c_0^2 + \frac{e_a}{c_0} A^{\mu}_{a}(x) \eta_{\mu\nu} \dot{x}^{\nu} + \tfrac{1}{2} \lambda_a \right], \tag{5.11}$$

where the λ's are the multipliers. A Birkhoffian generalization of the above system can be reached by first expressing H in the Minkowski-isotopic space

$$\hat{H} = \sum_{a=1}^{N} \left[p^{\mu}_{a} g_{\mu\nu} p^{\nu}_{a} - m_a c^2 + \frac{e_a}{c} A^{\mu}_{a}(x) g_{\mu\nu} \dot{x}^{\nu} + \tfrac{1}{2} \lambda_a \right] \qquad g = g(\dot{x}), \tag{5.12}$$

and then by generalizing the conventional, canonical, action principle to the

Pfaffian form (see ref. [11] for the Galilean case)

$$\hat{A} = \int_{s_1}^{s_2} \left[\sum_{a=1}^{N} \left(p^{\mu}_{a} g_{\mu\nu} \dot{x}^{\nu} - \Gamma_a \lambda_a \right) - \hat{H} \right] ds. \tag{5.13}$$

This guarantees the existence of potential as well as nonpotential (non-Hamiltonian) forces, the latter being due precisely to the Birkhoffian lifting of Hamiltonian (5.11) in isospace $\hat{M}$.

The Birkhoffian tensor of the theory is the symplectic tensor

$$\Omega_{ij} = \begin{pmatrix} 0 & -g_{\mu\nu} \\ g_{\mu\nu} & 0 \end{pmatrix}, \qquad i, j = 1, 2, \ldots, 8, \tag{5.14}$$

with contravariant Lie form

$$\Omega^{ij} = \begin{pmatrix} 0 & g^{\mu\nu} \\ -g^{\mu\nu} & 0 \end{pmatrix}, \tag{5.15}$$

which results to be exactly Lie-isotopic. The underlying brackets of the theory are therefore the generalized brackets

$$[A \hat{;} B] = \frac{\partial A}{\partial x^{\mu}} g^{\mu\nu} \frac{\partial B}{\partial p^{\nu}} - \frac{\partial B}{\partial x^{\mu}} g^{\mu\nu} \frac{\partial A}{\partial p^{\nu}}. \tag{5.16}$$

Finally, the equations of motion with subsidiary constraints are given by the following *relativistic Birkhoff's equations*

$$\dot{x}^{\mu}_{a} = g^{\mu\nu} \frac{\partial \hat{H}}{\partial p^{\nu}_{a}}, \qquad \dot{p}^{\mu}_{a} = -g^{\mu\nu} \frac{\partial \hat{H}}{\partial x^{\nu}_{a}}, \tag{5.17a}$$

$$\dot{\lambda}_a = \frac{\partial \hat{H}}{\partial \Gamma_a}, \qquad \dot{\Gamma}_a = -\frac{\partial \hat{H}}{\partial \lambda_a}, \tag{5.17b}$$

where, as familiar, the last two equations represent the subsidiary constraints $\dot{x}^2_a = -1$.

It is easy to prove that Pfaffian principle (5.13) or Birkhoff's equations (5.17) represent a closed-isolated system \ ith non-Hamiltonian (but Birkhoffian) internal forces. In fact, the system is manifestly invariant under the Lorentz-isotopic group $\hat{O}$ (3.1) by construction. This implies the preservation of the *conventional* generators $p^{\mu}_{\rm tot}$ and $M^{\mu\nu}_{\rm tot}$ of the Lorentz group as the generators of the isotope (Sect. 2). The use of the (Birkhoffian version of) Noether's theorem then guarantees the conservations

$$\dot{P}^{\mu}_{\rm tot} = 0 \qquad \dot{M}^{\mu\nu}_{\rm tot} = 0, \tag{5.18}$$

without subsidiary constraints, as desired.

The departures from orthodox lines of inquiry should be understood here. Traditionally, the stability of a closed-isolated system is conceived as being due to the stability of the orbit of each individual constituent. This is the case of the planetary system at the classical level, and of the atomic structure at the operator level (as well as the current conception of the structure of hadrons along quark hypothesis and QCD). This is the setting of unequivocal validity of Einstein's Special Relativity, trivially, because of its local validity at each point of the trajectories of the individual constituents.

The systems under study here are profoundly different than that, and structurally more complex. Systems (5.7) or (5.8) are indeed globally stable, but each constituent is conceived as being in the highest possible *nonconservative* conditions in order to maximize the internal interactions and to prevent unnecessary simplifications of nature. Evidently the nonconservations are merely internal and such to verify conventional total conservation laws. For these latter systems, the conventional Lorentz symmetry is broken at each and every point of the trajectories of the constituents (but it is exact for the center-of-mass trajectory). This is precisely the physical setting of the covering Lorentz-isotopic relativity proposed in ref. [9].

Classical examples are well known. In fact, a planet such as Jupiter, when considered as isolated from the rest of the universe, is precisely a closed non-Hamiltonian/Birkhoffian system. Generalizations of the Minkowski metric for the interior of hadrons of type (1.3) provide preliminary, yet quite clear phenomenological support to the hypothesis formulated in ref. [12] according to which hadrons are also closed non-Hamiltonian systems. According to this view, atoms are the operator image of planetary systems, while hadrons are the operator image of the much more complex interior problems such as that of Jupiter.

The classical local breaking of the rotational/Lorentz symmetry is clearly established by direct experimental observation, e.g., the vortices in Jupiter's atmosphere with their continuously varying angular momentum. The measure of the lifetime of unstable hadrons at different energies, in case different from the Einsteinian prediction, would establish the operator counterpart and the need to reinspect the hadronic structure from a new perspective.

We close this section with the indication that *free particles are not permitted in the Lorentz-isotopic relativity*. This is the opposite of the conventional setting whereby truly interacting particles are incompatible with Einstein Special Relativity (this is the so-called *No Interaction Theorem*, see, e.g., ref. [24]). In fact, we can prove the following.

THEOREM 5.1. Systems of particles verifying the Lorentz-isotopic relativity on the Minkowski-isotopic space $\hat{M}(x, g, \hat{\mathbb{R}})$ cannot be free, unless the generalized metric g is reduced to the conventional Minkowski metric η (up to trivial scalar isotopies $g = N\eta$).

Proof. Gasperini [16] has established that, under the conditions of the theorem, the motion cannot be geodesic. The property of the theorem then follows under the condition that $g \neq N\eta$. QED

Equivalently, one can prove the existence of an "isotopic transformation" of Birkhoff's equations (5.17) which reduces the Hamiltonian to a "free" form (that is, which eliminates the potential energy as in the conventional relativistic case). However, the lifting $\eta \to g$ of the metric persists, trivially, because the transformation theory cannot alter the unit of the theory, $\hat{I} = g^{-1}$. Under these conditions, the particles experience no potential force, but the contact, non-Hamiltonian forces represented by g cannot be eliminated. For numerous examples, see ref. [16–18]. Also equivalently, we can say that the use of the transformation theory can eliminate the SA forces of Eq. (5.10). But the NSA forces persist, thus characterizing a motion that is irreducibly nongeodesic and, thus, irreducibly interacting.

6. Some of the Problematic Aspects of Einstein's Gravitation

As indicated earlier, when Einstein's Special Relativity is violated because of motion of extended-deformable particles within an inhomogeneous and anisotropic material medium, it must still be considered as approximately valid.

The situation for Einstein's General Theory is different because of the existence of a truly large number of fundamental open problems which cast doubt even on the approximate validity of the theory.

It is time for the scientific community to finally confront these problems (which have been known for quite some time) and attempt their resolution.

First, let us go back to the original distinction between the *interior problem* of gravitation (that inside the surface encompassing all matter, including the atmosphere when it exists), and the *exterior problem* (that in the remaining region of space). This paper is devoted to the study of the interior relativistic problem. It is therefore natural to consider here some of the problematic aspects of Einstein's gravitation for the interior dynamics. The physical setting we are referring to is, again, the motion of an extended test particle moving within a physical medium (e.g., Earth's atmosphere), this time from a gravitational profile.

This author has contended since some time [25] that the background geometry underlying Einstein's Gravitation, the Riemannian geometry, even though so effective for the exterior problem, is ultimately inapplicable for the interior problem not even in a first approximation.

This is due to the incontrovertible evidence that the ultimate structure of matter (say, the core of a star undergoing gravitational collapse) is *nonlocal*, while the Riemannian geometry is strictly *local* in topological character. A suitable nonlocal, integrodifferential generalization of the Riemannian geometry was therefore advocated [25].

The above occurrence rules out the applicability of the Riemannian geometry even in first approximation. In fact, the insistence on the exact validity of Einstein's Gravitation for the interior problem literally and directly implies the acceptance of the perpetual motion in our environment thus shifting the issue from a scientific setting to an ethical profile which is inevitable whenever excessive approximations of nature are admitted.

In fact, to prevent perpetual-motion approximations, the applicable geometry must be able to recover *all* Newtonian forces at the PPN limit. The fundamental insufficiency by the Riemannian geometry of recovering all possible Newtonian forces is well known since quite time (and called Cartan's legacy).

Another aspect of Einstein's Gravitation that is in manifest conflict with physical evidence is its homogeneous and isotropic character. Again, such a character is fully admissible in the exterior problem because, there, motion occurs in empty space. But the insistence on the same character for the interior problem would literally prevent the representation of physical conditions such as the manifest local variation of the density of the sun with the distance from its center, etc.

Still another fundamental problematic aspect is the local Lorentz character of the theory. Such a character is manifestly valid for the exterior problem, e.g., because of the stability of the orbits of the planets, but its insistence as exactly valid also in the interior dynamics would imply again the acceptance of the perpetual motion in our environment. In fact, the rotational and Lorentz symmetry are manifestly broken for classical interior trajectories of extended test particles. The old claim that such a breaking is "apparent" because resolvable via the reduction to elementary constituents has been proved to be technically inconsistent [25], e.g., because of the impossibility of reducing a classical, noncanonical and non-Hamiltonian time evolution to a finite collection of quantum mechanical, unitary and Hamiltonian time evolutions. Besides, the physical problem is that of representing the *classical* reality of interior test particles with continuously decaying angular momenta, and prior to any operator reduction.

In view of this and other inconsistencies we are not reviewing here for brevity, the *theory of gravitation for the interior problem* which is advocated here should have the following main features:

1) The theory should represent the generally inhomegeneous and anisotropic character of material media, with the understanding that space itself remains homogeneous and isotropic.

2) The applicable geometry should be a nonlocal integrodifferential generalization of the Riemannian geometry. If a local approximation is used, it should be able to recover all possible Newtonian equations of motion under the usual PPN approximation.

3) The theory should be able to represent local deviations from the conventional rotational and Lorentz symmetries, in order to avoid perpetual-motion approximations, of course in a way compatible with total conservation laws (see below).

4) The theory should be locally Lorentz-isotopic in character. In particular, the local deformations of the Minkowski metric should preserve the underlying topological character in such a way that the local, abstract, Lorentz symmetry is preserved.

5) Finally, the theory should be essentially nonself-adjoint in variational character [11] as a necessary condition to represent the trajectories of the real interior world. As a result, the theory should violate the integrability conditions for the existence of a Hamiltonian in the frame of the experimenter. Nevertheless, the theory should admit an unambiguous Birkhoffian representation as a necessary basis for possible operator formulations.

We now consider the exterior problem of gravitation. In this case, as stressed earlier, motion occurs in empty space. The dimension of the test particle is therefore ignorable and the Riemannian geometry is indeed fully applicable. Also, the theory can indeed be locally Lorentz in character.

Despite the verification of these basic structural properties, Einstein's gravitation for the exterior problem is affected by such deep problematic aspects to cast serious doubts on its approximate validity.

One of them has been identified in ref. [26] and consists of an irreconcilable incompatibility of Einstein's gravitation with Maxwell's electrodynamics. Consider a celestial body with null total electric charge and null total electric and magnetic moments. Einstein's field equations for the exterior problem in this case are given by the familiar form

$$G_{\mu\nu} = 0, \tag{6.1}$$

where $G_{\mu\nu}$ is Einstein's tensor, and they embody the ultimate essence of Einstein's conception of gravitation: its reduction to pure geometry without source.

In ref. [26] one can see the explicit computation of the electromagnetic field produced by the elementary charged constituents of matter beginning with a classical analysis for the π^0. It turns out then, even though the π^0 has a null total electromagnetic phenomenology (as the celestial body considered), the total electromagnetic field produced by the charged constituents

$$F_{\pi^0}^{\mu\nu} = {}_qF^{\mu\nu} + {}_\mu F^{\mu\nu}, \tag{6.2}$$

(where q and μ represents the electric and magnetic contributions, respectively, computed via the Lienard-Wieckert potentials) is so high that the volume integral

$$\begin{aligned} m_{\pi^0}^{\mathrm{Elm}} &= \frac{1}{c_0^2} \int \theta_{\pi^0 0}{}^{0}\, dv, \\ \theta_{\pi^0}^{\alpha\beta} &= \frac{1}{4\pi}\left(F_{\pi^0}^{\alpha\mu} F_{\pi^0\mu}^{\beta} + \frac{1}{4}\eta^{\alpha\beta} F_{\pi^0}^{\mu\nu} F_{\pi^0\mu\nu} \right), \end{aligned} \tag{6.3}$$

can well account for the entire gravitational mass of the π^0.

A simple extrapolation of the model to a massive body with non-null total electromagnetic phenomenology produced a tensor of the type

$$\theta_{\mu\nu}^{\mathrm{Elm}} = \frac{1}{4\pi}\left(F_{\mu\alpha}F_{\nu}^{\alpha} + \frac{1}{4}\eta_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right),$$

$$F_{\mu\nu} = F_{\mu\nu}^{\mathrm{Elm}} = \sum_{q,\mu=1}^{N} \left({}_qF_{\mu\nu}^{\mathrm{Elm}} + {}_{\mu}F_{\mu\nu}^{\mathrm{Elm}}\right), \tag{6.4}$$

$$M^{\mathrm{Elm}} = \frac{1}{c_0^2}\int \theta_0^{\mathrm{Elm}\,0}\, dv,$$

which, again has such a high value to account in principle for the entire gravitational mass of the celestial body (see below for contributions from short range interactions).

The modification of Einstein's equations (6.1) recommended in ref. [26] are therefore given by

$$G_{\mu\nu} = \frac{8\pi G}{c_0^4}\theta_{\mu\nu}^{\mathrm{Elm}}. \tag{6.5}$$

Note that the tensor $\theta_{\mu\nu}^{\mathrm{Elm}}$ is nowhere reducible to zero (in the frame of the experimenter!), unless one works out a profound modification of Maxwell's theory that is certainly bound to be in conflict with experiments, or modifies the structure of matter (by reducing all elementary charges to an at rest condition at very small mutual distances).

Hypothesis (6.5) allowed the possibility of reducing the vexing problem of the "unification" of the gravitational and electromagnetic field, to their "identification". A number of experiments to test Eqs. (6.5) were also proposed [26].

The above inconsistency of Eqs. (6.1), by far, does not exhaust the problematic aspects of the theory. A leading expert in this issue is H. Yilmaz who, in a series of papers initiated back in 1958 [27–29] has pointed out a series of problematic aspects of Eqs. (6.1) due to the lack of the stress-energy tensor $t_{\mu\nu}^{\mathrm{Stress}}$ and the need to modify the exterior equations into the form

$$G_{\mu\nu} = \frac{8\pi G}{c_0^4}t_{\mu\nu}^{\mathrm{Stress}}. \tag{6.6}$$

We here mention apparent inconsistencies of Einstein's Gravitation in: recovering the relativistic limit and related conservation laws; representing the Newtonian 532" of advancement of the perihelium of Mercury; and others. In particular, Yilmaz has worked out a revision of Einstenian theory for the exterior problem along Eqs. (6.6) to a rather remarkable extent.

It is important to point out here that Yilmaz's tensor $t_{\mu\nu}^{\mathrm{Stress}}$ *is not* equivalent to the tensor $\theta_{\mu\nu}^{\mathrm{Elm}}$, trivially, because the latter is traceless, while the former is not. As a result, there are rather deep structural differences between Eqs. (6.5) and (6.6).

Yilmaz's work itself is not immune from criticism. In fact, his equations (6.6) are unable to account for the (primary) electromagnetic origin of the gravitational field, i.e., the tensor $t_{\mu\nu}^{\mathrm{Stress}}$ cannot include the tensor $\theta_{\mu\nu}^{\mathrm{Elm}}$ (e.g., because the contribution to the field by the former is much smaller than that by the latter).

Also, the basic problem which is still open in Yilmaz's work is the ultimate origin of the tensor $t_{\mu\nu}^{\mathrm{Stress}}$. The hypothesis which is submitted in this paper is that *Yilmaz's tensor $t_{\mu\nu}^{\mathrm{Stress}}$ sees its origin in the short range, weak and strong interactions in the structure of matter.*

To put it differently, all interactions in the interior of matter are expected to provide a contribution to the gravitational field. The electromagnetic interactions are expected to provide the quantitatively largest contribution along hypothesis (6.5), while the remaining short range (weak and strong interactions in nuclei as well as in the structure of hadrons) are expected to provide an additional (although smaller) contribution. The hypothesis hereby submitted is that, upon suitable classical averaging over the macroscopic body, such short range contributions are expected to result precisely into a modification of Eq. (6.1) along Yilmaz' form (6.6).

The modification of Einstein's equations here advocated for the exterior problem is therefore of the type

$$G_{\mu\nu} = \frac{8\pi G}{c_0^4}\left(\theta_{\mu\nu}^{\mathrm{Elm}} + t_{\mu\nu}^{\mathrm{Stress}}\right), \tag{6.7}$$

with the following main features:

6) The theory of the interior problem as per the properties 1-2-3-4-5 above should be able to recover conventional, total conservation laws, e.g., via a gravitational extension of the closed non-Hamiltonian systems of Section 5.

7) The theory for the interior problem should produce a conventional Riemannian geometry in the exterior problem, with its known locally Lorentz (and *not* Lorentz-isotopic) character.

8) The theory for the interior problem should produce in the exterior problem all conceivable source terms i.e., the sources due to the electromagnetic and short range fields along Eqs. (6.7); and, last but not least,

9) The gravitation theory should be compatible with all experimental evidence in both the exterior *and* interior problems.

We should stress here that a gravitational theory verifying all the above requirements does not exist at this time to our best knowledge. Nevertheless, significant progresses have been made during recent years toward a possible solution of at least part of the problematic aspects afflicting Einstein's Gravitation.

7. Comments on Gasperini's Gravitation

As indicated in Section 1, in a series of pioneering papers, Gasperini [16–18] constructed a step-by-step isotopic generalization of Einstein's Gravitation, hereinafter referred to as *Gasperini's Gravitation* which is precisely locally Lorentz-isotopic in character. It is important for the completeness of this paper to briefly review Gasperini's theory in order to point out its capability of resolving at least some of the problematic aspects of Einstein's Gravitation.

Let

$$S = \int\left(\tfrac{1}{4}R^{ab} \wedge V^c \wedge V^d \varepsilon_{abcd} - \tfrac{1}{3}M_a \wedge V^a\right), \tag{7.1}$$

be (the geometrical form of) the simplest possible action of Einstein's Gravitation in standard units, with matter sources (the M_a 3-form) minimally coupled to gravity, where

$$M^a = M^{a\mu}\varepsilon_{\mu\gamma\alpha\beta}\, dx^\nu \wedge dx^\alpha \wedge dx^\beta, \qquad x \in M, \tag{7.2a}$$

$$V^a = V^a_\mu\, dx^\mu, \tag{7.2b}$$

$$R^{ab} = dw^{ab} + w^a_c \wedge w^{cb}, \qquad w^{ab} = w^{ab}_\mu\, dx^\mu. \tag{7.2c}$$

The standard potential can be written

$$h = h^A X_A = V^a P_a + w^{ab} M_{ab}, \tag{7.3}$$

where the P_a and M_{ab} are the conventional generators of the Poincaré group.

Following the algebraic guidelines of ref. [10], Gasperini subjected the above theory to an isotopic lifting characterized by

$$\hat{h} = h^A T^B_A X_B$$

$$= V^a T^b_a P_b + V^a T^{bc}_a M_{bc} + w^{ab} T^c_{ab} P_c + w^{ab} T^{cd}_{ab} M_{cd}, \tag{7.4a}$$

$$\hat{V}^a = h^A T^a_A = V^b T^a_b + w^{bc} T^a_{bc}, \tag{7.4b}$$

$$\hat{W}^{ab} = h^B T^{ab}_B = V^c T^{ab}_c + w^{cd} T^{ab}_{cd}, \tag{7.4c}$$

$$\hat{R}^a = d\hat{V}^a + \hat{w}^a b \wedge \hat{V}^b, \tag{7.4d}$$

$$\hat{R}^{ab} = d\hat{w}^{ab} + \hat{w}^a_c \wedge \hat{w}^{cb}, \tag{7.4e}$$

where the T's are the *isotopic elements* which, as one can see, are generally different for different generators.

Gasperini's Gravitation can therefore be defined via the isotopic action

$$\hat{S} = \int\left(\tfrac{1}{4}\hat{R}^{ab} \wedge \hat{V}^c \wedge \hat{V}^d \varepsilon_{abcd} - \tfrac{1}{3}M_a \wedge \hat{V}^a\right). \tag{7.5}$$

The implications of the above lifting are truly intriguing and far reaching. To begin, the variation of action (7.5) with respect to T^a yields the field equations

$$\tfrac{1}{2}\hat{R}^{ab} \wedge T^c \varepsilon_{abcd} = \tfrac{1}{3}M_d, \tag{7.6}$$

which, in their explicit form, can be written

$$G^\beta_\alpha = M^\beta_\alpha + \Lambda^\beta_\alpha, \tag{7.7}$$

where G^β_α is Einstein's tensor, and, for the decomposition $T = V + \tau$ we have

$$\Lambda^\beta_\alpha = G^\beta_\alpha \tau^\beta_\alpha + R^\nu_\mu \tau^\mu_\nu \delta^\beta_\alpha + R^\beta_\nu \tau^\nu_\alpha + R^{\nu\beta}_{\mu\alpha}\tau^\mu_\nu, \tag{7.8}$$

By using these and other developments, Gasperini [loc. cit.] reached the following primary results:

The isotopic lifting of Einstein's Gravitation produces a generalized theory in which

1) *the torsion is irreducibly nonnull even though the original theory is torsion-free;*

2) *the motion is irreducibly nongeodesic, even though the original theory is strictly geodesic; and*

3) *the tangent space has a local Lorentz- (or Poincaré-) isotopic symmetry with local metric*

$$\hat{\eta}^{ab} = \eta^{cd} T^a_c T^b_d, \tag{7.9}$$

even though the original theory is strictly Lorentz in local character with conventional Minkowski metric η *of the tangent space.*

In particular, the isotopic theory can be characterized via the following

GASPERINI'S PRINCIPLE OF EQUIVALENCE. Gravitational effects may locally disappear when the metric of the space-time manifold approaches the metric of the tangent, Minkowski-isotopic space.

We refer the interested reader to the original papers [16–18] for numerous developments and particularizations, such as the realization of the theory for Nielsen-Picek metric (1.3); the explicit proof of the local Lorentz-isotopic character; the calculations of the generalization for the orbital motion under an isotopic metric of the type (1.3); the construction of an isotope of the Schwarzschild metric; and others.

The above features render Gasperini's Gravitation ideally suited for the interior problem only, under the condition of recovering conventional geometric approaches for the exterior problem. It is sufficient here to mention the experimental evidence that (at least locally within a planetary system) space is homogeneous and isotropic. This implies the local Lorentz character of the applicable geometry for the exterior problem. In the transition to the interior problem, we have instead extended test particles moving within a physical medium which is manifestly inhomogeneous and anisotropic (even though space itself remains homogeneous and isotropic). This fact alone is sufficient to require a deviation from the local Lorentz character as a necessary condition to avoid the perpetual-motion-type of excessive approximations which are inherent in the conventional theory.

In this paper, we therefore propose the *restriction of Gasperini's Gravitation to the interior problem*, in such a way that conventional local geometries and space-time symmetries are recovered for the exterior problem. This is readily accomplished by imposing the condition that all isotopic elements reduce to the unity for any position $\mathbf{r}$ in the exterior problem, e.g., when the celestial body considered is a sphere of radius $\mathbf{R}$, for

$$T_A|_{\mathbf{r}>\mathbf{R}} = 1. \tag{7.10}$$

The functional dependence of the isotopic elements T_A can be primarily on the local velocities $\dot{x}$ (along the preceding lines of inquiry of this paper), as well as on all physical characteristics of the medium in the interior problem, such as density μ, temperature t, etc, and any other needed physical quantity,

$$T_A = T_A(\dot{x}, \mu, t, \dots). \tag{7.11}$$

Boundary condition (7.10) can then be realized in a number of ways. If the celestial body considered is without atmosphere, a step function, e.g., in the density, is then expected to represent the discontinuous transition from the interior to the exterior problem. If, instead, the density goes continuously to zero (as in Earth's atmosphere), then a continuous transition from the interior to the exterior problem is more appropriate.

Secondly, we suggest the *restriction of Gasperini's Gravitation for the interior problem to such topological structure for which the local Lorentz-isotopic symmetry is isomorphic to the conventional symmetry*. This is readily achieved for isotopic metrics (7.9) which are topologically equivalent to the conventional Minkowski metric (Section 2), e.g., for isotopic elements of the diagonal type

$$\begin{aligned} \hat{\eta}^{ab} &= \eta^{cd} T^a_c T^b_d, \\ \eta &= \mathrm{Diag}(1, 1, 1, -1), \\ \hat{\eta} &= \mathrm{Diag}(b_1^2, b_2^2, b_3^2, -b_4^2), \\ T &= \mathrm{Diag}(b_1, b_2, b_3, b_4), \qquad b_a > 0. \end{aligned} \tag{7.12}$$

In this way, the fundamental Lorentz symmetry is not lost, but simply expressed in its algebraic and geometric most general possible way (while in Einstein's Gravitation it is expressed in its simplest possible algebraic and geometric way). The reader familiar with the analysis of the preceding sections will know that a condition of type (7.12) is necessary to ensure the local isomorphy of the conventional and isotopic Lorentz symmetries. In fact, the local symmetry of isotopic metric (7.9) can be isomorphic, e.g., to SO(2.2) or any other six parameter group of Cartan's classification, depending on the topology at hand.

Furthermore, we suggest the *condition that Gasperini's Gravitation for the interior problem verifies conventional total conservation laws*

$$M_{ab,c} = 0, \tag{7.13}$$

which are therefore imposed as subsidiary constraints. This results into the most general possible (classical) formulation of the notion of closed non-Hamiltonian systems.

Finally, and along the lines of paper [26], we suggest the rejection of any source term of unknown origin such as the traditional mass tensors M_{ab}^{Mass}, and identify instead the source tensor M_{ab} with those of all interactions occurring in the structure of matter, that is, the electromagnetic tensor produced by all charged constituents of matter, Eq. (6.4), and the additional tensor produced by the short range, weak and strong interactions. We shall furthermore assume that, upon suitable classical averaging, the latter tensor results into Yilmaz's tensor (6.6). We shall then write

$$M_{ab} \equiv M_{ab}^{\mathrm{Elm}} + M_{ab}^{\mathrm{Stress}}. \tag{7.14}$$

In summary, we here submit the restriction of Gasperini's theory of gravitation for the interior problem and the identification of the source terms to the form characterized by the following equations

$$\hat{S} = \int \left[\tfrac{1}{4} \hat{R}^{ab} \wedge \hat{V}^c \wedge \hat{V}^d \varepsilon_{abcd} - \tfrac{1}{3}\left(M_a^{\mathrm{Elm}} + M_a^{\mathrm{Stress}}\right) \wedge \hat{V}^a, \tag{7.15a}$$

$$T_A = T_A(\dot{x}, \mu, t, \dots), \tag{7.15b}$$

$$T_A|_{r>R} = 1, \tag{7.15c}$$

$$\left(\hat{\eta}^{ab}\right) = \left(\eta^{cd} T^a_c T^b_d\right) = \mathrm{Diag}\left(b_1^2, b_2^2, b_3^2 - b_4^2\right), \qquad b_a^2 > 0, \tag{7.15d}$$

$$M_{ab,c}^{\mathrm{Tot}} = \left(M_{ab}^{\mathrm{Elm}} + M_{ab}^{\mathrm{Stress}}\right)_{,c} = 0. \tag{7.15e}$$

The consistency of the above system can be readily proved. In particular, algebraic solutions exist, as it occurred for other closed non-Hamiltonian systems

(Section 5), because the degrees of functional freedom of the theory (the isotopic elements) are higher than the number of subsidiary constraints.

It should be stressed that the field equations for the exterior problem originating from system (7.15) *do not* coincide with Einstein's equations (6.1), but are given instead by [26]

$$G_{\mu\nu} = M_{\mu\nu}^{\mathrm{Elm}} + M_{\mu\nu}^{\mathrm{Stress}}, \tag{7.16}$$

that is, the gravitational field cannot be reduced to pure geometry, but admits instead sources that are nowhere null, even when the total electromagnetic phenomenology is null.

As a final comment, we would like to indicate that the Lie-isotopic lifting of the Lorentz symmetry can be applied in a two-fold way to system (7.15) for the explicit construction of symmetry transformations via expansions (2.11).

In fact, model (7.15) is a gravitational theory with two metrics [16]. First, there is a generalized gravitational metric g, and then there is a generalized tangent metric $\hat{\eta}$. Each of these metrics can be considered as a lifting of η. As a result, expansions (2.11) can be applied in each case to produce the explicit form of the symmetry transformations. (Recall that the only unknown in expansions (2.11) is the isotopic metric, and that the convergence of the series is ensured under the topological conditions considered).

Intriguingly, lifting (2.11) can be applied also to *conventional* theories of gravitation, by providing, apparently for the first time, methods for the explicit construction of the symmetry of familiar metrics, such as the Scharzschild metric.

Gravitation (7.15) resolves a number (but not all) of the problematic aspects of Einstein's Gravitation in both the interior and in the exterior problem. (Requirements 1 through 9 of Section 6).

To begin, the theory allows the representation of the inhomogeneous and anisotropic character of the physical media in the interior dynamics (Requirement 1).

Second, the theory is based on a genuine, topologically nontrivial generalization of the Riemannian geometry which is generally nonlocal. This is permitted by integrodifferential realizations of the isotopic elements that are topologically acceptable because Lie's theory is known to be insensitive to the topology of its unit, as recalled earlier. Furthermore, the underlying geometry allows the recovering under PPN approximation of all possible Newton's equations of motion, again, because of the unrestricted functional dependence of the isotopic elements in the velocities and other quantities. As a result, Requirement 2 is satisfied in its entirety.

Deviations from the conventional, local, rotational symmetry are also permitted precisely by the deviations of the isotopic tangent metric from the conventional one. This allows the representation of decaying orbits and other typical trajectories for the interior problem. Requirement 3 is therefore also satisfied.

Subsidiary condition (7.15d) ensures the local isomorphism between the isotopic and conventional Lorentz symmetries. The crucial Requirement 4 on the preservation of this fundamental space-time symmetry is therefore verified.

Furthermore, the theory is manifestly nonself-adjoint in character [11] (as a matter of fact, Eqs. (7.7) are essentially nonselfadjoint). Therefore, the theory violates, by construction, the integrability conditions for the existence of a *Hamiltonian* representation. Nevertheless, we do not know whether a consistent *Birkhoffian* representation exists. The fulfillment of Requirement 5 is therefore open at this time. This essentially requires the study whether generalized action (7.15a) admits a consistent and unambiguous Pfaffian (rather than canonical) first-order counterpart. The existence of such representation would then open the possibility for an unambiguous "hadronization" into the operator mechanics of refs. [12–14], at least for the interior problem.

Conventional, total, conservation laws are imposed as subsidiary constraints. Requirement 6 is then satisfied by the existence of algebraic solutions.

The conventional Riemannian structure in the exterior problem is automatically produced by condition (7.15c), and Requirement 7 is verified.

All conceivable source terms originating in the structure of matter are present by construction in the exterior problem, Eqs. (7.16), and Requirement 8 is also satisfied.

Finally, the crucial Requirement 9 on the verification of all available experimental data for both the exterior and the interior problem requires specific studies. At this point we can only say that the capability to represent experimental evidence of the *interior* problem is better for system (7.15) than for conventional Einstein's theories for the reasons indicated earlier. But, as far as the *exterior* problem is concerned, specific additional studies are needed. At a deeper level, we can say that, if Yilmaz's source term M_{ab}^{Stress} is eliminated, Eqs. (7.15) have the same predictive capacity of Einstein's equations, owing to the identification $M_{ab}^{\mathrm{Matter}} \equiv M_{ab}^{\mathrm{Elm}}$. Still unsettled aspects might emerge in the representation of experimental data for the exterior problem when both exterior terms M_{ab}^{Elm} and M_{ab}^{Stress} are admitted. This latter aspect does not appear to have been investigated in the existing literature to our best knowledge (Yilmaz's studies refer to the case in which $M_{ab}^{\mathrm{Stress}} \neq 0$ and $M_{ab}^{\mathrm{Elm}} = 0$). For precautional reasons we therefore leave this latter aspect open at this time.

8. Proposed Experiments

The fundamental tests recommended in this paper are now predictable, and have been actually suggested in the literature for decades but regrettably ignored:

Measure the behavior of the lifetime of unstable hadrons, such as pions and kaons, at a sufficient number of differen' speeds to allow the resolution whether Einstenian law (1.1) *holds*,

$$\tau = \tau_0 \gamma, \qquad \gamma = (1 - \beta^2)^{-1/2}, \qquad \beta^2 = v^2/c_0^2, \tag{8.1}$$

in which case conventional space-time symmetries and related relativities hold in the interior of hadrons, or generalized law (4.10) *holds,*

$$\tau = \tau_0 \hat{\gamma}, \qquad \hat{\gamma} = \left(1 - \hat{\beta}^2\right)^{-1/2}, \qquad \hat{\beta}^2 = \frac{vb^2v}{c^2}, \tag{8.2}$$

in which case the isotopic space-time symmetries and related relativities hold in the interior of hadrons.

As stressed in the Introduction, all available direct phenomenological investigations, e.g., refs. [3–8], show violation of law (8.1) in favor of isotopic law (8.2), thus confirming the rather plausible expectation that motion in the interior dynamics does not occur in vacuum but within a physical medium. All investigations [3–8] are a particular case of isotopic law (8.2), as stressed earlier, because of the arbitrariness of the generalized metric g. Finally, for all known studies, the topological character of the original Minkowski metric is preserved. As a consequence, the Lorentz symmetry remains exact for all known phenomenological papers [3–8]. Possible deviations, if experimentally confirmed, should therefore be specifically referred to Einstein's Special Relativity, and not to the Lorentz symmetry. Finally, the reader should keep in mind that the conceivable deviations from the Special Relativity under consideration here are specifically referred to the interior problem only, while the exact validity of the relativity is recovered in full for the center-of-mass trajectory of the particle in a particle accelerator.

The fundamental character of the experiments suggested here is clear and incontrovertible. In fact, if the deviations from the Special Relativity are experimentally established, this would provide experimental evidence for the need to generalize both Einstein Special and General Relativities for the interior problem along the lines discussed earlier. In such an event, it is hoped that the conjectural and tentative line of inquiry presented in this paper may serve as a possible starting point for the traditional scientific process of trial and error toward the future construction of covering relativities.

9. Concluding Remarks

We initiated this paper by noting the *unreassuring condition in theoretical particle physics* created by the ignorance of the historical legacy on the ultimate nonlocal nature of the strong interactions by quark theories and QCD. We would like to close this paper by noting the *unreassuring condition in experimental particle physics* created, this time, by the excessively protracted ignorance of the fundamental tests of Section 8, which were well known, and fully motivated since the early 60's. The lifetime of unstable hadrons continues to be measured and remeasured with the passing of the decades, but always for at rest conditions and as a refinement of already known data, while the comparatively much more important measures of

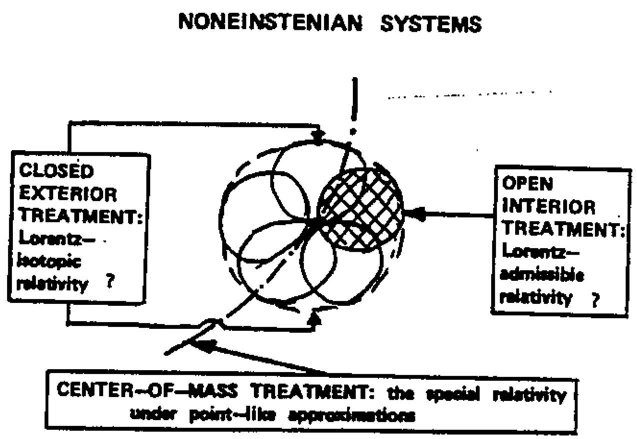

FIG. 2. A summary view of the various formulations that may result to be needed for the appropriate description of the dynamical behaviour of a hadron and of its individual constituents. Under conventional action-at-a-distance, potential interactions, only one description is needed, the Hamiltonian one, because of the local stability of the orbits of each individual constituent and, thus, of a composite system as a whole. When internal, non-Hamiltonian forces are admitted the situation changes quite deeply. First, the conventional formulations (e.g., the Special Relativity) remain fully applicable in an exact form for the center-of-mass trajectory, of course, under a point-like approximation of the composite system. For the description of the structure of the composite system when closed-isolated, the Lorentz-isotopic relativity considered in this paper is recommended as a first step, because of its intrinsic capability of incorporating conventional potential forces, as well as contact/zero-range/non-Hamiltonian forces due to motion within a physical medium, with consequential local instability of the trajectories of the individual constituents while the system as a whole verifies conventional total conservation laws. Finally, for the characterization of *one* constituent when all others are assumed as external, a still more general approach of the so-called Lie-admissible type is advocated [10] because of the need to represent, this time, the time-rate-of-variation of the physical quantities of the constituent considered while keeping the Hamiltonian (i.e., the total energy) of that constituent as the generator of the time evolution. These requirements demand the use of brackets that are Lie-admissible, i.e., they are not totally antisymmetric, although their attached antisymmetric brackets are Lie. This latter approach is not considered in this paper for brevity. In summary, the admission of contact/non-Hamiltonian internal forces implies profound methodological implications because, besides the evident need to construct covering descriptions for the representation of such forces, it implies, in addition, a structural differentiation in the transition from the description of the composite system as a whole, to that of one individual constituent.

the behaviour of the lifetime at different speeds continue to be ignored. The outcome of this scenario cannot but be unreassuring, and raise all sort of questions which, with the passing of the decades have predictably shifted the emphasis from a purely technical aspects, to issues of scientific ethics.

Physics is a discipline with an absolute standard of value: the experimental verification. No matter how important, theories remain tentative and conjectural until proved with direct experiments. Einstein's theories in the interior of hadronic matter cannot escape this rigid scientific fate despite their relevance.

Experiments themselves have their own absolute standard of value: the more fundamental the test, the more relevant is its conduction as compared to lesser fundamental tests. Refinements of already known physical quantities can at best provide refinements of established theories with at best marginal advances in physical knowledge. But basic tests can clearly provide the basis for truly fundamental advances, no matter what is their outcome.

By no means does this paper (and all refs. [3–8]) recommend the test of the *violation* of Einstein's Special Relativity inside hadrons. This paper (and all quoted references) merely insist on establishing the validity of the Special Relativity in the arena considered via direct experimental evidence, rather than via a plethora of individually unverified assumptions (such as the assumption that quarks are physical particles; the assumption that they are the ultimate elementary constituents of hadrons; the assumption that they confine; etc.).

The number of independent proposals on the lifetimes of unstable hadrons at different speeds has been simply overwhelming. For instance, paper [5] by Kim was written at SLAC in 1978, and concluded with the statement that the tests are "*unprocrastinable*." Paper [7] by Huerta-Quintanilla and Lucio was written at FERMILAB in 1983 and also concluded by favoring *deviations* from the Special Relativity, thus rendering the tests simply compelling. This author has been engaged in decades of actions to recommend the tests with no results.

In summary, we can conclude by saying that:

1) the clear plausibility of the violations;
2) the number and authority of the phenomenological papers predicting violations;
3) the manifestly fundamental character of the proposed tests;
4) the deep implications for all of the theoretical and experimental particle physics for whatever the outcome;
5) the clear feasibility of the tests with current accelerators at virtually all major laboratories around the world;
6) the truly moderate cost of the tests, particularly when compared to other lesser relevant, yet substantially more expensive tests currently preferred by the physics community;
7) the direct nature of the tests without theoretical assumptions in the data elaboration (direct measures of time), particularly when compared to the plethora of (often tacit) unverified assumptions needed for the data elaboration of currently preferred experiments;

and numerous other aspects, render the test truly "unprocrastinable." Any additional delay can only deepen the above indicated unreassuring condition of theoretical and experimental particle physics.

Note added in proof. Prior to the release of this manuscript for printing, we were informed of the ongoing preparation of the general review of Lie-isotopic theories by Aringazin, Jannussis, Lopez, Nishioka and Velijanoski [30]. Also, Prof. R. Mignani kindly telefaxed to me excerpts from the recent monograph by Lagunov and Mestvirshvili [31] on gravitation which has interesting connections with Gasperini's Isotopic Gravitation considered in Sections 6–7 of this paper. Prof. Mignani also telefaxed to me his paper [32] where he provides the application of Postulate 4 to the problem of the Doppler's shift of large quasars, exactly as suggested in Section 4, and with the expected results (lack of need for assuming that extremely massive objects such as large quasars have to move in our Universe under Einstenian conditions for speeds violating Einstenian laws). Finally, we should mention that A. K. Aringazin proved in paper [33] the direct universality of isotopic law (4.12) on the behaviour of the meanlife with speed for all various particularizations available in the literature, such as those of refs. [3–8], as anticipated in Section 4.

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