Iso-Representation of the Deuteron Spin and Magnetic Moment via Bohm's Hidden Variables

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In this paper, we review and upgrade the iso-representation of the spin 1/2 of nucleons according to the isotopic branch of hadronic mechanics, known as *hadronic spin,* which is characterized by an isotopy of Pauli's matrices with an explicit and concrete realization of Bohm's *hidden variable* λ and show, apparently for the first time, that it allows a consistent and time invariant representation of the spin $J_D = 1$ of the Deuteron in its true ground state, that with null angular contributions $L_D = 0$. We then show, also apparently for the first time, that the indicated hadronic spin allows a numerically exact and time invariant representation of the magnetic moment of the Deuteron with the numeric value $\lambda = 2.65557$.

1 The Einstein-Podolsky-Rosen argument

In the preceding paper [1], we have outlined the axiom-preserving completion of 20th century applied mathematics into *iso-mathematics,* (see [2] for an extended presentation and [5–7] for independent studies), and the related *iso-mechanical branch of hadronic mechanics* (see [3] for a detailed treatment, [8–10] for independent studies and [11–13] for recent reviews) which isotopic methods have been used for the verification in [14, 15] of the 1935 historical argument by A. Einstein, B. Podolsky and N. Rosen that *Quantum mechanics is not a complete theory* [16] (see [17] for the proceedings of the *2020 Teleconference in the EPR Argument*, and its overviews [18, 19]).

Via the use of said isotopic methods, [1] achieved, apparently for the first time, a non-relativistic and relativistic representation of *all* characteristics of the muons (including their recently measured anomalous magnetic moment) as an extended and naturally unstable hadronic bound state of electrons and positrons produced free in the spontaneous decay with the lowest mode.

In the subsequent paper [20], we showed that said isotopic methods confirm the 1983 experimentally unresolved deviations [21] from the conventional formulation of time dilation for composite particles such as the muons, in favor of its axiom-preserving isotopic completion. We indicated in [20] that said deviations are due to incompatibility of the conventional time dilation with the time-irreversible character of the muon decay voiced since 1967 by R. M. Santilli [22] (see the 1995 full treatment [3]) and independently voiced in 1968 by D. I. Blokhintsev [23] for the incompatibility of the conventional time dilation with internal non-local effects of composite particles.

In this paper, we review and upgrade the notion of *hadronic spin* first introduced in [3, Section 6.8, page 250] and then used for verification [14] of the EPR argument [16] as well as in other applications [4]. The new notion of hadronic spin is then used for the characterization of the spin 1/2 of the nucleons, and realized via an isotopy of Pauli's matrices with an explicit and concrete realization of Bohm's *hidden variable* λ [43]. We then show, apparently for the first time, that said hadronic spin allows the first known exact and time-invariant representation of the spin $S_D = 1$ of the deuteron in the true ground state, that with null contributions from angular momenta $L_D = 0$.

We then show, also apparently for the first time, that said hadronic spin allows a numerically exact and time-invariant representation of the magnetic moment of the Deuteron with $\lambda = 2.65557$.

A technical understanding of this paper requires a technical knowledge at least of [2, 3]. A preliminary understanding of this paper requires a knowledge of reviews [11–13].

2 Iso-representation of the Deuteron spin

As it is well known, the quantum mechanical spin 1/2 of nucleons is characterized by the fundamental irreducible representation of the special unitary Lie algebra SU(2) which is notoriously given by the celebrated *Pauli matrices*

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{1}$$

(where σ_3 is set hereon along the spin direction) with commutation rules

$$[\sigma_i, \sigma_j] = \sigma_i\sigma_j - \sigma_j\sigma_i = i2\epsilon_{ijk}\sigma_k\,. \tag{2}$$

The value $S = 1/2$ of the nucleon spin is characterized by the eigenvalue equations on a Hilbert space $\mathcal{H}$ over the field of complex numbers C with basis $|b\rangle$

$$\begin{aligned} &S_k = \tfrac{1}{2}\,\sigma_k\,, \\ &\sigma_3\,|b\rangle = \pm|b\rangle\,, \\ &\sigma^{\hat{2}}\,|b\rangle = (\sigma_1\sigma_1 + \sigma_2 v_2 + \sigma_3\sigma_3)\,|b\rangle = 3\,|b\rangle\,. \end{aligned} \tag{3}$$

A serious insufficiency of quantum mechanics in nuclear physics, which is fully supportive of the EPR argument [16], is that the representation of the spin 1/2 of nucleons via Pauli matrices does not allow a representation of the Deuteron spin $S_D = 1$ under the conditions of its experimental detection, that is, in its ground state with null orbital contributions $L_D = 0$. In fact, the sole possible *stable* bound state between a proton and a neutron permitted by quantum mechanics (qm) is the singlet

$$D = (n_\uparrow, n_\downarrow)_{qm}\,, \tag{4}$$

for which the total spin is null, $J_D = 0$. In an attempt of resolving this insufficiency while preserving quantum mechanics, nuclear physicists have assumed for about one century that the Deuteron is a bound state of a proton and a neutron in *excited orbits* such that $L_D = 1$ (see e.g. [25]).

When at Harvard University with DOE support, R. M. Santilli noted that the most effective way of resolving the above and other insufficiencies of quantum mechanics (see next section) is to *exit from its class of unitary equivalence.* Therefore, Santilli proposed in two 1978 memoirs [26, 27] and in two Springer Verlag monographs [28, 29], the EPR generalization / completion of quantum mechanics into a new discipline which he called *hadronic mechanics* (see the Abstract and [27, pages 684,749,777] and [29, page 112]).

Hadronic mechanics was conceived to be an *axiom-preserving, thus isotopic non-unitary image of quantum mechanics for the representation of the dimension, shape and density of hadrons in interior conditions with ensuing potential as well as non-potential interactions due to mutual penetration.*

The proposal voiced in [26]–[29] suggested the construction of the *time irreversible completion of quantum mechanics into hadronic mechanics with the basic time evolution* (see [27, (4.15.24), page 742], [29, (19), page 153] and [3, (4.3.1), page 154])

$$\begin{aligned} & i\,\frac{dA}{dt} = (A, H) = ARH - HSA = \\ & \qquad = (ATH - HTA) + (AJH + HJA)\,, \\ & A(t) = e^{HSti}\,A(0)\,e^{-itRH}\,, \\ & R = T + J\,, \quad s = -T + J\,, \end{aligned} \tag{5}$$

which is called Lie-admissible / Jordan-admissible since the bracket (A, H) clearly contains a Lie algebra $(ATH - HTA)$ and a Jordan algebra $(AJH + HJA)$ content.

By recalling that quantum mechanics can only represent systems whose time reversal images verify causality laws (because Heisenberg's equation is invariant under anti–Hermiticity), the aim of Santilli's proposal (stemming from his DOE support) was to achieve a consistent treatment of systems whose time reversal image violate causality, which is the case for all energy-releasing processes, with particular reference to nuclear fusions and fossil fuel combustion.

In this paper, we study *stable nuclei* that, as such, are time-reversal invariant. Consequently, our study requires the *Lie-isotopic branch of hadronic mechanics,* called for brevity *iso-mechanics,* which is based on the completion of the quantum mechanical enveloping associative algebra of Hermitean operators $A, B, ...$ on $\mathcal{H}$ over C with product $A \times B = AB$ and multiplicative unit I into the new product (first introduced in [26, (3.710), page 352] and [29, (5), page 71])

$$A \star B = A\hat{T}B, \quad \hat{T} > 0\,, \tag{6}$$

called *iso-product* because associativity-preserving, the positive-definite quantity $\hat{T}$ being called the *isotopic element* and new compatible multiplicative unit

$$\hat{I} = 1/\hat{T} > 0, \quad \hat{I} \star A = A \star \hat{I} \equiv A \;\forall\; A \;\in \mathcal{H}\,, \tag{7}$$

called *iso-unit* with ensuing basis time evolution first introduced in [27, (4.15.59), page 752] (see also [29, (18), page 163, Vol. II] and [3, (3.1.6), page 81])

$$i\,\frac{dA}{dt} = [A, H]^\dagger = A\hat{T}H - H\hat{T}A\,, \tag{8}$$

$$\begin{aligned} & A(t) = e^{H\hat{T}ti}\,A(0)\,e^{-it\hat{T}H} = W(t)\,A(0)\,W(t)^\dagger\,, \\ & WW^\dagger \neq I\,, \end{aligned} \tag{9}$$

which is called Lie-isotopic because of the clear verification of the Lie algebra axioms by the new brackets $[A, H]^\star$, although in a generalized form.

Following the identification of the basic structure (6) to (8), Santilli constructed in the 1983 monograph [29] the systematic isotopies in the 1983 volume [29] of the various branches of Lie's theory (universal enveloping associative algebra, Lie's theorems, Lie's transformation groups, etc.), resulting in a theory nowadays known as the *Lie-Santilli iso-theory* [5] (see also [30, 31]).

Santilli then constructed the isotopies of all known spacetime symmetries [32]–[42]. In particular, systematic studies were conducted on the construction, classification and verification isotopies of the SU(2)-spin symmetry which can be found in [3, Chapter 6, page 209 on], in papers [33]–[37] with a summary in Section 3 of [12].

The *hadronic spin* (first introduced in [3, Section 6.8]) is the characterization of the spin of hadrons under strong interactions via the iso-irreducible, iso-unitary, iso-representations of the Lie-Santilli iso-symmetry $\widehat{\mathrm{SU}}(2)$.

The simplest possible case of spin 1/2 of the nucleons can be outlined as following: all mathematical and physical aspects of the (regular [31]) isotopic branch of hadronic mechanics can be uniquely and unambiguously constructed via a simple, positive-definite *non-unitary transformation* set equal to the iso-unit of the new theory

$$UU^\dagger = \hat{I} > 0 \;\neq\; I, \;\; \hat{T} = 1/\hat{I} = (UU^\dagger)^{-1} > 0\,, \tag{10}$$

provided said non-unitary transformation is applied to the *totality* of the quantum mechanical, mathematical and physical quantities and their operations *with no exception known to the author*, to prevent insidious inconsistencies in mixing mathematics and iso-mathematics that generally remain undetected by non-experts in the field.

The indicated correct use of the above procedure permits the map of all quantum mechanical quantities, including unit, product, Lie algebras, etc., into their hadronic formulations that are generally denoted with a "hat"

$$\begin{aligned} I &\rightarrow UIU^\dagger = \hat{I}\,,\\ AB &\rightarrow U(AB)U^\dagger = \\ &\quad = (UAU^\dagger)(UU^\dagger)^{-1}(UBU^\dagger) = \hat{A} \star \hat{B}\,,\\ AB - BA &= [A,B] \rightarrow U(AB-BA)U^\dagger = \\ &\quad = \hat{A} \star \hat{B} - \hat{B} \star \hat{A} = [\hat{A},\hat{B}]^\star,\ \text{etc}\,. \end{aligned} \tag{11}$$

The hadronic spin $1/2$ for nuclear constituents is given by the iso-fundamental, iso-unitary, iso-irreducible iso-representation of the Lie-Santilli iso-algebra $\widehat{\mathrm{SU}}(2)$ under the condition of *iso-unimodularity*

$$\mathrm{Det}\,\hat{I} = 1\,. \tag{12}$$

The above condition allowed Santilli to characterize the basic iso-unit of iso-mechanics in terms of Bohm's *hidden variable* λ [43] which was presented for the first time in [3, (6.8.19), page 248], according to the rules

$$\begin{aligned} &\mathrm{Det}\,\hat{I} = \mathrm{Det}\,[(UU^\dagger)] = \mathrm{Det}\,[\mathrm{Diag}\,(g_{11}, g_{22})] = 1\,,\\ &g_{11} = g_{22}^{-1} = \lambda \geq 0\,, \end{aligned} \tag{13}$$

yielding the *iso-Pauli matrices* first proposed in [3, (6.8.20), page 248]

$$\begin{aligned} &\hat{\sigma}_k = U\sigma_k U^\dagger,\\ &UU^\dagger = \hat{I} = \mathrm{Diag}\,(\lambda^{-1}, \lambda),\ \ \hat{T} = \mathrm{Diag}\,(\lambda, \lambda^{-1}),\\ &\hat{\sigma}_1 = \begin{pmatrix} 0 & \lambda \\ \lambda^{-1} & 0 \end{pmatrix},\ \hat{\sigma}_2 = \begin{pmatrix} 0 & -i\lambda \\ i\lambda^{-1} & 0 \end{pmatrix},\\ &\hat{\sigma}_3 = \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & -\lambda \end{pmatrix}, \end{aligned} \tag{14}$$

and then used in [14] for the verification of the EPR argument thanks to the evident inapplicability of Bell's theorem [44] due to the non-unitary structure of the theory.

It is easy to see that the iso-Pauli matrices verify the Lie-Santilli iso-commutation rules

$$\begin{aligned} [\hat{\sigma}_i, \hat{\sigma}_j]^* &= \hat{\sigma}_i \star \hat{\sigma}_j - \hat{\sigma}_j \star \hat{\sigma}_i = \\ &= \hat{\sigma}_i \hat{T} \hat{\sigma}_j - \hat{\sigma}_j \hat{T} \hat{\sigma}_i = i2\epsilon_{ijk}\hat{\sigma}_k\,, \end{aligned} \tag{15}$$

showing the clear iso-morphism $\widehat{\mathrm{SU}}(2) \approx \mathrm{SU}(2)$.

The representation of the spin $1/2$ of nucleons despite its generalized structure is given by the iso-eigenvalues on an iso-state $|\hat{b}\rangle$ of the *Hilbert-Myung-Santilli iso-space* $\hat{\mathcal{H}}$ [45] over the iso-field of iso-complex iso-numbers $\hat{C}$ [46]

$$\begin{aligned} &\hat{S}_k = \hat{\tfrac{1}{2}} \star \hat{\sigma}_k = \tfrac{1}{2}\hat{\sigma}_k\,,\\ &\hat{\sigma}_3 \star |\hat{b}\rangle = \hat{\sigma}_3 \hat{T}\,|\hat{b}\rangle = \pm|\hat{b}\rangle\,,\\ &\hat{\sigma}^{\hat{2}} \star |\hat{b}\rangle = (\hat{\sigma}_1\hat{T}\hat{\sigma}_1 + \hat{\sigma}_2\hat{T}\hat{\sigma}_2 + \hat{\sigma}_3\hat{T}\hat{\sigma}_3)\hat{T}\,|\hat{b}\rangle = 3\,|\hat{b}\rangle\,. \end{aligned} \tag{16}$$

As it is well known, *non-unitary theories violate causality,* and that is the case for the hadronic spin when considered in its projection on a conventional Hilbert space $\mathcal{H}$ over a conventional field C. Additionally, non-unitary transforms generally change the numeric value of the isotopic element which represent physical, measurable quantities (see next section). These and other problems are resolved by the reformulation of non-unitary time evolution (9) into the *iso-unitary iso-transformations* [47]

$$\begin{aligned} &WW^\dagger = \hat{I}, \quad W = \hat{W}\hat{T}^{1/2}\,,\\ &WW^\dagger = \hat{W} \star \hat{W}^\dagger = \hat{W}^\dagger \star \hat{W} = \hat{I}\,, \end{aligned} \tag{17}$$

under which reformulation the iso-unit, iso-product, Lie-Santilli iso-algebras, etc., are invariant,

$$\hat{I} \rightarrow \hat{W} \star \hat{I} \star \hat{W}^\dagger = \hat{I}' \equiv \hat{I}\,, \tag{18}$$

$$\begin{aligned} &\hat{A} \star \hat{B} \rightarrow \hat{W} \star (\hat{A} \star \hat{B}) \star \hat{W}^\dagger = \\ &\quad = \hat{A}' \star \hat{B}' = \hat{A}'\hat{T}'\hat{B}',\ \ \hat{T}' \equiv \hat{T}\,,\\ &\hat{A}' = \hat{W} \star A \star \hat{W}^\dagger,\ \ \hat{B}' = \hat{W} \star \hat{B} \star \hat{W}^\dagger\,,\\ &\hat{T} = (W^\dagger \star \hat{W})^{-1}\,. \end{aligned} \tag{19}$$

It should be noted that, by no means, hadronic spin solely characterizes the spin $1/2$ because it was conceived [26, 27] for the characterization of the most general possible notion of spin for an extended particle such as a hadron in the core of a star with ensuing *non-local contributions from the star environment* (see [3, 14], [34]–[37]) according to the *de Broglie-Bohm non-local theory* [48]. The notion of hadronic spin was then specialized to the spin of nucleons because of clear experimental evidence, rather than popular views in nuclear physics, establishing its value $1/2$.

The iso-representation of the Deuteron spin $J_D = 1$ in its true bound state with $L_D = 0$ via the hadronic spin is elementary. To see it, let us call for clarity *iso-protons, iso-neutron, iso-nucleons, iso-Deuteron and iso-Helium* (with corresponding symbols $\hat{p}, \hat{n}, \hat{N}, \hat{D}, \widehat{He}$), the particles and nuclei characterized by the hadronic spin. With reference to [3, Section 2.11, page 265 on] on the addition of hadronic spins, the most stable hadronic bound state of the iso-Deuteron as a hadronic bound state of an iso-proton and an iso-neutron is given by the *axial triplet state.* The axial triplet coupling first identified in the new chemical species of magnecules (see [49, Chapter 8, page 303 on] and [50, 51]) and then used for the new Intermediate Controlled Nuclear Fusion [52–54] with iso-representation (Fig. 1)

$$\hat{D} = \begin{pmatrix} \hat{p}_{\uparrow} \\ \star \\ \hat{n}_{\uparrow} \end{pmatrix} . \tag{20}$$

3 Iso-representation of the Deuteron magnetic moment

Another serious limitation of quantum mechanics in nuclear physics has been the inability, in about one century of studies, to achieve an exact representation of nuclear magnetic moments via the tabulated values for the magnetic moments of the proton and of the neutron in vacuum [55]

$$\mu_p = +2.79285\,\mu_N, \quad \mu_n = -1.91304\,\mu_N\,, \tag{21}$$

where μ_N represents the *nuclear magneton.*

As an example, the magnetic moment predicted by quantum mechanics (qm) from values (21) for the magnetic moment of the Deuteron is given by

$$\mu_D^{qm} = \mu_p + \mu_n = (2.79285 - 1.91304)\,\mu_N = 0.87981\mu_N\,, \tag{22}$$

and does not represent the experimental value of the Deuteron magnetic moment

$$\mu_D^{ex} = 0.85647\,\mu_N\,, \tag{23}$$

due to a deviation in *excess* of about 3%,

$$\mu_D^{qm} - \mu_D^{ex} = 0.02334\,\mu_N \approx 2.95\%\,\mu_D^{ex}\,, \tag{24}$$

with larger deviations for heavier nuclei.

E. Fermi [56], V. F. Weisskopf [25] and other founders of nuclear physics formulated the hypothesis, hereon referred to as the *Fermi-Weisskopf hypothesis*, that in the transition from isolated particles in vacuum to members of a nuclear structure, protons and neutrons experience a deformation of their extended charge distribution with consequential change of their magnetic moments (21) while conserving their spin 1/2 (see the statement at the top of [25, page 31]).

The first numerically exact and time-invariant representation of the Deuteron magnetic moment (23) was achieved in 1994 by Santilli [57] (see also its subsequent extended study in [58]) thanks to the prior construction of the isotopic branch of hadronic mechanics for the representation of extended, thus deformable hadrons and related iso-symmetries [32]–[42] with the isotopic element

$$\hat{T} = \mathrm{Diag}\left(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, \frac{1}{n_4^2}\right), \tag{25}$$

in which $n_k^2,\ k = 1, 2, 3$, represent the semi-axes of the *deformable* proton and of the nucleon under strong nuclear forces and n_4^2 repesents their density. Under the assumption, for simplicity, that the proton and the neutron in the Deuteron structure have the same dimension, shape and density, [57] reached a numerically exact and time invariant representation of the magnetic moment of the Deuteron in [57, (3.6), page 124] with the following values of the characteristic n-quantities (that are denoted with the symbols $b_\mu = 1/n_\mu$ in [57])

$$\begin{aligned} b_1 &= \frac{1}{n_1} = b_2 = \frac{1}{n_2} = 1.0028\,, \\ b_3 &= \frac{1}{} \quad 1.662\,, \ \ b_4 = \frac{1}{n_4} = 1.653 \end{aligned} \tag{26}$$

(whose derivation is not reviewed here for brevity), by therefore confirming the 1981 preliminary experimental verification of the Fermi-Weisskopf hypothesis via neutron interferometry [59].

In this paper, we present, apparently for the first time, a second numerically exact and time invariant representation of the magnetic moment of the Deuteron (23), with spin $S_D = 1$ in its ground state via the representation of Santilli's iso-Pauli matrices (14) by using the Clifford's algebra representation of the conventional Pauli matrices [60]–[64], whose representation is here assumed to be known for brevity.

Note that, when formulated on their associative enveloping algebra, the iso-Pauli matrices satisfy all algebraic properties of the conventional Pauli matrices. Consequently, we can use the conventional representation in its entirety and introduce the *representation of iso-Pauli matrices (14) in terms of Clifford algebra* $\tilde{\mathbf{G}}_3 = \tilde{\mathbf{G}}_3(\mathbf{R^3})$ with the iso-basis

$$\tilde{\mathbf{G}}_3 : \{1, \hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3, \hat{\sigma}_1\hat{\sigma}_2, \hat{\sigma}_1\hat{\sigma}_3, \hat{\sigma}_2\hat{\sigma}_3, i := \hat{\sigma}_1\hat{\sigma}_2\hat{\sigma}_3\}\,, \tag{27}$$

and main properties

$$\begin{aligned} &\hat{\sigma}_1^2 = \hat{\sigma}_2^2 = \hat{\sigma}_3^2 = 1\,, \\ &\hat{\sigma}_{12} = \hat{\sigma}_1\hat{\sigma}_2 = -\hat{\sigma}_{21},\ \hat{\sigma}_{13} = \hat{\sigma}_1\hat{\sigma}_3,\ \hat{\sigma}_{23} = \hat{\sigma}_2\hat{\sigma}_3\,, \\ &\hat{\sigma}_{12}^2 = \hat{\sigma}_1\hat{\sigma}_2\hat{\sigma}_1\hat{\sigma}_2 = -\hat{\sigma}_1\hat{\sigma}_2\hat{\sigma}_2\hat{\sigma}_1 = -\hat{\sigma}_1^2\hat{\sigma}_2^2 = -1\,. \end{aligned} \tag{28}$$

The *standard basis* of unit iso-vectors $\{\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3\}$ define the x, y, z iso-coordinate axes, respectively. The *iso-spectral basis* is

$$\begin{pmatrix} \hat{u}_+ & \hat{\sigma}_1\hat{u}_- \\ \hat{\sigma}_1\hat{u}_+ & \hat{u}_- \end{pmatrix}, \tag{29}$$

where $\hat{u}_\pm := \frac{1}{2}(1 \pm \hat{\sigma}_3)$ are *mutually annihilating iso-idempotents*. In the standard iso-basis of $\hat{\mathbf{G}}_\mathbf{3}$,

$$\begin{aligned} &\{\hat{\sigma}_1,\ \hat{\sigma}_2 = i\hat{\sigma}_1\hat{\sigma}_3,\ \hat{\sigma}_3\}\,, \\ &\hat{i} = \hat{\sigma}_1\hat{\sigma}_2\hat{\sigma}_3\,, \end{aligned} \tag{30}$$

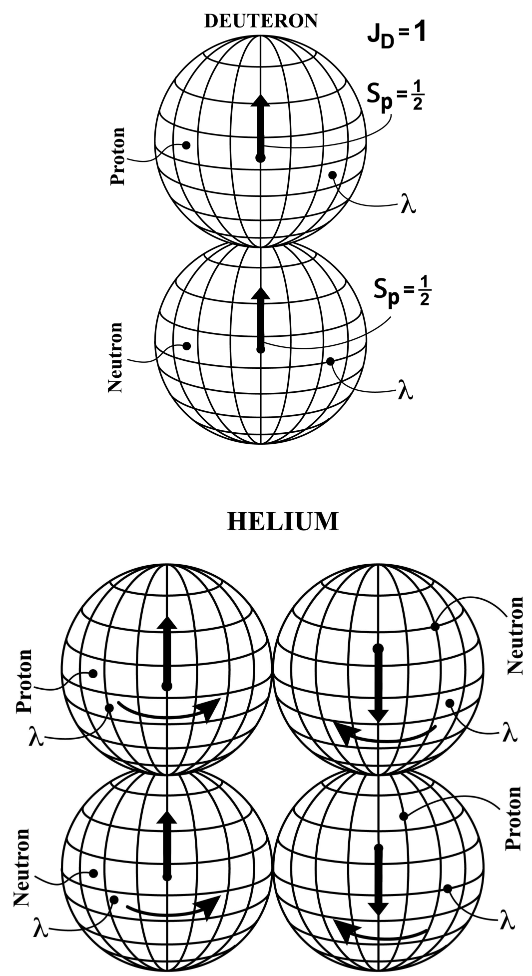

Fig. 1: In the top, we illustrate the structure of the iso-Deuteron as a hadronic bound state of an iso-proton and an iso-neutron in axial triplet coupling, thus representing for the first time the spin of the Deuteron $S_D = 1$ in its ground state, that with null angular contributions $L_D = 0$. The prefix "iso" represents the novel hadronic spin characterized by the iso-Pauli matrices, (14), with an explicit and concrete realization of Bohm's hidden variable λ. The axial triplet coupling was first identified in the new chemical species of magnecules (see [49, Chapter 8, page 303 on] and [50, 51]) and then used for the new Intermediate Controlled Nuclear Fusion [52–54]. In the bottom, we illustrate the structure model of the iso-Helium as a hadronc bound state under strong interactions of two iso-Deuterons in singlet coupling, which allows a representation of the null spin and magnetic moment of Helium in its ground state. It should be noted that the above model is not necessarily extendable to heavier stable nuclei due to the prior need of resolving the problem of nuclear stability caused by the natural instability of the neutron, which problem is planned for study in a subsequent paper (see [65] for a preliminary study).

is the *conventional unit* of the associative algebra $\tilde{\mathbf{G}}_3$. It must be remembered that $\hat{\sigma}_k$ verify the property

$$\hat{\sigma}_k^2 = \hat{\sigma}_k \star \hat{\sigma}_k = 1\,, \tag{31}$$

for $k = 1, 2, 3$, where the $\star$ denotes the iso-product.

We now show that the hidden variable λ of the iso-Pauli matrices (14) can provide a second representation of the deformation of the magnetic moment of nucleons of [57, 58] with consequential exact representation of nuclear magnetic moments.

By introducing the realization of the hidden variable λ

$$\lambda = e^{\phi} \geq 0\,, \tag{32}$$

with respect to the basis of the *standard unit* of the iso-Pauli matrices $\hat{I}$, the *iso-reciprocal* $\hat{T}$ and the *iso-vector basis* $\{\hat{\sigma}_k\}$, are given by

$$\begin{aligned} \hat{I} &= \cosh\phi + \sigma_3 \sinh\phi = e^{\phi\sigma_3}\,, \\ \hat{T} &= \cosh\phi - \sigma_3 \sinh\phi = e^{-\phi\sigma_3}\,, \end{aligned} \tag{33}$$

Consequently

$$\begin{aligned} \hat{\sigma}_1 &= \sigma_1 \hat{I} = \hat{T}\sigma_1\,, \\ \hat{\sigma}_2 &= \sigma_2 \hat{I} = \hat{T}\sigma_2\,, \\ \hat{\sigma}_3 &= \sigma_3 \hat{I} = \hat{I}\sigma_3\,. \end{aligned} \tag{34}$$

By recalling that σ_3 characterizes the nucleon spin $S = 1/2$, we reach the result that the replacement of the standard basis of the Clifford algebra $\mathbf{G_3}$ for Pauli matrices with the iso-Pauli matrices (14) implies the EPR completion of $\hat{\sigma}_3$ into the expression

$$\hat{\sigma}_3\,|\hat{b}\rangle = \sigma_3\,\hat{I}\,|\hat{b}\rangle = \sigma_3\,e^{\phi\sigma_3}\,|\hat{b}\rangle\,. \tag{35}$$

Recall that the quantum mechanical (qm) relationship between magnetic moments μ and spins S occurs via the gyromagnetic factor g,

$$\mu = gS\,, \tag{36}$$

and that the corresponding relation for the isotopic branch of hadronic mechanics (hm) is given by an expression of the type [57]

$$\mu_{hm}\,|\hat{b}\rangle = KgS\,|\hat{b}\rangle\,, \tag{37}$$

where K is an iso-renormalization constant of the gyromagnetic factor g created by the new notion of hadronic spin $1/2$. By using property (28), we reach the relation

$$\mu_{hm}\,|\hat{b}\rangle = e^{\phi\sigma_3}\mu_{qm}\,|\hat{b}\rangle = e^{\phi\sigma_3}gS\,|\hat{b}\rangle\,. \tag{38}$$

Recall also that: 1) Bohm's hidden variable λ is associated with the *spin* of a particle according to (14); 2) The proton and the neutron have the same spin $1/2$ and essentially the same mass, thus being characterized by the same λ; 3) The quantum mechanical representation of the magnetic moment of the Deuteron is *in excess* of about 3% according to (24). By selecting the value for conformity with the selected spin orientation (Fig. 1)

$$\sigma_3\,|\hat{b}\rangle = -|\hat{b}\rangle\,, \tag{39}$$

we can write the expression per each nucleon

$$\mu_{hm,k} \approx (1+\phi\sigma_3)\,\mu_{qm,k} = (1-\phi)\,\mu_{qm.k},\ \ k = p,n\,, \tag{40}$$

from which we obtain the iso-renormalized value of the magnetic moment of the proton and of the neutron

$$\hat{\mu}_p = +(1-\phi)\,2.79285\,\mu_N,\ \ \hat{\mu}_n = -(1-\phi)\,1.91304\,\mu_N\,, \tag{41}$$

with corresponding value for the magnetic moment of the Deuteron

$$\begin{aligned}\mu_D^{hm} &= (1-\phi)\,2.79285 - (1-\phi)\,1.91304\,\mu_N = \\ &= (1-\phi)\,0.87981\,\mu_N = \mu_D^{ex} = 0.85647\,\mu_N\,.\end{aligned} \tag{42}$$

From this, we obtain the numeric value

$$\phi = 1 - 0.87981/0.85647 = 1 - 0.02334 = 0.97666\,, \tag{43}$$

with corresponding *numeric value of Bohm's hidden variable for the Deuteron*

$$\lambda = e^{\phi} = e^{0.97666} = 2.65557\,, \tag{44}$$

by thereby achieving the desired exact representation of the magnetic moment of the Deuteron in terms of Bohm hidden variable λ. Its invariance over tine follows fom the derivation of iso-Pauli matrices (14) from the Lie-Santilli iso-symmetry $\hat{\mathcal{P}}(3.1)$ [39]–[41].

The iso-representation of the magnetic moment of $4-He-2$ as the *iso-Helium* $\widehat{He}(2)$ is a consequence (Fig. 1). The study of the iso-representation for heavier stable nuclei was initiated in [65], but its in-depth achievement requires the still missing consistent representation of nuclear stability against the natural instability of the neutron, which problem is planned for study in a subsequent paper.

We should finally note that in this section we have used the *standard Clifford algebra* and not the full isotopic Clifford algebra $\hat{\mathbf{G}}$ introduced by R. da Rocha and J. Vaz Jr. [66]. This is due to the fact that the full isotopy $\hat{\mathbf{G}}_3$ of $\mathbf{G_3}$ would have required the use of iso-product (6) with the isotopic element $\hat{T} = e^{-\phi\sigma_3} = 1/\hat{I}$, and the consequential lack of representation in (38) of the magnetic moment of the Deuteron for spin $S_D = 1$ in the ground state.

The understanding is however that the full iso-Clifford iso-algebra $\hat{\mathbf{G}}_{3N}$ is expected to be important for the numerically exact and time invariant representation of the spins and magnetic moments of nuclei with $A \geq 2$ nucleons.

In a nutshell, we can say that the Copenhagen interpretation of quantum mechanics deals with the simplest possible realization of quantum axioms, while the EPR completion of quantum into hadronic mechanics deals with progressively broader realizations of the same axioms for systems with progressively increasing complexity.

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