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Galleys to be corrected

QUANTUM-ISO-GRAVITY

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In this note we recall the open problems of quantum gravity and propose a new quantization of gravity via the generalization of the unit of relativistic quantum mechanics, show its axiomatic consistency, introduce its universal isopoincaré symmetry, prove its isomorphism to the conventional symmetry and point out a number of intriguing implications.

The historical open problem of quantum gravity (QG) is the need, on one side, for relativistic quantum mechanics (RQM) to have a meaningful Hamiltonian while, on the other side, Einstein's gravitation in vacuum has a null Hamiltonian[1]. A second open problem is the achievement of a QG which is axiomatically consistent as the conventional RQM, i.e., invariant under its own time evolution with physical quantities which are Hermitean-observable at all times, etc. A third open problem has emerged from recent studies in interior gravitational problems of quasars[1c], that QG should be a *nonunitary* image of conventional quantum theories, as needed, e.g., for a representation of irreversibility.

In this note we propose a new QG based on the generalization of the unit of RQM which, as such, requires no Hamiltonian at all, thus resolving the first historical problem. The axiomatic consistency of the proposed QG is guaranteed by the preservation of the abstract axioms of the RQM only realized in a more general way, including form-invariance, Hermiticity of observables at all times, etc. thus resolving the second problem. Finally, the proposed QG is a rather natural nonunitary image of conventional RQM, thus verifying the third condition.

Our model is based on the *isotopic methods* introduced by this author back in 1978[2], developed in the recent monographs[3] and independently studied by various authors[4] (see papers[5] for recent reviews). The main elements to render this note self-sufficient are reviewed below. Further developments and details are presented in the more detailed presentation[6].

The main idea of the isotopic methods is to lift the conventional associative product among operators AB into the form $A * B \equiv ATB$, where T is a fixed positive-definite operator called *isotopic element*, while jointly lifting the original unit I into the form $\hat{I} = T^{-1}$ which is the correct unit of the new theory $\hat{I} * A = A * \hat{I} \equiv A$, called *isounit.* Such dual lifting is *isotopic* in the sense of preserving the original axioms[2a], i.e., the isotopic images of fields, vector spaces, algebras, geometries, etc., remain isomorphic to the original structures by construction.

Isotopic liftings are mathematically nontrivial because they require the isotopies of the *entire* structure of the original theory into a simple yet unique and nontrivial form admitting of $\hat{I}$ as the new unit. This includes suitable isotopes of the number theory, functional analysis, algebras, geometries, etc.[3c,3e] The isotopies are also physically non-trivial because $\hat{I}$ possesses an unrestricted functional dependence on local space-time coordinates, wavefunctions and their derivartives, $\hat{I} = \hat{I}(x, \dot{x}, \ddot{x}, \psi, \partial\psi, \partial\partial\psi, \ldots)$. Such a dependence implies the mapping of linear-local-Lagrangian theories into nonlinear-nonlocal forms which are however such to reconstruct the linear, local and Lagrangian characters in isospace[3d,3f].

The most direct way to reach an isotopic structure is by submitting RQM to a *nonunitary* transformation $UU^\dagger = \hat{I} \neq 1$ on a conventional Hilbert space $\mathcal{H}$ under which we have, e.g., $U[x^\mu, p_\nu]U^\dagger = Ux^\mu p_\nu U^\dagger - Up_\nu x^\mu U^\dagger = \overline{X}^\mu T \overline{p}_\nu - \overline{p}_\nu T x^\mu = i\,\delta^\mu{}_\nu\, UIU^\dagger = i\,\delta^\mu{}_\nu \hat{I}$, $\overline{x}^\mu = Ux^\mu U^\dagger$, $\overline{p}_\nu = Up_\nu U^\dagger$, $T = (UU^\dagger)^{-1} = \hat{I}^{-1}$. However, it is easy to see that such an isotopic theory *is not* form-invariant under an additional unitary transform. It has also been proved by Lopez[5a] that such a theory does not preserve Hermiticity-observability at all times. In fact, the enveloping operator algebra ξ with elements A, B, and associative product AB is mapped under nonunitary transforms into the *enveloping isoassociative operator algebra* $\hat{\xi}$ with elements $\bar{A}, \bar{B}, \ldots$ and isotopic product $\bar{A}\ T\ \bar{B}$. Starting from the original condition of Hermiticity on $\mathcal{H}\{<|H^{\dagger}\}|>=<|\{H|>\}, H^{\dagger}=H$ that under nonunitary transforms still defined on a conventional Hilbert space $\mathcal{H}$ becomes $\{<|T\bar{H}^{\hat{\dagger}}\}|>=<|\{H\,T|>\}$, i.e., $\bar{H}^{\hat{\dagger}}=T^{-1}\bar{H}\,T$ which, as such, is generally violated. This general loss of form-invariance, Hermiticity-observability, etc. also holds for q-deformations, quantum groups and all theories of quantum gravity possessing nonunitary time evolutions yet defined on a conventional Hilbert space[5a].

The resolutions of the latter problems requires the *necessary* isotopies of the entire structure of RQM into a form called *relativistic hadronic mechanics* (RHM)[3f], including: A) the lifting of the field $F(a, +, x)$ of real numbers R or complex numbers C with conventional sum $a+b$ and product $a \times b + ab$ into the *isofields* $\hat{F}(\hat{a}, +, \hat{*})$ with *isonumbers* $\hat{a} = aI$ with sum $\hat{a}+\hat{b}=(a+b)\,\hat{I}$, product $\hat{a} * \hat{b} = \hat{a}\,T\,b)\,\hat{I}$, and all generalized operators (see[7] for details); B) the lifting of the conventional Minkowski space $M(x, \eta, R)$ in the chart $x=\{x^{\mu}\}$ $=\{r, x^4\}, x^4=c_0 t$, where c_0 is the speed of light in vacuum, η = diag, (1, 1, 1, −1), with invariant $x^2 = x^t\eta x$ on $R(n, +, x)$ into the *isominkowski* space[8a] $\hat{M}(x, \hat{\eta}, \hat{R}$ with *isometric* $\hat{\eta}=T(x, \dot{x}, \ddot{x}, \psi, \partial\psi, \partial\partial\psi, \ldots)\,\eta$, and *isoseparation* on $\hat{R}(\hat{n}, +, *)$ among two points x, y (see ref.[5c] for topological aspects)

$$x^{\hat{2}} = [(x^1-y^1)\,T_{11}(x,\ldots)\,(x^1-y^1) + (x^2-y^2)\,T_{22}(x,\ldots)\,(x^2-y^2) + (x^3-y^3)\,T_{33}(x,\ldots)\,(x^3-y^3) - (x^4-y^4)\,T_{44}(x,\ldots)\,(x^4-y^4)]\,\hat{I}; \tag{1}$$

C) the lifting of the original Hilbert space $\mathcal{H}$ with states $|>$ and inner product $<|> \varepsilon\, C(c, +, x)$ into the *isohilbert space* $\hat{\mathcal{H}}$ with *isoinner product* $<\hat{|}>=<|T|>\hat{I}\,\varepsilon\,\hat{C}(\hat{c}, +, *)$; D) the lifting of eigenvalue equations $H|>=E_0\,\|>$ into the isotopic form $H*|>=HT|>$

$= \hat{E} * | > = E \hat{I} T | > \equiv E | >, E \neq E_0$ indicating that the final numbers of the theory are the conventional ones; *E*) the lifting of the operator four-momentum $p_\mu | > = -i \partial_\mu | >$ into the isoform $p_\mu * | > = -\hat{\partial}_\mu | >$, where $\hat{dx} = Tdx$ is the *isodifferential* and $\hat{\partial}_\mu = \hat{I} \partial / \partial x^\mu$ is the *isoderivative*, *F*) the lifting of expectation values $< A > = < | A | > / < | >$ into the form $\hat{<} A \hat{>} = < | TAT | > / < | T >$; and the compatible liftings of the remaining aspects of RQM. Since $\hat{I} = UU^\dagger$ is Hermitean we can hereon assume it to be positive definite and diagonal.

The most important properties emerging from the above liftings are the following. First, RHM is a fully axiomatic theory because nonunitary transforms can always be written $U = \hat{U} T^{1/2}$, and therefore turned into the *isounitary transform* $\hat{U} * \hat{U}^{\hat{\dagger}} = \hat{U}^{\hat{\dagger}} * \hat{U} = \hat{I}$ under which RHM is form-invariant. In fact, the isounit of the theory is invariant $\hat{U} * \hat{I} * \hat{U}^{\hat{\dagger}} \equiv \hat{I}$ and so are the *fundamental isocommutation rules*, $\hat{U} * (x^\mu * p_\nu - p_\nu * x^\nu) \hat{U}^\dagger = \bar{x}^\mu * \bar{p}_\nu - \bar{p}_\nu * \bar{x}^\nu = i\delta^\mu{}_\nu \hat{U} * \hat{I} \times \hat{U}^{\hat{\dagger}} = i\delta^\mu{}_\nu \hat{I}$. Also all operators which are initially Hermitean remain so at all times. In fact, the condition of Hermiticity on $\mathcal{H}$ over $C(\hat{c}, +, *)$ now reads $\{< | T H^{\hat{\dagger}} \} | > = < | \{H T | > \}$ and, as such, it *coincides* with the Hermiticity on $\mathcal{H}$ over $C(c, +, \times)$, $H^{\hat{\dagger}} \equiv H^\dagger = H$. All observables of RQM therefore remain observables for RHM. Moreover, RHM and RQM coincide at the abstract level (for $\hat{I} > 0$) where $R(\hat{n}, +, *) \equiv R(n, +, x)$, $\hat{\xi} \equiv \xi$, $\hat{M}(x, \hat{\eta}, \hat{R}) \equiv M(x, \eta, R)$, $\hat{\mathcal{H}} \equiv \mathcal{H}$, etc. Also, RHM can approximate RQM as close as desired for $\hat{I} \approx I$ and admit the latter identically as a particular case for $\hat{I} \equiv I$. Finally, RHM admits all infinitely possible signature-preserving deformations $\hat{\eta} = T\eta$ of the Minkowski metric (universality) directly in the frame of the observer (direct university).

We now apply RHM to the *isoquantization of gravity* hereon called *quantum isogravity* (QIG). A first condition is the *representation of gravity via the isominkowskian, rather than the Riemannian space*. Let $\mathcal{A}(x, g, R)$ be a conventional (3+1)-Riemannian space with symmetric, nonsingular and real valued metric $g(x)$ and separation $x^2 = x^t gx \, \varepsilon \, R(n, +, \times)$. It is easy to see that $g(x)$ is *identically* admitted *as a particular case* of the isominkowski metric $\hat{\eta}(x, \dot{x}, \ddot{x}, ...)$ resulting in the local isomorphism $\mathcal{A}(x, g, R) \approx \hat{M}(x, \hat{\eta}, R)$, $g(x) \equiv \hat{\eta}(x)$.

The main idea of QIG is to embed gravitation *in the unit of conventional RQM.* This is permitted by the isotopic methods via the factorization of any given Riemannian metric in the form $g(x) = T_{gr}(x)\,\eta$, where $T_{gr}(x)$ is always positive-definite from the locally Minkowskian character of $\mathcal{A}$, and the joint lifting of the unit $I = \text{diag.}(1, 1, 1, 1)$ of RQM into the *gravitational isounit* $\hat{I}_{gr} = [\,T_{gr}\,x)]^{-1} = (\hat{I}_{gr}\,\mu_\nu) = (\hat{I}_\mu{}^\nu)$ resulting in RHM. Since the isometric of RHM in this case is the Riemannian metric, this results in a novel quantization of gravity which resolves the three basic problems indicated earlier. In fact, the quantization is via the *unit*, rather then the Hamiltonian; it is invariant under its own time evolution; and it is indeed a nonunitary image of the RQM. Moreover, the preservation of the basic axioms of RQM at the abstract level ensures the *mathematical* consistency of the theory, the understanding being that its *physical* consistency requires specific studies.

In short, the main conjecture submitted in this note is that a *consistent operator form of gravity already exists. It did creep in un-noticed until now because it is embedded in the unit of conventional RQM. In fact, the axioms of RHM imply that* $\hat{<}\hat{I}_{gr}\hat{>} = <|\,T_{gr}\,T_{gr}^{-1}\,T_{gr}\,|>/<|\,T_{gr}\,|> = 1$, *thus confirming the "hidden" character of gravitation in conventional RQM. As an illustration, the embedding of gravity in Dirac's equation can be written*

$$(\hat{\gamma}^\mu * p_\mu + i\hat{m}) * |> = [\hat{\hat{\gamma}}^\mu(x)\,T_{gr}(x)\,\hat{\eta}_{\mu\nu}(x)\,p^\nu - i\,m\,\hat{I}\,]\,T_{gr}(x)\,|> = 0\,, \tag{2a}$$

$$[\{\hat{\gamma}^\mu, \hat{\gamma}^\nu\} = \hat{\gamma}^\mu\,T_{gr}\,\hat{\gamma}^\nu + \hat{\gamma}^\nu\,T_{gr}\,\hat{\gamma}^\nu\,T_{gr}\,\hat{\gamma}^\mu = 2\hat{\eta}^{\nu\mu} \equiv 2g^{\mu\nu},\ \hat{\gamma}^\mu = T_{\mu\mu}^{1/2}\,\gamma^\mu\,\hat{I}_{gr}\,, \tag{2b}$$

where γ^μ are the conventional gammas and $\hat{\gamma}^\mu$ are called *isogamma matrices*. The important point is that at the abstract level the conventional and isogravitational Dirac equations coincide from the topological equivalence of I and $\hat{I}_{gr}$, $(\gamma^\mu p_\mu + im)\,|> \equiv (\hat{\gamma}^\mu * p_\mu + i\hat{m}) * |>$. Note that *the anticommutator of the isogamma matrices yields (twice) the Riemannian metric $g(x)$, thus confirming the full embedding of gravitation. As an example, the Dirac-Schwartzschild equation* (there presented for the first time) is given by equation (2) with $\hat{\gamma}_k = (1 - 2M/r)^{-1/2}\,\gamma_k\,\hat{I}_{gr}$ and $\hat{\gamma}_4 = (1 - 2M/r)^{1/2}\,\gamma_4\,\hat{I}_{gr}$. Similarly one can construct the *Dirac Krasner equation* and others or similar realizations for the Klein-Gordon, Weyl and any other relativistic field equation.

In order to initiate the appraisal of the possible physical relevance of QIG, we here identify the following primary implications:

Consequence 1: *QIG permits the introduction, apparently for the first time, of a universal symmetry for all possible gravitations called isopoincaré symmetry $\hat{P}(3.1)$[8], which results to be locally isomorphic to the conventional symmetry* P (3.1). The isosymmetry can be readily constructed via the Lie-isotopic theory[2a,3b,3d,3f,4,5b] and consists in the reconstruction of P (3.1) for the generalized unit $\hat{I}_{gr} = [\,T_{gr}(x)]^{-1}$, $g(x) = T_{gr}(x)\eta$. Since $\hat{I}_{gr} > 0$, one can see from the inception that $\hat{P}(3.1) \approx P(3.1)$. Under the lifting $P(3.1) \to \hat{P}(3.1)$ the original generators $X = \{X_k\} = \{M_{\mu\nu}, p_\alpha\}$, $M_{\mu\nu} = x_\mu p_\nu - x_\nu p_\mu$ $k = 1, 2, \ldots 10$, $\mu, \nu = 1, 2, 3, 4$, remain uncharged while the original parameters $w = \{w_k\} = \{(\theta, v), a\} \in R$ are lifted into the form $\hat{w} = w\hat{I} \in \hat{R}(\hat{n}, +, *)$. The connected component $\hat{P}_0(3.1)$ can be written via the exponentiation in ξ characterized by the *isotopic Poincaré-Birkhoff-Witt Theorem*[2a]

$$\hat{P}_0(3.1) : \hat{A}(\hat{w}) = \prod_k \hat{e}^{i\hat{X}*w} = \left(\prod_k e^{iXTw}\right)\hat{I}\,, \tag{3}$$

while the preservation of the original dimension is ensured by the *isotopic Baker-Campbell-Hausdorff Theorem*[2a]. It is easy to see that structure (3) forms a connected *Lie-isotopic transformation group* with *isogroup laws* $\hat{A}(\hat{w}) * \hat{A}(\hat{w}') = \hat{A}(\hat{w}') * \hat{A}(\hat{w}) = \hat{A}(\hat{w} + \hat{w}')$, $\hat{A}(\hat{w}) * \hat{A}(-\hat{w}) = \hat{A}(0) = \hat{I}_{gr} = [T_{gr}(x)]^{-1}$. Note that $\hat{P}(3.1)$ acts *isotransitively* in $\hat{M}(x, \hat{\eta}, \hat{R})$, i.e., $x' = \hat{A}(\hat{w}) * x$, because the preservation of the original transform $x' = Ax$ would now violate linearity in isospace. The isotopy of the discrete transforms is elementary, and reducible to the forms $\hat{\pi} * x = \pi x = (-r, x^4)$, $\hat{\tau} * x = \tau x = (r, -x^4)$, where $\hat{\pi} = \pi \hat{I}$, $\hat{\tau} = \tau \hat{I}$, where π, τ are the conventional inversion operators.

To identify the isoalgebra $\hat{p}_0$ (3.1) of $\hat{P}_0$ (3.1), we here introduce the *isodifferential calculus* on $\hat{M}$ characterized by the *isodifferentials* $\hat{d} x^\mu = T^\mu{}_\nu dx^\nu$ and *isoderivatives* $\hat{\partial}_\mu + \hat{\partial} / \hat{\partial} x^\mu = \hat{I}_\mu{}^\nu \partial_\nu$, $\partial_\nu = \partial / \partial x^\nu$ with isotopic forms of the various axioms of conventional differential calculus, e.g., $\hat{\partial}_\mu (f * g) = (\hat{\partial} f) * g + f * (\hat{\partial} \hat{g})$ $\hat{\partial}_\mu{}^{\hat{2}} = \hat{\partial}_\mu * \hat{\partial}_\mu = \hat{I}_\mu{}^\nu \hat{\partial}_\nu{}^2$, etc. By recalling that the coordinates in the covariant form in $\hat{M}$ are given by $x_\mu = \hat{\eta}_{\mu\nu} x^\nu$, we have the property $\hat{\partial} x_\mu / \hat{\partial} x^\nu = \hat{I}^\rho_\alpha \hat{\eta}_{\nu\rho} \partial x_\mu / \partial x_\alpha$ $= (T^0_\mu)^{-1} T_\nu{}^\delta \eta_{\delta\rho} \equiv \eta_{\mu\nu}$ (where the last term is the *conventional* Minkowskian metric). The *fundamental isocommutation rules* of RHM can therefore be written $[x_\mu, \hat{,} p_\nu] * | > = i\eta_{\mu\nu} | >$, namely, the Lie product is structurally generalized via nonunitary transforms, but the eigenvalues are the conventional ones. The isocommutation rules of p_0 (3.1) are then given by

$$[M_{\mu\nu} \hat{,} M_{\alpha\beta}] = i (\eta_{\nu\alpha} M_{\mu\beta} - \eta_{\mu\alpha} M_{\nu\beta} - \eta_{\nu\beta} M_{\mu\alpha} + \eta_{\mu\beta} M_{\alpha\nu}), \quad \text{(4a)}$$

$$[M_{\mu\nu} \hat{,} p_\alpha] = i (\eta_{\mu\alpha} p_\nu - \eta_{\nu\alpha} p_\mu), \; [p_\alpha \hat{,} p_\beta] = 0, \quad \text{(4b)}$$

where $[A, \hat{,} B] = AT_{gr}(x) B - BT_{gr}(x) A$ is the *Lie-isotopic product* (originally proposed in[2a]) which does indeed verify the Lie axioms as one can verify. The *isocasimir invariants* are then lifted into the forms

$$C^{(0)} = \hat{I}_{gr} = [T_{gr}(x)]^{-1}, \; C^{(1)} = p^{\hat{2}} = p_\mu * p^\mu = \hat{\eta}^{\mu\nu} p_\mu * p_\nu, \; C^{(3)}$$

$$= \tilde{W}_\mu * \hat{W}^\mu, \; \tilde{W}_\mu = \in_{\mu\alpha\beta\rho} M^{\alpha\beta} * p^0. \quad \text{(5)}$$

The local isomorphism $\hat{p}_0(3.1) \approx p_0(3.1)$ is transparently expressed by the fact that *the Poincaré algebra and its isotopic image possess the same structure constants.* This is sufficient, per se, to guarantee the axiomatic character of QIG. Note also that the *momentum operators become commutative in their isominkowskian representation* (while they are notoriously noncommutative in their Riemannian representation). This confirms the achievement of a *representation of gravitation in an isoflat space, i.e., a space possessing zero curvature in the isospace $\hat{M}$, but not in its projection in our space-time.*

Under sufficient boundedness, regularity and smoothness of the isotopic element, the space components $\hat{SO}(3)$, called *isorotations*[8b,8b] can be easily computed from isoexponentiations (3) yielding the explicit form in the (x, y)- plane

$$x' = x \cos (T_{11}^{1/2} T_{22}^{1/2} \theta_3) - y\, T_{11}^{-1/2} T_{22}^{1/2} \sin (T_{11}^{1/2} T_{22}^{1/2} \theta_3) , \qquad (6a)$$

$$y' = x\, T_{11}^{1/2} T_{22}^{-1/2} \sin (T_{11}^{1/2} T_{22}^{1/2} \theta_3) + y \cos (T_{11}^{1/2} T_{22}^{1/2} \theta_3) , \qquad (6b)$$

(see[3f] for general isorotations in all three Euler angles). Isotransforms (6) leave invariant all ellipsoidical deformations $xT_{11}x + yT_{22}y + zT_{33}z + r$ of the sphere $xx + yy + zz = r$ in the Euclidean space $E(r, \delta, R)$, $r = \{x, y, z)$, $\delta = \text{diag.}\ (1,\ 1,\ 1)$. Such ellipsoids become perfect spheres $\hat{r}^2 = (r^t \hat{\delta} r) \hat{I}_s$ in *isoeuclidean spaces* $\hat{E}(r, \hat{\delta}, \hat{R})$, $\hat{\delta} = T_s \delta, T_s = \text{Diag.}\ (T_{11}, T_{22}, T_{33})$, $\hat{I}_s = T_s^{-1}$, called *iso-spheres,*[9a] because of the deformation of the semiaxes $l_k \to T_{kk}$ while the related units are deformed of the inverse amounts $l_k \to T_{kk}^{-1}$. This isosphericity is the geometric origin of the isomorphism $\hat{O}(3) \approx O(3)$.

The connected space-time isosymmetry $\hat{SO}(3.1)$ is characterized by the isorotations and the *isolorentz boosts*[8a,8d] which can be written in the (3,~4)-plane

$$x^{3'} = x^3 \sinh (T_{33}^{1/2} T_{44}^{1/2} v) - x^4\, T_{33}^{-1/2} T_{44}^{1/2} \cosh (T_{33}^{1/2} T_{44}^{1/2} v)$$

$$= \hat{\gamma} (x^3 - T_{33}^{-1/2} T_{44}^{1/2} \hat{\beta} x^4) , \qquad (7a)$$

$$x^{4'} = x^3\, T_{33}^{1/2} c_0^{-1} T_{44}^{-1/2} \sinh (T_{33}^{1/2} T_{44}^{1/2} v) + x^4 \cosh (T_{33}^{1/2} T_{44}^{1/2} v)$$

$$= \hat{\gamma} (x^4 - T_{33}^{1/2} T_{44}^{-1/2} \hat{\beta} x^3), \tag{7b}$$

$$\hat{\beta} = v_k T_{kk}^{1/2} / c_0 T_{44}^{1/2}, \quad \hat{\gamma} = | 1 - \hat{\beta}^2 |^{-1/2}. \tag{7c}$$

Note that the above isotransforms are *nonlinear in x,* as expected for a correct symmetry of gravitation, and are formally similar to the Lorentz transforms, as expected from their isotopic character. Isotransforms (7) characterize the *gravitational isolight cone*,[9b] i.e., the perfect cone in isospace $\hat{M}(x, \hat{\eta}, \hat{R})$. In fact, in a way similar to the isosphere, we have the deformation of the original light cone $l_\mu \to T_{\mu\mu}$ while the corresponding units are deformed of the inverse amount $l_\mu \to T_{\mu\mu}^{-1}$ thus preserving the original geometry as a necessary condition for an isotopy. The abstract identity of the light and isolight cones is the geometric origin of the isomorphisms $\hat{O}(3.1) \approx O(3.1)$.

In particular, the isolight cone possesses all properties of the conventional light cone, including the characteristic angle. The maximal causal speed in isospace therefore remains the speed of light in vacuum c_0. This is an evident important property for the physically correct characterization of quantum gravity (although it is per se insufficient on physical grounds).

The *isotranslations* can be written $x' = (\hat{e}^{ipa}) * x = x + aA(x)$, $p' = (\hat{e}^{ipa}) * p = p$, where $A_\mu = T_{\mu\mu}^{1/2} + a^\alpha [T_{\mu\mu}^{1/2} \hat{,} p_\alpha] / 1! + \ldots$ with "gravitational isoplanewave" $\psi = \hat{e}^{kx} = \{\exp(k\, T_s r - k_4 T_t c_0 \hat{t})\} \hat{I}$. The isoinversions have been indicated earlier. The above results imply the following:

THEOREM : *The isopoincaré symmetry is the universal invariance of all infinitely possible separations* (1), *thus providing the universal symmetry of gravitational as a particular case.*

Note that *there is nothing to compute* in the sense that for any arbitrarily given (diagonal) Riemannian metric $g(x)$ (such as Schwartzschild, Krasner, etc, [1a] one merely *plots* the $T_{\mu\mu}$ terms in the decomposition $g_{\mu\mu} = T_{\mu\mu} \eta_{\mu\mu}$ (no sum) in the above isotransforms. The invariance of the separation $x^t gx$ is then ensured by the construction of the isosymmetry, as one can easily verify. At any rate Lie symmetries are known to leave invariance their own unit. Note also that the (2 + 2)-de Sitter or other cases can be derived from the theorem via mere changes of signature or dimension of the isounit. Note finally that the above theorem includes invariances for theories much broader than the Riemannian metric, such as the invariance for the *isoriemannian metrics* $\hat{g} = T(x, \dot{x}, \ddot{x}, \hat{\partial}\psi, \partial\partial\psi, \ldots)\, g(x)$ currently under study for interior gravitational problems[3f,9b].

CONSEQUENCE 2: *QIG implies the geometric unification of the special and general relativity.* This is evidently due to the fact that all distinctions between the special relativity in Minkowski space and the general relativity in isominkowski space are now lost owing to the abstract identities $\hat{R}(\hat{n}, +, *) \equiv R(n, +, R)$, $\hat{M}(x, \hat{\eta}, \hat{R}) \equiv M(x, \eta, R)$, $\hat{\mathcal{H}} \equiv \mathcal{H}$, $\hat{P}(3.1) \equiv P(3.1)$, etc. An important implication is the elimination of the historical difference between the special and general relativities whereby the former admits the universal Poincaré symmetry, while the latter does not[1a]. Isogravitation emerges from QIG as possessing a universal symmetry which turns out to be locally isomorphic to the conventional Poincaré symmetry. The gravitational field on $\hat{M}(x, \hat{\eta}, R)$ must now be isocovariant under $\hat{P}(3.1)$ in essentially the same way as the electromagnetic field on $M(x, \eta, R)$ must be covariant under $P(3.1)$. Note the *necessity* of the *isoflat* representation of gravity for the very formulation of its universal isopoincaré symmetry. In fact, no isosymmetry can be constructed in the Riemannian space.

CONSEQUENCE 3: *QIG permits a novel approach to the unification of weak, electromagnetic and gravitational interactions via the embedding of gravity in the unit of conventional unified guage theories here called "iso-grand-unification", which is planned for study elsewhere. The conjecture here submitted is therefore that gravitation is already* contained in the existing unified gauge theories. It did escape identification until now because it is embedded in the *unit* of the theory (for the isotopies of the electromagnetic interactions see Ref.[2b] for the isotopies if gauge theories see ref.[10] and review[4d]).

CONSEQUENCE 4 : *QIG permits a novel approach to gravitational horizons as the zeros of (the space component of) the isounit, and of gravitational singularities as the zeros (the space component of) the isotopic element.* In fact, at the Schwartzschild's horizon $r = 2M$ the space isounit $\hat{I}_s = (1 - 2M / r) \times \text{diag.}(1, 1, 1)$ of the isosphere $\hat{r}^2 = (r^t \hat{\delta} r) \hat{I}_s$ is null, while at $r = 0$ the space isotopic element $T_s = (1 - 2M / r)^{-1} \times$ diag. (1, 1, 1) is null. Recall in this respect that the restriction of the isounits/isotopic elements to a sole x-dependence is grossly un-necessary for isotopic theories as illustrated by the above theorem. The extension of the above *exterior quantum isogravity* to the corresponding interior quantum isogravity is merely given by admitting a *nonlinearity in the velocities and in the derivatives of the wavefunction*, $\hat{I}_{gr}(x, \dot{x}, \ddot{x}, \psi, \partial\psi, \partial\partial\psi, ...)$. A more adequate formulation of gravitational horizons and singularities is then given by the zeros of the (space component of) the latter isounits and isotopic elements. This extension evidently permits a second generation of studies on gravitational collapse, black holes and all that, because it permits a quantitative treatment of internal effects, such as interior nonlocal and non-(first)- order-Lagrangian effects expected in very high densities, etc. which are outside any realistic treatment via the Riemannian geometry[3f].

Also, recall that the "universal constancy of the speed of light" is a philosophical abstraction because in interior conditions (such as in our atmosphere) light has a locally varying speed. The isopoincaré symmetry can directly represent the *actual* speed of light in interior conditions via the more general isotopic elements $\hat{T}_{\mu\mu} = T_{\mu\mu} / n_\mu^2$ (no sum), where $T_{\mu\mu}$ the gravitational term. Isoinvariant (1), when projected in our space-time, then yields the local speed of light $c = c_0 / n_4$ where n_4 is the familiar local index of refraction with the understanding that in isospace the maximal casual speed remains c_0 as indicated earlier. The space components n_k are evidently requested by isolorentz covariance which essentially provides a *space-time symmetrization of the index of refraction.* The latter symmetrization is important for a direct geometrization of the *inhomogenous and aniso-*

tropic of physical media (such as our atmosphere), e.g., via a dependence of the n's from the local density, the differentiation of the value of their space and time components, the factorization of a preferred direction *in the medium* (the underlying empty space remaining perfectly homogeneous and isotropic under isotopies), etc. (see ref.[3b,3e] for specific applications and available experimental verifications)

These features are not merely formal because the immediate exterior of gravitational horizons is not empty, but composed of hyperdense chromospheres in which the speed of light is *not* constant, thus implying the inapplicability of the conventional light cone. Our QIG resolves these problems too via the direct representation of the local variation of the light speed and the reconstruction in isospace of the perfect light cones thus permitting quantitative studies.

CONSEQUENCE 5 : *Space and time in QIG have a local character in the sense that their isounits have an explicit dependence on the local gravitational field itself.* In fact, the isotopic reformulation of gravity implies the redefinition under the conventional unit $x^t g x = x^t \hat{\eta} x = \bar{x}^t \eta \bar{x}$, $g = T_{gr} \eta$, $\bar{x} = x T^{1/2}_{gr}$. Riemannian coordinates are equivalent to space-time coordinates in our Minkowski space with *space isounits* $\hat{I}_k = T^{-1/2}_{kk}$ and *time isounit* $\hat{I}_t = T^{-1/2}_{44}$. As an example, the space- time isounits for an observer in the exterior Schwartzschild field are given by $\hat{I}_k = (1 - 2M / r)^{1/2} I_k$ and $\hat{I}_t = (1 - 2M / r)^{-1/2} (M > r)$. Note that the isogravitational theory recovers the relativistic Einstein space-time for $M = 0$ or $r \to \infty$, for which $\hat{I}_k \equiv \hat{I}_t = 1$. However, for a non-null gravitational field the isounits are different than the conventional units. QIG therefore predicts the capability of *alterings space and time via the alteration of their units*, evidently in addition to the Einstein variation with speed.

The above results pose the intriguing *experimental* question whether time here on Earth's atmosphere and say, time on Jupiter's atmosphere are different due to the difference of their gravitational fields as predicted by the isounit $\hat{I}_t = (1 - 2M / r)^{1/2}$, $r > 2M$ (in addition to conventional gravitational corrections).

With the understanding that the *mathematical* consistency of QIG is established by its Poincaré covariance, the resolution of the *physical* consistency of QIG requires experiments measures as to whether

> we live in space-time as conventionally understood, in which case the Riemannian description of gravitation is the physically correct one and the equivalent isominkowskian formulation has a mere mathematical character, or

> we live in isospace and isotime, in which case the isominkowskian description is the physically correct one and the Riemannian description has a mathematical value.

The above issue can be resolved with current technology by sending a probe to Jupiter's atmosphere capable of conducting comparative measures of time with respect to Earth.

We finally note that the *Lie-isotopies* were proposed as *closed-reversible* particular cases of the more general *Lie-admissible genotopies* for *open-irreversible* conditions[2a,3]. The *Lie-admissible quantum gravity*, or *quantum genogravity* (QGG) can be constructed from the formalism of this note by merely relaxing the condition that the isotopic element T is symmetric. QGG, rather than QIG, is more appropriate for the geometrization of interior irreversible gravitational processes and, as such it is the geometrization more appropriate of the novel Lie-admissible black hole dynamics recently introduced in ref. [11] (see also ref.[3.f,12] for additional Lie-admissible studies of gravita- tional). The content of this note was first presented at the *Seventh Marcel Grossmann Meeting on General Relativity,* Stanford University, July 24-29, 1994 (see the proceedings[1c]).

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