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Introduction to Isodual Mathematics and its Application to Special Relativity

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Abstract. The classical treatment of antimatter cannot be undertaken by conventional mathematics as charge conjugation is anti-homomorphic at quantum level. Thus, anti-homomorphism or better anti-isomorphism is the basic requirement for new mathematics to be able to deal with antimatter. In this paper we introduce units, numbers and fields of Isodual mathematics from which all remaining formulations can be uniquely and unambiguously, derived via simple compatibility arguments. Isodual Functional analyses, Isodual differential and Integral Calculus are covered. It is found that Special Relativity is inapplicable for the classical treatment of anti-particles. Special relativity is also afflicted by the historical inability to represent irreversible processes. We have also considered the application of Isodual Mathematics in the field of Special Relativity.

Keywords: Anti-isomorphism, Isodual mathematics, Special Relativity
PACS: 03.30+P:

INTRODUCTION

Paul Dirac's equations describing electron [1] were thought to be incorrect due to its impossible solutions. This impossible solution later on was accepted and gave birth to an epoch making discover of anti particle (positron). A few years later positrons were found among the particles generated by cosmic gamma rays, the radiation from outer space [2]. The positron is mathematically best described as an electron moving backwards in time. A positron behaves just like an electron with all its attributes reversed. When an electron and a positron come together they annihilate each other, all their mass is converted into energy. The end result of antimatter meeting matter is a release of energy proportional to the mass as the mass-energy equivalence equation, $E=mc^2$. Recent observations by the European Space Agency's INTEGRAL satellite may explain the origin of a giant cloud of antimatter surrounding the galactic center. *Existance of galaxies, stars and planets indicates that there is a subtle asymmetry between matter and antimatter, and that nature somehow favours the former. Two such asymmetries have indeed been found. But neither is big enough to explain why so much matter has survived.* At this time, the apparent asymmetry of matter and antimatter in the visible universe is one of the greatest unsolved problems in physics[3]. It is well known that the laws of nature obey a fundamental symmetry called "CPT" (charge conjugation, parity, and time reversal), which postulates that if all the matter in the universe were replaced with antimatter, left and right get inverted as if looking into a mirror, and the flow of time reversed, this "anti-world" would be indistinguishable from our real matter world.

The classical treatment of antimatter cannot be undertaken by conventional mathematics as charge conjugation is anti-homomorphic at quantum level. Thus, anti-homomorphism or better anti-isomorphism is the basic requirement for new mathematics to be able to deal with antimatter [4]. Ruggero Maria Santilli, Institute for Basic Research, FL, USA has developed Isodual Mathematics [5] to deal with antimatter. Our presentation involves beginning with a study of the most fundamental elements of all mathematical and physical formulations, units, numbers and fields, from which all remaining formulations get uniquely and unambiguously derived via simple compatibility arguments. Isodual functional analysis, isodual differential and integral calculus have been described in brief. It is found that special relativity is inapplicable for the classical treatment of anti-particles. Special relativity is also afflicted by the historical inability to represent irreversible processes [4, 5]. We have also considered the application of isodual-mathematics in the field of special relativity.

UNITS, NUMBERS AND FIELDS OF ISODUAL MATHEMATICS

Let the symbol \dagger denotes Hermitean conjugation, Hence, for real numbers 'n' we have $n^\dagger = n$, for complex numbers a we have $a^\dagger = a^c$ and for quaternions q we have $q^\dagger = q^c$. Let $F = F(a, +, \times)$ be a field (of characteristic zero), Isodual number is defined as $a^d = -a^\dagger$, additive isodual unit $0^d = 0$, multiplicative isodual unit $1^d = -1^\dagger$,

$$a^d = -a^\dagger, \quad a^d \cdot F^d, \quad (1)$$

with associative and commutative isodual sum

$$a^d +^d b^d = -(a + b)^\dagger = c^d \cdot F^d, \quad (2)$$

associative and distributive isodual product

$$a^d \times^d b^d = a^d \times (1^d)^{-1} \times b^d = c^d \cdot F^d, \quad (3)$$

additive isodual unit $0^d = 0$,

$$a^d +^d 0^d = 0^d +^d a^d = a^d, \quad (4)$$

and multiplicative isodual unit $1^d = -1^\dagger$,

$$a^d \times^d 1^d = 1^d \times^d a^d = a^d, \text{ \& } a^d, b^d \cdot F^d \quad (5)$$

Thus, $F^d = F^d(a^d, +^d, \times^d)$ is a Field called as Santilli's isodual fields. Isodual fields have a negative-definite norm, called as isodual norm. Also when a quantity and its isodual are same it is called as isoself dual. Isoself duality is an important concept as it has important application in the field of cosmology with reference to equal distribution of matter and antimatter in the universe. If F is a field, then its image F^d under the isodual map is also a field. It can be seen that the field F and its isodual images F^d are anti-isomorphic. This is the basis of isodual mathematics which is required to understand antimatter. Santilli's isodual mathematics is the perfect mathematics to deal with the problem of antimatter. Further we observe that isodual mathematics is the dual of conventional mathematics.

ISODUAL FUNCTIONAL ANALYSIS

Isodual special functions and transforms get constructed from conventional functions by applying isoduality.

Isodual trigonometric functions are defined as $\sin^d \theta^d = -\sin(-\theta)$, $\cos^d \theta^d = -\cos(-\theta)$, satisfying $\sin^{d2d} \theta^d + \cos^{d2d} \theta^d = 1^d = -1$ also isodual hyper geometric functions $\sinh^d \omega^d = -\sinh(-\omega)$, $\cosh^d \omega^d = -\cosh(-\omega)$ satisfy $\cosh^{d2d} \omega^d - \sinh^{d2d} \omega^d = 1^d = -1$

Isodual logarithmic and exponential functions are similarly defined in isodual mathematics as $\log^d n^d = -\log(-n)$ and $(e^{-A})^d = 1^d + A^d / 1^d! + A^{d2d} / 2^d! + \dots = -e^A$

ISODUAL DIFFERENTIAL AND INTEGRAL CALCULUS

Isodual differential is defined as $d^d x^k = 1^d \times dx^k = -dx^k$ i.e. $d^d x_k = -dx_k$ corresponding isodual partial derivatives are defined as $\partial^{d/d} \partial^d x^k = -\partial / \partial x^k$ and $\partial^{d/d} \partial^d x_k = -\partial / \partial x_k$

It can be noted that conventional differential are isoselfdual i.e. $(dx^k)^d = d^d x^{kd} = dx^k$, but derivatives are not self dual i.e. $(\partial f / \partial x^k)^d = -\partial^d f^d / \partial^d x^{kd}$

ISODUAL MATHEMATICS IN THE FIELD OF SPECIAL RELATIVITY

The isodual numbers have been induced for the generalization of the conventional space-time, algebras, geometries and mechanics which are the images of the conventional structures under an anti-homomorphic conjugation called isoduality. It is shown that isoduality of different space-time, is equivalent to charge conjugation in our own space-time, which leads to characterization of anti-particle via isodual numbers, spaces, algebras, geometries and mechanics. This leads to isodual universe which is geometrically separate from our universe with characteristics of negative energy $|E^d| < 0$ and also evolving backward in time $|t^d| < 0$ [6]. It is well known that special and general relativity do not distinguish between matter and antimatter therefore entire antimatter content cannot be treated by special or general relativity. Universal constancy of speed of light, the basic postulate of special relativity does not stand as it is experimentally found that speed of light varies from medium to medium. The speed of light C is a local quantity dependent on the characteristics in which the propagation occurs, with speed $C = c$ in vacuum, speeds $C \ll c$ within physical media of low density and speeds $C \gg c$ within media of very high density. The variable

character of the speed of light then seals the lack of universal special relativity is also afflicted by the historical inability to represent irreversible processes [7]. Isodual special relativity developed by Prof. Santilli deals with the classical relativistic treatment of point like anti-particles. We know that special relativity is constructed on the fundamental 4-dimensional unit of Minkowski space.

$I = \text{Diag.}(1,1,1,1) > 0 \rightarrow I^d = \text{Diag.}(\{-1,-1,-1,-1\}) < 0$ antimatter relativity is based on negative units of space and time. This implies the reconstruction of the entire mathematics of the special relativity with respect to common isodual unit I^d , Isodual field R^d with isodual nos. $n^d = nI^d$, the isodual Minkowski space time $M^d(x^d, \eta^d, R^d)$ with isodual coordinates $x^d = xI^d$, isodual metric $\eta^d = -\eta$ and basic invariant on R^d ,

$$(x^\mu - y^\mu) \eta_{\mu\nu}^d (x^\nu - y^\nu) I^d R^d$$

The basic postulates of the isodual special relativity are also a simple isodual image of the conventional postulates [7].

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