Iso-Minkowskian Geometry For Interior Dynamical Problems

Ruggero Maria Santilli

Contributed paper in
Cosmology, Quantum Vacuum, and Zeta Functions,
Diego Sez-Gmez Sergei Odintsov Sebasti Xamb Editors,
Springer, 2011.

Abstract By recalling that the exact validity of special relativity in vacuum has been experimentally established beyond doubt, we indicate mathematical, physical, chemical experimental and industrial evidence according to which physical media alter the Minkowskian spacetime; we outline the novel *iso-Minkowskian geometry* specifically built for interior dynamical problems; and we point out its universality for all possible spacetimes characterized by a symmetric metric in (3+1)-dimensions.

Key words: light, isoredshift, deformed Lorentz symmetry,
PACS 42.25.Bs, 98.80.-k, 42.62.-b, 03.30.+p,

1 Apparent Lack of Exact Validity of the Minkowskian and Riemannian Geometries for Interior Problems

Research conducted by numerous scholars during the past fifty years (see general review [1] and specialized treatments [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]) has identified mathematical, physical, chemical, experimental and industrial evidence according to which the Minkowskian geometry, special relativity and the Lorentz-Poincaré (LP) symmetry are not exactly valid for *interior dynamical problems* (e.g.,

---

Ruggero Maria Santilli
Institute for Basic Research, P. O. Box 1577, Palm Harbor, FL 34682, USA, e-mail: ibr@verizon.net, http://www.santilli-foundation.org

extended-deformable particles and electromagnetic waves propagating within physical media). Needless to say, the *exact validity* of the Minkowskian geometry, special relativity and the LP symmetry for *exterior dynamical problems* (point particles and electromagnetic waves propagating in vacuum) and their *approximate validity* for interior problems remain beyond scientific doubt.

The reasons for said insufficiency are numerous indeed, and include: the impossibility of introducing inertial reference frames within physical media (such as air or water) due to known resistive forces, with consequential inability of formulating the very principle of relativity, let alone testing it experimentally; physical media solely admit the privileged frame at rest with themselves in direct conflict with relativity axioms; massive particles, such as electrons, can travel in water faster than the local speed of light, thus forcing the assumption in *water* of the speed of light in *vacuum* as the maximal causal speed, in which case the sum of two local speeds of light does not yield the local speed of light in disagreement with the relativistic sum of speeds.

During the 20th century, all these insufficiency have been generally dismissed via the reduction of light to photons traveling in empty space while experiencing scattering, absorption and remission by the atoms of the medium. However, such a reduction is afflicted by major insufficiencies. As an illustration, for the case of light propagating in water, we have: the impossibility of a numerical representation of the large angle of refraction (since photons must scatter in all directions at the impact with the water surface); the impossibility of a numerical representation of the large reduction of the speed of light by about $1/3$ (since scattering, absorption and re-emission of photons can at best account for a small reduction of speed); the impossibility of reducing to photons electromagnetic waves with a large wavelength, e.g., of one meter, that experience the same phenomenology as that of light; the impossibility of the very existence of light within opaque media with consequential obliteration of the entire conceptual, mathematical and physical framework of special relativity; and numerous other insufficiencies [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

It should be indicated that Einstein introduced the reduction to photons solely for light with such a frequency to admit quantized black-body effects, which reduction is still entirely valid to this day and permits a quantitative representation of the small percentage of the light beam in water lost to dispersion. Definitely, Einstein did not voice the reduction of light to photons to claim the validity of his theory within physical media. The latter claim has been proffered by Einstein's *followers* without any serious scrutiny and despite rather visible inconsistencies. In fact, the reduction to photons of the *entire* beam of light in water against Einstein's teaching implies that an extremely big number of photons must traverse an extremely big number of nuclei without any deflection, as an evident necessary condition to maintain the propagation of a light beam in water along a visible straight line.

It is today known that the sole scientific, that is, quantitative-numerical, representation of *all* experimental; evidence for *all* frequencies is given by a return to the Maxwell conception of light (as well as photons as wave packets) as being *transverse electromagnetic waves* propagating in the universal substratum known as the *ether* with historical expression for the speed within transparent physical media

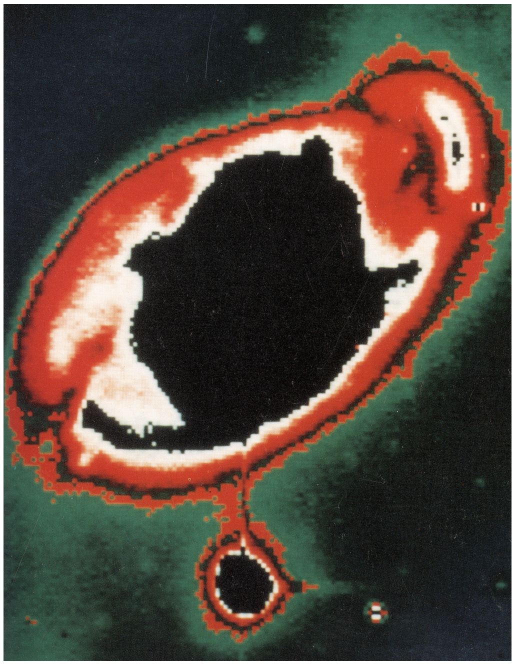

Fig. 1 A view of one of the first experimental deviations of the Minkowskian geometry within physical media identified in the 1970s by H. Arp [39] (see Sect. 3), the galaxy NGC 4319 (top) and the quasar Mark 205 (bottom) that result as being physically connected according to gamma spectroscopy, yet have largely different cosmological redshifts ($z = 0.0056$ and $z = 0.07$, respectively), whose quantitative representation requires different geometric features for the different interior physical media of the galaxy and the quasar

$$C(x, v, \omega, \delta, \tau, \dots) = \frac{c}{n(x, v, \omega, \delta, \tau, \dots)} , \tag{1}$$

where c is the speed of light in vacuum and n is the familiar index of refraction with an unrestricted dependence on all needed local variables, such as coordinates x, speed v, frequency ω, density δ, temperature τ, etc.

Evidently, the lack of exact character of the Minkowskian geometry necessarily implies the lack of exact character of the (pseudo-) Riemannian geometry for interior gravitational problems. Independently from physical and geometrical evidence, a most forceful argument is topological. The topology of exterior problems is notoriously local-differential, thus solely capable of representing a finite set of isolated points. By contrast, the correct topology for interior problems, especially interior gravitational problem with very high densities, must be of nonlocal-integral character due to the evident mutual penetration of extended and hyperdense charge distributions. Consequently, the Riemannian geometry cannot possibly be exactly valid for interior gravitational problems of stars, quasars and black holes beginning with its topological foundations.

Almost needless to say, the indicated limitations of the Riemannian geometry have no impact on its historical value because geometries can at best provide an approximation of out rather complex physical reality, while the *approximate* validity of the Riemannian geometry for interior gravitational problems remains beyond scientific doubt.

2 The Universal Iso-Minkowskian Geometry

The return to the Maxwellian conception of light as electromagnetic wave with local speed (1) brings into focus the so-called *Lorentz problem,* referred to the construction of the symmetry leaving invariant a locally varying speed of light. As well known to historians, Lorentz first attempted the achievement of the universal symmetry of local speed $C = c/n(x, v, \omega, \delta, \tau, \ldots)$, but encountered major technical difficulties that forced him to consider the simpler case of constant speed c by setting up in this way the foundation of special relativity.

The author has dedicated most of his research life to the Lorentz problem beginning with his Ph. D. studies in the mid 1960s. The first outcome of these studies is that Lorentz's inability to achieve the invariance of local speed (1) was due to *insufficiency of the basic theory, Lie's theory.* Independently from topological and other insufficiencies, Lie's theory is known as being *linear, local-differential and Hamiltonian.* By contrast, serious studies of interior problems ate large, including Lorentz problem, require a treatment which is *nonlinear, nonlocal-integral and non-Hamiltonian,* the latter characteristics being referred to the *variational nonselfadjointness* of interior problems [2] and the consequential impossibility of their representation via the sole knowledge of a Hamiltonian.

Hence, the solution of the Lorentz problem left no other alternative than that of working out a structural generalization of Lie's theory of nonlinear, nonlocal and non-Hamiltonian character. Along these lines, the author proposed in 1967 [16] a *Lie-admissible generalization of Lie's theory* subsequently specialized for the description of systems than, besides being nonlinear, nonlocal and non-Hamiltonian, are also *irreversible over time* (see memoir [17] of 1978, general presentation [18] of 2006 and review [4, 5]).

The Lie-admissible irreversible treatment of interior problems is excessively complex for the limited length of this note. Consequently, we have to restrict ourselves to a study of the subclass of *nonlinear, nonlocal and non-Hamiltonian interior problems that are reversible over time.* For the case of light propagating within a transparent medium such as water, the above subclass essentially requires ignoring in first approximation the percentage of the light beam lost to dispersion, under which light propagation is indeed reversible over time.

The latter class of systems can be quantitatively treated via the subclass of Lie-admissible theories known as *isotopies* (i.e., axiom-preserving lifting) of Lie's theory [2, 3, 4, 5, 6, 7], today known as the *Lie-Santilli isotheory,* [8, 9, 10, 11, 12, 13, 14, 15] which are based based on the lifting of the trivial unit of Lie's theory into the most general possible, integral-differential, positive-definite unit known as *Santilli isounit*

$$I = \text{Diag.}(1,1,1,\ldots) \to \hat{I}(x,v,\omega,\delta,\tau,\ldots) = 1/T > 0 \ , \tag{2}$$

with the joint lifting of the Lie algebras into the *Lie-Santilli algebras*

$$\left[J_i, J_j\right] = J_i J_j - J_j J_i = C_{ij}^k J_k \to \left[J_i \hat{,} J_j\right] = J_i T J_j - J_j T J_i = \hat{C}_{ij}^k J_k \ , \tag{3}$$

as well as of Lie's transformation groups into the *Lie-Santilli transformation isogroups*

$$A(w) = e^{iJw} A(0) e^{-iwJ} \to A(w) = e^{iJTw} A(0) e^{-iwTJ} \ , \tag{4}$$

with the Lie's parameters w and generators J remaining unchanged under isotopies.

To understand the complexity of the Lorentz problem, and the decades of "out of the mainstream" research required for its solution, let us recall that he invariance of the *constant* speed c is known as being canonical at the classical level and unitary at the operator lever, thus enjoying a majestic axiomatic and physical consistency, including: the same numerical predictions under the same conditions at different time; preservation over time of Hermiticity-observability; verification of causality and conservation laws; etc.

By contrast, the invariance of the locally varying speed of light (1) soon emerged as being *noncanonical* at the classical level and *nonunitary* at the operator level and, thus verifying the so-called *Theorems of Catastrophic Mathematical and Physical Inconsistencies of Noncanonical and Nonunitary Theories* (see original works [18, 19, 20, 21, 22, 23] and review [5]a, including: the prediction of different numerical values under the same conditions at different times (inconsistency that, alone, prevents any possible invariance of $C = c/n$); loss over time of Hermiticity and, therefore, of observability (an occurrence known as the *Lopez Lemma* [20]); violation of causality and conservation laws; and other inconsistencies.

The resolution of these inconsistency problems required decades of solitary studies and was solely achieved in 1996 in mathematical memoir [24] with t the correct formulation of the covering Lie-Santilli isotheory via a new mathematics, today known as *isomathematics* [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] consisting of the isotopies of the entire mathematics used in Lie's theory, including the lifting of numbers, functional analysis, differential calculus, etc. into such a form admitting $\hat{I}(x,v,\omega,\delta,\tau,\ldots)$, rather than I, as the correct left and right unit at all levels. Readers with a vast knowledge of Lie's theory, but not experts in the covering Lie-Santilli isotheory, should be alerted that, in the event only *one* of the isotopies is not used, e.g., the isotheory is treated with the conventional differential calculus, the inconsistency theorems are activated resulting in the absence of mathematical and physical maturity.

Following these decades in the prior construction of the isotopies of Lie's theory, the author constructed the srep-by-step isotopies of all aspects of special relativity, including th isotopies of: the rotational symmetry [25, 26]; the SU(2)-spin symmetry [27, 28]; the Lorentz symmetry at the classical [29] and operator [30] levels; the Poincaré symmetry [31]; the spinorial covering of the Poincaré symmetry [32];

including the isotopies of the axioms and physical laws of special relativity, first formulated in monographs [23] of 1991, and then studies in various works (see monographs [3, 4, 5, 6, 7] and vast literature quoted therein).

The main lines of spacetime isotopies are now elementary. The most fundamental geometric point of this paper is that *the alteration of any characteristics of light cannot occur without a modification of the Minkowskian spacetime.* Consequently, the alteration of the speed of light requires a corresponding lifting of the Minkowskian metric from its historical form with constant c, $\eta = \text{Diag.}(1,1,1,-c^2)$, to a generalized metric characterizing the local speed c^2/n^2 whose most general possible symmetric realization can be expressed with the lifting [29]

$$\begin{aligned} \eta &= \text{Diag.}\left(1,1,1,- \rightarrow c^2\right) \\ \hat{\eta} &= T(x,v,\omega,\delta,\tau,\ldots) \\ \eta &= \text{Diag.}\left(1/n_1^2, 1/n_2^2, 1/n_3^2, -c^2/n_4^2\right), \\ n_\mu &= n_\mu(x,v,\omega,\delta,\tau,\ldots) > 0\;, \end{aligned} \tag{5}$$

($\mu = 1,2,3,4$), the corresponding lifting of the spacetime invariant

$$\begin{aligned} x^2 &= \left(x^\mu \eta_{\mu\nu} x^\nu\right) \times I \\ &= \left(x_1^2 + x_2^2 + x_3^2 - t^2 c^2\right) \times I \in \mathscr{R} \rightarrow \hat{x}^{\hat{2}} \\ &= \left[x^\mu \left(T_\mu^\rho \eta_{\rho\nu}\right) x^\nu\right] \times \hat{I} \\ &= \left(\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2}\right) \times \hat{I} \in \hat{\mathscr{R}}\;, \end{aligned} \tag{6}$$

and of the lifting of the basic unit

$$I = \text{Diag.}(1,1,1,1) \rightarrow \hat{I} = \text{Diag.}\left(n_1^2, n_2^2, n_3^2, n_4^2\right)\;, \tag{7}$$

where: $n_4 = n$ acquires the meaning of the *time characteristic function* of the medium considered; the n_k, $k = 1,2,3$, are the *space characteristic functions;* $\mathscr{R}$ ($\hat{\mathscr{R}}$) is the field of real numbers (real isonumbers); and the multiplication by $\hat{I}$ is necessary for $\hat{x}^{\hat{2}}$ being an isonumber. Note that the characteristic functions n_μ are far from being the usual "free parameters" since they represent measurable characteristics of the medium, such as size, density, index of refraction, inhomogeneity and anisotropy normalized to the value $n_\mu = 1$, $\mu = 1,2,3,4$ for the vacuum (see Refs. [3, 4, 5, 6, 7] for details).

Since the generators are not altered in the transition from the Lie to the Lie-Santilli covering theory, the isotopies of all possible spacetime (as well as internal) symmetries are done via the use of now elementary rules (2)–(7) that lead Santilli [29] in 1983 to the first formulation of the universal symmetry of invariance (6) that we can write here for the simple case in $(3,4)$-dimension (see monograph [4]b for the general case)

$$x'^1 = x^1, \qquad x'^2 = x^2, \tag{8a}$$

$$x'^3 = \hat{\gamma}\left(x^3 - \hat{\beta}\frac{n_4}{n_3}x^4\right), \qquad x'^4 = \hat{\gamma}\left(x^4 - \hat{\beta}\frac{n_3}{n_4}x^3\right), \tag{8b}$$

$$\hat{\gamma} = \frac{1}{\sqrt{1-\hat{\beta}^2}}, \qquad \hat{\beta} = \beta\frac{n_4}{n_3} = \frac{v_3}{c}\frac{n_4}{n_3}. \tag{8c}$$

The corresponding isotopies of the Poincaré symmetry [24, 25, 26, 27, 28, 29, 30, 31, 32] are today known as the *Lorentz-Poincaré-Santilli (LPS) isosymmetry.*

On primitive grounds, the geometry underlying the isotopies of the Minkowskian spacetime is given by a novel geometry first formulated in Ref. [29] of 1983, then studied in various works, finalized in paper [33] of 1998, and today known as the *Minkowski-Santilli isogeometry,* or *isogeometry* for short, essentially consists in the reconstruction of the entire formulation of the Minkowskian geometry with respect to isounit (7). The proof of the following property is instructive for the non-initiated reader:

Lemma 1 ([29, 34, 35]). *The Minkowski-Santilli isogeometry is "directly universal" for symmetric* $(3+1)$*-dimensional spacetimes, in the sense of admitting as particular cases all possible spacetime geometries, thus including the Minkowskian, Riemannian, Finslerian and other geometries ("universality") directly in th isometric, thus without transforming the coordinates of the observer ("direct universality").*

Some of the features of the isogeometry should not appear unusual due to the novelty of the underlying isomathematics. For instance, the Minkowski-Santilli isogeometry admits the entire machinery of the Riemannian geometry (such as Christoffel's symbols, covariant derivative, etc.), trivially, due to the explicit dependence of the isometric $\hat{\eta}_{\mu\nu} = T_\mu^\rho \eta_{\rho\nu}$ in the local coordinates. Yet, *the novel isogeometry has null curvature,* trivially, as a central condition for being a correct isotopy of the Minkowski geometry. This occurrence too should not be surprising because, in the final analysis, the center of a massive body (with spherical symmetry) has null gravitational force.

As an illustrative example, one can *identically* reformulate any Riemannian metric, such as the Schwartzschild metric, in terms of the characteristic quantities $1/n_\mu^2$ (see Ref. [4]b for details). In this case, the geometry does indeed require curvature when the metric is referred to the trivial unit I. However, the same geometry show no curvature when formulated with respect to the isounit $\hat{I}$, trivially, because the latter has the *inverse* value n_μ^2, resulting in an invariant flatness under lifting (6). Note that the elimination of curvature is *necessary,* to our best understanding, to achieve the universal invariance of all Riemannian line elements, as well as to bypass the activation of the inconsistency theorems caused by the conventional covariance [18, 19, 20, 21, 22, 23]. At any rate,the lack of experimental detection of gravitational waves pointed out by C. Corda [36] appears to confirm all these lines.

In closing, mathematical inclined readers should be aware that all the above results are permitted by a structural generalization of the conventional, 20th century,

Fig. 2 Views taken by the author in Palm Harbor, Florida, of the horizon when the Sun is at the Zenith (left), at Sunset (top right) and Sunrise (bottom right), illustrating the predominant blue color when the Sun is at the Zenith and the predominant red color at both Sunset and Sunrise. As reviewed in Sect. 3, these colors constitute visible evidence of deviations from the Minkowskian geometry in our atmosphere in full agreement with Arp's discovery indicated in Fig. 1

local-differential topology into its isotopic covering initiated by the mathematicians Gr. T. Tsagas and D. S. Sourlas [37], completed by R. M. Falcon Ganfornina and J. Nunez Valdes [14, 38] and today known as the *Tsagas-Sourlas-Ganformina-Nunez (TSGN) isotopology.*

3 Experimental Verifications

The novel isomathematics, related geometries and physical formulations for non-relativistic and relativistic, classical and operator formulations for interior dynamical problems (reversible over time) have nowadays experimental verifications in all quantitative sciences, including classical physics, particle physics, nuclear physics, superconductivity, chemistry, biology, astrophysics and cosmology (see Refs. [2, 5]a for details). Evidently, we cannot possibly review in this short note all these verifications. Hence, we limit ourselves to the review of the verifications of direct geometric nature, those based on deviations from the Minkowskian geometry within physical media in the absence of gravitation.

Remember that Doppler's law is an ultimate manifestation of the Minkowskian geometry uniquely derivable from the LP symmetry. By contrast, the covering isogeometry and related LPS isosymmetry uniquely predict the following generalized law for the frequency of electromagnetic waves propagating within a physical medium, known as *Doppler-Santilli isoshift law*

$$\omega' = \frac{1 - \hat{\beta}\cos(\alpha)}{\sqrt{1 - \hat{\beta}^2}}\,\omega\,, \tag{9}$$

The above covering law predicts that the isoshift is not generally null for null speeds (due to the indicated dependence of the characteristic quantities on the speed v) and we write for null angle of aberration

$$\mathrm{Lim}\,\omega'_{v\to 0} \approx \mathrm{Lim}_{v\to 0}\left(1 - \frac{v_3}{c}\frac{n_4}{n_3} + \cdots\right)\omega = 1 - K(r, v, \omega.\delta, \ldots), \quad K > 0 \tag{10}$$

This novel event was predicted in Refs. [3]b of 1991, is today known as *Santilli isoredshift* and is referred to *a shift toward the red for light propagating within transparent physical media without any relative motion between the source, the medium and the observer.* We merely have inevitable interactions between light and the medium under which light loses energy $E = h\omega$ with consequential reduction of the frequency ω due to the impossibility of atoms in the medium of losing energy since they are generally in their stable ground state.

The first experimental evidence known to this author on the existence of the isoredshift (although not interpreted as such) has been the discovery by H. Arp [39] of quasars that, according to gamma spectroscopic evidence, are physically connected to an associated galaxy, yet their respective cosmological redshifts are dramatically different (see Fig. 1).

Such a difference clearly indicates a departure from the Minkowskian geometry of the vacuum because, under its validity, said large difference in cosmological redshifts would require that the quasar has at least 100 times the speed of the galaxy, under which conditions the quasar and its associated galaxy would have separated completely billions of years ago. Numerous hypotheses were formulated in order to resolve this “anomaly,” while maintaining the validity of special relativity, without achieving to date a resolution accepted by the scientific community at large.

Santilli’s proposal [3]b of the isotopies of special relativity was based on Arp’s discovery that was recommended for further study. In 1992, R. Mignani [40] provided a direct experimental verification of Santilli’s isorelativity and the isoshift law by showing that they apply for all known pairs of quasars and associated galaxies.

The isotopies of special relativity [3]b additionally identified the colors of our atmosphere as confirmatory experimental evidence and suggested the conduction of experiments. As established by the conventional quantum scattering theory as well as by evidence in air and water, red light is absorbed by physical media, resulting in the predominance of blue light that originates the color of the sky with the Sun at the Zenith (or the color of water at a sufficient depth).

The predominance of red for the Sun at Sunset and Sunrise was interpreted throughout the 20th century via the abrupt and unexplained assumption of the op-

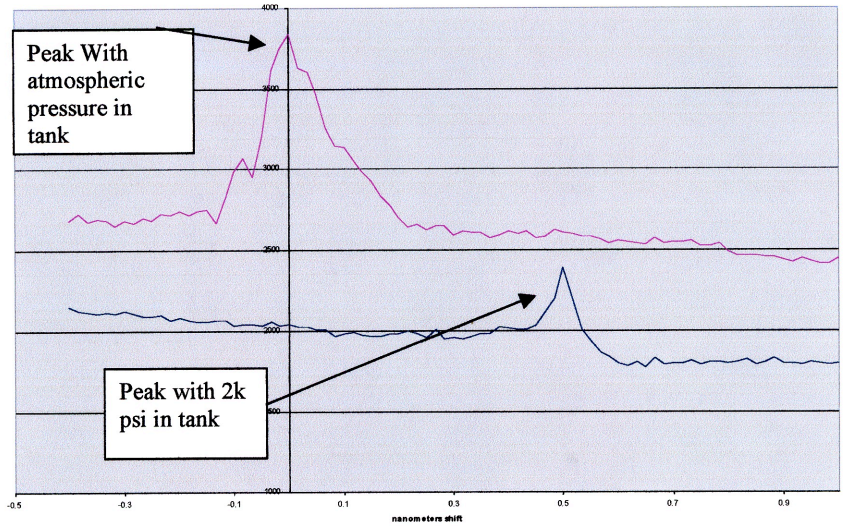

Fig. 3 The first scan confirming Santilli isoredshift obtained at the Isoredshift Testing Station of the Institute for Basic Research in Florida on June 27, 2009 (see Ref. [41] for details)

posite, namely, that blue light is absorbed by the medium with the increase of the trajectory resulting in the predominant red. Santilli [3]b pointed out that this interpretation is in violation of the quantum scattering theory as well as physical evidence, thus leaving as the sole plausible interpretation the isoredshift of light which is indeed proportional to the trajectory within [physical media.]

Additionally, Santilli pointed out that the predominance of red at Sunset and Sunrise occurs for *direct* sunlight, thus excluding possible interpretation via scattering (that refer to the diffused light); the scattering of photons cannot possibly provide a quantitative representation of the large change of wavelength from blue to red (of about 300 nm); and the presence of the isoredshift is rendered necessary by the fact that the predominance of red is essentially the same at Sunset, where we move *away* from the Sun, as well as Sunrise, where we move *toward* the Sun, thus establishing the isoredshift as dominant over the expected small contributions from the Doppler's law, of course, under a sufficiently long interior trajectory. In view of all the above, Santilli conclude Ref. [3]b suggesting the conduction of experiments on Earth, such as the measurement of a Fraunhofer line of the Sun while moving from the Zenith to the equator, and various other experiments.

Despite the passing of decades, the propagation of the information and the author solicitations for conducting the proposed experiments to various physical and astrophysical laboratories, the above experimental verifications (including Arp's discovery) remained vastly ignored by most physicists and astrophysicists to their evident peril. Consequently, in 2009, Santilli [41] conducted the measurement of the isoredshift in a 20 m long tube containing air at about 140 bars with the resulting measurement of about 0.5 nm isoshift of a blue laser light (see Fig. 3). Ref. [40] proved the capability of such an isoredshift to provide a numerical representation of Arp's discovery, the color of our atmosphere, and other interior events.

Ref. [41] also pointed out that, while being valid in locally empty spaces, *the Minkowskian geometry is nowhere exactly valid in the universe at large, because at cosmological distances the universe is a medium with high energy density, since it is everywhere filled up with light or stars* (see also Ref. [35] for cosmological imp[lications). Consequently, the intergalactic or galactic isoredshift can consequently imply the possible absence of universe expansion, big bang, dark matter and dark energy. Ref. [41] concluded with the need to conduct experiments on Earth, a number of them already under way, as the sole grounds for serious science along the teaching of Galileo Galilei.

Acknowledgements The author has no words to thank the organizers and participants of the *2010 Cosmology, the Quantum Vacuum, and Zeta Functions* for penetrating criticisms and invaluable comments.

References

1. Anderson, R., Cloonan, M., & Gandzha, D.: New Sciences for a New Era" Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli, to appear, preparation, preliminary version available in the website http://www.santilli-foundation.org/santilli-scientific-discoveries.html
2. Santilli, R.M., *Foundation of Theoretical Mechanics,* Volume I (1978) [2a], and Volume II (1982) [2b], Springer-Verlag, Heidelberg, Germany, http://www.santilli-foundation.org/docs/Santilli-209.pdf
3. Santilli, R.M., *Isotopic Generalization of Galileo and Einstein Relativities.* Hadronic Press 1991, Volumes I [3a] and II [3b], http://www.santilli-foundation.org/docs/Santilli-01.pdf http://www.santilli-foundation.org/docs/Santilli-35.pdf
4. Santilli, R.M., *Elements of Hadronic Mechanics.* Ukraine Academy of Sciences, Kiev, 1995, Vol. I [4a], Vol. II [4b], http://www.santilli-foundation.org/docs/Santilli-300.pdf http://www.santilli-foundation.org/docs/Santilli-301.pdf
5. Santilli, R.M., *Hadronic Mathematics, Mechanics and Chemistry.* International Academic Press 2008, Volumes I [5a], II [5b], III [5c], IV [5d] and V [5e], available as free downloads from http://www.i-b-r.org/Hadronic-Mechanics.htm
6. Santilli, R.M., *Foundations of Hadronic Chemistry, with Applications to New Clean Energies and Fuels,* Kluwer Academic Publishers (2001), http://www.santilli-foundation.org/docs/Santilli-113.pdf
7. Santilli, R.M., *Isodual Theory of Antimatter with Applications to Antigravity, Grand Unifications and Cosmology,* Springer (2006).
8. Aringazin, A.K., Jannussis, A., Lopez, F., Nishioka, M., & Veljanosky, B., *Santilli's Lie-Isotopic Generalization of Galileo and Einstein Relativities,* Kostakaris Publishers, Athens, Greece (1991), http://www.santilli-foundation.org/docs/Santilli-108.pdf
9. Myung, H.C., *Lie Algebras and Flexible Lie-Admissible Algebras,* Hadronic Press (1982), http://www.santilli-foundation.org/docs/Santilli-107.pdf
10. Sourlas, D.S., & Tsagas, G.T., *Mathematical Foundation of the Lie-Santilli Theory,* Ukraine Academy of Sciences 91993), http://www.santilli-foundation.org/docs/santilli-70.pdf

11. Lôhmus, J., Paal, E., & Sorgsepp, L., *Nonassociative Algebras in Physics* (Hadronic Press, Palm Harbor, 1994), http://www.santilli-foundation.org/docs/Lohmus.pdf
12. Kadeisvili, J.V., *Santilli's Isotopies of Contemporary Algebras, Geometries and Relativities,* Ukraine Academy of Sciences, Second edition (1997), http://www.santilli-foundation.org/docs/Santilli-60.pdf
13. Jiang, Chun-Xuan, *Foundations of Santilli Isonumber Theory,* International Academic Press (2001), http://www.i-b-r.org/docs/jiang.pdf
14. Ganfornina, Raul M. Falcon, & Valdes, Juan Nunez, *Fundamentos de la Isoteoria de Lie-Santilli,* International Academic Press (2001), http://www.i-b-r.org/docs/spanish.pdf
15. Davvaz, B., *Hyperrings Theory and Its Applications.* International Academic Press (2008), available as free download from http://www.santilli-foundation.org/docs/Davvaz.pdf
16. Santilli, R.M., "Embedding of Li algebras into Lie-admissible algebras," Nuovo Cimento **51**, 570 (1967), http://www.santilli-foundation.org/docs/Santilli-54.pdf
17. Santilli, R.M., "On a possible Lie-admissible covering of Galilei's relativity in Newtonian mechanics for nonconservative and Galilei form-noninvariant systems," Hadronic J. **1**, 223–423 (1978), http://www.santilli-foundation.org/docs/Santilli-58.pdf
18. Santilli, R.M., "Lie-admissible invariant representation of irreversibility for matter and anti-matter at the classical and operator levels," Nuovo Cimento B **121**, 443 (2006), available in free pdf download from the link http://www.santilli-foundation.org/docs/Lie-admiss-NCB-I.pdf
19. Jannussis, A., Mignani, R., & Santilli, R.M., Ann. Fond. L. De Broglie, **19**, 371 (1993), http://www.santilli-foundation.org/docs/Santilli-123.pdf
20. Lopez, D.F., In "Symmetry Methods in Physics", Volume 1, Joint Institute for Nuclear Research, Dubna, Russia (1994), http://www.santilli-foundation.org/docs/Santilli-121.pdf
21. Jannussis, A., & Skaltzas, D., Ann. Fond. L/ De Broglie **18**, 137 (1995), http://www.santilli-foundation.org/docs/Santilli-129.pdf
22. Santilli, R.M., Intern. J. Modern Phys. **14**, 3157 (1999, http://www.santilli-foundation.org/docs/Santilli-104.pdf
23. Santilli, R.M., Galilean Electrodynamics, Summer 2006, **17**, 43 (2006), http://www.i-b-r.org/Incons.GravFinalGED-I.pdf
24. Santilli, R.M., "Nonlocal-integral isotopies of differential calculus, algebras and geometries," Rendiconti Circolo Matematico Palermo, Suppl. 1996; **42**, 7, available as free download from http://www.santilli-foundation.org/docs/Santilli-37.pdf
25. Santilli, R.M., "Isotopies of Lie symmetries, I" Hadronic J. **8**, 36 (1985), http://www.santilli-foundation.org/docs/santilli-65.pdf
26. Santilli, R.M., "Isotopies of Lie symmetries, II" Hadronic J. **8**, 36 (1985), http://www.santilli-foundation.org/docs/santilli-65.pdf
27. Santilli, R.M., JINR rapid Comm. **6**. 24-38 (1993), http://www.santilli-foundation.org/docs/Santilli-19.pdf
28. Santilli, R.M., Acta Applicandae Mathematicae **50**, 177 (1998), http://www.santilli-foundation.org/docs/Santilli-19.pdf
29. Santilli, R.M., Lettere Nuovo Cimento **37** 545 (1983), http://www.santilli-foundation.org/docs/Santilli-50.pdf
30. Santilli, R.M., Lettere Nuovo Cimento **38**, 509 (1983), http://www.santilli-foundation.org/docs/Santilli-51.pdf
31. Santilli, R.M., Moscow Phys. Soc. **3**, 255 (1993), http://www.santilli-foundation.org/docs/Santilli-40.pdf
32. Santilli, R.M., Communication of the Joint Institute for Nuclear Research, Dubna, Russia, No. E4-93-252 (1993), published in the Chinese J. System Engineering and Electronics **6**, 177 (1995, http://www.santilli-foundation.org/docs/Santilli-18.pdf

33. Santilli, R.M., "Iso-Minkowskian geometry for the gravitational treatment of matter and its isodual for antimatter," Intern. J. Modern Phys. D **7**, 351 (1998), http://www.santilli-foundation.org/docs/Santilli-35.pdf
34. Kadeisvili, J.V., "Direct universality of the Lorentz-Poincare'-Santilli isosymmetry for extended-deformable particles, arbitrary speeds of light and all possible spacetimes" in *Photons: Old problems in Light of New Ideas,* V. V. Dvoeglazov Editor Nova Science (2000, http://www.santilli-foundation.org/docs/Santilli-25.pdf
35. Kadeisvili, J.V., "Implausibility of universe expansion, big bang, dark matter and dark energy," Hadronic Journal **32**, 647-657 (2010) http://www.santilli-foundation.org/docs/Kadeisvili-2.pdf
36. Corda, C., in the Proceedings of the Workshop on *Cosmology, the Quantum Vacuum, and Zeta Function,* Springer, to appear.
37. Tsagas, Gr.T., & Sourlas, D.S., Algebras, Groups and Geometries **12**, 1-65 and 67-88 (1995). http://www.santilli-foundation.org/docs/Santilli-324.pdf
38. Ganfornina, R.M. Falcon, & Valdes, J. Nunez, Algebras, Groups and Geometries **20**, 1 (2003), http://www.santilli-foundation.org/docs/isotopologia.pdf
39. Arp, H., *Quasars Redshift and Controversies.* Interstellar Media, Berkeley (1987).
40. Mignani, R., Physics Essay 1992; 5, 531, http://www.santilli-foundation.org/docs/Santilli-31.pdf
41. Santilli, R.M., "Experimental verifications of isoredshift with possible absence of universe expansion, big bang, dark matter and dark energy," The Open Astronomy Journal **3**, 126-144 (2010), http://www.santilli-foundation.org/docs/Santilli-isoredshift.pdf