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THERMODYNAMICS OF ANTIMATTER VIA SANTILLI'S ISODUALITIES

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For some time now, isodual mathematics, as devised and expounded by Santilli and others, has been well known and has been applied to many physical situations, most notably in creating an isodual theory of antimatter. However, the associated thermodynamics has not been considered. That defect is remedied here.

Key words: thermodynamics, antimatter, Santilli's isodualities.

1. INTRODUCTION

In this note the *classical* laws of thermodynamics are formulated, apparently for the first time, for antimatter. Since the *only* known *classical* formulation of antimatter consistent with charge conjugation at the *operator* level is *Santilli's isodual theory* [1 - 10], the formulation of the thermodynamics of antimatter presented here is based on Santilli's isodualities.

Santilli's main contribution in this field may be outlined as follows: the notion of isoduality as an anti-isomorphic map applicable at all classical and quantum levels of study was introduced for the first time in Ref. [1] in conjunction with the classification of all possible iso-

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topies of the rotational symmetry; the notion of isodual numbers (that is, numbers having a sign opposite to that of usual numbers but referred to a negative unit with compatible new product and negative norm) introduced in [1] was studied in detail in [2]; the first use of the isodual map for the characterisation of antimatter was in [3] which also contains the first proof that isoduality is equivalent to charge conjugation at the operator level; the all important isodualities of the fundamental Poincaré symmetry were introduced in [4]; the fundamental prediction that antimatter under isodual treatment experiences a repulsion in the gravitational field of matter (and vice-versa) was formulated first in [5] which also contains an explicit experimental proposal consisting in the use of a very low energy collimated beam of positrons travelling in a *horizontal* high vacuum tube at the end of which the displacement due to gravity, up or down, would be visible to the naked eye (e.g. via scintillators); mathematical maturity in the formulation of isodualities was achieved in [6]; the construction of the isodual image of quantum mechanics was presented in [7]; the prediction that antimatter under isoduality emits a *new* photon that is repelled by the gravitational field of matter — thus offering the first experimental possibility for distinguishing between distant matter and antimatter in due time — appeared in [8]; the fundamental role of isodualities for an axiomatically consistent grand unification of electroweak and gravitational interactions was formulated in [9]; a systematic classical isodual theory of antimatter, beginning from Newton's equations, is available also [10].

Among various independent contributions in this field, a most important one is provided by the ATT-Bell Lab. experimentalist, Mills [11], who proved that Santilli's proposal to test the gravity of positrons via a sufficiently long collimated horizontal tube [5] is feasible and can yield a resolutory answer with the use of positrons of Mev energy flying horizontally in a collimated high vacuum tube of about 100m length and 1m diameter.

A monograph [12] by Kadeisvili presents a comprehensive review, up to 1995, including a discussion of other independent studies.

2. ON ISODUAL THERMODYANMICS FOR ANTIMATTER

Isoduality for real quantities [1] — the only one needed in this paper — is the simplest conceivable map of the most fundamental quantity, the unit +1 of numbers with which quantities are measured, into its negative value:

$$+1 \to -1 = 1^d$$

and then the reconstruction of the entire formulation so as to admit -1 as the new left and right unit. For a real-valued vector V, the isodual map is

$$V \rightarrow V^d = -V^\dagger,$$

where † indicates a transpose; while, for an operator A on a Hilbert space, the isodual map is

$$A \rightarrow A^d = -A^*,$$

where $*$ represents Hermitian conjugation.

The first quantities to be reconstructed under isodualities are the ordinary numbers. In fact, negative numbers are not isodual numbers because their product $\times$ is conventional, so that -1 is not their unit; that is,

$$(-1) \times (-n) = +n.$$

Santilli's isodual numbers are, therefore, ordinary numbers n, m, $\ldots$ (both positive and negative) equipped with the new product

$$n \times^d m = n \times (-1) \times m = -n \times m,$$

under which 1^d is the correct new left and right unit. It should be noted also that

$$1^d \times^d n = (-1) \times (-1) n = n.$$

Isodual numbers are equipped then with an isodual quotient

$$n /^d m = -n/m,$$

an isodual norm

$$| \, n \, |^d = - \, | \, n \, | < 0,$$

and isodual images of all other operations. As a consequence, all characteristics of matter (not only the charge) change sign under isodualities to antimatter. However, now the new quantities are referred to negative units and are, therefore, as physically acceptable as conventional quantities. A good example is Santilli's isodual time $t^d = -t < 0$ which is negative definite. However, it is defined with respect to the negative unit — 1sec, and so is as causal as conventional time $t > 0$ referred to the positive unit of time +1sec. Hence, the use of negative units eliminates ab initio all problems of causality for negative times existing in the literature since they were tacitly referred to positive units.

For consistency, the entire formalism of thermodynamics must be subject to isoduality also — with no exceptions! As has been seen previously, the use of a mixture of isodual quantities and conventional quantities leads to catastrophic physical inconsistencies [8]. To proceed, the isodualities of all functions are needed:

$$f(t) \to f^d\left(t^d\right) = -f(-t),$$

as well as the isodualities of their operations, for example

$$df(t) \to d^d f^d\left(t^d\right) = -d[-f(-t)]$$

and

$$\partial f(t)/\partial t \to \partial^d f^d\left(t^d\right)/^d \partial^d t^d = -\partial[-f(-t)]\partial(-t).$$

Again, Santilli termed a quantity *isoselfdual* when it coincided with its isodual. There are several examples of both quantities and operations which fall into this category and the notion is of paramount importance for the study of antimatter since it identifies quantities and laws which apply irrespective of whether matter or antimatter is under consideration. It is evident, therefore, that, if a physical law is not isoselfdual, it is *different* for matter and antimatter.

When attention is turned specifically to thermodynamics, the quantities of immediate interest are heat Q, internal energy U, work W, entropy S, and absolute temperature T. Using the above results, it follows that

$$Q \to Q^d = -Q, U \to U^d = -U, W \to W^d = -W,$$

$$S \to S^d = -S, T \to T^d = -T,$$

$$dQ \to d^d Q^d = dQ, dU \to d^d U^d = dU, dW \to d^d W^d = dW,$$

$$dS \to d^d S^d = dS.$$

Thermodynamics is a subject based very closely on physical observation and experiment. It is governed by a number of 'laws' and, of these, the so- called First and Second Laws of Thermodynamics are probably the most powerful and far-reaching [13]. However, when the background to these two laws is examined in detail, it is found that both might quite reasonably be termed "facts of experience." Nevertheless, both have stood the test of time; neither has been found to be violated yet; until something to the contrary is proved, they must continue to form the basis for the subject. Having said that, it follows that, in any system, the First Law of Thermodynamics will hold and may be used to show that

$$dQ = dU - dW.$$

Using the above results, it follows that

$$dQ \rightarrow d^dQ^d = dQ = dU - dW \rightarrow d^dU^d - d^dW^d = dU - dW.$$

Hence, the First Law of Thermodynamics is *isoselfdual.*

The Second Law of Thermodynamics is used to show that an integrating factor exists for the inexact differential which represents an amount of heat added to a system. This finds expression in the equation

$$dQ = TdS.$$

Using the above results again, it is seen that

$$dQ \rightarrow d^dQ^d = dQ = TdS \rightarrow T^d \times^d d^dS^d = (-T) \times (-1)dS = TdS$$

and so, the Second Law of Thermodynamics is *isoselfdual* also.

It follows immediately that the combination of the First and Second Laws, represented by

$$TdS = dU - dW$$

must be isoselfdual as well. Usually, the work done is given by $-pdV$, where p represents pressure and V volume. If the expression is modified to apply to open systems, the above equation becomes

$$TdS = dU + pdV - \mu dN$$

where μ represents the chemical potential and N the number of particles. A moment's consideration, using the above results, shows this equation to be isoselfdual, as will be the so-called Euler relation

$$TS = U + pV - \mu N,$$

and this in turn guarantees the isoselfduality of the important Gibbs-Duhem relation:

$$SdT - Vdp + Nd\mu = 0.$$

The crucial conclusion to draw is that the Laws of Thermodynamics are isoselfdual and, therefore, the theory applies equally well for both matter and antimatter. Indeed, the above considerations appear to indicate that all the equations of conventional thermodynamics are isoselfdual, and it is trivial to check that the well-known thermodynamic potentials,

$$\text{Enthalpy, } H = U + pV,$$

$$\text{Helmholtz Free Energy, } F = U - TS,$$
$$\text{Gibbs Free Energy, } G = U + pV - TS,$$

are isoselfdual, and so it follows that the Maxwell thermodynamic relations are also.

3. CONCLUSIONS

The end result of this discussion is simply to conclude that all the well-known, powerful thermodynamic results are isoselfdual. All the above results follow very easily once the basic transformation equations are noted. However, the end result is an important one, even if not totally unexpected, in that it confirms that the various thermodynamic results may be used in conjunction with isodual mathematics and, possibly more importantly, it indicates once again the power of this very basic branch of physics. However, although the laws remain the same, they are subject to physically different interpretations for matter and antimatter; the two theories are anti-isomorphic to each other. For example, the entropy will *decrease* in time under isodualities, since it is negative in numerical value, but it is referred to a *negative unit* and is, therefore, completely equivalent to the conventional *increase* of the entropy with time when referred to conventional *positive units.*

It might be noted also that an important difference between the treatment of antimatter via charge conjugation and Santilli's isodualities is that *matter and antimatter are defined on the same space-time for the former while they exist in different space-times for the latter.* In particular, the formulations are such that these two different, yet coexisting, universes are *identical* when treated via their own mathematics. For example, if an electron has a counterclockwise directional motion in a given magnetic field, the isodual electron (positron) has exactly the same counterclockwise directional motion under the same field. The well-known inversion of rotation emerges only in the *projection* of the isodual space-time of the positron in the conventional space-time of the electron, that is, when isodual quantities are referred to positive unit. In turn, the above occurrence renders mandatory the repulsion of antiparticles in the field of matter, including the repulsion of the isodual photon.

As two final comments, it is worth realising that, for all its background in "facts of experience," thermodynamics continues to be applicable in all situations which come up for consideration; it certainly can lay claim to being a topic at the very heart of physics. Also, it seems an ideal opportunity to urge the conduction of experiments to measure the gravity of antiparticles [5, 11], as well to attempt to verify the existence of the isodual photon [7]. If positive results emerged, such experiments would make possible the identification of quasars and even entire galaxies as being composed entirely of antimatter.

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