HADRONIC MATHEMATICS, MECHANICS AND CHEMISTRY

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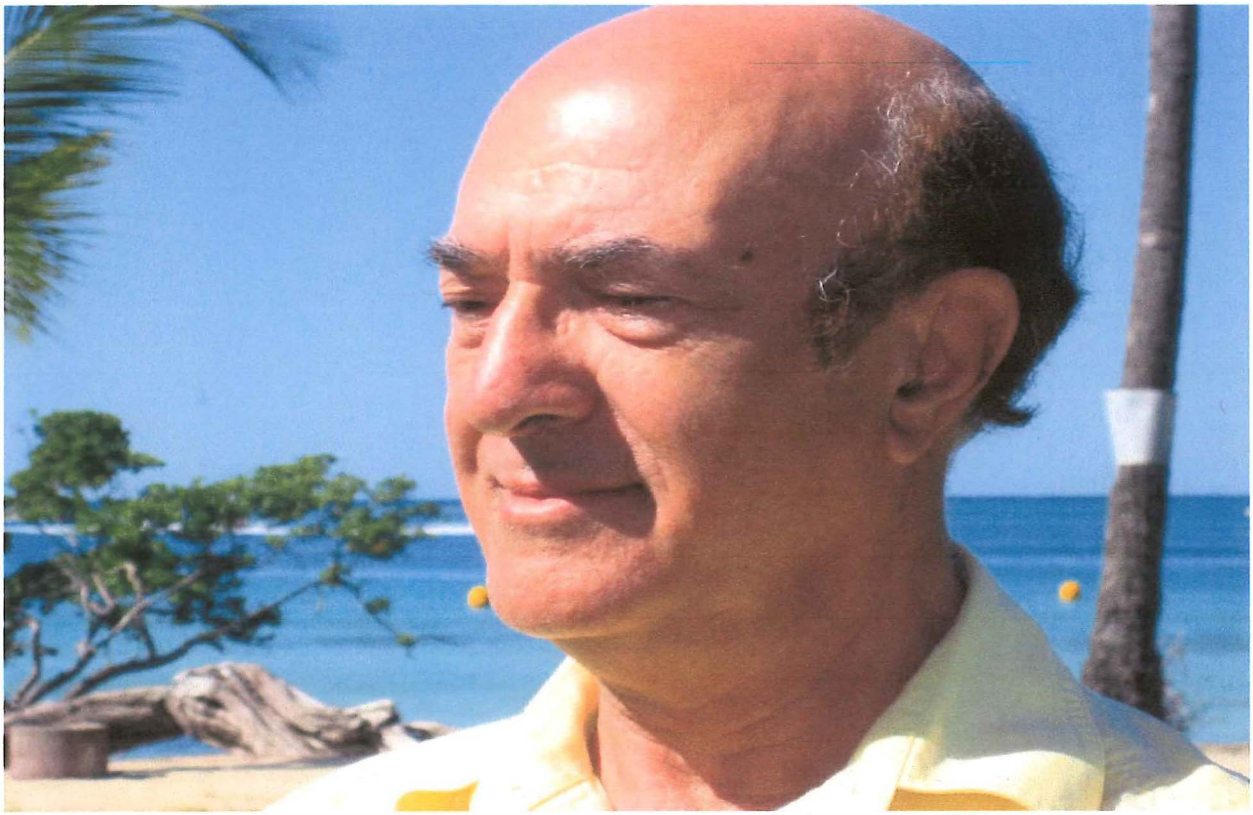

Dedicated to Sir Prof. Ruggero Maria Santilli
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PART 1:

FOUNDATIONS OF HADRONIC MATHEMATICS

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Special Issue:
Issue I: Foundations of Hadronic Mathematics

Lead Guest Editor

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Board of Trustees, The R. M. Santilli Foundation, Palm Harbor, Florida, USA

Introduction

20th century mathematics underlying mainstream physical and chemical theories is local-differential, thus solely permitting the representation of point-like masses. The Italian-American scientist R. M. Santilli accepted such a mathematics for the representation of particles when the masses are at large mutual distances, thus allowing point-like approximations, as it is the case for the atomic structure. Santilli then identified clear limitation of 20th century mathematics for the representation of extended charge distributions or wavepackets in conditions of partial or total mutual penetration, as it is the case for the synthesis of the neutron from a proton and an electron in the core of a star; for the structure of nuclei, stars and black holes; for the molecular bond of two identical valence electrons in singlet coupling; and other composite systems.

When at the Department of Mathematics of Harvard University in the late 1970s, Santilli developed a series of new mathematics for the representation of extended charge distributions or wavepackets when in condition of partial or total mutual penetration, resulting in:

1. The novel, single valued- isomathematics for the representation of composite matter-systems reversible over time of with extended constituents at short mutual distances;

2. The novel, single valued genomathematics for the representation of composite matter-systems or reactions irreversible over time with extended constituents at short mutual distance;

3. The novel multi-valued hypermathematics for the representation of biological matter-systems.

Additionally, Santilli constructed their anti-Hermitean isodual images for the representation of corresponding antimatter-systems in conditions of increasing complexity. These varieties of new mathematics are today collectively addressed by the name of hadronic mathematics, in view of their applications. The special issue of AJMP on the Foundations of Hadronic Mathematics shall review the above novel mathematics and present new advances for the use in subsequent special issues devoted to its applications.

For more information about the Special Issue, please pay a visit to the following website:
http://www.sciencepublishinggroup.com/specialissue/122013

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Outline of Hadronic Mathematics, Mechanics and Chemistry as Conceived by R. M. Santilli

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Abstract: In this paper, we outline the various branches of hadronic mathematics and their applications to corresponding branches of hadronic mechanics and chemistry as conceived by the Italian-American scientist Ruggero Maria Santilli. According to said conception, hadronic mathematics comprises the following branches for the treatment of matter in conditions of increasing complexity: 1) 20th century mathematics based on Lie's theory; 2) IsoMathematics based on Santilli's isotopies of Lie's theory; 3) GenoMathematics based on Santilli's formulation of Albert's Lie-admissibility; 4) HyperMathematics based on a multi-valued realization of genomathematics with classical operations; and 5) HyperMathematics based on Vougiouklis H_v hyperstructures expressed in terms of hyperoperations. Additionally, hadronic mathematics comprises the anti-Hermitean images (called isoduals) of the five preceding mathematics for the description of antimatter also in conditions of increasing complexity. The outline presented in this paper includes the identification of represented physical or chemical systems, the main mathematical structure, and the main dynamical equations per each branch. We also show the axiomatic consistency of various branches of hadronic mathematics as sequential coverings of 20th century mathematics; and indicate a number of open mathematical problems. Novel physical and chemical applications permitted by hadronic mathematics are presented in subsequent collections.

Keywords: Santilli Isomathematics, Genomathematics, Hypermathematics

1. 20th Century Mathematics, Mechanics and Chemistry

1.1. Represented Systems

Single-valued, closed-isolated, time-reversible systems of point-like particles moving in vacuum solely under action at a distance Hamiltonian interactions, such as the structure of atoms and molecules.

1.2. Main Mathematical Structure

Basic unit

$$I = +1 \tag{1}$$

Basic numeric fields n = real, complex, quaternionic numbers

$$F(n,\times,1),n \tag{2}$$

Basic Associative product

$$nm = n \times m, 1 \times n = n \times 1 = n \; \forall n \in F \tag{3}$$

Measurement units of time, energy, etc. all positive
Ordinary functional analysis $f(r) \in F$,
Ordinary differential calculus
Conventional Lie theory

$$[X_i, X_j] = X_i \times X_j - X_j \times X_i == C_{ij}^k \times X_k, \tag{4}$$

$$A(w) = e^{X \times w \times i} \times A(0) \times e^{-i \times w \times X}. \tag{5}$$

Euclidean geometry and topology

$$E(r,\delta,1), r = (r^k), k = 1,2,3, \delta = Diag.(1,1,1), \tag{6}$$

$$r^2 = r^i \times \delta_{ij} \times r^j = r_1^2 + r_2^2 + r_3^2 \in F, \tag{7}$$

Minkowskian geometry

$$M(x,\eta,I): x = (x^\mu), \mu = 1,2,3,4, x^4 = t, \tag{8}$$

$$\eta = Diag.(+1,+1,+1,-c^2), \tag{9}$$

$$x^2 = x^\mu \times \eta_{\mu\nu} \times x^\nu = x_1^2 + x_2^2 + x_3^2 - t^2c^2 \in F, \quad (10)$$

Riemannian geometry

$$R(x,g(x)I): x = (x^\mu), \mu = 1,2,3,4, x^4 = t, \quad (11)$$

$$x^2 = x^\mu \times g(x)_{\mu\nu} \times x^\nu \in F \quad (11)$$

$$x^2 = x^\mu \times g(x)_{\mu\nu} \times x^\nu \in F \quad (12)$$

Symplectic geometry.

$$\omega = dr^k \wedge dp_k \quad (13)$$

1.3. Dynamical equations

Newton equation

$$m \times \frac{dv}{dt} - F^{SA}(t,r,v,) = 0, \quad (14)$$

Variational principle

$$\delta A = \delta \int (p_k \times dr^k - H \times dt) = 0. \quad (15)$$

Hamilton's equations without external terms

$$\frac{dr^k}{dt} = \frac{\partial H(r,p)}{\partial p_k}, \quad \frac{dp_k}{dt} = -\frac{\partial H(r,p)}{\partial r^k}, \quad (16)$$

Hilbert space H over C with states $|\psi>$ over (C)
Expectation value of a Hermitean operator A

$$<A>=<\psi| \times A \times |\psi> \in C, \quad (17)$$

Heisenberg equation

$$i \times \frac{dA}{dt} = [A,H] = A \times H - H \times A, \quad (18)$$

Schrödinger equations

$$H \times |\psi> = E \times |\psi> \quad (19)$$

$$p \times |\psi> = -i \times \partial_r |\psi> \quad (20)$$

Dirac equation

$$(\eta^{\mu\nu} \times \gamma_\mu \times p_\nu - i \times m \times c) \times |\psi> = 0. \quad (21)$$

$$\{\gamma_\mu, \gamma_\nu\} = \gamma_\mu \times \gamma_\nu + \gamma_n u \times \gamma_\mu = 2 \times \eta_{\mu\nu}, \quad (22)$$

Comments and References

The literature on 20th century mathematics, mechanics and chemistry is so vast and so easily identifiable to discourage discriminatory partial listings.

2. Isomathematcs, Isomechanics and Isochemistry

2.1. Represented Systems [1-5]

Single-value, closed-isolated, time-reversible system of extended-deformable particles with action at a distance Hamiltonian and contact non-Hamiltonian interactions, such as the structure of hadrons, nuclei and stars, in the valence electron bonds and other systems.

2.2. Main Mathematical Structure s [1-5]

Santilli IsoUnit $\hat{I}$ and isotopic element $\hat{T}$[1]

$$\hat{I} = \hat{I}(r,p,a,\psi,....) = 1/\hat{T}(r,p,a,\psi,....) > 0, \quad (23)$$

Santilli IsoFields

$$\hat{F}(\hat{n},\hat{\times},\hat{I}), \hat{n} = n \times \hat{I}_{,,} \quad (24)$$

Santilli isoproduct

$$\hat{n} \,\hat{\times}\, \hat{m} = \hat{n} \times \hat{T} \times \hat{m} \in \hat{F}, \quad (25)$$

$$\hat{I} \,\hat{\times}\, \hat{n} = \hat{n} \,\hat{\times}\, \hat{I} = \hat{n} \; \forall \hat{n} \in \hat{F}, \quad (26)$$

Representation via the isotopic element of extended-deformable particles under non-Hamiltonian interactions

$$\hat{T} = Diag.(\tfrac{1}{n_1^2}, \tfrac{1}{n_2^2}, \tfrac{1}{n_3^2}) \times e^{\Gamma(r,p,\psi,\partial\psi,...)} \quad (27)$$

IsoCoordinates $\hat{r} = r \times \hat{I} \in \hat{F}$,
IsoFunctional analysis $\hat{f}(\hat{r}) = f(\hat{r}) \times \hat{I} \in \hat{F}$,
IsoDifferential Calculus

$$\hat{d}\hat{r} = dr + r \times \hat{T} \times d\hat{I}, \quad (28)$$

$$\frac{\hat{\partial} \hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \hat{I} \times \frac{\partial \hat{f}(\hat{r})}{\partial \hat{r}}, \quad (29)$$

Santilli Lie-Isotopic Theory

$$[X_i, \hat{X}_j] = X_i \,\hat{\times}\, X_j - X_j \,\hat{\times}\, X_i == C_{ij}^k(r,p,...) \times X_k, \quad (30)$$

$$A(w) = \hat{e}^{X \times w \times i} \,\hat{\times}\, A(0) \,\hat{\times}\, \hat{e}^{-i \times w \times X}. \quad (31)$$

Santilli Iso-Euclidean Geometry

$$\hat{E}(\hat{r},\hat{\delta},\hat{I}), \hat{\delta}(r,p,z,\psi,...) = \hat{T}(r,p,z,\psi,...) \times \delta, \quad (32)$$

$$\hat{T} = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2), \quad (33)$$

$$\hat{r}^{\hat{2}} = \hat{r}^i \,\hat{\times}\, \hat{\delta}_{ij} \,\hat{\times}\, \hat{r}^j = (\tfrac{r_1^2}{n_1^2} + \tfrac{r_2^2}{n_2^2} + \tfrac{r_3^2}{n_3^2}) \times \hat{I} \in \hat{F}, \quad (34)$$

Santilli Iso-Minkowskian Geometry

$$\hat{M}(\hat{x},\hat{\eta},\hat{I}): \hat{x} = (\hat{x}^\mu), \mu = 1,2,3,4, x_4 = t, \quad (35)$$

$$\hat{\eta}(x,\psi,...) = \hat{T}(x,\psi,...) \times \eta, \quad (36)$$

$$\hat{T} = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2), \quad (37)$$

$$\hat{x}^{\hat{2}} = \hat{x}^\mu \,\hat{\times}\, \hat{\eta}_{\mu\nu} \,\hat{\times}\, \hat{x}^\nu = (\tfrac{x_1^2}{n_1^2} + \tfrac{x_2^2}{n_2^2} + \tfrac{x_3^2}{n_3^2} - t^2 \tfrac{c^2}{n_4^2}) \times \hat{I} \in \hat{F}, \quad (38)$$

[1]See Santilli's curriculum
http://www.world-lecture-series.org/santilli-cv
Prizes and Nominations
http://www.santilli-foundation.org/santilli-nobel-nominations.html
and scientific archive
http://www.santilli-foundation.org/news.html

Santilli Iso-Riemannian Geometry

$$\hat{R}(\hat{x},\hat{g},\hat{I})\colon \hat{g} = \hat{T}(x,v,\dots)\times g(x), \tag{39}$$

$$\hat{x}^{\hat{2}} = (\frac{g_{11}}{n_1^2}+\frac{g_{22}}{n_2^2}+\frac{g_{33}}{n_3^2}-\frac{g_{44}}{n_4^2})\times \hat{I}\in \hat{F}, \tag{40}$$

Santilli Iso-Symplectic Geometry

$$\hat{\omega} = \hat{d}\hat{r}^k \hat{\wedge} \hat{d}\hat{p}_k \tag{41}$$

2.3. IsoDynamical IsoEquations s [1-5]

Newton-Santilli IsoEquation

$$\hat{m}\,\hat{\times}\,\frac{\hat{d}\hat{v}}{\hat{d}\hat{t}} - F^{SA}(t,r,p) = m\times\frac{dv}{dt} - F^{SA}(t,r,p) - F^{NSA}(t,r,p,\dots) = 0, \tag{42}$$

Iso Variational principle

$$\hat{\delta}\hat{A} = \hat{\delta}\hat{\int}\ (\hat{p}_k\,\hat{\times}\,\hat{d}\hat{r}^k - \hat{H}\,\hat{\times}\,\hat{d}\hat{t}) = 0. \tag{43}$$

Hamilton-Santilli IsoEquations

$$\frac{\hat{d}\hat{r}^k}{\hat{d}\hat{t}} = \frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{p}_k},\quad \frac{\hat{d}\hat{p}_k}{\hat{d}\hat{t}} = -\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{r}^k}, \tag{44}$$

Iso-Hilbert space $\hat{H}$ over C with states $|\hat{\psi}>$ over the isofield $\hat{C}$[2]

IsoExpectation value of a Hermitean operator $\hat{A}$ on $\hat{H}$

$$<\hat{A}>=<\hat{\psi}|\,\hat{\times}\,\hat{A}\,\hat{\times}\,|\hat{\psi}>\in\hat{C} \tag{45}$$

Heisenberg-Santilli IsoEquation

$$\hat{\imath}\,\hat{\times}\,\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = [\hat{A},\hat{H}] = \hat{A}\,\hat{\times}\,\hat{H} - \hat{H}\,\hat{\times}\,\hat{A} = \hat{A}\times\hat{T}(\hat{\psi},\dots)\times\hat{H}(\hat{r},\hat{p}) - \hat{H}(\hat{r},\hat{p})\times\hat{T}(\hat{\psi},\dots)\times\hat{A} \tag{46}$$

Schrödinger-Santilli IsoEquation

$$\hat{H}\,\hat{\times}\,|\hat{\psi}>=\hat{H}(\hat{r},\hat{p})\times\hat{T}(\hat{\psi},\hat{\partial}\hat{\psi},\dots)\times|\hat{\psi}>=\hat{E}\,\hat{\times}\,|\hat{\psi}>= E\times|\hat{\psi}>, \tag{47}$$

$$\hat{p}\,\hat{\times}\,|\hat{\psi}>=-\hat{\imath}\,\hat{\times}\,\hat{\partial}_{\hat{r}}|\hat{\psi}>=-i\times\hat{I}\times\partial_{\hat{r}}|\hat{\psi}>, \tag{48}$$

Dirac-Santilli IsoEquation

$$(\hat{\eta}^{\mu\nu}\,\hat{\times}\,\hat{\gamma}_\mu\,\hat{\times}\,\hat{p}_\nu - \hat{\imath}\,\hat{\times}\,\hat{m}\,\hat{\times}\,\hat{c})\,\hat{\times}\,|\hat{p}si>=0. \tag{49}$$

$$\{\hat{\gamma}_\mu,\hat{\gamma}_\nu\} = \hat{\gamma}_\mu\,\hat{\times}\,\hat{\gamma}_\nu + \hat{\gamma}_\nu\,\hat{\times}\,\hat{\gamma}_\mu = \hat{2}\,\hat{\times}\,\hat{\eta}_{\mu\nu} = 2\times\hat{\eta}_{\mu\nu}, \tag{50}$$

2.4. Comments and References

As it is well known, the local-differential calculus of 20th century mathematics can solely represent a finite set of isolated dimensionless points. In view of this structural feature, Newton formulated his celebrated equations (14) for *massive points,* resulted in a conception of nature that was adopted by Galileo and Einstein, became the dominant notion of 20th century sciences, and was proved to be valid for classical or quantum particles moving in vacuum at large mutual distances, such as for our planetary system or the atomic structure.

However, when bodies move within physical media, such as for a spaceship during re-entry in our atmosphere or for a proton in the core of a star, point-like abstractions of particles became excessive, e.g., because a macroscopic collection of point-particles cannot have entropy (since all known Hamiltonian interactions are invariant under time reversal), with consequential violation of thermodynamical laws and other insufficiencies.

Besides the clear identification of these insufficiencies, the first historical contribution by the Italian-American scientist Ruggero Maria Santilli (see Footnote 1) has been the *generalization of 20th century mathematics into such a form to admit a time invariant representation of extended, and therefore deformable particles under conventional Hamiltonian as well as contact non-Hamiltonian interactions, with implications for all quantitative sciences.*

The above central objective was achieved in monographs [1] originally written by Santilli during his stay at MIT from 1974 to 1977 (where they appeared as MIT preprints). Monographs [1] were then completed by Santilli during his stay at Harvard Universityfrom 1977 to 1982 under DOE support, and released for publication only following the delivery at Harvard of a post Ph. D. seminar Course in the field.

The representation of extended-deformable bodies moving within physical media was achieved via an axiom-preserving lifting, called *isotopy,* of the conventional associative product $AB = A\times B$ between generic quantities A, B (such as numbers,functions, matrices, operators, etc.) into the form $A\,\hat{\times}\,B = A\times\hat{T}\times B$, Eq. (25). Conventional interactions are represented via conventional Hamiltonian, while actual shape and non-Hamiltonian interactions are represented via realization of the quantity $\hat{T}$, called *isotopic element,* of the type (27).

Santilli then achieved in monographs [1] the axiom-preserving isotopies of the various branches of Lie's theory, e.g., Eqs. (30), (31,) including their elaboration via the initiation of the isotopies of functional analysis. In particular, Santilli showed that the isotopies of the rotational symmetry $SO(3)$ characterized by isotopic element (27) do represent extended, generally non-spherical and deformable bodies. Finally, Santilli proved in Vol. II of Ref. [1] the

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[2] As shown in the seminal paper [6] of 1982, but vastly ignored for the past four decades, isomechanics formulated on iso-Hilbert spaces over isofields eliminates the divergencies of quantum mechanics and related scattering theories. This important feature is primarily due to the fact that, for all physical and chemical applications worked out to date, the isounit $\hat{I} = 1/\hat{T} > 0$ must have a large value of the exponential type (27) and, consequently, the isotopic element $\hat{T}$ must have a very small value. This occurrence eliminates the singularity of the Dirac delta "distribution" when lifted to the Dirac-Myung-Santilli delta "isofunction" as shown by the realization of the type

$$\hat{\delta}(r-r_0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ik\hat{T}(r-r_0)}\,dk,$$

with $\hat{T} = \frac{N}{r-r_0}$, $N \ll 1$. Similarly, perturbative and other series with Hermitean operators that are divergent or slowly convergent in quantum mechanics can be lifted into isoseries of the type

$$A(w) = \hat{I} + \frac{w(A\hat{T}H - H\hat{T}A)}{1!} + \cdots$$

that are manifestly convergent for $w > 1$ but $\hat{T} \ll w$.As shown by A. O. E. Animalu and R. M. Santilli in five papers published proceedings [25], the above lack of divergences carries over to the covering of the scattering theory known as isoscattering theory, by therefore achieving numerical results without the use of infinities for the renormalization of divergent series.

significance of his Lie-isotopic theory by showing that it characterizes the Birkhoffian covering of classical Hamiltonian mechanics and its "direct universality" for the representation of all possible, non-singular, generally non-Hamiltonian Newtonian systems in the frame of the experimenter, which direct universality was subsequently proved to hold also for isotopic operator theories. The above advances were formulated on an ordinary numeric field.

Subsequently, Santilli discovered in 1993 [2] that the axioms of numeric fields with characteristic zero do not necessarily require that the basic multiplicative unit is the trivial number $+1$, since said axioms admit arbitrary generalized units, today called *Santilli isounits,* provided that they are positive-definite and are the inverse of the isotopic element, $\hat{I} = 1/\hat{T} > 0$. This second historical discovery identified new numbers today known as *Santilli isoreal, isocomplex and isoquaternionic numbers* of the First (Second) kind when the isounit is outside (an element of) the original field. This discovery prompted a flurry of reformulation over Santilli isofields of all preceding isotopies, including most importantly the reformulation of Santilli's Lie-isotopic theory.

Despite the above momentous advances, Santilli remained dissatisfied because the isotopic formulations of the early 1990s were not invariant under their time evolution, thus being unable to predict the same numerical values under the same conditions at different times. Since the entire 20th century mathematics had been isotonically lifted by the early 1990s, Santilli was left with no other choice than that of reinspecting the Newton-Leibnitz differential calculus by discovering that, contrary to a popular belief in mathematics and physics for some four centuries, the differential calculus is indeed dependent on the basic multiplicative unit. In this way, Santilli achieved in memoir [3] of 1996 the third historical discovery according to which the ordinary differential calculus needs generalizations of the type (28), (29) whenever the isounit depends on the local variable of differentiation. This discovery signaled the achievement of mathematical maturity of isomathematics that permitted numerous advances in physics and chemistry as well as novel industrial applications.

All in all, Santilli has written about 150 papers on the isotopies of all various aspects of 20th century mathematics. These contributions are reported in monographs [4] of 1995 that remain to this day the most comprehensive presentation on isotopies. In the subsequent series of monographs [5] of 2008, Santilli introduces the names of *Hadronic Mathematics, Mechanics and Chemistry* which have been adopted for this review due to their wide acceptance.

Numerous authors have made important contributions in Santilli isomathematics, among whom we quote: the mathematician H. C. Myung who initiated (with R. M. Santilli) [6] the isotopies of Hilbert Spaces, including the momentous elimination of the divergencies of quantum mechanics under sufficiently small values of the isotopic element $\hat{T}$; the mathematicians D. S. Sourlas and G. T. Tsagas [7] who conducted in 1993 the first comprehensive study of the Lie-Santilli isotheory; the theoretician J. V. Kadeisvili [8] who presented systematic studies of Santilli's isotopies of 20th century geometries and relativities; the mathematician Chun-Xuan Jiang [9] who conducted in 2001 systematic studies of Santilli IsoNumber Theory; the mathematicians R. M. Falcon Ganfornina and J. Nunez Valdes who wrote in 2001 the now historical, first mathematically rigorous treatment of Santilli isotopies [10], and the historical achieved isotopology [11] which provides the ultimate mathematical structure of the Newton-Santilli isoequations (42) for extended-deformable particles under Hamiltonian and non-hamiltonian interactions achieved in memoir [3]; the mathematician S. Georgiev who wrote one of the most monumental and important mathematical works in scientific history [12], by showing that Santilli's IsoDifferential Calculus implies a variety of fully consistent coverings of 20th century mathematics; the mathematician A. S. Muktibodh [13] who presented the first known generalization of Santilli isonumber theory for the case of characteristic $p \neq 0$; the physicists I. Gandzha and J. Kadeisvili who presented in 2011 [14] a comprehensive review of Santilli isomathematics and its applications in physics and chemistry; plus additional seminal advances presented in the subsequent papers of this collection.

3. Genomathematics, Genomechanics and Genochemistry

3.1. Represented Systems s [1-5]

Single-valued, time-irreversible system of extended-deformable particles under action at a distance Hamiltonian and contact non-Hamiltonian interactions, as occurring in nuclear reactions, biological structures and chemical reactions.

3.2. Main Mathematical Structure s [1-5]

Santilli Forward GenoUnit

$$\hat{I}^{>} = \hat{I}^{>}(t^{>} r^{>}, p^{>}, a^{>}, \psi^{>}, \partial^{>}\psi^{>}, \ldots.) = 1/\hat{T}^{>} > 0, \quad (51)$$

Santilli Backward GenoUnit

$$^{<}\hat{I} = {}^{<}\hat{I}({}^{<}r, {}^{<}p, {}^{<}a, {}^{<}\psi, {}^{<}\partial{}^{<}\psi, \ldots.) = 1/{}^{<}\hat{T} > 0, \quad (52)$$

Condition for time-irreversibility

$$\hat{I}^{>} \neq {}^{<}\hat{I} \quad (53)$$

Forward GenoFields

$$\hat{F}^{>}(\hat{n}^{>}, >, \hat{I}^{>}), \hat{n}^{>} = n \times \hat{I}^{>} \quad (54)$$

Backward GenoFields

$$^{<}\hat{F}({}^{<}\hat{n}, <, {}^{<}\hat{I}), \quad {}^{<}\hat{n} = {}^{<}\hat{I} \times n, \quad (55)$$

Forward GenoProduct

$$\hat{n} > \hat{m} = \hat{n}^{>} \times \hat{T}^{>} \times \hat{m}^{>} \in \hat{F}^{>}, \quad (56)$$

$$\hat{I}^{>} > \hat{n}^{>} = \hat{n}^{>} > \hat{I}^{>} = \hat{n}^{>} \; \forall \hat{n}^{>} \in \hat{F}^{>} \quad (57)$$

Backward Genoproduct

$$<\hat{n} <^{<} \hat{m} =^{<} \hat{n} \times^{<} \hat{T} \times^{<} \hat{m} \in^{<} \hat{F}, \quad (58)$$

$$<\hat{I} <^{<} \hat{n} =^{<} \hat{n} <^{<} \hat{I} =^{<} \hat{n} \forall \quad <\hat{n} \in^{<} \hat{F}, \quad (59)$$

Representation of forward extended-deformable particles under non-Hamiltonian interactions

$$\hat{T}^{>} = Diag.(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2})^{>} \times e^{\Gamma(t,r,p,\psi,\partial\psi,\ldots)^{>}} \quad (60)$$

Forward GenoCoordinates

$$\hat{r}^{>} = r \times \hat{I}^{>} \in \hat{F}^{>}, \quad (61)$$

Backward GenoCoordinates

$$<\hat{r} =^{<} \hat{I} \times r \in^{<} \hat{F}, \quad (62)$$

Forward GenoFunctional analysis

$$\hat{f}^{>}(\hat{r}^{>}) = f(\hat{r}^{>}) \times \hat{I}^{>} \in \hat{F}^{>}, \quad (63)$$

Backward GenoFunctional analysis

$$<\hat{f}(<\hat{r}) = f(<\hat{r}) \times^{<} \hat{I} \in^{<} \hat{F}, \quad (64)$$

Forward GenoDifferential Calculus

$$\hat{d}^{>}\hat{r}^{>} = dr + r \times \hat{I}^{>} \times d\hat{I}^{>}, \quad (65)$$

$$\frac{\hat{\partial}^{>}\hat{f}^{>}(\hat{r}^{>})}{\hat{\partial}^{>}\hat{r}^{>}} = \hat{I}^{>} \times \frac{\partial \hat{f}^{>}(\hat{r}^{>})}{\partial \hat{r}^{>}}, \quad (66)$$

Backward GenoDifferential Calculus

$$<\hat{d}<\hat{r} = dr + r \times^{<} \hat{T} \times d^{<}\hat{I}, \quad (67)$$

$$\frac{<\hat{\partial}<\hat{f}(<\hat{r})}{<\hat{\partial}<\hat{r}} =^{<} \hat{I} \times \frac{\partial^{<}\hat{f}(<\hat{r})}{\partial<\hat{r}}, \quad (68)$$

Santilli Lie-Admissible Theory

$$(X_i \hat{,} X_j) = X_i < X_j - X_j > X_i = C_{ij}^{k}(t,r,p,\psi,\ldots) \times X_k, \quad (69)$$

$$A(w) = \hat{e}_{>}^{X \times w \times i} > A(0) <_{<} \hat{e}^{-i \times w \times X}. \quad (70)$$

Santilli Forward Geno-Euclidean Geometry

$$\hat{E}^{>}(\hat{r}^{>}, \hat{\delta}^{>}, \hat{I}^{>}), \hat{\delta}^{>}(t,r,p,\psi,\ldots) = \hat{T}^{>}(t,r,p,\psi,\ldots) \times \delta, \quad (71)$$

$$\hat{r}^{>\hat{2}} = \hat{r}^{>i} > \hat{\delta}_{ij}^{>} > \hat{r}^{>j} \in F^{>}, \quad (72)$$

$$\hat{\delta}^{>} \neq \hat{\delta}^{>\,tranp} \quad (73)$$

Santilli Backward Geno-Euclidean Geometry

$$<\hat{E}(<\hat{r},<\hat{\delta},<\hat{I}), \quad <\hat{\delta}(t,r,p,\psi,\ldots) =^{<} \hat{T}(t,r,p,\psi,\ldots) \times \delta, \quad (74)$$

$$<\hat{2}\hat{r} =^{<i} \hat{r} <^{<} \hat{\delta}_{ij} <^{<j} \hat{r} \in^{<} F, \quad (75)$$

$$<\hat{\delta} \neq^{<\,transp} \hat{\delta} \quad (76)$$

Santilli Forward Geno-Minkowskian Geometry (μ = 1,2,3,4)

$$\hat{M}^{>}(\hat{x}^{>}, \hat{\eta}^{>}, \hat{I}^{>}): \hat{x}^{>} = (\hat{x}^{>\mu}), x_4^{>} = t^{>}, \quad (77)$$

$$\hat{\eta}^{>}(x,\psi,\ldots) = \hat{T}^{>}(x,\psi,\ldots) \times \eta, \quad (78)$$

$$\hat{x}^{>2} = \hat{x}^{>\mu} > \hat{\eta}_{\mu\nu}^{>} > \hat{x}^{>} \in \hat{F}^{>}, \quad (79)$$

$$\hat{\eta}^{>} \neq \hat{\eta}^{>\,transp} \quad (80)$$

Santilli Backward Geno-Minkowskian Geometry (μ = 1,2,3,4,)

$$<\hat{M}(<\hat{x},<\hat{\eta},<\hat{I}): \quad <\hat{x} = (\hat{x}^{\mu}), \quad <x_4 =^{<} t, \quad (81)$$

$$<\hat{\eta}(x,v,\ldots) =^{<} \hat{T}(x,v,\ldots) \times \eta, \quad (82)$$

$$<\hat{x}^{<2} =^{<\mu} \hat{x} <^{<} \hat{\eta}_{\mu\nu} <^{<\nu} \hat{x} \in^{<} \hat{F}, \quad (83)$$

$$<\hat{\eta} \neq^{<\,transp} \hat{\eta} \quad (84)$$

Santilli Forward Geno-Riemannian Geometry

$$\hat{R}^{>}(\hat{x}^{>}, \hat{g}^{>}, \hat{I}^{>}): \hat{g}^{>} = \hat{T}^{>}(x,v,\ldots) \times g(x), \quad (85)$$

$$\hat{x}^{>2} = \hat{x}^{>\mu} > \hat{g}_{\mu\nu}^{>} > \hat{x}^{>} \in \hat{F}^{>}, \quad (86)$$

$$\hat{g}^{>} \neq \hat{g}^{>\,transp} \quad (87)$$

Santilli Backward Geno-Riemannian Geometry

$$<\hat{R}(<\hat{x},<\hat{g},<\hat{I}): \quad <\hat{g} =^{<} \hat{T}(x,v,\ldots) \times g(x), \quad (88)$$

$$<\hat{x}^{<2} =^{<\mu} \hat{x} <^{<} \hat{g}_{\mu\nu} <^{<\nu} \hat{x} \in^{<} \hat{F}, \quad (89)$$

$$<\hat{g} \neq^{<\,transp} \hat{g} \quad (90)$$

Santilli Forward Geno-Symplectic Geometry

$$\hat{\omega}^{>} = \hat{d}^{>}\hat{r}^{>k} \hat{\wedge}^{>} \hat{d}^{>}\hat{p}_k^{>} \quad (91)$$

Santilli Backward Geno-Symplectic Geometry

$$<\hat{\omega} =^{<} \hat{d}^{<}\hat{r}^{<} \hat{\wedge}^{<} \hat{d}^{<}\hat{p}_k \quad (92)$$

3.3. GenoDynamical GenoEquations s [1-5]

Newton-Santilli Forward GenoEquation

$$\hat{m}^{>} > \frac{\hat{d}^{>}\hat{v}^{>}}{\hat{d}^{>}\hat{t}^{>}} - F^{>SA}(t,r,p) = [m \times \frac{dv}{dt}]^{>} - F^{SA>}(t,r,p) - F^{NSA>}(t,r,p,\ldots) = 0, \quad (93)$$

Newton-Santilli Backward GenoEquation

$$<\hat{m} < \frac{<\hat{d}^{<}\hat{v}}{<\hat{d}^{<}\hat{t}} -^{<SA} F(t,r,p) = =^{<} [m \times \frac{dv}{dt}] -^{<SA} F(t,r,p) -^{<NSA} F(t,r,p,\ldots) = 0, \quad (94)$$

Forward GenoVariational principle

$$\hat{\delta}^{>}\hat{A}^{>} = \hat{\delta}^{>} \hat{\int}^{>} (\hat{p}_k^{>} > \hat{d}^{>}\hat{r}^{>k} - \hat{H}^{>} > \hat{d}^{>}\hat{t}^{>}) = 0. \quad (95)$$

Backward GenoVariational principle

$$<\hat{\delta}^{<}\hat{A} =^{<} \hat{\delta}^{<} \hat{\int}^{<} (<\hat{p}_k <^{<} \hat{d}^{<}\hat{r}^{k} -^{<} \hat{H} <^{<} \hat{d}^{<}\hat{t}) = 0. \quad (96)$$

Forward Hamilton-Santilli GenoEquations

$$\left[\frac{d\hat{r}^k}{dt}\right]^{>} = \left[\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{p}_k}\right]^{>}, \quad \left[\frac{d\hat{p}_k}{dt}\right]^{>} = -\left[\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{r}^k}\right]^{>}, \tag{97}$$

Backward Hamilton-Santilli GenoEquations

$$<\left[\frac{\hat{d}\hat{r}^k}{\hat{d}t} = \frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{p}_k}\right], \quad ^{<}\left[\frac{\hat{d}\hat{p}_k}{\hat{d}t}\right] = -<\left[\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{r}^k}\right], \tag{98}$$

Forward Geno-Hilbert space $\hat{H}^{>}$ with states $|\hat{\psi}^{>}>$ over the isofield $\hat{C}^{>}$

GenoExpectation value of a Hermitean operator $\hat{A}$ on $\hat{H}^{>}$

$$<\hat{A}^{>}>=<^{<}\hat{\psi}|<\hat{A}^{>}>|\hat{\psi}^{>}>\in\hat{C} \tag{99}$$

Heisenberg-Santilli GenoEquation[3]

$$\hat{\imath}\hat{\times}\frac{\hat{d}\hat{A}}{\hat{d}t} = (\hat{A},\hat{H}) = \hat{A}<\hat{H}-\hat{H}>\hat{A} = A\times^{<}\hat{T}(\hat{\psi},\ldots)\times \hat{H}(\hat{r},\hat{p}) - \hat{H}(\hat{r},\hat{p})\times\hat{T}^{>}(\hat{\psi},\ldots)\times\hat{A} \tag{100}$$

Forward Schrödinger-Santilli GenoEquation

$$\hat{H}^{>}>|\hat{\psi}^{>}>=\hat{H}^{>}(\hat{r},\hat{p})\times\hat{T}^{>}(\hat{\psi},\hat{\partial}\hat{\psi},\ldots)\times|\hat{\psi}^{>}>=\hat{E}^{>}>|\hat{\psi}^{>}>=E^{>}\times|\hat{\psi}^{>}>, \tag{101}$$

$$\hat{p}^{>}>|\hat{\psi}^{>}>=-\hat{\imath}^{>}>\hat{\partial}_{\hat{r}}^{>}|\hat{\psi}^{>}>=-i\times\hat{I}^{>}\times\partial_{\hat{r}}|\hat{\psi}^{>}>, \tag{102}$$

Backward Schrödinger-Santilli GenoEquation

$$<^{<}\hat{\psi}|<^{<}\hat{H} =<^{<}\hat{\psi}|\times^{<}\hat{T}(\hat{\psi},\hat{\partial}\hat{\psi},\ldots)\times^{<}\hat{H}(\hat{r},\hat{p}) = <^{<}\hat{\psi}|<^{<}\hat{E} =<^{<}\hat{\psi}|\times^{<}E, \tag{103}$$

$$<^{<}\hat{\psi}|<^{<}\hat{p} = -<^{<}\hat{\psi}|<^{<}\hat{\imath}<_{\hat{r}}^{<}\hat{\partial} = -i\times<^{<}\hat{\psi}|_{\hat{r}}^{<}\partial\times^{<}\hat{I} \tag{104}$$

Forward Dirac-Santilli IsoEquation

$$(\hat{\eta}^{>\mu\nu}>\hat{\gamma}_{\mu}^{>}>\hat{p}_{\nu}^{>}-\hat{\imath}^{>}>\hat{m}^{>}>\hat{c}^{>})>|\hat{p}si^{>}>=0. \tag{105}$$

$$\{\hat{\gamma}_{\mu},\hat{\gamma}_{\nu}\}^{>} = [\hat{\gamma}_{\mu}\hat{\times}\hat{\gamma}_{\nu}+\hat{\gamma}_{\nu}\hat{\times}\hat{\gamma}_{\mu}]^{>} = \hat{2}^{>}>\hat{\eta}_{\mu\nu}^{>}, \tag{106}$$

Backward Dirac-Santilli GenoEquation

$$<^{<}\hat{\psi}|<(^{<}\hat{p}_{\nu}<^{<}\hat{\gamma}_{\mu}<^{<\mu\nu}\hat{\eta}-^{<}\hat{\imath}<^{<}\hat{m}<^{<}\hat{c}) = 0. \tag{107}$$

$$^{<}\{\hat{\gamma}_{\mu},\hat{\gamma}_{\nu}\} =^{<}[\hat{\gamma}_{\mu}\hat{\times}\hat{\gamma}_{\nu}+\hat{\gamma}_{\nu}\hat{\times}\hat{\gamma}_{\mu}] =^{<}\hat{2}<^{<}\hat{\eta}_{\mu\nu} = 2\times^{<}\hat{\eta}_{\mu\nu}, \tag{108}$$

3.4. Comments and References

As it is also well known, all 20th century mathematical, physical or chemical formulations are reversible over time. Following research over half a century initiated during his Ph. D. studies at the University of Torino, Italy, in the mid 1960s [15, 17-23,4,5], R. M., Santilli has made the additional historical discovery of the first and only known, axiomatically consistent, generalization of 20th century mathematics as well as of its covering isomathematcs into a form embedding irreversibility over time in ordered forward and backward units, in corresponding ordered forward and backward products and, consequently, in all subsequent mathematical structures, resulting in the new mathematics nowadays known as *Santilli forward and backward genomathematics* with corresponding physical and chemical theories for the representation of irreversible processes.

Since the reversibility over time of 20th century theories can be reduced to the invariance under anti-Hermiticity of the Lie product between Hermitean operators, $[a,b] = ab - ba = -[a,b]^{\dagger}$, Santilli presented in 1967 [15] the first known (p, q)-deformation of the Lie product $(a,b) = pab - qba$, where p, q are scalars and the product ab is generally non-associative. Following an intense search in European mathematical libraries, Santilli discovered that the new product verifies the axiom of *Lie-admissibility* by the American mathematician A, A, Albert [16] in the sense that the attached anti-symmetric product $[a,b] = (a,b) - (b,a)$ verifies the axioms of a Lie algebra.

Since spaceship during re-entry are notoriously irreversible over time, Santilli was invited by the Center for Theoretical Physics of the University of Miami, Florida, under NASA support, where he moved with his wife Carla and newly born daughter Luisa inAugust 1967, and published a number of additional works in Lie-admissibility, including the first known Lie-admissible generalization of Hamilton and Heisenberg equations [17,18], nowadays considered at the foundation of hadronic mechanics and chemistry, as well as the first and only known Lie-admissible formulation of dissipative plasmas surrounding spaceships during reentry [19].

Santilli then spent seven years , from 1968 to 1974, at the Department of Physics of Boston University, and then three years, from 1974 to 1977, at MIT, during which tine he wrote, in his words, *Phys.. Rev of career-oriented papers nobody reads.* InSeptember 1977, Santilli joined Harvard University and was invited by the DOE to study irreversible processes because all energy releasing processes are irreversible over time. In April 1978, Santilli published under his DOE support his most important mathematical contribution [20] (see also monographs [21]) in which he achieved a Lie-admissible covering of the various branches of Lie's theory, Eqs. (69), (70), including the most general known time evolution whose brackets characterize an algebra, Eqs. (1000). It should be indicated that the isotopies of Lie's theory outlined in the preceding section were derived by Santilli as a particular case of the broader Lie-admissible theory of Ref. [20], and then published in monographs [1].

Subsequently, Santilli discovered in paper [2] of 1993 that the axiom of a numeric field, besides admitting a generalization of the multiplicative unit, also admit the restriction of the associative product to an ordered form to the right and, separately,to the left. In this way, Santilli discovered two additional classes of new numbers, today known as *Santilli forward and backward genoreal, genocomplex and genoquarternionic numbers.* In the seminal memoir [3] of

[3] By including the multi-valued (Section 4) and hyperstructural formulations (Section 5), Lie-admissible equations (100) are so broad that it will take centuries for their generalizations. For this reason,Santilli has requested in his will that his tombstone should have the engraving

$$i\dot{A} = A<H-H>A$$

below his name.

1996 Santilli discovered two additional coverings of the ordinary differential calculus and of its isotopic covering, today known as *Santilli forward and backward genodifferential calculi,* Eqs. (65) to (68). Santilli called a *genotopy* [20] the lifting of isomathematics into ordered formulations to the right and to the left in the Greek sense of inducing a covering of Lie's axioms, Eqs. (69), (70).

As it is well known, thousands of papers have been published beginning from the late 1980s on the so-called q-deformations of Lie algebras with product $(a,b) = ab - qba$ which are an evident particular case of Santilli Lie-admissible product [15]. Whatit is lesser known, or not admitted, all q-deformations did not achieve invariance over time, thus being afflicted by serious inconsistencies, since they consisted of non-unitary theories formulated via the mathematics of unitary theories. Santilli solved this problem in 1997 by achieving the first and only known invariant formulation of q- as well as of (p, q)-deformations [22].

We should indicate that Santilli's conception of a genotopic lifting of his preceding isomathematcs (indicated in Section 2 by "hat" on symbols plus the "arrow of time") is necessary to achieve a consistent representation of irreversibility because point-like particles can only experience action-at-a-distance interactions that are reversible over time. Therefore, a simple genotopy of 20th century mathematics based on the conventional associative product would be axiomatically inconsistent. Consequently, to represent irreversibility it is first necessary to lift 20th century mathematics into isomathematcs, with consequential representation of extended-deformable particles via realizations of type (27) so that extended particles can experience non-Hamiltonian interactions needed for irreversibility. It is then necessary to add irreversibility via the ordering of all products. It should also be indicated that, when formulated via time-dependent isounits, isomathematics can becomes genomathematics via the identifications $\hat{I}(t,\dots) = \hat{I}^{\dagger}(t,\dots) = \hat{I}^{>}, \hat{I}(-t,\dots) = \hat{I}^{\dagger}(-t,\dots) =^{<} \hat{I}, \hat{I}(t,\dots) \neq \hat{I}(-t)$, and the judicious addition of ordered products.

Systematic studies on the Lie-Admissible treatment of irreversible systems were presented in memoir [3] and monographs [4]. Santilli's subsequent memoir [23] of 2006 remains to this day the most comprehensive presentation of Lie-admissible treatments of irreversibility at the classical and operator levels. Monographs [5] of 2008 presented an update. Paper collection [24[presents all available independent contributions in Lie-admissibility up to [1984. The Proceedings of the *Third International Conference on Lie-admissible Treatment of Irreversible Systems* [25] present numerous additional independent contributions as well as references for the five *Workshops on Lie-0admissible Algebras* organized by Santilli at Harvard University, and for the preceding two international conference in Lie-admissibility, the first at the Université d'Orleans, France, in 1981 and the second at the Castle Prince Pignatelli, Italy, in 1995 (see also the general review [14] and large literature quoted therein).

As it is well known, there exists a large number of papers on Lie-admissible algebras within the context of non-associative algebras (see Tomber's Bibliography [26] listing all significant papers in the field up to 1986). It should be indicated that, regrettably, these studies have no connection with Santilli genomathematics since the latter deals with the irreversible generalizations of all aspects of 20th century mathematics.

4. Classical Hypermathematcs, Hypermechanics and Hyperchemistry

4.1. Represented Systems s [1-5]

Multi-valued, time-irreversible systems of extended -deformable particles or constituents under the most general known Hamiltonian and non-Hamiltonian interaction, as occurring for multi-valued universes or the structure of the DNA.

4.2. Main Mathematical Structure s [1-5]

Basic HyperUnits and HyperProducts

$$\hat{I}^{>} = \{\hat{I}_1^{>}, \hat{I}_2^{>}, \hat{I}_3^{>}, \dots\} = 1/\hat{S}, \tag{109}$$

$$^{<}\hat{I} = \{^{<}\hat{I}_1, ^{<}\hat{I}_2, ^{<}\hat{I}_3, \dots\} = \frac{1}{\hat{R}}, \tag{110}$$

Forward and Backward HyperProducts

$$A > B = \{A \times \hat{S}_1 \times B, A \times \hat{S}_2 \times B, A \times \hat{S}_3 \times B, \dots\}, \hat{I}^{>} > A = A > \hat{I}^{>} = A \times I, \tag{111}$$

$$A < B = \{A \times \hat{R}_1 \times B, A \times hat R_2 \times B, A \times \hat{R}_3 \times B, \dots\}^{<}\hat{I} < A = -A <^{<} \hat{I} = I \times A, \tag{112}$$

$$A = A^{\dagger}, B = B^{\dagger}, \hat{R} = \hat{S}^{\dagger}. \tag{113}$$

Classical hypermathematcs then follow as for genomathematcs with multi-valued units, quantities and operations.

4.3. Classical Hyper-Dynamical Equations s [1-5]

The same as those for genomathematics, but with multi-valued hyperunits, quantities and operations.

Comments and References

The multi-valued three-dimensional (rather than multi-dimensional) realization of genomathematics outlined in Section 4 emerged from specific biological needs. The Australian biologist C. Illert [27] confirmed that the *shape* of seashells can indeed be represented in a three-0dimensional Euclidean space as known since Fourier's time, but proved that the *growth in time* of a seashell cannot any longer be consistently represented in a conventional, three-dimensional Euclidean space, and achieved a consistent representation via the doubling of the three reference axis.

Santilli [27,28] confirmed Illert's findings because the conventional Euclidean geometry has no time arrow and, consequently, cannot consistently represent a strictly irreversible system, such as the growth of seashells. Additionally, Santilli proved thathis geno-Euclidean geometry,

Eqs. (71) to (73), is equally unable to represent the growth in time of seashells despite its irreversible structure, however, an axiomatically consistent and exact representation of the growth of seashells was possible via the multi-valued realization of the forward geno-Euclidean geometry, thus beginning to illustrate the complexity of biological structures.

The multi-valued, rather than multi-dimensional character of classical hypermathematics is indicated by Santilli as follows [28] *We perceive the growth of a seashell specifically in three dimensions from our Eustachian lobes. Therefore, an irreversible mathematics suitable to represent the growth of sea shells must be perceived by us as being in three dimensions. However, Illert has shown the need to double the three Cartesian axis. Classical hypermathematics has been conceived and structured in such a way that the increase of the reference axes is complemented by a corresponding multi-valued hyperunit in such a way that a classical hyper-Euclidean geometry, when seen at the abstract level, remains indeed three-dimensional as necessary to achieve representation of biological structures compatible with our sensory perception.*

5. Hope Hypermathematics, Hypermechanics and Hyperchemistry

Represented Systems

The most complex known multi-valued, time-irreversible requiring extremely large number of data, such as the DNA code [31-35].

Comments and References

Despite the preceding structural generalization of 20th century mathematics, Santilli remained dissatisfied in view of the complexity of nature, particularly of biological entities because advances in the *structure* of the DNA are indeed possible via classical hypermathematics, as we shall see in the third collection of this series dedicated to chemistry (e.g., via Santilli hypermagnecules), but any attempt at representing the DNA *code* via any of the preceding mathematics can be proved to be excessively restrictive due to the volume, complexity, diversification and coordination of the information.

Therefore, Santilli approved one of the most important mathematicians in hyperstructures, T. Vougiouklis from Greece, and asked for his assistance in further generalizing the preceding mathematics via hyperstructures defined on hyperfields, as necessary for applications implying measurements, and formulated via hyperoperations (called "hope") permitting the needed broadening of the representational capability.

The above contact lead to the hypermathematics indicated in this section as presented in Refs. [29-33] which is based on Vougiouklis H_v hyperaxioms and which mathematics, in Santilli's words, *constitutes the most general mathematics that can be conceived nowadays by the human mind.*

6. Isodual Mathematics, Mechanics and Chemistry

6.1. Represented Systems

Single-valued, closed-isolated, time-reversible systems of classical and operatorpoint-like antiparticles moving in vacuum solely under action at a distance Hamiltonian interactions, such as the stricture of antimatter atoms and antimatter molecules [2,36-43].

6.2. Main Mathematical Structure [2,36-43]

Basic isodual unit

$$1^d = -1^\dagger = -1, \tag{114}$$

Isodual numeric fields

$$F^d(n^d, \times^d, 1^d), n^d = n \times 1^d, n^d \times^d m^d = n^d \times (1^d)_{-1} \times m^d \in F^d,$$

$$n^d = isodual\,!\,real, complex, quatern.\,!\,numbers, \tag{115}$$

Isodual functional analysis

$$f^d(r^d) = f(r^d) \times 1^d \in F^d \tag{116}$$

Isodual differential calculus

$$d^d r^d = (1)^{-1} \times dr^d = dr, \tag{117}$$

$$\frac{\partial^d f^d(r^d)}{\partial^d r^d} = 1^d \times \frac{\partial f^d(r^d)}{\partial r^d}, \tag{118}$$

Santilli Isodual Lie theory

$$[X_i, X_j]^d = (X_i \times X_j - X_j \times X_i)^d == -C_{ij}^k \times X_k, \tag{119}$$

$$A^d(w^d) = e_d^{X \times w \times i} \times^d A^d(0) \times^d e_d^{-i \times w \times X}. \tag{120}$$

Santilli isodual Euclidean geometry

$$E^d(^d, \delta^d, 1^d), r^d = (r^{dk}), k = 1,2,3,$$

$$\delta^d = Diag.(-1,-1,-1), \tag{121}$$

$$r^{d2d} = r^{di} \times \delta_{ij} \times^d r^{dj} = (r_1^2 + r_2^2 + r_3^2) \times 1^d \in F^d, \tag{122}$$

Santilli Isodual Minkowskian geometry ($\mu = 1,2,3,4,$)

$$M^d(x^d, \eta^d, I^d): x^d = (x^{d\mu}), x^{d4} = t^d = t \times 1^d = -t, \tag{123}$$

$$\eta^d = Diag.(-1,-1,-1,+c^{d2d}), \tag{124}$$

$$x^{d2d} = (x^\mu \times \eta_{\mu\nu} \times x^\nu)^d = (x_1^2 + x_2^2 + x_3^2 - t^2c^2) \times 1^d \in F^d, \tag{125}$$

Isodual Riemannian geometry, Santilli Isodual Symplectic Geometry.

6.3. Isodual Dynamical Equations [2,36-43

Newton-Santilli Isodual Equation

$$m^d \times^d \frac{d^d v^d}{d^d t^d} - F^{dSA}(t^d, r^d, v^d) = 0, \quad (126)$$

Isodual Variational Principle

$$\delta^d A^d = \delta^d \int^d (p_k^d \times^d d^d r^{dk} - H^d \times^d d^d t^d) = 0. \quad (127)$$

Hamilton-Santilli Isodual Equations without external terms

$$\frac{d^d r^{dk}}{d^d t^d} = \frac{\partial^d H^d(r^d, p^d)}{\partial^d p_k^d}, \quad \frac{d^d p_k^d}{d^d t^d} = -\frac{\partial^d H^d(r^d, p^d)}{\partial^d r^{dk}}, \quad (128)$$

Isodual Hilbert space H^d over C with states $|\psi^d >= -< \psi|$ over C^d

Expectation value of a Hermitean operator A

$$< A^d >=< \psi| \times A^d \times |\psi >\in C^d m \quad (129)$$

Heisenberg-Santilli Isodual Equations

$$i^d \times^d \frac{d^d A^d}{d^d t^d} = [A, H]^d = (A \times H - H \times A)^d, \quad (130)$$

Schrödinger-Santilli Isodual Equations

$$H^d \times^d |\psi^d >= E^d \times^d |\psi^d >= -E \times |\psi > \quad (131)$$

$$p^d \times^d |\psi^d >= +i^d \times^d \partial_{rd}^d |\psi^d > \quad (132)$$

Dirac-Santilli Isodual Equation

$$(\eta^{d\mu\nu} \times^d \gamma_\mu^d \times^d p_\nu^d + i^d \times^d m^d \times^d c^d) \times |\psi >= 0. \quad (133)$$

$$\{\gamma_\mu, \gamma_\nu\}^d = (\gamma_\mu \times \gamma_\nu + \gamma_n u \times \gamma_\mu)^d = 2^d \times^d \eta_{\mu\nu}^d, \quad (134)$$

Comments and References

In addition to the the study of irreversible processes and the representation of extended-deformable particles, during his Ph. D. studies of the md 1960s Santilli was interested to ascertain whether a far away galaxy is made up of matter or of antimatter. He soon discovered that none of the mathematics and physics he had learned during his graduate studies was applicable for a quantitative study of the problem considered since, at that time, antimatter was solely represented in second quantization, while the study of far away antimatter galaxies requested their representation at the purely *classical and neutral* level. In this way, Santilli initiated a solitary scientific journey that lasted for half a century.

This occurrence created one of the biggest imbalances in scientific history because matter was treated at all possible levels, from Newtonian mechanics to second quantization, while antimatter was solely treated in second quantization. The imbalance originated from the fact that special and general relativities had been conceived decades before the discovery of antimatter and, therefore, they had no possibility of representing antimatter at the classical and neutral (as well as charged) level.

It should be stressed that the ongoing trend to extend the application of special and general relativities to the classical treatment of antimatter is afflicted by a number of serious inconsistencies, such as the impossibility to achieve a consistent representation of neutral antimatter, the impossibility to reach a consistent representation of matter-antimatter annihilation (evidently due to the lack of a suitable conjugation from matter to antimatter), violation of the PCT theorem and other inconsistencies that remain generally ignored.

Being an applied mathematician by instinct and training, Santilli knew that the imbalance was the result of a purely mathematical insufficiency because the transition from matter to antimatter is an anti-homomorphism. Consequently, the description of antimatter required a mathematics which is anti-homomorphic to conventional mathematics.

Santilli dedicated a decade to the search of the needed mathematics for antimatter. Following an additional extended search done at the Department of Mathematics of Harvard University under DOE support in the early 1980s, *Santilli concluded that a mathematics suitable for the joint classical and operator treatment of antimatter did not exist and had to be constructed.*

In the early 1980s, Since he had introduced the isoproduct $A \hat{\times} B = A \times \hat{T} \hat{B}, \hat{T} > 0$, Eq. (25). Consequently, it was natural to introduce its *negative-definite* counterpart which he called *isodual* and denoted with theupper index ^{d}, namely $A \hat{\times}^d B = A \times \hat{T}^d \hat{B}, \hat{T}^d = (\hat{T}^d)^\dagger < 0$. While constructing the isotopies of 20th century mathematics presented in Section 2, Santilli initiated the construction of their isodual image but published no paper in the new mathematics for over a decade.

This caution was due to the fact that, despite the lack of any visible mathematical inconsistency, Santilli remained skeptical on a mathematics based on a negative-definite product is afflicted by known physical inconsistencies, such as the violation of causality for negative time, energies and other physical quantities.

A breakthrough occurred in paper [2] of 1993. During the achievement of the broadest possible realizations of the abstract axioms of a numeric field (of characteristic zero), Santilli discovered that realizations with negative-definite units were simply unavoidable. This lead to the discovery of additional new numbers, today known as *Santilli isodual real, isodual complex and isodual quaternionic numbers* occurring for $I^d = -1$, Eq. (14), with isodual products (5), which are at the foundation of the isodual mathematics of this section and the additional numbers known as *Santilli isodual iso- and isodual geno-real, complex and quaternionic numbers* which are at the foundation of the isodual isomathematics and isodual genomathematics of Sections 7 and 8m respectively [2].

The discovery of isodual numbers is truly historical in our view due to its far reaching implications. In fact, the discovery established the existence of the desired *isodual mathematics* as an anti-isomorphic image of 20th century mathematics for the representation of antimatter. Additionally, the discovery permitted the resolution of the problems of causality for negative values of physical quantities.

To avoid insidious inconsistencies generally not seen by non-experts in the field, the isodual map must be applied for consistency to the *totality* of quantities and their operations. This lead to Santilli's conception of antimatter as possessing it

negative-definite physical quantities for time, energy, momentum, frequency, etc, but such negative values are referred to *negative units* of measurements. Consequential a theory with negative time referred to negative units of time is as causal as our reality with a positive time referred to positive units, and the same holds for all other physical quantities.

Following the resolution of these basic issues, Santilli published in 1994 his first paper [36] specifically devoted to the isodual representation of antimatter. In mathematical memoir [3] of 1996, Santilli achieved the first isodual mathematical and physical representation of antimatter. In paper [37] of 1998, Santilli achieved his first goal of the early 1960s, namely, a consistent classical representation of neutral (as well as charged) antimatter.

By the early 1990s, Santilli had shown that *isodual mathematics represents all available experimental, data on antimatter at the classical and operator level.* Hence, he initiated the second phase of his studies, namely, the identification of new predictions for subsequent experimental verification.

A breakthrough occurred at the 1996 *First International Conference on Antimatter* help in Sepino, Italy [38]. By that time, Santilli had shown that the only conceivable representation of *neutral* antimatter required the conjugation of the sign of all physical quantities (jointly with the corresponding conjugation of their units of measurements). Since photons are neutral, the application of the same principle to light implies light emitted by antimatter, that Santilli called *isodual light,* is physically different than light emitted by matter in an experimentally verifiable way, e.g., because antimatter light is predicted to be *repelled* by a matter gravitational field.

Santilli then passed to a deeper geometric study of the gravitational field of antimatter. As indicated earlier, general relativity was formulated decades before the discovery of antimatter and, therefore, had no clue for the representation of the gravitational field of antimatter bodies. In Ref.[39] of 1998, Santilli conducted an in depth geometric study of antimatter, and in monograph [40) of 2006, Santilli completed the gravitational study of antimatter via the isodual Riemannian geometry.

All these studies concluded with the prediction of *gravitational repulsion* (antigravity) between matter and antimatter at all levels of analysis, from the isodual Newton-Santilli equations (26) to isodual second quantization. These aspects will be studied in the second collection of this series dedicated to hadronic mechanics.

Thanks to all the above advances, Santilli was finally in a position to address his original main aim of the 1960s, namely, ascertain whether a far away galaxy is made up of matter or of antimatter. The preceding studies had established that the light emitted by antimatter must have a *negative index of refraction* that, as such, require *concave* lenses for its focusing. Consequently, Santilli secured the construction of a revolutionary telescope with concave lenses. About fifty years following his original aim, Santilli finally published in 2013 [41[measurements of the night sky with his new telescope showing images that can be solely due to light with a negative index of refraction which light, in turn, can solely originate from far away antimatter stars or galaxies (see also the two independent confirmations [42,43]).

An intriguing aspect that should be of interest to pure mathematicians is the conclusion of these studies illustrating the power of new mathematics, to the effect that none of the large numbers of telescopes available nowadays can detect antimatter starsor galaxies since they all have *convex* lenses. Similarly, as humans evolved in a matter world, we will never be able to see antimatter with our eyes since our cornea is convex and, as such, it will disperse antimatter light all over the retina.

Needless to say, isodual mathematics and its application to antimatter have implications so intriguing that are stimulating the participation of a large number of scientists as we shall report in the second collection of this series

7. Isodual Isomathematics, Isodual Isomechanics and Isodual Isochemistry

7.1. Represented Systems [2,36-43

Single-value, closed-isolated, time-reversible system of classical or operator extended-deformable antiparticles with action at a distance Hamiltonian and contact non-Hamiltonian interactions, such as the structure of antimatter hadrons, nuclei and stars, in the antimatter valence electron bonds and other antimatter systems.

7.2. Main Mathematical Structure[2,36-43

Basic Isodual IsoUnit

$$\hat{I}^d = \hat{I}^d(r^d, p^d, a^d, \psi, {}^d\partial^d\psi^d, \dots) = 1^d/{}^d\hat{I}^d < 0, \quad (135)$$

Basic Isodual IsoFields

$$\hat{F}^d(\hat{n}^d, \hat{\times}^d, \hat{I}^d), \hat{n}^d = n \times \hat{I}^d, \hat{n}^d \,\hat{\times}^d\, \hat{m}^d = \hat{n}^d \times \hat{I}^d \times \hat{m}^d \in \hat{F}^d, \quad (136)$$

Isodual IsoCoordinates $\hat{r}^d = r \times \hat{I}^d \in \hat{F}^d$,

Isodual IsoFunctional analysis $\hat{f}^d(\hat{r}^d) == f(\hat{r}^d) \times \hat{I}^d \in \hat{F}^d$,

Isodual IsoDifferential Calculus

$$\hat{d}^d\hat{r}^d = dr - r^d \times \hat{I}^d \times d\hat{I}^d, \quad (137)$$

$$\frac{\hat{\partial}^d \hat{f}^d(\hat{r})}{\hat{\partial}^d \hat{r}^d} = \hat{I}^d \times \frac{\partial \hat{f}^d(\hat{r}^d)}{\partial \hat{r}^d}, \quad (138)$$

Santilli Isodual Lie-Isotopic Theory

$$[\hat{X_i}, X_j]^d = X_i \,\hat{\times}\, X_j - X_j \,\hat{\times}\, X_i)^d == -C_{ij}^k(r, p, \dots) \times X_k, \quad (139)$$

$$A^d(w^d) = \hat{e}_d^{X^d \times w^d \times i^d} \,\hat{\times}^d\, A^d(0^d) \,\hat{\times}^d\, \hat{e}_d^{-i^d \times w^d \times X^d}. \quad (140)$$

Santilli Isodual Iso-Euclidean Geometry

$$\hat{E}^d(\hat{r}^d, \hat{\delta}^d, \hat{I}^d), \hat{\delta}^d(r^d, p^d, a^d, \psi, \dots) =$$

$$\hat{T}^d(r^d, p,^d a^d, \psi^d, \ldots) \times \delta, \quad (141)$$

$$\hat{T}^d = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2)^d, \quad (142)$$

$$\hat{r}^{d\hat{2}d} = (\hat{r}^i \hat{\times} \hat{\delta}_{ij} \hat{\times} \hat{r}^j)^d = (\frac{r_1^2}{n_1^2} + \frac{r_2^2}{n_2^2} + \frac{r_3^2}{n_3^2})^d \times \hat{I}^d \in \hat{F}^d, \quad (143)$$

Santilli Isodual Iso-Minkowskian Geometry ($\mu = 1,2,3,4$)

$$\hat{M}^d(\hat{x}^d, \hat{\eta}^d, \hat{I}^d): \hat{x}^d = (\hat{x}^{d\mu}), \hat{x}_4^d = \hat{t}^d = t \times \hat{I}^d, \quad (144)$$

$$\hat{\eta}^d(x^d, \psi^d, \ldots) = \hat{T}^d(x^d, \psi^d, \ldots) \times \eta, \quad (145)$$

$$\hat{T}^d = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2)^d, \quad (146)$$

$$\hat{x}^{d\hat{2}d} = (\hat{x}^\mu \hat{\times} \hat{\eta}_{\mu\nu} \hat{\times} \hat{x}^\nu)^d == (\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2})^d \times \hat{I}^d \in \hat{F}^d, \quad (147)$$

Santilli Isodual Iso-Riemannian Geometry

$$\hat{R}^d(\hat{x}^d, \hat{g}^d, \hat{I}^d): \hat{g}^d = \hat{T}^d(x^d, v^d, \ldots) \times g(x), \quad (148)$$

$$\hat{x}^{d\hat{2}d} = (\frac{g_{11}}{n_1^2} + \frac{g_{22}}{n_2^2} + \frac{g_{33}}{n_3^2} - \frac{g_{44}}{n_4^2})^d \times \hat{I}^d \in \hat{F}^d, \quad (149)$$

Santilli Isodual Iso-Symplectic Geometry

$$\hat{\omega}^d = \hat{d}\hat{r}^{dk} \hat{\wedge}^d \hat{d}\hat{p}_k^d \quad (150)$$

7.3. Isodual IsoDynamical IsoEquation[2,36-43

Newton-Santilli Isodual IsoEquation

$$\hat{m}^d \hat{\times}^d \frac{\hat{d}^d \hat{v}^d}{\hat{d}^d \hat{t}^d} - F^{dSA}(r^d, p^d) == (m \times \frac{dv}{dt})^d - F^{dSA}(r^d, p^d) - F^{dNSA}(r^d, p^d, \ldots) = 0^d = 0, \quad (151)$$

Isodual IsoVariational principle

$$\hat{\delta}^d \hat{A}^d = \hat{\delta}^d \hat{\int}^d (\hat{p}_k^d \hat{\times}^d \hat{d}^d hat r^{dk} - \hat{H}^d \hat{\times}^d \hat{d}^d \hat{t}^d) = 0^d = 0. \quad (152)$$

Hamilton-Santilli Isodual IsoEquations

$$\frac{\hat{d}^d \hat{r}^{dk}}{\hat{d}^d \hat{t}^d} = \frac{\hat{\partial}^d \hat{H}^d(\hat{r}^d, \hat{p}^d)}{\hat{\partial}^d \hat{p}_k^d}, \quad \frac{\hat{d}\hat{p}_k}{\hat{d}^d \hat{t}^d} = + \frac{\hat{\partial}^d \hat{H}^d(\hat{r}^d, \hat{p}^d)}{\hat{\partial}^d \hat{r}^{dk}}, \quad (153)$$

Isodual iso-Hilbert space $\hat{H}^d$ over C with states $|\hat{\psi}^d >= -< \hat{\psi}|$ over $\hat{C}^d$

Expectation value of a Hermitean operator A

$$< A^d >=< \hat{\psi}| \hat{\times} A^d \hat{\times} |\hat{\psi} > \in C^d \quad (154)$$

Heisenberg-Santilli Isodual IsoEquation

$$\hat{\imath}^d \hat{\times}^d \hat{d}^d hat A^d over \hat{d}^d \hat{t}^d = [\hat{A}, \hat{H}]^d = (\hat{A} \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{A})^d = (\hat{A} \times \hat{T}(\hat{\psi}, \hat{\partial}\hat{\psi}, \ldots) \times \hat{H}(\hat{r}, \hat{p}) - \hat{H}(\hat{r}, \hat{p}) \times \hat{T}(\hat{\psi}, \hat{\partial}\hat{\psi}, \ldots) \times \hat{A})^d, \quad (155)$$

Schrödinger-Santilli Isodual IsoEquation

$$(\hat{H} \hat{\times} |\hat{\psi} >)^d =< \hat{\psi}^d| \hat{\times}^d \hat{H}^d = (\hat{H}(\hat{r}, \hat{p}) \times \hat{T}(\hat{\psi}, \hat{\partial}\hat{\psi}, \ldots) \times |\hat{\psi} >)^d = -< \hat{\psi}^d| \hat{\times}^d \hat{E}^d = -< \hat{\psi}^d| \times \hat{E}^d, \quad (156)$$

$$(\hat{p} \hat{\times} |\hat{\psi} >)^d =< \hat{\psi}^d| \hat{\times}^d \hat{\partial}_{\hat{r}^d} = -i \times< \hat{\psi}^d| \hat{\times}^d \hat{\partial}_{\hat{r}^d}^d, \quad (157)$$

Dirac-Santilli Isodual IsoEquation

$$[(\hat{\eta}^{\mu\nu} \hat{\times} \hat{\gamma}_\mu \hat{\times} \hat{p}_\nu - \hat{\imath} \hat{\times} \hat{m} \hat{\times} \hat{c}) \hat{\times} |\hat{p}si >]^d = 0. \quad (158)$$

$$\{\hat{\gamma}_\mu, \hat{\gamma}_\nu\}^d = (\hat{\gamma}_\mu \hat{\times} \hat{\gamma}_\nu + \hat{\gamma}_\nu \hat{\times} \hat{\gamma}_\mu)^d = \hat{2}^d \hat{\times}^d \hat{\eta}_{\mu\nu}^d, \quad (159)$$

Comments and References

See monograph [40] with particular reference to the use of the isodual isomathematics for the achievement of a grand unification of electroweak and gravitational interactions inclusive of matter and antimatter.

8. Isodual Genomathematics, Isodual Genomechanics and Isodual Genochemistry

8.1. Represented Systems [2,36-43

Single-valued, time-irreversible system of extended-deformable antiparticles under action at a distance Hamiltonian and contact non-Hamiltonian interactions, as occurring in antimatter nuclear reactions, antimatter biological structures and antimatter chemical reactions.

8.2. Main Mathematical Structure [2,36-43

Backward Isodual GenoUnit

$$\hat{I}^{>c} = \hat{I}^{>d}(t^{>}r^{>}, p^{>d}, a^{>d}, \psi^{>d}, \partial^{>d}\psi^{>d}, \ldots.) = 1/\hat{T}^{>d} > 0, \quad (160)$$

Forward Isodual GenoUnit

$$^{<d}\hat{I} =^{<d} \hat{I}(^{<d}r,^{<d} p,^{<d} a,^{<d} \psi,^{<d} \partial^{<d}\psi, \ldots.) = 1/^{<d}\hat{T} > 0, \quad (161)$$

Condition for time-irreversibility

$$\hat{I}^{>d} \neq {}^{<d}\hat{I} \quad (162)$$

Backward Isodual GenoFields

$$\hat{F}^{>d}(\hat{n}^{>d}, >, \hat{I}^{>d}), \hat{n}^{>d} = n \times \hat{I}^{>d}, \hat{n}^{>d} >^d \hat{m}^{>d} = \hat{n}^{>d} \times \hat{T}^{>d} \times \hat{m}^{>d} \in \hat{F}^{>d}, \quad (163)$$

Forward Isodual GenoFields

$$^{<d}\hat{F}(^{<d}\hat{n}, <,^{<d} \hat{I}), \quad ^{<d}\hat{n} =^{<d} \hat{I} \times n, \quad ^{<d}\hat{n} <^d \quad ^{<d}\hat{m} = =^{<d} \hat{n} \times^{<d} \hat{T} \times^{<d} \hat{m} \in^{<d} \hat{F}, \quad (164)$$

Backward Isodual GenoCoordinates

$$\hat{r}^{>d} = r \times \hat{I}^{>d} \in \hat{F}^{>d}, \quad (165)$$

Forward Isodual GenoCoordinates

$$^{<d}\hat{r} =^{<d} \hat{I} \times r \in^{<d} \hat{F}, \quad (166)$$

Backward Isodual GenoFunctional analysis

$$\hat{f}^{>d}(\hat{r}^{>d}) = f(\hat{r}^{>d}) \times \hat{I}^{>d} \in \hat{F}^{>d}, \quad (167)$$

Forward Isodual GenoFunctional analysis

$$^{<d}\hat{f}(^{<d}\hat{r}) = f(^{<d}\hat{r}) \times^{<d} \hat{I} \in^{d} \hat{F} \quad (168)$$

Backward Isodual GenoDifferential Calculus

$$\hat{d}^{>d}\hat{r}^{>d} = dr + r \times \hat{T}^{>d} \times d\hat{I}^{>d}, \quad (169)$$

$$\frac{\hat{\partial}^{>d}\hat{f}^{>d}(\hat{r}^{>d})}{\hat{\partial}^{>d}\hat{r}^{>d}} = \hat{I}^{>d} \times \frac{\partial \hat{f}^{>d}(\hat{r}^{>d})}{\partial \hat{r}^{>d}}, \quad (170)$$

Forward Isodual GenoDifferential Calculus

$$^{<d}\hat{d}^{<d}\hat{r} = dr + r \times^{<d} \hat{T} \times d^{<d}\hat{I}, \quad (171)$$

$$\frac{^{<d}\hat{\partial}^{<d}\hat{f}(^{<d}\hat{r})}{^{<d}\hat{\partial}^{<d}\hat{r}} =^{<dd} \hat{I} \times \frac{\partial^{<d}\hat{f}(^{<d}\hat{r})}{\partial^{<d}\hat{r}}, \quad (172)$$

Santilli Isodual Lie-Admissible Theory

$$(X_i \hat{,} X_j)^d = (X_i < X_j - X_j > X_i)^d = -C_{ij}^{dk}(t^d, r^d, p^d, \psi^d, \dots) \times X_k, \quad (173)$$

$$A^d(w^d) = \hat{e}_{>d}^{X \times w \times i} >^d A(0) <_{<d}^{d} \hat{e}^{-i \times w \times X}. \quad (174)$$

Santilli Backward Geno-Euclidean Geometry

$$\hat{E}^{>d}(\hat{r}^{>d}, \hat{\delta}^{>d}, \hat{I}^{>d}), \delta^{>d}(t, r, p, \psi, \dots) = \hat{T}^{>d}(t, r, p, \psi, \dots) \times \delta, \quad (175)$$

$$\hat{r}^{>d\hat{2}d} = (\hat{r}^{>di} >^{d} \hat{\delta}_{ij}^{>d} > \hat{r}^{>dj} =\in F^{>d}, \quad (176)$$

$$\hat{T}^{>d} \neq \hat{T}^{>d\,transp} \quad (177)$$

Santilli Forward Isodual Geno-Euclidean Geometry

$$^{<d}\hat{E}(^{<d}\hat{r}, ^{<d}\hat{\delta}, ^{<d}\hat{I}), \quad ^{<d}\hat{\delta}(t, r, p, \psi, \dots) =^{<d} \hat{T}(t, r, p, \psi, \dots) \times \delta, \quad (178)$$

$$^{<d\hat{2}d}\hat{r} =^{<di} \hat{r} <^{<d} \hat{\delta}_{ij} <^{d} \quad ^{<dj}\hat{r} \in^{<d} F, \quad (179)$$

$$^{<d}\hat{T} \neq^{<d\,transp} \hat{T} \quad (180)$$

Santilli Backward Isodual Geno-Minkowskian Geometry ($\mu = 1,2,3,4$)

$$\hat{M}^{>d}(\hat{x}^{>d}, \hat{\eta}^{>d}, \hat{I}^{>d}): \hat{x}^{>d} = (\hat{x}^{>d\mu}), x_4^{>d} = t^{>d}, \quad (181)$$

$$\hat{\eta}^{>d}(x, \psi, \dots) = \hat{T}^{>d}(x, \psi, \dots) \times \eta, \quad (182)$$

$$\hat{x}^{>d2d} = \hat{x}^{>d\mu} >^{d} \hat{\eta}_{\mu\nu}^{>d} >^{d} \hat{x}^{>dv} \in \hat{F}^{>d}, \quad (183)$$

$$\hat{\eta}^{>d} \neq \hat{\eta}^{>d\,transp} \quad (184)$$

Santilli Forward Isodual Geno-Minkowskian Geometry ($mu = 1,2,3,4$)

$$^{<d}\hat{M}(^{<d}\hat{x}, ^{<d}\hat{\eta}, ^{<d}\hat{I}): \quad ^{<d}\hat{x} = (\hat{x}^{\mu}), \quad ^{<d}x_4 =^{<d} t, \quad (185)$$

$$^{<d}\hat{\eta}(x, v, \dots) =^{<d} \hat{T}(x, v, \dots) \times \eta, \quad (186)$$

$$^{<d}\hat{x}^{<d2d} =^{<d\mu} \hat{x} <^{d} \quad ^{<d}\hat{\eta}_{\mu\nu} <^{d} \quad ^{<v}\hat{x} \in^{<d} \hat{F}, \quad (187)$$

$$^{<d}\hat{\eta} \neq^{<d\,transp} \hat{\eta} \quad (188)$$

Santilli Backward Isodual Geno-Riemannian Geometry

$$\hat{R}^{>d}(\hat{x}^{>d}, \hat{g}^{>d}, \hat{I}^{>d}): \hat{g}^{>d} = \hat{T}^{>d}(x, v, \dots) \times g(x), \quad (189)$$

$$\hat{x}^{>d2>} = x^{>d\mu} \hat{.}^{d} \hat{g}_{\mu\nu}^{>d} >^{d} x^{>dv} \in \hat{F}^{>d} \quad (190)$$

$$\hat{T}^{>d} \neq \hat{T}^{>d\,transp} \quad (191)$$

Santilli Forward Isodual Geno-Riemannian Geometry

$$^{<d}\hat{R}(^{<d}\hat{x}, ^{<}\hat{g}, ^{<d}\hat{I}): \quad ^{<d}\hat{g} =^{<d} \hat{T}(x, v, \dots) \times g(x), \quad (192)$$

$$^{<d}\hat{x}^{<d2d} =^{<d\mu} \hat{x} <^{d} \quad ^{<d}\hat{g}_{\mu\nu} <^{d} \quad ^{<dv}\hat{x} \in^{<d} \hat{F}, \quad (193)$$

$$^{<d}\hat{g} \neq^{<d\,transp} \hat{g} \quad (194)$$

Santilli Backward Isodual Geno-Symplectic Geometry

$$\hat{\omega}^{>d} = \hat{d}^{>d}\hat{r}^{>dk} \hat{\wedge}^{>d} \hat{d}^{>d}\hat{p}_k^{>d} \quad (195)$$

Santilli Forward Isodual Geno-Symplectic Geometry

$$^{<d}\hat{\omega} =^{<d} \hat{d}^{<dl}\hat{r}^{<d} \hat{\wedge}^{<d} \hat{d}^{<d}\hat{p}_k \quad (196)$$

8.3. Isodual GenoDynamical GenoEquations [2,36-43

Newton-Santilli Backward Isodual GenoEquation

$$\hat{m}^{>d} > \frac{\hat{d}^{>d}\hat{v}^{>d}}{\hat{d}^{>d}\hat{t}^{>d}} - F^{>dSA}(t, r, p) = [m \times \frac{dv}{dt}]^{>d} - F^{SA>d}(t, r, p) - F^{NSA>d}(t, r, p, \dots) = 0, \quad (197)$$

Newton-Santilli Forward Isodual GenoEquation

$$^{<d}\hat{m} < \frac{^{<d}\hat{d}^{<d}\hat{v}}{^{<d}\hat{d}^{<d}\hat{t}} -^{<dSA} F(t, r, p) =^{<d} [m \times \frac{dv}{dt}] -^{<dSA} F(t, r, p) -^{<dNSA} F(t, r, p, \dots) = 0, \quad (198)$$

Backward Isodual GenoVariational principle

$$\hat{\delta}^{>d}\hat{A}^{>d} = \hat{\delta}^{>d} \hat{\int}^{>d} (\hat{p}_k^{>d} > \hat{d}^{>d}\hat{r}^{>dk} - \hat{H}^{>d} > \hat{d}^{>d}\hat{t}^{>d}) = 0. \quad (199)$$

Forward Isodual GenoVariational principle

$$^{<d}\hat{\delta}^{<d}\hat{A} =^{<d} \hat{\delta}^{<d} \hat{\int} (^{<d}\hat{p}_k <^{<d} \hat{d}^{<d}\hat{r}^{k} -^{<d}\hat{H} <^{<d} \hat{d}^{<d}\hat{t}) = 0. \quad (200)$$

Backward Isodual Hamilton-Santilli GenoEquations

$$\frac{\hat{d}^{>d}\hat{r}^{>dl}}{\hat{d}^{>d}\hat{t}^{>d}} = [\frac{\hat{\partial}\hat{H}(\hat{r}, \hat{p})}{\hat{\partial}\hat{p}_k}]^{>d}, \quad [\frac{\hat{d}^{>d}\hat{p}_k^{>d}}{\hat{d}^{>d}\hat{t}^{>d}}] = -[\frac{\hat{\partial}\hat{H}(\hat{r}, \hat{p})}{\hat{\partial}\hat{r}^{k}}]^{>d}, \quad (201)$$

Forward isodual Hamilton-Santilli GenoEquations

$$\frac{^{<d}\hat{d}^{<d}\hat{r}^{k}}{^{<d}\hat{d}^{<d}\hat{t}} =^{<d} \frac{\hat{\partial}\hat{H}(\hat{r}, \hat{p})}{\hat{\partial}\hat{p}_k}], \quad \frac{^{<d}\hat{d}^{<d}\hat{p}_k}{^{<d}hatd^{<d}\hat{t}}] = -^{<}[\frac{\hat{\partial}\hat{H}(\hat{r}, \hat{p})}{\hat{\partial}\hat{r}^{k}}], \quad (202)$$

Heisenberg-Santilli IsoDual GenoEqutions

$$\hat{\imath} \hat{\times} \frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = (\hat{A}, \hat{H}) = \hat{A} < \hat{H} - \hat{H} > \hat{A} = A \times^{<} T(\hat{\psi}, \hat{\partial}\hat{\psi}, \dots) \times \hat{H}(\hat{r}, \hat{p}) - \hat{H}(\hat{r}, \hat{p}) \times \hat{T}^{>}(\hat{\psi}, \hat{\partial}\hat{\psi}, \dots) \times \hat{A} \quad (203)$$

Schrödinger-Santilli Backward Isodual GenoEquations

$$\hat{H}^{>d} >^{d} |\hat{\psi}^{>d} >= \hat{H}^{>d}(\hat{r}, \hat{p}) \times \hat{T}^{>d}(\hat{\psi}, \hat{\partial}\hat{\psi}, \dots) \times |\hat{\psi}^{>} >=$$

$$\hat{E}^{>d} >^{d} |\hat{\psi}^{>d} >= E^{>d} \times |\hat{\psi}^{>d} >, \quad (204)$$

$$\hat{p}^{>d} > |\hat{\psi}^{>d} >= -\hat{\imath}^{>d} > \hat{\partial}_{\hat{r}}^{>d} |\hat{\psi}^{>} >= -i \times \hat{I}^{>} \times \partial_{\hat{r}} |\hat{\psi}^{>d} >, \quad (205)$$

Schrödinger-Santilli Forward Isodual GenoEquations

$$<^{<d} \hat{\psi}| <^{d} \quad <\hat{H} =<^{<d} \hat{\psi}| \times^{<d} \hat{T}(\hat{\psi}, \hat{\partial}\hat{\psi}, \ldots) \times^{<} \hat{H}(\hat{r}, \hat{p}) = <^{<d} \hat{\psi}| <^{<} \hat{E} =<^{<d} \hat{\psi}| \times^{<d} E, \quad (206)$$

$$<^{<d} \hat{\psi}|<^{<d} \hat{p} = -<^{<d} \hat{\psi}| <^{d} \quad <^{d}\hat{\imath} <^{d} \quad \hat{r}^{<d}\hat{\partial} = -i \times <^{<d} \hat{\psi}|\hat{r}^{<d}\partial \times^{<d} \hat{I} \quad (207)$$

Dirac-Santilli Backward Isodual IsoEquation

$$\left(\hat{\eta}^{>d\mu\nu} >^{d} \hat{\gamma}_{\mu}^{>d} >^{d} \hat{p}_{\nu}^{>d} - \hat{\imath}^{>d} > \hat{m}^{>d} > \hat{c}^{>d}\right) > |\hat{p}\hat{s}i^{>d} >= 0. \quad (208)$$

$$\{\hat{\gamma}_{\mu}, \hat{\gamma}_{\nu}\}^{>d} = [\hat{\gamma}_{\mu} \hat{\times} \hat{\gamma}_{\nu} + \hat{\gamma}_{\nu} \hat{\times} \hat{\gamma}_{\mu}]^{>} = \hat{2}^{>d} > \hat{\eta}_{\mu\nu}^{>d}, \quad (209)$$

Dirac-Santilli Forward Isodual GenoEquation

$$<^{<d} \hat{\psi}|< (^{<d}\hat{p}_{\nu} <^{<d} \hat{\gamma}_{\mu} <^{d} \quad <^{d\mu\nu}\hat{\eta} -^{<}\hat{\imath} <^{d} \quad <^{d}\hat{m} <^{d} \quad <\hat{c}) = 0. \quad (210)$$

$$<^{d}\{\hat{\gamma}_{\mu}, \hat{\gamma}_{\nu}\} =<^{d} [\hat{\gamma}_{\mu} \hat{\times} \hat{\gamma}_{\nu} + \hat{\gamma}_{\nu} \hat{\times} \hat{\gamma}_{\mu}] =<^{d} \hat{2} <^{<} \hat{\eta}_{\mu\nu} = 2 \times^{<d} \hat{\eta}_{\mu\nu}, \quad (211)$$

Comments and References

See memoir [20] which constitutes the most comprehensive study of antimatter in irreducible conditions available at this writing.

9. Isodual Classical and Hope Isodual Hypermathematics

Isodual Hyper-Formulations are generally considered to be part of the Hyper-Formulations of Section 4 and 5 because the classification of ordered sets of hyperunits includes isodual realizations, as illustrated in the paper [44] and references quoted therein.

10. Simple Method for the Construction of Regular Hadronic Mathematics

10.1. Introduction [4.5]

Hadronic formulations are called *regular* when the *structure quantities* C_{ij}^{i} of Santilli's Lie-Isotopic algebras, Eqs. (3), Lie-admissible algebras, Eqs. (69) (zzz) and their isoduals, Eqs. (119-, (139), are *constant.* When the structure quantities are *functions of the local variables* $C_{ij}^{k}(t, r, p, \psi, \partial\psi, \ldots)$, hadronic formulations are called *irregular*.

In this section, we shall review a very simple method for the construction of regular hadronic formulations via the mere use of non-unitary transformations of the corresponding conventional formulations. We shall then review the axiomatic consistency of hadronic formulations by showing that Santilli iso-, geno-, hyper-units and their isoduals are invariant under the transformations, thus implying the crucial invariance over time of extended-deformable shapes and their non-Hamiltonian interactions that are invariantly represented precisely nwith such generalized units.

No method exists to our knowledge at this writing (June 2015) for the construction of irregular hadronic formulations via maps of conventional formulations and, therefore, irregular hadronic formulations characterize a new axiomatic structure still mostlyunexplored.

10.2. Simple Construction of Regular Iso-Formulations [4.5]

A simple method has been identified in Refs. [4,5] for the construction of the Lie-Santilli isotheory, all its underlying isomathematics and all physical methods This method is important because it permits a simple implementation of conventional models into their isotopic covering without the need for advanced mathematics. The method consists in:

(i) Representing all conventional potential interactions with a Hamiltonian $H(r, p)$ and all extended-deformable shapes and their non-Hamiltonian interactions and effects with Santilli's isounit $\hat{I}(r, p, \psi, \partial\psi, ./..)$;

(ii) Identifying the latter interactions with a nonunitary transform

$$U \times U^{\dagger} = \hat{I} \neq I \quad (212)$$

and

(iii) Subjecting the *totality* of conventional mathematical and physical quantities and all their operations to the above nonunitary transform, resulting in expressions of the type

$$I \to \hat{I} = U \times I \times U^{\dagger} = 1/\hat{T}, \quad (213)$$

$$a \to \hat{a} = U \times a \times U^{\dagger} = a \times U \times U^{\dagger} = a \times \hat{I}, a \in F, \quad (214)$$

$$e^{A} \to U \times e^{A} \times U^{\dagger} = \hat{I} \times e^{\hat{T} \times \hat{A}} = \left(e^{\hat{A} \times \hat{T}}\right) \times \hat{I}, \quad (215)$$

$$A \times B \to U \times (A \times B) \times U^{\dagger} = (U \times A \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times B \times U^{\dagger}) = \hat{A} \hat{\times} \hat{B}, \quad (216)$$

$$[X_i, X_j] \to U \times [X_i X_j] \times U^{\dagger} = [\hat{X}_i, \hat{X}_j] = U \times (C_{ij}^{k} \times X_k) \times U^{\dagger} = \hat{C}_{ij}^{k} \hat{\times} \hat{X}_k = C_{ij}^{k} \times \hat{X}_k, \quad (217)$$

$$< \psi| \times |\psi > \to U \times < \psi| \times |\psi > \times U^{\dagger} =< \psi| \times U^{\dagger} \times (U \times U^{\dagger})^{-1} \times U \times |\psi > \times (U \times U^{\dagger}) = < \hat{\psi}| \hat{\times} |\hat{\psi} > \times \hat{I}, \quad (218)$$

$$H \times |\psi > \to U \times (H \times |\psi >) = (U \times H \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times |\psi >) = \hat{H} \hat{\times} |\hat{\psi} >, etc. \quad (219)$$

Note that serious inconsistencies emerge in the event even 'one' single quantity or operation is not subjected to the above non-unitary map. In the absence of comprehensive liftings, we would have a situation equivalent to the elaboration of quantum spectral data of the hydrogen atom with isomathematics, resulting in large deviations from reality.

The construction of isodual iso-formulations is simply done via Santilli's isodual map, namely, via the simple anti-hermitean image of the above isotopic formulations.

10.3. Axiomatic consistency of Iso-Formulation [4.5]

Let us recall that Santilli's central assumption is the representation of extended-deformable shapes and their non-Hamiltonian interactions via the isounit. Therefore, any change of the numerical value of the isounit implies the inability to represent the same system over time, besides activating the *Theorem of Catastrophic Mathematical and Physical Inconsistencies of Non-Canonical and Non-Unitary Theories* when formulated via the mathematics of conventional canonical and unitary theories,respectively [23].

It is easy to see that the application of an additional nonunitary transform

$$W \times W^\dagger \neq I, \tag{220}$$

to the preceding expressions causes their *lack of invariance,* with consequential activation of the theorem of catastrophic inconsistencies. This is due to the *change of the value of the basic isounit* under additional non-unitary transformations

$$\hat{I} \to \hat{I}' = W \times \hat{I} \times W^\dagger \neq \hat{I}, \tag{221}$$

However, any given nonunitary transform can be identically rewritten in the isounitary form [3]

$$W \times W^\dagger = \hat{I}, \quad W = \widehat{W} \times \hat{T}^{1/2}, \tag{222}$$

$$W \times W^\dagger = \widehat{W} \,\hat{\times}\, \widehat{W}^\dagger = \widehat{W}^\dagger \,\hat{\times}\, \widehat{W} = \hat{I}, \tag{223}$$

under which we have the invariance of the isounit and isoproduct [7]

$$\hat{I} \to \hat{I}' = \widehat{W} \,\hat{\times}\, \hat{I} \,\hat{\times}\, \widehat{W}^\dagger = \hat{I}, \tag{224}$$

$$\hat{A} \,\hat{\times}\, \hat{B} \to \widehat{W} \,\hat{\times}\, (\hat{A} \,\hat{\times}\, \hat{B}) \,\hat{\times}\, \widehat{W}^\dagger = (\widehat{W} \times \hat{T} \times \hat{A} \times \hat{T} \times \widehat{W}^\dagger) \times (\hat{T} \times \widehat{W}^\dagger)^{-1} \times \hat{T} \times (\widehat{W} \times \hat{T})^{-1} \times (\widehat{W} \times \hat{T} \times \hat{B} \times \hat{T} \times \widehat{W}^\dagger) = \hat{A}' \times (\widehat{W}^\dagger \times \hat{T} \times \widehat{W})^{-1} \times \hat{B}' = \hat{A}' \times \hat{T} \times \hat{B}' = \hat{A}' \,\hat{\times}\, \hat{B}', \; etc. \tag{225}$$

from which the invariance of the entire isotopic formalism follows.

Note that the invariance is ensured by the *numerically invariant values of the isounit and of the isotopic element under non-unitary-isounitary transformations,*

$$\hat{I} \to \hat{I}' \equiv \hat{I}, \tag{226}$$

$$A \,\hat{\times}\, B \to A' \,\hat{\times}'\, B' \equiv A' \,\hat{\times}\, B', \tag{227}$$

in a way fully equivalent to the invariance of Lie's theory and quantum mechanics, as expected to be necessarily the case due to the preservation of the abstract axioms under isotopies. The resolution of the inconsistencies for non-invariant theories is then consequential.

The proof of the invariance of Santilli isodual iso-formulations is an interesting exercise for non-initiated readers.

10.4. Simple Construction of Regular GenoMathematics and its IsoDual [4.5]

An important feature of the Lie-Santilli genotheory is its *form invariance* under the appropriate geno-transformations in a way fully similar to the invariance of the mathematical and physical structures of quantum mechanics under unitary transformations.

This feature can be shown via a *pair* of non-unitary transformations

$$V \times V^\dagger \neq I, W \times W^\dagger \neq I, V \times W^\dagger \neq I, W \times V^\dagger \neq I, \tag{228}$$

under which we have the characterization of the forward and backward genounits and related genoproduct

$$I \to V \times I \times W^\dagger = \hat{I}^>, eqno \tag{229}$$

$$A \times B \to V \times (A \times B) \times W^\dagger = A^> > B^> \tag{230}$$

$$I \to W \times I \times V =^< \hat{I}, \tag{231}$$

$$A \times B \to W \times (A \times B) \times V =^< A <^< B/ \tag{232}$$

10.5. Axiomatic Consistency of GenoMathematics and its Isodual [4.5]

It is easy to see that the above dual non-unitary transformations can always be identically rewritten as the *geno-unitary transforms* on geno-Hilbert spaces over complex genofields,

$$V \times V^\dagger \neq 1, V =^< \hat{V} \times \hat{R}^{1/2}, V \times V^\dagger =^< \hat{V} <^< \hat{V}^\dagger =^< \hat{V}^\dagger <^< \hat{V} =^< \hat{I}, \tag{233}$$

$$W \times W^\dagger \neq 1, W = \widehat{W}^> \times \hat{S}^{1/2}, W \times W^\dagger = \widehat{W}^> > \widehat{W}^{>\dagger} = \widehat{W}^{>\dagger} > \widehat{W}^> = \hat{I}^>, \tag{234}$$

under which we have indeed the following forward geno-invariance laws [3j]

$$\hat{I}^> \to \hat{I}'^> = \widehat{W}^> > \hat{I}^> > \widehat{W}^{>\dagger} = \hat{I}^>, \tag{235}$$

$$\hat{A} > \hat{B} \to \widehat{W}^> > (\hat{A} > \hat{B}) > \widehat{W}^{>\dagger} = \hat{A}' > \hat{B}', \tag{236}$$

$$\hat{H}^> > | >= \hat{E}^> > | >= E \times | > \to \widehat{W}^> > \hat{H}^> > | >= \hat{H}'^> > | >' = \widehat{W}^> > \hat{E}^> > | >= E \times | >', \tag{237}$$

with corresponding rules for the backward and classical counterparts.

The above rules confirm the achievement of the *invariance of the numerical values of genounits, geno-products and geno-eigenvalues*, thus permitting physically consistent applications.

The invariance of the isodual geno-formulations can then be proved via the isodual map applied to the above procedure.

11. Open Mathematical Problems

Among a predictable large number of basic open problems, we list for the interested readers the following ones:

Study methods to transform nonlinear models on

conventional spaces into isolinear models on isospaces over isofields;

See whether simple solutions of isolinear equations on isospaces over isofields provide at least ä" solution of their nonlinear projection on conventional spaces over conventional fields;

Study the removal of divergencies in quantum mechanics and scattering theories (Footnote 2) by isomechanics on an iso-Hilbert space over an isofield.

Study the regular and irregular isorepresentations of the Lie-Santilli isotheory;

Study Santilli isoMinkowskian geometry via the machinery of the Riemannian geometry, yet lack of curvature [39];

Study the Lie-admissible theory in Santilli's sense, that is, as a generalization of Lie's theory elaborated via genomathematics;

Study Santilli geno-Euclidean, geno-Minkowskian and geno-Riemannian geometries where irreversibility is embedded in the non symmetric character of the metric [23];

extend the Tsagas, Ganformina-Nunez isotopology to the genotopic form and their isoduals.

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Santilli's Isoprime Theory

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Abstract: We study Santilli's isomathematics for the generalization of modern mathematics via the isomultiplication $a \hat{\times} a = ab\hat{T}$ and isodivision $a \hat{\div} b = \frac{a}{b}\hat{I}$, where the new multiplicative unit $\hat{I} \neq 1$ is called Santilli isounit, $\hat{T}\hat{I} = 1$, and $\hat{T}$ is the inverse of the isounit, while keeping unchanged addition and subtraction, , In this paper, we introduce the isoaddition $a \hat{+} b = a + b + \hat{0}$ and the isosubtraction $a \hat{-} b = a - b - \hat{0}$ where the additive unit $\hat{0} \neq 0$ is called isozero, and we study Santilli isomathem,atics formulated with the four isooperations $(\hat{+}, \hat{-}, \hat{\times}, \hat{\div})$. We introduce, apparently for the first time, Santilli's isoprime theory of the first kind and Santilli's isoprime theory of the second kind. We also provide an example to illustrate the novel isoprime isonumbers.

Keywords: Isoprimes, Isomultiplication, Isodivision, Isoaddition, Isosubtraction

1. Introduction

Santilli [1] suggests the isomathematics based on the generalization of the multiplication × division ÷ and multiplicative unit 1 in modern mathematics. It is epoch-making discovery. From modern mathematics we establish the foundations of Santilli's isomathematics and Santilli's new isomathematics. We establish Santilli's isoprime theory of both first and second kind and isoprime theory in Santilli's new isomathematics.

1.1. Division and Multiplican in Modern Mathematics

Suppose that

$$a \div a = a^0 = 1, \tag{1}$$

where 1 is called multiplicative unit, 0 exponential zero.

From (1) we define division ÷ and multiplication ×

$$a \div b = \frac{a}{b}, b \neq 0, a \times b = ab, \tag{2}$$

$$a = a \times (a \div a) = a \times a^0 = a \tag{3}$$

We study multiplicative unit 1

$$a \times 1 = a, a \div 1 = a, 1 \div a = 1/a \tag{4}$$

$$(+1)^n = 1, (+1)^{a/b} = 1, (-1)^n = (-1)^n, (-1)^{a/b} = (-1)^{a/b} \tag{5}$$

The addition $+$, subtraction $-$, multiplication × and division ÷ are four arithmetic operations in modern mathematics which are foundations of modern mathematics. We generalize modern mathematics to establish the foundations of Santilli's isomathematics.

1.2. Isodivision and Isomultiplication in Santilli's Isomathematics

We define the isodivision $\hat{\div}$ and isomultiplication $\hat{\times}$ [1-5] which are generalization of division ÷ and multiplication × in modern mathematics.

$$a \hat{\div} a = a^{\hat{0}} = \hat{I} \neq 1, \quad \hat{0} \neq 0, \tag{6}$$

where $\hat{I}$ is called isounit which is generalization of multiplicative unit 1, $\hat{0}$ expeoenential isozero which is generalization of exponential zero.

We have

$$a \hat{\div} b = \hat{I}\frac{a}{b}, b \neq 0, a \hat{\times} b = a\hat{T}b, \tag{7}$$

Suppose that

$$a = a\hat{\times}(a\hat{\div}a) = a\hat{\times}a^{\hat{0}} = a\hat{T}\hat{I} = a. \quad (8)$$

From (8) we have

$$\hat{T}\hat{I} = 1 \quad (9)$$

where $\hat{T}$ is called inverse of isounit $\hat{I}$.

We conjectured [1-5] that (9) is true. Now we prove (9). We study isounit $\hat{I}$

$$a\hat{\times}\hat{I} = a,\ a\hat{\div}\hat{I} = a,\ \hat{I}\hat{\div}a = a^{\hat{-I}} = \hat{I}^2/a, \quad (10)$$

$$(+\hat{I})^{\hat{n}} = \hat{I}, (+\hat{I})^{\frac{\hat{a}}{\hat{b}}} = \hat{I}, (-\hat{I})^{\hat{n}} = (-1)^n\hat{I}, (-\hat{I})^{\frac{\hat{a}}{\hat{b}}} = (-1)^{\frac{a}{b}}\hat{I} \quad (11)$$

Keeping unchanged addition and subtraction, $(+,-,\hat{\times},\hat{\div})$ are four arithmetic operations in Santilli's isomathematics, which are foundations of isomathematics. When $\hat{I}=1$, it is the operations of modern mathematics.

1.3. Addition and Subtraction in Modern Mathematics

We define addition and subtraction

$$x = a+b,\quad y = a-b \quad (12)$$

$$a+a-a = a \quad (13)$$

$$a-a = 0 \quad (14)$$

Using above results we establish isoaddition and isosubtraction

1.4. Isoaddition and Isosubtraction in Santilli's New Isomathematics

We define isoaddition $\hat{+}$ and isosubtraction $\hat{-}$.

$$a\hat{+}b = a+b+c_1,\ a\hat{-}b = a-b-c_2 \quad (15)$$

$$a = a\hat{+}a\hat{-}a = a+c_1-c_2 = a \quad (16)$$

From (16) we have

$$c_1 = c_2 \quad (17)$$

Suppose that $c_1 = c_2 = \hat{0}$,

where $\hat{0}$ is called isozero which is generalization of addition and subtraction zero

We have

$$a\hat{+}b = a+b+\hat{0},\quad a\hat{-}b = a-b-\hat{0} \quad (18)$$

When $\hat{0}=0$, it is addition and subtraction in modern mathematics.

From above results we obtain foundations of santilli's new isomathematics

$$\hat{\times} = \times\hat{T}\times,\ \hat{+} = +\hat{0}+;\ \hat{\div} = \times\hat{I}\div,\ \hat{-} = -\hat{0}-;\ a\hat{\times}b = ab\hat{T},\ a\hat{+}b = a+b+\hat{0};$$

$$a\hat{\div}b = \frac{a}{b}\hat{I},\ a\hat{-}b = a-b-\hat{0};\ a = a\hat{\times}a\hat{\div}a = a,\ a = a\hat{+}a\hat{-}a = a;$$

$$a\hat{\times}a = a^2T, a\hat{+}a = 2a+\hat{0};\ a\hat{\div}a = \hat{I}\neq 1,\ a\hat{-}a = -\hat{0}\neq 0;\ \hat{T}\hat{I} = 1. \quad (19)$$

$(\hat{+},\hat{-},\hat{\times},\hat{\div})$ are four arithmetic operations in Santilli's new isomathematics.

Remark, $a\hat{\times}(b\hat{+}c) = a\hat{\times}(b+c+\hat{0})$, From left side we have

$a\hat{\times}(b\hat{+}c) = a\hat{\times}b + a\hat{\times}\hat{+} + a\hat{\times}c) = a\hat{\times}(b+\hat{+}+c)$

$= a\hat{\times}(b+\hat{0}+c)$, where $\hat{+}=\hat{0}$ also is a number.

$a\hat{\times}(b\hat{-}c) = a\hat{\times}(b-c-\hat{0})$. From left side we have

$a\hat{\times}(b\hat{-}c) = a\hat{\times}b - a\hat{\times}\hat{-} - a\hat{\times}c)$

$= a\hat{\times}(b-\hat{-}-c) = a\hat{\times}(b-\hat{0}-c)$, where $\hat{-}=\hat{0}$ also is a number.

It is satisfies the distributive laws. Therefore $\hat{+},\hat{-},\hat{\times}$ and $\hat{\div}$ also are numbers.

It is the mathematical problems in the 21st century and a new mathematical tool for studying and understanding the law of world.

2. Santilli's Isoprime Theory of the First Kind

Let $F(a,+,\times)$ be a conventional field with numbers a equipped with the conventional sum $a+b\in F$, multiplication $ab\in F$ and their multiplicative unit $1\in F$. Santilli's isofields of the first kind $\hat{F} = \hat{F}(\hat{a},+,\hat{\times})$ are the rings with elements

$$\hat{a} = a\hat{I} \quad (20)$$

called isonumbers, where $a\in F$, the isosum

$$\hat{a}+\hat{b} = (a+b)\hat{I} \quad (21)$$

with conventional additive unit $0 = 0\hat{I} = 0,\ \hat{a}+0 = 0+\hat{a} = \hat{a}$, $\forall\hat{a}\in\hat{F}$ and the isomultiplications is

$$\hat{a}\hat{\times}\hat{b} = \hat{a}\hat{T}\hat{b} = a\hat{I}\hat{T}b\hat{I} = (ab)\hat{I}. \quad (22)$$

Isodivision is

$$\hat{a} \hat{\div} \hat{b} = \hat{I} \frac{a}{b} \tag{23}$$

We can partition the positive isointegers in three classes:

(1)The isouniti $\hat{I}$;

(2)The isonumbers: $\hat{1} = \hat{I}, \hat{2}, \hat{3}, \hat{4}, \hat{5}, \cdots$;

(3)The isoprime numbers: $\hat{2}, \hat{3}, \hat{5}, \hat{7}, \cdots$.

Theorem 1. Twin isoprime theorem

$$\hat{P}_1 = \hat{P} + \hat{2}. \tag{24}$$

Jiang function is

$$J_2(\omega) = \prod_{3 \le P} (P-2) \ne 0, \tag{25}$$

whre $\omega = \prod_{2 \le P} P$ is called primorial.

Since $J_2(\omega) \ne 0$, there exist infinitely many isoprimes $\hat{P}$ such that $\hat{P}_1$ is an isoprime.

We have the best asymptotic formula of the number of isoprimes less than $\hat{N}$

$$\pi_2(\hat{N}, 2) \sim \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{N}{\log^2 N}, \tag{26}$$

where

$$\phi(\omega) = \prod_{2 \le P} (P-1).$$

Let $\hat{I} = 1$. From (24) we have twin prime theorem

$$P_1 = P + 2 \tag{27}$$

Theorem 2. Goldbach isoprime theorem

$$\hat{N} = \hat{P}_1 + \hat{P}_2 \tag{28}$$

Jiang function is

$$J_2(\omega) = \prod_{3 \le P} (P-2) \prod_{P \mid N} \frac{P-1}{P-2} \ne 0 \tag{29}$$

Since $J_2(\omega) \ne 0$ every isoeven number $\hat{N}$ greater than $\hat{4}$ is the sum of two isoprimes.

We have

$$\pi_2(\hat{N}, 2) \sim \frac{J_2(\omega)}{\phi^2(\omega)} \frac{N}{\log^2 N}. \tag{30}$$

Let $\hat{I} = 1$. From (28) we have Goldbach theorem

$$N = P_1 + P_2 \tag{31}$$

Theorem 3. The isoprimes contain arbitrarily long arithmetic progressions. We define arithmetic progressions of isoprimes:

$$\hat{P}_1, \hat{P}_2 = \hat{P}_1 + \hat{d}, \hat{P}_3 = \hat{P}_1 + \hat{2} \hat{\times} \hat{d}, \cdots, \hat{P}_k = \hat{P}_1 + (\hat{k} - I) \hat{\times} \hat{d}, (\hat{P}_1, \hat{d}) = \hat{I}. \tag{32}$$

Let $\hat{I} = 1$. From (32) we have arithmetic progressions of primes:

$$P_1, P_2 = P_1 + d, P_3 = P_1 + 2d, \cdots, P_k = P_1 + (k-1)d, (P_1, d) = 1. \tag{33}$$

We rewrite (33)

$$P_3 = 2P_2 - P_1, P_j = (j-1)P_2 - (j-2)P_1, 3 \le j \le k. \tag{34}$$

Jiang function is

$$J_3(\omega) = \prod_{3 \le P} [(P-1)^2 - \chi(P)], \tag{35}$$

$\chi(P)$ denotes the number of solutions for the following congruence

$$\prod_{j=3}^{k} [(j-1)q_2 - (j-2)q_1] \equiv 0 (\mathrm{mod}\, P), \tag{36}$$

where $q_1 = 1, 2, \cdots, P-1; q_2 = 1, 2, \cdots, P-1$.

From (36) we have

$$J_3(\omega) = \prod_{3 \le P < k} (P-1) \prod_{k \le P} (P-1)(P-k+1) \ne 0. \tag{37}$$

We prove that there exist infinitely many primes P_1 and P_2 such that $P_3, \cdots, P_k$ are all primes for all $k \ge 3$.

We have the best asymptotic formula

$$\pi_{k-1}(N, 3) = \left| \{(j-1)P_2 - (j-2)P_1 = \text{prime}, 3 \le j \le k, P_1, P_2 \le N\} \right|$$

$$\sim \frac{J_3(\omega)\omega^{k-2}}{2\phi^k(\omega)} \frac{N^2}{\log^k N} = \frac{1}{2} \prod_{2 \le P < k} \frac{P^{k-2}}{(P-1)^{k-1}} \prod_{K \le P} \frac{P^{k-2}(P-k+1)}{(P-1)^{k-1}} \frac{N^2}{\log^k N}. \tag{38}$$

Theorem 4. From (33) we obtain

$$P_4 = P_3 + P_2 - P_1, \ P_j = P_3 + (j-3)P_2 - (j-3)P_1, 4 \le j \le k. \tag{39}$$

Jiang function is

$$J_4(\omega)=\prod_{3\le P}((P-1)^3-\chi(P)), \tag{40}$$

$\chi(P)$ denotes the number of solutions for the following congruence

$$\prod_{j=4}^{k}[q_3+(j-3)q_2-(j-3)q_1]\equiv 0(\mathrm{mod}\,P), \tag{41}$$

where $q_i=1,2,\cdots,P-1, i=1,2,3.$

Frome (41) we have

$$J_4(\omega)=\prod_{3\le P<(k-1)}(P-1)^2\prod_{(k-1)\le P}(P-1)[(P-1)^2-(P-2)(k-3)]\neq 0. \tag{42}$$

We prove there exist infinitely many primes P_1, P_2 and P_3 such that $P_4,\cdots,P_k$ are all primes for all $k\ge 4$.

We have the best asymptotic formula

$$\pi_{k-2}(N,4)=\left|\{P_3+(j-3)P_2-(j-3)P_1=\text{prime}, 4\le j\le k, P_1,P_2,P_3\le N\}\right|\sim\frac{J_4(\omega)\omega^{k-3}}{6\phi^k(\omega)}\frac{N^3}{\log^k N}$$

$$=\frac{1}{6}\prod_{2\le P<(k-1)}\frac{P^{k-3}}{(P-1)^{k-2}}\prod_{(k-1)\le P}\frac{P^{k-3}[(P-1)^2-(P-2)(k-3)]}{(P-1)^{k-1}}\frac{N^3}{\log^k N} \tag{43}$$

The prime distribution is order rather than random. The arithmetic progressions in primes are not directly related to ergodic theory, harmonic analysis, discrete geometry and combinatorics. Using the ergodic theory Green and Tao prove there exist arbitrarily long arithmetic progressions of primes which is false [6,7,8,9,10].

Theorem 5. Isoprime equation

$$P_2=\hat{P}_1+2=P_1\hat{I}+2. \tag{44}$$

Let $\hat{I}$ be the odd number. Jiang function is

$$J_2(\omega)=\prod_{3\le P}(P-2)\prod_{P\mid\hat{I}}\frac{P-1}{P-2}\neq 0. \tag{45}$$

Since $J_2(\omega)\neq 0$, there exist infinitely primes P_1 such that P_2 is a prime.

We have

$$\pi_2(N,2)\sim\frac{J_2(\omega)\omega}{\phi^2(\omega)}\frac{N}{\log^2 N}. \tag{46}$$

Theorem 6. Isomprime equation

$$P_2=(\hat{P}_1)^{\hat{2}}+2=P_1^2\hat{I}+2. \tag{47}$$

Let $\hat{I}$ be the odd number. Jiang function is

$$J_2(\omega)=\prod_{3\le P}(P-2-X(P)), \tag{48}$$

where

$$X(P)=\begin{cases}\left(\dfrac{-2I}{P}\right)\\ -1 \quad \text{if} \quad P\mid\hat{I}\end{cases}$$

If $\left(\frac{-2I}{3}\right)=-1$, there infinitely many primes P_1 such that P_2 is a prime. If $\left(\frac{-2I}{3}\right)=1, J_2(3)=0$, there exist finite primes P_1 such that P_2 is a prime.

3. Santilli'S Isoprime Theory of the Second Kind

Santilli's isofields of the second kind $\hat{F}=\hat{F}(a,+,\hat{\times})$ (that is, $a\in F$ is not lifted to $\hat{a}=a\hat{I}$) also verify all the axioms of a field.

The isomultiplication is defined by

$$a\hat{\times}b=a\hat{T}b. \tag{49}$$

We then have the isoquotient, isopower, isosquare root, etc.,

$$a\hat{\div}b=(a/b)\hat{I}, a^{\hat{n}}=a\hat{\times}\cdots\hat{\times}a\,(\text{ntimes})=a^n(\hat{T})^{n-1}, a^{\hat{1/2}}=a^{1/2}(\hat{I})^{1/2}. \tag{50}$$

Theorem 7. Isoprime equations

$$P_2=P_1^{\hat{2}}+6, P_3=P_1^{\hat{2}}+12, P_4=P_1^{\hat{2}}+18 \tag{51}$$

Let $T=1$. From (51) we have

$$P_2=P^2+6, P_3=P_1^2+12, P_4=P_1^2+18, \tag{52}$$

Jiang function is

$$J_2(\omega)=2\prod_{5\le P}(P-4-(\frac{-6}{P})-(\frac{-3}{P})-(\frac{-2}{P}))\neq 0, \quad (53)$$

where $(\frac{-6}{P}),(\frac{-3}{P})$ and $(\frac{-2}{P})$ denote the Legendre symbols.

Since $J_2(\omega)\neq 0$, there exist infinitely many primes P_1 such that P_2, P_3 and P_4 are primes.

$$\pi_4(N,2)\sim\frac{J_2(\omega)\omega^3}{8\phi^4(\omega)}\frac{N}{\log^4 N} \quad (54)$$

Let $\hat{T}=5$. From (51) we have

$$P_2=5P_1^2+6, P_3=5P_1^2+12, P_4=5P_1^2+18. \quad (55)$$

Jiang function is

$$J_2(\omega)=8\prod_{7\le P}(P-4-(\frac{-30}{P})-(\frac{-15}{P})-(\frac{-10}{P}))\neq 0. \quad (56)$$

Since $J_2(\omega)\neq 0$, there exist infinitely many primes P_1 such that P_2, P_3 and P_4 are primes.

We have

$$\pi_4(N,2)\sim\frac{J_2(\omega)\omega^3}{8\phi^4(\omega)}\frac{N}{\log^4 N}. \quad (57)$$

Let $\hat{T}=7$. From (51) we have

$$P_2=7P_1^2+6, P_3=7P_1^2+12, P_4=7P_1^2+18. \quad (58)$$

We have Jiang function

$$J_2(5)=0. \quad (59)$$

There exist finite primes P_1 such that P_2, P_3 and P_4 are primes.

Theorem 8. Isoprime equations

$$P_2=P_1^{\hat{2}}+30, P_2=P_1^{\hat{2}}+60, P_4=P_1^{\hat{2}}+90, P_5=P_1^{\hat{2}}+120. \quad (60)$$

Let $\hat{T}=7$. From (60) we have

$$P_2=7P_1^2+30, P_3=7P_1^2+60, P_4=7P_1^2+90, P_5=7P_1^2+120 \quad (61)$$

Jiang function is

$$J_2(\omega)=48\prod_{11\le P}(P-5-\sum_{j=1}^{4}(\frac{-210j}{P}))\neq 0. \quad (62)$$

Since $J_2(\omega)\neq 0$, there exist infinitely many primes P_1 such that P_2, P_3, P_4 and P_5 are primes.

We have

$$\pi_5(N,2)\sim\frac{J_2(\omega)\omega^4}{16\phi^5(\omega)}\frac{N}{\log^5 N}. \quad (63)$$

Let $\hat{T}\ge 7$ be the odd prime. From (60) we have

$$P_k=P_1^2\hat{T}+30(k-1), k=2,3,4,5. \quad (64)$$

Jiang function is

$$J_2(\omega)=8\prod_{7\le P}(P-5-\chi(P))\neq 0. \quad (65)$$

If $P|\hat{T}$, $\chi(P)=4$; $\chi(P)=\sum_{j=1}^{4}(\frac{-30\hat{T}j}{P})$ otherwise.

Since $J_2(\omega)\neq 0$, there exist infinitely many primes P_1 such that P_2, P_3, P_4 and P_5 are primes.

We have

$$\pi_5(N,2)\sim\frac{J_2(\omega)\omega^4}{16\phi^5(\omega)}\frac{N}{\log^5 N}. \quad (66)$$

Theorem 9. Isoprime equation

$$P_3=P_2\hat{\times}(P_1+b)-b. \quad (67)$$

Let $\hat{T}=1$ Jiang function is

$$J_2(\omega)=\prod_{3\le P\le P_l}(P^2+3P+3-\chi(P))\neq 0, \quad (68)$$

where $\chi(P)=-P+2$ if $P|b$; $\chi(P)=0$ otherwise.

Since $J_3(\omega)\neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

The best asymptotic formula is

$$\pi_2(N,3)=|\{P_1,P_2: P_1,P_2\le N; P_3=\text{prime}\}|\sim\frac{J_3(\omega)\omega}{4\phi^3(\omega)}\frac{N^2}{\log^3 N}. \quad (69)$$

Theorem 10. Isoprime equation

$$P_3=P_2\hat{\times}(P_1^{\hat{2}}+b)-b \quad (70)$$

Let $\hat{T}=1$ Jiang function is

$$J_3(\omega)=\prod_{3\le P\le P_l}(P^2-3P+3+\chi(P))\neq 0 \quad (71)$$

where $\chi(P)=P-2$ if $P|b$; $\chi(P)=(\frac{-b}{P})$ otherwise.

Since $J_3(\omega)\neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

The best asymptotic formula is

$$\pi_2(N,3)=\left|\{P_1,P_2:P_1,P_2\le N;P_3=\text{prime}\}\right|\sim\frac{J_3(\omega)\omega}{6\phi^3(\omega)}\frac{N^2}{\log^3 N}. \quad (72)$$

Theorem 11. Isoprime equation

$$P_3=P_2\hat{2}(P_1+1)-1. \quad (73)$$

Let $\hat{T}=1$. Jiang function is

$$J_2(\omega)=\prod_{3\le P\le P_1}(P^2-3P+4)\ne 0 \quad (74)$$

Since $J_3(\omega)\ne 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

The best asymptotic formula is

$$\pi_2(N,3)=\left|\{P_1,P_2:P_1,P_2\le N;P_3=\text{prime}\}\right|$$

$$\sim\frac{J_3(\omega)\omega}{6\phi^3(\omega)}\frac{N^2}{\log^3 N}. \quad (75)$$

4. Isoprime Theory in Santilli's New Isomathematics

Theorem 12. Isoprime equation

$$P_3=P_1\hat{+}P_2=P_1+P_2+\hat{0}. \quad (76)$$

Suppose $\hat{0}=1$. From (76) we have

$$P_3=P_1+P_2+1. \quad (77)$$

Jiang function is

$$J_3(\omega)=\prod_{3\le P}(P^2-3P+3)\ne 0. \quad (78)$$

Since $J_3(\omega)\ne 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

We have the best asymptotic formula is

$$\pi_2(N,3)\sim\frac{J_3(\omega)\omega}{2\phi^3(\omega)}\frac{N^2}{\log^3 N}. \quad (79)$$

Theorem 13. Isoprime equation

$$P_3=(P_1\hat{+}2)\hat{\times}(P_1\hat{-}2)\hat{+}P_2=\hat{T}[P_1^2-(2+\hat{0})^2]+P_2+\hat{0} \quad (80)$$

Suppose $\hat{T}=6$ and $\hat{0}=4$. From (80) we have

$$P_3=6(P_1^2-36)+P_2+4 \quad (81)$$

Jiang function is

$$J_3(\omega)=\prod_{3\le P}(P^2-3P+2)\ne 0. \quad (82)$$

Since $J_3(\omega)\ne 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

We have the best asymptotic formula is

$$\pi_2(N,3)\sim\frac{J_3(\omega)\omega}{4\phi^3(\omega)}\frac{N^2}{\log^3 N}. \quad (83)$$

5. An Example

We give an example to illustrate the Santilli's isomathematics.

Suppose that algebraic equation

$$y=a_1\times(b_1+c_1)+a_2\div(b_2-c_2) \quad (84)$$

We consider that (84) may be represented the mathematical system, physical system, biological system, IT system and another system. (84) may be written as the isomathematical equation

$$\hat{y}=a_1\hat{\times}(b_1\hat{+}c_1)\hat{+}a_2\hat{\div}(b_2\hat{-}c_2)=a_1\hat{T}(b_1+c_1+\hat{0})+\hat{0}+a_2/\hat{T}(b_2-c_2-\hat{0}). \quad (85)$$

If $\hat{T}=1$ and $\hat{0}=0$, then $y=\hat{y}$.

Let $\hat{T}=2$ and $\hat{0}=3$. From (85) we have the isomathematical subequation

$$\hat{y}_1=2a_1(b_1+c_1+3)+3+a_2/2(b_2-c_2-3). \quad (86)$$

Let $\hat{T}=5$ and $\hat{0}=6$. From (85) we have the isomathematical subequation

$$\hat{y}_2=5a_1(b_1+c_1+6)+6+a_2/5(b_2-c_2-6). \quad (87)$$

Let $\hat{T}=8$ and $\hat{0}=10$. From (85) we have the isomathematical subequation

$$\hat{y}_3=8a_1(b_1+c_1+10)+10+a_2/8(b_2-c_2-10) \quad (88)$$

From (85) we have infinitely many isomathematical subequations. Using (85)-(88), $\hat{T}$ and $\hat{0}$ we study stability and optimum structures of algebraic equation (84).

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Measurable Iso-Functions

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Abstract: In this article are given definitions definition for measurable is-functions of the first, second, third, fourth and fifth kind. They are given examples when the original function is not measurable and the corresponding iso-function is measurable and the inverse. They are given conditions for the isotopic element under which the corresponding is-functions are measurable. It is introduced a definition for equivalent iso-functions. They are given examples when the iso-functions are equivalent and the corresponding real functions are not equivalent. They are deducted some criterions for measurability of the iso-functions of the first, second, third, fourth and fifth kind. They are investigated for measurability the addition, multiplication of two iso-functions, multiplication of iso-function with an iso-number and the powers of measurable iso-functions. They are given definitions for step iso-functions, iso-step iso-functions, characteristic iso-functions, iso-characteristic iso-functions. It is investigate for measurability the limit function of sequence of measurable iso-functions. As application they are formulated the iso-Lebesgue's theorems for iso-functions of the first, second, third, fourth and fifth kind. These iso-Lebesgue's theorems give some information for the structure of the iso-functions of the first, second, third, fourth and fifth kind.

Keywords: Measurable Iso-Sets, Measurable Is-Functions, Is-Lebesgue Theorems

1. Introduction

Genious idea is the Santilli's generalization of the basic unit of quantum mechanics into an integro-differential operator $\hat{I}$ which is as positive-definite as +1 and it depends of local variables and it is assumed to be the inverse of the isotopic element $\hat{T}$

$$+1 > 0 \to \hat{I}(t,r,p,a,E,\cdots) = \frac{1}{\hat{T}} > 0$$

and it is called Santilli isounit. Santilli introduced a generalization called lifting of the conventional associative product ab into the form

$$ab \to a\hat{x}b = a\hat{T}b$$

Called isoproduct for which

$$\hat{I}\hat{x}a = \frac{1}{\hat{T}}\hat{T}a = a\hat{x}\hat{I} = a\hat{T}\frac{1}{\hat{T}} = a$$

For every element a of the field of real numbers, complex numbers and quaternions. The Santilli isonumbers are defi ned as follows: for given real number or complex number or quaternion a,

$$\hat{a} = a\hat{I}$$

With isoproduct

$$\hat{a}\hat{x}\hat{b} = \hat{a}\hat{T}\hat{b} = a\frac{1}{\hat{T}}\hat{T}b\frac{1}{\hat{T}} = ab\frac{1}{\hat{T}} = \widehat{ab}.$$

If $a \neq 0$, the corresponding isoelement of $\frac{1}{a}$ will be denoted with $\hat{a}^{-1}$or $\hat{I} \searrow \hat{a}$.

With $\hat{F}_{\mathbb{R}}$ we will denote the field of the is-numbers $\hat{a}$ for which $a \in \mathbb{R}$ and basic isounit $\hat{I}_1$.

In [1], [3]-[12] are defined isocontinuous isofunctions and isoderivative of isofunction and in [1] are proved some of their properties. If $\hat{D}_1$ is an isoset in $\hat{F}_{\mathbb{R}}$, the class of isofunctions is denoted by $\widehat{FC}_{D_1}$ and the class of isodifferentiable isofunctions is denoted by $\widehat{FC}^1_{D_1}$, with the same basic isounit $\hat{I} = \frac{1}{\hat{T}}$, it is supposed

$$\hat{T} \in C^1(D_1), \hat{T} > 0 \; in \; D_1.$$

Here D_1 is the corresponding real set of $\hat{D}_1$. If x is an independent variable, then the corresponding lift is $\frac{x}{\hat{T}(x)}$, if f is real-valued function on D_1, then the corresponding lift of first kind is defined as follows

$$\hat{f}^{\wedge}(\hat{x}) = \frac{f\left(\hat{t}(x)\frac{x}{\hat{T}(x)}\right)}{\hat{T}(x)} = \frac{f(x)}{\hat{T}(x)}$$

and we will denote it by $\hat{f}^{\wedge\wedge}$.

In accordance with [1], the isodifferential is defined as follows

$$\hat{d}(\cdot) = \hat{T}(x)d(\cdot).$$

Then

$$\hat{d}(\hat{x}) = \hat{T}(x)d(\hat{x}) = \hat{T}(x)d\left(\frac{x}{\hat{T}(x)}\right) = \hat{T}(x)\left(\frac{1}{\hat{T}(x)} - X\frac{\hat{T}'(x)}{\hat{T}^2(x)}\right)dx = \left(1 - x\frac{\hat{T}'(x)}{\hat{T}(x)}\right)dx,$$

$$\hat{d}\left(\hat{f}^{\wedge}(\hat{x})\right) = \hat{T}(x)d\left(\hat{f}^{\wedge}(\hat{x})\right) = T(x)\widehat{d\left(\frac{f(x)}{\hat{T}(x)}\right)} = \left(f'(x) - f(x)\frac{\hat{T}'(x)}{\hat{T}(x)}\right)dx.$$

In accordance with [1], the first is-derivative of the is-function $\hat{f}^{\wedge\wedge}$ is defined as follows

$$\left(\hat{f}^{\wedge\wedge}\ (x)\right)_x^{\otimes} = \hat{d}\left(\hat{f}^{\wedge}(\hat{x})\right) \nearrow \hat{d}(\hat{x}) = \frac{1}{\hat{T}(x)}\frac{\hat{d}\left(\hat{f}^{\wedge}(\hat{x})\right)}{\hat{d}(\hat{x})} = \frac{f'(x)\hat{T}(x) - f(x)\hat{T}'(x)}{\hat{T}^2(x) - x\hat{T}(x)\hat{T}'(x)}.$$

When $\hat{T}(x) \equiv 1$, then

$$\left(\hat{f}^{\wedge}(\hat{x})\right)_x^{\otimes} = f'(x).$$

Our aim in this article is to be investigated some aspects of theory of measurable iso-functions. The paper is organized as follows. In Section 2 are defined measurable iso-functions and they are deducted some of their properties. In Section 3 is investigated the structure of the measurable iso-functions.

2. The Definition and the Simplest Properties of Measurable Is-Functions

We suppose that A is a given point set, $\hat{T}: A \to \mathbb{R}$, $\hat{T}(x) > 0$ for every $x \in A$, $\hat{T}_1 > 0$ be a given constant, $f: A \to \mathbb{R}$ bea given real-valued function. With $\hat{f}$ we will denote the corresponding is-function of the first, second, third, fourth and fifth kind. More precisely,

1. $\hat{f}(x) \equiv \hat{f}^{\wedge}(\hat{x}) = \frac{f(x)}{\hat{T}(x)}$, when $\hat{f}$ is an is-function of the first kind.
2. $\hat{f}(x) = \hat{f}^{\wedge}(x) = \frac{f(x\hat{T}(x))}{\hat{T}(x)}$, when $\frac{x}{\hat{T}(x)} \in A$ for $x \in A$, when $\hat{f}$ is an is-function of the second kind.
3. $\hat{f}(x) = \hat{f}(\hat{x}) = \frac{f\left(\frac{x}{\hat{T}(x)}\right)}{\hat{T}(x)}$, $\frac{x}{\hat{T}(x)} \in A$ for $x \in A$, when $\hat{f}$ is an is-function of the third kind.
4. $\hat{f}(x) \equiv f^{\wedge}(x) = f\left(x\hat{T}(x)\right)$, when $x\hat{T}(x) \in A$ for $x \in A$, when $\hat{f}$ is an is-function of the fourth kind.
5. $\hat{f}(x) = f^{\vee}(x) = f\left(\frac{x}{\hat{T}(x)}\right)$, $\frac{x}{\hat{T}(x)} \in A$ for $x \in A$, when $\hat{f}$ is an is-function of the fifth kind.

For $a \in A$ with $A(\hat{f} > a)$ we will denote the set

$$A(\hat{f} > a) := \{x \in A: \hat{f}(x) > a\}.$$

We define the symbols $A(\hat{f} \geq a)$, $A(\hat{f} = a)$, $A(\hat{f} < a)$, $A(a < \hat{f} < b)$ and etc., in the same way.

If the set on which the is-function $\hat{f}$ is defined is designated by a letter C or D, we shall write $C(\hat{f}>a)$ or $D(\hat{f}>a)$.

Definition 2.1. The is-function $\hat{f}$ is said to be measurable if

1. The set A is measurable.
2. The set $A(\hat{f} > a)$ is measurable for all $a \in A$.

Theorem 2.3. Let $\hat{f}$ be a measurable is-function defined on the set A. If B is a measurable subset of A, then the is-function $\hat{f}(x)$, considered only for $x \in B$, is measurable.

Proof. Let $a \in \mathbb{R}$ be arbitrarily chosen and fixed. We will prove that

$$B(\hat{f} > a) = B \cap A(\hat{f} > a). \quad (1)$$

Really, let $x \in B(\hat{f} > a)$ be arbitrarily chosen. Then $x \in B$ and $\hat{f}(x)>a$. Since $B \subset A$, we have that $x \in A$. From $x \in A$ and $\hat{f}(x) > a$ it follows that $x \in A(\hat{f} > a)$. Because $x \in B(\hat{f} > a)$ was arbitrarily chosen and for it we get that it is an element of the set $B \cap A(\hat{f} > a)$, we conclude that

$$B \subset (\hat{f} > a) \subset B \cap A(\hat{f} > a). \quad (2)$$

Let now $x \in B \cap A(\hat{f} > a)$ be arbitrarily chosen. Then $x \in B$ and $x \in A(\hat{f} > a)$. Hence $x \in B$ and $\hat{f}(x) > a$. Therefore $x \in B(\hat{f} > a)$. Because $x \in B \cap A(\hat{f} > a)$ was arbitrarily chosen and we get that it is an element of $B(\hat{f} > a)$, we conclude that

$$B \cap A(\hat{f} > a) \subset B(\hat{f} > a).$$

From the last relation and from (2) we prove the relation (1).

Since the iso-function $\hat{f}$ is a measurable function on the set A, we have that $A(\hat{f} > a)$ is a measurable set. As the intersection of two measurable sets is a measurable set, we have that $B \cap A(\hat{f} > a)$ is a measurable set. Consequently, using (1), the set $B(\hat{f}>a)$ is measurable set. In this way we have

1. B is a measurable set,
2. $B(\hat{f} > a)$ is a measurable set for all $a \in \mathbb{R}$.

Therefore the iso-function $\hat{f}$, considered only for $x \in B$, is a measurable is-function.

Theorem 2.4. Let $\hat{f}$ be defined on the set A, which is the union of a finite or denumerable number of measurable sets A_K, $A = \bigcup_K A_k$. If $\hat{f}$ is measurable on each of the sets A_k, then it is also measurable on A.

Proof. Let $a \in \mathbb{R}$ be arbitrarily chosen. We will prove that

$$A(\hat{f} > a) = \cup_k A_k(\hat{f} > a). \quad (3)$$

Let $x \in A(\hat{f} > a)$ be arbitrarily chosen. Then $x \in A$ and $\hat{f}(x) > a$. Since $x \in A$ and $A = \cup_k A_k$, there exists k such that $x \in A_k$. Therefore $x \in A_k$ and $\hat{f}(x) > a$. Hence, $x \in A_k(\hat{f} > a)$ and $x \in \cup_k A_k(\hat{f} > a)$. Because $x \in A(\hat{f} > a)$ was arbitrarily chosen and for it we get that it is an element of $\cup_k A_k(\hat{f} > a)$, we conclude that

$$A(\hat{f} > a) \subset \cup_k A_k(\hat{f} > a). \quad (4)$$

Let now $y \in \cup_k A_k(\hat{f} > a)$ be arbitrarily chosen. Then there exists 1 such that $y \in A_l(\hat{f}>a)$. From here $x \in A_l$ and $\hat{f}(y) > a$. Hence, $y \in A = \cup_k A_k$ and $\hat{f}(y) > a$. Consequently $\in A(\hat{f} > a)$. Because $y \in \cup_k A_k(\hat{f} > a)$ was arbitrarily chosen and for it we get that it is an element of $A(\hat{f} > a)$ we conclude that

$$A(\hat{f} > a) \subset \bigcup_k A_k(\hat{f} > a).$$

From the last relation and from (4) we prove the relation (3).

Since the union of finite or denumerable number of measurable sets is a measurable set, using that the sets $A_k(\hat{f} > a)$ are measurable, we obtain that A and $A(\hat{f} > a)$ are measurable sets. Therefore $\hat{f}$ is a measurable is-function.

Definition 2.5. Two is-functions $\hat{f}$ and $\hat{g}$, defined on the same setr A, are said to be equivalent if

$$\mu\left(A(\hat{f} \neq \hat{g})\right) = 0.$$

We will write

$$\hat{f} \sim \hat{g}.$$

Remark 2.6. There is a possibility $f \nsim g$ and in the same time $\hat{f} \sim \hat{g}$.

Let

$$A = [1,2], f(x) = x, g(x) = x + 1,$$

$$\hat{T}(x) = \frac{-1+\sqrt{1+4x^2}}{2x}, x \in A.$$

Then

$$f \nsim g.$$

On the oth-1+er hand,

$$\hat{f}^{\wedge}(\hat{x}) = \frac{f(x)}{\hat{T}(x)} = \frac{x}{\frac{-1+\sqrt{1+4x^2}}{x}}$$

$$= \frac{2x^2}{-1+\sqrt{1+4x^2}} = \frac{2x^2(1+\sqrt{1+4x^2})}{(\sqrt{1+4x^2}-1)(\sqrt{1+4x^2}-1)}$$

$$= \frac{2x^2(1+\sqrt{1+4x^2})}{4x^2} = \frac{1+\sqrt{1+4x^2}}{2},$$

$$g^{\wedge}(x) = g(x\hat{T}(x)) = x\hat{T}(x) + 1$$

$$= x\frac{-1+\sqrt{1+4x^2}+1}{2x} = \frac{-1+\sqrt{1+4x^2}}{2} + 1$$

$$= \frac{1+\sqrt{1+4x^2}}{2}.$$

We have that

$$\mu\left(A(\hat{f}^{\wedge\wedge} \neq g^{\wedge})\right) = 0$$

Or

$$\hat{f}^{\wedge\wedge} \sim g^{\wedge}.$$

Remark 2.7. There is a possibility $f \sim g$ and in the same time $\hat{f} \nsim \hat{g}$. Let

$$A = [1,2], f(x) = g(x) = x^2, \hat{T}(x) = x + 1, x \in A.$$

Then

$$f \sim g.$$

On the other hand,

$$f^{\wedge}(x) = f(x\hat{T}(x)) = x^2\hat{T}^2(x) = x^2(x+1)^2, g^{\vee}(x) = g\left(\frac{x}{\hat{T}(x)}\right) = \frac{x^2}{\hat{T}^2(x)} = \frac{x^2}{(x+1)^2}.$$

Then

$$f^{\wedge}(x) = g^{\vee}(x) \Leftrightarrow x^2(x+1)^2 = \frac{x^2}{(x+1)^2} \Leftrightarrow (x+1)^4 = 1 \Leftrightarrow x = 0 \notin A.$$

Therefore

$$\mu(A(f^{\wedge} = g^{\vee})) = 0,$$

Hence,

$$\mu(A(f^{\wedge} \neq g^{\vee})) = 1.$$

Consequently

$$f^{\wedge} \sim g^{\vee}.$$

Proposition 2.8. The fuAnctions f and g are equivalent if and only if the functions $\hat{f}^{\wedge\wedge}$ and $\hat{g}^{\wedge\wedge}$ are equivalent

Proof. We have

$$\mu(A(f \neq g)) = 0 \Leftrightarrow \mu\left(A\left(\frac{f}{\hat{T}} \neq \frac{g}{\hat{T}}\right)\right) = 0$$

$$\Leftrightarrow \mu\left(A(\hat{f}^{\wedge\wedge} \neq \hat{g}^{\wedge\wedge})\right) = 0.$$

Definition 2.9. Let some property P holds for all the points of the set A, except for the points of a subset B of the set A. If $= 0$, we say that the property P holds almost everywhere on the set A, or for almost all points of A.

Definition 2.10. We say that two is-functions defined on

the set A are equivalent if they are equal almost everywhere on the set A.

Theorem 2.11. If $\hat{f}(x)$ is a measurable is-function defined on the set A, and if $\hat{f} \sim \hat{g}$, then the is-function $\hat{g}(x)$ is also measurable.

Proof. Let

$$B := A(\hat{f} \neq \hat{g}), D := A \setminus B.$$

Because $\hat{f} \sim \hat{g}$ we have that

$$\mu\left(A(\hat{f} \neq \hat{g})\right) = 0$$

or $\mu B = 0$.

Since every function, definite on a set with measure zero is measurable on it, we have that the is-function $\hat{g}$ is measurable on the set B.

We note that the is-functions $\hat{f}(x)$ and $\hat{g}(x)$ are identical on D and since the is0-function $\hat{f}$ is measurable on D, we get that the is-function $\hat{g}$ is measurable on D.

Consequently the is-function $\hat{g}$ is measurable on

$$B \cup D = A.$$

Theorem 2.12. If the is-function $\hat{f}(x)$, defined on the set A, is measurable, then the sets

$$A(\hat{f} \geq a), A(\hat{f} = a), A(\hat{f} \leq a), A(\hat{f} < a)$$

Are measurable for all $a \in \mathbb{R}$.

Proof. We will prove that

$$A(\hat{f} \geq a) = \prod_{n=1}^{\infty} A\left(\hat{f} > a - \frac{1}{n}\right). \quad (5)$$

Really, let $x \in A(\hat{f} \geq a)$ be arbitrarily chosen. Then $x \in A$ and $\hat{f}(x) \geq a$. Hence, for every $n \in \mathbb{N}$ we have $\hat{f}(x) > a - \frac{1}{n}$. Therefore

$$x \in \prod_{n=1}^{\infty} A\left(\hat{f} > a - \frac{1}{n}\right).$$

Because $x \in A(\hat{f} \geq a)$ was arbitrarily chosen and for it we obtain $x \in \prod_{n=1}^{\infty} A\left(\hat{f} > a - \frac{1}{n}\right)$,

We conclude that

$$A(\hat{f} \geq a) \subset \prod_{n=1}^{\infty} A\left(\hat{f} > a - \frac{1}{n}\right). \quad (6)$$

Let now $x \in \prod_{n=1}^{\infty} A\left(\hat{f} > a - \frac{1}{n}\right)$ be arbitrarily chosen. Then $x \in A\left(\hat{f} > a - \frac{1}{n}\right)$ for every natural number n. From here $x \in A$ and

$$\hat{f}(x) > a - \frac{1}{n}$$

For all natural number n. Consequently

$$\lim_{n\to\infty} \hat{f}(x) \geq \lim_{n\to\infty} \left(a - \frac{1}{n}\right)$$

or

$$\hat{f}(x) \geq a$$

and $x \in A(\hat{f} \geq a)$. Since $x \in \prod_{n=1}^{\infty} A\left(\hat{f} > a - \frac{1}{n}\right)$ was arbitrarily chosen and we get that $x \in A(\hat{f} \geq a)$, we conclude

$$\prod_{n=1}^{\infty} A\left(\hat{f} > a - \frac{1}{n}\right) \subset A(\hat{f} \geq a).$$

From the last relation and from (6) we obtain the relation (5).

Because the intersection of denumerable measurable sets is a measurable set, using the relation (5) and the fact that all sets $A\left(\hat{f} > a - \frac{1}{n}\right)$ are measurable for all natural numbers n, we conclude that the set $A(\hat{f} \geq a)$ is a measurable set.

The set $A(\hat{f} = a)$ is a measurable set because

$$A(\hat{f} = a) = A(\hat{f} \geq a) \setminus A(\hat{f} > a).$$

The set $A(\hat{f} \leq a)$ is measurable set since

$$A(\hat{f} \leq a) = A \setminus A(\hat{f} > a).$$

The set $A(\hat{f} < a)$ is measurable since

$$A(\hat{f} < a) = A \setminus A(\hat{f} \geq a).$$

Remark 2.13. We note that if at least one of the sets

$$A(\hat{f} \geq a), A(\hat{f} = a), A(\hat{f} \leq a), A(\hat{f} < a)$$

Is measurable for all $a \in \mathbb{R}$, then the iso-function $\hat{f}$ is measurable on the set A.

Really, let $A(\hat{f} \geq a)$ is measurable for all $a \in \mathbb{R}$. Then, using the relation

$$A(\hat{f} > a) = \prod_{n=1}^{\infty} A\left(\hat{f} \geq a - \frac{1}{n}\right), \quad (7)$$

we obtain that the set $A(\hat{f} > a)$ is measurable for all $a \in \mathbb{R}$.

If $A(\hat{f} \leq a))$ is measurable for all $a \in \mathbb{R}$, then using the relation

$$A(\hat{f} > a) = A \setminus A(\hat{f} \leq a),$$

we get that the set $A(\hat{f} > a)$ is measurable for all $a \in \mathbb{R}$.

If $A(\hat{f} < a)$ is measurable for all $a \in \mathbb{R}$, then using the relation

$$A(\hat{f} > a) = A \setminus A(\hat{f} \leq a),$$

We conclude that the set $A(\hat{f} > a)$ is measurable for all $a \in \mathbb{R}$.

Theorem 2.14. If $\hat{f}(x) = c = const$ for all points of a measurable set A, then the is-function $\hat{f}(x)$ is measurable.

Proof. For all $a \in \mathbb{R}$ we have that

$$A(\hat{f} > a) = A \text{ if } c > a \text{ and } A(\hat{f} > a) = \emptyset \text{ if } c \leq a.$$

Since the sets A and $\emptyset$ are measurable sets, then $A(\hat{f} > a)$

is measurable for all $a \in \mathbb{R}$. Therefore the is-function $\hat{f}(x)$ is measurable.

Definition 2.15. An is-function $\hat{f}(x)$ defined on the closed interval [a, b] is said to be a step is-function if there is a finite number of points

$$a = a_0 < a_1 < \cdots < a_{n-1} < a_n = b$$

Such that $\hat{f}(x)$ is a constant on (a_i, a_{i+1}), $i = 0, 1, 2, \cdots, n-1$.

Proposition 2.16. A step is-function is measurable.

Proof. Let $\hat{f}$(x) is a step is-function on the closed interval [a, b]. Let also,

$$a = a_0 < a_1 < a_2 < \cdots < a_{n-1} < a_n = b$$

be such that $\hat{f}(x)$ is a constant on (a_i, a_{i+1}), $i = 0, 1, 2, \cdots, n-1$. From the previous theorem we have that $\hat{f}(x)$ is measurable on (a_i, a_{i+1}), $i = 0, 1, 2, \cdots, n$. We note that

the sets $\{a_i\}$, $i = 0, 1, 2, \cdots, n-1$, are sets with measure zero. Therefore the is-function

$\hat{f}(x)$ is measurable on $\{a_i\}$, $i = 0, 1, 2, \cdots, n$. From here, using that

$$[a, b] = \bigcup_{i=0}^{n} (a_i, a_{i+1}) \bigcup \bigcup_{i=0}^{n} \{a_i\},$$

We conclude that the is-function $\hat{f}(x)$ is measurable on [a, b].

Theorem 2.17. If the is-function $\hat{f}(x)$, defined on the set A is measurable and $c \in \mathbb{R}, c \neq 0$, then the is-functions

1. $\hat{f}(x) + c$,
2. $c\hat{f}(x)$,
3. $|\hat{f}(x),|$
4. $\hat{f}^2(x)$,
5. $\frac{1}{\hat{f}(x)}$,

are also measurable.

Proof. Let $a \in \mathbb{R}$ be arbitrarily chosen. The assertion follows from the following relations.

1. $A(\hat{f} + c > a) = A(\hat{f} > c - a)$.
2. $A(c\hat{f} > a) = A\left(\hat{f} > \frac{a}{c}\right)$ if c>0, $A(c\hat{f} > a) = A\left(\hat{f} < \frac{a}{c}\right)$ if c<0.
3. $A(|\hat{f}| > a) = A$ if a<0, $A(|\hat{f}| > a) = A(\hat{f} > a) \cup A(\hat{f} < -a)$ if $a \geq 0$.
4. $A(\hat{f}^2 > a) = A$ if a<0, [$A(\hat{f}^2 > a) = A(|\hat{f}| > \sqrt{a})$ if $a \geq 0$.
5. $A\left(\frac{1}{\hat{f}} > a\right) = A(\hat{f} > 0) \cap A\left(\hat{f} < \frac{1}{a}\right)$ if a>0, $A\left(\frac{1}{\hat{f}} > a\right) = A(\hat{f} > 0) \cup \left(A(\hat{f} < 0) \cap A\left(\hat{f} < \frac{1}{a}\right)\right)$ if a<0, $A\left(\frac{1}{\hat{f}} > a\right) = A(\hat{f} > 0)$ if a=0.

Definition 2.18. An is-function $\hat{f}$, defined on the closed interval [pa, b], is said to be is-step is-function, if there is a finite number of points

$$a = a_0 < a_1 < \cdots < a_{n-1} < a_n = b,$$

such that

$$\hat{f}(x) = \frac{c_i}{\hat{T}(x)}, x \in [a_i, a_{i+1}), c_i = const, i = 0, 1, \cdots, n-1.$$

Theorem 2.19. Let $\hat{T}$(x)>0 for every x ∈ [a, b] and $\hat{T}$(x) is measurable on [a, b]. Let also, $\hat{T}$(x) is an iso-step is-function on [a, b]. Then $\hat{f}$(x) is measurable on [a, b].

Proof. Let

$$a = a_0 < a_1 < \cdots < a_{n-1} < a_n = b,$$

be such that

$$\hat{f}(x) = \frac{c_i}{\hat{T}(x)}, x \in [a_i, a_{i+1}), c_i = const, i = 0, 1, \cdots, n-1.$$

From the last theorem it follows that $\frac{c_i}{\hat{T}(x)}$ is a measurable is-function on $[a_i, a_{i+1})$, $i = 0, 1, 2, \cdots, n-1$. Fromn-1 here and from

$$[a, b] = \bigcup_{i=0}^{n-1} [a_i, a_{i+1}) \cup \{b\}.$$

Since {b} is a set with measure zero, we conclude that the is-step is-function $\hat{f}$ is measurable on [a, b].

Definition 2.20. Let M be a subset of the closed interval [a, b]. The function $\varphi_M(x) = 0$ for $x \in [a, b] \setminus M$ and $\varphi_M = 1$ for $x \in M$, is called the characteristic function of the set M.

Theorem 2.21. If the set M is a measurable subset of the closed interval A=[a, b], then the characteristic function $\varphi_M(x)$ is measurable on [a, b].

Proof. The assertion follows from the following relations. $A(\varphi_M > a) = \emptyset$ if $a \geq 1$, $A(\varphi_M > a) = M$ if $1 > a \geq 0, A(\varphi_M > a) = A$ if a<0.

Definition 2.22. Let M be a subset of the set A=[a, b]. The iso- function $\hat{\varphi}_M(x) = 0$ if $x \in A \setminus M$ and $\hat{\varphi}_M = \frac{1}{\hat{T}(x)}$ if $x \in M$, will be called characteristic is-function of the set M.

Theorem 2.23. Let $\hat{T}(x)$ be a measurable function on A=[a, b], M be a measurable subset of A. Then the characteristic is-function $\hat{\varphi}_M(x)$ of the set M is measurable.

Proof. Let $a \in \mathbb{R}$ be arbitrarily chosen. Then

$$A(\hat{\varphi}_M > a) = (A \setminus M)(0 > a) \cup M\left(\frac{1}{\hat{T}(x)} > a\right),$$

From here, using that the sets $(A \setminus M)(0 > a)$ and $M\left(\frac{1}{\hat{T}(x)} > a\right)$ are measurable sets, we conclude that $A(\hat{\varphi}_M > a)$ is a measurable set. Because the constant a was arbitrarily chosen, we have that the characteristic function $\hat{\varphi}_M$ is a measurable is-function.

Theorem 2.24. Let f and $\hat{T}$ are continuous functions on the closed set A. Then the is-function $\hat{f}^{\wedge}(\hat{x})$ is measurable.

Proof. Let $a \in \mathbb{R}$ be arbitrarily chosen. Since every closed set is a measurable set, we conclude that the set A is a measurable set.

We will prove that the set $A(\hat{f}^{\wedge\wedge} \le a)$ is a closed set.

Let $\{x_n\}_{n=1}^{\infty}$ be a sequence of elements of the set $A(\hat{f}^{\wedge\wedge} \le a)$ such that

$$\lim_{n\to\infty} x_n = x_0.$$

Since $A(\hat{f}^{\wedge\wedge} \le a)$ is a subset of the set A we have that $\{x_n\}_{n=1}^{\infty} \subset A$. Because the set A is a closed set, we obtain that $x_0 \in A$. From the definition of the set $A(\hat{f}^{\wedge\wedge} \le a)$ we have that

$$\hat{f}^{\wedge}(\hat{x}_n) = \frac{f(x_n)}{\hat{T}(x_n)} \le a,$$

Hence, when $n \to \infty$, using that f and $\hat{T}$ are continuous functions on the set A, we get

$$\lim_{n\to\infty} \hat{f}^{\wedge}(\hat{x}_n) = \lim_{n\to\infty} \frac{f(x_n)}{\hat{T}(x_n)} = \frac{f(x_0)}{\hat{T}(x_0)} = \hat{f}^{\wedge}(\hat{x}_0) \le a,$$

i.e., $x_0 \in A(\hat{f}^{\wedge\wedge} \le a)$. Therefore the set $A(\hat{f}^{\wedge\wedge} \le a)$ is a closed set. From here, the set $A(\hat{f}^{\wedge\wedge} \le a)$ is a measurable set. Because the difference of two measurable sets is a measurable set, we have that the set

$$A(\hat{f}^{\wedge\wedge} > a) = A \setminus A(\hat{f}^{\wedge\wedge} \le a)$$

Is a measurable set.

Since $a \in \mathbb{R}$ was arbitrarily chosen, we obtain that the is-function of the first kind $\hat{f}^{\wedge\wedge}$ is measurable.

Theorem 2.25. Let f and $\hat{T}$ are continuous functions on the closed set A. The the is-functions

$$\hat{f}^{\wedge}(x), \hat{f}(\hat{x}), f^{\wedge}(x), f^{\vee}(x)$$

are measurable on A.

Theorem 2.26. If two measurable is-functions $\hat{f}$ and $\hat{g}$ are defined on the set A, then the set $A(\hat{f} > \hat{g})$ is measurable.

Proof. We enumerate all rational numbers

$$r_1, r_2, r_3, \cdots.$$

We will prove that

$$A(\hat{f} > \hat{g}) = \cup_{k=1}^{\infty} \left(A(\hat{f} > r_k) \cap A(\hat{g} < r_k)\right). \quad (8)$$

Let

$$x \in A(\hat{f} > \hat{g})$$

Be arbitrarily chosen. Then

$$x \in A, \hat{f}(x) > \hat{g}(x).$$

There exists a rational number r_k such that

$$\hat{f}(x) > r_k > \hat{g}(x).$$

Therefore

$$x \in A \text{ and } \hat{f}(x) > r_k; x \in A \text{ and } r_k > \hat{g}(x),$$

i.e.,

$$x \in A(\hat{f} > r_k), x \in A(\hat{g} < r_k).$$

Consequently

$$x \in A(\hat{f} > r_k) \cap A(\hat{g} < r_k)$$

And

$$x \in \bigcup_{k=1}^{\infty} \left(A(\hat{f} > r_k) \cap A(\hat{g} < r_k)\right).$$

Because $x \in A(\hat{f} > \hat{g})$ was arbitrarily chosen and for it we get $x \in \cup_{k=1}^{\infty} \left(A(\hat{f} > r_k) \cap A(\hat{g} < r_k)\right)$, we conclude that

$$A(\hat{f} > \hat{g}) \subset \cup_{k=1}^{\infty} \left(A(\hat{f} > r_k) \cap A(\hat{g} < r_k)\right). \quad (9)$$

Let no

$$x \in \bigcup_{k=1}^{\infty} \left(A(\hat{f} > r_k) \cap A(\hat{g} < r_k)\right)$$

be arbitrarily chosen. Then there exists a natural k so that

$$x \in A(\hat{f} > r_k) \cap A(\hat{g} < r_k).$$

Hence,

$$x \in A(\hat{f} > r_k), x \in A(\hat{g} < r_k).$$

Then

$$x \in A, \hat{f}(x) > r_k, r_k < \hat{g}(x)$$

or

$$x \in A, \hat{f}(x) > r_k > \hat{g}(x).$$

Therefore

$$x \in A(\hat{f} > \hat{g}).$$

Because

$$x \in \bigcup_{k=1}^{\infty} \left(A(\hat{f} > r_k) \cap A(\hat{g} < r_k)\right)$$

Was arbitrarily chosen and for it we get that $x \in A(\hat{f} > \hat{g})$, we conclude that

$$\bigcup_{k=1}^{\infty} \left(A(\hat{f} > r_k) \cap A(\hat{g} < r_k)\right) \subset A(\hat{f} > \hat{g}).$$

From the last relation and from the relation (9) we get the relation (8).

Since $\hat{f}$ and $\hat{g}$ are measurable iso-functions on A, we have that the sets

$$A(\hat{f} > r_k), A(\hat{g} < r_k)$$

are measurable sets for every natural k, whereupon the sets

$$A(\hat{f} > r_k) \cap A(\hat{g} < r_k)$$

Are measurable sets for every natural k.

Therefore, using the relation (8), we obtain that the set $A(\hat{f} > \hat{g})$ is a a measurable set.

Theorem 2.27. Let $\hat{f}(x)$ and $\hat{g}(x)$ be finite measurable is-functions on the set A. Then each of the is-functions

1. $\hat{f}(x) - \hat{g}(x)$,
2. $\hat{f}(x) + \hat{g}(x)$,
3. $\hat{f}(x)\hat{g}(x)$,
4. $\frac{\hat{f}(x)}{\hat{g}(x)}$ if $\hat{g}(x) \neq 0$ on A,

Is measurable.

Proof.

1. Let $a \in \mathbb{R}$ be arbitrarily chosen. Since $\hat{g}(x)$ is measurable, then $a + \hat{g}(x)$ is measurable. From here and from the last theorem it follows that the set

$$A(\hat{f}(x) - \hat{g}(x) > a) = A(\hat{f}(x) > a + \hat{g}(x))$$

Is measurable. Because $a \in \mathbb{R}$ was arbitrarily chosen, we conclude that the function $\hat{f}(x) - \hat{g}(x)$ is measurable.

2. Since $\hat{g}$ is a measurable is-function, we have that the function $-\hat{g}$ is a measurable is-function. From here and from 1) we conclude that the is-function

$$\hat{f} + \hat{g} = \hat{f} - (-\hat{g})$$

Is measurable.

3. We note that

$$\hat{f}(x)\hat{g}(x) = \frac{1}{2}(\hat{f}(x) + \hat{g}(x))^2 - \frac{1}{2}(\hat{f}(x) - \hat{g}(x))^2. \quad (10)$$

Since $\hat{f}(x)$ and $\hat{g}(x)$ are measurable iso-functions, using 1) and 2) we have that

$$\hat{f}(x) + \hat{g}(x) and\ \hat{f}(x) - \hat{g}(x)$$

Are measurable is-functions. Hence the is-functions

$$(\hat{f}(x) + \hat{g}(x))^2, (\hat{f}(x) - \hat{g}(x))^2$$

Are measurable, whereupon

$$\frac{1}{2}(\hat{f}(x) + \hat{g}(x))^2 \ and\ \frac{1}{2}(\hat{f}(x) - \hat{g}(x))^2$$

Are measurable. From here, using 1) and (10), we conclude that $\hat{f}(x)\hat{g}(x)$ is measurable.

4. Since $\hat{g}(x)$ is measurable and $\hat{g}(x) \neq 0$ on A, we have that the is-function $\frac{1}{\hat{g}(x)}$ is measurable. From here and from 3) the is-function

$$\frac{\hat{f}(x)}{\hat{g}(x)} = \hat{f}(x)\frac{1}{\hat{g}(x)}$$

Is measurable.

Theorem 2.28. Let $\{\hat{f}_n(x)\}_{n=1}^{\infty}$ be a sequence of measurable is-functions defined on the set A. If

$$\lim_{n\to\infty} \hat{f}_n(x) = \hat{f}(x) \quad (11)$$

Exists for every $x \in A$, then the is-function $\hat{f}(x)$ is measurable.

Proof. Let $a \in \mathbb{R}$ be arbitrarily chosen. For $n, k, m \in \mathbb{N}$ we define the sets

$$A_{m,k} := A\left(\hat{f}_k > a + \frac{1}{m}\right), B_{m,n} := \prod_{k=n}^{\infty} A_{m,k}.$$

We will prove that

$$A(\hat{f} > a) = \cup_{n,m} B_{m,n}. \quad (12)$$

Let

$$x \in A(\hat{f} > a)$$

Be arbitrarily chosen. Then

$$x \in A\ and\ \hat{f}(x) > a.$$

Hence, there is enough large natural number m_1 such that

$$\hat{f}(x) > a + \frac{1}{m_1}.$$

Using (11), there are enough large natural numbers k and m such that

$$\hat{f}_k(x) > a + \frac{1}{m},$$

i.e., $x \in A_{m,k}$.

From here, it follows that there is enough large n so that $x \in A_{m,k}$ for every $k \geq n$, i.e., $x \in B_{m,n}$ and then $x \in \cup_{m,n} B_{m,n}$.

Since $x \in A(\hat{f} > a)$ was arbitrarily chosen and we get that it is an element of the set $\cup_{m,n} B_{m,n}$, we conclude that

$$A(\hat{f} > a) \subset \cup_{m,n} B_{m,n}. \quad (13)$$

Let now $x \in \cup_{m,n} B_{m,n}$ be arbitrarily chosen.

Then, there are $m_2, n \in \mathbb{N}$ so that

$$x \in B_{m_2,n_1} = \prod_{k=n_1}^{\infty} A_{m_2,k_1}$$

or

$$\hat{f}_{k_1}(x) > a + \frac{1}{m_2}\ for\ \forall k \geq n_1.$$

Hence,

$$\lim_{k_1\to\infty} \hat{f}_{k_1}(x) \geq \lim_{k_1\to\infty} \left(a + \frac{1}{m_2}\right)$$

or

$$\hat{f}(x) \geq a + \frac{1}{m_2} > a.$$

Therefore

$$x \in A(\hat{f} > a).$$

Since $x \in \bigcup_{m,n} B_{m,n}$ was arbitrarily chosen and for it we obtain $x \in A(\hat{f} > a)$, we conclude that

$$\bigcup_{m,n} B_{m,n} \subset A(\hat{f} > a).$$

From the last relation and from (13) it follows the relation (12).

Since $\hat{f}_k(x)$ are measurable, we have that the sets $A_{m,k}$ are measurable for every $m, k \in \mathbb{N}$, hence $B_{m,n}$ are measurable for every $m, n \in \mathbb{N}$ and then, using (12), the set $A(\hat{f} > a)$ is measurable. Consequently the is-function $\hat{f}$ is measurable.

Theorem 2.29. be a sequence of measurable is-functions defined on the set A. If

$$\lim_{n\to\infty} \hat{f}_n(x) = \hat{f}(x) \quad (14)$$

Exists for almost everywhere $x \in A$, then the is-function $\hat{f}(x)$ is measurable.

Proof. Let B be the subset of A so that the relation (14) holds for every $x \in B$. From the previous theorem it follows that the is-function $\hat{f}(x)$ is measurable on the set B.

We note that

$$\mu(A \setminus B) = 0.$$

Therefore the is-function $\hat{f}(x)$ is measurable on $A \setminus B$. Hence, the is-function $\hat{f}(x)$ is measurable on A.

Let

$$\hat{T}_n, \hat{T}: A \to (0, \infty), f_n, f: A \to \mathbb{R},$$

$$0 < q_1 \leq \hat{T}_n(x), \hat{T}(x) \leq q_2 for\ x \in A, n \in \mathbb{N}.$$

Then

1. $\hat{f}_n^{\wedge}(\hat{x}) = \frac{f_n(x)}{\hat{T}_n(x)}, \hat{f}^{\wedge}(x) = \frac{f(x)}{\hat{T}(x)},$
2. $\hat{f}_n^{\wedge}(x) = \frac{f_n(x\hat{T}_n(x))}{\hat{T}_n(x)}, \hat{f}^{\wedge}(x) = \frac{f(x\hat{T}(x))}{\hat{T}(x)}$

   If

$$x\hat{T}_n(x), x\hat{T}(x), x \in A,$$

3. $\hat{f}_n(\hat{x}) = \frac{f_n\left(\frac{x}{\hat{T}_n(x)}\right)}{\hat{T}_n(x)}, \hat{f}(\hat{x}) = \frac{f\left(\frac{x}{\hat{T}(x)}\right)}{\hat{T}(x)}$

   If

$$\frac{x}{\hat{T}_n(x)}, \frac{x}{\hat{T}(x)}, x \in A,$$

4. $f_n^{\wedge}(x) = f_n(x\hat{T}_n(x)), f^{\wedge}(x) = f(x\hat{T}(x)),$

   If

$$x\hat{T}_n(x), x\hat{T}(x), x \in A,$$

5. $f_n^{\vee}(x) = f_n\left(\frac{x}{\hat{T}_n(x)}\right), f^{\vee}(x) = f\left(\frac{x}{\hat{T}(x)}\right)$

   If

$$\frac{x}{\hat{T}_n(x)}, \frac{x}{\hat{T}(x)}, x \in A.$$

3. The Structure of the Measurable Is-Functions

Theorem 3.1. (is-Lebesgue theorem for is-functions of the first kind) Let there be given a sequence $\{f_n(x)\}_{n=1}^{\infty}$ of measurable functions on a set A, all of which are finite almost everywhere. Let also, $\{\hat{T}_n(x)\}_{n=1}^{\infty}$ be a sequence of measurable functions on the set A,

$$0 < q_1 \leq \hat{T}_n(x) \leq q_2$$

For all natural numbers n and for all $x \in A$, where q_1 and q_2 are positive constants. Suppose that

$$\lim_{n\to\infty} f_n(x) = f(x),$$

$$\lim_{n\to\infty} \hat{T}_n(x) = \hat{T}(x)$$

Almost everywhere on the set A, and $f(x)$ is finite almost everywhere on A,

$$q_1 \leq \hat{T}(x) \leq q_2$$

For all $x \in A$. Then

$$\lim_{n\to\infty} \mu A\left(\left|\hat{f}_n^{\wedge}(\hat{x}) - \hat{f}^{\wedge}(\hat{x})\right| \geq \sigma\right) = 0$$

For all $\sigma \geq 0$.

Proof. We will note that the limit functions f(x) and $\hat{T}(x)$ are measurable and the sets under considerations are measurable.

Let

$$A := A(|f| = \infty),$$

$$B_n := A(|f_n| = \infty),$$

$$C := A(f_n \nrightarrow f),$$

$$D := B \cup \left(\bigcup_{n=1}^{\infty} B_n\right) \cup C.$$

Since

$$\mu B = 0, \mu C = 0, \mu B_n = 0,$$

using the properties of the measurable sets, we have that

$$\mu\mathbb{Q} = 0.$$

Let

$$A_k(\sigma) = A\left(\left|\frac{f_k}{\hat{T}_k} - \frac{f}{\hat{T}}\right| \geq \sigma\right),$$

$$R_n(\sigma)=\bigcup_{k=n}^{\infty}A_k(\sigma),$$

$$M=\prod_{n=1}^{\infty}R_n(\sigma).$$

We have that

$$R_1(\sigma)\supset R_2(\sigma)\supset\cdots.$$

Hence,

$$\lim_{n\to\infty}\mu R_n(\sigma)=\mu M.$$

Let us assume that $x_0\notin\mathbb{Q}$. Then, using the definition of the set $\mathbb{Q}$, we have

$$\lim_{n\to\infty}\frac{f_k(x_0)}{\hat{T}_k(x_0)}=\frac{f(x_{0)}}{\hat{T}(x_0)}$$

Since

$$0<q_1\leq\hat{T}_n(x),\hat{T}(x)\leq q_2\ ,k=1,2,\cdots,n,$$

we have that

$$\frac{f_1(x_0)}{\hat{T}_1(x_0)},\frac{f_2(x_0)}{\hat{T}_2(x_0)},\cdots,\frac{f_k(x_0)}{\hat{T}_k(x_0)},\cdots$$

and their limit

$$\frac{f(x_{0)}}{\hat{T}(x_0)}$$

are finite.

Therefore there is an enough large natural n such that

$$\left|\frac{f_k(x_0)}{\hat{T}_k(x_0)}-\frac{f(x_{0)}}{\hat{T}(x_0)}\right|<\sigma$$

for every $k\geq n$. Then $x_0\notin A_k(\sigma)$, $k\geq n$, where $x_0\notin R_n(\sigma)$ and from here $x_0\notin M$.

Consequently $M\subset\mathbb{Q}$.

Because $\mu\mathbb{Q}=0$, from the last relation, we have that $\mu M=0$, i.e.,

$$\lim_{n\to\infty}R_n(\sigma)=0,$$

and since

$$A_n(\sigma)\subset R_n(\sigma),$$

$$\lim_{n\to\infty}R_n(\sigma)=0$$

or

$$\lim_{n\to\infty}\mu A\left(\left|\hat{f}_n^{\wedge}(\hat{x})-\hat{f}^{\wedge}(\hat{x})\right|\geq\sigma\right)=0.$$

As in above one can prove the following results for the other kinds of is-functions.

Theorem 3.2. (is-Lebesgue theorem for is-functions of the second kind) Let there be given a sequence $\{f_n(x)\}_{n=1}^{\infty}$ of measurable functions on a set A, all of which are finite almost everywhere. Let also, $\{\hat{T}_n(x)\}_{n=1}^{\infty}$ be a sequence of measurable functions on the set A,

$$0<q_1\leq\hat{T}_n(x)\leq q_2$$

For all natural numbers n and for all $x\in A$, where q_1 and q_2 are positive constants. Suppose that

$$\lim_{n\to\infty}f_n(x)=f(x),$$

$$\lim_{n\to\infty}\hat{T}_n(x)=\hat{T}(x)$$

Almost everywhere on the set A, and $f(x)$ is finite almost everywhere on A,

$$q_1\leq\hat{T}(x)\leq q_2$$

For all $x\in A$. Then

$$\lim_{n\to\infty}\mu A\left(\left|\hat{f}_n^{\wedge}(x)-\hat{f}^{\wedge}(x)\right|\geq\sigma\right)=0$$

for all $\sigma\geq 0$.

Theorem 3.3. (is-Lebesgue theorem for is-functions of the third kind) Let there be given a sequence $\{f_n(x)\}_{n=1}^{\infty}$ of measurable functions on a set A, all of which are finite almost everywhere. Let also, $\{\hat{T}_n(x)\}_{n=1}^{\infty}$ be a sequence of measurable functions on the set A,

$$0<q_1\leq\hat{T}_n(x)\leq q_2$$

For all natural numbers n and for all $x\in A$, where q_1 and q_2 are positive constants. Suppose that

$$\lim_{n\to\infty}f_n(x)=f(x),$$

$$\lim_{n\to\infty}\hat{T}_n(x)=\hat{T}(x)$$

Almost everywhere on the set A, and $f(x)$ is finite almost everywhere on A,

$$q_1\leq\hat{T}(x)\leq q_2$$

For all $x\in A$. Then

$$\lim_{n\to\infty}\mu A\left(\left|\hat{f}_n\ (\hat{x})-\hat{f}\ \ (\hat{x})\right|\geq\sigma\right)=0$$

for all $\sigma\geq 0$.

Theorem 3.4. (is-Lebesgue theorem for is-functions of the fourth kind) Let there be given a sequence $\{f_n(x)\}_{n=1}^{\infty}$ of measurable functions on a set A, all of which are finite almost everywhere. Let also, $\{\hat{T}_n(x)\}_{n=1}^{\infty}$ be a sequence of measurable functions on the set A,

$$0<q_1\leq\hat{T}_n(x)\leq q_2$$

For all natural numbers n and for all $x\in A$, where q_1 and q_2 are positive constants. Suppose that

$$\lim_{n\to\infty}f_n(x)=f(x),$$

$$\lim_{n\to\infty} \hat{T}_n(x) = \hat{T}(x)$$

Almost everywhere on the set A, and $f(x)$ is finite almost everywhere on A,

$$q_1 \leq \hat{T}(x) \leq q_2$$

For all $x \in A$. Then

$$\lim_{n\to\infty} \mu A(|f_n^{\wedge}(x) - f^{\wedge}(x)| \geq \sigma) = 0$$

For all $\sigma \geq 0$.

Theorem 3.5. (is-Lebesgue theorem for is-functions of the fifth kind) Let there be given a sequence $\{f_n(x)\}_{n=1}^{\infty}$ of measurable functions on a set A, all of which are finite almost everywhere. Let also, $\{\hat{T}_n(x)\}_{n=1}^{\infty}$ be a sequence of measurable functions on the set A,

$$0 < q_1 \leq \hat{T}_n(x) \leq q_2$$

For all natural numbers n and for all $x \in A$, where q_1 and q_2 are positive constants. Suppose that

$$\lim_{n\to\infty} f_n(x) = f(x),$$

$$\lim_{n\to\infty} \hat{T}_n(x) = \hat{T}(x)$$

Almost everywhere on the set A, and $f(x)$ is finite almost everywhere on A,

$$q_1 \leq \hat{T}(x) \leq q_2$$

For all $x \in A$. Then

$$\lim_{n\to\infty} \mu A(|f_n^{\vee}(x) - f^{\vee}(x)| \geq \sigma) = 0$$

for all $\sigma \geq 0$.

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Santilli Isomathematics for Generalizing Modern Mathematics

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Abstract: The establishment of isomathematics, as proposed by R. M. Santilli thirty years ago in the USA, and contributed to by Jiang Chun-Xuan in China during the past 12 years, is significant and has changed modern mathematics. At present, the primary teaching of mathematics is based on the simple operations of addition, subtraction, multiplication and division; a middle level teaching ofmathematics takes these four operations to a higher level, while the university teaching of mathematics extends them to an even higher level. These four arithmetic operations form the foundation of modern mathematics. Santilli isomathematics is a generalisation of these four fundamental operations and heralds a great revolution in mathematics. HIn this paper, we study the four generalized arithmetic operations of isoaddition, isosubtraction, isomultiplication and isodivision at the primary level of isomathematics. The material introduced here should be readily understandable by middle school pupils and university students.Santilli's isomathematics [1] ßßis based on a generalisation of modern mathematics. Isomultiplication is defined by $a \hat{\times} a = ab\hat{T}$, isodivision by $a \hat{\div} b = \frac{a}{b}\hat{I}$, where $\hat{I} \neq 1$ is called an isounit; $\hat{T}\hat{I} = 1$, where $\hat{T}$ is the inverse of the isounit. If addition and subtraction remain unchanged, $(\hat{+}, \hat{-}, \hat{\times}, \hat{\div})$ are the four arithmetic operations in Santilli's isomathematics[1-5]. Isoaddition $a\hat{+}b = a + b + \hat{0}$ and isosubtraction $a\hat{+}b = a + b + \hat{0}$, where $\hat{0} \neq 0$ is called the isozero, together with the operations of isomultiplication and isodivision introduced above, form the four arithmetic operations$(\hat{+}, \hat{-}, \hat{\times}, \hat{\div})$ in Santilli-Jiang isomathematics[6]. Santilli [1] suggests isomathematics based on a generalisation of multiplication ×, division ÷, and the multiplicative unit 1 of modern mathematics. It is an epoch making suggestion. From modern mathematics, the foundations of Santilli's isomathematics will be established.

Keywords: Santilli-Jiang Math, Isomultiplication, Isodivision, Isoaddition and Isosubtraction

1. Division and Multiplication in Modern Mathematics

Suppose that

$$a \div a = a^0 = 1 \tag{1}$$

where 1 is the multiplicative unit and 0 is exponent zero.

From (1), division ÷ and multiplication × are defined by

$$a \div b = \frac{a}{b}, b \neq 0, a \times b = ab \tag{2}$$

$$a = a \times (a \div a) = a \times a^0 = a \tag{3}$$

Now consider the multiplicative unit 1,

$$a \times 1 = a, a + 1 = a, 1 \div a = 1/a \tag{4}$$

$$(+1)^n = 1, (+1)^{a/b} = 1, (-1)^n = (-1)^n, (-1)^{a/b} = (-1)^{a/b} \tag{5}$$

Addition +, subtraction -, multiplication ×, and division ÷ are the four operations forming the foundation of modern mathematics. The modern mathematics is generalised to establish the foundations of Santilli-Jiang isomathematics.

2. Isodivision and Isomultiplication in Santilli's Isomathematics

Isodivision $\hat{\div}$ and isomultiplication $\hat{\times}$ [1 – 5], which are generalisations of the division ÷ and multiplication × of modern mathematics, are now defined.

$$a \hat{\div} a = a^{\bar{0}} = \hat{I} \neq 1, \quad \bar{0} \neq 0 \tag{6}$$

where $\hat{I}$ is called the isounit and is a generalisation of the multiplicative unit 1 and $\bar{0}$ is the isoexponent zero which is a generalisation of the exponent zero 0. Then,

$$a \hat{\div} b = \hat{I} \frac{a}{b}, b \neq 0, a \hat{\times} b = a \hat{T} b \tag{7}$$

It is seen that

$$a = a \hat{\times} (a \hat{\div} a) = a \hat{\times} a^{\bar{0}} = a \hat{T} \hat{I} = a \tag{8}$$

from which it follows that

$$\hat{T} \hat{I} = 1 \tag{9}$$

where $\hat{T}$ is the inverse of the isounit $\hat{I}$.

The isounit $\hat{I}$ has the following properties[5,p93-95,isoexponents]:

$$a \hat{\times} \hat{I} = a, \ a \hat{\div} \hat{I} = a, \ \hat{I} \hat{\div} a = a^{-\hat{I}} = \hat{I}^2 / a \tag{10}$$

$$(+\hat{I})^{\hat{n}} = \hat{I}, (+\hat{I})^{\frac{\hat{a}}{\hat{b}}} = \hat{I}, (-\hat{I})^{\hat{n}} = (-1)^n \hat{I}, (-\hat{I})^{\frac{\hat{a}}{\hat{b}}} = (-1)^{\frac{a}{b}} \hat{I} \tag{11}$$

With addition and subtraction unchanged, $(+, -, \hat{\times}, \hat{\div})$ are the four arithmetic operations in Santilli's isomathematics and these form the foundations of Santilli isomathematics. When $\hat{I} = 1$, the operations revert to being those of the modern mathematics.

3. Addition and Subtraction in Modern Mathematics

These are defined by

$$x = a + b \text{ and } y = a - b \tag{12}$$

$$a + a - a = a \tag{13}$$

$$a - a = 0 \tag{14}$$

Isoaddition and isosubtraction may be established using these results.

4. Isoaddition and Isosubtraction in Santilli-Jiang Isomathematics

Isoaddition $\hat{+}$ and isosubtraction $\hat{-}$ are defined by

$$a \hat{+} b = a + b + c_1, \ a \hat{-} b = a - b - c_2 \tag{15}$$

$$\therefore a = a \hat{+} a \hat{-} a = a + c_1 - c_2 = a \tag{16}$$

Then, from (16), it follows that

$$c_1 = c_2 \tag{17}$$

Suppose that $c_1 = c_2 = \hat{0}$, where $\hat{0}$ is called the isozero which is a generalisation of the zero 0 of addition and subtraction[6]. Hence,

$$a \hat{+} b = a + b + \hat{0}, \quad a \hat{-} b = a - b - \hat{0} \tag{18}$$

When $\hat{0} = 0$, these equations are the usual laws of addition and subtraction of modern mathematics.

From the above results, the foundations of Santilli-Jiang isomathematics are readily established:

$$\hat{\times} = \times \hat{T} \times, \ \hat{+} = +\hat{0} +; \ \hat{\div} = \times \hat{I} \div, \ \hat{-} = -\hat{0} -; \ a \hat{\times} b = ab\hat{T}, a \hat{+} b = a + b + \hat{0};$$

$$a \hat{\div} b = \frac{a}{b} \hat{I}, a \hat{-} b = a - b - \hat{0}; a = a \hat{\times} a \hat{\div} a = a, a = a \hat{+} a \hat{-} a = a;$$

$$a \hat{\times} a = a^2 T, a \hat{+} a = 2a + \hat{0}; a \hat{\div} a = \hat{I} \neq 1, a \hat{-} a = -\hat{0} \neq 0; \hat{T} \hat{I} = 1. \tag{19}$$

Here $(\hat{+}, \hat{-}, \hat{\times}, \hat{\div})$ are the four arithmetic operations in Santilli-Jiang isomathematics.

Remark:

$$a \hat{\times} (b \hat{+} c) = a \hat{\times} (b + c + \hat{0})$$

From the left-hand side, it follows

$$a \hat{\times} (b \hat{+} c) = a \hat{\times} b + a \hat{\times} \hat{+} + a \hat{\times} c) = a \hat{\times} (b + \hat{+} + c)$$
$$= a \hat{\times} (b + \hat{0} + c),$$

where $\hat{+} = \hat{0}$ is a number also.

Again,

$$a \hat{\times} (b \hat{-} c) = a \hat{\times} (b - c - \hat{0})$$

From the left-hand side of this relation, it is seen that

$$a \hat{\times} (b \hat{-} c) = a \hat{\times} b - a \hat{\times} \hat{-} - a \hat{\times} c)$$
$$= a \hat{\times} (b - \hat{-} - c) = a \hat{\times} (b - \hat{0} - c),$$

where $\hat{-} = \hat{0}$ is a number also.

The distributive laws are satisfied and $\hat{+}, \hat{-}, \hat{\times}, \hat{\div}$ are numbers.

This Santilli-Jiang isomathematics therefore, provides a new mathematical tool for studying the mathematical

problems of the 21st century and helping in the understanding the mysteries of our universe.

5. An Illustrative Example

Consider the algebraic equation

$$y = a_1 \times (b_1 + c_1) + a_2 \div (b_2 - c_2) \quad (20)$$

(20) may represent a mathematical system, physical system, biological system,cryptogram system, IT system, or some other system. It may be written as the isomathematical equation

$$\hat{y} = a_1 \hat{\times}(b_1 \hat{+} c_1) \hat{+} a_2 \hat{\div}(b_2 \hat{-} c_2) = a_1 \hat{T}(b_1 + c_1 + \hat{0}) + \hat{0} + a_2 / \hat{T}(b_2 - c_2 - \hat{0}) \quad (21)$$

If $\hat{T} = 1$ and $\hat{0} = 0$ then $y = \hat{y}$.

Let $\hat{T} = 2$ and $\hat{0} = 3$. From (21) we have the isomathematical subequation

$$\hat{y}_1 = 2a_1(b_1 + c_1 + 3) + 3 + a_2 / 2(b_2 - c_2 - 3). \quad (22)$$

Let $\hat{T} = 5$ and $\hat{0} = 6$. From (21) we have the isomathematical subequation

$$\hat{y}_2 = 5a_1(b_1 + c_1 + 6) + 6 + a_2 / 5(b_2 - c_2 - 6) \quad (23)$$

Let $\hat{T} = 8$ and $\hat{0} = 10$. From (21) we have the isomathematical subequation

$$\hat{y}_3 = 8a_1(b_1 + c_1 + 10) + 10 + a_2 / 8(b_2 - c_2 - 10) \quad (24)$$

Therefore, (21) has infinitely many isomathematical subequations. Using (21) – (24), $\hat{T}$ and $\hat{0}$, the stability and optimum structure of the algebraic equation (20) may be studied.

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Hypermathematics, H_v-Structures, Hypernumbers, Hypermatrices and Lie-Santilli Addmissibility

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Abstract: We present the largest class of hyperstructures called H_v-structures. In H_v-groups and H_v-rings, the fundamental relations are defined and they connect the algebraic hyperstructure theory with the classical one. Using the fundamental relations, the H_v-fields are defined and their elements are called hypernumbers or H_v-numbers. H_v-matrices are defined to be matrices with entries from an H_v-field. We present the related theory and results on hypermatrices and on the Lie-Santilli admissibility.

Keywords: Representations, Hope, Hyperstructures, H_v-Structures

1. Introduction to Hypermathematics, the H_v-Structures

Hyperstructure is called an algebraic structure containing at least one hyperoperation. More precisely, a set H equipped with at least one multivalued map $\cdot: H\times H \to P(H)$, is called hyperstructure and the map hyperoperation, we abbreviate hyperoperation by hope. The first hyperstructure was the hypergroup, introduced by F. Marty in 1934 [25], [26], where the strong generalized axioms of a group wrere used. We deal with the largest class of hyperstructures called H_v-structures introduced in 1990 [40],[44],[45] which satisfy the weak axioms where the non-empty intersection replaces the equality.

Some basic definitions:

Definitions 1.1 In a set H with a hope $\cdot: H\times H\to P(H)$, we abbreviate by WASS the weak associativity: $(xy)z\cap x(yz)\neq\emptyset$, $\forall x,y,z\in H$ and by COW the weak commutativity: $xy\cap yx\neq\emptyset$, $\forall x,y\in H$.

The hyperstructure $(H,\cdot)$ is called H_v-semigroup if it is WASS and is called H_v-group if it is reproductive H_v-semigroup:

$$xH=Hx=H,\ \forall x\in H.$$

The hyperstructure $(R,+,\cdot)$ is called H_v-ring if $(+)$ and $(\cdot)$ are WASS, the reproduction axiom is valid for $(+)$ and $(\cdot)$ is weak distributive with respect to $(+)$:

$$x(y+z)\cap(xy+xz)\neq\emptyset,\ (x+y)z\cap(xz+yz)\neq\emptyset,\ \forall x,y,z\in R.$$

For definitions, results and applications on H_v-structures, see books [44],[4],[10],[12] and papers [6],[7],[8],[9],[11], [17],[18],[19],[22],[24],[46]. An extreme class is defined as follows [41],[44]: An H_v-structure is very thin iff all hopes are operations except one, with all hyperproducts singletons except only one, which is a subset of cardinality more than one. Thus, a very thin H_v-structure is an H with a hope $(\cdot)$ and a pair $(a,b)\in H^2$ for which $ab=A$, with $cardA>1$, and all the other products, are singletons.

The main tools to study hyperstructures are the so called, fundamental relations. These are the relations β^* and γ^* which are defined, in H_v-groups and H_v-rings, respectively, as the smallest equivalences so that the quotient would be group and ring, respectively [38],[40],[44],[48],[49]. The way to find the fundamental classes is given as follows [44]:

Theorem 1.2 Let $(H,\cdot)$ be an H_v-group and let us denote by U the set of all finite products of elements of H. We define the relation β in H as follows: $x\beta y$ iff $\{x,y\}\subset u$ where $u\in U$. Then the fundamental relation β^* is the transitive closure of the relation β.

The main point of the proof is that β guaranties that the following is valid: Take elements x,y such that $\{x,y\}\subset u\in U$ and any hyperproduct where one of these elements is used. Then, if this element is replaced by the other, the new hyperproduct is inside the same fundamental class where the first hyperproduct is. Thus, if the 'hyperproducts' of the above

β-classes are 'products', then, they are fundamental classes. Analogously for the γ in H_v-rings.

An element is called single if its fundamental class is a singleton.

Motivation for H_v-structures:

1. The quotient of a group with respect to an invariant subgroup is a group.

2. Marty states that, the quotient of a group with respect to any subgroup is a hypergroup.

3. The quotient of a group with respect to any partition is an H_v-group.

In H_v-structures a partial order can be defined [44].

Definition 1.3 Let $(H,\cdot)$, $(H,\otimes)$ be H_v-semigroups defined on the same H. $(\cdot)$ is smaller than $(\otimes)$, and $(\otimes)$ greater than $(\cdot)$, iff there exists automorphism $f\in Aut(H,\otimes)$ such that $xy\subset f(x\otimes y)$, $\forall x\in H$.

Then $(H,\otimes)$ contains $(H,\cdot)$ and write $\cdot\leq\otimes$. If $(H,\cdot)$ is structure, then it is called basic and $(H,\otimes)$ is an H_b-structure.

The Little Theorem [26]. Greater hopes of the ones which are WASS or COW, are also WASS and COW, respectively.

The fundamental relations are used for general definitions of hyperstructures. Thus, to define the general H_v-field one uses the fundamental relation γ^*:

Definition 1.4 [40],[43],[44]. The H_v-ring $(R,+,\cdot)$ is an H_v-field if the quotient R/γ^* is a field.

The elements of an H_v-field are called hypernumbers. Let ω^* be the kernel of the canonical map and from H_v-ring R to R/γ^*; then we call it reproductive H_v-field if:

$$x(R-\omega^*) = (R-\omega^*)x = R-\omega^*, \forall x\in R-\omega^*.$$

From this definition a new class is defined [51],[56]:

Definition 1.5 The H_v-semigroup $(H,\cdot)$ is called h/v-group if the H/β^* is a group.

An H_v-group is called cyclic [33],[44], if there is an element, called generator, which the powers have union the underline set, the minimal power with this property is the period of the generator. If there exists an element and a special power, the minimum one, is the underline set, then the H_v-group is called single-power cyclic.

To compare classes we can see the small sets. To enumerate and classify H_v-structures, is complicate because we have great numbers. The partial order [44],[47], restrict the problem in finding the minimal, up to isomorphisms, H_v-structures. We have results by Bayon & Lygeros as the following [2],[3]: In sets with three elements: Up to isomorphism, there are 6.494 minimal H_v-groups. The 137 are abelians; 6.152 are cyclic. The number of H_v-groups with three elements is 1.026.462. 7.926 are abelians; 1.013.598 are cyclic, 16 are very thin. Abelian H_v-groups with 4 elements are, 8.028.299.905 from which the 7.995.884.377 are cyclic.

Some more complicated hyperstructures can be defined, as well. In this paper we focus on H_v-vector spaces and there exist an analogous theory on H_v-modules.

Definition 1.6 [44],[50]. Let $(F,+,\cdot)$ be an H_v-field, $(M,+)$ be COW H_v-group and there exists an external hope

$$F\times M\to P(M): (a,x)\to ax,$$

such that, $\forall a,b\in F$ and $\forall x,y\in M$ we have

$$a(x+y)\cap(ax+ay)\neq\varnothing,\ (a+b)x\cap(ax+bx)\neq\varnothing,\ (ab)x\cap a(bx)\neq\varnothing,$$

then M is called an H_v-vector space over F.

The fundamental relation ε^* is defined to be the smallest equivalence such that the quotient M/ε^* is a vector space over the fundamental field F/γ^*. For this fundamental relation there is an analogous to the Theorem 1.2.

Definitions 1.7 [51],[53],[55]. Let $(H,\cdot)$ be hypergroupoid. We remove $h\in H$, if we consider the restriction of $(\cdot)$ in the set $H-\{h\}$. We say that $h\in H$ absorbs $h\in H$ if we replace h by h and h does not appear in the structure. We say that $h\in H$ merges with $h\in H$, if we take as product of any $x\in H$ by h, the union of the results of x with both h, h, and consider h and h as one class, with representative h, therefore the element h does not appeared in the hyperstructure.

Let $(H,\cdot)$ be an H_v-group, then, if an element h absorbs all elements of its own fundamental class then this element becomes a single in the new H_v-group.

Theorem 1.8 In an H_v-group $(H,\cdot)$, if an element h absorbs all elements of its fundamental class then this element becomes a single in the new H_v-group.

Proof. Let $h\in\beta^*(h)$, then, by the definition of the 'absorb', h is replaced by h that means that $\beta^*(h)=\{h\}$. Moreover, for all $x\in H$, the fundamental property of the product of classes

$$\beta^*(x)\cdot\beta^*(h) = \beta^*(xh) \text{ becomes } \beta^*(x)\cdot h = \beta^*(xh),$$

and from the reproductivity ([44] p.19) we obtain $x\cdot h=\beta^*(xh)$, $\forall x\in\beta^*(x)$. This is the basic property that enjoys any single element [44].

Remark that in case we have a single element then we can compute all fundamental classes.

A well known and large class of hopes is given as follows [33],[37],[39],[44],[20]:

Definitions 1.9 Let $(G,\cdot)$ be a groupoid, then for every subset $P\subset G$, $P\neq\varnothing$, we define the following hopes, called P-hopes: $\forall x,y\in G$

$$P: xPy = (xP)y\cup x(Py),$$

$$P_r: xP_ry = (xy)P\cup x(yP),\ P_l: xP_ly = (Px)y\cup P(xy).$$

The (G,P), (G,P_r) and (G,P_l) are called P-hyperstructures. In the case of semigroup $(G,\cdot)$: $xPy=(xP)y\cup x(Py)=xPy$ and (G,P) is a semihypergroup but we do not know about (G,P_r) and (G,P_l). In some cases, depending on the choice of P, the (G,P_r) and (G,P_l) can be associative or WASS.

A generalization of P-hopes is the following [13],[14]: Let $(G,\cdot)$ be abelian group and P a subset of G with more than one elements. We define the hope $\times_P$ as follows:

$$x\times_P y = x\cdot P\cdot y = \{x\cdot h\cdot y \mid h\in P\} \text{ if } x\neq e \text{ and } y\neq e$$

$$x\cdot y \text{ if } x=e \text{ or } y=e$$

we call this hope, P_e-hope. The hyperstructure $(G,\times_P)$ is an abelian H_v-group.

A general definition of hopes, is the following [57],[58]:

Definitions 1.10 Let H be a set with n operations (or hopes) $\otimes_1,\otimes_2,\ldots,\otimes_n$ and one map (or multivalued map) $f:H\to H$, then n hopes $\partial_1,\partial_2,\ldots,\partial_n$ on H are defined, called ∂-hopes by putting

$$x\partial_i y = \{f(x)\otimes_i y,\ x\otimes_i f(y)\},\ \forall x,y\in H,\ i\in\{1,2,\ldots,n\}$$

or in case where $\otimes_i$ is hope or f is multivalued map we have

$$x\partial_i y = (f(x)\otimes_i y)\cup(x\otimes_i f(y)),\ \forall x,y\in H,\ i\in\{1,2,\ldots,n\}$$

Let $(G,\cdot)$ groupoid and $f_i:G\to G$, $i\in I$, set of maps on G. Take the map $f_U:G\to P(G)$ such that $f_U(x)=\{f_i(x)\,|\,i\in I\}$, call it the union of the $f_i(x)$. We call the union ∂-hope (∂), on G if we consider the map $f_U(x)$. An important case for a map f, is to take the union of this with the identity id. Thus, we consider the map $f\cong f\cup(id)$, so $f(x)=\{x,f(x)\}$, $\forall x\in G$, which is called b-∂-hope, we denote it by (∂), so we have

$$x\partial y = \{xy,\ f(x)\cdot y,\ x\cdot f(y)\},\ \forall x,y\in G.$$

Remark If $\otimes_i$ is associative then ∂_i is WASS. If ∂ contains the operation $(\cdot)$, then it is b-operation. Moreover, if $f:G\to P(G)$ is multivalued then the b-∂-hopes is defined by using the $f(x)=\{x\}\cup f(x)$, $\forall x\in G$.

Motivation for the definition of ∂-hope is the derivative where only multiplication of functions is used. Therefore, for functions s(x), t(x), we have $s\partial t=\{s't,st'\}$, $(')$ is the derivative.

Example. For all first degree polynomials $g_i(x)=a_ix+b_i$, we have

$$g_1\partial g_2 = \{a_1a_2x+a_1b_2,\ a_1a_2x+b_1a_2\},$$

so it is a hope in the set of first degree polynomials. Moreover all polynomials x+c, where c be a constant, are units.

There exists the uniting elements method introduced by Corsini–Vougiouklis [5] in 1989. With this method one puts in the same class, two or more elements. This leads, through hyperstructures, to structures satisfying additional properties.

Definition 1.11 The uniting elements method is the following: Let G be an algebraic structure and let d be a property, which is not valid. Suppose that d is described by a set of equations; then, consider the partition in G for which it is put together, in the same partition class, every pair of elements that causes the non-validity of the property d. The quotient by this partition G/d is an H_v-structure. Then, quotient out the H_v-structure G/d by the fundamental relation β^*, a stricter structure $(G/d)\beta^*$ for which the property d is valid, is obtained.

An interesting application of the uniting elements is when more than one property is desired, because some of the properties lead straight to the classes. The commutativity and the reproductivity property are easily applicable. The following is valid:

Theorem 1.12 [44] Let $(G,\cdot)$ be a groupoid, and

$$F = \{f_1,\ldots,f_m,f_{m+1},\ldots,f_{m+n}\}$$

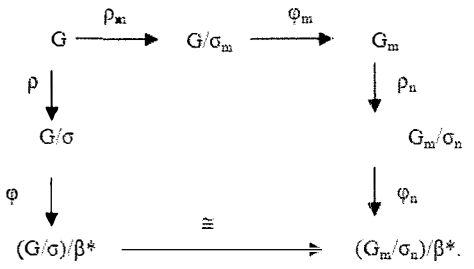
be a system of equations on G consisting of two subsystems

$$F_m = \{f_1,\ldots,f_m\}\ \text{and}\ F_n = \{f_{m+1},\ldots,f_{m+n}\}.$$

Let σ, σ_m be the equivalence relations defined by the uniting elements procedure using the systems F and F_m respectively, and let σ_n be the equivalence relation defined using the induced equations of F_n on the grupoid $G_m=(G/\sigma_m)/\beta^*$. Then

$$(G/\sigma)/\beta^* \cong (G_m/\sigma_n)/\beta^*.$$

i.e. the following diagram is commutative



From the above it is clear that the fundamental structure is very important, and even more so if this is known from the beginning. This is the problem to construct hyperstructures with desired fundamental structures [44].

Theorem 1.13 Let $(S,\cdot)$ be a commutative semigroup with one element $w\in S$ uch that the set wS is finite. Consider the transitive closure L^* of the relation L defined as follows: xLy iff there exists $z\in S$ such that $zx=zy$.

Then $<S/L^*,\circ>/\beta^*$ is finite commutative group, where $(\circ)$ is the induced operation on classes of S/L^*.

For the proof see [5],[44].

An application combining hyperstructures and fuzzy theory, is to replace the 'scale' of Likert in questionnaires by the bar of Vougiouklis & Vougiouklis [69],[70],[21],[27]:

Definition 1.14 In every question substitute the Likert scale with the 'bar' whose poles are defined with '0' on the left end, and '1' on the right end:

0 ——————————————————— 1

The subjects/participants are asked instead of deciding and checking a specific grade on the scale, to cut the bar at any point they feel expresses their answer to the question.

The use of the bar of Vougiouklis & Vougiouklis instead of a scale of Likert has several advantages during both the filling-in and the research processing. The final suggested length of the bar, according to the Golden Ratio, is 6.2cm. The hyperstructure theory, offer innovating new suggestions to connect finite groups of objects. These suggestions are obtained from properties and special elements inside the hyperstructure.

2. Hyper-Representations

Representations (abbreviate by rep) of H_v-groups can be faced either by generalized permutations or by H_v-matrices [34],[36],[39],[43],[44],[52],[54],[66]. Reps by generalized permutations can be achieved by using translations [42]. We

present an outline of the hypermatrix rep in H_v-structures and there exist the analogous theory for the h/v-structures.

Definitions 2.1 [44],[66] H_v-matrix is a matrix with entries elements of an H_v-field. The hyperproduct of two H_v-matrices $A=(a_{ij})$ and $B=(b_{ij})$, of type m×n and n×r respectively, is defined, in the usual manner,

$$A\cdot B = (a_{ij})\cdot(b_{ij}) = \{ C= (c_{ij}) \mid c_{ij}\in\oplus\Sigma a_{ik}\cdot b_{kj} \},$$

and it is a set of m×r H_v-matrices. The sum of products of elements of the H_v-field is the union of the sets obtained with all possible parentheses put on them, called n-ary circle hope on the hyperaddition.

The hyperproduct of H_v-matrices does not satisfy WASS.

The problem of the H_v-matrix reps is the following:

Definitions 2.2 For a given H_v-group $(H,\cdot)$, find an H_v-field $(F,+,\cdot)$, a set $M_R=\{(a_{ij}) \mid a_{ij}\in F\}$ and a map $T: H\to M_R: h\to T(h)$ such that

$$T(h_1h_2)\cap T(h_1)T(h_2) \neq \varnothing,\ \forall h_1,h_2\in H.$$

The map T is called H_v -matrix rep. If $T(h_1h_2)\subset T(h_1)T(h_2)$, $\forall h_1,h_2\in H$, then T is called inclusion rep. T is a good rep if $T(h_1h_2)=T(h_1)T(h_2)=\{T(h) \mid h\in h_1h_2\}, \forall h_1,h_2\in H$. If T is one to one and good then it is a faithful rep.

The problem of reps is complicated since the hyperproduct is big. It can be simplified in cases such as: The H_v-matrices are over H_v-fields with scalars 0 and 1. The H_v-matrices are over very thin H_v-fields. On 2×2 H_v-matrices, since the circle hope coincides with the hyperaddition. On H_v-fields which contain singles, which act as absorbings.

The main theorem of reps is the following [44],[52]:

Theorem 2.3 A necessary condition in order to have an inclusion rep T of an H_v-group $(H,\cdot)$ by n×n H_v-matrices over the H_v-field $(F,+,\cdot)$ is the following:

For all classes $\beta^*(x)$, $x\in H$ there must exist elements $a_{ij}\in H$, $i,j\in\{1,...,n\}$ such that

$$T(\beta^*(a)) \subset \{A=(a'_{ij}) \mid a'_{ij}\in \gamma^*(a_{ij}),\ i,j\in\{1,...,n\}\}$$

Thus, every inclusion rep $T: H\to M_R: a\to T(a)=(a_{ij})$ induces a homomorphic rep T* of the group H/β^* over the field F/γ^* by setting

$$T^*(\beta^*(a)) = [\gamma^*(a_{ij})],\ \forall \beta^*(a)\in H/\beta^*,$$

where $\gamma^*(a_{ij})\in R/\gamma^*$ is the ij entry of the matrix $T^*(\beta^*(a))$. T* is called fundamental induced rep of T.

Denote $tr_\varphi(T(x)) = \gamma^*(T(x_{ii}))$ the fundamental trace, then the mapping

$$X_T : H \to F/\gamma^*: x\to X_T(x) = tr_\varphi(T(x)) = trT^*(x)$$

is called fundamental character.

Using special classes of H_v-structures one can have several reps [52],[66]:

Definition 2.4 Let $M=M_{m\times n}$ be vector space of m×n matrices over a field F and take sets

$$S=\{s_k: k\in K\}\subseteq F,\ Q=\{Q_i: j\in J\}\subseteq M,\ P=\{P_i: i\in I\}\subseteq M.$$

Define three hopes as follows

$$S: F\times M\to P(M): (r,A)\to rSA=\{(rs_k)A: k\in K\}\subseteq M$$

$$Q_+: M\times M\to P(M): (A,B)\to AQ_+B=\{A+Q_j+B: j\in J\}\subseteq M$$

$$P: M\times M\to P(M): (A,B)\to APB=\{AP^t_iB: i\in I\}\subseteq M$$

Then (M,S,Q_+,P) is a hyperalgebra over F called general matrix P-hyperalgebra.

The bilinear hope P, is strong associative and the inclusion distributivity with respect to addition of matrices

$$AP(B+C)\subseteq APB+APC,\ \forall A,B,C\in M$$

is valid. So (M,+,P) defines a multiplicative hyperring on non-square matrices.

In a similar way a generalization of this hyperalgebra can be defined considering an H_v-field instead of a field and using H_v-matrices instead of matrices.

In the representation theory several constructions are used, one can find some of them as follows [43],[44],[52], [54]:

Construction 2.5 Let $(H,\cdot)$ be H_v-group, then for all $(\oplus)$ such that $x\oplus y\supset\{x,y\}$, $\forall x,y\in H$, the $(H,\oplus,\cdot)$ is an H_v-ring. These H_v-rings are called associated to $(H,\cdot)$ H_v-rings.

In rep theory of hypergroups, in sense of Marty where the equality is valid, there are three associated hyperrings $(H,\oplus,\cdot)$ to $(H,\cdot)$. The $(\oplus)$ is defined respectively, $\forall x,y\in H$, by:

type a: $x\oplus y=\{x,y\}$, type b: $x\oplus y=\beta^*(x)\cup\beta^*(y)$, type c: $x\oplus y=H$

In the above types the strong associativity and strong or inclusion distributivity, is valid.

Construction 2.6 Let $(H,\cdot)$ be an H_V-semigroup and $\{v_1,\ldots,v_n\}\cap H=\varnothing$, an ordered set, where if $v_i<v_j$, when $i<j$. Extend $(\cdot)$ in $H_n=H\cup\{v_1,v_2,\ldots,v_n\}$ as follows:

$$x\cdot v_i = v_i\cdot x = v_i,\ v_i\cdot v_j = v_j\cdot v_i = v_j,\ \forall i<j \text{ and}$$

$$v_i\cdot v_i = H\cup\{v_1,\ldots,v_{i-1}\},\ \forall x\in H,\ i\in\{1,2,\ldots,n\}.$$

Then $(H_n,\cdot)$ is an H_V-group, called Attach Elements Construction, and $(H_n,\cdot)/\beta^*\cong Z_2$, where v_n is single [51],[55].

Some problems arising on the topic, are:

Open Problems.

a. Find standard H_v-fields to represent all H_v-groups.

b. Find reps by H_v-matrices over standard finite H_v-fields analogous to Z_n.

c. Using matrices find a generalization of the ordinary multiplication of matrices which it could be used in H_v-rep theory (see the helix-hope [68]).

d. Find the 'minimal' hypermatrices corresponding to the minimal hopes.

e. Find reps of special classes of hypergroups and reduce these to minimal dimensions.

Recall some definitions from [68],[16],[32]:

Definitions 2.7 Let $A=(a_{ij})\in M_{m\times n}$ be m×n matrix and $s,t\in N$ be natural numbers such that $1\le s\le m$, $1\le t\le n$. Then we define a characteristic-like map cst: $M_{m\times n}\to M_{s\times t}$ by corresponding to the matrix A, the matrix $Acst=(a_{ij})$ where $1\le i\le s$, $1\le j\le t$. We call

it cut-projection of type st. We define the mod-like map st: $M_{m\times n}\to M_{s\times t}$ by corresponding to A the matrix $Ast=(a_{ij})$ which has as entries the sets

$$a_{ij}=\{a_{i+\kappa s,j+\lambda t} \mid 1\le i\le s, 1\le j\le t \text{ and } \kappa,\lambda\in N,\ i+\kappa s\le m,\ j+\lambda t\le n\}.$$

Thus we have the map

$$st: M_{m\times n}\to M_{s\times t}: A\to Ast=(a_{ij}).$$

We call this multivalued map helix-projection of type st. So Ast is a set of s×t-matrices $X=(x_{ij})$ such that $x_{ij}\in a_{ij}, \forall i,j$.

Let $A=(a_{ij})\in M_{m\times n}$, $B=(b_{ij})\in M_{u\times v}$ matrices and s=min(m,u), t=min(n,u). We define a hope, called helix-addition or helix-sum, as follows:

$$\oplus: M_{m\times n}\times M_{u\times v}\to P(M_{s\times t}):$$

$$(A,B)\to A\oplus B=Ast+Bst=(a_{ij})+(b_{ij})\subset M_{s\times t},$$

where

$$(a_{ij})+(b_{ij})=\{(c_{ij})=(a_{ij}+b_{ij}) \mid a_{ij}\in a_{ij} \text{ and } b_{ij}\in b_{ij}\}.$$

And define a hope, called helix-multiplication or helix-product, as follows:

$$\otimes: M_{m\times n}\times M_{u\times v}\to P(M_{m\times v}): (A,B)\to A\otimes B=Ams\cdot Bsv=(a_{ij})\cdot(b_{ij})\subset M_{m\times v},$$

where

$$(a_{ij})\cdot(b_{ij})=\{(c_{ij})=(\sum a_{it}b_{tj}) \mid a_{ij}\in a_{ij} \text{ and } b_{ij}\in b_{ij}\}.$$

Remark. In $M_{m\times n}$ the addition of matrices is an ordinary operation, therefore we are interested only in the 'product'. From the fact that the helix-product on non square matrices is defined, the definition of the Lie-bracket is immediate, therefore the helix-Lie Algebra is defined [62], as well. This algebra is an H_v-Lie Algebra where the fundamental relation ε^* gives, by a quotient, a Lie algebra, from which a classification is obtained.

For more results on the topic see [16],[32],[61],[62].

In the following we denote E_{ij} any type of matrices which have the ij-entry 1 and in all the other entries we have 0.

Example 2.8 Consider the 2×3 matrices of the following form,

$$A_\kappa=E_{11}+\kappa E_{21}+E_{22}+E_{23},\ B_\kappa=\kappa E_{21}+E_{22}+E_{23},\ \forall\kappa\in\mathbb{N}.$$

Then we obtain $A_\kappa\otimes A_\lambda=\{A_{\kappa+\lambda},A_{\lambda+1},B_{\kappa+\lambda},B_{\lambda+1}\}$

Similarly, $B_\kappa\otimes A_\lambda=\{B_{\kappa+\lambda},B_{\lambda+1}\}$, $A_\kappa\otimes B_\lambda=B_\lambda=B_\kappa\otimes B_\lambda$.

Thus the set $\{A_\kappa,B_\lambda \mid \kappa,\lambda\in\mathbb{N}\}$ becomes an H_v-semigroup which is not COW because for $\kappa\neq\lambda$ we have

$$B_\kappa\otimes B_\lambda=B_\lambda\neq B_\kappa=B_\lambda\otimes B_\kappa,$$

however

$$(A_\kappa\otimes A_\lambda)\cap(A_\lambda\otimes A_\kappa)=\{A_{\kappa+\lambda},B_{\kappa+\lambda}\}\neq\varnothing,\ \forall\kappa,\lambda\in\mathbb{N}.$$

All elements B_λ are right absorbing and B_1 is a left scalar, because $B_1\otimes A_\lambda=B_{\lambda+1}$ and $B_1\otimes B_\lambda=B_\lambda$, A_0 is a unit.

3. Hyper-Lie-Algebras

Lie-Santilli admisibility

The general definition of an H_v-Lie algebra over an H_v-field is given as follows [61],[62]:

Definition 3.1 (L,+) be H_v-vector space on H_v-field $(F,+,\cdot)$, $\varphi:F\to F/\gamma^*$ the canonical map and $\omega_F=\{x\in F:\varphi(x)=0\}$, where 0 is the zero of the fundamental field F/γ^*. Moreover, let ω_L be the core of the canonical map φ': $L\to L/\varepsilon^*$ and denote by the same symbol 0 the zero of L/ε^*. Consider the bracket (commutator) hope:

$$[\ ,\ ]: L\times L\to P(L): (x,y)\to[x,y]$$

then L is called an H_v-Lie algebra over F if the following axioms are satisfied:

(L1) The bracket hope is bilinear, i.e.

$$[\lambda_1x_1+\lambda_2x_2,y]\cap(\lambda_1[x_1,y]+\lambda_2[x_2,y])\neq\varnothing$$

$$[x,\lambda_1y_1+\lambda_2y]\cap(\lambda_1[x,y_1]+\lambda_2[x,y_2])\neq\varnothing,$$

$$\forall x,x_1,x_2,y,y_1,y_2\in L \text{ and } \lambda_1,\lambda_2\in F$$

(L2) $[x,x]\cap\omega_L\neq\varnothing$, $\forall x\in L$

(L3) $([x,[y,z]]+[y,[z,x]]+[z,[x,y]])\cap\omega_L\neq\varnothing$, $\forall x,y\in L$

Example 3.2 Consider all traceless matrices $A=(a_{ij})\in M_{2\times3}$, in the sense that $a_{11}+a_{22}=0$. In this case, the cardinality of the helix-product of any two matrices is 1, or 2^3, or 2^6. These correspond to the cases: $a_{11}=a_{13}$ and $a_{21}=a_{23}$, or only $a_{11}=a_{13}$ either only $a_{21}=a_{23}$, or if there is no restriction, respectively. For the Lie-bracket of two traceless matrices the corresponding cardinalities are up to 1, or 2^6, or 2^{12}, resp. We remark that, from the definition of the helix-projection, the initial 2×2, block guaranties that in the result there exists at least one traceless matrix.

From this example it is obvious the following:

Theorem 3.3 Using the helix-product the Lie-bracket of any two traceless matrices $A=(a_{ij})$, $B=(b_{ij})\in M_{m\times n}$, m<n, contain at least one traceless matrix.

Last years, hyperstructures have a variety of applications in mathematics and other sciences. The hyperstructures theory can now be widely applicable in industry and production, too. In several books [4],[10],[12] and papers [1],[11],[17],[23], [31],[35],[50],[67],[70] one can find numerous applications.

The Lie-Santilli theory on isotopies was born in 1970's to solve Hadronic Mechanics problems. Santilli proposed [28] a 'lifting' of the trivial unit matrix of a normal theory into a nowhere singular, symmetric, real-valued, new matrix. The original theory is reconstructed such as to admit the new matrix as left and right unit. The isofields needed in this theory correspond into the hyperstructures were introduced by Santilli and Vougiouklis in 1996 and they are called e-hyperfields [29],[30],[59],[60],[64],[13],[14],[15] which are used in physics or biology. The H_v-fields can give

e-hyperfields which can be used in the isotopy theory for applications.

The IsoMathematics Theory is very important subject in applied mathematics. It is a generalization by using a kind of the Rees analogous product on matrix semigroup with a sandwich matrix, like the P-hopes. It contains the classical theory but also can find easy solutions in different branches of mathematics. To compare this novelty we give two analogous examples: (1) The unsolved, from ancient times, problems in Geometry was solved in a different branch of mathematics, the Algebra with the genius Galois Theory. (2) With the Representation Theory one can solve problems in Lie Algebras and to transfer these in Lie Groups using the exponential map, and the opposite. One very important thing of the IsoMathematics Theory is that admits generalizations, as well. Two very important of them are the following: First, is the so called Admissible Lie-Santilli Algebras [28],[30], [62],[65] by using again a kind of Rees sandwich product. Second, is that one can extend this theory into the multivalued case, i.e. into H_v-structures.

Definitions 3.4 A hyperstructure $(H,\cdot)$ containing a unique scalar unit e, is called e-hyperstructure. We assume that $\forall x$, there is an inverse x^{-1}, i.e. $e\in x\cdot x^{-1}\cap x^{-1}\cdot x$. A hyperstructure $(F,+,\cdot)$, where $(+)$ is an operation and $(\cdot)$ is a hope, is called e-hyperfield if the following are valid:

$(F,+)$ is abelian group with the additive unit 0, $(\cdot)$ is WASS,

$(\cdot)$ is weak distributive with respect to $(+)$, 0 is absorbing: $0\cdot x=x\cdot 0=0$, $\forall x\in F$, there exist a multiplicative scalar unit 1, i.e. $1\cdot x=x\cdot 1=x$, $\forall x\in F$, and $\forall x\in F$ there exists a unique inverse x^{-1}, such that $1\in x\cdot x^{-1}\cap x^{-1}\cdot x$.

The elements of an e-hyperfield are called e-hypernumbers. In the case that the relation: $1=x\cdot x^{-1}=x^{-1}\cdot x$, is valid, then we say that we have a strong e-hyperfield.

A general construction based on the partial ordering of the H_v-structures:

Construction 3.5 [13],[14],[15],[30] Main e-Construction. Given a group $(G,\cdot)$, where e is the unit, then we define in G, a large number of hopes $(\otimes)$ by extended $(\cdot)$, as follows:

$$x\otimes y=\{xy,g_1,g_2,\ldots\},\ \forall x,y\in G-\{e\},\text{ and } g_1, g_2,\ldots\in G-\{e\}$$

Then $(G,\otimes)$ becomes an H_v-group, in fact is H_b-group which contains the $(G,\cdot)$. The H_v-group $(G,\otimes)$ is an e-hypergroup. Moreover, if $\forall x,y$ such that $xy=e$, so we have $x\otimes y=xy$, then $(G,\otimes)$ becomes a strong e-hypergroup.

Definition 3.6 Let $(H_o,+,\cdot)$ be the attached, by one element, H_v-field of the H_v-semigroup $(H,\cdot)$. Thus, for $(H,\cdot)$, take an element v outside of H, and extend $(\cdot)$ in $H_n=H\cup\{v\}$ by:

$$x\cdot v=v\cdot x=v,\ v\cdot v=H,\ \forall x\in H.$$

$(H_n,\cdot)$ is an H_V-group, called Attach Elements Construction, and $(H_n,\cdot)/\beta^*\cong Z_2$, where v, is single. If $(H,\cdot)$ has a left and right scalar unit e then $(H_o,+,\cdot)$ is an e-hyperfield, the attached H_v-field of $(H,\cdot)$.

Remark. The above main e-construction gives an extremely large class of e-hopes. These e-hopes can be used in the several more complicate hyperstructures to obtain appropriate e-hyperstructures. However, the most useful are the ones where only few products are enlarged.

Example 3.7 Take the finite-non-commutative quaternion group $Q=\{1,-1, i,-i, j,-j, k,-k\}$. Using this operation one can obtain several hopes which define very interesting e-groups. For example, denoting $i=\{i,-i\}$, $j=\{j,-j\}$, $k=\{k,-k\}$ we may define the $(*)$ hope by the Cayley table:

*	1	-1	i	-i	j	-j	k	-k
1	1	-1	i	-i	j	-j	k	-k
-1	-1	1	-i	i	-j	j	k	k
i	i	-i	-1	1	k	-k	-j	j
-i	-i	i	1	-1	-k	k	j	-j
j	j	-j	-k	k	-1	1	i	-i
-j	-j	j	k	-k	1	-1	-i	i
k	k	k	j	-j	-i	i	-1	1
-k	-k	k	-j	j	i	-i	1	-1

The hyperstructure $(Q,*)$ is strong e-hypergroup because 1 is scalar unit and the elements $-1,i,-i,j,-j,k$ and $-k$ have unique inverses the elements $-1,-i,i,-j,j,-k$ and k, resp., which are the inverses in the basic group. Thus, from this example one can have more strict hopes.

In [30],[62],[65] a kind of P-hopes was introduced which is appropriate to extent the Lie-Santilli admissible algebras in hyperstructures:

The general definition is the following:

Construction 3.8 Let $(L=M_{m\times n},+)$ be an H_v-vector space of $m\times n$ hyper-matrices over the H_v-field $(F,+,\cdot)$, $\varphi:F\to F/\gamma^*$, the canonical map and $\omega_F=\{x\in F:\varphi(x)=0\}$, where 0 is the zero of the fundamental field F/γ^*, ω_L be the core of the canonical map $\varphi':L\to L/\varepsilon^*$ and denote again by 0 the zero of L/ε^*. Take any two subsets $R,S\subseteq L$ then a Santilli's Lie-admissible hyperalgebra is obtained by taking the Lie bracket, which is a hope:

$$[,]_{RS}: L\times L\to P(L):\ [x,y]_{RS}=xR^ty-yS^tx.$$

Notice that $[x,y]_{RS}=xR^ty-yS^tx=\{\ r^ty-ys^tx\mid r\in R \text{ and } s\in S\}$.

Special cases, but not degenerate, are the 'small' and 'strict':

(a) $R=\{e\}$ then $[x,y]_{RS}=xy-yS^tx=\{xy-ys^tx\mid s\in S\}$

(b) $S=\{e\}$ then $[x,y]_{RS}=xR^ty-yx=\{xr^ty-yx\mid r\in R\}$

(c) $R=\{r_1,r_2\}$ and $S=\{s_1,s_2\}$ then

$$[x,y]_{RS}=xR^ty-yS^tx=$$

$$\{xr_1{}^ty-ys_1{}^tx,\ xr_1{}^ty-ys_2{}^tx,\ xr_2{}^ty-ys_1{}^tx,\ xr_2{}^ty-ys_2{}^tx\}$$

4. Galois H_v-Fields and Low Dimensional H_v-Matrices

Recall some results from [63], which are referred to finite H_v-fields which we will call, according to the classical theory, Galois H_v-fields. Combining the uniting elements procedure

with the enlarging theory we can obtain stricter structures or hyperstructures. So enlarging operations or hopes we can obtain more complicated structures.

Theorem 4.1 In the ring $(Z_n,+,\cdot)$, with n=ms we enlarge the multiplication only in the product of elements $0\cdot m$ by setting $0\otimes m=\{0,m\}$ and the rest results remain the same. Then

$$(Z_n,+,\otimes)/\gamma^* \cong (Z_m,+,\cdot).$$

Proof. First we remark that the only expressions of sums and products which contain more, than one, elements are the expressions which have at least one time the hyperproduct $0\otimes m$. Adding to this special hyperproduct the element 1, several times we have the equivalence classes modm. On the other side, since m is a zero divisor, adding or multiplying elements of the same class the results are remaining in one class, the class obtained by using only the representatives. Therefore, γ^*-classes form a ring isomorphic to $(Z_m,+,\cdot)$.

Remark. In the above theorem we can enlarge other products as well, for example $2\cdot m$ by setting $2\otimes m=\{2,m+2\}$, then the result remains the same. In this case the elements 0 and 1 remain scalars, so they are refered in e-hyperstructures.

From the above theorem it is immediate the following:

Corollary 4.2 In the ring $(Z_n,+,\cdot)$, with n=ps where p is a prime number, we enlarge the multiplication only in the product of the elements $0\cdot p$ by setting $0\otimes p=\{0,p\}$ and the rest results remain the same. Then the hyperstructure $(Z_n,+,\otimes)$ is a very thin H_v-field.

The above theorem provides the researchers with H_v-fields appropriate to the rep theory since they may be smaller or minimal hyperstructures.

Remarks 4.3 The above theorem in connection with Uniting Elements method leads to the fact that in H_v-structure theory it is able to equip algebraic structures or hyperstructures with properties as associativity, commutativity, reproductivity. This equipment can be applied independently of the order of the desired properties. This is crucial point since some properties are easy to be applied, so we can apply them first, and then the difficult ones. For example from an H_v-ring we first go to an H_v-integral domain, by uniting the zero divisors, and then to the H_v-field by reaching the reproductivity.

Construction 4.5 (Galois H_v-fields) In the ring $(Z_n,+,\cdot)$, with n=ps where p is prime, enlarge only the product of the elements 2 by p+2, i.e. $2\cdot(p+)$, by setting $2\otimes(p+2)=\{2,p+2\}$ and the rest remain the same. Then $(Z_n,+,\otimes)$ is a COW very thin H_v-field where 0 and 1 are scalars and we have:

$$(\mathbf{Z}_n,+,\otimes)/\gamma^* \cong (\mathbf{Z}_p,+,\cdot).$$

Proof. Straightforward.

Remark 4.6 Galois Hv-fields of the above type are the most appropriate in the representation theory since the cardinality of the products is low. Moreover, one can use more enlargements using elements of the same fundamental class, therefore, one can have several cardinalities. The low dimensional reps can be based on the above Galois Hv-fields, since they use infinite Hv-fields although the fundamental fields are finite.

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Santilli Autotopisms of Partial Groups

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Abstract: This paper deals with those partial groups that contain a given Santilli isotopism in their autotopism group. A classification of these autotopisms is explicitly determined for partial groups of order $n \leq 4$.

Keywords: Partial Group, Isotopism, Classification

1. Introduction

In 1942, Albert [1] introduced the concept of *isotopy* of algebras: Two algebras $(A_1, \cdot)$ and $(A_2, *)$ over a field K are said to be *isotopic* if there exist three regular linear transformations f, g and h from A_1 to A_2 such that

$$f(u) * g(v) = h(u \cdot v), \text{ for all } u, v \in A_1. \quad (1)$$

The algebra A_2 is then said to be *isotopic* to the algebra A_1, or, equivalently, A_2 *is* an *isotope* of A_1. The triple $\Theta = (f, g, h)$ is an *isotopy* or *isotopism* between both algebras A_1 and A_2. If $f = g = h$, then this is indeed an isomorphism. If the elements of A_1 and A_2 coincide, then the isotopism Θ is said to be *principal* if h is the trivial transformation Id, that is, if $h(u) = Id(u) = u$, for all $u \in A_1$. In this case, the algebra A_2 is said to be a *principal isotope* of A_1. In his original paper, Albert proposed the question as to whether a principal isotope of a Lie algebra is Lie. In this regard, he proved that a principal isotope A_2 of a Lie algebra A_1 with respect to a principal isotopism (f, g, Id) is a Lie algebra if and only if, for all $u, v, w \in A_1$, it is verified that

$$f(u) \cdot g(v) = -f(v) \cdot g(u). \quad (2)$$

$$f(f(u) \cdot g(v)) \cdot g(w) - f(f(u) \cdot g(w)) \cdot g(v) - f(u) \cdot g(f(v) \cdot g(w)) = 0. \quad (3)$$

In 1944, Bruck [2] introduced the concept of *isotopically simple algebra* as a simple algebra such that all their isotopic algebras are simple. He proved in particular that the Lie algebra of order $n \cdot (n-1)/2$, consisting of all skew-symmetric matrices over any subfield of the field of all reals, under the Lie product $[u, v] = u \cdot v - v \cdot u$, is isotopically simple. Further, the Lie algebra of order $n \cdot (n-1)$ consisting of all skew-Hermitian matrices in any field $R(i)$ (where R is a subfield of the reals and $i^2 = -1$), under the multiplication $[u, v] = u \cdot v - v \cdot u$, is an isotopically simple algebra over R.

More recently, in 1978, Santilli [3] generalized the associative product $u \cdot v$ between Hermitian generators of the universal enveloping associative algebra by considering the new product

$$u * v = u \cdot T \cdot v \quad (4)$$

where T is a positive-definite operator called the *isotopic element*, which can indeed depend on distinct factors

$$T = T(x, x', x', \ldots, \mu, \tau) \quad (5)$$

The product

$$[u, v] = u * v - v * u \quad (6)$$

preserves the Lie axioms and is called the *Lie-isotopic product*. The application to Lie's theory (enveloping algebras, Lie algebras and Lie groups) that emerges from this new product is the so-called *Lie-Santilli isotheory* (see [3, pp. 287-290 and 329-374] and also [4-9]).

In the development of the isotheory, Santilli extended the unit of the ground field to the generalized unit or *isounit*

$$I = I(x, x', x', \ldots, \mu, \tau) = T^{-1} \quad (7)$$

He defined then the isonumbers

$$u = u * I(x, x', x', \ldots, \mu, \tau), \text{ for all } u \in A. \quad (8)$$

and the *isoproduct*

$$[u, v] = u * v - v * u \quad (9)$$

This isoproduct constitutes the Lie product of an isomorphic Lie algebra of A whenever the isounit $\hat{I}$ is constant. In any other case, this determines a generalization of the classical notion (2) of isotopism. In order to analyze this fact, the authors [10] reinterpreted in 2006 the dependence on distinct factors of the isounit $\hat{I}$ as a family of classical Bruck's isotopisms. This reinterpretation became clearer shortly after [11] once the attention was focused not on isotopisms of algebras, but on isotopisms of partial quasigroups.

The term *quasigroup* was introduced in 1937 by Haussmann and Ore [12] to denote a nonempty set Q endowed with a product $\cdot$, such that if any two of the three symbols u, v and w in the equation $u \cdot v = w$ are given as elements of Q, then the third is uniquely determined as an element of Q. Its *order* is the cardinality of the underlying set, that is, the number of elements of the quasigroup Q. This is said to be a *loop* if it contains a unit element, that is, there exists an element $e \in Q$ such that $e \cdot u = u \cdot e = u$ for all $u \in Q$. Every associative loop is indeed a group. The multiplication table of a quasigroup of order n is a *Latin square of order n*, that is, an $n \times n$ array with elements chosen from a set of n distinct symbols such that each symbol occurs precisely once in each row and each column (see Figure 1).

2	3	4	1
3	4	1	2
4	1	2	3
1	2	3	4

Figure 1. *Latin square of order 4.*

A *partial Latin square of order n* is an $n \times n$ array with elements chosen from a set of n distinct symbols such that each symbol occurs at most once in each row and each column (see Figure 2). It constitutes the multiplication table of a *finite partial quasigroup* $(Q, \cdot)$ of order n. Let $u, v \in Q$. The product $u \cdot v$ is then an element of Q or it is undefined. This last case is denoted as $u \cdot v = \emptyset$. By abuse of notation, it is also considered that $u \cdot \emptyset = \emptyset \cdot u = \emptyset$, for all $u \in Q$ and hence, the partial quasigroup is *associative* if $(u \cdot v) \cdot w = u \cdot (v \cdot w)$, for all $u, v, w \in Q$. It is a *partial loop* if there exists an element $e \in Q$ such that $e \cdot u = u \cdot e \in \{u, \emptyset\}$ for all $u \in Q$ and there does not exist an element $e' \neq e$ such that $e' \cdot u = u$ or $u \cdot e' = u$. Every associative partial loop constitutes a *partial group*.

1			
	2		4
3			
4		3	

Figure 2. *Partial Latin square of order 4.*

In 1943-44, Albert [13, 14] together with Bruck [15] extended the definition of isotopy from algebras to quasigroups. Particularly, two quasigroups $(Q_1, \cdot)$ and $(Q_2, *)$ of the same order are said to be *isotopic* if there exist three bijections f, g and h between their sets of elements such that

$$f(u) * g(v) = h(u \cdot v), \text{ for all } u, v \in Q_1. \quad (10)$$

The definition can be naturally extended to partial quasigroups once it is considered $h(\emptyset) = \emptyset$. The triple $\Theta = (f, g, h)$ is said to be an *isotopism* between Q_1 and Q_2 and it is denoted $Q_2 = Q_1^{\Theta}$. If $Q_2 = Q_1$, then the isotopism Θ is said to be an *autotopism* of Q_1 and f, g and h are permutations of the elements of Q_1. The set of autotopisms of a (partial) quasigroup constitutes, therefore, a group with the component-wise composition of permutations.

In 2007, the authors [11] introduced the concept of Santilli isotopism between partial quasigroups. Specifically, an isotopism $\Theta = (f, g, h)$ between two partial quasigroups $(Q_1, \cdot)$ and $(Q_2, *)$ is said to be a *Santilli isotopism* if there exist three elements i_f, i_g and i_h in Q_1 such that

$$f(u) = u \cdot i_f,\ g(u) = u \cdot i_g \text{ and } h(u) = u \cdot i_h, \text{ for all } u \in P_1. \quad (11)$$

The triple (i_f, i_g, i_h) is denoted by $S(\Theta, Q_1)$. If $Q_2 = Q_1$, then the isotopism Θ is said to be a *Santilli autotopism* of Q_1.

In [11], there were exposed several properties of the set of partial quasigroups having a Santilli autotopism that fixes at least one of the symbols. An exhaustive study of Santilli autotopisms is, however, currently required. This paper is established as a first contribution in this regard. In Section 2, some new general properties of the set of Santilli isotopisms of (associative) partial quasigroups, partial loops and partial groups are analyzed. In Section 3, a classification of the Santilli autotopisms of groups of order $n \leq 6$ is explicitly given. Remark that, even if the number of quasigroups is known for order up to 11 [16, 17], that of partial quasigroups is only known for order up to four [18, 19].

2. Santilli Autotopisms

From now on, every partial quasigroup of order n is considered to be formed by the set of elements $\{1, \ldots, n\}$. The set of isotopisms of partial quasigroups of order n is then denoted as $I_n = S_n \times S_n \times S_n$, where S_n is the symmetric group on $\{1, \ldots, n\}$. The set of fixed symbols in a permutation $\pi \in S_n$ is denoted as

$$Fix(\pi) = \{u \in \{1, \ldots n\} \text{ such that } \pi(u) = u\}. \quad (12)$$

Let $\Theta \in I_n$ and let $SQ(\Theta)$, $SL(\Theta)$, $SAQ(\Theta)$ and $SG(\Theta)$ be, respectively, the sets of partial quasigroups, partial loops, associative partial quasigroups and partial groups that have Θ as a Santilli autotopism. The next results are satisfied.

Lemma 2.1. Let $\Theta = (f, g, h) \in I_n$ and $(Q, \cdot) \in SQ(\Theta)$ be such that $S(\Theta, Q) = (i_f, i_g, i_h)$. Then, $i_h = g(i_f)$. As a consequence,

$$(i \cdot i_f) \cdot (j \cdot i_g) = (i \cdot j) \cdot (i_f \cdot i_g), \text{ for all } i, j \in Q. \quad (13)$$

Proof. Given $v \in Q$, let $u \in Q$ be such that $f(u) = v$. Then, $v \cdot i_h = h(v) = h(f(u)) = h(u \cdot i_f) = f(u) \cdot g(i_f) = v \cdot g(i_f)$ and the result holds from the fact that Q is a partial quasigroup and $h(v) \in Q$.

Proposition 2.2. Let $\Theta = (f, g, h) \in I_n$ and $(Q, \cdot) \in SQ(\Theta)$ be such that $S(\Theta, Q) = (i_f, i_g, i_h)$. If $h = f$, then $i_f \in Fix(g)$.

Proof. The result follows straightforward from Lemma 2.1 and the fact of being $h = f$.

Lemma 2.3. Let $\Theta = (f, g, h) \in I_n$. If there exist two permutations $\alpha, \beta \in \{f, g, h\}$ such that $\alpha(u_0) = \beta(u_0)$ for some $u_0 \in Q$, then $\alpha = \beta$.

Proof. Let $(Q, \cdot)$ be a partial quasigroup in $SQ(\Theta)$ and let $i_\alpha, i_\beta \in Q$ be such that $\alpha(u) = u \cdot i_\alpha$ and $\beta(u) = u \cdot i_\beta$ for all $u \in Q$. Particularly, $u_0 \cdot i_\alpha = \alpha(u_0) = \beta(u_0) = u_0 \cdot i_\beta$. This product is not undefined because $\alpha(u_0) \in Q$. Since Q is a partial quasigroup, it must be then $i_\alpha = i_\beta$ and hence, $\alpha = \beta$.

Proposition 2.4. Let $\Theta = (f, g, h) \in I_n$ be such that $Fix(g) = \emptyset$. Then, $f(u) \neq h(u)$ *for all* $u \in Q$.

Proof. Let $u \in Q$ be such that $f(u) = h(u)$. From Lemma 2.3 it must be $f = h$. Thus, from Lemma 2.1, it is $i_f = i_h = g(i_f)$ and hence, $i_f \in Fix(g)$, which is a contradiction.

Lemma 2.5. Let $\Theta = (f, g, h) \in I_n$ and $(Q, \cdot) \in SQ(\Theta)$ be such that $S(\Theta,Q) = (i_f, i_g, i_h)$. If there exists $u_0 \in Q$ such that $h^m(g(u_0)) = g(f^m(u_0))$ for some positive integer m, then $i_g \in Fix(g^m)$. As a consequence, if $Fix(g^m) = \emptyset$ for some positive integer m, then $h^m(g(u)) \neq g(f^m(u))$, for all $u \in Q$.

Proof. Let m be such that $h^m(g(u_0)) = g(f^m(u_0))$ for some $u_0 \in Q$. It is then $f^m(u_0) \cdot g^m(i_g) = h^m(u_0 \cdot i_g) = h^m(g(u_0)) = g(f^m(u_0)) = f^m(u_0) \cdot i_g$. This product is not undefined because $h^m(g(u_0)) \in Q$. Since Q is a partial quasigroup, it must be then $i_g \in Fix(g^m)$. The consequence is immediate.

Lemma 2.6. Let $\Theta = (f, g, h) \in I_n$ be such that $|Fix(f)| \cdot |Fix(g)| \cdot |Fix(h)| > 0$. Let $(Q, \cdot) \in SQ(\Theta)$ be such that $S(\Theta,Q) = (i_f, i_g, i_h)$. If there exist $u_0 \in Fix(f)$, $w_0 \in Fix(h)$ and $\alpha \in \{f, g, h\}$ such that $\alpha(u_0) = w_0$, then $i_\alpha \in Fix(g)$. Further, if $i_g \in Fix(g)$, then $g(u) \in Fix(h)$ for all $u \in Fix(f)$.

Proof. It is satisfied that $u_0 \cdot i_\alpha = \alpha(u_0) = w_0 = h(w_0) = h(u_0 \cdot i_\alpha) = f(u_0) \cdot g(i_\alpha) = u_0 \cdot g(i_\alpha)$. Since $w_0 \in Q$ and Q is a quasigroup, it must be $i_\alpha \in Fix(g)$. Let us suppose now that $i_g \in Fix(g)$ and let us consider an element $u \in Fix(f)$. Then $g(u) = u \cdot i_g = f(u) \cdot g(i_g) = h(u \cdot i_g) = h(g(u))$ and hence, $g(u) \in Fix(h)$.

The next three results deal with the set of partial loops $SL(\Theta)$ having a Santilli isotopism Θ in their autotopism group.

Proposition 2.7. Let $\Theta = (f, g, h) \in I_n$ and $(Q, \cdot) \in SL(\Theta)$ be a partial loop with unit element e. Then, $S(\Theta,Q) = (f(e), g(e), g(f(e)))$.

Proof. Let $S(\Theta,Q) = (i_f, i_g, i_h)$. The result follows straightforward from Lemma 2.1 and the fact that $\pi(e) \in Q$. Hence, $\pi(e) = e \cdot i_\pi = i_\pi$, for all $\pi \in \{f, g\}$.

Lemma 2.8. Let $\Theta = (f, g, h) \in I_n$. If there exists a permutation $\pi \in \{f, g, h\}$ such that $Fix(\pi) \neq \emptyset$, then $\pi = Id$.

Proof. Let $(Q, \cdot) \in SL(\Theta)$ and $S(\Theta,Q) = (i_f, i_g, i_h)$. Let $\pi \in \{f, g, h\}$ and $u_0 \in Q$ be such that $\pi(u_0) = u_0$. Since $u_0 = u_0 \cdot i_\pi$, the element i_π is the unit element of the partial loop. Let $u \in Q$. Since $\pi(u) \in Q$, it is $\pi(u) = u \cdot i_\pi = u$ and hence, $\pi = Id$.

Lemma 2.9. Let $\Theta = (f, g, h) \in I_n$ and $(Q, \cdot) \in SL(\Theta)$ be a partial loop with unit element e. If $e \in Fix(f^m)$ for some positive integer m, then $h^m = g^m$. Similarly, if $e \in Fix(g^m)$, then $h^m = f^m$.

Proof. Let us suppose that $e \in Fix(f^m)$ for some positive integer m. Let $u \in Q$. It is $g^m(u) = e \cdot g^m(u) = f^m(e) \cdot g^m(u) = h^m(e \cdot u)$. Since $g^m(u) \in Q$, it must be $e \cdot u = u$ and hence, $g^m(u) = h^m(u)$. The last assertion follows analogously.

We focus now on the set $SAQ(\Theta)$ of associative partial quasigroups having a Santilli autotopism in their autotopism group.

Proposition 2.10. Let $\Theta = (f, g, h) \in I_n$. If $SAQ(\Theta) \neq \emptyset$, then $h = g \circ f$.

Proof. Let $(Q, \cdot) \in SAQ(\Theta)$ and $S(\Theta,Q) = (i_f, i_g, i_h)$. From Lemma 2.1, we know that $i_h = g(i_f)$. Hence, for all $u \in Q$, it is verified that $h(u) = u \cdot i_h = u \cdot g(i_f) = u \cdot (i_f \cdot i_g) = (u \cdot i_f) \cdot i_g = g(f(u))$.

Lemma 2.11. Let $\Theta = (f, g, h) \in I_n$ be such that $SAQ(\Theta) \neq \emptyset$ and let $m \leq n$ be a positive integer. Then

a) $SAQ(\Theta) \subseteq SAQ(\Theta^m)$.

b) $SAQ(\Theta) = SAQ((f, g \circ f^m, h \circ f^m))$.

Proof. Let $(Q, \cdot) \in SAQ(\Theta)$ be such that $S(\Theta,Q) = (i_f, i_g, i_h)$ and let $m \leq n$ be a positive integer. Then

1. The isotopism Θ^m is an autotopism of $(Q, \cdot)$ because $h^m(u \cdot v) = h^{m-1}(f(u) \cdot g(v)) = \ldots = f^m(u) \cdot g^m(v)$, for all $u, v \in Q$. Since the quasigroup $(Q, \cdot)$ is associative, this is indeed a Santilli autotopism for which $S(\Theta^m,Q) = (i_f^m, i_g^m, i_h^m)$.
2. The isotopism $(f, g \circ f^m, h \circ f^m)$ is an autotopism of $(Q, \cdot)$ because $h(f^m(u \cdot v)) = h((u \cdot v) \cdot i_f^m) = h(u \cdot (v \cdot i_f^m)) = h(u \cdot f^m(v)) = f(u) \cdot g(f^m(v))$, for all $u, v \in Q$. Since the quasigroup $(Q, \cdot)$ is associative, this is indeed a Santilli autotopism for which $S((f, g \circ f^m, h \circ f^m),Q) = (i_f^m, i_f^m \cdot i_g, i_f^m \cdot i_h)$. Hence, $SAQ(\Theta) \subseteq SAQ((f, g \circ f^m, h \circ f^m))$.

Let us consider now an associative partial quasigroup $(Q', *) \in SAQ((f, g \circ f^m, h \circ f^m))$ such that $S((f, g \circ f^m, h \circ f^m),Q') = (i_1, i_2, i_3)$. It is then verified that Θ is a Santilli autotopism of $(Q', *)$ because, since $f^m = Id$, it is $h(u * v) = h(f^m(u * v)) = h(f^m(f^{n-m}(u * v)) = h(f^m(u * f^{n-m}(v)) = f(u) * g(f^m(f^{n-m}(v)) = f(u) * g(f^n(v)) = f(u) * g(v)$, for all $u, v \in Q'$. Further, $S(\Theta,Q') = (i_1, i_2 * i_1^{n-m}, i_3 * i_1^{n-m})$. Hence, $SAQ((f, g \circ f^m, h \circ f^m)) \subseteq SAQ(\Theta)$.

In general, given a positive integer $m \leq n$, it is not true that $SAQ(\Theta^m) \subseteq SAQ(\Theta)$. Thus, for instance, the isotopism $\Theta = ((1234), (1234), (13)(24))$ is a Santilli autotopism of the associative quasigroup whose multiplication table is the Latin square exposed in Figure 1. Nevertheless, even if the isotopism $\Theta^2 = ((13)(24), (13)(24), Id)$ is a Santilli autotopism of the associative partial quasigroup whose multiplication table is exposed in Figure 3, this is not contained in $SAQ(\Theta)$.

	3		1
	4		2
	1		3
	2		4

Figure 3. *Partial Latin square of order 4.*

Let us finish with a result about the set $SG(\Theta)$ of partial groups having a Santilli isotopism in their autotopism group.

Theorem 2.12. Let $\Theta = (f, g, h) \in I_n$. If $SG(\Theta) \neq \emptyset$ and $Fix(f) \neq \emptyset$, then $g = h$ and $f = Id$.

Proof. The result follows straightforward from Lemma 2.8 and Proposition 2.10.

3. Santilli Autotopisms of Partial Groups of Order $n \leq 4$

The results that have been exposed in Section 2 can be taken into account in order to determine explicitly the set of Santilli isotopisms that are autotopisms of partial groups of a given order. To this end, we say that two isotopisms $\Theta_1 = (f_1, g_1, h_1)$ and $\Theta_2 = (f_2, g_2, h_2)$ in I_n are *equivalent* if $f_2 = f_1$ and there exists a positive integer $m \leq n$ such that $g_2 = g_1 \circ f_1^m$ and $h_2 = h_1 \circ f_1^m$. From assertion (b) in Lemma 2.11, it is verified that $SAQ(\Theta_1) = SAQ(\Theta_2)$. To be equivalent is then an equivalence relation in the set I_n. Let $[\Theta]$ denote the equivalence class of $\Theta \in I_n$. We expose in Table 1 these equivalence classes for Santilli autotopisms of partial groups of order $n \leq 4$. We focus on the case of non-trivial isotopisms, that is, those that do not coincide with (Id, Id, Id).

Table 1. Santilli autotopisms of partial groups.

n	$[\Theta]$	$SG(\Theta)$
2	[((12), (12), Id)] [(Id, (12),(12))]	A_2
3	[(123), (123), (132)] [(132), (132), (123))] [(Id, (123), (123))] [(Id, (132), (132)]	A_3
4	[(1234), (1234), (13)(24))] [(1432), (1432), (13)(24))] [((13)(24), (1234), (1432)] [(Id, (1234), (1234)] [(Id, (1432), (1432)] [(1243),(1243), (14)(23))] [(1342), (1342), (14)(23))] [((14)(23), (1243), (1342)] [(Id, (1243), (1243)] [(Id, (1342), (1342)] [(1324), (1324), (12)(34))] [(1423), (1423), (12)(34))] [((12)(34), (1324), (1423)] [(Id, (1324), (1324)] [(Id, (1423), (1423)] [((12)(34), (13)(24), (14)(23)] [((12)(34), (14)(23), (13)(24)] [((13)(24), (12)(34), (14)(23)] [((13)(24), (14)(23), (12)(34)] [((14)(23), (12)(34), (13)(24)] [((14)(23), (13)(24), (12)(34)] [((12)(34), (12)(34), Id] [(Id, (12)(34), (12)(34)] [((13)(24), (13)(24), Id] [(Id, (13)(24), (13)(24)] [((14)(23), (14)(23), Id] [(Id, (14)(23), (14)(23)]	A_4 B_4 C_4 D_4 C_4, D_4, E_4 A_4, D_4, F_4 B_4, D_4, G_4

We indicate for each class $[\Theta]$ in Table 1 the set $SG(\Theta)$ of partial groups that have all the isotopisms of the class in their corresponding autotopism group. The multiplication tables of the elements of these sets are described in Figures 4–12.

1	2	2	1
2	1	1	2

Figure 4. Partial Latin squares related to A_2.

1	2	3	3	1	2	2	3	1
2	3	1	1	2	3	3	1	2
3	1	2	2	3	1	1	2	3

Figure 5. Partial Latin squares related to A_3.

1	2	3	4	4	1	2	3	3	4	1	2	2	3	4	1
2	3	4	1	1	2	3	4	4	1	2	3	3	4	1	2
3	4	1	2	2	3	4	1	1	2	3	4	4	1	2	3
4	1	2	3	3	4	1	2	2	3	4	1	1	2	3	4

Figure 6. Partial Latin squares related to A_4.

1	2	3	4	3	1	4	2	2	4	1	3	4	3	2	1
2	4	1	3	1	2	3	4	4	3	2	1	3	1	4	2
3	1	4	2	4	3	2	1	1	2	3	4	2	4	1	3
4	3	2	1	2	4	1	3	3	1	4	2	1	2	3	4

Figure 7. Partial Latin squares related to B_4.

1	2	3	4	2	1	4	3	4	3	1	2	3	4	2	1
2	1	4	3	1	2	3	4	3	4	2	1	4	3	1	2
3	4	2	1	4	3	1	2	1	2	3	4	2	1	4	3
4	3	1	2	3	4	2	1	2	1	4	3	1	2	3	4

Figure 8. Partial Latin squares related to C_4.

1	2	3	4	2	1	4	3	3	4	1	2	4	3	2	1
2	1	4	3	1	2	3	4	4	3	2	1	3	4	1	2
3	4	1	2	4	3	2	1	1	2	3	4	2	1	4	3
4	3	2	1	3	4	1	2	2	1	4	3	1	2	3	4

Figure 9. Partial Latin squares related to D_4.

1	2			2	1					1	2			2	1
2	1			1	2					2	1			1	2
3	4			4	3					3	4			4	3
4	3			3	4					4	3			3	4

Figure 10. Partial Latin squares related to E_4.

1		3			1		3	3		1			3		1
2		4			2		4	4		2			4		2
3		1			3		1	1		3			1		3
4		2			4		2	2		4			2		4

Figure 11. Partial Latin squares related to F_4.

1			4		1	4			4	1		4			1
2			3		2	3			3	2		3			2
3			2		3	2			2	3		2			3
4			1		4	1			1	4		1			4

Figure 12. Partial Latin squares related to G_4.

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Hyper-Representations by Non Square Matrices Helix-Hopes

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Abstract: Hyperstructure theory can overcome restrictions which ordinary algebraic structures have. A hyperproduct on non-square ordinary matrices can be defined by using the so called helix-hyperoperations. We define and study the helix-hyperstructures on the representations and we extend our study up to Lie-Santilli theory by using ordinary fields. Therefore the related theory can be faced by defining the hyperproduct on the extended set of non square matrices. The obtained hyperstructure is an H_v-algebra or an H_v-Lie-alebra.

Keywords: Hyperstructures, H_v-Structures, H/V-Structures, Hope, Helix-Hope

1. Introduction

We deal with the largest class of hyperstructures called H_v-structures introduced in 1990 [23],[26], which satisfy the weak axioms where the non-empty intersection replaces the equality.

Basic definitions:

Definitions 1.1 In a set H equipped with a hyperoperation, which we abbreviate it by hope $\cdot: H\times H\to P(H)$, we abbreviate by WASS the weak associativity: $(xy)z\cap x(yz)\neq\emptyset$, $\forall x,y,z\in H$ and by COW the weak commutativity: $xy\cap yx\neq\emptyset$, $\forall x,y\in H$.

The hyperstructure $(H,\cdot)$ is called H_v-semigroup if it is WASS and is called H_v-group if it is reproductive H_v-semigroup: $xH=Hx=H$, $\forall x\in H$. $(R,+,\cdot)$ is called H_v-ring if $(+)$ and $(\cdot)$ are WASS, the reproduction axiom is valid for $(+)$ and $(\cdot)$ is weak distributive with respect to $(+)$:

$$x(y+z)\cap(xy+xz)\neq\emptyset,\ (x+y)z\cap(xz+yz)\neq\emptyset,\ \forall x,y,z\in R.$$

For more definitions and applications on H_v-structures, see books [26],[2],[8] and the survey papers [6],[25],[30]. An extreme class is the following [26]: An H_v-structure is very thin iff all hopes are operations except one, with all hyperproducts singletons except only one, which is a subset of cardinality more than one. Therefore, in a very thin H_v-structure in a set H there exists a hope $(\cdot)$ and a pair $(a,b)\in H^2$ for which $ab=A$, with $\text{card}A>1$, and all the other products, with respect to any other hopes (so they are operations), are singletons.

The fundamental relations β^* and γ^* are defined, in H_v-groups and H_v-rings, respectively, as the smallest equivalences so that the quotient would be group and ring, respectively [22],[23],[26],[27],[28],[35]. The way to find the fundamental classes is given by analogous theorems to the following:

Theorem 1.2 Let $(H,\cdot)$ be an H_v-group and let us denote by U the set of all finite products of elements of H. We define the relation β in H as follows: $x\beta y$ iff $\{x,y\}\subset u$ where $u\in U$. Then the fundamental relation β^* is the transitive closure of the relation β.

The main point of the proof of this theorem is that β guaranties that the following is valid: Take two elements x,y such that $\{x,y\}\subset u\in U$ and any hyperproduct where one of these elements is used. Then, if this element is replaced by the other, the new hyperproduct is inside the same fundamental class where the first hyperproduct is. Therefore, if the 'hyperproducts'of the above β-classes are 'products', then, they are fundamental classes. Analogously for the γ in H_v-rings.

An element is single if its fundamental class is a singleton.

Motivation for H_v-structures:

We know that the quotient of a group with respect to an invariant subgroup is a group.

Marty states that, the quotient of a group with respect to any subgroup is a hypergroup.

Now, the quotient of a group with respect to any partition is an H_v-group.

Definition 1.3 Let $(H,\cdot)$, $(H,\otimes)$ be H_v-semigroups defined on

the same set H. $(\cdot)$ is smaller than $(\otimes)$, and $(\otimes)$ greater than $(\cdot)$, iff there exists automorphism

$$f\in Aut(H,\otimes) \text{ such that } xy\subset f(x\otimes y),\ \forall x\in H.$$

Then $(H,\otimes)$ contains $(H,\cdot)$ and write $\cdot\leq\otimes$. If $(H,\cdot)$ is structure, then it is basic and $(H,\otimes)$ is an H_b-structure.

The Little Theorem [26]. Greater hopes of the ones which are WASS or COW, are also WASS and COW, respectively.

The fundamental relations are used for general definitions of hyperstructures. Thus, to define the general H_v-field one uses the fundamental relation γ^*:

Definition 1.4 [23],[26],[27]. The H_v-ring $(R,+,\cdot)$ is called H_v-field if the quotient R/γ^* is a field.

Let ω^* be the kernel of the canonical map from R to R/γ^*; then we call reproductive H_v-field any H_v-field $(R,+,\cdot)$ if the following axiom is valid:

$$x(R-\omega^*) = (R-\omega^*)x = R-\omega^*,\ \forall x\in R-\omega^*.$$

From the above a new class is introduced [31],[38]:

Definition 1.5 The H_v-semigroup $(H,\cdot)$ is called h/v-group if the H/β^* is a group.

Similarly the h/v-rings, h/v-fields, h/v-modulus, h/v-vector spaces etc, are defined. The h/v-group is a generalization of the H_v-group since the reproductivity is not necessarily valid. Sometimes a kind of reproductivity of classes is valid, i.e. if H is partitioned into equivalence classes $\sigma(x)$, then the quotient is reproductive $x\sigma(y)=\sigma(xy)=\sigma(x)y$, $\forall x\in H$ [31].

An H_v-group is cyclic [17],[26], if there is element, called generator, which the powers have union the underline set, the minimal power with this property is the period of the generator. If there exists an element and a special power, the minimum one, is the underline set, then the H_v-group is called single-power cyclic.

To compare classes we can see on small sets. The problem of enumeration and classification of H_v-structures, or of classes of them, is complicate in H_v-structures because we have great numbers. The partial order in H_v-structures, introduced in [26], restrict the problem in finding the minimal H_v-structures, up to isomorphism. We have results recently by Bayon & Lygeros as the following [1],[13]:

In sets with three elements: Up to isomorphism, there are 6.494 minimal H_v-groups. The 137 are abelians; the 6.152 are cyclic. The number of H_v-groups with three elements, up to isomorphism, is 1.026.462. The 7.926 are abelians; 1.013.598 are cyclic. 16 are very thin. Abelian H_v-groups with 4 elements are, 8.028.299.905, the 7.995.884.377.

Definitions 1.6 [25],[26],[38] Let $(R,+,\cdot)$ be H_v-ring, $(M,+)$ be COW H_v-group and there exists an external hope:

$$R\times M\to P(M):\ (a,x)\to ax,$$

such that, $\forall a,b\in R$ and $\forall x,y\in M$ we have

$$a(x+y)\cap(ax+ay)\neq\emptyset,\ (a+b)x\cap(ax+bx)\neq\emptyset,\ (ab)x\cap a(bx)\neq\emptyset$$

then M is called an H_v-module over R. In case of an H_v-field F instead of H_v-ring R, then the H_v-vector space is defined.

The fundamental relation ε^* is defined to be the smallest equivalence such that the quotient M/ε^* is a module (resp., a vector space) over the fundamental ring R/γ^* (resp. the fundamental field F/γ^*). The analogous to Theorem 1.2, is:

Theorem Let $(M,+)$ be H_v-module on the H_v-ring R. Denote by U the set of all expressions consisting of finite hopes either on R and M or the external hope applied on finite sets of elements of R and M. Define relation ε in M as follows: $x\varepsilon y$ iff $\{x,y\}\subset u$ where $u\in U$.

Then the relation ε^* is the transitive closure of the relation ε.

Definitions 1.7 [28],[29],[38]. Let $(H,\cdot)$ be hypergroupoid. We remove $h\in H$, if we consider the restriction of $(\cdot)$ in the $H-\{h\}$. We say that $h\in H$ absorbs $h\in H$ if we replace h by h and h does not appear in the structure. We say that $h\in H$ merges with $h\in H$, if we take as product of any $x\in H$ by h, the union of the results of x with both h, h, and consider h and h as one class, with representative h, therefore the element h does not appeared in the hyperstructure.

Let $(H,\cdot)$ be an H_v-group, then, if an element h absorbs all elements of its own fundamental class then this element becomes a single in the new H_v-group.

Definition 1.8 [35],[37] Let $(L,+)$ be H_v-vector space over the field $(F,+,\cdot)$, $\varphi:F\to F/\gamma^*$, the canonical map and $\omega_F=\{x\in F:\varphi(x)=0\}$, where 0 is the zero of the fundamental field F/γ^*. Similarly, let ω_L be the core of the canonical map φ': $L\to L/\varepsilon^*$ and denote by the same symbol 0 the zero of L/ε^*. Consider the bracket (commutator) hope:

$$[\ ,\ ]: L\times L\to P(L):\ (x,y)\to[x,y]$$

then L is an H_v-Lie algebra over F if the following axioms are satisfied:

(L1) The bracket hope is bilinear, i.e.

$$[\lambda_1x_1+\lambda_2x_2,y]\cap(\lambda_1[x_1,y]+\lambda_2[x_2,y])\neq\emptyset$$

$$[x,\lambda_1y_1+\lambda_2y]\cap(\lambda_1[x,y_1]+\lambda_2[x,y_2])\neq\emptyset,$$

$$\forall x,x_1,x_2,y,y_1,y_2\in L \text{ and } \lambda_1,\lambda_2\in F$$

(L2) $[x,x]\cap\omega_L\neq\emptyset$, $\forall x\in L$

(L3) $([x,[y,z]]+[y,[z,x]]+[z,[x,y]])\cap\omega_L\neq\emptyset$, $\forall x,y\in L$

A well known and large class of hopes is given as follows [17],[21]:

Definitions 1.9 Let $(G,\cdot)$ be a groupoid, then for every $P\subset G$, $P\neq\emptyset$, we define the following hopes, P-hopes: $\forall x,y\in G$

$$P:\ xPy=(xP)y\cup x(Py),$$

$$P_r:\ xP_ry=(xy)P\cup x(yP),\ P_l:\ xP_ly=(Px)y\cup P(xy).$$

The (G,P), (G,P_r) and (G,P_l) are called P-hyperstructures. For semigroup $(G,\cdot)$, we have $xPy=(xP)y\cup x(Py)=xPy$ and (G,P) is a semihypergroup but we do not know about (G,P_r) and (G,P_l). In some cases, depending on the choice of P, the (G,P_r) and (G,P_l) can be associative or WASS.

A generalization of P-hopes is the following [9], [10]:

Let $(G,\cdot)$ be abelian group and P a subset of G with more

than one elements. We define the hope $\times_P$ as follows:

$$x\times_P y = x\cdot P\cdot y = \{x\cdot h\cdot y \,|\, h\in P\} \text{ if } x\neq e \text{ and } y\neq e$$

$$x\cdot y \text{ if } x=e \text{ or } y=e$$

we call this, P_e-hope. The $(G,\times_P)$ is an abelian H_v-group.

A general definition of hopes, is the following [32],[35], [36],[37]:

Definitions 1.10 Let H be a set with n operations (or hopes) $\otimes_1,\otimes_2,\ldots,\otimes_n$ and one map (or multivalued map) $f: H\to H$, then n hopes $\partial_1,\partial_2,\ldots,\partial_n$ on H are defined, called ∂-hopes, by putting

$$x\partial_i y = \{f(x)\otimes_i y,\ x\otimes_i f(y)\},\ \forall x,y\in H,\ i\in\{1,2,\ldots,n\}$$

or in case where $\otimes_i$ is hope or f is multivalued map we have

$$x\partial_i y = (f(x)\otimes_i y)\cup(x\otimes_i f(y)),\ \forall x,y\in H,\ i\in\{1,2,\ldots,n\}$$

Let $(G,\cdot)$ groupoid and $f_i: G\to G$, $i\in I$, set of maps on G. Take the map $f_U: G\to P(G)$ such that $f_U(x)=\{f_i(x) \,|\, i\in I\}$, call it the union of the $f_i(x)$. We call the union ∂-hope (∂), on G if we consider the map $f_U(x)$. An important case for a map f, is to take the union of this with the identity id. Thus, we consider the map $f\equiv f\cup(id)$, so $f(x)=\{x,f(x)\}, \forall x\in G$, which is called b-$\partial$-hope, we denote it by (∂), so we have

$$x\partial y = \{xy,\ f(x)\cdot y,\ x\cdot f(y)\},\ \forall x,y\in G.$$

Remark. If $\otimes_i$ is associative then ∂_i is WASS. If ∂ contains the operation $(\cdot)$, then it is b-operation. Moreover, if $f: G\to P(G)$ is multivalued then the b-∂-hopes is defined by using the $f(x)=\{x\}\cup f(x)$, $\forall x\in G$.

Motivation for the definition of ∂-hope is the derivative where only multiplication of functions is used. Therefore, for functions s(x), t(x), we have $s\partial t=\{s't,st'\}$, $(')$ is the derivative.

Example. Take all polynomials of first degree $g_i(x)=a_ix+b_i$. We have

$$g_1\partial g_2 = \{a_1a_2x+a_1b_2,\ a_1a_2x+b_1a_2\},$$

so it is a hope in the set of first degree polynomials. Moreover all polynomials x+c, where c be a constant, are units.

In hyperstructures there is the uniting elements method. This is defined as follows [3],[26],[28]: Let G be a structure and d be a property, which is not valid, and d is described by a set of equations. Consider the partition in G for which it is put together, in the same class, every pair of elements that causes the non-validity of d. The quotient G/d is an H_v-structure. The quotient of G/d by β^*, is a stricter structure $(G/d)\beta^*$ for which d is valid.

2. Matrix Representations

H_v-structures are used in Representation (abbr. by rep) Theory. Reps of H_v-groups can be considered either by generalized permutations or by H_v-matrices [18],[20],[24], [25],[26],[38]. The reps by generalized permutations can be achieved by using left or right translations. We present here the hypermatrix rep in H_v-structures and there exist the analogous theory for the h/v-structures.

Definitions 2.1 [20],[26] H_v-matrix is called a matrix with entries elements of an H_v-ring or H_v-field. The hyperproduct of two H_v-matrices $A=(a_{ij})$ and $B=(b_{ij})$, of type $m\times n$ and $n\times r$ respectively, is defined, in the usual manner,

$$A\cdot B = (a_{ij})\cdot(b_{ij}) = \{\ C=(c_{ij}) \,|\, c_{ij}\in\oplus\Sigma a_{ik}\cdot b_{kj}\ \},$$

and it is a set of $m\times r$ H_v-matrices. The sum of products of elements of the H_v-field is the union of the sets obtained with all possible parentheses put on them, called n-ary circle hope on the hyperaddition.

The hyperproduct of H_v-matrices does not necessarily satisfy WASS.

The problem of the H_v-matrix representations is the following:

Definitions 2.2 Let $(H,\cdot)$ be an H_v-group. Find an H_v-ring or an H_v-field $(F,+,\cdot)$, a set $M_R=\{(a_{ij}) \,|\, a_{ij}\in R\}$ and a map

$$T: H\to M_R: h\to T(h)$$

such that

$$T(h_1h_2)\cap T(h_1)T(h_2)\neq\emptyset,\ \forall h_1,h_2\in H.$$

T is an H_v-matrix rep. If the $T(h_1h_2)\subset T(h_1)T(h_2)$, $\forall h_1,h_2\in H$ is valid, then T is an inclusion rep. If $T(h_1h_2)=T(h_1)T(h_2)=\{T(h) \,|\, h\in h_1h_2\}$, $\forall h_1,h_2\in H$, then T is a good rep and then an induced rep T* for the hypergroup algebra is obtained. If T is one to one and good then it is a faithful rep.

The problem of reps is complicated because the cardinality of the product of H_v-matrices is very big. It can be simplified in special cases such as the following: The H_v-matrices are over H_v-fields with scalars 0 and 1. The H_v-matrices are over very thin H_v-fields. On 2×2 H_v-matrices, since the circle hope coincides with the hyperaddition. On H_v-fields which contain singles, then these act as absorbing.

The main theorem of reps is the following [20],[25],[26]:

Theorem 2.3 A necessary condition in order to have an inclusion rep T of an H_v-group $(H,\cdot)$ by $n\times n$ H_v-matrices over the H_v-rind or H_v-field $(F,+,\cdot)$ is the following:

For all classes $\beta^*(x)$, $x\in H$ there must exist elements $a_{ij}\in H$, $i,j\in\{1,\ldots,n\}$ such that

$$T(\beta^*(a))\subset\{\ A=(a'_{ij}) \,|\, a'_{ij}\in\gamma^*(a_{ij}),\ i,j\in\{1,\ldots,n\}\ \}$$

So every inclusion rep $T: H\to M_R: a\to T(a)=(a_{ij})$ induces a homomorphic rep T* of the group H/β^* over the field F/γ^* by putting $T^*(\beta^*(a))=[\gamma^*(a_{ij})]$, $\forall\beta^*(a)\in H/\beta^*$, where the $\gamma^*(a_{ij})\in R/\gamma^*$ is the ij entry of the matrix $T^*(\beta^*(a))$. T* is called fundamental induced rep of T.

Denote $tr_\varphi(T(x))=\gamma^*(T(x_{ii}))$ the fundamental trace, then the mapping

$$X_T: H\to R/\gamma^*: x\to X_T(x) = tr_\varphi(T(x)) = trT^*(x)$$

is called fundamental character. There are several types of traces.

Using several classes of H_v-structures one can face several reps [26],[29],[30],[38]:

Definition 2.4 Let $M=M_{m\times n}$ be a module of m×n matrices over a ring R and take sets

$$S=\{s_k:k\in K\}\subseteq R,\ Q=\{Q_i;j\in J\}\subseteq M,\ P=\{P_i:i\in I\}\subseteq M.$$

Define three hopes as follows

$$S: R\times M\to P(M): (r,A)\to rSA=\{(rs_k)A: k\in K\}\subseteq M$$

$$Q_+: M\times M\to P(M): (A,B)\to AQ_+B=\{A+Q_j+B: j\in J\}\subseteq M$$

$$P: M\times M\to P(M): (A,B)\to APB=\{AP^t_iB: i\in I\}\subseteq M$$

Then (M,S,Q_+,P) is a hyperalgebra over R called general matrix P-hyperalgebra.

The hope P, which is a bilinear map, is a generalization of Rees' operation where, instead of one sandwich matrix, a set of sandwich matrices is used. The hope P is strong associative and the inclusion distributivity with respect to addition of matrices

$$AP(B+C)\subseteq APB+APC\ \forall A,B,C\in M$$

is valid. Thus, (M,+,P) defines a multiplicative hyperring on non-square matrices.

In a similar way a generalization of this hyperalgebra can be defined considering an H_v-ring or an H_v-field instead of a ring and using H_v-matrices instead of matrices.

Definition 2.5 Let $A=(a_{ij}),B=(b_{ij})\in M_{m\times n}$, we call (A,B) unitize pair of matrices if $A^tB=I_n$, where I_n denotes the n×n unit matrix.

The following theorem can be applied in the classical theory [37],[38].

Theorem 2.6 If m<n, then there is no unitize pair.

Proof. Suppose that n>m and that

$$A^tB=(c_{ij}),\ c_{ij}=\sum_{k=1}^{m} a_{ik}b_{kj}\ .$$

Denote by A_m the block of the matrix A such that $A_m=(a_{ij})\in M_{m\times m}$, i.e. we take the matrix of the first m columns. Then we suppose that we have $(A_m)^tB_m=I_m$, therefore we must have $\det(A_m)\neq 0$. Now, since n>m, we can consider the homogeneous system with respect to the 'unknowns' $b_{1n},b_{2n},\ldots,b_{mn}$:

$$c_{in}=\sum_{k=1}^{m} a_{ik}b_{kn}=0 \text{ for } i=1,2,\ldots,m.$$

From which, we obtain that $b_{1n}=b_{2n}=\ldots=b_{mn}=0$, since $\det(A_m)\neq 0$. Using this fact on the last equation, on the same unknowns,

$$c_{nn}=\sum_{k=1}^{m} a_{nk}b_{kn}=1$$

we have 0=1, absurd. ■

We recall some definitions from [18],[20],[25].

Definition 2.7 Let (G,·) hypergroupoid, is called set of fundamental maps on G, a set of onto maps

$$Q=\{\ q: G\times G\to G: (x,y)\xrightarrow{onto} z\,|\,z\in xy\ \}.$$

Any subset $Q_s\subset Q$ defines a hope ($\circ_s$) on G as follows

$$x\circ_s y=\{\ z\,|\,z=q(x,y) \text{ for some } q\in Q\ \}$$

$\circ_s\leq\cdot$, and $Q_s\subset Q_{os}$, where Q_{os} is the set of fundamental maps with respect to ($\circ_s$). A $Q_a\subset Q$ for which every $Q_s\subset Q_a$ has ($\circ_s$) associative (resp. WASS) is called associative (resp. WASS). A hypergroupoid (G,·) is q-WASS if there exists an element $q_o\in Q$ which defines an associative operation ($\circ$) in G. Remark that for H_v-groups we have $Q\neq\emptyset$.

If G is finite, $\text{card}G=|G|=n$, it is q-WASS with associative $q_o\in Q$. In the set K[G] of all formal linear combinations of elements of G with coefficients from a field K, we define an operation (+):

$$(f_1+f_2)(g)=f_1(g)+f_2(g),\forall g\in G,f_1,f_2\in K[G]$$

and a hope (*), the convolution,

$$f_1*f_2=\{\ f_q: f_q(g)=\sum_{q(x,y)=g} f_1(x)f_2(y),\ q\in Q\ \}.$$

(K[G],+,*) is a multiplicative H_v-ring where the inclusion distributivity is valid, which is called hypergroupoid H_v-algebra.

For all $q\in Q$, $g\in G$, we have

$$|Q|\leq\prod_{(x,y)\in G\times G}(|xy|),\ 1\leq|q^{-1}(g)|\leq n^2-n+1$$

$$\text{and}\ \sum_{g\in G}|q^{-1}(g)|=n^2.$$

The zero map f(x)=0 is a scalar element in K[G].

In the representation theory several constructions are used, some of them are the following [26],[28],[29],[30]:

Constructions 2.8 Let (H,·) be H_v-group, then for all (⊕) such that $x\oplus y\supset\{x,y\}$, $\forall x,y\in H$, the (H,⊕,·) is an H_v-ring. These H_v-rings are called associated to (H,·) H_v-rings.

In rep theory of hypergroups, in sense of Marty where the equality is valid, there are three associated hyperrings (H,⊕,·) to (H,·). The (⊕) is defined respectively, $\forall x,y\in H$, by: type a $x\oplus y=\{x,y\}$, type b $x\oplus y=\beta^*(x)\cup\beta^*(y)$, type c $x\oplus y=H$.

In the above types the strong associativity and strong or inclusion distributivity, is valid.

Let (H,·) be H_V-semigroup and $\{v_1,\ldots,v_n\}\cap H=\emptyset$, an ordered set, where if $v_i<v_j$, when i<j. Extend (·) in $H_n=H\cup\{v_1,v_2,\ldots,v_n\}$ as follows:

$$x\cdot v_i=v_i\cdot x=v_i,\ v_i\cdot v_j=v_j\cdot v_i=v_j,\ \forall i<j \text{ and}$$

$$v_i\cdot v_i=H\cup\{v_1,\ldots,v_{i-1}\},\ \forall x\in H,\ i\in\{1,2,\ldots,n\}.$$

Then $(H_n,\cdot)$ is an H_V-group (Attach Elements Construction). We have $(H_n,\cdot)/\beta^*\cong Z_2$ and v_n is single.

Some open problems arising on the topic of rep theory of hypergroups, are:

Open Problems.

a. Find standard H_v-rings or H_v-fields to represent all H_v-groups by H_v-matrices.

b. Find reps by H_v-matrices over standard finite H_v-rings analogous to Z_n.

c. Using matrices find a generalization of the ordinary multiplication of matrices which it could be used in H_v-rep theory (see the helix-hope [40]).

d. Find the 'minimal' hypermatrices corresponding to the minimal hopes.

e. Find reps of special classes of hypergroups and reduce these to minimal dimensions.

3. Helix-Hopes and Applications

Recall some definitions from [40],[16],[11]:

Definition 3.1Let $A=(a_{ij})\in M_{m\times n}$ be an m×n matrix and s,t∈N be natural numbers such that $1\le s\le m$, $1\le t\le n$. Then we define a characteristic-like map cst: $M_{m\times n}\to M_{s\times t}$ by corresponding to the matrix A, the matrix $Acst=(a_{ij})$ where $1\le i\le s$, $1\le j\le t$. We call this map cut-projection of type st. In other words Acst is a matrix obtained from A by cutting the lines, with index greater than s, and columns, with index greater than t.

We can use cut-projections on several types of matrices to define sums and products, however, in this case we have ordinary operations, not multivalued.

In the same attitude we define hopes on any type of matrices:

Definition 3.2 Let $A=(a_{ij})\in M_{m\times n}$ be an m×n matrix and s,t∈N, such that $1\le s\le m$, $1\le t\le n$. We define the mod-like map st from $M_{m\times n}$ to $M_{s\times t}$ by corresponding to A the matrix $Ast=(a_{ij})$ which has as entries the sets

$$a_{ij}=\{a_{i+\kappa s,j+\lambda t}\mid 1\le i\le s,\ 1\le j\le t.\ \text{and}\ \kappa,\lambda\in N,\ i+\kappa s\le m,\ j+\lambda t\le n\}.$$

Thus we have the map

$$st: M_{m\times n}\to M_{s\times t}: A\to Ast=(a_{ij}).$$

We call this multivalued map helix-projection of type st. Thus Ast is a set of s×t-matrices $X=(x_{ij})$ such that $x_{ij}\in a_{ij},\forall i,j$. Obviously Amn=A. We may define helix-projections on 'matrices' of which their entries are sets.

Let $A=(a_{ij})\in M_{m\times n}$ be a matrix and s,t∈N such that $1\le s\le m$, $1\le t\le n$. Then it is clear that we can apply the helix- projection first on the columns and then on the rows, the result is the same if we apply the helix-progection on both, rows and columns. Therefore we have

$$(Asn)st=(Amt)st=Ast.$$

Let $A=(a_{ij})\in M_{m\times n}$ be matrix and s,t∈N such that $1\le s\le m$, $1\le t\le n$. Then if Ast is not a set of matrices but one single matrix then we call A cut-helix matrix of type s×t. Thus the matrix A is a helix matrix of type s×t, if Acst= Ast.

Definitions 3.3 (a) Let $A=(a_{ij})\in M_{m\times n}$ and $B=(b_{ij})\in M_{u\times v}$ be matrices and s=min(m,u), t=min(n,u). We define a hope, called helix-addition or helix-sum, as follows:

$$\oplus: M_{m\times n}\times M_{u\times v}\to P(M_{s\times t}):$$

$$(A,B)\to A\oplus B=Ast+Bst=(a_{ij})+(b_{ij})\subset M_{s\times t},$$

where

$$(a_{ij})+(b_{ij})=\{(c_{ij})=(a_{ij}+b_{ij})\mid a_{ij}\in a_{ij}\ \text{and}\ b_{ij}\in b_{ij}\}.$$

(b) Let $A=(a_{ij})\in M_{m\times n}$ and $B=(b_{ij})\in M_{u\times v}$ be matrices and s=min(n,u). We define a hope, called helix-multiplication or helix-product, as follows:

$$\otimes: M_{m\times n}\times M_{u\times v}\to P(M_{m\times v}):$$

$$(A,B)\to A\otimes B=Ams\cdot Bsv=(a_{ij})\cdot(b_{ij})\subset M_{m\times v},$$

where

$$(a_{ij})\cdot(b_{ij})=\{(c_{ij})=(\sum a_{it}b_{tj})\mid a_{ij}\in a_{ij}\ \text{and}\ b_{ij}\in b_{ij}\}.$$

The helix-addition is an external hope since it is defined on different sets and the result is also in different set. The commutativity is valid in the helix-addition. For the helix-multiplication we remark that we have $A\otimes B=Ams\cdot Bsv$ so we have either Ams=A or Bsv=B, that means that the helix-projection was applied only in one matrix and only in the rows or in the columns. If the appropriate matrices in the helix-sum and in the helix-product are cut-helix, then the result is singleton.

Remark. In $M_{m\times n}$ the addition of matrices is an ordinary operation, therefore we are interested only in the 'product'. From the fact that the helix-product on non square matrices is defined, the definition of the Lie-bracket is immediate, therefore the helix-Lie Algebra is defined [36],[37], as well. This algebra is an H_v-Lie Algebra where the fundamental relation ε^* gives, by a quotient, a Lie algebra, from which a classification is obtained.

In the following we restrict ourselves on the matrices $M_{m\times n}$ where m<n. We have analogous results in the case where m>n and for m=n we have the classical theory. In order to simplify the notation, since we have results on modm, we will use the following notation:

Notation. For given $\kappa\in N-\{0\}$, we denote by $\bar{\kappa}$ the remainder resulting from its division by m if the remainder is non zero, and $\bar{\kappa}=m$ if the remainder is zero. Thus a matrix

$A=(a_{\kappa\lambda})\in M_{m\times n}$, m<n is a cut-helix matrix if $a_{\kappa\lambda}=a_{\bar{\kappa}\lambda}$, $\forall\kappa,\lambda\in N-\{0\}$.

Moreover let us denote by $I_c=(c_{\kappa\lambda})$ the cut-helix unit matrix which the cut matrix is the unit matrix I_m. Therefore, since $I_m=(\delta_{\kappa\lambda})$, where $\delta_{\kappa\lambda}$ is the Kronecker's delta, we obtain that, $\forall\kappa,\lambda$, we have $c_{\kappa\lambda}=\delta_{\bar{\kappa}\lambda}$.

Proposition 3.4 For m<n in $(M_{m\times n},\otimes)$ the cut-helix unit matrix $I_c=(c_{\kappa\lambda})$, where $c_{\kappa\lambda}=\delta_{\bar{\kappa}\lambda}$, is a left scalar unit and a right unit. It is the only one left scalar unit.

Proof. Let $A,B\in M_{m\times n}$ then in the helix-multiplication, since m<n, we take helix projection of the matrix A, therefore, the result $A\otimes B$ is singleton if the matrix A is a cut-helix matrix of type m×m. Moreover, in order to have $A\otimes B=Amm\cdot B=B$, the matrix Amm must be the unit matrix. Consequently, $I_c=(c_{\kappa\lambda})$, where $c_{\kappa\lambda}=\delta_{\bar{\kappa}\lambda}$, $\forall\kappa,\lambda\in N-\{0\}$, is necessarily the left scalar unit

element.

Now we remark that it is not possible to have the same case for the right matrix B, therefore we have only to prove that cut-helix unit matrix I_c is a right unit but it is not a scalar, consequently it is not unique.

Let $A=(a_{uv})\in M_{m\times n}$ and consider the hyperproduct $A\otimes I_c$. In the entry $\kappa\lambda$ of this hyperproduct there are sets, for all $1\le\kappa\le m$, $1\le\lambda\le n$, of the form

$$\Sigma a_{\kappa s}c_{s\lambda} = \Sigma a_{\kappa s}\delta_{s\lambda} = a_{\kappa\lambda}\ni a_{\kappa\lambda}.$$

Therefore $A\otimes I_c\ni A$, $\forall A\in M_{m\times n}$. ■

In the following examples of the helix-hope, we denote E_{ij} any type of matrices which have the ij-entry 1 and in all the other entries we have 0.

Example 3.5 [38] Consider the 2×3 matrices of the following form,

$$A_\kappa=E_{11}+\kappa E_{21}+E_{22}+E_{23},\ B_\kappa=\kappa E_{21}+E_{22}+E_{23},\ \forall\kappa\in\mathbb{N}.$$

Then we obtain $A_\kappa\otimes A_\lambda=\{A_{\kappa+\lambda}, A_{\lambda+1}, B_{\kappa+\lambda}, B_{\lambda+1}\}$.

Similarly we have $B_\kappa\otimes A_\lambda=\{B_{\kappa+\lambda}, B_{\lambda+1}\}$, $A_\kappa\otimes B_\lambda=B_\lambda=B_\kappa\otimes B_\lambda$.

Thus $\{A_\kappa, B_\lambda \mid \kappa,\lambda\in\mathbb{N}\}$ becomes an H_v-semigroup, not COW because for $\kappa\neq\lambda$ we have $B_\kappa\otimes B_\lambda=B_\lambda\neq B_\kappa=B_\lambda\otimes B_\kappa$, however

$$(A_\kappa\otimes A_\lambda)\cap(A_\lambda\otimes A_\kappa) = \{A_{\kappa+\lambda}, B_{\kappa+\lambda}\}\neq\emptyset,\ \forall\kappa,\lambda\in\mathbb{N}.$$

All B_λ are right absorbing and B_1 is a left scalar, because $B_1\otimes A_\lambda=B_{\lambda+1}$ and $B_1\otimes B_\lambda=B_\lambda$. The A_0 is a unit.

Example 3.6 Consider the 2×3 matrices of the forms,

$$A_{\kappa\lambda}=E_{11}+E_{13}+\kappa E_{21}+E_{22}+\lambda E_{23},\ \forall\kappa,\lambda\in\mathbb{Z}.$$

Then we obtain $A_{\kappa\lambda}\otimes A_{st}=\{A_{\kappa+s,\kappa+t}, A_{\kappa+s,\lambda+t}, A_{\lambda+s,\kappa+t}, A_{\lambda+s,\lambda+t}\}$.

Moreover $A_{st}\otimes A_{\kappa\lambda}=\{A_{\kappa+s,\lambda+s}, A_{\kappa+s,\lambda+t}, A_{\kappa+t,\lambda+s}, A_{\kappa+t,\lambda+t}\}$, so $A_{\kappa\lambda}\otimes A_{st}\cap A_{st}\otimes A_{\kappa\lambda}=\{A_{\kappa+s,\lambda+t}\}$, thus $(\otimes)$ is COW.

The helix multiplication $(\otimes)$ is associative.

Example 3.7 Consider all traceless matrices $A=(a_{ij})\in M_{2\times3}$, in the sence that $a_{11}+a_{22}=0$. In this case, the cardinality of the helix-product of any two matrices is 1, or 2^3, or 2^6. These correspond to the cases: $a_{11}=a_{13}$ and $a_{21}=a_{23}$, or only $a_{11}=a_{13}$ either only $a_{21}=a_{23}$, or if there is no restriction, respectively. For the Lie-bracket of two traceless matrices the corresponding cardinalities are up to 1, or 2^6, or 2^{12}, respectively. We remark that, from the definition of the helix-projection, the initial 2×2, block guaranties that in the result there exists at least one traceless matrix.

From this example it is obvious the following:

Theorem 3.8 The Lie-bracket of any two traceless matrices $A=(a_{ij})$, $B=(b_{ij})\in M_{m\times n}$, $m<n$, contain at least one traceless matrix.

Last years hyperstructures there is a variety of applications in mathematics and in other sciences. Hyperstructures theory can now be widely applicable in industry and production, too. In several books and papers [2],[4],[5],[7],[8],[10],[12], [19],[26],[33],[39] one can find numerous applications.

The Lie-Santilli theory on isotopies was born in 1970's to solve Hadronic Mechanics problems. The original theory is reconstructed such as to admit the new matrix as left and right unit. Isofields needed in this theory correspond into the hyperstructures were introduced by Santilli and Vougiouklis in 1996 and they are called e-hyperfields [9],[14],[15],[33], [36]. The H_v-fields can give e-hyperfields which can be used in the isotopy theory for applications.

Definitions 3.9 A hyperstructure $(H,\cdot)$ which contain a unique scalar unit e, is called e-hyperstructure, where we assume that $\forall x$, there exists an inverse x^{-1}, so $e\in x\cdot x^{-1}\cap x^{-1}\cdot x$. A hyperstructure $(F,+,\cdot)$, where $(+)$ is an operation and $(\cdot)$ is a hope, is called e-hyperfield if the following are valid:

$(F,+)$ is abelian group with the additive unit 0, $(\cdot)$ is WASS,

$(\cdot)$ is weak distributive with respect to $(+)$, 0 is absorbing: $0\cdot x=x\cdot 0=0$, $\forall x\in F$, exist a scalar unit 1, i.e. $1\cdot x=x\cdot 1=x$, $\forall x\in F$, $\forall x\in F$ there exists unique inverse x^{-1}, s.t. $1\in x\cdot x^{-1}\cap x^{-1}\cdot x$.

The elements of an e-hyperfield are called e-hypernumbers. In the case that the relation: $1=x\cdot x^{-1}=x^{-1}\cdot x$, is valid, then we say that we have a strong e-hyperfield.

A general construction based on the partial ordering of the H_v-structures:

Construction 3.10 [6],[36], Main e-Construction. Given a group $(G,\cdot)$, where e is the unit, then we define in G, a large number of hopes $(\otimes)$ by extended $(\cdot)$, as follows:

$x\otimes y = \{xy, g_1, g_2,\ldots\}$, $\forall x,y\in G-\{e\}$, and $g_1, g_2,\ldots\in G-\{e\}$

Then $(G,\otimes)$ becomes an H_v-group, in fact is H_b-group which contains the $(G,\cdot)$. The H_v-group $(G,\otimes)$ is an e-hypergroup. Moreover, if $\forall x,y$ such that $xy=e$, so we have $x\otimes y=xy$, then $(G,\otimes)$ becomes a strong e-hypergroup.

An application combining hyperstructures and fuzzy theory, is to replace the scale of Likert in questionnaires by the bar of Vougiouklis & Vougiouklis [41]:

Definition 3.11 In every question substitute the Likert scale with 'the bar' whose poles are defined with '0' on the left end, and '1' on the right end:

0 ——————————————————— 1

The subjects/participants are asked instead of deciding and checking a specific grade on the scale, to cut the bar at any point they feel expresses their answer to the question.

The use of the bar of Vougiouklis & Vougiouklis instead of a scale of Likert has several advantages during both the filling-in and the research processing [41]. The suggested length of the bar, according to the Golden Ratio, is 6.2cm.

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Rudiments of IsoGravitation for Matter and its IsoDual for AntiMatter

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Abstract: In this paper, we hope to initiate due scientific process on some of the historical criticisms of Einstein gravitation expressed by Einstein himself as well as by others. These criticisms have remained widely ignored for one century and deal with issues such as: the apparent lack of actual, physical curvature of space due to the refraction of star-light within the Sun chromosphere; the absence of a source in the field equations due to the electromagnetic origin (rather than the charge) of gravitational masses; the lack of clear compatibility of general relativity with special relativity, interior gravitational problems, electrodynamics, quantum mechanics and grand unifications; the lack of preservation over time of numerical predictions inherent in the notion of covariance; and other basic issues. We show that a resolution of these historical doubts can be apparently achieved via the use of the novel *isomathematics* and related *iso-Minkowskian geometry* based on the embedding of gravitation in *generalized isounits,* with isodual images for antimatter. Thanks to half a century of prior research, we then show that the resulting new theory of gravitation, known as *isogravitation,* preserves indeed Einstein's historical field equations although formulated on the iso-Minkowskian geometry over isofields *whose primary feature is to have null isocurvature.* We then show that isogravitation allows: Einstein field equations to achieve a unified treatment of generally inhomogeneous and anisotropic, exterior and interior gravitational problems; the achievement of a clear compatibility with 20th century sciences; the achievement of time invariant numerical predictions thanks to the strict invariance (rather than covariance) of gravitation under the Lorentz-Santilli isosymmetry; the apparent achievement of a consistent representation of the gravitational field of antimatter thanks ti the isodual iso-Minkowskian geometry; the apparent achievement of a grand unification inclusive of electroweak and gravitational interactions for matter and antimatter without known causality or structural inconsistencies; and other advances. We then present, apparently for the first time, the isogravitational isoaxioms characterized by the infinite family of isotopies of special relativity axioms as uniquely characterized by the Lorentz-Santilli isosymmetry which are applicable to both exterior and interior isogravitational problems of matter with their isodual for antimatter. We finally show, also for the first time, the apparent compatibility of isogravitation with current knowledge on the equivalence principle, matter black holes and other gravitational data.

Keywords: Gravitation, Isogravitation, Antimatter

1. Introduction

The author has stated several times in his writings that the theory developed by Lorentz [1], Poincaré [2], Einstein [3], Minkowski [4] and others, known as *special relativity,* has a majestic axiomatic structure and an impeccable body of experimental verifications under the conditions clearly stated by Einstein, namely, for: A) point-particles and electromagnetic waves; B) propagating in vacuum; and C) when referred to an inertial reference frame.

Whenever any of Einstein's conditions A), B), C) are violated, special relativity is at best approximately valid, and often it is completely inapplicable (rather than violated), in the sense that it produces no quantitative description at all, as it is the case for the synthesis of the neutron from the hydrogen in the core of a star for which any use of Dirac's equation has no scientific meaning [5].

By contrast, the author has stated various times that Einstein general relativity [6] is a scientific religion at this writing because of historical insufficiencies, some of which identified by Einstein himself, such as lack of clear compatibility of general relativity with special relativity, interior gravitational problems, electrodynamics, quantum mechanics and grand

unifications, which insufficiencies have remained unaddressed by the "mainstream physics" for one full century, let alone resolved in peer reviewed journals [7] (see also the view by the late J. V. Kadeisvili [8] and papers quoted therein).

In this paper, the author reports half a century of research toward a resolution of the historical insufficiencies of general relativity via the use of a basically new mathematics and its ensuing new physical vistas in the origin of gravitation, besides its description, for the exterior and interior gravitational problems of matter and antimatter.

It should be noted that the literature accumulated in the field is very large. To avoid a prohibitive length, we only list the references of direct relevance to the problems addressed. A comprehensive presentation and list of references up to 2011 is available in the independent general review [41] with the suggestive title of *New Sciences for a New Era.*

2. Historical Insufficiency of General Relativity

2.1. First Historical Insufficiency of General Relativity: Ignoring the Refraction of Star-light Passing Through the Sun Chromosphere, with Consequential Lack of Evidence that Space is Actually, Physicallys Curved

As it is well known, the conjecture of an actual, physical, curvature of space was inferred from the 1.75 arc-second "bending" of star-light passing near the Sun. Half of this value, 0.87 arc-seconds, is known to be due to a purely Newtonian attraction of light.

To see it, we first recall that for Newton gravitation to be "universal" it must also attract light, and that the source of gravitation is the energy of a body since mass is a measure of our ignorance on inertia. Hence, the author always wrote Newton's equation in the identical form in terms of the energy rather than mass

$$F = g\frac{m_1 m_2}{r^2} = G\frac{E_1 E_2}{r^2}, \quad G = \frac{g}{c^4}. \tag{1}$$

The calculation of the 0.87 arc-seconds deviation caused by Newton gravitation of star light passing near the Sun surface is then a good exercise for graduate students in physics by computing the energy equivalence $E_1 = mc^2$ of the Sun, and using the energy $E_2 = h\nu$ for a given frequency of visible light.

The remaining 0.87 arc-seconds deviation have been known for a century, *not* to be due to the curvature of space, but to the refraction of sta-light when passing through the Sun chromosphere (see, e.g., Ref. [10] and references quoted therein). Additionally, the refraction of light passing through gaseous media is inherent in the experimental confirmations of Santilli IsoRedShift (IsoBlueShift) of light traveling through cold (hot) gases [11-15] (see Figures 1, 2, 3).

Irrespective of the above, the conjecture of curvature of space has been unable to represent without ambiguities truly basic gravitational events, such as the free fall of masses that has to be necessarily along a "straight" radial line, the weight of bodies in a gravitational field, and other basic events that are clearly represented by Newton gravitation.

Figure 1. *According to the first and perhaps most important unresolved historical criticism of Einstein gravitation, Sunset is a visual evidence of the lack of actual, physical, curvature of space because we still see the Sun at the horizon, while in reality it is already below the horizon due to the refraction of light passing through our atmosphere. Exactly the same refraction without curvature of space occurs for star-light passing through the Sun chromosphere, in which case the only "bending of light" is that due to Newton's gravitation in a flat space (see Section 2). Note that Einstein gravitation cannot represent light refraction because it requires a locally varying speed of light within a medium, first with increasing and then decreasing density. Hence, the representation of refraction via the curvature of space violates visual evidence, physical laws and experimental data [111-15]. To achieve a credible proof that the bending of Star-light passing near the Sun is "evidence" of the curvature of space, Einstein supporters have to prove that star-light passing through the Sun chromosphere does not experience refraction. The impossible existence of such a proof is readily seen from the fact that Einstein gravitation was solely aimed at a description of "exterior gravitational problems in vacuum," while the propagation of star-light within the Sun chromosphere is strictly an "interior gravitational problem" treated later on in Section 5. Its description via the Riemannian geometry is beyond any realistic possibilities due to the need for a metric possessing a dependence on coordinates x, as well as density μ, temperature τ, frequency ω, etc. $g = g(x, \mu, \tau, \omega, \ldots)$ (see Sections 5-11 below).*

Despite one century of studies, the "actual" orbits of planets in our Solar system have not been represented in an accurate, unique and time invariant way via Einstein gravitation, while they are exactly and unambiguously represented by Newton's gravitation and Kepler's laws. In fact, calculations based on the Riemannian geometry of the actual orbits of planets, besides not being unique due to the non-linearity of the theory, are generally different than physical orbits, and are not the same over time (see below).

It should also be indicated that a concrete visualization of the curvature space require an increase of the number of space dimension. In fact, the curvature in a *two*-dimensional Riemannian space can only be seen in *three* dimensions, as well known. Consequently, a concrete visualization of the curvature of space in *three* dimensions requires the implausible assumption of a *fourth* space dimension.

Needless to say, gravitational waves [6] crucially depend on the curvature of space represented via the Riemannian geometry. Until we dismiss in peer reviewed journals the mathematical, theoretical, experimental and visual evidence against the curvature of space, studies on gravitational waves

may well remain in suspended animation.

It goes without saying that a critical inspection of the conjecture of curvature of space creates great emotions in colleagues who have spent their research life on curved spaces. Yet, serious appraisals should be voiced only after identifying the huge limitations caused by curvature and only after inspecting the vast advances permitted by novel theories of gravitation on a flat space treated with the appropriate novel mathematics (Section 5).

2.2. Second Historical Insufficiency of General Relativity: Ignoring the Electromagnetic Origin of the Mass, with Consequential Invalidation of Einstein's Reduction of Gravitation to Pure Curvature Without Sources

As it is well known, the contribution to gravity of the total electric and magnetic field of a body is of the order of 10^{-30} or smaller. Consequently, following the assumption of the curvature of space, Einstein was forced to avoid any source in the r.h.s. of his field equations and reduce gravitation to pure geometry according to the the celebrated equations

$$G_{ij} = R_{ij} - g_{ij}R/2 = 0, i, j = 1,2,3,4 \qquad (2)$$

In 1974, the author identified the electromagnetic origin of the mass via the full use of quantum electrodynamics, including advanced and retarded treatments, and showed that such an origin requires the necessary presence in the r.h.s. of the field equations of a source first order in magnitude, irrespective of whether the body is charged or neutral [16],

$$G_{ij} = R_{ij} - g_{ij}R/2 = kT_{ij,elm}, \qquad (3)$$

where k is a unit-dependent constant, and the terms "first order in magnitude" are referred to the condition of *entirely representing the gravitational mass* of the body considered [16]

$$m_{grav} = \int T_{00}dv. \qquad (4)$$

The most skeptical physicist should admit that the mass of the electron is of entire electromagnetic origin. Therefore, field equations (2) are insufficient to represent the gravitational field of the electron in favor of Eqs. (3)-(4).

But then, the same skeptical physicist should admit that exactly the same conclusion holds for the positronium, namely, the gravitational mass of the positronium is of entire electromagnetic origin despite the total charge and magnetic moment being null. Therefore, Einstein's field equations (2) are insufficient for the representation of the gravitational field of the positronium in favor of broader Eqs. (3)-(4).

Paper [16] essentially extended the above known reality to the π^0-meson under the assumption of being a bound state of a charged constituent and its anti-particle. Paper [16] then extended the results to all masses with null total charge and null total magnetic moments. The inclusion of gravitational contributions from total electromagnetic characteristics was trivial.

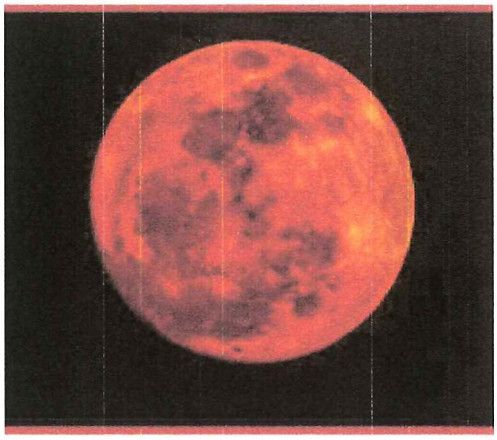

Figure 2. The "blood red moon" (top view) during a Lunar eclipse is an additional visual evidence of the lack of curvature of space because Sunlight reaches the Moon even when it should be in total darkness (bottom view). Note that for both Sunsets and Lunar eclipses the entire spectrum of Sunlight is redshifted without relative motion, merely due to loss of energy by light to a cold medium (IsoRedShift). Note also that we are dealing with "direct Sunlight" traveling in empty space for which scattering and other interpretations have been dismissed in peer refereed journals [11-15]. Note finally that the "blood red moon" confirms the view by Einstein, Hubble, Fermi, Zwicky, Hoyle, de Broglie and others on the lack of expansion of the universe because, when our Sun is seen millions of light years away, we merely have the replacement of Earth's atmosphere with very cold intergalactic gases under which the entire spectrum of visible Sunlight will appear redshifted without any relative motion [11-15].

In defense of Einstein, we have to recall that, contrary to his followers, Einstein always expressed serious doubts of field equations (2), for instance, by calling their r.h.s. *A house made of wood,* compared to the l.h.s. which he called *A house made of marble.* It is unfortunate for scientific knowledge that Einstein's own doubts have remained vastly ignored in the "mainstream literature" in gravitation.

We should also recall that, according to Ref. [16], the characterization of the inertial mass of a body requires the additional inclusion of all possible short range (e.g., weak and string) interactions, resulting in the need for an additional source in the r.h.s. of the equations whenever considering interior gravitational problems

$$G_{ij} = R_{ij} - g_{ij}R/2 = k_1T_{ij,elm} + k_2T_{ij,shortrange}, \qquad (5)$$

such that (c = 1)

$$m_{inert} = \int (T_{00,elm} + T_{00,shortrange})dv \qquad (6)$$

Consequently, the inertial mass is predicted as bigger than

the gravitational mass [16] (c = 1)

$$m_{inert} > m_{grav} \tag{7}$$

The expectation is that serious scientists will admit our lack of final experimental resolution on the relationship between the exterior gravitational and the interior inertial mass.

Besides the incontrovertible need for a source of first order in magnitude, the structure of Eqs. (5)-(6) is mandated by the fifth identity of the Riemannian geometry, the forgotten *Freud identity* [17] (see also the recent treatment by the late mathematician H. Rund [18]) which establishes the need on purely mathematical grounds of a source of first order in magnitude in the r.h.s of the field equations according precisely to Eqs. (5)-(6).

In fact, the source term of the Freud identity can be decomposed into a term with null trace, (evidently, the electromagnetic term), and a term with non-null trace (evidently, the source for short range interactions), thus providing a geometric confirmation of Eqs. (5)-(6).

We should indicate that the problem of a source in the gravitational field equations has been debated at length in the literature (see, e.g., Ref. [6]), although for its interpretation as a stress-energy tensor, or for other interpretations, while generally ignoring its electromagnetic origin.

Interested scholars should be aware of various claims in the literature that Einstein's gravitation verifies the Freud identity. These claims are based on the admission indeed of a source of electromagnetic nature, but restricted to the the total electromagnetic characteristics, thus violating condition (4) by a missing factor of 10^{30} or so.

Additionally, and perhaps more importantly, the Freud identity requires a source of first order in magnitude also for bodies with null total electromagnetic characteristics, thus confirming the lack of compliance of Einstein gravitation with the Freud identity.

Remember that gravitational waves are crucially dependent on Einstein's reduction of gravitation to pure geometry, Eqs, (2) [6]. However, physical and geometric needs mandate their extension to Eqs. (3), (4), for which gravitational waves cannot even be formulated, to our best knowledge at this writing.

Therefore, by noting the lack of independent addressing of the issues for the last four decades since the appearnce of paper [16], the theoretical prediction of gravitational waves will remain in suspended animation until the additional problem of theelectromagnetic origin of the gravitational mass is dismissed in refereed publications.

Again, the author has experienced over decades huge emotional reactions by colleagues at the instant of examining Einstein's reduction of gravitation to pure geometry, Eq. (2), without any in depth inspection of the advances permitted by a source term as in Eqs. (3)-(4). In a nutshell, the alternative between Eqs. (2) and (3), (4) bolls down to the belief of the existence of local infinities in the universe or not. Eqs. (2) do admit these local infinities, while covering Eqs. (3), (4) recover all main results of Eqs. (2) except replacing local infinities with large, yet finite values (Section 5 and Subsection 5.10 in particular).

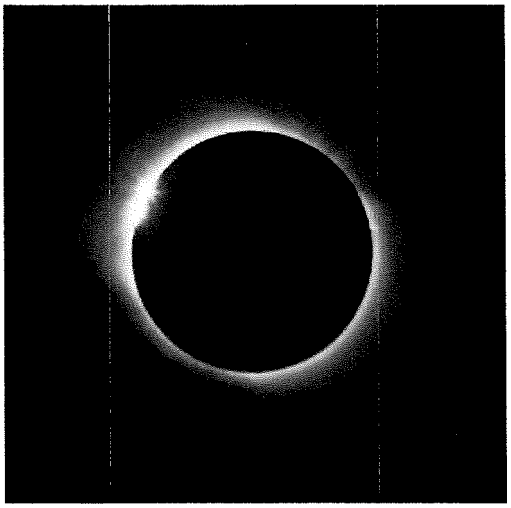

Figure 3. A view of a Solar eclipse showing no "bending of light" because the Newtonian attraction of light by the moon is extremely small and there is no refraction due to the lack of lunar atmosphere. The faint luminescence at sea level is due to the diffraction of light in our atmosphere. In conclusion, final claims of "bending of light due to curvature of space" must be based on star light passing tangentially on a body without atmosphere or chromosphere and be proved to be greater than the Newtonian attraction.

As a final note, the reader may have noted the lack of use of the *mathematical* terms "tensors" or "oseudotensors" and the use instead of the *physical* term "source." This is due to the fact that the clear physical content of the forgotten Freud identity is often dismissed on ground of purely mathematical differences in nomenclatures and personal mathematical interpretations without serious physical implications.

2.3. Third Historical Insufficiency of General Relativity: Abandoning the Majestic Lorentz and Poincaré "Invariance" of Special Relativity in Favor of the "Covariance" of General Relativity with Consequential Lack of Prediction of the Same Numerical Values under the Same Conditions at Different times

In our view, the above is perhaps the biggest insufficiency of Einstein gravitation because it implies the inability of gravitation to have *time invariance,* here referred to the prediction of the same numerical values under the same conditions at different times, while such a crucial requirement is verified by Galileo relativity and Einstein special relativity because of their Galilei and Poincare' symmetries, respectively.

In turn, the lack of time invariance establishes the lack of final character of all claims of "experimental verification of general relativity" [9] due to the absence of a physically consistent dynamical evolution.

In fact, "experimental verifications" of general relativity are done in ad hoc selected coordinate systems generally with no connection to the frame of the experimenter, thus prohibiting final experimental values, not only because said systems are different among themselves, but also because the needed experimental frame is generally not necessarily achievable via covariance.

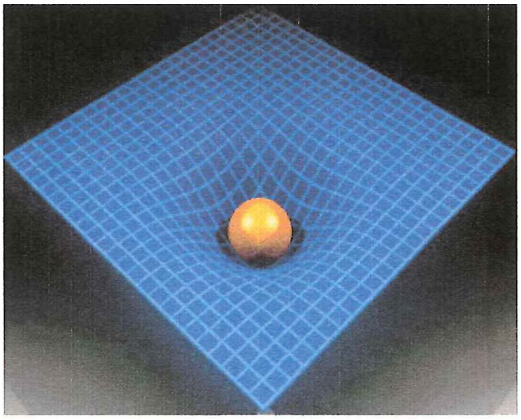

Figure 4. A typical representation of the claimed curvature of space caused by the gravitational field of a mass, which representation has been adopted for one full century. The historical, yet unresolved criticism is that the notion of physical curvature in one dimension requires a bigger dimension for its identification. In fact, the physical interpretation of the mathematical Riemannian curvature in two dimension can only be identified in three dimension as clearly illustrated by the above figure. Therefore, the additional historical criticism of Einstein gravitation that needs to be addressed is that the physical identification of the mathematical Riemannian curvature in three dimensions, as needed for realistic models of gravitations, requires four space dimensions that do not exist, thus confirming the lack of physical evidence for the actual physical curvature of space depicted in Figure 1, 2, 3. In any case, Einstein supporters are requested to illustrate with concrete geometric example the physical curvature needed for realistic models, not in two dimensions as done for one century, but in three dimensions.

Under the lack of invariance, general relativity could at best offer a kind of "polaroid picture" of gravitation [7,8]. However, such a static view of gravitation is dismissed by mathematical, physical, visual and experimental evidence on the lack of existence of the actual curvature of space.

Additional rather serious objections against published claims of "experimental verifications of Einstein gravitation" [9] stem from the fact that numerical predictions are, by far, not unique and/or unambiguous due to the non-linearity of the field equations. In fact, for any claim of "experimental verification" [9] we can assume a different PPN approximation with different expansions and show dramatic divergences with physical realities [7,8].

The lack of time invariance of Einstein's gravitation identifies an additional impossibility for gravitational waves to exist because any serious experimental verification should not only detect gravitational waves, which has been impossible for half a century despite the use of large public funds, but said gravitational waves should change in time without any change of the source, which is a blatant physical impossibility.

In defense of Einstein we should indicate that, once the Riemannian geometry is assumed for the representation of gravitation, no symmetry of the line element is possible for technical reasons similar to those of the *historical Lorentz problem.* We are here referring to Lorentz inability to achieve the invariance of the locally varying speeds of light of his time, that within physical media $C = c/n$, due to insurmountable technical difficulties in attempting to use Lie's theory for non-linear systems.

This is yet another case in which the author has experienced pre-judgments by colleagues mainly due to decades of research with covariance in gravitation without a serious inspection of qualified alternative views. In reality, serious judgments can only be expressed after a technical knowledge of the huge possibilities for further advances in gravitation permitted by alternative invariant theories (Section 5).

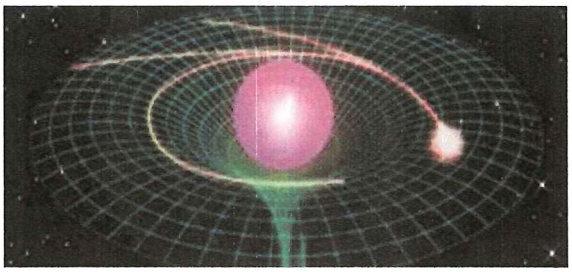

Figure 5. \ Another illustration of the insufficiencies of the one century old assumption that planets moving around the Sun in our Solar system actually move along a real, physical curvature of space. The historical criticism is that the above representation is purely mathematical because, to actually sense curvature in a three-dimensional space, the planet should move in a fort space dimension that does not exist.

2.4. Consequences of the Historical Insufficiencies of General Relativity: Incompatibility of Gravitation with Special Relativity, Interior Gravitational Problems, Electrodynamics, Quantum Mechanics, and Grand Unifications

There comes a point in the life of a scientist in which realities have to be admitted. The Riemannian geometry does indeed admit a unique and unambiguous reduction to the Minkowskian geometry via tangent, limit and other procedures.

However, it has been known for a century that general relativity does not admit a clear and unambiguous limit to special relativity of the type according to which special relativity uniquely and unambiguously admits a limit into the Galilei relativity. As one among many impossibilities, there exists no consistent limit of the covariance of general relativity into the fundamental Poincaré invariance of special relativity. The inconpatibilities that follow are then endless.

Another serious insufficiency is that the description by general relativity of "exterior gravitational problems" in vacuum is incompatible with "interior gravitational problems" that dominated the scientific scene in gravitation until the advent of Einstein's theory (e.g., Schwartzchild wrote *two* papers, one on the exterior and one on the interior gravitational problem [6], the second one being vastly ignored).

This is a serious incompatibility because its resolution prohibits the use of the Riemannian geometry due to the need of a geometry not only without curvature, but also (as indicated in Fig.1) with a metric having a dependence on coordinates x, as well as density μ, temperature τ, frequency ω, etc. $g = g(x, \mu, \tau, \omega, \ldots)$ (see Section 5 for details).

Another aspect that should be admitted to prevent exiting from physical reality is the irreconcilable incompatibility between Einstein gravitation and electrodynamics to such an extent that [16]:

2A) Either one assumes Einstein's gravitation as being valid, in which case electrodynamics must be revised from its foundations so as to eliminate the electromagnetic origin of the mass, or

2B) One assumes electrodynamics and its inherent origination of the gravitational mass as being valid, in which case, Einstein gravitation must be revised from its foundations.

Yet another reality that has to be faced following one century of wide oblivion, is that Einstein's gravitation is incompatible with quantum mechanics as conventionally understood, (that is, a unitary theory on a Hilbert soace) for several reasons. The reason most important in our view is that a gravitational theory formulated on a Riemannian space is necessarily *non-canonical* at the classical level (variationally non-self-adjoint [20]).

Therefore, any consistent "quantization" of Einstein gravitation must be *non-unitary,* with the consequential activation of the *Theorems of Catastrophic Inconsistencies of Non-Canonical and Non-Unitary Theories* [19] and ensuing loss of physical value, e.g., due to the violation of causality laws.

The moment of truth also implies the admission that Einstein gravitation is incompatible with grand unified theories, if nothing else, because of failed attempts o [6]ver one full century, beginning with the failed attempt of unifying gravitation and electromagnetism by Einstein himself.

2.5. Problems to be Solved for an Axiomatically Consistent Grand Unification

Following studies on grand unifications for decades, the incompatibilities of a grand unification of Einstein gravitation with electroweak interactions are the following (see, later on monograph [40]):

2.I. The physical consistency of electroweak interactions on a flat Minkowski space cannot be salvaged when joined to a theory on the curved Riemannian space because the insufficiencies of the latter carry over to the former;

2.II. Within a grand unification, the covariance of Einstein's gravitation carries over to electroweak interactions, by therefore destroying their gauge invariance and,consequently, the very structure of electroweak interactions;

2.III. Electroweak interactions represent both particles and antiparticles, while Einstein gravitation solely represent matter, thus rendering any grand unification technically impossible and catastrophically inconsistent if attempted.

We should mention a recent trend of extending the applicability of special and general relativities to the *classical* representation of antimatter. Serious scholars should be alerted that this trend is afflicted by serious inconsistencies, such as the impossibility of admitting the annihilation of matter and antimatter precisely due to the lack of a conjugation in the transition from matter to antimatter, violation of the PCT theorem and other inconsistencies.

Another reality that should be faced by serious scholars in the field is that a consistent representation of the gravitational field of antimatter cannot be achieved by Einstein gravitation and a new theory must be constructed from its mathematical foundations.

3. Rudiments of IsoMathematics

The most important lesson the author has learned in fifty years of research is that *the protracted lack of resolution of physical problems is generally due to the use of insufficient mathematics, rather than to physical issues.*

We believe that this is precisely the case for gravitation, namely, all problems treated above are caused by the use of an excessively insufficient mathematics, that based on the differential calculus that dates back to the Newton-Leibniz times. Only after the achievement of a more adequate mathematics, open physical problems can be quantitatively and effectively addressed.

To see the case, note that for a theory of gravitation to resist the test of time, it is expected to possess an invariance similar to that of the Poincaré symmetry in special relativity so as to predict the same numerical values under the same conditions at different times.

The best known way to achieve an invariant theory of gravitation is via the use of Lie's theory. But the latter theory solely applies to linear systems. The necessary non-linearity of gravitation then precludes any realistic possibility of achieving an invariance via the use of 20th century mathematics.

The above occurrence forced the author to construct the *isotopies* (intended as axiom-preserving) of 20th century applied mathematics [20], today known as *isomathematics,* that was initiated by when author was at the Department of Mathematics of Harvard University in the late 1970s under DOE support.

Isomathematics is based on the isotopic lifting of the conventional associative product AB between generic quantities A, B (such as numbers, functions, matrices, etc.) into the isoproduct [19b]

$$A \hat{\times} B = A\hat{T}B \tag{8}$$

where the quantity $\hat{T}$, called the *isotopic element,* is positive definite but otherwise posses an arbitrary functional dependence on all needed local quantities, such as time t, coordinates r, velocities v, accelerations a, density μ, temperature τ, frequency ω, wavefunction ψ, etc. $\hat{T} = \hat{T}(t, r, v, a, \mu, \tau, \omega, \psi, \ldots) > 0$.

Product (8) was introduced for the primary intent of achieving an invariant representation of interior dynamical problems referred to extended, non-spherical and deformable particles moving within physical media, which is notoriously impossible via 20th century mathematics, but possible via isomathematics (see below for examples).

Therefore, isomathematics was suggested for the primary intent of achieving a generalization of Lie's theory into a form applicable for the first time to non-linear, non-local and non-Hamiltonian systems (that is, variationally non-self-adjoint systems not representable with a Hamiltonian [20a]).

A systematic isotopic lifting of the various branches of Lie's theory was presented in monograph [20b]. The resulting theory is today known as the *Lie-Santilli IsoTheory,* and it is

based on the isoalgebra (the isotopues of Lie's second theorem)

$$[X_i,\hat{X}_j] = X_i \hat{\times} X_j - X_j \hat{\times} X_i = X_i\hat{T}X_j - X_j\hat{T}X_i = iC_{ij}^{k}X_k \quad (9)$$

and their integration into a finite isogroup here illustrated for simplicity via the one dimensional time evolution with the Hamiltonian $X = H$

$$A(t) = e^{x\hat{T}ti}A(0)e^{-it\hat{T}x} \quad (10)$$

with evident non-linear, non-local and non-Hamiltonian characters due to the presence of the isotopic element in the exponent.

In Vol. [20b], the author then presented a concrete realization of the Lie-Santilli isotheory given by the *Birkhoffian generalization of classical Hamiltonian mechanics* and its "direct universality," namely, the representation of all infinitely possible, well behaved, non-Hamiltonian systems directly in the frame of the experimenter.

However, the new mechanics activated the Theorems of Catastrophic Inconsistencies of Non-Canonical and Non-Unitary Theories when formulated via the mathematics of canonical and unitary theories, respectively [19].

Therefore, while visiting at the JINR in Dubna, Russia, the author was forced in 1993 [21] to reinspect the historical classification of numbers and discovered that the abstract axioms of a numeric field do not necessarily require that the basic multiplicative unit is the number +1, since they also admit realizations with arbitrary positive-definite units, provided that the associative product is lifted accordingly.

This lead to the discovery of new numbers, today known as *Santilli isonumbers*, with an arbitrary positive-=definite unit, called *Santilli isounit,* which is the inverse of the isotopic element of isoproduct (8)

$$\hat{I}(t,r,v,a,\mu,\tau,\nu,\psi,....) = 1/\hat{T}(t,r,v,a,\mu,\tau,\nu,\psi,.....) \quad (11)$$

Applied mathematics was then reformulated on isofields. Yet, the fundamental invariance under the time evolution remained elusive. This forced the author to lift the Newton-Leibniz differential calculus into the form today known as *Santilli IsoDifferential Calculus* first presented in mathematical memoir [22] of 1996, with basic isodifferential

$$\hat{d}\hat{r} = dr + r\hat{T}d\hat{I} \quad (12)$$

and related isoderivative

$$\frac{\hat{\partial}\hat{F}(\hat{r})}{\hat{\partial}\hat{r}} = \hat{I}\frac{\partial\hat{F}(\hat{r})}{\partial\hat{r}}, \quad (13)$$

where the realizations $\hat{F} = F\hat{I}, \hat{r} = r\hat{I}$, etc. are necessary for the values to be isonumbers.

The isodifferential calculus permitted the achievement of maturity for mathematical, physical, and chemical developments, with ensuing numerous scientific as well as industrial applications. *Isomathematics* is today referred to the isotopies of the *totality* of 20th century mathematics formulated via isofunctional analysis, isodiufferential calculus, isoalgebras, isosymmetries, isogeometriesc, etrc., on Santgilli isofields.

A comprehensive presentation of isomathematics for physicists has been provided by the author in monographs [23]. A presentation of isomathematiccs for mathematicians is available in monograph [23] by R. M. Falcon Ganfornina and J. Nunez Valdes, while a monumental work on the isodifferential calculus and its bimplications for all of mathematics is available in the five monographs [25] by S. Georgiev.

4. Rudiments of IsoMechanics

The primary physical application of isomathematics is the isotopic lifting of Newton's equations, first presented in Ref. [22]

$$\hat{m}\hat{\times}\frac{\hat{d}\hat{v}}{\hat{d}\hat{t}} = F^{sa}(t,r,v).principle \quad (14)$$

today known as the *Newton-Santilli IsoEquations.*

Eqs. (14) allow the first known representation of the actual extended shape of bodies, for instance, via the isounit for the velocities

$$\hat{I}(t,r,v,a,\mu,\tau,\omega,....) = Diag.(n_1^2,n_2^2,n_3^2)e^{\Gamma},$$
$$n_k = n_k(t,r,v,a,\mu,\tau,\nu,....) > 0,$$
$$\Gamma = \Gamma(t,r,v,a,\mu,\tau,\omega,....) > 0, k = 1,2,3 \quad (15)$$

as well as the representation of non-Hamiltonian (variationally non-self-adjoint [20]) forces via the exponent of the isounit (15) and their embedded in the isodifferential $\hat{d}\hat{v} = \hat{d}(v\hat{I})$ in such a way that only Hamiltonian (variationally self-adjoint [20]) forces appear in the r.h.s. of the equations.

In view of these features, the Newton-Santilli isoequations for non-Hamiltonian systems admit the first known representation via *isoaction principle* [22]

$$\hat{\delta}\hat{A} = \int (\hat{p}\hat{\times}\hat{d}\hat{r} - \hat{H}\hat{\times}\hat{t}) = 0 \quad (16)$$

thus permitting the first known use of the optimal control theory for the shape, e.g., of a wing moving within a fluid.

In turn, the availability of the isoaction principle has allowed the isotopic lifting of classical Hamiltonian mechanics into its covering *Hamilton-Santilli isomechanics* with basic isotopies of the conventional Lagrange and Hamilton equations here ignored for brevity as well as of the *Hamilton-Jacobi-Santilli isoequations* [22,23]

$$\frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{t}} + \hat{H} = 0, \quad \frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{r}} - \hat{p} = 0, \quad \frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{p}} = 0. \quad (17)$$

Still in turn, the availability of the latter isoequations has permitted the first known, axiomatically consistent, unique and unambiguous, operator map of non-Hamiltonian systems into a covering of quantum mechanics introduced in 1978 under the name of hadronic mechanics [20], with Schrödinger-Santilli Isoequations [22]

$$\hat{\imath} \hat{\times} \frac{\hat{\partial}\hat{\psi}}{\hat{\partial}\hat{t}} = \hat{H} \hat{\times} \hat{\psi} =$$

$$= \hat{H}(\hat{r}, \hat{p})\hat{T}(t, r, p, \mu, \tau, \nu, \psi, \ldots)\hat{\psi} =$$

$$= \hat{E} \hat{\times} \hat{\psi} = E\hat{\psi}, \tag{18}$$

related *isolinear momentum*

$$\hat{p} \hat{\times} \hat{\psi} = -\hat{\imath} \hat{\times} \frac{\hat{\partial}\hat{\psi}}{\hat{\partial}\hat{r}} = -i\hat{I}\frac{\partial\hat{\psi}}{\partial\hat{r}}, \tag{19}$$

and their isounitarily equivalent *Heisenberg-Santilli isoequations* [20,23] for the isotime evolution of an operator $\hat{A}$ in the infinitesimal form

$$\hat{\imath} \hat{\times} \frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = [\hat{A}, \hat{H}] = \hat{H} \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{A} =$$

$$= A\hat{T}(t, r, p, \mu, \tau, \nu, \psi, \ldots)\hat{H}(\hat{r}, \hat{p}) -$$

$$-\hat{H}(\hat{r}, \hat{p})\hat{T}(t, r, p, \omega, \tau, \omega, \psi, \ldots)A, \tag{20}$$

and integrated finite form (10), where the "hat" denotes formulation on an iso-Hilbert space over the isofield of isocomplex numbers [23].

For readers not familiar with the field, we should recall that *hadronic mechanics is a non-unitary "completion" of quantum mechanics much along the celebrated argument by Einstein-Podolsky and Rosen* (see later on Ref. [36]). However, non-unitary theories formulated on a conventional Hilbert space over a conventional field violate causality [9,19]. Hence, the reformulation of non-unitary theories via isomathematics is crucial for the mathematical and physical consistency of hadronic mechanics at large and its isomechanical branch in particular (see monographs [23] for a comprehensive presentation).

We should also mention that *hadronic mechanics eliminates the divergencies of quantum mechanics* because the value of the isounit (15) is generally very big. Consequently, the value of the isotopic element $\hat{T}$ is very small, thus permitting the conversion of divergent or weakly convergent quantum series into strongly convergent isotopic forms via the systematic use of isoproduct (8). Additionally, the functional dependence of the isotopic element is unrestricted, thus allowing the removal of the singularity of the Dirac delta distributions under isotopy, which feature persists for the isotopies of the scattering theory. The absence of divergencies is particularly important for approximate solutions of exterior and interior dynamical problems, as well as of non-linear gravitational equations when reformulated in terms of isomathematics.

Finally, the non-initiated reader should be aware that *quantum mechanics and hadronic mechanics coincide at the abstract level by conception and construction* to such an extent that they can be expressed via the same symbols and equations, merely subjected to *different realizations.* Following decades of research in the field, we believe that the above features are iportant to assure consistency and causality of hadronic mechanics and its applications.

5. Rudiments of IsoGravitation for Matter

5.1. Elementary Formulation of IsoGravitation

The main result of the studies in gravitation herein reported is that *the conjecture of the actial curvature of space is the dominant origin of all problematic aspects of Einstein gravitation, including all its incompatibilities with 20th century sciences,* besides being disproved by visual, mathematical and experimental evidence (Figure 1-5).

Therefore, the main objectives of the studies herein reported are: A) the reformulation of Einstein field equations via a basically new geometry admitting the invariance of line elements without curvature; B) show the compatibility of said reformulation with 20th century sciences; and C) provide at least preliminary experimental verifications.

Following decades of preparatory research on the new isomathematics and isomechanics, isogravitation for matter was presented for the first time at the 1992 *Marcel Grossmann Meeting in Gravitation* [26] via the following elementary rules:

RULE 5-I: Decompose any non-singular Riemannian metric g(x) in (3+1)-dimensions into the product of the the Minkowski metric $\eta = Diag.(1, 1, 1, -1)$ and the 4×4-dimensional *gravitational isotopic element* $\hat{T}_{gr}(x)$

$$g(x) = \hat{T}_{gr}(x)\eta \tag{21}$$

where the positive-definite character of $\hat{T}_{gr}(x)$ is assured by the topology of the Riemannian space;

RULE 5-II: Assume the inverse of the isotopic element as the *gravitational isounit*

$$\hat{I}_{gr}(x) = 1/\hat{T}_{gr}(x) > 0 \tag{22}$$

RULE 5-III: Reformulate the *totality* of Einstein gravitation into such a form admitting $\hat{I}_{gr}(x)$ as the correct left and right unit at all levels, including numbers, functional analysis, differential calculus, geometries, Christoffel symbols, etc.

As we shall see, the above simple rules will allow maintaining Einstein's field equations including its primary verifications, although formulated on a new geometry over new fields with null curvature.

5.2. Minkowski-Santilli IsoSpace

The spacetime of isogravitation verifying the above conditions is given by the infinite family of isotopies of the Minkowski space first introduced by the author in Ref. [26] of 1983 for the classical profile and Ref. [27] of the same year for the operator counterpart, and it is today known as the *Minkowski-Santilli IsoSpace.*

Consider the conventioinal Minkowski space $M(x, \eta, I)$ with spacetime coordinates $x = (\text{xi}), i = 1, 2, 3, 4$, metric $\eta = Diag.(1,1,1,-c^2)$ and unit $I = Diag.(1,1,1,1)$. The Minkowski-Santilli isospace is denoted $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$, and it is characterized by the infinite family of isotopies for which coordinates are lifted into isocoordinates (a necessary condition for their value to be isonumbers) [26]

$$x \to \hat{x} = x\hat{I} \tag{23}$$

the Minkowski metric is lifted into the infinite family of isometrics

$$\eta \to \hat{\eta} = \hat{T}_{gr}\eta \tag{24}$$

the Minkowski unit is lifted into the isounits with related isotopic elements

$$\hat{I}_{gr}(t,r,p,\mu,\tau,\omega,\psi,\ldots) =$$

$$= Diag.(n_1^2, n_2^2, n_3^2, n_4^2) > 0 \quad , n_i > 0, \tag{25}$$

$$\hat{T}_{gr}(t,r,p,\mu,\tau,\omega,\psi,\ldots) =$$

$$= Diag.(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, \frac{1}{n_4^2}) > 0, \tag{26}$$

and line Minkowski element into the infinite family of *isoline elements*

$$\hat{x}^{\hat{2}} = \hat{x}^i \hat{\times} \hat{\Xi}_{ij} \hat{\times} \hat{x}^j = (x^i \hat{\eta}_{ij} x^j)\hat{I} =$$

$$= (\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2})\hat{I}, \tag{27}$$

where: $\hat{\Xi} = \hat{\eta}\hat{I}$ is a condition is a condition to have correct isomatrices, that is, matrices whose elements are isonumbers; one should note the multiplication of the isoline elements by the isounit which is also a necessary condition forf the line element to be isonumbers; and we have ignored for simplicity the exponential factor in the isounits and isotopic elements representing non-Hamiltonian interactions as in Eqs. (15) (see Refs. [23] for the full treatment).

The n-quantities are called the *characteristic quantities* of the gravitational field and they are illustrated in the verifications below. Readers are suggested to exercise caution for the popular interpretation of the n-quantities as being "free parameters" since this would literally imply that, for instance, the terms characterizing the Schwartzchild metric are "free parameters."

It is easy to see that the projection of the isoline element (27) in conventional spacetime is the most general possible symmetric (thus diagonalized) and non-singular line element in (3+1)-dimensions, thus including as particular cases all possible Minkowskian, Riemannian, Fynslerian and other line elements (it should be noted that non-symmetric line elements for the geometric representation of irreversible gravitational events require the broader Lie-admissible genomathematics [19,23])

5.3. Minkowski-Santilli IsoGeometry

The geometry of isospace $\hat{M}(\hat{c},\hat{\eta},\hat{I})$ was first studied in memoir [28] of 1998 and it is today known as the *Minkowski-Santilli isogeometry.* Its first important feature is the admission of the entire machinery of the Riemannian geometry, such as covariant derivative, Christoffel symbols, etc. merely reformulated in terms of the isodifferential calculus, Eqs. (12)-(13).

This is evidently due to the fact that, unlike the Minkowski metric η, its isotopic covering $\hat{\eta}$ admits the most general possible functional dependence, under the sole condition of positive-definiteness of the isotopic element, Eq. (26). Regrettably, an outline of the new geometry would be excessively advanced for the elementary character of this presentation.

The second important feature of the Minkowski-Santilli isogeometry is that of being *isoflat,* that is, its curvature is identically null when elaborated via isomathematics and defined over isofields.

An elementary way of seeing the second features is to note that, under isotopies, we have the mutation of the Minkowskian coordinates while the corresponding unit is mutated by the *inverse* amount,

$$x_k \to \hat{x}_k \hat{I}_k = \frac{x_k}{n_k^2} \tag{28}$$

$$I_k \to \hat{I}_k = n_k^2, \tag{29}$$

thus preserving the original flatness.

In any case, isotopies must preserve the original axioms by central condition and technical realization. This means that, when properly treated, the isotopies of the Minkowski space must preserve the original flatness despite the dependence of the isometric on local coordinates.

5.4. Lorentz-Santilli IsoSymmetry

Thanks to the prior construction of the Lie-Santilli isotheory [20], the universal isosymmetry of all possible isoline elements (27) was constructed for the first time in only one page of Ref. [26]; it is today called the *Lorentz-Santilli isosymmetry;* it is characterized by the original symmetry plus the isotopic element (26); and can be written for isotransformations in the (3, 4)-plane (see Refs. [23] for the general case)

$$x'^3 = \hat{\gamma}[x^3 - \hat{\beta}\frac{n_3}{n_4}x^4], \tag{30}$$

$$x'^4 = \hat{\gamma}[x^4 - \hat{\beta}\frac{n_4}{n_3}x^3]. \tag{31}$$

where

$$\hat{\gamma} = \frac{1}{\sqrt{1-\hat{\beta}^2}}, \quad \hat{\beta} = \frac{v/n_3}{c/n_4}; \tag{32}$$

As one can see, it is evident that the Lorentz-Santilli isosymmetry is locally isomorphic to the original symmetry by conception and realization. It is also evident that this local isomorphism is crucial for achieving compatibility of isogravitation with 20th century theories and for attempting a consistent grand unification of gravitation and electroweak interactions, as outlined below.

Following the original isotopies of the Lorentz symmetry

[26,27], systematic studies were done by the author on the isotopies of all most significant spacetime and internal symmetries. In fact, Ref. [29] was devoted to the isotopies $\hat{O}(3)$ of the rotational symmetry $O(3)$ to achieve the invariance of all topology preserving deformations of the sphere; Refs. [30,31] were devoted to the isotopies $\hat{SU}(2)$ of the $SU(2)$ spin symmetry; Ref. [32] presented for the first time the isotopies $\hat{P}(3.1)$ of the Poincaré symmetry $P(3.1)$ with the first proof of the universal invariance of all possible non-singular, Riemannian line elements; and Ref. [33] was devoted to the isotopies $P(3.1)$ of the spinorial covering of the Poincaré symmetry $P(3.1)$. Independent papers [34,35] confirmed the universal character of the Lorentz-Santilli isosymmetry for the invariance of all infinitely possible symmetric line elements in (3+1)-dimensions.

5.5. IsoGravitational IsoEquations

Another important feature of isogravitation is that of preserving Einstein's field equations (2), although necessarily extended to forms (3)-(6) and reformulated on the Minkowski-Santilli isogeometry without curvature.

Along these lines, we have the *isoequations for exterior gravitational problems*

$$\hat{G}_{ij} = \hat{R}_{ij} - \hat{\Xi}_{ij}(\hat{x},\hat{v}) \hat{\times} \hat{R}/\hat{2} =$$

$$= \hat{k} \hat{\times} \hat{T}_{ij,elm}(\hat{x},\hat{v}), \tag{33}$$

under the condition

$$m_{grav} = \int \hat{T}_{00,elm}(\hat{x}) \hat{\times} \hat{d}\hat{v} \tag{34}$$

where one should note the dependence of the source on isocoordinates and isovelocities, as typical for electromagnetic source.

Consequently, the isometric is equally dependent on isocoordinates and isovelocities, $\hat{\Xi}(\hat{x},\hat{v}) = \hat{\eta}(\hat{x},\hat{v})\hat{I}$, a property forbidden by the Riemannian geometry but readily permitted by the Minkowski-Santilli isogeometry due to the unrestricted functional dependence of the isometric.

We also have the broader isoequations for the interior isogravitational problem

$$\hat{G}_{ij} = \hat{R}_{ij} - \hat{\Xi}_{ij}(\hat{t},\hat{r},\hat{v},\hat{a},\hat{\mu},\hat{t},\hat{\omega},\hat{\psi},\dots) \hat{\times} \hat{R}/\hat{2} =$$

$$\hat{I}_{sch} = Diag.[1,1,(1-\frac{r_{sch}}{r}),(1-\frac{r_{sch}}{r})^{-1}]. \tag{41}$$

or with the isogravitational characteristic quantities

$$n_r^2 = 1-\frac{r_{sch}}{r}, \quad n_4^2 = (1-\frac{r_{sch}}{r})^{-1}, \tag{42}$$

where one should note the suggestive reformwhere one should note the suggestive reformulation of gravitational singularities in terms of the zeros of the space component of the isounit.

We now consider the isotopies of the Dirac equations introduced in Ref. [33], now called the *Dirac-Santilli IsoEquations,* and specialize then to the Schwartzchild metric

$$(\hat{\Xi}_{st}^{\mu\nu} \hat{\times} \hat{\Gamma}_{\mu} \hat{\times} \hat{\partial}_{\nu} + \hat{\imath} \hat{\times} \hat{m} \hat{\times} \hat{c}) \hat{\times} \hat{\psi} = (\hat{\eta}_{sch}^{\mu\nu}\hat{\gamma}_{\mu}\hat{\partial}_{\nu} + imc)\hat{\psi} = 0 \tag{43}$$

where $\hat{\Xi} = \hat{\eta}\hat{I}, \hat{\Gamma} = \hat{\gamma}\hat{I}$ and the matrices $\hat{\gamma}$, known as the *Dirac-Santilli IsoGamma matrices,* are given by

$$\hat{\gamma}_k = \frac{1}{n_k}\begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix},$$

$$\hat{\gamma}_4 = i\frac{1}{n_4}\begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}, \tag{44}$$

with anti-isocommutation rules [25]

$$\{\hat{\gamma}_{\mu},\hat{\gamma}_{\nu}\} = \hat{\gamma}_{\mu}T_{st}\hat{\gamma}_{\nu} + \hat{\gamma}_{\nu}T_{st}\hat{\gamma}_{\mu} = 2\hat{\eta}_{\mu\nu,sch} \tag{45}$$

As one can see, Eqs. (43) did indeed succeed in embedding gravitation in the Dirac equation, for which reason Santilli proposed the name of the *Dirac-Schwartzchild IsoEquation* [25]. It's expected physical relevance is evident, e.g., as the first description on scientific records of an electron within an intense gravitational field in the surface of the Sun or near the event horizon of a black hole.

In closing, we would like to honor the memory of Einstein, Podolsky and Rosen [36] for their view on the "lack of completeness of quantum mechanics" which was instrumental for the birth of hadronic mechanics and its applications. In fact, operator isogravitation can be defined as an invariant non-unitary, axiom-preserving completion of relativistic quantum mechanics.

5.6. Compatibility of IsoGravitation with 20th Century Theories

The compatibility of isogravitation with 20th century sciences is direct and immediate. The compatibility of isogravitation with special relativity is immediately established by the fact that its universal isosymmetry is locally isomorphic to the conventional Poincaré symmetry. The compatibility of the physical laws of isogravitation with those of special relativity is then an immediate consequence.

The compatibility of isogravitation with the interior gravitational problem is established by the completely unrestricted functional dependence of the gravitational isometric. The compatibility of isogravitation with electromagnetism is established by the electromagnetic origin of the gravitational mass appearing in Eqs. (33).

The compatibility of isogravitation with quantum mechanics is inherent in the very notion of isotopies and it is used at the foundation of the very proposal of isogravitation [25]. The compatibility of isogravitation with grand unifications will be discussed in Section 7.

5.7. IsoGravitational IsoAxioms

The isotopies of the axioms of special relativity, today known as *IsoAxioms,* were initiated by Santilli in paper [26] of 1983; they received a first systematic formulation in

monographs [37] of 1991; and they were finalized in monographs [23] of 1995 jointly with the discovery of the isodifferential calculus (see Ref. [41] for an independent review).

In works [23,26,37], the isoaxioms were specifically conceived and technically developed for quantitative treatments of relativistic interior dynamical problems, such as for the propagation of light within gaseous media (Figure 1), in which application they have received numerous experimental verifications (see, e.g., Refs. [11-15] and general review [41]).

The isoaxioms presented in Refs. [23,26,37] had no gravitational content. The application of the isoaxioms for a representation of gravity is presented for the first time in this paper under the proposed name of *IsoGravitational IsoAxioms.*

The presentation of this subsection is the most general possible for both the exterior and the interior gravitational problems characterized by a non-singular, symmetric isometric in (3+1)-dimension. This general formulation is merely achieved without any specification of the functional dependence of the isometric. In the verifications of the isogravitational isoaxioms of the next subsection, we will be forced to specify the isoaxioms to exterior or interior gravitational problems.

The first implication of the isotopies of special relativity is the abandonment of the speed of light in vacuum as the maximal causal speed in favor of a covering geometric notion. This is necessary for isogravitation because light is expected not to propagate within the hyperdense media inside planets or stars.

This occurrence is easily seen by specializing the isoline element (27) to the isolight isocone [23, 37]

$$\hat{x}^{\hat{2}} = \frac{x_k^2}{n_k^2} - t^2\frac{c^2}{n_4^2}) = 0, \tag{46}$$

thus leading to the maximal Causal Speed V_{max} of IsoAxiom 5.1 below.

The remaining isoaxioms can be uniquely and unambiguously identified via a procedure parallel to the construction of the axioms of special relativity from the Poincaré symmetry [23,37].

The reader should be aware that isogravitation is generally *inhomogeneous* and *anisotropic* for both exterior and interior problems, as evidently intrinsic in the fact that the characteristic quantities n_k of isoelement (27) generally have different values for different space directions.

These features are necessary for a more realistic representation of exterior and interior gravitational fields of planets such as Earth. Inhomogeneity and anisotropy are then easily represented thanks to the arbitrary functional dependence of the characteristic quantities of the Minkowski-Santilli isogeometry.

A consequence of the inhomogeneity and anisotropy of isogravitation is that the isoaxioms are presented for one given direction in space, hereon denoted with the sub-index k, since the change of space direction generally implies a change in the explicit value of the characteristic quantities.

ISOAXIOM 5.I: The maximal causal speed in a given space direction k within an isogravitational field is given by

$$V_{max,k} = c\frac{n_k}{n_4}, \tag{47}$$

ISOAXIOM 5.II: The local isospeed of light within an isogravitational field is given by

$$\hat{c} = \frac{c}{n_4} \tag{48}$$

where c is the speed of light in intergalactic spaces w
where c is the speed of light in intergalactic spaces without any gravitational field.

ISOAXIOM 5.III: The addition of isospeeds in the k-direction within an isogravitational field follows the isotopic law

$$V_{tot,k} = \frac{v_{1,k}/n_k + v_{2,k/n_k}}{1 + \frac{v_{1,k}v_{2,k}}{c^2}\frac{n_4^2}{n_k^2}}. \tag{49}$$

ISOAXIOM 5.IV: The isodilatation of isotime, the isocontraction of isolengths, theiso variation of mass with isospeed, and the mass-energy isoequivalence principle follow the isotopic laws

$$\Delta t' = \hat{\gamma_k}\,\Delta t, \tag{50}$$

$$\Delta \ell' = \hat{\gamma_k}^{-1}\,\Delta \ell, \tag{51}$$

$$m' = \hat{\gamma_k}\,m, \tag{52}$$

$$E = mV_{max}^2 = mc^3\frac{n_k^2}{n_4^2} \tag{53}$$

where $\hat{\gamma}$ and $\hat{\beta}$ have values (32).
where $\hat{\gamma}$ and $\hat{\beta}$ have values (32).

ISOAXIOM 5.V: The frequency isoshift of light propagating within an isogravitational field in the k-direction follows the Doppler-Santilli isotopic law

$$\omega_e = \omega_o\hat{\gamma}_k\left[1 \pm \frac{v/n_k}{c/n_4}\cos\,\alpha\right] \tag{54}$$

where ω_e is the experimentally measured value, ω_o is the value at the origin, and we have ignored for simplicity the isotopies of trigonometry (see Refs. [23] for brevity).

A technical understanding of the isoaxioms requires a technical knowledge of isomathematics. In fact, the isoaxioms presented below are given by their projection from the Minkowski-Santilli isospace over an isofield with isounit (25) into the conventional Minkowski space over a conventional field with isounit 1.

A main feature is that, when the isoaxioms are represented on isospace over isofields, they coincide with the conventional axioms of special relativity by conception and technical realization. In particular, the maximal causal speed $V_{max} \neq c$ solely occurs in the projection of the isoaxioms on Minkowski

space because, at the isotopic level, the maximal causal speed is c for all possible isogravitational problems.

5.8. Verification of IsoGravitation for Exterior Problems without Source

It It is important for the self-consistency of this paper to initiate the appraisal of isogravitation via its application to the exterior gravitational problem without source in order to verify that Einctein field equations (2) can indeed be consistently formulated on a Minkowski-Santilli isospace.

In fact, all consistent experimental verifications of general relativity also apply to isogravitation without source because, for its own conception and technical realizations, isotopic liftings preserve all original numerical values (for brevity, see ref. [23b] with particular reference to the proof that the maximal causal speed on Minkowski-Santilli isospaces on isofields is the conventional speed of light in vacuum c).

In particular, it is easy to see that Einstein's Equivalence Principle [6,9] is maintained in its integrity for various independent reasons. First of all, the projection of isogravitation on the conventional Riemannian space over a conventional field coincides with Einstein gravitation with consequential trivial validity of Einstein';s Equivalence Principle. Additionally, the Equivalence Principle independently holds on the Minkowski-Santilli isospace over isofields by very conception of isotopies [23]. The verification of other serious experimental verifications of Einstein gravitation follows in the same way.

To verify the above general lines, let us assume the Schwartzchild metric (39) as a good *approximation* of the isometric for isoequations (33) for the case without source, and present the results for appraisal by interested readers.

Note that, under said assumption, we have the homogeneity and isotropy of the isogravitational field, thus eliminating the selected space direction \$k\$ of the general isoaxioms.

Note that, under said assumption, we have the homogeneity and isotropy of the isogravitational field, thus eliminating the selected space direction k of the general isoaxioms.

Let us begin by recalling values (42) of the characteristic quantities for the Schwartzchild metric for which

$$\frac{\hat{v}}{\hat{c}} = \frac{v}{n_r}\,\frac{n_4}{c} =$$

$$= \frac{v}{1 - r_{sch}/r}\,\frac{1}{c(1 - r_{sch}/r)} =$$

$$= \frac{r/c}{(1 - r_{sch}/r)^2}, \tag{55}$$

and consequential expressions for the isogamma (32)

$$\hat{\gamma} = \frac{1}{\sqrt{1 - \hat{\beta}^2}} = \frac{1}{\sqrt{1 - \frac{\hat{v}^2}{\hat{c}^2}}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}\,\frac{n_4^2}{n_r^2}}} =$$

$$= \frac{1}{\sqrt{1 - \left(\frac{v}{1 - r_{sch}/r}\right)^2 \left(\frac{1}{c(1 - r_{sch}/r)}\right)^2}} =$$

$$= \frac{1}{\sqrt{1 - \frac{v^2/c^2}{(1 - r_{sch}/r)^4}}} \approx$$

$$\approx \frac{1}{1 - \frac{v/c}{(1 - r_{sch}/r)^2}} \tag{56}$$

From the above values, we have the maximal causal speed in an isogravitational field

$$c = cn4 = c(1 - rschr) \tag{58}$$

which evidently tends to zero at the event horizon.

We believe that this occurrence is a significant confirmation of isogravitation because it provided a most effective, quantitative representation of the impossibility of matter to escape from a black hole.

Similarly, we have the expression for the isospeed of light

$$\hat{c} = \frac{c}{n_4} = c\left(1 - \frac{r_{sch}}{r}\right) \tag{58}$$

which also tends to zero at the event horizon and expectedly thereafter.

We believe that this is another supporting feature of isogravitation because the speed of light decreases for about $100{,}000 km/sec$ when propagating within water. It is then logical to assume that the speed of light is null when reaching the densest conceivable medium in the universe. The null value at the event horizon is also an effective way to represent the impossibility for light to escape from a black hole.

It should be noted that the conventional speed of light c is \ an invariant under the Lorentz-Santilli isosymmetry and related isogravitation because, e.g., the isosum of two light speeds c does not reproduce c as it is the case for special relativity.

However, isospeed (58) is indeed an isoinvariant because the isosum of two light isospeeds does indeed yield the light isospeed,

$$V_{tot,s} = \frac{c/n_4 + c/n_4}{1 + \frac{c^2}{c^2}} = \frac{c}{n_4} = c\left(1 - \frac{r_{sch}}{r}\right) \tag{59}$$

The reader should be aware of the fact that isogravitation predicts that the speed of light c in intergalactic spaces without any gravitational field is "bigger" then the speed of light $\hat{c}_{earth}$ measured on Earth, although for a very small amount,

$$\hat{c}_{earth} = \frac{c}{n_4} = c\left(1 - \frac{r_{sch}}{r}\right) < c, \tag{60}$$

By using isospeeds away from the observer, and values (42), we can write the first order approximation

$$\Delta t' = \Delta t\hat{\gamma} \approx \frac{\Delta t}{1 - \frac{v/c}{(1-r_{sch}/r)^2}} \tag{61}$$

which recovers the conventional time dilation of special relativity at a given distance r. However, the value of $\Delta t'$ within a gravitational field $(grav)$ is predicted to be smaller than that for special relativity (sr),

$$\Delta t'_{grav} < \Delta t'_{sr} \tag{62}$$

in such a way that time tends to zero at the event horizon, in full agreement with the behavior of the time component of the Schwartzchild metric (39),

$$Lim_{r\to sch}\Delta t = 0 \tag{63}$$

Similarly, we have the isolength isocontraction

$$\Delta \ell' = \frac{\Delta \ell}{\hat{\gamma}} \approx \Delta \ell 1 - \frac{v/c}{(1 - r_{sch}/r)^2} \tag{64}$$

which recovers the length contraction of special relativitywhich recovers the length contraction of special relativity for a given distance r. However, the value of $\Delta \ell'$ in the presence of a gravitational field is predicted to be bigger than that of special relativity

$$\Delta \ell'_{grav} > \Delta \ell'_{sr} \tag{65}$$

in such a manner that $\delta t'$ tends to infinity at the event horizon

$$Lim_{r\to sch}\Delta \ell = \infty \tag{66}$$

also in fiill agreement with the space component of the Schwartzchild metric (39).

We also have from isoaxiom (52) the isovariation of mass in an isogravitational field

$$m' = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}\frac{n_4^2}{n_r^2}}} \approx \frac{m}{\sqrt{1 - \frac{v^2/c^2}{(1-r_{sch}/r)^4}}} \tag{67}$$

illustrating the prediction based on the Schwartzchild metricthat the mass tends to zero at the event horizon.

Similarly, from the energy equivalence (53), we have in the vicinity of the event horizon

$$E' = m'V_{max}^2 = \frac{m}{\sqrt{1 - \frac{v^2/c^2}{(1-r_{sch}/r)^4}}} \text{ x}$$

$$x\, c^2(1 - \frac{r_{sch}}{r})^4 \approx$$

$$\approx mc^2 \frac{(1 - r_{sch}/r)^6}{(1 - r_{sch}/r)^2 - v/c'} \tag{68}$$

illustrating the prediction that the energy isoequivalence of a particle tends to zero at the event horizon much faster than that for the mass.

We believe that the above features are an important verification of the isoaxioms for various reasons. Firstly, the expectation that Newton's inertia and other laws are valid within a black hole is nowadays rejected by the vast majority of scientists. Secondly, any expectation that particles may experience inertia when constrained within the densest medium in the universe without any possibility of motion, is manifestly illogical. Thirdly, and perhaps most importantly, the limitation for the divergent increase of mass and energy within a black hole appears to be an important mechanism set by nature to prevent the achievement of infinite mass under which one single black hole would swallow the entire universe.

It should be stressed to prevent misrepresentations that the null limit of the mass at the event horizon is similar to the singularity of the Schwartzchild metric and solely occur for the case of field equations (2) without source. As indicated in the next subsection, the presence of a source of first order in magnitude, Eq. (4), appears to avoid both the null value of the mass and the singularity at6 the event horizon by turning them into more realistic finite values.

For IsoAxiom 5.V, we have the Doppler-Santilli isoshift of the frequency of light within an exterior isogravitational field for the simple case of null aberration in the space k-direction

$$\omega_e \approx \omega_o[1 \pm \frac{v}{c}(1 - \frac{r_{sch}}{r})], \tag{69}$$

clearly showing Santilli isoRedShift [11,37], namely a redshift of the entire spectrum of visible light without any relative motion between the light source and the origin of the gravitational field.

The energy lost by light for the isoredshift when traversing a gravitational field is expected to be one of the continuous sources of energy needed for the Cosmic Background Radiation to exist in view of its weakness, in addition to the energy originating from the de-excitation of hydrogen atoms in intergalactic spaces when hit by light [11-15] which appears to be an additional source of the energy needed to maintain in time the Cosmic Backgrtound Radiation [11-15].

Note also that all frequencies of visible light become identically null at the event horizon. This feature is necessary for compatibility with the null value of the speed of light at the event horizon, thus confirming the plausible expectation that the conventional notion of electromagnetic waves becomes meaningless within the densest media existing in the universe. Needless to say, the energy lost by light to the event horizon is absorbed by the black hole.

In conclusion, to our best understanding at this writing, the predictions of isogravitational isoaxioms for matter appear to be supported by the behavior of the isotopic reformulation of the Schwartzchild metric, although more studies are evidently needed to achieve any conclusion due to the complexity of the problem and our rather limited final knowledge of black holes.

5.9. Verification of IsoGravitation for Exterior Problems with Source

As indicated earlier, the Schwartzchild metric (39) has a just place in the history of gravitation because it achieved for

the first time a geometric understanding of gravitational singularities, besides other advances.

However, the Schwartzchild metric remains a first approximation of a rather complex physical reality because local infinities cannot exist in the universe as a condition for its continued existence.

Following decades of studies on the covering of the Schwartzchild metric suitable to avoid local infinities, the author has found no other consistent approach than that allowed by a first-order electromagnetic source in the r.h.s. of the field equations according to Eqs. (33).

This raises the question as to whether Einstein's Equivalence Principle also holds for exterior isogravitation with a source. Einstein supporters quickly voice their opinion that this is not the case for the intent of invalidating isogravitation. However, serious science is far from these unsubstantiated personal opinions because the problem is rather complex indeed and, to avoid a prohibitive length, it will be studied by the author in a subsequent paper.

At this moment, we limit ourselves to the indication that, apparently, the introduction of a source in the gravitational field equations implies numerical contributions in the verification of the Equivalence Principle well within experimental errors. Consequently, the introduction of a source *does not* invalidate the Equivalence Principle on serious scientific grounds until proved so with detailed calculations published in serious refereed journals.

The needed solution is scheduled for detailed studies in a subsequent paper. For the completeness of this paper, we limit ourselves to indicate that an approximate solutions of Eqs. (33) can be written

$$ds^2 = r^2(d\theta^2 + sin^2 d\theta^2 + d\phi^2) +$$

$$+(1 - \frac{r_{sch} + S(r,v)}{r})^{-1} dr^2 -$$

$$-(1 - \frac{r_{sch} + S(r,v)}{r}) dt^2 \equiv$$

$$\equiv \hat{T}_{sch} \times \eta \equiv \hat{\eta}_{sch}, \tag{70}$$

with characteristic quantities

$$= 1 - \frac{r_{sch} + S(r,v)}{r}, \quad n_4^2 =$$

$$= (1 - \frac{r_{sch} + S(r,v)}{r})^{-1}, \tag{71}$$

whose limit for $r \to 0$ (rather than for $r \to sch$) is such to avoid local singularities, e.g., of the type

$$Lim_{r\to 0}(1 - \frac{\hat{v}}{\hat{c}}) =$$

$$= Lim_{r\to 0}[1 - \frac{r/c}{1 - \frac{r_{sch}+S(r,v)}{r}}] =$$

$$= N \neq 0, \ N < \infty. \tag{72}$$

and the numerical value of N evidently requires the consideration of a specific black hole.

It then follows that isogravitational isoequations (33) with a first-order electromagnetic source recover all main historical results achieved by the Schwartzchild's metric, with the elimination of singularities that are not expected to exist in nature.

As a first illustration, the expected behavior of the isotime isodilation (61) acquires the form

$$Lim_{r\to 0}\Delta t' \approx Lim_{r\to 0} \frac{\Delta t}{(1 - \frac{\hat{v}}{\hat{c}})^2} =$$

$$= \frac{\Delta t}{N^2} > 0, \tag{73}$$

thus eliminating the singularity in time of the Schwartzchild metric (39)

Similarly, for the isolength isocontraction we have

$$Lim_{r\to 0}\Delta\ell \approx Lim_{r\to 0}\Delta\ell(1 - \frac{\hat{v}}{\hat{c}})^2 =$$

$$= N^2\Delta\ell < \infty, \tag{74}$$

thus eliminating jointly the local singularity of the Schwartzchild metric for the space component.

Similar corrections occur for the remaining physical quantities studied in the preceding subsection, as the reader can verify.

Note the truly crucial role of the first-order nature of the electromagnetic source, that is, such to represent the entire gravitational mass, Eq. (34). In fact, the standard consideration of the total electromagnetic characteristics of a body leaves Schwartzchild's singularities completely unaffected since their contribution to the gravitational field is of the order of 10^{-30} or less.

In conclusion, we can state that the inclusion in the r.h.s. of the field equation of a first order source of electromagnetic character, essentially along Einstein's own intuition, besides achieving compatibility of gravitation with electrodynamics, does indeed offer realistic possibilities of avoiding local infinities in the universe, with ensuing significant advances in various gravitational problems.

5.10. Verification of IsoGravitation for the Interior Problem

Contrary to isogravitation, Einstein gravitation cannot even formulate interior gravitational problems in any realistic way, e.g., due to the inability to represent a locally varying speed of light. In this case there is the loss of credibility for Einstein supporters who even mention experimental verifications of Einstein gravitation, for the evident reason that we have no direct experimental tests in the interior of the Sun or planet.

The interior character of the Doppler-Santilli isolaw has been extensively studied in Refs. [11-15]. We hereby limit ourselves to consider the interior gravitational case of light passing through the Sun chromosphere.

In this case, the characteristic n -quantities have a functional dependence on the speed v, the distance d

covered within the physical medium, etc. thus admitting the expansion in the travwersed distance d by light within the medium

$$n_4 n_k \approx 1 \pm$$

$$\frac{n_4}{n_k} \approx 1 \pm \frac{c}{v} Hd, \tag{75}$$

where H is the Hubble constant with resulting Doppler-Santilli isoshift law [11]

$$\omega_e \approx \omega_0[1 \pm vc(1 \pm Hd)] \tag{76}$$

Measurements [11-15] have established that, for a sufficiently dense chromosphere (or for a sufficiently long travel d in a thin atmosphere), the conventional Doppler term is ignorable, e.g., for the case of earth's rotation, and the Hubble term becomes dominant, resulting in the Hubble law for the cosmological shift

$$z \approx \pm Hd \tag{77}$$

which is uniquely and unambiguously characterized by the Poincaré-Santilli isosymmetry.

We hereby add, apparently for the first time, the extended version of the Doppler-Santilli isolaw within a transparent physical medium with the inclusion of a strong isogravitational, field from isoaxiom (64)

$$\omega_e \approx \omega_0[1 \pm \frac{v}{c}(1 - \frac{r_{sch}}{r}) \pm Hd). \tag{78}$$

As one can see, the isoredshift caused by isogravitation in the vicinity of the event horizon is dominant over all others, as expected.

We believe that the above features provide a significant additional verification of isogravitation at large, and of its isoaxioms in particular.

In conclusion, we can safely state that isogravitation does indeed allow meaningful models of interior gravitational problems that are notoriously impossible for Einstein gravitation.

It should be noted that these interior gravitational models are reached via the same axioms of exterior problems under the uncompromisable condition that the metric has an unrestricted functional dependence on all possible interior local variable, which dependence is solely admitted at this writing by isogeometries.

6. Rudiments of IsoDual IsoGravitation for AntiMatter

Despite all the above advances, attempts at an axiomatically consistent grand unification of electroweak and gravitational interaction continued to be inconsistent and not worth their presentation in a scientific paper, because Einstein gravitation, as well as isogravitation, solely apply for *matter-bodies,* thus preventing any consistent unification with electroweak theories that are bona-fide theories of particles and antiparticles.

A solution of the latter problem required the construction of yet another new mathematics, specifically conceived for the *classical* representation of *neutral* (or charged) *antimatter-bodies.*

The transition from matter to antimatter required the new mathematics to be anti-isomorphic in general and anti-Hermitean in particular, to isomathematics, as a condition to be consistent with charge conjugation and experimental data, including matter-antimatter annihilation.

Following numerous failed attempts, when being at he Department of Mathematics of Harvard University in the early 1980s, the author finally succeeded in identifying the needed mathematics, called isodual mathematics and denoted with the upper symbol ^{d}.

In view of the above aspects, Santilli constructed the isodual image of 20th century mathematics and quantum mechanics under the condition of admitting the *isodual unit*

$$I^d = -1 \tag{79}$$

at all its mathematical and physical levels [40].

The above studies remained grossly insufficient to initiate studies on possible grand unifications due to the need of the anti-isomorphic image of isomathematics for antimatter whose need emerges even stronger from the model of isogravitation presented in this paper.

The latter mathematics was built via the systematic application of the *following isodual map*

$$\hat{I}(t, r, p, \mu, \tau, \nu, \psi, ...) \to \hat{I}^d =$$

$$-\hat{I}^\dagger(-t^\dagger, -r^\dagger, -v^\dagger, -a^\dagger, -\mu^\dagger, -\tau^\dagger, -\nu^\dagger, -\psi^\dagger, ...) \tag{80}$$

to the totality of quantities and the totality of their operations used for matter.

The resulting new mathematics is today known as *Santilli isodual isomathematics* and includes isodual isonumbers, isodual isofunctions, isodual isodifferential calculus, isodual isoalgebras, isodual isogeometries, etc. (see monograph [40] for a comprehensive study and Ref. [41] for an independent general review).

Following the construction of the isodual isomathematics it was necessary to construct the isodual image of classical and operator theories, with particular reference to the isodual Lorentz-Santilli isosymmetry and the axiomatically consistent classical representation of the gravitational field of neutral (or charged) antimatter-bodies. The compatibility of the emerging isodual theory of antimatter with experiental data was assured by the equivalence of the isodual map with charge conjugation (for brevity, one may inspect monograph [23]).

7. Rudiments of IsoGrandUnification

In our view, a most important implication of the search for axiomatically consistent grand unifications is the shift from the *description* of gravitation to a study of its *origin.* In fact, Ref. [16] is crucially dependent on the abandonment of the standard "unification" of gravitation and electromagnetic interactions in favor of their "identification" under appropriate

field equations.

Ref. [16] also submitted experiments for the possible laboratory creation of a measurable gravitational field that appears feasible nowadays thanks to the availability of highly sensitive detectors, such as those based on neutron interferometry.

Only following the above scientific journey the author was finally in a position to present at the 1997 *Marcel Grossmann Meeting in Gravitatio,* a grand unification of electroweak and gravitational interactions with the inclusion of matter and antimatter at all classical and operator levels [38] (see also Ref. [39]).

The emerging grand unification essentially consistent in the embedding of gravitation in the gravitational isounit of electrostatic interactions under the universal isospinorial covering $P(3.1)$ of the Poincaré-Santilli isosymmetry $\hat{P}(3.1)$ the selected isotopic image of the selected gauge symmetry g for matter and their isodual for antimatter

$$\hat{S} = \{\hat{P}(3.1) \times \hat{G}\} \times \{\hat{P}^d(3.1) \times \hat{G}^d\} \tag{81}$$

which is the isosymmetry of the Dirac-Santilli isoequations (43) [33] and which, rather intriguingly, emerges as the isosymmetry of the universe at the limit of equal amounts of matter and antimatter (see monograph [40] for brevity).

Of course, we do not know whether the abovegrand unification is verified in nature, but we believe that the studies reported in this paper have provided at least much needed new vistas in gravitation [41] for further advances by interested colleagues.

To follow Albert Einstein teaching for powerful self-criticism, we note that the dynamics of test masses in a gravitational field is fully reversible in time. By contrast, the dynamics of a black holes is strictly irreversible over, since we are dealing with a *one way* absorption of matter and light.

By remembering that isomathematics and related isomechanics are reversible over time, \, a more accurate description of black holes may require a covering of isogravitation constructed via genomathematics with a Lie-admissible (rather than a Lie-isotopic) structure and related genogeometries with non-symmetric genometrics as a condition to embed irreversibility in the ultimate mathematical and physical structures [19,23].

All in all, the studies presented in this paper confirm that physics is a discipline that will never admit final theories.

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Comments on the Regular and Irregular IsoRepresentations of the Lie-Santilli IsoAlgebras

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Abstract: As it is well known, 20th century applied mathematics with related physical and chemical theories, are solely applicable to point-like particles moving in vacuum under Hamiltonian interactions (exterior dynamical problems). In this note, we study the covering of 20th century mathematics discovered by R. M. Santilli, today known as *Santilli isomathematics,* representing particles as being extended, non-spherical and deformable while moving within a physical medium under Hamiltonian and non-Hamiltonian interactions (interior dynamical problems). In particular, we focus the attention on a central part of isomathematics given by the isorepresentations of the Lie-Santilli isoalgebras that have been classified into *regular (irregular) isorepresentations* depending on whether the structure quantities of the isocommutation rules are constants (functions of local variables). The importance of the study of the isorepresentation theory for a number of physical and chemical applications is pointed out.

Keywords: Lie Theory, Lie-Santilli Isotheory, Isorepresentations

1. Introduction

As it is well known, 20th century applied mathematics at large, and the Lie theory in particular, can only represent *point-like particles moving in vacuum* (exterior dynamical problems), resulting in a body of methods that have proved to be effective whenever particles can be effectively abstracted as being point-like, such as for the structure of atoms, and crystals, particles moving in accelerators, and many other systems.

An important feature of exterior problems is that, being dimensionless, point-like particles can only experience *action-at-a-distance, potential and, therefore, Hamiltonian interactions,* which Hamiltonian character is a central condition for the very applicability of Lie's theory.

It is equally well known that point-like abstractions of particles are excessive for *extended, non-spherical and deformable particles moving within a physical medium* (interior dynamical problems), as it is the case for the structure of hadrons, nuclei and stars since their constituents are in a state of mutual penetration of their wavepackets and/or charge distribution.

An important feature of the finite size of particles in interior conditions is that they experience conventional action-a-distance, Hamiltonian interactions, as well as additional contact, non-potential and, therefore, non-Hamiltonian interactions, with the consequential inapplicability of 20th century applied mathematics at large, and of Lie's theory in particular.

In a series of pioneering works [1-11], R. M. Santilli has constructed a new mathematics, today known as *Santilli IsoMathematics,* for the representation of extended, non-spherical and deformable particles under Hamiltonian as well as non-Hamiltonian interactions, which new mathematics has seen contributions by numerous important mathematicians (see, e.g. Rfs., [12-21]).

In this note, the author would like to bring to the attention of the mathematical community the need for further studies on the central branch of isomathematics, namely, the *Lie-Santilli IsoTheory* [1], since the latter provides the only known time invariant methods for the lifting of the various applications of the conventional Lie theory from exterior to interior conditions.

In particular, we focus the attention on the

IsoRepresentations of the Lie-Santilli IsoAlgebras which have been classified into *regular* and *irregular*, depending on whether the structure quantities of the isocommutation rules are constant or functions of the local variables.

Besides Santilli's works, no study on the isorepresentation theory of the Lie-Santilli isoalgebras is on scientific record to our best knowledge, with consequential limitations on important applications, such as the search for much needed, new nuclear energies without the release of harmful radiations and other equally important applications outlined in Section 5.

It should be indicated that Santilli's pioneering works signal the historical transition from the notion of *massive point*, introduced by Newton, and adopted by Galileo and Einstein, to a new generation of physical and chemical theories representing particles as they are in the physical or chemical reality. This historical advance has so many implications for all of quantitative sciences that it has been referred to as characterizing *New Sciences for a New Era* in the title of Ref. [21].

2. The Lie-Santilli IsoTheory

Let L be an N -dimensional Lie algebra on a Hilbert space H over a field F(n,×,1) with elements n given by real, complex and quaternionic numbers, associative product hereon denoted $nm = n \times m \in F$, and multiplicative unit 1.

Let the generators of L be given by Hermitean operators $X_k, k = 1,2,...,N$, on H over F. Let ξ be the universal enveloping associative algebra characterized by the infinite-dimensional set of ordered monomials according to the Poincaré-Birkhoff-Witt Theorem.

Let the Lie algebra L be isomorphic to the anti-symmetric algebra attached to the enveloping algebra $L \simeq \xi^-$ with ensuing Lie's theorems and commutation rules. Let G be the Lie transformation group characterized by L.

In pioneering works done in 1978-1983 at the Department of Mathematics of Harvard University under DOE support, R. M. Santilli [1] proposed the axiom-preserving *isotopies* of 20th century applied mathematics at large, and of the Lie theory in particular, via the following isotropy of the associative product

$$X_i \hat{\times} X_j = X_i \times \hat{T} \times X_j \tag{1}$$

where $\hat{T}$, called the *isotopic element*, is solely restricted to be positive-definite, but otherwise possesses an arbitrary dependence on local variables such as time t, coordinates r, velocities v, density μ, temperature τ, index of refraction δ, frequency ω, wave functions Ψ, etc.

The fundamental significance of Santilli's infinite class of isotopies (1) of the associative product is that they permit the representation of the actual extended, and deformable shape of the body considered under Hamiltonian interactions represented via the conventional Hamiltonian, and contact non-Hamiltonian interactions via realizations of the isotopic element of the type

$$\hat{T} = Diag.\left(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}\right) \times e^{-\Gamma(t,r,v,\mu,\tau,\delta,\omega,\psi,\ldots)} \tag{2}$$

where $n_k^2 = n_k^2(t,r,v,\mu,\tau,\delta,\omega,\psi,\ldots) > 0, k = 1,2,3$, represents, in this case, the deformable semi-axes of the considered ellipsoid, and Γ is a positive-definite function representing all interactions not representable with the Hamiltonian.

Following the above basic assumptions, Santilli passed in monographs [1] to the construction of the isotopies of the various branches of Lie's theory over a conventional field F, and illustrated its significance via the *Birkhoffian generalization of Hamiltonian mechanics* which achieves "direct universality" for the representation of all possible (regular) non-Hamiltonian Newtonian systems directly in the frame of the experimenter. The resulting new theory is today known as the *Lie-Santilli IsoTheory*.

Following the above seminal advances, Santilli discovered that the original formulation [1] of the isotopies does not predict the same numerical values under the same conditions at different times (hereon referred to as *time invriance*), because the time evolution is *non-unitary* on H over F.

In summer 1993, while visiting the Joint Institute for Nuclear Research in Dubna, Russia, Santilli [2] discovered that the abstract axioms of a numeric field do not necessarily require that the basic unit be the number 1, since the multiplicative unit can be an arbitrary, positive definite quantity $\hat{1}$ irrespective of whether an element of the original field F or not, under the condition that it is the inverse of the isotopic element

$$\hat{I} = 1/\hat{T}, \tag{3}$$

and all possible associative products are lifted into form (1) under which $\hat{1}$ is the correct left and right multiplicative unit for all elements of the set considered

$$\hat{I} \hat{\times} X = X \hat{\times} \hat{I} = X \; \forall X \in L \tag{4}$$

This lead to the discovery of new fields, today known as *Santilli isofields* $\hat{F}(\hat{n},\hat{\times},\hat{1})$ with *isoreal, isocomplex and isoquaternionic isonumbers* $\hat{n} = n \times \hat{1}, n \in F$ equipped with the isoproduct $\hat{n}hat \times \hat{m} = n \times m \times \hat{1} \in \hat{F}$ [2].

Subsequently, Santilli discovered that, despite the reformulation over an isofield, the Lie-Santilli isotheory was still unable to achieve the crucial time invariance of the numerical prediction.

Following various trials and errors, while studying at the Institute for Basic research, Castle Prince Pignatelli in Italy, Santilli [3] discovered in 1995 that, contrary to a popular belief in mathematics and physics for centuries, the Newton-Leibnitz differential calculus depends indeed on the assumed basic multiplicative unit because, in the event such unit has a functional dependence on the differentiation variable, the conventional differential must be generalized into the *isodifferential*

$$\hat{d}\hat{r} = \hat{T} \times d[r \times \hat{I}(r,\ldots)] = dr + r \times \hat{T} \times d\hat{I}(r,\ldots), \tag{5}$$

with ensuing *isoderivative*

$$\frac{\hat{\partial}\hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \hat{I} \times \frac{\partial \hat{f}(\hat{r})}{\partial \hat{r}}. \quad (6)$$

where, for consistency, $\hat{f}$ is an *isofunction* with the structure $\hat{f} = f \times \hat{I}$ and $\hat{r}$ is the *isovariable* with the structure $\hat{r} = r \times \hat{I}$ as an evident condition to have values in the isofield $\hat{F}$.

The discovery of isofields and of the isodifferential calculus signed the achievement in memoir [3] of mathematical maturity in the formulations of the isotopies of 20th century applied mathematics at large, and of the Lie-Santilli isotheory in particular, which maturity stimulated seminal, advances in mathematics as well as in physics and chemistry, including novel industrial applications indicated later on.

Nowadays, the *Lie-Santilli IsoTheory* is referred to the infinite family of isotopies of Lie's theory as defined in memoir [3], namely, formulated on an *iso-Hilbert space* $\hat{H}$ over an isofield $\hat{F}$ with iso-Hermitean generators $X_k, k = 1,2,\ldots,N$, with all possible products lifted into the isoassociative form (1) and multiplicative isounit (3), the elaboration beng done via the *isofunctional analysis* and the *isodifferential calculus.*

A rudimentary outline of the Lie-Santilli isotheory comprises the following main branches [3,9]:

2.1) The *universal enveloping isoassociative isoalgebra* $\hat{\xi}$ with infinite-dimensional *isobasis* given by the *ordered isomonomials* of the *Poincaré-Birkhoff-Witt-Santilli isotheorem*

$$\hat{I}, X_k, X_i \times X_j, i \leq j, \ldots . \quad (7)$$

with related *isoexponentiation*

$$\hat{e}^X = \hat{I} + \frac{X}{\hat{1}!} + \frac{X \hat{\times} X}{\hat{2}!} + \cdots = \left(e^{X \times \hat{T}}\right) \times \hat{I} = \hat{I} \times \left(e^{\hat{T} \times X}\right) \quad (8)$$

and other isofunctions;

2.2) The *Lie-Santilli isoalgebras*

$$\hat{L} \approx \hat{\xi}^- \quad (9)$$

with *isocommutation rules*

$$[X_i, X_j] = X_i \hat{\times} X_j - X_j \hat{\times} X_i = \hat{C}_{ij}^k \hat{\times} X_k \quad (10)$$

where $\hat{C}_{ij}^k = C_{ij}^k \times \hat{I}$ are the *isostructure quantities* of $\hat{L}$ with values in $\hat{F}$;

2.3) The *Lie-Santilli isogroups* $\hat{G}$ with structure for the one dimensional case ()

$$\hat{A}(\hat{w}) = \hat{e}^{\hat{H} \hat{\times} \hat{w} \hat{\times} \hat{\imath}} \hat{\times} \hat{A}(\hat{0}) \hat{\times} \hat{e}^{(-\hat{\imath} \hat{\times} \hat{w} \hat{\times} \hat{H})} =$$

$$= e^{H \times \hat{T} \times w \times i} \times A(0) \times e^{-i \times w \times]\hat{T} \times H} \quad (11)$$

where $\hat{H} = H \times \hat{I}$ is an *isomatrix,* namely, a matrix whose elements are isoscalars. The remaining aspect of the Lie-Santilli isotheory can be then constructed via axiom preserving isotopies of the *totality* of the conventional formulations with no exception known to the author.

Following the achievement in memoir [3] of a consistent formulation of the isotopies, Santilli applied the isotopies of Lie's isotheory them to a number of physical and chemical problems that cannot be even formulated with conventional Lie theory due to the need to represent of extended bodies under non-Hamiltonian interactions (see applications [6.7.8] with corresponding independent verifications and industrial applications [12,13,14], monograph [9] for a general treatment of the Lie-Santilli isotheory, and monographs [10,11] for applications in physics and chemistry, respectively).

In the author's view, Santilli's most salient achievement has been, not only the transition from the massive points of Newton, Galileo and Einstein theories to extended bodies, but also their representation under the most general (but non-singular) known non-linear, non-local and non-Hamiltonian interactions in a way as *invariant* as Hamiltonian formulations are.

This historical result was achieved via *the embedding of all non-Hamiltonian quantities in the generalized unit of the theory* because, whether conventional or generalized, the unit is indeed the basic invariant of any theory.

3. Classification of IsoRepresentations

The isorepresentations of Lie-Santilli isoalgebras are classified into [4,5,9]:

3.1) *Regular isorepresentations* occurring when the C's of rules (5) are constant; and

3.2) *Irregular isorepresentations* occurring when the C's of rules (5) are functions of local variables.

We should recall that "structure functions" are impossible for Lie's theory, and they are solely possible for the covering Lie-Santilli isotheory, by therefore establishing the non-trivial character of Santilli isotopies.

4. Regular IsoRepresentations

Let us recall that a given Lie algebra admits an infinite family of isotopies because a point-like particle in vacuum admits an infinite number of generalizations to extended particles moving within physical media.

Let us also recall that the extended shape of a particle and its non-Hamiltonian interactions are represented by the basic isounit or, equivalently, by the isotopic element [2].

Therefore, the transition from the conventional representations of a Lie algebra to the isorepresentation of the covering Lie-Santilli isoalgebras represents extended particles moving within physical media under conventional Hamiltonian interactions, as well as the most general known non-linear, non-local and non-Hamiltonian interactions.

Consider a given Lie algebra L and one of its representations. Santilli [4,5,9] has identified a simple method for the construction of the infinite family of regular isorepresentations of the Lie-Santilli covering $\hat{L}$ of L based on non-unitary transformations of the original Lie formulation. The method consists in:

4.1) Identifying the extended character of the particle considered and its non-Hamiltonian interactions represented via Santilli's isounit.

4.2) The identification of a non-unitary transform representing said isounit according to the rule

$$U \times U^{\dagger} = \hat{I} \tag{12}$$

where

$$U \times U^{\dagger} \neq I, \tag{13}$$

4.3) The application of the above nonunitary transform to the *totality* of the mathematics underlying the original representation of L , thus including numbers, spaces, algebras, geometries, symmetries, etc, with no known exception.

The bove method is illustrated by the transformations:

$$I \to \hat{I} = U \times I \times U^{\dagger} = 1/\hat{T}, \tag{14a}$$

$$n \to \hat{n} = U \times n \times U^{\dagger} = n \times U \times U^{\dagger} = n \times \hat{I} \in \hat{F}, , n \in F, \tag{14b}$$

$$e^{A} \to U \times e^{A} \times U^{\dagger} = \hat{I} \times e^{\hat{T} \times \hat{A}} = (e^{\hat{A} \times \hat{T}}) \times \hat{I}, \tag{14c}$$

$$A \times B \to U \times (A \times B) \times U^{\dagger} = (U \times A \times U^{\dagger}) \times$$

$$(U \times U^{\dagger})^{-1} \times (U \times B \times U^{\dagger}) = \hat{A} \hat{\times} \hat{B}, \tag{14d}$$

$$[X_i, X_j] \to U \times [X_i X_j] \times U^{\dagger} = [\hat{X}_i \hat{,} \hat{X}_j] =$$

$$U \times (C_{ij}^{k} \times X_k) \times U^{\dagger} = \hat{C}_{ij}^{k} \hat{\times} \hat{X}_k = C_{ij}^{k} \times \hat{X}_k, \tag{14e}$$

$$< \psi| \times |\psi > \to U \times < \psi| \times |\psi > \times U^{\dagger} =<$$

$$\psi| \times U^{\dagger} \times (U \times U^{\dagger})^{-1} \times U \times |\psi > \times (U \times U^{\dagger}) =<$$

$$\hat{\psi}| \hat{\times} |\hat{\psi} > \times \hat{I}, \tag{14f}$$

$$H \times |\psi > \to U \times (H \times |\psi >) =$$

$$(U \times H \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times |\psi >) =$$

$$\hat{H} \hat{\times} |\hat{\psi} >, etc. \tag{14g}$$

As an illustration, Santilli considered in Refs. [4,5] the two-dimensional irreducible representation of the SU(2) Lie algebra, which is given by the known Pauli matrices.

The regular isorepresentations of the Lie-Santilli isoalgebras $\hat{SU}(2)$ can be constructed via the infinite family of non-unitary transformations with representative example

$$\hat{\sigma}_k = U \times \sigma_k \times U^{\dagger}, \tag{15a}$$

$$U = \begin{pmatrix} i \times g_1 & 0 \\ 0 & i \times g_2 \end{pmatrix},$$

$$U^{\dagger} = \begin{pmatrix} -i \times g_1 & 0 \\ 0 & -i \times g_2 \end{pmatrix}, \tag{15b}$$

$$g_1^2 = \frac{1}{g_2^2} = \lambda^2, \tag{15c}$$

where conditions (15c) is necessary for the isounitarity of the algebra and the g's are well behaved nowhere null functions. The application of transformations (14) yields the *regular Pauli-Santilli isomatrices* [4,5,9]

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & g_1^2 \\ g_2^2 & 0 \end{pmatrix},$$

$$\hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times g_1^2 \\ i \times g_2^2 & 0 \end{pmatrix},$$

$$\hat{\sigma}_3 = \begin{pmatrix} g_1^2 & 0 \\ 0 & g_2^2 \end{pmatrix}. \tag{16}$$

with isoalgebra isomorphic to the conventional $SU(2)$ algebra

$$[\hat{\sigma}_i, \hat{\sigma}_j] = \hat{\sigma}_i \times \hat{T} \times \hat{\sigma}_j - \hat{\sigma}_j \times \hat{T} \times \hat{\sigma}_i = 2 \times i \times \varepsilon_{ijk} \times \hat{\sigma}_k, \tag{17}$$

and consequential preservation of the conventional eigenvalues for spin $1/2$

$$\hat{\sigma}^{\hat{2}} \hat{\times} |\hat{\psi} >= (\hat{\sigma}_1 \times T \times \hat{\sigma}_1 + \hat{\sigma}_2 \times T \times \hat{\sigma}_2 +$$

$$+\hat{\sigma}_3 \times T \times \hat{\sigma}_3) \times T \times |\hat{\psi} >= 3 \times |\hat{\psi} >, \tag{18a}$$

$$\hat{\sigma}_3 \hat{\times} |\hat{\psi} >= \hat{\sigma}_3 \times T \times |\hat{\psi} >= \pm 1 \times |\hat{\psi} >, \tag{18b}$$

Despite the apparent triviality, Santilli's isotopies of the SU(2)-spin algebra are not trivial because they introduce a new degree of freedom in the conventional spin $1/2$ given by the non-singular, but unrestricted parameter (or function) λ^2 of Eqs. (15c).

In turn, this new degree of freedom has permitted a number of novel applications, such as [4,5,9]: the reconstruction of the exact isospin symmetry in nuclear physics which was believed to be broken by weak interactions; the achievement of a concrete and explicit realization of hidden variables in quantum mechanics via the degrees of freedom λ^2; and rather seminal implications for local realism (see Ref. [5] for brevity).

5. Irregular IsoRepresentations

Santilli has additionally constructed in Refs. [4,5] the following example of irregular isorepresentation of the $\hat{SU}(2)$ spin algebra

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & g_1^2 \\ g_2^2 & 0 \end{pmatrix},$$

$$\hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times g_1^2 \\ i \times g_2^2 & 0 \end{pmatrix},$$

$$\hat{\sigma}_3 = \begin{pmatrix} w \times g_1^2 & 0 \\ 0 & w \times g_2^2 \end{pmatrix}. \tag{19}$$

which are known as the *irregular Pauli-Santilli isomatry,* and cannot any longer be constructed via non-unitary transformations of the Pauli matrices, and.

The irregular character of isomatrices (19) is established by the appearance of *structure functions* in the isocommutation rules

$$[\tilde{\sigma}_1\hat{,}\tilde{\sigma}_2] = i \times w^{-1} \times \tilde{\sigma}_3, [\tilde{\sigma}_2\hat{,}\tilde{\sigma}_3] = i \times w \times \tilde{\sigma}_1,$$
$$[\tilde{\sigma}_3\hat{,}\tilde{\sigma}_2] = i \times w \times \tilde{\sigma}_1, \quad (20)$$

with the characterization of the following *mutation* (in Santilli's words) of the SU(2) -spin eigenvalues

$$\tilde{\sigma}^{\hat{2}} \hat{\times} |\hat{\psi}> =$$
$$(\tilde{\sigma}_1 \times T \times \tilde{\sigma}_1 + \tilde{\sigma}_2 \times T \times \tilde{\sigma}_2 + \tilde{\sigma}_3 \times T \times \tilde{\sigma}_3) \times T \times |\hat{\psi}> =$$
$$= (2 + w^2) \times |\hat{\psi}>, \quad (21a)$$
$$\tilde{\sigma}_3 \hat{\times} |\hat{\psi}> = \tilde{\sigma}_3 \times T \times |\hat{\psi}> = \pm w \times |\hat{\psi}>, w \neq 1, \quad (21b)$$

In essence, Santilli's irregular isorepresentation of $\hat{SU}(2)$ characterize a generalization of the conventional constant values of spin 1/2 into *locally variable spin isoeigenvalues.*

Rather than being a mathematical curiosity, the above spin mutation is expected to be important for a consistent representation of the spin of an electron, e.g., under the immense pressures, densities and temperature in the core of a star.

6. Independent Studies

Numerous mathematicians have made seminal contributions to the Lie-Santilli isotheory, among whom we quote: C-X, Jiang has conducted comprehensive studies [15] on the isonumber theory at the foundation of the Lie-Santilli isotheory; D. S. Sourlas and G. T. Tsagas have conducted the first comprehensive study of the Lie-Santilli theory [16], although prior to the discovery of isonumbers [2]; J. V. Kadeisvili has studied in detail the Lie-Santilli isotheory [17] following its formulation as in memoir [3]; R. M. Falcon and J. N. Valdés [18] have presented the most rigorous formulation to date of Santilli's isotopies; T. Vougiouklis [19] has developed the hyperstructural formulation of the Lie-Santilli isotheory which is the broadest possible formulation achievable with current mathematical knowledge; and S. Georgiev [20] has produced one of the most monumental works in mathematics showing the implications for all of mathematics of the isodifferential calculus which is nowadays called the *Santilli-Georgiev isodifferential calculus.* A comprehensive review with a large list of contributions has been produced by I. I. Gandzha and J. Kadeisvili, in monograph [21] with the suggestive title of *New Sciences for a New Era.*

7. Open Problems

The author has no words to recommend the study of regular and irregular isorepresentations of Lie-Santilli isoalgebras, with particular reference to the identification of a method for the construction of irregular isorepresentation parallel to that for the regular case of Section 4. The proposed study is important for a number of applications, such as:

7.1. Reconstruction of Exact Symmetries

Santilli has shown in Ref. [9] that the breaking of conventional spacetime and internal symmetries is the outcome of insufficient mathematics. because broken symmetries can be reconstructed as being exact at the covering isotopic level under the preservation of the conventional structure constants. This reconstruction has a number of important epistemological as well as technical implications. It is sufficient to note the reconstruction of parity under weak interactions or the maintaining of Einstein's abstract axioms of special relativity for interior conditions to illustrate the implications at hand. Their systematic study can be best done via the study of the isorepresentation of Lie-Santilli isoalgebras.

7.2. Invariant Representation of Hubble's Law

The regular Lorentz-Santilli isosymmetry has permitted an invariant derivation of the Hubble law on the cosmological redshift z = Hd via the mere admission that light loses energy to the cold intergalactic medium without any need for the hyperbolic conjecture of the expansion of the universe via the assumption z = Hd - v/c [6,12]. It is important to verify this occurrence via the study of the regular isorepresentations of the Lorentz-Santilli isosymmetry due to its implications for all of cosmology, since the elimination of the expansion of the universe will likely require the revision of all our cosmological knowledge.

7.3. Synthesis of the Neutron from the Hydrogen

In the author's view, the most important application and verification of isomathematics has been Santilli's exact and invariant representation at both the non-relativistic and relativistic levels of *all* characteristics of the neutron in its synthesis from the hydrogen (see review [21]). Such a synthesis is notoriously impossible for the conventional Hilbert space and related mathematics, e.g., because the rest energy of the neutron is *bigger* than the sum of the rest energies of the proton and the electron (a pure anathema for quantum mechanics); the Dirac equation, which is so effective for the representation of the electron orbiting around the proton in the hydrogen atom, becomes completely ineffective for the representation of the same electron when "compressed" (according to Rutherford) inside the proton; and for other reasons. The representation of the neutron synthesis was crucially dependent on the assumption of the proton and the electron as being *isoparticles,* that is, isounitary irreducible representations of the Galileo-Santilli or the Lorentz-Santilli isosymmetry whose study is evidently fundamental for true advances in particle physics, as well as in the structure of stars.

7.4. Nuclear Constituents as Extended Particles

One of the most important applications of isomathematics is the quantitative prediction of new nuclear energies without the release of harmful radiations (see review [21]). This prediction is based on the invariant representation of nuclear

constituents as being extended and deformable charge distributions. Such a representation has been instrumental for the first achievement of the exact representation of nuclear magnetic moments and spin [7,10]. This new conception of the nuclear structure requires the representation of protons and neutrons as *isoparticles.* It is evident that important advances in nuclear physics and new clean energies will be curtailed until there are systematic studies on the isorepresentations of the Lie-Santilli isosymmetry.

7.5. Elimination of the Divergencies of Quantum Mechanics

Some of the biggest insufficiencies of quantum mechanics in particle physics are due to the singular character of Dirac's delta distribution at the origin, with ensuing divergencies of perturbative series that requiring the achievement of numerical results via the unreassuring subtraction of infinities. Santilli [9,10] has shown that isotopies of Dirac's delta *distribution* into a *function* without singularities at the origin. Additionally, in all known applications the absolute value of the isotopic element (2) is *very small,* with the consequential capability of turning divergent or slowly convergent quantum series into rapidly convergent ones (see the infinite series of isomonomials (8) for comparison). Due to the implications of these features for all quantitative sciences, it appears recommendable that they are confirmed and further developed via the study of the isorepresentation of the Galileo-Santilli or Lorentz-Santilli isosymmetries.

7.6. Electron Valence Bonds

According to the axioms of quantum mechanics and chemistry, two valence electrons, rather than forming any molecular bond, should *repel* each other due to the Coulomb repulsion of their equal charges $F = ke^2 / r^2$ which becomes extremely strong at the distances 10^{-13}cm of valence bonds. Santilli [11] has achieved a *strongly attractive* force between two electrons in singlet valence coupling via the admission that their wavepackets is in condition of total mutual penetration, resulting in non-Hamiltonian interactions represented with isotopic elements of type (2). In view of the predictable advances for all of chemistry, it is important to verify Santilli's strong valence bond via the study of the regular isorepresentations of the Lorentz-Santilli isosymmetry characterizing the valence electrons.

7.7. Nuclear and Chemical Reactions

The preceding applications can be sufficiently treated via regular isorepresentations since they deal with systems of extended particles assumed as being isolated from the rest of the universe. Santilli [9,10,11] has pointed out the insufficiency of the regular isorepresentations for nuclear and chemical reactions because they are *irreversible over time,* a feature that can only be represented via structure functions with an explicit time dependence of the type $C_{ij}^{*}(t,...) \neq C_{ij}^{*}(-t,...)$. Therefore, advances on much needed new energies without harmful radiation and on clean burning fuels will crucially depend on the availability of mathematical studies on irreducible isorepresentation of Lie-Santilli isoalgebras.

Due to their relevance, the R. M. Santilli Foundation has research funds for the writing of papers on the isorepresentations of the Lie-Santilli isotheory and their applications.

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PART 2:

FOUNDATIONS OF HADRONIC MECHANICS

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Special Issue:
Issue II: Foundations of Hadronic Mechanics

Lead Guest Editor

Richard Anderson
Board of Trustees, the R. M. Santilli Foundation, Palm Harbor, Florida, USA

Introduction

In continuation of the new mathematics discussed in the preceding special issue entitled Foundations of Hadronic Mathematics, we recall that the Italian-American scientist R. M. Santilli proposed in 1978 the construction of a covering of quantum mechanics called hadronic mechanics, which is solely valid at mutual distances of one Fermi while recovering quantum mechanics identically and uniquely at larger mutual distances. By using the novel iso-, geno- and hyper-mathematics and their isoduals the Hadronic mechanics is divided into isomechanics, genomechanics and hypermechanics for the representation of single-valued, reversible, single-valued irreversible and multi-valued irreversible matter-system or reactions, respectively, with corresponding isodual for antimatter composite systems or reactions.

Thanks to the collaboration of numerous physicists, hadronic mechanics has now received applications and experimental verifications in classical mechanics, particle physics, nuclear physics, astrophysics, cosmology and other fields. The special issue of the AJMP entitled the Foundations of Hadronic Mechanics shall review some of these applications and present new advances that can potentially stimulate the birth of new technologies. It should be indicated that novel technologies solely predicted by hadronic mechanics have reached industrial applications, such as Thunder Energy Corporation, a U. S. publicly traded company with stock symbol TNRG, that has developed the first laboratory synthesis of neutrons from a hydrogen gas and is now entering into production and sale of the related equipment.

For more information about the Special Issue, please pay a visit to the following website:
http://www.sciencepublishinggroup.com/specialissue/122014

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Outline of Hadronic Mathematics, Mechanics and Chemistry as Conceived by R. M. Santilli

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Abstract: In this paper, we outline the various branches of hadronic mathematics and their applications to corresponding branches of hadronic mechanics and chemistry as conceived by the Italian-American scientist Ruggero Maria Santilli. According to said conception, hadronic mathematics comprises the following branches for the treatment of matter in conditions of increasing complexity: 1) 20th century mathematics based on Lie's theory; 2) IsoMathematics based on Santilli's isotopies of Lie's theory; 3) GenoMathematics based on Santilli's formulation of Albert's Lie-admissibility; 4) HyperMathematics based on a multi-valued realization of genomathematics with classical operations; and 5) HyperMathematics based on Vougiouklis H_v hyperstructures expressed in terms of hyperoperations. Additionally, hadronic mathematics comprises the anti-Hermitean images (called isoduals) of the five preceding mathematics for the description of antimatter also in conditions of increasing complexity. The outline presented in this paper includes the identification of represented physical or chemical systems, the main mathematical structure, and the main dynamical equations per each branch. We also show the axiomatic consistency of various branches of hadronic mathematics as sequential coverings of 20th century mathematics; and indicate a number of open mathematical problems. Novel physical and chemical applications permitted by hadronic mathematics are presented in subsequent collections.

Keywords: Santilli Isomathematics, Genomathematics, Hypermathematics

1. 20th Century Mathematics, Mechanics and Chemistry

1.1. Represented Systems

Single-valued, closed-isolated, time-reversible systems of point-like particles moving in vacuum solely under action at a distance Hamiltonian interactions, such as the structure of atoms and molecules.

1.2. Main Mathematical Structure

Basic unit

$$I = +1 \tag{1}$$

Basic numeric fields n = real, complex, quaternionic numbers

$$F(n,\times,1), n \tag{2}$$

Basic Associative product

$$nm = n \times m, 1 \times n = n \times 1 = n \; \forall n \in F \tag{3}$$

Measurement units of time, energy, etc. all positive
Ordinary functional analysis $f(r) \in F$,
Ordinary differential calculus
Conventional Lie theory

$$[X_i, X_j] = X_i \times X_j - X_j \times X_i == C_{ij}^k \times X_k, \tag{4}$$

$$A(w) = e^{X \times w \times i} \times A(0) \times e^{-i \times w \times X}. \tag{5}$$

Euclidean geometry and topology

$$E(r, \delta, 1), r = (r^k), k = 1,2,3, \delta = Diag.(1,1,1), \tag{6}$$

$$r^2 = r^i \times \delta_{ij} \times r^j = r_1^2 + r_2^2 + r_3^2 \in F, \tag{7}$$

Minkowskian geometry

$$M(x, \eta, I): x = (x^\mu), \mu = 1,2,3,4, x^4 = t, \tag{8}$$

$$\eta = Diag.(+1, +1, +1, -c^2), \tag{9}$$

$$x^2 = x^\mu \times \eta_{\mu\nu} \times x^\nu = x_1^2 + x_2^2 + x_3^2 - t^2c^2 \in F, \quad (10)$$

Riemannian geometry

$$R(x, g(x), I): x = (x^\mu), \mu = 1,2,3,4, x^4 = t, \quad (11)$$

$$x^2 = x^\mu \times g(x)_{\mu\nu} \times x^\nu \in F \quad (11)$$

$$x^2 = x^\mu \times g(x)_{\mu\nu} \times x^\nu \in F \quad (12)$$

Symplectic geometry.

$$\omega = dr^k \wedge dp_k \quad (13)$$

1.3. Dynamical equations

Newton equation

$$m \times \frac{dv}{dt} - F^{SA}(t, r, v,) = 0, \quad (14)$$

Variational principle

$$\delta A = \delta \int (p_k \times dr^k - H \times dt) = 0. \quad (15)$$

Hamilton's equations without external terms

$$\frac{dr^k}{dt} = \frac{\partial H(r,p)}{\partial p_k}, \quad \frac{dp_k}{dt} = -\frac{\partial H(r,p)}{\partial r^k}, \quad (16)$$

Hilbert space H over C with states $|\psi>$ over (C)
Expectation value of a Hermitean operator A

$$< A >=< \psi| \times A \times |\psi > \in C, \quad (17)$$

Heisenberg equation

$$i \times \frac{dA}{dt} = [A, H] = A \times H - H \times A, \quad (18)$$

Schrödinger equations

$$H \times |\psi >= E \times |\psi > \quad (19)$$

$$p \times |\psi >= -i \times \partial_r |\psi > \quad (20)$$

Dirac equation

$$(\eta^{\mu\nu} \times \gamma_\mu \times p_\nu - i \times m \times c) \times |\psi >= 0. \quad (21)$$

$$\{\gamma_\mu, \gamma_\nu\} = \gamma_\mu \times \gamma_\nu + \gamma_n u \times \gamma_\mu = 2 \times \eta_{\mu\nu}, \quad (22)$$

Comments and References
The literature on 20th century mathematics, mechanics and chemistry is so vast and so easily identifiable to discourage discriminatory partial listings.

2. Isomathematcs, Isomechanics and Isochemistry

2.1. Represented Systems [1-5]

Single-value, closed-isolated, time-reversible system of extended-deformable particles with action at a distance Hamiltonian and contact non-Hamiltonian interactions, such as the structure of hadrons, nuclei and stars, in the valence electron bonds and other systems.

2.2. Main Mathematical Structure s [1-5]

Santilli IsoUnit $\hat{I}$ and isotopic element $\hat{T}$[1]

$$\hat{I} = \hat{I}(r, p, a, \psi, \ldots.) = 1/\hat{T}(r, p, a, \psi, \ldots.) > 0, \quad (23)$$

Santilli IsoFields

$$\hat{F}(\hat{n}, \hat{\times}, \hat{I}), \hat{n} = n \times \hat{I}_{,,} \quad (24)$$

Santilli isoproduct

$$\hat{n} \,\hat{\times}\, \hat{m} = \hat{n} \times \hat{T} \times \hat{m} \in \hat{F}, \quad (25)$$

$$\hat{I} \,\hat{\times}\, \hat{n} = \hat{n} \,\hat{\times}\, \hat{I} = \hat{n} \; \forall \hat{n} \in \hat{F}, \quad (26)$$

Representation via the isotopic element of extended-deformable particles under non-Hamiltonian interactions

$$\hat{T} = Diag.(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}) \times e^{\Gamma(r,p,\psi,\partial\psi,\ldots)} \quad (27)$$

IsoCoordinates $\hat{r} = r \times \hat{I} \in \hat{F}$,
IsoFunctional analysis $\hat{f}(\hat{r}) = f(\hat{r}) \times \hat{I} \in \hat{F}$,
IsoDifferential Calculus

$$\hat{d}\hat{r} = dr + r \times \hat{T} \times d\hat{I}, \quad (28)$$

$$\frac{\hat{\partial} \hat{f}(\hat{r})}{\hat{\partial} \hat{r}} = \hat{I} \times \frac{\partial \hat{f}(\hat{r})}{\partial \hat{r}}, \quad (29)$$

Santilli Lie-Isotopic Theory

$$[\hat{X_i}, X_j] = X_i \,\hat{\times}\, X_j - X_j \,\hat{\times}\, X_i == C_{ij}^k(r, p, \ldots) \times X_k, \quad (30)$$

$$A(w) = \hat{e}^{X \times w \times i} \,\hat{\times}\, A(0) \,\hat{\times}\, \hat{e}^{-i \times w \times X}. \quad (31)$$

Santilli Iso-Euclidean Geometry

$$\hat{E}(\hat{r}, \hat{\delta}, \hat{I}), \hat{\delta}(r, p, z, \psi, \ldots) = \hat{T}(r, p, z, \psi, \ldots) \times \delta, \quad (32)$$

$$\hat{T} = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2), \quad (33)$$

$$\hat{r}^{\hat{2}} = \hat{r}^i \,\hat{\times}\, \hat{\delta}_{ij} \,\hat{\times}\, \hat{r}^j = (\frac{r_1^2}{n_1^2} + \frac{r_2^2}{n_2^2} + \frac{r_3^2}{n_3^2}) \times \hat{I} \in \hat{F}, \quad (34)$$

Santilli Iso-Minkowskian Geometry

$$\hat{M}(\hat{x}, \hat{\eta}, \hat{I}): \hat{x} = (\hat{x}^\mu), \mu = 1,2,3,4, x_4 = t, \quad (35)$$

$$\hat{\eta}(x, \psi, \ldots) = \hat{T}(x, \psi, \ldots) \times \eta, \quad (36)$$

$$\hat{T} = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2), \quad (37)$$

$$\hat{x}^{\hat{2}} = \hat{x}^\mu \,\hat{\times}\, \hat{\eta}_{\mu\nu} \,\hat{\times}\, \hat{x}^\nu = (\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2}) \times \hat{I} \in \hat{F}, \quad (38)$$

[1]See Santilli's curriculum
http://www.world-lecture-series.org/santilli-cv
Prizes and Nominations
http://www.santilli-foundation.org/santilli-nobel-nominations.html
and scientific archive
http://www.santilli-foundation.org/news.html

Santilli Iso-Riemannian Geometry

$$\hat{R}(\hat{x},\hat{g},\hat{I}):\hat{g}=\hat{T}(x,v,\dots)\times g(x), \quad (39)$$

$$\hat{x}^{\hat{2}}=(\frac{g_{11}}{n_1^2}+\frac{g_{22}}{n_2^2}+\frac{g_{33}}{n_3^2}-\frac{g_{44}}{n_4^2})\times\hat{I}\in\hat{F}, \quad (40)$$

Santilli Iso-Symplectic Geometry

$$\hat{\omega}=\hat{d}\hat{r}^k\,\hat{\wedge}\,\hat{d}\hat{p}_k \quad (41)$$

2.3. IsoDynamical IsoEquations s [1-5]

Newton-Santilli IsoEquation

$$\hat{m}\,\hat{\times}\,\frac{\hat{d}\hat{v}}{\hat{d}\hat{t}}-F^{SA}(t,r,p)=m\times\frac{dv}{dt}-F^{SA}(t,r,p)-F^{NSA}(t,r,p,\dots)=0, \quad (42)$$

Iso Variational principle

$$\hat{\delta}\hat{A}=\hat{\delta}\int\hat{}\;(\hat{p}_k\,\hat{\times}\,\hat{d}\hat{r}^k-\hat{H}\,\hat{\times}\,\hat{d}\hat{t})=0. \quad (43)$$

Hamilton-Santilli IsoEquations

$$\frac{\hat{d}\hat{r}^k}{\hat{d}\hat{t}}=\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{p}_k},\quad \frac{\hat{d}\hat{p}_k}{\hat{d}\hat{t}}=-\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{r}^k}, \quad (44)$$

Iso-Hilbert space $\hat{H}$ over C with states $|\hat{\psi}>$ over the isofield $\hat{C}$[2]

IsoExpectation value of a Hermitean operator $\hat{A}$ on $\hat{H}$

$$<\hat{A}>=<\hat{\psi}|\,\hat{\times}\,\hat{A}\,\hat{\times}\,|\hat{\psi}>\in\hat{C} \quad (45)$$

Heisenberg-Santilli IsoEquation

$$\hat{\imath}\,\hat{\times}\,\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}}=[\hat{A},\hat{H}]=\hat{A}\,\hat{\times}\,\hat{H}-\hat{H}\,\hat{\times}\,\hat{A}=\hat{A}\times\hat{T}(\hat{\psi},\dots)\times\hat{H}(\hat{r},\hat{p})-\hat{H}(\hat{r},\hat{p})\times\hat{T}(\hat{\psi},\dots)\times\hat{A} \quad (46)$$

Schrödinger-Santilli IsoEquation

$$\hat{H}\,\hat{\times}\,|\hat{\psi}>=\hat{H}(\hat{r},\hat{p})\times\hat{T}(\hat{\psi},\hat{\partial}\hat{\psi},\dots)\times|\hat{\psi}>=\hat{E}\,\hat{\times}\,|\hat{\psi}>=E\times|\hat{\psi}>, \quad (47)$$

$$\hat{p}\,\hat{\times}\,|\hat{\psi}>=-\hat{\imath}\,\hat{\times}\,\hat{\partial}_{\hat{r}}|\hat{\psi}>=-i\times\hat{I}\times\partial_{\hat{r}}|\hat{\psi}>, \quad (48)$$

Dirac-Santilli IsoEquation

$$(\hat{\eta}^{\mu\nu}\,\hat{\times}\,\hat{\gamma}_\mu\,\hat{\times}\,\hat{p}_\nu-\hat{\imath}\,\hat{\times}\,\hat{m}\,\hat{\times}\,\hat{c})\,\hat{\times}\,|\hat{p}si>=0. \quad (49)$$

$$\{\hat{\gamma}_\mu\hat{,}\hat{\gamma}_\nu\}=\hat{\gamma}_\mu\,\hat{\times}\,\hat{\gamma}_\nu+\hat{\gamma}_\nu\,\hat{\times}\,\hat{\gamma}_\mu=\hat{2}\,\hat{\times}\,\hat{\eta}_{\mu\nu}=2\times\hat{\eta}_{\mu\nu}, \quad (50)$$

2.4. Comments and References

As it is well known, the local-differential calculus of 20th century mathematics can solely represent a finite set of isolated dimensionless points. In view of this structural feature, Newton formulated his celebrated equations (14) for *massive points,* resulted in a conception of nature that was adopted by Galileo and Einstein, became the dominant notion of 20th century sciences, and was proved to be valid for classical or quantum particles moving in vacuum at large mutual distances, such as for our planetary system or the atomic structure.

However, when bodies move within physical media, such as for a spaceship during re-entry in our atmosphere or for a proton in the core of a star, point-like abstractions of particles became excessive, e.g., because a macroscopic collection of point-particles cannot have entropy (since all known Hamiltonian interactions are invariant under time reversal), with consequential violation of thermodynamical laws and other insufficiencies.

Besides the clear identification of these insufficiencies, the first historical contribution by the Italian-American scientist Ruggero Maria Santilli (see Footnote 1) has been the *generalization of 20th century mathematics into such a form to admit a time invariant representation of extended, and therefore deformable particles under conventional Hamiltonian as well as contact non-Hamiltonian interactions, with implications for all quantitative sciences.*

The above central objective was achieved in monographs [1] originally written by Santilli during his stay at MIT from 1974 to 1977 (where they appeared as MIT preprints). Monographs [1] were then completed by Santilli during his stay at Harvard Universityfrom 1977 to 1982 under DOE support, and released for publication only following the delivery at Harvard of a post Ph. D. seminar Course in the field.

The representation of extended-deformable bodies moving within physical media was achieved via an axiom-preserving lifting, called *isotopy,* of the conventional associative product $AB=A\times B$ between generic quantities A,B (such as numbers,functions, matrices, operators, etc.) into the form $A\,\hat{\times}\,B=A\times\hat{T}\times B$, Eq. (25). Conventional interactions are represented via conventional Hamiltonian, while actual shape and non-Hamiltonian interactions are represented via realization of the quantity $\hat{T}$, called *isotopic element,* of the type (27).

Santilli then achieved in monographs [1] the axiom-preserving isotopies of the various branches of Lie's theory, e.g., Eqs. (30), (31,) including their elaboration via the initiation of the isotopies of functional analysis. In particular, Santilli showed that the isotopies of the rotational symmetry $SO(3)$ characterized by isotopic element (27) do represent extended, generally non-spherical and deformable bodies. Finally, Santilli proved in Vol. II of Ref. [1] the

[2]As shown in the seminal paper [6] of 1982, but vastly ignored for the past four decades, isomechanics formulated on iso-Hilbert spaces over isofields eliminates the divergencies of quantum mechanics and related scattering theories. This important feature is primarily due to the fact that, for all physical and chemical applications worked out to date, the isounit $\hat{I}=1/\hat{T}>0$ must have a large value of the exponential type (27) and, consequently, the isotopic element $\hat{T}$ must have a very small value. This occurrence eliminates the singularity of the Dirac delta "distribution" when lifted to the Dirac-Myung-Santilli delta "isofunction" as shown by the realization of the type

$$\hat{\delta}(r-r_0)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{ik\hat{T}(r-r_0)}\,dk,$$

with $\hat{T}=\frac{N}{r-r_0}$, $N\ll 1$. Similarly, perturbative and other series with Hermitean operators that are divergent or slowly convergent in quantum mechanics can be lifted into isoseries of the type

$$A(w)=\hat{I}+\frac{w(A\hat{T}H-H\hat{T}A)}{1!}+\cdots$$

that are manifestly convergent for $w>1$ but $\hat{T}\ll w$.As shown by A. O. E. Animalu and R. M. Santilli in five papers published proceedings [25], the above lack of divergences carries over to the covering of the scattering theory known as isoscattering theory, by therefore achieving numerical results without the use of infinities for the renormalization of divergent series.

significance of his Lie-isotopic theory by showing that it characterizes the Birkhoffian covering of classical Hamiltonian mechanics and its "direct universality" for the representation of all possible, non-singular, generally non-Hamiltonian Newtonian systems in the frame of the experimenter, which direct universality was subsequently proved to hold also for isotopic operator theories. The above advances were formulated on an ordinary numeric field.

Subsequently, Santilli discovered in 1993 [2] that the axioms of numeric fields with characteristic zero do not necessarily require that the basic multiplicative unit is the trivial number $+1$, since said axioms admit arbitrary generalized units, today called *Santilli isounits*, provided that they are positive-definite and are the inverse of the isotopic element, $\hat{I} = 1/\hat{T} > 0$. This second historical discovery identified new numbers today known as *Santilli isoreal, isocomplex and isoquaternionic numbers* of the First (Second) kind when the isounit is outside (an element of) the original field. This discovery prompted a flurry of reformulation over Santilli isofields of all preceding isotopies, including most importantly the reformulation of Santilli's Lie-isotopic theory.

Despite the above momentous advances, Santilli remained dissatisfied because the isotopic formulations of the early 1990s were not invariant under their time evolution, thus being unable to predict the same numerical values under the same conditions at different times. Since the entire 20th century mathematics had been isotonically lifted by the early 1990s, Santilli was left with no other choice than that of reinspecting the Newton-Leibnitz differential calculus by discovering that, contrary to a popular belief in mathematics and physics for some four centuries, the differential calculus is indeed dependent on the basic multiplicative unit. In this way, Santilli achieved in memoir [3] of 1996 the third historical discovery according to which the ordinary differential calculus needs generalizations of the type (28), (29) whenever the isounit depends on the local variable of differentiation. This discovery signaled the achievement of mathematical maturity of isomathematics that permitted numerous advances in physics and chemistry as well as novel industrial applications.

All in all, Santilli has written about 150 papers on the isotopies of all various aspects of 20th century mathematics. These contributions are reported in monographs [4] of 1995 that remain to this day the most comprehensive presentation on isotopies. In the subsequent series of monographs [5] of 2008, Santilli introduces the names of *Hadronic Mathematics, Mechanics and Chemistry* which have been adopted for this review due to their wide acceptance.

Numerous authors have made important contributions in Santilli isomathematics, among whom we quote: the mathematician H. C. Myung who initiated (with R. M. Santilli) [6] the isotopies of Hilbert Spaces, including the momentous elimination of the divergencies of quantum mechanics under sufficiently small values of the isotopic element $\hat{T}$; the mathematicians D. S. Sourlas and G. T. Tsagas [7] who conducted in 1993 the first comprehensive study of the Lie-Santilli isotheory; the theoretician J. V. Kadeisvili [8] who presented systematic studies of Santilli's isotopies of 20th century geometries and relativities; the mathematician Chun-Xuan Jiang [9] who conducted in 2001 systematic studies of Santilli IsoNumber Theory; the mathematicians R. M. Falcon Ganfornina and J. Nunez Valdes who wrote in 2001 the now historical, first mathematically rigorous treatment of Santilli isotopies [10], and the historical achieved isotopology [11] which provides the ultimate mathematical structure of the Newton-Santilli isoequations (42) for extended-deformable particles under Hamiltonian and non-hamiltonian interactions achieved in memoir [3]; the mathematician S. Georgiev who wrote one of the most monumental and important mathematical works in scientific history [12], by showing that Santilli's IsoDifferential Calculus implies a variety of fully consistent coverings of 20th century mathematics; the mathematician A. S. Muktibodh [13] who presented the first known generalization of Santilli isonumber theory for the case of characteristic $p \neq 0$; the physicists I. Gandzha and J. Kadeisvili who presented in 2011 [14] a comprehensive review of Santilli isomathematics and its applications in physics and chemistry; plus additional seminal advances presented in the subsequent papers of this collection.

3. Genomathematics, Genomechanics and Genochemistry

3.1. Represented Systems s [1-5]

Single-valued, time-irreversible system of extended-deformable particles under action at a distance Hamiltonian and contact non-Hamiltonian interactions, as occurring in nuclear reactions, biological structures and chemical reactions.

3.2. Main Mathematical Structure s [1-5]

Santilli Forward GenoUnit

$$\hat{I}^{>} = \hat{I}^{>}(t^{>} r^{>}, p^{>}, a^{>}, \psi^{>}, \partial^{>}\psi^{>}, \ldots.) = 1/\hat{T}^{>} > 0, \quad (51)$$

Santilli Backward GenoUnit

$$^{<}\hat{I} = {}^{<}\hat{I}({}^{<}r, {}^{<}p, {}^{<}a, {}^{<}\psi, {}^{<}\partial {}^{<}\psi, \ldots.) = 1/{}^{<}\hat{T} > 0, \quad (52)$$

Condition for time-irreversibility

$$\hat{I}^{>} \neq {}^{<}\hat{I} \quad (53)$$

Forward GenoFields

$$\hat{F}^{>}(\hat{n}^{>}, >, \hat{I}^{>}), \hat{n}^{>} = n \times \hat{I}^{>} \quad (54)$$

Backward GenoFields

$$^{<}\hat{F}({}^{<}\hat{n}, <, {}^{<}\hat{I}), \quad {}^{<}\hat{n} = {}^{<}\hat{I} \times n, \quad (55)$$

Forward GenoProduct

$$\hat{n} > \hat{m} = \hat{n}^{>} \times \hat{I}^{>} \times \hat{m}^{>} \in \hat{F}^{>}, \quad (56)$$

$$\hat{I}^{>} > \hat{n}^{>} = \hat{n}^{>} > \hat{I}^{>} = \hat{n}^{>} \; \forall \, \hat{n}^{>} \in \hat{F}^{>} \quad (57)$$

Backward Genoproduct

$$<\hat{n} <^{<} \hat{m} =^{<} \hat{n} \times^{<} \hat{I} \times^{<} \hat{m} \in^{<} \hat{F}, \quad (58)$$

$$<\hat{I} <^{<} \hat{n} =^{<} \hat{n} <^{<} \hat{I} =^{<} \hat{n} \forall \quad <\hat{n} \in^{<} \hat{F}, \quad (59)$$

Representation of forward extended-deformable particles under non-Hamiltonian interactions

$$\hat{T}^{>} = Diag.(\tfrac{1}{n_1^2}, \tfrac{1}{n_2^2}, \tfrac{1}{n_3^2})^{>} \times e^{\Gamma(t,r,p,\psi,\partial\psi,\ldots)^{>}} \quad (60)$$

Forward GenoCoordinates

$$\hat{r}^{>} = r \times \hat{I}^{>} \in \hat{F}^{>}, \quad (61)$$

Backward GenoCoordinates

$$<\hat{r} =^{<} \hat{I} \times r \in^{<} \hat{F}, \quad (62)$$

Forward GenoFunctional analysis

$$\hat{f}^{>}(\hat{r}^{>}) = f(\hat{r}^{>}) \times \hat{I}^{>} \in \hat{F}^{>}, \quad (63)$$

Backward GenoFunctional analysis

$$<\hat{f}(<\hat{r}) = f(<\hat{r}) \times^{<} \hat{I} \in^{<} \hat{F}, \quad (64)$$

Forward GenoDifferential Calculus

$$\hat{d}^{>}\hat{r}^{>} = dr + r \times \hat{T}^{>} \times d\hat{I}^{>}, \quad (65)$$

$$\frac{\hat{\partial}^{>}\hat{f}^{>}(\hat{r}^{>})}{\hat{\partial}^{>}\hat{r}^{>}} = \hat{I}^{>} \times \frac{\partial \hat{f}^{>}(\hat{r}^{>})}{\partial \hat{r}^{>}}, \quad (66)$$

Backward GenoDifferential Calculus

$$<\hat{d}<\hat{r} = dr + r \times^{<} \hat{T} \times d^{<}\hat{I}, \quad (67)$$

$$-\frac{<\hat{\partial}<\hat{f}(<\hat{r})}{<\hat{\partial}<\hat{r}} =^{<} \hat{I} \times \frac{\partial^{<}\hat{f}(<\hat{r})}{\partial<\hat{r}}, \quad (68)$$

Santilli Lie-Admissible Theory

$$(X_i \hat{,} X_j) = X_i < X_j - X_j > X_i = C_{ij}^{k}(t,r,p,\psi,\ldots) \times X_k, \quad (69)$$

$$A(w) = \hat{e}_{>}^{X \times w \times i} > A(0) <_{<} \hat{e}^{-i \times w \times X}. \quad (70)$$

Santilli Forward Geno-Euclidean Geometry

$$\hat{E}^{>}(\hat{r}^{>}, \hat{\delta}^{>}, \hat{I}^{>}), \hat{\delta}^{>}(t,r,p,\psi,\ldots) = \hat{T}^{>}(t,r,p,\psi,\ldots) \times \delta, \quad (71)$$

$$\hat{r}^{>\hat{2}} = \hat{r}^{>i} > \hat{\delta}_{ij}^{>} > \hat{r}^{>j} \in F^{>}, \quad (72)$$

$$\hat{\delta}^{>} \neq \hat{\delta}^{>\, tranp} \quad (73)$$

Santilli Backward Geno-Euclidean Geometry

$$<\hat{E}(<\hat{r},<\hat{\delta},<\hat{I}), \quad <\hat{\delta}(t,r,p,\psi,\ldots) =^{<} \hat{T}(t,r,p,\psi,\ldots) \times \delta, \quad (74)$$

$$<\hat{2}\hat{r} =^{<i} \hat{r} <^{<} \hat{\delta}_{ij} <^{<j} \hat{r} \in^{<} F, \quad (75)$$

$$<\hat{\delta} \neq^{<\, transp} \hat{\delta} \quad (76)$$

Santilli Forward Geno-Minkowskian Geometry (μ = 1,2,3,4)

$$\hat{M}^{>}(\hat{x}^{>}, \hat{\eta}^{>}, \hat{I}^{>}): \hat{x}^{>} = (\hat{x}^{>\mu}), x_4^{>} = t^{>}, \quad (77)$$

$$\hat{\eta}^{>}(x,\psi,\ldots) = \hat{T}^{>}(x,\psi,\ldots) \times \eta, \quad (78)$$

$$\hat{x}^{>2} = \hat{x}^{>\mu} > \hat{\eta}_{\mu\nu}^{>} > \hat{x}^{>} \in \hat{F}^{>}, \quad (79)$$

$$\hat{\eta}^{>} \neq \hat{\eta}^{>\, transp} \quad (80)$$

Santilli Backward Geno-Minkowskian Geometry (μ = 1,2,3,4,)

$$<\hat{M}(<\hat{x},<\hat{\eta},<\hat{I}): \quad <\hat{x} = (\hat{x}^{\mu}), \quad <x_4 =^{<} t, \quad (81)$$

$$<\hat{\eta}(x,v,\ldots) =^{<} \hat{T}(x,v,\ldots) \times \eta, \quad (82)$$

$$<\hat{x}^{<2} =^{<\mu} \hat{x} <^{<} \hat{\eta}_{\mu\nu} <^{<\nu} \hat{x} \in^{<} \hat{F}, \quad (83)$$

$$<\hat{\eta} \neq^{<\, transp} \hat{\eta} \quad (84)$$

Santilli Forward Geno-Riemannian Geometry

$$\hat{R}^{>}(\hat{x}^{>}, \hat{g}^{>}, \hat{I}^{>}): \hat{g}^{>} = \hat{T}^{>}(x,v,\ldots) \times g(x), \quad (85)$$

$$\hat{x}^{>2} = \hat{x}^{>\mu} > \hat{g}_{\mu\nu}^{>} > \hat{x}^{>} \in \hat{F}^{>}, \quad (86)$$

$$\hat{g}^{>} \neq \hat{g}^{>\, transp} \quad (87)$$

Santilli Backward Geno-Riemannian Geometry

$$<\hat{R}(<\hat{x},<\hat{g},<\hat{I}): \quad <\hat{g} =^{<} \hat{T}(x,v,\ldots) \times g(x), \quad (88)$$

$$<\hat{x}^{<2} =^{<\mu} \hat{x} <^{<} \hat{g}_{\mu\nu} <^{<\nu} \hat{x} \in^{<} \hat{F}, \quad (89)$$

$$<\hat{g} \neq^{<\, transp} \hat{g} \quad (90)$$

Santilli Forward Geno-Symplectic Geometry

$$\hat{\omega}^{>} = \hat{d}^{>}\hat{r}^{>k} \hat{\wedge}^{>} \hat{d}^{>}\hat{p}_k^{>} \quad (91)$$

Santilli Backward Geno-Symplectic Geometry

$$<\hat{\omega} =^{<} \hat{d}^{<}\hat{r}^{<} \hat{\wedge}^{<} \hat{d}^{<}\hat{p}_k \quad (92)$$

3.3. GenoDynamical GenoEquations s [1-5]

Newton-Santilli Forward GenoEquation

$$\hat{m}^{>} > \frac{\hat{a}^{>}\hat{v}^{>}}{\hat{d}^{>}\hat{t}^{>}} - F^{>SA}(t,r,p) = [m \times \frac{dv}{dt}]^{>} - F^{SA>}(t,r,p) - F^{NSA>}(t,r,p,\ldots) = 0, \quad (93)$$

Newton-Santilli Backward GenoEquation

$$<\hat{m} < \frac{<\hat{a}^{<}\hat{v}}{<\hat{a}^{<}\hat{t}} -^{<SA} F(t,r,p) = =^{<} [m \times \frac{dv}{dt}] -^{<SA} F(t,r,p) -^{<NSA} F(t,r,p,\ldots) = 0, \quad (94)$$

Forward GenoVariational principle

$$\hat{\delta}^{>}\hat{A}^{>} = \hat{\delta}^{>} \hat{\int}^{>} (\hat{p}_k^{>} > \hat{d}^{>}\hat{r}^{>k} - \hat{H}^{>} > \hat{d}^{>}\hat{t}^{>}) = 0. \quad (95)$$

Backward GenoVariational principle

$$<\hat{\delta}^{<}\hat{A} =^{<} \hat{\delta}^{<} \hat{\int}^{<} (<\hat{p}_k <^{<} \hat{d}^{<}\hat{r}^{k} -^{<} \hat{H} <^{<} \hat{d}^{<}\hat{t}) = 0. \quad (96)$$

Forward Hamilton-Santilli GenoEquations

$$\left[\frac{d\hat{r}^k}{dt}\right]^{>} = \left[\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{p}_k}\right]^{>}, \quad \left[\frac{d\hat{p}_k}{dt}\right]^{>} = -\left[\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{r}^k}\right]^{>}, \tag{97}$$

Backward Hamilton-Santilli GenoEquations

$$<\left[\frac{d\hat{r}^k}{dt} = \frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{p}_k}\right], \quad <\left[\frac{d\hat{p}_k}{dt}\right] = -<\left[\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{r}^k}\right], \tag{98}$$

Forward Geno-Hilbert space $\hat{H}^{>}$ with states $|\hat{\psi}^{>}>$ over the isofield $\hat{C}^{>}$

GenoExpectation value of a Hermitean operator $\hat{A}$ on $\hat{H}^{>}$

$$<\hat{A}^{>}>=<^{<}\hat{\psi}|<\hat{A}^{>}>|\hat{\psi}^{>}>\in\hat{C} \tag{99}$$

Heisenberg-Santilli GenoEquation[3]

$$\hat{\imath}\,\hat{\times}\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = (\hat{A},\hat{H}) = \hat{A}<\hat{H} - \hat{H}>\hat{A} = A\times^{<}T(\hat{\psi},\ldots)\times \hat{H}(\hat{r},\hat{p}) - \hat{H}(\hat{r},\hat{p})\times\hat{T}^{>}(\hat{\psi},\ldots)\times\hat{A} \tag{100}$$

Forward Schrödinger-Santilli GenoEquation

$$\hat{H}^{>}>|\hat{\psi}^{>}>=\hat{H}^{>}(\hat{r},\hat{p})\times\hat{T}^{>}(\hat{\psi},\hat{\partial}\hat{\psi},\ldots)\times|\hat{\psi}^{>}>=\hat{E}^{>}>|\hat{\psi}^{>}>=E^{>}\times|\hat{\psi}^{>}>, \tag{101}$$

$$\hat{p}^{>}>|\hat{\psi}^{>}>=-\hat{\imath}^{>}>\hat{\partial}_{\hat{r}}^{>}|\hat{\psi}^{>}>=-i\times\hat{I}^{>}\times\partial_{\hat{r}}|\hat{\psi}^{>}>, \tag{102}$$

Backward Schrödinger-Santilli GenoEquation

$$<^{<}\hat{\psi}|<^{<}\hat{H} =<^{<}\hat{\psi}|\times^{<}\hat{T}(\hat{\psi},\hat{\partial}\hat{\psi},\ldots)\times^{<}\hat{H}(\hat{r},\hat{p}) = <^{<}\hat{\psi}|<^{<}\hat{E} =<^{<}\hat{\psi}|\times^{<}E, \tag{103}$$

$$<^{<}\hat{\psi}|<^{<}\hat{p} = -<^{<}\hat{\psi}|<^{<}\hat{\imath}<_{\hat{r}}^{<}\hat{\partial} = -i\times<^{<}\hat{\psi}|_{\hat{r}}^{<}\partial\times^{<}\hat{I} \tag{104}$$

Forward Dirac-Santilli IsoEquation

$$(\hat{\eta}^{>\mu\nu}>\hat{\gamma}_{\mu}^{>}>\hat{p}_{\nu}^{>} - \hat{\imath}^{>}>\hat{m}^{>}>\hat{c}^{>})>|\hat{p}si^{>}>=0. \tag{105}$$

$$\{\hat{\gamma}_{\mu},\hat{\gamma}_{\nu}\}^{>} = [\hat{\gamma}_{\mu}\,\hat{\times}\,\hat{\gamma}_{\nu} + \hat{\gamma}_{\nu}\,\hat{\times}\,\hat{\gamma}_{\mu}]^{>} = \hat{2}^{>}>\hat{\eta}_{\mu\nu}^{>}, \tag{106}$$

Backward Dirac-Santilli GenoEquation

$$<^{<}\hat{\psi}|<(<^{<}\hat{p}_{\nu}<^{<}\hat{\gamma}_{\mu}<^{<\mu\nu}\hat{\eta} - <^{<}\hat{\imath}<^{<}\hat{m}<^{<}\hat{c}) = 0. \tag{107}$$

$$<^{<}\{\hat{\gamma}_{\mu},\hat{\gamma}_{\nu}\} =<^{<}[\hat{\gamma}_{\mu}\,\hat{\times}\,\hat{\gamma}_{\nu} + \hat{\gamma}_{\nu}\,\hat{\times}\,\hat{\gamma}_{\mu}] =<^{<}\hat{2}<^{<}\hat{\eta}_{\mu\nu} = 2\times^{<}\hat{\eta}_{\mu\nu}, \tag{108}$$

3.4. Comments and References

As it is also well known, all 20th century mathematical, physical or chemical formulations are reversible over time. Following research over half a century initiated during his Ph. D. studies at the University of Torino, Italy, in the mid 1960s [15, 17-23,4,5], R. M., Santilli has made the additional historical discovery of the first and only known, axiomatically consistent, generalization of 20th century mathematics as well as of its covering isomathematcs into a form embedding irreversibility over time in ordered forward and backward units, in corresponding ordered forward and backward products and, consequently, in all subsequent mathematical structures, resulting in the new mathematics nowadays known as *Santilli forward and backward genomathematics* with corresponding physical and chemical theories for the representation of irreversible processes.

Since the reversibility over time of 20th century theories can be reduced to the invariance under anti-Hermiticity of the Lie product between Hermitean operators, $[a,b] = ab - ba = -[a,b]^{\dagger}$, Santilli presented in 1967 [15] the first known (p, q)-deformation of the Lie product $(a,b) = pab - qba$, where p, q are scalars and the product ab is generally non-associative. Following an intense search in European mathematical libraries, Santilli discovered that the new product verifies the axiom of *Lie-admissibility* by the American mathematician A, A, Albert [16] in the sense that the attached anti-symmetric product $[a,b] = (a,b) - (b,a)$ verifies the axioms of a Lie algebra.

Since spaceship during re-entry are notoriously irreversible over time, Santilli was invited by the Center for Theoretical Physics of the University of Miami, Florida, under NASA support, where he moved with his wife Carla and newly born daughter Luisa inAugust 1967, and published a number of additional works in Lie-admissibility, including the first known Lie-admissible generalization of Hamilton and Heisenberg equations [17,18], nowadays considered at the foundation of hadronic mechanics and chemistry, as well as the first and only known Lie-admissible formulation of dissipative plasmas surrounding spaceships during reentry [19].

Santilli then spent seven years , from 1968 to 1974, at the Department of Physics of Boston University, and then three years, from 1974 to 1977, at MIT, during which tine he wrote, in his words, *Phys.. Rev of career-oriented papers nobody reads.* InSeptember 1977, Santilli joined Harvard University and was invited by the DOE to study irreversible processes because all energy releasing processes are irreversible over time. In April 1978, Santilli published under his DOE support his most important mathematical contribution [20] (see also monographs [21]) in which he achieved a Lie-admissible covering of the various branches of Lie's theory, Eqs. (69), (70), including the most general known time evolution whose brackets characterize an algebra, Eqs. (1000). It should be indicated that the isotopies of Lie's theory outlined in the preceding section were derived by Santilli as a particular case of the broader Lie-admissible theory of Ref. [20], and then published in monographs [1].

Subsequently, Santilli discovered in paper [2] of 1993 that the axiom of a numeric field, besides admitting a generalization of the multiplicative unit, also admit the restriction of the associative product to an ordered form to the right and, separately,to the left. In this way, Santilli discovered two additional classes of new numbers, today known as *Santilli forward and backward genoreal, genocomplex and genoquarternionic numbers.* In the seminal memoir [3] of

[3] By including the multi-valued (Section 4) and hyperstructural formulations (Section 5), Lie-admissible equations (100) are so broad that it will take centuries for their generalizations. For this reasou,Santilli has requested in his will that his tombstone should have the engraving

$$i\dot{A} = A<H - H>A$$

below his name.

1996 Santilli discovered two additional coverings of the ordinary differential calculus and of its isotopic covering, today known as *Santilli forward and backward genodifferential calculi,* Eqs. (65) to (68). Santilli called a *genotopy* [20] the lifting of isomathematics into ordered formulations to the right and to the left in the Greek sense of inducing a covering of Lie's axioms, Eqs. (69), (70).

As it is well known, thousands of papers have been published beginning from the late 1980s on the so-called q-deformations of Lie algebras with product $(a,b) = ab - qba$ which are an evident particular case of Santilli Lie-admissible product [15]. Whatit is lesser known, or not admitted, all q-deformations did not achieve invariance over time, thus being afflicted by serious inconsistencies, since they consisted of non-unitary theories formulated via the mathematics of unitary theories. Santilli solved this problem in 1997 by achieving the first and only known invariant formulation of q- as well as of (p, q)-deformations [22].

We should indicate that Santilli's conception of a genotopic lifting of his preceding isomathematcs (indicated in Section 2 by "hat" on symbols plus the "arrow of time") is necessary to achieve a consistent representation of irreversibility because point-like particles can only experience action-at-a-distance interactions that are reversible over time. Therefore, a simple genotopy of 20th century mathematics based on the conventional associative product would be axiomatically inconsistent. Consequently, to represent irreversibility it is first necessary to lift 20th century mathematics into isomathematcs, with consequential representation of extended-deformable particles via realizations of type (27) so that extended particles can experience non-Hamiltonian interactions needed for irreversibility. It is then necessary to add irreversibility via the ordering of all products. It should also be indicated that, when formulated via time-dependent isounits, isomathematics can becomes genomathematics via the identifications $\hat{I}(t,\dots) = \hat{I}^{\dagger}(t,\dots) = \hat{I}^{>}, \hat{I}(-t,\dots) = \hat{I}^{\dagger}(-t,\dots) =^{<} \hat{I}, \hat{I}(t,\dots) \neq \hat{I}(-t)$, and the judicious addition of ordered products.

Systematic studies on the Lie-Admissible treatment of irreversible systems were presented in memoir [3] and monographs [4]. Santilli's subsequent memoir [23] of 2006 remains to this day the most comprehensive presentation of Lie-admissible treatments of irreversibility at the classical and operator levels. Monographs [5] of 2008 presented an update. Paper collection [24[presents all available independent contributions in Lie-admissibility up to [1984. The Proceedings of the *Third International Conference on Lie-admissible Treatment of Irreversible Systems* [25] present numerous additional independent contributions as well as references for the five *Workshops on Lie-0admissible Algebras* organized by Santilli at Harvard University, and for the preceding two international conference in Lie-admissibility, the first at the Université d'Orleans, France, in 1981 and the second at the Castle Prince Pignatelli, Italy, in 1995 (see also the general review [14] and large literature quoted therein).

As it is well known, there exists a large number of papers on Lie-admissible algebras within the context of non-associative algebras (see Tomber's Bibliography [26] listing all significant papers in the field up to 1986). It should be indicated that, regrettably, these studies have no connection with Santilli genomathematics since the latter deals with the irreversible generalizations of all aspects of 20th century mathematics.

4. Classical Hypermathematcs, Hypermechanics and Hyperchemistry

4.1. Represented Systems s [1-5]

Multi-valued, time-irreversible systems of extended -deformable particles or constituents under the most general known Hamiltonian and non-Hamiltonian interaction, as occurring for multi-valued universes or the structure of the DNA.

4.2. Main Mathematical Structure s [1-5]

Basic HyperUnits and HyperProducts

$$\hat{I}^{>} = \{\hat{I}_1^{>}, \hat{I}_2^{>}, \hat{I}_3^{>}, \dots\} = 1/\hat{S}, \tag{109}$$

$$^{<}\hat{I} = \{^{<}\hat{I}_1, ^{<}\hat{I}_2, ^{<}\hat{I}_3, \dots\} = \frac{1}{\hat{R}}, \tag{110}$$

Forward and Backward HyperProducts

$$A > B = \{A \times \hat{S}_1 \times B, A \times \hat{S}_2 \times B, A \times \hat{S}_3 \times B, \dots\}, \hat{I}^{>} > A = A > \hat{I}^{>} = A \times I, \tag{111}$$

$$A < B = \{A \times \hat{R}_1 \times B, A \times hat R_2 \times B, A \times \hat{R}_3 \times B, \dots\}^{<}\hat{I} < A = -A <^{<} \hat{I} = I \times A, \tag{112}$$

$$A = A^{\dagger}, B = B^{\dagger}, \hat{R} = \hat{S}^{\dagger}. \tag{113}$$

Classical hypermathematcs then follow as for genomathematcs with multi-valued units, quantities and operations.

4.3. Classical Hyper-Dynamical Equations s [1-5]

The same as those for genomathematics, but with multi-valued hyperunits, quantities and operations.

Comments and References

The multi-valued three-dimensional (rather than multi-dimensional) realization of genomathematics outlined in Section 4 emerged from specific biological needs. The Australian biologist C. Illert [27] confirmed that the *shape* of seashells can indeed be represented in a three-0dimensional Euclidean space as known since Fourier's time, but proved that the *growth in time* of a seashell cannot any longer be consistently represented in a conventional, three-dimensional Euclidean space, and achieved a consistent representation via the doubling of the three reference axis.

Santilli [27,28] confirmed Illert's findings because the conventional Euclidean geometry has no time arrow and, consequently, cannot consistently represent a strictly irreversible system, such as the growth of seashells. Additionally, Santilli proved thathis geno-Euclidean geometry,

Eqs. (71) to (73), is equally unable to represent the growth in time of seashells despite its irreversible structure, however, an axiomatically consistent and exact representation of the growth of seashells was possible via the multi-valued realization of the forward geno-Euclidean geometry, thus beginning to illustrate the complexity of biological structures.

The multi-valued, rather than multi-dimensional character of classical hypermathematics is indicated by Santilli as follows [28] *We perceive the growth of a seashell specifically in three dimensions from our Eustachian lobes. Therefore, an irreversible mathematics suitable to represent the growth of sea shells must be perceived by us as being in three dimensions. However, Illert has shown the need to double the three Cartesian axis. Classical hypermathematics has been conceived and structured in such a way that the increase of the reference axes is complemented by a corresponding multi-valued hyperunit in such a way that a classical hyper-Euclidean geometry, when seen at the abstract level, remains indeed three-dimensional as necessary to achieve representation of biological structures compatible with our sensory perception.*

5. Hope Hypermathematics, Hypermechanics and Hyperchemistry

Represented Systems

The most complex known multi-valued, time-irreversible requiring extremely large number of data, such as the DNA code [31-35].

Comments and References

Despite the preceding structural generalization of 20th century mathematics, Santilli remained dissatisfied in view of the complexity of nature, particularly of biological entities because advances in the *structure* of the DNA are indeed possible via classical hypermathematics, as we shall see in the third collection of this series dedicated to chemistry (e.g., via Santilli hypermagnecules), but any attempt at representing the DNA *code* via any of the preceding mathematics can be proved to be excessively restrictive due to the volume, complexity, diversification and coordination of the information.

Therefore, Santilli approved one of the most important mathematicians in hyperstructures, T. Vougiouklis from Greece, and asked for his assistance in further generalizing the preceding mathematics via hyperstructures defined on hyperfields, as necessary for applications implying measurements, and formulated via hyperoperations (called "hope") permitting the needed broadening of the representational capability.

The above contact lead to the hypermathematics indicated in this section as presented in Refs. [29-33] which is based on Vougiouklis H_v hyperaxioms and which mathematics, in Santilli's words, *constitutes the most general mathematics that can be conceived nowadays by the human mind.*

6. Isodual Mathematics, Mechanics and Chemistry

6.1. Represented Systems

Single-valued, closed-isolated, time-reversible systems of classical and operatorpoint-like antiparticles moving in vacuum solely under action at a distance Hamiltonian interactions, such as the stricture of antimatter atoms and antimatter molecules [2,36-43].

6.2. Main Mathematical Structure [2,36-43]

Basic isodual unit

$$1^d = -1^\dagger = -1, \tag{114}$$

Isodual numeric fields

$$F^d(n^d, \times^d, 1^d), n^d = n \times 1^d, n^d \times^d m^d = n^d \times (1^d)_{-1} \times m^d \in F^d,$$

$$n^d = isodual\,!\,real, complex, quatern.\,!\,numbers, \tag{115}$$

Isodual functional analysis

$$f^d(r^d) = f(r^d) \times 1^d \in F^d \tag{116}$$

Isodual differential calculus

$$d^d r^d = (1)^{-1} \times dr^d = dr, \tag{117}$$

$$\frac{\partial^d f^d(r^d)}{\partial^d r^d} = 1^d \times \frac{\partial f^d(r^d)}{\partial r^d}, \tag{118}$$

Santilli Isodual Lie theory

$$[X_i, X_j]^d = (X_i \times X_j - X_j \times X_i)^d == -C_{ij}^k \times X_k, \tag{119}$$

$$A^d(w^d) = e_d^{X \times w \times i} \times^d A^d(0) \times^d e_d^{-i \times w \times X}. \tag{120}$$

Santilli isodual Euclidean geometry

$$E^d(^d, \delta^d, 1^d), r^d = (r^{dk}), k = 1,2,3,$$

$$\delta^d = Diag.(-1,-1,-1), \tag{121}$$

$$r^{d2d} = r^{di} \times \delta_{ij} \times^d r^{dj} = (r_1^2 + r_2^2 + r_3^2) \times 1^d \in F^d, \tag{122}$$

Santilli Isodual Minkowskian geometry ($\mu = 1,2,3,4,$)

$$M^d(x^d, \eta^d, I^d): x^d = (x^{d\mu}), x^{d4} = t^d = t \times 1^d = -t, \tag{123}$$

$$\eta^d = Diag.(-1,-1,-1,+c^{d2d}), \tag{124}$$

$$x^{d2d} = (x^\mu \times \eta_{\mu\nu} \times x^\nu)^d = (x_1^2 + x_2^2 + x_3^2 - t^2c^2) \times 1^d \in F^d, \tag{125}$$

Isodual Riemannian geometry, Santilli Isodual Symplectic Geometry.

6.3. Isodual Dynamical Equations [2,36-43

Newton-Santilli Isodual Equation

$$m^d \times^d \frac{d^d v^d}{d^d t^d} - F^{dSA}(t^d, r^d, v^d) = 0, \quad (126)$$

Isodual Variational Principle

$$\delta^d A^d = \delta^d \int^d (p_k^d \times^d d^d r^{dk} - H^d \times^d d^d t^d) = 0. \quad (127)$$

Hamilton-Santilli Isodual Equations without external terms

$$\frac{d^d r^{dk}}{d^d t^d} = \frac{\partial^d H^d(r^d, p^d)}{\partial^d p_k^d}, \quad \frac{d^d p_k^d}{d^d t^d} = -\frac{\partial^d H^d(r^d, p^d)}{\partial^d r^{dk}}, \quad (128)$$

Isodual Hilbert space H^d over C with states $|\psi^d >= -< \psi|$ over C^d

Expectation value of a Hermitean operator A

$$< A^d >=< \psi| \times A^d \times |\psi > \in C^d m \quad (129)$$

Heisenberg-Santilli Isodual Equations

$$i^d \times^d \frac{d^d A^d}{d^d t^d} = [A, H]^d = (A \times H - H \times A)^d, \quad (130)$$

Schrödinger-Santilli Isodual Equations

$$H^d \times^d |\psi^d >= E^d \times^d |\psi^d >= -E \times |\psi > \quad (131)$$

$$p^d \times^d |\psi^d >= +i^d \times^d \partial_{rd}^d |\psi^d > \quad (132)$$

Dirac-Santilli Isodual Equation

$$(\eta^{d\mu\nu} \times^d \gamma_\mu^d \times^d p_\nu^d + i^d \times^d m^d \times^d c^d) \times |\psi >= 0. \quad (133)$$

$$\{\gamma_\mu, \gamma_\nu\}^d = (\gamma_\mu \times \gamma_\nu + \gamma_\nu u \times \gamma_\mu)^d = 2^d \times^d \eta_{\mu\nu}^d, \quad (134)$$

Comments and References

In addition to the the study of irreversible processes and the representation of extended-deformable particles, during his Ph. D. studies of the md 1960s Santilli was interested to ascertain whether a far away galaxy is made up of matter or of antimatter. He soon discovered that none of the mathematics and physics he had learned during his graduate studies was applicable for a quantitative study of the problem considered since, at that time, antimatter was solely represented in second quantization, while the study of far away antimatter galaxies requested their representation at the purely *classical and neutral* level. In this way, Santilli initiated a solitary scientific journey that lasted for half a century.

This occurrence created one of the biggest imbalances in scientific history because matter was treated at all possible levels, from Newtonian mechanics to second quantization, while antimatter was solely treated in second quantization. The imbalance originated from the fact that special and general relativities had been conceived decades before the discovery of antimatter and, therefore, they had no possibility of representing antimatter at the classical and neutral (as well as charged) level.

It should be stressed that the ongoing trend to extend the application of special and general relativities to the classical treatment of antimatter is afflicted by a number of serious inconsistencies, such as the impossibility to achieve a consistent representation of neutral antimatter, the impossibility to reach a consistent representation of matter-antimatter annihilation (evidently due to the lack of a suitable conjugation from matter to antimatter), violation of the PCT theorem and other inconsistencies that remain generally ignored.

Being an applied mathematician by instinct and training, Santilli knew that the imbalance was the result of a purely mathematical insufficiency because the transition from matter to antimatter is an anti-homomorphism. Consequently, the description of antimatter required a mathematics which is anti-homomorphic to conventional mathematics.

Santilli dedicated a decade to the search of the needed mathematics for antimatter. Following an additional extended search done at the Department of Mathematics of Harvard University under DOE support in the early 1980s, *Santilli concluded that a mathematics suitable for the joint classical and operator treatment of antimatter did not exist and had to be constructed.*

In the early 1980s, Since he had introduced the isoproduct $A \hat{\times} B = A \times \hat{T} \hat{B}, \hat{T} > 0$, Eq. (25). Consequently, it was natural to introduce its *negative-definite* counterpart which he called *isodual* and denoted with theupper index ^{d}, namely $A \hat{\times}^d B = A \times \hat{T}^d \hat{B}, \hat{T}^d = (\hat{T}^d)^\dagger < 0$. While constructing the isotopies of 20th century mathematics presented in Section 2, Santilli initiated the construction of their isodual image but published no paper in the new mathematics for over a decade.

This caution was due to the fact that, despite the lack of any visible mathematical inconsistency, Santilli remained skeptical on a mathematics based on a negative-definite product is afflicted by known physical inconsistencies, such as the violation of causality for negative time, energies and other physical quantities.

A breakthrough occurred in paper [2] of 1993. During the achievement of the broadest possible realizations of the abstract axioms of a numeric field (of characteristic zero), Santilli discovered that realizations with negative-definite units were simply unavoidable. This lead to the discovery of additional new numbers, today known as *Santilli isodual real, isodual complex and isodual quaternionic numbers* occurring for $I^d = -1$, Eq. (14), with isodual products (5), which are at the foundation of the isodual mathematics of this section and the additional numbers known as *Santilli isodual iso- and isodual geno-real, complex and quaternionic numbers* which are at the foundation of the isodual isomathematics and isodual genomathematics of Sections **7** and 8m respectively [2].

The discovery of isodual numbers is truly historical in our view due to its far reaching implications. In fact, the discovery established the existence of the desired *isodual mathematics* as an anti-isomorphic image of 20th century mathematics for the representation of antimatter. Additionally, the discovery permitted the resolution of the problems of causality for negative values of physical quantities.

To avoid insidious inconsistencies generally not seen by non-experts in the field, the isodual map must be applied for consistency to the *totality* of quantities and their operations. This lead to Santilli's conception of antimatter as possessing it

negative-definite physical quantities for time, energy, momentum, frequency, etc, but such negative values are referred to *negative units* of measurements. Consequential a theory with negative time referred to negative units of time is as causal as our reality with a positive time referred to positive units, and the same holds for all other physical quantities.

Following the resolution of these basic issues, Santilli published in 1994 his first paper [36] specifically devoted to the isodual representation of antimatter. In mathematical memoir [3] of 1996, Santilli achieved the first isodual mathematical and physical representation of antimatter. In paper [37] of 1998, Santilli achieved his first goal of the early 1960s, namely, a consistent classical representation of neutral (as well as charged) antimatter.

By the early 1990s, Santilli had shown that *isodual mathematics represents all available experimental, data on antimatter at the classical and operator level.* Hence, he initiated the second phase of his studies, namely, the identification of new predictions for subsequent experimental verification.

A breakthrough occurred at the 1996 *First International Conference on Antimatter* help in Sepino, Italy [38]. By that time, Santilli had shown that the only conceivable representation of *neutral* antimatter required the conjugation of the sign of all physical quantities (jointly with the corresponding conjugation of their units of measurements). Since photons are neutral, the application of the same principle to light implies light emitted by antimatter, that Santilli called *isodual light,* is physically different than light emitted by matter in an experimentally verifiable way, e.g., because antimatter light is predicted to be *repelled* by a matter gravitational field.

Santilli then passed to a deeper geometric study of the gravitational field of antimatter. As indicated earlier, general relativity was formulated decades before the discovery of antimatter and, therefore, had no clue for the representation of the gravitational field of antimatter bodies. In Ref.[39] of 1998, Santilli conducted an in depth geometric study of antimatter, and in monograph [40) of 2006, Santilli completed the gravitational study of antimatter via the isodual Riemannian geometry.

All these studies concluded with the prediction of *gravitational repulsion* (antigravity) between matter and antimatter at all levels of analysis, from the isodual Newton-Santilli equations (26) to isodual second quantization. These aspects will be studied in the second collection of this series dedicated to hadronic mechanics.

Thanks to all the above advances, Santilli was finally in a position to address his original main aim of the 1960s, namely, ascertain whether a far away galaxy is made up of matter or of antimatter. The preceding studies had established that the light emitted by antimatter must have a *negative index of refraction* that, as such, require *concave* lenses for its focusing. Consequently, Santilli secured the construction of a revolutionary telescope with concave lenses. About fifty years following his original aim, Santilli finally published in 2013 [41[measurements of the night sky with his new telescope showing images that can be solely due to light with a negative index of refraction which light, in turn, can solely originate from far away antimatter stars or galaxies (see also the two independent confirmations [42,43]).

An intriguing aspect that should be of interest to pure mathematicians is the conclusion of these studies illustrating the power of new mathematics, to the effect that none of the large numbers of telescopes available nowadays can detect antimatter starsor galaxies since they all have *convex* lenses. Similarly, as humans evolved in a matter world, we will never be able to see antimatter with our eyes since our cornea is convex and, as such, it will disperse antimatter light all over the retina.

Needless to say, isodual mathematics and its application to antimatter have implications so intriguing that are stimulating the participation of a large number of scientists as we shall report in the second collection of this series

7. Isodual Isomathematics, Isodual Isomechanics and Isodual Isochemistry

7.1. Represented Systems [2,36-43

Single-value, closed-isolated, time-reversible system of classical or operator extended-deformable antiparticles with action at a distance Hamiltonian and contact non-Hamiltonian interactions, such as the structure of antimatter hadrons, nuclei and stars, in the antimatter valence electron bonds and other antimatter systems.

7.2. Main Mathematical Structure[2,36-43

Basic Isodual IsoUnit

$$\hat{I}^d = \hat{I}^d(r^d, p^d, a^d, \psi, {}^d\partial^d\psi^d, \ldots.) = 1^d/{}^d\hat{I}^d < 0, \quad (135)$$

Basic Isodual IsoFields

$$\hat{F}^d(\hat{n}^d, \hat{\times}^d, \hat{I}^d), \hat{n}^d = n \times \hat{I}^d, \hat{n}^d \,\hat{\times}^d\, \hat{m}^d = \hat{n}^d \times \hat{I}^d \times \hat{m}^d \in \hat{F}^d, \quad (136)$$

Isodual IsoCoordinates $\hat{r}^d = r \times \hat{I}^d \in \hat{F}^d$,

Isodual IsoFunctional analysis $\hat{f}^d(\hat{r}^d) == f(\hat{r}^d) \times \hat{I}^d \in \hat{F}^d$,

Isodual IsoDifferential Calculus

$$\hat{d}^d\hat{r}^d = dr - r^d \times \hat{I}^d \times d\hat{I}^d, \quad (137)$$

$$\frac{\hat{\partial}^d \hat{f}^d(\hat{r})}{\hat{\partial}^d\hat{r}^d} = \hat{I}^d \times \frac{\partial \hat{f}^d(\hat{r}^d)}{\partial \hat{r}^d}, \quad (138)$$

Santilli Isodual Lie-Isotopic Theory

$$[X_i, \hat{X}_j]^d = X_i \,\hat{\times}\, X_j - X_j \,\hat{\times}\, X_i)^d == -C_{ij}^k(r, p, \ldots) \times X_k, \quad (139)$$

$$A^d(w^d) = \hat{e}_d^{X^d \times w^d \times i^d} \,\hat{\times}^d\, A^d(0^d) \,\hat{\times}^d\, \hat{e}_d^{-i^d \times w^d \times X^d}. \quad (140)$$

Santilli Isodual Iso-Euclidean Geometry

$$\hat{E}^d(\hat{r}^d, \hat{\delta}^d, \hat{I}^d), \hat{\delta}^d(r^d, p^d, a^d, \psi, \ldots) =$$

$$\hat{T}^d(r^d, p^d, a^d, \psi^d, \ldots) \times \delta, \quad (141)$$

$$\hat{T}^d = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2)^d, \quad (142)$$

$$\hat{r}^{d\hat{2}d} = (\hat{r}^i \hat{\times} \hat{\delta}_{ij} \hat{\times} \hat{r}^j)^d = (\frac{r_1^2}{n_1^2} + \frac{r_2^2}{n_2^2} + \frac{r_3^2}{n_3^2})^d \times \hat{I}^d \in \hat{F}^d, \quad (143)$$

Santilli Isodual Iso-Minkowskian Geometry ($\mu = 1,2,3,4$)

$$\hat{M}^d(\hat{x}^d, \hat{\eta}^d, \hat{I}^d): \hat{x}^d = (\hat{x}^{d\mu}), \hat{x}_4^d = \hat{t}^d = t \times \hat{I}^d, \quad (144)$$

$$\hat{\eta}^d(x^d, \psi^d, \ldots) = \hat{T}^d(x^d, \psi^d, \ldots) \times \eta, \quad (145)$$

$$\hat{T}^d = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2)^d, \quad (146)$$

$$\hat{x}^{d\hat{2}d} = (\hat{x}^\mu \hat{\times} \hat{\eta}_{\mu\nu} \hat{\times} \hat{x}^\nu)^d == (\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2})^d \times \hat{I}^d \in \hat{F}^d, \quad (147)$$

Santilli Isodual Iso-Riemannian Geometry

$$\hat{R}^d(\hat{x}^d, \hat{g}^d, \hat{I}^d): \hat{g}^d = \hat{T}^d(x^d, v^d, \ldots) \times g(x), \quad (148)$$

$$\hat{x}^{d\hat{2}d} = (\frac{g_{11}}{n_1^2} + \frac{g_{22}}{n_2^2} + \frac{g_{33}}{n_3^2} - \frac{g_{44}}{n_4^2})^d \times \hat{I}^d \in \hat{F}^d, \quad (149)$$

Santilli Isodual Iso-Symplectic Geometry

$$\hat{\omega}^d = \hat{d}\hat{r}^{dk} \hat{\wedge}^d \hat{d}\hat{p}_k^d \quad (150)$$

7.3. Isodual IsoDynamical IsoEquation[2,36-43

Newton-Santilli Isodual IsoEquation

$$\hat{m}^d \hat{\times}^d \frac{\hat{\partial}^d \hat{v}^d}{\hat{\partial}^d \hat{t}^d} - F^{dSA}(r^d, p^d) == (m \times \frac{dv}{dt})^d - F^{dSA}(r^d, p^d) - F^{dNSA}(r^d, p^d, \ldots) = 0^d = 0, \quad (151)$$

Isodual IsoVariational principle

$$\hat{\delta}^d \hat{A}^d = \hat{\delta}^d \hat{\int}^d (\hat{p}_k^d \hat{\times}^d \hat{d}^d hat r^{dk} - \hat{H}^d \hat{\times}^d \hat{d}^d \hat{t}^d) = 0^d = 0. \quad (152)$$

Hamilton-Santilli Isodual IsoEquations

$$\frac{\hat{d}^d \hat{r}^{dk}}{\hat{d}^d \hat{t}^d} = \frac{\hat{\partial}^d \hat{H}^d(\hat{r}^d, \hat{p}^d)}{\hat{\partial}^d \hat{p}_k^d}, \quad \frac{\hat{d}\hat{p}_k}{\hat{d}^d \hat{t}^d} = + \frac{\hat{\partial}^d \hat{H}^d(\hat{r}^d, \hat{p}^d)}{\hat{\partial}^d \hat{r}^{dk}}; \quad (153)$$

Isodual iso-Hilbert space $\hat{H}^d$ over C with states $|\hat{\psi}^d >= -< \hat{\psi}|$ over $\hat{C}^d$

Expectation value of a Hermitean operator A

$$< A^d >=< \hat{\psi}| \hat{\times} A^d \hat{\times} |\hat{\psi} >\in C^d \quad (154)$$

Heisenberg-Santilli Isodual IsoEquation

$$\hat{\imath}^d \hat{\times}^d \hat{d}^d hat A^d over \hat{d}^d \hat{t}^d = [\hat{A}, \hat{H}]^d = (\hat{A} \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{A})^d = (\hat{A} \times \hat{T}(\hat{\psi}, \hat{\partial}\hat{\psi}, \ldots) \times \hat{H}(\hat{r}, \hat{p}) - \hat{H}(\hat{r}, \hat{p}) \times \hat{T}(\hat{\psi}, \hat{\partial}\hat{\psi}, \ldots) \times \hat{A})^d, \quad (155)$$

Schrödinger-Santilli Isodual IsoEquation

$$(\hat{H} \hat{\times} |\hat{\psi} >)^d =< \hat{\psi}^d| \hat{\times}^d \hat{H}^d = (\hat{H}(\hat{r}, \hat{p}) \times \hat{T}(\hat{\psi}, \hat{\partial}\hat{\psi}, \ldots) \times |\hat{\psi} >)^d = -< \hat{\psi}^d| \hat{\times}^d \hat{E}^d = -< \hat{\psi}^d| \times \hat{E}^d, \quad (156)$$

$$(\hat{p} \hat{\times} |\hat{\psi} >)^d =< \hat{\psi}^d| \hat{\times}^d \hat{\partial}_{\hat{r}^d} = -i \times < \hat{\psi}^d| \hat{\times}^d \hat{\partial}_{\hat{r}^d}^d, \quad (157)$$

Dirac-Santilli Isodual IsoEquation

$$[(\hat{\eta}^{\mu\nu} \hat{\times} \hat{\gamma}_\mu \hat{\times} \hat{p}_\nu - \hat{\imath} \hat{\times} \hat{m} \hat{\times} \hat{c}) \hat{\times} |\hat{p}si >]^d = 0. \quad (158)$$

$$\{\hat{\gamma}_\mu, \hat{\gamma}_\nu\}^d = (\hat{\gamma}_\mu \hat{\times} \hat{\gamma}_\nu + \hat{\gamma}_\nu \hat{\times} \hat{\gamma}_\mu)^d = \hat{2}^d \hat{\times}^d \hat{\eta}_{\mu\nu}^d, \quad (159)$$

Comments and References

See monograph [40] with particular reference to the use of the isodual isomathematics for the achievement of a grand unification of electroweak and gravitational interactions inclusive of matter and antimatter.

8. Isodual Genomathematics, Isodual Genomechanics and Isodual Genochemistry

8.1. Represented Systems [2,36-43

Single-valued, time-irreversible system of extended-deformable antiparticles under action at a distance Hamiltonian and contact non-Hamiltonian interactions, as occurring in antimatter nuclear reactions, antimatter biological structures and antimatter chemical reactions.

8.2. Main Mathematical Structure [2,36-43

Backward Isodual GenoUnit

$$\hat{I}^{>c} = \hat{I}^{>d}(t^{>}r^{>}, p^{>d}, a^{>d}, \psi^{>d}, \partial^{>d}\psi^{>d}, \ldots.) = 1/\hat{T}^{>d} > 0, \quad (160)$$

Forward Isodual GenoUnit

$$^{<d}\hat{I} = ^{<d}\hat{I}(^{<d}r, ^{<d}p, ^{<d}a, ^{<d}\psi, ^{<d}\partial^{<d}\psi, \ldots.) = 1/^{<d}\hat{T} > 0, \quad (161)$$

Condition for time-irreversibility

$$\hat{I}^{>d} \neq {}^{<d}\hat{I} \quad (162)$$

Backward Isodual GenoFields

$$\hat{F}^{>d}(\hat{n}^{>d}, >, \hat{I}^{>d}), \hat{n}^{>d} = n \times \hat{I}^{>d}, \hat{n}^{>d} >^d \hat{m}^{>d} = \hat{n}^{>d} \times \hat{T}^{>d} \times \hat{m}^{>d} \in \hat{F}^{>d}, \quad (163)$$

Forward Isodual GenoFields

$$^{<d}\hat{F}(^{<d}\hat{n}, <, ^{<d}\hat{I}), \quad ^{<d}\hat{n} = ^{<d}\hat{I} \times n, \quad ^{<d}\hat{n} <^d \quad ^{<d}\hat{m} = = ^{<d}\hat{n} \times ^{<d}\hat{T} \times ^{<d}\hat{m} \in ^{<d}\hat{F}, \quad (164)$$

Backward Isodual GenoCoordinates

$$\hat{r}^{>d} = r \times \hat{I}^{>d} \in \hat{F}^{>d}, \quad (165)$$

Forward Isodual GenoCoordinates

$$^{<d}\hat{r} = ^{<d}\hat{I} \times r \in ^{<d}\hat{F}, \quad (166)$$

Backward Isodual GenoFunctional analysis

$$\hat{f}^{>d}(\hat{r}^{>d}) = f(\hat{r}^{>d}) \times \hat{I}^{>d} \in \hat{F}^{>d}, \quad (167)$$

Forward Isodual GenoFunctional analysis

$$^{<d}\hat{f}(^{<d}\hat{r}) = f(^{<d}\hat{r}) \times^{<d} \hat{I} \in^{d} \hat{F} \quad (168)$$

Backward Isodual GenoDifferential Calculus

$$\hat{d}^{>d}\hat{r}^{>d} = dr + r \times \hat{T}^{>d} \times d\hat{I}^{>d}, \quad (169)$$

$$\frac{\hat{\partial}^{>d}\hat{f}^{>d}(\hat{r}^{>d})}{\hat{\partial}^{>d}\hat{r}^{>d}} = \hat{I}^{>d} \times \frac{\partial \hat{f}^{>d}(\hat{r}^{>d})}{\partial \hat{r}^{>d}}, \quad (170)$$

Forward Isodual GenoDifferential Calculus

$$^{<d}\hat{d}^{<d}\hat{r} = dr + r \times^{<d} \hat{T} \times d^{<d}\hat{I}, \quad (171)$$

$$\frac{^{<d}\hat{\partial}^{<d}\hat{f}(^{<d}\hat{r})}{^{<d}\hat{\partial}^{<d}\hat{r}} = ^{<dd}\hat{I} \times \frac{\partial^{<d}\hat{f}(^{<d}\hat{r})}{\partial^{<d}\hat{r}}, \quad (172)$$

Santilli Isodual Lie-Admissible Theory

$$(X_i \hat{,} X_j)^d = (X_i < X_j - X_j > X_i)^d = -C_{ij}^{dk}(t^d, r^d, p^d, \psi^d, \ldots) \times X_k, \quad (173)$$

$$A^d(w^d) = \hat{e}_{>d}^{X \times w \times i} >^d A(0) <_{<d}^d \hat{e}^{-i \times w \times X}. \quad (174)$$

Santilli Backward Geno-Euclidean Geometry

$$\hat{E}^{>d}(\hat{r}^{>d}, \hat{\delta}^{>d}, \hat{I}^{>d}), \hat{\delta}^{>d}(t,r,p,\psi,\ldots) = \hat{T}^{>d}(t,r,p,\psi,\ldots) \times \delta, \quad (175)$$

$$\hat{r}^{>d\hat{2}d} = (\hat{r}^{>di} >^d \hat{\delta}_{ij}^{>d} > \hat{r}^{>dj} =\in F^{>d}, \quad (176)$$

$$\hat{T}^{>d} \neq \hat{T}^{>d\,transp} \quad (177)$$

Santilli Forward Isodual Geno-Euclidean Geometry

$$^{<d}\hat{E}(^{<d}\hat{r}, ^{<d}\hat{\delta}, ^{<d}\hat{I}), \quad ^{<d}\hat{\delta}(t,r,p,\psi,\ldots) = ^{<d}\hat{T}(t,r,p,\psi,\ldots) \times \delta, \quad (178)$$

$$^{<d\hat{2}d}\hat{r} = ^{<di}\hat{r} <^{<d}\hat{\delta}_{ij} <^{d} \quad ^{<dj}\hat{r} \in^{<d} F, \quad (179)$$

$$^{<d}\hat{T} \neq ^{<d\,transp}\hat{T} \quad (180)$$

Santilli Backward Isodual Geno-Minkowskian Geometry ($\mu = 1,2,3,4$)

$$\hat{M}^{>d}(\hat{x}^{>d}, \hat{\eta}^{>d}, \hat{I}^{>d}): \hat{x}^{>d} = (\hat{x}^{>d\mu}), x_4^{>d} = t^{>d}, \quad (181)$$

$$\hat{\eta}^{>d}(x,\psi,\ldots) = \hat{T}^{>d}(x,\psi,\ldots) \times \eta, \quad (182)$$

$$\hat{x}^{>d2d} = \hat{x}^{>d\mu} >^d \hat{\eta}_{\mu\nu}^{>d} >^d \hat{x}^{>dv} \in \hat{F}^{>d}, \quad (183)$$

$$\hat{\eta}^{>d} \neq \hat{\eta}^{>d\,transp} \quad (184)$$

Santilli Forward Isodual Geno-Minkowskian Geometry ($mu = 1,2,3,4$)

$$^{<d}\hat{M}(^{<d}\hat{x}, ^{<d}\hat{\eta}, ^{<d}\hat{I}): \quad ^{<d}\hat{x} = (\hat{x}^{\mu}), \quad ^{<d}x_4 = ^{<d} t, \quad (185)$$

$$^{<d}\hat{\eta}(x,v,\ldots) = ^{<d}\hat{T}(x,v,\ldots) \times \eta, \quad (186)$$

$$^{<d}\hat{x}^{<d2d} = ^{<d\mu}\hat{x} <^{d} \quad ^{<d}\hat{\eta}_{\mu\nu} <^{d} \quad ^{<v}\hat{x} \in^{<d} \hat{F}, \quad (187)$$

$$^{<d}\hat{\eta} \neq ^{<d\,transp}\hat{\eta} \quad (188)$$

Santilli Backward Isodual Geno-Riemannian Geometry

$$\hat{R}^{>d}(\hat{x}^{>d}, \hat{g}^{>d}, \hat{I}^{>d}); \hat{g}^{>d} = \hat{T}^{>d}(x,v,\ldots) \times g(x), \quad (189)$$

$$\hat{x}^{>d2>} = x^{>d\mu} \hat{.}^{d}\, \hat{g}_{\mu\nu}^{>d} >^{d} x^{>dv} \in \hat{F}^{>d} \quad (190)$$

$$\hat{T}^{>d} \neq \hat{T}^{>d\,transp} \quad (191)$$

Santilli Forward Isodual Geno-Riemannian Geometry

$$^{<d}\hat{R}(^{<d}\hat{x}, ^{<}\hat{g}, ^{<d}\hat{I}): \quad ^{<d}\hat{g} = ^{<d}\hat{T}(x,v,\ldots) \times g(x), \quad (192)$$

$$^{<d}\hat{x}^{<d2d} = ^{<d\mu}\hat{x} <^{d} \quad ^{<d}\hat{g}_{\mu\nu} <^{d} \quad ^{<dv}\hat{x} \in^{<d} \hat{F}, \quad (193)$$

$$^{<d}\hat{g} \neq ^{<d\,transp}\hat{g} \quad (194)$$

Santilli Backward Isodual Geno-Symplectic Geometry

$$\hat{\omega}^{>d} = \hat{d}^{>d}\hat{r}^{>dk} \hat{\wedge}^{>d} \hat{d}^{>d}\hat{p}_k^{>d} \quad (195)$$

Santilli Forward Isodual Geno-Symplectic Geometry

$$^{<d}\hat{\omega} = ^{<d}\hat{d}^{<dl}\hat{r}^{<d}\hat{\wedge}^{<d}\hat{d}^{<d}\hat{p}_k \quad (196)$$

8.3. Isodual GenoDynamical GenoEquations [2,36-43

Newton-Santilli Backward Isodual GenoEquation

$$\hat{m}^{>d} > \frac{\hat{d}^{>d}\hat{v}^{>d}}{\hat{d}^{>d}\hat{t}^{>d}} - F^{>dSA}(t,r,p) = [m \times \frac{dv}{dt}]^{>d} - F^{SA>d}(t,r,p) - F^{NSA>d}(t,r,p,\ldots) = 0, \quad (197)$$

Newton-Santilli Forward Isodual GenoEquation

$$^{<d}\hat{m} < \frac{^{<d}\hat{d}^{<d}\hat{v}}{^{<d}\hat{d}^{<d}\hat{t}} - ^{<dSA}F(t,r,p) = ^{<d}[m \times \frac{dv}{dt}] - ^{<dSA}F(t,r,p) - ^{<dNSA}F(t,r,p,\ldots) = 0, \quad (198)$$

Backward Isodual GenoVariational principle

$$\hat{\delta}^{>d}\hat{A}^{>d} = \hat{\delta}^{>d} \hat{\int}^{>d} (\hat{p}_k^{>d} > \hat{d}^{>d}\hat{r}^{>dk} - \hat{H}^{>d} > \hat{d}^{>d}\hat{t}^{>d}) = 0. \quad (199)$$

Forward Isodual GenoVariational principle

$$^{<d}\hat{\delta}^{<d}\hat{A} = ^{<d}\hat{\delta}^{<d} \hat{\int}^{} (^{<d}\hat{p}_k <^{<d}\hat{d}^{<d}\hat{r}^{k} -^{<d}\hat{H} <^{<d}\hat{d}^{<d}\hat{t}) = 0. \quad (200)$$

Backward Isodual Hamilton-Santilli GenoEquations

$$\frac{\hat{d}^{>d}\hat{r}^{>dl}}{\hat{d}^{>d}\hat{t}^{>d}} = [\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{p}_k}]^{>d}, \quad [\frac{\hat{d}^{>d}\hat{p}_k^{>d}}{\hat{d}^{>d}\hat{t}^{>d}}] = -[\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{r}^{k}}]^{>d}, \quad (201)$$

Forward isodual Hamilton-Santilli GenoEquations

$$\frac{^{<d}\hat{d}^{<d}\hat{r}^{k}}{^{<d}\hat{d}^{<d}\hat{t}} = ^{<d}\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{p}_k}], \quad \frac{^{<d}\hat{d}^{<d}\hat{p}_k}{^{<d_{hat}d}\hat{t}}] = -^{<}[\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{r}^{k}}], \quad (202)$$

Heisenberg-Santilli IsoDual GenoEqutions

$$\hat{\imath} \hat{\times} \frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{t}} = (\hat{A},\hat{H}) = \hat{A} < \hat{H} - \hat{H} > \hat{A} = A \times^{<} T(\hat{\psi},\hat{\partial}\hat{\psi},\ldots) \times \hat{H}(\hat{r},\hat{p}) - \hat{H}(\hat{r},\hat{p}) \times \hat{T}^{>}(\hat{\psi},\hat{\partial}\hat{\psi},\ldots) \times \hat{A} \quad (203)$$

Schrödinger-Santilli Backward Isodual GenoEquations

$$\hat{H}^{>d} >^{d} |\hat{\psi}^{>d} >= \hat{H}^{>d}(\hat{r},\hat{p}) \times \hat{T}^{>d}(\hat{\psi},\hat{\partial}\hat{\psi},\ldots) \times |\hat{\psi}^{>} >=$$

$$\hat{E}^{>d} >^{d} |\hat{\psi}^{>d}>= E^{>d} \times |\hat{\psi}^{>d}>, \quad (204)$$

$$\hat{p}^{>d} > |\hat{\psi}^{>d}>= -\hat{\imath}^{>d} > \hat{\partial}_{\hat{r}}^{>d} |\hat{\psi}^{>}>= -i \times \hat{I}^{>} \times \partial_{\hat{r}} |\hat{\psi}^{>d}>, \quad (205)$$

Schrödinger-Santilli Forward Isodual GenoEquations

$$<^{<d} \hat{\psi}| <^{d} \quad <\hat{H} =<^{<d} \hat{\psi}| \times^{<d} \hat{T}(\hat{\psi}, \hat{\partial}\hat{\psi}, \ldots) \times^{<} \hat{H}(\hat{r}, \hat{p}) = <^{<d} \hat{\psi}| <^{<} \hat{E} =<^{<d} \hat{\psi}| \times^{<d} E, \quad (206)$$

$$<^{<d} \hat{\psi}|<^{<d} \hat{p} = -<^{<d} \hat{\psi}| <^{d} \quad <^{d}\hat{\imath} <^{d} \quad {}^{<d}_{\hat{r}}\hat{\partial} = -i \times <^{<d} \hat{\psi}|^{<d}_{\hat{r}} \partial \times^{<d} \hat{I} \quad (207)$$

Dirac-Santilli Backward Isodual IsoEquation

$$\left(\hat{\eta}^{>d\mu\nu} >^{d} \hat{\gamma}_{\mu}^{>d} >^{d} \hat{p}_{\nu}^{>d} - \hat{\imath}^{>d} > \hat{m}^{>d} > \hat{c}^{>d}\right) > |\hat{p}si^{>d}>= 0. \quad (208)$$

$$\{\hat{\gamma}_{\mu}, \hat{\gamma}_{\nu}\}^{>d} = [\hat{\gamma}_{\mu} \hat{\times} \hat{\gamma}_{\nu} + \hat{\gamma}_{\nu} \hat{\times} \hat{\gamma}_{\mu}]^{>} = \hat{2}^{>d} > \hat{\eta}_{\mu\nu}^{>d}, \quad (209)$$

Dirac-Santilli Forward Isodual GenoEquation

$$<^{<d} \hat{\psi}|< (^{<d}\hat{p}_{\nu} <^{<d} \hat{\gamma}_{\mu} <^{d} \quad <^{d\mu\nu}\hat{\eta} -^{<}\hat{\imath} <^{d} \quad <^{d}\hat{m} <^{d} \quad <\hat{c}) = 0. \quad (210)$$

$$<^{d}\{\hat{\gamma}_{\mu}, \hat{\gamma}_{\nu}\} =<^{d} [\hat{\gamma}_{\mu} \hat{\times} \hat{\gamma}_{\nu} + \hat{\gamma}_{\nu} \hat{\times} \hat{\gamma}_{\mu}] =<^{d} \hat{2} <^{<} \hat{\eta}_{\mu\nu} = 2 \times^{<d} \hat{\eta}_{\mu\nu}, \quad (211)$$

Comments and References

See memoir [20] which constitutes the most comprehensive study of antimatter in irreducible conditions available at this writing.

9. Isodual Classical and Hope Isodual Hypermathematics

Isodual Hyper-Formulations are generally considered to be part of the Hyper-Formulations of Section 4 and 5 because the classification of ordered sets of hyperunits includes isodual realizations, as illustrated in the paper [44] and references quoted therein.

10. Simple Method for the Construction of Regular Hadronic Mathematics

10.1. Introduction [4.5]

Hadronic formulations are called *regular* when the *structure quantities* C_{ij}^{i} of Santilli's Lie-Isotopic algebras, Eqs. (3), Lie-admissible algebras, Eqs. (69) (zzz) and their isoduals, Eqs. (119-, (139), are *constant.* When the structure quantities are *functions of the local variables* $C_{ij}^{k}(t, r, p, \psi, \partial\psi, \ldots)$, hadronic formulations are called *irregular*.

In this section, we shall review a very simple method for the construction of regular hadronic formulations via the mere use of non-unitary transformations of the corresponding conventional formulations. We shall then review the axiomatic consistency of hadronic formulations by showing that Santilli iso-, geno-, hyper-units and their isoduals are invariant under the transformations, thus implying the crucial invariance over time of extended-deformable shapes and their non-Hamiltonian interactions that are invariantly represented precisely nwith such generalized units.

No method exists to our knowledge at this writing (June 2015) for the construction of irregular hadronic formulations via maps of conventional formulations and, therefore, irregular hadronic formulations characterize a new axiomatic structure still mostlyunexplored.

10.2. Simple Construction of Regular Iso-Formulations [4.5]

A simple method has been identified in Refs. [4,5] for the construction of the Lie-Santilli isotheory, all its underlying isomathematics and all physical methods This method is important because it permits a simple implementation of conventional models into their isotopic covering without the need for advanced mathematics. The method consists in:

(i) Representing all conventional potential interactions with a Hamiltonian $H(r, p)$ and all extended-deformable shapes and their non-Hamiltonian interactions and effects with Santilli's isounit $\hat{I}(r, p, \psi, \partial\psi, ./..)$;

(ii) Identifying the latter interactions with a nonunitary transform

$$U \times U^{\dagger} = \hat{I} \neq I \quad (212)$$

and

(iii) Subjecting the *totality* of conventional mathematical and physical quantities and all their operations to the above nonunitary transform, resulting in expressions of the type

$$I \to \hat{I} = U \times I \times U^{\dagger} = 1/\hat{T}, \quad (213)$$

$$a \to \hat{a} = U \times a \times U^{\dagger} = a \times U \times U^{\dagger} = a \times \hat{I}, a \in F, \quad (214)$$

$$e^{A} \to U \times e^{A} \times U^{\dagger} = \hat{I} \times e^{\hat{T} \times \hat{A}} = \left(e^{\hat{A} \times \hat{T}}\right) \times \hat{I}, \quad (215)$$

$$A \times B \to U \times (A \times B) \times U^{\dagger} = (U \times A \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times B \times U^{\dagger}) = \hat{A} \hat{\times} \hat{B}, \quad (216)$$

$$[X_i, X_j] \to U \times [X_i X_j] \times U^{\dagger} = [\hat{X}_i, \hat{X}_j] = U \times (C_{ij}^{k} \times X_k) \times U^{\dagger} = \hat{C}_{ij}^{k} \hat{\times} \hat{X}_k = C_{ij}^{k} \times \hat{X}_k, \quad (217)$$

$$< \psi| \times |\psi >\to U \times< \psi| \times |\psi > \times U^{\dagger} =< \psi| \times U^{\dagger} \times (U \times U^{\dagger})^{-1} \times U \times |\psi > \times (U \times U^{\dagger}) = < \hat{\psi}| \hat{\times} |\hat{\psi} > \times \hat{I}, \quad (218)$$

$$H \times |\psi >\to U \times (H \times |\psi >) = (U \times H \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times |\psi >) = \hat{H} \hat{\times} |\hat{\psi} >, etc. \quad (219)$$

Note that serious inconsistencies emerge in the event even 'one' single quantity or operation is not subjected to the above non-unitary map. In the absence of comprehensive liftings, we would have a situation equivalent to the elaboration of quantum spectral data of the hydrogen atom with isomathematics, resulting in large deviations from reality.

The construction of isodual iso-formulations is simply done via Santilli's isodual map, namely, via the simple anti-hermitean image of the above isotopic formulations.

10.3. Axiomatic consistency of Iso-Formulation [4.5]

Let us recall that Santilli's central assumption is the representation of extended-deformable shapes and their non-Hamiltonian interactions via the isounit. Therefore, any change of the numerical value of the isounit implies the inability to represent the same system over time, besides activating the *Theorem of Catastrophic Mathematical and Physical Inconsistencies of Non-Canonical and Non-Unitary Theories* when formulated via the mathematics of conventional canonical and unitary theories,respectively [23].

It is easy to see that the application of an additional nonunitary transform

$$W \times W^\dagger \neq I, \tag{220}$$

to the preceding expressions causes their *lack of invariance,* with consequential activation of the theorem of catastrophic inconsistencies. This is due to the *change of the value of the basic isounit* under additional non-unitary transformations

$$\hat{I} \to \hat{I}' = W \times \hat{I} \times W^\dagger \neq \hat{I}, \tag{221}$$

However, any given nonunitary transform can be identically rewritten in the isounitary form [3]

$$W \times W^\dagger = \hat{I}, \quad W = \hat{W} \times \hat{T}^{1/2}, \tag{222}$$

$$W \times W^\dagger = \hat{W} \hat{\times} \hat{W}^\dagger = \hat{W}^\dagger \hat{\times} \hat{W} = \hat{I}, \tag{223}$$

under which we have the invariance of the isounit and isoproduct [7]

$$\hat{I} \to \hat{I}' = \hat{W} \hat{\times} \hat{I} \hat{\times} \hat{W}^\dagger = \hat{I}, \tag{224}$$

$$\hat{A} \hat{\times} \hat{B} \to \hat{W} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{W}^\dagger = (\hat{W} \times \hat{T} \times \hat{A} \times \hat{T} \times \hat{W}^\dagger) \times (\hat{T} \times \hat{W}^\dagger)^{-1} \times \hat{T} \times (\hat{W} \times \hat{T})^{-1} \times (\hat{W} \times \hat{T} \times \hat{B} \times \hat{T} \times \hat{W}^\dagger) = \hat{A}' \times (\hat{W}^\dagger \times \hat{T} \times \hat{W})^{-1} \times \hat{B}' = \hat{A}' \times \hat{T} \times \hat{B}' = \hat{A}' \hat{\times} \hat{B}', \text{ etc.} \tag{225}$$

from which the invariance of the entire isotopic formalism follows.

Note that the invariance is ensured by the *numerically invariant values of the isounit and of the isotopic element under non-unitary-isounitary transformations,*

$$\hat{I} \to \hat{I}' \equiv \hat{I}, \tag{226}$$

$$A \hat{\times} B \to A' \hat{\times}' B' \equiv A' \hat{\times} B', \tag{227}$$

in a way fully equivalent to the invariance of Lie's theory and quantum mechanics, as expected to be necessarily the case due to the preservation of the abstract axioms under isotopies. The resolution of the inconsistencies for non-invariant theories is then consequential.

The proof of the invariance of Santilli isodual iso-formulations is an interesting exercise for non-initiated readers.

10.4. Simple Construction of Regular GenoMathematics and its IsoDual [4.5]

An important feature of the Lie-Santilli genotheory is its *form invariance* under the appropriate geno-transformations in a way fully similar to the invariance of the mathematical and physical structures of quantum mechanics under unitary transformations.

This feature can be shown via a *pair* of non-unitary transformations

$$V \times V^\dagger \neq I, W \times W^\dagger \neq I, V \times W^\dagger \neq I, W \times V^\dagger \neq I, \tag{228}$$

under which we have the characterization of the forward and backward genounits and related genoproduct

$$I \to V \times I \times W^\dagger = \hat{I}^>, eqno \tag{229}$$

$$A \times B \to V \times (A \times B) \times W^\dagger = A^> > B^> \tag{230}$$

$$I \to W \times I \times V =^< \hat{I}, \tag{231}$$

$$A \times B \to W \times (A \times B) \times V =^< A <^< B/ \tag{232}$$

10.5. Axiomatic Consistency of GenoMathematics and its Isodual [4.5]

It is easy to see that the above dual non-unitary transformations can always be identically rewritten as the *geno-unitary transforms* on geno-Hilbert spaces over complex genofields,

$$V \times V^\dagger \neq 1, V =^< \hat{V} \times \hat{R}^{1/2}, V \times V^\dagger =^< \hat{V} <^< \hat{V}^\dagger =^< \hat{V}^\dagger <^< \hat{V} =^< \hat{I}, \tag{233}$$

$$W \times W^\dagger \neq 1, W = \hat{W}^> \times \hat{S}^{1/2}, W \times W^\dagger = \hat{W}^> > \hat{W}^{>\dagger} = \hat{W}^{>\dagger} > \hat{W}^> = \hat{I}^>, \tag{234}$$

under which we have indeed the following forward geno-invariance laws [3j]

$$\hat{I}^> \to \hat{I}'^> = \hat{W}^> > \hat{I}^> > \hat{W}^{>\dagger} = \hat{I}^>, \tag{235}$$

$$\hat{A} > \hat{B} \to \hat{W}^> > (\hat{A} > \hat{B}) > \hat{W}^{>\dagger} = \hat{A}' > \hat{B}', \tag{236}$$

$$\hat{H}^> > | >= \hat{E}^> > | >= E \times | >\to \hat{W}^> > \hat{H}^> > | >= \hat{H}'^> > | >'= \hat{W}^> > \hat{E}^> > | >= E \times | >', \tag{237}$$

with corresponding rules for the backward and classical counterparts.

The above rules confirm the achievement of the *invariance of the numerical values of genounits, geno-products and geno-eigenvalues*, thus permitting physically consistent applications.

The invariance of the isodual geno-formulations can then be proved via the isodual map applied to the above procedure.

11. Open Mathematical Problems

Among a predictable large number of basic open problems, we list for the interested readers the following ones:

Study methods to transform nonlinear models on

conventional spaces into isolinear models on isospaces over isofields;

See whether simple solutions of isolinear equations on isospaces over isofields provide at least ä" solution of their nonlinear projection on conventional spaces over conventional fields;

Study the removal of divergencies in quantum mechanics and scattering theories (Footnote 2) by isomechanics on an iso-Hilbert space over an isofield.

Study the regular and irregular isorepresentations of the Lie-Santilli isotheory;

Study Santilli isoMinkowskian geometry via the machinery of the Riemannian geometry, yet lack of curvature [39];

Study the Lie-admissible theory in Santilli's sense, that is, as a generalization of Lie's theory elaborated via genomathematics;

Study Santilli geno-Euclidean, geno-Minkowskian and geno-Riemannian geometries where irreversibility is embedded in the non symmetric character of the metric [23];

extend the Tsagas, Ganformina-Nunez isotopology to the genotopic form and their isoduals.

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Studies on Santilli's Isonumber Theory

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Abstract: Beginning with studies in the 1980s at the Department of Mathematics of Harvard University, the Italian-American scientist R. M. Santilli discovered new realizations of the abstract axioms of numeric fields with characteristic zero, based on an axiom-preserving generalization of conventional associative product and consequential positive-definite generalization of the multiplicative unit, today known as Santilli *isonumbers* [1], and the resulting novel numeric fields are known as Santilli *isofields.* By remembering that 20th century mathematics was formulated on numeric fields, their generalization into isofields stimulated a corresponding generalization of all of 20th century mathematics and its application to mechanics, today known as Santilli *isomatheatics* and *isomechanics*, respectively, which is used for the representation of extended-deformable particles moving within physical media under Hamiltonian as well as contact non-Hamiltoian interactions. Additionally, Santilli discovered a second realization of the abstract axioms of a numeric field, this time with arbitrary (non-singular) negative definite generalized unit and related multiplication, today known as Santilli *isodual isonumber* [1] that have stimulated a second covering of 20th century mathematics and mechanics known as Santilli *isodual isomathematics* and *isodual isomechanics.* The latter methods are used for the classical as well as operator form of antimatter in full democracy with the study of matter. In this paper, we present a comprehensive study of Santilli's epoch making discoveries of isonumbers and their isoduals along with their application to isomechanics and its isodual for matter and antimatter, respectively.

Keywords: Isonumber, Isodual Number, Isodual-Isonumber, Genonumber

1. Introduction

As it is well known, modern mathematics has a strong foundation on number theory, algebraic structures such as groups, rings, algebra, vector spaces and related methods have found vast applications in all quantitative sciences. More general structures like groupoids, semigroups, monoids, quasigroups and loops were also being studied in 20th century, although their applications in quantitative sciences are under development. The detailed consolidated account of these generalized structures is found in Survey of Binary Systems by R.H.Bruck [2].

While the scientific discoveries and mathematical knowledge were moving hand in hand, towards the end of 20th century there were few mathematically unexplained physical phenomena in Quantum Physics and Quantum Chemistry. These new physical situations could not be faithfully described by the existing mathematical structures and called for more generalized mathematical structures.

It was Enrico Fermi, [3] beginning of chapter VI, p.111 said "..... there are some doubts as to whether the usual concepts of geometry hold for such small region of space." His inspiring doubts on the exact validity of quantum mechanics for the nuclear structure led to the genesis of the whole new kind of generalized mathematics, called isomathematics and generalized mechanics, called as Hadronic mechanics.

In fact, the prevailing Newtonian and Einsteinian 'Dynamical systems' called as 'Exterior Dynamical systems' which are characterized as 'local', 'linear' 'Lagrangian' and 'Hamiltonian' could not accommodate these obscure situations. *Thus it was the pressing demand of time to formulate new mathematical theory which could deal with the obscure phenomena and develop a new physical theory.* This stupendous task was taken up by the Italian-American theoretical physicist Ruggero Maria Santilli, President of Institute for Basic Research, Palm Harbor, Florida, USA and did the pioneering work by defining axiom-preserving, nonlinear, nonlocal and noncanonical isotopies of conventional mathematical structures, including units, fields,

vector spaces, transformation theory, algebras, groups, geometries, Hilbert spaces etc. while at Department of Mathematics of Harvard University in the early 80's. Prof. Santilli has rightly said;

"There can not be really new physical theories without

really new mathematics, and there can not be really

new mathematics without new numbers".

The founders of analytic mechanics, such as Lagrange, Hamilton [4] and others classified dynamical systems in to two kinds. First one is the 'Exterior Dynamical system' and the second one is the more complex but generalized 'Interior Dynamical system'.

However, over a period of time the the above distinction was abandoned preventing the identification of limitations of the prevailing mathematical and physical theories. One can easily notice that *Lie's Theory* is exactly applicable to the exterior dynamical systems. It was Prof. Santilli who at the Department of Mathematics of Harvard University, for the first time, drew the attention of the scientific community towards the crucial distinction between exterior and interior dynamical systems and presented insufficiencies of prevailing mathematical and physical theories by submitting the so-called *axiom-preserving, nonlinear, nonlocal, and noncanonical isotopies of Lie's theory* [5] under the name *Lie Isotopic theory.* Further generalization as *Lie-admissible theory* [6,7] was also achieved by him.

Exterior Dynamical Systems: In this system Point-like particles are moving in a homogeneous and isotropic vacuum with local-differential and potential-canonical equations of motion. These are linear, local, Newtonian Lagrangian and Hamiltonian. Conventional Mathematical structures such as Algebras, Geometries, Analytical Mechanics, Lie Theory can faithfully represent these systems.

Interior Dynamical Systems: In this system we consider extended non-spherical deformable particles moving within non-homogeneous anisotropic physical medium. These are non-linear, non-local, non-Newtonian, non-Lagrangian and non-Hamiltonian. The mathematical structures needed to describe these systems are most general possible which are axiom preserving; non-linear and non-local formulations of current mathematical structures.

During a talk at the conference *Differential Geometric Methods in Mathematical Physics* held in Clausthal, Germany, in 1980, Ruggero Maria Santilli submitted new numbers based on certain axiom preserving generalization of the multiplication, today known as *isotopic numbers* or *isonumbers*[1] in short. This generalization induced the so-called isotopies of the conventional multiplication with consequential generalization of the multiplicative unit, where the Greek word "isotopy" from the Greek word "ισος τοπος " implied the meaning "same topology" [8,9]. Subsequently, Ruggero Maria Santilli submitted a **new conjugation**, under the name *isoduality* which yields an additional class of numbers, today known as ***isodual isonumbers*** [1].

The discovery of isonumbers was made with the specific need of quantitative representation of the transition from Exterior Dynamical Systems to Interior Dynamical System.

It should be quite clear that there can not be new numbers without new fields. This led Santilli to define 'Isofield' which is the first new algebraic structure defined by him. This concept of 'Isofield' further led to a plethora of new isoalgebraic structures and a whole new 'Isomathematics' which is a step further in Modern Mathematics. Subsequently, 'Isomathematics' has grown in to a huge tree with various branches like 'Isofunctional Analysis', 'Isocalculus', 'Isoalgebra', isocryptography etc.

Prof. Santilli attracted great attention from academic community at Chinese Academy of Sciences during a workshop in China on August 23, 1997. Since then Prof Santilli and his associates in various countries around the world have produced numerous papers, monographs, conference proceedings which cover approximately 10,000 pages of research work.

Today Number theory has advanced as an important branch of axiomatized mathematics with highly sophisticated applications in the Modern world of computer science and information technology. After some advances in 19th century due to Gauss [10], Abel [11], Hamilton [4], Cayley [12], Galois [13] and others, major important advances were made during 20th century which included axiomatic formulation, the algebraic number theory [14].

The classification of all normed algebras with identity, over reals, in view of the previous studies by Hurwitz[15], Albert [16], and N.Jacobson [17] may be expressed in the following important Theorem.

Theorem 1.1. All possible normed algebras with multiplicative unit over the field of real numbers are given by algebras of dimension 1 (real numbers), 2 (complex numbers), 4 (quaternions), and 8 (octonians).

In this comprehensive presentation of the development of 'Isonumber theory' we cover the following important aspects of fundamental importance as formulated by Prof. R. M. Santilli [18], [1].

Starting with the brief background of the origin of 'isounit' and isofield, we present the theory of isonumbers, pseudoisonumbers, "hidden numbers" and their isoduals. Genonumbers, pseudogenonumbers and their isoduals are also of fundamental importance. We will study the isotopies and isodualities of the notions of numbers, fields and normed algebras with unit ref.[1]. In short, in this paper we are going to study the properties of isonumbers and their isoduals [1].

In his study Santiili has taken into account the four normed algebras over reals as given in the above theorem. The isotopic lifting of these algebras give rise to isotopies of normed algebras with multiplicative unit of dimension 1,2,4 and 8 which includes realization of 'isoreal numbers', 'isocomplex numbers', 'isoquaternions' and 'isooctonions'. Isodualities of these structures give isodual isonumbers.

The mathematical non-triviality of these structures is evident due to lack of unitary equivalence of isotopic and genotopic theories to conventional ones, non-applicability of

trigonometry and some other aspects. On the other hand, the physical non-triviality of these structures emerges from the fact that this theory of isonumbers is at the foundations of Li-isotopic theory used successfully to study nonlinear, nonlocal, and nonhamiltonian dynamical systems. The more general Lie-admissible theory emerges from the more general genonumbers.

In a nutshell, the theory of isonumbers is at the foundation of current studies of nonlinear-nonlocal-nonhamiltonian systems in nuclear, particle and statistical physics, superconductivity and other fields.

1.1. Origin of Isonumbers

The concept of 'Isotopy' plays a vital role in the development of this new age mathematics ref. R. H. Bruck [2] and [19].

The first and foremost algebraic structure defined by Santilli is 'isofield'. Elements of an isofield are called as 'isonumbers'. The conversion of unit 1 to the isounit $\hat{1}$ is of paramount importance for further development of 'Isomathematics'.

The reader should be aware that there are various definitions of "fields" in the mathematical literature [20], [21], [22] and [14] with stronger or weaker conditions depending on the given situation. Often "fields" are assumed to be associative under the multiplication.

i.e.

$$a\times(b\times c)=(a\times b)\times c \quad \forall a,b,c\in F$$

We formally define an isofield [23], [24] as follows.

Definition 1.1 *Given a "field" F, here defined as a ring with with elements $a,b,c\ldots$, sum $a+b$, multiplication ab, which is commutative and associative under the operation of conventional addition $+$ and (generally nonassociative but) alternative under the operation of conventional multiplication $\times$ and respective units 0 and 1, "Santilli's isofields" are rings of elements $\hat{a}=a\hat{1}$ where a are elements of F and $\hat{1}=T^{-1}$ is a positive- definite $n\times n$ matrix generally outside F equipped with the same sum $\hat{a}+\hat{b}$ of F with related additive unit $\hat{0}=0$ and a new multiplication $\hat{a}*\hat{b}=\hat{a}T\hat{b}$, under which $\hat{1}=T^{-1}$ is the new left and right unit of F in which case $\hat{F}$ satisfies all axioms of the original field.*

T is called the isoelement. In the above definitions we have assumed "fields" to be alternative, i.e.

$$a\times(b\times b)=(a\times b)\times b, \quad (a\times a)\times b=a\times(a\times b) \quad \forall a,b\in F$$

Thus, "isofields" as per above definition are not in general isoassociative, i.e. they generally violate the isoassociative law of the multiplication, i.e.

$$\hat{a}\hat{\times}(\hat{b}\hat{\times}\hat{c})=(\hat{a}\hat{\times}\hat{b})\hat{\times}\hat{c}, \quad \forall \hat{a},\hat{b},\hat{c}\in\hat{F}$$

but rather satisfy isoalternative laws.

The specific need to generalize the definition of "number" to 'real numbers', complex numbers, 'quaternions' and 'octonians' suggested the above definition. The resulting new numbers are 'isoreal numbers', isocomplex numbers, 'isoquaternions' and 'isooctonians' respectively, where 'isooctonians' are alternative but not associative.

The 'isofields' $\hat{F}=\hat{F}(\hat{a},+,\hat{\times})$ are given by elements $\hat{a},\hat{b},\hat{c}\ldots$ characterized by one-to-one and invertible maps $a\to\hat{a}$ of the original element $a\in F$ equipped with two operations $(+,\hat{\times})$, the conventional addition $+$ of F and a new multiplication $\hat{\times}$ called "isomultiplication" with corresponding conventional additive unit 0 and a generalized multiplicative unit $\hat{1}$, called "multiplicative isounit" under which all the axioms of the original field F are preserved.

Santilli has shown that the transition from exterior dynamical system to interior dynamical system can be effectively represented via the isotopy of conventional multiplication of numbers a and b from its simple possible associative form $a\times b$ in to the isotopic multiplication, or isomultiplication for short, as introduced in [8].

The lifting of the product $ab=a\times b$ of conventional numbers in to the form

$$a\hat{\times}b:=a\times T\times b \tag{1}$$

denoted by $\hat{\times}=\times T\times$, where T is a fixed invertible quantity for all possible a,b called isotopic element.

This isomultiplication then lifts the conventional unit 1 defined by $1\times a=a\times 1=a$ to the multiplicative isounit $\hat{1}$ defined by

$$\hat{1}\hat{\times}a=a\hat{\times}\hat{1}=a, \text{where} \quad \hat{1}=T^{-1} \tag{2}$$

Under the condition that $\hat{1}$ preserves all the axioms of 1 the lifting $1\to\hat{1}$ is an isotopy, i.e. the conventional unit 1 and the iso unit $\hat{1}$ (as well as the conventional product $a\times b$ and its isotopic form $a\hat{\times}b$) have the same basic axioms and coincide at the abstract level by conception. The isounit $\hat{1}$ is so chosen that it follows the axioms of the unit 1 namely; boundedness, smoothness, nowhere degeneracy, hermiticity and positive-definiteness. This ensures that the lifting $1\to\hat{1}$ is an isotopy and conventional unit 1 and the isounit $\hat{1}$ coincide at the abstract level of conception.

Thus, the isonumbers are the generalization of the conventional numbers characterized by the isounit and the isoproduct as defined above.

The liftings $a\to\hat{a}$, and $\times\to\hat{\times}$ can be used jointly or individually.

It is important to note that unlike isotopy of multiplication $\times\to\hat{\times}$, the lifting of the addition $+\to\hat{+}$ implies general loss of left and right distributive laws. Hence the study of such a lifting is the question of independent mathematical investigation.

The first generalization was introduced by Prof. Santilli when he generalized the real, complex and quaternion numbers [23], [24] based on the lifting of the unit 1 into isounit $\hat{1}$ as defined above. Resulting numbers are called isorealnumbers, isocomplex numbers and isoquaternion numbers.

In fact, this lifting leads to a variety of algebraic structures which are often used in physics. The following flowchart is self explanatory.

Isonumbers → Isofields → Isospaces → Isotransformations → Isoalgebras → Isogroups → Isosymmetries → Isorepresentations → Isogeometries etc.

The isounit is generally assumed to be outside the original field with all the possible compatible conditions imposed on it. For rudiments of isomathematics reader can refer to [1, 6, 7, 25].

The lifting of unit I to isounit $\hat{I}$ may be represented as, $I \to \hat{I}(t,r,\dot{r},p,T,\psi,\psi^{\dagger},\partial\psi,\partial\psi^{\dagger},\ldots)$. where t is time, r is the position vector, p is the momentum vector, ψ is the wave function and $\psi^{\dagger}$ are the corresponding partial differentials. The positive definiteness of the isounit $\hat{I}$ is assured by, $\hat{I}(t,r,\dot{r},p,T,\psi,\psi^{\dagger},\partial\psi,\partial\psi^{\dagger},\ldots) = \frac{1}{T} > 0$ where T is called the isotopic element, a positive definite quantity. The isonumbers are generated as, $\hat{n} = n \times \hat{I}$, $n = 0,1,2,3,\ldots$.

Isofields are of two types, isofield of first kind; wherein the isounit does not belong to the original field, and isofield of second kind; wherein the isounit belongs to the original field. The elements of the isofield are called as isonumbers. This leads to number of new terms and parallel developments of conventional mathematics.

2. Isounits and Their Isoduals

As stated earlier, the isonumbers and their product can first be introduced as the generalization of conventional numbers by equations (1) and (2) as above.

Prof. Santilli further, introduced *isodual isonumbers* [26, 27, 28] by lifting the isounit into the form

$$\hat{1}^{d} \hat{\times}^{d} a = a \hat{\times}^{d} \hat{1}^{d} = a, \text{where} \quad \hat{1}^{d} := -\hat{1} \qquad (3)$$

called the *isodual isounit* following lifting of iso multiplication defined in (1) into the *isodual multiplication* called *isoduality* as

$$a\hat{\times}b \to a\hat{\times}^{d} b := a \times T^{d} \times b = -a \times T \times b = -a\hat{\times}b \quad \text{where} T^{d} = -T \qquad (4)$$

The *isodual isonumbers* were first conceived as characterized by isodual multiplication (4) with respect to the multiplicative *isodual isounit* $\hat{1}^{d} = -\hat{1}$.

The significance of isonumbers and isodual isonumbers lies in fulfilling the specific physical needs refs [18, 29, 30, 31] as given below;

- In the exterior dynamical system ordinary particles moving in the vacuum are characterized by conventional numbers.
- In the interior dynamical system ordinary particles moving in the physical medium are characterized by isonumbers.
- In the exterior dynamical system ordinary antiparticles moving in vacuum are characterized by isodual numbers.

In the interior dynamical system the antiparticles moving in the physical medium are characterized by isodual isonumbers.

Interpretation of customary characterization of antiparticles via negative-energy solutions of Dirac's equations behave in an un-physical way when interpreted with respect to the same numbers and unit 1 of particles, forcing various hypothetical assumptions and postulates, where as, reinterpretation of antiparticles with same negative energy solutions when interpreted as belonging to the field of isodual numbers behave in a fully physical way ref [1]. This treatment of antiparticles with isodual numbers also leads to intriguing geometrical implications which predict another universe, called as *isodual universe*, interconnected to our universe via isoduality and identified by the isodualities of Riemannian geometry and their isoduals refs.[31, 24, 32]. Thus, the isodual theory emerged from the identification of negative units in the antiparticle component of the conventional Dirac equation and the reconstruction of the theory with respect to this new negative unit. Hence isoduality provides a mere reinterpretation of Dirac's original notion of antiparticle leaving all numerical predictions electro-weak interactions essentially unchanged.

In view of the definition of an isofield [1], we can say that an isofield is an additive abelian group equipped with a new unit (called isounit) and isomultiplication defined appropriately so that the resulting structure becomes a field.

If the original field is alternative then the isofield also satisfies weaker isoalternative laws as follows. $\hat{a}\hat{\times}(\hat{b}\hat{\times}\hat{b}) = (\hat{a}\hat{\times}\hat{b})\hat{\times}\hat{b}$ and $\hat{a}\hat{\times}(\hat{a}\hat{\times}\hat{b}) = (\hat{a}\hat{\times}\hat{a})\hat{\times}\hat{b}$.

We mention two important propositions by Santilli.

Proposition 2.1. The necessary and sufficient condition for the lifting (where the multiplication is lifted but elements not the elements) $F(a,+,\times) \to \hat{F}(a,+,\hat{\times}), \hat{\times} = \times T \times, \hat{1} = T^{-1}$ to be an isotopy (that is for $\hat{F}$ to verify all axioms of the original field F) is that T is a non-null element of the original field F.

Proposition 2.2. The lifting (where both the multiplication and the elements are lifted)

$$F(a,+,\times) \to \hat{F}(\hat{a},+,\hat{\times}), \hat{a} = a \times \hat{1}, \hat{\times} = \times T \times, \hat{1} = T^{-1}$$

constitutes an isotopy even when the multiplicative isounit $\hat{1}$ is not an element of the original field.

The above proposition guarantees the physically fundamental capability of generating Plank's unit ν of quantum mechanics into an integro-differential operator $\hat{1}$ for quantitative treatment of nonlocal interactions [33].

As the first application of the isotopies of numbers Santilli

considers the set $S=<in>$, the set of all purely imaginary numbers. This set is not closed ($i^2=-1\notin S$). On the other hand, the same set S represented as $\hat{S}(\hat{n},+,\hat{\times})$ with $\hat{n}=in$ constitutes an isofield. i.e. it verifies all the axioms of a field including closure under isomultiplication because $T=i^{-1}$ and $\hat{n}\hat{\times}\hat{m}=in\hat{\times}im=inm\in\hat{S}$.

This illustrates an important fact that, even when a given set does not constitute a field, there may exist an isotopy under which it verifies the axioms of a field.

As stated earlier the lifting of $+$ to $\hat{+}$ does not necessarily produce an isotopy of a given field. This lifting does not preserve the distributivity in the resulting set as stated in the following proposition 2.3.

Proposition 2.3 The lifting $F(a,+,\times)\to\hat{F}(\hat{a},\hat{+},\hat{\times})$ where

$$\hat{a}=a\times 1,\quad \hat{+}=+\hat{K}+,\quad \hat{0}=-\hat{K}=-\hat{K}\times\hat{1},\quad \hat{\times}=\times T\times,\quad \hat{1}=T^{-1}$$

where K is an element of the original field F and T is an arbitrary invertible quantity, is not an isotopy for all nontrivial values of the quantity $K\neq 0$, because it preserves all the axioms of proposition 2.1 except the distributive law.

Based on the failure of distributivity Santilli defines "pseudoisofields" as follows.

Definition 2.1 Let $\hat{F}(\hat{a},+,\hat{\times})$ be an isofield as defined above. Then the "pseudoisofields" $\hat{F}(\hat{a},\hat{+},\hat{\times})$ are given by the images of $\hat{F}(\hat{a},+,\hat{\times})$ under all possible liftings of the addition $+\to\hat{+}=+K+$, with additive isounit $\hat{0}=-\hat{K}=-K\times\hat{1}, K\neq 0$ in which case the elements $\hat{a}$ are called the "pseudoisonumbers".

For the algebra of isonumbers and isodual numbers readers are advised to refer [1, 34].

Images of field, isofield and pseudoisofield under the change of sign of the isounit $\hat{1}\to\hat{1}^d=-1$ is called the *Isotopic conjugation* or *isoduality* ref. [28, 29, 30].

Definition 2.2 Let $F(a,+,\times)$ be a field as per definition 1.1. Then the isodual field $F^d(a^d,+,\times^d)$ is constituted by the elements called "isodual numbers"

$$a^d:=a\times 1^d=-a \tag{5}$$

defined with respect to the "isodual multiplication" and related "isodual unit"

$$\times^d:=\times 1^d\times=-\times,\quad 1^d=-1. \tag{6}$$

Definition 2.3 Let $F(\hat{a},+,\hat{\times})$ be an isofield as per definition 1.1. Then the isodual isofield $F^d(\hat{a}^d,+,\hat{\times}^d)$ is constituted by the elements called "isodual isonumbers"

$$\hat{a}^d:=a^c\times 1^d=-a^c\times 1 \tag{7}$$

where a^c is the conventional conjugation of F (e.g. complex conjugation) defined in terms of the " isodual isomultiplication"

$$\hat{\times}^d:=\times T^d\times=-\hat{\times},\quad T^d=-T. \tag{8}$$

Definition 2.4 Let $F(\hat{a},\hat{+},\hat{\times})$ be a pseudofield $\hat{F}(\hat{a},\hat{+},\hat{\times})$ as per definition 2.1. Then the "isodual pseudofield" $F(\hat{a}^d,\hat{+}^d,\hat{\times}^d)$ is given by the image of the original isofield under isodualities (6) and (7) plus the additional isoduality

$$\hat{0}\to\hat{0}^d=-0 \tag{9}$$

and its elements $\hat{a}^d$ are called "isodual pseudonumbers.

2.1. Classes of Isofields

Kadeisvilli [35] classified isounits into five primary classes according to their usefulness.

- **CLASS I:** *Isounits*:- These are the isounits when they are sufficiently smooth, bounded, nowhere singular, Hermitian and positive-definite. This class is of primary use in physics for characterization of ordinary particles moving in interior physical conditions. This class represents the isotopy of the conventional unit.
- **CLASS II:** *Isodual Isounits*:- They are same as isounits except that they are negative-definite. Isodual isounits are used in physics to characterize antiparticles via reinterpretation of the negative energy solutions of Dirac's equation [31, 36]. They represent *isodual isotopy* according to *isodual conjugation.*
- **CLASS III:** *Singular Isounits*:- These occur when isounits are considered as a divergent limit, $1\Rightarrow\pm\infty$. These are used to represent gravitational collapse into a singularity and other limit conditions ref.[37, 23].
- **CLASS IV:** *Indefinite Isounits* :- This class represents isounits which are sufficiently smooth, bounded, nowhere singular, Hermitian and can smoothly interconnect positive definite with negative definite values. These are particularly used in mathematics.
- **CLASS V:** *General Isounits*, when they are solely Hermitian:- This is the most general class which includes preceding ones and permits a large variety of additional realizations including those in terms of discrete structures, discontinous functions, distributions etc.

Isofields can be classified according to the isounits as defined above. They are;

1. Isofields.
2. Isodual isofields.
3. Singular isofields.
4. Indefinite isofields.
5. General isofields.

The following four fundamental numbers are generated depending upon the isofield we consider;

1. (a) Ordinary numbers: real numbers $R(n,+,\times)$,

complex numbers $C(c,+,\times)$, quaternions $Q(q,+,\times)$ and octonians $O(o,+,\times)$ which are used in the characterization of particles in vacuum.

(b) Isonumbers: isoreal numbers $\hat{R}(\hat{n},+,\hat{\times})$, isocomplex numbers $\hat{C}(\hat{c},+,\hat{\times})$, isoquaternions $\hat{Q}(\hat{q},+,\hat{\times})$ and isooctonians $\hat{O}(\hat{o},+,\hat{\times})$ which are used for the characterization of particles within the physical media.

(c) Isodual numbers: isodual real numbers $R^d(n^d,+,\times^d)$, isodual complex numbers $C^d(c^d,+,\times^d)$, isodual quaternions $Q^d(q^d,+,\times^d)$ and isodual octonians $O^d(o^d,+,\times^d)$ which are used in the characterization of antiparticles in vacuum.

(d) Isodual isonumbers: isodual isoreal numbers $\hat{R}^d(\hat{n}^d,+,\hat{\times}^d)$, isodual isocomplex numbers $\hat{C}^d(\hat{c}^d,+,\hat{\times}^d)$, isodual isoquaternions $\hat{Q}^d(\hat{q}^d,+,\hat{\times}^d)$ and isodual isooctonians $\hat{O}^d(\hat{o}^d,+,\hat{\times}^d)$ which are used for the characterization of particles within the physical media.

2. Genofield is the generalization of isofield with the selection of an ordering of the multiplication to the left or to the right and applied for the more general Lie-admissible branch of hadronic mechanics.
3. Pseudofields, and
4. Pseudogenofields are the further generalization based on lifting of addition which relaxes at least one axiom of conventional fields, and which do have applications in other fields.
5. Hyper numbers can be constructed from hyperstructures ref.[35].

2.2. Isospaces

Let $S(x,g,R(n,+,\times))$ be a metric (or pseudo metric) n-dimensional space with local coordinates x and (Hermitean) metric $g = g^\dagger$ over the field of reals $R(n,+,\times)$. Then the *isospace* $\hat{S}(x,\hat{g},\hat{R}(n,+,\hat{\times}))$ first introduced in [38] is characterized by;

$$\begin{aligned} &\hat{S}(x,\hat{g},\hat{R}(n,+,\hat{\times})): \quad \hat{g} = T \times g, \\ &\hat{\times} = \times T, \quad \hat{1} = T^{-1}. \end{aligned} \tag{10}$$

Also the *isodual isospace* [28] is given by;

$$\hat{S}^d(x,\hat{g}^d,\hat{R}^d(n^d,+,\hat{\times}^d)): \quad \hat{g}^d = T^d \times g = -T \times g,$$

$$\hat{\times}^d = \times T^d \times = -\times T \times, \quad \hat{1}^d = -\hat{1}. \tag{11}$$

Note that isospaces $\hat{S}(x,\hat{g},\hat{R}(n,+,\hat{\times}))$ coincide with spaces $S(x,g,R(n,+,\times))$ at the abstract level of conception. Spaces have the most general known curvature and integral character owing to the arbitrariness in the isotopic element T. The isometries $\hat{g} = T \times g$ have the most general possible, nonlinear, nonlocal, noncanonical dependence in all variables,

$$g = g(x) \to \hat{g} = T(t,x,\dot{x},\ddot{x},\ldots) \times g(x) = \hat{g}(t,x,\dot{x},\ddot{x},\ldots). \tag{12}$$

The isospaces which are most important for physical and mathematical applications are *isoeuclidean spaces* $\hat{E}(x,\hat{\delta},\hat{R})$, *isominkowski spaces* $\hat{M}(x,\hat{\eta},\hat{R})$ and *isoriemanian spaces* $\hat{R}(x,\hat{g},\hat{R})$. These are the foundations of the representation of nonlinear, nonlocal, and noncanonical interior systems in nonrelativistic and gravitational interior problems [31, 23].

Also, *pseudoisospaces* can be defined as the images $\hat{S}(x,\hat{g},\hat{R}(n,\hat{+},\hat{\times}))$ of the original space characterized by further lifting $+ \to \hat{+} = +K+$, $0 \to \hat{0} = -K$. Subsequently, *isodual pseudoisospaces* are also defined.

2.4. Isoalgebras

The concept of isoalgebra was fundamental in the correct description of interior dynamical systems. As conventional numbers constitute normed algebras with unit, isoalgebras were defined to represent isonumbers ref. [21, 8, 39]. An isovector space $\hat{U}$ with elements $A,B,C\ldots$ and isomultiplication $\hat{e}$ over an isofield $\hat{F}(a,+,\hat{\times})$ with elements a,b,c and isomultiplication $a\hat{\times}b$ with multiplicative isounit $\hat{1} = T^{-1}$ is called (associative or nonassociative) *isoalgebra* when it satisfies right and left scaler and distributive laws;

$$(a\hat{\times}A)\hat{\odot}B = A\hat{\odot}(a\hat{\times}B) = a\hat{\times}(A\hat{\odot}B). \tag{15}$$

$$(A\hat{\times}a)\hat{\odot}B = A\hat{\odot}(B\hat{\times}a) = (A\hat{\odot})B\hat{\times}a \tag{16}$$

$$\begin{aligned} &A\hat{\odot}(B+C) = A\hat{\odot}B + A\hat{\odot}C, \\ &(B+C)\hat{\odot}A = B\hat{\odot}A + C\hat{\odot}A \end{aligned} \tag{17}$$

for all the elements $A,B,C \in \hat{U}$ and $a,b,c \in \hat{F}$.

Note that the isoalgebra $\hat{U}$ may contain the matrices where as the iso field $\hat{F}$ can contain ordinary numbers.

The isoalgebra $\hat{U}$ is an *isodivision algebra* if the equation $A\hat{\times}x = B$ always admits a solution in $\hat{U}$, for nonzero A.

Isonorm can be defined in the following manner;

Let $\hat{e}_k$ be an "isobasis" of $\hat{U}$ over the isofield $\hat{F}(a,+,\hat{\times})$. Then the generic element $A \in \hat{U}$ can be written as $A = \sum_{k=1,\ldots,m} n_k \hat{\times} \hat{e}_k$, with $n_k \in \hat{F}$ and $\hat{e}^{\hat{2}} = \sum_k \hat{e}_k \hat{\odot} \hat{e}_k = 1$. The *isonorm* of $\hat{U}$ in the isobasis considered, is then given by;

$$\hat{A} = (\sum_{k=1...,m} n_k^{\hat{2}})^{\hat{\frac{1}{2}}} \times \hat{1} = (\sum_{k=1...,m} n_k \hat{\times} n_k)^{\hat{\frac{1}{2}}} \times \hat{1} \in \hat{F} \quad (18)$$

The isoalgebra $\hat{U}$ is said to be isoassociative if;

$$A \hat{\odot} (B \hat{\odot} C) = (A \hat{\odot} B) \hat{\odot} C, \forall A, B, C \in \hat{U} \quad (19)$$

(isoassociative law)

and

$$A^{\hat{2}} \hat{\odot} B = A \hat{\odot} (A \hat{\odot} B), A \hat{\odot} B^{\hat{2}} = (A \hat{\odot} B) \hat{\odot} B \quad (20)$$

(isoalternative laws)

The isoalgebra $\hat{U}$ is said to be *Lie-isotopic* when the isoproduct $\hat{e}$ satisfies Lie-algebra axioms (anticommutativity and Jacobi laws) in the following form;

$$A \hat{\odot} B = ATB - BTA, \quad A, T, B, \textbf{etc.} = \textbf{assoc.} \quad (21)$$

It is said to be *lie-admissible* when the antisymmetric bracket product is;

$$[A; B] := A \hat{\odot} B - B \hat{\odot} A \quad (22)$$

and is *Lie-isotopic* as in the realization;

$$A \hat{\odot} B = ARB - BSA. \quad (23)$$

We shall be mainly interested in the isoassociative isonormed algebras with isounit $\hat{1}$ which can be extended to isoalternative algebras in order to include isooctonians.

Extension of U and $\hat{U}$ under the pseudofield $\hat{F}(a, \hat{+}, \hat{\times})$ implies loss of distributive laws and hence do not remain algebras in the real sense, however, we call them *pseudoisoalgebras* ref.[39].

2.5. Isoreal Numbers and Their Isoduals

2.5.1. Real Numbers

Real numbers constitute a one-dimensional normed associative and commutative algebra $U(1)$ ref.[1].

Real numbers are realized ref.[8] as a one-dimensional real Euclidean space $E_1(x, , R(n, +, \times))$ which represents a straight line with origin at 0, local coordinates x, metric $\delta = 1$, additive unit 0 and multiplicative unit 1. Another characterization of real numbers is defined by the isomorphism of the reals $R(n, +, \times)$ into the commutative one-dimensional multiplicative group of dilations $G(1)$ defined by;

$$x' = n \times x, \quad n \in R(n, +, \times), \quad x, x' \in E_1(x, \delta, R). \quad (24)$$

The basis is given by

$$e = 1 \quad (25)$$

with the norm defined by

$$|n| = (n \times n)^{\frac{1}{2}} > 0 \quad (26)$$

and

$$|n \times n'| = |n| \times |n'|. \quad (27)$$

2.5.2. Isodual Real Numbers

Isodual Real numbers constitute a one-dimensional isodual associative and commutative normed algebra $U^d(1)$ which is anti-isomorphic to $U(1)$ ref.[1].

Isodual real numbers are the conventional numbers n defined with respect to the isodual unit $1^d = -1$. The isodual conjugation of real numbers is then written as

$$n \simeq n \times 1 \to n^d = n \times 1^d = -n. \quad (28)$$

Note that, such a sign inversion occurs when the isodual real numbers are projected in the field of conventional real numbers. As a result, all the numerical values change sign under isoduality.

The one-dimensional real isodual Euclidean space $E^d(x, \delta^d, R^d(n^d, +, \times^d))$ is a straight line, with conventional additive unit 0, and isodual multiplicative unit $1^d = -1$. The $R^d(n^d, +, \times^d)$ represents the Euclidean space $E^d(x, \delta^d, R^d(n^d, +, \times^d))$. Also, the isodual dilations are defined by

$$x' = n^d \times^d x = n \times x \quad (29)$$

This establishes an isomorphism between $R^d(n^d, +, \times^d)$ and the isodual group of dilations $G^d(1)$ (the conventional group reformulated according to the multiplicative unit 1^d). Santilli points out that $E_1(x, \delta, R)$ and $E_1^d(x, \delta^d, R^d)$ are antiisomorphic and the same property holds for $G(1)$ and $G^d(1)$. Also, the isodual dilations coincide with dilations as defined above. Santilli further says that *"this could be the a reason for the lack of detection of isodual numbers until then."* ref.s [38, 27, 28].

In the isodual case, the isodual basis is given by

$$e^d = 1^d \quad (30)$$

with isodual norm

$$|n| = (n \times n)^{\frac{1}{2}} \times 1^d = |n| \times 1^d = -|n| < 0 \quad (31)$$

satisfying the axioms

$$|n^d \times n'^d|^d = |n^d|^d \times^d |n'^d|^d . \quad (32)$$

2.5.3. Isoreal Numbers

Isoreal numbers constitute a one-dimensional, isonormed isoassociative and isocommutative isoalgebra $\hat{U}(1) \approx U(1)$ ref.[1].

Isoreal numbers are the numbers $\hat{n} = n \times \hat{1}$ of an isofield of Class I, with isomultiplication defined by $\hat{x} = \times T \times$ and isounit $\hat{1} = T^{-1} > 0$, generally outside the original field $R(n,+,\times)$. These can be represented as the isoeuclidean spaces $\hat{E}_{1,1}(x,\hat{\delta},\hat{R}(\hat{n},+,\hat{x}))$ with $\hat{\delta} = T\delta$, over $\hat{R}(\hat{n},+,\hat{x})$ the isotopes of conventional one-dimensional Euclidean spaces $E_1(x,\delta,R)$.

Some of the important remarks are as follows.

- The conventional Euclidean space $E_1(x,\delta,R)$ and its isotopic covering $\hat{E}_{1,1}(x,\hat{\delta},\hat{R})$ are locally isomorphic due to the joint liftings $\delta \to \hat{\delta} = T \times \delta$ and $1 \to \hat{1} = T^{-1}$.
- $\hat{E}_{1,1}(x,\hat{\delta},\hat{R})$ is not a Riemannian space because of the intrinsic dependence of the isometric $\hat{\delta}$ on the derivatives $\dot{x}, \ddot{x}, \ldots$ as well as the fact that the basic unit is not the conventional quantity 1.
- However, $\hat{E}_{1,1}(x,\hat{\delta},\hat{R})$ is a simple, yet bona-fide isoriemannian space [24], because $\hat{\delta} = T \times \delta = \hat{\delta}(t,x,\dot{x},\ddot{x},\ldots)$, where the local dependence is generally nonlinear, nonlocal and noncanonical in all variables.

In fact, the *one-dimensional isospace* $\hat{E}_{1,1}(x,\hat{\delta},\hat{R})$ represents a one-dimensional generalization of conventional straight line, called as *isoline*. This is because of its intrinsically nonlinear, nonlocal and noncanonical metric $\hat{\delta}(t,x,\dot{x},\ddot{x},\ldots)$ with multiplicative isounit $\hat{1} = \hat{1}(t,x,\dot{x},\ddot{x},\ldots)$. Then $\hat{R}_1(\hat{n},\hat{+},\hat{x})$ can be realized via *isodilations* on $\hat{E}_1(x,\hat{\delta},\hat{R})$ as;

$$x' = \hat{n}\,\hat{x}\,x = n \times x, \quad (33)$$

which is isodual dilation and represents one-dimensional isogroup of isodilations $\hat{G}(1)$ same as the group $G(1)$ realized with respect to isounit $\hat{1}$.

Again, the *isobasis* is given by

$$\hat{e} = \hat{1} \quad (34)$$

with *isonorm* defined as;

$$\|\hat{n}\| := (n \times n)^{\frac{1}{2}} \times \hat{1} = |n| \times \hat{1} \quad (35)$$

which is the conventional norm only rescaled to the new unit $\hat{1}$. We then also have

$$\|\hat{n}\,\hat{x}\,\hat{n}'\| = \|\hat{n}\|\,\hat{x}\,\|\hat{n}'\|. \quad (36)$$

2.5.4. Isodual Isoreal Numbers

The isodual isoreal numbers are the realization of the one-dimensional isodual, isonormed, isoassociative and isocommutative isoalgebra $\hat{U}^d(1) \approx U^d(1)$ ***ref.[1].***

These are the isodual numbers

$$\hat{n}^d = n \times \hat{1}^d, \quad \hat{1}^d = -\hat{1} \quad (37)$$

in the isodual isofield $\hat{R}_{11}^d(\hat{n}^d,+,\times^d)$. These correspond to $\hat{E}_{11}^d(x,\hat{\delta}^d,\hat{R}^d)$ the isoeuclidean space of Class II $\hat{E}_{11}^d(x,\hat{\delta}^d,\hat{R}^d)$ of dimension one with *isodual isodilations*

$$x' = \hat{n}^d\,\hat{\times}^d\,x \quad (38)$$

coinciding with dilations (24). This also characterizes an isomorphism isodual isoreal numbers with the one-dimensional isodual isogroup $\hat{G}^d(1)$. The underlying isomorphism

$$E_1^d(x,\delta^d,R^d(n^d,+,\times^d)) \approx E_{11,1}^d(x,\hat{\delta}^d,\hat{R}^d(\hat{n}^d,+,\hat{\times}^d))$$

implies the $\hat{G}^d(1) \approx G^d(1)$.

The *isodual isobasis* is defined by

$$\hat{e}^d = \hat{1}^d \quad (39)$$

The isodual isonorm

$$\|\hat{n}^d\|^d := (n \times n)^{\frac{1}{2}} \times \hat{1}^d = -\|\hat{n}\| \quad (40)$$

verifies the axioms

$$\|\hat{n}^d\,\hat{\times}^d\,\hat{n}'\|^d = \|\hat{n}^d\|^d\,\hat{\times}^d\,\|\hat{n}'\|^d \quad (41)$$

2.6. Isocomplex Numbers and Their Isoduals

2.6.1. Complex Numbers

Complex numbers constitute a two-dimensional, normed associative and commutative algebra $U(2)$ *ref.[1].*

Complex numbers $c = n_0 + n_1 i$ where n_0 and n_1 are real numbers and i is an imaginary unit, are represented in a Gauss plane which is a realization of two-dimensional Euclidean space $E_2(x,\delta,R(n,+,\times))$ satisfying

$$x^2 = x^t \delta_{ij} x_j = x_1^2 + x_2^2 \in R(n,+,\times) \quad (42)$$

whose group of isometries is one dimensional Lie Group $O(2)$, the invariance of the circle. Hence, complex numbers can be represented via fundamental representation of $O(2)$ as follows.

A one-to-one correspondence between complex numbers and points in the Gauss plane can be obtained by following *dilative rotations*

$$z' = (x_1 + x_2 i)' = c \circ z = (n_0 + n_1 i) \circ (x_1 + x_2 i) \quad (43)$$

and multiplication

$$c \circ z = (n_0, n_1) \circ (x_1, x_2) = (n_0 x_1 - n_1 x_2, n_0 x_2 + n_1 x_1) \quad (44)$$

which preserve all the properties of a field.

Representation of a complex number via matrices has the following form

$$c := n_0 \times I_0 + n_1 \times I_1 = \begin{pmatrix} n_0 & n_1 \times i \\ n_1 \times i & n_0 \end{pmatrix} \quad (45)$$

where

$$-\bar{c}^{d} = c, i^{d} = i \quad (46)$$

which are well known as the identity and fundamental representation of $O(2)$.

Norm can also be defined as

$$| c | = | n_0 + n_1 \times i | := (\mathrm{Det} c)^{\frac{1}{2}} = (n_0^2 + n_1^2)^{\frac{1}{2}} \quad (47)$$

Also, the identification of basis in terms of matrices is $e_1 = I_0$ and $e_2 = I_1$.

2.6.2. Isodual Complex Numbers

Isodual complex numbers constitute a two-dimensional isodual, normed, associative and commutative algebra $U^d(2)$ ***anti-isomorphic to*** $U(2)$ *ref.[1].*

Isodual complex numbers are given by

$$C^d = \{(c^d, +, \times^d) | \times^d = -\times, i^d = -i; c^d = \bar{c} \times i^d = -\bar{c}, \bar{c} \in \bar{C}\} \quad (48)$$

where $\bar{c}$ is the complex conjugation. Thus, given a complex number $c = n_0 + n_1 \times i$, its isodual is given by

$$c^d = -\bar{c} = n_o^d + n_1^d \times \bar{i} = -n_0 - n_1 \times \bar{i} = -n_0 + n_1 \times i \in C^d. \quad (49)$$

Considering the group of isometries, the one-dimensional isodual Lie group $O^d(2)$ i.e. the image of O_2 under the lifting $I = diag.(1,1) \to I^d = diag.(-1,-1)$ of the two-dimensional isodual Euclidean space

$E_2^{\ d}(x, \delta^d, R^d(n^d, +, \times^d))$ with basic invariant

$$x^{2d} = x^t \delta^d x = x_i \delta_{ij}^d x_j = x_1^{2d} + x_2^{2d} = x_1 \times^d x_1 + x_2 \times^d x_2 = -x_1^2 - x_2^2 \in R^d(n^d, +, \times^d) \quad (50)$$

isodual complex numbers can be characterized by the isorepresentation of $O^d(2)$.

Now, the image of the conventional plane under isoduality is the isodual Gauss plane. Also, a one-to-one correspondence between the points $P = (x_1, x_2)$ and complex numbers can be defined by *isodual dilative rotations* as

$$z' = (x_1 + x_2 \times i)' = c^d \circ^d z = (-n_0 + n_1 \times i) \circ^d (x_1 + x_2 \times i) \quad (51)$$

following the multiplication rules

$$c^d \circ^d z = (-n_0, n_1) \circ^d (x_1, x_2) = (-n_0 \times x_1 \times n_1 \times x_2, -n_0 \times x_2 + n_1 \times x_1) \quad (52)$$

which preserve all the properties of a field.

Isodual transformations form an isodual group $G^d(2)$ antiisomorphic to $G(2)$. Even the one-to-one correspondence between complex numbers and Gauss plane continues under isoduality.

Matrix representation of isodual complex numbers can be defined as

$$c^d := n_0^d \times i_0^d + n_1^d \times i_1^d = \begin{pmatrix} -n_0 & n_1 \times i \\ n_1 \times i & -n_0 \end{pmatrix}, \quad (53)$$

$$i_0^d = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, i_1 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \quad (54)$$

with the isodual unit and isodual representations of $O^d(2)$ respectively.

The *isodual norm* can be defined as $| c^d |^d = | -n_0 + n_1 \times i |^d :=$

$$[\mathbf{Det}_R (c^d \times T^d)]^{\frac{1}{2}} \times i_0^d = (c^{-d} \times^d c^d)^{\frac{1}{2}} \times i_0^d$$

which may be written as

$$| c^d |^d = (c \times \bar{c}) \times i_0^d = (n_0^2 + n_1^2) \times i_0^d \quad (55)$$

and verifies the axioms

$$| c^d \circ c'^d |^d = | c^d |^d \times^d | c'^d |^d \in R^d, \quad c^d, c'^d \in C^d. \quad (56)$$

The *isodual basis* in terms of matrices is given by

$$e_i^d = i_0^d, \quad e_2^d = i_1^d. \quad (57)$$

2.6.3. Isocomplex Numbers

Isocomplex numbers constitute a two-dimensional, isonormed, isoassociative and isocommutative isoalgebras over the isoreals $\hat{U}(2) \approx U(2)$ *ref.[1].*

In this case we consider the isofield of isocomplex numbers

$$\hat{C} = \{(\hat{c}, +, \hat{\times}) \mid \hat{x} = \times T \times, \hat{1} = T^{-1}, \hat{c} = c \times \hat{1}, \\ c \in C(c, +, \times)\} \tag{58}$$

withe the generic element $\hat{c} = \hat{n}_0 + \hat{n}_1 \times i$. Here we need the two-dimensional isoeuclidean space of class I, $\hat{E}_{1,2}(x, \hat{\delta}, \hat{R}(\hat{n}, +, \hat{\times}))$. The most important realization used in the physical literature has the diagonalized and positive-definite isotopic element and isounit

$$T = diag.(b_1^2, b_2^2), \hat{1} = diag.(b_1^{-1}, b_2^{-2}), b_k > 0, k = 1,2. \tag{59}$$

with basic isoseparation

$$\hat{e}_1 = \hat{I}_0, \quad \hat{e}_{k+1} \\ = \hat{i}_k, \quad k = 1,2,3. \tag{60}$$

The group of isometries of this space is the Lie group $\hat{O}(2) \approx O(2)$, the group constructed with respect to the multiplicative isounit $\hat{1} = diag.(b_1^{-2}, b_2^{-2})$ which provides the invariance of all possible ellipses with semiaxes $a = b_1^{-2}, b = b_2^{-2}$ as the infinitely possible deformation of the circle $x^2 = x_1^2 + x_2^2 \in R(n, +, \times)$. Thus, isocomplex numbers are characterizable via fundamental representation of $\hat{O}(2)$.

Isocomplex numbers $\hat{c} = (\hat{n}_0, \hat{n}_1)$ can also be characterized to be the set of points $P = (\hat{x}_1, \hat{x}_2)$ on the *isogauss plane* on $\hat{E}_{1,2}(x, \hat{\delta}, \hat{R}(\hat{n}, +, \hat{\times}))$.

In fact, a one-to one correspondence between isocomplex numbers $\hat{C}(\hat{c}, +, \hat{\times})$ and the points on the isogauss plane can be defined via following *isodilative isorotations*

$$z' = (x_1 + x_2 \times i)' = \hat{c} \hat{\circ} z \tag{61}$$

characterized by the isomultiplication defined as

$$\hat{c} \hat{\circ} z = (\hat{n}_0, \hat{n}_1) \hat{\circ} (x_1, x_2) =$$

$$= ([(n_0 \times x_0) \times \hat{1} - \hat{\Delta}^{\frac{1}{2}} \times (n_1 \times n_2) \times \hat{1}], [(n_0 \times x_2) \times \hat{1} + (n_1 \times x_1) \times \hat{1}]),$$

with

$$\Delta = DetT = b_1^2 \times b_2^2 \tag{62}$$

Isocomplex numbers also admit following two-by-two matrix representation.

$$\hat{c} = \hat{n}_0 \times \hat{i}_0 \hat{+} n_1 \hat{i}_1 = \\ \begin{pmatrix} n_0 \times b_1^{-2} & i \times n_1 \times b_1^2 \times \Delta^{-\frac{1}{2}} \\ i \times n_1 \times b_2^2 \times \Delta^{-\frac{1}{2}} & n_0 \times b_1^{-2} \end{pmatrix} \tag{63}$$

where

$$\hat{1} = \hat{I}_0 = \begin{pmatrix} b_1^{-2} & 0 \\ 0 & b_2^{-2} \end{pmatrix}, \hat{I}_1 = \Delta^{-\frac{1}{2}} \begin{pmatrix} 0 & i \times b_1^2 \\ i \times b_2^2 & 0 \end{pmatrix} \tag{64}$$

and

$$\Delta = \mathbf{Det}T = b_1^2 b_2^2 \tag{65}$$

which characterize the isounit and the fundamental (adjoint)representation of $\hat{O}(2)$ respectively.

The set of matrices (63) is closed under addition and isomultiplication. Also, each element possesses the isoinverse

$$\hat{c}^{-\hat{1}} = \hat{c}^{-1} \times \hat{1} \tag{66}$$

where $\hat{c}^{-\hat{1}}$ is the ordinary inverse. As a result, $\hat{S}(\hat{c}, +, \hat{\times})$ is an isofield with the local isomorphism $\hat{S}(\hat{c}, +, \hat{\times}) \approx \hat{C}(\hat{c}, +, \hat{\times})$. We note that the one-to-one correspondence between complex numbers and Gauss plane is preserved under isotopy. It is important know that the realization of complex numbers as matrices is not unique.

The *isonorm* is defined as

$$\|\hat{c}\| = [Det_R(\hat{c} \times T)]^{\frac{1}{2}} \times \hat{I}_0 = (n_0^2 + \Delta n_1^2)^{\frac{1}{2}} \times \hat{I}_0 \tag{67}$$

which readily verifies the axiom

$$\|\hat{c} \hat{\circ} \hat{c}'\| = \|\hat{c}\| \hat{\times} \|\hat{c}'\| \in \hat{R}, \quad \hat{c}, \hat{c}' \in \hat{C}. \tag{68}$$

The *isobasis* is given by

$$\hat{e}_1 = \hat{I}_0 \quad \hat{e}_2 = \hat{1}. \tag{69}$$

2.6.4. Isodual Isocomplex Numbers

The isodual isocomplex numbers constitute a two-dimensional, isodual, isonormed, isoassociative and isocommutative isoalgebras over the isodual isoreals $\hat{U}^d(2) \approx U^d(2)$ ref.[1].

Now the isodual isocomplex numbers are defined as

$$\hat{C}^d = \{(\hat{c}^d, +, \times^d) \mid \hat{c} = -\bar{c}\hat{1}^d, \hat{x}^d = \times T^d \times, T^d \\ = -T, \hat{1}^d = T^{-1d}, c \in C(c, +, \times)\} \tag{70}$$

with generic element $\hat{c}^d = \hat{n}^d + \hat{n}_1{}^d \times i^d = -\hat{n}_0 + \hat{n}_1 \times i.$ Here we need a two-dimensional isodual isoeuclidean space $E^d_{II,2}(x, \delta^d, \hat{R}^d(n^d, +, \hat{\times}^d))$ with the realization

$$T^d = diag.(-b_1^2, -b_2^2), \hat{1}^d = \\ diag.(-b_1^{-2}, -b_2^{-2}), b_k > 0, k = 1,2, \tag{71}$$

with basic isoseparation

$$x^{2d} = (x^t \delta^d x) \times \hat{1}^d = (x_i \delta^d_{ij} x_j) \times \hat{1}^d = \\ (-x_1 b_1^2 x_1 - x_2 b_2^2 x_2) \times \hat{1}^d \in \hat{R}(\hat{n}^d, +, \times^d), \tag{72}$$

whose group of isosymmetries is the isodual isoorthogonal group $\hat{O}^d(2) \tilde{} O^d(2)$.

The *isodual isogauss plane* is defined as the set of points $P = (\hat{x}_1, x_2)$ on $\hat{E}_{1,2}(x, \delta^d, \hat{R}^d(\hat{n}^d, +, \hat{\times}^d))$ which characterize the isocomplex numbers $\hat{c} = (-\hat{n}_0, \hat{n}_1)$.

The correspondence between the isodual isocomplex numbers $\hat{C}^d(\hat{c}^d, +, \times^d)$ and the isodual gauss plane can be made one-to-one by the isodual isodilative isorotations

$$z' = (x_1 + x_2 \times i)' = \hat{c}^d \circ^d z \tag{73}$$

having rule for multiplication as

$$\hat{c} \hat{\circ}^d z = (\hat{n}_0, \hat{n}_1) \hat{\circ}^d (x_1, x_2) = \tag{74}$$

$$= [(-n_0 \times x_0) \times \hat{1} + \hat{\Delta}^{\frac{1}{2}} \times (n_1 \times x_2) \times \hat{1}], \\ [(-n_0 \times x_2) \times \hat{1} + (n_1 \times x_1) \times \hat{1}].$$

Isodual isoguass planes characterizes isodual isofield. Also the isodual isotransformations forms an isodual isogroup $\hat{G}^d(2) \approx G^d(2)$.

Isodual isocomplex numbers also admit the following two-by-two matrix representation.

$$\hat{c}^d = \hat{n}_0{}^d \times^d \hat{I}_0{}^d + n_1^d \times^d \hat{I}^d = \\ \begin{pmatrix} -n_0 \times b_1^{-2} & i \times n_1 \times b_1^2 \times \Delta^{-\frac{1}{2}} \\ i \times n_1 \times b_2^2 \times \Delta^{-\frac{1}{2}} & -n_0 \times b_2^{-2} \end{pmatrix} \tag{75}$$

where

$$\hat{1}^d = \hat{I}_0{}^d = \begin{pmatrix} -b_1^{-2} & 0 \\ 0 & -b_2^{-2} \end{pmatrix}, \\ \hat{I}^d = \begin{pmatrix} 0 & -i \times b_1^2 \times \Delta^{-\frac{1}{2}} \\ -i \times b_2^2 \times \Delta^{-\frac{1}{2}} & 0 \end{pmatrix}. \tag{76}$$

This satisfies isomultiplication rule (74) characterizing the isodual isounit and fundamental representation of $\hat{O}^d(2)$.

The set of matrices representing isodual complex numbers $\hat{S}^d(\hat{c}^d, +, \times^d)$, is closed under addition and isomultiplication. Each element possesses the isodual isoinverse

$$(\hat{c}^{-1})^d = (\hat{c}^d)^{-1} \times \hat{I}^d. \tag{77}$$

As a result we get a local isomorphism $\hat{S}^d(\hat{c}^d, +, \times^d) \approx \hat{C}^d(\hat{c}^d, +, \times^d)$.

Now, the *isodual isonorm* can be defined as

$$\|\hat{c}\|^d = [Det_R(\hat{c}^d \times T^d)]^{\frac{1}{2}} \times \hat{I}_0{}^d = (n_0^2 + \Delta n_1^2)^{\frac{1}{2}} \times \hat{I}^d, \tag{78}$$

which verifies

$$\left\| \hat{c}^d \hat{\circ}^d \hat{c}'^d \right\|^d = \left\| \hat{c}^d \right\|^d \hat{\times}^d \left\| \hat{c}'^d \right\|^d \in \hat{R}^d, \quad \hat{c}^d, \hat{c}'^d \in \hat{C}^d. \tag{79}$$

The isodual isobasis is given by

$$\hat{e}_1{}^d = \hat{I}_0{}^d \quad \hat{e}_2{}^d = \hat{1}^d. \tag{80}$$

2.7. Isoquaternions and Their Isoduals

2.7.1. Quaternions

Quaternions constitute a normed, associative, non-commutative algebra of dimension 4 over reals $U(4)$ *ref.[1].*

Quaternions $q \in Q(q, +, \times)$ admit a realization in the complex Hermitean plane $E_2(z, \delta, C)$ with separation

$$E_2(z, \delta, C): \quad z ? z \quad z^{-i} \delta_{ij} z^j = z^{-1} z^1 + z^{-2} z^2, \\ \delta ? \equiv \delta \tag{81}$$

with basic (unimodular) invariant $SU(2)$. Hence quaternions have a fundamental representation $SU(2)$ by *Pauli's matrices.*

Quaternions Q can be realized as the pairs of complex numbers, $q = (c_1, c_2)$, $q \in Q$ and $c_1, c_2 \in C$ with multiplication $\circ$. Hermitean dilative rotation on $E_2(z, \delta, C)$ which leaves $z \dagger z$ invariant is given by

$$z'^1 = c_1 \circ z^1 + c_2 \circ z^2, \quad z'^2 = -\bar{c}_2 \circ z^1 + \bar{c}_1 \circ z^2, \tag{82}$$

where the dilation is represented by $\bar{c}_1 \circ c_1 + \bar{c}_2 \circ c_2 \neq 1$. These transformations form a group $G(4)$. This group is associative but noncommutative resulting into a one-to-one correspondence with quaternions.

Quaternions can be represented via matrices over the field of complex numbers $C(c, +, \times)$ as

$$q=\begin{pmatrix} c_1 & c_2 \\ -\bar{c}_2 & \bar{c}_1 \end{pmatrix} \tag{83}$$

with

$$c_1 = n_0 + n_3 \times i, \quad c_2 = n_1 + n_2 \times i \tag{84}$$

The matrix q admits the representation

$$q = n_0 \times I_0 + n_1 \times i_1 + n_2 \times i_2 + n_3 \times i_3 \tag{85}$$

where I_0, i_1, i_2, i_3 are the Pauli's matrices

$$I_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, i_1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad i_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, i_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \tag{86}$$

with fundamental relations

$$i_n \times i_m = -\varepsilon_{nmk} \times i_k, \quad n \neq m, \quad n,m = 1,2,3. \tag{87}$$

where ε_{nmk} is the tensor of rank three. The norm of the quaternion can be defined as

$$|q| = (q^{\dagger} q')^{\frac{1}{2}} = (\sum_{k=1,2,3} n_k^2)^{\frac{1}{2}}, \tag{88}$$

satisfying

$$|q \circ q'| = |q| \times |q'| \in R, \quad q,q' \in Q \tag{89}$$

The basis is defined by

$$e_1 = I_0, \quad e_{k+1} = i_k, \quad k = 1,2,3. \tag{90}$$

2.7.2. Isodual Quaternions

Isodual quaternions constitute an isodual four-dimensional, normed associative and noncommutative algebra over the isodual reals $U^d(4)$ which is anti-isomorphic to $U(4)$ ref.[1].

Isodual quaternions $q^d \in Q^d(q^d,+,\times^d)$ can be represented via the isodual Hermitean Euclidean space

$$E_2^d(z^d,\delta^d,C^d(c^d,+,\times^d)) : (z^{-id}\delta_{ij}^d z^{jd}) \times I^d = (-z^{-1}z^1 - z^{-2}z^2) \times I^d \in R^d. \tag{91}$$

Isodual complex numbers can also be realized via pairs of isodual complex numbers as $q^d = (c^d, c2^d), q^d \in Q^d, \quad c_1^d, c_2^d \in C^d$.

Also, the *isodual Hermitean dilative rotation* on $E_2^d(z^d,\delta^d,C^d(c^d,+,\times^d))$ leaving invariant $z^d \dagger \delta^d z^d$ is given by

$$z'^{1d} = c_1^d \circ^d z^{1d} - \bar{c}_2^d \circ^d z^{2d}, \quad z'^{2d} = c_2^d \circ^d z^{1d} + \bar{c}_1^d \circ^d z^{2d} \tag{92}$$

where the dilation is represented by the value $\bar{c}_1^d \circ^d c_1^d + \bar{c}_2^d \circ^d c_2^d \neq -1$.

These transformations form an associative but noncommutative isodual group $G^d(4)$ which is in one-to-one correspondence with isodual quaternions $Q^d(q^d,+,\times^d)$.

As a result there is a matrix representation of isodual complex numbers over the field of isodual complex numbers $C^d(c^d,+,\times^d)$ as

$$q^d = \begin{pmatrix} c_1^d & -\bar{c}_2^d \\ c_2^d & \bar{c}_1^d \end{pmatrix} \tag{93}$$

under the condition

$$c_1^d = -n_0 + n_3 \times i, \quad c_2^d = -n_1 + n_2 \times i \tag{94}$$

where $-\bar{c}^d = c, \quad i^d = i$.

We can represent q^d as

$$q^d = n_0^d \times^d I_0^d + n_1^d \times^d i_1^d + n_2^d \times^d i_2^d + n_3^d \times^d i_3^d = = -n_0 \times I_0 + n_1 \times i_1 + n_2 \times i_2 + n_3 \times i_3 \tag{95}$$

where i's are the Pauli's matrices. Note that Pauli's matrices change sign under isoduality although their product with isodual numbers is isoselfdual.

Isodual norm is then defined as

$$|q^d| = [Det_C(q^d \times T^d)] \times I^d = (-\sum_{k=0,1,2,3} n_k^2)^{\frac{1}{2}} \times I^d \tag{96}$$

satisfying

$$|q^d \circ^d q'^d|^d = |q^d|^d \times^d |q'^d|^d \in R^d, \quad q^d, q'^d \in Q^d. \tag{97}$$

The *isodual basis* is defined as

$$e_1^d = I_0^d, e_{k+1}^d = i_k, \quad k = 1,2,3. \tag{98}$$

2.7.3. Isoquaternions

Isoquaternions constitute a four-dimensional, isonormed, isoassociative, non-isocommutative isoalgebra over the

isoreals $\hat{U}(4) \approx U(4)$, *ref.[1].*

Isoquatrnions $\hat{q} \in \hat{Q}(\hat{q},+,\hat{\times})$ can be represented using two-dimensional, complex Hermitean isoeuclidean space of class I, $\hat{E}_{I,2}(\hat{z},\hat{\delta},\hat{C}), \hat{z}^k = z^k, \hat{z}_k = \hat{\delta}_{ki}\hat{z}^i, \hat{\delta} = \hat{T}\hat{\delta} = \hat{\delta}_{ij}$ on the isofield $\hat{C}(\hat{c},+,\hat{\times})$ with real separation given by

$$\hat{z}\hat{\dagger}\,\hat{\delta}\hat{z} = \bar{z}^1 b_1^2 z^1 + \bar{z}^2 b_2^2 z^2, \quad \hat{\delta}\hat{\dagger} \equiv \delta > 0, \tag{99}$$

with basic isotopic element and isounit

$$T = Diag.(b_1^2, b_2^2), \quad \hat{1} = Diag.(b_1^{-2}, b_2^{-2}), b_k > 0, \tag{100}$$

The (unimodular) invariance group of this space is the Lie-isotopic group $S\hat{U}(2)$. Isoquaternions can also be characterized by fundamental representation of $S\hat{U}(2)$ algebra. A Hermitean isodilative isorotation on $\hat{E}_{I,2}(\hat{z},\hat{\delta},\hat{C}(\hat{c},+,\hat{\times}))$ is given by

$$\hat{z}'^1 = \hat{c}_1 \hat{\circ} \hat{z}^1 + \hat{c}_2 \hat{\circ} \hat{z}^2, \quad \hat{z}'^2 = -\hat{c}_2 \hat{\circ} \hat{z}^1 + \hat{c}_1 \hat{\circ} \hat{z}^2, \tag{101}$$

where the dilation is represented by the value $\hat{c}_1 \hat{\circ} \hat{c}_1 + \hat{c}_2 \hat{\circ} \hat{c}_2 \neq \hat{1}$. Representation of isoquaternions into two-by-two matrices on $\hat{C}(\hat{c},+,\hat{\times})$ is characterized by the isorepresentations of the Lie-isotopic algebra $S\hat{U}(2)$ ref. [40, 41, 42]. These can be expressed in terms of the basic isounit

$$\hat{I} = \hat{1}_0 = \begin{pmatrix} b_1^{-2} & 0 \\ 0 & b_2^{-2} \end{pmatrix} \tag{102}$$

and fundamental representation of $S\hat{U}(2)$ as

$$\hat{i}_1 = \Delta^{-\frac{1}{2}} \begin{pmatrix} 0 & ib_1^2 \\ ib_2^2 & 0 \end{pmatrix}, \quad \hat{i}_2 = \Delta^{-\frac{1}{2}} \begin{pmatrix} 0 & b_1^2 \\ -b_2^2 & 0 \end{pmatrix}, \quad \hat{i}_3 = \Delta^{-\frac{1}{2}} \begin{pmatrix} ib_2^2 & 0 \\ 0 & -ib_1^2 \end{pmatrix} \tag{103}$$

Note that the matrices above satisfy the properties of isotopic image

$$\hat{i}_n \hat{\circ} \hat{i}_m = \Delta^{-\frac{1}{2}} \varepsilon_{nmk} \hat{1}_k, \quad n \neq m, \quad n,m = 1,2,3, \quad \Delta = b_1^2 b_2^2, \tag{104}$$

and hence are closed under commutators, which is a necessary condition for the existence of an isotopy. This results into a Lie-isotopic $S\hat{U}(2)$ algebra

$$[\hat{i}_n, \hat{i}_m] := \hat{i}_n \hat{\circ} \hat{i}_m - \hat{i}_m \hat{\circ} \hat{i}_n = -2\Delta^{-\frac{1}{2}} \varepsilon_{nmk} \hat{i}_k. \tag{105}$$

Isoquaternions can be represented in the form

$$\hat{q} = n_0 I_0 + n_1 \hat{i}_1 + n_2 \hat{i}_2 + n_3 \hat{i}_3 = \begin{pmatrix} (n_0 b_1^{-2} + \Delta^{-\frac{1}{2}} i n_3 b_2^2) & \Delta^{-\frac{1}{2}}(-n_2 + i n_1) b_1^2 \\ \Delta^{-\frac{1}{2}}(n_2 + i n_1) b_2^2 & (n_0 b_2^{-2} - \Delta^{-\frac{1}{2}} i n_3 b_1^2) \end{pmatrix}. \tag{106}$$

Note that the set $\hat{S}(\hat{q},+,\hat{\times})$ is a four dimensional vector space over the isoreals $\hat{R}(\hat{n},+,\hat{\times})$ which is closed under the operation of conventional addition and isomultiplication and hence, is an isofield. Thus, $\hat{S}(\hat{q},+,\hat{\times}) \approx \hat{Q}(\hat{q},+,\hat{\times})$.

The *isonorm* of the isoquaternions is defined as follows

$$\|\hat{q}\| = [Det_R(\hat{q}T)]^{\frac{1}{2}} \hat{I}_0, \tag{107}$$

and may be written as

$$\|\hat{q}\| = [n_0^2 + \Delta(n_1^2 + n_2^2 + n_3^2)]\hat{I}_0, \tag{108}$$

and then

$$\|\hat{q} \hat{\circ} \hat{q}'\| = \|\hat{q}\| \hat{\times} \|\hat{q}'\| \in \hat{R}, \quad \hat{q},\hat{q}',\hat{\circ} \in \hat{Q} \tag{109}$$

The *isobasis* is defined as

$$\hat{e}_1 = \hat{I}_0, \quad \hat{e}_{k+1} = \hat{i}_k, \quad k = 1,2,3. \tag{110}$$

2.7.4. Isodual Isoquaternions

The isodual isoquaternions constitute a four-dimensional, isodual, isonormed, isoassociative, non-isocommutative isoalgebra over the isodual isoreals $\hat{U}(4) \approx U^d(4)$ *ref. [1].*

The *isodual isoquaternions* $\hat{q}^d \in \hat{Q}^d(\hat{q}^d,+\hat{\circ}^d)$ by a two-dimensional isodual complex Hermitean isoeuclidean space of class II over the isodual isocomplex field as

$$E^d_{II,2}(\hat{z}^d, \delta^d, \hat{C}^d(c^d,+,\hat{\times}^d)) : \hat{z}^d \,?\, \hat{\delta}^d z^d \quad z^{-1d} \hat{\times}^d z^{1d} + z^{-2d} \hat{\times}^d z^{2d} = -z^{-1} b_1^2 z^1 - z^{-2} b_2^2 z^2. \tag{111}$$

having basic isodual isotopic element and *isodual isounit*

$$T^d = Diag.(-b_1^2, -b_2^2), \hat{1}^d = Diag.(-b_1^{-2}, -b_2^{-2}) \tag{112}$$

having invariance as the isodual Lie-isotopic group $S\hat{U}^d$. An *isodual Hermitean isodilative isorotation* on $E^d_{II,2}(\hat{z}^d, \delta^d, \hat{C}^d(c^d,+,\hat{\times}^d))$ is given by

$$\hat{z}'^{1d} = \hat{c}_1^d \hat{\circ}^d \hat{z}^{1d} - \bar{c}_2^{\,d} \hat{\circ}^d z^{2d}, \quad \hat{z}'^{2d} = \hat{c}_2^d \hat{\circ}^d \hat{z}^{1d} + \bar{c}_1^{\,d} \hat{\circ}^d \hat{z}^{2d}, \tag{113}$$

where dilation is represented by

$\hat{c}_1^d \hat{\circ}^d \hat{c}_1^d + \bar{c}_2^d \hat{\circ}^d \hat{c}_2^d \neq \hat{1}^d$.

Isodual Isoquaternions can also be realized as the isodual isorepresentation of $S\hat{U}^d(2)$ and can be written as

$$\hat{q}^d = \hat{n}_0^d + \hat{n}_1^d \hat{\times}^d \hat{i}_1^d + \hat{n}_2^d \hat{\times}^d \hat{i}_2^d + \hat{n}_3^d \hat{\times}^d \hat{i}_3^d =$$

$$-\hat{n}_0 + \hat{n}_1\hat{i}_1 + \hat{n}_2\hat{i}_2 + \hat{n}_3\hat{i}_3 =$$

$$= \begin{pmatrix} (-n_0 b_1^{-2} + \Delta^{-\frac{1}{2}} i n_3 b_2^2) & \Delta^{-\frac{1}{2}}(-n_2 + i n_1) b_1^2 \\ \Delta^{-\frac{1}{2}}(n_2 + i n_1) b_2^2 & (-n_0 b_2^{-2} - \Delta^{-\frac{1}{2}} i n_3 b_1^2) \end{pmatrix} \quad (114)$$

Note that the set of all the matrices ${}^d(\hat{q}^d, +, \times^d)$ is an isofield and hence ${}^d(\hat{q}^d, +, \times^d) \approx \hat{Q}^d(\hat{q}^d, +, \hat{\times}^d)$.

The *isodual isonorm* is defined as

$$\|\hat{q}^d\|^d = [Det_R(\hat{q}^d T^d)]^{\frac{1}{2}} \hat{I}_0^d =$$

$$= [-n_0^2 - \Delta(n_1^2 + n_2^2 + n_3^2)]\hat{I}_0^d, \quad (115)$$

$$\left\|\hat{q}^d \hat{\circ}^d \hat{q}'^d\right\|^d = \left\|\hat{q}^d\right\|^d \hat{\times}^d \left\|\hat{q}'^d\right\|^d \in \hat{R}^d, \quad (116)$$

$$\hat{q}^d, \hat{q}'^d, \hat{\circ}^d \in \hat{Q}^d$$

The *isodual isobasis* is defined as

$$\hat{e}_1^d = \hat{I}_0^d \quad \hat{e}_{k+1}^d = \hat{i}_k^d, \quad k = 1,2,3. \quad (117)$$

2.8. Isooctonians and Their Isoduals

2.8.1. Octonians

Octonians constitute and eight-dimensional normed, non-associative and non-commutative , alternative algebra $U(8)$ ***over the field of reals*** $R(n,+,\times)$***.ref.[20, 21].***

Octonians $o \in O(o,+,\times)$ can be realized as two-dimensional quaternions $o = (q_1, q_2)$ with multiplication rules

$$o \circ o' = (q_1, q_2) \circ (q_1', q_2') =$$
$$(q_1 \circ q_1' + q_1 \circ q_2', -\bar{q}_1 \circ q_2' + \bar{q}_1 \circ q_2). \quad (118)$$

The *antiautomorphic conjugation* of an octonian is defined as

$$\bar{o} = (\bar{q}_1, -q_2). \quad (119)$$

The *norm* of an octonian is defined as

$$|o| := (\bar{o} \circ o)^{\frac{1}{2}} = |q_1| + |q_2|, \quad (120)$$

with the basic axioms

$$|o \circ o'| = |o| \times |o'| \in R, \quad o, o' \in O. \quad (121)$$

It is important to note that Octonions do not constitute a realization of the abstract axioms of a numeric field and, therefore, they do not constitute numbers as conventionally known in mathematics due to the non-associative character of their multiplication (see ref. [1]).

2.8.2. Isodual Octonians

The isodual octonians constitute an eight-dimensional isodual, normed, non-associative, and non-commutative algebra $U^d(8)$ ***over the isodual real numbers*** $R^d(n^d, +, \times^d)$ *ref. [1].*

Isodual octonians are defined as

$$o^d = (q_1^d, q_2^d) \quad (122)$$

over the isodual reals $R^d(n^d, +, \times^d)$. The isodual multiplication of isodual octonians is defined by

$$o^d \circ^d o'^d = (q_1^d, q_2^d) \circ^d (q_1'^d, q_2'^d) =$$
$$(q_1^d \circ^d q_1'^d - \bar{q}_1^d \circ^d q_2'^d, q_1^d \circ^d q_2'^d + \bar{q}_1^d \circ^d q_2^d). \quad (123)$$

The isodual antiautomorphic conjugation of an octonian is defined as

$$\bar{o}^d = (\bar{q}_1^d, -q_2^d). \quad (124)$$

The *isodual norm* of an octonian is defined as

$$|o| := (\bar{o} \circ o)^{\frac{1}{2}} = |q_1| + |q_2|, \quad (125)$$

with the basic axioms

$$|o^d \circ^d o'^d| = |o^d|^d \times^d |o'^d|^d \in R^d, \quad o^d, o'^d \in O. \quad (126)$$

2.8.4. Isodual Isooctonians

Isodual isooctonians form an eight-dimensional isodual, isonormed, non-isoassociative, non-isocommutative, but isoalternative isoalgebra $\hat{U}^d(8) \approx U^d(8)$ ***over the isodual isofield*** $\hat{R}^d(\hat{n}^d, +, \times^d)$***, ref. [43].***

Isodual isooctonians $\hat{o}^d \in \hat{O}^d(\hat{o}^d, +, \hat{\times}^d)$ can be defined as the pair of isoquaternions $\hat{o}^d = (\hat{q}_1^d, \hat{q}_2^d)$ over the isodual isoreals $\hat{R}^d(\hat{n}^d, \hat{\times}^d)$ with the multiplication rule

$$\hat{o}^d \hat{\circ}^d \hat{o}'^d = (\hat{q}_1^d, \hat{q}_2^d) \hat{\circ}^d (\hat{q}_{1'}^d, \hat{q}_{2'}^d) =$$
$$(\hat{q}_1^d \hat{\circ} \hat{q}_1'^d - \tilde{q}_1^d \hat{\circ}^d \hat{q}_{2'}^d, q_1^d \hat{\circ}^d \hat{q}_{2'}^d + \hat{q}_1^d \hat{\circ}^d q_2^d) \quad (131)$$

The isodual isoantiautomorphism is defined as

$$\tilde{o}^d = (\tilde{q}^d, -\hat{q}_2^d) \quad (132)$$

The *isodual isonorm* is defined as

$$\left\|\hat{o}^d\right\|^d := (\tilde{o}^d \circ^d \hat{o}^d)^{\frac{1}{2}} \times \hat{1}^d = \left\|\hat{q}_1^d\right\|^d + \left\|\hat{q}_2^d\right\|^d \quad (133)$$

which readily verifies

$$\left\|\hat{o}^d \hat{\circ}^d \hat{o}'^d\right\| = \left\|\hat{o}^d\right\|^d \hat{\times}^d \left\|\hat{o}'^d\right\|^d \in \hat{R}^d, \quad \hat{o}^d, \hat{o}'^d \in \hat{O}^d. \quad (134)$$

Again it is important to note that Isodual isooctonians do not constitute a realization of the abstract axioms of a numeric field and, therefore, they do not constitute numbers as conventionally known in mathematics due to the non-associative character of their multiplication (see Ref. [1]).

3. Grand Unification of Numeric Fields

Isotopic generalization has brought about a grand unification of the conventional numbers into one single, abstract notion of isonumber. It is important to note that the unification of all numbers was conjectured by Prof. Santilli in numerous publications through out his research for many years. Finally it was proved by Kadeisville, Kamiya and Santilli ref.[40]. The following theorem is the main result in this regard.

Theorem 3.1. *Let* $\mathrm{F}(\mathrm{a},+,\times)$ *be the fields of real numbers, complex numbers and quaternions, respectively,* $\mathrm{F}^{\mathrm{d}}(\mathrm{a}^{\mathrm{d}},+,\times^{\mathrm{d}})$ *the isodual fields,* $\mathrm{a}^{\mathrm{d}} := \mathrm{a} \times 1^{\mathrm{d}} = -\mathrm{a}$ *the isofields, and* $\times^{\mathrm{d}} := \times 1^{\mathrm{d}} \times = -\times$, $1^{\mathrm{d}} = -1$. *the isodual isofields as defined in the preceding section. Then all these fields can be constructed with the same methods for the construction of* $\mathrm{F}(\hat{\mathrm{a}},+,\hat{\times})$ *from* $\mathrm{F}^{\mathrm{d}}(\hat{\mathrm{a}}^{\mathrm{d}},+,\hat{\times}^{\mathrm{d}})$, *under the relaxation of the condition of positive-definiteness of the isounit, thus achieving a unification of all the fields, isofields and their isoduals into the single, abstract isofield of Class III, denoted by* $\boldsymbol{R}$.

3.1. Hidden Numbers of Dimension 3, 5, 6, 7

Based on the historical problem 'The four and eight square problem and division algebras' ref.[21], Prof. Santilli conjectured the possibility of 'Hidden numbers' of dimension 3, 5, 6 and 7'. The numbers studied by Santilli, namely, reals, complex, quaternions and octonians are the solution of the following problem.

$$(a_1^2 + a_2^2 + \ldots + a_n^2) \times (b_1^2 + b_2^2 + \ldots + b_n^2) = A_1^2 + A_2^2 + \ldots + A_n^2$$

with

$$A_k = \sum_{r,s} c_{krs} \times a_r \times b_s \quad (135)$$

where all the a's, b's and c's are elements of a field $F(a,+,\times)$ with conventional operations $+$ and $\times$. It is well known that the only possible solutions of the problem are of dimension 1, 2, 4 and 8. These facts are in corporated in the theorem 1.1, restated here

Theorem 3.2 All possible normed algebras with multiplicative unit over the field of real numbers are given by algebras of dimension 1 (real numbers), 2 (complex numbers), 4 (quaternions), and 8 (octonians).

The question posed by Santilli: Is 'Does the classification according to above theorem persist under isotopies, pseudoisotopies and their isodualities ?' or 'Is it incomplete ?' First, we investigate this problem for isotopies of the multiplication. The above problem, equation (135) is reformulated under the isotopies of the multiplication as follows.

The isotopic lifting of the multiplication

$$\times \to \hat{\times} = \times T \times, 1 \to \hat{1} = T^{-1} \quad (136)$$

transforms the problem (135) in to

$$(a_1^{\hat{2}} + a_2^{\hat{2}} + \ldots + a_n^{\hat{2}}) \hat{\times} (b_1^{\hat{2}} + b_2^{\hat{2}} + \ldots + b_n^{\hat{2}}) = A_1^{\hat{2}} + A_2^{\hat{2}} + \ldots + A_n^{\hat{2}} \quad (137)$$

with

$$A_k = \sum_{r,s} c_{krs} \hat{\times} a_r \hat{\times} b_s \quad (138)$$

where all the a's, b's and c's are elements of an isofield $\hat{F}(a,+,\hat{\times})$ in which $\hat{1}$ is an element of the original field, can be simplified to the conventional operations as

$$(a_1^2 + a_2^2 + \ldots + a_n^2) \times (b_1^2 + b_2^2 + \ldots + b_n^2) = T^{-2} \times (A_1^2 + A_2^2 + \ldots + A_n^2) \quad (139)$$

with

$$A_k = T^2 \times \sum_{r,s} c_{krs} \times a_r \times b_s. \quad (140)$$

Comparing the original problem and its isotopic conversion as formulated above, we observe that the reformulation of the problem is same as the original problem and hence the isotopic lifting and isoduality of the field $F(a,+,\times) \to \hat{F}(\hat{a},+,\hat{\times})$ does not change the solution of the problem. As the result we get the following theorem.

Theorem 3.3. *All possible isonormed isoalgebras with multiplicative isounit over the field of the isoreals are the isoalgebras of dimension 1 (isoreals), 2 (isocomplex), 4 (isoquaternions), and 8 (isooctonians) and the classification persists under isoduality.*

Further, lifting of addition gives the third formulation which is pseudoisotopic type

$$+ \to \hat{+} = +\hat{K}, \quad 0 \to \hat{0} = -\hat{K}, \quad \hat{K} = K \times \hat{1} \quad (141)$$

under which (137), (138) can be written over the pseudoisofield $\hat{F}(\hat{a},\hat{+},\hat{\times})$ as

$$(\hat{a}_1^{\hat{2}} \hat{+} \hat{a}_2^{\hat{2}} \hat{+} \ldots \hat{+} \hat{a}_n^{\hat{2}}) \hat{\times} (\hat{b}_1^{\hat{2}} \hat{+} \hat{b}_2^{\hat{2}} \hat{+} \ldots \hat{+} \hat{b}_n^{\hat{2}}) = \hat{A}_1^{\hat{2}} \hat{+} \hat{A}_2^{\hat{2}} \hat{+} \ldots \hat{+} \hat{A}_n^{\hat{2}} \quad (142)$$

with

$$\hat{A}_k = \sum_{r,s} \hat{c}_{krs} \hat{\times} \hat{a}_r \hat{\times} \hat{b}_s = (\sum_{r,s} c_{krs} a_r b_s) \hat{1} = A_k \times \hat{1} \quad (143)$$

This can be written in the conventional operations as

$$[(a_1^2 + a_2^2 + \ldots + a_n^2)\hat{1} + (n-1)K\hat{1}]T$$
$$[(b_1^2 + b_2^2 + \ldots + b_n^2)\hat{1} + (n-1)K\hat{1}] =$$
$$= (A_1^2 + A_2^2 + \ldots + A_n^2)\hat{1} + (n-1)K\hat{1}, \quad \hat{A}_k = A_k\hat{1} \quad (144)$$

The solution to (144) of dimension other than 1,2,4,8 under the pseudoisofield $\hat{F}(\hat{a},\hat{+},\hat{\times})$ was envisaged by prof.Santilli as a conjecture under the loss of the needed axioms of a field, such as distributive laws.

It was found that the solution do exist, but under the loss of number of axioms of the original field, in addition to the loss of distributivity. We consider a representative example of "Hidden numbers" of dimension 3 as follows

$$(\hat{1}^{\hat{2}} + \hat{2}^{\hat{2}} + \hat{3}^{\hat{2}}) * (\hat{5}^{\hat{2}} + \hat{6}^{\hat{2}} + \hat{7}^{\hat{2}}) = \hat{1}2^{\hat{2}} + \hat{2}4^{\hat{2}} + \hat{3}0^{\hat{2}} \quad (145)$$

Note that also the condition on $\hat{A}_k$ is true, that the elements in the r.h.s can be written as the combinations of the elements on the l.h.s as

$$12 = 2\times 6, \quad 24 = 2\times 5 + 2\times 7, \quad 30 = 3\times 3 + 3\times 7. \quad (146)$$

Hence we can rewrite the problem as

$$[(1^2 + 2^2 + 3^2)\hat{1} + 2K]T[(5^2 + 6^2 + 7^2)\hat{1} + 2K\hat{1}] = (12^2 + 24^2 + 30^2)\hat{1} + 2K\hat{1} \quad (147)$$

which on simplification gives a quadratic equation in K as

$$4K^2 + 246K - 80 = 0 \quad (148)$$

with solution

$$K = 0.325\ldots \quad (149)$$

Thus the solution exists, but is not an integer. This implies the loss of closure under isoaddition for the case of integers. However, the closure can be regained if the original field is enlarged to include all real numbers. The issue whether such solutions do indeed form a pseudoisofield is open for the mathematicians.

As algebras of dimensions higher than 8 are not alternative [21], also, as this property persists under isotopies and pseudoisotopies, leads to the fact that formulations (137)and (142) are restricted to dimensions $n \leq 8$.

Prof. Santilli ref.[1] identified following open problems with regards to the notion of isofields.

- Investigative study of "number with singular unit", i.e. isofields of class IV which are at the foundations of the isotopic studies of gravitational collapse.
- The study of isofields of characteristic $p \neq 0$, to see whether new fields and therefore new Lie-algebras are permitted by isotopies.

Author of this article has defined 'Iso-Galois fields' ref.[44] which are basically finite isofields essentially of nonzero characteristic. As predicted by Santilli these isofields have important applications in Cryptography, Genetics, Fractal geometry etc.

- The study of the integro-differential topology characterized by isofields with local differential structure and integral isounits.

3.2. Genonumbers and Their Isoduals

We have seen that the two degrees of freedom due to isotopic lifting of *addition* and *multiplication* give rise to isofields and pseudoisofields respectively. These fields are at the foundation of the *Lie-isotopic theory* [8, 9, 45].

Also, there exists a third degree of freedom caused by the *ordering* of the above operations which leads to further generalization of a field which is at the foundation of *Lie-admissible algebras* [8, 9, 18].

Given a field $F(a,+,\times)$ of ordinary numbers with generic elements $a,b,c\ldots$, with addition $a+b=b+a$ and multiplication $a\times b$, we can define the following.

Genoaddition: Addition of a to b from the left, denoted by $a +^{>} b$ and addition of b to a from the right denoted by $a^{<} + b$ are called *genoadditions.*

Genomultiplication: Multiplication of a times b from the left denoted by $a \times^{>} b$, and multiplication b times a from the right denoted by $a^{<} \times b$ are called *genomultiplications*.

It is worthwhile to note that ordering of multiplication is fully compatible with its basic axioms, such as commutativity for real and complex numbers, associativity for quaternions, and alternativity for the octonions. In the case of real and complex numbers we will have

$$a \times^{>} b \equiv b \times^{>} a, \quad a^{<} \times b \equiv b^{<} \times a \quad (150)$$

The identity of multiplication from left and right can be different and hence two genomultiplications can very well be different i.e.

$$a \times^{>} b \neq a^{<} \times b \quad (151)$$

with realization,

$$a \times^{>} b := aRb, \quad a^{<} \times b := aSb, \quad R \neq S, \tag{152}$$

where R and S are fixed isotopic elements, called the genotopic elements. These are sufficiently smooth, bounded and nowhere singular (not necessarily Hermitean) outside the original field.

The left and right generalized *genounits* can be defined in the following manner

$$a \hat{\times} b \tag{153}$$

$$\hat{1} = T^{-1} \tag{154}$$

Note that all the axioms and properties of the original field are preserved under the mentioned left or right multiplication and multiplicative units under the appropriate ordering for all the dimensions 1,2,4,8. This procedure leads to new fields called as *genofield* denoted by $\hat{F}^{>}(\hat{a}^{>},+,\hat{x}^{>})$ (right genofield) or ${}^{<}\hat{F}({}^{<}\hat{a},+,{}^{<}\hat{x})$ (left genofield) or ${}^{<}\hat{F}^{>}({}^{<}\hat{a}^{>},+,{}^{<}\hat{x}^{>})$. Also, *isodual genofields* are defined by the antiautomorphic conjugations

$$R \to R^{d} = -R, \quad S \to S^{d} = -S \tag{155}$$

denoted by ${}^{<}\hat{F}^{>d}({}^{<}\hat{a}^{>d},+,{}^{<}\hat{x}^{>d})$.

Note that isofields are the particular case of genofields where the genotopic elements coincide. i.e.

$${}^{<}\hat{F}^{>d}({}^{<}\hat{a}^{>d},+,{}^{<}\hat{x}^{>d})_{|R=S=T} = F(\hat{a},+,\hat{x}). \tag{156}$$

R-S mutation of the Lie product: is defined as

$$(A,B) = ARB - BSA \tag{157}$$

which is Lie-admissible via the attached antisymmetric product

$$[A,\hat{B}] = (A,B) - (B,A) = ATB - BTA, T = R - S \tag{158}$$

which is Lie-isotopic.

The lifting $[A,B] \to [A,\hat{B}]$ is called an *isotopy*. The lifting $[A,B] \to (A,B)$ is called a *genotopy*, ref. [8, 1].

The Lie-isotopic algebras are defined by one single isotopy of the enveloping associative algebra and related unit

$$AB = A \times B \to A \hat{\times} B = ATB, \quad 1 \to \hat{1} = T^{-1}. \tag{159}$$

For the consistent formulation of Lie-isotopic algebras they must be defined over an isofield $\hat{F}(\hat{a},+,\hat{x})$ with isounit $\hat{1} = T^{-1}$.

Note that for the conventional multiplication $\times$ there is no ordering as $1^{>} = {}^{<}1 = 1$. The above ordering can be defined for isomultiplication $\hat{\times}$ wherein we can have different isounits.

The Lie-admissible algebras can be generated by two different isotopies of the original associative algebra using left and right isounits with corresponding isotopies as

$$AB \to ARB := A \times^{>} B, \quad 1 \to \hat{1}^{>} = R^{-1}, \tag{160}$$

$$BA \to BSA := B^{<} \times A, \quad 1 \to {}^{<}\hat{1} = S^{-1}. \tag{161}$$

which must be defined over the genofields ${}^{<}\hat{F}^{>}({}^{<}\hat{a}^{>},+,{}^{<}\hat{x}^{>})$ with isounits ${}^{<}\hat{1}^{>}$. Here, the isounits related with the left and right isomultiplication are disjoint and can indeed be Hermitean and real-valued, which admit Kadeisville classification into classes I. II, III, IV and V.

However, in physics the isounits (left and right) used have a real physical significance when they are inter-related by a Hermitean conjugation as

$$\hat{1}^{>} = ({}^{<}\hat{1})^{\dagger} \tag{162}$$

This representation of the genounits (and hence genofields) provides approximation of irreversibility ref.[18].

It is important to note that conventional addition admits no meaningful ordering as $0^{>} = {}^{<}0 \equiv 0$. However, the ordering exists for the isoaddition $\hat{+} = +K+$ as $\hat{+}^{>} \neq {}^{<}\hat{+}$ with $K^{>} \neq {}^{<}K$. But there is loss of distributive law for the resulting genofield under genoadditions ${}^{<}\hat{+}^{>}$.

All the above discussion leads to a broadest generalization of the existing theory of numbers through

1. *pseudogenofields* ${}^{<}\hat{F}^{>}({}^{<}\hat{a}^{>},{}^{<}\hat{+}^{>},{}^{<}\hat{x}^{>})$ defined via genotopies of all aspects of conventional fields $F(a,+,\times)$ and
2. *isodual pseudogenofields* ${}^{<}\hat{F}^{>d}({}^{<}\hat{a}^{>d},{}^{<}\hat{+}^{>d},{}^{<}\hat{x}^{>d})$ defined via isoduality of pseudogenofields.

This new generalization of the conventional numbers leads to the following categorization of numbers:

- Conventional numbers of dimension 1,2,4,8 and their isoduals;
- Isonumbers of the same dimension and their isoduals;
- Genonumbers of the same dimensions and their isoduals;
- Pseudoisonumbers of the same dimension and their isoduals;
- Pseudogenonumbers of the same dimension and their isoduals;
- "Hidden pseudoisonumbers" of dimension 3,4, 5,7 and their osoduals;
- "Hidden pseudogenonumbers" of dimension 3,4,5,7 and their isoduals.

Note that each of these can be defined for the fields of characteristic 0 or for $p \neq 0$.

In addition to above generalization, we can have an ordered set of values for the multiplicative unit such as

$1^{>} = \{2, \frac{4}{3}, 6, \ldots\}$ defined as applicable or to the right or left. This possibility leads to the new numbers called as *hyper-Santillian numbers*. These include hyper-real, hyper complex, hyper-quaternion numbers which have vast applications in biological sciences.

In the further generalization, the multiplicative unit can very well have non-zero negative values. This leads to a new class of numbers called *iso-dual Santillian numbers.* This further leads to a new kinds of conventional iso-dual numbers called as *iso-topic isodual numbers, geno-topic iso-dual numbers* and *hyper-structural isodual*. These numbers have applications for antimatter.

The above generalization of the conventional numbers gives us, in all, eleven classes of new numbers namely, *the iso-topic numbers, genotopic to the right and left, right and left hyper-structural numbers, iso-dual conventional numbers, iso-dual iso-topic numbers, iso-dual geno-topic to the right and left numbers and hyper-structural iso-dual to the right and left numbers.* Each class is applicable to the real, complex and quaternion numbers where each of the applications have infinite number of possible units.

4. Applications and Advances

Quantum mechanics was sufficient to deal with 'Exterior Dynamical systems' which are liner, local, lagrangian and hamiltonian. The main purpose of formulating the new generalized mathematics was to deal with the insufficiencies in the modern mathematics to describe 'Interior Dynamical systems' which are intrinsically non-linear, non-local, non-hamiltonian and non-lagrangian. The axiom-preserving generalization of quantum mechanics which can also deal with non-linear, non-local non-hamiltonian and non-lagrangian systems is called the *Hadronic mechanics.* The mechanics; built specifically to deal with 'hadrons' (strongly interacting particles) ref. [18]. Prof. Santilli, in 1978 when at Harvard University, proposed 'Hadronic mechanics' under the support from U. S. Department of Energy, which was subsequently studied by number of mathematicians, theoreticians and experimentalists. Hadronic mechanics is directly universal; that is, capable of representing all possible nonlinear, nonlocal, nonhamiltonian, continuous or discrete, inhomogeneous and anisotropic systems (universality), directly in the frame of the experimenter (direct universality). In particular the hadronic mechanics has shown that quantum mechanics is completely inapplicable to the synthesis of neutron [46], as mass of the neutron is greater than the sum of the masses of proton and electron (called "mass defect") of which it is made. In this case quantum equations are completely inconsistent. Hadronic mechanics has achieved numerically exact results in the cases in which quantum mechanics results are not valid. For further details of isonumber theory we recommend refs. [47, 1, 48, 46, 49].

As far as mathematics is concerned, one of the major applications of isonumber theory is in Cryptography, ref. [50]. Cryptograms can be lifted to iso-cryptograms which render highest security for a given crypto-system. Isonumbers, hypernumbers and their pseudo-formulations can be used effectively for the tightest security via new disciplines, *isocryptology, genocryptology, hypercryptology, pseudocryptology* etc. More complex cryptograms can be achieved using *pseudocryptograms* in which we have the additional hidden selection of addition and multiplication to the left and those to the right whose results are generally different among themselves. Yet more complex pseudocryptograms can be achieved in which the result of each individual operations of addition and multiplication is given by a set of numbers [50]. *Santillian iso-crypto systems* have maximum security due to a large variety of isounits which can be changed automatically and continuously, achieving maximum possible security needed for the modern age banking and other systems related with information technology.

Reformulations of conventional numbers to the most generalized isonumbers and subsequently to genonumbers and hypernumbers led to a vast variety of parallel developments in the conventional mathematics including hyperstructures [51] and its various branches such as 'iso-functional analysis' ref [35], iso-calculus ref [52], iso-cryptography [50] etc.

Iso-Galois fields [53], Iso-permutation groups [54, 53] have been defined by this author, which can play an important role in cryptography and other branches of mathematics where finite fields are used. Investigations are underway.

Isomathematics can also explain complex biological structures and hence has applications in Fractal geometry. Further applications in Neuroscience and Genetics can provide new insight in these disciplines.

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Santilli Synthesis of the Neutron According to Hadronic Mechanics

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Abstract: In 1920 H. Rutherford conjectured that neutron is a compressed hydrogen atom in the core of the stars. W. Pauli noted that such synthesis of neutron violates the conservation of the angular momentum. Therefore, E. Fermi proposed emission of massless particle, called "neutrino". However, R.M. Santilli more recently noted that, even though the angular angular momentum would be conserved, the neutrino hypothesis does not allow non-relativistic quantum mechanics to be valid because the rest energy of the neutron is bigger than the sum of the rest energies of the proton and electron, under these conditions Schrodinger equation becomes inconsistent. Similarly, Santilli showed that relativistic quantum mechanics is also inapplicable (rather than violated) because, even though exactly valid for the electron at large distance from the proton in the hydrogen atom, the celebrated Dirac's equation is clearly inapplicable for the representation of electron when immersed inside the proton. In this paper, we study Santilli's decades of mathematical, theoretical and experimental research, first for the construction of the covering hadronic mechanics, and then the resulting numerically exact and time invariant representation at the non-relativistic and relativistic levels of "all" chararcteristics of the neutron in its synthesis from a proton and an electron. In particular, we show that, within said covering context, the representation of proton as an extended particle implies the existence of an orbital angular momentum of the electron within the hyperdense proton which is totally non-existence for quantum mechanics, under which the total angular momentum is conserved without any need for the conjectural neutrino. We finally study Santilli's suggestive hypothesis of the "etherino" as a longitudinal impulse (rather than particle) from the ether as a universal substratum that delivers missing energy for the synthesis of the neutron.

Keywords: Neutron, Binding Energy, Isoelectron, Hulthen Potential, Lie-Santilli Isoalgebras

1. Introduction

In 1920, Rutherford [1] submitted the hypothesis that hydrogen atoms in the core of stars are compressed into new neutral particles having the size of the proton that he called neutrons (**Figure 1**), according to the synthesis

$$p^+ + e^- \to n. \tag{1}$$

The existence of the neutron was confirmed in 1932 by Chadwick [2]. However, Pauli [3] noted that the spin $1/2$ of the neutron cannot be represented via a quantum state of proton and electron, each having spin $1/2$. Fermi [4] adopted Pauli's objection and, he then developed the theory of weak interactions according to which the synthesis of the neutron is characterized by either the emission of a neutral and massless particle, named neutrino (ν) or absorption of antineutrino $(\bar{\nu})$. The particle reactions as per proposed theory of weak interaction are given by

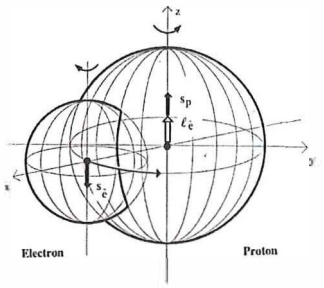

Figure 1. *A schematic view of the the deep overlapping of the wavepackets of electrons into hyperdense medium of proton with singlet coupling.*

$$p^+ + e^- \rightarrow n + \nu, \quad or \tag{2}$$

$$p^+ + e^- + \bar{\nu} \rightarrow n. \tag{3}$$

However, Santilli [5-7] has dismissed the Fermi's version of synthesis of neutron on following grounds:

1. the sum of the rest energies of the proton and of the electron,

$$m_p + m_e = 938.272MeV + 0.511MeV = 938.783MeV \tag{4}$$

is smaller than the rest energy of the neutron,

$$m_n = 939.565MeV \tag{5}$$

with positive energy (binding energy) difference of 0.78 MeV,
2. Schrödinger equation does not admit positive binding energy for quantum bound states when electron totally immersed within the hyper-dense medium inside the proton structure,
3. classical theory of antimatter requires that the anti-neutrino has a negative energy, although, eq.(3) is needed positive energy to supply the missing energy, 0.78 MeV,
4. neither, antineutrino can deliver the 0.78 MeV needed for the neutron synthesis because the cross section of former with electron or proton is null, and
5. the proton and the electron are the only experimentally discovered stable massive particles. Hence, emission of neutrino in neutron formation does not have any relevance. Moreover, it cannot be directly detected.

2. Hadronic Energy

The only bound state of a proton and an electron predicted by quantum mechanics is the hydrogen atom, with smallest orbit (Bohr's orbit) of the order of 10^{-8} cm. Santilli's hadronic mechanics has identified the existence of an additional bound state when the electron orbits within the proton structure at distances of the order of 10^{-13} cm or less. The difference between these two bound states is depicted in "Figure 2".

Remarkably, Santilli has proved that the hadronic state is one and only one, the neutron, and its first excited state is the Hydrogen atom which is formed when the electron leaves the proton structure, thus recovering all conventional quantum states. In this sense, the energy levels of the hydrogen atom are the excited states of the neutron.

The mutual overlapping of the charge distribution or wavepackets of electron and proton leads to new interactions of contact type. However, it is not possible via conventional quantum mechanics to represent these new interactions for various reasons, such as:

1. quantum mechanics can only represent particles as dimensionless point masses; quantum mechanics has a local-differential structure ruling out any consistent treatment of the nonlocal integral interactions;
2. quantum mechanics can only represent interactions derivable from a local potential, while contact interactions of the type required to be considered herein can be represented with anything except a potential or a Hamiltonian.

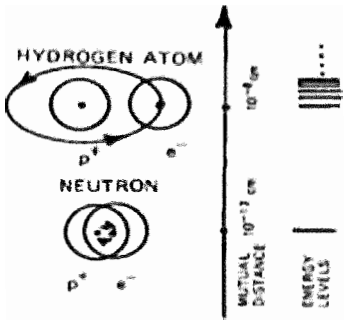

Figure 2. *A schematic comparison of Bohr's orbit and hadronic structure identified by Santilli.*

In this event, Santilli's isomechanics is ideally suited for a quantitative study of the neutron synthesis because, in addition to all interactions characterizing the hydrogen atom, it allows the new interactions caused by deep mutual penetration of the constituents. This method has been used by Santilli in numerous applications. Santilli [5-8] obtained an isoequation for the neutron by isotopically lifting of Schrödinger equation introducing additional potential term of Coulomb nature that reads as,

$$\left[\frac{1}{m}\hat{p}\times\hat{T}\times\hat{p}\times\hat{T} - \frac{ze^2}{r} + \frac{e^2}{r}\hat{T}\right]\times\hat{\psi}(r) = E\times\hat{\psi}(r) \tag{6}$$

with isounit,

$$\hat{I} = U\times I\times U^{\dagger} = 1/\hat{T} > 0. \tag{7}$$

The suitable isounit to represent the two particle penetration (now termed as an isoelectron), is defined as, with isounit,

$$\hat{I} = Diag\left(n_1^2(1), n_2^2(1), n_3^2(1), n_4^2(1)\right) \times Diag\left(n_1^2(2), n_2^2(2), n_3^2(2), n_4^2(2)\right) \times \exp\left((\psi/\hat{\psi})\times\int\hat{\psi}(r)_{1\downarrow}^{\dagger}\times\hat{\psi}(r)_{2\uparrow}\,dr\right) \tag{8}$$

where the two diagonal matrices represent the shapes (assumed to be spheroids) and the densities of the particles considered, while the last term represents the non-Hamiltonian interactions. For spherical point-like charge particle, such as electrons, the diagonal matrices get reduced to 1. Next, the evaluation of the volume integral into a constant,

$$N = \int\hat{\psi}(r)_{1\downarrow}^{\dagger}\times\hat{\psi}(r)_{2\uparrow}\,dr\,, \tag{9}$$

and the expansion of the isoexponent up to the second term, yields,

$$\hat{I} \approx e^{N\times\psi/\hat{\psi}} \approx 1 + N\times\psi/\hat{\psi} \tag{10}$$

$$\hat{T} \approx e^{-N\times\psi/\hat{\psi}} \approx 1 - N\times\psi/\hat{\psi} \tag{11}$$

$$|\hat{I}| \gg 1, \quad |\hat{T}| \ll 1, \quad \lim_{r\to 1fm} \hat{I} = 1. \tag{12}$$

In above equations ψ and $\hat{\psi}$ behave respectively as

$$\psi \approx P\times e^{-br} \tag{13}$$

$$\hat{\psi} \approx Q\times(1-e^{-br})/r \tag{14}$$

where P and Q are constants and b is inverse of hadronic horizon, r_h. Using eqs.(13) and (14), the isotopic element depicted in eq.(11) reads as

$$\hat{T} \approx 1 - N\times\psi/\hat{\psi} = 1 - r\times V_0 \frac{e^{-br}}{(1-e^{-br})}, \tag{15}$$

where $V_0 = NP/Q$. Now, by introducing Hulthen potential,

$$V_{Hulthen} = V_0 \frac{e^{-br}}{1-e^{-br}} \tag{16}$$

where V_0 is the Hulthen's constant, the isotopic element can be written as

$$\hat{T} \approx 1 - N\times\psi/\hat{\psi} = 1 - r\times V_{Hulthen}. \tag{17}$$

Further, at small distances, the Hulthen potential behaves like Coulomb potential,

$$V_{Hulthen} \approx \frac{V_0}{b\times r}. \tag{18}$$

which is very strong as the quantity b in the denominator is of the order of 10^{-13} cm, thus resulting the multiplicative factor of the order of 10^{13}. As a result, inside the hadronic horizon, the Coulomb potential is absorbed by the Hulthen potential, thus we can write

$$\begin{aligned} &\frac{e^2}{r}\times\hat{T} - \frac{ze^2}{r} \\ &\approx \frac{e^2}{r}\times(1 - r\times V_{Hulthen}) - \frac{ze^2}{r} \\ &= -V\times\frac{e^{-br}}{(1-e^{-br})} \end{aligned} \tag{19}$$

where $z = 1$ and $V = e^2 V_0$.

Using eqs.(6) and (19), Santilli obtained the nonrelativistic radial isoequation of the hadronic two-body structure model that reads as

$$\begin{aligned} &\left[\frac{1}{r^2}\left(\frac{d}{dr}r^2\frac{d}{dr}\right)\right]\times\hat{\psi}(r) \\ &+\left[\frac{m}{\rho^2\hbar^2}\left(E_{hb} + V\frac{e^{-br}}{1-e^{-br}}\right)\right]\times\hat{\psi}(r) = 0 \end{aligned} \tag{20}$$

where E_{hb} is hadronic binding energy. Assuming the change of variable, $x = 1 - e^{-br}$, eq.(20) can be written as

$$\begin{aligned} &\left[x(1-x)\frac{d^2}{dx^2}\right]S(x) \\ &-\left[\left(2|A|^{1/2}+1\right)\frac{d}{dx} + \beta^2\right]S(x) = 0 \end{aligned} \tag{21}$$

where

$$\begin{aligned} k_2 &= \frac{mV_0}{\hbar^2\rho^2 b^2}, \\ A &= \frac{m}{\hbar^2\rho^2 b^2}E_b < 0, \end{aligned} \tag{22}$$

The solution of eq.(21) is then given by

$$\begin{aligned} &G_n(x) \\ &= \sum_{k=1}^{k=n}\binom{n-1}{k-1}\binom{n+k+2|A|^{1/2}-1}{k}x^k \end{aligned} \tag{23}$$

with isonormalized isoeigenfunction

$$\begin{aligned} \hat{\psi}(r) = &\left[\frac{\Gamma\left(2|A|^{1/2}+3\right)}{\Gamma(3)\Gamma\left(2|A|^{1/2}\right)}\right]^{1/2} \\ &\times \frac{1-e^{-br}}{r} e^{-|A|^{1/2}br} \end{aligned} \tag{24}$$

the expression for hadronic binding energy is then obtained as

$$|E_{hb}| = E^{Bind} = \frac{V_0}{4k_2}\left[\frac{k_2}{n} - n\right]^2. \tag{25}$$

The boundary conditions demand that $k_2 > n$. This indicates the finite value of eigenvalues for Hulthen potential. This is in concurrence with the hadronic bound state. Further, for an isoparticle to be bounded inside the hadronic horizon b^{-1}, its wavelength, λ must be proportional to the horizon itself, and we shall write

$$\lambda = \frac{1}{2\pi k_1 b} \tag{26}$$

where k_1 is a positive quantity that must be constant for a stationary state. Next the hadronic kinetic energy E_{hk} is given by

$$E_{hk} = \frac{\hat{p}^2}{2m} \approx \frac{\hbar^2 \rho^2 b^2}{2m}. \tag{27}$$

Now, from eqs.(22) and (27), we have

$$V_0 = 2k_2 E_{hk}. \tag{28}$$

Hence, the hadronic total energy of the hadronic bound state is given by

$$\begin{aligned} E_{ht} &= 2E_{hr} + 2E_{hk} - E_{hb} \\ &\approx 2k_1\left[1-(k_2-1)^2\right]\hbar c_0. \end{aligned} \tag{29}$$

where c_0 is the speed of light in vacuum, and note that the last approximation holds for hadronic bound states where the rest energy is insignificant with respect to the kinetic energy. Thus, at this point we obtained the expression for the total energy of the two-body hadronic bound state which is depend on two unknown quantities, k_1 and k_2. To achieve a numerical solution, we now introduce second expression, the meanlife, τ of the unstable hadron

$$\tau^{-1} = \lambda^2 \left|\hat{\psi}(0)\right|^2 \frac{\alpha^2 E_{hk}}{\pi\hbar}, \tag{30}$$

where α is the fine structure constant. By using the above expressions, we can write

$$\begin{aligned} \hat{\psi}(0)\left[\frac{1}{2}\frac{(k_2-1)\Gamma\left[\frac{1}{2}(k_2-1)+2\right]}{3!\Gamma\left[\frac{1}{2}(k_2-1)+2\right]}\right]^{1/2} \\ \times b = \frac{(k_2-1)^{3/2}}{(48)^{1/2}} b \end{aligned} \tag{31}$$

The meanlife of the hadronic bound state then becomes

$$\tau^{-1} = \frac{4\pi}{48(137)^2}\frac{(k_2-1)^3}{k_1} b c_0 \tag{32}$$

Thus, we obtained a system of two equations in terms of two unknown quantities k_1 and k_2, total rest energy, E_{tot}, the meanlife, τ and the charge radius, R_c of the two-body hadronic bound state, that it is reproduced identically below:

$$k_1\left[1-(k_2-1)^2\right] = \frac{E_{ht}}{2\hbar b c_0} \tag{33}$$

$$\frac{(k_2-1)^3}{k_1} = \frac{48(137)^2}{4\pi b c_0}\tau^{-1}. \tag{34}$$

On substituting $b = 10^{-13}$, $\tau^{-1} = 10^{-3}$ and $E_{th} = 939$ in eqs.(33) and (34), we extract

$$k_1 = 2.6, \quad k_2 = 1 + 0.81\times 10^{-8} \approx 1. \tag{35}$$

For admissible state, $n = 1$, we further have

$$\frac{k_2}{n} - n \approx 0, \tag{36}$$

$$E_{hb} = -\frac{V_0}{4k_2}\times\left(\frac{k_2}{n} - n\right) \approx 0 \tag{37}$$

Thus, this proves that the in nonrelativistic approximation the hadronic binding energy is insignificant. Further, the numerical value of the hadronic kinetic energy is obtained as

$$E_{hk} \approx k_1 \hbar b c_0 = 6.63\times 10^{-23} MeV \approx 0 \tag{38}$$

which is also insignificant. The reason for being very small hadronic binding energy and ignorable in first approximation is due to the fact that contact resistive forces have no potential energy. The main physical origin of hadronic structure is the contact, zero-range, interaction due to the complete immersion of one wavepacket within the other.

Finally, Santilli arrives at the following result namely the total hadronic energy of the neutron is primarily characterized by the rest energy of the proton and the isonormalized rest energy of the isoelectron,

$$\begin{aligned} E_n &\approx E_p + E_{hr,\hat{e}} = E_p + \frac{m_e c_0^2}{\rho^2} \\ &= 938.272 + 1.293 = 939.565 MeV \end{aligned} \tag{39}$$

where $\rho^2 = 0.3952$ is a geometrization of the departure of the interior of hadrons from our space-time. Since the proton is not mutated in this first approximation as per our assumption, have

$$b_1 = b_2 = b_3 = 1, \tag{40}$$

$$\rho^2 = n_4^2 = b_4^{-2} = \frac{0.511}{1.293} = 0.3952 \tag{41}$$

$$\rho = n_4 = b_4^{-1} = 0.6286. \tag{42}$$

Notice that the above value for the characterization of the density of the neutron coincides with the experimental value of the density of the fireball of the Bose-Einstein correlation.

3. The Neutron Spin

The conceptual interpretation of the observed spin $1/2$ of the neutron, for the first, was successfully explained by Santilli as follows. Considering the initiation of Rutherford's process of compression of the isoelectron within the proton in singlet coupling, it is evident that, as soon as the penetration begins, the isoelectron is trapped inside the hyperdense medium inside the proton, thus resulting in a constrained orbiting motion of the isoelectron that must superpose on the proton spin (FIGURE 2). Santilli stresses that the proton is not mutated because it is 2000 times heavier than the electron, and that the coupling must be in singlet for stability. This implies

that, for the case of the neutron structure, the spin of the electron is also not mutated. However, the angular momentum of electron is mutated inside the hadronic sphere. The needed mutation of the quantum into the hadronic angular momentum is trivially given by the nonunitary transforms

$$U \times U^{\dagger} = \hat{I} = \frac{1}{2}, \qquad \hat{T} = 2, \tag{43}$$

The mutation is supported by the isotopic invariance of the Hilbert space. Nonunitary lifting of angular momentum, in this case, reads

$$\begin{aligned} &\langle l,m|\times L_3 \times|l,m\rangle \times I \\ &\to U \times \left[\langle l,m|\times L_3 \times|l,m\rangle\right] \times U^{\dagger} \\ &= \langle \hat{l},\hat{m}|2\times\hat{L}_3\times 2|\hat{l},\hat{m}\rangle \times \frac{1}{2}, \end{aligned} \tag{44}$$

In order to represent the spin of neutron Santilli (1990) used irregular isorepresentations of Lie-Santilli isoalgebras [9-11], namely, isorepresentations characterized by nonunitaryisounitary transforms for the generators different than those for the product. This difference is rather natural for the structure of the neutron, since the basic nonunitary transform for the rest energy has already been selected for calculation of binding energy. This irregular isopresentation of $\hat{SO}(3)$ based on the the isodifferential calculus and isolinear momentum is given below [9-11]:

$$\left[\hat{r}_i,\hat{\hat{r}}_j\right] = \left[\hat{p}_i,\hat{\hat{p}}_j\right] = 0, \tag{45}$$

$$\left[\hat{r}_i,\hat{\hat{p}}_j\right] = \hat{\delta}_{ij} = \hat{I}\times\delta_{ij} = \rho\delta_{ij} \tag{46}$$

$$\hat{L}^2 \hat{\times} \hat{Y}_{\hat{l}\hat{m}}(\hat{\theta},\hat{\phi}) = \rho^2 \times \hat{l}(\hat{l}+1)\hat{Y}_{\hat{l}\hat{m}}(\hat{\theta},\hat{\phi}), \tag{47}$$

$$\hat{L}_3 \hat{\times} \hat{Y}_{\hat{l}\hat{m}}(\hat{\theta},\hat{\phi}) = \rho \times \hat{m} \times \hat{Y}_{\hat{l}\hat{m}}(\hat{\theta},\hat{\phi}), \tag{48}$$

$$\hat{l} = 1,2,3,..... \qquad \hat{m} = \hat{l},\hat{l}-1,.....,-l. \tag{49}$$

Notice that the isotopic lifting of the integer value of the angular momentum, $l = 1,2,3,...$ into the value $\rho\times\hat{l}$, where, again, $\hat{l} = 1,2,3....$, the value $\hat{l} = 0$ being excluded by boundary conditions, ρ being a variable depending on the local conditions. For the study of the neutron spin on the line of hadronic mechanics, Santilli selected the following two-dimensional irregular isorepresentation of $\widehat{SU}(2)$:

$$\hat{I} = \begin{pmatrix} g_{11} & 0 \\ 0 & g_{22} \end{pmatrix}, \ \hat{T} = \begin{pmatrix} g_{11}^{-1} & 0 \\ 0 & g_{22}^{-1} \end{pmatrix} \tag{50}$$

$$\begin{aligned} \hat{J}_1 &= \frac{1}{2}\begin{pmatrix} 0 & g_{11}^{-1/2} \\ g_{22}^{-1/2} & 0 \end{pmatrix}, \\ \hat{J}_2 &= \frac{1}{2}\begin{pmatrix} 0 & -ig_{11}^{-1/2} \\ ig_{22}^{-1/2} & 0 \end{pmatrix} \end{aligned} \tag{51}$$

$$\begin{aligned} &\left[\hat{J}_1,\hat{\hat{J}}_2\right] = i\hat{J}_3 \\ &= \frac{1}{2}\times\frac{\Delta^{1/2}}{2}\begin{pmatrix} g_{11}^{-1} & 0 \\ 0 & g_{22}^{-1} \end{pmatrix} \end{aligned} \tag{52}$$

$$\hat{J}_3 \times |\hat{j},\hat{s}\rangle = \hat{J}_3 \times \hat{T}\,|\hat{j},\hat{s} = \pm\frac{\Delta}{2}|\hat{j},\hat{s}\rangle \tag{53}$$

In this case, Santilli [9-11] has selected the two-dimensional irregular isorepresentation of $\widehat{SU}(2)$ and then computed the total angular momentum of the neutron model, $n = (p^+,\hat{e}^-)_{hm}$ as,

$$J_n = J_p + \hat{L}_{\hat{e}}^{orb} + \hat{J}_{\hat{e}}^{int} = \frac{1}{2} + \rho - \frac{\Delta}{2} \tag{54}$$

resulting in the values anticipated above, namely:

$$\rho = \frac{1}{2}, \qquad \Delta = 1. \tag{55}$$

It shows that the spin of the isoelectron is not mutated and the angular momentum is mutated in such a way that the isoelectron is merely carried out by the proton spin.

4. The Neutron Magnetic Moment

In view of the hadronic orbiting motion of isoelectron, the magnetic moment of the neutron was generated by Santilli by considering the following three contributions,

$$\mu_n = \mu_p^{int} - \mu_{\hat{e}}^{orb} + \mu_{\hat{e}}^{int} \tag{56}$$

The observed values of magnetic moment of neutron and proton are respectively,

$$\mu_n = -1.9\times\frac{e}{2m_p c_0}, \mu_p = 2.7\times\frac{e}{2m_p c_0}. \tag{57}$$

Now, on rearranging magnetic moment of neutron as

$$\begin{aligned} \mu_n &= -1.9\times\frac{e}{2m_p c_0} \\ &= 2.7\times\frac{e}{2m_p c_0} - 4.6\times\frac{e}{2m_p c_0}, \end{aligned} \tag{58}$$

and comparing with eq.(56), we obtain following identity:

$$-\mu_{\hat{e}}^{orbital} + \mu_{\hat{e}}^{intrinsic} = -4.6\times\frac{e}{2m_p c_0}. \tag{59}$$

This is equivalent to

$$-\hat{\mu}_e^{orbital} + \hat{\mu}_e^{intrinsic} = \frac{-2.5\times10^{-3} e}{2m_e c_0} = -2.5\times10^{-3}\mu_e. \quad (60)$$

From eq.(60), Santilli derived the desired value of, $\hat{\mu}_e^{orbital}$, that is

$$\hat{\mu}_e^{orbital} = (1+2.5\times10^{-3})\times\mu_e. \quad (61)$$

The small value of the total magnetic moment of the isoelectron is fully compatible with the null value of its total angular momentum.

5. Santilli Aetherino Hypothesis

Santilli replaces the neutrino as a physical particle in our space-time with a longitudinal impulse originated by the ether as a universal substratum that he calls "etherino" [12]. In this view, all physical quantities missing in the neutron synthesis, such as energy and spin, are delivered by said impulse.

A particular motivation for the etherino hypothesis is due to the evident difficulties in accepting that neutrino now believed to have mass could traverse entire planets and stars without appreciable scattering. By contrast, this difficulty is resolved by the propagation of a longitudinal impulse in the universal substratum because it would underlie matter.

Additionally, the replacement of the neutrino with the etherino appears to preserve the experimental evidence in the field because what is today detected and interpreted as a "neutrino scattering" could in reality be due to the scattering of the longitudinal impulse with targets.

Hence, the etherino hypothesis appears to resolve some of the insufficiencies of the neutrino conjecture, may eventually resulting to be fully compatible with available experimental data, and is already stimulating rather intriguing research on superluminal communications, that are the only possible for interstellar contact [12] due to evident insufficiencies of electromagnetic waves for galactic distances.

6. Don Borghi Experiment on the Synthesis of Neutrons

The first experiment on the synthesis of neutrons from protons and electrons was conducted by Carlo Borghi, C. Giori and A. Dall'Olio in the 1960 at the CEN Laboratories in Recife, Brazil [13], [14]. Hydrogen gas at fraction of 1 bar pressure was obtained from the electrolytical separation of water and was placed in the interior of a cylindrical metal chamber (called klystron) and kept mostly ionized by an electric arc with about 500 V and 10 mA. Additionally, the gas was traversed by microwaves with 10s frequency. Suitable materials which are vulnerable to nuclear transmutation when exposed to a neutron flux, were placed exterior of the chamber. Following exposures of the order of days or weeks, the experimentalists reported nuclear transmutations that were based on the observed neutron counts of up to 104 cps. Don Borghi experiment has been strongly criticized by academia on pure theoretical grounds without the actual repetition of the tests. Note that experiment makes no claim of direct detection of neutrons, and only claims the detection of clear nuclear transmutations.

To verify the claim of Don Borghi's experiment, Santilli repeated this experiment in large number of laboratories and institutions the world over.

7. Santilli Experiment on the Synthesis of Neutrons

Santilli conceived his experiment [15], [16] as being solely based on the use of an electric arc within a cold (i.e., at atmospheric temperature) hydrogen gas without any use of microwave at all. Three different klystrons were manufactured, tested and used for the measurements. The specifications of detectors were used for measurements are given below:

1. A detector model PM1703GN manufactured by Polimaster, Inc., with sonic and vibration alarms as well as memory for printouts, with the photon channel activated by CsI and the neutron channel activated by LiI.
2. A photon-neutron detector SAM 935 manufactured by Berkeley Nucleonics, Inc., with the photon channel activated by NaI and the neutron channel activated by He-3 also equipped with sonic alarm and memory for printouts of all counts. This detector was used to verify the counts from the preceding one.
3. A BF3 activated neutron detector model 12-4 manufactured by Ludlum Measurements, Inc., without counts memory for printouts. This detector was used to verify the counts by the preceding two detectors.

Electric arcs were powered by welders manufactured by Miller Electric, Inc., including a Syncrowave 300, a Dynasty 200, and a Dynasty 700 capable of delivering an arc in DC or AC mode, the latter having frequencies variable from 20 to 400 Hz.

Klystron-I was cylindrical and sealed, of about 6" outside diameter and 12" height, made of commercially available, transparent, PolyVinyl Chloride (PVC) housing along its symmetry axis a pair of tungsten electrodes. The electrodes gap was controllable by sliding the top conducting rod through the seal of the flange. The klystron cylindrical wall was transparent so as to allow a visual detection of arc. After initiation of DC arc there was no detection for hours. However, shaking of klystron the neutrons were detected in a systematic and repetitive way. The detection was triggered by a neutron-type particle, excluding contributions from photons. However, these detections were anomalous, that is, they did not appear to be due to a flux of actual neutrons originating from the klystron. This anomaly is established by the repeated "delayed detections," that is, exposure of the detector to the

klystron with no counts of any type, moving the detector away from the klystron (at times for miles), then seeing the detectors enter into off-scale vibrations and sonic alarms with zero photon counts.

Klystron-II was a rectangular, transparent, made up of PVC of dimension. This klystron was small in size than earlier one to avoid implosion caused by combustion with atmospheric oxygen. This test was conducted only once because of instantaneous off-scale detection of neutrons by all detectors which led to evacuation of the laboratory. Hence, this test was not repeated for safety.

Klystron-III was cylindrical made up of carbon steel pipe with 12" outer diameter, 0.5" wall thickness, 24" length and 3" thick end flanges to sustain hydrogen pressure up to 500 psi with the internal arc between throated tungsten electrodes controlled by outside mechanisms. This test was conceived for the conduction of the test at bigger hydrogen pressure compared to that of Klystron I. The test was conducted only once at 300 psi hydrogen pressures because of instantaneous, off-scale, neutron detections such to cause another evacuation of the laboratory.

The main purpose of Santilli's of conducting these tests was to establish the production of neutron-type particles via a DC arc within a hydrogen gas. He has experimented identical tests with other gases, but no meaningful counts were detected other than hydrogen. No neutron, photon or other radiation was measured from electric arcs submerged within liquids. Hence, the reported findings appear to be specific for electric arcs within a hydrogen gas under the conditions stated above.

8. The Don Borghi-Santilli Neutroid

Santilli [5,15] excludes that the entities produced in the tests with Klystron I are true neutrons for various reasons, such as:

1. The anomalous behavior of the detector, in the case of the 15 minute delay, namely the self-activated detection indicates first the absorption of "entities" producing nuclear transmutations that, in turn release ordinary neutrons.
2. The environment inside stars can indeed provide the missing energy of 0.78 MeV for the neutron synthesis, but the environment inside Klystron-I cannot do the same due to the very low density of the hydrogen gas.
3. The physical laws of hadronic mechanics do not allow the synthesis of the neutron under the conditions of Klystron-I because of the need of the trigger, namely, an external event permitting the transition from quantum to hadronic conditions. In fact, the tests with Klystrons-II and III do admit the trigger required by hadronic mechanics. However, Santilli did not discard that the "entities" produced in the tests with Klystrons-II and III are indeed actual neutrons, due to the instantaneous, off-scale nature of the neutron alarms in clear absence of photon or vibrations.

In view of above reasons, Don Borghi [13], [14] submitted the hypothesis that the "entities" are neutron-type particles called "neutroids". Santilli adopted this hypothesis and presented the first technical characterization of neutroids with the symbol, $\tilde{n}$ and the characteristics in conventional nuclear units, $A=1, Z=0, J=0, amu=0.008$. Hence, Santilli assumed that in Klystron-I, he produced the following reaction precisely along Rutherford's original conception

$$p^+ + e^- \to \tilde{n}(1.0,0,1.008) \tag{62}$$

where the value $J=0$ is used for the primary purpose of avoiding the spin anomaly in the neutron synthesis as indicated above and the rest energy of the neutroids is assumed as being that of the hydrogen atom.

9. Interpretation of Don Borghi and Santilli Experiments

In Don Borghi's and Santilli's experiments the various substances placed in the exterior of the klystrons did indeed experience nuclear transmutations. If we discard the Don Borghi's klystron and Santilli's Klystron-I to produce actual neutrons, then the main question arises from where the neutrons originated and detected. Evidently, only two possibilities remain, namely, that the detected neutrons were actually synthesized in the walls of the klystrons, or by the activated substances themselves following the absorption of the neutroids produced by the klystrons. Considering the neutrino hypothesis has no sense for the neutron synthesis for various reasons, Santilli [5, 15] assumes that the energy, spin and magnetic anomalies in the neutron synthesis are accounted for by their transfer either from nuclei or from the aether via his etherino hypothesis

$$\tilde{n}(1.0,0,1.008) + a \to n(1.0,0,1.008). \tag{63}$$

Assuming the binding energy of a neutroid is similar to that of an ordinary nucleon (since neutroids are assumed to be converted into neutrons when inside nuclei, or to decompose into protons and electrons, thus recovering again the nucleon binding energy), Santilli indicates the following possible nuclear reaction for one of the activated substances in Don Borghi's tests

$$\begin{aligned} &Au(197,79,3/2,196.966) + \tilde{n} + a \\ &\quad \to Au(198,79,2,197.972), \end{aligned} \tag{64}$$

produces known nuclide, hence it indicates that neutrons were synthesized by the activating substances themselves on absorption of neutroid. The nuclear reaction with steel wall of the klystron,

$$\begin{aligned} &Fe(57,26,1,57.935) + \tilde{n} + a \\ &\quad \to Fe(58,26,1,57.941), \end{aligned} \tag{65}$$

yields an unknown nuclide, $Fe(58,26,1,57.941)$ because the known nuclide is $Fe(58,26,0,57.933)$. This indicates that the neutrons in Don Borghi experiment were not

synthesized in the walls of his klystron. Eq.(2) also allow an interpretation of some of Santilli detections [5], [15], with the understanding that the anomalous behavior of the detectors, such as the delayed neutron counts, requires special studies and perhaps the existence of some additional event not clearly manifested in Don Borghi's tests.

To initiate the study, Santilli considered the first possible reaction inside the klystron

$$H(1,1,1/2,1.008)+\tilde{n}+a \rightarrow H(1,1,1,2.014), \tag{66}$$

delivers ordinary deuteron on coupling of hydrogen atom and neutroid. This indicates neutrons cannot be originated inside the klystron-I. Next, Santilli considered following nuclear reactions with the polycarbonate of Klystron-I wall containing about 75 percent carbon and 18.9 percent oxygen

$$\begin{aligned} &C(12,6,0,12.00)+\tilde{n}+a \\ &\rightarrow C(13,6,1/2,13.006) \\ &\rightarrow C(13,6,1/2,13.006)+\gamma, \end{aligned} \tag{67}$$

$$\begin{aligned} &O(16,8,0,16.00)+\tilde{n}+a \\ &\rightarrow O(17,8,1/2,17.006), \end{aligned} \tag{68}$$

do not give conventional activation processes. Thus, in Santilli's experiment too, it does not appear that the detected neutrons are synthesized by the walls of klystron. The above analysis leads us to the only remaining possibility that in Santilli tests, the neutrons are synthesized by the detectors themselves. To study this possibility, Santilli considered the reaction using Li-activated detectors,

$$\begin{aligned} &Li(7,3,3/2,7.016)+\tilde{n}+a \\ &\rightarrow Li(8,3,2,8.022) \\ &\rightarrow Be(8,4,0,8.005)+e^{-} \rightarrow 2\alpha, \end{aligned} \tag{69}$$

that behaves fully equivalent to detection of neutriods or neutrons. This indicated that neutrons detected in Santilli experiment were synthesized by the substance used for detection after absorption of neutriods.

10. Concluding Remarks

It is observed that Santilli's discovery of hadronic mechanics appropriately represents, at both non-relativistic and relativistic levels, "all" characteristics of neutron according to Rutherford's conjecture of its synthesis from hydrogen atom in the core of a star. A first implication of the studies is that the orbital motion of the electron within the hyperdense proton allows the conservation of the total angular momentum without any need for the conjecture of the hypothetical neutrino. Another important implication is the dismissal of quarks as the actual physical constituents of the neutron since the proton and the electron cannot "disappear" at the time of the neutron synthesis to be replaced by the hypothetical quarks, and then "reappear" at the time of the neutron decay. We show that, besides the above mathematical and theoretical studies, Santilli has provided numerous experimental verification of the laboratory synthesis of the neutron from a hydrogen gas in support of Rutherford's historical hypothesis.

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Studies on Santilli Three-Body Model of the Deuteron According to Hadronic Mechanics[i]

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Abstract: In this paper, we outline the inapplicability (rather than the violation) of quantum mechanics for the representation of the synthesis of the neutron from the Hydrogen atom in the core of a star, and we outline the corresponding inability of quantum mechanics for a consistent representation of all characteristics of the deuteron as a two-body state of one proton and one neutron in its ground state. We then outline the first representation of all characteristics of the neutron achieved by R. M. Santilli via a a generalized two-body bound state of one proton and one electron in conditions of total mutual penetration according to the laws of hadronic mechanics, thus implying the mutation of particles into isoparticles under the Lorentz-Santilli isosymmetry. We then outline the first representation of all characteristics of the deuteron also achieved by R. M. Santilli via a generalized three-body bound state of two isoprotons and one isoelectron, including the first known exact and time invariant representation of the deuteron spin, magnetic moment, binding energy, stability, charge radius, dipole moment, etc. We finally study further advances of Santilli three-body model of the deuteron in preparation of its extension to all nuclei, such as: the admission of exact analytic solution for the structure of the deuteron as a restricted three-body system; the validity in first approximation of the structure of the deuteron as a two-body system of one isoproton and one iso neutron; the importance for the representation of experimental data of the deformability of the charge distribution of the proton and the neutron which is prohibited by quantum mechanics but readily permitted by hadronic mechanics in the notion of isoparticle; and other aspects.

Keywords: Neutron, Deuteron, Hadronic Mechanics

1. Introduction

The nucleus of deuterium is called a deuteron and it contains one proton and one neutron, whereas the far more common hydrogen nucleus contains no neutron. The isotope name is formed from the Greek deuterons meaning "second", to denote the two particles composing the nucleus. Thus Deuteron is normally considered as the combination of proton and neutron and thus it is considered as a two body system by quantum mechanical bound state. It is the simplest bound state of nucleons and therefore gives us an ideal system for studying the nucleon-nucleon interaction. In analogy with the ground state of the hydrogen atom, it is reasonable to assume that the ground state of the deuteron also has zero orbital angular momentum L = 0. However the measured total angular momentum is J = 1 (one unit of $h/2\pi$) thus it obviously follows that the proton and neutron spins are parallel: $s_n + s_p = 1/2 + 1/2 = 1$. On the other hand, its high stability is to the tune of 2.2 MeV. The stability of deuteron plays a very important part of the existence of the universe.

The structure of deuteron and its physical properties were first proposed by Santilli [1, 2]. Although Deuteron is a simple molecule, quantum mechanics has been unable to explain its different properties like the spin, magnetic moment, binding energy, stability, charge radius, dipole moment, etc. The magnetic moment of deuteron was for the first time represented exactly by Santilli [3]. Also for the first time the notion of isoproton and isoelectron was introduced by Santilli [4, 5], which was further elaborated by him [6, 7]. He made Rutherford's conjecture of neutron a quantitative description based on his Hadronic Mechanics [8-10]. Santilli

under the covering laws of Hadronic Mechanics has demonstrated and established that all nuclei and therefore all the matter at large are supposed to be composed of protons and electrons in their isoprotons and isoelectrons realization characterized by Lorentz-Santilli isosymmetry [4, 5, 8]. The conception of nuclei as quantum mechanical bound states of proton and neutron remains valid but only as a first approximation. Thus, Santilli's reduction of the neutron to a hadronic bound state of a proton and an electron suggests the reduction of all nuclei and, therefore, all matter in the universe, to protons and electrons. However, on technical grounds, the constituents of nuclei are given by protons and electron in their form mutated by contact non-Hamiltonian, thus nonunitary interactions called isoprotons and isoelectrons [5, 11] (for further details see [6, 7] and technically defined as isounitary irreducible representations of the Lorentz-Poincare-Santilli isosymmetry.

Hadronic mechanics not only allows the reduction of a nuclei into (iso) protons and (iso) electrons, but also achieves, for the first time, a numerically exact and invariant representation of various nuclear data beyond any dream of representation via quantum mechanics.

For the sake of some sort of continuity we start in the next Section with a very brief description of neutron structure based on Santilli hadronic mechanics and then would devote all succeeding Sections to hadronic mechanics of deuteron as developed by Santilli.

2. A Brief Review of Neutron Structure Based on Santilli's Hadronic Mechanics

In the history of science Santilli for the first time quantified the Rutherford conjecture that a neutron is indeed a compressed hydrogen atom using his hadronic mechanics. The main motivation to develop corresponding hadronic mechanics has been the inadequacy of quantum mechanics to arrive at experimentally established properties of neutron e.g. its spin, magnetic moment, its stability within nucleus (an isolated neutron is unstable having half life of about 10 min), etc. For the details of all these aspects can be found in [8-10]. However, herein we recall only the main features of Santilli's quantification of neutron structure and synthesis to illustrate the continuity of nuclear structure from neutron to deuteron according to hadronic mechanics.

In order to make Rutherford's conjecture a quantitative one he proposed a model in which the wave packets of an electron and a proton mutually overlap to form a dynamic union such that electron revolves around proton as shown in Figure 1.

In other words, the proton and the electron are actual physical constituents of the neutron in our space-time, not in their conventional quantum mechanical states, but in generalized states due to the total penetration of the wave packet of the electron within the hyperdense proton, for which Santilli has suggested the names of "isoproton, "here denoted $\hat{p}^+$, and "isoelectron," here denoted $\hat{e}^-$, these new states are technically realized as irreducible isorepresentation of the Lorentz-Poincaré-Santilli isosymmetry. In this way he studied the representation of "Rutherford's compression" of the Hydrogen atom into a neutron inside a star via a non-unitary transform of the conventional structure of the Hydrogen atom (HA).

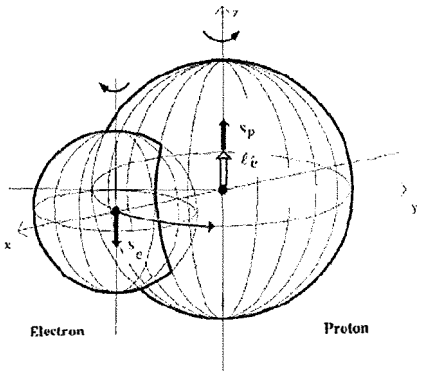

Figure 1. *A conceptual view of Rutherford's compression of the electron inside the hyperdense proton in singlet coupling (necessary for stability), resulting in the constrained orbital angular momentum of the electron under which the total angular momentum of the electron is zero and the spin of the neutron coincides with that of proton.*

Thus the mutated electron and proton as shown in Figure 1 are termed as *isoelectron* and *isoproton* respectively. The iso-prefix stems from the need of Santilli isomathematics [12] to describe the process of the said mutation. The said mutation gets mathematically expressed as,

$$HA \equiv (p^+,e^-)_{QM} \rightarrow n = (\hat{p}^+,\hat{e}^-)_{HM} \quad (1)$$

where subscripts QM and HM stands for the horizons of quantum mechanics and hadronic mechanics respectively. From the model of Figure 1 it is evident that the dimensions of interaction between isoelectron and isoproton are of 1 fm or less. But to maintain an electron within such a short nuclear volume very strong attractive force is needed because the conventional electrostatic attraction at such a short distances turns out to be grossly inadequate. This then indicated that an external trigger is operating that forces an electron to penetrate within the hyperdense medium of a proton. This in hadronic mechanics has been quantified through corresponding Hulthén potential, which produces very large attractive force compared to the conventional electrostatic force.

The reader is advised to refer to the references cited herein for the details of the Rutherford-Santilli model of neutron and its synthesis both in Stars and in laboratory.

3. Santilli's Structured Model of Deuteron as a Hadronic Bound State of Two Protons and One Electron

Santilli considerd deuteron as a hadronic bound state of

two protons and one electron verifying the laws and symmetries of hadronic mechanics. According to him:

1. The deuteron is a stable light, natural isotope that, as such, is reversible over time.
2. Thus Santilli assumes the quantum mechanical structure less of the deuteron (denoted as " d ")

$$d \approx (p^+, n)_{QM} \tag{2}$$

as valid in first approximation, and reduces the deuteron to two protons and one electron according to the structure:

$$d = (\hat{p}^+, \hat{e}^-, \hat{p}^+)_{HM} \tag{3}$$

In the above equation all the constituents are isoparticles, namely, two iso- protons and one isoelectron. Their iso-character has been depicted by (^) over the symbols.

3. Contrary to expectations, contact interactions generate a special version of restricted three body system that admits an exact analytic solution.

In this communication we intend to review the insufficiencies of quantum mechanics for a quantitative representation of experiential data on the deuteron and then review their exact and invariant representation via Santilli's isomechanics and underlying isomathematics.

3.1. Insufficiencies of Quantum Mechanics to Adequately Describe the Structure of Deuteron

3.1.1. Quantum Mechanics has been Unable to Represent or Explain the Stability of the Deuteron

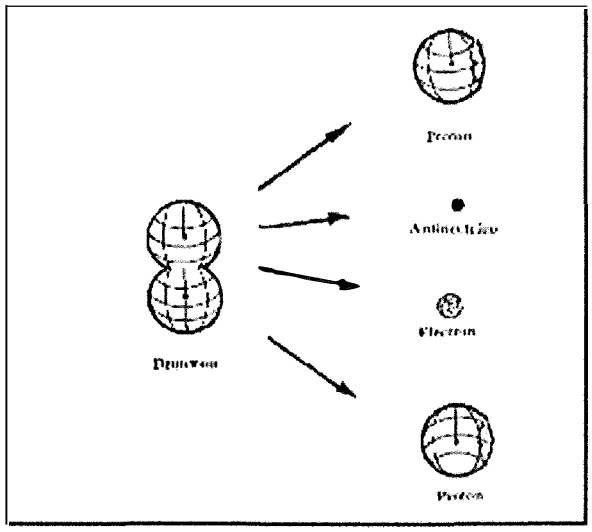

Figure 2. *Three body model of the deuteron.*

This problem might be also due to unavailability of the technical literature of quantitative numerical proofs that, when bonded to a proton, the neutron cannot decay, as an evident condition for stability. Thus the stability of the deuteron has been left fundamentally unexplained by quantum mechanics till date. Santilli illustrated the inability by quantum mechanics to represent the stability of the deuteron, since the neutron is naturally unstable and, therefore, the deuteron should decay into two protons, an electron and the hypothetical antineutrino. Even today, no reason is known that why neutron should become stable when coupled to a proton. Santilli represented three body model of the deuteron and its stability as shown in Figure 2.

3.1.2. Quantum Mechanics has been Unable to Represent the Spin 1 of the Ground State of the Deuteron

According to quantum mechanics the most stable bound state of two particles is with the opposite spins and hence should have SPIN ZERO. No such state has been detected in the deuteron. Thus quantum mechanics has been unable to represent the spin 1 of the ground state of the deuteron. This is illustrated in Figure 3.

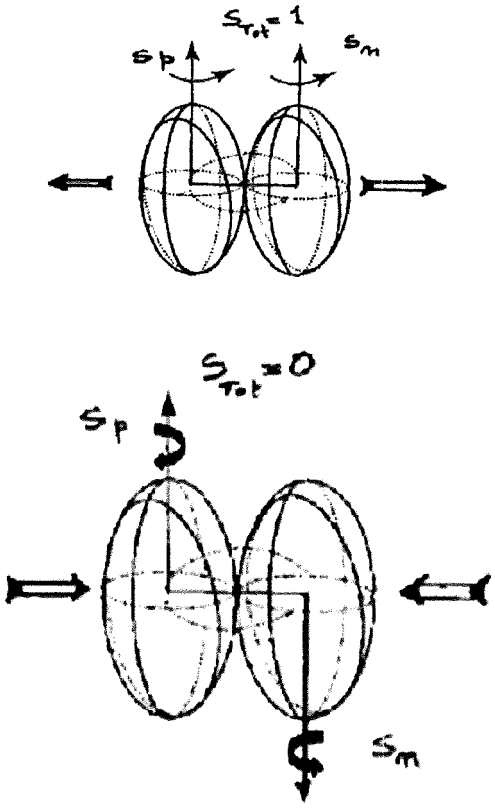

Figure 3. *Figures above represent the impossibility of quantum mechanics to represent the spin 1 of the deuteron in a way compatible with its size. First figure explains how spin 1 can solely be achieved with a triplet coupling in which case no stable nucleus is conceivable due to very strong repulsive forces at the distance of nuclear forces. Thus only stable state is the singlet but in this case the total angular momentum is zero, in disagreement with experimental evidence.*

3.1.3. Quantum Mechanics has been Unable to Reach an exact Representation of the Magnetic Moment of the Deuteron

It has been observed that non-relativistic quantum mechanics misses 0.022 Bohr units corresponding to 2.6% of the experimental value. Relativistic corrections reduce the error down to about 1% but under highly questionable

theoretical assumptions, such as the use for ground state of a mixture of different energy levels that are assumed to exist without any emission or absorption of quanta as expected by quantum mechanics. The situation becomes worst for the magnetic moments of heavier nuclei.

3.1.4. Quantum Mechanics has been Unable to Identify the Physical Origin of the Attractive Force that Binds Together the Proton and the Neutron in the Deuteron

Since the neutron is neutral, there is no known electrostatic origin of the attractive force needed for the existence of the deuteron. The only Coulomb force for the proton-neutron system is that of the magnetic moments, which force is REPULSIVE for the case of spin 1 with parallel spin. Therefore, a "strong" force was conjectured and its existence was subsequently proved to be true.

3.1.5. Quantum Mechanics has also been Unable to Treat the Deuteron Space Parity in a Way Consistent with the Rest of the Theory

The experimental value of the space parity of the deuteron is positive for the ground state, because the angular momentum L is null. However, nuclear physicists assume for the calculation of the magnetic moment of deuteron that the ground state is a mixture of the lowest state with L = 0 with other states in which the angular momentum is not null. This produces incompatibility of these calculations with the positive parity of the ground state.

3.2. Inferences

Thus from above discussion we can infer that, after about one century of research, quantum mechanics has left unresolved fundamental problems even for the case of the smallest possible nucleus, the deuteron, with progressively increasing unresolved problems for heavier nuclei. Following these insufficiencies, any additional belief on the final character of quantum mechanics in nuclear physics is a sheer political posture in disrespect of the societal need to search for a more adequate mechanics.

Not only quantum mechanics is not exactly valid in nuclear physics, but the very assumption of neutrons as nuclear constituents is approximately valid since neutrons are composite particles. Therefore, the main objective of this chapter is the identification of stable, massive physical constituents of nuclei and their theoretical treatment that admits in first approximation the proton-neutron model, while permitting deeper advances.

The replacement of protons and neutrons with the hypothetical quark is mathematically significant, with the clarification that, in Santilli's view, *quarks cannot be physical particles* because, as stresses several times by Santilli, *quarks are purely mathematical representations of a purely mathematical symmetry realized in a purely mathematical internal unitary space without any possible formulation in our spacetime* (because of the O'Rafearthaigh's theorem).

Consequently, *quark masses are purely mathematical parameters and cannot be physical inertial masses.* As also stressed several times, on true scientific grounds, inertial masses can only be defined as the eigenvalues of the second order Casimir ìnvariant of the Lorentz-Poincaré symmetry. But this basic symmetry is notoriously inapplicable for the representation of quarks because of their particular features. Therefore, quark "masses" cannot have inertia. Additionally, Santilli points out that the hypothetical orbits of the hypothetical quarks are excessively small to allow an exact representation of nuclear magnetic moments via their polarization. In fact, various attempts have been made in representing magnetic moments when reducing nuclei to quarks with the result of bigger deviations from experimental data than those for the proton-neutron structure. Similar increases of the problematic aspects occur for all other insufficiencies of quantum mechanics in nuclear physics. Consequently, the reduction of nuclei to quarks will be ignored hereon because of its excessive deviation from solid physical foundations as well as experimental data.

In conclusion, quarks can indeed be considered as replacements of protons and neutrons, with the understanding that nuclei made up of *quarks cannot have any weight*, since, according to Albert Einstein, gravity can solely be defined for bodies existing in our spacetime.

4. Deuteron and Hadronic Mechanics

It is evident from the above facts that quantum mechanics has been unable to treat the deuteron space parity, in a way consistent with the rest of the theory [1, 8, 10]. Thus quantum mechanics has not been able to solve fundamental problems even for the case of the smallest possible nucleus, the deuteron, with progressively increasing unresolved problems for heavier nuclei.

4.1. Deuteron Structure

The nuclear force solely applies up to the distance of 10^{-13} cm, which distance coincides with the charge radius of the proton as well as the electron wavepacket, and that the sole stable orbit for the two protons under contact strong interactions is the circle. The size of the deuteron then forces the charge distribution of two protons as essentially being in contact with each other. It can be said that the electron is totally immersed within a proton, expectedly exchanging its penetration from one proton to the other.

Now the spin of the deuteron in its ground state is 1; the spin of the protons is 1/2; the spin of the isoelectron is 1/2; and that the mutated angular momentum of the isoelectron is -1/2. So Santilli assumed the structure of the deuteron as being composed of two un-mutated protons with parallel spins rotating around the central isoelectron to allow the triplet coupling of protons, and then the two coupled particles in line have an orbital motion around the isoelectron at the center, resulting in the first approximation in the following hadronic structure model of the deuteron [2].

$$d = (p^+_\uparrow, \hat{e}^-_\downarrow, p^+_\uparrow)_{HM} \tag{4}$$

Thus, proton is the only stable particle and neutron is unstable, comprising of proton and electron. Santilli assumed that nuclei are a collection of protons and neutrons, in first approximation, while at a deeper level a collection of mutated protons and electrons. It has been proved that a three-body structure provides the only known consistent representation of all characteristics of the deuteron, first achieved by R. M. Santilli. Thus Coulomb and contact attractive forces in pair-wise singlet couplings proton-isoelectron are so strong to overcome Coulomb repulsion among the two protons and form a bound state that is permanently stable when isolated, as already established for the valence bond and Cooper pairs of identical electrons.

Volodymyr Krasnoholovets has tried to resolve the above anomalies in his recent paper [13]. He analyzed the problem of the deuteron from the viewpoint of the constitution of the real space that he developed. He concluded that the nucleus does not hold the electrons in the orbital position and polarized inertons [14-16] of atomic electrons directly interact with the nucleus. He also analyzed the problem of the motion of nucleons in the deuteron, which takes into account their interaction with the space and concluded that nucleons in the deuteron oscillate along the polar axis and also undergo rotational oscillations. In other words, the nucleons execute radial and rotationally oscillatory motions. Trying to account for the reasons for nuclear forces, he has analyzed major views available in the literature including quantum field theories, hadronic mechanics, and even the Vedic literature.

R. M. Santilli in 1998 provided the consistent representation of all the characteristics of the deuteron using its three body model [2] that involves isomathematics based methods of hadronic mechanics. His hadronic mechanics method explains the strong attraction between protons and neutrons via the Hulthén potential concept [17]. Thus the hadronic mechanics:

1. could successfully explain the experimental value of spin 1 of the deuteron;
2. offered the exact and invariant representation of the total magnetic moment of the deuteron;
3. provided a physical insight into the deuteron size and charge.

4.2. Size of Deuteron

It has been observed experimentally that the proton has the following values for the charge radius and diameter (size) $R_p = 0.8 \times 10^{-13} = 0.8$ fm; $D_p = 1.6$ fm. Whereas, the value of the size of the deuteron given in literature is: $D_d = 4.31$ fm.

Structure model represented by equation 4 does indeed fully justifies the above data in accordance with Figure 4. In fact, the above data indicate that the charge radii of the two protons are separated by approximately 1.1 fm, namely, an amount that is fully sufficient, on one side, to allow the triplet alignment of the two protons as in the upper part of Figure 4 and, on the other side, to generate contact nonlocal effects from the penetration of the wave packet (here referred to the square of the probability amplitude) of the central spinning electron within the two peripheral protons.

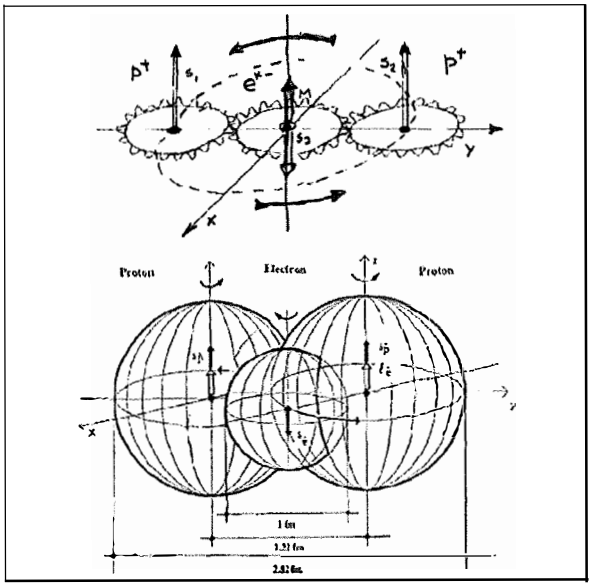

Figure 4. Represents the structure of the deuteron as a restricted three body of two un-mutated protons (due to their weight) and one mutated electron. The top view uses the very effective "gear model" to avoid the highly repulsive triplet couplings, while the bottom view is the same as the top view, the particles being represented with overlapping spheres.

4.3. Representation of the Stability of the Deuteron

As indicated earlier, the lack of a quantitative representation of the stability of the deuteron when composed by the stable proton and the unstable neutron has been one of the fundamental problems left unsolved by quantum mechanics in about one century of research.

By comparison, protons and electrons are permanently stable particles. Therefore, structure model equation (4) resolves the problem of the stability of the deuteron in a simple, direct, and visible way. The deuteron has no unstable particle in its structure and, consequently it is stable due to the strength of the nuclear force.

In fact, as shown below, the Coulomb and contact attractive forces in pair-wise singlet couplings proton-isoelectron are so "strong" to overcome Coulomb repulsion among the two protons and form a bound state that is permanently stable when isolated, as already established for the valence bond and Cooper pairs of identical electrons.

4.4. Deuteron Charge

Model given by equation 4 represents the deuteron positive charge +e. This is due to the fact that hadronic mechanics generally implies the mutation of all characteristics of particles, thus including the mutation of conventional charges Q, and so that mutated charge of the deuteron constituents

$$\hat{Q}_{p1} = ae, \quad \hat{Q}_e = be, \quad \hat{Q}_{p2} = ce \tag{5}$$

where a, b, c are positive-definite parameters, and e is the

elementary charge. These mutations are necessary for consistency with other aspects, such as the reconstruction of the exact isospin symmetry in nuclear physics. However, these mutations are only internal, under the condition of recovering the conventional total charge +e for the system as a whole, as it is the case for closed non-Hamiltonian systems. Consequently, the charge mutations are subject to cancelation in such a way to yield the total charge +e, i.e.,

$$Q_d = (a+b+c)e = e;\ a+b+c=1 \tag{6}$$

However, the mutations of the charge is expected to be quite small in value as being a second order effect ignorable at a first approximation, the deuteron structure does not require the mutual penetration of the charge distribution of protons.

4.5. Representation of the Deuteron Spin

According to quantum mechanics the most stable state between two particles with spin 1/2 is the singlet, for which the total spin is zero. Thus for the ground state of the deuteron as a bound state of a proton and a neutron should have spin zero. This is exactly contrary to the experimental value of spin 1. When the deuteron is assumed to be a three-body bound state of two protons with an intermediate electron, hadronic mechanics achieves the exact and invariant representation of the spin 1 of model represented by equation 4.

It can be seen that the electron is trapped inside one of the two protons, thus being constrained to have an angular momentum equal to the spin of the proton itself. In this case, with reference to Figure 4 the total angular momentum of the isoelectron is null. Thus the ground state has null angular momentum, the total angular momentum of the deuteron is given by the sum of the spin 1/2 of the two isoprotons.

According to quantum mechanics fractional angular momenta are prohibited because they violate the crucial condition of unitarity, with consequential violation of causality, probability laws, and other basic physical axioms.

For hadronic mechanics, the isotopic lifting and of the spin S and angular momentum L of the electron when immersed within a hyperdense hadronic medium are characterized by

$$\hat{S}^2\hat{T}\,|\,\hat{s}\rangle = (PS)(PS+1)\,|\,\hat{s}\rangle \tag{7}$$

$$\hat{S}_3\hat{T}\,|\,\hat{s}\rangle = \pm(PS)\,|\,\hat{s}\rangle \tag{8}$$

$$\hat{L}^2\hat{T}\,|\,\hat{a}\rangle = (QL)(QL+1)\,|\,\hat{a}\rangle \tag{9}$$

$$\hat{Q}_3\hat{T}\,|\,\hat{a}\rangle = \pm(QL)\,|\,\hat{a}\rangle \tag{10}$$

where $S=1/2$ $L=0,1,2,\cdots$, where P and Q are arbitrary (non-null) positive parameters and isotopically lifted S and L are $\hat{S}$ and $\hat{L}$ respectively.

Santilli introduced the above isotopy of SU(2)-spin to prevent the belief of the perpetual motion that is inherent when the applicability of quantum mechanics is extended in the core of a star.

In fact, quantum mechanics predicts that an electron moves in the core of a star with an angular momentum that is conserved in exactly the same manner as when the same electron orbits around proton in vacuum, thus an electron in the core of a star can only have a locally varying angular momentum and spin as represented by Eqs. 7 - 10.

In case of the isoelectron in the deuteron, we have the constraint that the orbital angular momentum must be equal but opposite to that of the spin:

$$\hat{S} = (P)\frac{1}{2} = -\hat{L} = Q,\quad Q = -\frac{P}{2},\quad \hat{J}_{tot} = 0 \tag{11}$$

The exact and invariant representation of the spin 1 of the ground state of the deuteron then follows according to the rule

$$J_d = S_{p1} + S_{p2} = 1 \tag{12}$$

Now suppose that the quantum mechanical angular momentum operator L has expectation value 1, then

$$\langle a\,|\,L\,|\,a\rangle = 1 \tag{13}$$

Under isotopic lifting the above expression easily acquires the value 1/2 for $\hat{T}=1/2$, $\hat{L}=2$.

$$\langle \hat{a}\,|\,\hat{T}\hat{L}\hat{T}\,|\,\hat{a}\rangle = 1/2 \tag{14}$$

However, in this case the isounit is given by $\hat{I}=1/\hat{T}=2$. Therefore, when the isoeigenvalue of the angular momentum is properly represented as an isonumber (an ordinary number multiplied by the isounit), one recovers the original value 1.

$$\langle \hat{a}\,|\,\hat{T}\hat{L}\hat{T}\,|\,\hat{a}\rangle\hat{I} = 1 \tag{15}$$

thus recovering causality and other laws.

It should be noted that there is no violation of Pauli's exclusion principle in this case since that principle only applies to "identical" particles and does not apply to protons and neutrons, as well known (more explicitly, one of the two protons of Eq. 4 is in actuality the neutron since it has embedded in its interior, the isoelectron).

4.6. Magnetic Moment of Deuteron

The experimental values of magnetic moment of deuteron and its constituents are:

$$\mu_d = \frac{0.8754eh}{2\pi M_p c};\quad \mu_p = \frac{2.795782eh}{4\pi M_p c} \tag{16}$$

and

$$\mu_e = \frac{eh}{4\pi M_e c} = \frac{eh}{4\pi M_p c}\cdot\frac{M_p}{M_e} = \frac{938.272}{0.511}\cdot\frac{eh}{4\pi M_p c}$$

$$= 1.836\times10^{3}\cdot\frac{eh}{4\pi M_p c} \tag{17}$$

We know that deuteron is in its ground state with null angular momentum and there is no orbital contribution to the total magnetic moment from the two protons. Thus the exact and invariant representation of the total magnetic moment of the deuteron is then given by:

$$\mu_d = 2\mu_p + \mu_{tot,e} = 2\times2.792\frac{eh}{4\pi M_p c} + \mu_{tot,e}$$

$$= 0.8754\frac{eh}{4\pi M_p c} \tag{18}$$

$$\mu_{tot,e} = 0.8754\frac{eh}{4\pi M_p c} - 5.584\frac{eh}{4\pi M_p c}$$

$$= -4.709\frac{eh}{4\pi M_p c} = -4.709\frac{eh}{4\pi M_e c}\cdot\frac{M_e}{M_p}$$

$$= -8.621\times10^{-4}\frac{eh}{4\pi M_e c} = \mu_{e,orb} - \mu_{e,spin} \tag{19}$$

In the above equation, missing contribution is provided by the total magnetic moment of the isoelectron. The latter numerical value is given by the difference between the orbital and the intrinsic magnetic moment that is very small (per electron's standard) since the total angular momentum of the isoelectron is indeed small. Also note the correct value of the sign because the isoelectron has the orbital motion in the direction of the proton spin. But the charge is of opposite sign.

Thus the direction of the orbital magnetic moment of the isoelectron is opposite to that of the proton, as represented in equation 4. The small value of the total magnetic moment of the isoelectron for the case of the deuteron is close to the corresponding value for the neutron.

4.7. Deuteron Force

The assumption that the deuteron is a bound state of a proton and a neutron does not provide any explanation for physical origin of the nuclear forces. Quantum mechanics provides mathematical description of the attractive force via number of potentials, although none of them admits a clear physical explanation of the strong attraction between protons and neutrons. Santilli has always tried to generalize quantum mechanics for nuclear physics by providing fundamentally different notions and representations by using hadronic mechanics principles.

We have seen that Model represented by equation 4 permits a clear resolution of this additional insufficiency of quantum mechanics via the precise identification of two types of nuclear forces, the first derivable from a Coulomb potential and the second of contact type represented with the isounit. On the inspection of Figure 4 we see that the constituents of deuteron are in specific configuration such that there we have short range pair-wise opposite signs of charges and magnetic moments with long range identical signs of charges and magnetic moments. Thus it implies that the net attractive Coulomb force in the deuteron is determined by the following expression of potential:

$$V_d = \frac{e^2}{0.6\,\text{fm}} + \frac{e^2}{1.2\,\text{fm}} - \frac{\mu_p\cdot\mu_e}{0.6\,\text{fm}} + \frac{\mu_p\cdot\mu_e}{1.2\,\text{fm}} \tag{20}$$

In addition, the constituents admit an attractive force not derivable from a potential due to the deep penetration of their wavepackets in singlet pair-wise couplings, which force is the same as that of the two identical electrons in the Cooper and valence pairs, the structure of mesons, the structure of the neutron, and can be represented via the isounit:

$$\hat{I} = \exp\left(F(r)\int\psi_{\downarrow}^{\dagger}(r)\times\psi_{\uparrow}^{\dagger}(r)d^3r\right) \tag{21}$$

The projection of the above force chracterizes a strongly attractive Hulthen potential, that behaves at short distances like the Coulomb potential, thereby absorbing the latter and resulting in a single, dominating, attractive Hulthen well with great simplification of the calculations. Thus it can be seen that besides the above potential and contact force, no additional nuclear force is needed for an exact and invariant representation of the remaining characteristics of the deuteron, such as binding and total energies. It can be proved that the isoelectron is not restricted to exist within one of the two protons, because there lies a 50% isoprobability of moving from the interior of one proton to that of the other proton. Therefore, the proton-neutron exchange is confirmed by model given by equation 4.

4.8. Deuteron Binding Energy

We know that quantum mechanics is a purely Hamiltonian theory in the sense that the sole admitted forcers are those derivable from a potential. So direct and immediate consequence is the impossibility of quantitative representation of the deuteron binding energy. The the experimental binding energy of deuteron is

$$E_d = -2.26\,\text{MeV} \tag{22}$$

that is, a representation via equations, rather than via the existing epistemological arguments. Thus the mathematics underlying quantum mechanics, being local differential, can only represent the proton and the neutron of model as being point-like particles. As a result of this fact quantum mechanics admits no binding energy at all for the Deuteron, including the absence of binding energy of Coulomb type, because the neutron is abstracted as a neutral massive point. The lack of a quantum mechanical binding energy for the Deuteron persists even under the assumption that the Deuteron is composed of six hypothetical quarks because attractive and repulsive contributions between the

hypothetical quarks of the proton and those of the neutron cancel out, resulting in no force acting at all between the proton and the neutron, irrespective of whether attractive or repulsive.

Model given by equation 4, under the covering laws of hadronic mechanics has permitted the achievement of the first quantitative representation of the binding as well as the total energy of the Deuteron in scientific history, thus illustrating the validity of Santilli's original proposal of 1978 [18] to build the covering hadronic mechanics.

According to hadronic mechanics, the binding energy is mainly characterized by forces derivable from a potential since the contact forces due to mutual wave-overlapping of wave packets have no potential energy. Hence, the binding energy of the deuteron is due to the potential component of the deuteron binding force given by equation 20. This can be verified by using known values of charges and magnetic moments for the two electron-proton pairs of the deuteron and their mutual distances.

Now, Hadronic mechanics also permits the exact and invariant representation of the total energy of the deuteron, that is direct verification of model given by equation 4.

Now 1 amu = 941.49432 MeV gives,

$$M_p = \frac{938.265\,\text{MeV}}{c^2} = 1.00727663\,\text{amu}$$

$$M_e = \frac{0.511\,\text{MeV}}{c^2} = 5.48597 \times 10^{-4} \quad \text{amu}$$

The mass of a nucleus with A nucleons and Z protons without the peripheral atomic electrons is characterized by

$$M_{nucleus} = M_{amu} - Z \times M_e + 15.73 \times Z^{-1/3} \times 10^{-6}\,\text{amu} \qquad (23)$$

and thus for deuteron

$$M_d = 2.1035\,\text{amu} = 1875.563\,\text{MeV} \qquad (24)$$

The iso-Schrödinger equation for model given by equation 4 can be reduced to that of the neutron, under the assumption that the isoelectron spends 50% of the time within one proton and 50% within the other, thus reducing model (equation 4) in first approximation to a two-body system of two identical particles with un-isorenormalized mass given by

$$\hat{M} = 937.782\ \text{amu} \qquad (25)$$

The main differences are given by different numerical values for the energy, meanlife and charge radius. Thus Santilli derived the structured equation of the deuteron as a two-body nonrelativistic approximation

$$d = (\hat{p}_\uparrow, \hat{p}_\uparrow)_{hm} \qquad (26)$$

$$\left(-\frac{\hbar^2}{2M_{\hat{p}}} \nabla^2 - V \times \frac{\exp(-r/R)}{1-\exp(-r/R)} \right) |\hat{p}\rangle = E\,|\hat{p}\rangle \qquad (27)$$

$$E_d = 2E_{\hat{p}} - |E| = 1875\text{MeV} \qquad (28)$$

$$\tau_d^{-1} = 2\lambda^2\,|\hat{e}(0)|^2\,\alpha^2 E_{\hat{e}} / h = \infty \qquad (29)$$

$$R_d = 4.32 \times 10^{-13}\,\text{cm} \qquad (30)$$

The above equations admit a consistent solution reducible to the algebraic expressions as for the case of Rutherford-Santilli neutron,

$$k_2 = 1, \quad k_1 = 2.5 \qquad (31)$$

It is worth noting that, in the above model, the deuteron binding energy is zero,

$$E = -V\left(\frac{k_2 - 1}{4k_2}\right)^2 \approx 0 \qquad (32)$$

because all potential contributions have been included in the structure of p and, for the binding of the two p all potential forces have been absorbed by the nonlocal forces and k_2 has now reached the limit value of 1 (while being close to but bigger than 1). It has been observed that a more accurate description can be obtained via the restricted three-body configuration of Figure 4. This model gives an exact solution. The model can be constructed via a nonunitary transform of the conventional restricted three-body Schrödinger equation for two protons with parallel spin 1/2 and one isoelectron with null total angular momentum as per Figure 4 with conventional Hamiltonian $H = T + V_{Coul}$, where V_{Coul} is given by equation 20. The nonunitary transforms then produces an additional strong Hulthèn potential that can absorb the Coulomb potential resulting in a solvable equation.

4.9. Electric Dipole Moment and Parity of Deuteron

It is well known that the electric dipole moment of the proton, neutron and Deuteron are null. The preservation of these values by hadronic mechanics is assured by the general property that axiom-preserving lifting preserves the original numerical values, and the same holds for parity. The positive parity of the deuteron is represented by hadronic mechanics via the expression

$$\text{Isoparity} = (-1)^{\hat{L}} \qquad (33)$$

The value for unperturbed deuteron in its ground state $\hat{L} = L = 0$. It should be noted that on one hand, the parity of the deuteron is positive $(L = 0)$, while on the other hand, in order to attempt a recombination of deuteron magnetic moments and spin, the unperturbed deuteron is assumed as being a mixture of different levels, some of which have non-null values of L, thus implying the impossibility of a positive parity.

Thus Santilli has shown that the isotopic branch of nonrelativistic hadronic mechanics permits the exact and invariant representation of "all" the characteristics of the deuteron composed of two isoprotons and one isoelectron, at

the same time resolving all quantum insufficiencies spelled out in the main text above.

4.10. Reduction of Matter to Isoproton and Isoelectrons

It is evident that, following the reduction of the neutron to a proton and an electron and the reduction of the deuteron to two protons and one electron, Santilli has indeed achieved the important reduction of all matter to protons and electrons, since the reduction of the remaining nuclei to protons and electron is consequential, e.g., as a hadronic bound state of two mutated deuterons represents Helium nucleus.

We would like to close our discussion by indicating Santilli's additional astro- physical contribution given by the fact that the so-called "neutron stars" are in reality an extremely high density and high temperature fluid composed by the original constituents of the star, protons and electrons in their isoprotons and isoelectrons realization, in conditions of deep mutual penetration under the laws of hadronic mechanics.

5. Conclusion

As it is well known, the local-differential structure of quantum mechanics solely permits the representation of p[articles as being massive points. This abstraction has been proved to be effective for the representation of the structure of atoms, since the atomic constituents are at very large mutual distances compared to the size of charge distributions or wave packets of particles.

As shown by R. M. santilli in mathematical and physical details, the insufficiency of quantum mechanics to represent the characteristics of the neutron in its synthesis from the hydrogen atom in the core of a star are due precisely to the insufficiency of the representation of the proton and electron as massive points.

In fact, the representation of the proton as an extended charge distribution of $1\,fm$ radius has permitted the representation of all characteristics of the neutron as a compressed hydrogen atom in the core of stars [8]. As an illustration, the anomalous magnetic moments of the neutron is readily represented by a contribution which is impossible for quantum mechanics, but intrinsic in the very conception of hadronic mechanics, namely, the contribution from the orbital motion of the electron when totally compressed inside the proton.

The same advances have shown that the characteristics of the electron change in the transition from isolated conditions in vacuum to the condition of total penetration within the hyperdense proton.

This difference has been quantitatively and invariantly represented by Santilli via, firstly, the transition from Lie' theory to the covering lie-Santilli isotheory, and, secondly, via the transition from particles to isoparticles, namely, the transition from irreducible unitary representations from the conventional Lorentz symmetry to those of the covering Lorentz-Santilli isosymmetry. An exact and time invariant representation of all characteristic of the neutron as a generalized bound state of one isoproton and one isoelectron then follow.

Following, and only following the achievement of a constant, exact and invariant representation of the structure of the neutron Santilli has applied the results to the structure of the deuteron conceived as a three-body generalized bound state of two isoprotons and one isoelectron [2].

This has permitted the exact and invariant representation of all characteristics of the deuteron, with intriguing implications, such as the reduction of all matter in the universe, to protons and electrons in various dynamical conditions.

As an illustration, Santilli's astrophysical contributions finds their root in the fact that the so-called "neutron stars" are in reality an extremely high density and high temperature fluid composed by the original constituents of the star, protons and electrons, in conditions of deep mutual penetration under the laws of hadronic mechanics.

Needless to say, a virtually endless list of intriguing open problems have emerged from the above new vistas in nuclear physics,m among which we mention: the need to reexamine from its foundation the notion of nuclear force due to the emergence of a component not derivable from a potential whose control may lead to new clean nuclear energies; the implications of Santilli's deuteron structure on the natural radioactivity elsewhere; the exact and invariant representation of the spin and magnetic moments of all nuclei; and others.

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Exact and Invariant Representation of Nuclear Magnetic Moments and Spins According to Hadronic Mechanics

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Abstract: In order to render this paper minimally self-sufficient, we review and specialize the main structure of the isomathematics to nuclear constituents as extended and deformable charge distributions under linear and non-linear, local and non-local and Hamiltonian as well non-Hamiltonian interactions; we then review and specialize for the nuclear structure the main laws of the isotopic branch of hadronic mechanics known as isomechanics; we review and specialize the method for turning quantum mechanical nuclear models for point-like nucleons into covering isomechanical models for extended and deformable constituents under the most general known realization of strong interactions; we then review and specialize to nuclear structures the consequential notion of isoparticles; we then review the ensuing, first known, numerically exact and time invariant representation of the magnetic moments of stable nuclides; we then review the structure of the neutron as a bound state according to isomechanics of an isoproton and an isoelectron; and we finally review the ensuing three-body structure of the Deuteron. Via the use of the preceding advances. We then present, apparently for the first time, a numerically exact and time invariant representation of the spin of stable nuclides, firstly, via their approximation as isotopic bound states of isodeuterons, isoneutron and isoprotons, and secondly, via their reduction to isobound states of isoprotons and isoelectrons. Some observations on the nuclear configurations so obtained have also been presented in the case of the first model and in view of the second option we have identified in isoelectrons the nuclear glue which tightly holds isonucleons of stable nuclide in the atomic nucleus in the preferred orientation of their intrinsic spins. In Appendix A, we provide a technical review specialized for the first time to nuclear physics of the Lie-Santilli theory and its main application to the notion of isoparticles as isoirreducible isounitary isorepresentations of the Lorentz-Poincaré-Santilli isosymmetry.

Keywords: Hadronic Mechanics, Nuclear Magnetic Moments, Nuclear Spins

PACS: 21.10. Hw; 21.30.-x; 21.60. De; 21.30. Fe

This paper is dedicated to the memory of Enrico Fermi who: expressed doubts as to whether conventional geometries apply to the structure of particles; supported the introduction of the size of nucleons for basic advances in nuclear physics; and suggested that the anomalous magnetic moment of nuclei may be due to the deformation of their charge distributions under the strong nuclear forces [1]*, all visions that are quantitatively studies in this paper.*

1. Introduction

In the authors view, quantum mechanics is exactly valid for the atomic structure, but it is only approximately valid for the nuclear structure because quantum mechanics achieved a very accurate representation of atomic data, compared to the known inability by quantum mechanics to achieve an accurate representation of nuclear data, thus supporting the historical argument by Einstein, Podolsky and Rosen according to which quantum mechanics is "incomplete" [2].

A first reason for the above dichotomy is the fact that the mathematics underlying quantum mechanics (including the local-differential calculus, functional analysis, Hilbert spaces, Lie algebras, etc.) can only represent a finite number of isolated point-particles moving in vacuum, which conditions are known as characterizing *exterior dynamical problems*. The abstraction of particles into dimensionless points is evidently effective for the atomic structure due to the large mutual

distances of the atomic constituents, but the same abstraction is ineffective for the nuclear structure because nuclear constituents consist of extended charge distributions in conditions of partial mutual penetration, which conditions are known as characterizing broader interior dynamical problems (Figure 1).

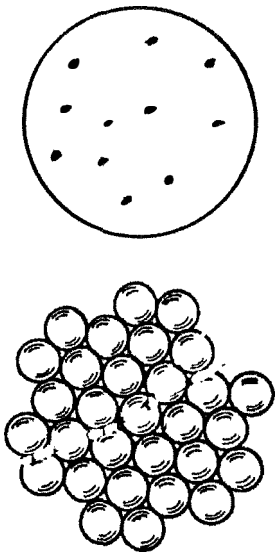

Figure 1. *A conceptual rendering (top view) of the abstraction of the nuclear structure as a sphere with isolated point-particles in its interior which is necessary for the applicability of the mathematics underlying quantum mechanics, namely, the local-differential calculus with related Hilbert spaces and Lie's theory. By contrast (bottom view), the nuclear structure consists of hyperdense, extended charge distributions in conditions of partial mutual penetration, as one can verify by the fact that nuclear volumes are generally smaller than the sum of the volumes of the constituent protons and neutrons. In the authors view, the inability by quantum mechanics to achieve an exact representation of nuclear data is due to the evident insufficiency of the abstraction of the nuclear structure depicted in the top when compared to the physical reality of the bottom view. This first insufficiency establishes the fundamental need of a basically new mathematics for the representation of extended charge distributions as they occur in the nuclear reality (Section 2).*

A second reason for the above dichotomy is that the fundamental symmetries of non-relativistic and relativistic quantum mechanics, the Galileo and Poincaré symmetries respectively, are solely valid for a *Keplerian system,* namely, for a system of particles orbiting around a heavier center, as it is the case indeed for atomic structures. By contrast, as stressed in the recent literature, *nuclei do not have nuclei* and, therefore, the symmetries valid for systems of point particles with a Keplerian nucleus cannot possibly be exactly valid for structurally different systems of extended particles without a Keplerian nucleus (Figure 2).

A third reason for the above dichotomy is that none of the 20th century sciences, including quantum mechanics and special relativity, can represent Fermi's historical hypothesis that the deviations of the values of nuclear magnetic moments from the predictions of relativistic quantum mechanics are due to deformations of the charge distributions of protons and neutrons (nucleons) when under the strong interactions of a nuclear structure, with consequential alteration (called in this paper *mutation*) of their intrinsic magnetic moments (Figure 3). This insufficiency is evidently due to the fact that *dimensionless points cannot experience deformations.* Therefore, a mathematics which can solely represent dimensionless points is structurally unable to represent the deformation of extended charge distributions as they occur in the nuclear reality.

A fourth reason for the above dichotomy is the fact that *dimensionless points can only experience interactions at a distance, thus derivable from a potential* (interactions technically known as *variationally self-adjoint* [3a]). In view of this basic feature, recent representations of the strong nuclear force have reached un-reassuring limits, such as a Hamiltonian with forty or so potentials, without the desired achievement of an exact representation of nuclear data. In the authors view, it is necessary to complement these conventional studies with the admission that the interactions between extended charge distributions under conditions of partial mutual penetrations are of *contact* type, thus not being derivable from a potential (interactions technically known as *variationally non-selfadjoint* [3a]). Consequently, it is recommendable to ascertain whether some of the potential components of nuclear Hamiltonians should be replaced with non-Hamiltonian representations.

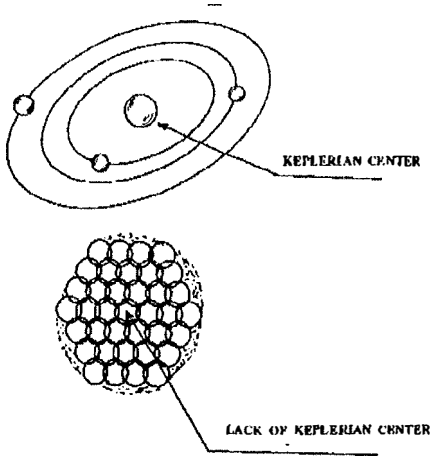

Figure 2. *A conceptual rendering of the second impossibility for quantum mechanics to be exactly valid for the nuclear structure, which is given by the fact that the basic symmetries of non-relativistic and relativistic quantum mechanics, the Galileo and Poincaré symmetries respectively, only apply for Keplerian systems of particles orbiting around a heavier nucleus. By contrast, R. M. Santilli has stated several times in his writings that "nuclei do not have nuclei," thus implying a necessary breaking of said fundamental symmetries, with consequential lack of exact character of non-relativistic and relativistic quantum mechanics for the nuclear structure. The same breaking is confirmed by numerous additional evidences, such as the fact that the partial mutual penetration of nucleons in a nuclear structure implies the presence of contact interactions not representable with a Hamiltonian, thus implying the inapplicability of the entire Lie theory, let alone of Lie's symmetries, due to its strictly Hamiltonian character. This second insufficiency establishes the need for a covering of Lie's theory for the construction of the symmetries of systems of extended particles without Keplerian nuclei under Hamiltonian as well as non-Hamiltonian internal forces (Section 2 and Appendix A).*

A fifth reason for the above dichotomy is that quantum mechanics is certainly effective for the description of nuclear *fissions* due to the effective representation of the fission debris as point particles, but quantum mechanics has proved to be ineffective for the achievement of nuclear *fusions* for all the above indicated reasons, plus the fact that *nuclear fusions are structurally irreversible over time while quantum mechanics is structurally reversible,* hence the need for a covering of quantum mechanics that can represent extended charge distributions with Hamiltonian and non-Hamiltonian interactions in generally irreversible conditions.

In this paper, we shall briefly outline decades of research by one of us (R. M. Santilli) [3-33] for: the construction of a generalization of 20^{th} century mathematics suitable to represent extended particles (Figure 1); the generalization of Lie's theory for the construction of symmetries of systems of extended particles without Keplerian center under Hamiltonian and non-Hamiltonian internal forces (Figure 2); the representation of Fermi's historical hypothesis on the deformability of nucleons; and the consequential, first known, exact and time invariant representation of nuclear magnetic moments (Figure 3).

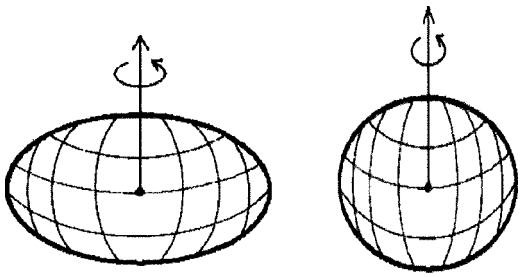

Figure 3. *A third insufficiency of quantum mechanics for nuclear structures is given by the historical prediction by Enrico Fermi [1] that the anomalous values of nuclear magnetic moments is due to deformations of the charge distribution of protons and neutrons when under the strong interactions of the nuclear structure, with consequential alteration of their conventional magnetic moments. In fact, a quantitative treatment of Fermi's teaching requires the use of the deformation theory which is known to be incompatible with quantum mechanics. This third insufficiency establishes the need that the novel mathematics and Lie's theory for extended charge distributions should be constructed in such a way to be compatible with the deformation theory "ab initio" (Section 2 and Appendix A).*

Since the advances considered here [3-33] are only known to a restricted number of experts, and they are generally unknown to the nuclear physics community, in order to render minimally understandable the advances presented in this paper, it has been necessary to: outline in Section 2 the novel mathematics (known as *isomathematcs* for the reversible case and *genomathematics* for the irreversible form); outline in Section 3 the corresponding invariant branches of hadronic mechanics (known as *isomechanics* and *genomechanics* respectively); outline in Section 3.1 the non-relativistic nuclear isomechanics; outline is Section 3.2 the relativistic nuclear isomechanics; outline in Section 4 a simple construction of iso- and gene-mechanics; outline in subsequent sections the exact and time invariant representation of nuclear magnetic moments (Section 5), the test of spinorial symmetry by neutron interferometry (Section 6), and then outline the emerging new structure of the neutron (Section 7), deuteron and nuclei at large (Section 8). Above all, it has emerged as recommendable to formulate advances [3-33] in a form directly applicable to nuclear physics, rather than leaving such an adaptable to the imagination of non-initiated readers.

We shall then present, apparently for the first time, the achievement of an exact and time invariant representation of the spin of stable nuclide which, thanks to the above advances, is compatible with the mutation of the intrinsic magnetic moments of nucleons, and then indicate the implications of these advances in nuclear physics for basically new, environmentally acceptable forms of nuclear energies. For the sake of self sufficiency of this presentation we start with a very brief description of stable and unstable nuclides in Section 9, a brief description of new and old vistas of nuclear forces with the earlier conjectural assertions of the stability of nucleons in Section 10. In Section 11 we have developed notations to represent isoneutrons and isodeuterons. We have presented in Section 12 two models of nuclear configuration, namely (i) considering isodeuterons, isoneutrons and isoprotons as isonucleons (Section 12.1) and (ii) isoprotons and isoelectrons as isonucleons (Section 12.2). These nuclear configurations were written down in a way to be commensurate with the experimental nuclear spins and tabulated in Section 13 for both the nuclear models stated above. We have also presented our observations in Section 14 on these nuclear configurations with an idea to provide the facts about the isonucleons within the nuclides that would help in developing corresponding theories of nuclear stability and generate new explanations of other nuclear properties (Sections 14.1 and 14.2). In the second model of nuclear configuration arrived at in this paper we propose, apparently for the first time, that the isoelectrons serve as the nuclear glue that tightly holds the nuclear isoprotons together in the atomic nucleus (Section 14.2). For the sake of ready reference we have also presented the Lorentz-Poincaré-Santilli Isosymmetry and its characterization of isoparticles in Appendix A.

2. Elements of Iso-Mathematics and Geno-Mathematics

The first known time-invariant representation of extended and deformable charge distributions in interior dynamical conditions was proposed by Santilli in the early 1980s [3b] via the *isotopic* (in the sense of being axiom-reserving) lifting of the associative product AB between generic quantities (numbers, functions, matrices, operators, etc.) into the form, todays known as *Santilli isoproduct,*

$$AB = AB \rightarrow A\hat{T}B = A\hat{\times}B \tag{1}$$

where $\hat{T}$ is solely restricted to be invertible, but otherwise possesses an arbitrary dependence on local variables such as:

time t, coordinates r, velocities v, density μ, temperature τ, index of refraction ρ, frequency ω, wave functions ψ, etc., $\hat{T} = \hat{T}(t, r, v, \mu, \tau, \rho, \omega, \psi, ...)$.

When $\hat{T}$ is positive-definite and invariant under time-reversal $t \to -t$, it is called isotopic element, and when it is positive-definite (or merely Hermitean) but non-invariant under time reversal, it is called the genotopic element.

The representation of extended and deformable charge distributions is then immediately achieved via realizations of $\hat{T}$ of the type [3]

$$\hat{T} = Diag.\left(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, \frac{1}{n_4^2}\right) e^{-\Gamma(\psi,...)\int \psi^\dagger \psi dr^3} \tag{2}$$

where: $n_k = n_k(t, r, v, \mu, \tau, \delta, \omega, \psi, ...), k = 1, 2, 3,$ represent, in this simple case, the deformable semi-axes of a nucleon assumed for simplicity to be an ellipsoid; $n_4 = \rho$ characterizes the density of the nucleon considered; all quantities $n_\mu, \mu = 1, 2, 3, 4$, called characteristic quantities of the nucleon considered, are normalized to the value $n_\mu = 1$ for exterior conditions in vacuum; $\Gamma(\psi, ...)$ is a positive definite function or operator characterizes all non-linear interactions not representable with the conventional Hamiltonian; and the integral in the exponent of Eq. (2) tends to zero at mutual distances of particles much bigger than their charge radius (about $1\,\text{fm} \simeq 10^{-13}$ cm), thus implying the limit

$$\lim_{r \gg 1\,\text{fm}} \hat{T} = 1, \tag{3}$$

for which

$$\lim_{r \gg 1\,\text{fm}} \left(A \hat{\times} B\right) = AB. \tag{4}$$

When $\hat{T}$ verifies the conditions

$$\hat{T}(t, ...) = \hat{T}^\dagger(t, ...) = \hat{T}(-t, ...) = \hat{T}^\dagger(-t, ...) \tag{5}$$

it is called the *isotopic element,* while under the verification of the conditions

$$\hat{T}(t, ...) = \hat{T}^\dagger(t, ...) \neq \hat{T}(-t, ...) = \hat{T}^\dagger(-t, ...) \tag{6}$$

$\hat{T}$ is called the genotopic element. Conditions (5) characterize the use of isomathematics, while conditions (6) characterize the use of the broader genomathematics. The most important mathematical difference is that the conventional Lie theory with historical product between Hermitean operators

$$[A, B] = AB - BA, \tag{7}$$

at the foundations of quantum mechanics is lifted in the former case into *Santilli Lie-isotopic theory* with basic product

$$[A \hat{,} B] = A \hat{\times} B - B \hat{\times} A = A\hat{T}B - B\hat{T}A, \tag{8}$$

while in the latter case Lie's theory is lifted into the broader *Santilli Lie-admissible theory* with covering product

$$(A \hat{,} B) = A\hat{T}(-t, ...)B - B\hat{T}(t, ,, ,)A = A\hat{R}B - B\hat{S}A, \tag{9}$$

$$R = R^\dagger, \; S = S^\dagger, \; R \neq S, \tag{10}$$

according to conceptions, formulations and terminologies first introduced by Santilli in Ref. [3b].

It should be indicated from the upset the importance of conditions (5) and (6) for nuclear physics. In fact, conditions (5) characterize a *stable nuclide* composed by extended nucleons when isolated from the rest of the universe, thus being reversible over time. By contrast, conditions (6) characterize *irreversible nuclear reactions,* such as nuclear syntheses.

In fact, as it is well known, the time reversibility of quantum mechanics is ultimately due to the *invariance of the Lie product under anti-Hermiticity* (for hermitean operators A and B)

$$[A, B] \equiv -[A, B]^\dagger. \tag{11}$$

It is then easy to see that isomathematics and its ensuing physical formulations are also time reversal invariant due to the *invariance of the Lie-Santilli isoproduct under anti Hermiticity,*

$$[A \hat{,} B] \equiv -[A \hat{,} B]^\dagger. \tag{12}$$

By contrast, genomathematics and its related physical formulations are irreversible over time precisely because *Santilli's Lie-admissible product violates, by central conception, the invariance under anti-Hermiticity*

$$(A \hat{,} B) \neq -(A \hat{,} B)^\dagger. \tag{13}$$

Monograph [3b] presented the lifting of most 20th century applied mathematics via the systematic lifting of all products into the isotopic form (1), although all liftings were formulated on conventional numeric fields.

Since this paper deals with magnetic moments and spins of stable, thus reversible nuclides, we shall mainly use isomathematics under basic conditions (5). However, it is recommendable for the non-initiated reader to know that that the extension to irreversible nuclear processes is immediate, thus being recommendable when applicable.

Subsequently, Santilli discovered that the emerging formulations were not invariant over time, that is, they failed to predict the same numerical values under the same conditions at different times. In order to resolve this basic insufficiency, Santilli re-examined in 1993 [4] conventional numeric fields $F(n, \times, 1)$ with classification of numbers n into real, complex or quaternionic numbers n, conventional associative product $nm = n \times m \in F$ and basic multiplicative unit 1, $1 \times n = n \times 1 = n \quad \forall n \in F$.

In this way, Santilli [4] discovered that the axioms of

numeric field also admit solution with an arbitrary basic unit $\hat{I}$, under the conditions that: 1) all numbers are lifted in the isonumbers

$$n \to \hat{n} = n\hat{I}; \tag{14}$$

2) all products are lifted into the *isoproduct* (1),

$$nm \to n \hat{\times} m = n\hat{T}m; \tag{15}$$

and 3) the conventional unit 1 of 20th century numeric field is lifted into the isounit under the sole conditions of being positive-definite and being the inverse of isotopic element $\hat{T}$,

$$\hat{I} = \frac{1}{\hat{T}} \tag{16}$$

Under these conditions all axioms of a numeric field are verified and $\hat{I}$ is the correct left and right multiplicative unit,

$$\hat{I} \hat{\times} \hat{n} = \hat{n} \hat{\times} \hat{I} = \hat{n} \quad \forall \hat{n} \in \hat{F}. \tag{17}$$

Under conditions (4), $\hat{I}$ is called Santilli isounit, while under broader conditions (5) it is called Santilli genounit [4].

This lead to the discovery of new numeric fields $\hat{F}(\hat{n}, \hat{\times}, \hat{I})$ called isofields under conditions (4) and genofields under conditions (5) with corresponding novel isoreal, isocomplex and isoquaternionic numbers and general, genocomplex and genoquarternionic genonumbers $\hat{n} = n\hat{I}$.

Following the discovery of isonumbers and genonumbers, all theories originally formulated on conventional fields [3] where lifted into formulations defined over isofields and genofields [5, 6], but the crucial time invariance of the numeric predictions was still missing.

In order to resolve this impasse, Santilli reinspected in 1995 the Newton-Leibnitz differential calculus and discovered that, contrary to popular beliefs in mathematics and physics for centuries, the Newton-Leibnitz differential calculus depends on the assumed basic multiplicative unit because, in the event said unit has a functional dependence on the differentiation variable, the ordinary differential dr must be generalized into the form first introduced in memoir [7]

$$\hat{d}\hat{r} = \hat{T}d[r\hat{I}(\hat{r},...)] = dr + r\hat{T}d\hat{I}(\hat{r},...), \tag{18}$$

and called *isodifferential* under conditions (4) and *genodifferential* under conditions (5), with corresponding *isoderivatives* (and *genoderivatives*) [7]

$$\frac{\hat{\partial}\hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \frac{\partial \hat{f}(\hat{r})}{\partial \hat{r}} + \hat{f}(\hat{r})\hat{T}\frac{\partial \hat{I}(\hat{r},...)}{\partial \hat{r}}, \tag{19}$$

where, for consistency, coordinates and functions must be isoscalars, that is, have values in $\hat{F}$ with structures

$$\hat{r} = r\hat{I}(\hat{r},...), \ \hat{f}(\hat{r},...) = f(\hat{r},...)\hat{I}(\hat{r},...). \tag{20}$$

It should be stressed that the representation of nuclear magnetic moments and spin presented in this paper depends crucially on a non-potential component of the nuclear force due to partial mutual penetration of the charge distribution of nucleons, which non-potential components is represented precisely via the isodifferential calculus and, therefore, with the novel additional terms in the r.h.s. of Eqs. (18) and (19).

In memoir [7] Santilli introduced a third broader mathematics under the name of *hypermathematics* which is given by a covering of genomathematics when the genounit is multi-valued (rather than multi-dimensional), e.g. of the ordered type $\hat{I} = \{\hat{I}_1, \hat{I}_2, ..., \hat{I}_n\}$ where n can assume an arbitrarily larger values such as $n = 10^{50}$ as needed for biological structures.

The discovery of the generalized differential calculus signed the achievement in memoir [7] of mathematical maturity in the generalizations of 20th century applied mathematics at large, that stimulated seminal advances in mathematics (see representative monographs [8-11]) as well as generalized physical and chemical theories, including novel industrial applications indicated below.

The above studies lead to the following chain of generalized mathematics:

1. *IsoMathematics*, which is used for the representation of stable and isolated, thus time-reversible nuclei composed by extended nucleons in conditions of partial mutual penetration and is characterized by the lifting of the totality of 20th century applied mathematics in such a way to admit a positive-definite and time-reversal invariant isounit (5) at all levels of treatment.
2. GenoMathematics, which is used for the representation of time-irreversible nuclear reactions and it is characterized by a dual lifting of the totality of 20th century mathematics in such a way to admit a positive-definite time-noninvariant genounit (6) at all levels of treatment, one genounit, $\hat{I}(t,...) = 1/\hat{T}(t,...)$ characterizes motion forward in time, and its time reversal image $\hat{I}(-t,...) = 1/\hat{T}(-t,...)$ characterizes motion backward in time, irreversibility over time being assured by inequivalent forward and backward genounits $\hat{I}(t,...) \neq \hat{I}(-t,...)$.

The knowledge of the above distinct mathematics is important for researchers to prevent the use of time non-invariant isounits that may eventually imply irreversible contributions for the structure of isolated and stable nuclei, with evident inconsistencies.

Important independent contributions on the foundations of isomathematics and genomathematics can be found in monographs [8-11] and in their bibliographies.

The main methodological problems for the representation of nuclear magnetic moments and spins are the following:

2.I: The representation of the deformation of the charge distribution of protons and neutrons when members of a nuclear structure and the ensuing mutation of their intrinsic magnetic moments according to Fermi's historical hypothesis [2]. This first central problem was solved by Santilli via the use of the isotopies of the rotational symmetry [12], as

reviewed in the next section and in Appendix A.

2.II: The representation of the mutation of the intrinsic magnetic moments of nuclear constituents in a way compatible with the conventional ten conservation laws of total physical quantities (the conservation of the total angular momentum, total linear momentum, the center of motion, and the total energy), which must hold for all isolated bound states of particles. This problem was solved by Santilli by showing the isotopies of the Lorentz and of the Poincaré symmetry [15-17] do verify indeed said conventional total conservation laws because in the lifting of Lie's theory into the Lie-Santilli isotheory the generators of Lie algebras (that represent said conservation laws) remain unchanged, and only their products lifted for the representation of extended shapes and non-Hamiltonian interactions.

2.III: The representation of the spin of stable nuclides in a way compatible with mutation of the magnetic moments of protons and neutrons under strong nuclear interactions. This problem will be solved, apparently for the first time in this paper, by showing that the isotopies of the $SU(2)$ -spin symmetry do indeed admit a "hidden" degree of freedom directly connected to the mutation of spin.

The central physical notion used in this paper for the solution of the above problems and for the characterization of extended-deformable nuclear constituents in conditions of partial mutual penetration is that of *isoparticle,* specialized to *isoprotons, isoneutron and isoelectrons.*

The understanding of the notion of isoparticle and, therefore, of this paper, requires at least some knowledge of the central branch of isomathematics used for the derivation of the new notion of isoparticle, which is given by the isotopies of Lie's theory, originally proposed by Santilli in monograph [3b], including the isotopies of universal enveloping associative algebras, Lie's theorem and Lie's transformation groups.

Among a rather large literature in the field, Santilli's papers specifically devoted to the notions of isoparticle are given by the isotopies of: the rotational symmetry $O(3)$ [12]; the $SU(2)$ spin symmetry [13, 14]; the Lorentz symmetry $O(3.1)$ in classical [15] and operator [16] forms; the isotopies of the Poincaré symmetry $P(3.1)$ [17]; the spinorial covering of the Poincaré symmetry [18]; and the isotopies of the Minkowskian geometry [19]. The notion of isoparticle was then studied in details in Refs. [20-23].

In view of these advances, the isotopies of Lie's theory are today called the *Lie-Santilli isotheory* (see independent studies [24-33]).

Due to its fundamental character for the exact and time invariant representation of magnetic moments and spins, the notion of isoparticle will be reviewed in detail in Appendix A.

3. Elements of Nuclear IsoMechanics and GenoMechanics

The non-unitary covering of quantum mechanics was proposed under the name of *hadronic mechanics* by R. M. Santilli in monograph [3b] of 1981 (see page 112 for the proposal of the name of the new mechanics.) The original proposal comprised two branches, the isotopic branch with Lie-isotopic structure (8) and in the genotopic branch with Lie-admissible structure (9).

A fundamental contribution to hadronic mechanics (which is fully valid nowadays) was provided in paper [34] of 1982 by the mathematician (late) H. C. Myung and R. M. Santilli via the isotopies and genotopies of the Hilbert space (today known as the *Hilbert-Myung-Santilli isospace and genospace* respectively) and the indication that hadronic mechanics removes the divergencies of quantum mechanics via the isotopies of Dirac Delta "distribution" (today known as the *Dirac-Myung-Santilli isodelta "function"* and the fast convergence of isotopic series (see, e.g., Ref. [35]).

These initial studies were formulated on a conventional field and elaborated via the conventional differential calculus. Hadronic mechanics achieved a mature formulation only following the discovery of the novel isonumbers and genonumbers [4] in 1993 and of the isodifferential and genodifferential calculus [7] in 1996 (see monographs [22] for a general presentation of hadronic mechanics, including the fundamental notion of iso- and geno-particles).

With the passing of time, the above indicated two branches of hadronic mechanics acquired the names of *isomechanics* and *genomechanics,* respectively. Since these names have received a rather wide acceptance by the physics community, they have been adopted in this paper.

The reader should be aware that hadronic mechanics has a variety of applications in disparate fields all dealing with interior dynamical problems. The main reference for the specialization of isomechanics to nuclear physics is given by memoir [26] of 1998, while the main reference for genomechanics is given by memoir [37] of 2006.

Since hadronic mechanics at large, as well as isotopic and genotopic branches are essentially unknown to the nuclear physics community, it appears recommendable to provide in this section an elementary review specialized to nuclear physics sufficient for the understanding of the derivation of exact and invariant magnetic moments and spins, with the understanding that an in depth study of memoirs [35, 36] is essential for serious knowledge.

3.1. Elements of Non-relativistic Nuclear IsoMechanics

Non-relativistic nuclear isomechanics is characterized by the lifting of Planck's constant $\hbar$ into a 3×3 -dimensional, positive-definite *space isounit* [4, 7]

$$\hbar \to \hat{I}_r = 1/\hat{T}_r = Diag.(n_1^2, n_2^2, n_3^2) > 0 \tag{21}$$

where the quantities $n_k^2, k = 1,2,3$: represents ab initio the semiaxes of the extended-deformable shape of nucleons when members of a nuclear structure (see the l.h.s. of Figure 3); are normalized to the perfect sphere $n_k^2 = 1$ in empty space; are restricted to be positive-definite and time-reversal invariant; and possess an unrestricted functional dependence on all needed local variables (see Section 1)

$n_k^2 = n_k^2(\hat{t},\hat{r},\hat{v},\hat{\mu},\hat{\tau},\hat{\rho},\omega,\hat{\psi},..) > 0$, where the "hat" denotes the referral to *internal* variables, while variables without a "hat" refer to those of the *external* observer.

Note that we have ignored in Eq.(21) for simplicity the multiplicative exponential term representing internal non-linear, non-local and non-Hamiltonian interactions as in Eq. (2) since this term can be embedded into the n_k^2 via their simple redefinition.

Assumption (21) implies that all possible products AB of conventional nuclear formulations (including the product of numbers, functions, matrices, etc.) have to be lifted to the isoproduct [4],

$$AB \to A \hat{\times} B = A\hat{T}_{\hat{r}} B. \tag{22}$$

with Lie-isotopic structure (8) [4].

Assumptions (21) and (22) also implies that conventional numeric fields $F(n,\times,I)$ are lifted into isofields $\hat{F}(\hat{n},\hat{\times},\hat{I}_{\hat{r}})$ with isonumbers $\hat{n} = n\hat{I}_{\hat{r}}$ [4].

Note that, in the event the characteristic quantities $n's$ depend on time in a way not invariant under time-reversal, instead of the single unit (21) and product (22) for action to the right and to the left, we would have the genoproduct and genounit for motion forward in time [36]

$$AB \to A > B = AT(t,...)B, \ \hat{I}^{>} = 1/T(t,...), \tag{23}$$

and the genoproduct and genounit for motion backward in time

$$AB \to A < B = AT(-t,...)B, \ T(t,...) \neq T(-t), \ ^{<}\hat{I} = 1/T(-t,...) \tag{24}$$

with Lie-admissible structure (9).

Therefore, the use of time-reversal non-invariant quantities $n_k, k = 1,2,3$ for the study of a stable, time-reversal invariant nuclear structure would imply the inclusion of un-warranted irreversible contributions that should solely be admitted for irreversible nuclear reactions [36].

Nuclear isomechanics is additionally characterized by the lifting of time t into the isotime

$$t = t_{ext} \to \hat{t} = t_{int}\hat{I}_{\hat{t}} \tag{25}$$

where t_{ext} is the time of the external observer, t_{int} is the intrinsic time in the interior of nuclei, and $\hat{I}_t$ is different than $\hat{I}_r$, both dimensionally and numerically.

The representation of nuclear magnetic moments and spins has been achieved in the above stated paper via the simpler case in which $\hat{I}_{\hat{t}} = 1$ and the sole use of the time of the external observer $t = t_{ext}$. Consequently, isotime will be ignored for simplicity.

Nevertheless, the non-initiated reader should be aware that, on strict technical grounds, isomechanics implies that the time in the interior of nuclei is generally different than the external time [22].

The carrier isospace of isocoordinates $\hat{r} = r\hat{I}_{\hat{r}}$ is given by the Euclid-Santilli isospace [7] $\hat{E}(\hat{r},\hat{\delta},\hat{I})$ with isometric $\hat{\delta} = \hat{T}_{\hat{r}}\delta$ where $\delta = Diag.(1,1,1)$ the conventional Euclidean metric, with isoline element

$$\hat{r}^{\hat{2}} = \hat{r}^i \hat{\times} \hat{\delta}_{ij} \hat{\times} \hat{r}^j = (r^i \hat{T}_i^k \delta_{kj} r^j)\hat{I}_{\hat{r}} =$$

$$= \left(\frac{r_1^2}{n_1^2} + \frac{r_2^2}{n_2^2} + \frac{r_3^2}{n_3^2}\right)\hat{I}_{\hat{r}} \tag{26}$$

where one should keep in mind that the elements of the isometric must be isonumbers as a condition for the isoline element to be an isoscalar with value in the isoreal isofield R.

The understanding of nuclear isomechanics requires the knowledge that the isotopies map ellipsoids on conventional Euclidean space into the perfect sphere in the Euclid-Santilli isospace. This is due to the fact that the deformation of the semiaxes $1_k \to 1/n_k^2$ is done with the joint inverse deformation of the isounit $1_k \to n_k^2$, by therefore yielding the original value of the perfect sphere 1_k in isospace.

The reconstruction of the perfect sphere in isospace is essential for the isomorphism of the Lie-Santilli isorotations $\hat{O}(3)$ with the conventional rotations $O(3)$ under the central condition of including the deformation theory (Appendix A).

The isooperator isospace is given by the Hilbert-Myung-Santilli isospace [34] H defined on isofields of isocomplex isonumbers C with isounit (21), isostates $|\hat{\psi}(\hat{t},\hat{r})\rangle$ and isoinner isoproduct

$$\langle\hat{\psi}|\hat{\times}|\hat{\psi}\rangle\hat{I}_{\hat{r}} = \langle\hat{\psi}|\hat{T}_r|\hat{\psi}\rangle\hat{I}_{\hat{r}}, \tag{27}$$

isonormalization

$$\langle\hat{\psi}|\hat{\times}|\hat{\psi}\rangle I_{\hat{r}} = \hat{I}_{\hat{r}} \tag{28}$$

and isoexpectation values for an iso-Hermitean isooperator, $\hat{Q}$,

$$\langle\hat{Q}\rangle = \langle\hat{\psi}|\hat{\times}\hat{Q}\hat{\times}|\hat{\psi}\rangle\hat{I}_{\hat{r}} = \langle\hat{\psi}|\hat{T}_{\hat{r}}\hat{Q}\hat{T}_{\hat{r}}|\hat{\psi}\rangle\hat{I}_{\hat{r}} \tag{29}$$

with particular properties

$$\hat{I}_{\hat{r}}\hat{\times}|\hat{\psi}\rangle = |\hat{\psi}\rangle, \ \langle\hat{\psi}|\hat{\times}\hat{I}_{\hat{r}}\hat{\times}|\hat{\psi}\rangle\hat{I}_{\hat{r}} = \hat{I}_{\hat{r}}. \tag{30}$$

confirming that $\hat{I}_r$ is the correct isounit of the theory.

The dynamical equations of non-relativistic nuclear isomechanics are given by the Schrödinger-Santilli isoequation on H over C [3, 7]

$$-\hat{\imath}\hat{\times}\hat{\partial}_{\hat{t}}|\hat{\psi}\rangle = \hat{H}\hat{\times}|\hat{\psi}\rangle = \hat{H}(\hat{r},\hat{p})\hat{T}_{\hat{r}}(\hat{\psi},\hat{\partial}\hat{\psi},...)|\hat{\psi}\rangle =$$

$$= \hat{E}\hat{\times}|\hat{\psi}\rangle = E|\hat{\psi}\rangle \tag{31}$$

the *isolinear isomomentum*, introduced for the first time in memoir [7] following the discovery of the isodifferential

calculus

$$\hat{p}\hat{\times}|\hat{\psi}\rangle = -\hat{i}\,\hat{\times}\,\hat{\partial}_{\hat{r}}\,|\hat{\psi}\rangle = -i\hat{I}_{\hat{r}}\partial_{\hat{r}}\,|\hat{\psi}\rangle, \tag{32}$$

the *Heisenberg-Santilli IsoEquation* in the infinitesimal version [3, 7]

$$\hat{i}\,\hat{\times}\frac{\hat{d}\hat{Q}}{\hat{dt}} = [\hat{Q}\,\hat{;}\,\hat{H}] = \hat{Q}\hat{\times}\hat{H} - \hat{H}\,\hat{\times}\hat{Q} = \hat{Q}\hat{T}_{\hat{r}}\hat{H} - \hat{H}\hat{T}_{\hat{r}}\hat{Q} \tag{33}$$

the integrated version to a finite transform (see Refs [22, 36] for the correct formulation in isomechanics)

$$\hat{Q}(\hat{t}) = UQ(0)U^{\dagger} = e^{\hat{H}\hat{T}\hat{t}i}Q(0)e^{-i\hat{t}\hat{T}\hat{H}}, \tag{34}$$

$$UU^{\dagger} \neq I, \tag{35}$$

and the isocommutation rules

$$[\hat{r}_i\,\hat{;}\,\hat{p}_j] = \delta_{ij}\hat{I}_{\hat{r}}, \quad [\hat{p}_i\,\hat{;}\,\hat{p}_j] = [\hat{r}_i\,\hat{;}\,\hat{r}_j] = 0. \tag{36}$$

where the "hat" on operators denotes their definition on $\hat{\mathrm{H}}$ over $\hat{\mathrm{C}}$

3.2. Elements of Relativistic Nuclear Isomechanics

Relativistic nuclear isomechanics is characterized by the lifting of Planck's constant $\hbar$ into a 4×4 -dimensional, positive-definite, thus diagonalizable isounit (see Refs. [22] and the memoir [44])

$$\hbar \to \hat{I} = 1/\hat{T} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2) > 0 \tag{37}$$

where: the $n_k^2, k = 1,2,3$ continue to represent deformed nucleons; n_4 is a geometrization of the hyperdense medium inside nucleons' the characteristics quantities $n_\mu, \mu = 1,2,3,4$ are subjected to the normalization for the vacuum $n_\mu^2 = 1$; the multiplicative exponential term as in Eq. (1) is absorbed by the n_μ which have an arbitrary functional dependence on local internal variables solely subjected to be invariant under time reversal.

Assumption (35) implies that the totality of all products AB of relativistic nuclear isoformulations are lifted into the isoproducts $A\,\hat{\times}\,B = ATB$ defined on isofields $\hat{I}(\hat{n},\hat{\times},\hat{I})$.

Again, care must be exercised in the study of stable nuclei in order to prevent the transition from isomechanics to genomechanics that occurs whenever the characteristic quantities n_μ are not invariant under time reversal.

Let $M(x,\eta,I)$ be the conventional Minkowski space with coordinates $x = (x^1, x^2, x^3, x^4 = t)$, metric $\eta = Diag.(1,1,1,-c^2)$ and unit $I = Diag.(1,1,1,1)$. Then, the relativistic isospace of the isocoordinates $\hat{x} = x\hat{I}$ is given by the Minkowski-Santilli isospace $\hat{M}(\hat{x},\hat{\eta},\hat{I})$ [15, 19] over the isofield of isoreal isonumbers R with isometric

$$\hat{\eta} = \hat{T}\eta = Diag.\left(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, -\frac{c^2}{n_4^2}\right)\hat{I}, \tag{38}$$

where the multiplication by $\hat{I}$ is necessary for the elements of the isometric to be isoscalars, with isoinvariant

$$\hat{x}^{\hat{2}} = \hat{x}^{\mu}\,\hat{\times}\,\hat{\eta}_{\mu\nu}\,\hat{\times}\,\hat{x}^{\nu} = (x^{\mu}\hat{\eta}_{\mu\nu}x^{\nu})\hat{I} =$$

$$= \left(\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \times \frac{c^2}{n_4^2}\right)\hat{I}, \tag{39}$$

It is evident that, according to then original conception [15], the isotopies of the Minkowski space represent locally varying speeds of light $C = c/n_4$, with consequent mutation of the conventional light cone, which features have been shown in memoir [38] to be compatible with the abstract axions of special relativity.

However, non-initiated readers should be aware that the isotopies reconstruct the perfect light cone in isospace $\hat{M}$ including c as the maximal causal speed. This is due to the fact that the speed of light is mutated in the value $c^2 \to c^2/n_4^2$, while the corresponding unit is mutated by the inverse amount $1_4 \to n_4^2$, thus preserving the maximal causal speed c in isospace $\hat{M}$ over the isofield $\hat{\mathrm{R}}$.

By linearizing the second order isoinvariant of the Poincaré-Santilli isosymmetry $P(3.1)$ as in the conventional case (see Appendix A), one reaches the fundamental equations of relativistic nuclear isomechanics which is given by the Dirac-Santilli isoequation [18]

$$(\hat{\eta}^{\mu\nu}\hat{\gamma}_{\mu}\hat{p}_{\nu} + \hat{i}\,\hat{\times}\,\hat{m}\hat{\times}\hat{C})\,\hat{\times}\,|\hat{\psi}(\hat{x})\rangle =$$

$$= (-i\hat{I}\hat{\eta}^{\mu\nu}\hat{\gamma}_{\mu}\partial_{\nu} + imC)\,|\hat{\psi}(\hat{x})\rangle = 0. \tag{40}$$

which clearly illustrate the lifting of Plank's constant (35) when compared to the conventional equation, where the *Dirac-Santilli isogamma matrices* have a structure

$$\hat{\gamma}_k = \frac{1}{n_k}\begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \quad \hat{\gamma}_4 = i\frac{1}{n_4}\begin{pmatrix} I_{2\times2} & 0 \\ 0 & -I_{2\times2} \end{pmatrix}, \tag{41}$$

with anti-isocommutation rules

$$\{\hat{\gamma}_{\mu}\,\hat{;}\,\hat{\gamma}_{\nu}\} = \hat{\gamma}_{\mu}\hat{T}\hat{\gamma}_{\nu} + \hat{\gamma}_{\nu}\hat{T}\hat{\gamma}_{\mu} = 2\hat{\eta}_{\mu\nu}, \tag{42}$$

Note in Eq. (38) the replacement of the speed of light c with the isospeed $C = c/n_4$. This is necessary because c is no longer invariant under the Poincaré-Santilli isosymmetry, while C is indeed invariant (Appendix A).

It should be indicated that the above formulation of the Dirac-santilli isoequation is solely based on the isotopies of spacetime without the isotopies of the spin of nucleons, since such an isotopy is sufficient for the derivation of nuclear magnetic moments and spins.

For a general study of the Dirac-Santilli isoequation,

including the mutation of spacetime and spins as well as in regular and irregular realizations, we refer the interested reader to memoir [36].

The following comments are now in order:

3.1. By conception and construction, nuclear isomechanics is solely valid within regions of space of the nuclear radius (the order of 1 fm), because at larger distances the isounit recovers the convectional Planck's constant and, consequently, isomechanics recovers quantum mechanics uniquely and unambiguously (see Figure 4).

3.2. Also by conception and construction, nuclear isomechanics preserves the axioms of quantum mechanics and merely realize them via a broader mathematics. In fact, isomechanics and quantum mechanics coincide at the abstract realization-free level, to such an extent that they be expressed via the same equations only subjected to different realizations.

3.3. The name "hadronic mechanics" was suggested by Santilli [3b] for the representation of "hadrons" at large, thus including the representation of protons and neutrons. Consequently, nuclear isomechanics has been specifically constructed for the study of the nuclear stricture, while its covering genomechanics has been constructed to study nuclear reactions.

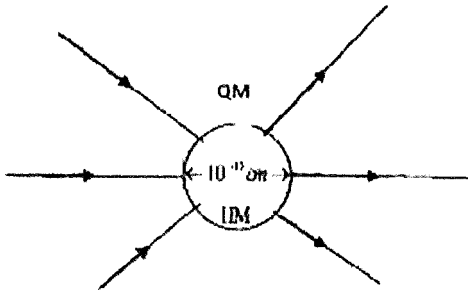

Figure 4. *A central feature of hadronic mechanics verified by all its branches is that the new mechanics is solely valid at distances of the order of 1 fm = 10^{-13} cm because at larger distances it recovers quantum mechanics uniquely and unambiguously since at larger distances the isounit recovers Planck's constant.*

3.4. As it is well known, non-linear interactions (here referred to nonlinearity in the wave-functions) cannot be consistently represented via quantum mechanics since, in this case, they can be solely represented with a Hamiltonian $H(r, p, \psi)$, with the ensuing violation of the superposition and other laws. Consequently, quantum mechanics cannot consistently define nuclear constituents under non-linear terms of the nuclear force. By contrast, nuclear isomechanics can consistently represent non-linear terms in the nuclear force because all non-linear contributions are embedded in the isounit (or the isotopic element), by therefore maintaining the superposition principle on isospace over isofields. Additionally, nuclear isomechanics reconstructs linearity on isospaces over isofields with evident computational advantages.

3.5. The elementary review of this section has been necessarily incomplete to avoid excessive length. Therefore, interested readers are suggested to study memoir [36] for more technical details and monographs [22] for a comprehensive presentation. Particularly important is the acquisition of technical knowledge on properties such as: iso-Hermiticity coincides with conventional Hermiticity, as a result of which all observables of quantum mechanics remain observable for nuclear isomechanics; nuclear isomechanics eliminates the divergencies of quantum mechanics because all products of divergent series are lifted into the form given in Eq. (22) where the absolute value of the isotopic element $\hat{T}_{\hat{r}}$ is very small (see the negative sign of the exponent of Eq. (1); nuclear isomechanics is a "completion" of quantum mechanics according to the Einstein-Podolsky-Rosen argument, thus providing a concrete and explicit realization of "hidden variables" λ via the isotopic element $\hat{T}_{\hat{r}}$; and other important properties [22, 36].

3.6. The replacement of Planck's constant $\hbar$ into the integro-differential operator $\hat{I}$ is a representation of the expectation that, when nucleons are represented as expended charge distributions in conditions of partial mutual penetration, the energy exchange is at least in part continuous. However, the deviations from discrete energy exchanges in nuclear is very small due to the very small absolute value of the isotopic element 2. By contrast, the deviation of quantized energy exchanges for a proton in the core of a star are expected to be finite due to its total immersion with a hyperdense hadronic media for which quantized energy exchanges cannot be even defined.

4. Simple Construction of Isomechanics and Genomechanics

For the benefit of experimental nuclear physicists, it is important to note that any given quantum mechanical nuclear model can be lifted via an elementary procedure into the corresponding isomechanical form, by therefore performing the transition from the point-like abstraction of nucleons, to extended-deformable nucleons under potential as well as contact non-potential interactions.

Isomechanics is a structurally *non-unitary* theory when formulated on a conventional Hilbert space over a conventional numeric field, Eq. (35), while quantum mechanics is unitary. Therefore, the novel isomechanical contributions due to the extended-deformable character of nucleons as well as to the non-potential component of the nuclear force can be represented, from Eq. (21), with a non-unitary transform of the type

$$UU^{\dagger} = \hat{I} = Diag.(n_1, n_2^2, n_3^2, n_4^2) \times e^{\Gamma(r,...)} \int r^{\dagger} r \, dr^3 \qquad (43)$$

It is then easy to see that the application of the above non-unitary transform to the "totality" of the formalist of a quantum nuclear model characterizes its isomechanical formulation in its entirety

$$I \to \hat{I} = U \times I \times U^{\dagger} = 1/\hat{T}, \qquad (44a)$$

$$n \to \hat{n} = U \times n \times U^{\dagger} = n \times U \times U^{\dagger} = \\ = n \times \hat{I} \in \hat{F}, , n \in F, \quad (44b)$$

$$e^{A} \to U \times e^{A} \times U^{\dagger} \simeq \hat{I} \times e^{\hat{T} \times \hat{A}} = (e^{\hat{A} \times \hat{T}}) \times \hat{I}, \quad (44c)$$

$$A \times B \to U \times (A \times B) \times U^{\dagger} = \\ = (U \times A \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times B \times U^{?}) = \hat{A} \hat{\times} \hat{B}, \quad (44d)$$

$$[X_i, X_j] \to U \times [X_i X_j] \times U^{\dagger} = \\ = [\hat{X}_i \hat{,} \hat{X}_j] = U \times (C_{ij}^{k} \times X_k) \times U^{\dagger} = \hat{C}_{ij}^{k} \hat{\times} \hat{X}_k = \\ = C_{ij}^{k} \times \hat{X}_k, \quad (44e)$$

$$\langle \psi | \times | \psi \rangle \to U \times \langle \psi | \times | \psi \rangle \times U^{\dagger} = \\ = \langle \psi | \times U^{\dagger} \times (U \times U^{\dagger})^{-1} \times U \times | \psi \rangle \times (U \times U^{?}) = \\ = \langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle \times \hat{I}, \quad (44f)$$

$$H \times | \psi \rangle \to U \times (H \times | \psi \rangle) = \\ = (U \times H \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times | \psi \rangle) = \\ = \hat{H} \hat{\times} | \hat{\psi} \rangle, etc. \quad (44g)$$

It is easy to see that the application of an additional non-unitary transform causes the lack of time invariance of the isounit

$$W \times W^{\dagger} \neq I, \quad (45a)$$

$$\hat{I} \to \hat{I}' = W \times \hat{I} \times W^{\dagger} \neq \hat{I}, \quad (45b)$$

with consequential lack of invariance of the numeric predictions, with activation of the catastrophic inconsistencies [37], as well as the loss of the represented system.

However, any given nonn-unitary transform can be identically rewritten in the isounitary form on $\hat{\mathrm{H}}$ over $\hat{\mathrm{C}}$

$$W \times W^{\dagger} = \hat{I}, \quad W = \hat{W} \times \hat{T}^{1/2}, \quad (46)$$

$$W \times W^{\dagger} = \hat{W} \hat{\times} \hat{W}^{\dagger} = \hat{W}^{\dagger} \hat{\times} \hat{W} = \hat{I}, \quad (47)$$

under which we have the invariance of the isounit and isoproduct [22, 36, 37]

$$\hat{I} \to \hat{I}' = \hat{W} \hat{\times} \hat{I} \hat{\times} \hat{W}^{\dagger} = \hat{I}, \quad (48a)$$

$$\hat{A} \hat{\times} \hat{B} \to \hat{W} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{W}^{\dagger} = \\ = (\hat{W} \times \hat{T} \times \hat{A} \times \hat{T} \times \hat{W}^{\dagger}) \times (\hat{T} \times \hat{W}^{\dagger})^{-1} \times \hat{T} \times (\hat{W} \times \\ \times \hat{T})^{-1} \times (\hat{W} \times \hat{T} \times \hat{B} \times \hat{T} \times \hat{W}^{?}) = \\ = \hat{A}' \times (\hat{W}^{\dagger} \times \hat{T} \times \hat{W})^{-1} \times \hat{B}' = \hat{A}' \times \hat{T} \times \hat{B}' = \hat{A}' \hat{\times} \hat{B}', etc. \quad (48b)$$

from which the invariance of the entire isotopic formalism follows.

Note that the invariance is ensured by the *numerically invariant values of the isounit and of the isotopic element under isounitary transforms,*

$$\hat{I} \to \hat{I}' \equiv \hat{I}, \quad (49a)$$

$$A \hat{\times} B \to A' \hat{\times}' B' \equiv A' \hat{\times} B', \quad (49b)$$

in a way fully equivalent to the invariance of quantum mechanics, as expected to be necessarily the case due to the preservation of the abstract axioms under isotopies. The resolution of the inconsistencies for non-invariant theories is then consequential.

It should be indicated that the above lifting of quantum into isomechanical models solely apply for the so-called *regular representations of the Lie-Santilli isotheory* (see Appendix A), that can be essentially expressed as representations preserving the conventional value of the spin, thus being sufficient for nuclear constituents.

Howsoever, the reader should be aware of the existence of *irregular representations of the Lie-Santilli isotheory* (see also Appendix A), which can be indicated as realizations of the axioms causing anomalous values of the spin, as expected for a proton when in the core of a star subjected to enormous pressures under which the very definition of conventional spin is technically flawed.

The lifting of a quantum mechanical nuclear model into the covering genomechanical version can be equally done via an elementary procedure, by performing the transition from time-reversible description to an irreversible one when applicable, e.g., for nuclear reactions.

Recall that genomathematics represent irreversibility by embedding the direction of time in the most ultimate quantities, the unit and related product. Therefore, the creation of a time ordering requires *two* different non-unitary transforms

$$UU^{\dagger} \neq I, \ WW^{\dagger} \neq I, \ UW^{\dagger} \neq I, \quad (50)$$

Then Planck's constant can be lifted in the form applicable for motion forward in time

$$\hbar = I \to \hat{I}^{>} = UIW^{?} = 1/\hat{T} \ > 0, \quad (51)$$

with corresponding lifting of all products AB into the ordered genoproduct to the right

$$AB \to A > B = A\hat{T}^{>}B, \quad (52)$$

and lifting of $\hbar$ for motion backward in time

$$\hbar = I \to^{<} \hat{I} = WIU^{?} = 1/ \ \hat{T} > 0, \quad (53)$$

and corresponding lifting of all quantum products into the form ordered to the left

$$AB \to A < B = A^{<}\hat{T}B. \quad (54)$$

The irreversible character of the representation is then assured by the different values of the forward and backward genounits, with consequential incoherence of the related

geno-Hilbert spaces (see memoir [37] for details).

5. Exact and Invariant Representation of Nuclear Magnetic Moments

Following the preparatory advances outlined in the preceding sections [3-37], the representation of Fermi's historical hypothesis on the representation of nuclear magnetic moments via the deformation of the charge distribution of nucleons (Section 1), becomes direct and immediate.

The first exact and time invariant representation of the anomalous magnetic moment of the Deuteron (where the term "anomalous" refers to deviations from quantum predictions) was achieved by R. M. Santilli in 1993 while visiting the JINRT in Dubna, Russia, and was presented at the local *International Symposium Deuteron-1993* [39]. The results were then extended to the representation of the anomalous magnetic moments of all stable nuclides in memoir [36] of 1998.

Let us recall from Refs. [2] that the magnetic moment of nucleons can be expressed in terms of their spin

$$\mu = g^S S + g^L L \tag{55}$$

where the g 's are the spin and orbital gyro-magnetic factors with values in unit of nuclear magnetons for protons and neutrons

$$g_p^S = 5.585\text{nm}, \quad g_n^S = -3.826\text{nm}, \tag{56}$$

$$g_p^L = 1, \quad g_n^L = 0. \tag{57}$$

By assuming that $L = 0$ for the ground state, the quantum mechanical (qm) prediction of the magnetic moment of the Deuteron is given by

$$\mu_D^{qm} = g_p^S S + g_n^S S = 0.879, \tag{58}$$

while the experimental value is given by '

$$\mu_D^{exp} = 0.857, \tag{59}$$

thus implying a deviation of 0.02 nm *in excess* between the prediction of quantum mechanics from experimental values.

It should be stressed that the "small" character of the deviation 0.02 nm may be misleading because it refers to the smallest nucleus, with increasingly embarrassing deviations for heavier nuclei, thus establishing the need for the exact and invariant representation of all nuclear magnetic moments, and not only that for the deuteron (Figure 5).

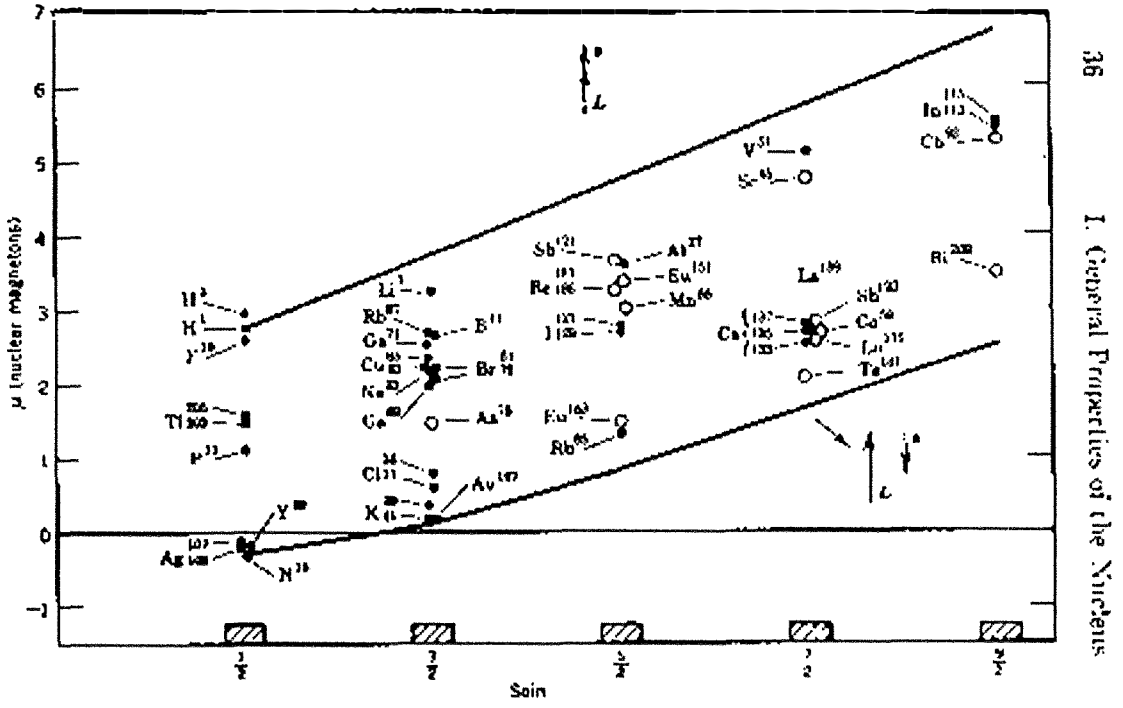

Figure 5. *On rigorous scientific grounds, a theory can be considered as being "exactly valid" for given physical conditions when it represents the entirety of the experimental data from unadulterated first principles. In this figure we reproduce the so-called "Schmidt limits" representing minimal and maximal values of nuclear magnetic moments. In the authors view, the Schmidt limits are a direct representation of the "deviations" of quantum mechanics from nuclear experimental data because they represent the deviation from quantum predictions for the simplest possible nucleus, the Deuteron, with increasingly embarrassing deviations for heavier nuclei. The achievement of an exact and invariant representation of nuclear magnetic moments according to Fermi's teaching (Section 1) has been a main motivation for the construction of the new isomathematics and isomechanics, as shown in Section 5.*

Attempts at the achievement of an exact representation of the anomalous magnetic moment of the Deuteron have been attempted for about one century via the use of quantum mechanics without any result that will resist the test of time.

The first attempts have been done by using an *ad hoc* combination of orbital angular momenta $L \neq 0$ of the proton and the neutron. However, the assumption $L \neq 0$ is in contradiction with the experimental evidence that the isolated Deuteron is in its ground state and, therefore, the orbital angular momenta of its constituents must be $L = 0$.

Numerous additional attempts have been done via relativistic corrections and relativistic field theory, by achieving the needed exact representation of the Deuteron magnetic moment with the introduction of arbitrary parameters or special form factors, thus, without deriving the needed value from first adulterated principles.

Additionally, it should be indicated that the reduction of protons and neutrons to the hypothetical quarks creates additional problems and solves none, because the hypothetical orbits of the hypothetical quarks inside nucleons are too small to admit a hypothetical polarization suitable for the representation of the Deuteron magnetic moment.

In conclusion, in 1993 the exact representation of the magnetic moment of the simplest nucleus, the Deuteron, let alone those of heavier nuclei (see Figure 5) had remained elusive because the proposed representations have contradictions or manipulations that will not resist the test of time.

In this way, Fermi's historical hypothesis acquires its full light when represented via isomathematics and isomechanics. The central conceptual and technical notion of nuclear isomechanics is that *the constituents of nuclei are "isoparticles"* (Ref. [40] and Appendix A), namely, *ordinary particles experience a mutation of their "intrinsic" characteristics when in conditions of partial penetration of their charge distributions (and/or wavepacket) as occurring in the nuclear structure, with ensuing exposure to the strong nuclear force.*[1]

The first intrinsic characteristics of particles experiencing a mutation under nuclear conditions is that of their intrinsic magnetic moments. Its explicit expression can be easily derived from the Dirac-Santilli isoequation (40) by repeating the corresponding procedure for the quantum case, yield the following *mutation of the intrinsic magnetic moment in the transition from particles to isoparticles* (see Ref. [39] as well as, for more details, Ref. [18])

$$\tilde{\mu}^{is} = \frac{n_4}{n_3}\mu, \tag{60}$$

where: (is) stands for isomechanics; we consider the magnetic moment along its symmetry axis, as usual; n_3 is the deformed semiaxis in the third direction; and n_4 a geometrization of the hyperdense medium inside nucleons.

We should note the use the upper symbol $\tilde{\mu}$, rather than $\hat{\mu}$, since the latter indicates elements of isofields because the use of the symbol $\hat{\mu}$ would indicate the transition from a scalar to an isoscalar (Section 2), which is merely given by the multiplication of the conventional magnetic moment and the isounit,

$$\hat{\tilde{\mu}} = \tilde{\mu}\hat{I}, \tag{61}$$

Due to the lack of impact to numerical values, the above isoscalar extension will be ignored hereon for simplicity.

From Eq. (60), we have the following mutation of the quantum mechanical magnetic moment (μ)

$$\tilde{\mu}_D^{is} = \tilde{g}_p^S S + \tilde{g}_n^S S = \frac{n_4}{n_3}\left(g_p^S S_p + g_n^S S\right) \tag{62}$$

where we have assumed for simplicity that the mutations of the charge distributions of the proton and the neutron are the same, since they have essentially the same volume and the same hyperdensity.

The numeric value of n_4 has been the subject of extensive phenomenological and experimental studies via the Bose-Einstein correlation and other particle experiments, resulting in the value (see Refs. [41-44] and Eqs. (6.1.101), page 864, Vol. IV of monographs [23])

$$n_4 = 0.654, \; n_4^2 = 0.355, \tag{63}$$

Consequently, under the numeric value of the third semiaxes

$$n_3 = 0.670, \; n_3^2 = 0.449, \tag{64}$$

we reach the following numerically exact and time invariant representation of the anomalous magnetic moment of the Deuteron according to isomathematics and isomechanics

$$\tilde{\mu}_D^{is} = \tilde{g}_p^S S + \tilde{g}_n^S S = 0.857\text{nm} \tag{65}$$

where we should note the use of slightly different numerical values than those used in the original derivation [39] due to advances occurred since 1993.

As one can see, the proton and the neutron are mutated from perfect spherical shapes when in vacuum under sole action-a-distance electromagnetic interactions, to a *oblate* charge distributions when constituents of the Deuteron, as expected in view of their high rotational conditions.

Note that *the deformability of nucleons under strong interactions does not imply the alteration of their volume due to their hyperdense character*. By assuming that the original semiaxes are normalized to 1, we then have the restriction on the numeric value of the remaining semiaxes

[1] The condition of partial mutual penetration of the charge distributions of protons and neutrons when nuclear constituents, can be easily derived by comparing the experimental values of nuclear volumes with those of protons and neutrons.

$$\frac{1}{n_1^2}+\frac{1}{n_2^2}+\frac{1}{n_3^2}=3, \tag{66}$$

under which we obtain the value of the remaining semiaxes of the oblate spheroid under the evident identification

$$n_1=n_2=1.635,\ n_1^2=n_2^2=2.574 \tag{67}$$

Even though the above values are certainly not suggested to be final, we can state that isomathematics and isomechanics provides the first known numerical values of the semiaxes of the proton and the neutron when constituents of the Deuteron, in a way compatible with the numerically exact and time invariant representation of the anomalous magnetic moment of the Deuteron (Figure 6).

As indicated earlier, the extension of the above representation for the Deuteron was extended to all stable nuclei in memoir [36] via the general *isomechanics representation of nuclear magnetic moments* [36].

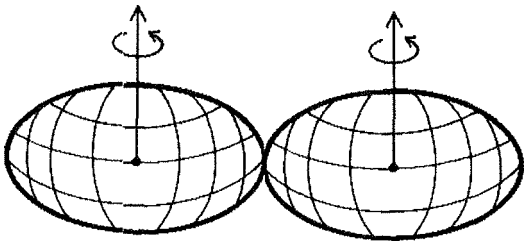

Figure 6. A conceptual rendering of the oblate shape of the proton and the neutron when constituents of the Deuteron in its ground state achieved, for the first time to our knowledge, by nuclear isomechanics from the anomalous magnetic moment of the Deuteron, with values of the semiaxes $n_3^2=0.449$, $n_1^2=n_2^2=2.574$ *(see Section 5). It should be stressed that the construction has been done along the conventional conception of the Deuteron, namely, with the proton and the neutron with parallel spins in order to represent the spin 1 of the Deuteron. However, quantum mechanical axioms predicts that the sole stable bound state between two particles with spin 1/2 is the singlet with antiparallel spins, since couplings with parallel spins are predicted to be highly unstable due to strong "repulsive" forces. Therefore, in Section 8 we shall first review the structure of the Deuteron according to nuclear isomechanics with a representation of the spin 1 without structural inconsistencies, and then provide a more accurate representation of the magnetic moment of the Deuteron.*

$$\tilde{\mu}_N^{is}=\sum_{k=1,...,Z}\left[\frac{n_4}{n_{3kp}}\left(g_{pk}^S S+g_{pk}^L L\right)\right]+\sum_{k=1,...,A-Z}\left[\frac{n_4}{n_{3kn}}\left(g_{nk}^S S+g_{nk}^L L\right)\right], \tag{68}$$

where we have assumed that: all nucleons as nuclear constituents nuclei have the same hyper density geometrized by n_4; the deformation of the charge distributions may vary with the increase of the constituents; and anomalous orbital contributions may eventually emerge for heavier nuclei.

The verification that Eq. (68) provides indeed a representation of the magnetic moment of all stable nuclei will be shown in a subsequent paper. At this moment we merely limit ourselves with the representation later on of the magnetic moment light stable nuclei. The following comments are now in order:

5.1. Representation (65) is invariant over time because the mutation of intrinsic magnetic moments, Eq. (61) is a consequence of the Dirac-Santilli isoequation (which is invariant under the Poincaré-Santilli isosymmetry (Refs. [15-18] and Appendix A).

5.2. As one can see, representation (65) does not require the mutation of the spin of nuclear constituents because the sole mutation of the Minkowski spacetime into isospace (39) underlying the Dirac-Santilli isoequaiton (40) has been sufficient. This does not exclude extreme physical conditions, such as those at the core of stars that may require the mutation also of the spin.

5.3. As clearly shown by Eq. (62), the mutation of the intrinsic magnetic moment of nucleons under their conventional value of the spin creates the problem of the intrinsic compatibility of the approach. This problem is solved in Appendix A, where we show that the degree of freedom of regular isotopies of the SU(2) spin identified in Refs. [13, 14] can represent indeed the mutation of spin, thus achieving full compatibility under isomathematics and isomechanics between mutation of intrinsic magnetic moments and conventional values of spins.

5.4. We should indicate that value (63) for the geometrization of the hyperdense medium inside nucleons has been derived via experimental data on different events, such as the fireball of proton-Antiproton annihilation in the Bose-Einstein correlation, while direct experimental data for nucleons are not available at this writing. Therefore, it is possible that value (63) and, consequently, value (64), may need revisions following direct test on the density of the medium inside nucleons.

6. Test of the Spinorial Symmetry Via Neutron Interferometry

It is evident that the deformability of protons and neutrons under sufficient external forces requires a direct experimental verification. The ideally suited test is the so-called 4π neutron interferometric experiment which consists of (see Figure 7): a thermal neutron beam which is first coherently split into two beams by a perfect crystal; one of the two split beams passes through the gap of an electromagnet with the magnetic field calibrated to such the value 7,496 G causing two complete spin flips (720° from which the name 4π) of the neutron on account of its intrinsic magnetic moment $-1.913148\pm0.000066\mu_N$. The two beams are then coherently recombined. Various analysis are then conducted between the original beam and the recombined one.

When electromagnet gap is empty and, therefore, the split neutron beam travels in empty space, all known tests confirm

the achievement of two complete spin flips in full agreement with quantum mechanics. However, in order to avoid stray fields, the electromagnet gap is filled up with Mu-metal or other heavy metal sheets. In the latter cased, the test essentially provides a *test of the spinorial symmetry of neutrons under the intense electric and magnetic fields in the vicinity of Mu-metal nuclei, without any appreciable contributions from the strong interactions of Mu- metal nuclei.*

The rather bizarre history of this fundamental test can be summarized as follows. The Austrian neutron interferometric experimentalist H. Rauch and his Austrian associate A. Zeilinger participated to the 1981 *Third Workshop on Lie-Admissible Formulations*, and presented preliminary results of a 4π neutron interferometric experiment that was going on via thermal neutron beams available at the nuclear facilities in Grenoble, France.

In particular, Rauch and Zeilinger reported at said 1981 meeting that they were *not* measuring 720° rotations, by rather the following values of minimal and maximal rotations [45]

$$\theta_{min} = 715.87°,\ \theta_{max} = 719.67°,\ \theta_{aver} = 712.07° \qquad (69)$$

which evidently *do not* contain 720°. In particular, Rauch and Zeilinger reported an angle of rotation systematically *smaller* then that expected, a feature referred to as the *angle slow-down effect* and expected to be due to a *decrease* of the intrinsic magnetic moment of the neutron under the strong fields of the Mu-metal nuclei.

The Austrian theoretical physicist G. Eder [46] who also attended the indicated 1981 workshop by presenting a Lie-admissible mutation of the rotational symmetry representing the decrease of the intrinsic magnetic moment of the neutron under the considered conditions in agreement with data (69).

Based on these studies, Santilli [47] presented at the same 1981 workshop the notion of *Lie-admissible mutation of elementary particles* (also called *genoparticles* under strong nuclear interactions considered as external (which is a condition to sue the irreversible Lie-admissible formulations). It should be noted that the Lie-isotopic notion of isoparticle presented in this paper is an evident particular case of the notion of genoparticles presented in 1981.

Immediately following the announcement of the above studies, H. Feshback, then chairman of the Department of Physics at MIT, strenuously opposed the completion of the 4π neutron interferometric experiment by Rauch and Zeilinger. the opposition, first by Feshback and then by his world wide collective was such that Rauch was prohibited the access at his own laboratory in Grenoble and was, therefore, prohibited its completion (see Refs. [48] and their three volumes of documentations).

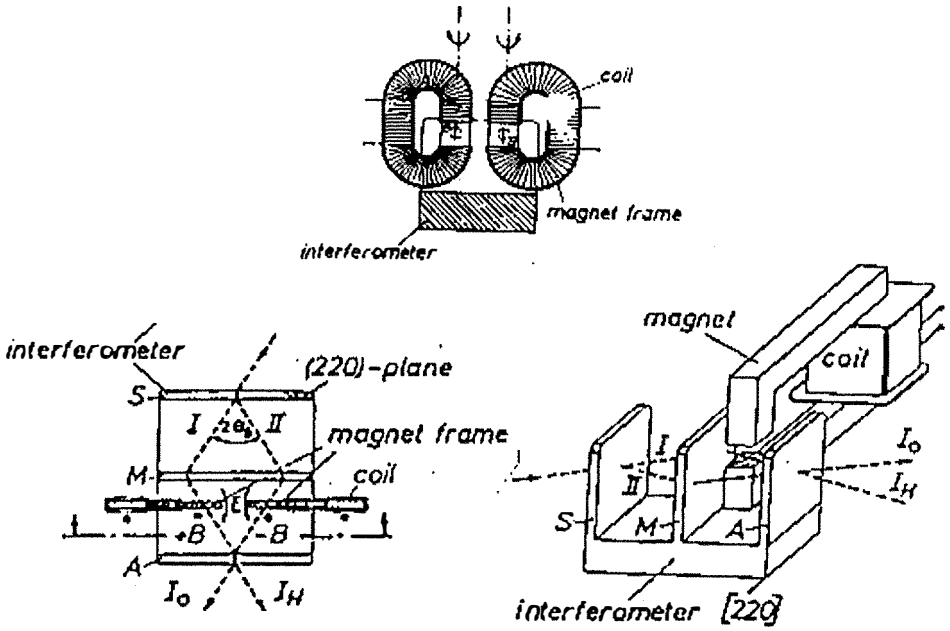

Figure 7. *A schematic view of one of the most fundamental experiments for the past half a century, the 4π neutron interferometric spinorial symmetry experiments described in Section 6 [45-51]. The lack of resolution of this experiments due to political obstructions [48] is one of the reasons fueling the growing view according to which we are currently eyewitnessing one of the biggest scientific obscurantism in the history of mankind.*

Subsequently, Rauch was offered the position of Director of the Atominstitut in Wien, Austria, while Zeilinger was invited for a one year stay at MIT after which he received a chair in physics at an Austrian university.

Following the above events in the early 1980s, the 4π neutron interferometric experiment was occasionally repeated, but either without heavy metal sheets in the electromagnet gap, by splitting the gap into two opposite contributions or in other versions essentially assuring the verification of the exact spinorial symmetry.

To our best knowledge, the current situation (October 2015) is the following. On one side, Rauch and Zeilinger have dismissed measurements (69) and claim the exact validity of the 4π symmetry (without any systematic experimental resolution on record), as reported by D. Kendellen [49] (see also book [50]).

By contrast, Santilli [51] claims that; 1) an accurate and unbiassed comparative analysis of the original and the recombined neutron beams show clear deviations from 4π rotations of at least 1%, even though the neutrons are solely exposed to *electromagnetic* interactions, thus expecting bigger deviations under nuclear strong interactions; 2) the deformability of the neutron is such a fundamental physical problem to require a systematic repetition of the 4π tests; and 3) Nowadays, the experiment can be repeated for a multiple of two complete rotations, with ensuing resolutory results (see Ref. [51] for details).

In the authors opinion, a reason for the incredible c hostility by the nuclear physics community against this fundamental experiment is the lack of technical knowledge of the Lie-Santilli isotheory according to which their fear of the violation of the "spinorial" symmetry in the 4π tests has no technical foundations because the experiment here considered deals with the *deformation of the charge distribution of the neutron while fully preserving its spin* $1/2$. In fact, the authors believe that the very name "spinorial" symmetry experiment is erroneous and misleading, since the Fermi-Dirac character of the neutrons remains fully valid under a deformation of their charge distribution (Appendix A).

In the final analysis, the serious scientist should keep in mind that perfectly rigid bodies solely exist in academic environments but they do not exist in nature. Therefore, the serious scientific issue is the measurement of the deformation of the charge distribution of neutrons for given sufficiently strong external forces, with the understanding that the deformability itself should be outside credible doubts.

7. The Synthesis of the Neutron from the Hydrogen

As it is well known, stars initiate their life as an aggregate of Hydrogen. The first nuclear synthesis in the core of a star is that of the neutron from the Hydrogen atom according to the historical reaction [2]

$$p^+ + e^- \to n + \nu \tag{70}$$

Deuterium, Tritium and other nuclei are synthesized only following the synthesis of the neutron. It is then evident that the understanding of the first and most basic synthesis of the neutron is crucial for a deeper understanding of the subsequent nuclear syntheses.

Unfortunately, the synthesis of the neutron is vastly ignored even at the most important Ph. D. courses in nuclear physics because it is *incompatible with quantum mechanics and special relativity.* This is due to the fact that *the rest energy of the neutron is bigger than the sum of the rest energies of the proton and of the electron,* as established by the known data

$$E_p = 938.272\,\text{MeV},\ E_e = 0.511\,\text{MeV},\ E_n = 939.565\,\text{MeV}, \tag{71a}$$

$$E_n - (E_p + E_e) = 0.782\,\text{MeV} > 0, \tag{71b}$$

Under these conditions, the Schrödinger equation does not yield physically consistent results due to the need for a "positive binding energy" resulting in a "mass excess" that are beyond any descriptive capacity of non-relativistic quantum mechanics.

Synthesis (70) is also incompatible with special relativity and relativistic quantum mechanics because the conventional Dirac equation, which is so effective for the description of the electron orbiting around the proton in the Hydrogen atom, becomes completely ineffective for the description of the same electron when "compressed" inside the proton in the core of a star according to Rutherford.

The proposal to build a non-unitary covering of quantum mechanics under the name of *hadronic mechanics,* including its isotopic and genotopic branches, was submitted in monograph [3b] precisely for the achievement of a quantitative representation of the synthesis of the neutron from the Hydrogen, and then apply the results to other nuclear syntheses.

Following decades of preparatory research [3-51], a numerically exact and time invariant representation of *all* characteristics of the neutron in its synthesis form the Hydrogen atom was achieved at the non-relativistic level via the Schrödinger-Santilli isoequation (31) in Refs. [52-54], and at the relativistic level via the Dirac-Santilli isoequations (40) in Refs. [18, 54].

The first laboratory synthesis of the neutron from a Hydrogen gas was done by the Italian priest-physoicist Don Carlo Borghi and his associates in the mid 1960s [55]. Santilli conducted comprehensive tests for the laboratory synthesis of the neutron from the Hydrogen reported in Refs. [56-60]. The above body of scientific knowledge is now used by the U. S. publicly traded company *Thunder Energies Corporation* for the industrial production of a *Thermal Neutron Source* (see the, e.g., Ref.[61] video [62]. Excellent reviews of the mathematical, theoretical and experimental aspects for the synthesis of the neutron from the Hydrogen are available in Refs.[63, 64].

The following comments are in order:

7.1. *Refs. [52-64] imply that the proton and the electron are actual physical constituents of the neutron, although in their mutated form known as "isoproton" and "isoelectron"* [40] (see Appendix A). In fact, one of the necessary condition to achieve a numerical representation of all characteristics of the neutron in its synthesis from the Hydrogen is that the electron rest energy is mutated according to a mechanism today known as *isorenormalization.*

It should be indicated that these results turn the conjecture of undetectable and unconfinable "point-like" quarks to a mathematical abstraction of the structure of hadrons because the proton and the electron are the only massive permanently

stable particles detected to date. As such, they cannot "disappear" (sic) at the time of the neutron synthesis to be replaced by the hypothetical quarks. Additionally, at the time of the neutron decay, quarks cannot "disappear" (sic) while the emitted proton and electron "reappear"(sic).

The name "hadronic mechanics" was suggested in Ref. [3b] precisely to permit a basically new structure model of all unstable particles with actual physical constituents, generally given by massive physical particles produced in their decay with the lowest mode. Advances along these lines have been reported in memoir [43].

It should be stressed that this new structure model of hadrons *is not* in conflict with the standard model of elementary particles because quarks remain necessary for its elaboration, although in their true scientific meaning of being purely mathematical representations of a purely mathematical internal symmetry formulated in a purely mathematical complex-valued unitary space.

We merely return to the teaching of all classifications that have historically required *two* different but compatible models, one model for the *classification* into families, and a different model for the *structure* of each element of a given family. The same historical teaching is confirmed by the fact that, in the transition from the classification to the structure of atoms there was the need for a new mathematical and physical theories. Similarly, in the transition from the classification of hadrons to their structure there is also the need for new mathematics and physical theories for the reasons indicated in Sections 1 - 3.

As a final comment, the serious scholar should be made aware of potentially large environmental and societal implications in abandoning the conjecture of the hypothetical and unconfinable quarks as actual physical constituents of hadrons in favor of physical particles in their isotopic form. For instance, the admission of the isoelectron as a physical constituent of the neutron allows the conception and experimental study of a number of basically new clean nuclear energies, originally proposed in Refs. [65] and currently under study at *Thunder Energies Corporation* as well as at other companies. By contract, the admission of the hypothetical quarks as the physical constituents of hadrons prohibits such possible environmentally large advances.

7.2. *Refs. [52-64] imply that the neutrino does not appear to exist as physical particles, thus creating the intriguing problem of seeking alternative conceptions.*

In his studies of synthesis (70), Enrico Fermi [1] had no other choice than that of representing the proton as a dimensionless point, resulting in the consequentially *necessary* hypothesis of the "neutrino" (meaning "little neutron" in Italian).

Thanks to the availability of the novel isomathematics (Section 2), in Refs. [52-64] we were able to represent the proton in its actual shape and dimension. This permitted the discovery of a new angular motion and related magnetic moment for the constrained rotation of the isoelectron when compressed in the hyperdense medium inside the proton (Figure 8), which new angular momentum is completely absent when the proton is abstracted as a dimensionless particle.

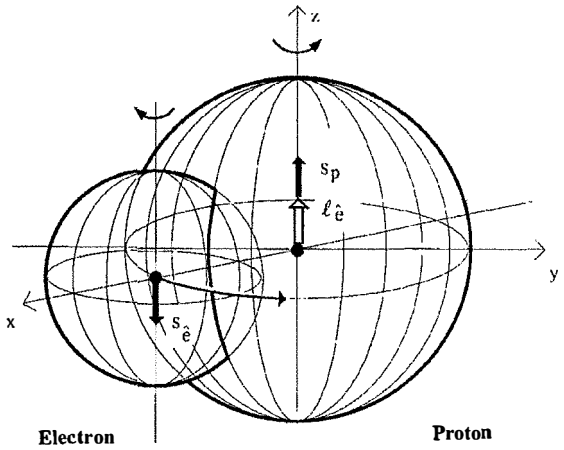

Figure 8. *A basic novelty in Santilli's synthesis of the neutron from the Hydrogen atom is the appearance of a constrained angular motion of the electron when totally immersed within the hyperdense proton. This orbital motion eliminates the need for the emission of the hypothetical neutrino; is solely permitted by the representation of the proton as extended according to hadronic mechanics; and did not exist during Fermi's time since quantum mechanics can solely represents the proton as a massive point [52-63].*

In turn, the constrained orbital motion of the isoelectron inside the proton must be equal to the proton spin (evidently to prevent that the extended wave-packet of the isoelectron moves within and against the hyperdense medium inside the problem), resulting in a null total angular momentum of the isoelectron in synthesis (70) as a result of which the spin of the neutron coincides with the spin of the proton.[2]

The conclusion is that *studies* [52-64] *eliminate any possibility for the production of a neutrino in synthesis* (70). In fact, the emission of a neutrino would *violate,* rather than verify, the conservation of the total angular momentum since the spin $1/2$ of the neutrino is represented by the constrained orbital angular moment of the isoelectron inside the proton. Additionally, reaction (70) already misses 0.782 MeV for the synthesis of the neutron. Any need for the additional energy to produce the hypothetical neutrino would cause catastrophic inconsistencies.

In a nutshell, Enrico Fermi did salvage the conservation of the angular momentum in the synthesis of the neutron with the hypothesis of the neutrino, but he did not salvage quantum mechanics and special relativity in the same synthesis.

7.3. *Refs.* [52-64] *have the intriguing implications of implying the apparent return to the "continuous creation" in the universe as the most plausible way at this moment to explain the missing* 0.782 MeV *for the synthesis of the*

[2] It should be recalled that half-odd-integer angular momenta are prohibited in quantum mechanics because they violate the unitarity of the theory, but they are fully allowed for the covering isomechanics precisely in view of its non-unitary structure (see Refs. [18, 22, 52-54] and Appendix A).

neutron from the Hydrogen atom.

One of the biggest mysteries in the synthesis of the neutron from the Hydrogen is the origin of the missing 0.782 MeV (assuming that the neutrino does not exist, otherwise the missing energy would be much bigger). This energy cannot be provided by the relative kinetic energy between the proton and the electron because at that energy value their cross section is virtually null, thus prohibiting any synthesis.

Additionally, the missing energy of 0.782 MeV cannot be provided by the star because, at the initiation of nuclear syntheses, stars synthesize up to 10^{50} neutrons per second. The assumption that the missing energy is provided by the star would then imply that the star *loses* about 10^{50} MeV per second, under which conditions a star would never initiate the majestic event of producing light.

In an attempt to initiate the solution of this mystery, Santilli has suggested that the missing energy of 0.782 MeV is provided by space conceived as a universal substratum with a very high energy density. via a "longitudinal impulse" (rather than a particle) submitted under the name of "etherino" with the symbol "a" (from the Latin aether), thus implying the replacement of the quantum mechanical reaction (70) with the isomechanical reaction [66]

$$p^+ + a + e^- \to n, \tag{72}$$

where one should note the need for the energy carrying impulse to be in the left (rather than the right) of the reaction, and that the use in the left of the antineutrino would increase the missing energy due to its negative energy state [*loc. cit.*].

It should be noted that the historical hypothesis of the neutrino was essentially dismissed by the lack of detection of the "solar neutrinos" (namely, neutrinos emitted by the Sun during its synthesis of the neutron), according to which our particle laboratories should be traversed by an extremely large flux of neutrinos none of which has been detected with such evidence to be acceptable by the scientific community at large.

The advent of the standard model has produced additional reasons for the dismissal of neutrinos since the standard model requires a variety of different neutrinos without clear physical differences, all neutrinos being assumed to have a mass. It is now widely accepted that particles with mass simply cannot traverse nuclei, planets and stars with a very small of no scattering, thus mandating a basically new interpretation of physical reality.

The hypothesis of the etherino has been submitted because of a possible resolution of these insufficiencies via a more realistic interrelation of experimental data. In fact, the traversing to nuclei, planets and stars without appreciable scattering is more plausibly interpreted by the etherino rather than by the neutrino, since the former refers to a longitudinal impulse propagating through the universal substratum, while the latter is assumed to be a massive particle that should traverse without appreciable scattering hyperdense media inside nuclei, planets and stars.

We should also clarify that a number of claimed "experimental verifications" of the neutrino do not refer to the direct detection of the neutrino which is impossible, but refer to the detection of ordinary particles predicted as being emitted under the neutrino hypothesis. The point is that the emission of exactly the same particles is predicted by the etherino and perhaps other hypotheses. Finally, we should indicate that the claimed "experimental verifications" of the neutrino hypothesis are based on very few events out of billions of events, thus lacking the credibility needed to resist the test of time.

In summary, the lack of existence of the neutrino as a physical particle emitted in the synthesis of the neutron creates one of the most fascinating scientific problems in history, that of the possible continuous creation in the universe (see, e.g., the historical paper [67]), since the missing energy for the neutron synthesis is "created" in the core of stars in the sense that it is acquired from the universal substratum. In turn such a fascinating problem has implications for virtually all quantitative sciences, including lack of expansion of the universe due to loss of energy by galactic light to the intergalactic medium [68], possible future interstellar travel at arbitrary speeds whose energy source would be permitted by a universal substratum with very high energy density [38], and other intriguing open problems.

8. Three-Body Structure of the Deuteron According to IsoMechanics

There comes a moment in the life of a serious scientist at which physical realities have to be admitted, no matter how against preferred doctrines, as a condition not to exit from the boundaries of science.

The physical reality here referred to is that *despite more than half a century of attempts, quantum mechanics has failed to achieve a constant representation of the structure of the simplest nucleus, the Deuteron, with embarrassing deviations for heavier nuclei,* in view of the following insufficiencies [69]:

8.1. *Quantum mechanics has been unable to represent the stability of the Deuteron.* As it is well known, the neutron is naturally unstable when isolated. Therefore, quantum mechanics has failed to explain how the neutron becomes permanently stable when bonded to the proton in the structure of the Deuteron.

8.2. *Quantum mechanics has been unable to achieve a consistent representation of the spin* 1 *of the ground state of the Deuteron.* The basic axioms of quantum mechanics require that the stable bound state of one proton and one neutron is the singlet with total spin zero, while the spin of the Deuteron is 1 . For the intent of maintaining quantum mechanics, 20^{th} century nuclear physics has assumed a combination of orbital states requiring excited conditions which are in direct contradiction with the physical evidence that the spin 1 occurs for the Deuteron in its "ground" state.

8.3. *Quantum mechanics has been unable to identify the physical origin of the attractive force binding the proton and the neutron in the Deuteron.* Since the neutron is neutral, there

is no known electrostatic origin of the attractive force needed for the existence of the Deuteron, while their magnetostatic force is "repulsive" in their triplet coupling. As a result of these occurrences, a "strong" force was conjectured for the bond of nuclear constituents [2] and its existence was subsequently confirmed. Nevertheless, the physical origin of the strong nuclear force has remained unidentified by quantum mechanics to this writing.

8.4. *Quantum mechanics has been unable to achieve a consistent representation of the Deuteron space parity.* According to experimental evidence, of the space parity is positive for the deuteron in its ground state because the angular momentum is null, while the quantum mechanical representation of the spin 1 of the Deuteron requires excited orbital states, resulting in an additional direct conflict between quantum predictions and experimental realities.

8.5. *Quantum mechanics has been unable to reach an exact representation of the magnetic moment of the Deuteron,* as discussed in Section 5.

Following the achievement of the non-relativistic and relativistic presentation of the structure of the neutron as a bound state of one isoproton and one isoelectron (Refs. [51-54] and Section 7), Santilli proposed in Part V of monograph [69] the *structure of the deuteron according to isomechanics as a three body bound state of two isoprotons in triplet coupling and one isoelectrons withy null total angular momentum which is exchanged in between the two isoprotons as a kind of isogluon, hereon referred to as the "iso-Deuteron"* (see Figure 9).

The new three-body structure model of the Deuteron achieves a numerically exact and time invariant representation of *all* characteristics of the Deuteron, including its binding energy, charge radius, stability, spin, parity, etc., which representation is here assumed as known for brevity from Ref. [69] (see also the excellent reviews [33, 70]).

The conceptual and, therefore, the most important reasons for the proposal of the iso-Deuteron were several [69]. The first origination is that *the reduction of the Deuteron to protons and electrons (although in a mutated form) sets clear foundations for stability* since the proton and the electron are the only stable massive particle known to mankind.

The second origination of the iso-Deuteron is that *the spin 1 of the Deuteron is direct evidence that it is a "three-body," rather than a two-body state,* because the configuration of two nucleons in triplet coupling, which is necessary for the representation of the spin 1 in the ground state, can only be achieved in a consistent way via the addition of a third particle with null total angular momenta as in Figure 9.

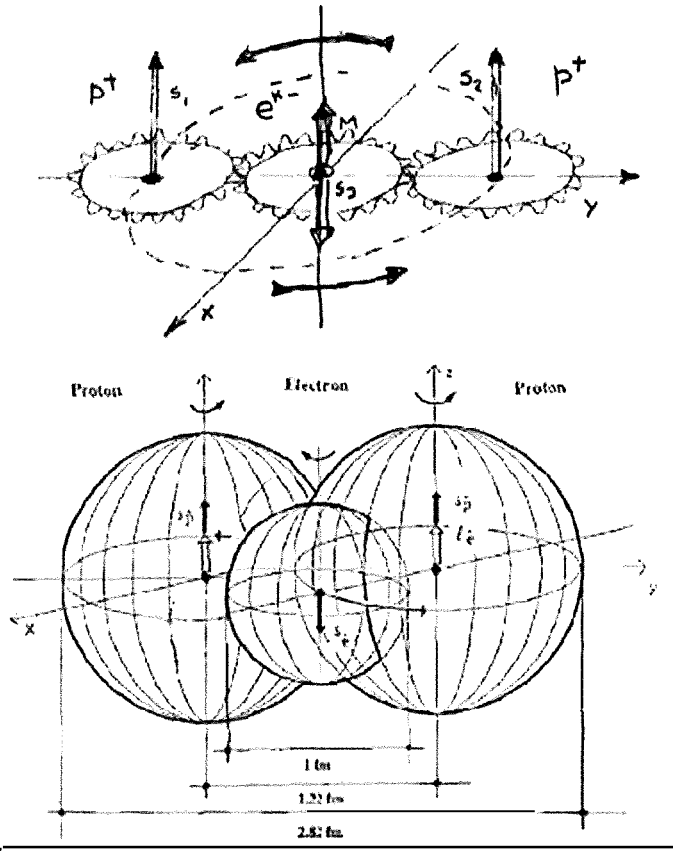

Figure 9. *A schematic view from Part V of Ref. [69] on the structure of the Deuteron following the reduction of the neutron to a hadronic bound state of an isoproton and an isoelectron. Note from the top view that the two isoprotons are in triplet coupling, while the isoelectron with null total angular momentum is exchanged between them, thus allowing the first known representation of the spin 1 of the Deuteron in its true ground state.*

The third origination of the iso-Deuteron is that *isomathematics and isomechanics are the only known methods achieving an explicit and concrete strongly "attractive" force in the Deuteron structure.* In the transition from quantum mechanical to isomechanical nuclear models via the non-unitary transform of Section 4, and realization of the isotopic element of type (1), there is the emergence of a strongly attractive Hulthén potential (see Ref. [69] for details) originating from the partial mutual penetration of the deformed charge distributions of the constituents. Note that in the structure of the iso-Deuteron there is *no* repulsive electrostatic force due to the continuous exchange of the isoelectron between the two isoprotons.

Particularly significant for this paper is the deeper representation of the anomalous magnetic moment of the Deuteron which is permitted by its three-body isotopic structure. In Section 5, we presented a first representation of the magnetic moment of the Deuteron based on its representation as a bound state of an isoproton and an isoneutron in triplet coupling to represent the spin 1 (see Figure 9).

However, as also indicated in Section 5, this representation is basically insufficient because the triplet coupling of Figure 3 generates strongly *repulsive* forces under which no stable bound state is possible. Santilli's three-body model of the iso-Deuteron allows an exact and time invariant representation of the magnetic moment without any known inconsistencies, which is essentially given by the muted magnetic moments of the two isoproton, plus a contributions from the isoelectron (see Ref. [69] for details).

This section concludes the review of past advances in nuclear physics permitted by isomathematics and isomechanics that are necessary for an understanding of the numerically exact and time invariant representation of the spin of stable nuclides presented in the following sections.

9. Stable and Unstable Nuclides

Notice that deuteron is the simplest neuclide having one proton and one neutron and is stable. However, we see that it, in fact, is an *isonuclide.* When we survey the elements of the periodic table we find that out of 289 primordial nuclides 254 are stable ones. The stability of nuclides depends also on evenness or oddness of its atomic number Z, neutron number N and, consequently, of their sum, the mass number A. Oddness of both Z and N tends to lower the nuclear binding energy, making odd nuclei, generally, less stable. This fact we have depicted [71] in Table 1.

However, in this paper, we are presenting, apparently for the first time, a structure model of stable nuclides of the first three rows of the periodic table, hereon called *stable isonuclides,* as bound states of extended, thus deformable isoprotons and isoelectrons according to the laws of hadronic mechanics, under the condition of recovering in first approximation the conventional structure model of nuclides as quantum mechanical bound states of point-like protons and neutrons.

We shall then show, also apparently for the first time, that the reduction of nuclides to isoprotons and isoelectrons allows the first known achievement of an exact representation of the spin of all stable nuclides.

Table 1. *Even and odd nucleon numbers. A is the atomic mass number, Z is the atomic number, N is the number of neutrons in the nucleus, EE is the even-even proton-neutron combination, OO is the odd-odd proton-neutron combination, EO is the even-odd proton-neutron combination and OE is the odd-even proton-neutron combination.*

A	Even		Odd		Total
Z,N	EE	OO	EO	OE	
Stable	148	5	53	48	254
	153		101		
Long-lived	22	4	4	5	35
	26		9		
All primordial	170	9	57	53	289
	179		110		

Next we will indicate without treatment that the reduction of nuclides to isoprotons and isoelectrons puts the foundations for an exact representation of the magnetic moment of all nuclides for studies to be presented in a subsequent paper. We shall also indicate, for studies in a subsequent paper, that the transition of the nuclear structure from that in terms of point-like protons and neutrons to that in terms of is extended, thus deformable isoprotons and isoelectrons offiers realistic possibilities for studying basically new forms of clear nuclear energies.

10. Old and New Vistas in Nuclear Forces

For the semi-quantitative discussion conventionally one uses the following expression of nuclear binding energy, namely:

$$\frac{BE}{\text{MeV}} = 931.4\left(Z\times m_{\text{H}} + (A-Z)\times m_n - M\right) \qquad (73)$$

where m_H and m_n are the masses on amu scale of hydrogen and neutron respectively and M is atomic mass on amu scale of the given element. Notice that the mass of electrons has not been included separately in the above expression because it remains included in m_H. The standard plot of binding energies of all nuclides is shown in Figure 10.

Glasstone [72] further asserts that the nuclear binding energy is the result of ($n-n$), ($n-p^+$) and (p^+-p^+) forces operating within the nucleus. The experimental data on the nuclear scattering and correspondence of binding energies of the identically same mass number elements (isobars) it was concluded that the magnitudes of ($n-n$), ($n-p^+$) and (p^+-p^+) forces of attraction are almost equal [72].

In view of the above assertion it was expected that the diproton and the dineutron nuclei should be stable as deuteron is a stable nucleus (which consists of one proton and one neutron). But so far neither of the former two particles have

been detected as stable particles. However, in proton-proton chain reaction it is suspected that a di-proton is formed in the first step which immediately disintegrates into two protons ($>99.99\%$) and to deuteron plus β^+ ($<0.01\%$) (however the measurement of corresponding half-lives could not succeed) [73]. Of course, it is certain that there is no nucleus made up only of two neutrons because it doesn't constitute a chemical element.

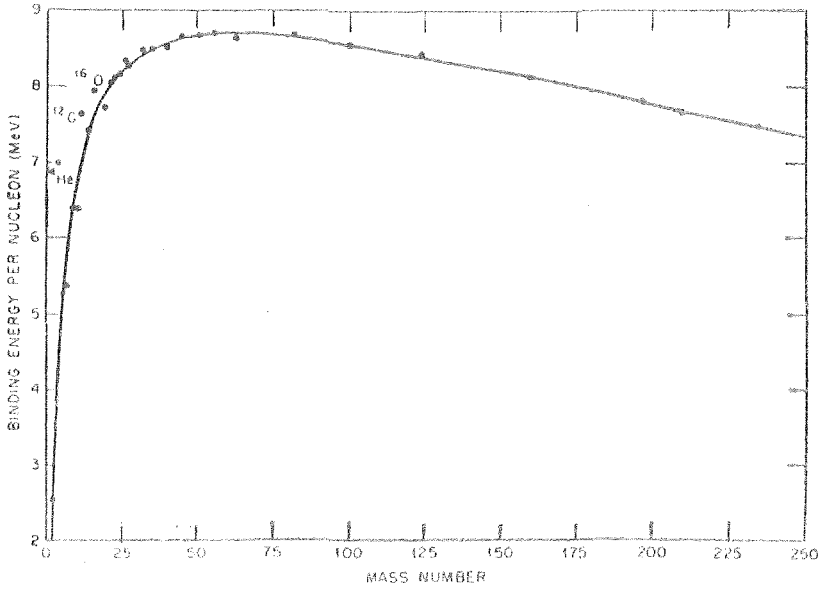

Figure 10. *Binding Energy per nucleon as a function of mass number of stable nuclides.*

On the other hand, the existence of a strong attraction between the pair ($n-p^+$) is exemplified by the stability of deuteron and the stable nuclides He-4, Li-6, B-10, C-12, N-14, O-16, Ne-20, Mg-24, Si-28, S-32, Ar-36 and Ca-40, they all have equal number of protons and neutrons. Besides these nuclides in other stable nuclides we do have neutrons and none of the neutrons disintegrates. The stabilization of neutrons in a nucleus is a subject matter of nuclear physics and the 20th century attempts to explain the said stability are based on quantum mechanics but a satisfactory quantum mechanical description still eludes [18, 22, 23, 33, 36].

The reader may very well notice that the structures of neutron and deuteron that Santilli had proposed, which we have described in brief in Sections 7 and 8 respectively, in fact, are in the form of isoneutron and isodeuteron respectively. These structures involve the mutual deep but partial penetration of the wave packets of electron and proton(s) (c.f. Figures 8 and 9). Thus the quantum mechanical perception of these particles as the point particles has been replaced by the respective tiny but finite size particles all well within the hadronic horizon. However, when we go beyond deuteron the size of the nucleus grows but less significantly (the standard cube root formula [74] estimates the nuclear radius of 1.25 fm for hydrogen nucleus to 4.275 fm for calcium-40 nucleus. Thus the nucleons have increased 40 times but the nuclear radius has increased only 3.4 times).

Therefore, on the lines of the structure of a neutron and a deuteron proposed by Santilli we hereby, apparently for the first time, propose that,

1. an atomic nucleus is composed of nucleons as particles of tiny volume of hadronic dimensions,
2. the wave packets of nucleons penetrate mutually but partially that produces strong nuclear force and
3. the said mutual penetration of wave packets between heteronucleons perhaps produces very strong attractive force compared to that between homonucleons.

This is what has been indicated in Section 1 and shown in Figure 1, that is — the nucleons within a nucleus are in a state of mutual but partial penetration of their wave packets. Thus all nucleons in a nucleus, in fact, are the isonucleons, namely isoneutrons, isodeuterons, isoelectrons and isoprotons.

Of course, one needs to investigate and evaluate quantitatively the magnitude of nuclear forces so generated via the methods of hadronic mechanics but at this juncture we consider that it would be profitable first to generate nuclear configuration of stable nuclides as if the nucleus of all stable nuclides are composed of isonucleons, which is likely to present enough ground for carrying out the detailed investigation of the corresponding quantitative haronic physics. Indeed, we have presented in Section 3 a brief description of Santilli's initial work on nuclear isomechanics and genomechanics.

In the following Section 11 we will see that there are two options for developing nuclear configuration. The first one, the model-I, is through the isodeuterons, isoneutrons and isoprotons as the building nucleons and the second option, the model-II, is through the isoprotons and isoelectrons as the building nucleons, both of them are easily interconvertible. We will also discuss the advantages and limitations of each.

11. Notations for Representation of IsoNeutronand IsoDeuteron

In order to develop the nuclear configuration of nuclides the first logical option is offered by the fact that the deuteron is a stable nuclide similar to a proton. In Section 8 we have described that the deuteron is a hadronic bound state of an isoneutron and an isoproton. But as described in Section 7 the neutron is indeed an isoneutron, which is a hadronic bound state of one isoproton and one isoelectron. However, the isoneutron is an unstable nuclide, which decays radiatively by β^- emission with half-life of 614.6 s [75] (In 1967 experiment the half-life of free neutron was recorded as 10.8 min [76]). But when it makes a union with an isoproton its instability vanishes altogether. Hence in this hadronic choice we have developed nuclear configuration of stable nuclides commensurate with the observed nuclear spin using isodeuterons, isoneutrons and if required used isoprotons. However, recall that each isodeuteron is made up of 2 isoprotons of parallel spin and one isoelectron of zero spin, and the isoneutron consists of one isoproton of half spin and one isoelectron of zero spin hence it is easy to convert the nuclear configuration of the first choice into the one in terms of isoprotons of 1/2 spin, isoprotons of -1/2 spin and isoelectrons of zero spin, that is our second choice. However, we can directly write the nuclear configuration in the second choice just by choosing correct number of isoprotons with 1/2 and -1/2 spin commensurate with the experimental nuclear spin, because isoelectron doesn't contribute to the nuclear spin.

A simple notation to represent Santilli's isoneutron, $\hat{n}$, is as given below as a compressed hydrogen atom, namely:

$$ha = (p^+, e^-)_{qm} \to \left(\hat{p}^+(\uparrow), \hat{e}^-(J=0)\right)_{hm} \equiv \hat{n}(\uparrow) \qquad (74)$$

where ha denotes the hydrogen atom; qm denotes quantum mechanics; p^+ denotes the conventional proton; e^- denotes the conventional electron; hm denotes hadronic mechanics; $\hat{p}^+$ denotes the isoproton; $\hat{e}^-$ denotes an isoelectron; J is the spin and $\uparrow$ denotes spin 1/2. The total angular momentum of the isoelectron is null because the particle is constrained to rotate within the hyperdense proton in singlet coupling, thus acquiring a value of the orbital angular momentum equal but opposite to its spin (Figure 8).

Similarly, the notation of an isodeuteron, $\hat{d}$, is obtained as given below, namely:

$$d(J=?) = \left(p^+(\uparrow), n(\downarrow)\right)_{qm} \to \left(\hat{p}^+(\uparrow), \hat{e}^-(J=0), \hat{p}^+(\uparrow)\right)_{hm}$$
$$\equiv \hat{d}(J=1) = \hat{d}(\uparrow\uparrow) \qquad (75)$$

where $\downarrow$ denotes the spin -1/2. The spin 1 of the isodeuteron is because of two up spins, $\uparrow\uparrow$, of two isoprotons.

The stability of deuteron gets excellently explained by the Santilli iso-deuteron model, Eq. (75). Namely, as the structure $\left(\hat{p}^+(\uparrow), \hat{e}^-(J=0)\right)_{hm}$ is unstable, there is a natural tendency of the bound electron in $\left(\hat{p}^+(\uparrow), \hat{e}^-(J=0), \hat{p}^+(\uparrow)\right)_{hm}$ to get released from the grip of its isoproton to which it is bound at the given instant of time, but no sooner it succeeds in getting released it immediately gets trapped into the hyper-dense medium of the other very closely placed proton. This is how isodeuteron enjoys its stability against radioactivity. This interpretation of nuclear stability and instability reasonably good.

In the next Section 12 we consider only the stable nuclides of periodic table up to the atomic number 82.

12. Proposed Nuclear Configuration of Stable IsoNuclides

We adopt ${}^{A}_{Z}\mathrm{X}_{N}(J) = \mathrm{X}(A,Z,J)$ to represent nuclides, where X represents the symbol of the chemical element, A is the mass number i.e. the total number of protons and neutrons, Z is the atomic number i.e. the total number of protrons, N is the total number of neutrons, and J is the nuclear spin. Obviously $(A-Z)$ is the total number of neutrons, N, in the nucleus. Notice that we have incorporated nuclear spin, J, in the conventional representation of nuclide.

In this paper we propose, apparently for the first time, the extension of Santilli isodeuteron to all stable nuclides under the proposed name of IsoNuclides with the symbol ${}^{A}_{Z}\hat{\mathrm{X}}_{N}(J)$. Notice that in this notation we have still retained the symbols A,N,Z because it would be easy to correlate with conventional description.

Now as stated in preceding sections there are two options for developing nuclear configuration of nuclides.

In the *model-I* the adopted working rule is that we are bound by the requirement of producing that nuclear configuration which predicts correctly the experimental nuclear spin. Our method is further based on the observed stability of an isodeuteron that indicates that the isonucleons of a nuclide first prefer to adopt the isodeuteron structures and in this way the unaccounted neutrons and protons stay in the nucleus as isoneutrons and isoprotons with appropriate spin orientation.

In the *model-II* we fix the number of isoelectrons equal to the number of neutrons (because in a nucleus an isoelectron with null spin is carried into through the neutron as isoneutron) and obviously the number of isoprotons of a nucleus equals to the mass number, A, of the nuclide. Thus our method is then to choose the number of isoprotons with spin 1/2 and -1/2 that correctly predicts the experimental nuclear spin of the nuclide.

12.1. Isodeuteron, Isoneutron and Isoproton as Constituents of Atomic Nuclei. Model-I

In this first option with the guidelines described above in this Section 12 we note that ${}^{3}_{2}\hat{\mathrm{He}}_{1}(0)$ can be readily interpreted as a hadronic bound state of an isodeuteron and an isoproton in singlet coupling (perhaps necessary for stability). Accordingly it gets represented as under,

$$ {}^{3}_{2}\hat{\mathrm{He}}_1(1/2) = \left(\hat{d}\left(\uparrow\uparrow\right),\ \hat{p}^{+}\left(\downarrow\right)\right)_{hm} \equiv \left({}^{2}_{1}\hat{d}_1(1), \hat{p}^{+}(-1/2)\right)_{hm} \quad (76) $$

Notice that in this isonuclide there we have one separate isoproton and two mutated protons as isoprotons in the form of $\hat{d}\left(\uparrow\uparrow\right)$.

Similarly, ${}^{4}_{2}\hat{\mathrm{He}}_2(0)$ can be readily interpreted as a hadronic bound state of two isodeuterons in singlet coupling, namely,

$$ {}^{4}_{2}\hat{\mathrm{He}}_2(0) = \left(\hat{d}\left(\uparrow\uparrow\right), \hat{d}\left(\downarrow\downarrow\right)\right)_{hm} \equiv \left({}^{2}_{1}\hat{d}_1(1), {}^{2}_{1}\hat{d}_1(-1)\right)_{hm}. \quad (77) $$

Along the same linens, ${}^{6}_{3}\hat{\mathrm{Li}}_3(1)$ can be readily interpreted as a hadronic bound state of ${}^{4}_{2}\hat{\mathrm{He}}_2(0)$ and one isodeuteron ${}^{2}_{1}\hat{d}_1(1)$

$$ {}^{6}_{3}\hat{\mathrm{Li}}_3(1) = \left({}^{4}_{2}\hat{\mathrm{He}}_2(0), {}^{2}_{1}\hat{d}_1(1)\right)_{hm} \quad (78) $$

and similarly for the remaining stable nuclides (see Table 2).

For the isotopic structure model of ${}^{7}_{3}\hat{\mathrm{Li}}_4(3/2)$ we have the more complex model

$$ {}^{7}_{3}\hat{\mathrm{Li}}_4(3/2) = \left({}^{4}_{2}\hat{\mathrm{He}}_2(0), {}^{2}_{1}\hat{d}_1(1), \hat{n}(1/2)\right)_{hm} \quad (79) $$

Therefore, we can symbolically write the *nuclear configuration of stable isonuclies* as under,

$$ {}^{A}_{Z}\hat{\mathrm{X}}_N(J) = [x_1({}^{2}_{1}\hat{d}_1(1)),\ x_2({}^{2}_{1}\hat{d}_1(-1)), x_3(\hat{n}(1/2)), x_4(\hat{n}(-1/2)), x_5(\hat{p}^{+}(1/2)), x_6(\hat{p}^{+}(-1/2))] \quad (80) $$

where $\hat{\mathrm{X}}$ denotes the isonuclide, x_i's are the number of the isonuclear or nuclear species depicted in the braces next to them. Notice that in this model-I in any nucleus the isoprotons would be in the form of isoneutrons, isodeuterons and remaining as separate isoprotons hence if the atomic number of a nuclide demands more protons than those accounted by isodeuterons and isoneutrons (the striking example is that of He-3, c.f. Eq. (76) they will be separate mutated proptons (i.e. the isoprotons). In view of this in above expression (80) the last two terms on the right hand side account for the separate isoprotons that are demanded by its atomic number, Z.

In this way the expression of the atomic mass number, A, is obtained as,

$$ A = 2x_1 + 2x_2 + x_3 + x_4 + x_5 + x_6 \quad (81) $$

the atomic number, Z, is given by,

$$ Z = x_1 + x_2 + x_5 + x_6 \quad (82) $$

Therefore, obviously the number of nuclear neutrons, N, is given by,

$$ N = A - Z = x_1 + x_2 + x_3 + x_4. \quad (83) $$

Whereas, the total number of isoprotons $\mathbb{P}_{\hat{p}^+}$, get computed as,

$$ \mathbb{P}_{\hat{p}^+} = 2x_1 + 2x_2 + x_3 + x_4 + x_5 + x_6 \quad (84) $$

and the total number of isoelectrons, $\mathbb{E}_{\hat{e}^-}$, get computed as,

$$ \mathbb{E}_{\hat{e}^-} = x_1 + x_2 + x_3 + x_4 \quad (85) $$

It is no wonder that $N = \mathbb{E}_{\hat{e}^-}$ because with each isoneutron there is associated one isoelectron. Moreover, the nuclear spin J gets computed as,

$$ J = x_1 - x_2 + \frac{1}{2}x_3 - \frac{1}{2}x_4 + \frac{1}{2}x_5 - \frac{1}{2}x_6 \quad (86) $$

Therefore, the isonuclide, ${}^{A}_{Z}\hat{\mathrm{X}}_N(J)$, gets reduced to isoprotons, $\hat{p}^{+}$ and isoelectrons, $\hat{e}^{-}$, that gets expressed as,

$$ {}^{A}_{Z}\hat{\mathrm{X}}_N(J) = \left(\mathbb{P}_{\hat{p}^+},\ \mathbb{E}_{\hat{e}^-}\right) \quad (87) $$

12.2. Isoprotons and Isonelectrons as Constituents of Atomic Nuclei. Model-II

Recall that all nuclear protons are indistinguishable whereas the isoprotons of the nuclear isoneutrons too remain indistinguishable because the isoneutrons have a natural tendency to get converted to protons. Therefore, we cannot label which proton out of the available nuclear protons at a given instant of time is actually bound to an isoelectron. In this way there must be on an average at a given moment of time a fixed number of isoprotons and the same number of isoelectrons, and remaining number of nucleons are the protons and are equal to the atomic number of the chemical element. However, in view of the housing of all protons and neutrons in extremely small nuclear volume (see also Section 10) there must be at least partial mutual penetration of wave packets of protons besides in addition to that with the wave packets of electrons that describe the isoneutron and isodeuteron. Hence all nuclear protons and neutrons taken together need to be treated as an assemblage of isoprotons and isoelectrons. Of course, the mutual penetration of wave packets of protons and the mutual penetration of wave packets of electrons and protons would definitely produce different hadronic effects hence needs to be quantitatively investigated

by the tools of hadronic mechanics. Therefore, in this *model-II* we treat every nucleon of a nucleus an isonucleon.

Thus the counter part of Eq. (80) in this case would read as,

$$\begin{aligned} {}^{A}_{Z}\hat{X}_{N}(J) = \big[&2x_1\big(\hat{p}^{+}(1/2)\big), 2x_2\big(\hat{p}^{+}(-1/2)\big), \\ &x_3\big(\hat{p}^{+}(1/2)\big), x_4\big(\hat{p}^{+}(-1/2)\big), \\ &x_5\big(\hat{p}^{+}(1/2)\big), x_6\big(\hat{p}^{+}(-1/2)\big), \\ &(x_1+x_2+x_3+x_4)\big(\hat{e}^{-}(0)\big)\big] \end{aligned} \tag{88}$$

which gets simplified to,

$$\begin{aligned} {}^{A}_{Z}\hat{X}_{N}(J) = \big[&(2x_1+x_3+x_5)\big(\hat{p}^{+}(1/2)\big), \\ &(2x_2+x_4+x_6)\big(\hat{p}^{+}(-1/2)\big), \\ &(x_1+x_2+x_3+x_4)\big(\hat{e}^{-}(0)\big)\big] \end{aligned} \tag{89}$$

where the number of isoprotons with spin 1/2, $\mathbb{P}(1/2)$, is given by,

$$\mathbb{P}(1/2) = 2x_1 + x_3 + x_5, \tag{90}$$

number of the isoprotons with spin -1/2, $\mathbb{P}(-1/2)$, is given by,

$$\mathbb{P}(-1/2) = 2x_2 + x_4 + x_6, \tag{91}$$

and number of the isoelectrons with spin 0, $\mathbb{E}(0) = N$, is given by,

$$\mathbb{E}(0) = x_1 + x_2 + x_3 + x_4. \tag{92}$$

Alternatively, we can directly express ${}^{A}_{Z}\hat{X}_{N}(J)$ as follows,

$${}^{A}_{Z}\hat{X}_{N}(J) = \big[\mathbb{P}(1/2), \mathbb{P}(-1/2), \mathbb{E}(0)\big] \tag{93}$$

where $\mathbb{P}(1/2)+\mathbb{P}(-1/2) = A$ and $Z = \mathbb{P}(1/2)+\mathbb{P}(-1/2)-N$. Since, all nuclear spins are null or positive numbers we have $\mathbb{P}(1/2) > \mathbb{P}(-1/2)$.

We would like to stress that the methods of writing nuclear configuration described above are entirely general that make no distinction between stable and unstable nuclides. However, with the above adopted notations we are now well equipped to build the nuclear configuration of stable nuclides as isonuclides ${}^{A}_{Z}\hat{X}_{N}(J)$, that we present in the next Section 13.

13. Hadronic Mechanics Based Configuration of Stable Nuclides

In this paper we are primarily presenting the nuclear configuration of the stable nuclides. The nuclides of atomic number higher than 82 are all radioactive therefore we have developed the nuclear configuration up to the chemical element Pb. Now onwards we will use the short hand notation of an isoneutron and an isodeuteron given in the extreme right hand side of Eqs. (74) and (75), namely $\hat{n}(\uparrow)$ and $\hat{d}(\uparrow\uparrow)$ respectively. Moreover, henceforth all nuclear protons would be treated as isoprotons whether the wave packet of any one of them penetrates with that of an isoelectron or not. This is so because as discussed in Section 10, in view of the extremely small size of atomic nuclei, all nuclear protons indeed get transformed to isoprotons.

13.1. Nuclear Configuration of Stable Isotopes as Isonuclides. Model-I

The observed stability of deuteron does indicate that the stable nuclides first prefer to have the isodeuteron structure from the available number of neutrons and protons. Whereas the remaining unaccounted neutrons and protons stay in the nucleus as isoneutrons and isoprotons.

Thus we have followed a nuclear version of the Aufbau type principle with the requirement that the resulting nuclear configuration should correctly predict the observed nuclear spin of each isotope of the elements. We are presenting in column 3 of Table 2 the so arrived at nuclear configuration of the stable isonuclides up to the element Pb of the periodic table along with the observed nuclear spin (in colum 5) against each isonuclide for the ready reference. All the nuclear spins reported now onwards are taken from the Ref. [80] unless otherwise other sources are cited.

13.2. Nuclear Configuration of Stable Isotopes as Isonuclides. Model-II

The nuclear configuration in terms of isoprotons and isoelectrons that replicate the observed nuclear spin is easy to write. We first write number of isoelectrons equal to the number of neutrons, N, in the nucleus and then write the number of isoprotons equal to the mass number, A, of the nuclide, which then is distributed in up and down spin isoprotons so that the net spin of the combination equals the experimental nuclear spin.

Equivalently, on realizing that each isodeuteron has two isoprotons of same spin and one isoelectron of null spin, and the isoproton of each isoneutron has the same spin as that of the latter. The total number of nuclear isoelectrons is given by the sum of the number of isodeuterons and isoneutrons in a given isonuclide. The nuclear configuration of the *model-II* has been listed in the column 4 of Table 2. Notice that the nuclear configuration in this option of all nuclei correctly predicts the respective observed nuclear spin.

Table 2. *Nuclear configuration of stable, primordial and very long lived isonuclides for nuclear model-I and model-II*

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
1	${}_{1}^{1}\mathrm{H}_{0}(1/2)$ Proton (not an isonuclide)	$p^{+}(\uparrow)$	$p^{+}(\uparrow)$	1/2
	${}_{1}^{2}\hat{\mathrm{H}}_{1}(1)$ (isodeuteron)	$\hat{d}(\uparrow\uparrow)$	$2\hat{p}^{+}(\uparrow),\hat{e}^{-}(\uparrow\downarrow)$	1
2	${}_{2}^{3}\hat{\mathrm{He}}_{1}(1/2)$	$\hat{d}(\uparrow\uparrow),\hat{p}^{+}(\downarrow)$	$2\hat{p}^{+}(\uparrow),\hat{p}^{+}(\downarrow),$ $\hat{e}^{-}(\uparrow\downarrow)$	1/2
	${}_{2}^{4}\hat{\mathrm{He}}_{2}(0)$	$\hat{d}(\uparrow\uparrow),\ \hat{d}(\downarrow\downarrow) = \left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right]$	$2\hat{p}^{+}(\uparrow),2\hat{p}^{+}(\downarrow),$ $2\hat{e}^{-}(\uparrow\downarrow)$	0
3	${}_{3}^{6}\hat{\mathrm{Li}}_{3}(1)$	$\left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow)$	$4\hat{p}^{+}(\uparrow),2\hat{p}^{+}(\downarrow),$ $3\hat{e}^{-}(\uparrow\downarrow)$	1
	${}_{3}^{7}\hat{\mathrm{Li}}_{4}(3/2)$	$\left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\hat{n}(\uparrow)$	$5\hat{p}^{+}(\uparrow),2\hat{p}^{+}(\downarrow),$ $4\hat{e}^{-}(\uparrow\downarrow)$	3/2
4	${}_{4}^{9}\hat{\mathrm{Be}}_{5}(3/2)$	$\left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right],2\hat{d}(\uparrow\uparrow),\ \hat{n}(\downarrow)$	$6\hat{p}^{+}(\uparrow),3\hat{p}^{+}(\downarrow),$ $5\hat{e}^{-}(\uparrow\downarrow)$	3/2
	${}_{5}^{10}\hat{\mathrm{B}}_{5}(3)$	$\left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right],3\hat{d}(\uparrow\uparrow)$	$8\hat{p}^{+}(\uparrow),2\hat{p}^{+}(\downarrow),$ $5\hat{e}^{-}(\uparrow\downarrow)$	3
5	${}_{5}^{11}\hat{\mathrm{B}}_{5}(3/2)$	$2\left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\hat{n}(\uparrow)$	$7\hat{p}^{+}(\uparrow),4\hat{p}^{+}(\downarrow),$ $6\hat{e}^{-}(\uparrow\downarrow)$	3/2
	${}_{6}^{12}\hat{\mathrm{C}}_{6}(0)$	$2\left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{d}(\downarrow\downarrow)$ $= 3\left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right]$	$6\hat{p}^{+}(\uparrow),6\hat{p}^{+}(\downarrow),$ $6\hat{e}^{-}(\uparrow\downarrow)$	0
6	${}_{6}^{13}\hat{\mathrm{C}}_{7}(1/2)$	$3\left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right],\ \hat{n}(\uparrow)$	$7\hat{p}^{+}(\uparrow),6\hat{p}^{+}(\downarrow),$ $7\hat{e}^{-}(\uparrow\downarrow)$	1/2
	${}_{7}^{14}\hat{\mathrm{N}}_{7}(1)$	$3\left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow)$	$8\hat{p}^{+}(\uparrow),6\hat{p}^{+}(\downarrow),$ $7\hat{e}^{-}(\uparrow\downarrow)$	1
7	${}_{7}^{15}\hat{\mathrm{N}}_{7}(1/2)$	$3\left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{n}(\downarrow)$	$8\hat{p}^{+}(\uparrow),7\hat{p}^{+}(\downarrow),$ $8\hat{e}^{-}(\uparrow\downarrow)$	1/2
	${}_{8}^{16}\hat{\mathrm{O}}_{8}(0)$	$3\left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right]\ \hat{d}(\uparrow\uparrow),\ \hat{d}(\downarrow\downarrow)$ $= 4\left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right]$	$8\hat{p}^{+}(\uparrow),8\hat{p}^{+}(\downarrow),$ $8\hat{e}^{-}(\uparrow\downarrow)$	0
8	${}_{8}^{17}\hat{\mathrm{O}}_{9}(5/2)$	$3\left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right],2\hat{d}(\uparrow\uparrow),\ \hat{n}(\uparrow)$	$11\hat{p}^{+}(\uparrow),6\hat{p}^{+}(\downarrow),$ $9\hat{e}^{-}(\uparrow\downarrow)$	5/2
	${}_{8}^{18}\hat{\mathrm{O}}_{10}(0)$	$4\left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right],\ \hat{n}(\uparrow),\ \hat{n}(\downarrow)$	$9\hat{p}^{+}(\uparrow),9\hat{p}^{+}(\downarrow),$ $10\hat{e}^{-}(\uparrow\downarrow)$	0
9	${}_{9}^{19}\hat{\mathrm{F}}_{10}(1/2)$	$4\left[{}_{2}^{4}\hat{\mathrm{He}}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{n}(\downarrow)$	$10\hat{p}^{+}(\uparrow),9\hat{p}^{+}(\downarrow),$ $10\hat{e}^{-}(\uparrow\downarrow)$	1/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
10	$^{20}_{10}\hat{Ne}_{10}(0)$	$4\left[{}^{4}_{2}\hat{He}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{d}(\downarrow\downarrow) \equiv 5\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$	$10\hat{p}^{+}(\uparrow),10\hat{p}^{+}(\downarrow),\ 10\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{21}_{10}\hat{Ne}_{11}(3/2)$	$4\left[{}^{4}_{2}\hat{He}_{2}(0)\right],2\,\hat{d}(\uparrow\uparrow),\ \hat{n}(\downarrow)$	$12\hat{p}^{+}(\uparrow),9\hat{p}^{+}(\downarrow),\ 11\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{22}_{10}\hat{Ne}_{12}(0)$	$5\left[{}^{4}_{2}\hat{He}_{2}(0)\right],\ \hat{n}(\uparrow),\ \hat{n}(\downarrow)$	$11\hat{p}^{+}(\uparrow),11\hat{p}^{+}(\downarrow),\ 12\hat{e}^{-}(\uparrow\downarrow)$	0
11	$^{23}_{11}\hat{Na}_{12}(3/2)$	$5\left[{}^{4}_{2}\hat{He}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\hat{n}(\uparrow)$	$13\hat{p}^{+}(\uparrow),10\hat{p}^{+}(\downarrow),\ 12\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{24}_{12}\hat{Mg}_{12}(0)$	$5\left[{}^{4}_{2}\hat{He}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{d}(\downarrow\downarrow) \equiv 6\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$	$12\hat{p}^{+}(\uparrow),12\hat{p}^{+}(\downarrow),\ 12\hat{e}^{-}(\uparrow\downarrow)$	0
12	$^{25}_{12}\hat{Mg}_{13}(5/2)$	$5\left[{}^{4}_{2}\hat{He}_{2}(0)\right],2\,\hat{d}(\uparrow\uparrow),\ \hat{n}(\uparrow)$	$15\hat{p}^{+}(\uparrow),10\hat{p}^{+}(\downarrow),\ 13\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{26}_{12}\hat{Mg}_{14}(0)$	$6\left[{}^{4}_{2}\hat{He}_{2}(0)\right],\ \hat{n}(\uparrow),\ \hat{n}(\downarrow)$	$13\hat{p}^{+}(\uparrow),13\hat{p}^{+}(\downarrow),\ 14\hat{e}^{-}(\uparrow\downarrow)$	0
13	$^{27}_{13}\hat{Al}_{14}(5/2)$	$5\left[{}^{4}_{2}\hat{He}_{2}(0)\right],3\,\hat{d}(\uparrow\uparrow),\ \hat{n}(\downarrow)$	$16\hat{p}^{+}(\uparrow),11\hat{p}^{+}(\downarrow),\ 14\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{28}_{14}\hat{Si}_{14}(0)$	$6\left[{}^{4}_{2}\hat{He}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{d}(\downarrow\downarrow) \equiv 7\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$	$14\hat{p}^{+}(\uparrow),14\hat{p}^{+}(\downarrow),\ 14\hat{e}^{-}(\uparrow\downarrow)$	0
14	$^{29}_{14}\hat{Si}_{15}(1/2)$	$7\left[{}^{4}_{2}\hat{He}_{2}(0)\right],\ \hat{n}(\uparrow)$	$15\hat{p}^{+}(\uparrow),14\hat{p}^{+}(\downarrow),\ 15\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{30}_{14}\hat{Si}_{16}(0)$	$7\left[{}^{4}_{2}\hat{He}_{2}(0)\right],\ \hat{n}(\uparrow),\ \hat{n}(\downarrow)$	$15\hat{p}^{+}(\uparrow),15\hat{p}^{+}(\downarrow),\ 16\hat{e}^{-}(\uparrow\downarrow)$	0
15	$^{31}_{15}\hat{P}_{16}(0)$	$7\left[{}^{4}_{2}\hat{He}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{n}(\downarrow)$	$16\hat{p}^{+}(\uparrow),15\hat{p}^{+}(\downarrow),\ 16\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{32}_{16}\hat{S}_{16}(0)$	$7\left[{}^{4}_{2}\hat{He}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{d}(\downarrow\downarrow) \equiv 8\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$	$16\hat{p}^{+}(\uparrow),16\hat{p}^{+}(\downarrow),\ 16\hat{e}^{-}(\uparrow\downarrow)$	0
16	$^{33}_{16}\hat{S}_{17}(3/2)$	$7\left[{}^{4}_{2}\hat{He}_{2}(0)\right],2\,\hat{d}(\uparrow\uparrow),\ \hat{n}(\downarrow)$	$18\hat{p}^{+}(\uparrow),15\hat{p}^{+}(\downarrow),\ 17\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{34}_{16}\hat{S}_{18}(3/2)$	$8\left[{}^{4}_{2}\hat{He}_{2}(0)\right],\ \hat{n}(\uparrow),\ \hat{n}(\downarrow)$	$17\hat{p}^{+}(\uparrow),17\hat{p}^{+}(\downarrow),\ 18\hat{e}^{-}(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
17	$^{36}_{16}\hat{S}_{20}(3/2)$	$8\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 2\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$18\hat{p}^{+}(\uparrow), 18\hat{p}^{+}(\downarrow), 20\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{35}_{17}\hat{Cl}_{18}(3/2)$	$8\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), \hat{n}(\uparrow)$	$19\hat{p}^{+}(\uparrow), 16\hat{p}^{+}(\downarrow), 18\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{37}_{17}\hat{Cl}_{20}(3/2)$	$8\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 2\hat{n}(\uparrow), \hat{n}(\downarrow)$	$20\hat{p}^{+}(\uparrow), 17\hat{p}^{+}(\downarrow), 20\hat{e}^{-}(\uparrow\downarrow)$	3/2
18	$^{36}_{18}\hat{Ar}_{18}(0)$	$8\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow) = 9\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$	$18\hat{p}^{+}(\uparrow), 18\hat{p}^{+}(\downarrow), 18\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{38}_{18}\hat{Ar}_{20}(0)$	$9\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$19\hat{p}^{+}(\uparrow), 19\hat{p}^{+}(\downarrow), 20\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{40}_{18}\hat{Ar}_{22}(0)$	$9\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 2\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$20\hat{p}^{+}(\uparrow), 20\hat{p}^{+}(\downarrow), 22\hat{e}^{-}(\uparrow\downarrow)$	0
19	$^{39}_{19}\hat{K}_{20}(3/2)$	$9\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), \hat{n}(\uparrow)$	$21\hat{p}^{+}(\uparrow), 18\hat{p}^{+}(\downarrow), 20\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{40}_{19}\hat{K}_{21}(4)$	$8\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 3\hat{d}(\uparrow\uparrow), 2\hat{n}(\uparrow)$	$24\hat{p}^{+}(\uparrow), 16\hat{p}^{+}(\downarrow), 21\hat{e}^{-}(\uparrow\downarrow)$	4
	$^{41}_{19}\hat{K}_{22}(3/2)$	$9\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 2\hat{n}(\uparrow), \hat{n}(\downarrow)$	$22\hat{p}^{+}(\uparrow), 19\hat{p}^{+}(\downarrow), 22\hat{e}^{-}(\uparrow\downarrow)$	3/2
20	$^{40}_{20}\hat{Ca}_{20}(0)$	$9\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow) = 10\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$	$20\hat{p}^{+}(\uparrow), 20\hat{p}^{+}(\downarrow), 20\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{42}_{20}\hat{Ca}_{22}(0)$	$10\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$21\hat{p}^{+}(\uparrow), 21\hat{p}^{+}(\downarrow), 22\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{43}_{20}\hat{Ca}_{23}(7/2)$	$9\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 2\hat{d}(\uparrow\uparrow), 3\hat{n}(\uparrow)$	$25\hat{p}^{+}(\uparrow), 18\hat{p}^{+}(\downarrow), 23\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{44}_{20}\hat{Ca}_{24}(0)$	$10\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 2\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$22\hat{p}^{+}(\uparrow), 22\hat{p}^{+}(\downarrow), 24\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{46}_{20}\hat{Ca}_{26}(0)$	$10\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 3\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$23\hat{p}^{+}(\uparrow), 23\hat{p}^{+}(\downarrow), 26\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{48}_{20}\hat{Ca}_{28}(0)$	$10\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 4\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$23\hat{p}^{+}(\uparrow), 23\hat{p}^{+}(\downarrow), 26\hat{e}^{-}(\uparrow\downarrow)$	0

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21	$^{45}_{21}\hat{Sc}_{24}(7/2)$	$9\left[\hat{^4_2He_2}(0)\right], 3\,\hat{d}(\uparrow\uparrow), 2\,\hat{n}(\uparrow), \hat{n}(\downarrow)$	$26\hat{p}^+(\uparrow), 19\hat{p}^+(\downarrow), 24\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{46}_{22}\hat{Ti}_{24}(0)$	$11\left[\hat{^4_2He_2}(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$23\hat{p}^+(\uparrow), 23\hat{p}^+(\downarrow), 24\hat{e}^-(\uparrow\downarrow)$	0
	$^{47}_{22}\hat{Ti}_{25}(5/2)$	$10\left[\hat{^4_2He_2}(0)\right], 2\,\hat{d}(\uparrow\uparrow), 2\,\hat{n}(\uparrow), \hat{n}(\downarrow)$	$26\hat{p}^+(\uparrow), 21\hat{p}^+(\downarrow), 25\hat{e}^-(\uparrow\downarrow)$	5/2
22	$^{48}_{22}\hat{Ti}_{26}(0)$	$11\left[\hat{^4_2He_2}(0)\right], 2\,\hat{n}(\uparrow), 2\,\hat{n}(\downarrow)$	$24\hat{p}^+(\uparrow), 24\hat{p}^+(\downarrow), 26\hat{e}^-(\uparrow\downarrow)$	0
	$^{49}_{22}\hat{Ti}_{27}(7/2)$	$10\left[\hat{^4_2He_2}(0)\right], 2\,\hat{d}(\uparrow\uparrow), 4\,\hat{n}(\uparrow), \hat{n}(\downarrow)$	$28\hat{p}^+(\uparrow), 21\hat{p}^+(\downarrow), 27\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{50}_{22}\hat{Ti}_{28}(0)$	$11\left[\hat{^4_2He_2}(0)\right], 3\,\hat{n}(\uparrow), 3\,\hat{n}(\downarrow)$	$25\hat{p}^+(\uparrow), 25\hat{p}^+(\downarrow), 28\hat{e}^-(\uparrow\downarrow)$	0
	$^{50}_{23}\hat{V}_{27}(6)$	$9\left[\hat{^4_2He_2}(0)\right], 5\,\hat{d}(\uparrow\uparrow), 3\,\hat{n}(\uparrow), \hat{n}(\downarrow)$	$31\hat{p}^+(\uparrow), 19\hat{p}^+(\downarrow), 27\hat{e}^-(\uparrow\downarrow)$	6
23	$^{51}_{23}\hat{V}_{28}(7/2)$	$11\left[\hat{^4_2He_2}(0)\right], \hat{d}(\uparrow\uparrow), 5\,\hat{n}(\uparrow)$	$29\hat{p}^+(\uparrow), 22\hat{p}^+(\downarrow), 28\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{50}_{24}\hat{Cr}_{26}(0)$	$12\left[\hat{^4_2He_2}(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$25\hat{p}^+(\uparrow), 25\hat{p}^+(\downarrow), 26\hat{e}^-(\uparrow\downarrow)$	0
	$^{52}_{24}\hat{Cr}_{28}(0)$	$12\left[\hat{^4_2He_2}(0)\right], 2\,\hat{n}(\uparrow), 2\,\hat{n}(\downarrow)$	$26\hat{p}^+(\uparrow), 26\hat{p}^+(\downarrow), 28\hat{e}^-(\uparrow\downarrow)$	0
24	$^{53}_{24}\hat{Cr}_{29}(3/2)$	$12\left[\hat{^4_2He_2}(0)\right], 4\,\hat{n}(\uparrow), \hat{n}(\downarrow)$	$28\hat{p}^+(\uparrow), 25\hat{p}^+(\downarrow), 29\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{54}_{24}\hat{Cr}_{30}(0)$	$12\left[\hat{^4_2He_2}(0)\right], 3\,\hat{n}(\uparrow), 3\,\hat{n}(\downarrow)$	$27\hat{p}^+(\uparrow), 27\hat{p}^+(\downarrow), 30\hat{e}^-(\uparrow\downarrow)$	0
25	$^{55}_{25}\hat{Mn}_{30}(5/2)$	$12\left[\hat{^4_2He_2}(0)\right], \hat{d}(\uparrow\uparrow), 4\,\hat{n}(\uparrow), \hat{n}(\downarrow)$	$30\hat{p}^+(\uparrow), 25\hat{p}^+(\downarrow), 30\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{54}_{26}\hat{Fe}_{28}(0)$	$13\left[\hat{^4_2He_2}(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$27\hat{p}^+(\uparrow), 27\hat{p}^+(\downarrow), 28\hat{e}^-(\uparrow\downarrow)$	0
	$^{56}_{26}\hat{Fe}_{30}(0)$	$13\left[\hat{^4_2He_2}(0)\right], 2\,\hat{n}(\uparrow), 2\,\hat{n}(\downarrow)$	$28\hat{p}^+(\uparrow), 28\hat{p}^+(\downarrow), 30\hat{e}^-(\uparrow\downarrow)$	0
26	$^{57}_{26}\hat{Fe}_{31}(1/2)$	$13\left[\hat{^4_2He_2}(0)\right], 3\,\hat{n}(\uparrow), 2\,\hat{n}(\downarrow)$	$29\hat{p}^+(\uparrow), 28\hat{p}^+(\downarrow), 31\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{58}_{26}\hat{Fe}_{32}(0)$	$13\left[\hat{^4_2He_2}(0)\right], 3\,\hat{n}(\uparrow), 3\,\hat{n}(\downarrow)$	$29\hat{p}^+(\uparrow), 29\hat{p}^+(\downarrow), 32\hat{e}^-(\uparrow\downarrow)$	0

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27	$^{59}_{27}\hat{Co}_{32}$ (7/2)	$13\left[\frac{4}{2}\hat{He}_2(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $5\hat{n}(\uparrow)$	$33\hat{p}^+(\uparrow), 26\hat{p}^+(\downarrow)$, $32\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{58}_{28}\hat{Ni}_{30}$ (0)	$14\left[\frac{4}{2}\hat{He}_2(0)\right]$, $\hat{n}(\uparrow)$, $\hat{n}(\downarrow)$	$29\hat{p}^+(\uparrow), 29\hat{p}^+(\downarrow)$, $30\hat{e}^-(\uparrow\downarrow)$	0
	$^{60}_{28}\hat{Ni}_{32}$ (0)	$14\left[\frac{4}{2}\hat{He}_2(0)\right], 2\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$30\hat{p}^+(\uparrow), 30\hat{p}^+(\downarrow)$, $32\hat{e}^-(\uparrow\downarrow)$	0
28	$^{61}_{28}\hat{Ni}_{33}$ (3/2)	$14\left[\frac{4}{2}\hat{He}_2(0)\right], 4\hat{n}(\uparrow)$, $\hat{n}(\downarrow)$	$32\hat{p}^+(\uparrow), 29\hat{p}^+(\downarrow)$, $33\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{62}_{28}\hat{Ni}_{34}$ (0)	$14\left[\frac{4}{2}\hat{He}_2(0)\right], 3\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$31\hat{p}^+(\uparrow), 31\hat{p}^+(\downarrow)$, $34\hat{e}^-(\uparrow\downarrow)$	0
	$^{64}_{28}\hat{Ni}_{36}$ (0)	$14\left[\frac{4}{2}\hat{He}_2(0)\right], 4\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$32\hat{p}^+(\uparrow), 32\hat{p}^+(\downarrow)$, $36\hat{e}^-(\uparrow\downarrow)$	0
29	$^{63}_{29}\hat{Cu}_{34}$ (3/2)	$14\left[\frac{4}{2}\hat{He}_2(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $3\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$33\hat{p}^+(\uparrow), 30\hat{p}^+(\downarrow)$, $34\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{65}_{29}\hat{Cu}_{36}$ (3/2)	$14\left[\frac{4}{2}\hat{He}_2(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $4\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$34\hat{p}^+(\uparrow), 31\hat{p}^+(\downarrow)$, $36\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{64}_{30}\hat{Zn}_{34}$ (0)	$15\left[\frac{4}{2}\hat{He}_2(0)\right], 2\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$32\hat{p}^+(\uparrow), 32\hat{p}^+(\downarrow)$, $34\hat{e}^-(\uparrow\downarrow)$	0
	$^{66}_{30}\hat{Zn}_{36}$ (0)	$15\left[\frac{4}{2}\hat{He}_2(0)\right], 3\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$33\hat{p}^+(\uparrow), 33\hat{p}^+(\downarrow)$, $36\hat{e}^-(\uparrow\downarrow)$	0
30	$^{67}_{30}\hat{Zn}_{37}$ (5/2)	$15\left[\frac{4}{2}\hat{He}_2(0)\right], 6\hat{n}(\uparrow)$, $\hat{n}(\downarrow)$	$36\hat{p}^+(\uparrow), 31\hat{p}^+(\downarrow)$, $37\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{68}_{30}\hat{Zn}_{38}$ (0)	$15\left[\frac{4}{2}\hat{He}_2(0)\right], 4\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$34\hat{p}^+(\uparrow), 34\hat{p}^+(\downarrow)$, $38\hat{e}^-(\uparrow\downarrow)$	0
	$^{68}_{30}\hat{Zn}_{38}$ (0)	$15\left[\frac{4}{2}\hat{He}_2(0)\right], 5\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$35\hat{p}^+(\uparrow), 35\hat{p}^+(\downarrow)$, $40\hat{e}^-(\uparrow\downarrow)$	0
31	$^{69}_{31}\hat{Ga}_{38}$ (3/2)	$15\left[\frac{4}{2}\hat{He}_2(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $4\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$36\hat{p}^+(\uparrow), 33\hat{p}^+(\downarrow)$, $38\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{71}_{31}\hat{Ga}_{40}$ (3/2)	$15\left[\frac{4}{2}\hat{He}_2(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $5\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$37\hat{p}^+(\uparrow), 34\hat{p}^+(\downarrow)$, $40\hat{e}^-(\uparrow\downarrow)$	3/2
32	$^{70}_{32}\hat{Ge}_{38}$ (0)	$16\left[\frac{4}{2}\hat{He}_2(0)\right], 3\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$35\hat{p}^+(\uparrow), 35\hat{p}^+(\downarrow)$, $38\hat{e}^-(\uparrow\downarrow)$	0
	$^{72}_{32}\hat{Ge}_{40}$ (0)	$16\left[\frac{4}{2}\hat{He}_2(0)\right], 4\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$36\hat{p}^+(\uparrow), 36\hat{p}^+(\downarrow)$, $40\hat{e}^-(\uparrow\downarrow)$	0

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	$^{73}_{32}\hat{G}e_{41}$ (9/2)	$16\left[\frac{1}{2}\hat{H}e_2(0)\right], 9\hat{n}(\uparrow)$	$41\hat{p}^+(\uparrow), 32\hat{p}^+(\downarrow), 41\hat{e}^-(\uparrow\downarrow)$	9/2
	$^{74}_{32}\hat{G}e_{42}$ (0)	$16\left[\frac{1}{2}\hat{H}e_2(0)\right], 5\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$37\hat{p}^+(\uparrow), 37\hat{p}^+(\downarrow), 42\hat{e}^-(\uparrow\downarrow)$	0
	$^{76}_{32}\hat{G}e_{44}$ (0)	$16\left[\frac{1}{2}\hat{H}e_2(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$38\hat{p}^+(\uparrow), 38\hat{p}^+(\downarrow), 44\hat{e}^-(\uparrow\downarrow)$	0
33	$^{75}_{33}\hat{A}s_{42}$ (3/2)	$16\left[\frac{1}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 5\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$39\hat{p}^+(\uparrow), 36\hat{p}^+(\downarrow), 42\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{74}_{34}\hat{S}e_{40}$ (0)	$17\left[\frac{1}{2}\hat{H}e_2(0)\right], 3\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$37\hat{p}^+(\uparrow), 37\hat{p}^+(\downarrow), 40\hat{e}^-(\uparrow\downarrow)$	0
	$^{76}_{34}\hat{S}e_{42}$ (0)	$17\left[\frac{1}{2}\hat{H}e_2(0)\right], 4\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$38\hat{p}^+(\uparrow), 38\hat{p}^+(\downarrow), 42\hat{e}^-(\uparrow\downarrow)$	0
	$^{77}_{34}\hat{S}e_{43}$ (0)	$17\left[\frac{1}{2}\hat{H}e_2(0)\right], 5\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$39\hat{p}^+(\uparrow), 38\hat{p}^+(\downarrow), 43\hat{e}^-(\uparrow\downarrow)$	1/2
34	$^{78}_{34}\hat{S}e_{44}$ (0)	$17\left[\frac{1}{2}\hat{H}e_2(0)\right], 5\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$39\hat{p}^+(\uparrow), 39\hat{p}^+(\downarrow), 44\hat{e}^-(\uparrow\downarrow)$	0
	$^{80}_{34}\hat{S}e_{46}$ (0)	$17\left[\frac{1}{2}\hat{H}e_2(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$40\hat{p}^+(\uparrow), 40\hat{p}^+(\downarrow), 46\hat{e}^-(\uparrow\downarrow)$	0
	$^{82}_{34}\hat{S}e_{48}$ (0)	$17\left[\frac{1}{2}\hat{H}e_2(0)\right], 7\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$41\hat{p}^+(\uparrow), 41\hat{p}^+(\downarrow), 48\hat{e}^-(\uparrow\downarrow)$	0
	$^{79}_{35}\hat{B}r_{44}$ (3/2)	$17\left[\frac{1}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 5\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$41\hat{p}^+(\uparrow), 38\hat{p}^+(\downarrow), 44\hat{e}^-(\uparrow\downarrow)$	3/2
35	$^{81}_{35}\hat{B}r_{46}$ (3/2)	$17\left[\frac{1}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 6\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$42\hat{p}^+(\uparrow), 39\hat{p}^+(\downarrow), 46\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{78}_{36}\hat{K}r_{42}$ (0)	$18\left[\frac{1}{2}\hat{H}e_2(0)\right], 3\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$39\hat{p}^+(\uparrow), 39\hat{p}^+(\downarrow), 42\hat{e}^-(\uparrow\downarrow)$	0
	$^{80}_{36}\hat{K}r_{44}$ (0)	$18\left[\frac{1}{2}\hat{H}e_2(0)\right], 4\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$40\hat{p}^+(\uparrow), 40\hat{p}^+(\downarrow), 44\hat{e}^-(\uparrow\downarrow)$	0
36	$^{82}_{36}\hat{K}r_{46}$ (0)	$18\left[\frac{1}{2}\hat{H}e_2(0)\right], 5\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$41\hat{p}^+(\uparrow), 41\hat{p}^+(\downarrow), 46\hat{e}^-(\uparrow\downarrow)$	0
	$^{83}_{36}\hat{K}r_{47}$ (9/2)	$18\left[\frac{1}{2}\hat{H}e_2(0)\right], 10\hat{n}(\uparrow), \hat{n}(\downarrow)$	$46\hat{p}^+(\uparrow), 37\hat{p}^+(\downarrow), 47\hat{e}^-(\uparrow\downarrow)$	9/2

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37	$^{84}_{36}\hat{K}r_{48}$ (0)	$18\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$42\hat{p}^{+}(\uparrow), 42\hat{p}^{+}(\downarrow), 48\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{86}_{36}\hat{K}r_{50}$ (0)	$18\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 7\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$43\hat{p}^{+}(\uparrow), 43\hat{p}^{+}(\downarrow), 50\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{85}_{37}\hat{R}b_{48}$ (5/2)	$18\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 7\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$45\hat{p}^{+}(\uparrow), 40\hat{p}^{+}(\downarrow), 48\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{87}_{37}\hat{R}b_{50}$ (3/2)	$18\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 7\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$45\hat{p}^{+}(\uparrow), 42\hat{p}^{+}(\downarrow), 50\hat{e}^{-}(\uparrow\downarrow)$	3/2
38	$^{84}_{38}\hat{S}r_{46}$ (0)	$19\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 4\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$42\hat{p}^{+}(\uparrow), 42\hat{p}^{+}(\downarrow), 46\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{86}_{38}\hat{S}r_{48}$ (0)	$19\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 5\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$43\hat{p}^{+}(\uparrow), 43\hat{p}^{+}(\downarrow), 48\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{87}_{38}\hat{S}r_{49}$ (9/2)	$19\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 10\hat{n}(\uparrow), \hat{n}(\downarrow)$	$48\hat{p}^{+}(\uparrow), 39\hat{p}^{+}(\downarrow), 49\hat{e}^{-}(\uparrow\downarrow)$	9/2
	$^{88}_{38}\hat{S}r_{50}$ (0)	$19\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$44\hat{p}^{+}(\uparrow), 44\hat{p}^{+}(\downarrow), 50\hat{e}^{-}(\uparrow\downarrow)$	0
39	$^{89}_{39}\hat{Y}_{50}$ (1/2)	$19\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 5\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$45\hat{p}^{+}(\uparrow), 44\hat{p}^{+}(\downarrow), 50\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{90}_{40}\hat{Z}r_{50}$ (0)	$20\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 5\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$45\hat{p}^{+}(\uparrow), 45\hat{p}^{+}(\downarrow), 50\hat{e}^{-}(\uparrow\downarrow)$	0
40	$^{91}_{40}\hat{Z}r_{51}$ (5/2)	$20\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 8\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$48\hat{p}^{+}(\uparrow), 43\hat{p}^{+}(\downarrow), 51\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{92}_{40}\hat{Z}r_{52}$ (0)	$20\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$46\hat{p}^{+}(\uparrow), 46\hat{p}^{+}(\downarrow), 52\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{94}_{40}\hat{Z}r_{54}$ (0)	$20\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 7\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$47\hat{p}^{+}(\uparrow), 47\hat{p}^{+}(\downarrow), 54\hat{e}^{-}(\uparrow\downarrow)$	0

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	$^{96}_{40}\hat{Z}r_{56}$ (0)	$20\left[\frac{1}{2}\hat{H}e_2(0)\right], 8\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$48\hat{p}^+(\uparrow), 48\hat{p}^+(\downarrow), 56\hat{e}^-(\uparrow\downarrow)$	0
41	$^{93}_{41}\hat{N}b_{52}$ (9/2)	$20\left[\frac{1}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 9\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$51\hat{p}^+(\uparrow), 42\hat{p}^+(\downarrow), 52\hat{e}^-(\uparrow\downarrow)$	9/2
	$^{92}_{42}\hat{M}o_{50}$ (0)	$21\left[\frac{1}{2}\hat{H}e_2(0)\right], 4\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$46\hat{p}^+(\uparrow), 46\hat{p}^+(\downarrow), 50\hat{e}^-(\uparrow\downarrow)$	0
	$^{94}_{42}\hat{M}o_{52}$ (0)	$21\left[\frac{1}{2}\hat{H}e_2(0)\right], 5\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$47\hat{p}^+(\uparrow), 47\hat{p}^+(\downarrow), 52\hat{e}^-(\uparrow\downarrow)$	0
	$^{95}_{42}\hat{M}o_{53}$ (5/2)	$21\left[\frac{1}{2}\hat{H}e_2(0)\right], 8\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$50\hat{p}^+(\uparrow), 45\hat{p}^+(\downarrow), 53\hat{e}^-(\uparrow\downarrow)$	5/2
42	$^{96}_{42}\hat{M}o_{54}$ (0)	$21\left[\frac{1}{2}\hat{H}e_2(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$48\hat{p}^+(\uparrow), 48\hat{p}^+(\downarrow), 54\hat{e}^-(\uparrow\downarrow)$	0
	$^{97}_{42}\hat{M}o_{55}$ (5/2)	$21\left[\frac{1}{2}\hat{H}e_2(0)\right], 9\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$51\hat{p}^+(\uparrow), 46\hat{p}^+(\downarrow), 55\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{98}_{42}\hat{M}o_{56}$ (0)	$21\left[\frac{1}{2}\hat{H}e_2(0)\right], 7\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$49\hat{p}^+(\uparrow), 49\hat{p}^+(\downarrow), 56\hat{e}^-(\uparrow\downarrow)$	0
	$^{100}_{42}\hat{M}o_{58}$ (0)	$21\left[\frac{1}{2}\hat{H}e_2(0)\right], 8\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$50\hat{p}^+(\uparrow), 50\hat{p}^+(\downarrow), 58\hat{e}^-(\uparrow\downarrow)$	0
43	$^{xx}_{43}\hat{T}c_{yy}$ (?)	No Stable Nuclide	No Stable Nuclide	
	$^{96}_{44}\hat{R}u_{52}$ (0)	$22\left[\frac{1}{2}\hat{H}e_2(0)\right], 4\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$48\hat{p}^+(\uparrow), 48\hat{p}^+(\downarrow), 52\hat{e}^-(\uparrow\downarrow)$	0
44	$^{98}_{44}\hat{R}u_{54}$ (0)	$22\left[\frac{1}{2}\hat{H}e_2(0)\right], 5\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$49\hat{p}^+(\uparrow), 49\hat{p}^+(\downarrow), 54\hat{e}^-(\uparrow\downarrow)$	0
	$^{99}_{44}\hat{R}u_{55}$ (5/2)	$22\left[\frac{1}{2}\hat{H}e_2(0)\right], 8\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$52\hat{p}^+(\uparrow), 47\hat{p}^+(\downarrow), 55\hat{e}^-(\uparrow\downarrow)$	5/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
	$^{100}_{44}\hat{R}u_{56}$ (0)	$22\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$50\hat{p}^{+}(\uparrow), 50\hat{p}^{+}(\downarrow), 56\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{101}_{44}\hat{R}u_{57}$ (5/2)	$22\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 9\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$53\hat{p}^{+}(\uparrow), 48\hat{p}^{+}(\downarrow), 57\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{102}_{44}\hat{R}u_{58}$ (0)	$22\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 7\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$51\hat{p}^{+}(\uparrow), 51\hat{p}^{+}(\downarrow), 58\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{104}_{44}\hat{R}u_{60}$ (0)	$22\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$52\hat{p}^{+}(\uparrow), 52\hat{p}^{+}(\downarrow), 60\hat{e}^{-}(\uparrow\downarrow)$	0
45	$^{103}_{45}\hat{R}h_{58}$ (1/2)	$22\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 6\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$52\hat{p}^{+}(\uparrow), 51\hat{p}^{+}(\downarrow), 58\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{102}_{46}\hat{P}d_{56}$ (0)	$23\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 5\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$51\hat{p}^{+}(\uparrow), 51\hat{p}^{+}(\downarrow), 56\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{104}_{46}\hat{P}d_{58}$ (0)	$23\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$52\hat{p}^{+}(\uparrow), 52\hat{p}^{+}(\downarrow), 58\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{105}_{46}\hat{P}d_{59}$ (5/2)	$23\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 9\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$55\hat{p}^{+}(\uparrow), 50\hat{p}^{+}(\downarrow), 59\hat{e}^{-}(\uparrow\downarrow)$	5/2
46	$^{106}_{46}\hat{P}d_{60}$ (0)	$23\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 7\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$53\hat{p}^{+}(\uparrow), 53\hat{p}^{+}(\downarrow), 60\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{108}_{46}\hat{P}d_{62}$ (0)	$23\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 8\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$54\hat{p}^{+}(\uparrow), 54\hat{p}^{+}(\downarrow), 62\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{110}_{46}\hat{P}d_{64}$ (0)	$23\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 9\hat{n}(\uparrow), 9\hat{n}(\downarrow)$	$55\hat{p}^{+}(\uparrow), 55\hat{p}^{+}(\downarrow), 64\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{107}_{47}\hat{A}g_{60}$ (1/2)	$23\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 6\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$54\hat{p}^{+}(\uparrow), 53\hat{p}^{+}(\downarrow), 60\hat{e}^{-}(\uparrow\downarrow)$	1/2
47	$^{109}_{47}\hat{A}g_{62}$ (1/2)	$23\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 7\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$55\hat{p}^{+}(\uparrow), 54\hat{p}^{+}(\downarrow), 62\hat{e}^{-}(\uparrow\downarrow)$	1/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
48	$^{106}_{48}\hat{C}d_{58}$ (0)	$24\left[\frac{4}{2}\hat{H}e_2(0)\right], 5\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$53\hat{p}^+(\uparrow), 53\hat{p}^+(\downarrow), 58\hat{e}^-(\uparrow\downarrow)$	0
	$^{108}_{48}\hat{C}d_{60}$ (0)	$24\left[\frac{4}{2}\hat{H}e_2(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$54\hat{p}^+(\uparrow), 54\hat{p}^+(\downarrow), 60\hat{e}^-(\uparrow\downarrow)$	0
	$^{110}_{48}\hat{C}d_{62}$ (0)	$24\left[\frac{4}{2}\hat{H}e_2(0)\right], 7\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$55\hat{p}^+(\uparrow), 55\hat{p}^+(\downarrow), 62\hat{e}^-(\uparrow\downarrow)$	0
	$^{111}_{48}\hat{C}d_{63}$ (1/2)	$24\left[\frac{4}{2}\hat{H}e_2(0)\right], 8\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$56\hat{p}^+(\uparrow), 55\hat{p}^+(\downarrow), 63\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{112}_{48}\hat{C}d_{64}$ (0)	$24\left[\frac{4}{2}\hat{H}e_2(0)\right], 8\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$56\hat{p}^+(\uparrow), 56\hat{p}^+(\downarrow), 64\hat{e}^-(\uparrow\downarrow)$	0
	$^{113}_{48}\hat{C}d_{65}$ (1/2)	$24\left[\frac{4}{2}\hat{H}e_2(0)\right], 9\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$57\hat{p}^+(\uparrow), 56\hat{p}^+(\downarrow), 65\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{116}_{48}\hat{C}d_{68}$ (0)	$24\left[\frac{4}{2}\hat{H}e_2(0)\right], 10\hat{n}(\uparrow), 10\hat{n}(\downarrow)$	$58\hat{p}^+(\uparrow), 58\hat{p}^+(\downarrow), 68\hat{e}^-(\uparrow\downarrow)$	0
49	$^{113}_{49}\hat{I}n_{64}$ (9/2)	$24\left[\frac{4}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 11\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$61\hat{p}^+(\uparrow), 52\hat{p}^+(\downarrow), 64\hat{e}^-(\uparrow\downarrow)$	9/2
	$^{115}_{49}\hat{I}n_{66}$ (9/2)	$24\left[\frac{4}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 12\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$62\hat{p}^+(\uparrow), 53\hat{p}^+(\downarrow), 66\hat{e}^-(\uparrow\downarrow)$	9/2
50	$^{112}_{50}\hat{S}n_{62}$ (0)	$25\left[\frac{4}{2}\hat{H}e_2(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$56\hat{p}^+(\uparrow), 56\hat{p}^+(\downarrow), 62\hat{e}^-(\uparrow\downarrow)$	0
	$^{114}_{50}\hat{S}n_{64}$ (0)	$25\left[\frac{4}{2}\hat{H}e_2(0)\right], 7\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$57\hat{p}^+(\uparrow), 57\hat{p}^+(\downarrow), 64\hat{e}^-(\uparrow\downarrow)$	0
	$^{115}_{50}\hat{S}n_{65}$ (1/2)	$25\left[\frac{4}{2}\hat{H}e_2(0)\right], 8\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$58\hat{p}^+(\uparrow), 57\hat{p}^+(\downarrow), 65\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{116}_{50}\hat{S}n_{66}$ (0)	$25\left[\frac{4}{2}\hat{H}e_2(0)\right], 8\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$58\hat{p}^+(\uparrow), 58\hat{p}^+(\downarrow), 66\hat{e}^-(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
51	$^{117}_{50}\hat{S}n_{67}$ (1/2)	$25\left[\frac{4}{2}\hat{H}e_2(0)\right], 9\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$59\hat{p}^+(\uparrow),58\hat{p}^+(\downarrow), 67\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{118}_{50}\hat{S}n_{68}$ (0)	$25\left[\frac{4}{2}\hat{H}e_2(0)\right], 9\hat{n}(\uparrow), 9\hat{n}(\downarrow)$	$59\hat{p}^+(\uparrow),59\hat{p}^+(\downarrow), 68\hat{e}^-(\uparrow\downarrow)$	0
	$^{119}_{50}\hat{S}n_{69}$ (1/2)	$25\left[\frac{4}{2}\hat{H}e_2(0)\right], 10\hat{n}(\uparrow), 9\hat{n}(\downarrow)$	$60\hat{p}^+(\uparrow),59\hat{p}^+(\downarrow), 69\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{120}_{50}\hat{S}n_{70}$ (0)	$25\left[\frac{4}{2}\hat{H}e_2(0)\right], 10\hat{n}(\uparrow), 10\hat{n}(\downarrow)$	$60\hat{p}^+(\uparrow),60\hat{p}^+(\downarrow), 70\hat{e}^-(\uparrow\downarrow)$	0
	$^{122}_{50}\hat{S}n_{72}$ (0)	$25\left[\frac{4}{2}\hat{H}e_2(0)\right], 11\hat{n}(\uparrow), 11\hat{n}(\downarrow)$	$61\hat{p}^+(\uparrow),61\hat{p}^+(\downarrow), 72\hat{e}^-(\uparrow\downarrow)$	0
	$^{124}_{50}\hat{S}n_{74}$ (0)	$25\left[\frac{4}{2}\hat{H}e_2(0)\right], 12\hat{n}(\uparrow), 12\hat{n}(\downarrow)$	$62\hat{p}^+(\uparrow),62\hat{p}^+(\downarrow), 74\hat{e}^-(\uparrow\downarrow)$	0
	$^{121}_{51}\hat{S}b_{70}$ (5/2)	$25\left[\frac{4}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 11\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$63\hat{p}^+(\uparrow),58\hat{p}^+(\downarrow), 70\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{123}_{51}\hat{S}b_{72}$ (7/2)	$25\left[\frac{4}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 13\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$65\hat{p}^+(\uparrow),58\hat{p}^+(\downarrow), 72\hat{e}^-(\uparrow\downarrow)$	7/2
52	$^{120}_{52}\hat{T}e_{68}$ (0)	$26\left[\frac{4}{2}\hat{H}e_2(0)\right], 8\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$60\hat{p}^+(\uparrow), 60\hat{p}^+(\downarrow), 68\hat{e}^-(\uparrow\downarrow)$	0
	$^{122}_{52}\hat{T}e_{70}$ (0)	$26\left[\frac{4}{2}\hat{H}e_2(0)\right], 9\hat{n}(\uparrow), 9\hat{n}(\downarrow)$	$61\hat{p}^+(\uparrow),61\hat{p}^+(\downarrow), 70\hat{e}^-(\uparrow\downarrow)$	0
	$^{123}_{52}\hat{T}e_{71}$ (1/2)	$26\left[\frac{4}{2}\hat{H}e_2(0)\right], 10\hat{n}(\uparrow), 9\hat{n}(\downarrow)$	$62\hat{p}^+(\uparrow),61\hat{p}^+(\downarrow), 71\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{124}_{52}\hat{T}e_{72}$ (0)	$26\left[\frac{4}{2}\hat{H}e_2(0)\right], 10\hat{n}(\uparrow), 10\hat{n}(\downarrow)$	$62\hat{p}^+(\uparrow),62\hat{p}^+(\downarrow), 72\hat{e}^-(\uparrow\downarrow)$	0
	$^{125}_{52}\hat{T}e_{73}$ (1/2)	$26\left[\frac{4}{2}\hat{H}e_2(0)\right], 11\hat{n}(\uparrow), 10\hat{n}(\downarrow)$	$63\hat{p}^+(\uparrow),62\hat{p}^+(\downarrow), 73\hat{e}^-(\uparrow\downarrow)$	1/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
	$^{126}_{52}\hat{T}e_{74}$ (0)	$26\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 11\,\hat{n}(\uparrow), 11\,\hat{n}(\downarrow)$	$63\hat{p}^{+}(\uparrow), 63\hat{p}^{+}(\downarrow), 74\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{128}_{52}\hat{T}e_{76}$ (0)	$26\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 12\,\hat{n}(\uparrow), 12\,\hat{n}(\downarrow)$	$64\hat{p}^{+}(\uparrow), 64\hat{p}^{+}(\downarrow), 76\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{130}_{52}\hat{T}e_{78}$ (0)	$26\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 13\,\hat{n}(\uparrow), 13\,\hat{n}(\downarrow)$	$63\hat{p}^{+}(\uparrow), 63\hat{p}^{+}(\downarrow), 74\hat{e}^{-}(\uparrow\downarrow)$	0
53	$^{127}_{53}\hat{I}_{74}$ (5/2)	$26\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 12\,\hat{n}(\uparrow), 9\,\hat{n}(\downarrow)$	$66\hat{p}^{+}(\uparrow), 61\hat{p}^{+}(\downarrow), 74\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{124}_{54}\hat{X}e_{70}$ (0)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 8\,\hat{n}(\uparrow), 8\,\hat{n}(\downarrow)$	$62\hat{p}^{+}(\uparrow), 62\hat{p}^{+}(\downarrow), 70\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{126}_{54}\hat{X}e_{72}$ (0)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 9\,\hat{n}(\uparrow), 9\,\hat{n}(\downarrow)$	$63\hat{p}^{+}(\uparrow), 63\hat{p}^{+}(\downarrow), 72\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{128}_{54}\hat{X}e_{74}$ (0)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 10\,\hat{n}(\uparrow), 10\,\hat{n}(\downarrow)$	$64\hat{p}^{+}(\uparrow), 64\hat{p}^{+}(\downarrow), 74\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{129}_{54}\hat{X}e_{75}$ (1/2)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 11\,\hat{n}(\uparrow), 10\,\hat{n}(\downarrow)$	$65\hat{p}^{+}(\uparrow), 64\hat{p}^{+}(\downarrow), 75\hat{e}^{-}(\uparrow\downarrow)$	1/2
54	$^{130}_{54}\hat{X}e_{76}$ (0)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 11\,\hat{n}(\uparrow), 11\,\hat{n}(\downarrow)$	$65\hat{p}^{+}(\uparrow), 65\hat{p}^{+}(\downarrow), 76\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{131}_{54}\hat{X}e_{77}$ (3/2)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 13\,\hat{n}(\uparrow), 10\,\hat{n}(\downarrow)$	$67\hat{p}^{+}(\uparrow), 64\hat{p}^{+}(\downarrow), 77\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{132}_{54}\hat{X}e_{78}$ (0)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 12\,\hat{n}(\uparrow), 12\,\hat{n}(\downarrow)$	$66\hat{p}^{+}(\uparrow), 66\hat{p}^{+}(\downarrow), 78\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{134}_{54}\hat{X}e_{80}$ (0)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 13\,\hat{n}(\uparrow), 13\,\hat{n}(\downarrow)$	$67\hat{p}^{+}(\uparrow), 67\hat{p}^{+}(\downarrow), 80\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{136}_{54}\hat{X}e_{82}$ (0)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 14\,\hat{n}(\uparrow), 14\,\hat{n}(\downarrow)$	$68\hat{p}^{+}(\uparrow), 68\hat{p}^{+}(\downarrow), 82\hat{e}^{-}(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
55	$^{133}_{55}\hat{C}s_{78}$ (7/2)	$27\left[\frac{4}{2}\hat{H}e_2(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $14\,\hat{n}(\uparrow)$, $9\,\hat{n}(\downarrow)$	$70\hat{p}^{+}(\uparrow),63\hat{p}^{+}(\downarrow)$, $78\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{130}_{56}\hat{B}a_{74}$ (0)	$28\left[\frac{4}{2}\hat{H}e_2(0)\right]$, $9\,\hat{n}(\uparrow)$, $9\,\hat{n}(\downarrow)$	$65\hat{p}^{+}(\uparrow),65\hat{p}^{+}(\downarrow)$, $74\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{132}_{56}\hat{B}a_{76}$ (0)	$28\left[\frac{4}{2}\hat{H}e_2(0)\right]$, $10\,\hat{n}(\uparrow)$, $10\,\hat{n}(\downarrow)$	$66\hat{p}^{+}(\uparrow),66\hat{p}^{+}(\downarrow)$, $76\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{134}_{56}\hat{B}a_{78}$ (0)	$28\left[\frac{4}{2}\hat{H}e_2(0)\right]$, $11\,\hat{n}(\uparrow)$, $11\,\hat{n}(\downarrow)$	$67\hat{p}^{+}(\uparrow),67\hat{p}^{+}(\downarrow)$, $78\hat{e}^{-}(\uparrow\downarrow)$	0
56	$^{135}_{56}\hat{B}a_{79}$ (3/2)	$28\left[\frac{4}{2}\hat{H}e_2(0)\right]$, $13\,\hat{n}(\uparrow)$, $10\,\hat{n}(\downarrow)$	$69\hat{p}^{+}(\uparrow),66\hat{p}^{+}(\downarrow)$, $79\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{136}_{56}\hat{B}a_{80}$ (0)	$28\left[\frac{4}{2}\hat{H}e_2(0)\right]$, $12\,\hat{n}(\uparrow)$, $12\,\hat{n}(\downarrow)$	$68\hat{p}^{+}(\uparrow),68\hat{p}^{+}(\downarrow)$, $80\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{137}_{56}\hat{B}a_{81}$ (3/2)	$28\left[\frac{4}{2}\hat{H}e_2(0)\right]$, $14\,\hat{n}(\uparrow)$, $11\,\hat{n}(\downarrow)$	$70\hat{p}^{+}(\uparrow),67\hat{p}^{+}(\downarrow)$, $81\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{138}_{56}\hat{B}a_{82}$ (0)	$28\left[\frac{4}{2}\hat{H}e_2(0)\right]$, $13\,\hat{n}(\uparrow)$, $13\,\hat{n}(\downarrow)$	$69\hat{p}^{+}(\uparrow),69\hat{p}^{+}(\downarrow)$, $82\hat{e}^{-}(\uparrow\downarrow)$	0
57	$^{138}_{57}\hat{L}a_{81}$ (5)	$28\left[\frac{4}{2}\hat{H}e_2(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $16\,\hat{n}(\uparrow)$, $8\,\hat{n}(\downarrow)$	$74\hat{p}^{+}(\uparrow),64\hat{p}^{+}(\downarrow)$, $81\hat{e}^{-}(\uparrow\downarrow)$	5
	$^{139}_{57}\hat{L}a_{82}$ (7/2)	$28\left[\frac{4}{2}\hat{H}e_2(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $15\,\hat{n}(\uparrow)$, $10\,\hat{n}(\downarrow)$	$73\hat{p}^{+}(\uparrow),66\hat{p}^{+}(\downarrow)$, $82\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{136}_{58}\hat{C}e_{78}$ (0)	$29\left[\frac{4}{2}\hat{H}e_2(0)\right]$, $10\,\hat{n}(\uparrow)$, $10\,\hat{n}(\downarrow)$	$68\hat{p}^{+}(\uparrow),68\hat{p}^{+}(\downarrow)$, $78\hat{e}^{-}(\uparrow\downarrow)$	0
58	$^{138}_{58}\hat{C}e_{80}$ (0)	$29\left[\frac{4}{2}\hat{H}e_2(0)\right]$, $11\,\hat{n}(\uparrow)$, $11\,\hat{n}(\downarrow)$	$69\hat{p}^{+}(\uparrow),69\hat{p}^{+}(\downarrow)$, $80\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{140}_{58}\hat{C}e_{82}$ (0)	$29\left[\frac{4}{2}\hat{H}e_2(0)\right]$, $12\,\hat{n}(\uparrow)$, $12\,\hat{n}(\downarrow)$	$70\hat{p}^{+}(\uparrow),70\hat{p}^{+}(\downarrow)$, $82\hat{e}^{-}(\uparrow\downarrow)$	0

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	$^{142}_{58}\hat{C}e_{84}$ (0)	$29\left[\frac{1}{2}\hat{H}e_2(0)\right], 13\hat{n}(\uparrow), 13\hat{n}(\downarrow)$	$71\hat{p}^+(\uparrow), 71\hat{p}^+(\downarrow), 84\hat{e}^-(\uparrow\downarrow)$	0
59	$^{141}_{59}\hat{P}r_{82}$ (5/2)	$29\left[\frac{1}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 13\hat{n}(\uparrow), 10\hat{n}(\downarrow)$	$73\hat{p}^+(\uparrow), 68\hat{p}^+(\downarrow), 82\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{142}_{60}\hat{N}d_{82}$ (0)	$30\left[\frac{1}{2}\hat{H}e_2(0)\right], 11\hat{n}(\uparrow), 11\hat{n}(\downarrow)$	$71\hat{p}^+(\uparrow), 71\hat{p}^+(\downarrow), 82\hat{e}^-(\uparrow\downarrow)$	0
	$^{143}_{60}\hat{N}d_{83}$ (7/2)	$30\left[\frac{1}{2}\hat{H}e_2(0)\right], 15\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$75\hat{p}^+(\uparrow), 68\hat{p}^+(\downarrow), 83\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{144}_{60}\hat{N}d_{84}$ (0)	$30\left[\frac{1}{2}\hat{H}e_2(0)\right], 12\hat{n}(\uparrow), 12\hat{n}(\downarrow)$	$72\hat{p}^+(\uparrow), 72\hat{p}^+(\downarrow), 84\hat{e}^-(\uparrow\downarrow)$	0
60	$^{145}_{60}\hat{N}d_{85}$ (7/2)	$30\left[\frac{1}{2}\hat{H}e_2(0)\right], 16\hat{n}(\uparrow), 9\hat{n}(\downarrow)$	$76\hat{p}^+(\uparrow), 69\hat{p}^+(\downarrow), 85\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{146}_{60}\hat{N}d_{86}$ (0)	$30\left[\frac{1}{2}\hat{H}e_2(0)\right], 13\hat{n}(\uparrow), 13\hat{n}(\downarrow)$	$73\hat{p}^+(\uparrow), 73\hat{p}^+(\downarrow), 86\hat{e}^-(\uparrow\downarrow)$	0
	$^{148}_{60}\hat{N}d_{88}$ (0)	$30\left[\frac{1}{2}\hat{H}e_2(0)\right], 14\hat{n}(\uparrow), 14\hat{n}(\downarrow)$	$74\hat{p}^+(\uparrow), 74\hat{p}^+(\downarrow), 88\hat{e}^-(\uparrow\downarrow)$	0
	$^{150}_{60}\hat{N}d_{90}$ (0)	$30\left[\frac{1}{2}\hat{H}e_2(0)\right], 15\hat{n}(\uparrow), 15\hat{n}(\downarrow)$	$75\hat{p}^+(\uparrow), 75\hat{p}^+(\downarrow), 90\hat{e}^-(\uparrow\downarrow)$	0
61	$^{xxx}_{61}\hat{P}m_{yy}$ (?)	No Stable Nuclide	No Stable Nuclide	
	$^{144}_{62}\hat{S}m_{82}$ (0)	$31\left[\frac{1}{2}\hat{H}e_2(0)\right], 10\hat{n}(\uparrow), 10\hat{n}(\downarrow)$	$72\hat{p}^+(\uparrow), 72\hat{p}^+(\downarrow), 82\hat{e}^-(\uparrow\downarrow)$	0
62	$^{147}_{62}\hat{S}m_{85}$ (7/2)	$31\left[\frac{1}{2}\hat{H}e_2(0)\right], 15\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$77\hat{p}^+(\uparrow), 70\hat{p}^+(\downarrow), 85\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{148}_{62}\hat{S}m_{86}$ (0)	$31\left[\frac{1}{2}\hat{H}e_2(0)\right], 12\hat{n}(\uparrow), 12\hat{n}(\downarrow)$	$74\hat{p}^+(\uparrow), 74\hat{p}^+(\downarrow), 86\hat{e}^-(\uparrow\downarrow)$	0

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63	$^{149}_{62}\hat{S}m_{87}$ (7/2)	$31\left[\frac{4}{2}\hat{H}e_2(0)\right], 16\hat{n}(\uparrow), 9\hat{n}(\downarrow)$	$78\hat{p}^+(\uparrow), 71\hat{p}^+(\downarrow), 87\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{150}_{62}\hat{S}m_{88}$ (0)	$31\left[\frac{4}{2}\hat{H}e_2(0)\right], 13\hat{n}(\uparrow), 13\hat{n}(\downarrow)$	$75\hat{p}^+(\uparrow), 75\hat{p}^+(\downarrow), 88\hat{e}^-(\uparrow\downarrow)$	0
	$^{152}_{62}\hat{S}m_{90}$ (0)	$31\left[\frac{4}{2}\hat{H}e_2(0)\right], 14\hat{n}(\uparrow), 14\hat{n}(\downarrow)$	$76\hat{p}^+(\uparrow), 76\hat{p}^+(\downarrow), 90\hat{e}^-(\uparrow\downarrow)$	0
	$^{154}_{62}\hat{S}m_{92}$ (0)	$31\left[\frac{4}{2}\hat{H}e_2(0)\right], 15\hat{n}(\uparrow), 15\hat{n}(\downarrow)$	$77\hat{p}^+(\uparrow), 77\hat{p}^+(\downarrow), 92\hat{e}^-(\uparrow\downarrow)$	0
	$^{151}_{63}\hat{E}u_{88}$ (5/2)	$31\left[\frac{4}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 14\hat{n}(\uparrow), 11\hat{n}(\downarrow)$	$78\hat{p}^+(\uparrow), 73\hat{p}^+(\downarrow), 88\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{153}_{63}\hat{E}u_{90}$ (5/2)	$31\left[\frac{4}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 15\hat{n}(\uparrow), 12\hat{n}(\downarrow)$	$79\hat{p}^+(\uparrow), 74\hat{p}^+(\downarrow), 90\hat{e}^-(\uparrow\downarrow)$	5/2
64	$^{152}_{64}\hat{G}d_{88}$ (0)	$32\left[\frac{4}{2}\hat{H}e_2(0)\right], 12\hat{n}(\uparrow), 12\hat{n}(\downarrow)$	$76\hat{p}^+(\uparrow), 76\hat{p}^+(\downarrow), 88\hat{e}^-(\uparrow\downarrow)$	0
	$^{154}_{64}\hat{G}d_{90}$ (0)	$32\left[\frac{4}{2}\hat{H}e_2(0)\right], 13\hat{n}(\uparrow), 13\hat{n}(\downarrow)$	$77\hat{p}^+(\uparrow), 77\hat{p}^+(\downarrow), 90\hat{e}^-(\uparrow\downarrow)$	0
	$^{155}_{64}\hat{G}d_{91}$ (3/2)	$32\left[\frac{4}{2}\hat{H}e_2(0)\right], 15\hat{n}(\uparrow), 12\hat{n}(\downarrow)$	$79\hat{p}^+(\uparrow), 76\hat{p}^+(\downarrow), 91\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{156}_{64}\hat{G}d_{92}$ (0)	$32\left[\frac{4}{2}\hat{H}e_2(0)\right], 14\hat{n}(\uparrow), 14\hat{n}(\downarrow)$	$78\hat{p}^+(\uparrow), 78\hat{p}^+(\downarrow), 92\hat{e}^-(\uparrow\downarrow)$	0
	$^{157}_{64}\hat{G}d_{93}$ (3/2)	$32\left[\frac{4}{2}\hat{H}e_2(0)\right], 16\hat{n}(\uparrow), 13\hat{n}(\downarrow)$	$80\hat{p}^+(\uparrow), 77\hat{p}^+(\downarrow), 93\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{158}_{64}\hat{G}d_{94}$ (0)	$32\left[\frac{4}{2}\hat{H}e_2(0)\right], 15\hat{n}(\uparrow), 15\hat{n}(\downarrow)$	$79\hat{p}^+(\uparrow), 79\hat{p}^+(\downarrow), 94\hat{e}^-(\uparrow\downarrow)$	0
	$^{160}_{64}\hat{G}d_{96}$ (0)	$32\left[\frac{4}{2}\hat{H}e_2(0)\right], 16\hat{n}(\uparrow), 16\hat{n}(\downarrow)$	$80\hat{p}^+(\uparrow), 80\hat{p}^+(\downarrow), 96\hat{e}^-(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
65	$^{159}_{65}\hat{T}b_{94}$ (3/2)	$32\left[\frac{1}{2}\hat{H}e_2(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $15\hat{n}(\uparrow)$, $14\hat{n}(\downarrow)$	$81\hat{p}^+(\uparrow), 78\hat{p}^+(\downarrow)$, $94\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{156}_{66}\hat{D}y_{90}$ (0)	$33\left[\frac{1}{2}\hat{H}e_2(0)\right]$, $12\hat{n}(\uparrow)$, $12\hat{n}(\downarrow)$	$78\hat{p}^+(\uparrow), 78\hat{p}^+(\downarrow)$, $90\hat{e}^-(\uparrow\downarrow)$	0
	$^{158}_{66}\hat{D}y_{92}$ (0)	$33\left[\frac{1}{2}\hat{H}e_2(0)\right]$, $13\hat{n}(\uparrow)$, $13\hat{n}(\downarrow)$	$79\hat{p}^+(\uparrow), 79\hat{p}^+(\downarrow)$, $92\hat{e}^-(\uparrow\downarrow)$	0
	$^{160}_{66}\hat{D}y_{94}$ (0)	$33\left[\frac{1}{2}\hat{H}e_2(0)\right]$, $14\hat{n}(\uparrow)$, $14\hat{n}(\downarrow)$	$80\hat{p}^+(\uparrow), 80\hat{p}^+(\downarrow)$, $94\hat{e}^-(\uparrow\downarrow)$	0
66	$^{161}_{66}\hat{D}y_{95}$ (5/2)	$33\left[\frac{1}{2}\hat{H}e_2(0)\right]$, $17\hat{n}(\uparrow)$, $12\hat{n}(\downarrow)$	$83\hat{p}^+(\uparrow), 78\hat{p}^+(\downarrow)$, $95\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{162}_{66}\hat{D}y_{96}$ (0)	$33\left[\frac{1}{2}\hat{H}e_2(0)\right]$, $15\hat{n}(\uparrow)$, $15\hat{n}(\downarrow)$	$81\hat{p}^+(\uparrow), 81\hat{p}^+(\downarrow)$, $96\hat{e}^-(\uparrow\downarrow)$	0
	$^{163}_{66}\hat{D}y_{97}$ (5/2)	$33\left[\frac{1}{2}\hat{H}e_2(0)\right]$, $18\hat{n}(\uparrow)$, $13\hat{n}(\downarrow)$	$84\hat{p}^+(\uparrow), 79\hat{p}^+(\downarrow)$, $97\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{164}_{66}\hat{D}y_{98}$ (0)	$33\left[\frac{1}{2}\hat{H}e_2(0)\right]$, $16\hat{n}(\uparrow)$, $16\hat{n}(\downarrow)$	$82\hat{p}^+(\uparrow), 82\hat{p}^+(\downarrow)$, $98\hat{e}^-(\uparrow\downarrow)$	0
67	$^{165}_{67}\hat{H}o_{98}$ (7/2)	$33\left[\frac{1}{2}\hat{H}e_2(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $18\hat{n}(\uparrow)$, $13\hat{n}(\downarrow)$	$86\hat{p}^+(\uparrow), 79\hat{p}^+(\downarrow)$, $98\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{162}_{68}\hat{E}r_{94}$ (0)	$34\left[\frac{1}{2}\hat{H}e_2(0)\right]$, $13\hat{n}(\uparrow)$, $13\hat{n}(\downarrow)$	$81\hat{p}^+(\uparrow), 81\hat{p}^+(\downarrow)$, $94\hat{e}^-(\uparrow\downarrow)$	0
	$^{164}_{68}\hat{E}r_{96}$ (0)	$34\left[\frac{1}{2}\hat{H}e_2(0)\right]$, $14\hat{n}(\uparrow)$, $14\hat{n}(\downarrow)$	$82\hat{p}^+(\uparrow), 82\hat{p}^+(\downarrow)$, $96\hat{e}^-(\uparrow\downarrow)$	0
68	$^{166}_{68}\hat{E}r_{98}$ (0)	$34\left[\frac{1}{2}\hat{H}e_2(0)\right]$, $15\hat{n}(\uparrow)$, $15\hat{n}(\downarrow)$	$83\hat{p}^+(\uparrow), 83\hat{p}^+(\downarrow)$, $98\hat{e}^-(\uparrow\downarrow)$	0
	$^{167}_{68}\hat{E}r_{99}$ (7/2)	$34\left[\frac{1}{2}\hat{H}e_2(0)\right]$, $19\hat{n}(\uparrow)$, $12\hat{n}(\downarrow)$	$87\hat{p}^+(\uparrow), 80\hat{p}^+(\downarrow)$, $99\hat{e}^-(\uparrow\downarrow)$	7/2

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69	${}^{168}_{68}\hat{E}r_{100}$ (0)	$34\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 16\hat{n}(\uparrow), 16\hat{n}(\downarrow)$	$84\hat{p}^{+}(\uparrow), 84\hat{p}^{+}(\downarrow), 100\hat{e}^{-}(\uparrow\downarrow)$	0
	${}^{170}_{68}\hat{E}r_{102}$ (0)	$34\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 17\hat{n}(\uparrow), 17\hat{n}(\downarrow)$	$86\hat{p}^{+}(\uparrow), 86\hat{p}^{+}(\downarrow), 102\hat{e}^{-}(\uparrow\downarrow)$	0
	${}^{169}_{69}\hat{T}m_{100}$ (1/2)	$34\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 16\hat{n}(\uparrow), 15\hat{n}(\downarrow)$	$85\hat{p}^{+}(\uparrow), 84\hat{p}^{+}(\downarrow), 100\hat{e}^{-}(\uparrow\downarrow)$	1/2
70	${}^{168}_{70}\hat{Y}b_{98}$ (0)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 14\hat{n}(\uparrow), 14\hat{n}(\downarrow)$	$84\hat{p}^{+}(\uparrow), 84\hat{p}^{+}(\downarrow), 98\hat{e}^{-}(\uparrow\downarrow)$	0
	${}^{170}_{70}\hat{Y}b_{100}$ (0)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 15\hat{n}(\uparrow), 15\hat{n}(\downarrow)$	$85\hat{p}^{+}(\uparrow), 85\hat{p}^{+}(\downarrow), 100\hat{e}^{-}(\uparrow\downarrow)$	0
	${}^{171}_{70}\hat{Y}b_{101}$ (1/2)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 16\hat{n}(\uparrow), 15\hat{n}(\downarrow)$	$86\hat{p}^{+}(\uparrow), 85\hat{p}^{+}(\downarrow), 101\hat{e}^{-}(\uparrow\downarrow)$	1/2
	${}^{172}_{70}\hat{Y}b_{102}$ (0)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 16\hat{n}(\uparrow), 16\hat{n}(\downarrow)$	$86\hat{p}^{+}(\uparrow), 86\hat{p}^{+}(\downarrow), 102\hat{e}^{-}(\uparrow\downarrow)$	0
	${}^{173}_{70}\hat{Y}b_{103}$ (5/2)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 19\hat{n}(\uparrow), 14\hat{n}(\downarrow)$	$89\hat{p}^{+}(\uparrow), 84\hat{p}^{+}(\downarrow), 103\hat{e}^{-}(\uparrow\downarrow)$	5/2
	${}^{174}_{70}\hat{Y}b_{104}$ (0)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 17\hat{n}(\uparrow), 17\hat{n}(\downarrow)$	$87\hat{p}^{+}(\uparrow), 87\hat{p}^{+}(\downarrow), 104\hat{e}^{-}(\uparrow\downarrow)$	0
	${}^{176}_{70}\hat{Y}b_{106}$ (0)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 18\hat{n}(\uparrow), 18\hat{n}(\downarrow)$	$88\hat{p}^{+}(\uparrow), 88\hat{p}^{+}(\downarrow), 106\hat{e}^{-}(\uparrow\downarrow)$	0
71	${}^{175}_{71}\hat{L}u_{104}$ (7/2)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 19\hat{n}(\uparrow), 14\hat{n}(\downarrow)$	$91\hat{p}^{+}(\uparrow), 84\hat{p}^{+}(\downarrow), 104\hat{e}^{-}(\uparrow\downarrow)$	7/2
	${}^{176}_{71}\hat{L}u_{105}$ (7)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 23\hat{n}(\uparrow), 11\hat{n}(\downarrow)$	$95\hat{p}^{+}(\uparrow), 81\hat{p}^{+}(\downarrow), 105\hat{e}^{-}(\uparrow\downarrow)$	7
72	${}^{174}_{72}\hat{H}f_{102}$ (0)	$36\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 15\hat{n}(\uparrow), 15\hat{n}(\downarrow)$	$87\hat{p}^{+}(\uparrow), 87\hat{p}^{+}(\downarrow), 102\hat{e}^{-}(\uparrow\downarrow)$	0

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	$^{176}_{72}\hat{H}f_{104}$ (0)	$36\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 16\hat{n}(\uparrow), 16\hat{n}(\downarrow)$	$88\hat{p}^{+}(\uparrow), 88\hat{p}^{+}(\downarrow), 104\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{177}_{72}\hat{H}f_{105}$ (7/2)	$36\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 20\hat{n}(\uparrow), 13\hat{n}(\downarrow)$	$92\hat{p}^{+}(\uparrow), 85\hat{p}^{+}(\downarrow), 105\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{178}_{72}\hat{H}f_{106}$ (0)	$36\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 17\hat{n}(\uparrow), 17\hat{n}(\downarrow)$	$89\hat{p}^{+}(\uparrow), 89\hat{p}^{+}(\downarrow), 106\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{179}_{72}\hat{H}f_{107}$ (9/2)	$36\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 22\hat{n}(\uparrow), 13\hat{n}(\downarrow)$	$94\hat{p}^{+}(\uparrow), 85\hat{p}^{+}(\downarrow), 107\hat{e}^{-}(\uparrow\downarrow)$	9/2
	$^{180}_{72}\hat{H}f_{108}$ (0)	$36\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 18\hat{n}(\uparrow), 18\hat{n}(\downarrow)$	$90\hat{p}^{+}(\uparrow), 90\hat{p}^{+}(\downarrow), 108\hat{e}^{-}(\uparrow\downarrow)$	0
73	$^{181}_{73}\hat{T}a_{108}$ (7/2)	$36\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 20\hat{n}(\uparrow), 15\hat{n}(\downarrow)$	$94\hat{p}^{+}(\uparrow), 87\hat{p}^{+}(\downarrow), 108\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{180}_{74}\hat{W}_{106}$ (0)	$37\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 16\hat{n}(\uparrow), 16\hat{n}(\downarrow)$	$90\hat{p}^{+}(\uparrow), 90\hat{p}^{+}(\downarrow), 106\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{182}_{74}\hat{W}_{108}$ (0)	$37\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 17\hat{n}(\uparrow), 17\hat{n}(\downarrow)$	$91\hat{p}^{+}(\uparrow), 91\hat{p}^{+}(\downarrow), 108\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{183}_{74}\hat{W}_{109}$ (1/2)	$37\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 18\hat{n}(\uparrow), 17\hat{n}(\downarrow)$	$92\hat{p}^{+}(\uparrow), 91\hat{p}^{+}(\downarrow), 109\hat{e}^{-}(\uparrow\downarrow)$	1/2
74	$^{184}_{74}\hat{W}_{110}$ (0)	$37\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 18\hat{n}(\uparrow), 18\hat{n}(\downarrow)$	$92\hat{p}^{+}(\uparrow), 92\hat{p}^{+}(\downarrow), 110\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{186}_{74}\hat{W}_{112}$ (0)	$37\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 19\hat{n}(\uparrow), 19\hat{n}(\downarrow)$	$93\hat{p}^{+}(\uparrow), 93\hat{p}^{+}(\downarrow), 112\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{185}_{75}\hat{R}e_{110}$ (5/2)	$37\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 19\hat{n}(\uparrow), 16\hat{n}(\downarrow)$	$95\hat{p}^{+}(\uparrow), 90\hat{p}^{+}(\downarrow), 110\hat{e}^{-}(\uparrow\downarrow)$	5/2
75	$^{187}_{75}\hat{R}e_{112}$ (5/2)	$37\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 20\hat{n}(\uparrow), 17\hat{n}(\downarrow)$	$96\hat{p}^{+}(\uparrow), 91\hat{p}^{+}(\downarrow), 112\hat{e}^{-}(\uparrow\downarrow)$	5/2

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76	$^{184}_{76}\hat{O}s_{108}$ (0)	$38\left[\frac{4}{2}\hat{H}e_2(0)\right], 16\hat{n}(\uparrow), 16\hat{n}(\downarrow)$	$92\hat{p}^+(\uparrow), 92\hat{p}^+(\downarrow), 108\hat{e}^-(\uparrow\downarrow)$	0
	$^{186}_{76}\hat{O}s_{110}$ (0)	$38\left[\frac{4}{2}\hat{H}e_2(0)\right], 17\hat{n}(\uparrow), 17\hat{n}(\downarrow)$	$93\hat{p}^+(\uparrow), 93\hat{p}^+(\downarrow), 110\hat{e}^-(\uparrow\downarrow)$	0
	$^{187}_{76}\hat{O}s_{111}$ (1/2)	$38\left[\frac{4}{2}\hat{H}e_2(0)\right], 18\hat{n}(\uparrow), 17\hat{n}(\downarrow)$	$94\hat{p}^+(\uparrow), 93\hat{p}^+(\downarrow), 111\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{188}_{76}\hat{O}s_{112}$ (0)	$38\left[\frac{4}{2}\hat{H}e_2(0)\right], 18\hat{n}(\uparrow), 18\hat{n}(\downarrow)$	$94\hat{p}^+(\uparrow), 94\hat{p}^+(\downarrow), 112\hat{e}^-(\uparrow\downarrow)$	0
	$^{189}_{76}\hat{O}s_{113}$ (3/2)	$38\left[\frac{4}{2}\hat{H}e_2(0)\right], 20\hat{n}(\uparrow), 17\hat{n}(\downarrow)$	$96\hat{p}^+(\uparrow), 93\hat{p}^+(\downarrow), 113\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{190}_{76}\hat{O}s_{114}$ (0)	$38\left[\frac{4}{2}\hat{H}e_2(0)\right], 19\hat{n}(\uparrow), 19\hat{n}(\downarrow)$	$95\hat{p}^+(\uparrow), 95\hat{p}^+(\downarrow), 114\hat{e}^-(\uparrow\downarrow)$	0
	$^{192}_{76}\hat{O}s_{116}$ (0)	$38\left[\frac{4}{2}\hat{H}e_2(0)\right], 20\hat{n}(\uparrow), 20\hat{n}(\downarrow)$	$96\hat{p}^+(\uparrow), 96\hat{p}^+(\downarrow), 116\hat{e}^-(\uparrow\downarrow)$	0
77	$^{191}_{77}\hat{I}r_{114}$ (3/2)	$38\left[\frac{4}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 19\hat{n}(\uparrow), 18\hat{n}(\downarrow)$	$97\hat{p}^+(\uparrow), 94\hat{p}^+(\downarrow), 114\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{193}_{77}\hat{I}r_{116}$ (3/2)	$38\left[\frac{4}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 20\hat{n}(\uparrow), 19\hat{n}(\downarrow)$	$98\hat{p}^+(\uparrow), 95\hat{p}^+(\downarrow), 116\hat{e}^-(\uparrow\downarrow)$	3/2
78	$^{192}_{78}\hat{P}t_{114}$ (0)	$39\left[\frac{4}{2}\hat{H}e_2(0)\right], 18\hat{n}(\uparrow), 18\hat{n}(\downarrow)$	$96\hat{p}^+(\uparrow), 96\hat{p}^+(\downarrow), 114\hat{e}^-(\uparrow\downarrow)$	0
	$^{194}_{78}\hat{P}t_{116}$ (0)	$39\left[\frac{4}{2}\hat{H}e_2(0)\right], 19\hat{n}(\uparrow), 19\hat{n}(\downarrow)$	$97\hat{p}^+(\uparrow), 97\hat{p}^+(\downarrow), 116\hat{e}^-(\uparrow\downarrow)$	0
	$^{195}_{78}\hat{P}t_{117}$ (1/2)	$39\left[\frac{4}{2}\hat{H}e_2(0)\right], 20\hat{n}(\uparrow), 19\hat{n}(\downarrow)$	$98\hat{p}^+(\uparrow), 97\hat{p}^+(\downarrow), 117\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{196}_{78}\hat{P}t_{118}$ (0)	$39\left[\frac{4}{2}\hat{H}e_2(0)\right], 20\hat{n}(\uparrow), 20\hat{n}(\downarrow)$	$98\hat{p}^+(\uparrow), 98\hat{p}^+(\downarrow), 118\hat{e}^-(\uparrow\downarrow)$	0

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79	$^{198}_{78}\hat{P}t_{120}$ (0)	$39\left[\frac{4}{2}\hat{H}e_2(0)\right], 21\hat{n}(\uparrow), 21\hat{n}(\downarrow)$	$99\hat{p}^+(\uparrow),99\hat{p}^+(\downarrow), 120\hat{e}^-(\uparrow\downarrow)$	0
	$^{197}_{79}\hat{A}u_{118}$ (3/2)	$39\left[\frac{4}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 20\hat{n}(\uparrow), 19\hat{n}(\downarrow)$	$100\hat{p}^+(\uparrow),97\hat{p}^+(\downarrow), 118\hat{e}^-(\uparrow\downarrow)$	3/2
80	$^{196}_{80}\hat{H}g_{116}$ (0)	$40\left[\frac{4}{2}\hat{H}e_2(0)\right], 18\hat{n}(\uparrow), 18\hat{n}(\downarrow)$	$98\hat{p}^+(\uparrow),98\hat{p}^+(\downarrow), 116\hat{e}^-(\uparrow\downarrow)$	0
	$^{198}_{80}\hat{H}g_{118}$ (0)	$40\left[\frac{4}{2}\hat{H}e_2(0)\right], 19\hat{n}(\uparrow), 19\hat{n}(\downarrow)$	$99\hat{p}^+(\uparrow),99\hat{p}^+(\downarrow), 118\hat{e}^-(\uparrow\downarrow)$	0
	$^{199}_{80}\hat{H}g_{119}$ (1/2)	$40\left[\frac{4}{2}\hat{H}e_2(0)\right], 20\hat{n}(\uparrow), 19\hat{n}(\downarrow)$	$100\hat{p}^+(\uparrow),99\hat{p}^+(\downarrow), 119\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{200}_{80}\hat{H}g_{120}$ (0)	$40\left[\frac{4}{2}\hat{H}e_2(0)\right], 20\hat{n}(\uparrow), 20\hat{n}(\downarrow)$	$100\hat{p}^+(\uparrow),100\hat{p}^+(\downarrow), 120\hat{e}^-(\uparrow\downarrow)$	0
	$^{201}_{80}\hat{H}g_{121}$ (3/2)	$40\left[\frac{4}{2}\hat{H}e_2(0)\right], 22\hat{n}(\uparrow), 19\hat{n}(\downarrow)$	$102\hat{p}^+(\uparrow),99\hat{p}^+(\downarrow), 121\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{202}_{80}\hat{H}g_{122}$ (0)	$40\left[\frac{4}{2}\hat{H}e_2(0)\right], 21\hat{n}(\uparrow), 21\hat{n}(\downarrow)$	$101\hat{p}^+(\uparrow),101\hat{p}^+(\downarrow), 122\hat{e}^-(\uparrow\downarrow)$	0
	$^{204}_{80}\hat{H}g_{124}$ (0)	$40\left[\frac{4}{2}\hat{H}e_2(0)\right], 22\hat{n}(\uparrow), 22\hat{n}(\downarrow)$	$102\hat{p}^+(\uparrow),102\hat{p}^+(\downarrow), 124\hat{e}^-(\uparrow\downarrow)$	0
81	$^{203}_{81}\hat{T}l_{122}$ (1/2)	$40\left[\frac{4}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 20\hat{n}(\uparrow), 21\hat{n}(\downarrow)$	$102\hat{p}^+(\uparrow),101\hat{p}^+(\downarrow), 122\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{205}_{81}\hat{T}l_{124}$ (1/2)	$40\left[\frac{4}{2}\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 21\hat{n}(\uparrow), 22\hat{n}(\downarrow)$	$103\hat{p}^+(\uparrow),102\hat{p}^+(\downarrow), 124\hat{e}^-(\uparrow\downarrow)$	1/2
82	$^{204}_{82}\hat{P}b_{122}$ (0)	$41\left[\frac{4}{2}\hat{H}e_2(0)\right], 20\hat{n}(\uparrow), 20\hat{n}(\downarrow)$	$102\hat{p}^+(\uparrow),102\hat{p}^+(\downarrow), 122\hat{e}^-(\uparrow\downarrow)$	0
	$^{206}_{82}\hat{P}b_{124}$ (0)	$41\left[\frac{4}{2}\hat{H}e_2(0)\right], 21\hat{n}(\uparrow), 21\hat{n}(\downarrow)$	$103\hat{p}^+(\uparrow),103\hat{p}^+(\downarrow), 124\hat{e}^-(\uparrow\downarrow)$	0
	$^{207}_{82}\hat{P}b_{125}$ (1/2)	$41\left[\frac{4}{2}\hat{H}e_2(0)\right], 22\hat{n}(\uparrow), 21\hat{n}(\downarrow)$	$104\hat{p}^+(\uparrow),103\hat{p}^+(\downarrow), 125\hat{e}^-(\uparrow\downarrow)$	1/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
	$^{208}_{82}\hat{P}b_{126}$ (0)	$41\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 22\,\hat{n}(\uparrow), 22\,\hat{n}(\downarrow)$	$104\hat{p}^{+}(\uparrow), 104\hat{p}^{+}(\downarrow), 126\hat{e}^{-}(\uparrow\downarrow)$	0

14. Some Observations

The purpose of presenting the nuclear configuration in terms of isonucleons in this paper is to make available adequate ground so that one can attempt to (i) develop a theory of nuclear stability and (ii) acquire understanding of other nuclear properties of the stable nuclides. In Table 2 we have proton as the first entry and 278 stable, primordial and very long lived isonuclides up to and including atomic number 82 of the periodic table. Beyond Pb all elements are radioactive. There are two elements namely Tc (Z=43) and Pm (Z = 61) having no stable isotopes, that has also been mentioned in Table 2. The columns 3, 4 and 5 of Table 2 depict respectively the hadronic mechanics based nuclear configuration of models I and II, and experimental nuclear spin of the isonuclide. In the present Section 14 we summarize our observations on them.

14.1. Nuclear Configuration of Model-I

14.1.1. Proton

In Table 2 there are two entries corresponding to Z=1. They are the isotopes of hydrogen, the first element of the periodic table.

Thus, ${}^{1}_{1}H_{0}$, in fact, is the proton, the fundamental particle, which is a stable particle. For its description no hadronic mechanics is required, hence it is not an isonuclide.

14.1.2. Isodeuteron

Hydrogen of mass number 2 is conventionally termed as deuterium. Its nucleus, indeed, is an isonucleus hence it is termed as isodeuteron that gets represented as ${}^{2}_{1}\hat{H}_{1}$. We represent this system in our proposed notation as,

$$\begin{aligned} &{}^{2}_{1}H_{1} : n = 1,\ p^{+} = 1\} \Rightarrow \\ &\left[\left(\hat{p}^{+}(\uparrow), \hat{e}^{-}(J=0), \hat{p}^{+}(\uparrow)\right)\right]_{hm} \\ &\equiv {}^{2}_{1}H_{1}(1) \equiv \hat{d}(\uparrow\uparrow),\ \text{stable},\ J=1 \end{aligned} \tag{94}$$

How the nuclear spin of value 1 for isodeuteron originates gets easily understood from Figure 9.

14.1.3. Other Stable Isonuclides of Table 2

All the stable isonuclides of Table 2 beyond hydrogen are the combination of isodeuterons and isoneutrons except He-3 which consists of an isodeuteron and an isoproton.

I. Stable Isonuclides with Null Nuclear Spin

Out of total of 278 isonuclides of Table 2 there we have 163 isonuclides having nuclear spin of 0. From this group 9 isonuclides consists only of all spin paired isodeuterons, and they can be considered as possessing 1, 3, 4, 5, 6, 7, 8, 9 and 10 ${}^{4}_{2}\hat{B}e_{2}(0)$ centers. Notice that the number 2 is notoriously missing in this list. That corresponds to the isonuclide ${}^{8}_{4}\hat{B}e_{4}(0)$ that we know is unstable and instantaneously disintegrates to α-particles. The same observation in terms of isodeuterons speaks as follows. Recall that the isodeuteron is a stable combination of isonucleons. However, two isodeuterons in the singlet coupling are also stable, which actually is the α-particle. Next on addition of one isodeuteron to it there we form an isonuclide of Li-6, which also is a stable isonuclide. But on further adding one more isodeuteron with total nuclear spin zero we obtain Be-8 isonuclide which is unstable. Thus we see that three isodeuteron in low spin state is stable but the four isodeuteron in zero spin state is unstable (But notice that in the case of stable Be-9 there we have two parallel spin isodeuterons coupled with one isoneutron of opposite spin. It means that the addition of one isoneutron to Be-8 forces one spin paired isodeuterons to assume parallel spin and itself combines to them with opposite spin that imparts stability to Be-9 with net nuclear spin of 3/2.). However, the next stable isonuclide is B-10 consisting of 5 isodeuterons. But in this case there we have two spin paired isodeuterons and three unpaired ones, ironically which is not a combination of 2α-particles and one isodeuteron similar to Li-6. The next stable isonuclide is C-12 that consists of 6 isodeuterons in the spin paired state, which is equivalent to strongly bound combination of 3α-particles. It is surprising that the combination of 2α-particles is unstable but the combination of 3α-particles is stable one. Here onwards 4 to 10α-particles combination are all stable ones.

The remaining 154 isonuclides with null nuclear spin consist of even number of isodeuterons and even number of isoneutrons and they are all spin paired. Notice that not only the isoneutrons get stabilized but also the zero spin di-isoneutrons are getting stabilized in the environment of zero spin isodeuterons. Amongst them from Ca-42 and onwards we have the combination of di-isoneutrons and isodeuterons. The number of spin paired di-isoneuterons continuously increases and rises ultimately to 22 in number in the case of Pb-208 that consists of 82 spin paired isodeuterons. Recall that a dineutron is not a stable entity but 22 spin paired di-isoneutrons of Pb-208 in the presence of 41 α-particles are stable. We need to investigate further what interactions are responsible for this extraordinary stability. However, we also need to take into account the nuclear configuration of adjacent unstable isonuclide. For example, Ca-40 and Ca-42 are both stable but Ca-39 and Ca-41 are unstable nuclides and Ca-43 is stable one. The nuclear configuration commensurate with the observed nuclear spin of Ca-39 and Ca-41 respectively are:

$$9\left[{}^{4}_{2}\hat{He}_{2}(0)\right],\hat{p}^{+}(\uparrow),\hat{d}(\uparrow\uparrow)$$

and

$$8\left[{}^{4}_{2}\hat{He}_{2}(0)\right],4\hat{d}(\uparrow\uparrow),\hat{n}(\downarrow)$$

They both are unstable nuclear configurations. Thus in going from Ca-39 to Ca-40 the nuclear configuration transforms as

$$9\left[{}^{4}_{2}\hat{He}_{2}(0)\right],\hat{p}^{+}(\uparrow),\hat{d}(\uparrow\uparrow)$$

$$\downarrow$$

$$9\left[{}^{4}_{2}\hat{He}_{2}(0)\right],\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\equiv 10\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$$

That is, in Ca-39 last pair of an isodeuteron and a proton are in high spin state but on the availability of one more isoneutron in case of Ca-40 not only one additional isodeuteron gets formed but also both the isodeuterons get spin paired, imparting stability.

Whereas in going from Ca-40 to Ca-41 the nuclear configuration transforms as

$$10\left[{}^{4}_{2}\hat{He}_{2}(0)\right]\rightarrow 8\left[{}^{4}_{2}\hat{He}_{2}(0)\right],4\hat{d}(\uparrow\uparrow),\hat{n}(\downarrow)$$

That is, 2α-particles out of 10 in Ca-40 become 4 spin unpaired isodeuterons on the availability of an additional isoneutron in the case of Ca-41 and at the same time the additional isoneutron orients its spin opposite to that of the isodeuterons and the result is nuclear instability of Ca-41. Next we observe that when we add one isodeuteron to Ca-41 to form Ca-42 strikingly the four spin unpaired isodeuterons of the former isonuclide as well as the two isoneutrons get spin paired to form Ca-42, a stable isonuclide.

Further, on going from Ca-42 to Ca-43 we see from Table 2 that one α-particle out of 10 of Ca-42 gets spin unpaired providing two isodeuterons and simultaneously the spin paired isoneutrons of Ca-42 become spin unpaired to form three spin unpaired isoneutrons of Ca-43. Still Ca-43 is a stable isonuclide. Moreover, the reason of the nuclear stability of Ca-44 seems to be the same as that of Ca-42 because in the former we have one spin paired di-isoneutron whereas in the latter case we have two spin paired di-isoneutrons.

It seems from the above observations that the spin pairing of the isonucleons is not the only parameter that determines the nuclear stability. Other factors need to be identified. This would get further substantiated by considering the stable isonuclides with non-zero nuclear spin in next subsection.

II. Stable Isonuclides with Non-zero Nuclear Spin

Moreover, there are 104 stable isonuclides (in addition to isodeuteron) in Table 2 with non-zero nuclear spin. All have the combination of isodeuterons and isoneutrons except He-3, which consists of one isodeuteron with both its spins up and an isoproton with spin down.

1. Notice that in Table 2 there we have highest nuclear spin of 7 (Lu-176). There also we have isonuclides with nuclear spins 6 (V-50), 5 (La-138) and 9/2 (Ge-73, Kr-83, Sr-87, Nb-93, In-113, In-115 and Hf-179). That is even though the spins are parallel the isonuclides are stable.
2. Also we notice that three parallel spin isodeuterons in the environment of α-particles are also stable they are B-10, K-40 (it also has 2 parallel spin isoneutrons) and Sc-45 (it also consists of one parallel spin isoneutron and one spin zero di-isoneutron).
3. In addition to these parallel spin high spin states there we have various combination of parallel spin isodeuterons combined with parallel or opposite spin isoneutrons resulting in the intermediate nuclear spins from 1/2 to 7/2.
4. As we know that an isolated single isoneutron is unstable but it gets stabilized in the form of an isodeuteron on the one hand but on the other hand it also gets stabilized in the environment of spin paired isodeuterons. This is the case of the nuclear spin of 1/2 of the isonuclides due only to a single isoneutron. From Table 2 we find that-
   a) in the cases of C-13 and Si-29 we have a single isoneutron in the environment of 3 and 7α-particles and both the isonuclides are stable.
   b) Another set of stable isonuclides with a single isoneutron consist of Fe-57, Se-77, Sn-115, Sn-117, Sn-119, Te-125, Xe-129, Xe-131, Yb-171, W-183, Os-187, Hg-199 and Pb-207. These isonuclides offer the environment of spin paired isodeuterons along with the spin paired isoneutrons to the last isoneutron resulting in the stability of the last isoneutron.
5. Another set of stabilized single isoneutron is in combination with high spin state of isodeuterons in the environment of α-particles. We list them as follows.
   (a). The cases of a single isoneutron in the environment of α-particles along with a single isodeuteron are of two types. The high spin (that is the net nuclear spin of 3/2) states are Li-7, B-11, Na-23, Cl-35 and K-39. The low spin (that is the net nuclear spin of 1/2) stable isonuclides are N-15, F-19 and P-31.
   (b). The cases of a single isoneutron in the environment of α-particles along with two parallel spin isodeuterons are also of two types. The high spin (that is the net nuclear spin of 5/2) stable states are O-17 and Mg-25. The low spin (that is the net nuclear spin of 3/2) stable states are Be-9, Ne-21 and S-33.
   (c). The cases of a single isoneutron in the environment of α-particles along with three parallel spin isodeuterons are two in number. The high spin (that is net nuclear spin of 7/2) state is Sc-45 and the low spin (that is net nuclear spin of 5/2) state is Al-27.
   (d). We have already seen in Section 14.1.3.1 that spin paired isoneutrons get stabilized in the environment of α-particles. Now we find that the combination of one isodeuteron and one isoneutron also get stabilized in the environment of α-particles when accompanied by

the zero spin di-isoneutrons. The corresponding isotopes with high spin (the net nuclear spin of 3/2) are Cl-37, K-41, Cu-63, Cu-65, Ga-71, As-75, Br-79, Br-81, Tb-159, Ir-191, Ir-193 and Au-197. Whereas the low spin (the net nuclear spin of 1/2) isonuclides are Y-89, Rh-103, Ag-107, Ag-109, Tm-169, Tl-203 and Tl-205.

(e). We have seen above that K-40 is a stable isonuclide. Herein di-isoneutron of spin 1 is getting stabilized in the environment offered by α-particles and three parallel spin isodeuterons. Three parallel spin isoneutrons get stabilized in Ca-43 that offers the environment of α -particles and two parallel spin isodeuterons.

(f). We also see that up to the net 9 parallel spin isoneutrons get stabilized in the environment of 36 α-particles and 13 di-isoneutrons in the case of Hf-179 whereas in the case of Ge-73 the 9 parallel spin isoneutrons get stabilized in the environment of 16 α-particles, no isodeuterons are required for this stabilization.

We have described above certain representative observations but on closer scrutiny of Table 2 we can spell out many more observations. However, the main task of presenting the nuclear configurations of Table 2 has been to provide ample facts that would provide base to evolve a comprehensive theory of nuclear stability against radioactivity and find out the factors that lead to nuclear instability.

While attempting to explain the nuclear stability we definitely need to consider unstable isonuclides in the immediate vicinity of the stable isonuclides along with their nuclear configurations commensurate with their observed spins. For example let us consider the cases of stable Nb-93 and In-113. We know that Nb-92 and Nb-94 are unstable isotopes and their experimentally observed nuclear spins are 7 and 6 respectively whereas that of Nb-93 it is 9/2. That is Nb-93 lies in between the higher nuclear spin isotopes. The nuclear configuration of Nb-92 is

$$19\left[{}^{4}_{2}\hat{He}_2(0)\right],3\left[\hat{d}(\uparrow\uparrow)\right],\left[\hat{n}(\uparrow)\hat{n}(\downarrow)\right],8\left[\hat{n}(\uparrow)\right]$$

that on adding one isoneutron changes to

$$20\left[{}^{4}_{2}\hat{He}_2(0)\right],\left[\hat{d}(\uparrow\uparrow)\right],2\left[\hat{n}(\uparrow)\hat{n}(\downarrow)\right],7\left[\hat{n}(\uparrow)\right]$$

That is the addition of one isoneutron forces two isodeuterons out of three parallel spin isodeuterons to get spin paired and simultaneously itself gets spin paired with one isoneutron leaving 7 parallel spin isoneutrons. The outcome is the stable Nb-93. Now to this stable isotope on adding one isoneutron it forces one pair of spin paired isoneutrons to become spin unpaired resulting in total number of 10 parallel spin isoneutrons. The resultant nuclear configuration obtained is

$$20\left[{}^{4}_{2}\hat{He}_2(0)\right],\left[\hat{d}(\uparrow\uparrow)\right],\left[\hat{n}(\uparrow)\hat{n}(\downarrow)\right],10\left[\hat{n}(\uparrow)\right]$$

which is unstable Nb-94.

Similarly, in the sequence In-112, In-113 and In-114 the nuclear configuration transforms as

$$24\left[{}^{4}_{2}\hat{He}_2(0)\right],\left[\hat{d}(\uparrow\uparrow)\right],7\left[\hat{n}(\uparrow)\hat{n}(\downarrow)\right]$$

$$\downarrow$$

$$24\left[{}^{4}_{2}\hat{He}_2(0)\right],\left[\hat{d}(\uparrow\uparrow)\right],4\left[\hat{n}(\uparrow)\hat{n}(\downarrow)\right],7\left[\hat{n}(\uparrow)\right]$$

$$\downarrow$$

$$24\left[{}^{4}_{2}\hat{He}_2(0)\right],\left[\hat{d}(\uparrow\uparrow)\right],8\left[\hat{n}(\uparrow)\hat{n}(\downarrow)\right]$$

Notice that in this sequence spin 1 states are unstable and 9/2 spin state is a stable one.

The above described are a few representative examples but they adequately pose the kind of challenge we need to undertake in order to explain nuclear stability/instability. One may think that an answer may be found through developing corresponding shell model and corresponding magic numbers.

In order to check if magic numbers play any role in nuclear configuration through isoneucleons we have also compiled the nuclear configuration in terms of isodeuterons as the only constituent and depicted in Table 3.

Table 3. *Isonuclides composed only of isodeuterons*

Nuclear Configuration	Isonuclide ${}^{A}_{Z}\hat{X}_N(J)$	Isodeuterons and Nuclear Stability / Instability	Nuclear Magnetic dipole Moment μ / μ_N	Nuclear Electric Quadrupole Moment Q/eb
$\hat{d}(\uparrow\uparrow)$	Isodeuteron	1 (odd) stable	0.85743823	+0.00286
$\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$	${}^{4}_{2}\hat{He}_2(0)$	2 (even) stable	0	N/A
$\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right],\ \hat{d}(\uparrow\uparrow)$	${}^{6}_{3}\hat{Li}_3(1)$	2, 1 (odd) stable	0.8220473	-0.00083
$2\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$	${}^{8}_{4}\hat{Be}_4(0)$ $\equiv 2\left[{}^{4}_{2}\hat{He}_2(0)\right]$	2, 2 (even) unstable	0	N/A

Nuclear Configuration	Isonuclide $\hat{A}_{\hat{Z}}\hat{X}_N(J)$	Isodenterons and Nuclear Stability / Instability	Nuclear Magnetic dipole Moment μ / μ_N	Nuclear Electric Quadrupole Moment Q/eb
$[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$, $\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow)$	$^{10}_{5}\hat{B}_5(3)$	2, 3 (odd) stable	1.8006448	0.08472
$3[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$	$^{12}_{6}\hat{C}_6(0)$	2, 4 (even) stable	0	N/A
$3[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$, $\hat{d}(\uparrow\uparrow)$	$^{14}_{7}\hat{N}_7(1)$	2, 5 (odd) stable	0.403761	0.0193
$4[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$	$^{16}_{8}\hat{O}_8(0)$	2, 6 (even) stable	0	N/A
$4[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$ $\hat{d}(\uparrow\uparrow)$	$^{18}_{9}\hat{F}_9(0)$	2, 6, 1 (odd) unstable	N/A	N/A
$5[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$	$^{20}_{10}\hat{N}e_{10}(0)$	2, 6, 2 (even) stable	0	N/A
$4[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$, $\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow)$	$^{22}_{11}\hat{N}a_{11}(3)$	2, 6, 3 (odd) unstable	1.746	N/A
$6[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$	$^{24}_{12}\hat{M}g_{12}(0)$	2, 6, 4 (even) stable	0	N/A
$4[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$, $\hat{d}(\uparrow\uparrow)$, $\hat{d}(\uparrow\uparrow)$, $\hat{d}(\uparrow\uparrow)$, $\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow)$	$^{26}_{13}\hat{A}l_{13}(5)$	2, 6, 5 (odd) unstable	N/A	N/A
$7[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$	$^{28}_{14}\hat{S}i_{14}(0)$	2, 6, 6 (even) stable	0	N/A
$7[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$, $\hat{d}(\uparrow\uparrow)$	$^{30}_{15}\hat{P}_{15}(1)$	2, 6, 6, 1 (odd) unstable	N/A	N/A
$8[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$	$^{32}_{16}\hat{S}_{16}(0)$	2, 6, 6, 2 (even) stable	0	N/A
$8[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$, $\hat{n}(\uparrow),\hat{p}(\downarrow)$	$^{34}_{17}\hat{C}l_{17}(0)$	2, 6, 6, 2 (even) unstable	0	N/A
$9[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$	$^{36}_{18}\hat{A}r_{18}(0)$	2, 6, 6, 4 (even) stable	0	N/A
$8[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$, $\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow)$	$^{38}_{19}\hat{K}_{19}(3)$	2, 6, 6, 5 (odd) unstable	1.371	N/A
$10[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$	$^{40}_{20}\hat{C}a_{20}(0)$	2, 6, 6, 6 (even) stable	0	N/A
$10[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$, $\hat{n}(\uparrow),\hat{p}(\downarrow)$	$^{42}_{21}\hat{S}c_{21}(0)$	2, 6, 6, 6 (even) unstable	0	N/A
$11[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$	$^{44}_{22}\hat{T}i_{22}(0)$	2, 6, 6, 8 (even) unstable	0	N/A
$11[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$, $\hat{n}(\uparrow),\hat{p}(\downarrow)$	$^{46}_{23}\hat{V}_{23}(0)$	2, 6, 6, 8 (even) unstable	0	N/A
$12[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)]$	$^{48}_{24}\hat{C}r_{24}(0)$	2, 6, 6, 8, 2 (even) unstable	0	N/A

From the column 3 of Table 3 we learn that the stable nuclides consists of odd number i.e. 1, 3, 5, 7 and 9, and even number i.e. 2, 6, 8, 10, 12, 14, 16, 18 and 20 isodeuterons. We also notice that there are no nuclides, stable or unstable, with 17, 21 and 23 isodeuterons this we have depicted in Table 3 by including Cl-34 (consisting of 16 spin paired isodeuterons and spin paired one isoneutron and one isoproton), Sc-42 (consisting of 20 spin paired isodeuterons and spin paired one isoneutron and one isoproton) and V-46 (consisting of 22 spin paired isodeuterons and spin paired one isoneutron and one isoproton). All of them are unstable isonuclides. The nearest stable isonuclides are Cl-35 (consisting of 16 spin paired isodeuterons, and one each isodeuteron and isoneutron with parallel spins), Sc-45 (consisting of 18 spin paired isodeuterons, two spin paired isoneutrons and parallel spin 3 isodeuterons and one isoneutron) and V-51 (consisting of 22 spin paired isodeuterons, and parallel spin one isodeuteron and 5 isoneutrons) respectively (can be seen in Table 2).

Thus, the column 3 of Table 3 doesn't appear to point out the existence of a system of magic numbers for isodeuterons when housed in a nucleus. Moreover, in Table 3 we have also depicted the nuclear magnetic dipole and electric quadrupole moments [77, 78] in columns 4 and 5 respectively. That reveals the nonspherical nuclear charge distribution in the case of non-zero nuclear quadrupole moments.

14.2. Nuclear Configuration of Model-II

In this model we treat atomic nuclei as constituted of up-spin isoprotons, down-spin isoprotons and null-spin isoelectrons. The column 4 of Table 2 list nuclear configuration of all stable nuclides in terms of these isonucleons. The striking feature of these nuclear configuration is that the number of isoelectrons, $\mathbb{E}(0)$, is equal to the number of neutrons (c.f. Eqs. (84) and (91)) and in this sense we may say that the isoelectrons have *replaced* neutrons.

Traditionally the nuclear stability is described by the ratio N/Z (number of neutrons to atomic number) and it is argued that this ratio increases from value 1 at lower atomic numbers to 1.537 in the case of Pb-208 (the last stable nuclide) because with the increase of atomic number the nuclear charge increases, which results in tremendous increase in repulsive force amongst nuclear protons. This repulsion gets minimized by the presence of neutrons. At higher atomic number the neutrons in a nuclide need to out number protons to attain nuclear stability, hence the said ratio increases up to 1.537 for Pb-208. Of course, there is a limitation on the effectiveness of neutrons to overcome the nuclear repulsive force otherwise by mere increase of number of neutrons all nuclide could have been stabilized. Hence, other explanation of nuclear stability were looked for. That resulted in postulation of a host of new subatomic particles. In layman's language scientists were looking for a *nuclear glue* which is responsible for tightly holding nucleons together within an atomic nucleus.

With this background we now interpret the stability of an isodeuteron in terms of the ability of an isoelectron to effectively hold two isoprotons together. Hence, we are in a position to say that the isoelectron acts in this case as an effective nuclear glue that holds tightly two isoprotons together. In view of this interpretation we now, apparently for the first time, hypothesize that *in all stable nuclides their isoelectrons act as effective glue that tightly hold their isoprotons together in the nucleus, of course, with appropriate distribution of up and down spins amongst isoprotons*. We caution the reader that the isoelectrons as nuclear glue has entirely different base than what the nuclear glue is described in the conventional nuclear physics. In the present model we have not postulated any new subatomic particle. Our proposal is that the conventional electrons and protons get transformed respectively to isoelectrons and isoprotons by way of mutual partial penetration of their wave packets in view of their very close proximity and that acts as the nuclear glue.

In view of the rôle of the nuclear glue played by isoelectrons we hereby propose that instead of N/Z ratio it would be more appropriate to use the ratio $\mathbb{E}(0)/Z$ to qualitatively describe nuclear stability.

Of course, the proposal of nuclear configuration in terms of isoprotons and isoelectrons with latter as the nuclear glue, opens up new vistas for further investigations on the topics of nuclear stability and understanding of all other nuclear properties.

The nuclear configuration of nuclides of *model-II* are listed in column 4 of Table 2. The observations and analysis of these nuclear configurations are described in Section 14.2.1.

14.2.1. Observations and Analysis of Nuclear Configuration of Model-II

In the *model-II we view the nucleus as a pool of isoprotons with the isoelectrons immersed in it.* In light of this we are presenting our preliminary visualization of only a few nuclides of lower atomic numbers and for the time being we are postponing our analysis of higher atomic number nuclides.

1. In the case of *isodeuteron* there we have one isoelectron and two isoprotons (both with up spin). Hence the isoelectron acts as a solitary nuclear glue that tightly holds both the up spin isoprotons. The most obvious geometry of these three isonucleons is linear that perfectly matches with the structure proposed by Santilli (see Figure 9). Thus oblate elliptical shape of isodeuteron described in Figure 6 perfectly matches with the present description. It seems that the zero spin isodeuteron is energetically unstable hence even if it is formed in some nuclear transmutations that gets quickly converted to the spin 1 isodeuteron.
2. The next entry in Table 2 is **He-3** with the nuclear spin 1/2. It is the case of a pool of 3 isoprotons and in that one isoelectron is immersed. The obvious minimum energy geometry is the one in which the isoelectron is at the center of an equilateral triangle and the three isoprotons situated at the vertices of it. Again in this case too the shape would be elliptical due to its overall

spinning motion. He-3 with nuclear spin of 3/2 has not been observed so far, which must be energetically unstable nuclide. Therefore, even if it is formed in some nuclear transmutations it gets quickly transformed to He-3 of 1/2 spin. Thus we learn that the low nuclear spin state He-3 is the preferred one. Moreover, when we add one down spin isoproton to an isodeuteron nucleus we get He-3 nucleus but we see that this addition does not disturbed the nuclear stability of., though the geometry changes from linear to planner.

3. Just for comparison with He-3 nuclide let us consider **H-3** *(triton)* nuclide. The latter nucleus too possesses 3 isoprotons with the net spin of 1/2 but consists of 2 isoelectrons. The minimum energy arrangement would be trigonal-bipyramidal in that the two isoelectrons occupy axial positions above and below the horizontal plane of symmetry. However, in this arrangement the penetration of wave packets of isoelectrons into those of isoprotons would not be as deep as one isoelectron in He-3 achieves. This perhaps leads to instability. It decays with β^- emission to the stable He-3 nuclide and its half life is 12.329 y that is it is not highly unstable nuclide. This is understandable because by emission of one isoelectron a stable He-3 geometrical arrangement is achived.
4. The **He-4** nuclide is the case of a pool of 4 isoprotons and immersed in it are two isoelectrons. The minimum energy shape in this case would be that of an octahedron in which isoelectrons occupy the two diagrammatically opposite axial positions and 4 isoprotons occupy the remaining 4 vertices. The spins of isoprotons would be alternately up and down so that the net nuclear spin is null. The charge distribution would be spherically symmetric in view of the repulsion between axial isoelectrons.
5. The **Li-6** nuclide is a case of a pool of 6 isoprotons and 3 isoelectrons immersed in it. The minimum energy shape seems to be the two trigonal-pyramids in a staggered geometry with 6 isoprotons at the vertices. All the 3 isoelectrons occupy axial positions and out of them one is at the center holding tightly both the trigonal pyramids. The observed spin 1 originate from the one up spin isoproton on each side of the central isoelectron. If one isoelectron is added to Li-6 arrangement described herein then we will have to house 2 isoelectrons at the center of the axial position. An equally probable geometry could be one He-4 arrangement and one isodeuteron moiety oriented above one of the axial isoelectrons such that the isodeuteron moeity and the axial isoelectrons of He-4 moiety form a straight line. Such an arrangement would not be stable because of the strong electrostatic repulsion between two central isoelectrons. The resultant nuclide would be He-6. However, it has two decay paths with half life of 806.7 ms. One is the obvious decay to Li-6 just by getting rid of the extra electron and in the second path simultaneously an α -particle is emitted, the daughter nuclides are a deuteron and He-4 nuclides.
6. The **Li-7** nuclide is a case of a pool of 7 isoprotons and 3 isoelectrons immersed in it. The obvious minimum energy geometry would be having two H-3 trigonal-bipyramids fused by one isoproton at the center such that its wave packet simultaneously allows penetration of wave packets of two adjacent central isoelectrons on its left and right hand sides. The observed spin 3/2 is because of the two up spin isoprotons on each trigonal plane and one up spin isoproton of the fusing isoproton. If we add one isoelectron to Li-7 the resultant nuclide would be He-7, which decays to He-6 by neutron emission which in turn decays by two simultaneous paths to Li-6 and He-4 along with a deuteron by β^- emission.
7. The case of **Be-8** is unique. It has a pool of 8 isoprotons and 4 isoelectrons immersed in it. The minimum energy shape would be two compressed octahedrons in the staggered orientation one above the other. Thus the four vertices of each octahedron would be alternately occupied by up and down spin isoprotons and the two axial vertices of each octahedron are occupied by one isoelectrons each. However, in this way middle two isoelectrons would come close to each other hence this arrangement cannot sustain itself. As a result of it the two octahedrons get separated. This is the reason why Be-8 is not a stable nuclide disintegrating to α-particles. If we add 1 isoelectron to Be-8 the resultant nuclide would be Li-8 which in turn disintegrates to Be-8 by β^- emission with half life of 840.3 ms.
8. The **Be-9** nuclide is a case of a pool of 9 isoprotons and 5 isoelectrons immersed in it. The obvious minimum energy geometrical arrangement of isonucleons consist of 2 H-3 trigonal-bipyramids fused by the triagonal planar geometry of He-3 in a staggered orientation with respect to both the trigonal-bipyramids. The observed spin of 3/2 is due to the spin 1/2 of one He-3 and two H-3 geometries. Notice that the wave packet of the isoelectron of the central He-3 geometry will be effectively shielded by the wave packets of its three isoprotons hence the wave packets of the isoelectrons of both the H-3 geometries oriented towards the central H-3 geometry wold penetrate into the wave packets of the central H-3 isoprotons. This seems to impart stability to Be-9. If we add 1 isoelectron to Be-9 nuclide the resultant nuclide would be Li-9 which disintegrates by two paths to Be-9 and Be-8 along with a neutron with the half life of 178.3 ms.
9. The **B-10** is the case of a pool of 10 isoprotons and 5 isoelectrons immersed in it. The minimum energy packing of isonucleons would be two He-4 type

octahedrons in the staggered orientation one above the other and one isodeuteron fusing them so that 2 central isoprotons and five axial isoelectrons are in a straight line. The spin 3 of B-10 originates from 8 up spin isoprotons 3 in each octahedron plus two in the fusing isodeuteron leaving two down spin isoprotons one each in the octahedron geometry. If we add one isoelectron to B-10 nuclide the resultant nuclide would be Be-10 nuclide which disintegrates back to B-10 by β^- emission with half life of 1.39×10^6 y.

10. The **B-11** is the case of a pool of 11 isoprotons and 6 isoelectrons immersed in it. The minimum energy packing of the isonucleons would be 2 He-4 structures in staggered orientation and the remaining 3 isoprotons and 2 isoelectrons linearly and alternately coupled acts as the fusing chain of the two octahedral structures. The nuclear spin of 3/2 is due to 3 up spin central isoprotons. If we add one isoelectron to the B-11 nuclide the resultant nuclide would be Be-11 which partly decays back to B-11 and partly to Li-7 and α-particle by β^- emission with half life of 13.81 s.

The above presented visualization of isonucleons in nuclides appears to be satisfactorily reasonable. We would extend the work on the same lines for all stable and unstable nuclides.

15. Concluding Remarks

In this paper, we have reviewed the numerous insufficiencies of quantum mechanics for the representation of the structure of stable nuclides, and the ensuing greater insufficiencies for the representation of the structure of unstable nuclides and nuclear reactions at large due to their structural irreversibility over time compared to the strict reversibility of quantum mechanical axions.

We have pointed out that the origin of the insufficiencies rests primarily in the *mathematics* of quantum mechanics, rather than in its axioms, due to its local-differential character with consequential abstraction of nuclear constituents as being point-like particles, compared to the evident need for the nuclear structure to represent nucleons as they are in the nuclear reality: extended charge distributions.

We have then reviewed the rudiments of the novel *isomathematics* which has been constructed precisely for the representation of nuclei as being composed by extended constituents in conditions of partial mutual penetration, thus resulting in the most general known interactions of linear and non-linear, local and non-local as well as Hamiltonian and non-Hamiltonian type.

We have then reviewed the rudiments of the covering of quantum mechanics known as *isomechanics* specifically formulated for the nuclear structure, by stressing that it essentially consists in an axiom-preserving "completion" of quantum mechanics along the historical argument by Einstein, Podolsky and Rosen, which is solely valid at one fermi distances while recovering quantum mechanics uniquely and identically for bigger distances.

We have then reviewed the use of the above new formulations for the first and only achievement on scientific records of an exact and time invariant representation of the magnetic moments of stable nuclei via the implementation of Fermi's historical hypothesis that the charge distributions of protons and neutrons is deformed when they are members of a nuclear structure, with a consequential deformation of their intrinsic magnetic moments (see Figures 8 and 9 for neutron as isoneutron and deuteron as isodeuteron respectively).

The conceptual and technically most dominant aspect of the above advances is that the admission of contact, non-linear, non-local and non-Hamiltonian interactions causes alterations of the *intrinsic* characteristics of particles called *isorenormalizations* that are simply beyond any possible quantitative treatment via 20th century knowledge.

Consequently, we reviewed in the Appendix A the rudiments of the covering of Lie's theory known as the *Lie-Santilli isotheory* which has been specifically constructed for the invariant treatment of systems with extended-deformable constituents with the most general known interactions.

The most prominent salient part of the Appendix A is the review of the *Lorentz-Poincaré-Santilli isosymmetry* and its characterization of isoparticles, with particular emphasis in the characterization of nuclear constituents as extended-deformable isoparticles.

We finally review the use of all the above knowledge for the first and only known numerically exact and time invariant representation of *all* characteristics of the neutron in its synthesis from the hydrogen atom as being composed by one isoproton and one isoelectron, with the consequential representation of *all* characteristics of the deuteron as being composed by two isoprotons and one isoelectron.

By using the above advances, we then present, apparently for the first time, two exact and invariant representations of the nuclear spin of the stable nuclides. The model-I is based on nuclear structures composed by isoprotons, isoneutrons and isodeuterons as isonucleons and the model-II is based on the final reduction of nuclides to isomechanical bound states of the respective isoprotons and isoelectrons.

In the *former model* we have considered that with the available neutrons and protons of the nuclide they first prefer to have the stable isodeuteron structure and the remaining nucleons stay as isoneutrons and isoprotons in the nucleus. In doing so the rule followed is that the so generated nuclear configuration should correctly reproduce the experimental nuclear spin of the given nuclide. Thus in Table 2 we have listed nuclear configuration of all stable nuclides up to the atomic number 82, that is up to Pb-208. Then we have analyzed these nuclear configurations and presented our observations in terms of the number of isodeuterons (both their low spin and high spin combinations) and their rôle in stabilization of various combination of spin paired and/or parallel spin isoneutrons. We have tried to look if these nuclear configurations indicate corresponding magic numbers but the data in Table 3 fail to provide any indication. However, it

seems that unless we systematically compare the nuclear configuration based on model-I of neighbouring unstable nuclides about the stable nuclides considered in Table 2 we may not be able to throw much light on the factors responsible for nuclear stability/instability. Indeed, our data of Tables 2 and 3 of model-I has opened up an entirely a new line of research in the fields of nuclear stability/instability and nuclear magnetic moments including nuclear electric quadrupole moments.

Whereas in arriving at the *model-II* we have first reinterpreted the stable structure of an isodeuteron in the sense that the isoelectron of it acts as a nuclear glue that tightly holds its two isoprotons. This proposal of isoelectrons acting as the nuclear glue we have, perhaps for the first time, extended to all stable nuclides. There we have assumed that a given nuclide consists of a pool of isoprotons and the isoelectrons are immersed in it, which in essence is the model-II of this paper. The working rule is that the number of isoelectrons is equal to the number of neutrons in the nuclide and the number of isoprotons is equal to the mass number of the nuclide. Next these isprotons are distributed in two groups of up and down spins in such a way to correctly predict the experimental nuclear spin of the given nuclide. The resulting nuclear configurations of all stable nuclides are listed in the column 4 of Table 2. Herein we have presented our preliminary observations on the so developed nuclear configuration. Of course, we have so far analyzed only a very few light nuclides in terms of geometrical arrangements of isoprotons and isoelectrons of H-2, H-3 (unstable), He-3, He-4, Li-6, Li-7, Be-8 (unstable), Be-9, B-10 and B-11 nuclides. Our assigned geometrical arrangements of isonucleons seem to provide reasonably satisfactory rational behind nuclear stability/instability. Particularly the reason of instability of H-3 and Be-8 so obtained seems to be rationally correct and encouraging.

The remarkable feature of both the models of nuclear configuration presented in this paper is that we need not to invent nuclear particles other than the basic subatomic particles, namely electrons, protons and neutrons.

Moreover, as stated in the main text of this paper the methods of writing nuclear configuration of a nuclide in both the models are equally applicable to unstable nuclides too hence while dealing with the nuclear stability/instability one can easily write down nuclear configurations of neighbouring unstable nuclides about a given stable one with identically the same rules as those we have followed in the case of stable nuclides and then attempt to rationalize nuclear stability/instability meaningfully.

Both the models promise new vistas of nuclear physics that lays a foundation of carrying out further investigations based on hadronic mechanics to strengthen our knowledge of nuclear physics.

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Appendix A

The Lorentz-Poincaré-Santilli IsoSymmetry and its Characterization of IsoParticles

A.1 Definition of IsoParticles

The 20th century definition of *particles* is that of unitary irreducible representation of the Lorentz-Poincaré (LP) symmetry on a Hilbert space over the field of complex numbers. This definition implies that all interactions are derivable from a potential and representable with a Hamiltonian as a central condition for the very applicability of Lie's theory at large, and that of the LP symmetry in particular. In turn, the local-differential mathematics underlying Lie's theory implies that the particles are abstracted as being point-like, as it is evident from the restriction of the interactions to actions-at-a-distance.

A central aim of this paper is the representation of nuclear constituents as they are in the physical reality, namely, extended, non-spherical and deformable charge distributions according to the representation of Eq. (1) which is structurally non-Hamiltonian, in the sense that it cannot be represented with a Hamiltonian, thus requiring a new quantity other than the Hamiltonian. In order to achieve a time invariant representation, isomathematics selects Santilli isounit Eq. (37), $\hat{I} = 1/\hat{T} > 0$ for the representation of the new interactions.

Additionally, the comparison of experimental data on nuclear volumes with those on the volume of protons and neutrons, establishes that, when they are members of a nuclear structure, protons and neutrons are in conditions of partial mutual penetration of their charge distributions.

These data imply the emergence of new nuclear interactions that are non-existence in the 20th century notion of particles, which are given by non-linear (in the wave functions), non-local (of integral and other type) and variationally non-selfadjoint [3a]. The latter interactions are also not representable with a Hamiltonian and can be invariantly represented with the exponent in the isotopic element, Eq. (1), or in the isounit.

The above basic assumptions imply the applicability of the Lie-Santilli isotheory [3b, 7, 22, 24-33] at large that was constructed precisely for the representation of non-Hamiltonian systems under the most general known linear and non-linear, local and non-local and Hamiltonian as well as non-Hamiltonian interactions,

Finally, the above basic assumptions imply that the universal symmetry for the non-relativistic treatment of isolated and stable nuclei is the *Galileo-Santilli isosymmetry* [21, 22], while that for the relativistic treatment is the *Lorentz-Poincaré-Santilli isosymmetry* [12-23]. We can, therefore, introduce the following:

DEFINITION A.1 [18, 21, 22]: A non-relativistic

(relativistic) isoparticle is an isounitary, isoirreducible isorepresentation of the Galileo-Santilli (Lorentz-Poincaré-Santilli) isosymmetry on a Hilbert-Myung-Santilli isospace over a Santilli isofields.

Within the context of this paper, whenever nuclear constituents are called "protons" and "neutrons" we refer to their quantum mechanical characterization as *point-like particles* under sole action-at-a-distance, potential-Hamiltonian interactions. Nuclear constituents according to this paper must necessarily be isoparticles at large, and isoprotons, isoneutron and isoelectrons in particular. In this Appendix we provide a summary characterization of relativistic isoparticles, while the particular case of non-relativistic isoparticles is referred to Refs. [18, 21] for brevity.

It should be stressed that a technical knowledge of the notion of isoparticle can solely be acquired from the study of Refs. [18, 21, 22]. In particular, a necessary pre-requisite for a technical characterization is the knowledge of Kadeisvili isofunctional analysis [22] we cannot possibly review to prevent excessive length.

A.2 The Lie-Santilli IsoTheory

The main branches of the Lie-Santilli isotheory can be outlined as follows (see the original proposal [3a] for the isotopies of enveloping algebras, Lie algebras and Lie group; Ref. [7] for their upgrading in terms of the isodifferential calculus over isofields; the final formulation in Ref. [22]; and Refs. [24-33] for independent studies):

Universal Enveloping Isoassociative Algebras

Let $E = E(L)$ be the universal enveloping associative algebra of an N-dimensional Lie algebra L with ordered (Hermitean) generators $X_k, k = 1,2,..,N$, and attached antisymmetric algebra isomorphic to the Lie algebra, $[E(L)]^- \approx L$ over a field F (of characteristic zero), and let the infinite-dimensional basis I, X_k, $X_i \times X_j$, $i \leq j,...$ of $E(L)$ be characterized by the Poincaré-Birkhoff-Witt theorem. We then have the following:

THEOREM A.1 [3b, 7]: (Poincaré-Birkhoff-Witt-Santilli theorem): The isocosets of the isounit and of the standard isomonomials

$$\hat{I},\ X_k,\ \hat{X}_i \hat{\times} \hat{X}_j, i \leq j;\ \ \hat{X}_i \hat{\times} \hat{X}_j \hat{\times} \hat{X}_k; i \leq j \leq k,..., \quad (A.1)$$

form an infinite dimensional basis of the universal enveloping isoassociative algebra $\hat{E}(\hat{L})$ (also called isoenvelope for short) of a Lie-Santilli isoalgebra $\hat{L}$.

The first application of the above theorem, also formulated in Ref. [3b] and then reexamined by various authors, is a rigorous characterization of the *isoexponentiation,* i.e.,

$$\hat{e}^{\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}} =$$
$$= \hat{I} + \hat{i} \hat{\times} \hat{w} \hat{\times} \hat{X} \hat{/} \hat{1}! + (\hat{i} \hat{\times} \hat{w} \hat{\times} \hat{X}) \hat{\times} (\hat{i} \hat{\times} \hat{w} \hat{\times} \hat{X}) \hat{/} \hat{2}! + ... =$$
$$= \hat{I} \times (e^{i \times w \times T \times X}) = (e^{i \times w \times X \times T}) \times \hat{I}, \quad (A.2a)$$

$$\hat{i} = i \times \hat{I}, \hat{w} = w \times \hat{I} \in \hat{F}. \quad (A.2b)$$

where quantities with a "hat" are formulated on isospaces over isofields and those without are their projection on conventional spaces over conventional fields.

The non-triviality of the Lie-Santilli isotheory is established by the emergence of the isotopic element T directly in the exponent, thus ensuring the desired generalization, thus establishing "ab initio" that while Lies theory can solely characterize linear, local-dofferential and Hamiltonian systems, the covering Lie-Santilli isotheory characterize the most general known non-linear, non-local and non-canonical or non-unitary systems.

LIE-SANTILLI ISOALGEBRAS.

As it is well known, Lie algebras are the antisymmetric algebras $L \approx [\xi(L)]^-$ attached to the universal enveloping algebras $\xi(L)$. This main characteristic is preserved although enlarged under isotopies as expressed by the following:

THEOREM A.2 [3b, 7] (Lie-Santilli Second theorem): The antisymmetric isoalgebras $\hat{L}$ attached to the isoenveloping algebras $\hat{E}(\hat{L})$ verify the isocommutation rules.

$$[\hat{X}_i \hat{,} \hat{X}_j] = \hat{X}_i \hat{\times} \hat{X}_j - \hat{X}_j \hat{\times} \hat{X}_i =$$
$$= X_i \times T(x,v,\xi,\omega,\psi,\partial\psi,...) \times X_j - X_j \times T(x,v,\xi,\omega,\psi,\partial\psi,...) \times X_i =$$
$$= \hat{C}_{ij}^k(x,v,\xi,\omega,\psi,\partial\psi,...) \hat{\times} \hat{X}_k = C_{ij}^k(x,v,\xi,\omega,\psi,\partial\psi,...) \times X_k,, \quad (A.3)$$

where T is the projection of the isotopic element $\hat{T}$ on a conventional space over a conventional field, and the C's, called the "structure isofunctions" of $\hat{L}$, generally have an explicit dependence on local variables, and are restricted by the conditions (Lie-Santilli Third Theorem)

$$[X_i \hat{,} X_j] + [X_j \hat{,} X_i] = 0, \quad (A.4a)$$

$$[[X_i \hat{,} X_j] \hat{,} X_k] + [[X_j \hat{,} X_k] \hat{,} X_i] + [[X_k \hat{,} X_i] \hat{,} X_j] = 0. \quad (A.4b)$$

It was stated in the original proposal [3b, 7] that all isoalgebras $\hat{L}$ are isomorphic to the original algebra L for all positive-definite isotopic elements. In other words, the isotopies cannot characterize any new Lie algebras because all possible Lie algebras are known from Cartan classification. Therefore, Lie-Santilli isoalgebras merely provide new non-linear, non=local and non=canonical or non-unitary realizations of existing Lie algebras.

LIE-SANTILLI ISOGROUPS.

Under certain integrability and smoothness conditions

hereon assumed, Lie algebras L can be "exponentiated" to their corresponding Lie transformation groups G and, vice-versa, Lie transformation groups G admit corresponding Lie algebras L when computed in the neighborhood of the unit I.

These basic properties are preserved under isotopies although broadened to the most general possible, axiom-preserving nonlinear, nonlocal and noncanonical transformations groups according to the following:

THEOREM A.3 [3b, 7] (Lie-Santilli isogroups): The isogroup characterized by finite (integrated) form $\hat{G}$ of isocommutation rules (1.12) on an isospace $\hat{S}(\hat{x},\hat{F})$ over an isofield $\hat{F}$ with common isounit $\hat{I}=1/\hat{T}>0$ is a group mapping each element $\hat{x}\in\hat{S}$ into a new element $\hat{x}'\in\hat{S}$ via the isotransformations

$$\hat{x}'=\hat{g}(\hat{w})\hat{\times}\hat{x},\ \hat{x},\hat{x}'\in\hat{S},\ \hat{w}\in\hat{F}, \tag{A.5}$$

with the following isomodular action to the right:

1) The map $\hat{g}\hat{\times}\hat{S}$ into $\hat{S}$ is isodifferentiable $\forall\hat{g}\in\hat{G}$;

2) $\hat{I}$ is the left and right unit

$$\hat{I}\hat{\times}\hat{g}=\hat{g}\hat{\times}\hat{I}\equiv\hat{g},\ \forall\hat{g}\in\hat{G}; \tag{A.6}$$

3) the isomodular action is isoassociative, i.e.,

$$\hat{g}_1\hat{\times}(\hat{g}_2\hat{\times}\hat{x})=(\hat{g}_1\hat{\times}\hat{g}_2)\hat{\times}\hat{x},\ \forall\hat{g}_1,\hat{g}_2\in\hat{G}; \tag{A.7}$$

4) in correspondence with every element $\hat{g}(\hat{w})\in\hat{G}$ there is the inverse element $\hat{g}^{-\hat{I}}=\hat{g}(-\hat{w})$ such that

$$\hat{g}(\hat{0})=\hat{g}(\hat{w})\hat{\times}\hat{g}(-\hat{w})=\hat{I}; \tag{A.8}$$

5) the following composition laws are verified

$$\hat{g}(\hat{w})\hat{\times}\hat{g}(\hat{w}')=\hat{g}(\hat{w}')\hat{\times}\hat{g}(\hat{w})=\hat{g}(\hat{w}+\hat{w}'),\forall\hat{g}\in\hat{G},\ \hat{w}\in\hat{F}; \tag{A.9}$$

with corresponding isomodular action to the left, and general expression

$$\hat{g}(\hat{w})=\prod_k\hat{e}^{\hat{I}\hat{\times}\hat{w}_k\hat{\times}\hat{X}_k}\ \hat{\times}\ \hat{g}(0)\ \hat{\times}\prod_k\hat{e}^{-\hat{I}\hat{\times}\hat{w}_k\hat{\times}\hat{X}_k}, \tag{A.10}$$

Another important property is that conventional group composition laws admit a consistent isotopic lifting, resulting in the following

THEOREM A.4 [3b, 7] (Baker-Campbell-Hausdorff-Santilli theorem):

$$(\hat{e}^{\hat{X}_1})\hat{\times}(\hat{e}^{\hat{X}_2})=\hat{e}^{\hat{X}_3}, \tag{A.11a}$$

$$\hat{X}_3=\hat{X}_1+\hat{X}_2+[\hat{X}_1,\hat{X}_2]\hat{/}\hat{2}+[(\hat{X}_1-\hat{X}_2),[\hat{X}_1,\hat{X}_2]]\hat{/}\hat{12}+\ldots. \tag{A.11b}$$

Let $\hat{G}_1$ and $\hat{G}_2$ be two isogroups with respective isounits $\hat{I}_1$ and $\hat{I}_2$. The direct isoproduct $\hat{G}_1\hat{\times}\hat{G}_2$ is the isogroup of all ordered pairs

$$(\hat{g}_1,\hat{g}_2),\ \hat{g}_1\in\hat{G}_1,\hat{g}_2\in\hat{G}_2, \tag{A.12}$$

with isomultiplication

$$(\hat{g}_1,\hat{g}_2)\hat{\times}(\hat{g}_1',\hat{g}_2')=(\hat{g}_1\hat{\times}\hat{g}_1',\hat{g}_2\hat{\times}\hat{g}_2'), \tag{A.13}$$

total isounit $(\hat{I}_1,\hat{I}_2)$ and inverse $(\hat{g}_1^{-\hat{I}_1},\hat{g}_2^{-\hat{A}.13})$.

The following particular case is important for the isotopies of inhomogeneous groups. Let $\hat{G}$ be an isogroup with isounit $\hat{I}$ and $\hat{G}_{\hat{a}}$ the group of all its inner automorphisms. Let $\hat{G}_{\hat{a}}^{o}$ be a subgroup of $\hat{G}_{\hat{a}}$ with isounit $\hat{I}^{\bullet}$, and let $\Lambda(\hat{g})$ be the image of $\hat{g}\in\hat{G}$ under $\hat{G}_{\hat{a}}$. The semi-direct isoproduct $\hat{G}\hat{\times}\hat{G}_{\hat{a}}^{o}$ is the isogroup of all ordered pairs $(\hat{g},\hat{\Lambda})\hat{\times}(\hat{g}^{o},\hat{\Lambda}^{o})$ with total isounit

$$I_{tot}=\hat{I}\times\hat{I}^{o}. \tag{A.14}$$

The studies of the isotopies of the remaining aspects of the structure of Lie groups is then consequential. It is hoped the reader can see from the above elements that the entire conventional Lie theory does indeed admit a consistent and nontrivial lifting into the covering Lie-Santilli formulation.

A.3 Classification of Lie-Santilli IsoTheories

The Lie-Santilli isotheories are classified into [7]:

3.1) *Regular isotheories* when the C's of rules (A.3) are constant; and

3.2) *Irregular isotheories* when the C's of rules (A.3) are functions of local variables.

We should recall for the benefit of concrete applications in nuclear physics that *all regular Lie-Santilli isotheories can be constructed via the application of a non-canonical or non-unitary transformation to the totality of the conventional formulation of Lie's theory,* according to the rule of Section 4.

From now on, except for an illustration in Section 16.13, we should solely consider regular realizations of the Lie-santilli isotheories because amply sufficient for nuclear applications, although the use of irregular realizations appear to be necessary for astrophysical applications.

We should also recall that "structure functions" are impossible for Lie's theory, and they are solely possible for the covering Lie-Santilli isotheory, by therefore establishing the non-trivial character of Santilli isotopies.

A.4 The Fundamental Theorem on IsoSymmetries

As recalled in Section 16.1, the fundamental symmetries of the 20th century physics characterize point-like abstractions of particles in vacuum under linear, local and potential interactions, and are given by the *Galilei symmetry* $G(3.1)$ for non-relativistic treatment, the *Lorentz-Poincaré symmetry* $P(3.1)$ or relativistic formulations, the rotational symmetry $O(3)$, the $SU(2)$ symmetries and others.

A central objective of hadronic mechanics is the broadening of these fundamental symmetries to represent extended, non-spherical and deformable particles under linear and non-linear, local and non-local and potential as well as non-potential interactions *in such a way to preserve the original symmetries at the abstract level* as a necessary condition to maintain the conventional total conservation laws for isolated stable systems.

This central objective is achieved by the following property first proved by Santilli in Ref. [22b]:

THEOREM A.5: Let G be an N-dimensional Lie symmetry of a K-dimensional metric or pseudo-metric space $S(x,m,F)$ over a field F,

$$G:\ x' = \Lambda(w)\times x,\ \ y' = \Lambda(w)\times y,\ \ x,y\in\hat{S}, \tag{A.15a}$$

$$(x'-y')^\dagger \times \Lambda^\dagger \times m\times\Lambda\times(x-y) \equiv (x-y)^\dagger\times m\times(x-y), \tag{A.15b}$$

$$\Lambda^\dagger(w)\times m\times\Lambda(w)\equiv m. \tag{A.15c}$$

Then, all infinitely possible isotopies $\hat{G}$ of G acting on the isospace $\hat{S}(\hat{x},\hat{M},\hat{F})$, $\hat{M}=\hat{m}\times\hat{I}=(\hat{T}_i^{ik}\times m_{kj})\times\hat{I}$ characterized by the same generators and parameters of G and the infinitely possible, common isounits $\hat{I}=1/\hat{T}>0$ leave invariant the isocomposition

$$\hat{G}:\ x' = \hat{\Lambda}(w)\times x,\ \ y' = \hat{\Lambda}(w)\times y,\ \ x,y\in\hat{S}, \tag{A.16a}$$

$$(x'-y')^\dagger \times \hat{\Lambda}^\dagger\times\hat{m}\times\hat{\Lambda}\times(x-y) \equiv (x-y)^\dagger\times\hat{m}\times(x-y) \tag{A.16b}$$

$$\hat{\Lambda}^\dagger(\hat{w})\times\hat{m}\times\hat{\Lambda}(\hat{w})\equiv\hat{m}. \tag{A.16c}$$

and all infinitely possible so constructed isosymmetries $\hat{G}$ are locally isomorphic to the original symmetry G.

For a proof of the above theorem, one may inspect Section 1.2, Vol. II of Ref. [22].

To achieve a technical understanding of the Lie-Santilli isotheory and its applications in nuclear physics, the reader should note that, while a given Lie symmetry G is unique as well known, there can be an infinite number of covering isosymmetries $\hat{G}$ with generally different explicit forms of the transformations due to the infinite number of possible isotopic elements.

In fact, systems are characterized by the Hamiltonian H in the conventional scattering theory with trivial unit $I=Diag.(1,1,...,1)$. In this case, changing the Hamiltonian implies the referral to a different system, but the symmetry transformations remain the same. In the isoscattering theory, systems are characterized by the Hamiltonian H plus the isotopic element T. In this case, changing the isotopic element implies the referral to a different system as well as the characterization of generally different transformations due to the appearance of the isotopic element in the very structure of the isosymmetry.

Note also that all possible isosymmetries can be explicitly and uniquely constructed via the sole knowledge of the conventional symmetry and the isotopic element (1). in fact, as implied by Theorem A.5, the existence of the original symmetry plus the condition $\hat{I}>0$ ensure verification of the integrability conditions for the existence of finite transformations, a property hereon tacitly implied.

Recall that all quantities that are Hermitean in quantum mechanics are iso-Hermitean in hadronic mechanics as one can verify via Eq. (29), to such as extent that Hermiticity and iso-Hermiticity coincide at the abstract realization-free level,

$$\hat{X}^{\hat{\dagger}}\equiv\hat{X}^\dagger. \tag{A.17}$$

The following property is then crucial for the physical consistency of the nuclear applications of hadronic mechanics, particularly the isomechanical models of closed-isolated stable nuclei:

THEOREM A.6 [22]: Physical quantities that are Hermitean and conserved in quantum mechanics remain iso-Hermitean and iso-conserved in isomechanics.

The proof of the theorem can be easily done via the local isomorphism of conventional Lie algebras L and their isotopic covering $\hat{L}$ since isotopies do not change the generators, and merely generalize their associative products.

Recall that the basic space time symmetries, the Galileo and the Lorentz-Poincaré symmetries, characterize ten total conservation laws for the total linear momentum P, the total angular momentum J, the tonal energy H, the uniform motion of the center of mass M.

Theorem A.6 then assures that all total-external quantities that are conserved for quantum mechanical models remain conserved for their covering isomechanical form achieved via the rules of Section 4.

A.5 The Minkowski-Santilli IsoGeometry

Let $M(x,\eta,I)$ be the conventional Minkowski space over the field of real numbers R, with coordinates $x=(x^\mu)=(x^1,x^2,x^3,x^4=t),\mu=1,2,3,4$, metric $\eta=Diag.(1,1,1,-c^2)$, unit $I=Diag.(1,1,1,1)$ and line element

$$(x-y)^2\to(\hat{x}-\hat{y})^{\hat{2}}=$$
$$=[(x_1-y_1)^2+(x_2-y_2)^2+(x_3-y_3)^2-(t_1-t_2)^2c^2]I, \tag{A.18}$$

As it is well known, the Lorentz-Poincaré symmetry, hereon denoted $P(3.1)$, leaves invariant the above line element and constitutes the ultimate structural foundations of special relativity because it permits the unique and unambiguous characterization of its basic axioms and physical laws for exterior problems of point-particles moving in vacuum.

The fundamental isospace of relativistic isomechanics is the Minkowski-Santilli isospace [15] $\hat{M}(\hat{x},\hat{\eta},\hat{I})$ over the isoreals R, with isocoordinates $\hat{x}=x\hat{I}$, isometric from Eq. (37) is

$$\hat{\eta} = \hat{T}\eta,\ \hat{T} = Diag\left(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, -\frac{c^2}{n_4^2}\right), \qquad (A.19)$$

isounit $\hat{I} = 1/\hat{T} > 0$, and isoline element

$$(\hat{x} - \hat{y})^{\hat{2}} = \left[(\hat{x} - \hat{y})^{\mu} \hat{\times} \hat{\eta}_{\mu\nu} \hat{\times} (\hat{x} - \hat{y})^{\nu}\right] = \left[\frac{(x_1 - y_1)^2}{n_1^2} + \frac{(x_2 - y_2)^2}{n_2^2} + \frac{(x_3 - y_3)^2}{n_3^2} - \frac{(t_1 - t_2)^2 c^2}{n_4^2}\right] \hat{I}, \qquad (A.20)$$

where the isometric characteristic quantities n_μ are positive-definite but have otherwise an unrestricted functional dependence on all needed quantities, such as space-time coordinates x, velocities v, accelerations a, energy E, distance d, frequencies ω, temperature τ, wavefunction ψ, their derivatives $\partial\psi$, etc.

$$n_\mu = n_\mu(x, v, a, E, d, \omega, \tau, \psi, \partial\psi, ...) > 0,\ \mu = 1,2,3,4, \qquad (A.21)$$

Isoprotons, isoneutrons and isoelectrons are defined on isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$ over the isoreals. As one can see, isometric (A.19) is the most general possible metric with signature $(+,+,+,-)$, thus including as particular case the Riemannian, Fynslerian, Minkowskian and other possible metric.

The Minkowski-Santilli isogeometry [19] is the geometry of isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$ and can be conceptually identified as the Riemannian geometry reformulated with respect to the isofields of isoreals because the isometric is indeed dependent on local coordinates, thus requiring the machinery of the Riemannian geometry, such as Christoffel symbols, covariant derivatives, etc., although reformulated with respect to isomathematics.

The intriguing part of the Minkowski-Santilli isogeometry is that it has zero curvature as necessary from the local isomorphism of isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$ with the conventional space $M(x,\eta,I)$. It should be stressed that the lack of curvature was a necessary prerequisite for the construction of the symmetry of isoinvariant (A.20) (see Refs. [22] for details).

A.6 The Lorentz-Poncaré-Santilli IsoSymmetry

Following the prior construction of the isotopies of Lie's theory [3b], the universal isosymmetry of all infinitely possible isoline elements (A.19) was first identified by Santilli in 1983 [15], subjected to systematic studies in Refs. [15-19], and presented in a systematicw ay in monographs [21, 22], resulting in a new isosymmetry today known as the Lorentz-Poincaré-Santilli isosymmetry (LPD) and denoted with the symbol $\hat{P}(3.1)$.

The isosymmetry $\hat{P}(3.1)$ can be defined as the isotransformations on Minkowski-Santilli isospaces over isoreals

$$\hat{x}' = \hat{\Lambda}(\hat{w}) \hat{\times} \hat{x},\ \hat{x}' = \hat{x} + \hat{A}(\hat{x}, ...), \qquad (A.22a)$$

$$\hat{\Lambda}^{\dagger} \hat{\times} \hat{\eta} \hat{\times} \hat{\Lambda} = \Lambda \times \hat{\eta} \times \Lambda^{\dagger} = \hat{I} \times \hat{\eta} \times \hat{I}, \qquad (A.22b)$$

where we shall preserve the symbol $\times$ of ordinary multiplications hereon, under the *isomodularity condition*

$$\hat{Det}(\hat{\Lambda}) = \pm\hat{I}, \qquad (A.23)$$

where the quantity $\hat{A}$ is identified below and $\hat{w} = w \times \hat{I}$ represents isoparameters.

The regular isoconnected component of the LPS isosymmetry $\hat{P}^0(3.1)$ is characterized by the condition

$$\hat{Det}\,\hat{\Lambda} = +\hat{I}, \qquad (A.24)$$

and can be written

$$\hat{P}^0(3.1) = \hat{SO}(3.) \times \hat{T}(3.1) \times \hat{D}, \qquad (A.25)$$

where D is the 11^{th} dimensionality of the LPS isosymmetry identified below.

By expanding the preceding finite isotransforms (A.22) in terms of the isounit, the *regular LPS isoalgebra* is characterized by the conventional generators of the LP algebra and the isocommutation rules [21, 22, 25]

$$[J_{\mu\nu} \hat{,} J_{\alpha\beta}] = i(\hat{\eta}_{\nu\alpha} J_{\beta\mu} - \hat{\eta}_{\mu\alpha} J_{\beta\nu} - \hat{\eta}_{\nu\beta} J_{\alpha\mu} + \hat{\eta}_{\mu\beta} J_{\alpha\nu}), \qquad (A.26a)$$

$$[J_{\mu\nu} \hat{,} P_\alpha] = i \times (\hat{\eta}_{\mu\alpha} \times P_\nu - \hat{\eta}_{\nu\alpha} \times P_\mu), \qquad (A.26b)$$

$$[P_\mu \hat{,} P_\nu] = 0. \qquad (A.26c)$$

The iso-Casimir isoinvariants of $\hat{P}(3.1)$ are given by [loc. cit]

$$\hat{C}_1 = \hat{I}(x, ...), \qquad (A.27a)$$

$$\hat{C}_2 = P^{\hat{2}} = P_\mu \hat{\times} P^\mu = P^\mu \times \hat{\eta}_{\mu\nu} \times P^\nu = P_k \times g_{kk} \times P_k - p_4 \times g_{44} \times P_4, \qquad (A.27b)$$

$$\hat{C}_3 = W^{\hat{2}} = W_\mu \hat{\times} W^\mu,\ W_\mu = \hat{\varepsilon}_{\mu\alpha\beta\rho} \hat{\times} J^{\alpha\beta} \hat{\times} P^\rho, \qquad (A.27c)$$

and they are at the foundation of classical and operator *isorelativistic kinematics* [43].

It is easy to prove that *the LPS isosymmetry is locally isomorphic to the conventional LP symmetry.* It then follows that the isotopies increase significantly the arena of applicability of the LP (as well as any Lie symmetry) by lifting the Minkowskian spacetime (A.18) to all infinitely possible isospacetime (A.20).

Note that *isolinear isomomenta isocommute, Eqs. (A.26c), that is, they commute in isospace over isoreals, but they do not*

generally commute when projected in the ordinary Minkowski space. This occurrence is a clear confirmation of a nonlinear structure of isorelativity with rather deep gravitational implications not considered in this paper.

Yet, this property is significant because it appears, for the first time to our knowledge, the possibility of identifying a possible gravitational component in the structure of nuclei, as studied preliminarily in Refs. [79].

The *isoirregular LPS isoalgebra* is characterized by *structure functions*, thus no longer being locally isomorphic to the conventional LP symmetry. The study of the irregular realization is left to the interested reader for brevity.

By using the original generators of the LP symmetry, the isotopic element (37) and Lie-Santilli isotheory, regular LPS isotransformations can be easily identified as outlined below.

A.7 Regular IsoRotations

The regular isorotations, first presented in Ref. [12], and then treated in details in Refs. [22] via isofunctional analysis in general, and isotrgonometric functions in particular. Since the isounit $\hat{I} = Diag(n_1^2, n_2^2, n_3^2)$ is positive-definite, the isosymmetry $\hat{SO}(3)$ is locally isomorphic to the conventional rotational symmetry $O(3)$ (Figure 11).

Isorotations provide the technical characterization of the deformation of protons and neutrons when members of a nuclear structure under strong interactions. In their projection on an ordinary Euclidean space, isorotations can be written in the (1-2)-plane (see Ref. [22] for the general case).

$$x^{1'} = x^1 \cos[[\theta(n_1 n_2)^{-1}] - \\ -x^2 \frac{n_1^2}{n_2^2} \sin[\theta(n_1 \times n_2)^{-1}], \tag{A.28a}$$

$$x^{2'} = x^1 \frac{n_2^2}{n_1^2} \sin[\theta(n_1 n_2)^{-1}] + \\ +x^2 \cos[\theta(n_1 n_2)^{-1}], \tag{A.28b}$$

The isomorphism $\hat{SO}(3) \approx O(3)$ is due to the fact that ellipsoid deformations of the semiaxes of the perfect sphere are compensated on isospaces over isofields by the inverse deformation of the related unit

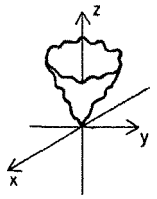

Figure 11. *It was popularly believed in the 20th century physics that the Lorentz symmetry is broken for locally varying speeds of light within physical media, here represented with a wiggly light cone. the Lie-Santilli isosymmetries have restored the exact validity of the Lorentz symmetry for all possible subluminal and superluminal speeds, thus confirming the preservation of the abstract axioms of special relativity for interior dynamical problems [15, 22].*

$$Radius \quad 1_k \to 1/n_k^2, \; Unit \quad 1_k \to n_k^2. \tag{A.29a}$$

$$\hat{r}^{\hat{2}} = \hat{r}_1^{\hat{2}} + \hat{r}_2^{\hat{2}} + \hat{r}_3^{\hat{2}}. \tag{A.29b}$$

resulting in the reconstruction of the perfect sphere on isospace called the *isosphere*, (A.29b), with consequential reconstruction of the exact rotational symmetry.

A.8 Regular Lorentz-Santilli IsoTransformations

The regular Lorentz-Santilli (LS) isotransforms were first identified in Ref. [15] and then studied in details in monographs [22]. Their elaboration also requires the use of the isofunctional analysis we cannot possibly review in this paper for brevity. It is easy to prove from the positive-definite character of the isounit $\hat{I} = Diag.\left(n_1^2, n_2^2, n_3^2, n_4^2\right)$ that the Lorentz-Santilli isosymmetry $\hat{SO}(3.1)$ is locally isomorphic to the conventional symmetry $SO(3.1)$ (Figure 11).

The LS isotransformations are at the foundations of the relativistic results of this paper as well as of their invariance over time. They were first derived in Ref. [15] of 1983 and can be presented projected in the conventional Minkowski (3-4)-plane (see monograph [22b] for the general case)

$$x^{1'} = x^1, \; x^{2'} = x^2, \tag{A.30a}$$

$$x^{3'} = \hat{\gamma}\left(x^3 - \hat{\beta} \times \frac{n_3}{n_4} x^4\right), \tag{A.30b}$$

$$x^{4'} = \hat{\gamma}\left(x^4 - \hat{\beta} \times \frac{n_4}{n_3} x^3\right), \tag{A.30c}$$

where

$$\hat{\beta} = \frac{v_3 / n_3}{c_o / n_4}, \tag{A.31a}$$

$$\hat{\gamma} = \frac{1}{\sqrt{1 - \hat{\beta}^2}}. \tag{A.31b}$$

It should be indicated that the main aim of Ref. [15] was the solution of the historical Lorentz probem, namely, the achievement of the universal symmetry for locally varying speeds of light within physical media $C = c / n_4$. Since this problem is highly non-linear, its solution could not be derived via the conventional Lie's theory. For this reason, Santilli conducted decades of studies for the generalization of Lie's theory into a form valid for nonlinear systems, first presented in monograph [3b], as a prerequisite for the solution of Lorentz's historical problem.

The isomorphism $\hat{SO}(3.1) \approx SO(3.1)$ is due to the reconstruction of the exact light cone on isospace over isofields called the light isocone. In fact, jointly with the deformation of the light cone

$$x^2 = x_3^2 - t^2c^2 = 0 \to \frac{x_3^2}{n_3^2} - t^2\frac{c^2}{n^4} = 0, \tag{A.32}$$

we have the corresponding *inverse* deformations of the units,

$$d_3 = 1 \to \hat{I}_3 = n_3^3 \;\; d_4 = 1 \to \hat{I}_4 = n_4^2, \tag{A.33}$$

thus reconstructing the original light cone on isospaces over isofields.

The reader should be aware that the above reconstruction includes the preservation on isospace over isofields of the original characteristic angle of the conventional light cone, namely, *the maximal causal speed on isospace over isofields is the conventional speed of light* c *in vacuum* [22].

A.9 Regular IsoTranslations

The regular isotranslations $T(4)$ were first studied in Ref. [16] and then studied in details in monographs [22]. and can be expressed in their projection in the conventional Minkowski space with the following lifting of the conventional translations $x^{\mu'} = x^\mu + a^\mu, \mu = 1,2,3,4,$ and a^μ constants,

$$x^{\mu'} = x^\mu + A^\mu(a,\ldots), \tag{A.34}$$

where

$$A^\mu = a^\mu(n_\mu^{-2} + a^\alpha[n_\mu^{-2}\,\hat{,}\,P_\alpha]/1!+\ldots), \tag{A.35}$$

and there is no summation on the μ indices.

Note the high nonlinearity of the isotranslations. This is due to the fact that the above expressions are the projection in the conventional spacetime since, when written on a Minkowski-Santilli isospace over isofields, isotransformations coincide with conventional translations.

A.10 Regular IsoDilations and IsoContractions

The *regular isodilations and isoContractions* $D(1)$ were first identified in Ref. [16] and then studied in details in monographs [22]. They constitute a *basically new spacetime symmetry* with vast implications, e.g., for grand unified theories [71], and can be expressed via the transformation

$$\hat{\eta} \to \hat{\eta}' = w^{-1}\hat{\eta}, \hat{I} \to \hat{I}' = w\hat{I}, \tag{A.36}$$

with ensuing invariance

$$(x^\mu\hat{\eta}_{\mu\nu}x^\nu)\hat{I} \equiv \left[x^\mu(w^{-1}\hat{\eta}_{\mu\nu})x^\nu\right](w\times\hat{I}) = \\ = (x^\mu\hat{\eta}_{\mu\nu}x^\nu)\times\hat{I}', w\in R. \tag{A.37}$$

It was popularly believed in the 20th century that the LP symmetry was 10 -dimensional. The above invariance establishes that, instead, *the LPS isosymmetry as well as the LP conventional symmetry are* 11 *-dimensional.*

A.11 Regular IsoInversions

The regular isoinversions are given by [22b]

$$\hat{\pi}\hat{\times}x = \pi x = (-r,tc), \tag{A.38a}$$

$$\hat{\tau}\hat{\times}x = \tau x = (r,-tc). \tag{A.38b}$$

where π and τ are the conventional space and time inversion operators.

A.12 Regular $\hat{S}U(2)$ *IsoSymmetry*

In this section we provide the solution, apparently for the first time, of a central problem for the consistent and time invariant representation of nuclear magnetic moments via the deformations of the charge distributions of nucleons with consequential mutation of their intrinsic magnetic moment, under the conservation of conventional, values of the spins.

By remembering the lack of uniqueness of the isounits and related isotopic element, the simplest regular two-dimensional irreducible isorepresentations of $\hat{SU}(2)$ are characterized by the lifting of the two-dimensional complex-valued unitary space with metric $\delta = Diag.(1,1)$ into the isotopic image [12, 15, 22]

$$\hat{I} = Diag.(n_1^2,n_2^2), \;\; \hat{T} = Diag.(1/n_1^2,1/n_2^2), \tag{A.39a}$$

$$\hat{\delta} = \hat{T}\times\delta = Diag.(1/n_1^2,\, 1/n_2^2), \tag{A.39b}$$

$$Det\,\hat{\delta} = (n_1n_2)^{-2} = 1, \tag{A.39c}$$

The basic non-unitary transform (43) of Section 4 us then given by

$$U\times U^\dagger = \hat{I} = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, \quad T = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix} \tag{A.40a}$$

$$U = \begin{pmatrix} i\times n_1 & 0 \\ 0 & i\times n_2 \end{pmatrix}, \quad U^\dagger = \begin{pmatrix} -i\times n_1 & 0 \\ 0 & -i\times n_2 \end{pmatrix}, \tag{A.40b}$$

here the n 's are well behaved nowhere null functions, resulting in the *regular Pauli-Santilli isomatrices* [*loc. cit*]

The related lifting of Pauli's matrices are then given by the *Regular Paili-Santilli isomatrices* [13, 14]

$$\sigma_k \to \hat{\sigma}_k = U\times\sigma_k\times U^\dagger, \tag{A.41a}$$

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1^2 \\ n_2^2 & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -in_1^2 \\ in_2^2 & 0 \end{pmatrix},$$

$$\hat{\sigma}_3 = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}. \tag{A.41b}$$

Another realization of the regular hadronic spin 1/2 is given by *non diagonal* nonunitary transforms [*loc. cit.*].

$$U=\begin{pmatrix}0 & n_1\\ n_2 & 0\end{pmatrix},\qquad U^{\dagger}=\begin{pmatrix}0 & n_2\\ n_1 & 0\end{pmatrix},$$
$$\hat{I}=\begin{pmatrix}n_1^2 & 0\\ 0 & n_2^2\end{pmatrix},\qquad \hat{T}=\begin{pmatrix}n_1^{-2} & 0\\ 0 & n_2^{-2}\end{pmatrix}, \tag{A.42}$$

with corresponding alternative version of the regular Pauli-Santilli isomatrices,

$$\hat{\sigma}_1=\begin{pmatrix}0 & n_1n_2\\ n_1n_2 & 0\end{pmatrix},\qquad \hat{\sigma}_2=\begin{pmatrix}0 & -in_1n_2\\ in_1n_2 & 0\end{pmatrix},$$
$$\hat{\sigma}_3=\begin{pmatrix}n_1^2 & 0\\ 0 & n_2^2\end{pmatrix}, \tag{A.43}$$

or by more general realizations with Hermitean non diagonal isounits $\hat{I}$ [15b].

All regular Pauli-Santilli isomatrices verify the following *isocommutation rules and isoeigenvalue equations* on H over C

$$[\hat{\sigma}_i,\hat{\sigma}_j]=$$
$$=\hat{\sigma}_i\hat{T}\hat{\sigma}_j-\hat{\sigma}_j\hat{T}\hat{\sigma}_i=2i\varepsilon_{ijk}\hat{\sigma}_k, \tag{A.44a}$$

$$\hat{\sigma}^{\hat{2}}\hat{\times}|\hat{\psi}\rangle=$$
$$(\hat{\sigma}_1T\times\hat{\sigma}_1+\hat{\sigma}_2T\times\hat{\sigma}_2+\hat{\sigma}_3T\times\hat{\sigma}_3)T|\hat{\psi}\rangle=3\times|\hat{\psi}\rangle, \tag{A.44b}$$

$$\hat{\sigma}_3\hat{\times}|\hat{\psi}\rangle=\hat{\sigma}_3\times T\times|\hat{\psi}\rangle=\pm1\times|\hat{\psi}\rangle, \tag{A.44c}$$

thus preserving conventional structure constants and eigenvalues for spin $1/2$ under non-Hamiltonian/nonunitary interaction, while adding the degree of freedom

$$n_1^2=\lambda,\ n_2^2=\lambda^{-1}, \tag{A.45}$$

That indeed is fully compatible with the mutation of intrinsic magnetic moments of spin $1/2$ particles, Eq. (60).

Additionally, *the regular Pauli-Santilli isomatrices provide an explicit and concrete realization of hidden variables,* with intriguing implications for local realism studied in detail in ref. [14]. In turn, the above aspect confirm the origination of isomechanics as a concrete and explicit realization of the "incompleteness" of quantum mechanics according to Einstein, Podolsky and Rosen [1].

A.13 Irregular $\hat{SU}(2)$ IsoSymmetry

As indicated throughout this paper, there appears to be no need for a mutation of the spin of nuclear constituents to achieve an exact representation of nuclear magnetic moments and spins.

Nevertheless, the issue persists as to whiter a proton in the core of a star should have the same spin when member of a nuclear structure. Santilli has introduced the irregular isotopies of the $SU(2)$ spin precisely for future studies of this important problem for the structure of stars.

One illustrative example of *irregular Pauli-Santilli isomatrices* is given by [12-14]

$$\tilde{\sigma}_1=\begin{pmatrix}0 & n_1^2\\ n_2^2 & 0\end{pmatrix},\qquad \tilde{\sigma}_2=\begin{pmatrix}0 & -in_1^2\\ in_2^2 & 0\end{pmatrix},$$
$$\tilde{\sigma}_3=\begin{pmatrix}wn_1^2 & 0\\ 0 & wn_2^2\end{pmatrix}. \tag{A.46}$$

where w is the *mutation parameter*; with isocommutation rules and eigenvalue equations

$$[\tilde{\sigma}_1,\tilde{\sigma}_2]=iw^{-1}\tilde{\sigma}_3,\ [\tilde{\sigma}_2,\tilde{\sigma}_3]=iws\tilde{\sigma}_1,$$
$$[\tilde{\sigma}_3,\tilde{\sigma}_2]=isw\tilde{\sigma}_1, \tag{A.47a}$$

$$\tilde{\sigma}^{\hat{2}}\hat{\times}|\hat{\psi}\rangle=$$
$$(\tilde{\sigma}_1T\tilde{\sigma}_1+\tilde{\sigma}_2T\tilde{\sigma}_2+\tilde{\sigma}_3T\tilde{\sigma}_3)T|\hat{\psi}\rangle=(2+w^2)\times|\hat{\psi}\rangle, \tag{A.47b}$$

$$\tilde{\sigma}_3\hat{\times}|\hat{\psi}\rangle=\tilde{\sigma}_3T|\hat{\psi}\rangle=\pm w|\hat{\psi}\rangle,\ w\neq1, \tag{A.47c}$$

Additional examples of irregular Pauli-Santilli isomatrices can be found in Refs. [12-14].

The assumption of a mutated spin in hyperdense interior conditions evidently implies the inapplicability (rather than the violation) of the Fermi-Dirac statistics, Pauli's exclusion principle and other quantum mechanical laws, with the understanding that, by central assumption, non-Hamiltonian bound states of particles as a whole must have conventional total quantum values. Therefore, *we are here referring to possible internal exchanges of angular momentum and spin always in such a way as to cancel out and yield total conventional values.*

A.14 IsoRelativity IsoAxioms

As shown in this paper, a numerically exact and time invariant representation of nuclear magnetic moments and spins has required the isotopies of 20th century mathematics, with ensuing isotopies of quantum mechanics into isomechanics.

Interested readers should be aware that the above isotopies imply the inapplicability of special relativity for the nuclear structure in favor of a covering relativity known as isorelativity [15, 21-23]. The central aim of special relativity is the invariance of the speed of light in vacuum. A central aim of isorelativity is the invariance of local varying speeds of light $C=c/n$ within physical media as shown in Appendix A.8.

A rudimentary knowledge of the covering relativity is important to prevent major misrepresentations of the results of this paper as well as in possible further advances because the the appraisals of the new nuclear structure provided by isomechanics via special relativity would be equivalent to the appraisal of the results by special relativity via Newtonian mechanics.

The isotopies of the axioms of special relativity, today known as *IsoAxioms,* were initiated by Santilli in paper [15] of

1983; they received a first systematic formulation by Santilli in monographs [21] of 1991; they were finalized in monographs [22] of 1995 jointly with the discovery of the isodifferential calculus; and their experimental verifications were presented ion Refs. [23].

In this paper we specialize, apparently for the first time, the isoaxioms for the isomechanical structure of stable and isolated nuclei whose constituents are isoparticles. The gravitational formulation of the isoaxioms of Reg. [71] should be kep in mind because it offers, also apparently for the first time, the possibility of addressing the origin of gravitational field in the structure of nuclei.

The first implication of the isotopies of special relativity is the *abandonment of the speed of light in vacuum as the maximal causal speed in favor of a covering geometrization of physical media.* This occurrence is easily seen by specializing the isoline element (27) to the *isolight isocone* [23, 37]

$$\hat{x}^{\hat{2}} = \left(\frac{x_k^2}{n_k^2} - t^2 \frac{c^2}{n_4^2} \right) = 0, \tag{A.48}$$

thus leading to the Maximal Causal Speed V_{max} of IsoAxiom 5.1 below.

The remaining isoaxioms can be uniquely and unambiguously identified via a procedure parallel to the construction of the axioms of special relativity from the Lorentz-Poincaré symmetry.

Another departure from 20th century views is that isoaxioms refer to generally inhomogeneous and anisotropic physical media, as it os typically the case of the medium within spinning charge distributions., Therefore, the isoaxioms are formulated below for a generic space direction k,

ISOAXIOM A. I: The maximal causal speed in a given space direction k in the interior of nuclei is given by

$$V_{max,k} = c \frac{n_k}{n_4}, \tag{A.49}$$

ISOAXIOM A. II: The local isospeed of light is given by

$$\hat{c} = \frac{c}{n_4} \tag{A.50}$$

where c is the speed of light in vacuum.

ISOAXIOM A. III: The addition of isospeeds in the k -direction follows the isotopic law

$$V_{tot,k} = \frac{v_{1,k} / n_k + v_{2,k/n_k}}{1 + \frac{v_{1,k} v_{2,k}}{c^2} \frac{n_4^2}{n_k^2}}. \tag{A.51}$$

ISOAXIOM A. IV: The isodilatation of isotime, the isocontraction of isolengths, the variation of mass with isospeed, and the mass-energy isoequivalence principle follow the isotopic laws

$$\Delta t' = \hat{\gamma}_k \, \Delta t, \tag{A.52a}$$

$$\Delta \ell' = \hat{\gamma}_k^{-1} \, \Delta \ell, \tag{A.52b}$$

$$m' = \hat{\gamma}_k \, m, \tag{A.52c}$$

$$E = m V_{max}^2 = m c^3 \frac{n_k^2}{n_4^2} \tag{A.52d}$$

where $\hat{\gamma}$ and $\hat{\beta}$ have values (32).

ISOAXIOM A. V: The frequency isoshift of light propagating within a nucleus in the k -direction follows the Doppler-Santilli isotopic law

$$\omega_e = \omega_o \hat{\gamma}_k \left[1 \pm \frac{v / n_k}{c / n_4} \cos \alpha \right] \tag{A.53}$$

where ω_e is the frequency experimentally measured in the outside, ω_o is the frequency at the origin inside a nucleus, and we have ignored for simplicity the isotopies of trigonometry (see Refs. [23] for brevity).

It should be stressed that in the above formulations as well as in the next section we present the isoaxioms in their *projection on the conventional Minkowski space.* while their technical treatment requires the full use of the various branches of isomathematcs, including the formulation of the isoaxioms on a Minkowski-Santilli isos; ace over an isofield.

A main feature is that, when the isoaxioms are represented on isospace over isofields, they coincide with the conventional axioms of special relativity by conception and technical realization. In particular, the maximal causal speed $V_{max} \neq c$ solely occurs in the projection of the isoaxioms on Minkowski space because, at the isotopic level, the maximal causal speed is c for all possible isogravitational problems.

A.15 Predicted Implications of the IsoAxioms for the Nuclear Structure

In this final section, we identify the most important predictions of isorelativity [15, 21, 22] emerging as a consequence of our exact and invariant representation of nuclear magnetic moments and spins, and present their preliminary appraisals by soliciting comments from interested colleagues.

Isoaxioms clearly imply *two* different representations of the nuclear structure, the first is the representation of nuclear characteristics as measured from outside observer here indicated with the subindex "ext," and the second representation is that in the interior of nuclei here indicated with the subindex "int."

These two representations are necessary for the evident reason that the exterior observer is assumed as being in vacuum thus obeying conventional relativity axioms while the second representation occurs within hyperdense physical media, here assumed as obeying the covering isorelativity axioms.

A first implications of isorelativity is that *the time of the*

exterior observer is not necessarily the same as that in the interior of nuclei. In fact, by recalling the isodilation and isocontraction of Appendix A.10, we can write the identity

$$t_{ext}1 = t_{int}\hat{I}_t. \tag{A.54}$$

Since for the nuclear structures considered in this paper $\hat{I}_t = n_4^2 < 1$ as in Eq. (63), one can see that the interior time evolution of nucleons is *predicted to be "faster" than that of an outsider observer.*

Note that at the abstract realization-free level there is no distinction between interior and exterior times as typical for all isotopies [22] since Eq. (A.54) can be written

$$t \equiv \hat{t} \tag{A.55}$$

where t is an ordinary scalar, while $\hat{t}$ is an isoscalar (Section 2). Therefore, t_{ext} and t_{int} are the projection of Eq. (A.55) in our spacetime.

For the case of distances, we can write the corresponding differentiations between external and internal distances according to the isotopic law

$$r_{ext}1 = r_{int,k}\hat{I}_k. \tag{A.56}$$

Since the space isounits are generally smaller than one from Eqs. (64), one can see again that space distances perceived in the outside observer are predicted to be bigger than the actual distances in the interior.

Intriguingly, isolaw (A.56) is verified in ordinary water where, as we all know, dimensions perceived from the outsider are bigger than those actually occurring within water (Figure 12). Therefore, our argument is that, since isolaw (A.55) is verified in a medium with relatively big density such as water, the possibility of a similar occurrence in much denser media such as nuclei deserves due scientific process.

Figure 12. *This picture illustrates the representation by isorelativity of the known effect that dimensions in water appears as being bigger then their actual dimensions when seen fro an outside observer, thus warranting the study of the corresponding effect within nuclei.*

Next, the speed of light in vacuum c has no mathematical or physical meaning for isorelativity and, in particular, it is not invariant under the time evolution. The sole mathematically and physically accepted quantity is Lorentz locally varying speed $C = c / n_4$.

In fact, the relativistic sum of two ordinary speeds of light does not yield the speed of light within physical media such as water and the same is expected within nuclei. By contrast, the isorelativistic sum of two locally varying speeds of light does indeed yield the local speed of light according to isoaxioms III,

$$V_{tot} = \frac{c/n_4 + c/n_4}{1 + \frac{c^2/n_4^2}{c^2/n_4^2}} = \frac{c}{n_4}. \tag{A.57}$$

In particular, one should note that a necessary condition for the isorepresentation of nuclear magnetic moments is that the local speed of light in the interior of nuclei is bigger than that in vacuum, see Eq. (64). This is a confirmation of the similar condition for $C > c$ which is necessary for the synthesis of the neutron from bthe hydrogen (Section 7).

Yet another prediction of isorelativity according to isoaxioms A. IV is that *the energy isoequivalent according to isoaction (A.52d) is "bigger" than that described from the outside.* This is a typical occurrence for all structure models of hadrons, nuclei and stars according to iksomathematics, and it is nowadays known as *isorenormalization.*

Consequently, in considering the structure model of nuclei as isobound states of isoprotons and isoelectrons, the reader should be aware that the rest energy of the isoelectron is isorenormalized to a minimum value of 1.293 MeV in the first approximation of ignoring Coulomb interactions, with bigger predicted values of the rest energy of the isoelectrons when including Coulomb interactions (due to the Coulomb attraction between isoprotons and isoelectrons).

As an illustration, a necessary condition for the achievement of an exact representation of the synthesis of the neutron from the hydrogen is that (by ignoring coulomb interactions) the isoremnormalized rest energy of the electron is 1.293 MeV.

Finally, we mention the prediction of isorelativity according to which *the frequency of the photons emitted by nuclei and measured in the outside is bigger than that at the point of emission in the interior ofg nuclei.* This additional effect is due to the *isoblueshift,* namely, the acquisition of energy by photons from hot environments without any relative motion, which was predicted by Santilli in 1992 [21], and experimentally verified in hot gases in 2010 [80] (see Refs. [68] for a comprehensive bibliography).

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Hadronic Nuclear Energy: An Approach Towards Green Energy

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Abstract: Nuclear energy is undoubtedly the largest energy source capable of meeting the total energy requirements to a large extent in long terms. However the conventional nuclear energy involves production of high level radioactive wastes which possesses threat, both to the environment and mankind. The modern day demand of clean, cheap and abundant energy gets fulfilled by the novel fuels that have been developed through hadronic mechanics / chemistry. In the present paper, a review of Prof. R. M. Santilli's Hadronic nuclear energy by intermediate controlled nuclear synthesis and particle type like stimulated neutron decay and double beta decay has been presented.

Keywords: Intermediate Controlled Nuclear Synthesis, Stimulated Neutron Decay

1. Introduction

The ever increasing demand for good quality of livelihood has ultimately culminated in increasing global energy demands. The demand can be met conventionally by either molecular combustion or nuclear fission. The former is achieved by combustion of fossil fuel or hydrogen which produces large amount of green house gases as well as depletes breathable oxygen from the environment. The latter does not generate green house gases or depletes breathable oxygen but creates large amount of radioactive wastes. Moreover, the shielding from the high energy ionizing and non-ionizing radiations is cumbersome and expensive. The handling of the highly radioactive wastes posses environmental as well as security threat. Thus, handling of these wastes requires great deal of safety requirements. There are several ways that are used to curb the either menace such as using better furnace design, improvising fuels and additives for molecular combustion or improvising fuel geometry and reactor design for efficient nuclear fission. In either case the perilous waste products are not completely eliminated. Although there are energy sources that have zero emissions like the energy harnessed from renewable sources like solar, wind, tidal, geo-thermal, wave, ocean-thermal and so on but are mainly time and location dependent. Hence cannot be universally employed for harnessing energy or power generation.

On the other hand, the nucleus of an atom has always been considered to be the source of unlimited energy since its discovery in 1911 by Ernest Rutherford [1]. The basic nuclear processes are of two types viz., fission and fusion. Both these processes generate large amount of energy which can be conveniently harnessed for useful work. The fission reaction is exoergic and criticality can be attained easily but fusion is endoergic and achieving criticality is comparatively difficult. Hence fission has been extensively explored for destructive as well as constructive work.

The unlimited source of the atomic nucleus due to fission process was initially exclusively exploited for destructive purpose. However, post World War II the focus shifted more towards constructive work. Attention was turned to the peaceful and directly beneficial application of nuclear energy. In the course of developing nuclear weapons the Soviet Union and rest of the Western world had discovered range of new technologies. Scientists also realized that the tremendous heat produced in the process could be tapped either for direct use or for generating electricity. It was also clear that this new form of energy had tremendous potential for the development of compact long-lasting power sources which could have various applications.

The world's first artificial nuclear reactor was Chicago Pile-1. It was a research reactor. Its construction was a part of the Manhattan Project. It was carried out by the Metallurgical

Laboratory, University of Chicago under the supervision of Enrico Fermi, alongwith Leó Szilárd (discoverer of the chain reaction), Martin Whittaker, Walter Zinn and George Weil [2, 3]. The first man-made self-sustaining nuclear chain reaction was initiated in Chicago Pile-1 on December 2, 1942. The apparatus was described as *a crude pile of black bricks and wooden timbers* by Fermi. The pile contained large amount of graphite (771,000 lbs) and uranium (80,590 lbs of uranium oxide and 12,400 lbs of uranium metal). The pile was in the form of flattened ellipsoid measuring 25 feet wide and 20 feet high. The neutron producing uranium pellets were separated from one another by graphite blocks in the pile. Some of the free neutrons produced by the natural decay of uranium were absorbed by other uranium atoms, causing nuclear fission of those atoms and the release of additional free neutrons. The graphite between the uranium pellets was neutron moderator that thermalized neutrons, increasing fission cross-section. The control rods were of cadmium, indium (for preventing uncontrolled chain reaction) and silver (measuring the flux). Unlike the modern reactors, it lacked radiation shield or cooling system.

The first nuclear reactor to produce electricity by fission was the Experimental Breeder Reactor-I or Chicago Pile-4 designed and operated by Argonne National Laboratory, Idaho, USA under the supervision of Walter Zinn. This LMFBR went critical in December 1951. It produced 0.8 kW in a test run on December 20, 1951[4] and 100 kW of electrical power the following day, [5] having a design output of 200 kW of electrical power.

With advancement of technologies, the modern fission reactors have high energy output but have disadvantages such as enrichment of fuel and / or moderator; disposal of high energy radioactive waste and cumbersome shielding from high energy ionizing radiations. Thus, the energy harnessed is not completely green. On the other hand fusion process does not generate large amount of nuclear waste and if nuclei combine at threshold energy then the chances of crossing the fission barrier and emission of ionizing radiation are reduced considerably.

2. Nuclear Fusion

The nuclear fusion has always been considered the holy grail of unlimited clean energy. The reason for this is probably the thermonuclear reactions taking place in the sun and other stars [6]. In this case, nuclear fusion is achieved by using extremely high temperatures. The average kinetic energy of the combining nuclei increases proportionately with temperature. The temperature is determined by Lawson criteria [7] as given by expression 1.

$$n\tau_E \geq L \equiv \frac{12}{E_{ch}} \frac{k_B T}{\langle \sigma v \rangle} \qquad (1)$$

where, n , τ_E , E_{ch} and k_B are particle density, confinement time, energy of charged fusion product and Boltzmann constant respectively.

The quantity $T/\langle \sigma v \rangle$ is a function of temperature with an absolute minimum. Replacing the function with its minimum value provides an absolute lower limit for the product $n\tau_E$. This is the Lawson criterion. At the temperature predicted by the Lawson criterion the energy of the colliding particles confined within the plasma are high enough to overcome the Coulomb barrier and chances of fusion increases. The colliding nuclei are confined within the plasma by gravitational or magnetic or inertial confinement. The controlled thermonuclear fusion reactions take place in an environment allowing some of the resulting energy to be harnessed for constructive purposes. Since this reaction takes place at very high temperature, so is popularly known as Hot Fusion. The major drawback is that it is not self sustaining and compound nucleus undergoes fission leading to formation of radioactive wastes. This is because the atomic electron clouds are completely stripped off. Kinetic energies of the colliding nuclei are increased to overcome the coulombic barrier and the energy attained by the compound nucleus is generally higher than the fission barrier which results in fission reaction or nuclear decay as prominent exit channels.

Fleischmann, Pons and Hawkins [8] in the year 1989 reported their historic but the most debatable findings. They observed unusual excess heat in the electrolysis of heavy water using deuterium loaded palladium electrodes. This they presumed to be due nuclear fusion reaction. Since the reaction taking place is at low temperature, they termed it as cold fusion on similar terms as hot fusion. Cold fusion or low energy nuclear fusion is known to occur under certain conditions in metal hydrides. It produces excess heat and nuclear ash such as helium, charged particles and sometimes very low level of neutrons. In certain cases the host metal has been found to be transmuted into other elements. The cold fusion reaction has been reported with palladium, titanium, nickel, some superconducting ceramics and so on. It has been observed due to varied triggers like ultrasonic waves, laser beam, electrical current. The major explanation for this phenomenon is reported to be the induction of electrostatic pressure to the reacting nuclei within the lattice of the metal. This environment is difficult to achieve and hence the phenomenon is non-reproducible. This could be due to insufficient energy required to expose the atomic nuclei from within the covering atomic electron cloud.

2.1. Nuclear Processes and Quantum Mechanics

Quantum mechanics is based on Galilei and Poincaré symmetries [9]. They are applicable only for Keplerian systems, where the various particles orbit around a centrally located nucleus, such as planets around central star / sun or electrons around nucleus. However, quantum mechanics is not applicable in understanding interaction between those particles which lack such symmetries like interaction between two electrons in a sigma bond or lateral overlap as in π-bonds. The Hamiltonian nature of quantum mechanics restricts the understanding of nuclear forces. Hence, to represent the nuclear force with a potential up to 35 different potentials have been added without achieving the required exact representation. The linear, local and Hamiltonian character of quantum mechanics is effective for the classification of

hadrons under their point-like approximation, but is inadequate for structure related problems due to expected nonlinear, nonlocal and non-Hamiltonian effects occurring within the hyper dense media inside hadrons.

Thus, Prof. Santilli [10] states: *According to the standard model, at the time of the neutron synthesis from protons and electrons inside a star, the permanently stable protons and electrons simply disappear from the universe to be replaced by conjectural quarks, and then the proton and the electron simply reappear at the time of the neutron decay. These beliefs are simply repugnant to me because excessively irrational, thus showing the conduction of particle physics via academic authority, rather than scientific veritas.*

The quantum theory fails to explain the following even for the simplest nucleus of deuterium [9, 10]-

1. The spin 1 of deuterium since quantum axioms require that the single stable bound state of two particles with spin ½, (proton and neutron) must be the singlet state with spin zero.
2. To represent the magnetic moment of deuterium.
3. The stability of unstable neutron when coupled to proton in a nucleus (e.g. deuterium).
   $T_{½}$ of neutron ≅ 15 minutes.
4. Quantum Mechanics is inapplicable for explaining the synthesis of neutron from a proton and an electron as occurring in stars because; in this case the Schrödinger equation becomes inconsistent.

It is unsuitable for all processes that are irreversible over time, like nuclear fusions, because quantum mechanics is reversible over time, thus admitting the time reversal event which violates energy conservation, causality and other basic laws.

2.2. Hadronic Mechanics

Quantum mechanics was conceived for the study of interactions among particles at large mutual distances which is represented with differential equations defined over a finite set of isolated points. It does not have the scope for the study of the additional nonlocal-integral interactions due to mutual wave overlapping. These interactions are defined over an entire volume and cannot be effectively approximated by their abstraction into finite number of isolated points. Thus, the same cannot be derived from a Hamiltonian or their derivatives [9].

Thus, Prof. Santilli has founded more fundamental theory of the universe, named after the composite nuclear particle *hadron* as *Hadronic Mechanics*. Hadronic mechanics was formulated as an extension of quantum mechanics, encompassing its insufficiencies for the study of the additional nonlocal-integral interactions due to mutual wave overlapping. Thus the range of hadronic, quantum and classical mechanics can be depicted as in Figure 1.

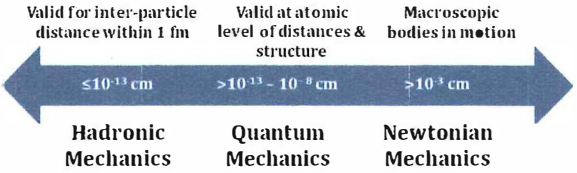

Figure 1. *Various range of validity for Hadronic, Quantum and Newtonian Mechanics.*

The emergence of strongly attractive force for deeply overlapping particles is one of the fundamental contributions of hadronic mechanics. There are varied instances where hadronic mechanics could satisfactorily explain the interactions such as quantitative treatment of neutron synthesis from protons and electrons (as occurring in stars), nuclear fusion, explanation of nuclear structure, strong nuclear binding energy, strong interaction between two electrons in a sigma bond, formation of magnecular bonds, formation of cooper pair in superconductors, and so on. Thus, hadronic mechanics could provide a quantitative treatment for the possible utilization of inextinguishable energy contained inside the neutron and formation of light nuclei. In other words, the study of new clean energies and fuels that cannot even be conceived with the 20th century doctrines and other basic advances can be done with the new sciences. So, hadronic mechanics is rightly called as new sciences for new era [10].

The modern day demand of clean, cheap and abundant energy source can be fulfilled by changing the approach from quantum mechanics to hadronic mechanics to hadronic chemistry. In view of this, Prof. Ruggero Maria Santilli proposed various types of new non-nuclear as well as nuclear fuels. Non-nuclear fuels are basically magnecules that show magnecular combustion similar to conventional molecular combustion albeit cleaner, greener and with higher calorific values probably due to stored magnetostatic energy within the magnecules [12, 13]. The hadronic fuels are summarized in Figure 2.

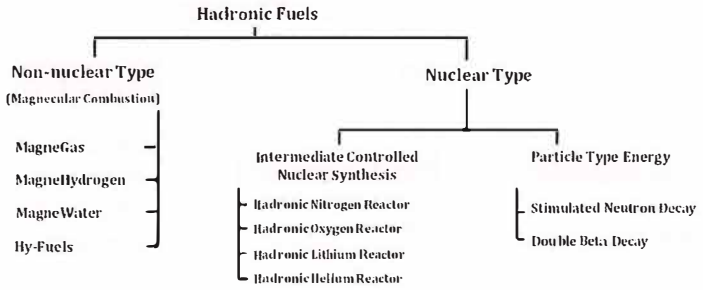

Figure 2. *The classification of various hadronic fuels.*

2.3. Nuclear Type Hadronic Fuels

The nuclear fuels proposed by Prof. Santilli under hadronic mechanics are controlled nuclear reactions (fusion as well as fission) without ionizing radiations and radioactive waste. The nuclear fission could be adequately explained by quantum mechanics by considering the fragments as point mass. However, the same theory fails to explain nuclear fusion because considering the reacting nuclei as point mass is impractical [10]. Nucleus is a hyper dense medium containing protons and neutrons. Since neutrons are made up of protons and electron, hence Prof. Santilli projects nucleus of an element as collection of mutated protons and electrons. The basic assumptions [11,12] proposed by Prof. Santilli are-

1. *Nuclear force:* Nuclear force was initially considered to be derived completely from a potential. So it was represented with a Hamiltonian. However, Prof. Santilli assumed that nuclear force is partly of action-at-a-distance, potential type that can be represented with a Hamiltonian and partly is of contact, non-potential type that cannot be represented with a Hamiltonian. This assumption implies that the time evolution of nuclear structure and processes is essentially of non-unitary type. So the use of hadronic mechanics is mandatory as it is the only known axiomatically consistent and time invariant non-unitary formulations of nuclear structures and their processes.
2. *Stable nuclei:* Nuclei have no nuclei of their own and are composed of particles in contact with each other having mutual penetration of about 10^{-3} of their charge distributions shown in Figure 3. So, the nuclear force is expected to be partially of potential and partially of non-potential type, with ensuing non-unitary character of the theory, and related applicability of hadronic mechanics.

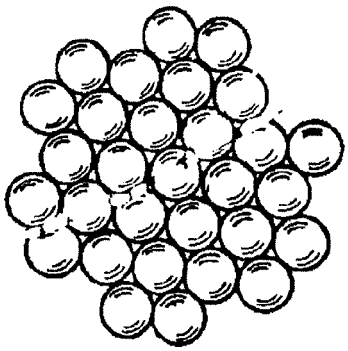

Figure 3. *Schematic representation used by Prof. Santilli to illustrate that nuclei have no nuclei of their own and are composed of particles in contact with each other.*

Let A be the time evolution of a Hermitean operator in the infinitesimal and finite forms derived from *Heisenberg-Santilli Lie-isotopic equations* proposed in 1978 by Prof. Santilli for stable, reversible, interior dynamical problems.

$$i\frac{dA}{dt}=[A, H]=ATH-HTA$$
$$A(t)=\exp(HTti)A(0)\exp(-itTH) \quad (2)$$

where the Hermitean Hamiltonian

$H=\frac{p^2}{2m}+V(r)$ represents all possible nuclear forces truly derived from potential V (r);

Isotopic element T represents all contact non-potential interactions allowing the nuclear structure with all constituents in actual contact with each other, and the simplest possible realizations of type

$$T=\exp\left(-F(r)\int \psi^{\dagger}(r)\psi(r)d^3r\right)>0 \quad (3)$$

which recovers quantum mechanics when there is no appreciable overlapping of the wavefunctions of nuclear constituents; and the inverse of isotopic element

$$I=\frac{1}{T}>0 \quad (4)$$

represents the basic, right and left unit of the theory at all levels, non-zero values of T depicts non-Hamiltonian interactions (presence of contact).

The stability of the nucleus (reversibility over time) is represented by the identity of the basic iso-unit to the right and to the left, namely, for motions forward and backward in time.

3. *Unstable nuclei and nuclear fusion:* According to the *Heisenberg-Santilli Lie-admissible equations* for the time evolution of Hermitean operator A in their infinitesimal and finite forms

$$i\frac{dA}{dt}=(A; H)=ARH-HSA \quad (5)$$

where H is Hermitean representing the non-conserved total energy
genotopic elements R and S represents the non-potential interactions.

Irreversibility is depicted by the different values of the genounit for forward (f) and backward (b) motions in time

$$I=\frac{1}{R}\neq I=\frac{1}{S} \quad (6)$$

Here, the Lie-admissible branch of hadronic mechanics is ideally suited to represent the decay of unstable nuclei as well as nuclear fusions, since both are irreversible over time.

4. *Neutron synthesis:* Rutherford's conjecture on neutron as a compressed hydrogen atom was experimentally verified later by Don Borghi's experiment. It is also well-known that synthesis of neutron from the compressed hydrogen gas is precursor to synthesis of all natural elements in a star. So, the synthesis of the neutron is the most fundamental nuclear synthesis. As shown in Figure 4 (a) the original drawing used by Prof. Santilli to illustrate the physical difference between the hydrogen atom and neutron synthesis from proton and

electron. The figure 4 (b) depicts the additional non-linear, non-local and non-potential interactions due to deep wave overlapping of proton and electron in neutron which is otherwise absent in hydrogen atom. This non-Hamiltonian character requires a non-unitary extension of quantum mechanics.

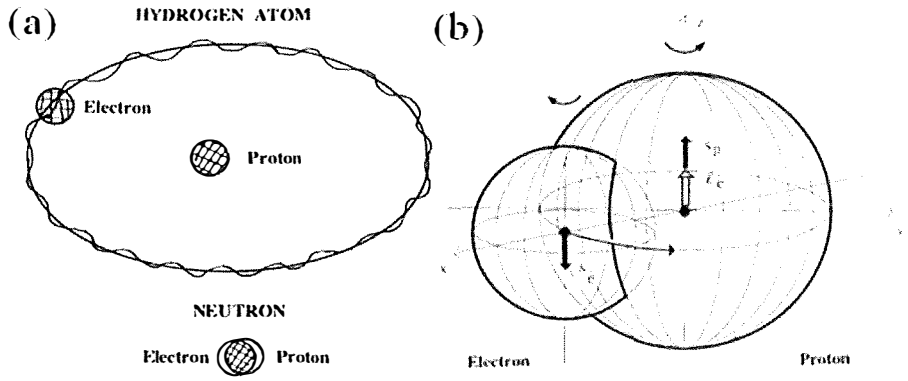

Figure 4. (a) Original drawing used by Prof. Santilli to illustrate the physical difference between the hydrogen atom and neutron. (b) additional non-linear, non-local and non-potential interactions due to deep wave overlapping of proton and electron in neutron.

Consequently, Prof. Santilli quantified neutron synthesis using hadronic mechanics as

$$p^+ + a + e^- \rightarrow n \tag{7}$$

where 'a' is Santilli's etherino which is a conventional Hilbert space the transfer of 0.782 MeV and spin 1/2 missing in the neutron synthesis from the environment to the neutron structure.

The etherino disappears at the covering level of hadronic mechanics and the neutron synthesis on a iso-Hilbert space over an iso-fields. Finally, the missing 0.782 MeV energy required for the synthesis of the neutron is provided by the environment. For instance, a star would not start producing light due to huge amount of energy needed for the synthesis of large number of neutrons. Thus, it was concluded that for continuous creation of neutron in the universe the missing energy is provided by the ether as a universal substratum.

5. *Nuclear Structure:* Prof. Santilli assumes the unitary classification of baryons as valid, but introduces new structure models of each member of the baryonic family with physical constituents that can be produced free, thus being detected in the spacetime. Resolution of historical objections is merely achieved by assuming that, when in interior conditions (only), barionic constituents obey hadronic mechanics and symmetries with subsequent mutations (denoted by hat) of their intrinsic characteristics. Proton is assumed to be an elementary stable particle without known structure and neutron to be an unstable particle comprising of a proton $\hat{p}^+$ and an electron $\hat{e}^-$ in mutated conditions due to their total mutual immersion and resulting synthesis

$$n = (\hat{p}^+, \hat{e}^-)_{hm} \tag{8}$$

As a result, it is assumed that nucleus is a collection of protons and neutrons in first approximation, while being at a deeper level it is a collection of mutated protons and electrons.

2.3.1. Controlled Nuclear Synthesis (CNS)

The hot fusion or cold fusion reactions are difficult to achieve. The high temperature required for hot fusion and random occurrence of cold fusion limits their use for economic energy output. One of the major successes of hadronic mechanics and iso sciences is their ability to obtain industrial realization of fusion reactions without any ionizing radiations. These reactions are controlled as well as have intermediate energy requirements than hot or cold fusions hence are called as controlled nuclear synthesis (CNS) or intermediate controlled nuclear synthesis (ICNS). *Controlled Nuclear Synthesis (CNS) are given by systematic energy releasing nuclear fusions whose rate of synthesis (or of energy output) is controllable via one or more mechanisms capable of performing the engineering optimization of the applicable laws* [11, 12].

There are various physical laws which are to be obeyed by all nuclear fusions to occur in a systematic way rather than in a random way. The CNS is governed by Santilli's laws for controlled nuclear synthesis [11, 12]:

1. *The orbitals of peripheral atomic electrons are controlled such that nuclei are systematically exposed.* Nuclei are shielded by the electron cloud. It is obvious that nuclear synthesis between two atoms is impossible at low energies because the electron cloud restricts approachability of the interacting nuclei. This law explains the inability of the cold fusions to achieve energy output of industrial significance because in this case the energy necessary for systematic exposure of nucleus from electron cloud is low. This law also emphasises the need for the proposed intermediate synthesis in which the first energy requirement is precisely the control of atomic clouds.
2. *CNS occurs when nuclei spins are either in singlet planar coupling or triplet axial coupling.* This law shows the structural difference between quantum and

hadronic mechanics. The constituents of a bound state of two quantum particles must be point-like to avoid structural inconsistencies such as local-differential topology. As a result, as per quantum mechanics singlet and triplet couplings are equally possible. However, when the actual extended character of the constituents is taken into account, it is clear that triplet planar couplings of extended particles at short distances are strongly repulsive, while singlet planar couplings are strongly attractive. Planar means that the two nuclei have a common median plane and axial means a common axial symmetry as shown in figure 5. This law was the basis to build hadronic mechanics via gear model. In fact, the coupling of gears in triplet (parallel spins) causes extreme repulsion, while the only possible coupling of gears is in singlet (antiparallel spins). The emergence of strongly attractive force for the singlet planar or triplet axial couplings is one of the fundamental contributions of hadronic mechanics to fusion processes since such a force is totally absent for quantum mechanics, while it appears naturally in all spinning and deeply overlapping particles.

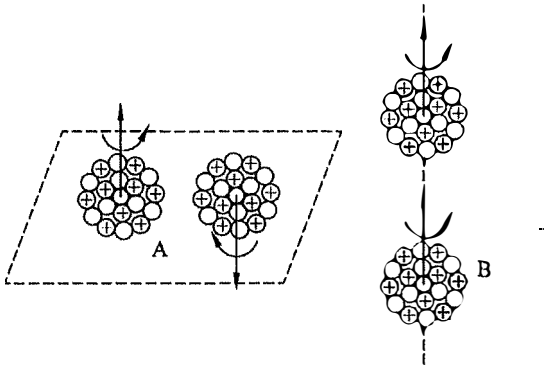

Figure 5. *Schematic representations of the only two stable couplings permitted by hadronic mechanics for nuclear synthesis; the singlet planar coupling (A) and the triplet axial coupling (B). All other spin configurations have been proved to produce strongly repulsive forces under which no CNS is possible.*

3. The most probable CNS are those occurring at threshold energies and without the release of massive particles or ionizing radiations. In other words, CNS occurring at threshold energies are green in nature as they do not emit ionizing radiations or ejectiles. The threshold energy mostly hinders fusion reaction. If the energy is lower than the threshold energy then industrially meaningful nuclear syntheses is not possible as in case for cold fusions, although random synthesis may occur due to tunnelling effect. On the other hand, if the energy of the interacting nuclei is higher than the threshold energy then the excess energy is reflected as excitation energy of the resulting compound nucleus. Thus, excitation energy of the compound nucleus is directly proportional to the energy of the interacting nuclei. The excitation energy is dissipated by emission of gamma photon or particles or fission of the resulting compound nucleus as shown in figure 6.

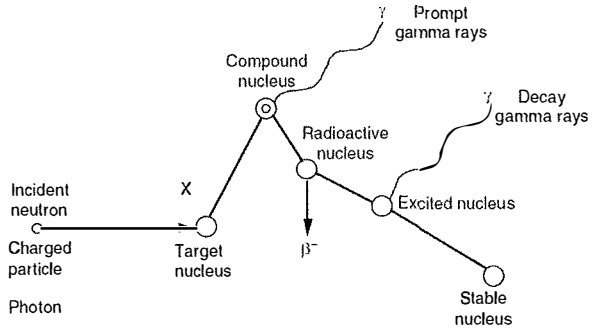

Figure 6. *Formation of compound nucleus having high excitation energy.*

The calculations based on hadronic mechanics indicate that the probability of a nuclear synthesis with the release of neutrons is much smaller than that of synthesis without the emission of massive particles. This law has been verified by ICNS data and also it appears to be verified by nuclear syntheses spontaneously occurring in nature. However, this does not mean that CNS with secondary emission is impossible. This only suggests that the nuclear synthesis could be green which was earlier unimaginable.

4. *CNS requires trigger, an external mechanism that forces exposed nuclei to come in fm range (hadronic horizon).* All nuclei are positively charged, thus repel each other at distances bigger than one Fermi. Nuclear synthesis is impossible without overcoming the coulombic repulsion that brings nuclei inside the hadronic horizon. Inside the hadronic horizon, the preceding laws are verified (particularly second law on spin couplings). The synthesis is inevitable due to the activation of the strongly attractive hadronic forces (typical non-potential interaction) that overcome the repulsive Coulomb force.

Considering the Fleishmann-Pons electrolytic cell in purview of Santilli's Law of CNS, it is clear that this cell does verify the conservation of the energy, angular momentum and has a trigger characterized by the electrostatic pressure compressing deuterium inside the palladium. However, Fleishmann-Pons electrolytic cell does not verify first law (control of atomic clouds to expose nuclei) and second law (control of spin couplings). Here the nuclear spin couplings occur at random; there is lack of identified mechanism for systematic exposure of the interacting nuclei and optimization of the verified laws. Consequently, nuclear syntheses occur at random, preventing economic values of the energy output. Thus, it is evident that for nuclear synthesis of economic value to occur all the above laws should be verified.

2.3.2. Magnecules: A Precursor for Nuclear Synthesis

Magnecules proposed by Prof. R. M. Santilli is a novel chemical species that have at least one magnecular bond between two atoms or radicals or molecules. The atoms are

held together by magnetic fields originating due to toroidal polarization of the atomic electron orbits [13, 14]. The rotation of the electrons within the toroid creates the fifth field force, the magnetic field, which cannot originate for the same atom if the conventional spherical electron distribution in orbitals is a physical reality. When two such polarized atoms are sufficiently close to each other and in north-south north-south alignment, the resulting total force between the two atoms is attractive as shown in Fig.7.

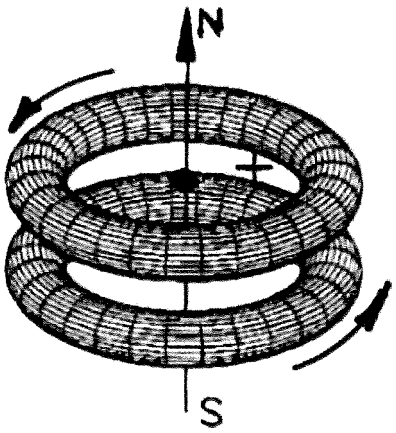

Figure 7. *Conceptual depiction of an elementary magnecule comprising two identical or different atoms whose bond is entirely of magnecular character, namely, originating from opposing polarities North-South-North-South of the toroidal distributions of orbitals, as well as the polarization of nuclear and electron magnetic moments.*

Thus, the simple principle of synthesizing magnecules is similar to the magnetization of a ferromagnet where the orbits of unbounded electrons are polarized. The added beauty of magnecules is that the nucleus is systematically exposed and the two nuclei can approach each other without appreciable columbic barrier. The internuclear distance is governed by the bond energy of the magnecular bond. The nuclei of the interacting atoms approach more closely than in case of conventional molecular bond allowing the required singlet planar or triplet axial coupling for nuclear synthesis. Thus, when a trigger brings the two nuclei within 1 fm range (hadronic horizon) the fusion becomes inevitable and a new nucleus is formed. Since the parent nuclei are not having high energy the resulting daughter nuclei also does not possess high excitation energy, consequently there is no nuclear emission. Thus the process is green.

Thus, the ICNS proposed by Santilli are of the generic type [11, 12]

$$\begin{aligned} &N_1(A_1,Z_1,J_1^{P_1},u_1)+N_2(A_2,Z_2,J_2^{P_2},u_2)+TR \\ &\qquad \rightarrow N_3(A_3,Z_3,J_3^{P_3},u_3)+\text{Heat} \\ &\text{where } A_1+A_2=A_3,\quad Z_1+Z_2=Z_3, \\ &J_1+J_2=J_3,\quad P_1+P_2=P_3 \end{aligned} \tag{9}$$

A is the atomic number

Z is the nuclear charge

J^P is the nuclear angular momentum with parity

u is the nuclear energy in amu units

TR is trigger mechanism (high voltage DC arc in hadronic reactor) and mass defect is observed in form of heat

Nuclear synthesis via green mechanism is known to occur silently in nature [11]. This can be verified from the chemical analyses of about one hundred million years old amber sample. The trapped air bubbles showed 40% nitrogen, whereas the current percentage of nitrogen in atmosphere is approximately 80%. Other chemical analyses verify the above analysis that the increase of nitrogen in our atmosphere has been gradual.

According to Prof. Santilli, these data indicate *the natural synthesis of nitrogen from lighter elements.* The most probable mode of nitrogen synthesis in nature seems to be initiated by *lightning* as quantitative explanation of *thunder* is impossible by conventional chemical reactions, thus requiring nuclear syntheses. Numerical explanation of thunder requires energy equivalent to hundreds of tons of explosives that simply cannot be explained via conventional processes due to the very small cylindrical volume of air affected by lightning and its extremely short duration of the order of nanoseconds. However, the nitrogen syntheses by lightning provide numerical explanation of thunder as well as the gradual increase of nitrogen in the atmosphere. Among all possible syntheses, the most probable one results in being the *synthesis of nitrogen from carbon and deuterium.*

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$$\begin{aligned} &C(12,6,0^+,12.0000)+D(2,1,1^+,2.0141) \\ &\quad +TR \rightarrow N(14,7,1^+,14.0030)+\Delta E \\ &\qquad \Delta E = 0.0111\text{amu} = 10.339\text{MeV} \end{aligned} \tag{10}$$

However, the amount of deuterium present in the atmosphere is negligible to justify thunder quantitatively. Here, Prof. Santilli emphasizes the synthesis of neutrons by lightning from protons and electrons.

The neutron synthesis is expected to be a pre-requisite for the synthesis of deuterium in atmosphere which in turn synthesizes nitrogen which justifies energy of the thunder quantitatively. The same synthesis has been reproduced in laboratory quantitatively by Prof. Santilli using hadronic reactor.

2.4. Synthesis of Nitrogen from Carbon and Deuterium by ICNS

The fusion reaction/nuclear synthesis taking place in hadronic reactor using deuterium as fuel and carbon electrode have shown to yield clean energy without formation of any radioactive species or ionizing radiations [15, 16]. This synthesis is of industrial importance because it yields 10^{10} BTU of energy per hour which is equivalent to 10^{30} ICNS per hour. The electric arc of the hadronic reactor polarizes carbon and hydrogen atoms by forming C × H × H magnecule, having triplet axial spin coupling. Under a suitable trigger (either high DC voltage or any other suitable means) the magnecule C×H×H yield a nucleus with A=14, Z=8, $J^P=1^+$. However, this is impossible as O (14, 8) has spin J = 0 and any other nucleus of the above mentioned type does not exist.

So, Prof. Santilli postulated that the nature synthesizes a neutron from proton, electron and etherino as

$$\begin{aligned} C \times H \times H &\rightarrow C(12,6,0) + 2 \times p^+ + e^- + a \\ &\rightarrow C(12,6,0) + H(2,1,1) \rightarrow N(14,7,1) \end{aligned} \tag{11}$$

Other Examples of ICNS

(i). Synthesis of Silicon from Oxygen and Carbon

$$\begin{aligned} &O(18,8,0^+,17.9991) + C(12,6,0^+,12.0000) \\ &+ TR \rightarrow Si(30,14,0^+,29.9737) + \Delta E \\ &\Delta E = 0.0254u \end{aligned} \tag{13}$$

This nucleosynthetic reaction verifies all conservation laws. The controlled fusion of oxygen and carbon into silica was done using CO_2 (green house gas) as hadronic fuel for the production of clean energy [11]. The whitish powder formed on the edge of carbon electrodes of the hadronic reactor suggests synthesis of silica. Hadronic reactor was filled up with CO_2 at pressure. The DC arc efficiently separates it into O_2 and C. O_2 and C burns to produce CO which in the presence of oxygen and an arc, reproduces CO_2.Thus recovering the energy used for the separation of CO_2. However, along with the conventional combustion, the hadronic reactor produces a net positive energy output due to the fusion of oxygen and carbon into silica [17].

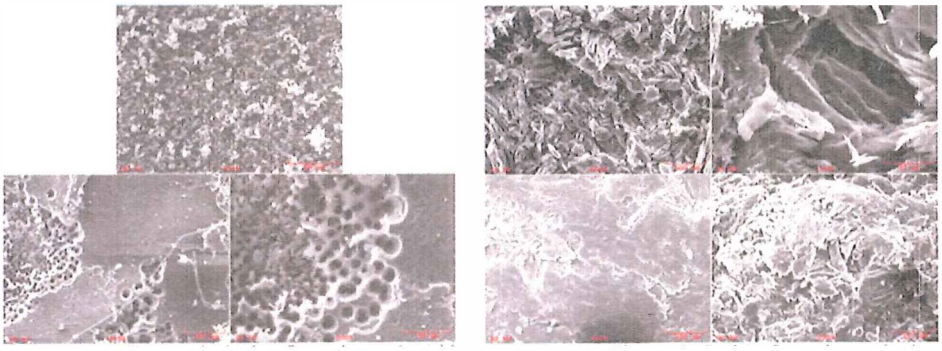

Figure 8. *Semi-quantitative chemical compositions of the used electrodes were obtained using an Amray Scanning Electron Microscope equipped with an iXRF energy dispersive spectrometer (SEM-EDS) by Constellation Technology on 15th October 2013. The elements detected were aluminium, silicon, calcium, chlorine and iron which were initially absent or if present were in trace amount.*

(ii). Synthesis of Oxygen from Carbon and Helium

$$\begin{aligned} &C(12,6,\bullet^-,12.0000) + He(4,2,\bullet^-,4.0026) \\ &+ TR \rightarrow O(16,8,0^+,15.9949) + \Delta E \\ &\Delta E = 0.0077u \end{aligned} \tag{12}$$

This nucleosynthetic reaction also verifies all conservation laws. Here, the interior of the reactor was cleaned, and various components replaced. A vacuum was pulled out of the inner chamber and the reactor was filled up with commercial grade helium at 100 psi. It was found that oxygen content decreased to a non-detectable amount but the CO increased from a non-detectable amount to 4.24%.

The formation of CO depicts synthesis of oxygen at the tip of the DC arc hitting the carbon in the cathode surface. The resulting large local heat rapidly expels the synthesized oxygen from the DC arc, preventing any additional nuclear synthesis. However, high affinity of carbon and oxygen results in formation of carbon monoxide.

(iii). Synthesis Silver from Palladium and Hydrogen

$$\begin{aligned} &Pd(106,46,0^+,105.9034) + H(1,1,1/2^+,1.0078) \\ &+ TR \rightarrow Ag(107,47,1/2^+,106.90509) \end{aligned} \tag{14}$$

This nucleosynthetic reaction depicts the basic difference between pre-existing studies on cold fusion and the proposed ICNS. In this reaction palladium 106 is used as cathode and reactor is filled with hydrogen at a certain pressure.

If cold fusion occurs, then fusion reactions should take place inside the palladium cathode. However, the engineering implementation of the new CNS laws inside the palladium electrodes is virtually impossible, thus explaining the reason for the lack of its consideration in the industrial research. According to Prof. Santilli the nuclear fusions may occur in such conditions at random, thus preventing the controlled energy output necessary of industrial relevance.

Thus, albeit this reaction verifies conventional nuclear conservation laws but is not of industrial relevance owing to

its random nature.

2.5. Santilli Hadronic Reactors

Hadronic reactors are the upgraded hadronic refineries originally designed by Prof. Santilli [11]. They use magnecular fuels for production of heat that can be used for power generation. The reactors house trigger mechanisms like high voltage DC arc or pressure impulse, etc as shown in Fig. 9 to facilitate controlled nuclear synthesis. Hadronic reactors can withstand higher pressure as compared to the hadronic refineries.

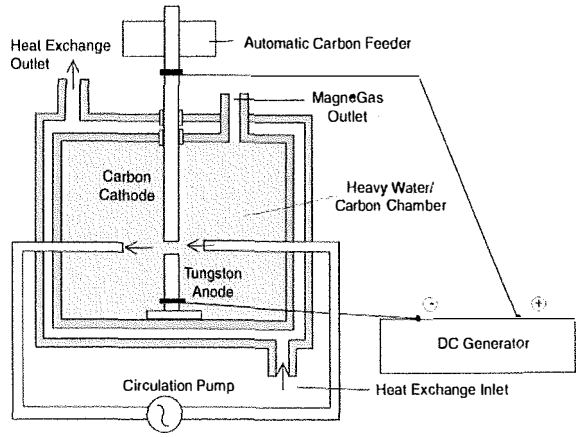

Figure 9. Schematic diagram of hadronic reactor based on an upgradation of the hadronic refineries showing emphasis on the production and use of a magnecular fuel in the latter, to the production and use of heat in the former.

These reactors are named based on the product or the fuel used. Hadronic nitrogen reactor is the most primitive type of hadronic reactor.

2.5.1. Hadronic Nitrogen Reactor

The reactor is filled with D_2 gas at 3,000 psi and is re-circulated through graphite electrodes. The trigger mechanism is by pulse DC arc of 100,000 V, 5 mA and other means. The heat is dissipated by the external heat exchanger. The heat is due to the nucleosynthetic reaction between deuterium and carbon occurring in reactor

$$\begin{aligned} &C(12,6,0^+,12.0000)+D(2,1,1^+,2.0141)+ \\ &TR \rightarrow N(14,7,1^+,14.0030) \end{aligned} \quad (15)$$

2.5.2. Hadronic Oxygen Reactor

It is one of the simplest reactors as the reaction does not require spin polarizations for conservation of the angular momentum. So the reactor is similar to the one shown in Fig. 5 housing carbon electrodes. The vessel is filled up with a 50-50 mixture of oxygen 16 and helium at 3,000 psi, which is re-circulated through 50 kW electric arc to create magnecules of the type O He. The trigger is DC pulses of 100,000 V and 5 mA, or by impulse pressures or other mechanisms. The heat produced is dissipated by the external heat exchangers.

Nucleosynthetic reaction occurring in the reactor

$$\begin{aligned} &O(16,8,0^*,15.9949)+H(2,1,1^-,2.0141) \\ &+TR \rightarrow F(18,9,1^-,18.0009)+\Delta E \\ &\Delta E = 0.0081u = 7.545MeV \end{aligned} \quad (16)$$

The instability of $F(18, 9, 1^+, 18.0009)$ results in secondary process

$$\begin{aligned} &F(18,9,1^+,18.0009)+EC \rightarrow \\ &O(18,8,1^+,17.9991)+1.656\,MeV \end{aligned} \quad (17)$$

Thus the total energy output per synthesis is equivalent to 9.201 MeV 1.30×10^{-15} BTU.

If 10^{30} syntheses occur per hour then amount of green energy yielded would be substantial.

2.5.3. First Hadronic Lithium Reactor

First lithium reactor is the same as that of the oxygen reactor. The only difference is that the vessel is filled with 50-50 mixture of hydrogen and helium gases at 3,000 psi. The mixture is also recirculated through a 50 kW electric arc that creates magnecules H He. The trigger is given by a high voltage pulse DC current or impulse pressure or other mechanism.

Nucleosynthetic reaction occurring in the reactor is

$$\begin{aligned} &H(2,1,1^+,2.0141)+He(4,2,0^+,4.0026)+ \\ &TR \rightarrow Li(6,3,1^+,6.0151)+\Delta E \\ &\Delta E = 0.0016u = 2.50\times10^{-16}BTU \end{aligned} \quad (18)$$

2.5.4. Second Hadronic Lithium Reactor

It is more complex than the first hadronic lithium reactor because of the need of lithium nuclei and a beam of protons with opposite polarization to avoid random reactions. The current technology allows a variety of engineering realizations of the needed polarization where a proton beam with down polarization enters a chamber of lithium with up polarization. Both polarizations are achieved via magnetic fields. The efficiency of the hadronic reactor depends on the geometry of the proton beams, the lithium chamber as well as required trigger.

Nucleosynthetic reaction occurring in the reactor

$$\begin{aligned} &Li(7,3,3/2^-,7.0160)+H(1,1,1/2^+,1.0078)+ \\ &TR \rightarrow 2\times He(4,2,0^-,4.0026)+\Delta E \\ &\Delta E = 2.74\times10^{-15}BTU \end{aligned} \quad (19)$$

Assuming efficiency of 10^{16} per minute one mole of lithium would produce energy equivalent to 1.7×10^6 J hour^{-1}.

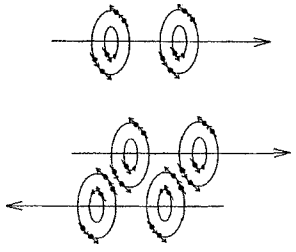

Figure 10. Schematic view of singlet (antiparallel) spin coupling required to synthesize helium from deuterium.

2.5.5. Hadronic Helium Reactor

It is one of the most difficult as it requires the application of a trigger to two different beams of deuterium gas with opposite spin polarizations as depicted in Fig. 10. The reactor as shown in Fig. 11 is a metal vessel that houses two parallel but separate electric arcs with opposing polarities so as to produce opposite polarizations of the deuterium gas. The flow of the gas through said arcs from opposite directions creates the superposition of the beams in the area located between said arcs with spin couplings as shown in Fig. 11. The trigger seems to be the impulse pressure.

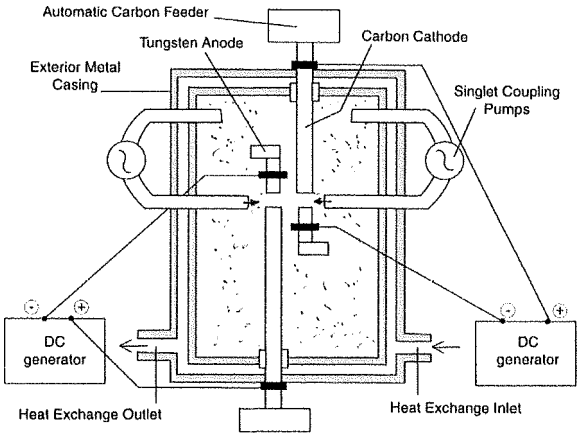

Figure 11. Schematic representation of hadronic reactor for nucleosynthesis of helium.

The nucleosynthetic reaction is-

$$D(2,1,1^{+}\uparrow,2.0141)+D(2,1,1^{+}\downarrow,2.0141)+TR\rightarrow He(4,2,0^{+}\uparrow,4.0026)+\Delta E \quad (20)$$

2.6. Particle Type Hadronic Energy: Stimulated Decay of Energy

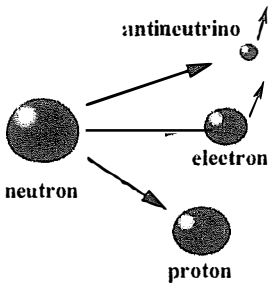

Figure 12. Schematic representation of stimulted neutron decay.

Hadronic nuclear energy can also be obtained by fission reactions or decay of stable nuclei. Theoretically any stable nuclei can be disintegrated into its nuclear constituents by photons having higher energy than the binding energy of the nuclei to be disintegrated. With higher stable nuclei the energy of the photons required to disintegrate also increases. The low binding nuclei like ${}_{1}^{2}H$ and ${}_{4}^{9}\mathrm{Be}$ are well-known to undergo photo-disintegration with 2.22 MeV and 2.62 MeV photons respectively [1]. Similarly, stimulated decay of neutrons as represented in Fig. 12 is also a well-known phenomenon. The prediction and its quantitative treatment can be done by hadronic mechanics.

According to Prof. Santilli, neutron is an unlimited source of energy because it decays releasing highly energetic electron and neutrino that can be easily trapped with a metal shield. It is known that an isolated neutron is highly unstable and has half life of approximately15 minutes. However, as a constituent of nuclei, it shows high stability which has been attributed to a strong nuclear force of attraction. The neutron shows stimulated decay as

$$TR+n\rightarrow p^{+}+\beta^{-} \quad (21)$$

where β– has spin zero for the conservation law of the angular momentum. β– can also be considered either as an electron and a neutrino or as an electron and an antietherino with opposing spin ½. However, this difference is irrelevant for the stimulated decay of the neutron.

When a resonating photon hits a nucleus, it excites the isoelectron inside a neutron irrespective of whether the photon penetrates or not inside the neutron. The excited isoelectron leaves the neutron structure, thus causing its stimulated decay. This is due to the fact that hadronic mechanics predicts only one energy level for the proton and the electron in conditions of total mutual immersion (as incase of neutron). Range of hadronic mechanics is given by the radius of neutron that is 1 fm. Thus, the excited isoelectron excites the proton and reassumes its conventional quantum features when moving in vacuum.

Numerous additional triggers are predicted by hadronic mechanics such as photons with a wavelength equal to the neutron size. Here, the whole neutron is excited, rather than the isoelectron in its interior, but the result is always the stimulated decay.

Double Beta Decay

In this typical example of double decay first reaction is stimulated and the second is spontaneous [11].

$$\gamma_{r}(0,0,1)+N(A,Z,J)\rightarrow N(A,Z+1,J+1)+\beta^{-}(0,-1,0)$$
$$N(A,Z+1,J+1)\rightarrow N(A,Z+2,J+2)+\beta^{-}(0,-1,0) \quad (22)$$

The original isotope should admit stimulated decay of at least one of its peripheral neutrons via one photon with a resonating frequency verifying all conservation laws of the energy, angular momentum, etc. The new nucleus formed should undergo spontaneous beta decay so that with one resonating photon there is production of two electrons whose kinetic energy is trapped with a metal shield to produce heat. The original isotope is metallic so that, following the emission of two electrons, it acquires an electric charge suitable for the production of a DC current between the metallic isotope and the metallic shield. The energy balance is positive. The initial and final isotopes are light, natural and stable elements so that the new energy is *clean* (since the electrons can be easily

trapped with a thin metal shield), and produce non-radioactive waste.

E.g. double beta decay of the Mo(100, 42, 0)

$$\begin{aligned}&\gamma_r(0,0,1)+\mathrm{Mo}(100,42,0)\\&\quad\rightarrow \mathrm{Tc}(100,43,1)+\beta^-(0,-1,0)\\&\mathrm{Tc}(100,43,1)\rightarrow\\&\quad \mathrm{Ru}(100,44,0)+\beta^-(0,-1,0)\end{aligned}\tag{23}$$

Mo(100, 42, 0) is naturally stable with mass 99.9074771 amu. Tc(100, 43) has mass 99.9076576 amu and is naturally unstable with spontaneous decay into Ru (100, 44, 0) and half life of 15.8 s. Ru (100, 44) is naturally stable with mass 99.9042197 amu. Although the mass of Mo(100, 42, 0) is smaller than that of Tc(100, 43, 1), yet the conservation of energy can be verified with a resonating frequency of 0.16803 MeV (obtained for n=1/7) where n is normalization contant.

But the mass of the original isotope is bigger than that of the final isotope for a value much bigger than that of the resonating photon, with usable hadronic energy (HE) power nuclear reaction

$$\begin{aligned}\mathrm{HE}&=\mathrm{M}(100,42)-\mathrm{M}(100,44)-\\&\quad\mathrm{E}(\gamma)-2\times\mathrm{E}(e)\\&=3.034-0.184-1.022\mathrm{MeV}\\&=1.828\,\mathrm{MeV}\end{aligned}\tag{24}$$

where Santilli subtracts the conventional rest energy of the two electrons because it is not usable as a source of energy in this case.

Under the assumptions of using a coherent beam with resonating photons hitting a sufficient mass of Mo(100, 42, 0) suitable to produce 10^{20} stimulated nuclear transmutations per hour, we have the following:

Hadronic production of heat :

$$2\times10^{20}\ \mathrm{MeV/h}=3\times10^{4}\ \mathrm{BTU/h},\tag{25}$$

Hadronic production of electricity :

$$2\times10^{20}\ \mathrm{e/h}=200\mathrm{C/h}=55\ \mathrm{mA}.\tag{26}$$

3. Applications of Hadronic Nuclear Energy

3.1. Intermediate Controlled Nuclear Synthesis

1. Green power generation source
   ICNS can be industrially exploited for power generation. Since there are no ionizing radiations or particular emissions, it is green and can be used for sustainable development.
2. Synthesis of heavy and super heavy elements
   Synthesis of heavy elements particularly of the seventh period is conventionally done by bombarding two heavy nuclei. The reacting rather bombarding nuclei have high energy to overcome the coulombic barrier. This results in high excitation energy of the resulting daughter nuclei which is often higher than the fission barrier. Thus, fission is one of the pre-dominant exit channel.
   However, if ICNS is used then the nuclei of the participating atoms can be exposed in controlled manner as well as can be brought near each other to a considerable extent without initiating coulombic repulsion. This is due to magnetic bond in the participating nuclei which in this case are magnecules rather than mere atoms or ions. The trigger mechanism then pushes the participating nuclei within hadronic radius where fusion is inevitable. Consequently formation of heavy nucleus takes place. Converting heavy nucleus into respective magnecules requires high magnetic field and would be a costly affair. However, the daughter heavy nuclei produced would be considerable stable making the synthesis green and viable.
   Stable heavy daughter nuclei formed would allow study of its actual chemical characteristic instead of predicting on the basis of periodic table and spectroscopic studies.

3.2. Particulate Type Nuclear Energy

1. Green power generation source
   Stimulated decay of neutron and double beta decay can be used for power generation. The by product is electron which can be stopped with a metal sheet. This results in a clean and green power source.
2. Recycling of nuclear waste
   It may also be used for recycling of nuclear waste generated due to existing conventional nuclear energy facility by stimulated neutron decay using photon with resonating frequency (or energy) of 1.294 MeV.

4. Conclusion

ICNS seems to be more promising than hot or cold fusion in terms of reproducibility and energy input to output ratio. The successful achievement of ICNS with industrial relevance depends on the proper selection of the *hadronic fuel.* The hadronic fuel is mainly due to-

a) The original and final nuclides are light, natural and stable isotope.
b) The nuclear syntheses cause no emission of ionizing radiations.
c) The energy produced ΔE is much bigger than the total energy used by the equipment for its production.

Stimulated beta decay and double beta decay also seems to be promising prospect for green power generation. Apart from power generation, ICNS and stimulated decay holds promising prospect for synthesis of heavy elements and recycling of nuclear wastes respectively.

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Possibilities for the Detection of Santilli Neutroids and Pseudo-protons

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Abstract: Following systematic mathematical, theoretical and experimental studies on the synthesis of the neutron from hydrogen, R. M. Santilli noted delayed neutron detections following the termination of tests, and attempted to represent them with the hypothesis of a new state of the hydrogen with spin zero called *neutroid* consisting of a proton and an electron at 1 fm mutual distance in singlet coupling. More recently, Santilli predicted the possible existence of a second new particle called *pseudo-proton* characterized by the synthesis of the electron with the neutron, therefore resulting in a negatively charged unstable particle with a mean life expected in the range of that of the neutron and a mass of the order of the hydride ion. Subsequently, Santilli has indicated that, in the event confirmed, the pseudo-proton could eliminate the Coulomb barrier for nuclear syntheses and trigger nuclear transmutations with large release of heat without neutron emission, thus identifying a possible novel use of hydrogen for the industrial production of a basically new clean nuclear energy. In view of the latter possibility, in this paper specific experiments are proposed for the verification or denial of the existence of Santilli neutroid and pseudo-proton and, in case of confirmation, accurate measurements of their characteristics and production in numbers sufficient for industrial application.

Keywords: Pseudo-proton, Low Energy Nuclear Transmutation, Hydrogen Energy Source, Neutron, Pseudo-protonium, Detection, Spectroscopy

1. Introduction

Recently, Santilli et al. called for the experimental verification of the so-called *pseudo-proton* [1]. The pseudo-proton is a particle predicted by Santilli [2] based on the hadronic mechanics he has derived since the 1970's (for an overview see [3,4]). The pseudo-proton is expected to play an important role in the upcoming field of low energy nuclear transmutations due to its ability to penetrate the atomic core with relative ease due to its negative charge [5]. It is expected that it plays a crucial role in the synthesis of hydrogen ions [6], [7]. Hence the experimental detection of the pseudo proton is important for future development of hydrogen energy.

2. Synthesis of Neutron from Hydrogen

The synthesis of the neutron from the hydrogen inside a star was proposed in 1920 by H. Rutherford [8], experimentally established in 1932 by J. Chadwick [9] and formalized by E. Fermi [10] via the familiar reaction

$$p^+ + e^- \rightarrow n + \nu \quad (1)$$

The properties of proton, electron and neutron are well known and a summary is given in table 1. The rest mass of the neutron, n is 0.782 MeV larger than the sum of the rest energies of both proton and electron, so that this reaction can only occur in sufficiently strong fields, for instance in the inside of stars or in the arc of a very strong discharge [1]. In 1978, R. M. Santilli [11] pointed out that quantum mechanics is not applicable to the above neutron synthesis because the rest energy of the neutron is 0.782 MeV bigger than the sum of the rest energies of the proton and the electron, as shown by the energy of the rest mass for proton, electron and neutron respectively E_p = 938.272 MeV, E_e = 0.511 MeV and

$E_n = 939.565$ MeV,

$$E_n - (E_p + E_e) = 0.782\,\text{MeV} > 0 \qquad (2)$$

thus requiring a 'positive binding energy' with a resulting 'mass excess' under which the Schrödinger equation no longer admits physically consistent solutions.

In order to explain neutron synthesis, Santilli [11] proposed the construction of a non-unitary covering of quantum mechanics under the name of *Hadronic Mechanics*. However, non-unitary theories on a conventional Hilbert space over a conventional numeric field are known to violate causality. Only following the achievement of sufficient mathematical and theoretical maturity (for more details see [5]), while visiting in 1991 the ICTP in Trieste, Italy, Santilli [12] achieved the first known, non-relativistic, exact representation of *all* characteristics of the neutron in its synthesis from the hydrogen in the core of a star. A relativistic representation was achieved in 1992 when visiting the JINR in Dubna, Russia [13].

Laboratory synthesis of the neutron from a hydrogen gas was achieved in the 1960s by the Italian priest-physicist Don Carlo Borghi and his associates [14] via a metal chamber containing hydrogen at pressure traversed by microwaves and an electric discharge used to keep hydrogen ionized.

Santilli could not account for the synthesis of the neutron via microwaves. However, the same formulations predicted the laboratory synthesis of the neutron under an electric discharge suitable to force the penetration of the electron within the proton following their strong Coulomb attraction (according to Rutherford [8]) and provide 0.782 MeV needed for the synthesis according to equation (2). Extensive tests apparently confirmed the laboratory synthesis of the neutron according to the predictions of Santilli's work [15]. Thereafter, Santilli continued systematic experimental research on the laboratory synthesis of the neutron from a hydrogen gas at the laboratory of the Institute for Basic Research, Palm Harbor, Florida (see the extensive data in Ref. [16]).

Systematic experimentations and industrial realizations are now conducted by the U. S. publicly traded company Thunder Energies Corporation (TEC) [17], which is currently organizing the production and sale of a *Thermal Neutron Source* (TEC domain names and patents pending) essentially consisting of a pressure vessel containing commercially available hydrogen traversed by a suitable discharge and control of hydrogen pressure, electric power and other characteristics to obtain the desired neutron flux output (see corporate paper [18] and movie [19] on a pre-production prototype.)

A recent technical review of the synthesis of the neutron has been provided by U. Abundo [20]; a review recommended to the general physics audience was provided in 2006 by the late J. V. Kadeisvili [21]; while a general review of the various mathematical, theoretical and experimental aspects has been provided in 2011 by I. Gandzha and the late J. V. Kadeisvili [22].

3. Neutroid and Pseudo-proton

Essential the bases for the prediction of the pseudo-proton is the natural assumption that mathematical point particles do not exist in nature. In real life particles will have a finite size and there is a finite probability that particles will overlap. When particles overlap, the intrinsic properties of the particles can change and can recover when they are released again in vacuum. A more elaborate overview is given in [22]. Hence, when a hydrogen atom consisting of a proton, p^+ and an electron, e^- is sufficiently compressed (so that its radius is of the order of 1 fm) they can form a neutron via the above described procedure.

Table 1. Properties for several particles. For the pseudo-proton the range in which the property is expected is indicated. [1] in units of $1.602177 \cdot 10^{-19}$ C.

Name	Sym	Description
Electron	e^-	Elementary particle
Proton	p^+	Elementary particle
Neutron	n	Elementary particle (standard model) or bound proton and electron (Santilli)
Hydrogen atom	H	Proton + orbit electron
Hydrogen ion	H_2^+	Ionized hydrogen molecule
Pseudo-proton	$\tilde{p}^-$	Synthesis of neutron and electron (Santilli)
Hydride ion	H^-	Proton + 2 electrons in orbit
Anti-proton	$\bar{p}^-$	Anti particle of proton

	Charge[1]	Mass 10^{-27}kg	MeV	Magnetic mom. 10^{26} J/T
e^-	-1	0.00091	0.511	928.477
p^+	1	1.67262	938.272	1.410608
n	0	1.67493	939.565	0.966237
H	0	1.67353	938.783	
H_2^+	-1	3.34615	1877.055	
$\tilde{p}^-$	-1	1.672...1.677	937...941	≈1
H^-	-1	1.67444	939.294	
$\bar{p}^-$	-1	1.67262	938.272	1.410608

Both tests by Don Borghi [14] and Santilli [15,16] indicated the existence of nuclear transmutations that could solely be explained via the absorption of neutrons or 'neutron-type' particles. Santilli conducted extensive studies of this occurrence based on the detection of neutron counts that were delayed for up to 15 minutes following the termination of the tests.

These delayed neutron counts occurred under certain characteristics of the reactor, such as power, hydrogen pressure and others, insufficient to provide 0.782 MeV needed for the synthesis of the neutron according to equation (2). By contrast, neutrons were detected under sufficient power, hydrogen density and other characteristics, to such an extend to require various evacuations of the laboratory.

In this way, Santilli proposed the possible existence of a new particle called *neutroid* [16] with essentially the characteristics of the neutron except having spin zero which is created by the reaction

$$p^+ + e^- \rightarrow \tilde{n} \qquad (3)$$

under insufficient conditions to achieve the full reaction according to equation (1). A number of nuclear

transmutations caused by the absorption of neutroids could then allow a quantitative representation of the delayed detection of neutrons (see Ref. [16] for brevity).

Abundo [20] noted that, in the event confirmed, Santilli neutroids would not experience the notorious Coulomb barrier opposing nuclear syntheses and can, therefore, trigger nuclear transmutations with large release of heat without the emission of neutrons, such as

$$^{1}\mathrm{H} + {}^{7}Li + TR \rightarrow {}^{7}Li + \tilde{n} \rightarrow {}^{8}\tilde{L}i \rightarrow {}^{8}\tilde{B}e + \beta^{-} \rightarrow 2\,{}^{4}He + \Delta E \quad (4)$$

where $\Delta E > \approx 15.3$ MeV per ^{7}Li nucleus and TR represents Santilli's trigger, namely, a mechanisms triggering the transmutation, such as instantaneous increase of power or pressure. Note that the symbol ^{1}H in the above reaction represents the hydrogen atom. Therefore, to our best knowledge, Abundo's transmutation [20] is the first in scientific records in which *the hydrogen atom, rather then the proton, triggers a nuclear transmutation, thus stimulating potential new technologies centrally based on hydrogen.*

To understand the occurrence, one should note that Santilli neutroid is essentially a new (unstable) form of the hydrogen atom with the proton and electron in singlet coupling kept together by the very strong Coulomb attraction at 1 fm mutual distance. Santilli's trigger is, therefore, a mechanism essentially triggering the transition from the hydrogen to the neutroid that, as such, has a number of possible engineering realizations.

Following the systematic studies on the synthesis of the neutron from hydrogen, Santilli predicted in the Appendix of Ref.. [2] the possible existence of a second new particle under the name of *pseudo-proton* with symbol $\tilde{p}^{-}$, generated by the synthesis of the electron, this time, with the hydrogen atom, according to the reaction

$$\left(p^{+}_{\downarrow} + e^{-}_{\uparrow}\right) + e^{-}_{\downarrow} \rightarrow \tilde{p}^{-} \quad (5)$$

Note that, in comparison with Fermi's reaction (equation (1)), Santilli's reaction does not require the emission of a neutrino. The pseudo-proton will have a spin 1/2, a negative unit charge, essentially the same charge radius and rest energy of the proton, and a mean life (when isolated) of the order of that of the neutron. The properties of the pseudo-proton are summarized in table 1. It is expected that the magnetic moment is of the same order as that of the proton and neutron as the charge radius and rest energy are also of the same order. It will be determined by the precise shape and velocity of the constituting charge distribution. The charge and mass of the pseudo-proton will be similar to that of the anti-proton or hydride ion.

More recently, Santilli [5] indicated that, in the event confirmed, *the pseudo-proton would not only eliminate the Coulomb barrier for nuclear syntheses, but could actually be attracted by nuclei, thus triggering esoenergetic nuclear transmutations without neutron emission and significantly increase the possibilities for the industrial development of a basically new and clean nuclear energy* [5] (TEC domain names and patents pending).

In order to illustrate the significance of the possibility, Santilli indicated the possible triggering by pseudo-protons of nuclear transmutations, such as [5] the transmutation of Li-7

$$\tilde{p}^{-} + {}^{7}Li \rightarrow {}^{8}\tilde{L}i + \beta^{-} \rightarrow {}^{8}Be + \beta^{-} \rightarrow 2\,{}^{4}He + \beta^{-} + \Delta E_{1} \quad (6)$$

with $\Delta E_1 = 2.887 \cdot 10^{-12}$ J, or Pd-106

$$\tilde{p}^{-} + {}^{106}Pd \rightarrow {}^{107}\tilde{P}d + \beta^{-} \rightarrow {}^{107}Ag + \beta^{-} + \Delta E_{2} \quad (7)$$

or Au-197

$$\tilde{p}^{-} + {}^{197}Au \rightarrow {}^{198}\tilde{A}u + \beta^{-} \rightarrow {}^{198}Au + \beta^{-} + \Delta E_{3} \quad (8)$$

where Au-198 is unstable and decays in 2.69 days into betas and Hg-198 with the release of 1.372 MeV, and others, all releasing a rather significant amount of heat, none of them emitting neutrons, while beta emission being easily trapped by a metal shield.

In summary, the above studies indicate possible, basically new and clean nuclear energies centered on the use of hydrogen, and its processing via suitable reactors first into neutroids and/or pseudo-protons and then their use to trigger esoenergetic nuclear transmutations. Due to the evident significance of these studies, both per se, as well as with respect to the emerging new hydrogen era, in this paper experiments are studied aimed at the confirmation or denial of the existence of Santilli neutroids and pseudo-protons and, in case affirmative, reach accurate measurements of their characteristics and their production in number sufficient for industrial applications. It should be noted that in this paper the possible connection between Santilli pseudo-proton and the anti-protons produced at various laboratories is ignored, since such a topic has been preliminarily studied in Refs. [2,5], although the technologies for the production of the two particles can benefit each other, thus warranting additional studies.

From the above it is clear that it is expected that neutrons and pseudo-protons will be produced in a sufficiently strong discharge in hydrogen gas as has been performed by Santilli [1]. That neutrons can be produced in hydrogen gas is well known and experimentally verified by many experiments [23-26] and developed for industrial applications [27]. In general it is assumed that neutrons are produced as a by-product of the fusion of two protons, deuterons, tritons or their combinations resulting in the production of neutrons with high kinetic energies, so-called fast neutrons. The discharge acts as a means to accelerate the hydrogen ions to such high energies that the Coulomb repulsion can be overcome so that the fusion reaction can occur. However, the energy needed to overcome the Coulomb repulsion so that two ions can come as close to each other as several femtometer (the size of the proton, or range of the strong

force) is of the order of 1.4 MeV which is much larger than the kinetic energy available. As a recourse for this problem it is assumed that quantum tunneling exists. In that case the ion nuclei need to overlap and the tunneling probability will be large enough to give a reasonable yield. In such a case also the possibility for the above mentioned reactions exists and neutron or pseudo-proton production can occur.

4. Detection Possibilities

In the above it has been made clear that it is possible that neutrons or pseudo-protons can be created in a hydrogen gas discharge. In the following it is discussed how these particle might be detected.

4.1. Neutrons

Neutrons are neutral and have limited interaction with most materials. One should differentiate between fast neutrons that are created during fusion of fission reactions and thermal neutrons that are created by the Santilli synthesis or by moderation (i.e. slowing down) of fast neutrons. The kinetic energy of fast neutrons is of the order of MeV while the kinetic energy of thermal neutrons is of the order of several tens of meV.. When fast neutrons have collisions with cores of the material constituting atoms or molecules, part of their energy will be released to the core and the neutrons energy decreases. After many collisions the fast neutron is slowed down to a speed comparable to the speed of the atoms or molecules of the material. At room temperature this corresponds to an average kinetic energy of 25 meV.. Only after the neutrons are slowed down their energy is low enough to be absorbed during the interaction with the nucleus.

Thermal neutrons are detected by a nuclear absorption process inside the detector. The nuclear absorption creates a charged particle pair that is detected by means of an appropriate mechanism. This can be an avalanche detector in case of a gas-filled detector like a ^3He or BF_3 detector. This can also be a photo-multiplier in case of a scintillator detector like a LiF crystal or Li-glass. The key point is that one can discriminate between fast and slow neutrons by means of a time-of-flight technique or by means of an appropriate thermal neutron absorbing material or fast neutron moderator.

The time-of-flight technique is based on the measurement of the arrival time of the neutron in the detector after it has been created. During this time the neutron has to travel the known distance from source to detector. As the travel time and travel distance are measured, the velocity of the neutrons can be determined. Slow neutrons will travel with speeds of the order of 1 km/s while fast neutrons will travel with speed of at least 10000 km/s. To determine the travel time it is needed to know when the neutrons are emitted form the source. One method is by using a pulsed source so that the time interval in which the neutrons are created is known.

When the source is surrounded by a thermal neutron absorber (like for instance Cadmium, Gadolinium, Lithium or Boron) no thermal neutrons can escape the source. Then any excess neutron detected by the detector after turning on the source is due to the moderation of fast neutrons created in the source. When the source is also surrounded by a fast neutron moderator the number of excess neutrons in the detector will increase as more neutrons will be moderated. With this scheme it is possible to determine a) whether or not neutrons are created and b) whether fast neutrons or slow neutrons are created and c) when slow neutrons are created a indication of their energy distribution can be measured. The cost for a time-of-flight detection including the needed materials for absorption and moderation is estimated between 20 and 50 kUSD.

4.2. Pseudo-protons

Pseudo protons are negatively charged particle with a mass comparable with that of the hydrogen atom or proton. It can be discriminated from an electron in a mass spectrometer due to the large mass difference and from a proton because of its reversed charge. In a hydrogen discharge also hydride ions will be produced. The mass and charge of the hydride ion is comparable to that of the pseudo-proton. Hence, it is possible that already in experiments that have been performed in the past the pseudo-proton was detected, but interpreted as hydride ion [25-28]. The mass difference between the hydride ion and the pseudo-proton is expected to be very small of the order of 0.1 % (see table 1). Hence, when using a mass spectrometer to determine what kind of particle is detected, the accuracy should be high enough to measure this small difference. An additional test might be done by measuring the magnetic moment of the detected negatively charged particle by means of an appropriate device as for example a penning trap. Mass spectrometers can be bought from the shelve, but one with an accuracy of 0.1 % for a mass of approximately 1 amu is not readily available and must be developed. In such a case it can be combined with the magnetic moment measurements. Estimated costs for such a development are between 500 kUSD and 1 MUSD.

4.3. Pseudo-protonium Atom

The pseudo-proton can quickly capture a proton to form a *pseudo-protonium* atom, $\tilde{p}p$ where the proton and pseudo-proton circle around each other like in a 2-body system.

Hence, one can imagine that in a strong electrical discharge in hydrogen gas the following reaction might occur

$$H_2 \rightarrow \mathrm{H} + \mathrm{H} \rightarrow \left(p^+_{\downarrow} + e^-_{\uparrow}\right) + \left(p^+_{\uparrow} + e^-_{\downarrow}\right) \rightarrow \left(p^+_{\downarrow} + e^-_{\uparrow} + e^-_{\downarrow}\right) + p^+_{\uparrow} \rightarrow \tilde{p}^-_{\downarrow} + p^+_{\uparrow} \rightarrow \tilde{p}p \tag{10}$$

Hence, in the event confirmed, pseudo-protons can form pseudo-protonium atoms during the discharge process. These atoms have energy levels determined by the energy levels of a two body system. The reduced mass of this system is about 1836 times larger than that of the hydrogen atom, comparable to those of protonium [32], resulting in energy levels in the keV range. These levels can be detected by means of X-ray spectrometry. The Lymann [33] (transitions form level n > 1

to 1) and Balmer [33] series (transitions form level n > 2 to 2) for this atom are shown in table 2.

The intensity of the spectral lines depends on the number density of the pseudo-protonium atoms and hence will depend strongly on the discharge parameters and the pressure of the hydrogen gas. If these series can be found in the spectra of hydrogen gas discharges with sufficient accuracy, then also the reduced mass of the pseudo-protonium atom and hence the mass of the pseudo-proton can be measured. It should be noted that in some Energy-dispersive X-ray spectroscopy experiments peaks are recorded that are close to the estimated value giving clues under what conditions pseudo-protonium might be formed [34]. The estimated costs of such an experiment are between 100 and 200 kUSD.

Table 2. The energy differences and wavelength in nm of the Lymann and Balmer series for pseudo-protonium atom.

Lymann Transition of n	2→1	3→1	4→1	5→1	∞→1
Energy difference in keV	9.367	11.102	11.71	11.99	12.49
Wavelength in nm	0.1318	0.1112	0.1054	0.1030	0.0989

Balmer Transition of n	3→2	4→2	5→2	6→2	7→2	o∞→2
Energy difference in keV	1.735	2.342	2.623	2.776	2.868	3.122
Wavelength in nm	0.712	0.527	0.471	0.445	0.431	0.395

5. Conclusions

Following lifelong mathematical, theoretical, and experimental studies on the synthesis of the neutron from hydrogen [19] Santilli proposed the possible existence of two new unstable particles: the neutroid [16], which is essentially a new, spin zero bound state of a proton and an electron in singlet coupling at 1 fm mutual distance, and the pseudo-proton [2,5], which is characterized by the 'compression' of two electrons with opposite spins inside the proton, rather than a single 'compression' according to Rutherford [8].

By remembering that the synthesis of the neutron from the hydrogen exists in nature, and that the probability for the synthesis of the pseudo-proton is smaller yet close to that of the neutron, Santilli neutroids and/or pseudo-protons may generate basically new applications of hydrogen for industrially meaningful, and environmentally acceptable new energies. Since these possibilities are already under industrial development [34], it appears advisable to conduct joint scientific studies.

In this paper experimental means to verify or deny the existence of Santilli neutroids and pseudo-protons are described. In the affirmative case, these means can achieve accurate measurements of the particles characteristics and provide information to produce them in sufficient number to have industrial significance. Since the charge and mass of the pseudo-proton are comparable to those of the hydride ion, it is possible that in previous experiments the pseudo-proton has already been measured but not identified.

For the identification, sufficiently accurate measurements of its mass are needed (accuracy at least 0.1 %) or a measurement of the magnetic moment by means of a penning trap should be performed. It is also possible that another kind of hydrogen molecule, called a pseudo-protonium atom, is formed when a pseudo-proton and proton are in orbit around each other. When a sufficient density of pseudo-protonium is realized, it is possible to measure its Lymann and Balmer series. When this property can be measured with high accuracy, it can be used to determine the reduced mass of the pseudo-protonium atom and hence the mass of the pseudo-proton.

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Study of Bose-Einstein Correlation Within the Framework of Hadronic Mechanics

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Abstract: The Bose-Einstein correlation is the phenomenon in which protons and antiprotons collide at extremely high energies; coalesce one into the other resulting into the fireball of finite dimension. They annihilate each other and produces large number of mesons that remain correlated at distances very large compared to the size of the fireball. It was believed that special relativity and relativistic quantum mechanics are the valid frameworks to represent this phenomenon. Although, R.M. Santilli showed that the Bose-Einstein correlation requires four arbitrary parameters (chaoticity parameters) to fit the experimental data which parameters are prohibited by the basic axioms of relativistic quantum mechanics, such as that for the vacuum expectation values. Moreover, Santilli showed that correlated mesons can not be treated as a finite set of isolated point-like particles as required for the exact validity of the Lorentz and Poincare's symmetries, because the event is non-local due to overlapping of wavepackets and consequential non-Hamiltonian effects. Therefore, the Bose-Einstein correlation is incompatible with the axiom of expectation values of quantum mechanics. In this paper, we study Santilli's exact and invariant representation of the Bose-Einstein correlation via relativistic hadronic mechanics including the exact representation of experimental data from the first axiomatic principles without adulterations, and consequential exact validity of the Lorentz-Santilli and Poincare-Santilli isosymmetries under non-local and non-Hamiltonian internal effect. We finally study the confirmation of Santilli's representation of the Bose-Einstein correlation by F. Cardone and R. Mignani.
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Keywords: Bose-Einstein Correlation, Special Relativity, Lorentz-Santilli Isosymmetry

1. Introduction

The main ingredient of hadronic mechanics [1], [2] is that strong interactions have a nonlocal component of contact, due to deep wave-overlappings at mutual distances of 1 Fermi. This nonlocal component can not be represented by the conventional quantum mechanics. However, novel hadronic mechanics encompass entire local and nonlocal effects with remarkable experimental evidences. Thus, the most fundamental experimental verifications of hadronic mechanics are, those which manifested the expected nonlocality of the strong interactions. Among them, the most important tests are those on the Bose-Einstein correlation [3], [4], [5], [6], [7], in which protons and antiprotons are made to collide at very big or very small energies and annihilate each other in a region called the fireball. The annihilation produces various unstable hadrons whose final states are given by correlated mesons which are "in phase" with each other despite large mutual distances compared to the size of the fireball. Correlated mesons can not be treated as a finite set of isolated point-like particles. It is non-local event due overlapping of wavepackets. There are several nonlocal theories which attempted to reduce nonlocal event into a finite set of isolated points distributed over the finite volume of the fireball. However, these theories are discarded by Santilli for the fact that the Bose-Einstein correlation is incompatible with the axiom of expectation values of quantum mechanics. It is purely manipulated nonlocal interaction to verify the quantum laws.

The first exact and invariant formulation of the

Bose-Einstein correlation via relativistic hadronic mechanics was done by R. M. Santilli [8] in 1962. F. Cardone and R. Mignani [9], [10] was the first to verify Santilli's theoretical isorelativistic calculation with experimental data (Figure 1) and they published their result in 1996.

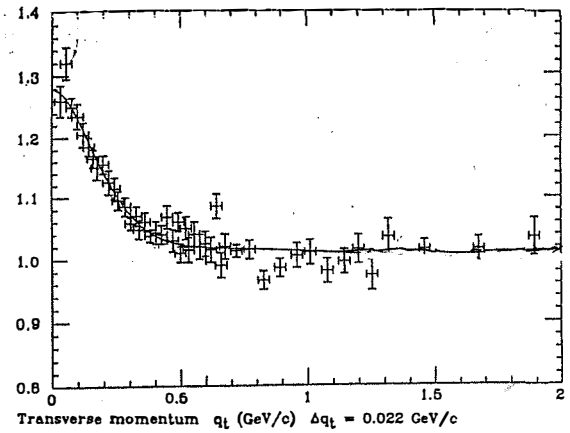

Figure. 1. The exact fit of Santilli's two-point Bose-Einstein isocorrelation function at high energy.

2. Conventional Treatment of the Bose-Einstein Correlation

Consider a quantum system in 2-dimensions represented on a Hilbert space H with initial and final states $|a_k\rangle, |b_k\rangle, k=1,2$. The vacuum expectation values of an operator A are given by [3]

$$\langle A\rangle = \langle a_k | \times A \times | b_k\rangle = \sum_{k=1,2} a_k \times A_{kk} \times b_k \tag{1}$$

which is necessarily diagonal, to fulfill the condition that operator corresponds to observable quantity must be Hermitian. The two-points correlation function of the Bose-Einstein correlation is defined by

$$C_2 = \frac{P(p_1,p_2)}{P(p_1)\times P(p_2)} \tag{2}$$

where $P(p_1,p_2)$ is the two particles probability density subjected to Bose-Einstein symmetrization, and $P(p_k), k=1,2$ is the corresponding quantity for the k^{th} particle with 4-momentum, p_k . The two-particles density is computed via the vacuum expectation value

$$P(p_1,p_2) = \int \psi_{12}^{\dagger}(x_1,x_2;r_1,r_2)\psi_{12}(x_1,x_2;r_1,r_2) \times F(r_1)F(r_2)\times d_{r_1}^4 d_{r_2}^4 \tag{3}$$

where ψ_{12} is the probability amplitude to produce two bosons at r_1 and r_2 that are detected at x_1 and x_2, given by

$$\psi_{12} = \frac{1}{\sqrt{2}}\left[e^{ip_1(x_1-r_1)+ip_2(x_2-r_2)}\right] + \frac{1}{\sqrt{2}}\left[e^{ip_1(x_1-r_2)+ip_2(x_2-r_1)}\right] \tag{4}$$

With the use of above equations, we obtain the final expression for the two-point correlation function

$$C_2 = 1 + e^{-Q_{12}^2 R^2}, \tag{5}$$

where $Q_{12} = p_1 - p_2$ is the momentum transfer, where R is the Gaussian width and r is generally assumed to be the radius of the fireball.

3. Incompatibility of the Bose-Einstein Correlation with Relativistic Quantum Mechanics

The Bose-Einstein correlation given by eq.(5) derived from conventional quantum mechanics, deviates from experimental results. This tempt to the introduction of a first, completely unknown parameter λ , called "chaoticity parameter", namely;

$$C_2 = 1 + \lambda e^{-Q_{12}^2 R^2}. \tag{6}$$

Note that the introduction of chaoticity parameter is quite arbitrary and it is impossible to derive the above parameter from any axiom of relativistic quantum mechanics. Hence, the chaoticity parameter λ introduced in eq.(6) is the first direct evidence of the incompatibility of the Bose-Einstein correlation with quantum axioms. Although, the modified eq.(6) too deviates dramatically from experimental data. In order to fit the desired experimental data eq.(6) was further modified by introducing an increasing number of completely unknown and arbitrary parameter, namely,

$$C_2 = 1 + \lambda_1 e^{-Q_{12}^2 R^2} + \lambda_2 e^{-Q_{12}^2 R^2} + \lambda_3 e^{-Q_{12}^2 R^2} + \lambda_4 e^{-Q_{12}^2 R^2} \tag{7}$$

which is strongly objected by Santilli because the only scientific route of achieving the addition terms in Bose-Einstein correlation function is by a formulation of nondiagonal elements of the expectation values. However, latter are prohibited by relativistic quantum mechanics for observable quantity (Hermitian operator). Thus, in the context of study of Bose-Einstein correlation, relativistic quantum mechanics has the following insufficiencies:

1. The theory can only represent the proton and the antiprotons as dimensionless points. Hence, the particle correlation and then the existence of the fireball is impossible with assumption of dimensionless points;

2. The point-like abstraction of particles has a number of technical consequences, such as the independent terms of the densities in eq.(3). That directly prohibit the boson correlation;

and

3. Relativistic quantum mechanics must insist the fireball to be necessarily spherical, so as to prevent the loss of the rotational symmetry to protect from the violation of spacetime symmetries.

4. Representation of the Bose-Einstein Correlation with Relativistic Hadronic Mechanics

In regard of derivation of Bose-Einstein correlation function and its experimental validity, Santilli observed that:

1. The Bose-Einstein correlation is incompatible with the axisms of relativistic quantum mechanics because of the impossibility to admit off-diagonal terms in the two-point correlation function from unadulterated first principles, and other reasons; and

2. The Bose-Einstein correlation is directly compatible with the axioms of the covering relativistic hadronic mechanics because of the admission of nonlocal non-Hamiltonian interactions and the appearence of off-diagonal terms from first principles.

The axiom of isoexpectation value for relativistic hadronic mechanics [8] is given by

$$\begin{aligned}&\langle \hat{A}\rangle\langle \hat{a}_k\,|\times\hat{T}\times\hat{A}\times\hat{T}\times|\,\hat{b}_k\rangle\\&=\Sigma_{ijk}\,\hat{a}_i\times\hat{T}^j_i\times\hat{A}_{ij}\times\hat{T}^k_j\times\hat{b}_k\end{aligned}\tag{8}$$

where $\hat{T}$ is the isotopic element, and the "hat" denotes quantities defined on isospaces over isofields. The main new feature is that the operator $\hat{A}$ must be Hermitian, thus diagonal, to be observable, but the isotopic element does not need to be diagonal. Santilli isorelativity with Minkowski-Santilli isospace $\widehat{M}(\hat{x},\hat{\eta},\hat{R})$, isoinvariant, isometric, isotopic element and isounit given respectively by [8]

$$\begin{aligned}\hat{x}^2&=\left(\hat{x}^\mu\times\hat{\eta}_{\mu\nu}\times\hat{x}^\nu\right)\times\hat{I}\\&=\left[x^\mu\times\left(\hat{T}^\nu_\mu\times\eta_{\nu\rho}\right)\times x^\rho\right]\times\hat{I}\in\hat{R},\end{aligned}\tag{9}$$

$$\begin{aligned}\hat{\eta}&=Diag\left(b_1^2,b_2^2,b_3^2,-b_4^2\right)\times\Gamma\\&=Diag\left(1/n_1^2,1/n_2^2,1/n_3^2,-1/n_4^2\right)\times\Gamma,\end{aligned}\tag{10}$$

$$\begin{aligned}\hat{T}&=Diag\left(b_1^2,b_2^2,b_3^2,b_4^2\right)\times\Gamma\\&=Diag\left(1/n_1^2,1/n_2^2,1/n_3^2,1/n_4^2\right)\times\Gamma,\end{aligned}\tag{11}$$

$$\begin{aligned}\hat{I}&=Diag\left(1/b_1^2,1/b_2^2,1/b_3^2,1/b_4^2\right)\times\Gamma^{-1}\\&=Diag\left(n_1^2,n_2^2,n_3^2,n_4^2\right)\times\Gamma^{-1},\end{aligned}\tag{12}$$

$$\begin{aligned}b_\mu&=b_\mu\left(t,x,p,E,....\right)>0,\\n_\mu&=n_\mu\left(t,x,p,E,....\right)>0,\end{aligned}\tag{13}$$

$$\begin{aligned}\hat{T}&=\hat{T}\left(t,x,p,E,....\right),\\\hat{I}&=\hat{I}\left(t,x,p,E,....\right)=\hat{T}^{-1}.\end{aligned}\tag{14}$$

It observed that unlike chaoticity parameter, the characteristic quantities must represent physically measurable quantities, namely, $1/b_k^2=n_k^2,\ \ k=1,2,3$ must characterize the semiaxes of the Bose-Einstein fireball according to a proper normalization and $1/b_4^2=n_4^2$ must characterize the density of the fireball in a way compatible with other experiments.

Note that the value of the type $b_1^2=b_2^2=b_3^2$ emerging from isorepresentation would be inconsistant because the Bose-Ein stein fireball cannot possibly be a sphere due to the extreme energies of the collision. The fireball must be a very elongated sheroidalellipsoid, for instance, of the type $b_3^2\gg b_1^2=b_2^2$. Moreover, the numerical value of the density $b_4^2=1/n_4^2$ must be compatible with numerical values from different experiments on comparable densities, such as we observed in case of protons and neutrons.

The correlation function on an iso-Hilbert space $\widehat{H}$ with initial and final isostates $|\,\hat{a}_k\rangle,|\,\hat{b}_k$; $k=1,2$ and the non-diagonal isotopic element in the explicit form is given by

$$\begin{aligned}\hat{T}&=Diag\left(b_1^2,b_2^2,b_3^2,b_4^2\right)\times\Gamma\\&=Diag\left(1/n_1^2,1/n_2^2,1/n_3^2,1/n_4^2\right)\times\Gamma,\end{aligned}\tag{15}$$

$$\Gamma=\begin{pmatrix}A & B\\ C & D\end{pmatrix}\tag{16}$$

where

$$B=B'\left|1-\exp\left(\int\psi^\dagger_{b2}\psi_{a1}dx\ \right)\right|$$

$$C=C'\left|1-\exp\left(\int\psi^\dagger_{a2}\psi_{b1}dx\ \right)\right|$$

and values of constants, A,B,C and D are determined by the condition of normalization, that is

$$Det\left(\Gamma\right)=1.\tag{17}$$

Notice that isoexpectation value of eq.(11), isotopic element

1. Allows indeed off-diagonal terms in the isoexpectation values;

2. Represents the overlapping of the wavepackets of particles via the integrals in the exponents of Γ ; and

3. Eliminates all correlations when said overlapping is null.

Next, the isorepresentation is given by a trivial isotopy of the conventional treatment, with the use now of the nontrivial

isoexpectation values of eq.(8). We then have the two-points isocorrelation function

$$\hat{C}_2 = \frac{\hat{P}(p_1, p_2)}{\hat{P}(p_1) \times \hat{P}(p_2)}, \tag{18}$$

where $\hat{P}(p_1, p_2)$ is the two-particle isoprobability density subjected to proper symmetrization, $\hat{P}(p_1)$ and $\hat{P}(p_2)$ are the corresponding quantity for the k particle with 4-momentum. The two-particles isoprobability density is now given by the isoeigenvalue expression

$$\begin{aligned} \hat{P}(p_k) = \int & \hat{\psi}^{\dagger}_{12}(x_k; r_k) \hat{\psi}_{12}(x_k; r_k) \\ & \times F(r_k) \times d^4_{r_1} d^4_{r_2}, \quad k = 1, 2 \end{aligned} \tag{19}$$

Note that the crucial difference between eqs.(3) and (19) given by the isotopic lifting of all quantities and their operations and the appearance in the former of the isotopic element allowing the mixing of nondiagonal terms. Another major difference between conventional and isotopic treatments is that the probability densities for particles 1 and 2 are factorized in the conventional treatment, eq.(3), while they cannot be factorized in the isotopic treatment. This is due to the fact that protons, antiprotons, and all produced mesons are pointlike for relativistic quantum mechanics, while they are extended in case Santilli's treatment. Hence, the separation of the densities would be equivalent to ceasing all correlations. The isotopy of of the conventional treatment referred to isoexpectation values of eq.(11), including the symmetrization of the isotopic element and isowavefunctions for all possible directions, plus the assumed normalizations then leads to isodensity, that is

$$\hat{F}(r_1, r_2) = \sum_{\mu} \hat{\eta}_{\mu\mu} \frac{b_{mu}{}^2}{4\pi^2} e^{\frac{-r^2 b_{\mu}^2}{2}} \tag{20}$$

where r is the radius of the sphere in which the correlated mesons are detected.

The continuation of calculations via a simple isotopy of the conventional treatment, the final expression of the two-points isocorrelation function, derived for the first time by Santilli is given by [8]

$$\begin{aligned} \hat{C}_2 &= 1 + \frac{1}{3} \sum_{\mu} b_{\mu}^2 \times e^{-q_i^2 K^2 / b_{\mu}^2} \\ &= 1 + \frac{1}{3} b_1^2 \times e^{\left(-q_i^2 K^2 / b_1^2\right)} + \frac{1}{3} b_2^2 \times e^{\left(-q_i^2 K^2 / b_2^2\right)} \\ &+ \frac{1}{3} b_3^2 \times e^{\left(-q_i^2 K^2 / b_3^2\right)} - \frac{1}{3} b_4^2 \times e^{\left(-q_i^2 K^2 / b_4^2\right)}, \end{aligned} \tag{21}$$

$$K^2 = b_1^2 + b_2^2 + b_3^2 = 3. \tag{22}$$

In the above isorepresentations, all operations are now conventional. Hence, the above expressions are the projections in our spacetime of the isocorrelation functions on isospace.

5. Exact Poincare Symmetry under Nonlocal and Non-Hamiltonian Interaction

As indicated earlier, a crucial insufficiency of the conventional treatment of the Bose-Einstein correlation, is the inability to provide an invariant representation of the fireball, due to its prolate character under which the conventional rotational symmetry no longer applies. The Bose-Einstein correlation creates a fireball characterized by a spheroid prolated in the direction of the proton-antiproton flight. Following its creation, the fireball expands rapidly, resulting in the correlated mesons. Consequently, the original characteristic quantities, here denoted $b_k'^2 = 1/n_k^2$, have an explicit dependence on time. By assuming that the prolateness is along the third axis, we have

$$\begin{aligned} K^2(t) &= b_1'^2(t) + b_2'^2(t) + b_3'^2(t) \\ &\neq const, \quad b_3'^2(t) \gg b_1'^2(t) = b_2'^2(t). \end{aligned} \tag{23}$$

However, the fireball must preserve its shape during its expansion when considered as isolated from the rest of the universe. This implies that all characteristic quantities have the same factorizable time dependence. In conclusion, the fireball can be studied at the time of its formation with constant characteristic quantities $b_k^2 = 1/n_k^2$ and the following isoinvariant formulated on the Euclide-Santilli isospace [11], [12] with isounit

$$\begin{aligned} \hat{R}^2 &= \left(x_1^2 b_1^2 + x_2^2 b_2^2 + x_3^2 b_3^2\right) \\ &\times \hat{I} = \left(\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2}\right) \times \hat{I}, \end{aligned} \tag{24}$$

$$\begin{aligned} \hat{I} &= Diag\left(1/b_1^2, 1/b_2^2, 1/b_3^2\right) \\ &= Diag\left(n_1^2, n_2^2, n_3^2\right). \end{aligned} \tag{25}$$

The reconstruction of the exact Lorentz symmetry $\hat{O}(3)$ [11], [12] for the Bose-Einstein correlation follows the same lines. Since the speed of light is assumed to be locally varying, we have mutated light cones of the type,

$$\hat{n}^2 = \left(x_3^2 \times b_3^2 - x_4^2 \times b_4^2\right) \times \hat{I} = \frac{x_3^2}{n_3^2} - \frac{x_4^2}{n_4^2}, \tag{26}$$

$$\hat{I} = Diag\left(1/b_3^2, 1/b_4^2\right) = Diag\left(n_3^2, n_4^2\right). \tag{27}$$

It is again easy to see that the mutated light cone in our spacetime is the perfect light cone in isospace, called light isocone, because, again, the mutation of each axis is complemented by the inverse mutation of the corresponding

unit. Recall that isorelativity and special relativity coincide at the abstract, realization free level, as confirmed by the speed of light in vacuum to be the constant maximal causal speed in isospace. Consequently, the understanding of the isorepresentation of the Bose-Einstein correlation requires the knowledge that, rather than "violating" special relativity as at times perceived, in reality allows the maximal possible enlargement of the arena of applicability of Einsteinian axioms.

6. Theoretical Prediction

It is important now to identify the theoretical prediction of isorepresentation so that we can compared them below with experimental data.

Prediction 1: The minimum value of the two-points isocorrelation function, first identified by Santilli, evidently holding for infinite momentum transfer.

$$\hat{C}_2^{Min} = 1 \tag{28}$$

Prediction 2: The maximal value is predicted to be evidently holding for null momentum transfer. The isorepresentation to be valid if all data must remain between the above minimum and maximum values.

$$\hat{C}_2^{Min} = 1 + \frac{1}{3} + \frac{1}{3} = 1.67 \tag{29}$$

Prediction 3: Isorepresentation also predicts the maximum value of the isodensity, occurring for $\hat{C}_2^{Max}$'. In fact, for $q_t = 0$ we have no correlations, in which case we have

$$\begin{aligned} b_k^2 &= 1, \quad k = 1,2,3, \\ K^2 &= b_1^2 + b_2^2 + b_3^2 = 3, \end{aligned} \tag{30}$$

$$\hat{C}_2^{Max} = 1 + \frac{K^4}{3} - \frac{K^2 b_4^2}{3} = 1.67, \tag{31}$$

$$\begin{aligned} b_4^2 &= 2.33, \quad n_4^2 = 0.429, \\ b_4 &= 1.526, \quad n_4 = 0.654. \end{aligned} \tag{32}$$

Prediction 4: By assuming that $K^2 = 3$ and that the fireball is very prolate, with $b_3^2 = 30 b_1^2 = 30 b_2^2$, we obtain the following prediction on the remaining characteristic quantities From the isoaxioms, Santilli also have the following additional predictions:

$$b_1^2 = b_2^2 = 0.043, \quad b_3^2 = 2.816, \tag{33}$$

$$b_1^2 = n_1^2 = n_2^2 = 10.666, \quad n_3^2 = 0.355. \tag{34}$$

Prediction 5: The maximal causal speed within the fireball is bigger than that in vacuum,

$$V_{max} = c_0 \left(\frac{b_4}{b_3} \right) > c_0. \tag{35}$$

Prediction 6: Time t within the fireball flows faster than time predicted by special relativity,

$$t = \hat{\gamma} \times t_0 > \gamma \times t_0. \tag{36}$$

Prediction 7: Lengths 'l inside the fireball are smaller than lengths predicted by special relativity,

$$l = \hat{\gamma}^{-1} \times l_0 < \gamma^{-1} \times l_0. \tag{37}$$

Prediction 8: Mass behavior with speed is bigger than that predicted by special relativity,

$$m = \hat{\gamma} \times m_0 > \gamma \times m_0. \tag{38}$$

Prediction 9: The energy equivalence of the fireball is bigger than that predicted by special relativity or, equivalently, for a given energy, the mass is smaller,

$$E = m \times V_{max} > E_0 = m \times c_0^2. \tag{39}$$

Prediction 10 : Frequencies of light emitted inside the fireball, exist the same isoblueshifted, namely, with an increase of frequency as compared to the corresponding behavior predicted by special relativity,

$$\omega = \hat{\gamma} \times \omega_0. \tag{40}$$

Prediction 11: The speed of light within the fireball is bigger than that in vacuum, $c = c_0 > b_4 > c_4$, by smaller than the maximal causal speed,

$$c = c_0 \times b_4 < V_{max} = c_0 \left(\frac{b_4}{b_3} \right). \tag{41}$$

The isoblueshift of light is nothing mysterious because it is a mere manifestation of the high energy density of the medium in which light propagates. Isoblueshift, as the increase of frequencies as predicted by special relativity in vacuum, is then a mere consequence of the medium transfer energy to light. A similar situation occurs for all other predictions.

7. Experimental Verification

F. Cardone and R. Mignani [9], [10] in 1992 had contested the eq.(15) for actual experimental data. The Bose-Einstein two-point correlation function derived by Santilli is perfectly matched with experimental results at high energy. The numerical values of the characteristic functions for the fireball of the Bose-Einstein correlation resulting from this exercise are

$$b_1 = 0.267 \pm 0.054, \quad b_2 = 0.437 \pm 0.035,$$

$$b_3 = 1.661 \pm 0.013, \quad b_4 = 1.653 \pm 0.015. \tag{42}$$

A most important feature of the above data is that they characterize the medium inside the fireball as being iso-Minkowskian of Group III, Type 9, thus confirming that all hadrons heavier than kaons have the same iso-Minkowskian features. The fit of FIGURE 1 and the above values provide the following experimental verifications:

1. The experimental data do indeed lie between the theoretically minimum and maximal value;

2. The experimental data confirm all eleven theoretical predictions;

3.The experimental proof confirms the reconstruction of the exact character of the Poincare symmetry for the Bose-Einstein correlation.

F. Cardone and R. Mignani investigation provides remarkable experimental verification of Santilli isorelativity and relativistic hadronic mechanics. This experimental verification on Bose-Einstein Correlation reveals the nonlocality of strong interactions of correlated mesons.

8. Concluding Remarks

Santilli's thorough investigation found that special relativity, the Lorentz and Poincare's symmetries, and relativistic quantum mechanics are not exactly valid to represent the Bose-Einstein correlation because their predictions from a large deviation from experimental data. Moreover, the resulting representations fail to tender non-explanation for the introduction of chaoticity parameters which are needed to fit the experimental data. Furthermore, it is observed that there are flaws in treating correlated mesons as a finite set of isolated point-like particles, sincethe Bose-Einstein correlation is purely non-local event with deep overlapping of wavepackets that cannot be treated by conventional quantum mechanics.

We have shown in this paper that Santilli's representation of the Bose-Einstein correlation via relativistic hadronic mechanics has indeed achieved a numerically exact and and time invariant representation of experimental data from unadulterated first axioms, with consequential exact validity of the Lorentz-Santilli and Poincare-Santilli isosymmetry that are known to be locally isomorphic to the corresponding conventional symmetry. Hence, relativistic hadronic mechanics restores indeed the validity of the conventional spacetime symmetries, although at the covering isotopic level.

We conclude the paper with a study of the confirmation of all results by Santilli independently provided by F. Cardone and R. Mignani, including the all important exact and invariant representation experimental data.

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Compatibility of Arbitrary Speeds with Special Relativity Axioms for Interior Dynamical Problems

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Abstract: In this paper, we outline the rapidly growing literature on arbitrary speeds within physical media and show that, contrary to a widespread belief for one century, arbitrary speeds for interior dynamical problems are compatible with the abstract axioms of special relativity, provided that they are realized with the covering isomathematics specifically developed for the conditions considered. We finally point out a number of intriguing implications in cosmology, particle physics, nuclear physics, chemistry, gravitation, and mathematical models of interstellar travel.

Keyworks: Special Relativity, Superluminal Speeds, Isorelativity

1. Maximal Speeds for Exterior Problems in Vacuum

As it is well known, the advent of the Lorentz transformations [1]

$$x'^1 = x^1, x'^2 = x^2, \tag{1a}$$

$$x'^3 = \gamma(x^3 - \beta x^4), x'^4 = \gamma(x^4 - \beta x^3), \tag{1b}$$

$$\beta = \frac{v}{c}, \gamma = \frac{1}{\sqrt{1-\beta^2}}. \tag{1c}$$

and their extension by Poincaré [2] (hereon referred to as the *Lorentz-Poincaré (LP) symmetry*) leave invariant the line element in Minkowski space-time $M(x, \eta, I)$

$$x^2 = x_1^2 + x_2^2 + x_3^2 - t^2c^2 = x^\mu \eta_{\mu\nu} x^\nu, \tag{2a}$$

$$x = (x^\mu) = (x_1^\mu - x_2^\mu), x^4 = t, \mu, \nu = 1,2,3,4, \tag{2b}$$

$$\eta = Diag.(1,1,1,-c^2), I = Diag.(1,1,1,1), \tag{2c}$$

and are at the foundation of axioms of Special Relativity (SR) [3].

As it is also well known, symmetry (1) identifies the *maximal causal speed* for the conditions clearly expressed by Lorentz, Poincaré and Einstein [1-3] and experimentally confirmed, namely, for *exterior dynamical problems,* consisting of *point particles and electromagnetic waves propagating in vacuum* (conceived as empty space) when represented in an inertial reference frame.

In fact, the light cone, e.g., for infinitesimal displacements in (3, 4)-dimensions

$$(\delta x^3)^2 - (\delta t)^2 c^2 = 0, \tag{3}$$

establishes the maximal causal speed in vacuum

$$\frac{\delta x^3}{\delta t} = V_{max}^{vacuum} = c. \tag{4}$$

For decades, *faster than light speeds* (also called "superluminal speeds") were essentially ignored because they would violate causality and other physical laws. Nevertheless, with the passing of time the study of superluminal speeds became inevitable.

Nowadays, a search on superluminal speeds at the various archives in the internet shows the existence of a large number of papers published in refereed journals, thus suggesting a study on the problem of the causal and time invariant formulation of superluminal speeds.

Under a literature on superluminal speeds of such a dimension, we regret being unable to provide a comprehensive review, and are forced to quote a few representative illustrations of the studies considered in this

paper, *superluminal speeds of ordinary masses or electromagnetic waves,* by deferring the study of tachyons (see, e.g., contributions by E. Recami and his group [47, 48]) to a separate paper.

2. Superluminal Speeds in the Expansion of the Universe

To our knowledge, studies of superluminal speeds were first motivated by the Doppler interpretation of the Hubble law [4] on the cosmological redshift of light

$$z = \frac{\lambda_o}{\lambda_e} - 1 \approx Hd \equiv \frac{v}{c}. \tag{5}$$

where λ_o (λ_e) is the wavelength of light at the origin (that observed on Earth), d is the distance of a galaxy from Earth, and H is the Hubble constant.

In fact, with the passing of the decades and the advances in telescopes, it became evident that the galaxies at the edge of the universe have values $z > 1$ with consequential superluminal speeds. This occurrence can be assumed as signaling the initiationof studies in faster than light speeds. As an example, in 1966, Rees [6] attempted the reconciliation of superluminal galactic speeds with special relativity limit (4) by studying the possibility that superluminal speeds are illusory.

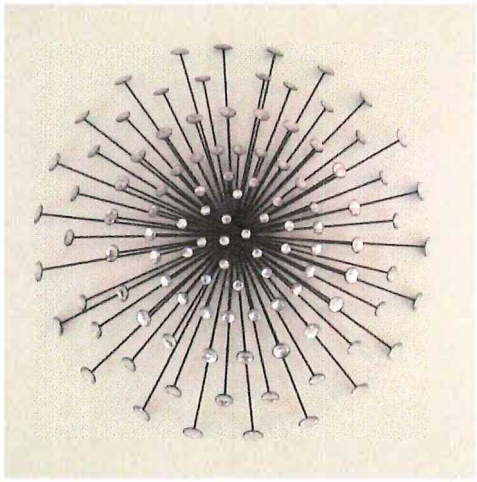

Figure 1. *An artist rendering of the conjecture of the expansion of the universe (5) showing Earth necessarily at the center of the universe due to the dependence of the expansion speed $v = Hcd$ on the distance d in all "radial" directions from Earth. This return to the Middle Ages, which is inherent ion the conjecture of the expansion of the universe, is the historical reason for which Einstein, Hubble, Hoyle, Zwicky, de Broglie, Fermi and other famous scientists died without accepting the Doppler interpretation of the cosmological redshift [43-47]. Said interpretation became fashionable despite such authoritative oppositions because of the tacit intent of imposing the validity of the conventional interpretation of special relativity for the large scale structure of the universe.*

It should be recalled that Einstein, Hubble, Hoyle, Zwicky, de Broglie, Fermi and other famous scientists died without accepting conjecture (5) on the Doppler interpretation of the cosmological redshift because the interpretation $Hd = v/c$ holds in all possible *radial* directions from Earth, thus implying a necessary return to the Middle Ages with Earth at the center of the universe (Figure 1).

The same conclusion is inevitable for the conjecture of the big bang because, as a necessary condition to represent experimental data on the radial character of the cosmological redshift, the big bang must have occurred in our galactic vicinity, thus implying an "explosion" of the type depicted in Figure 1.

For the intent of avoiding Earth at the center of the universe, supporters of special relativity ventured the additional conjecture that space itself is expanding. However, it is known that this conjecture would have achieved its intent in the event the expansion of the universe were uniform. In reality, conjecture (5) intrinsically implies the *acceleration of the expansion,* that is, the increase of the speed of galaxies with the increase of their distance, $v = Hcd$. This acceleration also occurs in all radial directions from Earth, thus implying again Earth at the center of the universe (see Figure 2 and Refs. [43-45]).

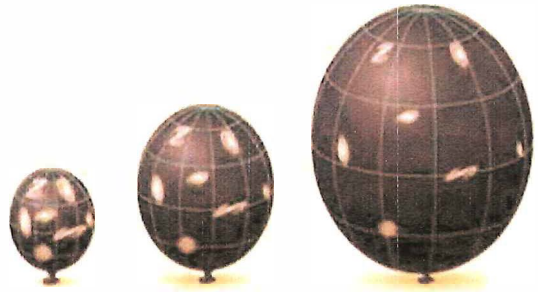

Figure 2. *An illustration of the lack of the evidence that the conjecture of the expansion of space itself would provide a consistent representation of conjecture (5) in the event the expansion were uniform. However, conjecture (5) implies the increase of the speed $v = Hcd$ in all radial directions from Earth that cannot possibly be consistent represented via the inflation of balloon [43-47].*

In any case, it is easy to see that the sole geometry representing conjecture (5) is that with the shape of a funnel (Figure 3). However, a necessary condition to represent experimental data is that Earth is at the tip of the funnel evidently because all speeds v are measured from Earth, thus implying again Earth at the center of the universe. Also, the funnel-type geometry causes a rather drastic departure from general relativity due to its irreconcilable incompatibility with the Riemannian geometry (Figure 3).

To confirm the sound vision by Einstein, Hubble, Hoyle, Zwicky, de Broglie, Fermi and other famous scientists, the implausibility of the expansion of the universe, is confirmed by the fact that the energy needed to accelerate billions and billions of galaxies is so disproportionately large to prevent any realistic model for its physical origin.

The yet additional conjecture that the universe is filled up with the mysterious and invisible dark energy was ventured to represent the expansion and inherent acceleration of the universe. However, no quantitative model has appeared to date

in the refereed scientific literature achieving such a goal due to several technical insufficiencies, including the fact that, according to Albert Einstein, energy is the source of gravitational *attraction,* and certainly not of *repulsion.*

The above orthodox models are based on the *conception of the large scale structure of the universe as an exterior dynamical problem consisting of particles and electromagnetic waves traveling in vacuum.* This conception is evidently mandatory for the tacit intent of imposing the cosmological validity of the conventional interpretation of special relativity.

Following decades of cosmological studies, the author's conclusion is that the inconsistencies or sheers insufficiencies of the conjecture on the expansion of the universe are due to the fact that *the cosmological redshift of galactic light characterizes a strictly interior dynamical problem consisting of particles and electromagnetic waves propagating within the intergalactic medium* [9-16], mostly composed of hydrogen and other gases at absolute zero degree temperature, dust, cosmic rays, besides including light emitted by all stars in the universe.

As established in Refs. [43-47], the latter conception of the universe implies necessary, experimentally established deviations from the conventional interpretation of special relativity, with particular reference to deviations from the Doppler shift. However, as we shall see in this paper, the abstract axioms of special relativity remain valid provided that they are elaborated with a mathematics more appropriate for interior conditions.

The conception of the universe as an interior dynamical problem was pioneered in 1929 by Zwicky [5] who suggested the interpretation of the cosmological redshift $z = Hd$ via the hypothesis that light loses energy during its long travel to reach Earth due to scattering with the intergalactic medium.

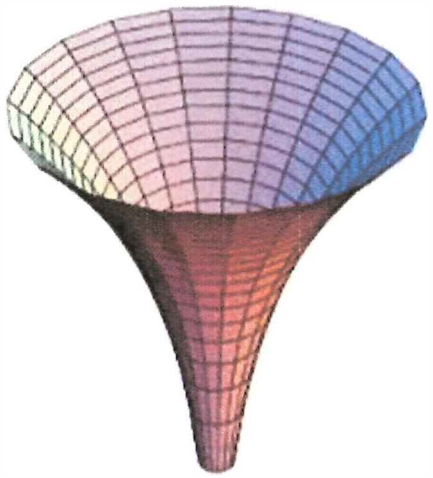

Figure 3. *An illustration of the only known consistent representation of the expansion of the universe according to assumption (5) that represents the linear increase of the speed with the distance. However, a necessary condition for consistency is that Earth must be at the tip of the funnel, thus implying again Earth at the center of the universe. Additionally, the funnel geometry is irreconcilably incompatible with the Riemannian geometry of general relativity [43=47].*

Unfortunately for scientific knowledge, Zwicky's hypothesis was "disqualified" by the orthodox physics community, and remains "disqualified" to this day, because it clearly violates Einstein's special relativity, evidently due to the fact that, for Zwicky's hypothesis, light is no longer immutable as required by special relativity axioms.

However, Zwicky's hypothesis is experimentally verifiable on Earth, while all conjectures on the expansion of the universe are individually conceived not to be testable on Earth so that they can be imposed via abuses of academic credibility. The author has spent decades of research in the field and confirmed experimentally the validity of Zwicky's hypothesis with the consequential lack of the universe (see for brevity Refs. [43-45]).

The conception of cosmology as an interior dynamical problem is best illustrated by the redshift of galactic stars, which is anomalous in the sense that it is generally smaller (bigger) than the redshift of the galaxy as a whole for stars near (far away from) the galactic center.

Always for the intent of reconciling physical evidence with special relativity, the scientific community coordinated the conjecture that galaxies (as well as their clusters) are filled up with the mysterious, invisible and undetectable dark matter.

However, no quantitative model has been published in the refereed literature showing that dark matter achieves a *quantitative* representation of the anomalous galactic redshift (Figure 5). Besides, according to Newton, galaxies should contract in the event they are filled up with any type of matter, contrary to astrophysical evidence.

In papers [46,47], the author has shown that the problem of the anomalous redshift of galactic stars is indeed a fully interior dynamical problem because the origin of the anomalies is entirely due to the loss (acquisition of energy of star light to (from) the actual material gas filling up all galaxies which is cold at the galactic periphery (very hot near its center).

The resulting frequency shift without any appreciable Doppler's contribution are today known as *Santilli isoredshift* (*isoblueshift*), where the prefix "iso" stands to denote their derivation via the coveting of 20th century mathematics known as *isomathematics*

Ref. [46,47] show in particular that, unlike the case for the conjecture of dark matter, the loss or acquisition of energy of star light from the innergalactic medium does indeed achieve a numerically exact and time invariant representation of the anomalous galactic star redshift without any appreciable Doppler contribution (Figure 6).

The above studies appear to provide sufficient experimental evidence acquired on Earth on the interior character of the large scale structure of the universe. In the next section, we shall show corresponding experimental evidence on the interior character of the structure of hadrons, nuclei and stars, thus suggesting the need to study interior problems for both the large and small scale structures of the universe.

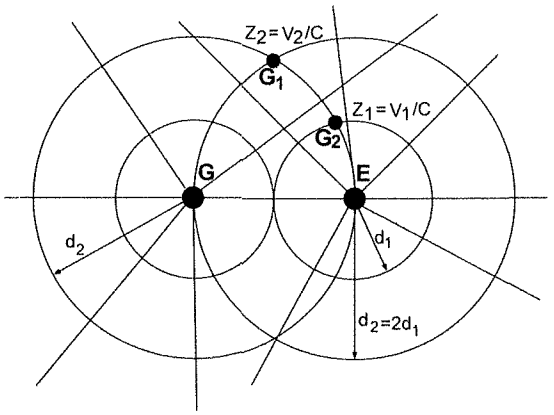

Figure 4. An illustration of the geometric inconsistency of all possible models on the expansion of the universe because galaxies G_1 and G_2 have a relative acceleration when seen from Earth E due to the double distance, while there exists an infinite number of observers in the universe for which the same galaxies G_1 and G_2 have the same distance, as it is the case for galaxy G, in which case there is no relative acceleration, with ensuing clear inconsistency of the very conjecture of the expansion of the universe [43-47].

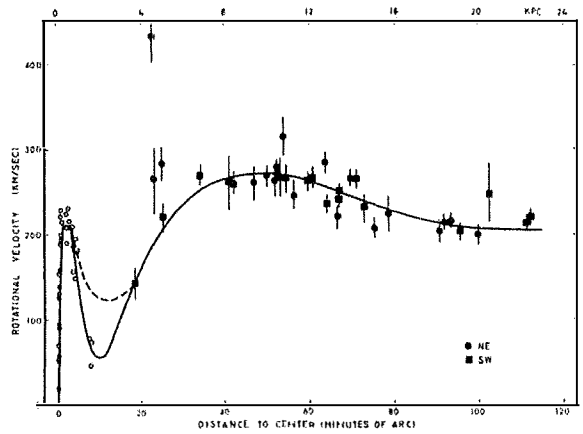

Figure 5. A plot of the anomalous redshift of galactic stars showing the decrease of the redshift for stars near the galactic center and its increase for peripheral stars. The conjecture of dark matter was ventured to represent theanomalous galactic stars, but no such a quantitative representation has appeared in the refereed literature to date, besides intrinsic in consistencies the conjecture of dark matter identified in Refs. [43-47].

3. Maximal Speeds for Interior Problems Within Physical Media

The author has essentially devoted fifty years of research to the mathematical, theoretical and experimental studies of interior dynamical problems beginning with his Ph. D. theses in the 1960s [9]. The general *irreversibility over time of interior dynamical problems* has requested the introduction since the mid 1960s of the *Lie-admissible covering of Lie's theory* which is at the foundation of the 20th century interpretation of special relativity [10-16].

The covering Lie-admissible formulations admit a particular case known as *Santilli Lie0isotopic formulations,* that apply for interior dynamical problems when considered as isolated from the rest of the universe, thus being reversible over time [23-34].

The conceptual foundations of these studies can be summarized as follows: when elementary particles move in vacuum as empty space, their only possible acceleration is that via action-at-a-distance potential interactions (more technically known as *variationally selfadjoint (SA) interactions* [15a]). In this case, it is easy to see that the achievement in vacuum of the speed of light c requires infinite energy and, therefore, the surpassing of the speed c in vacuum by ordinary masses or electromagnetic waves is excluded.

However, Santilli [10] showed in 1981 that the situation is substantially different when elementary particles move in interior conditions because, in this case, accelerations are the result, not only of conventional SA interactions, but also of *contact* non-potential interactions (technically known as *variationally nonselfadjoint (NSA) interactions,* [15a]) for which the notion of "potential energy" has no physical value or meaning.

It was then easy to see already in the 1980s that *under NSA interactions the local speeds of ordinary masses within physical media are unrestricted,* thus being arbitrarily bigger (or smaller) than c depending on local conditions of density, temperature, frequencies and other physical data.

The analytic background of the studies on interior conditions is given by the "true Hamilton's equations," those with external terms not derivable from a Hamiltonian

$$\frac{dr}{dt} = \frac{\partial H(r,p)}{\partial p}, \frac{dp}{dt} = -\frac{\partial H(r,p)}{\partial r} + F^{NSA}(t,r,v,\ldots), \qquad (6)$$

and their operator counterpart, which are specifically set for the representation of open irreversible processes we cannot possibly review here [16].

The mathematical backgrounds of the studies is given by the *Lie-admissible covering of Lie's theory* since the true Hamilton's equations emerged since the 1960s [9] as admitting a Lie-admissible algebra in the brackets of their time evolution when properly written (see the more recent memoir [13] for details).

To understand the complexity of the problem, let us recall that physical theories can be claimed to have physical value if and only if they verify the *invariance over time,* namely, they predict the same numerical values under the same conditions at different times. It is easy to see that the true Hamilton's equations and their operator counterpart violate this crucial condition because they are non-canonical and non-unitary by conception.

The achievement of invariance over time for non-canonical and non-unitary theories required the construction of a new mathematics, today known as *Lie-admissible genomathematics* we cannot possibly review here (see mathematics studies [11-13] and monographs [16]).

Figure 6. *An illustration of the evidence visible via telescopes that all galaxies are filled up indeed of matter, but of a conventional, real, gaseous medium which is very hot near the galactic center and very cold for peripheral stars. The deviations from the conventional interpretation of special relativity measured by Santilli's isoblueshift for hot gases and isoredshift for cold gases [43-45] has permitted a numerically exact and invariant representation of the anomalous redshift of galactic stars depicted in Figure 5 [46,47]. This and other representations, such as that of Arp's pair of connected quasars and galaxies with largely different cosmological redshifts, establishes that the large scale structure of the universe is an interior dynamical problem in which all cosmological redshifts are reduced to the experimentally established loss (absorption) of energy to (from) cold (hot) intergalactic or innergalactic media without any appreciable contributions from the Doppler's shift [43-47].*

A main application of these studies has been the first achievement at both nonrelativistic and relativistic levels of an *exact* representation of *all* characteristics of the neutron in its synthesis insider stars according to Rutherford's "compression of the hydrogen atom," namely, from a proton and an electron according to the known reaction (for brevity, see review [14])

$$p^+ + e^- \to n + \nu, \tag{7}$$

The main technical difficulty was due to the fact that the rest energy of the neutron is $0.872\ MeV$ *bigger* than the sum of the rest energies of the proton and of the electron, under which condition we would need "positive binding energies" which are anathema for quantum mechanics, since they cause the physical inconsistency of the Schrödinger equation.

Santilli's main point is that, even though there exist indeed particles with "point-like charges" (such as the electron), *there exist no "point-like wavepackets" in nature.* Therefore, Rutherford's compression of the extended wavepacket of the electron within the hyperdense medium inside the proton generates NSA interactions under which a solution of synthesis (7) has been indeed found [14].

The main mechanism is that contact interactions are NOSA and, therefore, they are non-unitary. The non-unitary image of Schrödinger equation achieves consistency under "positive binding energies" thanks to a new renormalization of the rest energies of the constituents (called *isorenormalization*) [14, 16, 34]).

The aspect important for this paper is that an apparent necessary condition for the representation of *all* characteristics of the neutron in synthesis (7) is that the constituents of the neutron travel at (tangential) superluminal speeds.

Intriguingly, systematic plots of experimental data in hadron physics conducted in monograph [16d] *without* the aprioristic assumption of the Lorentz symmetry have confirmed superluminal speeds within the interior of hadrons in numerous cases, such as: phenomenological fits via gauge theories; anomalous meanlives of unstable hadrons with speed; the Bose-Einstein correlation; and other cases directly relevant for the study of superluminal speeds.

We regret not being able to review these phenomenological fits and related literature to avoid a prohibitive length of this paper. Nevertheless, their knowledge is important for a technical understanding of this paper.

We should add that, as clarified by Wall [17], the superluminal speeds of hadronic constituents here considered are not referred to tachyons as conventionally defined (see, e,g, Ref. [42]) because, as we shall see in the next sections, interior physical media cause a deformation (called "mutation") of the light cone with superluminal speeds of real valued masses. Therefore, the existence of tachyons (called in the field of this paper *isotachyons*) is shifted for speed beyond the maximal causal speed within physical media which are generally bigger than c, as shown in the next section.

More specifically, we are not excluding possible tachyonic contributions in the structure of hadrons or in other physical conditions [42]. The only point we would like to clarify is that, under the validity of isotopic theories for the hadronic structure, the speed characterizing tachyons has to be shifted beyond c (see, later on, Eq. (21) and related arguments).

Independently from Santilli's research, additional relevant studies on superluminal speeds are the experimental works initiated in 1992 by Enders and Nimtz [18] (see also the more recent paper [19] for additional references and paper [20]) suggesting apparent superluminal propagation of electromagnetic waves within certain physical guides.

The reconciliation of superluminal speeds with special relativity limit (4) is generally attempted by assuming that we are dealing with a "tunnel effect." However, in our view, tunnel effects generally refer to passages through a barrier, thus for distances of $1\,fm$ covered by the uncertainty principle, and not for travel over lengths of several meters, as occurring for Refs. [18-20].

Hence, it is well possible that, in reality, Refs. [18-20] deal with an interior dynamical problem in which case superluminal speeds are due to NSA interactions of electromagnetic waves with the guides (including the so-called "stray fields" that are known to be NSA) under which interactions superluminal speeds are quite natural. Additional cases of superluminal speeds of ordinary masses can also be treated as interior dynamical problems, but we regret not being able to treat them here for brevity.

Yet an additional case relevant for this paper is the recent

controversy at CERN on superluminal or sub-luminal neutrinos, since the original 2011 announcements [21] indicated the detection of superluminal neutrino speeds, while the subsequent paper [22] indicated subluminal speeds.

We would like to point out that the truly fundamental issue for Refs. [21, 22] is the *identification of a causal and time invariant formulation of particle motion in interior conditions,* since for both views [21, 22] neutrinos travel *underground* from CERN to the Gran Sasso Laboratory.

The orthodox position is that neutrinos are point-like and travel underground without collisions, for which view interior conditions do not exist, and special relativity applies exactly.

However, *there exist no point-like wavepackets in nature;* to be physical, neutrinos must have an extended wavepacket that does interact with the wavepackets of peripheral atomic electrons in the dense underground conditions; special relativity cannot be even marginally formulated undergrounds., e.g., because light does not propagate there; and, therefore, the interior conditions of experiments [21, 22] are unavoidable on serious scientific grounds.

In the absence of the appropriate basic theory, the underground speed of neutrinos remains unsettled because the slightest modification of any form factor, parameter or data elaboration can produce subluminal results in Ref. [21] and superluminal results in Ref.. [22], as experts in the field can verify.

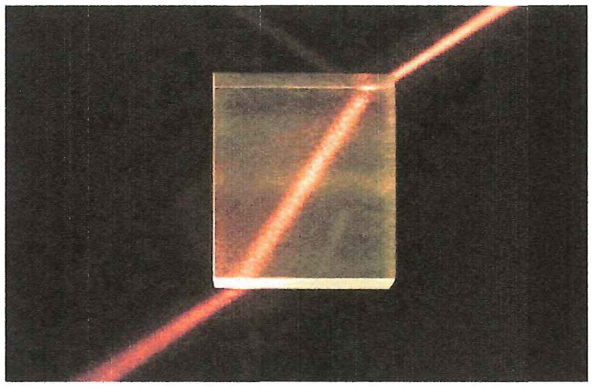

Figure 7. *An illustration of the historical Lorentz problem at the foundation of this paper, the invariance of locally varying speeds of light within transparent physical media, here illustrated with the variation of speed from air to water and then back to air. The orthodox view is that light can be reduced to photons scattering among the water molecules, thus propagating in vacuum without any interior conditions. However, such an abstraction is known not to represent experimental data such as: the angle of refraction (because photons will scatter in all directions at the impact with water); the large reduction of speed for about 1/3 (because photons could at best account for a few percentages of speed reduction); the propagation of light in water as a straight beam (because photons would scatter light in all directions and the view of this picture would be impossible; and other experimental data. Hence, on serious scientific grounds, the representation of the propagation of light within transparent physical media requires a covering of special relativity specifically conceived for interior conditions.*

Besides superluminal speeds, the study of interior dynamical problems inevitably requires the inclusion of *subluminal speeds,* namely, speeds of electromagnetic waves within transparent physical media *smaller* than c. This is typically the case for the propagation in water of infrared or radio waves for which the reduction to photon is not possible (Figure 1) [16a].

In this paper, we shall show that, when formulated with the appropriate mathematics, *Special relativity axioms do indeed admit fully causal and time invariant arbitrary, that is whether superluminal or subluminal speeds depending on the physical characteristics of the medium at hand.*

Since the Lie-admissible genomathematics is excessively advanced for the limited scope of this paper, we shall restrict our study to the particular case of interior events that are reversible over time, namely, events in which we ignore in first approximation the dispersion of light (Figure 1). This class of reversible interior events can be well treated via *isomathematics,* with particular reference to *isorelativity,* referred to the *Lie-isotopic genomathematics of special relativity.*

Nevertheless, the reader should keep in mind that *the propagation of light within transparent physical media is an intrinsically irreversible event,* as established by the simple evidence that media become warmer when traversed by light. Consequently, we should stress that the isotopic formulations presented in this paper only permit a first approximation of interior conditions.

That the proper causal and time invariant representation of irreversible interior events requires the covering *genomathematics, and related genorelativity, referred to the Lie-admissible covering of isorelativity.* To avoid a prohibitive length, this broader treatment will be merely indicated for intended presentations in subsequent papers.

We should finally recall historical contributions [67-71] dealing with superluminal speeds (although not within the context of interior dynamical problems),

4. Solution of the Historical Lorentz Problem

As it is well known to physics historians, Lorentz first attempted the achievement of the invariance of the speed of electromagnetic waves of his time, namely, the locally *varying* speed within physical media here referred to infrared, radio and other large wavelengths not admitting a consistent reduction to photons (see Section 4 for the general case)

$$C = \frac{c}{n(t,r,v,e,\rho,\omega,\tau,...)}, \tag{8}$$

where n is the familiar index of refraction with a rather complex functional dependence on local variables, such as time t, coordinates r, speeds v, energy e, density ρ, frequency ω, temperature τ and other variables.

Due to insurmountable technical difficulties, Lorentz was solely able to achieve invariance for the *constant* speed c of electromagnetic waves in vacuum, resulting in the celebrated transformations (1) leaving invariant line element (2a).

Santilli has studied for decades the solution of the *historical Lorentz problem,* namely, the achievement of the universal

invariance of all possible locally varying speeds of electromagnetic waves within physical media, Eq. (8), which case evidently admits the constant speed c in vacuum as a particular case.

As a first step, when a member of MIT from 1974 to 1978, Santilli realized that Lorentz's inability to achieve the invariance of speeds (8) was due to insufficiencies of the basic theory, Lie's theory, because such a theory is strictly linear, local-differential and potential-Hamiltonian, while the invariance of speeds (8) is a strictly non-linear, non-local/integral and non-potential, thus non-Hamiltonian problem.

The results of these initial studies were released in monographs [15] (that originally appeared as MIT preprints to be subsequently published under affiliation to Harvard University). A main aspect of these studies is their conception as *isotopic* (intended in the Greek meaning of being "axiom preserving") lifting of the various branches of Lie's theory into such a form to admit the treatment of non-linear, non-local and non-Hamiltonian systems.

The proposal was centered in the isotopic lifting of the unit of the Lorentz symmetry, $I = Diag.(1,1,1,1)$, into a quantity $\hat{I}$ (such as a function, a matrix, an operator, etc.) with an arbitrary functional dependence on all needed local variables, under the sole condition of being positive-definite, thus invertible,

$$I = Diag.(1,1,1,1) \to \hat{I} = \hat{I}(t,r,v,e,\rho,\omega,\tau,\dots) = 1/\hat{T}(t,r,v,\rho,\omega,\tau,\dots) > 0, \quad (9)$$

which lifting remains fixed for the interior problem considered.

For consistency, the lifting of the unit required the compatible lifting of the conventional associative product between arbitrary quantities A and B of the type

$$AB \to A \hat{\times} B = A\hat{T}B, \quad (10)$$

under which $\hat{I}$ is indeed to the right and left unit of the theory,

$$\hat{I} \hat{\times} A = A \hat{\times} \hat{I} \equiv A, \quad (11)$$

for all elements A of the set considered.

Following basic assumptions (9)-(11), Santilli constructed a step by step isotopic generalization of the various branches of Lie's theory, including [15b]:

1) The isotopic lifting $\hat{\xi}(L)$ of the universal enveloping associative algebra $\xi(L)$ of a n-dimensional Lie algebra L with (Hermitean) generators $X_i, i = 1,2,\dots,n$, and infinite-dimensional isotopic basis (today known as the *Poincaré-Birkhoff-Witt-Santilli isotheorem* [35-42]):

$$\hat{I},\ X_k,\ \hat{X}_i \hat{\times} \hat{X}_j,\ i \le j,\ \hat{X}_i \hat{\times} \hat{X}_j \hat{\times} \hat{X}_k,\ i \le j \le k,\ \dots; \quad (12)$$

2) The isotopic liftings of Lie algebras with closure rules (today called *Lie-Santilli isoalgebras* [*loc. cit.*]

$$[X_i,\hat{X}_j] = X_i \hat{\times} X_j - X_j \hat{\times} X_i = C_{ij}^k X_k' \quad (13)$$

3) The corresponding isotopic lifting of Lie's transformation groups (today called *Lie-Santilli isogroups* [*loc. cit.*]), e.g., here expressed for the time evolution

$$A(t) = U(t)A(0)U(t)^\dagger = [e^{H\hat{T}ti}]A(0)[e^{-it\hat{T}H}]; \quad (14)$$

and the isotopies of the representation theory.

The above isotopies clearly show the non-linear, non-local (integral) and non-Hamiltonian character of the isotopic theory due to the appearance of a positive-definite but otherwise arbitrary quantity $\hat{T}$ in the *exponent* of the group structure.

The representation of interior systems is then achieved via the representation of all SA interactions by means of the conventional Hamiltonian $H(r,p)$, and the representation of all NSA interactions by means of the generalized unit $\hat{I}$ (see Refs. [16, 34] for concrete examples in classical and operator mechanics).

Following the construction of the isotopies of Lie's theory, Santilli introduced in letter [23] of 1983 the following isotopies of Minkowski space (2) (today known as the *Minkowski-Santilli isospace* [*loc. cit.*]) with the most general possible nonsingular and symmetric line element (thus including all possible Minkowskian, Riemannian, Fynslerian and other line elements in (3+1)-dimensions)

$$\hat{x}^{\hat{2}} = x^\mu(\hat{T}_\mu^\rho \eta_{\rho\nu} x^\nu = x^\mu \hat{\eta}_{\mu\nu} x^\nu = \frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2\frac{c^2}{n_4^2}, \quad (15a)$$

$$n_\mu = n_\mu(t,r,v,e,\rho,\omega,\tau,\dots) > 0, \mu = 1,2,3,4, \quad (15b)$$

$$\hat{T} = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2) > 0, \quad (15c)$$

$$\hat{I} = 1/\hat{T} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2) > 0, \quad (15d)$$

where: the n's are called the *characteristic quantities* of the medium considered; n_4 is the conventional index of refraction providing a geometrization of the density of the medium normalized to the value $n_4 = 1$ for the vacuum; n_1, n_2, n_3 provide a geometrization of the shape of the medium considered normalized to the values $n_1 = n_2, n_3 = 1$ for the sphere; the general *inhomogeneity* of the medium is represented by the dependence of the characteristic quantity on the local variables (e.g., the elevation for the case of our atmosphere); and the general *anisotropy* of the medium (e.g., the anisotropy of our atmosphere caused by Earth's rotation) is represented by different values of the type $n_4 \neq n_s$.[1]

Note that, for exterior dynamical problems, homogeneity and isotropy equally occur in all directions. By contrast, within a physical medium inhomogeneity and anisotropy requires the selected of a given direction in space n_s due to variations for different directions.

Following the construction of the isotopies of Lie's theory and of Minkowski space-time, Santilli solved the historical Lorentz problem in page 551 of letter [23] via the lifting of the

[1] Scientific caution is suggested before dubbing the n-characteristic quantities as "free parameters" because that would imply that, e.g., the index of refraction n_4 is a free parameter when in reality it is measured for a given medium, or that the $g_{u\mu}$ elements of the Schwartzchild metric (see Eqs. (17) below) are free parameters, etc.

Lorentz symmetry characterized by the isotopic element (15c). This resulted in the generalized transformations (Eqs. (15) of Ref. [23]), today known as the *Lorentz-Santilli (LS) isotransforms* [35-42] which we write in the currently used symmetrized form

$$x^{`1} = x^1, x^{`2} = x^2, \tag{16a}$$

$$x^{`3} = \hat{\gamma}(x^3 - \hat{\beta}\,\frac{n_3}{n_4}x^4), x^{`4} = \hat{\gamma}(x^4 - \hat{\beta}\,\frac{n_4}{n_3}x^3), \tag{16b}$$

$$\hat{\beta} = \frac{v_3/n_3}{c_o/n_4}, \hat{\gamma} = \frac{1}{\sqrt{1-\hat{\beta}^2}}, \tag{16c}$$

leaving invariant the isoline element (15a), thus providing indeed the invariance of the varying speeds of light (8) (see Ref. [34b] for the general treatment).

Jointly with the above classical formulation, Santilli constructed the corresponding operator image [24] of the above isotransformations, and then constructed the isotopies of every main aspect of the LP symmetry, including the isotopies of: the rotational symmetry [25]; the SU(2)-spin symmetry [26]; the Poincaré symmetry [27]; the spinorial covering of the Poincaré symmetry [28]; the SU(2)-isospin symmetry and local realism [29]; and the isotopies of the Minkowskian geometry [30].

The resulting isosymmetry, today known as the *Lorentz-Poincaré-Santilli (LPS) isosymmetry,* was proved in Refs. [31,32] to be "directly universal" for all infinitely possible non-singular and symmetric space-times in (3 + 1)-dimensions, thus providing the universal symmetry of all possible Riemannian, Fynslerian, and other possible line elements, with a trivial extension to arbitrary space-time dimensions, such as those for the De Sitter symmetry.

Systematic studies can be found in monographs [33, 34], while independent studies can be found in monographs [35-42] and references quoted therein. A few comments are now in order to prevent possible misrepresentations that generally remain undetected by non-experts in the field.

To illustrate the universality of isoinvariant (15) and related isosymmetry (16) for all possible, symmetric and non-singular space-times in (3+1)-dimensions, let us note that they include as particular case all possible Riemannian line elements, such as the Schwartzchild's line element [27]

$$ds^2 = r^2(d\theta^2 + sin^2 d\theta^2 + d\phi^2) + (1 - \frac{2\times M}{r})^{-1} \times dr^2 - (1 - \frac{2\times M}{r}) \times dt^2 \tag{17a}$$

$$\hat{T}_{sch} = Diag.[1,1,(1 - \frac{2\times M}{r})_r^{-1}, (1 - \frac{2\times M}{r})_t], \tag{17b}$$

$$\hat{I}_{sch} = Diag.[1,1,(1 - \frac{2\times M}{r})_r, (1 - \frac{2\times M}{r})_t^{-1}], \tag{17c}$$

where one can see that the gravitational singularity is that of the isotopy, namely, the infinite value of the isotopic element and the null value of the isounit. In fact, Ref. [27] was primarily intended to indicate the achievement of the universal symmetry for all possible (non-singular) Riemannian line elements.

A rather popular belief during the 20th century physics was that interior dynamical systems are not essential because they can be reduced to elementary particles moving in vacuum, thus recovering at the elementary level exterior conditions without non-linear, non-local and non-Hamiltonian interactions. This belief was disproved by the following:

NO REDUCTION THEOREM [13, 34]: A macroscopic, non-conservative system cannot be consistently reduced to a finite number of elementary particles all in conservative conditions and, vice versa, a finite number of elementary particles all in conservative conditions cannot consistently yield a macroscopic non-conservative system under the correspondence or other principles.

Stated in different terms, the reduction of interior to exterior systems implies the belief that entropy and thermodynamical laws are "illusory" because, when interior systems are reduced to elementary particle constituents, entropy and thermodynamical laws "disappear." The above No Reduction Theorem establishes that non-conservative (thus, NSA) interactions, rather than "disappearing," originate at the most elementary level of nature.

As a concrete example, the above No Reduction Theorem is verified by a spaceship during reentry in atmosphere because its non-linear, non-local and non-Hamiltonian interactions originate at the most elementary level, that of the deep mutual penetration of the wavepackets of peripheral atomic electrons of the spaceship with the wavepackets of the electrons of atmospheric atoms.

The next aspect needed for a serious understanding of the content of this paper is the necessity to verify the time invariance indicated in the preceding section, namely, the prediction of the same numbers under the same conditions at different times.

Santilli selected a generalization of the unit for the representation of non-linear, non-local and non-Hamiltonian interactions for the intent of achieving the much needed time invariance, since the unit is the basic invariant of any theory.

However, it is easy to see that the representation of non-Hamiltonian interactions via the isounit is not sufficient *per se* to achieve the needed time invariance because isotopic theories are non-canonical at the classical level and non-unitary at the operator level by conception and construction [16,33,34]. It is then easy to see that the isounit is not preserved by the time evolution of the theory, e.g., Eq. (14)

$$\hat{I} \to \hat{I}' = U\,\hat{I}\,U^\dagger \neq \hat{I}, UU^\dagger \neq I \tag{18}$$

But the isounit represent interior conditions. Therefore, its lack of conservation in time implies the transition over time from one interior system to another (e.g., from the synthesis of the neutron (7), for instance, to a nuclear fusion).

This occurrence is known under the name of *Theorem of Catastrophic Mathematical and Physical Inconsistencies of Non-Canonical and Non-Unitary Theories* when formulated with the mathematics of canonical and unitary theories, respectively. Regrettably, we cannot review this theorem for brevity and must refer the reader to works [13, 33, 34].

The resolution of the inconsistencies caused by the lack of

time invariance required decades of additional research by Santilli and a number of colleagues. The solution was finally achieved following the construction of a new mathematics, today known as *Santilli isomathematics,* characterized by the isotopic lifting of the *totality* of the quantities and their operation of the mathematics used for exterior problems.

When classical non-canonical and operator non-unitary theories are elaborated with the appropriate classical and operator isomathematics, respectively, the invariance over time of numerical predictions is regained, thus offering the mathematical and physical consistency needed for applications.

5. Compatibility of Arbitrary Interior Speeds with Special Relativity

The locally varying speeds of electromagnetic waves propagating within physical media left invariant by the LPS isosymmetry (16) are completely unrestricted and can, therefore, be smaller, equal or bigger than the speed of light in vacuum,

$$C = \frac{c}{n_4} <=> c. \tag{19}$$

This is due to the unrestricted functional dependence of the isotopic element (15c), except for the condition of being non-singular.

It is then easy to see that the *maximal causal speed* in Minkowski-Santilli isospaces is arbitrarily bigger, equal or smaller than c. In fact, the mutated light cone, called *light isocone,* in the $(s, 4)$-dimensions is given by

$$\hat{x}^{\hat{2}} = \frac{x_s^2}{n_s^2} - t^2 \frac{c^2}{n_4^2} = 0, \tag{20}$$

and evidently characterizes the maximal causal speed within physical media

$$V_{max,s}^{media} = c \frac{n_s}{n_4} <=> c. \tag{21}$$

where, as indicated in Section 1, the selection of a space direction s is necessary since physical media are generally inhomogeneous and anisotropic.

Note the need to use a covering of the speed of light for maximal causal speed under the LPS isosymmetry because interior dynamical problems are generally opaque to light. Note also that the causal character of speeds (21) is guaranteed by the LPS isosymmetry in exactly the same way as the LP symmetry guarantees the causal character of c.

It has been popularly believed throughout the 20th century that any deviation from the speed of electromagnetic waves in vacuum implies a "violation of Einstein's special relativity." In this section, we show that this belief is not technically correct, because *Einstein's special relativity axioms do admit arbitrary causal speeds,* provided that they are realized via the appropriate mathematics.

To begin, Lorentz transformations (1) provide the invariance of the "constant" speed c without any identification of its numerical value, which value is set by measurements. Therefore, *Lorentz transformations equally apply for an arbitrary constant speed* $C = c/n_4$ *within physical media,* with known value in water

$$C_{water} = \frac{c}{n_4} = \frac{2}{3}c, n_4 = \frac{3}{2}. \tag{22}$$

in which case no violation of Einstein special relativity axioms can be claimed.

Additionally, the replacement in the *conventional* transforms (1) of the speed of light c with maximal causal speed (21) yields, identically, the LPS isotransformations (16), as one can readily verify.

$$x'^1 = x^1, x'^2 = x^2, \tag{23a}$$

$$x'^3 = \gamma(x^3 - \beta x^4), \tag{23b}$$

$$x'^4 = \gamma(x^4 - \beta x^3), \tag{23c}$$

$$\beta = \frac{v}{V_{max}^{media}}, \gamma = \frac{1}{\sqrt{1-\beta^2}}. \tag{23d}$$

Also, *the Lorentz-Santilli isosymmetry (16) is locally isomorphic to the conventional Lorentz symmetry by conception and construction* to the point of preserving the original structure constants [23, 33, 34]. Therefore, no claim that isotransforms (16) violate Einstein's special relativity axioms can be consistently voiced due to the very conception and technical realization of the isotopies of special relativity.

More technically, when represented on Minkowski-Santilli isospace over the isofield of real numbers [11], light isocone (20) becomes the perfect cone with the same maximal causal speed c as that valid in empty space [33, 34].[2]

This is due to the fact that the cone axes are indeed mutated under isotopies from their original unit value to new values

$$(1_s, 1_4) \to (1/n_s^2, 1/n_4^2), \tag{24}$$

but, jointly, the related units are mutated by the *inverse* amounts,

$$(1_s, 1_4) \to (n_s^2, n_4^2), \tag{25}$$

thus preserving the original Minkowskian light cone identically.

We finally illustrate the compatibility of arbitrary speeds with special relativity axioms via the following transformation of the Minkowskian coordinates

$$x^\mu \to \frac{x'^\mu}{n_\mu}, \tag{26}$$

under which the Minkowski line element (2a) is transformed into the isotopic image (15a), and Lorentz transformations (1) are turned into the LS transform by keeping in mind transformation of the speed, e.g., along the third space

[2]The mathematically correct formulation of the Minkowski-Santilli isospace is given by the isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$ defined over the field of isoreal numbers with unit $\hat{I}$ given by Eq. (15c), with $\hat{x} = x\hat{I}$ as a condition to be an isonumber and $\hat{\eta} = \hat{T}\eta$ (see Ref. [30] for details).

direction

$$v = \frac{\delta x^3}{\delta t} \to v\frac{n_4}{n_3}. \tag{27}$$

By recalling that the metric of isotopic theories $\hat{\eta} = \hat{T}\eta$ includes as particular case all possible Riemannian metrics g, in this section we have attempted to indicate that *Einstein's special relativity axioms have a representational capability dramatically broader than that believed in the 20th century because, in addition to admitting arbitrary maximal causal speeds, they also admit interior and exterior gravitational models.*

In fact, in Ref. [30] Santilli has shown the treatment of *exterior gravitation* via special relativity axioms on the metric $\hat{\eta}(x) = \hat{T}(x)\eta = g(x)$, while maintaining the machinery of the Riemannian geometry (covariant derivative, Christoffel's symbols, etc.) and Einstein-Hilbert field equations under the universal LPS symmetry as a condition to achieve the above indicated invariance over time of numerical predictions. In this exterior case, the maximal causal speed is evidently that in vacuum c.

In the same Ref. [3], Santilli has shown that special relativity axioms equally admit *interior gravitational models,* this time, with an unrestricted functional dependence of the metric $\hat{\eta}(x,\rho,-\tau,\omega,\dots) = \hat{T}(x,\rho,\tau,\omega,\dots)\eta = g(x,\rho,\tau,\omega,\dots)$ equally under the universal LPS symmetry, in which case the maximal local causal speed is arbitrarily bigger or smaller than c depending on local conditions.

Note that, under the full use of isomathematics, all the preceding enlargements of the conditions of applicability of Einstein's special relativity axioms can be formulated via conventional symbols as used in Eqs. (1)-(4), and merely subject them to different interpretations.

In fact, the variables x^μ can be interpreted as representing physical space-time coordinates with respect to the Lorentz unit $I = Diag.(1,1,1,1)$, in which case we have the 20th century formulation of special relativity for exterior problems invacuum.

Alternatively, we can consider the coordinates x^μ as being purely mathematical and assume the realization $x^\mu = x'^\mu/n_\mu$. In this case, when x'^μ are assumed as the physical coordinates and are referred to the isounit (15d), we realize special relativity axioms in such a way to have interior dynamical conditions with maximal causal speeds 921), exterior gravitation with the characteristic quantities representing conventional Riemannian metrics, interior gravitation, and other possibilities.

It should be stressed that the above studies apply to *interior dynamical problems that are reversible over time.* Their extension to irreversible problems can be first studied via isounit with an explicit but not invariant time dependence of the type

$$\hat{I}(t,\dots) - \hat{I}^\dagger(t\dots) \neq \hat{I}(-t,\dots.) = \hat{I}^\dagger(-t,\dots), \tag{28}$$

The reader should however be aware that under the above realization of the isounit the Lie-isotopic formulation is turned into a bimodular formulation with a Lie-admissible structure [9-16].

We should finally indicate that the entire body of mathematical, theoretical and experimental formulations studied in this paper are nowadays known under the name of *isorelativity for matter* [50-54] and the *isodual isorelativity for antimatter*[55].

6. Expected Implications

6.1. Cosmology

As reviewed in Section 2, the conjecture of the expansion of the universe and its endless chain of subsequent conjectures were aimed at the unspoken intent (or *de facto* primary implication) of maintaining the validity of special relativity for the large scale structure of the universe because the alternative offered by Zwicky [5], that of galactic light losing energy to the intergalactic medium, would violate the special relativity.

Despite one century of efforts, the conjecture of the expansion of the universe is nowadays discredited in serious scientific circles due to excessive insufficiencies or sheer inconsistencies published in refereed journals, but none of them disproved also in refereed journals.

In fact, all current dominating cosmological models imply a return to the Middle Ages with Earth at the center of the universe due to the *radial* character in all directions from Earth of the conjectured expansion of the universe and its conjectured acceleration.

Isomathematics [38], the Lorentz-Santilli isosymmetry [23-30], its related isorelativity [51-54], and the vast experimental verifications on Earth of Zwicky hypothesis [43-47] are expected to restore the validity of special relativity for the large scale structure of the universe without inconsistent assumptions on the expansion of the universe.

This important occurrence is due to the fact that the Lorentz-Santilli isosymmetry uniquely and unambiguously characterize the *Doppler-Santilli isoshift law* [51, 43]

$$z \approx \pm\frac{v}{c}\frac{n_4}{n_v} \tag{29}$$

where the characteristic quantities n_4, n_v depend on all local variables, including the speed v, the distance d traveled by light, etc., thus allowing the expansion

$$z \approx \pm\frac{v}{c}\frac{n_4}{n_v} \approx \pm\frac{v}{c}(1 \pm \frac{Hc}{v}d) = \pm\frac{v}{c} \pm Hd, \tag{30}$$

where the first term is the conventional Doppler term, the second term is the Santilli isoredshift, and H is a constant (in first approximation) that can be assumed to be Hubble's constant for cosmological applications.

But experiments [43-47] have established that the conventional Doppler contribution is ignorable with respect to Santilli isoredshift for the propagation of light within physical media either of large density and short distances as it is the case for our atmosphere, or of low density but extremely large distances, as it is the case for intergalactic media.

Consequently, the original, experimentally verified

Hubble's law [4]

$$z = Hd \tag{31}$$

is uniquely, unambiguously and invariantly represented by the Lorentz-Santilli isosymmetry. But the Lorentz-Santilli isosymmetry is locally isomorphic to the conventional symmetry, and the axioms of isorelativity are identical to those of special relativity [51-54], thus implying the restoration of special relativity for the large scale structure of the universe, although in its broader isotopic realization, without any expansion of the universe.

However, the cosmological implications of the above possibilities are far reaching because all current numerical values in cosmology, beginning with currently assumed distances of stars and galaxies, should be re-inspected for possible revisions whenever admitting the absorption of light by innergalactic or intergalactic physical media.

In fact, current distances are estimated based on the luminosity of supernovae or large stars under the current assumption, necessary to maintain special relativity in cosmology, that light propagates in vacuum.

It is evident that the admission of innergalactic and intergalactic media as well as the admission of the experimental evidence [43-47] on the loss of energy by light to said media, are expected to imply a revision of the value of the intensity at the origin, with consequential revision of current estimates on cosmological distances.

The same implications are expected for other numerical values in cosmology since they are all derived via *theoretical conjectures,* since the same experimentally known values are those of the cosmological redshift.

6.2. Particle Physics

The unspoken intent of maintaining special relativity in cosmology was also applied to the structure of strongly interacting particles (hadrons) via the same approach, that based on conjecture that could not be directly verified in experiments, so that the conjectures would achieve collegial acceptance via coordinated supports.

In fact, the structure of hadrons was reduced to the hypothetical quarks that, by conception have to be permanently confined inside hadrons, thus reducing the structure of hadrons to imaginary spheres containing point-like hypothetical quarks moving in vacuum, as necessary for the validity of special relativity.

In particular, for the first time in history, the same model was used for both the *classification* of hadrons into family and the *structure* of individual hadrons belonging to a given multiplet. By contrast, atoms required *two* models, the Mendeleev classification of atoms into families, and the structure of individual atoms belonging to a given family.

Also, the classification of atoms was achieved via classical methods, while the structure of individual atoms required the construction of a generalization of classical theories, quantum mechanics.

The insufficiencies of such an *ad hoc* conduction of physical research are numerous (see, e.g., paper [56] of 1981). Unfortunately for scientific knowledge, said insufficiencies are not disproved in refereed journals exactly as it has been the case for the insufficiencies of cosmological conjectures.

The most striking insufficiency with large societal implication is that belief, implicit in quark conjectures, that the permanently stable proton and electron "disappear" (sic) at the time of the synthesis of the neutron (7) top be replaced by the hypothetical quarks and, then, at the time of spontaneous decays of the neutron, the hypothetical quarks "disappear" and the permanently stable proton and electron "reappear."

Additionally, quarks cannot be defined in our spacetime due to incompatibilities with the Poincaré symmetry, as well known to quark experts. Therefore, any claim that quarks are "physical particles" has no serious physical ground. Additionally, the theoretical impossibility for quarks to be seriously confined inside hadrons (that is, to have an identical null probability of tunnel effects) is readily established by Heisenberg's uncertainty principle. In the final analysis, quarks are mere mathematical representations of a purely mathematical symmetry solely defined in a purely mathematical, unitary, complex-valued space.

In view of all the above insufficiencies (and numerous others), the view suggested by the author when at Harvard University in the 1980s [56] is that the $SU(3)$ -color classification of hadrons (now extended to the "Standard Model") is excellent; quarks are necessary for the mathematical elaboration of said classification; and the structure of individual hadrons requires a generalization of quantum mechanics into hadronic mechanics.

The latter view was suggested in the early 1980s because *there exist no point-like wavepackets in nature.* Consequently, hadrons are the densest medium identified by mankind to date caused by the total mutual penetration of the wavepackets of the constituents, with inevitable non-linear, non-local and non-Hamiltonian internal effects. Under these conditions, the intents at preserving the 20th century formulation of special relativity and quantum mechanics are non-scientific, besides being disproved by experimental evidence on the behavior of the main-life of unstable Kaons from $1q0$ to $100\ GeV$ showing clear deviations from special relativity [57].

Isomathematics [38], the Lorentz-Santilli isosymmetry [23-30], and the additional experimental evidence on deviations from special relativity within physical media [43-47], offer realistic possibilities of restoring the validity within hadrons of the abstract axioms of special relativity and quantum mechanics, although in their generalized isotopic realizations, without the assumption of hypothetical constituents that have not been produced free even at the extremely high energies available at CERN.

According to this view, the constituents of all unstable particles are generally the massive particles produced in the spontaneous decays, although in a generalized form characterized by the Poincaré-Santilli isosymmetry under the law of relativistic hadronic mechanics due to their total mutual immersion (see review [42] and original papers quoted therein).

As it was the case for cosmology, the implications of the above new vistas in particle physics are rather deep because they require the re-inspection of the entire 20th century particle physics for possible revision.

As an example, the Paulì-Fermi conjecture of the neutrino, voiced to conserved the angular momentum in synthesis (7), is fundamentally dependent on the abstraction of the proton as a point-particle because necessary for the very applicability of quantum mechanics, as it is well known.

The representation of the proton as an extended charge distribution has eliminated the need for the neutrino to conserve the angular momentum because, during the "Rutherford's compression" of the hydrogen atom inside a star, the electron is constrained to orbit within the hyperdense medium inside the proton in such a fashion to have a null total angular momentum, in which case the spin of the neutron in synthesis (7) is the spin of the proton [14].

In any case, the Pauli-Fermi conjecture of the neutrino did not salvage quantum mechanics because the rest energy of the neutron is $0.782\,MeV$ *bigger* than the sum of the rest energies of the proton and the electrons, in which case the Schrödingerequation is completely ineffective (for a bound states). The covering hadronic mechanics was proposed precisely for the representation of synthesis (7) and it proved to be correct at both the non-relativistic and relativistic level [14].

Despite the latter advances, our knowledge of the synthesis of the neutron remains Lilliputian at best and so much remains to be discovered by young minds of any age. For instance, we remain with the mystery of the origin of the missing $0.782\,MeV$ neededto create the neutron which energy cannot be due to relative kinetic energy (because in this case the cross section of the scattering of protons and electrons is virtually null).

The author has submitted the hypothesis that the missing energy originates from space conceived as a universal substratum for all electromagnetic waves and all particles with a verity high energy density (due to the high value of the speed of light). In this case, the energy is expected to be transmitted from space to the neutron via a *longitudinal impulse* the author has called the *etherino,* by turning the neutron synthesis into a Lie-admissible interior dynamical problem [58].

The aspect intriguing for this paper is that, being a *longitudinal* impulse in a universal medium with physical characteristics similar to that of "rigidity" (due to the transversal character of electromagnetic waves), *the speed of transmission of energy from space to the neutron is expected to be a large multiple of the speed of light.* This illustrates again the possible existence of arbitrary speeds for interior dynamical problems in a way fully compatible with the abstract axioms of special relativity and relativistic quantum mechanics, although realized via the covering Lie-isotopic and then Lie-admissible mathematics.

As a final comment, it appears that the etherino hypothesis can represent experimental data on the so-called "neutrino experiments" in a more credible way than that provides by contemporary neutrinos that are believed to have mass. In fact, the idea that massive particles can travel long distances within dense matter without collision is not plausible since a large number of neutrinos have to travel through a large number of nuclei without collision. By contrast, the same long travel through matter is more plausible for the etherino because the travel occurs *within the underlying universe substratum*, rather than through matter.

6.3. Nuclear Physics

The scientific scene depicted above for cosmology and particle physics occurred again in nuclear physics due to the restriction of nuclear physics research over one century to the unspoken (intent to be compatible with the conventional interpretation of special relativity and relativistic quantum mechanics despite truly vast insufficiencies.

In serious scientific circles, a theory can be assumed as being *exactly valid* for given, well defined conditions, if and only if said theory represents all experimental data from un-adulterated first principles. This has been indeed the case for the validity of quantum mechanics for the structure of the hydrogen atom.

Whenever a theory does not represent at least the main experimental data, said theory can be at best claimed to be *approximately valid.* In this conditions considered. Scientific ethics then requires the search for covering theories, as it was historically done for the atomic structure.

The conventional interpretation of special relativity and relativistic quantum mechanics can at best be considered as being approximately valid in nuclear physics because, following attempts over one century, they failed to achieve a consistent representation of the experimental data of the simplest nucleus, the deuteron, with embarrassing deviations for heavy nuclei such as the zirconium.

It is today known that the insufficiencies of standard theories in nuclear physics are due to the point-like abstraction of particles which is very effective for structures at large mutual distances of particles, such as the atomic structure, while being manifestly insufficient for the nuclear structure where the constituents are extended charge distributions in conditions of partial mutual penetration.

Isomathematics [38], the Lorentz-Santilli isosymmetry [23-30], and the experimental evidence on deviations from special relativity within physical media [43-47], offer realistic possibilities for new vistas in nuclear physics while preserving the abstract axioms of special relativity and quantum mechanics.

The central notion deals with the representation of nucleons as *extended, therefore deformable charge distribution* according to a conception dating back to Enrico Fermi, who indicated that the representation of nuclear magnetic moments may require the deformation of nucleons with consequential alteration of their magnetic moments. In view of their compatibility with the deformation theory [24-26], Santilli's Lie-isotopic formulations have provided a technical realization of Fermi's view, by achieving indeed the first known exact and invariant representations of nuclear magnetic moments [59,60] and spins [61].

Independently from the above, a structural insufficiency of standard theories is their lack of a "time arrow," with the consequential sole capability of representing isolated stable bound states that, as such, are reversible over time. Consequently, quantum mechanics is structurally insufficient for the representation of nuclear energies, because they are all based on *time irreversible processes.* Santilli Lie-Isotopic and, more appropriately, Lie-Admissible reformulations of special relativity and quantum mechanics offer a realistic possibility for quantitative studies of new nuclear energies that are already under way at U. S, corporations [52].

A novel aspect important for this presentation is that the above new vista in nuclear physics is based, not only in the representation of nucleons as extended and deformable under partial mutual penetration, but also in the consequential emergence of a new component in the nuclear force of contact, non-Hamiltonian type invariantly represented by Santilli isounit. In turn, the sole emergence of this new component in the nuclear forces implies possible advances in nuclear physics simply beyond our appraisal capabilities at this writing.

6.4. Chemistry

The author has always accepted the historical advances permitted by quantum chemistry at molecular distances, but could not accept quantum chemistry for the valence bond because valence electron pairs in singlet couplings should *repel,* rather than attract each other, due to the extremely large values of their Coulomb repulsion which is of the order of $10^{28}N$ at bond distances of $1\,fm$.

It is today known that this insufficiency is due to the impossibility of a consistent reduction of the universe to exterior conditions, intended to be everywhere composed of point-particles moving in vacuum (empty space). Chemistry establishes alone the impossibility in view of the non-Hamiltonian interaction originating from the deep mutual penetration of the wavepackets of valence electrons establishing the valence bond is an interior dynamical problem.

In fact, said contact non-Hamiltonioan interactions have been able to overcome the repulsive Coulomb force and produce an *attraction* between the identical valence electrons in singlet coupling achieving an exact representation of the binding energies of the hydrogen and water molecules, as well as other chemical data [63].

Rather than being a mere academic curiosity, the achievement of an attractive force between valence electron pairs has permitted the identification of new bonds, called *Santilli magnecular bonds,* that, being *weaker* than the valence by conception and technical realization, allow full combustion, i.e., the absence of combustible contaminants in the exhaust such as CO, HT, etc., with evident environmental advances [64].

6.5. Biology

The insistence in the use of special relativity and quantum mechanics in biology implies that biological structures are perfectly rigid, evidently due to the known incompatibility of said theories with the deformation theory, and eternal, evidently due to the known lack of a time arrow in the basic axioms.

By contrast, the Lie-isotopic, Lie-admissible and hyper-structural coverings of special relativity and quantum mechanics [50] are compatible with the deformation theory by conception and technical realization [25-27], and contain a time arrow in their ultimate foundations, the basic units [65].

It is hoped that these new vistas may permit more realistic representations of biological structures with expected advances perhaps not entirely conceivable at this writing.

6.6. Gravitation

Recent studies [54] have reviewed the historical objection against Einstein's gravitation, with particular reference to:

1. The lack of experimental evidence on the actual curvature of space since the bending of star light passing near the Sun is entirely due to the refraction of light in the Sun chromosphere and Newton's gravitation.
2. The impossibility to achieve a universal symmetry for Einstein's gravitation caused by the curvature of space, as studied in Section 3, with consequential adoption of "covariance" and ensuing lack of prediction of the same numerical values under the same conditions at different times.
3. The incompatibility of Einstein's gravitation with 20th century doctrines, including special relativity, electrodynamics, and quantum mechanics, which is also caused by the curvature of space.

The return of gravitation to a flat space via its formulation in the Minkowski-Santilli isospace and isogeometry presented in paper [54] under the name of *isogravitation,* has permitted the achievement of the universal invariance for all gravitational line elements [27], with a direct and unequivocal compatibility of isogravitation with 20th century theories.

The aspect of this new vista in gravitation important for this paper is the transition from the failed "unification" of the gravitational and electromagnetic fields, to their "identification" with ensuing possibility of the laboratory creation of the gravitational field according to proposals dating back to 1974 [66].

In turn, the capability to create an attractive gravitational field, combined with the predicted gravitational repulsion between matter and antimatter, is expected to initiate the study of fundamentally new propulsions beyond the rather primitive Newtonian propulsion of current use nowadays that are known as *isogeometric propulsion* [55].

6.7. Interstellar Travel

6.7.1. Foreword

As it is well known, the initiation of scientific studies on interstellar travel has been opposed by the orthodox physics community via discreditation and other means because interstellar travel requires arbitrary speeds that are popularly believed not to be permitted by Einstein's theories.

This posture is essentially based on the belief of the universal validity of Einstein's theories for all possible conditions existing in the large and small scale structure of the universe, expectedly, until the end of time.

In reality, it is the fate of all theories to have precise conditions of exact applicability beyond which the theories are only approximately valid or totally inapplicable. In the author's view, this is precisely the case of interstellar travel.

In this subsection we shall outline our current mathematical knowledge of interstellar travel, with a warning that its serious understanding requires a technical knowledge of our most advanced known mathematics for matter and antimatter, known as *hypermathematics* [50].

6.7.2. Inapplicability of Einstein's Theories for Interstellar Travel

Interstellar travel cannot be achieved in the event the necessary fuel has to be carried along in tanks. Consequently, in the author's view, interstellar travel can only be achieved by continuously extracting the necessary energy from space conceived as a *universal substratum* with a very large energy density, also known as *zero point energy* [72].

The process of energy extraction from space renders interstellar travel a strictly *interior* dynamical problem irreversible over time for which Einstein's theories cannot be minimally formulated in a consistent form due to the need for its Lie-admissible formulation for quantitative treatments.

In addition, we expect the need of the antimatter-type energy, known as *isodual energy,* and other aspects require the joint treatment of matter and antimatter which can be best done via of the most advanced mathematics known by mankind, the hypermathematics [50].

It should be indicated that, rather than being far fetched as popularly believed, the extraction of energy from space has resulted to be necessary for a consistent representation of the synthesis of the neutron from the hydrogen via a *longitudinal* impulse, called the *etherino* which is itself superluminal [58].

6.7.3. The Universal Substratum for Particles and Electromagnetic Waves

The first scientific work written by the author [73] (in 1957 when attending high school) dealt with the resolution of the historical problem of the "ethereal wind," namely, the expectation that, in the event space has physical characteristics similar to medium, the orbiting of Earth around the Sun would be slowed down due to the resistance caused by said medium, which is contrary to evidence.

The argument t of paper [73] is that electromagnetic waves as well as the elementary particles composing matter are "pure oscillations" of the universal substratum in the sense that the oscillations occur at specific points of space without any oscillating "little mass."

This is evidently the case for electromagnetic "waves" that, in the author opinion, cannot exist without a medium, but we have a similar occurrence also for the structure of the electron which is known to be a "pure oscillation" of a point of space with a frequency of $0.829 \times 10^{10}\, Hz$ without any little mass" oscillating in its interior.

Consequently, the main point of Ref., [73] is that, when we move an object, we have no solid substance at all because we merely have the propagation through space of the oscillations characterizing the object without any possible "ethereal wind."

Paper 73 then concluded that space should have characteristics similar to "rigidity" due to the *transverse* character of electromagnetic waves that prevent other structures of space, such as that as a "fluid" for which transverse waves cannot notoriously exist. The very high value of the speed of electromagnetic waves then suggested for space to have an extremely high energy density.

A central conception underlying the view here presented is therefore that *space is totally full and matter is totally empty.* This conception, which is clearly against our sensory perception, can be visualized by assuming the capability of "seeing" an electron. Under our ordinary time the electron appears as solid sphere due to the high value of the frequency of the oscillations. However, in the event said frequency goes to zero the electron would disappear. The same holds for the other elementary particles and, therefore, for all matter.

It should be noted that the conception of matter as being "pure oscillations" of space seems to be needed for other features of interstellar travel indicated below. It should be also noted that, according to a widely adopted view (see, e.g., Ref. [72]), matter is conceived as being "immersed" in the universal substratum.

6.7.4. The Universal Substratum for Matter and Antimatter

The above conception implies that the energy contained in space can be essentially conceived to be similar to energies in our environment, such as the potential energy, thus being a *positive* energy E measured with the conventional *positive* units, such as erg. This view is confirmed in particle laboratories since we can extract from space photons and/or particles via the use of conventional energies.

In the author's view, the above conception of space is basically insufficient to achieve interstellar travel due to the need for space to be the universal substratum also for antimatter photons and antimatter particles [74].

In the latter case, a number of consistency conditions require that the antimatter substratum should have *negative* energy, called *isodual energy* indicated as E^d although referred to *negative* units, known as *isodual units* indicated erg^d, where the upper symbol denotes the conjugation from matter to antimatter called *isoduality* and given by an anti-= Hermitean map.

Additional requirements of compatibility with our current knowledge of matter and antimatter (such as the capability of extracting from space both, particles and antiparticles) imply that *space is a superposition of equal amounts conventional and isodual energy, each with extremely high density,* which can be best represented as a hyperstructure [75].

6.7.5. Hyperspeeds

A main assumption of the propulsion for interstellar travel, here [presented is *the capability by a "matter-spaceship" to extract from space antimatter/isodual energy* [74] because

matter and antimatter are predicted to experience gravitational repulsion at all levels of study from Newtonian mechanics to second quantization [55].

In fact, the author has not been able to achieve a mathematical propulsion model when a matter-spaceship extract conventional energy from space, since conventional (positive) energy is the origin of *gravitational attraction* according to Einstein.

By contrast, in the event a matter-spaceship is capable of extracting antimatter-energy from space propulsion in all directions *opposite* said extraction become possible at arbitrary speeds evidently proportional to the extracted antimatter energy.

It should be noted that this view is not generally adopted in current studies on interstellar travel, since they assume the achievement of propulsion via the extraction form space of positive energy from a matter-spaceship [72].

At this point, the mathematical, treatment of the propulsion here suggested under the name of *hypergeometric propulsion* becomes rather complex (see Chapter 4 of monograph [55] for technical details of the sublease of isogeometric propulsion, and the various references on antimatter quoted in this paper).

The main assumption of the proposed *hypergeometric propulsion* is that the spaceship 'achieves propulsion buy changing the geometry in its environment without any motion being perceived by its operators, thus allowing instantaneous acceleration, singular trajectories, lack of sonic boom when traveling in atmosphere at supersonic speeds and other anomalies. Hypermathematics is needed for the joint use of matter ands antimatter energies.

The simpler case of the *isogeometric propulsion* studied in details in Ref.[55] can be mathematically represented via the invariant ;lifting of the spacetime line element of special relativity

$$\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2 \equiv$$

$$\equiv \Delta x'^2\hat{I}_x + \Delta y'^2\hat{I}_y + \Delta z'^2]hat I_z - c^2\Delta t'^2\hat{I}_t \equiv \quad (32)$$

where

$$\Delta x = x_1 - x_2, \Delta y = y_1 - Y_2, \Delta z = z_1, \Delta t = -t_r 1 - t_2 \quad (33)$$

represents the motion in space and time.as seen by an outside observer.

Isogeometric propulsion (for a spaceship composed of matter)occurs via the extraction from space of antimatter energy in a given direction, say Δx, resulting in a propulsion in the direction $-\Delta x$ opposite that of said extraction due to the repulsion of matter and antimatter.

The extraction of antimatter energy, say, in the x=direction, is mathematically represented via the transition from the conventional unit 1 of our spacetime to the isounit in the indicated direction, $\hat{I}_x$ with consequential decrease of the distance $\Delta x'$ in the indicated direction due to the invariance $\Delta x = \Delta x'\hat{I}_x$.

Mathematically, there is no limit in the acquired speeds, which can be millions of times the speed of light c, since the isounit in the x-direction can be extremely large in which cased, the actual distance $\Delta x'$ covered by the spaceship is infinitesimally small.

Some of the consequences of the above isogeometric locomotion can be expressed as follow:

A. The extraction of any energy from space causes a deformation of the stricture of space, called *mutation.*

B. The size of the spaceship perceived by external and internal observers can be dramatically different.

C.There cannot be mutation of space without a corresponding mutation of time, thus illustrating the name of ßpacetime machine" in Ref. [55].

D. The visual detection of a spaceship by an external observer does not mean that the spaceship exists at his/her time.

E. The lapse of time measured by the spaceship operator can be much smaller than the lapse of time perceived by an external observer.

The generalization of the above isogeometric propulsion to the hypergeometric form is excessively complex for the limited scope of this presentation and will be presented elsewhere.

6.7.6. Hypercommunications

At interstellar distances, electromagnetic communications are like the smoke signals of the American Indians because they are excessively slow compared to the distances at hand. Since interstellar travel cannot even be conceived without effective communications, we find again the need of surpassing the speed of light which has been opposed for one century by the orthodox physics community.

Yet, there already exist in the refereed literature *experiments* showing the propagation of signals faster than light (see, e.g., Ref. [19]); signals propagating at a large multiple the speed of light have already been computed for the *longitudinal* propagation of energy in the synthesis of the neutron [58]; and, according to (unpublished) initial calculations, the Lie-Isotopic and Lie-admissible coverings of Maxwell's equations predict virtually instantaneous longitudinal signals particularly if space is assumed to have a characteristic similar to external high "rigidity."

6.7.7. Hyperfading

Trajectory corrections to avoid collisions can indeed be done for spaceships traveling at interplanetary speeds, but said corrections cannot be done when traveling at interstellar speeds that are expected to be millions of time the speed of light.

A conceivable solution, here suggested by the author under the name of *hyperfading,* is the capability of interstellar spaceships to alter their physical structure in such a wave to cross through astrophysical bodies without collisions or damage. This also implies the capability to enter oceans without creating any wave.

Rather than being far fetched, this possibility is within current *mathematical* possibilities. In fact, the mutation of space which is inherent in the hypergeometric locomotion implies the alteration of the spacetime of the spaceship and of its inhabitants. Hyperfading is then mathematically possible

via the control of time permitted by antimatter, a conceivable dramatic reduction of the characteristic frequencies of the spaceship and its inhabitants under which the spaceship literally "fades away" from the universe, and other means.

Thanks to advances in antimatter with the ensuing *isogeometric propulsion* [55] in which hyperfading could be achieved due to the structural alteration of the local spacetime, the control of time predicted from a suitable use of *isodual energies* (negative energies referred to negative units), and other means.

It is hoped that young minds of any age will ignore biased obstructions from the orthodox physics community and initiate indeed scientific studies on the fascinating open problems connected to interstellar travel because, in the author's view, the achievement by mankind of interstellar travel is only a question of time.

7. Concluding Remarks

During the studies on axiom-preserving isotopies and genotopies of 20th century theories, the author has stated various times that:

Rather than abusing the names of Lorentz, Poincaré, Einstein and other founders of 20th century physics by applying their theories under interior dynamical conditions they were not intended for, and cannot be properly formulated and tested, the best way to honor their names is to maintain their axioms, and enlarge their conditions of exact applicability via the use of broader mathematics specifically built for interior conditions.

The implications in surpassing 20th century theories for interior dynamical conditions are so deep for all quantitative sciences to imply a "new scientific era," as indicated in the tile of Ref. [42].

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Santilli's Isodual Mathematics and Physics for Antimatter

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Abstract: In this paper, we study the isodual branch of hadronic mechanics achieved by the Italian-American scientist R. M. Santilli following decades of research. We show that, thanks to the background isodual mathematics (whose knowledge is assumed), Santilli's isodual theory of antimatter appears to be the only axiomatically consistent and time invariant theory permitting a {\it classical} representation of {\it neutral} (as well as charged) antimatter whose operator image is equivalent to charge conjugation, thus verifying all known experimental data in the field. We then show the important prediction of {\it gravitational repulsion} between matter and antimatter, with particular reference to the prediction that light emitted by antimatter, known as {\it isodual light,} experiences gravitational repulsion when in the field of matter. We finally point out that recent detections of antimatter galaxies via Santilli telescope with concave lenses may eventually result to be the first experimental evidence of antigravity.

Keywords: Antimatter, Isodual Light, Antigravity

1. Introduction

After being conjectured by A. Schuster in 1898, antimatter was predicted by P. A. M. Dirac [1] in the late 1920's in the negative-energy solutions of his celebrated equation. Dirac himself soon discovered that particles with negative-energy do not behave in a physical way and, for this reason, he submitted his celebrated "hole theory," which subsequently restricted the study of antimatter to the sole level of second quantization [2]. This occurrence created an imbalance in the physics of this century because matter is described at all levels of study, from Newtonian mechanics to quantum field theory, while antimatter is solely treated at the level of second quantization.

To initiate the study for the future removal of this imbalance in due time, Santilli presented a theory of antimatter which has been conceived to begin at the purely classical Newtonian level, and then to admit corresponding images at all subsequent levels of study [3] in which the guiding principle is to identify a map which possesses the main mathematical structure of charge conjugation, yet it is applicable at all levels, and not solely at the operator level.

The main characteristic of charge conjugation is that of being antiautomorphic (where the term "automorphic" is referred to the map of a given space onto itself). After studying a number of possibilities, Santilli has selected a map which is anti-isomorphic (where the term "isomorphic" is referred to a map from one space onto another of equivalent topological characteristics to be identified later on) applicable at all levels of study, and given by the following isodual map here generically expressed to an arbitrary quantity Q (i.e. a function, or a matrix or an operator)

$$Q(x, \varphi, \ldots) \rightarrow Q^d(x^d, \varphi^d, \ldots) = -Q^\dagger(-x^\dagger, -\varphi^\dagger, \ldots) \quad (1)$$

which, for consistency, must be applied to the totality of the mathematical structure of the conventional theory of matter, including numbers, fields, spaces, geometries, algebras, etc. This results in a new mathematics, called isodual mathematics, which is at the foundation of the classical isodual theory of antimatter of this paper.

The isodual map was first proposed by Santilli [4] in 1985 and then remained ignored for several years. According to it the isoduality can be represented as $+1 \rightarrow 1^d = -1$. The first hypothesis on the isodual theory of antimatter appeared for the operator version in 1993 [5] which also contains an initial study of the equivalence between isoduality and charge conjugation. The prediction of the isodual theory that antimatter in the field of matter experiences antigravity was first submitted in 1994 [6] which also proposed an experiment

for the measure of the gravity of elementary antiparticles in the gravitational field of earth. The experiment essentially consists of comparative measurements under the gravity of collimated, low energy beams of positrons and electrons in horizontal flight on a tube with sufficiently high vacuum as well as protection from stray and patch fields and of sufficient length to permit a definite result, e.g. the view by the naked eye of the displacements due to gravity of the positron and electron beams on a scintillator at the end of the flight.

Theoretical and experimental studies on the isodual theory of antimatter were conducted at the International Workshop on Antimatter Gravity and Anti-Hydrogen Atom Spectroscopy, held in Sepino, Italy, in May 1996 [7].

The motivations for the classical isodual theory of antimatter are rather numerous. First, there is the need indicated earlier to achieve a full equivalence in the treatment of matter and antimatter beginning at the classical level. In fact, far away galaxies and quasars may well be made up of antimatter. The absence of a classical theory of antimatter therefore implies the evident impossibility of quantitative studies of this important astrophysical issue.

Second, the current gravitational treatment of antimatter is afflicted by a number of problematic aspects. Current theories are based on only one map from classical to operator settings, the naive or symplectic quantization. Therefore, conventional classical representations of antimatter via positive energies do not yield antiparticles under quantization, but conventional particles with the mere reversal of the sign of the charge.

Third, there is a fundamental incompatibility between current theories of gravitation and unified gauge theories of electroweak interactions which is due precisely to antimatter. In fact, current gravitational theories characterize antimatter via a positive-definite energy-momentum tensor, while electroweak theories characterize antiparticles via negative energy states. Additional motivations have been identified in [8-10]. The need for a systematic study aiming at a resolution of these issues is then beyond scientific doubts.

The isodual theory emerged from the identification of negative units in the antiparticle component of the conventional Dirac equation and the reconstruction of the theory with respect to that unit. Isoduality therefore provides a mere reinterpretation of Dirac's original notion of antiparticle, while leaving all numerical predictions under electroweak interactions essentially unchanged.

Santilli was able to construct the isodual theory of antimatter which characterizes antimatter at all possible levels, from Newtonian mechanics to second quantization on the basis of his isodual mathematics [11-13].

In particular, isodual mathematics permitted the first known geometrically consistent representation of the gravitational field of a neutral antimatter body via the Riemann-Santilli isodual geometry defined over isodual fields [11, 13].

It should be stressed that the isodual theory of antimatter requires, for consistency requirement, the conjugation of all physical quantities of matter as well as, most importantly, all their units of measurements. Consequently, antimatter evolves along a time moving backward, the $t^d = - t$, and has negative-definite energy $E^d = - E$ along Dirac's original conception [14]. The historical inconsistencies are resolved via the joint conjunction of the related units; in fact, negative time and negative energy referred to negative units of time and energy are as causal as our positive time and energies referred to positive units of time and energy.

Eventually, in this paper the author tried to focus on some important features of Santilli's Isodual Physics for antimatter based on Santilli's Isodual Mathematics.

2. Salient Features of Santilli's Isodual Mathematics

The Newton's equations, Galileo's relativity and Einstein's special and general relativities were conceived before the discovery of antimatter and therefore no classical representation of "neutral" antimatter could be generated since at that time only conjugation from matter to antimatter was the change of sign of the charge [15], which created one of the biggest scientific imbalance in the history because throughout the 20th century matter was studied at all possible levels, from Newtonian mechanics to second quantization, while antimatter was solely studied at the particle level. In essence, the prevalent stand still adopted is that, since Einstein's special and general relativities do not provide a proper description, antimatter does not exist in the universe in any appreciable amount. The sole generally admitted exception is that of man-made antiparticles created in laboratory, since their existence cannot be denied.

The above scientific imbalance was identified, apparently for the first time, by the Italian-American scientist Ruggero Maria Santilli. Santilli has been interested since his graduate studies to ascertain whether a far away galaxy is made up of matter or of antimatter. He soon learned that Newtonian, Galilean and Einsteinian theories had no value for the indicated problem since far away galaxies must be assumed to be neutral, in which case said theories had no distinction whatsoever between matter and antimatter. For this reason, Santilli initiated a long journey that first required the identification of mathematical means for the consistent classical distinction between neutral matter and antimatter prior to any possible physical application. Santilli discovered that a mathematics for the consistent classical treatment of neutral (or charged) antimatter did not exist and had to be built.

The 20th century position on antimatter implied the rather general belief that antimatter galaxies do not exist. This stringent stand eliminated altogether the problem of detecting antimatter asteroids on grounds that they do not exist due to the absence of the antimatter galaxies and related antimatter supernovas needed for their origination. This position was evidently based in the unspoken intent of maintaining the validity of Einstein's theories for all of the universe via the denial of the existence of antimatter galaxies, despite it being disproved by evidence since our Earth has indeed been hit in the past by devastating antimatter asteroids, and similar

asteroids have been detected by various observatories.

In fact, the catastrophic 1908 Tunguska explosion in Siberia with the power of one thousand Hiroshima nuclear bombs can be solely interpreted in a scientific way as being due to an antimatter asteroid annihilating in our atmosphere [15, 16]. This is due to various reasons, such as the complete absence of debris, let alone of a crater, in the ground. The Tunguska explosion excited the entire Earth's atmosphere for days, to such an extent those two days following the explosion; people could read newspapers in Sydney, Australia, at midnight without artificial light; and other reasons. Such a large excitation of the atomic and molecular constituents of our atmosphere can only be scientifically (i.e. quantitatively) represented as being due to huge radiations that, in turn, can only originate from the annihilation of an antimatter asteroid with our matter atmosphere. The widely accepted "interpretation" of the Tunguska explosion as being due to a (matter) comet has no scientific credibility due to the impossibility of such an origination to excite the entire Earth's atmosphere for days, and occur with the absence of debris in the grounds, let alone with the absence of a crater.

NASA has also reported explosions in our upper atmosphere that can only be due to small antimatter asteroids because annihilating at the time of contact with the upper portion of our matter atmosphere. Similarly, astronauts and cosmonauts have observed ashes in our upper atmosphere when on the dark side with respect to our Sun; these ashes can be best interpreted as being due to antimatter cosmic rays that annihilate in our atmosphere, because the only cosmic rays that can reach us at sea level being those due to matter cosmic rays.

As indicated above as well as earlier by Santilli and others, the existence of antimatter stars and galaxies is imperative and should not be ignored. As a representative example out of many, that recall antimatter is thought to exist in the Oort cloud in view of a possible explanation for gamma ray bursts. In fact, these phenomena can be explained by the annihilation of matter and antimatter asteroids or small comets. The explosion would create powerful gamma ray bursts and accelerate matter [17].

Besides antimatter asteroids, it is possible that Earth has been hit in the past by antimatter comets as indicated by the old observations, since the biblical times, not only of excessive brilliance but also of trajectories in our atmosphere that cannot be interpreted as being due to matter comets, e.g., because of slow penetration of the said objects in our atmosphere. In conclusion, the evidence on the existence of antimatter asteroids as well as of antimatter comets and their possibility of hitting Earth again is sufficiently serious [18].

Scientific studies in the detection of antimatter asteroids requires mathematical and physical theories suitable for the classical treatment of neutral antimatter evidently because antimatter asteroids are too large to be treated via operator theories and they must be assumed as being neutral since they are isolated in space. Santilli has repeatedly stated in his writings that: A protracted lack of solution of physical problems is generally due to the use of insufficient or inadequate mathematics [15]. Additionally, Santilli stated that: There cannot exist a really new physical theory without a really new mathematics, and there cannot exist a really new mathematics without new numbers [loc. cit.]. For this reason, Santilli had spent decades in purely mathematical research, firstly, to identify new numbers and, secondly, to develop new mathematics that would allow a classical treatment of neutral or charged antimatter, because the entire body of applied mathematics is built on numbers.

Along these lines, the most fundamental and very first paper published by Santilli on antimatter is Ref. [19] of 1993 that introduced for the first time new numbers called "isodual" where the prefix "iso" was introduced in the Greek sense of indicating the preservation of conventional axioms used for matter and the term "dual" stands to indicate the map from matter to antimatter. The role of Santilli isodual numbers is such that his entire theory of antimatter that is called "isodual" precisely because of the main character of the new basic numbers.

It should be noted that Santilli discovered the new isodual number in Ref. [19] as a particular case of much more general numbers he called "isonumbers" and "genonumbers" and their isoduals.

Following the discovery of new numbers, Santilli constructed in Ref. [20] also of 1993 the isodualities of the Euclidean and Minkowski spaces which were evidently needed for any possible physical applications. He then proceeded in Ref. [21] to the construction of the isodual image of Lie's theory because it is evidently necessary for the construction of basic symmetries for antimatter, viz. the isodual images of the rotation, Galileo and Lorentz symmetries.

Finally, in the mathematical memoir [22] Santilli made a second fundamental mathematical discovery, a new formulation of the ordinary differential calculus that resulted in being crucial for the achievement of the first known formulation of Newton's equations for neutral or charged antiparticles. The complete formulation of the novel isodual mathematics was first presented by Santilli in monograph [8] of 1994 and then updated in monographs [15] of 2001.

Following the achievement of structural consistency of the new isodual mathematics, and only thereafter, Santilli initiated his physical studies with paper [23] of 1993 written on his original aim of the 1960s as a graduate student, how to detect possible antimatter stars and galaxies.

It is evident that the problem of detecting possible antimatter asteroids is of such a magnitude that it cannot be left unaddressed just to maintain the validity of Einstein's theories for antimatter. In an event North America is hit by an antimatter asteroid even with the size of a football, all North American communications will be disrupted, while the Military will be inoperative for days, due to extreme radiations absorbed and re-emitted by Earth's atmosphere. The same holds in the unfortunate event an antimatter asteroids hits India, Russia, China or other regions. Consequently, the problem of possible antimatter asteroids requires attention not only by the people at large, but also by the scientific and

military communities.

The multitude of open problems created by the detection of antimatter have been studied for decades by Santilli who has provided scientific arguments establishing that the threat to Earth caused by antimatter asteroids is more serious than what popularly believed in contemporary academia.

Therefore, Santilli had to confront the problem of identifying a mathematical conjugation (also called map or duality) capable of performing the transition from matter to antimatter at the purely classical Newtonian level, irrespectively of whether matter and antimatter are neutral or charged, under the condition that such a map recovers charge conjugation at the quantum level for the sake of consistency, which is evidently needed.

Following a decade of unpublished trials and errors, Santilli selected the following main assumption for the construction of the needed new mathematics. Recall that the conventional charge conjugation is defined on a Hilbert space H with states $\psi(r)$ over the field of complex numbers C and can be characterized by a conjugation of the type applied to the quantum representation of matter

$$C\,\psi(r) = -\,\psi^{\dagger}(r); \qquad (2)$$

where r is the coordinate of the Euclidean representation space.

Consequently, Santilli introduced an anti-Hermitean conjugation called isoduality and denoted with the upper index d that, by central condition, has to be applied to all physical quantities, to all their units and to all their operations, and can be written as,

$$Q(t, r, v,\ldots) \rightarrow Q^{d}(t^{d}, r^{d}, v^{d}\ldots) = -\,Q^{\dagger}(-t^{\dagger}, -r^{\dagger}, -v^{\dagger},\ldots) \quad (3)$$

where Q denotes a generic quantity depending on time t, coordinates r, velocities v, and any other needed variables [15].

For the trivial case of real numbers, isoduality reduces to the mere change of the sign of all quantities, all their units and the related operations. In the event a given body is charged, the isoduality evidently also applies by changing the sign of the charge. Hence, Santilli's isoduality applies irrespective of whether the body is charged or not.

The main difference between conventional charge conjugation and Santilli's isoduality is that the former solely applies at the quantum level, while the latter applies, by central conception, at the classical Newtonian level, as well as at all subsequent levels of study, including the quantum level in which charge conjugation and isoduality are equivalent [15].

In terms as simple as permitted by the advanced nature of the topic, the classical isodual map from neutral matter to the corresponding neutral antimatter requires that all physical quantities change their sign. Consequentially, under the isodual map, time, energy, linear momentum, entropy, and other positive-definite physical quantities, become negative. There have been a number of proposals that antimatter must move backward in time as a condition to admit annihilation into light. This rather natural assumption has been dismissed due to the violation of causality when motion backward in time is treated with the mathematics used for matter [15].

However, for Santilli's isodual mathematics, antimatter's motion backward in time $t^{d} = -t < 0$, when referred to a negative unit of time (e.g., $s^{d} = -1$ s), is as causal as matter moving forward in our time $t > 0$ when referred to the usual positive units of time (e.g., +1 s) [19].

Similarly, it is known since Dirac's time that negative energies also violate causality laws. However, Santilli has shown that negative-definite isodual energies $E^{d} = -E < 0$ referred to negative units (e.g., $erg^{d} = -1$ erg) are as physical as conventional positive energies $E > 0$ referred to positive units (e.g., +1 erg). Inconsistencies emerge only under crossovers of the two worlds, such as when positive energies are measured with negative units and vice versa [19].

In going deeper into the problem, Santilli discovered that the correct formulation of antimatter requires an entire new mathematics, today known as Santilli's isodual mathematics, which can be defined as the anti-Hermitean image of the entire mathematics used for matter, thus including isodual numbers, isodual functional analysis, isodual differential calculus, isodual Lie's theory, etc. In fact, any mix-up, even minute, of the mathematics for matter and that for antimatter leads to catastrophic inconsistencies that are generally not realized by non-experts in the field [19].

Santilli's research in antimatter was delayed for years by the classification of all "numbers" (namely, sets verifying the axioms of a numeric field) into real, complex and quaternion numbers that had been achieved by historic masters such as Gauss, Cayley, Hamilton and others. Finally, in 1993 Santilli re-inspected this historical classification and discovered that the axioms of a numeric field do not require the basic multiplicative unit to be necessarily positive definite, since said unit can be negative definite as well, provided that the conventional associative product of numbers n x m is redefined in such a way to admit the newly assumed negative unit [19].

We have in this way Santilli's isodual fields $F^{d}(n^{d}, \times^{d}, 1^{d})$ consisting of isodual real, isodual complex and isodual quaternion numbers with negative-definite isodual multiplicative unit and related isodual (associative) multiplication [19]

$$F^{d}(n^{d}, \times^{d}, 1^{d}) : n^{d} = -n^{\dagger},\ n^{d} \times^{d} m^{d} = n^{d} \times (1^{d})^{-1} \times m^{d},\ 1^{d} = -1^{\dagger}, \quad (4)$$

under which 1^{d} is indeed the basic multiplicative unit at all levels (the additive unit 0 of a field remains evidently unchanged under isoduality because $0^{d} \equiv 0$ [19]),

$$1^{d} \times^{d} n^{d} \equiv n^{d} \times^{d} 1^{d} \equiv n^{d} \in F^{d}, \qquad (5)$$

Note, to prevent insidious misinterpretations, that is for the evident mathematical consistency, all real numerical values in isodual mathematics must be elements of Santilli's isodual fields and, as such, must be given by ordinary numbers multiplied by the isodual unit.

Consequently, Santilli undertook to the construction of

isodual functional analysis and isodual metric spaces, such as the Euclid-Santilli isodual space with line element [20]

$$E^d(r^d, \delta^d, I^d): r^d = r \times I^d, \delta^d = Diag.(-1, -1, -1) = \delta \times I^d, I^d = Diag.(-1, -1, -1), \quad (6)$$

$$r^{d2d} = (r^{di} \times^d \delta^d_{ij} \times^d r^{dj}) \times I^d \equiv r^2 = (r^i \times \delta_{ij} \times r^j) \times I^d, \quad (7)$$

where one should note the necessity of multiplying the line element by the isodual unit as a condition to be an isodual scalar, that is, an element of the isodual field. It then follows that the line element of the Euclid-Santilli isodual space coincides with that of the conventional Euclidean space, Eq. (7), and this explains the reason for the lack of detection of the isodual spaces for centuries [20].

Note that the study of the isodualities of the Euclidean space was a necessary pre-requisite to reach the yet unknown (in 1993) formulation of Newton's equation for neutral or charged antimatter [20].

Additionally, Santilli constructed the isoduality of Lie's theories, Minkowskian and Riemannian geometries and of virtually all mathematics used for the study of matter [21].

Thanks to his keen self-criticism, the emerging new mathematics continued to have hidden inconsistencies whose solution required additional years of study. Finally, in 1995, Santilli had the courage to re-inspect another pillar of 20th century applied mathematics, the ordinary differential calculus, by discovering that, contrary to popular belief since Newton's time, the differential calculus does indeed depend on the assumed basic unit and related field. We reach in this way the discovery of Santilli's isodual differential calculus with isodual differentials and isodual derivatives that, for the case of a real-valued new unit acquires the simple form [22]

$$d^d r^d = 1^d dr^d = 1^d d(r1^d) \equiv dr, \quad (8)$$

$$\delta^d F^d(r^d) /^d \delta^d r^d = -\delta F/\delta r, \quad (9)$$

where one can see that (again for the case of a real-valued new unit) Santilli's isodual differential coincides with the conventional differential, and this explains the reason the isodual differential calculus remained unidentified for centuries.

Following the discovery of the isodual differential calculus, Santilli completed the construction of the isodual mathematics with a rigorous structural consistency, and passed only thereafter to physical applications.

A fundamental prediction of the isodual theory of antimatter is that light emitted by antimatter (antimatter light) is physically different than light emitted by matter (matter light) in a number of experimentally verifiable ways [9]. This important prediction was presented by Santilli at the International Conference on Antimatter held in Sepino, Italy, on June 1996.

Recall that light has no charge. But the isodual theory has been constructed to provide a differentiation between neutral matter and antimatter. Therefore, the physical distinction between matter and antimatter light is a direct and unavoidable consequence of the physical distinction between neutral matter and antimatter, as reflected in the distinction between their representations.

Besides, the physical differentiations at advanced level we cannot review here, the most visible difference is the prediction by the isodual theory of antimatter that antimatter light is repelled by a matter gravitational field (see Figure 9).

The simplest way to illustrate this prediction is that at the primitive Newtonian level since all subsequent levels of study are merely consequential. Let us recall that Santilli has formulated Newtonian gravitation in a truly "universal" way via the "identical" representation of the historical equation in terms of "energy," rather than mass [8, 24].

Following the study of a number of alternatives, Santilli gave priority to the search for new numbers since all mathematics used for physics must be based on a numeric field as a condition for experimental verifications and, in any case, all aspects of applied mathematics can be built on a given numeric field via simple compatibility arguments. In 1993, Santilli [19, 25] finally identified the desired new number under the name of isodual real, complex and quaternionic numbers [17], which verify the condition of being anti-isomorphic to the conventional real, complex and quaternionic numbers, respectively. The word "isodual" was suggested to indicate a duality under the preservation of the conventional abstract axioms of numeric fields. The crucial condition of anti-isomorphism was achieved via the anti-Hermitean conjugation of all elements of a numeric field and all its operations. This implies that, given a field $F(n,\times,1)$ with elements $n, m, \ldots$, conventional associative product $n \times m = nm$ and trivial unit 1, Santilli isodual fields are indicated with the upper symbols d, $F^d(n^d, \times^d, 1^d)$, and are characterized by a negative basic unit $1^d = -1^\dagger = -1$, isodual numbers $n^d = n1^d$ and isodual product $n^d \times^d m^d = n^d(1/1^d)m^d = nm1^d$.

Following the identification of the desired numbers, Santilli passed to the systematic construction of the isodual image of all main mathematics used for the study of matter, including functional analysis, differential calculus, metric spaces, Lie algebras, symmetries, Euclidean, Minkowskian and Riemannian geometries, etc. These isodual formulations were first presented in the mathematical memoir [22] and first treated systematically in monographs [8]. The resulting mathematics is today known as Santilli isodual mathematics. It may be of some value to indicate that isoduality is a new transformation not reducible to parity and/or other conventional transformations. We should also recall the new symmetry identified by the isodual mathematics, called isoselfduality [8, 22], namely, the invariance under the isodual transformation, which is verified by the imaginary number $i \equiv i^d$ as well as by Dirac's equation.

Contrary to a possible perception of mathematical complexities, the isodual mathematics needed for applications can be constructed via the application of the simple anti-Hermitean map $Q \to Q^d = -Q^\dagger$, provided it is applied to the totality of quantities and to the totality of their operations

used for the treatment of matter. Readers should be alerted that, in the absence of even one isodual map, there are inconsistencies that generally remain undetected to non-experts in the field [22].

3. Santilli's Isodual Physics

3.1. Apparent Lack of Visibility of Antimatter Asteroids with Sun Light

Santilli has achieved a representation of antimatter at all possible levels, from Newtonian mechanics to second quantization and for conditions of increasing complexity, from fully conservative conditions to the most general possible irreversible non-Hamiltonian conditions, as well as hyperstructural conditions expected in possible antimatter living structures. These studies are far from trivial and have direct implications for the very safety of our planet, since they predict that antimatter asteroids are not visible with the light of our matter Sun. In fact, the studies predict that light emitted by a matter star annihilates when hitting an antimatter body without any refraction. Alternatively, the studies predict that light emitted by an antimatter star, called by Santilli isodual light, annihilates when hitting matter, thus not reaching us on Earth due to annihilation in the upper atmosphere, as it is the case for antimatter cosmic rays. In short, Santilli has initiated an entire new field called antimatter astrophysics whose primary aim is the identification of methods for the detection of antimatter stars, by noting that their isodual light is expected to annihilate even in lenses of telescopes orbiting in space, thus requiring a basically new conception of antimatter telescopes.

It should be noted that, Einstein special and general relativity have no means for differentiating between neutral matter and antimatter as expected for asteroids and stars. As a consequence, antimatter has been assumed as being nonexistent in the universe in any appreciable amount. Santilli's discoveries indicates that antimatter has not been detected because of the above indicated occurrences, namely, the annihilation of our Sun light in an antimatter asteroid, or the annihilation of light from an antimatter star in our atmosphere or in orbiting telescopes [26].

3.2. Newton-Santilli Isodual Equation for Antimatter

No consistent classical theory of antimatter existed prior to Santilli's research, to our best knowledge as yet. For instance, by resuming the use of the conventional associative multiplication a x b = ab, the celebrated Newton's equation,

$$m \times dv/dt = F(t, r, v, \ldots..) \tag{10}$$

or the celebrated Newton's gravitation law

$$F = g \times m_1 \times m_2/r^2 \tag{11}$$

solely apply for matter, and have no means whatsoever to distinguish between matter and antimatter for the very simple reason that antimatter was inconceivable at Newton's times.

Prior discovery of his isodual mathematics, Santilli developed the isodual theory of antimatter that holds at all levels of study, thus restoring full democracy between matter and antimatter. In essence, in the 20th century antimatter was empirically treated by merely changing the sign of the charge, under the tacit assumption that antimatter exists in the same space as that for matter. Thus, both matter and antimatter were studied with respect to the same numbers, fields, spaces, etc. However, a correct classical representation of antimatter required a mathematics that is antiisomorphic to that used for matter as a necessary condition to admit a charge conjugated operator image.

Santilli represents antimatter via his anti-Hermitean isodual map that must be applied to the totality of quantities used for matter and all their operations. Hence, under isoduality, we have not only the change of the sign of the charge, but also the isodual conjugation of all remaining physical quantities (such as coordinates, momenta, energy, spin, etc.) and all their operations. This is the crucial feature that allows Santilli to achieve a consistent representation of antimatter also for neutral bodies.

We have in this way the Newton-Santilli isodual equation for antiparticles that we write in the simplified form

$$m^d \times^d d^d v^d /^d d^d t^d = F^d(t^d, r^d, v^d, \ldots..) \tag{12}$$

where "d" denotes isodual map, and the same conjugation holds for gravitation too.(see below).

Note that, after working out all isodual maps, antiparticle equation (12) merely yields minus the value of the conventional equation for particles in both the l.h.s. and the r.h.s, thus appearing to be trivial. However, a most important feature of the above equation is that it defines antiparticles in a new space, the Euclid-Santilli isodual space, which is coexistent but different than our own space. The Euclidean space and its isodual then form a two-valued hyperspace. In this section we describe how Santilli showed that, starting from the fundamental equation (12), the isodual theory of antimatter is consistent at all subsequent levels, including quantization and at that level it is equivalent to charge conjugation.

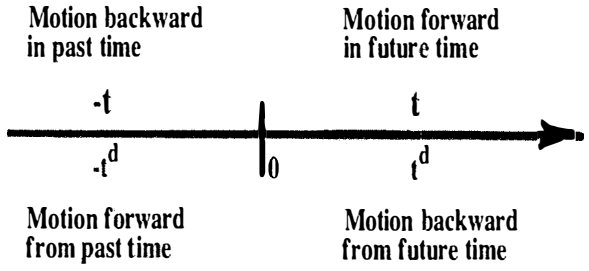


Figure 1. *Contrary to popular beliefs, time has four directions as depicted by Santilli in this figure to illustrate the need for isoduality. In fact, time reversal can only allow the representation of two time directions. The remaining two time directions can solely be represented via the isodual map [26].*

According to Santilli the isodual antiparticles have a negative energy. This feature is dismissed by superficial

inspections as being nonphysical, thus venturing judgments prior to the acquisition of technical knowledge. In fact, negative energies are indeed nonphysical, only when referred to our space time, that is, with respect to positive unit of time. By contrast, when referred to negative unit of time, all known objections on negative energies become inapplicable, let alone resolved.

Note also that isodual antiparticles move backward in time. This view was originally suggested by Stueckelberger in the early 1900s, and then adopted by various physicists, such as Feynman, but dismissed because of causality problems when treated with our own positive unit of time. Santilli has shown that the motion backward in time referred to a negative unit of time $t^d = - t$ is as causal as motion forward in time referred to a positive unit of time t, and this illustrates the nontriviality of the isodual map.

Moreover, the assumption that particles and antiparticles have opposing directions of time is the only one known aspect giving hopes for the understanding of the process of annihilation of particles and their antiparticles, a mechanism utterly incomprehensible for the 20th century physics [26].

3.3. Isodual Representation of the Coulomb Force

Santilli's assertion is that the isodual theory of antimatter verifies all classical experimental evidence on antimatter because it recovers the Coulomb law in a quite elementary way. Consider the case of two particles with the same negative charge and Coulomb Law

$$F = (-q_1) \times (-q_2) / (r \times r) \tag{13}$$

where the positive value of the r.h.s is assumed as representing repulsion, and the constant is assumed to have the value 1 for simplicity.

Under isoduality, the above expression becomes

$$F^d = (-q_1)^d \times^d (-q_2)^d /^d (r^d \times^d r^d) \tag{14}$$

thus reversing the sign of the equation for matter, $F^d = - F$. However, antimatter is referred to a negative unit of the force, charge, coordinates, etc. Hence, a positive value of the Coulomb force referred to a positive unit representing repulsion is equivalent to a negative value of the Coulomb force referred to a negative unit, and the latter also represents repulsion.

For the case of the electrostatic force between one particle and an antiparticle, the Coulomb law must be projected either in the space of matter

$$F = (-q_1) \times (-q_2)^d / (r \times r) \tag{15}$$

representing attraction, or in that of antimatter

$$F = (-q_1)^d \times^d (-q_2) /^d (r^d \times^d r^d) \tag{16}$$

in which case, again, we have attraction, thus representing classical experimental data on antimatter [26].

3.4. Hamilton-Santilli Isodual Mechanics

To proceed in his reconstruction of full democracy in the treatment of matter and antimatter, Santilli had constructed the isodual image of Hamiltonian mechanics because it is essential for all subsequent steps. In this way he reached what is today called the Hamilton-Santilli isodual mechanics based on the isodual equations

$$d^d r^d /^d d^d t^d = \hat{\partial}^d H^d(r^d, p^d) /^d \hat{\partial}^d p^d,$$
$$d^d p^d /^d d^d t^d = - \hat{\partial}^d H^d(r^d, p^d) / \hat{\partial} r \tag{17}$$

and their derivation from the isodual action A^d (a feature crucial for quantization), from which the rest of the Hamilton-Santilli isodual mechanics follows [26].

3.5. Isodual Special and General Relativities

The special and general relativities are basically unable to provide a consistent classical treatment of antimatter. Santilli has resolved this insufficiency by providing a detailed, step by step isodual lifting of both relativities with a mathematically consistent representation of antimatter in agreement with classical experimental data. The reader should be aware that the above liftings required the prior isodual images of the Minkowskian geometry, the Poincare symmetry and the Riemannian geometry, as well as the confirmation of the results with experimental evidence [26].

3.6. Prediction of Antigravity

Studies on antigravity were dismissed and disqualified in the 20th century on grounds that "antigravity is not admitted by Einstein's general relativity." According to Santilli this posture resulted in a serious obscurantism because general relativity cannot represent antimatter, thus being disqualified for any serious statement pertaining to the gravity between matter and antimatter.

With his isodual images of special and general relativity, Santilli has restored a serious scientific process in the field, by admitting quantitative studies for all possibilities, and has shown that once antimatter is properly represented, matter and antimatter must experience antigravity (defined as gravitational repulsion) because of supporting compatible arguments at all levels of study, with no known exclusion. Thereby, all known "objections" against gravitational repulsion between matter and antimatter become inapplicable under Santilli isoduality. As a trivial illustration, in Santilli's isodual theory there have the repulsive Newton-Santilli force between a particle and an isodual particle (antiparticle) both treated in our space

$$F = g \times m_1 \times m^d_2 / r^2 = -g \times m_1 \times m_2 . r^2 \tag{18}$$

which is indeed repulsive. The same conclusion is reached at all levels of study.

Santilli further asserts that a very compelling aspect supporting antigravity between matter and antimatter is his identification of gravity and electromagnetism. In fact, the

electromagnetic origin of exterior gravitation mandates that gravity and electromagnetism must have similar phenomenologies, thus including both attraction and repulsion [26].

3.7. Test of Antigravity

Santilli has proposed an experiment for the final resolution as to whether antiparticles in the gravitational field of Earth experience attraction or repulsion. The experiment consists in the measure of the gravitational force of a beam of positrons in flight on a horizontal vacuum tube 10 m long at the end of which there is a scintillator. Then, the displacement due to gravity is visible to the naked eye under a sufficiently low energy (in the range of the 10^{-3} eV). The experiment was studied by the experimentalist Mills and shown to be feasible with current technologies and resolutory [12, 26].

3.8. Isodual Quantum Mechanics

Next, Santilli constructed a step-by-step image of quantum mechanics under his isodual map based on the Heisenberg-Santilli isodual time evolution for an observable Q

$$i^d x^d d^d Q^d / d^d t^d = [Q, H]^d = H^d x^d Q^d - Q^d x^d H^d \quad (19)$$

and related isodual canonical commutation rules, Schrodinger-Santilli isodual equations, etc.

He then proved that, at the operator level, isoduality is equivalent to charge conjugation. Consequently, the isodual theory of antimatter verifies all experimental data at the operator level too. Nevertheless, there are substantial differences in treatment, such as:

1) Quantum mechanics represents antiparticles in the same space of particles, while under isoduality particles and antiparticles exist in different yet coexisting spaces;

2) Quantum mechanics represents antiparticles with positive energy referred to a positive unit, while isodual antiparticles have negative energies referred to a negative unit;

3) Quantum mechanics represents antiparticles as moving forward in time with respect to our positive time unit, while isodual antiparticles move backward in time referred to a negative unit of time [26].

3.9. Santilli's Comparative Test of the Gravity of Electrons and Positrons in a Horizontal Supercooled and Supervacuum Tube: Proposed Experiments on the Gravity of Antimatter

The gravitational repulsion (antigravity) between matter and antimatter was suspected immediately following the discovery of antimatter, although without any possible theoretical treatment due to the absence of a theory capable of representing the gravitational field of neutral antimatter [15]. This insufficiency has been resolved by Santilli's works on antimatter. In fact, the isodual theory of antimatter predicts in a consistent and systematic way at all levels of study, from Newtonian mechanics to the Riemannian geometry, that matter and antimatter must experience gravitational repulsion [6, 15].

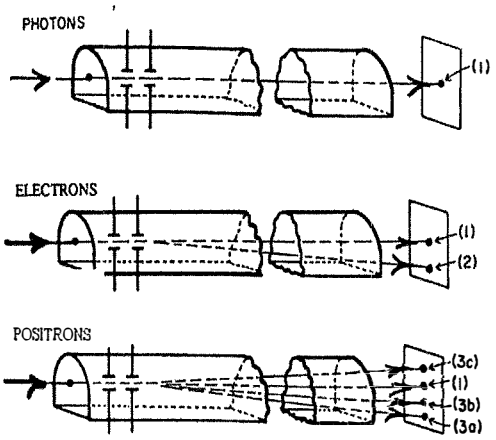


Figure 2. The original illustration used by Santilli for the 1994 proposal to test the gravity of positrons in horizontal light in a vacuum tube. The proposal has been qualified by experimentalists as being technically feasible nowadays and resolutory because the displacement due to gravity on a scintillator at the end of a 10 m light for positrons with milli-eV energy is visible to the naked eye. The usual criticisms based on disturbances caused by stray fields have been disqualified as political for a tube with at least 50 cm diameter. Virtually all major physics laboratories around the world have rejected even the consideration of the test, despite its dramatically lower cost and superior scientific relevance compared to preferred tests, on grounds that "Einstein theories do not admit antigravity," although with documented knowledge that said theories cannot consistently represent antimatter as reviewed in the test [26].

It can conceptually said that antigravity between matter and antimatter is a necessary consequence of the very existence of a "classical" gravitational representation of "neutral" antimatter because, since the charge is null, such a representation requires the sign conjugation of all physical quantities, thus including the sign of the gravitational force and, therefore, of the curvature tensor. On quantitative grounds, we refer to monograph [15] for the gravitational representation of antigravity via the Riemannian geometry for matter and its isodual for antimatter. For this writing it may sufficient to recall the most primitive prediction of antigravity, that in Newtonian mechanics, since all subsequent levels of study are evidently compatible to such a primitive one.

In fact, the Newton-Santilli isodual equation clearly predict gravitational repulsion between matter and antimatter both in our space as well as in the isodual space, according to the respective the laws,

$$F = g \times m_1 m^d_2 / r^2 < 0, \quad (20)$$

$$F^d = g^d x^d m^d_1 x^d m_2 / r^{d2d} > 0, \quad (21)$$

where in our world we have a repulsion because the gravitational force is negative, $F < 0$, and referred to a positive unit of force, while in the isodual world we equally have a repulsion because the gravitational force is positive, $F^d > 0$, but it is referred to a negative unit of force.

Safety of our planet and, consequently, this class of experiments will not be considered herein.

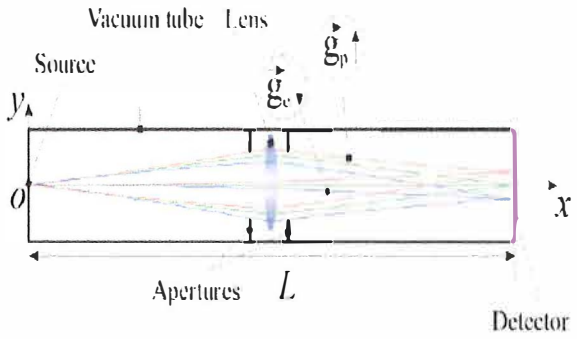


Figure 3. *Principle set-up of Mills's adaptation of Santilli's comparative test of the gravity of electrons and Positrons. :→ shows the gravitational attraction on a collimated beam of electrons that, when having a very low energy of the order of meV, is of the order of 1 cm following a flight of 10 m, thus being visible to the naked eye [12].*

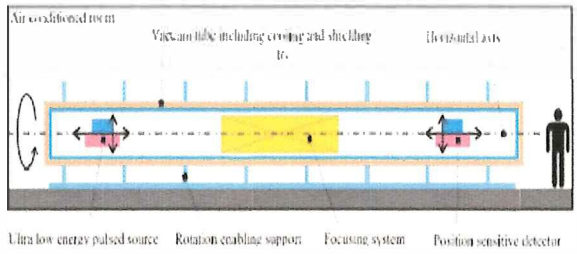


Figure 4. *The possible alternatives for a collimated beam of positrons. Santilli's isodual theory of antimatter predicts gravitational repulsion (antigravity) at all its levels for positrons in a horizontal flight on Earth that, for very low energy of the order of meV, is of the order of 1 cm following a 10 m flight, thus being visible to the naked eye on the scintillator at the end of the tube. For that reason, Santilli's proposed experiment has been stated to be "resolutory" by experimentalists in the field [12, 27]. The lower two renderings are from the technical realization of the test [28] by the R. M. Santilli Foundation on the technical realization of proposal [6] (forth view from the top) and illustration of its size compared to a person [29].*

The first experimental test of the gravity of positrons was formulated by W. E. Fairbanks and E. C. Witteborn at SLAC in 1967 [30] via the use of low energy positrons in vertical upward flight in a vacuum and cooled tube. Regrettably, the experiment could not be completed due to the unavailability at that time of detectors with the extreme sensitivity needed for meaningful measurements. Numerous additional experiments have been proposed to test the gravity of positrons in vertical flights, either upwards or downwards, such as the tests of Refs. [31, 32] and others. However, the gravitational force on particles is notoriously very weak, as a consequence of that the measurements with the most sophistical neutron interferometric or other techniques are expected to remain ambiguous.

Thus, the class of proposed experiments to measure the gravity of positrons in vertical flight cannot possibly be as resolutory as necessary for the

In view of the indicated limitations of testing the gravity of positrons in a vertical flight, Santilli proposed in paper [6] of 1994 the experimental verification or dismissal of the predicted gravitational repulsion between matter and antimatter via measurements of the comparative behavior of very low energy electrons and positrons moving in a 10 m long horizontal supercooled and super-vacuum tube (Figures 2 to 4).

It is evident that Santilli's gravity experiment via positrons in horizontal flight is strikingly better than preceding proposed tests [30-32] via positrons in a vertical flight. While the measurements in the latter tests are expected to remain ambiguous due to the smallness of the effect, in Santilli's experiment [6], for very low energy electrons and positrons of the order of meV in horizontal flight in a 10 m long supercooled and super-vacuum tube, the displacement due to gravity detected on a scintillator at the end of the tube is of the order of 1 cm, thus being visible to the naked eye. The preference of Santilli's test [6] over the tests of Refs. [30-32] is confirmed by a number of experimentalists in the field. For instance, during the International Conference on Antimatter held in Sepino, Italy, in June 1996, the experimentalist A. P. Mills declared Santilli's gravity experiment as being "resolutory" [12] and, therefore, is preferable over the others not equally resolutory experiments. Similarly, during the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes, held at the University of Kathmandu, Nepal, in January 2011, the experimentalist V. de Haan [27] confirmed Mills analysis and also declared Santilli's gravity experiment as being "resolutory".

Besides the above proposed experiments via the use of positrons, the only remaining proposed experiments are those based on anti-hydrogen atoms produced at CERN. Among the latter tests, we have pointed out the test proposed in Ref. [33] by the AEGIS Collaboration outlined in Figures 5 and 6, and the test proposed in Ref. [34] by the ALPHA Collaboration outlines in Figures 5 and 6. By assuming a technical knowledge of these proposed experiments, we here limit ourselves to the following comments.

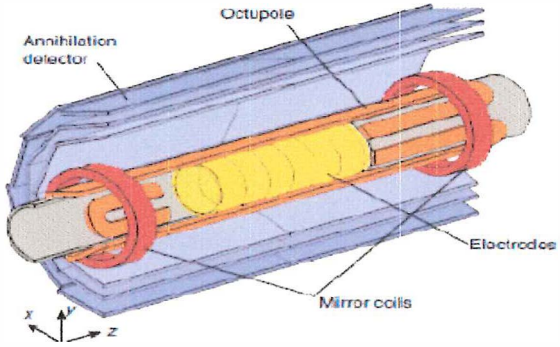


Figure 5. *An illustration from Ref. [34] providing a cut-away diagram of the antihydrogen production and trapping of the ALPHA Collaboration, showing the relative positions of the cryogenically cooled Penning- Malmberg trap electrodes and other features.*

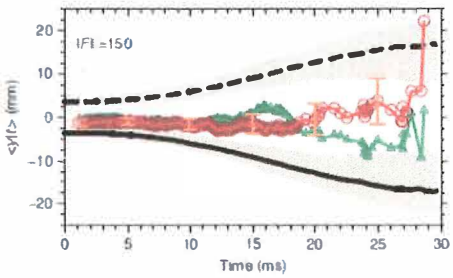


Figure 6. *A second illustration from Ref. [34] on the proposed test of the gravity of antihydrogen atoms via their fall downward when released from the ALPHA antihydrogen trap of the preceding figures. The illustration depicts one of several simulated reverse cumulative average analysis. Compared to the even data to the reverse cumulative average. The green-triangle line is the reverse cumulative average of the x annihilation positions of the event data, and is included as a comparison. The black solid line is the represents 900,000 simulated antihydrogen atoms. The black dashed line mirrors the black-solid line, and is equivalent to a simulation study of antigravity. The grey bands separate the 90% confidence region. Again, the complexity of the apparatus and the high sensitivity of the detectors should be compared to corresponding data of Santilli's gravity test [6, 28].*

To begin, the tests of Refs. [33, 34] have the same ambiguities in measurements as those of the tests with vertically moving positrons [30-32], since the former too deal with extremely small effects requiring extremely sensitive detectors under these conditions. The "experimental results" are inevitably prone to the approximations and/or manipulations that occurred in similar tests.

Besides that, the main problematic aspect of tests of Refs. [33, 34] is the one identified by Santilli [15] according to which, despite a popular beliefs at CERN and elsewhere, the "antiprotons" produced at CERN are not necessarily antiparticles, unless verified as such via annihilation processes, because at least in part, they can be anomalous protonic states created by the embedding of a singlet electron pairs inside ordinary protons. These states are called by Santilli the pseudoproton and denoted with the symbol $\hat{p}^-$.

Consequently, no gravity experiment based on "antihydrogen atoms" produced at CERN can be considered as being resolutory under such a serious ambiguity. Besides the study of antimatter, Santilli has dedicated decades of his research life also to the synthesis of neutrons inside a star according to Rutherford's historical conception that neutrons are synthesized by the "compression" of hydrogen atoms in the core of a star, nowadays represented with reaction

$$p^+ + e^- \rightarrow n + \nu \tag{22}$$

It is well known that the energies needed to achieve the synthesis of the neutron are fully available at CERN. In particular, Santilli has shown that neutrons can also be synthesized in laboratory from a hydrogen gas traverses by a DC arc, thus taking place at energies much smaller than those available at CERN. The experimental information important for the test of the gravity of antimatter obtained by Santilli is that Rutherford's compression is also achievable for an electron pair in singlet coupling (that occurs for valence electron pairs) resulting in the creation of pseudoproton according to the reaction

$$p^+ + (e_\uparrow^- + e_\downarrow^-) \rightarrow \hat{p}^- \tag{23}$$

where $\hat{p}^-$ is predicted to have a mean life essentially similar (if not longer) than that of the neutron due to the similarities of the two syntheses.

As a matter of fact, Santilli has shown that synthesis [28] is more probable than synthesis [26] for various reasons, such as: synthesis [28] does not require the emission of a neutrino for the conservation of the total angular momentum as necessary for synthesis [30], Rutherford's compression of a single electron pair inside the proton is statistically more probable than the compression of the electron due to spin zero of the electron pairs (thus requiring no special proton-pair coupling), compared to the need for a singlet proton-electron coupling for synthesis [26] and other reasons.

It should be stressed that quantum mechanics does not allow a quantitative representation of synthesis [26] because the rest energy of the neutron is bigger than the sum of the rest energies of the proton and the electron, thus requiring a "positive binding energy" which is anathema for quantum mechanics, since in this case the Schrodinger equation no longer admits physically meaningful solutions [8]. Thanks to its non-unitary invariant character, hadronic mechanics has resolved these insufficiencies by achieving, for the first time to our knowledge, a numerically exact representation of "all" characteristics of the neutrons in synthesis [26] at both non-relativistic and relativistic levels [8, 26].

In particular, the use of Santilli's non-unitary invariant methods that have permitted a representation of synthesis [26] when applied to synthesis [28], show that the rest energy of the pseudoproton can be close to that of the antiproton, although expecting of exact numerical values are premature at this time since the sole experimentations to date have been conducted is by Santilli.

Therefore, Santilli stresses that the distinction between the antiproton and the pseudoproton cannot be solely based on their charge and rest energy, their only resolutory distinction being that based on annihilation processes. Needless to say, the antimatter nature of the "antiprotons" claimed at CERN cannot be denied. The point is that the antimatter character has to be proved beyond doubt prior to any true scientific claim. Now, as it is well known, the production of "antiprotons" at CERN is based on hitting a target with the 26 GeV proton beam produced by the old Proton Synchrotron (PS). It is then evident to all that, during the collision of protons with matter target, Santilli synthesis [28] is indeed possible, resulting in the synthesis of the pseudoproton. In fact, at the time of the impact, protons collide first with electrons clouds in general, including precisely the valence electron pairs of synthesis [28]. Once the pseudoproton has been synthesized, its capability to capture a positron in the anti-hydrogen trap is established by quantum mechanical laws, resulting in a neutral state ($\hat{p}^-$; e^+) which is similar to, but not necessarily, the anti-hydrogen atom ($\hat{p}^-$, e^+).

In short, the mathematical, theoretical and experimental

studies illustrate Santilli's main objection against the test of the gravity of antimatter via "antihydrogen atoms" currently produced at CERN because of the lack of clear proof that they are indeed antimatter and the absence of experiments for the resolution of the ambiguities because, being necessarily beyond quantum mechanics, the said experiments are notoriously not even plausible at CERN under current control. In conclusion, both classes of tests of the gravity of antimatter, those based on vertical motion of positrons and those based on the "anti-hydrogen atoms" produced at CERN, are not resolutory on grounds of our current knowledge. Consequently, Santilli's gravity test is and remains the best measurement of the gravity of antimatter since it is the only experiment whose results would be visible to the naked eye [29].

3.10. Experimental Detection of Antimatter Galaxies

The isodual theory of antimatter was born out of Santilli's frustration as a physicist for not being able to ascertain whether a far away star, galaxy or quasar is made up of matter or of antimatter. Santilli has resolved this uneasiness via his isodual photon γ^d namely, photons emitted by antimatter that have a number of distinct, experimentally verifiable differences with respect to photons γ emitted by matter, thus allowing, in due time, experimental studies on the nature of far away astrophysical objects.

$$\gamma^d \neq \gamma \tag{24}$$

A most important difference between photons and their isoduals is that the latter have negative energy, as a result of which, isodual photons emitted by antimatter are predicted to be repelled in the gravitational field of matter. A possibility for the future ascertaining of the character of a far away star or quasar is, therefore, the test via neutron interferometry or other sensitive equipment, whether light from a far away galaxy is attracted or repelled by the gravitational field of Earth [26].

3.11. The New Isoselfdual Invariance of Dirac's Equation

Santilli has released the following statement on the Dirac equation: I never accepted the interpretation of the celebrated Dirac equation as presented in the 20th century literature, namely, as representing an electron, because the (four-dimensional) Dirac's gamma matrices are generally believed to characterize the spin 1/2 of the electron. But Lie's theory does not allow the SU (2)-spin symmetry to admit an irreducible 4-dimensional representation for spin 1/2, and equally prohibits a reducible representation close to the Dirac's gamma matrices. Consequently, Dirac equation cannot represent an electron intended as an elementary particle since elementarily requires the irreducible character of the representation. In the event Dirac's gamma matrices characterize a reducible representation of the SU (2)-spin, Dirac's equation must represent a composite system.

I discovered the isodual theory of antimatter by examining with care Dirac's equation. In this way, I noted that its gamma matrices contain a conventional two-dimensional unit I_{2x2} = Diag. (1, 1), as well as a conjugate negative-definite unit - I_{2x2}. That suggested me to construct a mathematics based on a negative definite unit. The isodual map come from the connection between the conventional Pauli matrices σ_k, k = 1, 2, 3, referred to I_{2x2} and those referred to $-I_{2x2}$. In this way I reached the following interpretation of Dirac's gamma matrices as being the tensorial product of I_{2x2}, σ_k times their isoduals,

$$\{I_{2x2}, \sigma_k, k = 1, 2, 3\} \text{ X } \{I^d_{2x2}, \sigma_k^d, k = 1, 2, 3\} \tag{25}$$

Therefore, I reached the conclusion that the conventional Dirac equation represents the tensorial product of an electron and its isodual, the positron. In particular, there was no need to use the "hole theory" or second quantization to represent antiparticles since the above re-interpretation allows full democracy between particles and antiparticles, thus including the treatment of antiparticles at the classical level, let alone in first quantization.

By continuing to study Dirac's equation without any preconceived notion learned from books, I discovered yet another symmetry I called isoselfduality, occurring when a quantity coincides with its isodual, as it is the case for the imaginary unit $i^d = i$. In fact, Dirac's gamma matrices are isoselfdual,

$$\gamma_\mu{}^d = \gamma_\mu, \ \mu = 0, 1, 2, 3. \tag{26}$$

This new invariance can have vast implications, all the way to cosmology, because the universe itself could be isoselfdual as Dirac's equation, in the event composed of an equal amount of matter and antimatter. In conclusion, Dirac's equation is indeed one of the most important discoveries of the 20th century with such a depth that it could eventually represent features at the particle level that actually hold for the universe as a whole [26].

3.12. Dunning-Davies Thermodynamics for Antimatter

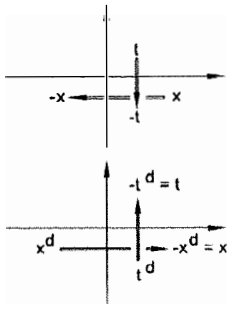


Figure 7. *A schematic view of the additional peculiar property that the projection in our spacetime of the isodual space inversion appears as a time inversion and vice versa. In fact, a point in the isodual spacetime is given by $(x^d, t^d) = (-x, -t)$. The projection in our spacetime of the isodual space inversion $(x^d, t^d) \rightarrow (-x^d, t^d)$ is then given by $(x, -t)$, thus appearing as a time (rather than a space) inversion. Similarly, the projection in our spacetime of the isodual time inversion $(x^d, t^d) \rightarrow (x^d, -t^d)$ appears as $(-x, t)$, that is, as a space (rather than time) inversion. Despite its simplicity, the above occurrence has rather deep implications for all discrete symmetries in particle physics [35].*

As well known, the sole formulation of thermodynamics of the 20th century was for matter. The first consistent formulation of thermodynamics for antimatter has been reached by J. Dunning-Davies with intriguing implications for astrophysics and cosmology yet to be explored, (see the original contribution by Dunning Davies quoted below) [26].

An important contribution to the isodual theory has been made by J. Dunning-Davies [35] who introduced in 1999 the first, and only known consistent thermodynamics for antimatter, here called Dunning-Davies antimatter thermodynamics with intriguing results and implications.

As conventionally done in the field, let us represent heat with Q, internal energy with U, work with W, entropy with S, and absolute temperature with T. Dunning-Davies isodual thermodynamics of antimatter is evidently defined via the isodual quantities

$$Q^d = -Q,\ U^d = -U,\ W^d = -W,\ S^d = -S,\ T^d = -T \quad (27)$$

on isodual spaces over the isodual field of real numbers $R^d = R^d(n^d, +^d, \times^d)$ with isodual unit $I^d = -1$.

It is also seen that isodual differentials are isoselfdual (that is, invariant under isoduality). Dunning-Davies then has the following theorem:

THEOREM [36]: Thermodynamical laws are isoselfdual.

Proof: For the First Law of thermodynamics we have

$$dQ = dU - dW \equiv d^dQ^d = d^dU^d - d^dW^d. \quad (28)$$

Similarly, for the Second Law of thermodynamics we have

$$dQ = T \times dS \equiv d^dQ^d = T^d \times^d S^d, \quad (29)$$

and the same occurs for the remaining laws.

Despite their simplicity, Dunning-Davies results [36] have rather deep implications. First, the identity of thermodynamical laws, by no means, implies the identity of the thermodynamics of matter and antimatter. In fact, in Dunning-Davies isodual thermodynamics the entropy must always decrease in time, since the isodual entropy is always negative and is defined in a space with evolution backward in time with respect to us. However, these features are fully equivalent to the conventional increase of the entropy tacitly referred to positive units.

Also, Dunning-Davies results indicate that antimatter galaxies and quasars cannot be distinguished from matter galaxies and quasars via the use of thermodynamics, evidently because their laws coincide, in a way much similar to the identity of the trajectories of particles and antiparticles of *Lemma: The trajectories under the same magnetic field of a charged particle in Euclidean space and of the corresponding antiparticle in isodual Euclidean space coincide* [8].

This result indicates that the only possibility known at this writing to determine whether far away galaxies and quasars are made up of matter or of antimatter is that via the predicted gravitational repulsion of the light emitted by antimatter called isodual light.

3.13. Isoselfdual Spacetime Machine

A "spacetime machine" is generally referred to a *mathematical* process dealing with a closed loop in the forward spacetime cone, thus requiring motions forward as well as backward in time. As such, the "machine" is not permitted by causality under conventional mathematical treatment, as well known.

Santilli discovered that *isoselfdual matter*, namely, matter composed by particles and their antiparticles such as the positronium, have a null intrinsic time, thus acquiring the time of their environment, namely, evolution forward in time when in a matter field, and motion backward in time when in an antimatter field.

Consequently, Santilli showed that *isoselfdual systems can indeed perform a closed loop in the forward light cone without any violation of causality laws,* because they can move forward when exposed to a matter and then move backward to the original starting point when exposed to antimatter [26].

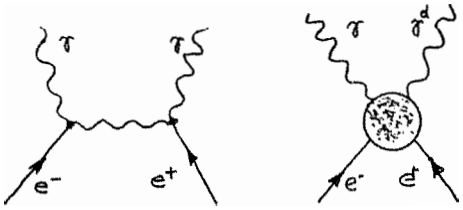


Figure 8. *An illustration of the serious implications of Santilli's isodual theory of antimatter: the need for a revision of the scattering theory of the 20th century due to its violation of the isoselfdual symmetry of Dirac's equation. The diagram in the left illustrates the isoselfduality of the initial particles (an electron and a positron) but its violation in the final particles (two identical photons). The diagram in the right illustrates one of the several needed revisions, the use for final particles of a photon and its isodual as a necessary condition to verify the new isoselfdual symmetry. Additional dramatic revisions are due to the purely action-at-a-distance, potential interactions of the conventional scattering theory (represented with a waving central line in the left diagram), compared to the non-Hamiltonian character of the scattering region caused by deep penetrations of the wave packets of particles (represented with a circle in the right diagram) [26].*

3.14. Original Literature

Santilli's first paper on the isodual theory of antimatter is the one dating to 1994 [37] (following the 1993 paper on isodual numbers).

The first presentations of the classical isodual theory, antigravity, the isodual photon and the isoselfdual spacetime machine appeared in papers [3, 9, 38, and 39]. An independent study by an experimentalist on the feasibility and resolutory character of the proposed measurements of the gravity of positron in horizontal flight on Earth can be found in paper [12].

Comprehensive presentations of the isodual theory of antimatter are available in the monographs [8, 11]. The first formulation of thermodynamics for antimatter was reached by J. Dunning Davies in paper [35, 40].

3.15. Main Features of Santilli's Isodual Theory of Antimatter

Santilli initiated systematic applications of isodual mathematics to the study of antimatter resulting in the new theory today called *isodual theory of antimatter (or Santilli's Isodual Physics)* as one of the branches of the broader *hadronic mechanics* [11, 26]. A main feature is that all quantities that are positive (negative) for the study of matter become negative (positive) for the study of antimatter, with the clarification that all positive and negative matter quantities are referred to positive units of measurements for matter, while all negative and positive antimatter quantities are referred to negative units. In particular, antimatter is predicted to have negative energy $E^d = -E$ exactly as conceived by Dirac [41] and evolve along a negative time $t^d = -t$ according to an old attempt to understand annihilation of matter and antimatter. Causality and other physical problems are resolved by the isodual mathematics, since negative quantities are measured in terms of negative units. Hence, antimatter evolving backward in time with respect to negative units of time is as causal as matter evolving forward in time with respect to positive units of time. The same holds for negative energy referred to negative units, and of other negative quantities.

The first known formulation of Newton equation for antiparticles is based on the Newton-Santilli isodual equations, and confirmed their verification of all known experimental data on the classical behavior of antiparticles [22].

A systematic presentation of the isodualities of Euclidean, Minkowskian and Riemannian geometries, Lie theory, rotational, Galilean, Lorentz and Poincare´ symmetries, Galilean and special relativities, and other basic formulations is provided which in particular, presented the first known consistent representation of the gravitational field of an antimatter body via the Riemann-Santilli isodual geometry [8].

New isoselfdual cosmology at the limit of equal amounts of matter and antimatter, in which case all total quantities of the universe, such as total time, total mass, total energy, etc., are identically null to avoid a discontinuity at creation and set up the basis for continuous creation [23].

The light emitted by antimatter, also called isodual light, resulting in a prediction of main character for the detection of antimatter galaxies according to which antimatter light is physically different than matter light in an experimentally verifiable way. Since the photon has no charge, the only possible conjugation is that for all other physical quantities. As a result, antimatter light is predicted to possess negative energy while all other characteristics are opposite to those of matter light. In particular, antimatter light is predicted to be repelled by matter gravity (Fig. 9), thus permitting the conception of experiments, e.g. via neutron interferometry, to verify whether one of the two photons emitted in electron-positron annihilation experiences repulsion in our gravitational field [9, 42-47].

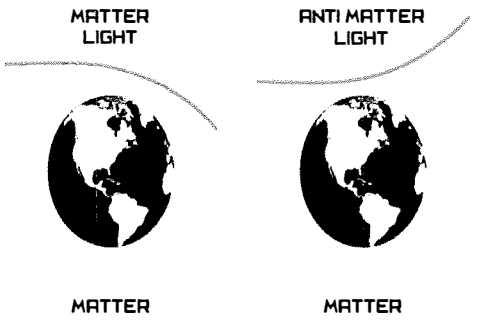


Figure 9. *A view of the repulsion of antimatter light by a matter gravitational field predicted by the isodual theory of antimatter: The repulsion of antimatter light by a matter gravitational field which is a consequence of the classical conjugation of neutral matter into antimatter.*

The first known hypothesis presented that the antimatter light possesses a negative index of refraction $n^d = -n$ when propagating within a transparent matter medium. Again, the consistent characterization of neutral antimatter requires the conjugation of all quantities with no exclusion to avoid catastrophic inconsistencies. This implies the necessary conjugation of the index of refraction into a negative value referred to our positive units of measurements since it is observed in our matter world (Fig. 10) [42-47].

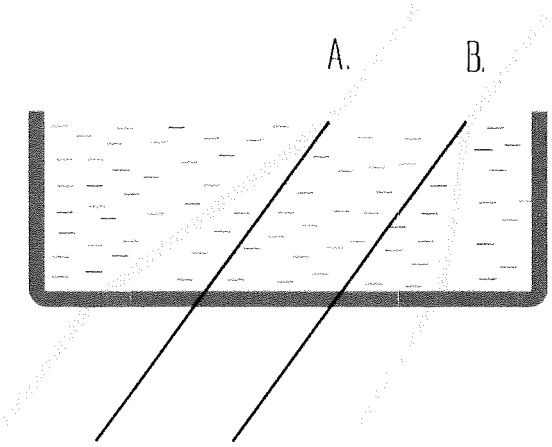


Figure 10. *The prediction of negative index of refraction of antimatter light within matter water: The negative index of refraction of antimatter light which is a consequence of the repulsion of antimatter light from a matter gravitational field.*

An important implication of the isodual theory of antimatter is the clarification that the conventional Dirac equation characterizes the tensorial product of one point--like particle with spin ½ and its antiparticle without any need for second quantization [11]. Santilli could not accept the conventional 20th century view that Dirac's equations represents only one particle with spin ½ because there exists no irreducible or reducible representation of the SU(2)-spin symmetry with the structure of Dirac's gamma matrices. Therefore, the author re-inspected Dirac's equation and showed that $\gamma^k = \sigma^k \times \sigma^{dk}$. And γ^4 = Diag. $(I_{2x2}, -I_{2x2})$ thus yielding the indicated

characterization of a spin ½ particle and its antiparticle.

Dirac himself provided the true foundation of the isodual theory of antimatter by characterizing antiparticles with the negative unit $-I_{2x2}$. Dirac merely missed the mathematics for the consistent physical treatment of negative energies. Note that there is no contradiction for a representation of antiparticle at the quantum mechanical level because the isodual theory of antiparticles applies at the classical level, let alone that of first quantization.

It should be aware that a negative index of refraction implies that antimatter light propagates within a transparent matter medium at superluminal speeds. A conceptual interpretation of this prediction is that the ordinary (positive) index of refraction for matter light propagating within a transparent matter medium is due to various, ultimately attractive interactions that slow down the speed of matter light. By contrast, when antimatter light propagates within a transparent matter medium, for consistency, all features of matter have to be conjugated, resulting in new repulsive interactions between antimatter light and the matter medium that, as such, accelerate antimatter light to superluminal speeds.

4. Application of Santilli's Isodual Theory for Detection of Antimatter Galaxies

During his Ph. D. in physics in the mid 1960s, the Italian American scientist Ruggero Maria Santilli decided to ascertaining whether a far away galaxy was made up of matter or of antimatter and, in this way, initiated a fifty year long scientific journey. As a first step, Santilli proved that none of the 20th century mathematics, physics and optics were applicable for a classical study of antimatter, because the annihilation of matter and antimatter into light (when in contact with each other) requires a conjugation of all physical characteristics in the transition from matter to antimatter. Such a conjugation was absent in all 20th century sciences, since they were specifically built to treat matter. As an example, Einstein special and general relativities were conceived decades before the discovery of antimatter and, therefore, they were unable to represent matter-antimatter annihilation. Also, far away antimatter stars and galaxies have to be assumed as being neutral, thus implying the complete "inapplicability" (and not the "violation") of Einstein theories for the study of antimatter, since said theories only had the sign of the charge for conjugation.

In the early 1980s, Santilli constructed a new mathematics via a conjugation of conventional mathematics that was suitable for the "classical" description of "neutral" (or charged) antimatter bodies, technically known as anti-Hermiticity and called Santilli isoduality. Physical applications of conventional mathematics are based on positive units (such as +1 sec, +1 meter, etc.). In order to conjugate from neutral matter to neutral antimatter, Santilli constructed his new mathematics based on negative units (such as -1 sec, -1 meter, etc.). Since the charge cannot be used for conjugation of neutral bodies, Santilli achieved a consistent representation of antimatter by conjugating all physical characteristics of matter, such as mass, energy, angular momentum, etc. and by conjugating for consistency also their units.

Santilli then spent decades of studies for the construction of the isodual image of the main aspects of 20th century mathematics, including the conjugation of number theory, functional analysis, differential calculus, symmetries, etc. The new mathematics has such a form as to admit negative left and right units at all levels [22]. The resulting new mathematics is today known as Santilli isodual mathematics [11].

Following the achievement of the appropriate new mathematics, Santilli conducted decades of studies on the construction of the corresponding physical theory, today known as Santilli isodual theory of antimatter, which includes the isodual image of all main parts of 20th century physics, including the isodual image of special and general relativities, by achieving in particular the first known consistent classical representation of the gravitational field of neutral (or charged) antimatter bodies. Additionally, Santilli constructed the isodual image of quantum mechanics, namely, an image of quantum mechanics compatible with isodual relativities. As a central part of the above studies, Santilli proved that the isodual theory of antimatter verifies "all" known experimental data on antimatter at both the classical and quantum levels [13].

Following, decades of research for the achievement of the appropriate mathematical and physical treats, Santilli initiated experimental test of his 50 year old dream: ascertain whether a far away star or galaxy is made up of matter or of antimatter. As an invited keynote speaker at the International Conference on Antimatter held in Sepino, province of Isernia, Italy in May 1996, Santilli presented the historical discovery that light emitted by antimatter (called antimatter light) is physically different than light emitted by matter (called matter light) in an experimentally verifiable way [9]. In particular, matter light is attracted by a matter gravitational field, while antimatter light is repelled by a matter field, namely, it experiences gravitational repulsion (Figure 9).

In 2012, at International Conference on Numerical Analysis and Applied Mathematics ICNAAM in Kos, Greece, Santilli presented a second historical discovery according to which, when propagating within a matter transparent medium such as glass, antimatter light has an index of refraction opposite that of matter light (see Ref. [42] and Figure 10). This property was derived as a consequence of the gravitational repulsion of Figure 9.

This second historical discovery established that a conventional Galileo refractive telescope cannot focus images from antimatter stars because its convex lenses, such as a Steinheil achromatic convex doublet, will disperse antimatter light in all directions as shown in Figure 11.

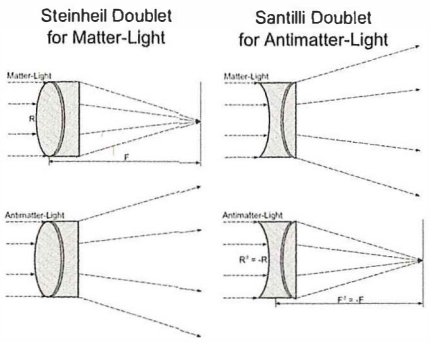


Figure 11. *A view of the lenses used in the Galileo telescope to detect matter galaxies and their conjugate version used in the Santilli telescope to detect antimatter galaxies.*

Consequently, Santilli conceived a conjugated doublet, called Santilli Achromatic Double Concave Doublet (international patent is pending), to focus images caused by antimatter light. Since antimatter light has an index of refraction in glass opposite that of matter light, the curvature of the lenses has to be conjugated from matter light, that is, has to be concave (see also Figures 13 and 14 of next section for details). In this way, Santilli established that none of the available telescopes can focus images of antimatter stars or galaxies because they are all based on the conventional law of refraction and related convex lenses. Consequently, images from far away antimatter stars or galaxies are dispersed in all directions by convex lenses without any focusing. Similarly, concave lenses will disperse in all directions images from mater light but they will converge images from antimatter light. Santilli also proved that we will never see antimatter images with our eyes because our iris is convex, thus dispersing antimatter light all over our retina without any focused view.

In 2012, Santilli constructed the first telescope with concave lenses, today known as Santilli Refractive Telescope or antimatter telescope (see Figure 14 of next section), and conducted systematic views of the night sky in the region of the Vega star, by achieving the first detection in scientific history of antimatter galaxies, antimatter cosmic rays and antimatter asteroids [43]. The above historical discovery has been confirmed twice by independent scientists [44] and [47].

This fifty years of mathematical, theoretical and experimental research of Santilli can provide an answer to his question of the mid 1960s, with the conclusions that: 1) All galaxies we see in the universe with the various available telescopes are solely made up of matter; 2) There exist indeed antimatter galaxies in the universe, but they are solely visible via special telescopes with concave lenses; and 3) We will never be able to focus images of antimatter with our eyes because our iris is convex.

The details of actual detections of far away antimatter galaxies and antimatter cosmic radiations are as given in the following sections.

4.1. Santilli's Refractive Telescope with Concave Lenses or Antimatter Telescope and Experimental Method

Santilli has been constructed a new refracting telescope with "concave" lenses; for detection of antimatter light from distant sources, because a conventional telescope with convex lenses (Galileo telescope) will disperse light with a negative index of refraction. For that Santilli secured the design and fabrication of two identical Galileo refracting telescopes; without the star diagonal viewer to avoid any unnecessary reflection of antimatter light.

Figure 12. *The two identical Galileo telescopes and the camera [43].*

One of the two telescopes converted to a concave version with identical but conjugated foci. The transformation of the telescope from the Galileo form with 100 mm effective convex primary lenses, to the Santilli's antimatter telescope with features identical to those the Galileo one but conjugated based on Santilli's isodual mathematics as described above. Since the camera is directly attached to the telescope without the eyepiece, this conversion essentially consisted in the fabrication and assembly of concave lenses as per the data of Figure 13 and Figure 14 provides a comparative view of the Galileo and the Santilli's antimatter telescope.

He secured one single suitably selected camera (Cannon: model EOS 600D with image sensor of type CMOS, and Bayer Filter) to obtain pictures from both the Galileo and the Santilli telescopes. He also secured a tripod with mount suitable for the parallel housing of the two telescopes. He optically aligned the two telescopes on the tripod by keeping in mind the evident impossibility of doing visual alignments with the antimatter telescope and conducted a number of day views with the so mounted and aligned pair of Galileo and Santilli telescopes to verify that astronomical objects visible in the former are not visible in the latter [43].

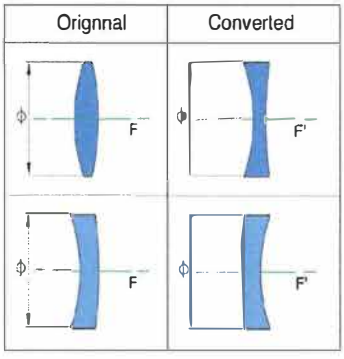


Figure 13. *Main characteristics of the Galileo and antimatter primary lenses [43].*

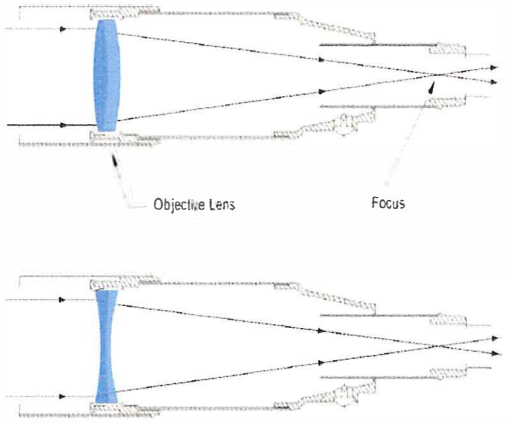


Figure 14. Schematic view of the telescopes with convex and concave lense [43].

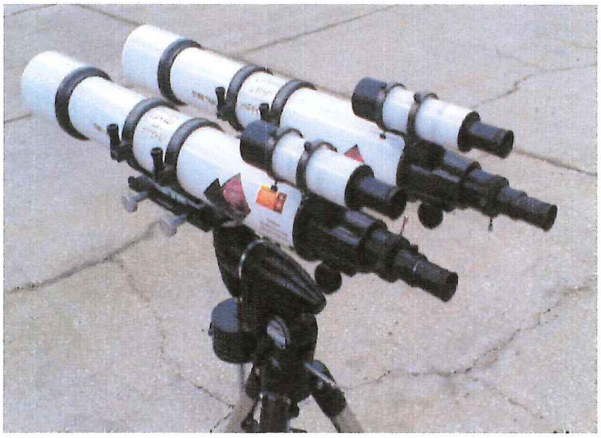


Figure 15. The parallel mount of the Galileo and antimatter telescopes with related finder scopes [43].

Figure 16. A close up view of the mounting of the camera directly in the telescope in place of the eyepieces [43].

A number of night views of the same region of the sky via the so mounted and aligned Galileo and Santilli telescopes was conducted and obtained a number of pictures from both telescopes via the selected camera; and finally conducted a comparative inspection of the pictures from both telescopes under a variety of enlargements and contrasts to see whether the pictures from the antimatter telescope contained focused images absent in the pictures from the Galileo telescope under the same enlargement and contrast.

Following the availability of the so mounted and aligned pair of telescopes, Santilli initiated night views by first confirming that, as expected, any celestial object visibly focused by the Galileo telescope was not focused at all with the antimatter telescope. In particular, the view of details of our Moon, which were very nicely focused by the Galileo telescope, resulted in a diffuse light when seen from the antimatter telescope without any possible identification. The same occurred for planets and nearby matter stars. Then Santilli finally initiated preliminary views of the sky at night with said pair of telescopes. He reported the tests conducted at the Gulf Anclote Park, Holiday, Florida, and GPS Coordinates: Latitude = 28.193, Longitude = - 82.786. The camera was set at the exposure of 15 seconds for the specific intent of having streaks of light from far away matter stars caused by Earth rotation, since streaks can be better identified with the limited capabilities of the available telescopes compared to individual dots of light in the pictures [43]. Additionally, streaks from matter stars have a clear orientation as well as length that are important for the identification of possible streaks from antimatter light. Following various tests, he selected the 10 setting of the camera at ISO 1600 because various tests with smaller and bigger ISO resulted inconclusive and ambiguous for various reasons. All pictures were analyzed (for details refer Ref. No. 43) with particular reference to the identification of the background as well as impurities in the camera sensors that are evidently present in both pictures from the Galileo and the Santilli telescope. The magnification has been obtained by Santilli via the Gimp 2.8 software [43]. In the succeeding sections pictures obtained by this pair of telescopes (Galileo telescope and Santilli telescope); in original (i.e. of Santilli) and two confirmative tests of independent researchers; are shown for the convenience of the readers. The focusing of images of antimatter galaxies, antimatter cosmic radiations etc. via a telescope with concave lenses is the first known experimental indication on the existence of antigenicity because a negative index of refraction is solely possible for the repulsion of antimatter light from matter.

4.2. Original Pictures of Santilli's Apparent Detection of Antimatter Galaxies

Following these preliminaries, Santilli oriented both telescopes at the indicated location and time toward the star Vega, and then specialized the orientation for the pair of matter stars Epsilon Alpha and Epsilon Beta near Vega. Some of the pictures taken by Galileo and Santilli telescopes of matter and antimatter galaxies respectively and of annihilation of antimatter cosmic rays [43] are as shown below.

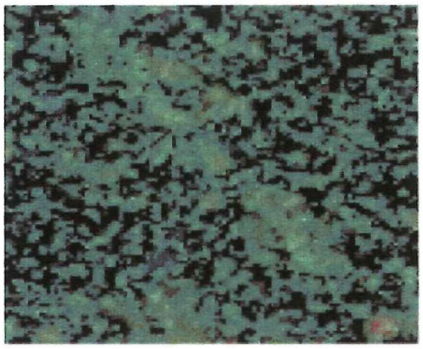


Figure 17. *View of one of the streaks of matter light representing a far away matter star or galaxy identified in the Epsilon Alpha and Beta region of the night sky near Vega via the Galileo telescope [43].*

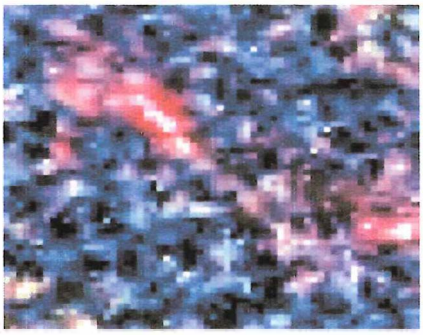


Figure 18. *View of (a) First Streak of light detected in the Epsilon Alpha and Beta region with the Santilli telescope of antimatter galaxies [43].*

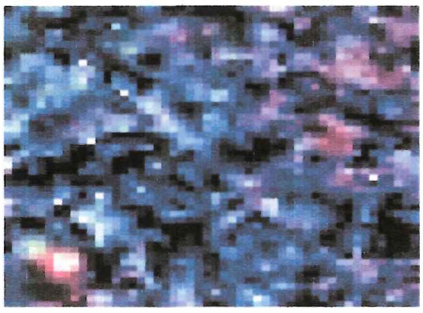


Figure 19. *View of Second streak of light detected in the Epsilon Alpha and Beta region with the Santilli telescope of antimatter galaxies [43].*

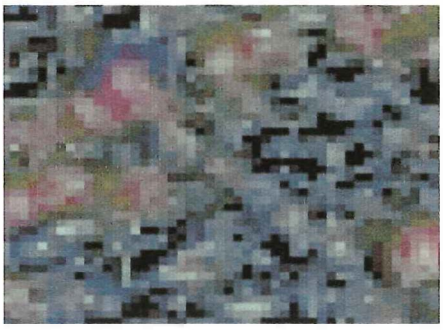


Figure 20. *View of Third streak of light detected in the Epsilon Alpha and Beta region with the Santilli telescope of antimatter galaxies [43].*

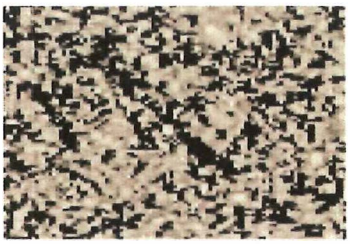


Figure 21. *The first streak of darkness identified in the picture of the Epsilon Alpha and Beta region of the night sky taken with the antimatter telescope providing possible evidence of a far away antimatter star or galaxy as an alternative for the streaks of light [43].*

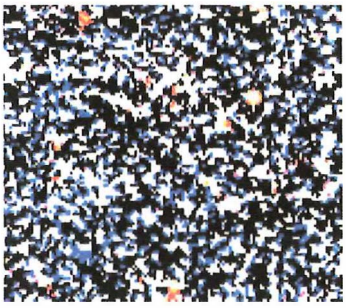


Figure 22. *Another representative streak of darkness present in the antimatter telescope but absent in the Galileo telescope that may constitute an alternative to the streak of light [43].*

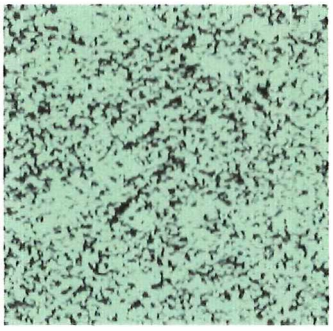


Figure 23. *Seemingly connected streaks of darkness identified in a picture of the Vega region of the night sky taken with the antimatter telescope that could be due to the annihilation of a shower of small antimatter asteroids in our atmosphere, in a way much similar but the conjugate of the frequent view in the night sky of the streaks of light caused by the annihilation of a shower of small matter asteroids in our atmosphere [43].*

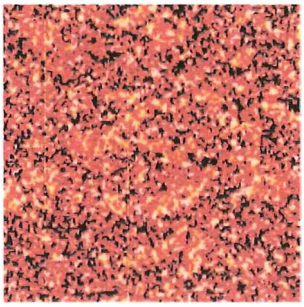


Figure 24. *The first of numerous circular traces identified in a picture of Vega regions of the night sky on November 7, 2013, with the Santilli's antimatter telescope that could be due to the annihilation of an antimatter cosmic ray [43].*

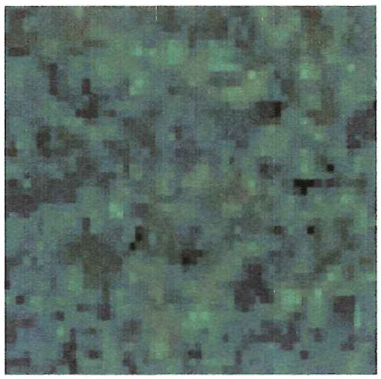


Figure 25. *View of a circular trace identified in a picture of Deneb regions of the night sky with the Santilli's antimatter telescope [43].*

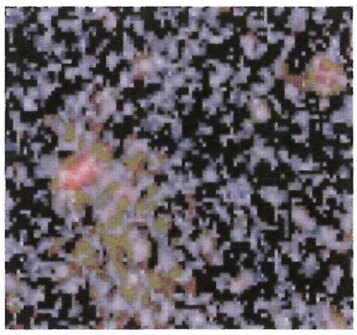


Figure 26. *View of a circular trace identified in a picture of Altair regions of the night sky with the Santilli's antimatter telescope [43].*

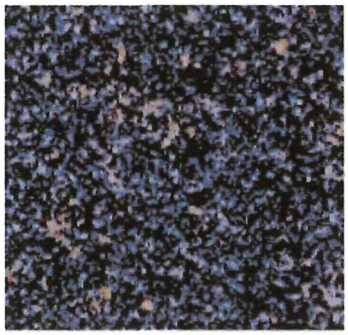


Figure 27. *View of a circular trace identified in a picture of Sadr regions of the night sky With the Santilli's antimatter telescope [43].*

Figure 28. *View of a circular trace identified in a picture of Gienah Cyngi regions of the Night sky with the Santilli's antimatter telescope [43].*

4.3. Images of Antimatter for Preliminary Confirmations of Santilli's Apparent Detection of Antimatter

By using the same pair of Galileo and Santilli telescopes, the same camera, the same exposure of 15 seconds for ISO 1600, a team of scientist [44] went to Sebring, Florida and Enclote Gulf Park in Holiday, Florida and obtained pictures from both telescopes of the same region of the night sky studied by Santilli (that of Epsilon Alpha and Beta stars). Some of the original pictures are available from Ref. [48] in raw and tiff formats under the markings "Galileo-Epsilon-Sebring" and "Santilli-Epsilon-Sebring for readers". Figures 29 to 43 reports selected joint views from the Galileo and the Santilli telescopes showing clearly anomalous streaks that are present in the Santilli telescope but absent in the Galileo telescope, which streaks have essentially the same orientation and length of the streaks caused by matter stars, thus confirming the corresponding anomalous streaks first obtained by Santilli in a telescope with concave lenses i.e. Santilli's antimatter telescope.

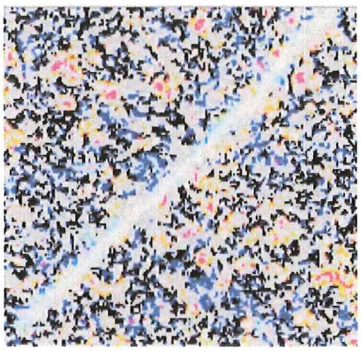


Figure 29. *Picture from the Galileo Telescope of a star in the Epsilon region of the sky from Sebring, Florida [44].*

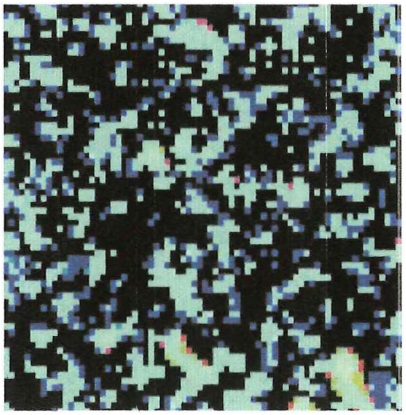


Figure 30. *Picture from the Santilli telescope of a black streak in the Epsilon region of the sky from Sebring, Florida [44].*

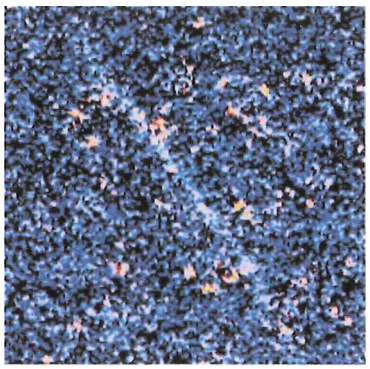


Figure 31. *Picture from the Galileo telescope of a star from the Epsilon region of the sky from Sebring, Florida [44].*

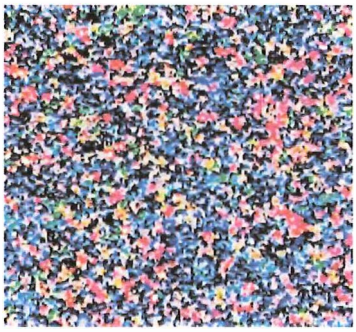


Figure 32. *Picture from the Santilli telescope of a black streak in the Epsilon region of the sky from Sebring, Florida [44].*

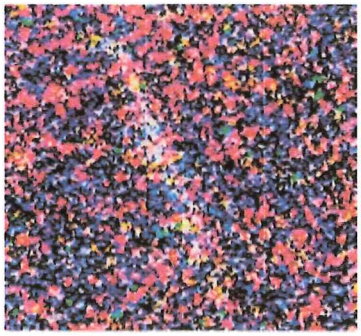


Figure 33. *Picture from the Galileo Telescope of a streak of a matter star in the Vega region of the sky from Holiday, Florida [44].*

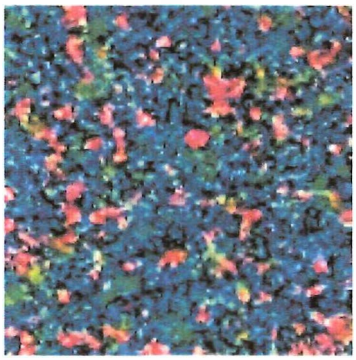


Figure 34. *Picture from the Santilli telescope of a black streak in the Vega region of the sky from Holiday, Florida [44].*

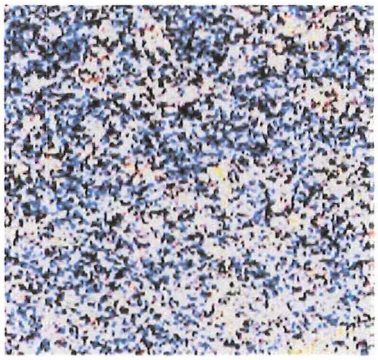


Figure 35. *Picture from the Santilli telescope of another streak in the Vega region of the sky from Holiday, Florida [44].*

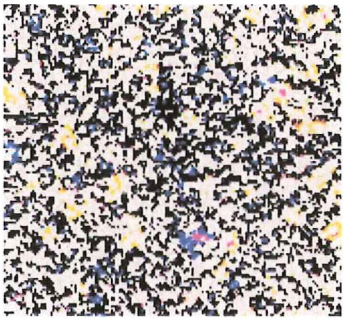


Figure 36. *Picture from the Santilli telescope of an unknown event in the Epsilon sky region from Sebring, Florida [44].*

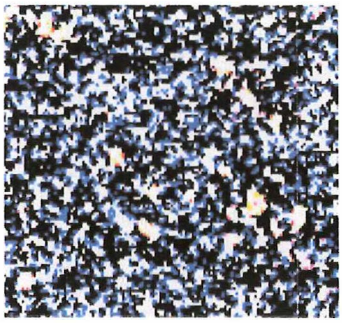


Figure 37. *Picture from the Santilli telescope of a circular trace [44].*

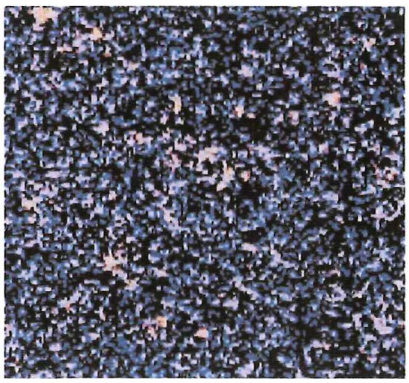


Figure 38. *Picture from the Santilli telescope of another circular trace [44].*

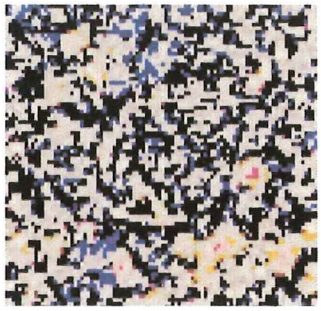


Figure 39. Picture from the Santilli telescope of yet another circular trace [44].

Figure 40. Picture from the Santilli telescope of yet another circular trace [44].

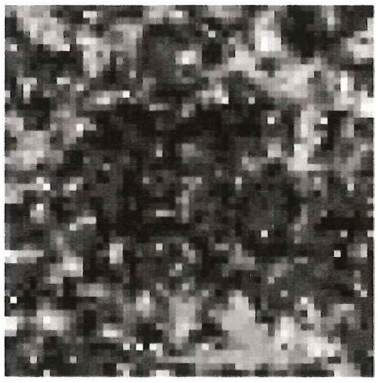


Figure 41. Picture from the Santilli telescope of yet another circular trace [44].

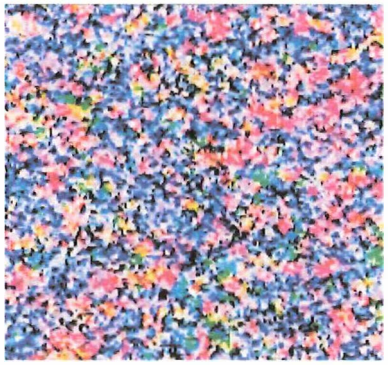


Figure 42. Picture from the Santilli telescope of yet another circular trace [44].

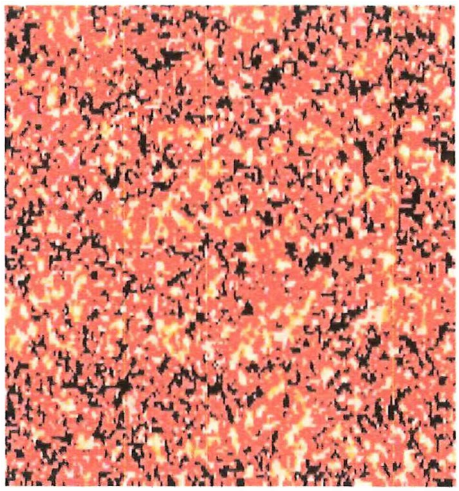
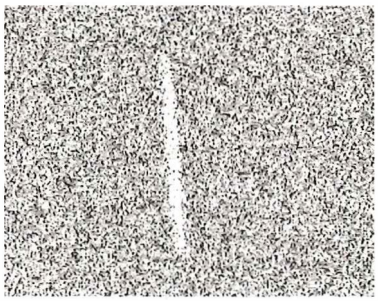


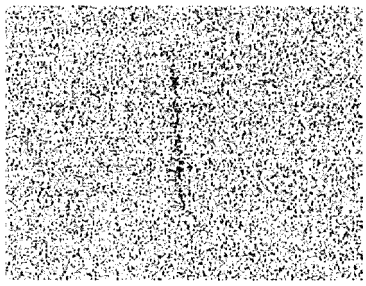
Figure 43. Picture from the Santilli telescope of yet another circular trace [44].

4.4. Pictures of Antimatter for Confirmation of Santilli's Detection of Antimatter Galaxies

Again some researchers [47] went to the same location of the preceding detections [43, 44], Gulf Anclote Park, Holiday, Florida, GPS: 28.193461, -82.786184, with the same pair of parallel Galileo and Santilli telescopes, and inspected the same region of the night sky, but this time via the use of a 35 mm Canon F-lN camera with SLR film, shutter speed B, Fujifilm roll ASA 400, and exposure Compensation 1. These telescopes were oriented toward the Draco and Vega regions of the night sky under a camera exposure of both telescopes for 15 seconds and captured numerous images on 35 mm Fujifilm Provia 400X ISO 400 out of which, for brevity we report in the link of Ref. [49] the following images: Vega-Gal-ISO400-019.tif, Vega-Sant-ISO400-020.tif, Draco-Gal-iso200-004.tif, Draco-Sant-Iso-ISO200-005.tif. The rolls containing all original images were developed by Zebra Color Company, 1763 1st Ave. North, St. Petersburg, FL 33713 (http://zebracolor.com/index.html) and the developed images were scanned at 5760 dpi. The scanned images were enlarged and inspected via the use of paint.net software for PC. These representative images of are listed as below, in Figures 44 to 50.



(a)



(b)

Figure 44. Images from the Vega region of the night sky showing a streak due to matter light from the Galileo telescope (a) and a streak of darkness from the Santilli telescope caused by antimatter light (b) [47].

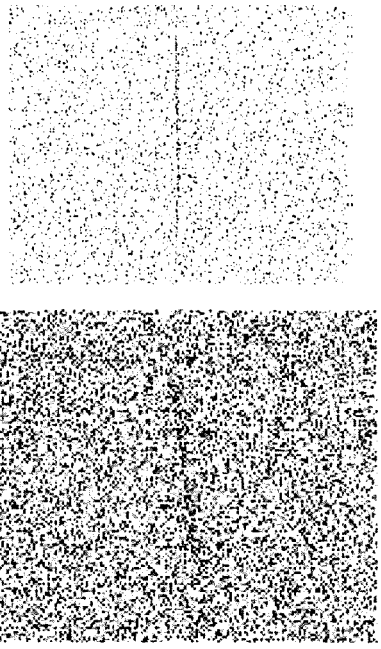
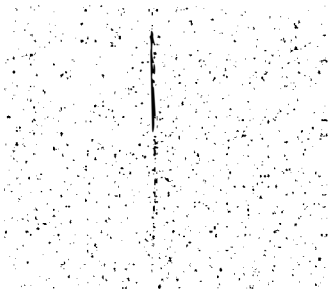


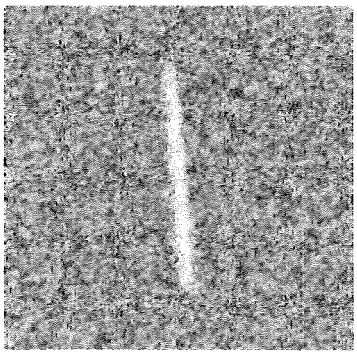
Figure 45. Images from the Vega region of the night sky from the Santilli telescope showing streaks of darkness caused by antimatter light [47].



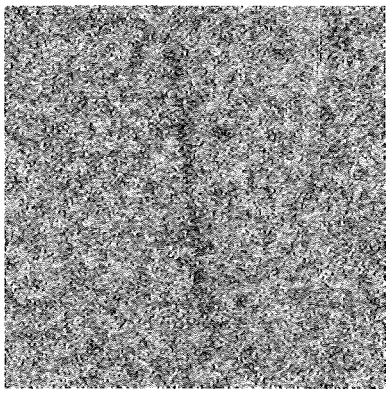
(a)

(b)

Figure 46. Images from the Vega region of the night sky showing a streak of darkness expectedly from a far away antimatter light (a), and a streak of darkness expectedly from a small antimatter asteroid annihilating in our atmosphere (b) [47].



(a)



(b)

Figure 47. Images from the Draco region of the night sky showing a streak caused by matter light from the Galileo telescope (a) and a streak of darkness from the Santilli telescope caused by antimatter light (b) [47].

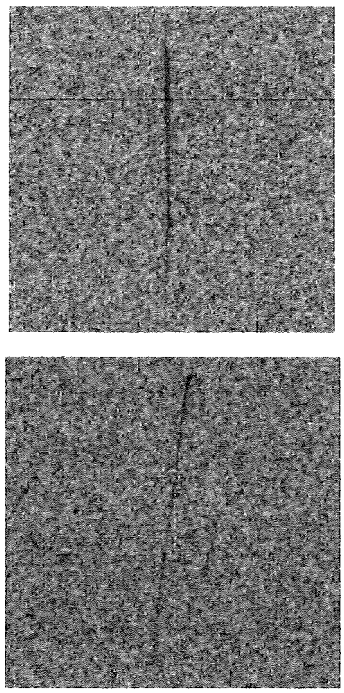
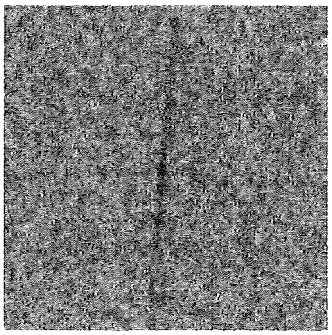
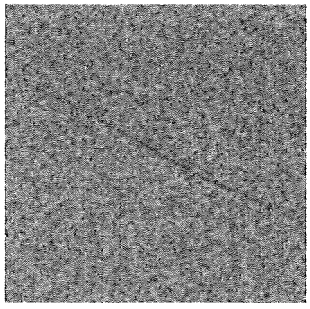


Figure 48. Images from the Draco region of the night sky via the Santilli telescope showing streaks of darkness caused by antimatter light [47].



(a)



(b)

Figure 49. Images from the Draco region of the night sky showing a streak of darkness expectedly from a far away antimatter light (a), and a streak of darkness expectedly from a small antimatter asteroid annihilating in our atmosphere (b) [47].

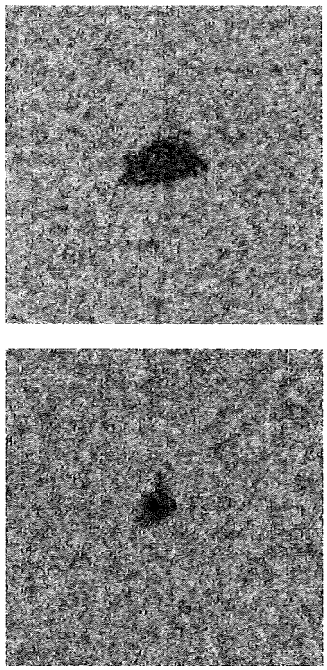


Figure 50. Images from the Draco region of the night sky via the Santilli telescope showing dots expected from the annihilation of antimatter cosmic rays in the upper regions of the atmosphere [47].

Figure 50 indicates anomalous dots (antimatter light produced by antimatter cosmic rays annihilating in the upper region of our atmosphere) present in the Santilli telescope but not in the Galileo telescope [47].

5. Conclusion

Based on the above analysis it is concluded that Santilli's isodual theory does indeed provide a consistent and time invariant, classical and operator description antimatter in a way compatible with available experimental data at both, the classical and quantum levels.

In particular, our analysis confirms that; 1) a consistent classical representation of neural antiparticles in a way compatible with the known quantum description can be achieved via negative-definite physical quantities such as energy, momentum, time, etc., under the consistency condition that they are measured with negative-definite units and 2) the focusing of images by a telescope with concave lenses appears to be the first experimental evidence of antigravity between matter and antimatter because the negative index of refraction for isodual light propagating within a matter medium necessary for said focusing can only be explained via repulsive interactions between the isodual light and the matter field.

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Possible Role of Antimatter Galaxies for the Stability of the Universe

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Abstract: Recent mathematical, theoretical and experimental studies have confirmed via measurements on Earth Zwicky's hypothesis according to which the cosmological redshift is due to galactic light losing energy to intergalactic media without the expansion of the universe. The main problem of the ensuing return to a static universe is the inevitable prediction that the universe should collapse due to gravitational attractions among galaxies. In this paper, we review the historical inability by general relativity to achieve a stable universe solely composed of matter, and present apparently for the first time a cosmological model in which the universe achieves stability under the condition of admitting an equal number of matter and antimatter galaxies at such a large mutual distance for which gravitational interactions are ignorable.

Keywords: Antimatter, Static Universe, Universe Stability

1. Historical Notes

The study of the origin, evolution, and eventual fate of the universe has always been debated by philosophers and cosmologists, beginning with Aristarchus, Aristotle and Ptolemy. In particular, the geocentric theory of Ptolemy has been the dominant paradigm until it was put into question by Copernicus, Kepler, Galilei and Newton. But it was Albert Einstein, who essentially set up the foundations of what we nowadays call modern cosmology [1].

Given the relative very small velocities of stars in the Milky Way, the general belief in the early part of the 20th century was that we were living in a static universe. The main assumptions by Einstein were that there is a system of reference relatively to which matter may be looked upon as being permanently at rest, and that the large scale structure and evolution of such a universe should be determined by its finite matter density.

These assumptions implied a difficulty in the determination of border conditions at infinity, a difficulty that was solved by Einstein with the hypothesis of a closed universe without borders. But to achieve a finite matter density in a static universe, Einstein was forced to introduce the cosmological constant in his field equations for the intent of achieve stability via a repulsive force counter-balancing the gravitational attraction.

Einstein himself didn't like the addition of the cosmological constant in his equations because it looked to him just like an ad-hoc hypothesis that achieved the desired stability but without explaining how the universe works. Numerous additional doubts on the validity of the model were expressed, such as those by Willem de Sitter who suggested Einstein to get rid of the lambda-term in his field equations [2, 3]. In essence, de Sitter argued that the cosmological constant is arbitrary and detracts from the elegance of Einstein's original theory of gravitation.

De sitter also proposed a model that bypassed the problem of border conditions at infinity in which space is basically empty, with zero matter density, zero curvature and no division between time and space (unlike Einstein's model where an absolute time existed), thus achieving isotropy and homogeneity throughout space and time.

This solution was considered quite seriously by Einstein. De Sitter had used lambda to derive an empty model (so no finite density of matter) that actually satisfies boundary conditions at infinity. So the lambda-term went against Einstein's own beliefs (the field should be due to the matter, without which it cannot exist), and he persuaded himself he

needed to get rid of it one way or the other.

With the passing of time, experimental measurements started to become available, with particular reference to the historical measurements by Hubble [4] on the redshift of light coming from distant "nebulas," later on called galaxies, according to the law z = Hd, where z is the redshift, H is the Hubble constant, and d is the distance of the galaxy from Earth. This view was embraced by Eddington [5], Weyl [6, 7], Slipher [8], Friedmann [9, 10], Lemaitre [11] and others.

It appears that, quite likely, Einstein's main desire was the removal of the cosmological constant from his field equations because his main concern was to account for a finite density of matter in the universe, and since it seemed he could do that via other models, he was happy to cast away what he thought was a troublesome addition to his theory [12]. This doesn't mean he approved the expansion and related Big Bang, but he didn't like the use of an arbitrary *ad-hoc* term to obtain equilibrium in a matter-only universe.

Einstein and de Sitter eventually came up with a new model, known as Einstein-de Sitter model [13], that influenced cosmology for the next 50 years. The solution implied a homogeneous and isotropic universe, with zero space curvature, zero cosmological constant, and zero pressure, with asymptotically null expansion (see for more details, e.g., Refs. [13-15].

The aspect most important for this paper is that, to our best understanding of historical profiles, Einstein never explicitly acknowledged the representation of the cosmological redshift via the conjecture of the expansion of the universe. Although he didn't participate actively to the cosmological debate after his 1917 paper, he continued to prefer a stationary universe, as it is clearly demonstrated by an unpublished paper of his [16], written in 1931 and recently rediscovered [17, 18], where, after Hubble's paper, he developed another steady-state model of the universe keeping into account Hubble's results, but, considering a pure matter universe, he apparently faced mathematical inconsistencies. The same position was adopted by Hubble and other cosmologists, such as Fritz Zwicky [19] who proposed the hypothesis that the redshift of galactic light is due to loss of energy by light to inter-galactic gases.

In summary, during the first part of the 20th century, jointly with the initiation of studies on the interpretation of Hubble's law via the expansion of the universe, there were also authoritative views that essentially implied a static conception of the universe, mostly according to Hoyle's cosmology [20]. It is important to note that Einstein himself basically anticipated Hoyle's view in his 1931 unpublished paper.

2. Recent Debates in Cosmology

Even today, after so many years the debate is still open, with the so-called Standard model suffering of multiple flaws, and many scientists looking back at models of the universe that don't need a Big Bang, if not completely static.

On May 22, 2004, an open letter was signed by many scientists and published in New Scientist [21]. In this letter they complained about the many inconsistencies of the Big Bang theory and its corollaries, stating that "*the successes claimed by the Big Bang theory's supporters consist of its ability to retrospectively fit observations with a steadily increasing array of adjustable parameters, just as the old Earth-centered cosmology of Ptolemy needed layer upon layer of epicycles. Yet the big bang is not the only framework available for understanding the history of the universe. Plasma cosmology and the steady-state model both hypothesize an evolving universe without beginning or end. These and other alternative approaches can also explain the basic phenomena of the cosmos.*"

Since publication, the letter has been signed by more than 500 researchers worldwide.

A group of those researchers created an association called "Alternative Cosmology Group" [22], that, as stated in their website, "*is an open society of scientists from all over the world, dedicated to the advance in cosmology and basic research*". In this site it's possible to find a wide variety of papers and articles presenting both observational results and theoretical research suggesting an alternative point of view on the evolution and fate of our universe.

As an example, in one of the most recent articles [23], the authors compared the size and brightness of many galaxies at different distances from us, considering the most luminous spiral galaxies for comparisons, finding that, contrary to the prediction of the Big Bang theory, near and far galaxies have similar surface brightness. The authors "*conclude that available observations of galactic SB* (surface brightness) *are consistent with a static Euclidean model of the universe*", therefore "*the redshift is due to some physical process other than expansion*".

It may be also interesting for the purposes of this paper to mention some other researchers that, even in the "mainstream" interpretation of measured redshifts, thus considering an expanding universe, completely disagree with the idea of a Big Bang and consequent dark energy and matter, and find in the repulsive gravity of antimatter a possible solution of the inconsistencies met by Einstein and others. This is the case of Dr. Massimo Villata of the Italian National Institute for Astrophysics (INAF) and Dr. Dragan Hajdukovic, physicist at CERN.

The first one has developed a theory according to which repulsive gravity between matter and antimatter located in cosmic voids (and, important to stress, not detectable by our standard instruments) can account for universe expansion without need of dark energy or matter and Big Bang.

It is interesting to notice also that according to his calculations a reasonable antimatter mass, located in a particular void, could account for a recorded local velocity anomaly of the "Local Sheet," the part of the universe that includes the Milky Way and other nearby galaxies, by the mechanism of repulsive gravity. This local anomaly apparently couldn't be explained by a "dark energy" that acts uniformly throughout the space [24, 25]. As the author states "*Through simple dynamical considerations, we find that the*

Local Void could host an amount of antimatter roughly equivalent to the mass of a typical supercluster, thus restoring the matter-antimatter symmetry. Like matter, antimatter is self-attractive, so we can expect that it forms anti-galaxies and anti-stars, which would emit electromagnetic radiation, we could then detect. However antimatter, if emitting, should emit advanced radiation, which can be undetectable".

The second researcher also considers a repulsive gravitational interaction between matter and antimatter as an alternative to dark energy, dark matter and Big Bang, but he focuses on the microscopic interactions at the particle level. In his view repulsive gravity of virtual particles and antiparticles in quantum vacuum can explain several observations, including effects usually attributed to dark matter [26].

3. The Zwicky-Santilli Effect

The historical accounts outlined in the preceding section indicate that Einstein, Hubble, Zwicky, Hoyle, and other cosmologists died without accepting the conjecture of the expansion of the universe apparently because Hubble's law z = Hd clearly establishes the same redshift for all galaxies at the same distance d, in all possible *radial* directions from Earth, thus essentially implying a return to a Ptolemaic view of the universe (Fig. 1).

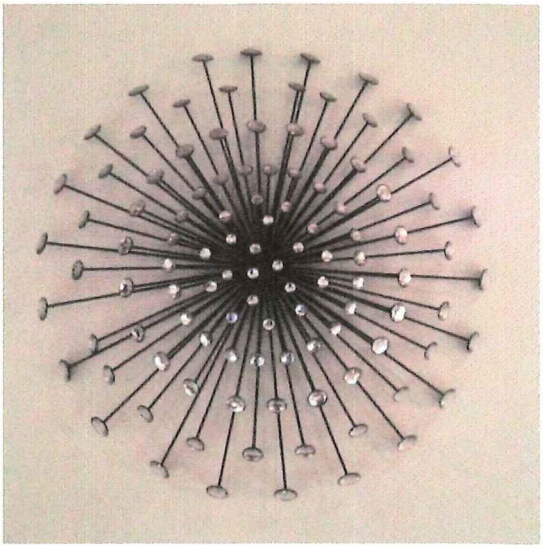

Figure 1. *A sculpture illustrating the radial character of the interpretation of the Hubble Law z=Hd, via the expansion of the universe with an evident return to the Ptolemaic conception of Earth at the center of the universe.*

In order to honor the above view, one of us (R. M. Santilli) conducted decades of mathematical, theoretical and experimental studies that have confirmed Zwicky's hypothesis via measurements on Earth resulting in an effect known as the Zwicky-Santilli effect (for brevity, see the collection of references [27]).

It may be interesting to mention that studies [27] are complemented by the suggestive possibility of a *continuous creation of matter* in the universe occurring in the core of stars during their synthesis of the neutron from the hydrogen atom [28].

4. The Proposed Isoselfdual Universe

The main problem for a return to a static universe is its inherent prediction that the universe should collapse due to gravitational attractions between galaxies.

By remembering from Section 1 that could not be resolved via theories for matter, in this paper, we present apparently for the first time a cosmological model achieving stability of the universe via an appropriate distribution of matter and antimatter galaxies.

To avoid a prohibitive length, the understanding of the proposed model requires a knowledge of: the *isodual theory* for the *classical* representation of *neutral or charged antimatter* [29]; the prediction of matter-antimatter gravitational repulsion occurring at all levels of study, the recent apparent detection of antimatter galaxies via telescopes with concave lenses and the recent apparent detections of antimatter galaxies, antimatter asteroids, and antimatter cosmic rays (for brevity, see list [30] referred publications in the field).

Let us consider a distribution of point-like matter and antimatter masses in vacuum only subject to gravitational interactions. We can then study the stability of this simplified system in order to appraise the correctness of our hypothesis. As further simplification, we first consider masses in a one-dimensional space, and then extend the model to three dimensions.

In order to look for an equilibrium of the system, we have to study the first derivative of the state vector, which is characterized by position and velocity of the point masses. The derivative of position at the initial time is assumed to be zero, since masses are assumed to be initially at rest, so the only significant component to look at is the derivative of velocity.

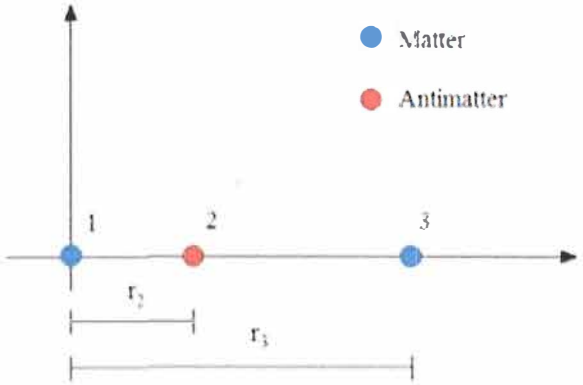

Figure 2. *We illustrate the case of three unit masses in a one-dimensional, according to the distribution of one matter mass, one antimatter mass and one matter mass under the assumption of matter-antimatter gravitational repulsion.*

Considering all possible permutations of up to 12 masses (half of them matter and half antimatter) at uniform finite distances, we easily find out that they are never in equilibrium, because the resultant acceleration on each mass is always different than zero. The use of variable distances for the same masses shows that equilibrium can be achieved only when distances themselves tend to infinity.

To better investigate the latter aspect, we study the equilibrium of 3 masses only (Fig 2), so that we can plot the results in a single graph. Being in one-dimension, there are only 2 free coordinates, which are the positions of the second and third mass with respect to the first one, and the accelerations can be plotted as a 3-D surface (Fig 3).

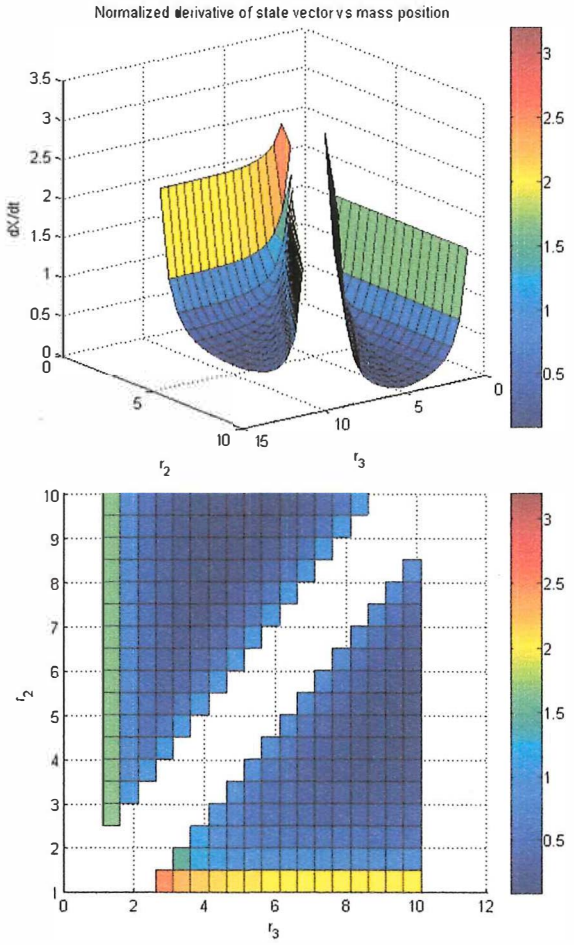

Figure 3. *The graph shows the normalized gravity acceleration acting on the three unit masses in the one-dimensional case of Figure 2 versus the positions of the masses. The values shown are the sum of absolute values of the resultant accelerations acting on each mass, due only to gravitational interactions among the masses themselves. From the graph it is possible to see that the global acceleration never crosses the zero plane, but it tends to decrease asymptotically with the increasing of the distances. In the diagonal and at the borders the acceleration tends to infinity because in these cases the positions of two of the masses tend to coincide. We have the lowest accelerations in the blue parts of the graph, where the masses tend to have large equal distances.*

As we can see from Figures 3 to 5, the resultant accelerations between matter and antimatter decreases with the increase of their distance, and reach equilibrium at a mutual distance for which the gravitational repulsion is ignorable.

Therefore, for better readability, we did plot also the accelerations in the case of equidistant masses in the sequence matter-antimatter-matter (Fig 4). Having only one variable, the resulting graph (Fig 5) is two-dimensional, and it clearly shows that the accelerations decrease with the increasing of distance, tending asymptotically to zero at infinity.

Needless to say, the proposed cosmological model needs considerable additional studies. However, it is rather easy to show that the extension of the model to two and three dimensions readily the same results.

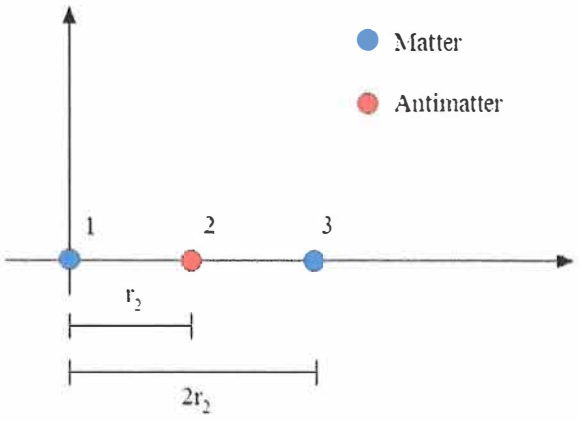

Figure 4. *In this case the three u unit masses are considered again in one-dimension, but they are equidistant.*

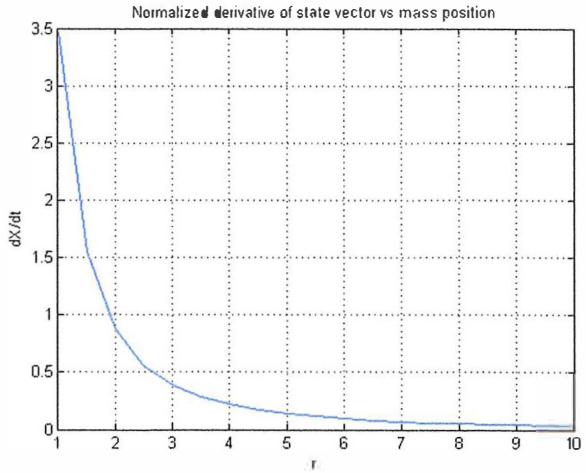

Figure 5. *The graph shows the normalized gravity acceleration acting on the three equidistant unit masses with the sequence matter-antimatter-matter in one-dimension versus the positions of the masses themselves. We know from the previous graph that in this case the acceleration tends to be a minimum. From this figure, it is possible to see more clearly that the sum of absolute values of accelerations tends asymptotically to zero with the increasing of the distances among the masses themselves.*

5. Conclusions

By remembering from Section 1 that the stability of the universe when solely composed by matter could not be consistently achieved via general relativity or other theories, we can conclude by saying that the universe can indeed achieve stability under the conditions of being composed by an equal number of matter and antimatter galaxies having a null total mass, matter and antimatter galaxies being at such large mutual distances caused by their gravitational repulsion at which all gravitational interactions are ignorable.

Needless to say, the above conditions merely represent limit values with large possibilities of local variations, such as: individual pairs of matter and antimatter galaxies need not necessarily have a null total mass since the gravitational repulsion exists also for different masses in absolute value; the conditions that the gravitational repulsion between matter and antimatter galaxies is ignorable implies very large variations of relative distances as well as (moderate) speeds, etc.

On more advanced grounds, we should note that the proposed cosmological model is isoselfdual, in the sense that it coincides with its isodual image [29], thus verifying the strict invariance under Santilli isoselfdual symmetry given by the Poincare' symmetry times its isodual [29]. The most rigorous proof of the consistency of our cosmological model (we can now call isoselfdual cosmology) can then be proved via the invariance under Santilli isoselfdual symmetry.

Recall that, in order to achieve a classical representation of neutral antimatter, it resulted necessary to assume that all characteristics of antimatter are opposite those of matter. Consequently, it is suggestive to note that all physical characteristics of the universe are identically null in our isoselfdual cosmology, thus including null total time, null total energy, etc.

The latter feature appears intriguing for quantitative studies on the origin of the universe without all calculations being lost at the act of creation due to singularities, since in the isoselfdual cosmology there is no change of numerical value of total physical quantities before and after creation. In turn, such a feature appears to support the suggestive hypothesis of a continuous creation in the core of matter and antimatter stars [28].

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PART 3:

FOUNDATIONS OF HADRONIC CHEMISTRY

reprinted from the American Journal of Modern Physics,
Volume 6, Issue 4-1, August 2017

Special Issue:
Issue II: Foundations of Hadronic Mechanics

Lead Guest Editor

Richard Anderson
Board of Trustees, the R. M. Santilli Foundation, Palm Harbor, Florida, USA

Introduction

In continuation of the new mathematics discussed in the preceding special issue entitled Foundations of Hadronic Mathematics, we recall that the Italian-American scientist R. M. Santilli proposed in 1978 the construction of a covering of quantum mechanics called hadronic mechanics, which is solely valid at mutual distances of one Fermi while recovering quantum mechanics identically and uniquely at larger mutual distances. By using the novel iso-, geno- and hyper-mathematics and their isoduals the Hadronic mechanics is divided into isomechanics, genomechanics and hypermechanics for the representation of single-valued, reversible, single-valued irreversible and multi-valued irreversible matter-system or reactions, respectively, with corresponding isodual for antimatter composite systems or reactions.

Thanks to the collaboration of numerous physicists, hadronic mechanics has now received applications and experimental verifications in classical mechanics, particle physics, nuclear physics, astrophysics, cosmology and other fields. The special issue of the AJMP entitled the Foundations of Hadronic Mechanics shall review some of these applications and present new advances that can potentially stimulate the birth of new technologies. It should be indicated that novel technologies solely predicted by hadronic mechanics have reached industrial applications, such as Thunder Energy Corporation, a U. S. publicly traded company with stock symbol TNRG, that has developed the first laboratory synthesis of neutrons from a hydrogen gas and is now entering into production and sale of the related equipment.

For more information about the Special Issue, please pay a visit to the following website:
http://www.sciencepublishinggroup.com/specialissue/122014

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Outline of Hadronic Mathematics, Mechanics and Chemistry as Conceived by R. M. Santilli

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Abstract: In this paper, we outline the various branches of hadronic mathematics and their applications to corresponding branches of hadronic mechanics and chemistry as conceived by the Italian-American scientist Ruggero Maria Santilli. According to said conception, hadronic mathematics comprises the following branches for the treatment of matter in conditions of increasing complexity: 1) 20th century mathematics based on Lie's theory; 2) IsoMathematics based on Santilli's isotopies of Lie's theory; 3) GenoMathematics based on Santilli's formulation of Albert's Lie-admissibility; 4) HyperMathematics based on a multi-valued realization of genomathematics with classical operations; and 5) HyperMathematics based on Vougiouklis H_v hyperstructures expressed in terms of hyperoperations. Additionally, hadronic mathematics comprises the anti-Hermitean images (called isoduals) of the five preceding mathematics for the description of antimatter also in conditions of increasing complexity. The outline presented in this paper includes the identification of represented physical or chemical systems, the main mathematical structure, and the main dynamical equations per each branch. We also show the axiomatic consistency of various branches of hadronic mathematics as sequential coverings of 20th century mathematics; and indicate a number of open mathematical problems. Novel physical and chemical applications permitted by hadronic mathematics are presented in subsequent collections.

Keywords: Santilli Isomathematics, Genomathematics, Hypermathematics

1. 20th Century Mathematics, Mechanics and Chemistry

1.1. Represented Systems

Single-valued, closed-isolated, time-reversible systems of point-like particles moving in vacuum solely under action at a distance Hamiltonian interactions, such as the structure of atoms and molecules.

1.2. Main Mathematical Structure

Basic unit

$$I = +1 \tag{1}$$

Basic numeric fields n = real, complex, quaternionic numbers

$$F(n,\times,1), n \tag{2}$$

Basic Associative product

$$nm = n \times m, 1 \times n = n \times 1 = n \; \forall n \in F \tag{3}$$

Measurement units of time, energy, etc. all positive
Ordinary functional analysis $f(r) \in F$,
Ordinary differential calculus
Conventional Lie theory

$$[X_i, X_j] = X_i \times X_j - X_j \times X_i == C_{ij}^k \times X_k, \tag{4}$$

$$A(w) = e^{X \times w \times i} \times A(0) \times e^{-i \times w \times X}. \tag{5}$$

Euclidean geometry and topology

$$E(r, \delta, 1), r = (r^k), k = 1,2,3, \delta = Diag.(1,1,1), \tag{6}$$

$$r^2 = r^i \times \delta_{ij} \times r^j = r_1^2 + r_2^2 + r_3^2 \in F, \tag{7}$$

Minkowskian geometry

$$M(x, \eta, I): x = (x^\mu), \mu = 1,2,3,4, x^4 = t, \tag{8}$$

$$\eta = Diag.(+1, +1, +1, -c^2), \tag{9}$$

$$x^2 = x^\mu \times \eta_{\mu\nu} \times x^\nu = x_1^2 + x_2^2 + x_3^2 - t^2c^2 \in F, \quad (10)$$

Riemannian geometry

$$R(x, g(x), I): x = (x^\mu), \mu = 1,2,3,4, x^4 = t, \quad (11)$$

$$x^2 = x^\mu \times g(x)_{\mu\nu} \times x^\nu \in F \quad (11)$$

$$x^2 = x^\mu \times g(x)_{\mu\nu} \times x^\nu \in F \quad (12)$$

Symplectic geometry.

$$\omega = dr^k \wedge dp_k \quad (13)$$

1.3. Dynamical equations

Newton equation

$$m \times \frac{dv}{dt} - F^{SA}(t, r, v,) = 0, \quad (14)$$

Variational principle

$$\delta A = \delta \int (p_k \times dr^k - H \times dt) = 0. \quad (15)$$

Hamilton's equations without external terms

$$\frac{dr^k}{dt} = \frac{\partial H(r,p)}{\partial p_k}, \quad \frac{dp_k}{dt} = -\frac{\partial H(r,p)}{\partial r^k}, \quad (16)$$

Hilbert space H over C with states $|\psi>$ over (C)
Expectation value of a Hermitean operator A

$$< A >=< \psi| \times A \times |\psi > \in C, \quad (17)$$

Heisenberg equation

$$i \times \frac{dA}{dt} = [A, H] = A \times H - H \times A, \quad (18)$$

Schrödinger equations

$$H \times |\psi >= E \times |\psi > \quad (19)$$

$$p \times |\psi >= -i \times \partial_r |\psi > \quad (20)$$

Dirac equation

$$(\eta^{\mu\nu} \times \gamma_\mu \times p_\nu - i \times m \times c) \times |\psi >= 0. \quad (21)$$

$$\{\gamma_\mu, \gamma_\nu\} = \gamma_\mu \times \gamma_\nu + \gamma_n u \times \gamma_\mu = 2 \times \eta_{\mu\nu}, \quad (22)$$

Comments and References

The literature on 20th century mathematics, mechanics and chemistry is so vast and so easily identifiable to discourage discriminatory partial listings.

2. Isomathematcs, Isomechanics and Isochemistry

2.1. Represented Systems [1-5]

Single-value, closed-isolated, time-reversible system of extended-deformable particles with action at a distance Hamiltonian and contact non-Hamiltonian interactions, such as the structure of hadrons, nuclei and stars, in the valence electron bonds and other systems.

2.2. Main Mathematical Structure s [1-5]

Santilli IsoUnit $\hat{I}$ and isotopic element $\hat{T}$[1]

$$\hat{I} = \hat{I}(r, p, a, \psi, \ldots.) = 1/\hat{T}(r, p, a, \psi, \ldots.) > 0, \quad (23)$$

Santilli IsoFields

$$\hat{F}(\hat{n}, \hat{\times}, \hat{I}), \hat{n} = n \times \hat{I}_{,,} \quad (24)$$

Santilli isoproduct

$$\hat{n} \hat{\times} \hat{m} = \hat{n} \times \hat{T} \times \hat{m} \in \hat{F}, \quad (25)$$

$$\hat{I} \hat{\times} \hat{n} = \hat{n} \hat{\times} \hat{I} = \hat{n} \forall \hat{n} \in \hat{F}, \quad (26)$$

Representation via the isotopic element of extended-deformable particles under non-Hamiltonian interactions

$$\hat{T} = Diag.(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}) \times e^{\Gamma(r,p,\psi,\partial\psi,\ldots)} \quad (27)$$

IsoCoordinates $\hat{r} = r \times \hat{I} \in \hat{F}$,
IsoFunctional analysis $\hat{f}(\hat{r}) = f(\hat{r}) \times \hat{I} \in \hat{F}$,
IsoDifferential Calculus

$$\hat{d}\hat{r} = dr + r \times \hat{T} \times d\hat{I}, \quad (28)$$

$$\frac{\hat{\partial} \hat{f}(\hat{r})}{\hat{\partial} \hat{r}} = \hat{I} \times \frac{\partial \hat{f}(\hat{r})}{\partial \hat{r}}, \quad (29)$$

Santilli Lie-Isotopic Theory

$$[\hat{X_i}, X_j] = X_i \hat{\times} X_j - X_j \hat{\times} X_i == C_{ij}^k(r, p, \ldots) \times X_k, \quad (30)$$

$$A(w) = \hat{e}^{X \times w \times i} \hat{\times} A(0) \hat{\times} \hat{e}^{-i \times w \times X}. \quad (31)$$

Santilli Iso-Euclidean Geometry

$$\hat{E}(\hat{r}, \hat{\delta}, \hat{I}), \hat{\delta}(r, p, z, \psi, \ldots) = \hat{T}(r, p, z, \psi, \ldots) \times \delta, \quad (32)$$

$$\hat{T} = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2), \quad (33)$$

$$\hat{r}^{\hat{2}} = \hat{r}^i \hat{\times} \hat{\delta}_{ij} \hat{\times} \hat{r}^j = (\frac{r_1^2}{n_1^2} + \frac{r_2^2}{n_2^2} + \frac{r_3^2}{n_3^2}) \times \hat{I} \in \hat{F}, \quad (34)$$

Santilli Iso-Minkowskian Geometry

$$\hat{M}(\hat{x}, \hat{\eta}, \hat{I}): \hat{x} = (\hat{x}^\mu), \mu = 1,2,3,4, x_4 = t, \quad (35)$$

$$\hat{\eta}(x, \psi, \ldots) = \hat{T}(x, \psi, \ldots) \times \eta, \quad (36)$$

$$\hat{T} = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2), \quad (37)$$

$$\hat{x}^{\hat{2}} = \hat{x}^\mu \hat{\times} \hat{\eta}_{\mu\nu} \hat{\times} \hat{x}^\nu = (\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2}) \times \hat{I} \in \hat{F}, \quad (38)$$

[1]See Santilli's curriculum
http://www.world-lecture-series.org/santilli-cv
Prizes and Nominations
http://www.santilli-foundation.org/santilli-nobel-nominations.html
and scientific archive
http://www.santilli-foundation.org/news.html

Santilli Iso-Riemannian Geometry

$$\hat{R}(\hat{x},\hat{g},\hat{\int}):\hat{g}=\hat{T}(x,v,\dots)\times g(x), \quad (39)$$

$$\hat{x}^{\hat{2}}=(\frac{g_{11}}{n_1^2}+\frac{g_{22}}{n_2^2}+\frac{g_{33}}{n_3^2}-\frac{g_{44}}{n_4^2})\times\hat{I}\in\hat{F}, \quad (40)$$

Santilli Iso-Symplectic Geometry

$$\hat{\omega}=\hat{d}\hat{r}^k\,\hat{\wedge}\,\hat{d}\hat{p}_k \quad (41)$$

2.3. IsoDynamical IsoEquations s [1-5]

Newton-Santilli IsoEquation

$$\hat{m}\,\hat{\times}\,\frac{\hat{d}\hat{v}}{\hat{d}\hat{t}}-F^{SA}(t,r,p)=m\times\frac{dv}{dt}-F^{SA}(t,r,p)-F^{NSA}(t,r,p,\dots)=0, \quad (42)$$

Iso Variational principle

$$\hat{\delta}\hat{A}=\hat{\delta}\hat{\int}\,(\hat{p}_k\,\hat{\times}\,\hat{d}\hat{r}^k-\hat{H}\,\hat{\times}\,\hat{d}\hat{t})=0. \quad (43)$$

Hamilton-Santilli IsoEquations

$$\frac{\hat{d}\hat{r}^k}{\hat{d}\hat{t}}=\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{p}_k},\quad \frac{\hat{d}\hat{p}_k}{\hat{d}\hat{t}}=-\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{r}^k}, \quad (44)$$

Iso-Hilbert space $\hat{H}$ over C with states $|\hat{\psi}>$ over the isofield $\hat{C}$[2]

IsoExpectation value of a Hermitean operator $\hat{A}$ on $\hat{H}$

$$<\hat{A}>=<\hat{\psi}|\,\hat{\times}\,\hat{A}\,\hat{\times}\,|\hat{\psi}>\in\hat{C} \quad (45)$$

Heisenberg-Santilli IsoEquation

$$\hat{\imath}\,\hat{\times}\,\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}}=[\hat{A},\hat{H}]=\hat{A}\,\hat{\times}\,\hat{H}-\hat{H}\,\hat{\times}\,\hat{A}=\hat{A}\times\hat{T}(\hat{\psi},\dots)\times\hat{H}(\hat{r},\hat{p})-\hat{H}(\hat{r},\hat{p})\times\hat{T}(\hat{\psi},\dots)\times\hat{A} \quad (46)$$

Schrödinger-Santilli IsoEquation

$$\hat{H}\,\hat{\times}\,|\hat{\psi}>=\hat{H}(\hat{r},\hat{p})\times\hat{T}(\hat{\psi},\hat{\partial}\hat{\psi},\dots)\times|\hat{\psi}>=\hat{E}\,\hat{\times}\,|\hat{\psi}>=E\times|\hat{\psi}>, \quad (47)$$

$$\hat{p}\,\hat{\times}\,|\hat{\psi}>=-\hat{\imath}\,\hat{\times}\,\hat{\partial}_{\hat{r}}|\hat{\psi}>=-i\times\hat{I}\times\partial_r|\hat{\psi}>, \quad (48)$$

Dirac-Santilli IsoEquation

$$(\hat{\eta}^{\mu\nu}\,\hat{\times}\,\hat{\gamma}_\mu\,\hat{\times}\,\hat{p}_\nu-\hat{\imath}\,\hat{\times}\,\hat{m}\,\hat{\times}\,\hat{c})\,\hat{\times}\,|\hat{p}si>=0. \quad (49)$$

$$\{\hat{\gamma}_\mu,\hat{\gamma}_\nu\}=\hat{\gamma}_\mu\,\hat{\times}\,\hat{\gamma}_\nu+\hat{\gamma}_\nu\,\hat{\times}\,\hat{\gamma}_\mu=\hat{2}\,\hat{\times}\,\hat{\eta}_{\mu\nu}=2\times\hat{\eta}_{\mu\nu}, \quad (50)$$

2.4. Comments and References

As it is well known, the local-differential calculus of 20th century mathematics can solely represent a finite set of isolated dimensionless points. In view of this structural feature, Newton formulated his celebrated equations (14) for *massive points,* resulted in a conception of nature that was adopted by Galileo and Einstein, became the dominant notion of 20th century sciences, and was proved to be valid for classical or quantum particles moving in vacuum at large mutual distances, such as for our planetary system or the atomic structure.

However, when bodies move within physical media, such as for a spaceship during re-entry in our atmosphere or for a proton in the core of a star, point-like abstractions of particles became excessive, e.g., because a macroscopic collection of point-particles cannot have entropy (since all known Hamiltonian interactions are invariant under time reversal), with consequential violation of thermodynamical laws and other insufficiencies.

Besides the clear identification of these insufficiencies, the first historical contribution by the Italian-American scientist Ruggero Maria Santilli (see Footnote 1) has been the *generalization of 20th century mathematics into such a form to admit a time invariant representation of extended, and therefore deformable particles under conventional Hamiltonian as well as contact non-Hamiltonian interactions, with implications for all quantitative sciences.*

The above central objective was achieved in monographs [I] originally written by Santilli during his stay at MIT from 1974 to 1977 (where they appeared as MIT preprints). Monographs [I] were then completed by Santilli during his stay at Harvard Universityfrom 1977 to 1982 under DOE support, and released for publication only following the delivery at Harvard of a post Ph. D. seminar Course in the field.

The representation of extended-deformable bodies moving within physical media was achieved via an axiom-preserving lifting, called *isotopy,* of the conventional associative product $AB=A\times B$ between generic quantities A,B (such as numbers,functions, matrices, operators, etc.) into the form $A\,\hat{\times}\,B=A\times\hat{T}\times B$, Eq. (25). Conventional interactions are represented via conventional Hamiltonian, while actual shape and non-Hamiltonian interactions are represented via realization of the quantity $\hat{T}$, called *isotopic element,* of the type (27).

Santilli then achieved in monographs [I] the axiom-preserving isotopies of the various branches of Lie's theory, e.g., Eqs. (30), (31,) including their elaboration via the initiation of the isotopies of functional analysis. In particular, Santilli showed that the isotopies of the rotational symmetry $SO(3)$ characterized by isotopic element (27) do represent extended, generally non-spherical and deformable bodies. Finally, Santilli proved in Vol. II of Ref.. [I] the

[2]As shown in the seminal paper [6] of 1982, but vastly ignored for the past four decades, isomechanics formulated on iso-Hilbert spaces over isofields eliminates the divergencies of quantum mechanics and related scattering theories. This important feature is primarily due to the fact that, for all physical and chemical applications worked out to date, the isounit $\hat{I}=1/\hat{T}>0$ must have a large value of the exponential type (27) and, consequently, the isotopic element $\hat{T}$ must have a very small value. This occurrence eliminates the singularity of the Dirac delta "distribution" when lifted to the Dirac-Myung-Santilli delta "isofunction" as shown by the realization of the type

$$\delta(r-r_0)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{ik\hat{T}(r-r_0)}\,dk,$$

with $\hat{T}=\frac{N}{r-r_0}$, $N\ll 1$. Similarly, perturbative and other series with Hermitean operators that are divergent or slowly convergent in quantum mechanics can be lifted into isoseries of the type

$$A(w)=\hat{I}+\frac{w(A\hat{T}H-H\hat{T}A)}{1!}+\cdots$$

that are manifestly convergent for $w>1$ but $\hat{T}\ll w$.As shown by A. O. E. Animalu and R. M. Santilli in five papers published proceedings [25], the above lack of divergences carries over to the covering of the scattering theory known as isoscattering theory, by therefore achieving numerical results without the use of infinities for the renormalization of divergent series.

significance of his Lie-isotopic theory by showing that it characterizes the Birkhoffian covering of classical Hamiltonian mechanics and its "direct universality" for the representation of all possible, non-singular, generally non-Hamiltonian Newtonian systems in the frame of the experimenter, which direct universality was subsequently proved to hold also for isotopic operator theories. The above advances were formulated on an ordinary numeric field.

Subsequently, Santilli discovered in 1993 [2] that the axioms of numeric fields with characteristic zero do not necessarily require that the basic multiplicative unit is the trivial number $+1$, since said axioms admit arbitrary generalized units, today called *Santilli isounits,* provided that they are positive-definite and are the inverse of the isotopic element, $\hat{I} = 1/\hat{T} > 0$. This second historical discovery identified new numbers today known as *Santilli isoreal, isocomplex and isoquaternionic numbers* of the First (Second) kind when the isounit is outside (an element of) the original field. This discovery prompted a flurry of reformulation over Santilli isofields of all preceding isotopies, including most importantly the reformulation of Santilli's Lie-isotopic theory.

Despite the above momentous advances, Santilli remained dissatisfied because the isotopic formulations of the early 1990s were not invariant under their time evolution, thus being unable to predict the same numerical values under the same conditions at different times. Since the entire 20th century mathematics had been isotonically lifted by the early 1990s, Santilli was left with no other choice than that of reinspecting the Newton-Leibnitz differential calculus by discovering that, contrary to a popular belief in mathematics and physics for some four centuries, the differential calculus is indeed dependent on the basic multiplicative unit. In this way, Santilli achieved in memoir [3] of 1996 the third historical discovery according to which the ordinary differential calculus needs generalizations of the type (28), (29) whenever the isounit depends on the local variable of differentiation. This discovery signaled the achievement of mathematical maturity of isomathematics that permitted numerous advances in physics and chemistry as well as novel industrial applications.

All in all, Santilli has written about 150 papers on the isotopies of all various aspects of 20th century mathematics. These contributions are reported in monographs [4] of 1995 that remain to this day the most comprehensive presentation on isotopies. In the subsequent series of monographs [5] of 2008, Santilli introduces the names of *Hadronic Mathematics, Mechanics and Chemistry* which have been adopted for this review due to their wide acceptance.

Numerous authors have made important contributions in Santilli isomathematics, among whom we quote: the mathematician H. C. Myung who initiated (with R. M. Santilli) [6] the isotopies of Hilbert Spaces, including the momentous elimination of the divergencies of quantum mechanics under sufficiently small values of the isotopic element $\hat{T}$; the mathematicians D. S. Sourlas and G. T. Tsagas [7] who conducted in 1993 the first comprehensive study of the Lie-Santilli isotheory; the theoretician J. V. Kadeisvili [8] who presented systematic studies of Santilli's isotopies of 20th century geometries and relativities; the mathematician Chun-Xuan Jiang [9] who conducted in 2001 systematic studies of Santilli IsoNumber Theory; the mathematicians R. M. Falcon Ganfornina and J. Nunez Valdes who wrote in 2001 the now historical, first mathematically rigorous treatment of Santilli isotopies [10], and the historical achieved isotopology [11] which provides the ultimate mathematical structure of the Newton-Santilli isoequations (42) for extended-deformable particles under Hamiltonian and non-hamiltonian interactions achieved in memoir [3]; the mathematician S. Georgiev who wrote one of the most monumental and important mathematical works in scientific history [12], by showing that Santilli's IsoDifferential Calculus implies a variety of fully consistent coverings of 20th century mathematics; the mathematician A. S. Muktibodh [13] who presented the first known generalization of Santilli isonumber theory for the case of characteristic $p \neq 0$; the physicists I. Gandzha and J. Kadeisvili who presented in 2011 [14] a comprehensive review of Santilli isomathematics and its applications in physics and chemistry; plus additional seminal advances presented in the subsequent papers of this collection.

3. Genomathematics, Genomechanics and Genochemistry

3.1. Represented Systems s [1-5]

Single-valued, time-irreversible system of extended-deformable particles under action at a distance Hamiltonian and contact non-Hamiltonian interactions, as occurring in nuclear reactions, biological structures and chemical reactions.

3.2. Main Mathematical Structure s [1-5]

Santilli Forward GenoUnit

$$\hat{I}^{>} = \hat{I}^{>}(t^{>} r^{>}, p^{>}, a^{>}, \psi^{>}, \partial^{>}\psi^{>}, \ldots.) = 1/\hat{T}^{>} > 0, \quad (51)$$

Santilli Backward GenoUnit

$$^{<}\hat{I} = {}^{<}\hat{I}({}^{<}r, {}^{<}p, {}^{<}a, {}^{<}\psi, {}^{<}\partial {}^{<}\psi, \ldots.) = 1/{}^{<}\hat{T} > 0, \quad (52)$$

Condition for time-irreversibility

$$\hat{I}^{>} \neq {}^{<}\hat{I} \quad (53)$$

Forward GenoFields

$$\hat{F}^{>}(\hat{n}^{>}, >, \hat{I}^{>}), \hat{n}^{>} = n \times \hat{I}^{>} \quad (54)$$

Backward GenoFields

$$^{<}\hat{F}({}^{<}\hat{n}, <, {}^{<}\hat{I}), \quad {}^{<}\hat{n} = {}^{<}\hat{I} \times n, \quad (55)$$

Forward GenoProduct

$$\hat{n} > \hat{m} = \hat{n}^{>} \times \hat{I}^{>} \times \hat{m}^{>} \in \hat{F}^{>}, \quad (56)$$

$$\hat{I}^{>} > \hat{n}^{>} = \hat{n}^{>} > \hat{I}^{>} = \hat{n}^{>} \; \forall \; \hat{n}^{>} \in \hat{F}^{>} \quad (57)$$

Backward Genoproduct

$$^{<}\hat{n} <^{<} \hat{m} =^{<} \hat{n} \times^{<} \hat{T} \times^{<} \hat{m} \in^{<} \hat{F}, \quad (58)$$

$$^{<}\hat{I} <^{<} \hat{n} =^{<} \hat{n} <^{<} \hat{I} =^{<} \hat{n} \forall \quad ^{<}\hat{n} \in^{<} \hat{F}, \quad (59)$$

Representation of forward extended-deformable particles under non-Hamiltonian interactions

$$\hat{T}^{>} = Diag.(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2})^{>} \times e^{\Gamma(t,r,p,\psi,\partial\psi,\ldots)^{>}} \quad (60)$$

Forward GenoCoordinates

$$\hat{r}^{>} = r \times \hat{I}^{>} \in \hat{F}^{>}, \quad (61)$$

Backward GenoCoordinates

$$^{<}\hat{r} =^{<} \hat{I} \times r \in^{<} \hat{F}, \quad (62)$$

Forward GenoFunctional analysis

$$\hat{f}^{>}(\hat{r}^{>}) = f(\hat{r}^{>}) \times \hat{I}^{>} \in \hat{F}^{>}, \quad (63)$$

Backward GenoFunctional analysis

$$^{<}\hat{f}(^{<}\hat{r}) = f(^{<}\hat{r}) \times^{<} \hat{I} \in^{<} \hat{F}, \quad (64)$$

Forward GenoDifferential Calculus

$$\hat{d}^{>}\hat{r}^{>} = dr + r \times \hat{T}^{>} \times d\hat{I}^{>}, \quad (65)$$

$$\frac{\hat{\partial}^{>}\hat{f}^{>}(\hat{r}^{>})}{\hat{\partial}^{>}\hat{r}^{>}} = \hat{I}^{>} \times \frac{\partial \hat{f}^{>}(\hat{r}^{>})}{\partial \hat{r}^{>}}, \quad (66)$$

Backward GenoDifferential Calculus

$$^{<}\hat{d}^{<}\hat{r} = dr + r \times^{<} \hat{T} \times d^{<}\hat{I}, \quad (67)$$

$$\frac{^{<}\hat{\partial}^{<}\hat{f}(^{<}\hat{r})}{^{<}\hat{\partial}^{<}\hat{r}} =^{<} \hat{I} \times \frac{\partial^{<}\hat{f}(^{<}\hat{r})}{\partial^{<}\hat{r}}, \quad (68)$$

Santilli Lie-Admissible Theory

$$(\hat{X_i}, X_j) = X_i < X_j - X_j > X_i = C_{ij}^{k}(t,r,p,\psi,\ldots) \times X_k, \quad (69)$$

$$A(w) = \hat{e}_{>}^{X \times w \times i} > A(0) <_{<} \hat{e}^{-i \times w \times X}. \quad (70)$$

Santilli Forward Geno-Euclidean Geometry

$$\hat{E}^{>}(\hat{r}^{>}, \hat{\delta}^{>}, \hat{I}^{>}), \hat{\delta}^{>}(t,r,p,\psi,\ldots) = \hat{T}^{>}(t,r,p,\psi,\ldots) \times \delta, \quad (71)$$

$$\hat{r}^{>\hat{2}} = \hat{r}^{>i} > \hat{\delta}_{ij}^{>} > \hat{r}^{>j} \in F^{>}, \quad (72)$$

$$\hat{\delta}^{>} \neq \hat{\delta}^{>\, tranp} \quad (73)$$

Santilli Backward Geno-Euclidean Geometry

$$^{<}\hat{E}(^{<}\hat{r}, ^{<}\hat{\delta}, ^{<}\hat{I}), \quad ^{<}\hat{\delta}(t,r,p,\psi,\ldots) =^{<} \hat{T}(t,r,p,\psi,\ldots) \times \delta, \quad (74)$$

$$^{<\hat{2}}\hat{r} =^{<i} \hat{r} <^{<} \hat{\delta}_{ij} <^{<j} \hat{r} \in^{<} F, \quad (75)$$

$$^{<}\hat{\delta} \neq^{<\, transp} \hat{\delta} \quad (76)$$

Santilli Forward Geno-Minkowskian Geometry (μ = 1,2,3,4)

$$\hat{M}^{>}(\hat{x}^{>}, \hat{\eta}^{>}, \hat{I}^{>}): \hat{x}^{>} = (\hat{x}^{>\mu}), x_4^{>} = t^{>}, \quad (77)$$

$$\hat{\eta}^{>}(x,\psi,\ldots) = \hat{T}^{>}(x,\psi,\ldots) \times \eta, \quad (78)$$

$$\hat{x}^{>2} = \hat{x}^{>\mu} > \hat{\eta}_{\mu\nu}^{>} > \hat{x}^{>} \in \hat{F}^{>}, \quad (79)$$

$$\hat{\eta}^{>} \neq \hat{\eta}^{>\, transp} \quad (80)$$

Santilli Backward Geno-Minkowskian Geometry (μ = 1,2,3,4,)

$$^{<}\hat{M}(^{<}\hat{x}, ^{<}\hat{\eta}, ^{<}\hat{I}): \quad ^{<}\hat{x} = (\hat{x}^{\mu}), \quad ^{<}x_4 =^{<} t, \quad (81)$$

$$^{<}\hat{\eta}(x,v,\ldots) =^{<} \hat{T}(x,v,\ldots) \times \eta, \quad (82)$$

$$^{<}\hat{x}^{<2} =^{<\mu} \hat{x} <^{<} \hat{\eta}_{\mu\nu} <^{<\nu} \hat{x} \in^{<} \hat{F}, \quad (83)$$

$$^{<}\hat{\eta} \neq^{<\, transp} \hat{\eta} \quad (84)$$

Santilli Forward Geno-Riemannian Geometry

$$\hat{R}^{>}(\hat{x}^{>}, \hat{g}^{>}, \hat{I}^{>}): \hat{g}^{>} = \hat{T}^{>}(x,v,\ldots) \times g(x), \quad (85)$$

$$\hat{x}^{>2} = \hat{x}^{>\mu} > \hat{g}_{\mu\nu}^{>} > \hat{x}^{>} \in \hat{F}^{>}, \quad (86)$$

$$\hat{g}^{>} \neq \hat{g}^{>\, transp} \quad (87)$$

Santilli Backward Geno-Riemannian Geometry

$$^{<}\hat{R}(^{<}\hat{x}, ^{<}\hat{g}, ^{<}\hat{I}): \quad ^{<}\hat{g} =^{<} \hat{T}(x,v,\ldots) \times g(x), \quad (88)$$

$$^{<}\hat{x}^{<2} =^{<\mu} \hat{x} <^{<} \hat{g}_{\mu\nu} <^{<\nu} \hat{x} \in^{<} \hat{F}, \quad (89)$$

$$^{<}\hat{g} \neq^{<\, transp} \hat{g} \quad (90)$$

Santilli Forward Geno-Symplectic Geometry

$$\hat{\omega}^{>} = \hat{d}^{>}\hat{r}^{>k} \hat{\wedge}^{>} \hat{d}^{>}\hat{p}_k^{>} \quad (91)$$

Santilli Backward Geno-Symplectic Geometry

$$^{<}\hat{\omega} =^{<} \hat{d}^{<}\hat{r}^{<} \hat{\wedge}^{<} \hat{d}^{<}\hat{p}_k \quad (92)$$

3.3. GenoDynamical GenoEquations s [1-5]

Newton-Santilli Forward GenoEquation

$$\hat{m}^{>} > \frac{\hat{d}^{>}\hat{v}^{>}}{\hat{d}^{>}\hat{t}^{>}} - F^{>SA}(t,r,p) = [m \times \frac{dv}{dt}]^{>} - F^{SA>}(t,r,p) - F^{NSA>}(t,r,p,\ldots) = 0, \quad (93)$$

Newton-Santilli Backward GenoEquation

$$^{<}\hat{m} < \frac{^{<}\hat{d}^{<}\hat{v}}{^{<}\hat{d}^{<}\hat{t}} -^{<SA} F(t,r,p) = =^{<} [m \times \frac{dv}{dt}] -^{<SA} F(t,r,p) -^{<NSA} F(t,r,p,\ldots) = 0, \quad (94)$$

Forward GenoVariational principle

$$\hat{\delta}^{>}\hat{A}^{>} = \hat{\delta}^{>} \hat{\int}^{>} (\hat{p}_k^{>} > \hat{d}^{>}\hat{r}^{>k} - \hat{H}^{>} > \hat{d}^{>}\hat{t}^{>}) = 0. \quad (95)$$

Backward GenoVariational principle

$$^{<}\hat{\delta}^{<}\hat{A} =^{<} \hat{\delta}^{<} \hat{\int} (^{<}\hat{p}_k <^{<} \hat{d}^{<}\hat{r}^{k} -^{<}\hat{H} <^{<} \hat{d}^{<}\hat{t}) = 0. \quad (96)$$

Forward Hamilton-Santilli GenoEquations

$$\left[\frac{d\hat{r}^k}{d\hat{t}}\right]^{>} = \left[\frac{\partial \hat{H}(\hat{r},\hat{p})}{\partial \hat{p}_k}\right]^{>}, \quad \left[\frac{d\hat{p}_k}{d\hat{t}}\right]^{>} = -\left[\frac{\partial \hat{H}(\hat{r},\hat{p})}{\partial \hat{r}^k}\right]^{>}, \tag{97}$$

Backward Hamilton-Santilli GenoEquations

$$<\left[\frac{d\hat{r}^k}{d\hat{t}} = \frac{\partial \hat{H}(\hat{r},\hat{p})}{\partial \hat{p}_k}\right], \quad <\left[\frac{d\hat{p}_k}{d\hat{t}}\right] = -<\left[\frac{\partial \hat{H}(\hat{r},\hat{p})}{\partial \hat{r}^k}\right], \tag{98}$$

Forward Geno-Hilbert space $\hat{H}^{>}$ with states $|\hat{\psi}>$ over the isofield $\hat{C}^{>}$

GenoExpectation value of a Hermitean operator $\hat{A}$ on $\hat{H}^{>}$

$$<\hat{A}> = <^{<} \hat{\psi}| < \hat{A}^{>} > |\hat{\psi}^{>} > \in \hat{C} \tag{99}$$

Heisenberg-Santilli GenoEquation[3]

$$\hat{\imath} \hat{\times} \frac{d\hat{A}}{d\hat{t}} = (\hat{A}, \hat{H}) = \hat{A} < \hat{H} - \hat{H} > \hat{A} = A \times^{<} T(\hat{\psi}, \ldots) \times \hat{H}(\hat{r},\hat{p}) - \hat{H}(\hat{r},\hat{p}) \times \hat{T}^{>}(\hat{\psi}, \ldots) \times \hat{A} \tag{100}$$

Forward Schrödinger-Santilli GenoEquation

$$\hat{H}^{>} > |\hat{\psi}^{>} >= \hat{H}^{>}(\hat{r},\hat{p}) \times \hat{T}^{>}(\hat{\psi}, \hat{\partial}\hat{\psi}, \ldots) \times |\hat{\psi}^{>} >= \hat{E}^{>} > |\hat{\psi}^{>} >= E^{>} \times |\hat{\psi}^{>} >, \tag{101}$$

$$\hat{p}^{>} > |\hat{\psi}^{>} >= -\hat{\imath}^{>} > \hat{\partial}_{\hat{r}}^{>} |\hat{\psi}^{>} >= -i \times \hat{I}^{>} \times \partial_{\hat{r}} |\hat{\psi}^{>} >, \tag{102}$$

Backward Schrödinger-Santilli GenoEquation

$$<^{<} \hat{\psi}| <^{<} \hat{H} =<^{<} \hat{\psi}| \times^{<} \hat{T}(\hat{\psi}, \hat{\partial}\hat{\psi}, \ldots) \times^{<} \hat{H}(\hat{r},\hat{p}) = <^{<} \hat{\psi}| <^{<} \hat{E} =<^{<} \hat{\psi}| \times^{<} E, \tag{103}$$

$$<^{<} \hat{\psi}| <^{<} \hat{p} = -<^{<} \hat{\psi}| <^{<} \hat{\imath} <_{\hat{r}}^{<} \hat{\partial} = -i \times <^{<} \hat{\psi}|_{\hat{r}}^{<} \partial \times^{<} \hat{I} \tag{104}$$

Forward Dirac-Santilli IsoEquation

$$(\hat{\eta}^{>\mu\nu} > \hat{\gamma}_{\mu}^{>} > \hat{p}_{\nu}^{>} - \hat{\imath}^{>} > \hat{m}^{>} > \hat{c}^{>}) > |\hat{p}si^{>} >= 0. \tag{105}$$

$$\{\hat{\gamma}_{\mu}, \hat{\gamma}_{\nu}\}^{>} = [\hat{\gamma}_{\mu} \hat{\times} \hat{\gamma}_{\nu} + \hat{\gamma}_{\nu} \hat{\times} \hat{\gamma}_{\mu}]^{>} = \hat{2}^{>} > \hat{\eta}_{\mu\nu}^{>}, \tag{106}$$

Backward Dirac-Santilli GenoEquation

$$<^{<} \hat{\psi}| < (<^{<} \hat{p}_{\nu} <^{<} \hat{\gamma}_{\mu} <^{<\mu\nu} \hat{\eta} -<^{<} \hat{\imath} <^{<} \hat{m} <^{<} \hat{c}) = 0. \tag{107}$$

$$<\{\hat{\gamma}_{\mu}, \hat{\gamma}_{\nu}\} =^{<} [\hat{\gamma}_{\mu} \hat{\times} \hat{\gamma}_{\nu} + \hat{\gamma}_{\nu} \hat{\times} \hat{\gamma}_{\mu}] =^{<} \hat{2} <^{<} \hat{\eta}_{\mu\nu} = 2 \times^{<} \hat{\eta}_{\mu\nu}, \tag{108}$$

3.4. Comments and References

As it is also well known, all 20th century mathematical, physical or chemical formulations are reversible over time. Following research over half a century initiated during his Ph. D. studies at the University of Torino, Italy, in the mid 1960s [15, 17-23,4,5], R. M., Santilli has made the additional historical discovery of the first and only known, axiomatically consistent, generalization of 20th century mathematics as well as of its covering isomathematcs into a form embedding irreversibility over time in ordered forward and backward units, in corresponding ordered forward and backward products and, consequently, in all subsequent mathematical structures, resulting in the new mathematics nowadays known as *Santilli forward and backward genomathematics* with corresponding physical and chemical theories for the representation of irreversible processes.

Since the reversibility over time of 20th century theories can be reduced to the invariance under anti-Hermiticity of the Lie product between Hermitean operators, $[a,b] = ab - ba = -[a,b]^{\dagger}$, Santilli presented in 1967 [15] the first known (p, q)-deformation of the Lie product $(a,b) = pab - qba$, where p, q are scalars and the product ab is generally non-associative. Following an intense search in European mathematical libraries, Santilli discovered that the new product verifies the axiom of *Lie-admissibility* by the American mathematician A, A, Albert [16] in the sense that the attached anti-symmetric product $[a,b] = (a,b) - (b,a)$ verifies the axioms of a Lie algebra.

Since spaceship during re-entry are notoriously irreversible over time, Santilli was invited by the Center for Theoretical Physics of the University of Miami, Florida, under NASA support, where he moved with his wife Carla and newly born daughter Luisa inAugust 1967, and published a number of additional works in Lie-admissibility, including the first known Lie-admissible generalization of Hamilton and Heisenberg equations [17,18], nowadays considered at the foundation of hadronic mechanics and chemistry, as well as the first and only known Lie-admissible formulation of dissipative plasmas surrounding spaceships during reentry [19].

Santilli then spent seven years , from 1968 to 1974, at the Department of Physics of Boston University, and then three years, from 1974 to 1977, at MIT, during which tine he wrote, in his words, *Phys.. Rev of career-oriented papers nobody reads.* InSeptember 1977, Santilli joined Harvard University and was invited by the DOE to study irreversible processes because all energy releasing processes are irreversible over time. In April 1978, Santilli published under his DOE support his most important mathematical contribution [20] (see also monographs [21]) in which he achieved a Lie-admissible covering of the various branches of Lie's theory, Eqs. (69), (70), including the most general known time evolution whose brackets characterize an algebra, Eqs. (1000). It should be indicated that the isotopies of Lie's theory outlined in the preceding section were derived by Santilli as a particular case of the broader Lie-admissible theory of Ref. [20], and then published in monographs [1].

Subsequently, Santilli discovered in paper [2] of 1993 that the axiom of a numeric field, besides admitting a generalization of the multiplicative unit, also admit the restriction of the associative product to an ordered form to the right and, separately,to the left. In this way, Santilli discovered two additional classes of new numbers, today known as *Santilli forward and backward genoreal, genocomplex and genoquarternionic numbers.* In the seminal memoir [3] of

[3] By including the multi-valued (Section 4) and hyperstructural formulations (Section 5), Lie-admissible equations (100) are so broad that it will take centuries for their generalizations. For this reasou,Santilli has requested in his will that his tombstone should have the engraving

$$i\dot{A} = A < H - H > A$$

below his name.

1996 Santilli discovered two additional coverings of the ordinary differential calculus and of its isotopic covering, today known as *Santilli forward and backward genodifferential calculi,* Eqs. (65) to (68). Santilli called a *genotopy* [20] the lifting of isomathematics into ordered formulations to the right and to the left in the Greek sense of inducing a covering of Lie's axioms, Eqs. (69), (70).

As it is well known, thousands of papers have been published beginning from the late 1980s on the so-called q-deformations of Lie algebras with product $(a,b) = ab - qba$ which are an evident particular case of Santilli Lie-admissible product [15]. Whatit is lesser known, or not admitted, all q-deformations did not achieve invariance over time, thus being afflicted by serious inconsistencies, since they consisted of non-unitary theories formulated via the mathematics of unitary theories. Santilli solved this problem in 1997 by achieving the first and only known invariant formulation of q- as well as of (p, q)-deformations [22].

We should indicate that Santilli's conception of a genotopic lifting of his preceding isomathematcs (indicated in Section 2 by "hat" on symbols plus the "arrow of time") is necessary to achieve a consistent representation of irreversibility because point-like particles can only experience action-at-a-distance interactions that are reversible over time. Therefore, a simple genotopy of 20th century mathematics based on the conventional associative product would be axiomatically inconsistent. Consequently, to represent irreversibility it is first necessary to lift 20th century mathematics into isomathematcs, with consequential representation of extended-deformable particles via realizations of type (27) so that extended particles can experience non-Hamiltonian interactions needed for irreversibility. It is then necessary to add irreversibility via the ordering of all products. It should also be indicated that, when formulated via time-dependent isounits, isomathematics can becomes genomathematics via the identifications $\hat{I}(t,\ldots) = \hat{I}^{\dagger}(t,\ldots) = \hat{I}^{>}, \hat{I}(-t,\ldots) = \hat{I}^{\dagger}(-t,\ldots) = {}^{<}\hat{I}, \hat{I}(t,\ldots) \neq \hat{I}(-t)$, and the judicious addition of ordered products.

Systematic studies on the Lie-Admissible treatment of irreversible systems were presented in memoir [3] and monographs [4]. Santilli's subsequent memoir [23] of 2006 remains to this day the most comprehensive presentation of Lie-admissible treatments of irreversibility at the classical and operator levels. Monographs [5] of2008 presented an update. Paper collection [24[presents all available independent contributions in Lie-admissibility up to [1984. The Proceedings of the *Third International Conference on Lie-admissible Treatment of Irreversible Systems* [25] present numerous additional independent contributions as well as references for the five *Workshops on Lie-0admissible Algebras* organized by Santilli at Harvard University, and for the preceding two international conference in Lie-admissibility, the first at the Université d'Orleans, France, in 1981 and the second at the Castle Prince Pignatelli, Italy, in 1995 (see also the general review [14] and large literature quoted therein).

As it is well known, there exists a large number of papers on Lie-admissible algebras within the context of non-associative algebras (see Tomber's Bibliography [26] listing all significant papers in the field up to 1986). It should be indicated that, regrettably, these studies have no connection with Santilli genomathematics since the latter deals with the irreversible generalizations of all aspects of 20th century mathematics.

4. Classical Hypermathematcs, Hypermechanics and Hyperchemistry

4.1. Represented Systems s [1-5]

Multi-valued, time-irreversible systems of extended -deformable particles or constituents under the most general known Hamiltonian and non-Hamiltonian interaction, as occurring for multi-valued universes or the structure of the DNA.

4.2. Main Mathematical Structure s [1-5]

Basic HyperUnits and HyperProducts

$$\hat{I}^{>} = \{\hat{I}_1^{>}, \hat{I}_2^{>}, \hat{I}_3^{>}, \ldots\} = 1/\hat{S}, \tag{109}$$

$${}^{<}\hat{I} = \{{}^{<}\hat{I}_1, {}^{<}\hat{I}_2, {}^{<}\hat{I}_3, \ldots\} = \frac{1}{\hat{R}}, \tag{110}$$

Forward and Backward HyperProducts

$$A > B = \{A \times \hat{S}_1 \times B, A \times \hat{S}_2 \times B, A \times \hat{S}_3 \times B, \ldots\}, \hat{I}^{>} > A = A > \hat{I}^{>} = A \times I, \tag{111}$$

$$A < B = \{A \times \hat{R}_1 \times B, A \times hat R_2 \times B, A \times \hat{R}_3 \times B, \ldots\}^{<}\hat{I} < A = -A <^{<} \hat{I} = I \times A, \tag{112}$$

$$A = A^{\dagger}, B = B^{\dagger}, \hat{R} = \hat{S}^{\dagger}. \tag{113}$$

Classical hypermathematcs then follow as for genomathematcs with multi-valued units, quantities and operations.

4.3. Classical Hyper-Dynamical Equations s [1-5]

The same as those for genomathematics, but with multi-valued hyperunits, quantities and operations.

Comments and References

The multi-valued three-dimensional (rather than multi-dimensional) realization of genomathematics outlined in Section 4 emerged from specific biological needs. The Australian biologist C. Illert [27] confirmed that the *shape* of seashells can indeed be represented in a three-0dimensional Euclidean space as known since Fourier's time, but proved that the *growth in time* of a seashell cannot any longer be consistently represented in a conventional, three-dimensional Euclidean space, and achieved a consistent representation via the doubling of the three reference axis.

Santilli [27,28] confirmed Illert's findings because the conventional Euclidean geometry has no time arrow and, consequently, cannot consistently represent a strictly irreversible system, such as the growth of seashells. Additionally, Santilli proved thathis geno-Euclidean geometry,

Eqs. (71) to (73), is equally unable to represent the growth in time of seashells despite its irreversible structure, however, an axiomatically consistent and exact representation of the growth of seashells was possible via the multi-valued realization of the forward geno-Euclidean geometry, thus beginning to illustrate the complexity of biological structures.

The multi-valued, rather than multi-dimensional character of classical hypermathematics is indicated by Santilli as follows [28] *We perceive the growth of a seashell specifically in three dimensions from our Eustachian lobes. Therefore, an irreversible mathematics suitable to represent the growth of sea shells must be perceived by us as being in three dimensions. However, Illert has shown the need to double the three Cartesian axis. Classical hypermathematics has been conceived and structured in such a way that the increase of the reference axes is complemented by a corresponding multi-valued hyperunit in such a way that a classical hyper-Euclidean geometry, when seen at the abstract level, remains indeed three-dimensional as necessary to achieve representation of biological structures compatible with our sensory perception.*

5. Hope Hypermathematics, Hypermechanics and Hyperchemistry

Represented Systems

The most complex known multi-valued, time-irreversible requiring extremely large number of data, such as the DNA code [31-35].

Comments and References

Despite the preceding structural generalization of 20th century mathematics, Santilli remained dissatisfied in view of the complexity of nature, particularly of biological entities because advances in the *structure* of the DNA are indeed possible via classical hypermathematics, as we shall see in the third collection of this series dedicated to chemistry (e.g., via Santilli hypermagnecules), but any attempt at representing the DNA *code* via any of the preceding mathematics can be proved to be excessively restrictive due to the volume, complexity, diversification and coordination of the information.

Therefore, Santilli approved one of the most important mathematicians in hyperstructures, T. Vougiouklis from Greece, and asked for his assistance in further generalizing the preceding mathematics via hyperstructures defined on hyperfields, as necessary for applications implying measurements, and formulated via hyperoperations (called "hope") permitting the needed broadening of the representational capability.

The above contact lead to the hypermathematics indicated in this section as presented in Refs. [29-33] which is based on Vougiouklis H_v hyperaxioms and which mathematics, in Santilli's words, *constitutes the most general mathematics that can be conceived nowadays by the human mind.*

6. Isodual Mathematics, Mechanics and Chemistry

6.1. Represented Systems

Single-valued, closed-isolated, time-reversible systems of classical and operatorpoint-like antiparticles moving in vacuum solely under action at a distance Hamiltonian interactions, such as the stricture of antimatter atoms and antimatter molecules [2,36-43].

6.2. Main Mathematical Structure [2,36-43]

Basic isodual unit

$$1^d = -1^\dagger = -1, \tag{114}$$

Isodual numeric fields

$$F^d(n^d, \times^d, 1^d), n^d = n \times 1^d, n^d \times^d m^d = n^d \times (1^d)_{-1} \times m^d \in F^d,$$

$$n^d = isodual\ real, complex, quatern.\ numbers, \tag{115}$$

Isodual functional analysis

$$f^d(r^d) = f(r^d) \times 1^d \in F^d \tag{116}$$

Isodual differential calculus

$$d^d r^d = (1)^{-1} \times dr^d = dr, \tag{117}$$

$$\frac{\partial^d f^d(r^d)}{\partial^d r^d} = 1^d \times \frac{\partial f^d(r^d)}{\partial r^d}, \tag{118}$$

Santilli Isodual Lie theory

$$[X_i, X_j]^d = (X_i \times X_j - X_j \times X_i)^d == -C_{ij}^k \times X_k, \tag{119}$$

$$A^d(w^d) = e_d^{X \times w \times i} \times^d A^d(0) \times^d e_d^{-i \times w \times X}. \tag{120}$$

Santilli isodual Euclidean geometry

$$E^d(^d, \delta^d, 1^d), r^d = (r^{dk}), k = 1,2,3,$$

$$\delta^d = Diag.(-1, -1, -1), \tag{121}$$

$$r^{d2d} = r^{di} \times \delta_{ij} \times^d r^{dj} = (r_1^2 + r_2^2 + r_3^2) \times 1^d \in F^d, \tag{122}$$

Santilli Isodual Minkowskian geometry ($\mu = 1,2,3,4,$)

$$M^d(x^d, \eta^d, I^d): x^d = (x^{d\mu}), x^{d4} = t^d = t \times 1^d = -t, \tag{123}$$

$$\eta^d = Diag.(-1, -1, -1, +c^{d2d}), \tag{124}$$

$$x^{d2d} = (x^\mu \times \eta_{\mu\nu} \times x^\nu)^d = (x_1^2 + x_2^2 + x_3^2 - t^2 c^2) \times 1^d \in F^d, \tag{125}$$

Isodual Riemannian geometry, Santilli Isodual Symplectic Geometry.

6.3. Isodual Dynamical Equations [2,36-43

Newton-Santilli Isodual Equation

$$m^d \times^d \frac{d^d v^d}{d^d t^d} - F^{dSA}(t^d, r^d, v^d) = 0, \quad (126)$$

Isodual Variational Principle

$$\delta^d A^d = \delta^d \int^d (p_k^d \times^d d^d r^{dk} - H^d \times^d d^d t^d) = 0. \quad (127)$$

Hamilton-Santilli Isodual Equations without external terms

$$\frac{d^d r^{dk}}{d^d t^d} = \frac{\partial^d H^d(r^d, p^d)}{\partial^d p_k^d}, \quad \frac{d^d p_k^d}{d^d t^d} = -\frac{\partial^d H^d(r^d, p^d)}{\partial^d r^{dk}}, \quad (128)$$

Isodual Hilbert space H^d over C with states $|\psi^d >= -< \psi|$ over C^d

Expectation value of a Hermitean operator A

$$< A^d >=< \psi| \times A^d \times |\psi >\in C^d m \quad (129)$$

Heisenberg-Santilli Isodual Equations

$$i^d \times^d \frac{d^d A^d}{d^d t^d} = [A, H]^d = (A \times H - H \times A)^d, \quad (130)$$

Schrödinger-Santilli Isodual Equations

$$H^d \times^d |\psi^d >= E^d \times^d |\psi^d >= -E \times |\psi > \quad (131)$$

$$p^d \times^d |\psi^d >= +i^d \times^d \partial_{rd}^d |\psi^d > \quad (132)$$

Dirac-Santilli Isodual Equation

$$(\eta^{d\mu\nu} \times^d \gamma_\mu^d \times^d p_\nu^d + i^d \times^d m^d \times^d c^d) \times |\psi >= 0. \quad (133)$$

$$\{\gamma_\mu, \gamma_\nu\}^d = (\gamma_\mu \times \gamma_\nu + \gamma_n u \times \gamma_\mu)^d = 2^d \times^d \eta_{\mu\nu}^d, \quad (134)$$

Comments and References

In addition to the the study of irreversible processes and the representation of extended-deformable particles, during his Ph. D. studies of the md 1960s Santilli was interested to ascertain whether a far away galaxy is made up of matter or of antimatter. He soon discovered that none of the mathematics and physics he had learned during his graduate studies was applicable for a quantitative study of the problem considered since, at that time, antimatter was solely represented in second quantization, while the study of far away antimatter galaxies requested their representation at the purely *classical and neutral* level. In this way, Santilli initiated a solitary scientific journey that lasted for half a century.

This occurrence created one of the biggest imbalances in scientific history because matter was treated at all possible levels, from Newtonian mechanics to second quantization, while antimatter was solely treated in second quantization. The imbalance originated from the fact that special and general relativities had been conceived decades before the discovery of antimatter and, therefore, they had no possibility of representing antimatter at the classical and neutral (as well as charged) level.

It should be stressed that the ongoing trend to extend the application of special and general relativities to the classical treatment of antimatter is afflicted by a number of serious inconsistencies, such as the impossibility to achieve a consistent representation of neutral antimatter, the impossibility to reach a consistent representation of matter-antimatter annihilation (evidently due to the lack of a suitable conjugation from matter to antimatter), violation of the PCT theorem and other inconsistencies that remain generally ignored.

Being an applied mathematician by instinct and training, Santilli knew that the imbalance was the result of a purely mathematical insufficiency because the transition from matter to antimatter is an anti-homomorphism. Consequently, the description of antimatter required a mathematics which is anti-homomorphic to conventional mathematics.

Santilli dedicated a decade to the search of the needed mathematics for antimatter. Following an additional extended search done at the Department of Mathematics of Harvard University under DOE support in the early 1980s, *Santilli concluded that a mathematics suitable for the joint classical and operator treatment of antimatter did not exist and had to be constructed.*

In the early 1980s, Since he had introduced the isoproduct $A \hat{\times} B = A \times \hat{T} \hat{B}, \hat{T} > 0$, Eq. (25). Consequently, it was natural to introduce its *negative-definite* counterpart which he called *isodual* and denoted with theupper index ^{d}, namely $A \hat{\times}^d B = A \times \hat{T}^d \hat{B}, \hat{T}^d = (\hat{T}^d)^\dagger < 0$. While constructing the isotopies of 20th century mathematics presented in Section 2, Santilli initiated the construction of their isodual image but published no paper in the new mathematics for over a decade.

This caution was due to the fact that, despite the lack of any visible mathematical inconsistency, Santilli remained skeptical on a mathematics based on a negative-definite product is afflicted by known physical inconsistencies, such as the violation of causality for negative time, energies and other physical quantities.

A breakthrough occurred in paper [2] of 1993. During the achievement of the broadest possible realizations of the abstract axioms of a numeric field (of characteristic zero), Santilli discovered that realizations with negative-definite units were simply unavoidable. This lead to the discovery of additional new numbers, today known as *Santilli isodual real, isodual complex and isodual quaternionic numbers* occurring for $I^d = -1$, Eq. (14), with isodual products (5), which are at the foundation of the isodual mathematics of this section and the additional numbers known as *Santilli isodual iso- and isodual geno-real, complex and quaternionic numbers* which are at the foundation of the isodual isomathematics and isodual genomathematics of Sections **7** and 8m respectively [2].

The discovery of isodual numbers is truly historical in our view due to its far reaching implications. In fact, the discovery established the existence of the desired *isodual mathematics* as an anti-isomorphic image of 20th century mathematics for the representation of antimatter. Additionally, the discovery permitted the resolution of the problems of causality for negative values of physical quantities.

To avoid insidious inconsistencies generally not seen by non-experts in the field, the isodual map must be applied for consistency to the *totality* of quantities and their operations. This lead to Santilli's conception of antimatter as possessing it

negative-definite physical quantities for time, energy, momentum, frequency, etc, but such negative values are referred to *negative units* of measurements. Consequential a theory with negative time referred to negative units of time is as causal as our reality with a positive time referred to positive units, and the same holds for all other physical quantities.

Following the resolution of these basic issues, Santilli published in 1994 his first paper [36] specifically devoted to the isodual representation of antimatter. In mathematical memoir [3] of 1996, Santilli achieved the first isodual mathematical and physical representation of antimatter. In paper [37] of 1998, Santilli achieved his first goal of the early 1960s, namely, a consistent classical representation of neutral (as well as charged) antimatter.

By the early 1990s, Santilli had shown that *isodual mathematics represents all available experimental, data on antimatter at the classical and operator level.* Hence, he initiated the second phase of his studies, namely, the identification of new predictions for subsequent experimental verification.

A breakthrough occurred at the 1996 *First International Conference on Antimatter* help in Sepino, Italy [38]. By that time, Santilli had shown that the only conceivable representation of *neutral* antimatter required the conjugation of the sign of all physical quantities (jointly with the corresponding conjugation of their units of measurements). Since photons are neutral, the application of the same principle to light implies light emitted by antimatter, that Santilli called *isodual light,* is physically different than light emitted by matter in an experimentally verifiable way, e.g., because antimatter light is predicted to be *repelled* by a matter gravitational field.

Santilli then passed to a deeper geometric study of the gravitational field of antimatter. As indicated earlier, general relativity was formulated decades before the discovery of antimatter and, therefore, had no clue for the representation of the gravitational field of antimatter bodies. In Ref.[39] of 1998, Santilli conducted an in depth geometric study of antimatter, and in monograph [40) of 2006, Santilli completed the gravitational study of antimatter via the isodual Riemannian geometry.

All these studies concluded with the prediction of *gravitational repulsion* (antigravity) between matter and antimatter at all levels of analysis, from the isodual Newton-Santilli equations (26) to isodual second quantization. These aspects will be studied in the second collection of this series dedicated to hadronic mechanics.

Thanks to all the above advances, Santilli was finally in a position to address his original main aim of the 1960s, namely, ascertain whether a far away galaxy is made up of matter or of antimatter. The preceding studies had established that the light emitted by antimatter must have a *negative index of refraction* that, as such, require *concave* lenses for its focusing. Consequently, Santilli secured the construction of a revolutionary telescope with concave lenses. About fifty years following his original aim, Santilli finally published in 2013 [41[measurements of the night sky with his new telescope showing images that can be solely due to light with a negative index of refraction which light, in turn, can solely originate from far away antimatter stars or galaxies (see also the two independent confirmations [42,43]).

An intriguing aspect that should be of interest to pure mathematicians is the conclusion of these studies illustrating the power of new mathematics, to the effect that none of the large numbers of telescopes available nowadays can detect antimatter starsor galaxies since they all have *convex* lenses. Similarly, as humans evolved in a matter world, we will never be able to see antimatter with our eyes since our cornea is convex and, as such, it will disperse antimatter light all over the retina.

Needless to say, isodual mathematics and its application to antimatter have implications so intriguing that are stimulating the participation of a large number of scientists as we shall report in the second collection of this series

7. Isodual Isomathematics, Isodual Isomechanics and Isodual Isochemistry

7.1. Represented Systems [2,36-43

Single-value, closed-isolated, time-reversible system of classical or operator extended-deformable antiparticles with action at a distance Hamiltonian and contact non-Hamiltonian interactions, such as the structure of antimatter hadrons, nuclei and stars, in the antimatter valence electron bonds and other antimatter systems.

7.2. Main Mathematical Structure[2,36-43

Basic Isodual IsoUnit

$$\hat{I}^d = \hat{I}^d(r^d, p^d, a^d, \psi, {}^d\partial^d\psi^d, \dots.) = 1^d/{}^d\hat{T}^d < 0, \quad (135)$$

Basic Isodual IsoFields

$$\hat{F}^d(\hat{n}^d, \hat{\times}^d, \hat{I}^d), \hat{n}^d = n \times \hat{I}^d, \hat{n}^d \,\hat{\times}^d\, \hat{m}^d = \hat{n}^d \times \hat{T}^d \times \hat{m}^d \in \hat{F}^d, \quad (136)$$

Isodual IsoCoordinates $\hat{r}^d = r \times \hat{I}^d \in \hat{F}^d$,

Isodual IsoFunctional analysis $\hat{f}^d(\hat{r}^d) == f(\hat{r}^d) \times \hat{I}^d \in \hat{F}^d$,

Isodual IsoDifferential Calculus

$$\hat{d}^d\hat{r}^d = dr - r^d \times \hat{T}^d \times d\hat{I}^d, \quad (137)$$

$$\frac{\hat{\partial}^d \hat{f}^d(\hat{r})}{\hat{\partial}^d \hat{r}^d} = \hat{I}^d \times \frac{\partial \hat{f}^d(\hat{r}^d)}{\partial \hat{r}^d}, \quad (138)$$

Santilli Isodual Lie-Isotopic Theory

$$[X_i, \hat{X}_j]^d = X_i \,\hat{\times}\, X_j - X_j \,\hat{\times}\, X_i)^d == -C_{ij}^k(r, p, \dots) \times X_k, \quad (139)$$

$$A^d(w^d) = \hat{e}_d^{X^d \times w^d \times i^d} \,\hat{\times}^d\, A^d(0^d) \,\hat{\times}^d\, \hat{e}_d^{-i^d \times w^d \times x^d}. \quad (140)$$

Santilli Isodual Iso-Euclidean Geometry

$$\hat{E}^d(\hat{r}^d, \hat{\delta}^d, \hat{I}^d), \hat{\delta}^d(r^d, p^d, a^d, \psi, \dots) =$$

$$\hat{T}^d(r^d, p,^d a^d, \psi^d, \ldots) \times \delta, \quad (141)$$

$$\hat{T}^d = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2)^d, \quad (142)$$

$$\hat{r}^{d\hat{2}d} = (\hat{r}^i \hat{\times} \hat{\delta}_{ij} \hat{\times} \hat{r}^j)^d = (\frac{r_1^2}{n_1^2} + \frac{r_2^2}{n_2^2} + \frac{r_3^2}{n_3^2})^d \times \hat{I}^d \in \hat{F}^d, \quad (143)$$

Santilli Isodual Iso-Minkowskian Geometry ($\mu = 1,2,3,4$)

$$\hat{M}^d(\hat{x}^d, \hat{\eta}^d, \hat{I}^d): \hat{x}^d = (\hat{x}^{d\mu}), \hat{x}_4^d = \hat{t}^d = t \times \hat{I}^d, \quad (144)$$

$$\hat{\eta}^d(x^d, \psi^d, \ldots) = \hat{T}^d(x^d, \psi^d, \ldots) \times \eta, \quad (145)$$

$$\hat{T}^d = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2)^d, \quad (146)$$

$$\hat{x}^{d\hat{2}d} = (\hat{x}^\mu \hat{\times} \hat{\eta}_{\mu\nu} \hat{\times} \hat{x}^\nu)^d == (\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2})^d \times \hat{I}^d \in \hat{F}^d, \quad (147)$$

Santilli Isodual Iso-Riemannian Geometry

$$\hat{R}^d(\hat{x}^d, \hat{g}^d, \hat{I}^d): \hat{g}^d = \hat{T}^d(x^d, v^d, \ldots) \times g(x), \quad (148)$$

$$\hat{x}^{d\hat{2}d} = (\frac{g_{11}}{n_1^2} + \frac{g_{22}}{n_2^2} + \frac{g_{33}}{n_3^2} - \frac{g_{44}}{n_4^2})^d \times \hat{I}^d \in \hat{F}^d, \quad (149)$$

Santilli Isodual Iso-Symplectic Geometry

$$\hat{\omega}^d = \hat{d}\hat{r}^{dk} \hat{\wedge}^d \hat{d}\hat{p}_k^d \quad (150)$$

7.3. Isodual IsoDynamical IsoEquation[2,36-43

Newton-Santilli Isodual IsoEquation

$$\hat{m}^d \hat{\times}^d \frac{\hat{d}^d \hat{v}^d}{\hat{d}\hat{t}^d} - F^{dSA}(r^d, p^d) == (m \times \frac{dv}{dt})^d - F^{dSA}(r^d, p^d) - F^{dNSA}(r^d, p^d, \ldots) = 0^d = 0, \quad (151)$$

Isodual IsoVariational principle

$$\hat{\delta}^d \hat{A}^d = \hat{\delta}^d \hat{\int}^d (\hat{p}_k^d \hat{\times}^d \hat{d}^d hatr^{dk} - \hat{H}^d \hat{\times}^d \hat{d}^d \hat{t}^d) = 0^d = 0. \quad (152)$$

Hamilton-Santilli Isodual IsoEquations

$$\frac{\hat{d}^d \hat{r}^{dk}}{\hat{d}^d \hat{t}^d} = \frac{\hat{\partial}^d \hat{H}^d(\hat{r}^d, \hat{p}^d)}{\hat{\partial}^d \hat{p}_k^d}, \quad \frac{\hat{d}\hat{p}_k}{\hat{d}\hat{t}^d} = + \frac{\hat{\partial}^d \hat{H}^d(\hat{r}^d, \hat{p}^d)}{\hat{\partial}^d \hat{r}^{dk}}, \quad (153)$$

Isodual iso-Hilbert space $\hat{H}^d$ over C with states $|\hat{\psi}^d >= -< \hat{\psi}|$ over $\hat{C}^d$

Expectation value of a Hermitean operator A

$$< A^d >=< \hat{\psi}| \hat{\times} A^d \hat{\times} |\hat{\psi} > \in C^d \quad (154)$$

Heisenberg-Santilli Isodual IsoEquation

$$\hat{\imath}^d \hat{\times}^d \hat{d}^d hatA^d over \hat{d}^d \hat{t}^d = [\hat{A}, \hat{H}]^d = (\hat{A} \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{A})^d = (\hat{A} \times \hat{T}(\hat{\psi}, \hat{\partial}\hat{\psi}, \ldots) \times \hat{H}(\hat{r}, \hat{p}) - \hat{H}(\hat{r}, \hat{p}) \times \hat{T}(\hat{\psi}, \hat{\partial}\hat{\psi}, \ldots) \times \hat{A})^d, \quad (155)$$

Schrödinger-Santilli Isodual IsoEquation

$$(\hat{H} \hat{\times} |\hat{\psi} >)^d =< \hat{\psi}^d| \hat{\times}^d \hat{H}^d = (\hat{H}(\hat{r}, \hat{p}) \times \hat{T}(\hat{\psi}, \hat{\partial}\hat{\psi}, \ldots) \times |\hat{\psi} >)^d = -< i\hat{\psi}^d| \hat{\times}^d \hat{E}^d = -< \hat{\psi}^d| \times \hat{E}^d, \quad (156)$$

$$(\hat{p} \hat{\times} |\hat{\psi} >)^d =< \hat{\psi}^d| \hat{\times}^d \hat{\partial}_{\hat{r}^d} = -i \times< \hat{\psi}^d| \hat{\times}^d \hat{\partial}_{\hat{r}^d}^d, \quad (157)$$

Dirac-Santilli Isodual IsoEquation

$$[(\hat{\eta}^{\mu\nu} \hat{\times} \hat{\gamma}_\mu \hat{\times} \hat{p}_\nu - \hat{\imath} \hat{\times} \hat{m} \hat{\times} \hat{c}) \hat{\times} |\hat{p}si >]^d = 0. \quad (158)$$

$$\{\hat{\gamma}_\mu, \hat{\gamma}_\nu\}^d = (\hat{\gamma}_\mu \hat{\times} \hat{\gamma}_\nu + \hat{\gamma}_\nu \hat{\times} \hat{\gamma}_\mu)^d = \hat{2}^d \hat{\times}^d \hat{\eta}_{\mu\nu}^d, \quad (159)$$

Comments and References

See monograph [40] with particular reference to the use of the isodual isomathematics for the achievement of a grand unification of electroweak and gravitational interactions inclusive of matter and antimatter.

8. Isodual Genomathematics, Isodual Genomechanics and Isodual Genochemistry

8.1. Represented Systems [2,36-43

Single-valued, time-irreversible system of extended-deformable antiparticles under action at a distance Hamiltonian and contact non-Hamiltonian interactions, as occurring in antimatter nuclear reactions, antimatter biological structures and antimatter chemical reactions.

8.2. Main Mathematical Structure [2,36-43

Backward Isodual GenoUnit

$$\hat{I}^{>c} = \hat{I}^{>d}(t^{>}r^{>}, p^{>d}, a^{>d}, \psi^{>d}, \partial^{>d}\psi^{>d}, \ldots.) = 1/\hat{T}^{>d} > 0, \quad (160)$$

Forward Isodual GenoUnit

$$^{<d}\hat{I} =^{<d} \hat{I}(^{<d}r,^{<d} p,^{<d} a,^{<d} \psi,^{<d} \partial^{<d}\psi, \ldots.) = 1/^{<d}\hat{T} > 0, \quad (161)$$

Condition for time-irreversibility

$$\hat{I}^{>d} \neq {}^{<d}\hat{I} \quad (162)$$

Backward Isodual GenoFields

$$\hat{F}^{>d}(\hat{n}^{>d}, >, \hat{I}^{>d}), \hat{n}^{>d} = n \times \hat{I}^{>d}, \hat{n}^{>d} >^d \hat{m}^{>d} = \hat{n}^{>d} \times \hat{I}^{>d} \times \hat{m}^{>d} \in \hat{F}^{>d}, \quad (163)$$

Forward Isodual GenoFields

$$^{<d}\hat{F}(^{<d}\hat{n}, <,^{<d} \hat{I}), \quad ^{<d}\hat{n} =^{<d} \hat{I} \times n, \quad ^{<d}\hat{n} <^d \quad ^{<d}\hat{m} = =^{<d} \hat{n} \times^{<d} \hat{I} \times^{<d} \hat{m} \in^{<d} \hat{F}, \quad (164)$$

Backward Isodual GenoCoordinates

$$\hat{r}^{>d} = r \times \hat{I}^{>d} \in \hat{F}^{>d}, \quad (165)$$

Forward Isodual GenoCoordinates

$$^{<d}\hat{r} =^{<d} \hat{I} \times r \in^{<d} \hat{F}, \quad (166)$$

Backward Isodual GenoFunctional analysis

$$\hat{f}^{>d}(\hat{r}^{>d}) = f(\hat{r}^{>d}) \times \hat{I}^{>d} \in \hat{F}^{>d}, \quad (167)$$

Forward Isodual GenoFunctional analysis

$$^{<d}\hat{f}(^{<d}\hat{r}) = f(^{<d}\hat{r}) \times^{<d} \hat{I} \in^{d} \hat{F} \quad (168)$$

Backward Isodual GenoDifferential Calculus

$$\hat{d}^{>d}\hat{r}^{>d} = dr + r \times \hat{T}^{>d} \times d\hat{I}^{>d}, \quad (169)$$

$$\frac{\hat{\partial}^{>d}\hat{f}^{>d}(\hat{r}^{>d})}{\hat{\partial}^{>d}\hat{r}^{>d}} = \hat{I}^{>d} \times \frac{\partial \hat{f}^{>d}(\hat{r}^{>d})}{\partial \hat{r}^{>d}}, \quad (170)$$

Forward Isodual GenoDifferential Calculus

$$^{<d}\hat{d}^{<d}\hat{r} = dr + r \times^{<d} \hat{T} \times d^{<d}\hat{I}, \quad (171)$$

$$\frac{^{<d}\hat{\partial}^{<d}\hat{f}(^{<d}\hat{r})}{^{<d}\hat{\partial}^{<d}\hat{r}} =^{<dd} \hat{I} \times \frac{\partial^{<d}\hat{f}(^{<d}\hat{r})}{\partial^{<d}\hat{r}}, \quad (172)$$

Santilli Isodual Lie-Admissible Theory

$$(\hat{X_i}\hat{X_j})^d = (X_i < X_j - X_j > X_i)^d = -C_{ij}^{dk}(t^d, r^d, p^d, \psi^d, \dots) \times X_k, \quad (173)$$

$$A^d(w^d) = \hat{e}_{>d}^{X \times w \times i} >^d A(0) <_{<d}^{d} \hat{e}^{-i \times w \times X}. \quad (174)$$

Santilli Backward Geno-Euclidean Geometry

$$\hat{E}^{>d}(\hat{r}^{>d}, \hat{\delta}^{>d}, \hat{I}^{>d}), \hat{\delta}^{>d}(t,r,p,\psi,\dots) = \hat{T}^{>d}(t,r,p,\psi,\dots) \times \delta, \quad (175)$$

$$\hat{r}^{>d\hat{2}d} = (\hat{r}^{>di} >^d \hat{\delta}_{ij}^{>d} > \hat{r}^{>dj} =\in F^{>d}, \quad (176)$$

$$\hat{T}^{>d} \neq \hat{T}^{>d\,transp} \quad (177)$$

Santilli Forward Isodual Geno-Euclidean Geometry

$$^{<d}\hat{E}(^{<d}\hat{r}, ^{<d}\hat{\delta}, ^{<d}\hat{I}), \quad ^{<d}\hat{\delta}(t,r,p,\psi,\dots) =^{<d} \hat{T}(t,r,p,\psi,\dots) \times \delta, \quad (178)$$

$$^{<d\hat{2}d}\hat{r} =^{<di} \hat{r} <^{<d} \hat{\delta}_{ij} <^{d} \quad ^{<dj}\hat{r} \in^{<d} F, \quad (179)$$

$$^{<d}\hat{T} \neq^{<d\,transp} \hat{T} \quad (180)$$

Santilli Backward Isodual Geno-Minkowskian Geometry ($\mu = 1,2,3,4$)

$$\hat{M}^{>d}(\hat{x}^{>d}, \hat{\eta}^{>d}, \hat{I}^{>d}): \hat{x}^{>d} = (\hat{x}^{>d\mu}), x_4^{>d} = t^{>d}, \quad (181)$$

$$\hat{\eta}^{>d}(x,\psi,\dots) = \hat{T}^{>d}(x,\psi,\dots) \times \eta, \quad (182)$$

$$\hat{x}^{>d2d} = \hat{x}^{>d\mu} >^d \hat{\eta}_{\mu\nu}^{>d} >^d \hat{x}^{>d\nu} \in \hat{F}^{>d}, \quad (183)$$

$$\hat{\eta}^{>d} \neq \hat{\eta}^{>d\,transp} \quad (184)$$

Santilli Forward Isodual Geno-Minkowskian Geometry ($mu = 1,2,3,4$)

$$^{<d}\hat{M}(^{<d}\hat{x}, ^{<d}\hat{\eta}, ^{<d}\hat{I}): \quad ^{<d}\hat{x} = (\hat{x}^{\mu}), \quad ^{<d}x_4 =^{<d} t, \quad (185)$$

$$^{<d}\hat{\eta}(x,v,\dots) =^{<d} \hat{T}(x,v,\dots) \times \eta, \quad (186)$$

$$^{<d}\hat{x}^{<d2d} =^{<d\mu} \hat{x} <^{d} \quad ^{<d}\hat{\eta}_{\mu\nu} <^{d} \quad ^{<\nu}\hat{x} \in^{<d} \hat{F}, \quad (187)$$

$$^{<d}\hat{\eta} \neq^{<d\,transp} \hat{\eta} \quad (188)$$

Santilli Backward Isodual Geno-Riemannian Geometry

$$\hat{R}^{>d}(\hat{x}^{>d}, \hat{g}^{>d}, \hat{I}^{>d}): \hat{g}^{>d} = \hat{T}^{>d}(x,v,\dots) \times g(x), \quad (189)$$

$$\hat{x}^{>d2>} = x^{>d\mu} \overset{d}{.} \hat{g}_{\mu\nu}^{>d} >^d x^{>d\nu} \in \hat{F}^{>d} \quad (190)$$

$$\hat{T}^{>d} \neq \hat{T}^{>d\,transp} \quad (191)$$

Santilli Forward Isodual Geno-Riemannian Geometry

$$^{<d}\hat{R}(^{<d}\hat{x}, ^{<}\hat{g}, ^{<d}\hat{I}): \quad ^{<d}\hat{g} =^{<d} \hat{T}(x,v,\dots) \times g(x), \quad (192)$$

$$^{<d}\hat{x}^{<d2d} =^{<d\mu} \hat{x} <^{d} \quad ^{<d}\hat{g}_{\mu\nu} <^{d} \quad ^{<d\nu}\hat{x} \in^{<d} \hat{F}, \quad (193)$$

$$^{<d}\hat{g} \neq^{<d\,transp} \hat{g} \quad (194)$$

Santilli Backward Isodual Geno-Symplectic Geometry

$$\hat{\omega}^{>d} = \hat{d}^{>d}\hat{r}^{>dk}\hat{\wedge}^{>d}\hat{d}^{>d}\hat{p}_k^{>d} \quad (195)$$

Santilli Forward Isodual Geno-Symplectic Geometry

$$^{<d}\hat{\omega} =^{<d} \hat{d}^{<dl}\hat{r}^{<d}\hat{\wedge}^{<d}\hat{d}^{<d}\hat{p}_k \quad (196)$$

8.3. Isodual GenoDynamical GenoEquations [2,36-43

Newton-Santilli Backward Isodual GenoEquation

$$\hat{m}^{>d} > \frac{\hat{d}^{>d}\hat{v}^{>d}}{\hat{d}^{>d}\hat{t}^{>d}} - F^{>dSA}(t,r,p) = [m \times \frac{dv}{dt}]^{>d} - F^{SA>d}(t,r,p) - F^{NSA>d}(t,r,p,\dots) = 0, \quad (197)$$

Newton-Santilli Forward Isodual GenoEquation

$$^{<d}\hat{m} < \frac{^{<d}\hat{d}^{<d}\hat{v}}{^{<d}\hat{d}^{<d}\hat{t}} -^{<dSA} F(t,r,p) =^{<d} [m \times \frac{dv}{dt}] -^{<dSA} F(t,r,p) -^{<dNSA} F(t,r,p,\dots) = 0, \quad (198)$$

Backward Isodual GenoVariational principle

$$\hat{\delta}^{>d}\hat{A}^{>d} = \hat{\delta}^{>d} \hat{\int}^{>d} (\hat{p}_k^{>d} > \hat{d}^{>d}\hat{r}^{>dk} - \hat{H}^{>d} > \hat{d}^{>d}\hat{t}^{>d}) = 0. \quad (199)$$

Forward Isodual GenoVariational principle

$$^{<d}\hat{\delta}^{<d}\hat{A} =^{<d} \hat{\delta}^{<d} \hat{\int} (^{<d}\hat{p}_k <^{<d} \hat{d}^{<d}\hat{r}^k -^{<d} \hat{H} <^{<d} \hat{d}^{<d}\hat{t}) = 0. \quad (200)$$

Backward Isodual Hamilton-Santilli GenoEquations

$$\frac{\hat{d}^{>d}\hat{r}^{>dl}}{\hat{d}^{>d}\hat{t}^{>d}} = [\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{p}_k}]^{>d}, \quad [\frac{\hat{d}^{>d}\hat{p}_k^{>d}}{\hat{d}^{>d}\hat{t}^{>d}}] = -[\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{r}^k}]^{>d}, \quad (201)$$

Forward isodual Hamilton-Santilli GenoEquations

$$\frac{^{<d}\hat{d}^{<d}\hat{r}^k}{^{<d}\hat{d}^{<d}\hat{t}} =^{<d} \frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{p}_k}], \quad \frac{^{<d}\hat{d}^{<d}\hat{p}_k}{^{<d}hatd^{<d}\hat{t}}] = -^{<}[\frac{\hat{\partial}\hat{H}(\hat{r},\hat{p})}{\hat{\partial}\hat{r}^k}], \quad (202)$$

Heisenberg-Santilli IsoDual GenoEqutions

$$\hat{\imath} \hat{\times} \frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = (\hat{A}, \hat{H}) = \hat{A} < \hat{H} - \hat{H} > \hat{A} = A \times^{<} T(\hat{\psi}, \hat{\partial}\hat{\psi}, \dots) \times \hat{H}(\hat{r},\hat{p}) - \hat{H}(\hat{r},\hat{p}) \times \hat{T}^{>}(\hat{\psi}, \hat{\partial}\hat{\psi}, \dots) \times \hat{A} \quad (203)$$

Schrödinger-Santilli Backward Isodual GenoEquations

$$\hat{H}^{>d} >^d |\hat{\psi}^{>d} >= \hat{H}^{>d}(\hat{r},\hat{p}) \times \hat{T}^{>d}(\hat{\psi}, \hat{\partial}\hat{\psi}, \dots) \times |\hat{\psi}^{>} >=$$

$$\hat{E}^{>d} >^{d} |\hat{\psi}^{>d} >= E^{>d} \times |\hat{\psi}^{>d} >, \quad (204)$$

$$\hat{p}^{>d} > |\hat{\psi}^{>d} >= -\hat{\imath}^{>d} > \hat{\partial}_{\hat{r}}^{>d} |\hat{\psi}^{>} >= -i \times \hat{I}^{>} \times \partial_{\hat{r}} |\hat{\psi}^{>d} >, \quad (205)$$

Schrödinger-Santilli Forward Isodual GenoEquations

$$<^{<d} \hat{\imath}\hat{\psi}| <^{d} \quad <^{<d} \hat{H} = <^{<d} \hat{\imath}\hat{\psi}| \times^{<d} \hat{T}(\hat{\psi}, \hat{\partial}\hat{\psi}, \ldots) \times^{<} \hat{H}(\hat{r}, \hat{p}) = <^{<d} \hat{\imath}\hat{\psi}| <^{<} \hat{E} = <^{<d} \hat{\psi}| \times^{<d} E, \quad (206)$$

$$<^{<d} \hat{\psi}| <^{<d} \hat{p} = -<^{<d} \hat{\psi}| <^{d} \quad <^{d} \hat{\imath} <^{d} \quad \hat{r}^{<d} \hat{\partial} = -i \times <^{<d} \hat{\psi}| \hat{r}^{<d} \partial \times^{<d} \hat{I} \quad (207)$$

Dirac-Santilli Backward Isodual IsoEquation

$$\left(\hat{\eta}^{>d\mu\nu} >^{d} \hat{\gamma}_{\mu}^{>d} >^{d} \hat{p}_{\nu}^{>d} - \hat{\imath}^{>d} > \hat{m}^{>d} > \hat{c}^{>d}\right) > |\hat{p}s\hat{\imath}^{>d} >= 0. \quad (208)$$

$$\{\hat{\gamma}_{\mu}, \hat{\gamma}_{\nu}\}^{>d} = [\hat{\gamma}_{\mu} \hat{\times} \hat{\gamma}_{\nu} + \hat{\gamma}_{\nu} \hat{\times} \hat{\gamma}_{\mu}]^{>} = \hat{2}^{>d} > \hat{\eta}_{\mu\nu}^{>d}, \quad (209)$$

Dirac-Santilli Forward Isodual GenoEquation

$$<^{<d} \hat{\psi}| < (<^{d} \hat{p}_{\nu} <^{<d} \hat{\gamma}_{\mu} <^{d} \quad <^{d\mu\nu} \hat{\eta} - <^{\hat{\imath}} <^{d} \quad <^{d} \hat{m} <^{d} \quad <^{\hat{c}}) = 0. \quad (210)$$

$$<^{d}\{\hat{\gamma}_{\mu}, \hat{\gamma}_{\nu}\} = <^{d} [\hat{\gamma}_{\mu} \hat{\times} \hat{\gamma}_{\nu} + \hat{\gamma}_{\nu} \hat{\times} \hat{\gamma}_{\mu}] = <^{d} \hat{2} <^{<} \hat{\eta}_{\mu\nu} = 2 \times^{<d} \hat{\eta}_{\mu\nu}, \quad (211)$$

Comments and References

See memoir [20] which constitutes the most comprehensive study of antimatter in irreducible conditions available at this writing.

9. Isodual Classical and Hope Isodual Hypermathematics

Isodual Hyper-Formulations are generally considered to be part of the Hyper-Formulations of Section 4 and 5 because the classification of ordered sets of hyperunits includes isodual realizations, as illustrated in the paper [44] and references quoted therein.

10. Simple Method for the Construction of Regular Hadronic Mathematics

10.1. Introduction [4.5]

Hadronic formulations are called *regular* when the *structure quantities* C_{ij}^{i} of Santilli's Lie-Isotopic algebras, Eqs. (3), Lie-admissible algebras, Eqs. (69) (zzz) and their isoduals, Eqs. (119-, (139), are *constant.* When the structure quantities are *functions of the local variables* $C_{ij}^{k}(t, r, p, \psi, \partial\psi, \ldots)$, hadronic formulations are called *irregular.*

In this section, we shall review a very simple method for the construction of regular hadronic formulations via the mere use of non-unitary transformations of the corresponding conventional formulations. We shall then review the axiomatic consistency of hadronic formulations by showing that Santilli iso-, geno-, hyper-units and their isoduals are invariant under the transformations, thus implying the crucial invariance over time of extended-deformable shapes and their non-Hamiltonian interactions that are invariantly represented precisely nwith such generalized units.

No method exists to our knowledge at this writing (June 2015) for the construction of irregular hadronic formulations via maps of conventional formulations and, therefore, irregular hadronic formulations characterize a new axiomatic structure still mostlyunexplored.

10.2. Simple Construction of Regular Iso-Formulations [4.5]

A simple method has been identified in Refs. [4,5] for the construction of the Lie-Santilli isotheory, all its underlying isomathematics and all physical methods This method is important because it permits a simple implementation of conventional models into their isotopic covering without the need for advanced mathematics. The method consists in:

(i) Representing all conventional potential interactions with a Hamiltonian $H(r, p)$ and all extended-deformable shapes and their non-Hamiltonian interactions and effects with Santilli's isounit $\hat{I}(r, p, \psi, \partial\psi, ./..)$;

(ii) Identifying the latter interactions with a nonunitary transform

$$U \times U^{\dagger} = \hat{I} \neq I \quad (212)$$

and

(iii) Subjecting the *totality* of conventional mathematical and physical quantities and all their operations to the above nonunitary transform, resulting in expressions of the type

$$I \to \hat{I} = U \times I \times U^{\dagger} = 1/\hat{T}, \quad (213)$$

$$a \to \hat{a} = U \times a \times U^{\dagger} = a \times U \times U^{\dagger} = a \times \hat{I}, a \in F, \quad (214)$$

$$e^{A} \to U \times e^{A} \times U^{\dagger} = \hat{I} \times e^{\hat{T} \times \hat{A}} = \left(e^{\hat{A} \times \hat{T}}\right) \times \hat{I}, \quad (215)$$

$$A \times B \to U \times (A \times B) \times U^{\dagger} = (U \times A \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times B \times U^{\dagger}) = \hat{A} \hat{\times} \hat{B}, \quad (216)$$

$$[X_i, X_j] \to U \times [X_i X_j] \times U^{\dagger} = [\hat{X}_i, \hat{X}_j] = U \times (C_{ij}^{k} \times X_k) \times U^{\dagger} = \hat{C}_{ij}^{k} \hat{\times} \hat{X}_k = C_{ij}^{k} \times \hat{X}_k, \quad (217)$$

$$< \psi| \times |\psi > \to U \times < \psi| \times |\psi > \times U^{\dagger} = < \psi| \times U^{\dagger} \times (U \times U^{\dagger})^{-1} \times U \times |\psi > \times (U \times U^{\dagger}) = < \hat{\psi}| \hat{\times} |\hat{\psi} > \times \hat{I}, \quad (218)$$

$$H \times |\psi > \to U \times (H \times |\psi >) = (U \times H \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times |\psi >) = \hat{H} \hat{\times} |\hat{\psi} >, etc. \quad (219)$$

Note that serious inconsistencies emerge in the event even 'one' single quantity or operation is not subjected to the above non-unitary map. In the absence of comprehensive liftings, we would have a situation equivalent to the elaboration of quantum spectral data of the hydrogen atom with isomathematics, resulting in large deviations from reality.

The construction of isodual iso-formulations is simply done via Santilli's isodual map, namely, via the simple anti-hermitean image of the above isotopic formulations.

10.3. Axiomatic consistency of Iso-Formulation [4.5]

Let us recall that Santilli's central assumption is the representation of extended-deformable shapes and their non-Hamiltonian interactions via the isounit. Therefore, any change of the numerical value of the isounit implies the inability to represent the same system over time, besides activating the *Theorem of Catastrophic Mathematical and Physical Inconsistencies of Non-Canonical and Non-Unitary Theories* when formulated via the mathematics of conventional canonical and unitary theories,respectively [23].

It is easy to see that the application of an additional nonunitary transform

$$W \times W^\dagger \neq I, \tag{220}$$

to the preceding expressions causes their *lack of invariance,* with consequential activation of the theorem of catastrophic inconsistencies. This is due to the *change of the value of the basic isounit* under additional non-unitary transformations

$$\hat{I} \to \hat{I}' = W \times \hat{I} \times W^\dagger \neq \hat{I}, \tag{221}$$

However, any given nonunitary transform can be identically rewritten in the isounitary form [3]

$$W \times W^\dagger = \hat{I}, \quad W = \hat{W} \times \hat{T}^{1/2}, \tag{222}$$

$$W \times W^\dagger = \hat{W} \hat{\times} \hat{W}^\dagger = \hat{W}^\dagger \hat{\times} \hat{W} = \hat{I}, \tag{223}$$

under which we have the invariance of the isounit and isoproduct [7]

$$\hat{I} \to \hat{I}' = \hat{W} \hat{\times} \hat{I} \hat{\times} \hat{W}^\dagger = \hat{I}, \tag{224}$$

$$\hat{A} \hat{\times} \hat{B} \to \hat{W} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{W}^\dagger = (\hat{W} \times \hat{T} \times \hat{A} \times \hat{T} \times \hat{W}^\dagger) \times (\hat{T} \times \hat{W}^\dagger)^{-1} \times \hat{T} \times (\hat{W} \times \hat{T})^{-1} \times (\hat{W} \times \hat{T} \times \hat{B} \times \hat{T} \times \hat{W}^\dagger) = \hat{A}' \times (\hat{W}^\dagger \times \hat{T} \times \hat{W})^{-1} \times \hat{B}' = \hat{A}' \times \hat{T} \times \hat{B}' = \hat{A}' \hat{\times} \hat{B}', \text{ etc.} \tag{225}$$

from which the invariance of the entire isotopic formalism follows.

Note that the invariance is ensured by the *numerically invariant values of the isounit and of the isotopic element under non-unitary-isounitary transformations,*

$$\hat{I} \to \hat{I}' \equiv \hat{I}, \tag{226}$$

$$A \hat{\times} B \to A' \hat{\times}' B' \equiv A' \hat{\times} B', \tag{227}$$

in a way fully equivalent to the invariance of Lie's theory and quantum mechanics, as expected to be necessarily the case due to the preservation of the abstract axioms under isotopies. The resolution of the inconsistencies for non-invariant theories is then consequential.

The proof of the invariance of Santilli isodual iso-formulations is an interesting exercise for non-initiated readers.

10.4. Simple Construction of Regular GenoMathematics and its IsoDual [4.5]

An important feature of the Lie-Santilli genotheory is its *form invariance* under the appropriate geno-transformations in a way fully similar to the invariance of the mathematical and physical structures of quantum mechanics under unitary transformations.

This feature can be shown via a *pair* of non-unitary transformations

$$V \times V^\dagger \neq I, W \times W^\dagger \neq I, V \times W^\dagger \neq I, W \times V^\dagger \neq I, \tag{228}$$

under which we have the characterization of the forward and backward genounits and related genoproduct

$$I \to V \times I \times W^\dagger = \hat{I}^>, eqno \tag{229}$$

$$A \times B \to V \times (A \times B) \times W^\dagger = A^> > B^> \tag{230}$$

$$I \to W \times I \times V =^< \hat{I}, \tag{231}$$

$$A \times B \to W \times (A \times B) \times V =^< A <^< B/ \tag{232}$$

10.5. Axiomatic Consistency of GenoMathematics and its Isodual [4.5]

It is easy to see that the above dual non-unitary transformations can always be identically rewritten as the *geno-unitary transforms* on geno-Hilbert spaces over complex genofields,

$$V \times V^\dagger \neq 1, V =^< \hat{V} \times \hat{R}^{1/2}, V \times V^\dagger =^< \hat{V} <^< \hat{V}^\dagger =^< \hat{V}^\dagger <^< \hat{V} =^< \hat{I}, \tag{233}$$

$$W \times W^\dagger \neq 1, W = \hat{W}^> \times \hat{S}^{1/2}, W \times W^\dagger = \hat{W}^> > \hat{W}^{>\dagger} = \hat{W}^{>\dagger} > \hat{W}^> = \hat{I}^>, \tag{234}$$

under which we have indeed the following forward geno-invariance laws [3j]

$$\hat{I}^> \to \hat{I}'^> = \hat{W}^> > \hat{I}^> > \hat{W}^{>\dagger} = \hat{I}^>, \tag{235}$$

$$\hat{A} > \hat{B} \to \hat{W}^> > (\hat{A} > \hat{B}) > \hat{W}^{>\dagger} = \hat{A}' > \hat{B}', \tag{236}$$

$$\hat{H}^> > | >= \hat{E}^> > | >= E \times | >\to \hat{W}^> > \hat{H}^> > | >= \hat{H}'^> > | >'= \hat{W}^> > \hat{E}^> > | >= E \times | >', \tag{237}$$

with corresponding rules for the backward and classical counterparts.

The above rules confirm the achievement of the *invariance of the numerical values of genounits, geno-products and geno-eigenvalues*, thus permitting physically consistent applications.

The invariance of the isodual geno-formulations can then be proved via the isodual map applied to the above procedure.

11. Open Mathematical Problems

Among a predictable large number of basic open problems, we list for the interested readers the following ones:

Study methods to transform nonlinear models on

conventional spaces into isolinear models on isospaces over isofields;

See whether simple solutions of isolinear equations on isospaces over isofields provide at least ä" solution of their nonlinear projection on conventional spaces over conventional fields;

Study the removal of divergencies in quantum mechanics and scattering theories (Footnote 2) by isomechanics on an iso-Hilbert space over an isofield.

Study the regular and irregular isorepresentations of the Lie-Santilli isotheory;

Study Santilli isoMinkowskian geometry via the machinery of the Riemannian geometry, yet lack of curvature [39];

Study the Lie-admissible theory in Santilli's sense, that is, as a generalization of Lie's theory elaborated via genomathematics;

Study Santilli geno-Euclidean, geno-Minkowskian and geno-Riemannian geometries where irreversibility is embedded in the non symmetric character of the metric [23];

extend the Tsagas, Ganformina-Nunez isotopology to the genotopic form and their isoduals.

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The Informational Magnecule: The Role of Aqueous Coherence and Information in Biological Dynamics and Morphology

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Abstract: Biological systems are dependent upon and intertwined with aqueous systems. We will present empirically derived evidence of the unique properties of water and demonstrate the efficacious role of molecular electromagnetic informational encoding as mediated through aqueous dynamics and mnemic properties. Working theory will then be articulated from quantum, thermodynamic and Hadronic aspects. An aqueous molecular species of dynamic magnecule will then be defined. Implications are drawn which point to a possible nontoxic, purely informational potential for future medical and pharmacological science. Magnecules and aqueous informational magnecular dynamics may one day redefine energy storage and production, as well as medical practice.

Keywords: Magnecule, Liquid Water, Aqueous Systems, Information, Electromagnetism

1. Introduction

Water is a highly complex and unique molecule which is basic to life as we understand it. However, at the very outset it is important to note that, as far as standard textbooks of chemistry are concerned, all theory is totally reliant on electrostatics and avoids all mention of electrodynamics and the consequent radiation field. It is this crucial point which contributes to the inability to recognize phenomena which are dependent on that field. In Hadronic science liquid water itself by way of its H bridges is understood as a magnecular structure, with a Curie temperature of 100°C (Santilli, 2005, 2008, 2012 and others). Interestingly and importantly, Santilli has pointed out in a private communication that much of his theory of magnecules could be derived via quantum electrodynamics instead of by the methods he espoused in his book *Foundations of Hadronic Chemistry* (2001). We will first articulate some of the special properties of water as they imply functional effects related to informational processes, EM fields, and aqueous dynamics within biology.

Water has many unique properties and is experimentally derivable as the primary interface with biological systems. Water state, pH, hydrogen bonding, and water magnetic 'memory' are affected by electromagnetism (Fesenko & Gluvstein 1995; Yamashita et al., 2003; Dunning-Davies 2012; Zhao et al., 2015). Extremely Low Frequency Electromagnetic Fields (ELF-EMF) affect water via alteration of the lower energy part of the stretching absorption band (~3250 cm^{-1}) relating to coherent fully hydrogen bonded populations (De Ninno & Castellano 2011). There is a great deal of evidence from experimental physics, chemistry and biology supporting the notion of water as a primary mediator of biological effects induced via electromagnetic means into living systems. Pollen germination, tobacco plant resistance to pathogens, seed hydration and germination, techniques such as photoluminescence spectroscopy used to determine the activity of pulsed fields on the "bubble/water interface," magnetic field effect simulation, and other experiments abound (Betti et al., 2011; Trebbi et al., 2007; Amyan 2004, 2004*a*, 2006; Vallée et al., 2005, 2005*a*; Chang & Weng 2006, 2008; Pang & Deng 2008). It seems clear from this

vantage, that water may well be the primary mnemic interface for electromagnetic/photonic quantum informational transfer from active compounds into biological systems. This effect takes place at room temperatures.

Quantum *entangled processes* and *informational exchange* are now known to be dynamic contributors in biological systems at room temperatures (Cai et al. 2010; Cifra et al. 2010; Rosi et al. 2011; Prasad et al. 2014) and by way of empirically rigorous Time Dependent Density Theory models, have been demonstrated as primary contributors to the evolution of life itself from photosynthetic prebiotic kernel systems in the Isua Greenstone Belt in Greenland some 3.7 - 3.85 billion years past (Tamulis et al. 2016; Norman and Tamulis 2016). There is further longstanding evidence of the delicate connectivity between photonic expression and biological processes. What is now known as the coherent biophoton field (please think of the life's work of Fritz Popp), was first discovered by Alexander Gurwitsch while working with onion roots in 1922 as "mitogenetic radiation" in the UV range, exemplifying his concept of "morphogenetic fields." In Popp (1999) we read: "...a single photon may suffice to trigger about 10^9 reactions per second since the average reaction time is of the order of 10^{-9} seconds and provided—in addition—that it is directed in a way that it delivers the right activation energy as well as the right momentum at the right time to the right place. This means that a surprisingly low photon intensity may suffice to trigger all the chemical reactions in a cell." Electromagnetic fields can be mathematically defined as informationally interactive with biological systems (Brizhik et al. 2003; Brizhik and Foletti 2014). Based upon this evidence, we conclude that given the correct conditions (at room temperatures) *photons can be informationally encoded and via quantum processes can and do affect biological systems.*

As we propose that (molecular) *information alone* can affect biological morphofunctional outcomes through aqueous memory, we must detail the history and mathematical evidence supporting the proposed notion of memory capacity in water.

2. History, Mathematics and General Theory in Support of Aqueous Memory and Coherent Biological Informational Distribution

Historical Background:

People have speculated for some time over whether substances, such as water, actually have a memory. However, it was in 1988 that a truly staggering article appeared in the journal *Nature* purporting to report the experimental observation of this property assumed by many to be merely an attribute of animals, particularly humans. The article in question (Davenas 1988) by a team, headed by Dr. Jacques Benveniste, claimed to have observed that extremely dilute biological agents were still capable of triggering relevant biological systems. In fact, they even claimed this to be so in the absence of actual physical molecules of the agents concerned. Some of the experiments had been reproduced in laboratories other than Benveniste's and members of these laboratories co-signed the article. However, this article provoked a flurry of comment and resulted in the experiments being rerun under the 'scientific' eyes of a fraud detector, a journalist and a magician. Presumably by 'a journalist' was meant the editor of *Nature*, but that person was by training a physicist and might have been expected to have had some elementary knowledge of information theory and that it had been applied to physical systems. Although a relatively old subject in its own right at that time, information theory had been coming into physics via such books as that of Brillouin (1962). It might have been thought by some that this fact would have introduced a more cautious note into some of the condemnation of Benveniste's work.

The article itself appeared in the issue of the journal for the 30th June 1988 and the ensuing furor was such that the then editor of *Nature* summed up his reading of the situation and called a halt to further correspondence in the issue of 27th October 1988, after allowing Dr. Benveniste a chance to answer his critics. What really caused the furor? The answer is best summed up by the 'Editorial Reservation' which appeared with the original article. This said that "readers of this article may share the incredulity of the many referees who have commented on several versions of it during the past several months. The essence of the result is that an aqueous solution of an antibody retains its ability to evoke a biological response even when diluted to such an extent that there is negligible chance of there being a single molecule in any sample. There is no physical basis for such an activity." In the later commentary, attention was drawn to the fact that one of the concerns of the editor of *Nature* was that the publication of the paper was "certain to excite the interest of the homeopathic community". Given this, therefore, it is surprising the article ever appeared in print, but appears it did even though it was stated there was no physical basis to explain the claimed phenomena.

It is this final statement which is now called into question with the appearance of an article purporting to give the biophysical basis of the Benveniste experiments (Widom et al., 2010). From this vantage, the general mechanism of Quantum Information Medicine may be implied.

Theoretical background:

The basis of information theory is now well-established. Following the approach of Brillouin (1962), if P denotes the number of states in a system, then the information memory capacity (denoted by I) in 'bits' is defined to be

$$I = \ln P, \tag{1}$$

where, if a problem is considered with N different independent selections, each corresponding to a binary choice (0 or 1), the total number of possibilities is

$$P = 2^N \tag{2}$$

and so the information is:

$$I = N\ln 2. \quad (3)$$

Alternatively, the entropy function of statistical thermodynamics is given by

$$S = k\ln P, \quad (4)$$

where k is Boltzmann's constant.

It follows that, for the above expression for P,

$$S = kln(2^N) = kN\ln 2 \quad (5)$$

Further, it may be noted that the first and second laws of thermodynamics may be combined into the equation

$$dU = TdS + d'W, \quad (6)$$

where dU denotes the internal energy change of the system, T the absolute temperature and $d'W$ the work done on or by the system. In terms of memory capacity, this becomes

$$dU = (kNT\ln 2)dN + d'W \quad (7)$$

and it is seen immediately that the energy required to add one bit of memory to the system is given by

$$kTln2 = \frac{\partial U}{\partial N} \quad (8)$$

where the partial derivative is evaluated with the work term held constant.

It might be noted that heat capacity is necessarily a positive quantity (Lavenda & Dunning-Davies 1990) and, therefore, this last equation leads to the realization (Widom et al., 2010) that a program written using ΔN bits of system memory dissipates energy of at least $[kNT\ln 2]\Delta N$. As noted previously, this constitutes an irreversible bound on a classical computation imposed by the second law of thermodynamics, although great care should always be exercised when applying results of classical thermodynamics in either statistical mechanics or information theory as it is not clear that the functions termed *entropy* in each of those three disciplines are always identical (Sands, 2016 and references cited there).

This brief introduction to some of the basic ideas of information theory and the link with statistical thermodynamics provides one part of the basis for the promotion of the idea that water possesses memory. The second part derives from a detailed study of some of the properties of water itself.

Properties of water:

Water is such a commonly available and apparently straightforward liquid that most take for granted and the popular picture, derived from standard chemistry, of it being composed of an oxygen atom attached to two hydrogen atoms belies a quite detailed, complex structure. Standard textbook chemistry has an enviable history of genuine scientific success but it is actually confined by a simple scheme of charges interacting via static Coulomb forces; that is, it is totally reliant on electrostatics and omits all mention of electrodynamics and the consequent radiation field. It is this basic neglect which is responsible for the inability to recognize phenomena which are, in fact, dependent on that radiation field. This is doubly unfortunate since physicists and engineers are only too aware of this cause and effect since it is due to this dynamical effect that so many modern-day appliances work; for example, the electric light on which we all depend and the wifi connections which are assuming increasing importance in our lives. It has been speculated that a goodish percentage of effects in condensed matter physics make use of the radiation field in one way or another but it still doesn't seem to have found a place in much of basic chemistry.

This paper [(Widom et al., 2010) and references cited there] draws attention to the fact that water has been shown to contain electric dipole ordered domains due to a condensation of photons interacting with molecular dipole moments. These ordered domains yield an unusually high heat of vaporisation of water per molecule and this has been shown to imply a high degree of memory storage capacity. In a similar manner, it has been shown that the partial entropy per molecule of an ionic species dissolved in an aqueous electrolyte implies a large number of bits of information per ion. This number is, in fact, so high as to lead to the expectation of such ions being attached to an ordered water domain. This state of affairs allows for semi-permeable membranes which may either permit or forbid the passage of an ion through a small gap. This would be expected to depend in part on the state of order in the ion attachment. Such a situation, based on information or, equivalently, entropy, indicates a program for biological cells analogous to polymer DNA based programs. It is ion flows through membranes in nerve cells which allow human memory storage in nerve cell networks in the human brain. These possess roughly the same magnitude for biological information capacity density and it well surpasses the comparable figure for commercial computer memory devices.

It should be noted also that the magnetic properties of water are again of great interest. In fact, a coherent ordered domain in water shows almost perfect diamagnetism, although the total diamagnetism in water is weak. This follows due to the magnetic flux tubes being capable of permeating normal water regions just as they can permeate type two superconductors via their normal regions. Trapped magnetic flux tubes may also carry information and give some directionality to what would otherwise be isotropic pure water.

The domains in water also exhibit a rotating electric dipole moment. If an electric field is applied, strings of electric dipole aligned water domains are formed and many such strings form a dipolar field bundle of strings. If the field is applied by employing a voltage between two electrodes then the bundle will start at one electrode and continue to the other. These strings will have an effect on the entropy and, therefore, on the information capacity of

the water memory. Further, according to the two fluid model of water structure, an ion could flow with virtually no friction through the bundle of strings from one electrode to the other.

Finally, it should be noted that, if the bundles of these strings are orthogonal to an applied magnetic field, ionic transport resonance effects can occur between the time varying part of the magnetic field and the cyclotron frequency associated with the uniform part of that field.

Implications:

It follows that the ordering of water through coherent domains yields sufficient structure for truly significant memory capacity. This view receives support from statistical thermodynamics and information theory. It is seen that ordered water domain polarized string bundles affect ionic motion and this can act as switches in networks of nerve cells. Many of these actions should be measurable by employing magnetic resonance imaging techniques.

What are the consequences of all this? To answer the objection: *"There appears to be no active chemical producing the effect,"* we need but remember the possibility of dynamic effects having a part to play, a point well illustrated by the case of a magnetic recording tape. In the investigation (Widom et al., 2010) it was found that, using electromagnetic theory, the existence of electromagnetic domains in water was confirmed. These are actually small ferro-electric structures within which electric fields are trapped. Hence, water is ferro-electric and it is this which is fundamentally responsible for many intriguing properties of water, including its memory. This general theoretic approach appears to be indicative of the likely mechanism responsible for the proposed mnemic effects within the idea of *Quantum Information Medicine*.

Nobel Laureate Dr. L. Montagnier and associates have suggested that quantum electromagnetic informational effects sustain many disease processes (Montagnier et al., 2011). These quantum processes involve the idea of a *Coherence Domain*, (CD). Please think of a CD as a dynamic aqueous structure, which uses the special properties of water, such as its electron dynamics and organized response to electromagnetic fields, to receive electromagnetically encoded information at a low frequency, and sum the resultant excitations, so as to foster the redistribution of that information at frequencies which may affect biological systems. A CD is a pool of quasi-free electrons, functioning as a semi-conductor, where coherent excitation creates a spectrum of coherent excited levels from resultant coherent quasi-free electron vortices, the magnetic dipoles of which are aligned with external/terrestrial magnetic fields. Coherent vortices have no internal friction and hence a long lifespan, so additive excitations sum to a vortex whose rotational frequency is the sum of the frequencies of the component vortices. The CD is thereby able to transform ambient noise, namely an ensemble of a large number of low frequency excitations, into a unique high frequency excitation (Del Giudice et al., 2013). "When the oscillation frequency of the CD matches the oscillation frequency of some non aqueous molecular species present on the CD boundaries, these "guest" molecules become members of the CD and are able to catch the whole stored energy, which becomes activation energy of the guest molecules; consequently, the CD gets discharged and a new cycle of oscillation could start" (Montagnier et al., 2011). Here, it appears we may have the mechanism whereby the correct frequencies (Brizhik 2003) for biological interactivity are achieved and distributed.

> "The CD is a self-produced cavity for the em field because of the well known Anderson-Higgs-Kibble mechanism . . . which implies that the photon of the trapped em field acquires an imaginary mass, becoming therefore unable to leave the CD. It is just this self-trapping of the em field that guarantees that the CD energy has a finite lower bound. Because of this self-trapping the frequency of the CD em field becomes much smaller than the frequency of the free field having the same wavelength. . . . In the case of liquid water, the CD . . . includes an ensemble of almost free electrons which are able to accept externally supplied energy and transform it into coherent excitations (vortices) whose entropy is much lower than the entropy of the incoming energy" (Montagnier et al., 2011).

In biological systems almost all water is within a fraction of a micron or less from a surface or molecular backbone and so is: *interfacial water*, which behaves in a quantum way, where the Coulomb law of electrostatics does not apply. In these circumstances, like charges attract. Biology itself depends on this, so as to allow the accumulation of tissues from negatively charged cell bodies (Del Giudice et al., 2013). Further depth will be afforded this conclusion in the Hadronic analysis section of this paper.

In (Heinze et al., 2013; de Riedmatten, 2013) we can see a simplified artificial example of what happens when encoded photons are trapped. Those trapped photons and their encoding are converted into collective coherent electron excitations within the medium: *spin waves*. Stored thus, the encoded information can then be retrieved. Please think of this same process as taking place within the more dynamic coherence domains of aqueous systems, which also act to further sum energy and frequency in order to distribute their stored information at appropriate target energies and frequencies to affect biological systems.

2.1. Experimental Evidence

Now that the basic aspects of the working theory have been articulated, we will turn to those replicable experiments which evidence the effects that the above mentioned theory describes. Although the unfortunate falsehoods of human history have left Benveniste's good name in shambles, his work itself is not rightly defined through such a-priori subjective devaluation, for science is or is not demonstrably, empirically correct: scientific truth being in all cases an objective proposition. We have collected replicable experiments demonstrative of just the sort of effects Benveniste had envisioned. Those experiments will be presented in highly condensed form here, then, a deeper underlying quantum/hadronic interpretation will be advanced.

It was claimed that Benveniste's work was false and could not be repeated. As referenced above, this appears to be an incorrect assertion. Although orthodox science shuts its eyes to the fact, the following experiments are now part of the valid scientific record (Norman, et al. 2016). We will select a small representative sample of the important work which has been conducted, and very briefly condense the results and conclusions.

1. In (Foletti et al. 2012) *Experimental Finding on the Electromagnetic Information Transfer of Specific Molecular Signals Mediated Through the Aqueous System on Two Human Cellular Models*, a 7Hz carrier frequency modulated at 3kHz is encoded with molecular information electromagnetically derived from retinoic acid, a known cell differentiation agent. The same expected effects of the actual molecule were evidenced from only the information with which it is associated, as demonstrated upon LAN-5 neuroblastoma and NT2/D1 stem teratocarcinoma cells in both the cell growth and morphology of cells seeded and cultured in aqueous informationally encoded preparations.

"Methods: Retinoic acid, a well-known chemical differentiating agent, was placed at room temperature in the input coil connected to an oscillator (VEGA select 719), while culture medium for human neuroblastoma cell (LAN-5) and NT2/D1 stem teratocarcinoma human cells was placed into the output coil and exposed to signals for 1 hour. At the end the oscillator was switched off and LAN-5 neuroblastoma and NT2/D1 stem teratocarcinoma cells were seeded, respectively, into the medium conditioned as reported into an incubator under controlled conditions. After 5 days of incubations, cells were examined by different strategies such as morphological and biochemical parameters.

Results: It was demonstrated that the electromagnetic signals coming from the retinoic acid molecule could be recorded and stored by the aqueous system of the cell culture medium. Cells seeded in the electronically conditioned medium received physical information generating a statistically significant decrease in metabolic activity and changes in phenotypical structure with protrusion typical of differentiated neuronal cells.

Conclusions: These experimental results provide some evidence that water could be tuned in a resonant manner by the Electro Magnetic Information Transfer procedure appropriately carried through a carrier frequency provided by the oscillator in a manner that seems related to the chemical structure of the source molecule as, in this case, retinoic acid."

2. In (Foletti et al. 2014) *Electromagnetic information delivery as a new tool in translational medicine,* we see a 7Hz carrier frequency modulated at 3 kHz was encoded with molecular information electromagnetically derived from retinoic acid, a known cell differentiation agent. The results: "LAN-5 neuroblastoma cell line was grown up for 4 days in standard medium (CTR) or in the presence of shielded retinoic acid signal (Shielded RA-ECM); Retinoic Acid molecule was used as positive control (RA). Cell proliferation was then analyzed by direct cell count. The results showed that LAN-5 cultured with the shielded electronically conditioned medium didn't present any changes in the proliferation rate compared to control.

Electromagnetic signals from Retinoic Acid do not affect cell viability

. . . reduction in cell proliferation rate is correlated with the electromagnetic information system, while it did not correlate with an increase in cell death. LAN-5 neuroblastoma cell line was grown up for 4 days in standard medium (CTR) or in the presence of Retinoic Acid signals (RA-ECM) while Retinoic Acid molecule was used as a positive control (RA). Cellular mortality was analyzed by Trypan blue exclusion test. . . . The results showed a sustained increase of cellular mortality in Retinoic Acid treated cells as compared to control ones. Moreover the cells cultured in the electronically conditioned medium, receiving physical electro-magnetic information from RA, displayed no differences in cellular mortality compared to control" and "Interestingly, cells grown in the presence of the electro-magnetic signal from RA (RA-ECM), showed a statistically significant decrease of cell growth, similarly to RA treatment, but no changes in cellular mortality . . . These findings suggest that the electromagnetic information system is able to induce the decrease of cell growth without affecting cell viability."

[Please do note the *presence* of active informational field effects demonstrably akin to the known biological activity of the source molecule, and the *absence* of associated chemical toxicity—the latter quite unlike the chemical molecule from which the information was derived.]

3. In (Foletti et al. 2011) "*Differentiation of human LAN-5 neuroblastoma cells induced by extremely low frequency electronically transmitted retinoic acid*," we see the same highly replicable results, this time using the field directly:

"METHODS: Retinoic acid was placed at room temperature on one coil attached to an oscillator (VEGA select 719), while LAN-5 neuroblastoma cells were placed on another coil and incubated under controlled condition. The oscillator was then turned on for 12 hours a day for 5 days, after which cells were counted and morphology studied by contrast microscopy.

RESULTS: The effect of the differentiating agent added to the cell culture by physical means generates a decrease in cell growth, metabolic activity, and the protrusion of a neuritelike structure typical of the differentiated cells.

CONCLUSIONS: These preliminary results suggest that retinoic acid molecules emit signals that can be transferred to LAN-5 neuroblastoma cells by artificial physical means in a manner that seems related to the chemical structure of the source molecules."

Just as important as these studies clearly demonstrating vital effects upon malignant cells, are others of equal reliability and replicability which demonstrate effective informational influence upon various types of infectious pathogens by way of extracted antibiotic molecular *information alone.* How potent is the effect? It works on many common, problematic infectious bacteriological pathogens, and also on the modern scourge of stubborn

treatment resistant MRSA!

4. In (Heredia-Rojas et al. 2015) Antimicrobial Effect Of Vancomycin Electro-Transferred Water Against *Methicillin-Resistant Staphylococcus Aureus* Variant, we may watch as MRSA is curtailed:

"Material and Methods: MRSA cultures were treated with vancomycin electro-transferred water samples, vancomycin (4.0 and 8.0 μg/mL), sham electro-transferred (water to water) and non-transferred water samples (medium alone). Growth inhibition was evaluated in liquid and solid culture medium, spectrophotometrically and by CFU determination respectively.

Results: The obtained data showed that by transferring vancomycin (4.0 and 8.0 μg/mL) information to water samples, the growth of cultured MRSA was significantly ($p< 0.05$) inhibited (up to 35%), compared with those cultures treated with electro-transferred water to water or cultured in medium alone (0% growth inhibition).

Conclusion: This in vitro study suggests that water samples that are electronically transferred with vibration sustained information of vancomycin are capable of inhibiting growth of axenically cultured *methicillin resistant S. aureus*."

5. In (Heredia-Rojas et al, 2011) *Entamoeba histolytica* and *Trichomonas vaginalis*: Trophozoite growth inhibition by metronidazole electro-transferred water, we see the same yet again, now working to ameliorate the proliferation of *Entamoeba histolytica* and *Trichomonas vaginalis*.

"This paper demonstrates that by transferring metronidazole information to water samples by an electronic amplifier (BRT device), the growth of axenically cultured trophozoites of *Entamoeba histolytica* and *Trichomonas vaginalis* is significantly inhibited, compared with those cultures treated with non and sham electro-transferred water samples. A positive control of metronidazole, a well known cytotoxic drug against parasites, was used as a reference."

"In conclusion, our in vitro study suggests that water samples that are electronically-transferred with vibrational information of metronidazole are capable of inhibiting trophozoite growth of axenically cultured *E. histolytica* and *T. vaginalis*."

6. In (Heredia-Rojas et al, 2012) "Antimicrobial effect of amphotericin B electronically activated water against *Candida albicans,"* we see the same informational effects demonstrated yet again upon a different pathogen.

"It was demonstrated that by transferring amphotericin B (125 $\mu g \cdot ml^{-1}$) information to water samples by an electronic amplifier (BRT device), the growth of cultured *Candida albicans* was significantly ($P < 0.05$) inhibited (46% growth inhibition), compared with those cultures treated with sham electro-activated water samples (0% growth inhibition), and a positive control of amphotericin B (125 $\mu g \cdot ml^{-1}$; 80% growth inhibition). Evidence for a measurable biological effect by electro-activated water samples that somehow acquires, or at least mimics, the antifungal property of amphotericin B has been demonstrated in the present study."

We invite the reader to examine closely each study referenced above in detail. Please recall that we have selected to represent here but a small representative sample of a larger body of work. [See: (Norman et al., 2016 and the references therein; Endler et al. 1995; Thomas et al., 2000).] Please examine the many various methods of evaluation used in the condensed studies above, including accurate measures such as reverse transcription PCR, contrast microscopy and others. Over and over the same effect is demonstrated. Benveniste was correct. The ugly accusations which ruined his career and good reputation may be left aside as false.

2.2. Magnecular Analysis and the H Bridge: Polarized Toroidal Orbits

It may be possible to apply a magnecular analysis and gain even deeper insight into the phenomena and effects we have demonstrated. Toward that end we will first articulate some of the particular specifications of magnecules.

". . . magnegas has a variable energy content, a variable specific weight, and a variable Avogadro number." (Santilli 2005 p. 101)

"Alternatively, the magnecular structure can be also interpreted as an unusual form of "semi-liquid" in the sense that the magnecular bond is much closer to the so called "H-bridges" of the liquid state of water. The increase of pressure evidently brings magnegas progressively closer to the liquid state, which continuous process can only occur for a variable Avogadro number." (Santilli 2005 p. 101)

"Magnecules have anomalous average atomic weights in the sense that they are bigger than that of any molecular constituent . . . ." (Santilli 2005 p. 23)

"Santilli magnecules in gases, liquids, and solids consist of stable clusters composed of conventional molecules, and/or dimers, and/or individual atoms bonded together by opposing magnetic polarities of toroidal polarizations of the orbits of at least the peripheral atomic electrons when exposed to sufficiently strong external magnetic fields, as well as the polarization of the intrinsic magnetic moments of nuclei and electrons." (Santilli 2005 p. 21)

"Magnecules can break down into fragments under sufficiently energetic collisions, with subsequent recombination with other fragments and/or conventional molecules, resulting in variations in time of spectrographic peaks (called time mutations of magnecular weights)" (Santilli 2005 p. 22)

"Substances with magnecular structure have anomalous physical characteristics, such as anomalous specific density, viscosity, surface tension, etc., as compared to the characteristics of the conventional molecular constituents" (Santilli 2005 p. 23).

"Magnecules can accrue or lose during collision individual atoms, dimers or molecules" (Santilli 2005 p. 22)

"Magnecules release in thermochemical reactions more energy than that released by the same reactions among unpolarized molecular constituents" (Santilli 2005 p. 23)

". . . the test at PCFL provided the first experimental evidence of mutation in time of the atomic weight of magnecules. In fact, the peak . . . is macroscopically

different . . . This difference provides evidence that, when colliding, magnecules can break down into ordinary molecules, atoms, and fragments of magneclusters, which then recombine with other molecules, atoms, and/or magnecules to form new clusters." (Santilli 2005 p. 82)

We propose that the coherent Exclusion Zone (EZ) water along hydrophilic surfaces that is so essential for biological processes is demonstrative of a particular type of magnecular structure. Unlike in the case of Hydrogen accretion in gasses which requires large magnetic fields along the lines of 10 to the 12 Gauss (Santilli, 2012, p. 3), the dynamism within the existent aqueous system of aqueous magnecular creation may be observed with low em field strength, and, the EZ coherent zone itself may be created, extended and fed, by relatively low levels of IR radiation (Pollack 2013, 2013*a*). The instantiation of molecular information into the aqueous medium may be accomplished through the encoding of a low energy 7Hz carrier frequency to supply the small perturbations to be summed in coherent CD processes which yield a fairly long lived and stable vortical structure, and then, the distributed information functions 'epigenetically' if you will, meaning that it affects how genes are expressed creating manifest morphology, functional development and manifest proliferation, yet does not affect genetic encoding itself (Borghini et al., 2012). All this activity, including chemically derived em encoded informational distribution acting as a sort of epigenetic informational allocation, happens by way of physical dynamics which can be expressed within the framework of Hadronic theory and QED. Del Giudice (2012) states: "According to Quantum Electro Dynamics (QED) these fields are able to attract co-resonating molecules giving rise to selective chemical attractions governed by specific chemical codes." In close analogy to the way magnecules seem to alter the expressed combustive properties of specific fuels, so does this informational epigenetic effect create alteration in the expressed result of biological energies and forms. In this case, the new particular magnecular species represented in coherent exclusion zone water is of the form H_3O_2, according to the experimental evidence gathered by Dr. Gerald Pollack (2013; 2013*a* and others). Here we see a new, larger, composite non-valence structuralization created by way of H bonds. This appears to be an informational magnecule with dynamic distributional functionality. Its collective *viscosity is divergent* from the 'parent' water molecule by a factor of as much as ten (Karbowski and Persinger, 2015, p. 6), and its properties and size change under conditions of IR exposure, under particular em and magnetic fields as referenced above and/or, in some instances 'spontaneously' over time (Persinger, 2015). However, the informational content and expression which stems from the near frictionless coherent CD process, are remarkably stable (Del Guidice et al., 2013; Monagnier et al., 2011). Another anomalous aspect: the refractive index, and hence one might infer the density of EZ water, is ten percent higher than bulk water (Pollack, 2013). As to the effects of pressure Dr. Pollack states: "EZ water has a higher density than bulk water. If you take H_2O and you put it under pressure, it should give you H_3O_2 because the EZ structure is denser than the H_2O. We did the experiments and we found, indeed, that's the case. If you put H_2O under pressure, you get more EZ water." (Mercola 2013) (Please note the similarity to above referenced magnecular processes).

Clearly, time reversal does not play a role in biological systems. No human or biological cell gets younger with time, any more than one might expect the droplets sprayed from a perfume bottle might somehow re-coalesce back into the container from which they originated. However, there MUST be a full spectrum of temporal exchange beyond the limits of relativity evidenced in biological systems (Santilli, 2008 p. 517). Think of the Wheeler delayed choice experiment, which has turned out so as to demonstrate temporal influence extending into the past (Manning et al., 2015), and also Predictive Anticipatory Activity (Mossbridge et al. 2014), which demonstrates human, biological, physiological evidence of the clear influence and presence of future events represented *before their temporal actualization*, as visible effectors in the present. We may therefore define the magnecule in question as a hypermagnecule (Santilli, 2005 p. 23; 2008 p. 511). We will invite the reader to consider the work of Montagnier to see the clearest example of biological structure created through encoded low strength EM within the context of aqueous informational mnemic capability (Montagnier, et al. 2011; 2014). DNA itself may be formed up from raw PCR ingredients with no trace of a physical DNA template! Only encoded EM is needed to affect the aqueous mnemic system, and so, we understand the processes which sustain disease themselves and those of health as well are deeply connected to informational dynamics and distribution stemming from magnecular aqueous processes. (Please keep in mind also that the DNA double helical structure itself is largely due to H bonding between base pairs).

After years of detailed research and experimental examination the coherent fraction of exclusion zone water to which Del Giudice refers has been derived and articulated by Dr. Pollack as being of the structure H_3O_2. The H bonded water structuralization of H_3O_2 had a molar mass of 35.02262 ± 0.00081 g/mol. Do keep in mind the familiar fact that H_2O has a molar mass of 18.01528 ± 0.00044 g/mol. These facts permits the following analysis:

1. In Del Giudice (2013) we read: "Consequently at each T there is a coherent fraction Fc(T) of molecules and a non-coherent fraction Fnc(T) whose sum is 1. Molecules cross over continuously between the two fractions leaving constant the total number of coherent and non-coherent molecules."

2. Dr. Pollack has clearly demonstrated the increase and shrinking of the exclusion zone layer in response to conditions including infrared exposure.

3. Furthermore, Del Giudice (2013) states (see original article for embedded references): "Water close to surfaces should therefore exhibit a much larger coherent fraction than bulk water. Its coherent inner structure should remain stable in time, allowing, contrary to bulk water where a continuous

cross-over between the two fractions occurs, a direct observation of the consequences of the presence of coherence. The depth of the coherent layer close to a surface is governed, according to [7], by the strength of the electric field emitted by the surface, which correlates coherently the CD electric dipoles. The depth of the layer could therefore reach values as high as hundreds of microns, much larger than the depths of a few molecular layers predicted by conventional theories based on H-bonding [9]."

4. Between the spontaneous bulk water's coherent non-coherent cross-over and the many various dynamic influences and effects upon exclusion zone size, we may conclude that *the entire aqueous system is variable over time as to its mass per mole.*

5. Temporal variability in molar mass (Avogadro number) is a hallmark of magnecular structure.

6. It appears that the hydrophilic/bulk aqueous bio-system corresponds to the pure gaseous species of magnecule by way of variability in the Avogadro number.

Condensation of findings:

a. Biological aqueous systems demonstrate variance in their Avogadro number.

b. Water structure may vary its H bonds 'spontaneously' or as a function of specific known field effects to yield anomalous changes in viscosity.

c. Coherent encoded water affects biological energetic expression and morpho-functionality.

d. Unique/anomalous spectral peaks, some of which indicate the presence of the aqueous informational magnecules' hypothesized coherence domains, are demonstrated by water affected/structured by fields (Persinger 2015; Murugan et al., 2015; Karbowski and Persinger, 2015; and references cited above).

e. Water loses its internal magnetic properties at 100 degrees C. indicating the presence of a Curie Temperature.

f. Water (H_2O) becomes structured into H_3O_2 via intermolecular variation in H bonds forming a different, heavier mass per mole. Liquid H_2O itself is given its special characteristics such as high vaporization temperature by way of intermolecular H bridges, which may well be nothing but pieces of polarized electromagnetic structure (see below).

g. The electromagnetically encoded information distributed via H bond dependent water structure and resultant coherent dynamics onto biological systems affects systemic energetic expression, proliferation and form as an 'epigenetic effect.'

h. The refractive index, and hence the implied density of EZ water, is ten percent higher than bulk water (Pollack, 2013). In another anomalous instance, the viscosity of coherent EZ water may be up to 10 times that of H_2O (Karbowski and Persinger, 2015, p. 6).

Ergo, Liquid Water, and Biological Aqueous Systems in particular may therefore be defined as: *magnecules.*

With a few more facts, the future implications will become clear.

From Dr. Pollack's (2013) book *The Fourth Phase of Water*: "EZ charge separation closely resembles the initial step of photosynthesis, which entails the splitting of water next to some hydrophilic surface. This resemblance may be auspicious: if that first step works as effectively as it does in photosynthesis, then some kind of water-based harvesting of light energy may have a promising future. Designs built around water might one day replace current photovoltaic designs." (p. 336).

Santilli (2005) writes:

"Recall that quantum chemistry was unable to achieve an exact and invariant representation of the main characteristics of the water molecule from unadulterated first principles despite efforts over the past century. In fact, a historical 2% has been missing in the representation of the water binding energy, while the representation of its electric and magnetic moments was embarrassingly wrong even in the signs." (Santilli 2005, p. 142).

A new approach is required. Hadronic mechanics is that approach. Dr. Santilli has also derived a novel underlying quantitative explanation for H bonds which is explicable entirely within the known confines of QED (Santilli, 2005, 2008, 2012 and others). As is known, liquid water may be ascribed its unique characteristics such as high vaporization temperature to the part played by H bridges. In place of the familiar description of H bond formation involving uneven molecular charge distribution and proton exchange, Dr. Santilli offers up a quantitatively specific model based in the forming up of H bonds and intermolecular adherence via the primary attraction of actual *polarized* toroidal electron orbits (as distinct from the orbitals, which are abstract mathematical objects rather than physical objects). (Santilli 2005, p. 31; 2012).

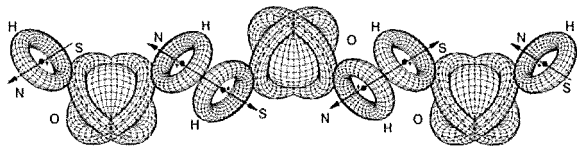

Figure 1. *One of the possible magnecular bonds of* H-*atoms in the liquid state of water.*

Pictorial representation of liquid water as hypothesized in Santilli's polarized toroidal orbit model of the H bridge. Figure used with permission of Dr. Santilli.

Due to the fact that within aqueous informational systems we are dealing with a ferro-electric structure with mnemic properties, we may not be surprised to discover the internal magnetic organization of the system is disrupted by increased temperature just as with a magnet, and so, the *Curie temperature* is that of the boiling point.

It is now possible to hypothesize from the theory of Santilli (Santilli, 2005, 2008, 2012 and others): Toroidal polarized (primarily) electron orbits induced/affected to specific geometric form and subsequent dynamics through external IR, ELF and also other molecular based fields, affect coherent hydrophilic surface water structure to induce hydrogen bonds yielding a magnecular species of the form

H_3O_2. That charge separation, which may well be the primary basis of photosynthesis itself, is therefore likely to be defined as a magnecular phenomenon. Life requires more than valence bonds. Life may also depend on the molecular species Santilli has named the *Magnecule*.

2.3. Possible Implications and Discussion Points

1. Should the effects evidenced in the above cited and condensed experiments be bought to practical fruition, the result could be of substantial importance. It is clear that information costs little and may be electromagnetically derived from molecular structure. Once stored within computer memory and geographically distributed as binary information converted at the receiving end into encoded em, it could help many at little cost, lowering drug prices by substituting cheap information for expensive drugs, and perhaps also reducing treatment toxicity. It appears at this early stage that the effects are akin to the molecule from which the information is derived, and may be nontoxic, unlike the toxic source molecule. As fields are not restrained by the Blood-Brain Barrier, molecules like dopamine and 5-HT could possibly be encoded and used in treatments, perhaps affecting a reduction in manifest symptoms of Parkinson's and OCD respectively. Chronic pain may be treated without recourse to addictive drugs and perhaps, addiction may also be ameliorated without recourse to drugs (Norman et al., 2016).

2. As aqueous informational memory is ferro-electric, perhaps as one creates a better recording using blank magnetic tape, the Curie temperature could be used to entirely replace the internal magnetic properties of water with those of an externally encoded field, using pressure to keep water in a liquid state while applying said field, or, by creating condensation of gas from temperatures over 100 degrees C cooled to encourage the emergent liquid state under the application of a suitable informational field.

3. Could Santilli's theory provide insight past the effects of 'like likes like' phenomena, and see into their cause? Could it thereby explain the quantitative mechanics which bind the organization of negatively charged water droplets into clouds, and the negatively charged conglomerative adherence of bodily cells and tissues? Are clouds and biological systemic forms intra-conglomerative magnecules?

4. Could the basic aqueous charge separation upon which photosynthesis is based be used to harvest radiant/solar energy? Could the increased viscosity evidenced by water left in the dark be interpreted as an energetic instantiation (Persinger, 2015; Karbowski and Persinger, 2015) and if so, could the effect be used to harvest vacuum energy?

5. Are aqueous systems and therefore biological systems, magnecular? Are coherent aqueous magnecular processes those upon which photosynthesis and life itself are dependent?

6. As the coherent fraction in aqueous hydrophilic systems is of a magnecular form (H_3O_2), do magnecular processes mediate coherence, and so, coherent informational aspects? If so, in what way?

7. Montagnier has demonstrated the CDs within water may be encoded so as to coherently sum and distribute information and create rightly sequenced DNA from raw ingredients sans any chemical template. Will science acknowledge the truth of this, and place the aqueous informational magnecule rightly at the center of disease processes and medical diagnosis and then, look to these dynamics in order to advance medical treatment past its current dependence upon dangerous, costly modes of radiation and drug therapies?

3. Conclusion

At this juncture in human knowledge, science has a choice; it may either acknowledge the limitations of relativistic and quantum theories as they relate to aqueous and biological systems, or it may not. If science chooses to stay the course and now, as before, run headlong after theories based upon a flawed axiomatic basis, little will change. However, should the pursuit of scientific knowledge be unfettered from its errors and begin to consider the possible, human misery may soon be transformed. It appears to us that the evolution of life itself is based on magnecular processes. The fact of liquid water is the fact of the magnecule. Within this molecular species is the hope for clean energy, clean burning fuels and, as this paper points out, the potential for non-toxic medical practice *based on information* and its dynamic magnecular distribution through aqueous systems as they affect biological systems. In defining a quantitative functional basis of the H bridge, Santilli may have placed a vital brick in the edifice of human knowledge. The deepest mystery in science is that of the dynamic relation between hypermagnecules and the aqueous/biological systems they define. In this mystery, our better future rests. Clean energy, human health and inexpensive, freely available nontoxic medical practice are to be found in the answer to an unacknowledged question, *What is the magnecule*?

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Two Body IsoElectronium Model of the Heliumic Systems

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Abstract: In preceding works, one of us (R. M. Santilli) has shown that, according to quantum chemistry, identical electrons cannot create the strong bond occurring in molecular structures due to their strongly repulsive Coulomb interaction; has constructed hadronic chemistry as a non-unitary covering of quantum chemistry solely valid at mutual distances of 10^{-13}cm; has introduced contact non-Hamiltonian interactions in the deep penetration of the wavepackets of valence electrons that overcomes said Coulomb repulsion, resulting in a strongly attractive bond of valence electron pairs in singlet called 'isoelectronium' and shown that the new valence bond allows an exact and time invariant representation of the binding energy of the hydrogen and water molecules. By using these advances and our inference that (from the fact that an atomic lone pair of electrons form a coordinate covalent bond identified by G. N. Lewis) the lone pairs of electrons are indeed isoelectronium, in this paper we present, apparently for the first time, a new structure model of the Helium atom under the name of Iso-Helium, in which the two electrons of a given orbital are strongly coupled into the isoelectronium that provided a quantitative description of Pauli exclusion principle. In particular, as a result of the strongly bound state of Santilli isoelectronium, the iso-Helium reduces to be a two-body system, thus admitting exact analytic solution. The presented analytic solution is applicable to all Helium-like systems. Using it we have calculated effective charge on the nuclei of Helium-like systems that are in excellent agreement with the literature values.

Keywords: Covalence, Isoelectronium, Helium, Hadronic Chemistry

1. Introduction

In quantum mechanics, the Helium atom (a two electron system) is cited as a first case wherein no analytic solution of the Schrödinger equation is possible. Therefore, in conventional quantum mechanics one has to go for approximate methods, such as perturbation and variation methods [1-4]. Indeed, each one of these two methods is a cumbersome mathematical technique obviously yielding merely approximate results. On the other hand, the solution of Schrödinger equation of Hydrogenic systems yields the concept of orbitals whose energy depends on the principal quantum number. As a result the energy gap between $1s$ and $2s$ orbitals is obtained as being quite large. Hence, as we go from a hydrogen atom to a Helium atom the second electron either should go to the $1s$ or to the $2s$ orbital. If it goes to the $1s$ orbital, then very strong electrostatic repulsion would make it an unstable atom. Whereas, if the second electron goes to the $2s$ orbital, then most probably they would remain spin unpaired (Ortho-helium). In the latter event again it would be a chemically very reactive atom being a diradical. Even if we assume that the second electron of $2s$ orbital remains paired with $1s$ electron (Para-helium) still they being separated electrons would be vulnerable to chemical reactivity besides this state has been experimentally found as a higher excited state than the triplet first excited state. However, Helium is a noble atomic gas having practically no chemical reactivity. Hence, on both the counts of energetics and chemical reactivity there is only one choice that is of housing both the electrons in the $1s$ orbital. In quantum mechanics, this state is prescribed as allowed by the Pauli exclusion principle requiring the two electrons in the $1s$ orbital to be coupled in the singlet state (that is with spins antiparallel). But how this spin paring imparts Helium atom the inertness towards chemical reactivity has not been so far addressed to. Unless the two electrons in the same orbital present themselves as one entity, say as a quasi-particle, they are no different than when they occupy two different orbitals with

opposite spins as far as chemical reactivity is concerned. However, the quantum mechanics remains silent on how two electrons of a given orbital remain stable in spite of very strong electrostatic repulsion between them. In fact, two electrons at a mutual distance of 62 pm will experience a *repulsive* force of about 60 nN [1].

Another fact that we need to consider is the case of lone pairs of electrons in the valence shell of the central atom of molecules. For example, in the case of NH_3 molecule there we have one lone pair of electrons on N atom. The electrons constituting the pair have never been observed to form two separate covalent bonds with other atoms. On the contrary the said lone pair forms another type of covalent bond named by G. N. Lewis as the directed or coordinate covalent bond (1913-1919). This led him to classify acids and bases as Lewis acids and Lewis bases [5]. The Lewis bases are the donor of a pair of electrons. Some of the examples of Lewis acids are BF_3, $AlCl_3$, SO_3, etc. and those of Lewis bases are NH_3, HO^-, etc. In inorganic chemistry the Lewis bases are also termed as ligands that form complexes with transition metal ions, which are known as coordination compounds.

From the above mentioned facts we assert that the coordinate covalent bond formation can take place only if the lone pair of electrons exist as a quasiparticle. Otherwise there should have been at least a few examples of formation of two separate covalent bonds from a lone pair of electrons. This then implies that every spin paired electrons in a given atomic orbital must be existing as a quasiparticle.

We recall that the strong electrostatic repulsion between two electrons in an orbital was, for the first time, pointed out by Santilli (see monograph [6-9] and original papers quoted therein) in connection with his poineering research on chemical bonding of hydrogen and water molecules. In essence, Santilli showed that, according to quantum chemistry, two identical electrons in the singlet coupling cannot create a valence bond because of the above indicated, very strong, repulsive Coulomb force. Therefore, Santilli constructed an invariant non-unitary covering of quantum chemistry under the name of *hadronic chemistry* and showed that such a covering theory does indeed permit the achievement for the first time of an *attractive* force between valence electron pairs in singlet coupling at mutual distances of the order of 10^{-13} cm. Santilli then proved that such an attractive force does indeed permit an essentially exact representation of the binding energies of the Hydrogen and water molecules. The central mechanism is provided by the first consistent representation of the contact, non-linear, non-local and non-potential interactions caused by the deep overlapping of the wave packets of the identical valence electrons which are outside the representational capabilities of quantum chemistry. The evident non-Hamiltonian character of the new interaction then mandates a *non-unitary* covering of quantum chemistry. When the said non-Hamiltonian interaction is treated with the suitable covering of the mathematics underlying quantum chemistry, known as *isomathematics*, the deep mutual overlapping of the wave packets of the valence electrons in singlet coupling creates a force representable with the Hulthén potential which in the model adopted by Santilli is so strongly *attractive* to "absorb" the repulsive Coulomb potential, resulting in the first known *attractive* force between identical valence electron in singlet coupling[2], resulting in a new state nowadays known as *Santilli Isoelectronium* (IE). Santilli also indicated that, electrons are predicted to experience a "mutation" of their conventional characteristics when they are the constituents of the isoelectronium that he named *isoelectrons* technically due to the fact that they are no longer characterized by the Poincaré symmetry but by its non-unitary covering known as the *Poincaré-Santilli isosymmetry*. It should be stressed to avoid misinterpretations that *hadronic chemistry* solely applies at mutual distances of 10^{-13} cm called the "hadronic horizon" and less; it recovers uniquely and identically quantum chemistry for all its molecular studies for distances larger than 10^{-13} cm; and the basic axioms of hadronic chemistry are exactly those of quantum chemistry, solely subjected to a broader representation permitted by the novel isomathematics (see Ref. [6, 7] for details).

As stated above to achieve the chemical inertness it is not sufficient that two electrons be housed in the same atomic orbital with opposite spin but also they should exist as a one entity. This strongly points out that each pair of two electrons with opposite spin housed in an orbital necessarily exist as a single entity, which in all probability is none else but the isoelectronium proposed by Santilli in describing the covalent bonds of water and hydrogen molecules [10, 11]. For example, in the case of ammonia molecule there we have one lone pair of electrons in the valence shell of nitrogen which is chemically inert in the sense of conventional covalent bonding because it must be an isoelectronium (IE).

Hence, with the coining of the concept of isoelectronium by Santilli as described above we hereby assert that all spin paired electrons of atomic orbitals are individually isoelectronium (IE).

Thus our task is to incorporate the concept of IE in Helium and Helium-like systems. To avoid any ambiguity to persist, we have taken this opportunity to first take a stock of the interactions that are supposed to operate in these systems. We recall Santilli's proposal and assertions that at atomic scales an IE may be treated as a quasi-particle hence Helium-like systems turns out as a two body problem namely the nucleus consisting of neutrons and protons as a point mass and the IE an another point mass. Both the nucleus and IE individually are of hadronic dimensions. On the lines of the conventional quantum mechanical approach we would be tempted to identify only two interactions, namely:

1) the electrostatic attraction between the IE, the proposed quasi-particle, and the nuclear protons and;

2) the electrostatic repulsion between the two isoelectrons of the IE.

But on little deeper pondering over the physical state of the

[1] The distance of 62 pm is the diameter of 1*s* orbital of Helium atom and if the two electrons are assumed to remain farthest away of each other then roughly they would be mutually 62 pm away (c.f. Tables 8 – 10 and also Appendix A).

[2] It should be recalled that according to Santilli's model the triplet coupling of two electrons of a given orbital creates strongly *repulsive* hadronic forces.

system one easily realizes that because the size of an IE is of the order of 1 fm or even less there should exist an equally important and strong magnetostatic attraction between the two electrons of IE (so far in none of the atomic systems this interaction has been separately dealt with in determining the total potential energy of the system. Most probably because it is expected that the two electrons of a given orbital would try to lie as far distance apart as possible to minimize electrostatic repulsion but then at such separation distances the magnetostatic interaction would be insignificantly small to reckon with (c.f. Appendix A)). This interaction originates because each spinning electron acts as a tiny magnet (refer also to Appendix A). In addition to this we also realize that the formation of an IE is through a deep overlap of the wave packets of two electrons which, indeed, is a new phenomena. Because of it a new type of interaction comes into play and would obviously contribute in determining the total potential energy of the system. This latter contribution we have quantified using Hulthén potential through the tools of hadronic mechanics and it turns out as a repulsive hadronic contribution (c.f. Section 5). On the face of it, this appears to be in contradiction to the demonstration of Santilli that the corresponding hadronic force is attractive in nature. However, it gets reconciled on realizing that, in fact, Santilli is not treating the above stated magnetostatic interaction separately hence his hadronic contribution is bound to be attractive in nature[3].

Thus in this paper, we apply the above advances to Helium and Helium-like systems apparently for the first time. As a result of the above considerations the IE can be considered in first approximation to be a single stable quasi-particle with spin 0, charge $2e$, rest energy of the order of 1 MeV, and an essentially null magnetic moment. Therefore, the structure model of the Helium and Helium-like systems, here proposed under the name of *iso-Helium-like systems*, has been earlier treated in all *quantum chemistry* texts with the sole exception of the short range interactions of the electron pair, which in this presentation has been treated for the first time by recognizing the short range very strong magnetostatic interaction and the covering known as *hadronic chemistry* to deal with the new hadronic force.

The advantages of this conception of the iso-Helium atom and hence iso-Helium-like systems are the following:

1) The hadronic model of the iso-Helium-like systems permits one of the few quantitative representations of Pauli exclusion principle because for the triplet coupling of two identical electrons in the same orbital there would be a strong repulsive magnetostatic interaction that would not allow IE formation hence no hadronic chemistry is warranted, thus only possibility that is left is the strong attraction due to the singlet coupling of two electrons in an orbital in excellent agreement with Pauli's exclusion principle.

2) To the best of our knowledge, the proposed model permits a quantitative representation of the lack of chemical reactivity of Helium evidently because either the unpaired electrons or the paired but un-united electrons are not statistically available for conventional chemical reactions.

3) The proposed model permits, apparently for the first time, the reduction of the Helium atom and Helium-like systems to a *two-body system*, with ensuing analytic solutions. It should be, however, recalled that the IE is permanently stable only in first approximation in view of Heisenberg's uncertainty principle and other factors [6].

A brief description of isomathematics and hadronic mechanics we have given in our preceding paper on nuclear spins [12] and for the sake of brevity we are not repeating that subject matter herein. For details of hadronic chemistry the reader is advised to refer to the excellent reviews by Santilli [6] and Tangde [7-9].

In Section 2 we provide a brief description of the concept of IE. In Section 3 we have presented the conventional quantum mechanical model of Helium and the Helium-like system, in Section 4 we have reformulated the Helium and Helium-like system by incorporating Santilli's IE and studied the corresponding implications both within and outside the hadronic horizon. In Sections 5 and 6 we have quantified the isopotential originating from the deep mutual overlap of the wave packets of the two isoelectrons of IE. Since we are treating IE as a quasi-particle which implies that it contributes either the null potential energy or a constant and presumably very small value of it to the net potential energy of the system[4]. It, therefore, legitimizes the assumption that the isopotential nullifies the algebraic sum of electrostatic and magnetostatic potentials of the two isoelectrons of the IE or results into a very small but constant value of it. As a result of it there ensues the quantum mechanical analytic solution of Helium-like systems. The details of this latter part has been described in Section 6. The wave functions so generated consists of radial and angular parts and consequently provide three quantum numbers namely principal, azimuthal and magnetic ones. The expression of energy so obtained is dependent on the principal quantum number, in exactly the same way as in the case of Hydrogenic system. The expressions so obtained are the same as those we obtain in the case of Hydrogenic system except that, instead of the reduced mass of the electron, there we have the reduced mass of the IE and instead of the nuclear charge, Z, there we have the effective nuclear charge, Z^*.

The results of our studies are tested with the estimation of the effective nuclear charge for the iso-Helium-like systems by using corresponding experimental values for ionization energies, which are presented in Section 6.1. Our calculations match exceedingly well with the effective nuclear charges obtainable from the Slater rule [13] and the more recent ones provided by Clementi et al using SCF theory [14-17]. However, our estimated values of effective nuclear charge are a shade superior over the above two earlier methods. Section

[3] In the case of π^0 meson formation from an electron and a positron obviously the magnetostatic interaction would be strongly repulsive and it gets countered by the electrostatic attraction and new hadronic attraction originating from deep overlap of wave packets of these two particles.

[4] Recall that in hydrogenic atoms the potential energy of nucleons to its net potential energy is assumed to contribute either null or a constant value not dependent on the distance between nucleus and extra-nuclear electron. Still it produces excellently well all the frequencies of emission spectra of such atoms.

6.2 presents our calculations of most probable radius of Helium-like systems that has been compared with the radius of hydrogenic systems. In the final Section 7 we have placed our concluding remarks.

This paper also includes three Appendices. In Appendix A we present the comparative calculations of electrostatic and magnetostatic potentials with varying distance between the two electrons. In Appendix B we illustrate the notion of the "trigger" needed to bring the two electrons inside the hadronic horizon and in Appendix C we present the energy of simple harmonic motion of the IE within the hadronic horizon.

2. The Concept of Isoelectronium

The Pauli exclusion principle states that no two electrons in an atom can have all the four quantum numbers identically same [1-4]. It, therefore, means that in an orbital two electrons can be housed only if their spin quantum numbers are different. However, the spin quantum number can only have two values +1/2 and -1/2, that we conventionally represent as up and down arrows because basically it is a vectorial quantity originating from the spinning of electrons (the charge particle), hence the dictate of the Pauli principle is that the two electrons of a given orbital are spin paired. Since the spinning electron generates a magnetic field perpendicular to the plane of the spin the net spin magnetic moment of two electrons remains zero in an orbital. The spins of the electrons, the corresponding spin quantum number values ($m_s = 1/2$ and $-1/2$), their magnetic moment vectors and their pairing with opposite spins are shown in Figure 1.

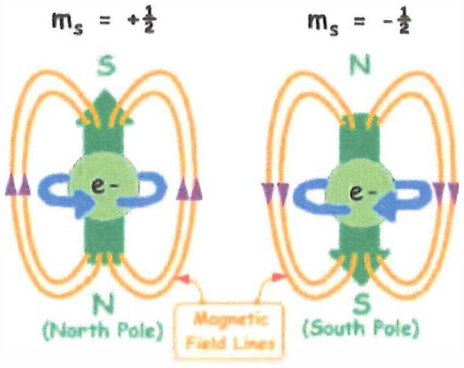

Figure 1. *Schematic representation of the spins of two electrons, correspondingly generated magnetic fields and magnetic moment vectors and the spin pairing. The values of the spin quantum number, m_s are also shown.*

Thus the net magnetic moment equals to zero is evident from Figure 1.

Recall that, the strong hold of spin pairing is meticulously maintained in all quantum mechanical descriptions wherever two spinning fundamental particles are described in quantum mechanics but the physics of concomitantly implied strong interaction between the two particles is not at all attended in any quantum mechanical description. From Figure 1 it is evident that when two electrons are spin paired there originates strong magnetostatic attraction and simultaneously due to their identical charges they must experience very strong electrostatic repulsion. Now for the time being if it is considered that the electrostatic repulsion nullifies the magnetostatic attraction when two electrons are housed in an orbital then it is easy to realize that the nullification of the said attractive and repulsive potentials would take place at a particular distance between two electrons, hence the assumption of their free movement about the nucleus within the extranuclear space cannot be guaranteed. Though, on an average for a collection of atoms the net magnetic moment would be zero. In Appendix A we have presented our calculations of the potential energies of electrostatic repulsion and magnetostatic attraction to illustrate their relative magnitudes.

Of course, on the one hand one would be tempted to assert that to minimize the electrostatic repulsion between the two electrons of 1s orbital they need to be farthest apart from each other, which according to the average diameter of 1s orbital of a Helium atom (that is 62pm) should be 62pm away. In this case to maintain the spin pairing of two electrons situated such a long distance apart would be extremely difficult (How can the Spin entanglement be considered at such a very short distances between the two spin paired electrons is debatable because there also exist very strong electrostatic and magnetostatic interactions.). On the other hand, in Appendix A we have seen that the null potential energy position (without involving Hulthén potential) is at $r_{null} = 0.2731\,\text{pm} = 273.1$ fm (which, indeed, is a very small a distance compared to the average diameter, 62pm, of $1s$ orbital of He atom). This would imply that in all doubly filled orbitals the two electrons remain situated at identically the same distance away from each other to fulfill the Pauli principle, which indeed is a too demanding a requirement as well as is a non-realistic one. Moreover, by considering the rest mass of an electron is entirely electromagnetic in origin J. J. Thomson in 1881 had calculated the radius of electron equal to 2.82×10^{-13} cm = 2.82fm, which is termed as the classical radius of an electron [18]. Thus we see that the wave packets of two spin paired electrons would not even touch each other but should be about 265 fm apart to have null potential energy. Therefore, as stated above to maintain spin pairing at such a distance apart for two electrons is too stringent a requirement to be followed in reality.

From the above discussion it is clear that the Pauli principle cannot hold if the two electrons of the same atomic orbital are allowed to remain a distance away. Precisely for this reason Santilli proposed that the two electrons come so much close to one another that their wave packets overlap, that is, the centers of two electrons achieve a distance of about 1 fm or less (1fm is less than half of the predicted radius (2.82fm) of the electron by Thomson). Of course, this assertion of Santilli is in connection with two spin paired electrons of the covalent bonds of hydrogen and water molecules[5]. Then he succeeded

[5] At a first glance one may have apprehension about the physical reality of the overlap of the wave packets of two electrons because of their identical charge that should cause strong repulsion. Hence, the concept of a trigger has been coined by Santilli (c.f. Appendix B). This proposed mutual overlap of wave packets of electrons can be compared with the quantum mechanics given concept of overlap of electronic atomic orbitals that forms a chemical bond. If one considers the electrostatic repulsion between any two electrons then the said overlap cannot be a

in unearthing a new kind of interaction that comes into play, which is of non-potential, non-local and non-Hamiltonian type originating from the physical contact between the two electrons by way of deep overlap of their wave packets. We recall that the small volume of the wave packets of electrons behave as a hyper-dense medium as asserted by Santilli.

This proposed mutual penetration of the wave packets of electrons has been termed as the mutation of electrons. Thus the union of mutated electrons, as stated above, has been given the name of IE and the two electrons are now termed as *isoelectrons*. The prefix iso- originates because the physics of this union can only be described by using Santilli isomathematics [19, 20]. Indeed, the IE formation has already been demonstrated by Santilli in molecular bonding of hydrogen and water molecules [6, 10, 11] and in Cooper pair [21, 22]. The schematic representation of coupling that forms IE is given in Figure 2.

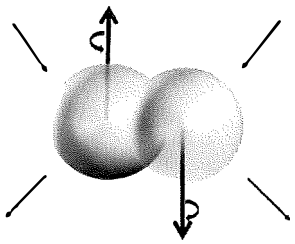

Figure 2. *The schematic representation of isoelectronium by way of deep overlap of wave packets of two electrons at a distance of 1 fm or less.*

An experimental evidence asserted by Santilli in support of the formation of IE is that in the Helium atom the two electrons are bounded most of the time, to such an extent that they are emitted in such a bonded form during photo-disintegrations, and in other events [6].

For detailed discussion on the concept of formation of IE the reader is directed to read the original sources namely [6, 10, 11]. However, in brief we state that the IE formation provides, for the first time, a quantitative theory of Pauli principle. *Due to the deep mutual overlap of wave packets of two electrons of an IE an entirely new type of interaction comes into play that Santilli identified as being of non-Hamiltonian, non-local and non-potential character.* To quantify it he has formulated hadronic mechanics by using his isomathematics. Consequently, its quantitative expression is obtained as Hulthén type potential, which at short distances behaves as *constant/r*, that is as Coulomb type.

This is the conceptual basis of the Pauli principle applied to the two spin paired electrons in a molecular orbital. Accordingly, Santilli and co-workers have described the hadronic mechanics (a quantitative description) of π^0 meson, the IE of Cooper pairs and the covalent bonds of hydrogen and water molecules.

Now in this paper we are presenting our investigations using an approximation of IE formation (of the two spin paired electrons) in atomic systems of Helium and Helium-like atoms (see for example [23] and the original references cited therein).

3. Helium and Helium-Like Systems According to Quantum Chemistry

A Helium atom consists of two extranuclear electrons and, two protons and 2 neutrons in its nucleus as depicted in Figure 3 without neutrons as they are of no relevance in the present discussion.

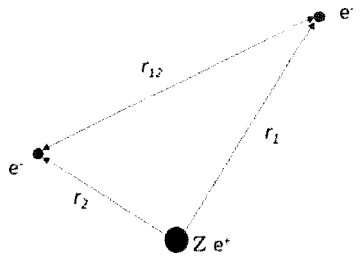

Figure 3. *Schematic representation of Helium like atoms (Ions).*

Therefore, there we have the following three potential energy terms, namely:

$$-\frac{Ze^2}{r_1}, \quad -\frac{Ze^2}{r_2}, \quad +\frac{e^2}{r_{12}} \tag{1}$$

where as depicted in Figure 3 r_1 and r_2 are the distances of the electron 1 and the electron 2 from the nucleus and r_{12} is the distance between two electrons. Thus in eq.(1) the first two terms are the attractive potentials and the third one is the repulsive one. Also as the distances r_1, r_2 and r_{12} are of comparable magnitude thereby all the terms of eq.(1) are also of comparable magnitude. Notice that it is indeed a three body problem.

4. IE in Helium and Helium-Like Systems According to Hadronic Chemistry

Recall that, according to Santilli, an IE is formed when two electrons in singlet spin coupling occupy the same molecular orbital. The same proposal we now adopt also in the case of two spin paired electrons of an atomic orbital.

Recall also that the IE, by definition, is formed by the deep mutual overlap of the wave packets of two electrons. In singlet coupling (only), this wave overlap produces a new

reality. However, in chemical bonding this apprehension is set aside on the grounds that the electrostatic attraction exerted by two nuclei on the two electrons forces the two electronic orbitals to overlap. In Santilli's language one can now say that it is the required trigger for forming a chemical bond. Moreover, as we have described in Appendix A when two electrons come close within hadronic volume then very strong magnrtostatic force of attraction starts exerting making the said overlap a reality. At the same time now a new hadronic effect comes into play that works to maintain balance between the attractive and repulsive forces within an IE. Another implication of IE when incorporated in the description of chemical bonding would be that the conventional overlap of the orbitals gets described as IE formation by the two valence electrons of the two atoms forming chemical bond. This subject matter we will discuss separately.

contribution of non-potential, non-local and non-Hamiltonian type to the net potential energy of the system.

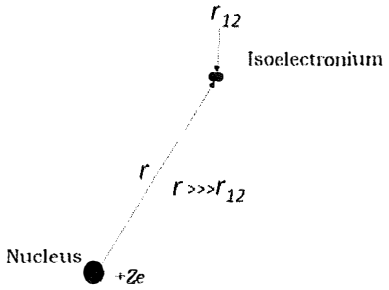

Figure 4. *A schematic representation of two body model of Helium atom* $(Z = 2)$ *with isoelectronium.*

When we treat two spin paired electrons in an atomic orbital as a tiny quasi-particle its implication is that for all practical considerations we have $r_{12} \lll r_1$ and r_2 . In the conventional quantum mechanical approach the potential energy terms of this system would get represented as,

$$V = -\frac{2Ze^2}{r} + \frac{e^2}{r_{12}} - \frac{\mu_e^2}{r_{12}^3} \tag{2}$$

where we have used $r(= r_1 \equiv r_2)$ which is the distance between the IE and the nucleus and r_{12} is the internal distance of IE and by definition is many fold smaller compared to r. The last term on the right hand side of eq.(2) originates from the magnetostatic interaction that start significantly exerting at short distances between the two spin paired electrons of the IE because they behave as two tiny magnets (due to their spinning) obviously placed at a short distance apart[6] and the magnitude of the corresponding interaction steeply increases as the distance between the two electrons decreases towards hadronic dimension (c.f. Appendix A).

In this way, we have approximated the three body model of the Helium into a two body model that we call IsoHelium. This gets pictorially represented as shown in Figure 4. Thus we see that, because of $r_{12} \lll r$ the second and third terms on the right hand side of eq.(2) completely over power the first electrostatic term due to attraction between the IE and the protons of the nucleus and the net effect turns out as a strong attraction. *Even though there emerges a strong magnetostatic attractive potential on bringing two electrons within the hadronic distances the story of interaction still remains incomplete. This is so because the new phenomena of the deep mutual overlap of the wave packets of two electrons, bound to bring in the corresponding entirely new source of potential that in essence implies having the non-potential, non-local and non-Hamiltonian character, that imparts the status of a quasi-particle to so situated spin paired two electrons, which Santilli christened as an IE. However, as asserted by Santilli the state of deep mutual overlap of wave packets of two electrons implies their mutation that in his words is the realization of iso-electrons characterized by the Lorentz-Santilli isosymmetry* [24, 25].

In order to quantify the said new potential originating within an IE we recall that the situation in Helium and Helium-like atomic systems is some what like an IE of hydrogen and water molecules described earlier by Santilli. In the present case the electrostatic attraction between the nucleus and IE serves as the required *trigger* [6, 10, 11] that forces two orbital electrons to form an IE (c.f. Appendix B).

In this model of iso-Helium atom and iso-Helium-like systems we have the following situation. The electrostatic and magnetostatic interactions between the two isoelectrons of IE still remain governed by the corresponding conventional laws. Whereas the attraction between the nucleus and the IE, and the kinetic energy of IE relative to the stationary nucleus involve distances many fold higher than 1 fm. Hence, the latter two aspects are needed to be treated by quantum mechanics and related conventional mathematics. That is they get adequately described by the conventional quantum mechanical tools. However, we have to quantify the new interaction originating from the deep overlap of the wave packets of two electrons of IE by the methods of hadronic mechanics. Thus we have a mixed situation of requiring to handle both the hadronic and the conventional interactions and two distances of non-comparable magnitudes. We recall that the case of π^0 meson is different in which we remain entirely well within the hadronic horizon because it has been postulated by Santilli as the hadronic union of isoelectron and isopositron [26].

However, as IE behaves as a quasiparticle all the interactions between two electrons of it should produce either a null potential or a very small value of it not dependent on the distance between nucleus and extra-nuclear IE (c.f. Section 5).

Accordingly, for the time being if we assume that the attractive, repulsive and hadronic potential energies within IE exactly get nullified, then for the system under consideration there are two inputs to the Hamiltonian, H, namely:

1. the kinetic energy, T, of IE relative to the nucleus,

$$T = \frac{p^2}{2\mu_{IE}} \tag{3}$$

where p is the momentum of the IE and its reduced mass, μ_{IE}, reads as,

$$\mu_{IE} = \frac{m_{IE} \times M_{nucleus}}{m_{IE} + M_{nucleus}} \tag{4}$$

where m_{IE} is the rest mass of IE and $M_{nucleus}$ is the rest mass of the nucleus.

2. the potential energy, V_i, is determined by the attraction between the effective nuclear charge and an electron of the

[6] This magnetostatic potential has not been incorporated in any of the earlier quantum mechanical descriptions. Perhaps because its magnitude remains insignificantly small compared to the electrostatic repulsion between two free electrons of an orbital as the average distance between them is expected to be of the order of atomic dimension (c.f. Table 11 of Appendix A). Whereas in Santilli's model of a molecular orbital housing an IE it seems that by default this magnetostatic interaction gets covered within the hadronic Hulthén potential that he calculated hence it is no wonder that the result is an attractive hadronic potential.

IE[7],

$$V_t = -\frac{Z^* e^2}{r} \tag{5}$$

where Z^* is the effective nuclear charge experienced by the IE.

It should be noted that, according to the model under consideration, the IE is seen from the nucleus of nuclear charge, Z^*, as one single entity with effective charge e and is treated quantum mechanically. Hadronic chemistry solely holds for the structure of the IE that necessitates the use of isomathematics. The rudiments of it we have described in the following Section 5.

5. The Isopotential in the Semi-IsoHelium-Like Systems

In order to understand the quantitative treatment of the strong hadronic *contact* force between two electrons in singlet coupling at very small mutual distances, it is essential to recall that such a force simply cannot be formulated via quantum mechanics and its known mathematical structure, e.g., via the conventional Hilbert space. This is due to the fact that the latter formulations can only represent interactions derivable from a potential, while *contact interactions have no potential by their very conception.* Consequently, the sole mathematically and physically consistent representation of an isolated IE is that via isomathematics with particular reference to the use of the Schrödinger-Santilli isoequation on an iso-Hilbert space over Santilli isocomplex isonumbers (see for example [6] and references cited therein). Therefore, we have the following three contributions in iso-Hilbert space, namely:

1. The non-linear, non-local, non-potential and non-Hamiltonian contribution due primarily to the deep mutual overlap of wave packets of two electrons we represent as,

$$\hat{\mathrm{g}} = \hat{\mathrm{g}}(\hat{r}_{12}) \tag{6}$$

2. The repulsive electrostatic interactions between the identical charge of two isoelectrons;

$$\hat{V}_{electrostatic} = +\hat{V}_e \hat{\times}_e \left(\hat{e}^{\hat{2}} \hat{\div} \hat{r}_{12}\right) \tag{7}$$

where $\hat{V}_e = V_e \times \hat{I}_e$ and

3. The attractive magnetostatic potential due to the two identical tiny isomagnetes in singlet coupling, namely:

$$\hat{V}_{magnetostatic} = -\hat{V}_\mu \hat{\times}_\mu \left(\hat{\mu}^{\hat{2}} \hat{\div} \hat{r}_{12}^{\hat{3}}\right) \tag{8}$$

where $\hat{V}_\mu = V_\mu \times \hat{I}_\mu$ and $\hat{\mu}$ is the iso-magnetic moment of the isoelectron and μ is the conventional magnetic moment of an electron.

Recall that isoquantities are equal to the quantum quantities multiplied by the isounits $\hat{I}$, $\hat{I}_e$, $\hat{I}_\mu$, etc., all isoproducts $\hat{\times}$, $\hat{\times}_e$, $\hat{\times}_\mu$, etc. are given by $\times\hat{T}\times$, $\times\hat{T}_e\times$, $\times\hat{T}_\mu\times$, etc., where $\times$ is the conventional associative product, and the same holds for the isofractisons [19, 20].

Notice that the above three interactions are taking place within the hadronic space of an IE, which we are treating as a quasi-particle. However, this quasi-particle when treated as a part of a heliumic system it is justified to treat it as a point mass because the distance between the nucleus and the IE is many fold larger than the size of an IE (c.f. $r \ggg r_{12}$). As stated above to the first approximation the interactions within a quasi-particle by its definition would not contribute to the potential energy of the heliumic system (even if it is a non-null contribution the net contribution would be a very small constant in the sense not dependent on r, the distance between nucleus and IE. Hence, our main result would not get altered in any way). Therefore, we are led to assume,

$$\hat{\mathrm{g}}(\hat{r}_{12}) = \hat{V}_\mu \hat{\times}_\mu \left(\hat{\mu}^{\hat{2}} \hat{\div} \hat{r}_{12}^{\hat{3}}\right) - \hat{V}_e \hat{\times}_e \left(\hat{e}^{\hat{2}} \hat{\div} \hat{r}_{12}\right) \tag{9}$$

However, it is profitable to use the projected version of eq.(9) on the conventional Hilbert space (the space to which the Helium-like system belongs). The said projections of each term of eq.(9) are[8]:

$$\hat{\mathrm{g}}(\hat{r}_{12}) \to \mathrm{g}(r_{12} \times \hat{T})$$

$$\hat{V}_\mu \hat{\times}_\mu \left(\hat{\mu}^{\hat{2}} \hat{\div} \hat{r}_{12}^{\hat{3}}\right) \to \frac{\mu_e^2}{r_{12}^3}$$

$$\hat{V}_e \hat{\times}_e \left(\hat{e}^{\hat{2}} \hat{\div} \hat{r}_{12}\right) \to \frac{e^2}{r_{12}}$$

Therefore, the projected version on the conventional Hilbert space of eq.(9) reads as,

$$\mathrm{g}(r_{12} \times \hat{T}') = \mathrm{g}\left(\hat{T}(r_{12})\right) = h \times \hat{T}(r_{12}) = \frac{\mu_e^2}{r_{12}^3} - \frac{e^2}{r_{12}} \tag{10}$$

Notice that we have used $\hat{r}_{12} = r_{12} \times \hat{I}'$ and then absorbed the r_{12} dependence of g entirely in the isotopic element by defining new isotopic element, $\hat{T}$, equal to $r_{12} \times \hat{T}'$ that allowed us to adopt,

$$\mathrm{g}(r_{12} \times \hat{T}') = \mathrm{g}\left(\hat{T}(r_{12})\right) = h \times \hat{T}(r_{12}) \tag{11}$$

where $\mathrm{g}(r_{12} \times \hat{T}')$ is the projected function on the Hilbert

[7] Though we are considering the attractive electrostatic interaction between the nuclear charge and the two electrons of an IE but the use of the effective nuclear charge, Z^*, in eq.(5) implies that one of the electrons of the IE joins into the act of shielding of nuclear charge hence the factor 2 in the numerator of this equation is not appearing.

[8] We have chosen isotopic elements, $\hat{T}_e$ and $\hat{T}_\mu$, such that when $\hat{V}_e$ and $\hat{V}_\mu$ are projected on the conventional Hilbert space the result is unity i.e. $V_e \times \hat{T}_e = V_\mu \times \hat{T}_\mu = 1$. This choice of mathematical handling of electrostatic and magnetostatic interactions between the two electrons of an IE implicitly assigns the entire quantification of hadronic effects to the term $\hat{\mathrm{g}}(\hat{r}_{12})$ or equivalently to $\mathrm{g}(r_{12} \times \hat{T})$.

space of $\hat{g}(\hat{r}_{12})$. Thus h is obtained as independent of r_{12}. In this way we need to quantify two quantities $\hat{T}$ and h each positive definite.

Although the approximation of eq.(9) or that of eq.(10) seems to be purely ad hoc but its physical reality appears to be as described below. Let us consider the case of a diatomic molecule as an analogy. The separated atoms have three translational degrees of freedom individually. On forming the union, similar to a diatomic molecule, IE possesses only three translational degrees of freedom. Hence the remaining three translational degrees of freedom generate new degrees of freedom. They are two rotational and one vibrational (symmetric). The physical existence of both the new types of degrees of freedom in a molecule have been experimentally and unequivocally proved by IR, UV and microwave spectroscopies. On the same lines the separated electrons in an orbital will have in all six translational degrees of freedom corresponding to their motion within an orbital. However, on forming IE there still remains three translational degrees of freedom for its motion within the volume of an orbital. Hence the remaining three translational degrees of freedom get transformed to one vibrational and two rotational degrees of freedom of the IE. The reason of vibration of an IE can be traced out from the fact that at very short distances of hadronic scales the magnetostatic attraction between two isoelectrons over powers the electrostatic repulsion between them (c.f. Appendix A). In this way ultimately a dielectron would be formed by way of the complete superposition of the wave packets of the two electrons. But so far there is no experimental evidence of such a species. The physical non-realization of such a species lead us to conjecture and believe that there originates a new form of interaction coming into play due to the deep overlap of the wave packets of two isoelectrons of an IE. That starts countering the very strong magnetostatic interaction at such small distances i.e. 1 fm and less. That on the lines of Santilli we have quantified via the Hulthén potential. Because of this mutual countering of attractive and repulsive interactions within an IE it maintains the vibrational mode. Similarly, each IE will have two rotational modes. But we must realize that all the degrees of freedom of IE would be constrained by the very strong electrostatic attraction between the nuclear charge and IE. Perhaps still it would be permissible to use the tools of hadronic mechanics for describing vibrational motion of an IE on the lines of the quantum mechanics of the vibrational motion of a diatomic molecule and that we have described in Appendix C. On the other hand, the rotational motion of IE takes place in the conventional Hilbert space, that is within the entire volume of an orbital. Hence, this motion needs to be computed using quantum mechanical tools on the similar lines of rotational motion of a diatomic molecule. The striking difference would be the drastic reduction of moment of inertia in going from a molecule to an IE. This aspect we have not investigated yet. However, it is clear that an IE in a Helium-like systems possesses (and for that matter all spin paired electrons of atomic and molecular orbitals) vibrational, rotational and translational energies[9].

In view of the above approximations the net potential energy of the system is solely determined by,

$$V = -\frac{Z^* e^2}{r} \tag{12}$$

originating from electrostatic attraction between the nuclear charge and one of the electrons of the IE in extra-nuclear region. Of course, under the condition of eq.(10) we need to evaluate $\hat{T}(r_{12})$ and also assign a suitable expression to h.

Since, the IE formation is assumed as triggered by the electrostatic attraction between nuclear charge and the two electrons (c.f. Appendix B) hence to the first approximation we adopt the following expression,

$$h = \frac{Z^* e^2}{r} \tag{13}$$

Of course, h has been assumed as dependent on r and the latter is independent of r_{12}, hence the expression on the right hand side of eq.(13) serves as practically of constant magnitude within the hadronic volume.

The isounit $\hat{I}'$ and its corresponding isotopic element are computed on the similar lines as it has been done in the case of an IE of hydrogen and water molecules [6, 10], that is,

$$\hat{I}' = e^{N \times \psi/\hat{\psi}} \approx 1 + N \times \frac{\psi}{\hat{\psi}},\ N = \int \hat{\psi}^{\dagger}_{1\downarrow}(r_{12}) \times \hat{\psi}^{\dagger}_{2\uparrow}(r_{12})\, d^3 r_{12} \tag{14}$$

that gives,

$$\hat{T}' \approx 1 - N \times \frac{\psi}{\hat{\psi}} \tag{15}$$

where ψ is the conventional wave function of an electron, $\hat{\psi}$ i s the iso-wave function of an isoelectron, the subscripts 1 an d 2 refer to the two isoelectrons of IE and their spin paired s tate has been shown by up and down arrows in the subscript. Moreover, the wave functions ψ, $\hat{\psi}$, $\hat{\psi}^{\dagger}_{1\downarrow}$ and $\hat{\psi}_{2\uparrow}$ are the fun ctions of r_{12}. That is the computation of isounit and the isoto pic element is being carried out within the hadronic volume. Therefore, from the preceding equations, we have,

$$\hat{I}' \gg 1,\ \hat{T}' \ll 1,\ \lim_{r_{12} \gg 1\text{fm}} \hat{I}' = I \tag{16}$$

As the explicit form of ψ is of Coulomb type, it behaves as,

$$\psi(r_{12}) \approx N \times e^{-(b \times r_{12})} \tag{17}$$

where N is approximately constant at distances termed as the coherence length, r_c, of the of IE ($= 1/b$), while $\hat{\psi}$ behaves like,

$$\hat{\psi}(r_{12}) \approx M \times \left(\frac{1 - e^{-(b \times r_{12})}}{r_{12}}\right) \tag{18}$$

[9] Among them the translational energy appears to overwhelmingly dominate that gets quantified and is contained in the electronic energy of the Helium-like system given by eq.(24).

where M is also approximately constant within the same range of applicability [6].

Therefore, the isotopic element $\hat{T}'$ of eq.(15) on using eqs.(17) and (18) gets expressed as,

$$\hat{T}' = 1 - \frac{N^2}{M} \times r_{12} \times \frac{e^{-(b\times r_{12})}}{1-e^{-(b\times r_{12})}} = 1 - r_{12} \times V_0 \times \frac{e^{-(b\times r_{12})}}{1-e^{-(b\times r_{12})}} = 1 - r_{12} \times V_{Hulthén} > 0 \quad (19)$$

where we have the constant $V_0 = N^2 / M$ and the Hulthén potential is given by,

$$V_{Hulthén} = V_0 \times \frac{e^{-(b\times r_{12})}}{1-e^{-(b\times r_{12})}} \quad (20)$$

Yet another expression of $V_{Hulthén}$ under the condition of eq.(11) in combination with eqs.(10) and (19) is obtained that reads as,

$$V_{Hulthén} = \left(\frac{r}{\left(r_{12}^{\dagger}\right)^3} \times \frac{1}{Z^*} + \frac{1}{r_{12}^{\dagger}}\right) - \frac{\mu_e^2}{Z^* e^2} \times \left(\frac{r}{\left(r_{12}^{\dagger}\right)^5}\right) \quad (21)$$

where $r_{12}^{\dagger}$ in eq.(21) is that value of r_{12} at which the net null potential is achieved.

Notice that r represents the most probable distance of IE from the nucleus. Obviously it would be different for different orbitals. That is the fixing of r implies considering IE in a given orbital. Thus we learn from eq.(21) that the magnitude of Hulthén potential varies from orbital to orbital and thereby the hadronic and conventional interactions between the two isoelectrons of IE also change from orbital to orbital implying that r_{12} also remains deferent for deferent orbitals.

6. Solution of Schrödinger Equation for the Two Body Approximation of Helium-Like Systems

Under the above two body model approximation in which we have adopted the nullification of attractive and repulsive potentials of two isoelectrons of an IE by the new isopotential, the Schrödinger equation reads as,

$$\left(\frac{1}{2\mu_{IE}} p \times p - \frac{Z^* e^2}{r}\right) \times |\psi\rangle = E \times |\psi\rangle \quad (22)$$

It is to be noted that as we are working within the conventional Euclidean space there is no need to use iso-wave function. The use of isotopy is required only for the quantification of the isopotential between the spin-paired isoelectrons in an orbital. Moreover, in eq.(22) there we have kinetic energy of IE with respect to nucleus hence there appears reduced mass of the IE.

Now eq.(22) gets further transformed to,

$$\nabla^2 |\psi\rangle + \frac{2\mu_{IE}}{\hbar^2}\left(E + \frac{Z^* e^2}{r}\right) \times |\psi\rangle = 0 \quad (23)$$

Notice that eq.(23) resembles the Schrödinger equation for a hydrogenic atom.

Therefore, we can directly use the standard solutions already available in literature for hydrogen atom or for a single electron systems [27, 28], of course, with appropriate changes. Thus the energy E is given by,

$$E = -\frac{2\pi^2 \mu_{IE} (Z^*)^2 e^4}{n^2 h^2} \quad (n = 1, 2, 3, \cdots) \quad (24)$$

where n is the radial or principal quantum number that can only have integer values. Notice that the energy given by eq.(24) is for the double occupancy of an orbital because in its numerator we have reduced mass of IE (c.f. eq.(4)) which is composed of two isoelectrons. Similarly, the total wave function gets determined by three quantum numbers n, l and m_l, the conventional principal, azimuthal and magnetic quantum numbers respectively. As usual the solution of eq.(23) gives,

$$n = 1, 2, 3, \cdots \quad (25)$$

$$l = 0, 1, 2, 3, \cdots, (n-1) \quad (26)$$

$$m_l = l, l-1, l-2, \cdots, 1, 0, -1, \cdots, -(l-1), -l \quad (27)$$

We are tabulating some of the normalized spin paired doubly occupied wave functions in Tables 1 – 4 each one of them are obtained for the spin paired occupancy by two electrons.

Table 1. Normalized spin paired doubly occupied wave functions of Helium atom/Helium-like systems, ψ_{nlm_l} with $a_0 = \frac{\hbar^2}{2\pi^2 e^2 \mu_{IE}}$, the conventional Bohr radius, where μ_{IE} is the reduced mass of the isoelectronium and is related as $\mu_{IE} = 2\times\mu_e$ for K shell.

Normalized spin paired doubly occupied wave functions
K Shell: $n = 1$
$l = 0$, $m_l = 0$
$\psi_{100} = \psi_{1s}$ or $1s = \frac{1}{\sqrt{\pi}}\left(\frac{2Z^*}{a_0}\right)^{3/2} \exp\left(-\frac{2Z^* r}{a_0}\right)$

Table 2. Normalized spin paired doubly occupied wave functions of Helium atom/Helium-like systems, ψ_{nlm_l} with $a_0 = \frac{\hbar^2}{2\pi^2 e^2 \mu_{IE}}$, the conventional Bohr radius, where μ_{IE} is the reduced mass of the isoelectronium and is related as $\mu_{IE} = 2\times\mu_e$ for L shell.

Normalized spin paired doubly occupied wave functions
L Shell: $n = 2$
$l = 0$, $m_l = 0$
$\psi_{200} = \psi_{2s}$ or $2s = \frac{1}{4\sqrt{2\pi}}\left(\frac{2Z^*}{a_0}\right)^{3/2} \cdot \left(2 - \frac{2Z^* r}{a_0}\right) \exp\left(-\frac{Z^* r}{a_0}\right)$
$l = 1$, $m_l = 0$
$\psi_{210} = \psi_{2p_z}$ or $2p_z = \frac{1}{4\sqrt{2\pi}}\left(\frac{2Z^*}{a_0}\right)^{5/2} \cdot z \cdot \exp\left(-\frac{Z^* r}{a_0}\right)$
$l = 1$, $m_l = 1$

Normalized spin paired doubly occupied wave functions
$\psi_{211} = \psi_{2p_x}$ or $2p_x = \frac{1}{4\sqrt{2\pi}}\left(\frac{2Z^*}{a_0}\right)^{5/2} \cdot x \cdot \exp\left(-\frac{Z^* r}{a_0}\right)$
$l=1,\ m_l=-1$
$\psi_{21(-1)} = \psi_{2p_y}$ or $2p_y = \frac{1}{4\sqrt{2\pi}}\left(\frac{2Z^*}{a_0}\right)^{5/2} \cdot y \cdot \exp\left(-\frac{Z^* r}{a_0}\right)$

Table 3. Normalized spin paired doubly occupied wave functions of Helium atom/Helium-like systems, ψ_{nlm_l} with $a_0 = \frac{\hbar^2}{2\pi^2 e^2 \mu_{IE}}$, the conventional Bohr radius, where μ_{IE} is the reduced mass of the isoelectronium and is related as $\mu_{IE} = 2\times\mu_e$ for M shell with $l = 0$ and 1.

Normalized spin paired doubly occupied wave functions M Shell: n=3
$l=0,\ m_l=0$
$\psi_{300} = \psi_{3s}$ or $3s = \frac{1}{81\sqrt{3\pi}}\left(\frac{2Z^*}{a_0}\right)^{3/2}\left(8\frac{(Z^*)^2 r^2}{a_0^2} - 36\frac{Z^* r}{a_0} + 27\right)\exp\left(-\frac{2Z^* r}{3a_0}\right)$
$l=1,\ m_l=0$
$\psi_{310} = \psi_{3p_z}$ or $3p_z = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{2Z^*}{a_0}\right)^{5/2} \cdot z \cdot \left(6 - \frac{2Z^* r}{a_0}\right)\exp\left(-\frac{2Z^* r}{3a_0}\right)$
$l=1,\ m_l=1$
$\psi_{311} = \psi_{3p_x}$ or $3p_x = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{2Z^*}{a_0}\right)^{5/2} \cdot x \cdot \left(6 - \frac{2Z^* r}{a_0}\right)\exp\left(-\frac{2Z^* r}{3a_0}\right)$
$l=1,\ m_l=-1$
$\psi_{31(-1)} = \psi_{3p_y}$ or $3p_y = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{2Z^*}{a_0}\right)^{5/2} \cdot y \cdot \left(6 - \frac{2Z^* r}{a_0}\right)\exp\left(-\frac{2Z^* r}{3a_0}\right)$

Table 4. Normalized spin paired doubly occupied wave functions of Helium atom/Helium-like systems, ψ_{nlm_l} with $a_0 = \frac{\hbar^2}{2\pi^2 e^2 \mu_{IE}}$, the conventional Bohr radius, where μ_{IE} is the reduced mass of the isoelectronium and is related as $\mu_{IE} = 2\times\mu_e$ for M shell with $l = 2$.

Normalized spin paired doubly occupied wave functions M Shell: n=3
$l=2,\ m_l=0$
$\psi_{320} = \psi_{3d_{z^2}}$ or $3d_{z^2} = \frac{1}{81\sqrt{6\pi}}\left(\frac{2Z^*}{a_0}\right)^{7/2}\left(3z^2 - r^2\right)\exp\left(-\frac{2Z^* r}{3a_0}\right)$
$l=2,\ m_l=1$
$\psi_{321} = \psi_{3d_{xz}}$ or $3d_{xz} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{2Z^*}{a_0}\right)^{7/2} \cdot xz \cdot \exp\left(-\frac{2Z^* r}{3a_0}\right)$
$l=2,\ m_l=-1$
$\psi_{321} = \psi_{3d_{yz}}$ or $3d_{yz} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{2Z^*}{a_0}\right)^{7/2} \cdot yz \cdot \exp\left(-\frac{2Z^* r}{3a_0}\right)$
$l=2,\ m_l=-2$
$\psi_{32(-2)} = \psi_{3d_{xy}}$ or $3d_{xy} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{2Z^*}{a_0}\right)^{7/2} \cdot xy \cdot \exp\left(-\frac{2Z^* r}{3a_0}\right)$
$l=2,\ m_l=2$

Normalized spin paired doubly occupied wave functions
$\psi_{322} = \psi_{3d_{x^2-y^2}}$ or $3d_{x^2-y^2} = \frac{1}{81\sqrt{2\pi}}\left(\frac{2Z^*}{a_0}\right)^{7/2} \cdot (x^2 - y^2) \cdot \exp\left(-\frac{2Z^* r}{3a_0}\right)$

Moreover, the shapes and orientations of these doubly occupied orbitals remains identically same as that is given by quantum mechanics for hydrogen-like systems (c.f. Figure 5). The only difference that results is the significant contraction of the orbitals in going from hydrogenic to heliumic systems (c.f. Tables 8 – 10 of Subsection 6.2.).

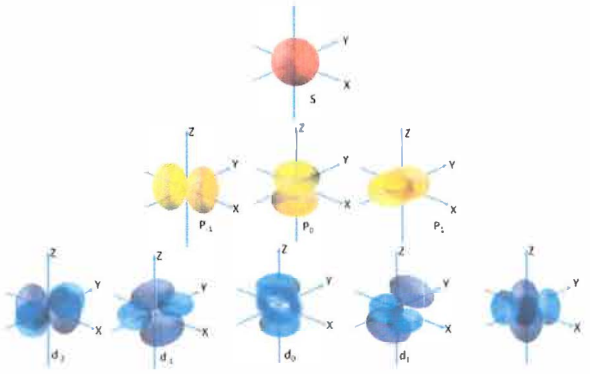

Figure 5. *Various orbital shapes of hydrogenic systems.*

The inspection of the expressions of wave functions given in Tables 1 – 4 establish that on occupation by spin paired two electrons none of the hydrogen like atomic orbitals change their shapes and spatial orientations. Only the size gets reduced appropriately. Hence, the change of terminology of orbitals in going from single electron orbitals to two spin paired electrons orbitals is not warranted. This is what is also reflected in Tables 1 – 4.

As the wave functions listed in Tables 1 – 4 are the double occupancy wave functions hence in going from $1s$ to any higher energy orbital represents the corresponding excited state. Thus the electronic configurations $2s^2$, $2p_x^2$, $2p_y^2$, $2p_z^2$, $3s^2$, $3p_x^2$, $3p_y^2$, $3p_z^2$, etc. are the various excited states. The term symbols for each of them is $^1S_0$. Notice that these excited states would be formed by simultaneous excitation of two spin paired electrons to a single higher energy orbital retaining spin pairing. But electronic transitions are governed by Franck-Condon prnciple [27] hence as such, such excitations would be a rare phenomena thus they would be extremely unstable even if they are formed in traces. However, the utility of these Helium-like orbitals is in the building up of electronic configuration of successive elements of the periodic table according to the Aufbau principle, Pauli principle and Hunds rule (see for example the reference [27]).

The energy of an orbital is determined primarily by the principal quantum number, n, therefore, the energy sequence in increasing order of doubly occupied orbitals, from eq.(24), gets established as,

$$1s < 2s = 2p_x = 2p_y = 2p_z < 3s = 3p_x = 3p_y = 3p_z = 3d_{z^2} = 3d_{x^2-y^2} = 3d_{xy} = 3d_{yz} = 3d_{zx} < 4s \cdots\cdots \quad (28)$$

which is identically same as that of Hydrogen-like systems.

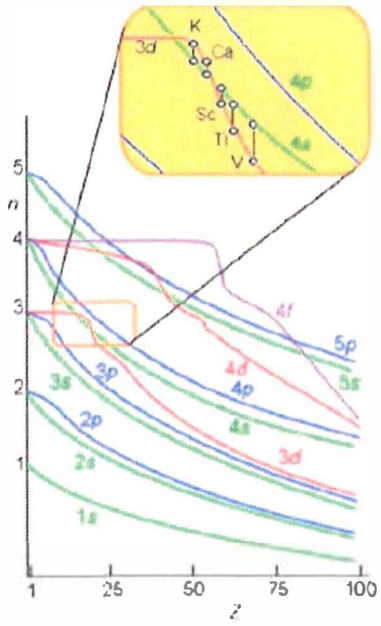

Figure 6. *Energy of Hydrogen-like atomic orbitals with increasing atomic number, Z, which is identical for Helium-like atomic orbitals.*

However, the helium-like wave functions given in Tables 1 – 4 are similar to the hydrogen-like wave functions and hence in the former the penetration of $2s, 2p, 3s, 3p, 3d$, etc. towards the nucleus would be similar to that we have in the latter case (for the said penetration see for example [27]). Hence, on the same lines the degeneracy of orbitals in the given shell K, L, M, etc. gets split and the above sequence of energy broadly transforms to,

$$1s < 2s < 2p_x = 2p_y = 2p_z < 3s < 3p_x = 3p_y = 3p_z < 3d_{xy} = 3d_{yz} = 3d_{xz} = 3d_{x^2-y^2} = 3d_{z^2} < 4s \cdots\cdots \tag{29}$$

Again this sequence is identically same which we have for hydrogen-like orbitals. More rigourously we will have the same energy sequence with increasing atomic number for Helium-like orbitals as that we have for Hydrogen-like orbitals depicted in Figure 6.

6.1. Effective Nuclear Charge of Helium Atom and Helium-Like Systems for 1s Occupancy

From eq.(4) in view of $M_{nucleus} \gg m_{IE}$ we practically have $\mu_{IE} \equiv m_{IE} = 2m_e$. Hence, eq.(24) effectively reads as,

$$E = -\frac{2\pi^2 m_e e^4}{n^2 h^2} \times 2(Z^*)^2 \qquad (n = 1, 2, 3, \cdots\cdots) \tag{30}$$

Therefore, for $n = 1$ eq.(27) would read as,

$$E_{1s} = E_{H(1s)} \times 2(Z^*)^2 \qquad \text{where} \qquad E_{H(1s)} = -\frac{2\pi^2 m_e e^4}{h^2} \tag{31}$$

which on rearrangement gives an expression of effective nuclear charge, namely:

$$Z^* = \sqrt{\frac{E_{1s}}{2E_{H(1s)}}} \tag{32}$$

In terms of experimental inonization energies eq.(32) then reads as,

$$Z^* = \sqrt{\frac{I_{n+1} + I_n}{2 \times I_{H(1s)}}} \tag{33}$$

where I_{n+1} and I_n are the last two ionization energies.

Therefore, we have substituted experimental values of the relevant ionization energies [29] in eq.(33) that produced corresponding values of effective nuclear charges which we have tabulated in Tables 5 – 7 along with the values that is given by Slater rule [13] and those computed by Clementi et al [14-17] for direct comparison.

Table 5. Effective Nuclear Charge (Z^*) of Helium and Helium-like Systems for 1s Spin-paired Electrons from H *to* Ne.

Element/ Ion	Z^* Equation (33)	Z^* Slater Rule §	Z^* Clementi et al ‡
H^-	0.686	0.7	Not available
He	1.704	1.7	1.6875
Li^+	2.698	2.7	2.691
Be^{2+}	3.696	3.7	3.685
B^{3+}	4.695	4.7	4.680
C^{4+}	5.69	5.7	5.673
N^{5+}	6.695	6.7	6.665
O^{6+}	7.695	7.7	7.658
F^{7+}	8.696	8.7	8.650
Ne^{8+}	9.698	9.7	9.642

§ [13], ‡ [17]

Table 6. EffiectiveNuclear Charge (Z^*) of Helium and Helium-like Systems for 1s Spin-paired Electrons from Na *to* Ca.

Element/ Ion	Z^* Equation (33)	Z^* Slater Rule §	Z^* Clementi et al ‡
Na^{9+}	10.699	10.7	10.626
Mg^{10+}	11.702	11.7	11.609
Al^{11+}	12.704	12.7	12.591
Si^{12+}	13.708	13.7	13.575
P^{13+}	14.711	14.7	14.558
S^{14+}	15.716	15.7	15.541
Cl^{15+}	16.721	16.7	16.524
Ar^{16+}	17.727	17.7	17.508
K^{17+}	18.733	18.7	18.490
Ca^{18+}	19.740	19.7	19.473

§ [13], ‡ [17]

Table 7. EffiectiveNuclear Charge (Z^*) of Helium and Helium-like Systems for 1s Spin-paired Electrons from Sc *to* Cu.

Element/ Ion	Z^* Equation (33)	Z^* Slater Rule §	Z^* Clementi et al ‡
Sc^{19+}	20.748	20.7	20.457
Ti^{20+}	21.756	21.7	21.441
V^{21+}	22.766	22.7	22.426
Cr^{22+}	23.776	23.7	23.414
Mn^{23+}	24.788	24.7	24.396
Fe^{24+}	25.800	25.7	25.381
Co^{25+}	26.814	26.7	26.367
Ni^{26+}	27.828	27.7	27.353
Cu^{27+}	28.844	28.7	28.339

§ [13], ‡ [17]

The data in Tables 5, 6 and 7 clearly demonstrates that the effective nuclear charge for two isoelectrons as IE in 1*s* orbital in each case using eq.(33) excellently matches with that obtained using Slater rule and it nicely correlates with that of Clementi et al. Hence, it serves to demonstrate the credibility of our ad hoc assumptions that imparts a physical reality to our structured model of Helium atom and Helium-like systems. If one carefully compares the values of Z^* given in the second column with those listed in the fourth column of Tables 5, 6 and 7 it would be noticed that the values of the second column are higher than those of fourth column and the difference between them increases as atomic number increases. To understand this trend we need to first realize that the Clementi et al values are obtained using SCF theory of quantum mechanics and in doing so a union of two electrons is not involved. Indeed, for a many body system one does not have analytic solution of Schrödinger equation and hence the SCF theory has been developed that involves the iteration technique and hence it is an approximate method. Still one can understand the said increasing difference between Z^* values. The separated two spin paired electrons of a given orbital would produce little higher shielding than the two spin paired electrons in the form of an IE. Therefore, as atomic number increases the the two electrons of 1*s* orbital become more tightly bound to nucleus and as a result their average distance from the nucleus continuously decreases (c.f. Tables 8 – 10 of Subsection 6.2). Therefore, it is easy to realize that if we assume the two spin paired electrons as unbound they would be more effective in shielding compared to the spin paired IE.

6.2. The Most Probable Radius of Helium Atom and Helium-Like Systems

The most probable radius, r^*, of 1*s* orbital of Helium atom and that of Helium-like systems is given by,

$$r^*(\uparrow\downarrow) = \frac{a_0}{2Z^*} \tag{34}$$

We have used the standard value of Bohr radius a_0 = 52.9 pm and the Z^* values calculated by eq.(33) using experimental ionization energies and tabulated in Tables 8 – 10. For the sake of comparison we have also tabulated the average radius $r^*(\uparrow) = a_0 / Z$ computed for the Hydrogen-like systems and tabulated in Tables 8 – 10.

Table 8. Most probable radius of Helium atom and Helium-like Systems and that of Hydrogenic Systems from H *to* Ne.

Element/ Ion	$r^*(\uparrow\downarrow)$ /pm Equation (34)	Element/ Ion	§ $r^*(\uparrow)$ /pm
H^-	77.11	H	52.9
He	31.04	He^+	26.45
Li^+	19.71	Li^{2+}	17.63
Be^{2+}	14.31	Be^{3+}	13.225
B^{3+}	11.27	B^{4+}	10.58
C^{4+}	9.30	C^{5+}	8.817
N^{5+}	7.90	N^{6+}	7.557
O^{6+}	6.87	O^{7+}	6.613
F^{7+}	6.08	F^{8+}	5.878
Ne^{8+}	5.45	Ne^{9+}	5.29

§ [27]

Table 9. Most probable radius of Helium atom and Helium-like Systems and that of Hydrogenic Systems from Na *to* Ca.

Element/ Ion	$r^*(\uparrow\downarrow)$ /pm Equation (34)	Element/ Ion	§ $r^*(\uparrow)$ /pm
Na^{9+}	4.94	Na^{10+}	4.809
Mg^{10+}	4.52	Mg^{11+}	4.408
Al^{11+}	4.16	Al^{12+}	4.069
Si^{12+}	3.86	Si^{13+}	3.779

Element/	$r^*(\uparrow\downarrow)$ /pm	Element/	§$r^*(\uparrow)$ /pm
P^{13+}	3.60	P^{14+}	3.527
S^{14+}	3.37	S^{15+}	3.306
Cl^{15+}	3.16	Cl^{16+}	3.112
Ar^{16+}	2.98	Ar^{17+}	2.939
K^{17+}	2.82	K^{18+}	2.784
Ca^{18+}	2.68	Ca^{19+}	2.645

§ [27]

Table 10. Most probable radius of Helium atom and Helium-like Systems and that of Hydrogenic Systems from Sc *to* Cu.

Element/ Ion	$r^*(\uparrow\downarrow)$ /pm Equation (34)	Element/ Ion	§$r^*(\uparrow)$ /pm
Sc^{19+}	2.55	Sc^{20+}	2.519
Ti^{20+}	2.43	Ti^{21+}	2.405
V^{21+}	2.32	V^{22+}	2.30
Cr^{22+}	2.23	Cr^{23+}	2.204
Mn^{23+}	2.13	Mn^{24+}	2.116
Fe^{24+}	2.05	Fe^{25+}	2.035
Co^{25+}	1.97	Co^{26+}	1.959
Ni^{26+}	1.90	Ni^{27+}	1.889
Cu^{27+}	1.83	Cu^{28+}	1.824

§ [27]

If no screening effect operates then $r^*(\uparrow)$ should be equal to $2\times r^*(\uparrow\downarrow)$. But from Tables 8 – 10 we see that it is not so. Actually we find that in all cases $2\times r^*(\uparrow\downarrow) > r^*(\uparrow)$, that is the shrinking of the electron charge is less than if no shielding effect operated. However, as atomic number increases $2\times r^*(\uparrow\downarrow)$ tends to approach $r^*(\uparrow)$.

It seems that our method of calculation of effective nuclear charge, that is experienced by two $1s$ electrons, via eq.(33) is superior over that of Clementi et al because the former method is based on the experimental ionization energies.

Yet another outcome of this presentation is that when we start filling atomic orbitals in accordance with the Aufbau principle we first place one electron in a hydrogen-like orbital and then when we fill the second electron guided by the Pauli principle it becomes a Helium-like orbital. Thus for example, in the case of Na the electronic configuration is $1s^2 2s^2 2p^6 3s^1$, in that $3s^1$ is the Hydrogen-like wave function and all the lower energy ones are the Helium-like wave functions. That is, in $1s^2 2s^2 2p_x^2 2p_y^2 2p_z^2$ there we have the occupancy by an IE in each orbital.

6.3. Lower Excited States of Helium Atom

We recall that energy of the three lower electronic states of He atom follows the following order, namely:

$$1s^2(\uparrow\downarrow)(^1S) < 1s^1(\uparrow)2s^1(\uparrow)(^3S) < 1s^1(\uparrow)2s^1(\downarrow)(^1S)$$

$$< 1s^1(\uparrow)2p^1(\uparrow)(^3P) < 1s^1(\uparrow)2p^1(\downarrow)(^1P) < \cdots\cdots \quad (35)$$

We notice that the triplate state $1s^2(\uparrow\uparrow)(^3S)$ is not amongst the lower excited states. This is so because in this case there would be very strong magnetostatic repulsion. Hence both the electrostatic and magnetostatic strong repulsions would not allow electrons to come close to form a union i.e. an IE. On the contrary the repulsive potential energy would be very high if two electrons are forced to occupy the small volume of the $1s$ orbital with parallel spins. However, from the next higher energy states the ^3S state is of lower energy than the ^1S state. To explain it we need to realize that in these two states the two electrons are in two different spherically symmetric hydrogenic orbitals hence no IE formation could take place. Therefore, no hadronic mechanics based explanation can be applied. Thus the conventional quantum mechanical explanation remains in order.

7. Concluding Remarks

This paper we dedicate to the memory of G. N. Lewis who coined the term coordinate covalent bond, which is formed by the donation of a lone pair of electrons to an accepter molecule (1923) that led us to conclude that all spin paired electrons of atomic orbitals are isoelectronium.

In this paper first we have discussed in Section 1 the chemical reactivity and chemical bonding, particularly the coordinate covalent bond (G. N. Lewis), bases that strongly indicate that all spin paired electrons in each atomic orbital, indeed, exist as a quasistatic particle formed by the union of two electrons, which was named earlier by Santilli as isoelectronium.

As a result of this from hadronic physics and chemistry point of view the iso-Helium atom and iso-Helium-like systems constitute a problem in which we have to deal with two different levels of lengths and corresponding interactions. The IE formation is a phenomena occurring within the hadronic volume and because of it, not only there occurs electrostatic interaction between two electrons but also a very strong magnetostatic interaction comes into play between them. However, at the same time a new phenomena comes into play because of deep overlap of wave packets of two electrons that obviously produces correspondingly additional interaction, which has been demonstrated by Santilli as getting quantified by Hulthén potential. Whereas the kinetic energy of IE and its electrostatic interaction with the nucleus both involve the distances of atomic dimensions.

The crucial assumption permitted by the quasi-particle status imparted to an IE is that — on formation of an IE the electrostatic, the magnetostatic and Hulthén potential between the two isoelectrons of an IE produce an algebraic sum equal to zero or very small but of a constant magnitude potential. In view of it we have termed our model as an approximation. However, in this way the problem simplifies to a great extent that it resembles hydrogenic systems with a difference that instead of a single electron now we have an IE. That is, now we can use quantum mechanical tools to solve the corresponding Schrödinger equation. Thus it becomes a very simple system in which we need not to mathematically tackle various potentials between the two isoelectrons in solving Schrödinger equation. Of course, we have derived an expression of Hulthén potential under the condition of the said nullification of potentials of two isoelectrons (c.f. eq.(21)).

Indeed, under this assumption any variation in r_{12} would be insignificant compared to the magnitude of r that amounts to treating IE as a rigid union. However, it is unlikely that they would form a rigid union. That is, an IE must be an oscillating union about r_{12}^0, the distance at which the said complete nullification of potentials takes place. This oscillatory motion we need to quantify using Schrödinger-Santilli isoequation. This we have described in Appendix C. We also see that according to eq.(21) r_{12}^0 will not be of same magnitude in different Helium-like orbitals.

Under this simplification the corresponding derived expression of energy of the atomic quantum states offered us to calculate the effective nuclear charge that is experienced by two electrons of $1s$ orbital of various Helium-like systems. Amazingly, our values match exceedingly well with earlier estimates based on SCF theory and Slater rule. Also we have calculated the most probable radius of $1s$ orbital of various Helium-like systems that as expected are smaller than those for Hydrogenic $1s$ orbitals and compares well with the radius of the latter in the sense that as atomic number increases the $1s$ orbital continuously shrinks with increasing nuclear charge.

The credibility of the approximations used in the two body model approximation of heliumic systems presented herein gets demonstrated by the calculated values of the effective nuclear charge using the input of the experimental ionization energies into eq.(33) — because the so calculated values are fantastically realistic. We have also explained the physical origin of the said assumption of null potential contribution to the total potential energy of the system.

With the demonstration herein of the credibility of Helium-like systems treated using IE and adopting corresponding implied approximations it is worth to extend the same approximation to describe

1. atomic structure of all elements of periodic table by considering every spin paired extranuclear electrons as an IE.
2. chemical bonding starting with hydrogen molecule, the simplest molecule housing an IE.

Finally a word on the difference between the Hadronic potentials of this paper and that appearing in Santilli's earlier works. In Santilli's works the hadronic potential is obtained as strongly attractive whereas that of the present paper it is strongly repulsive. As stated in the main text of this paper this seemingly opposite nature of the Hadronic potential gets easily reconciled. In the present model we have treated magnetostatic (strongly attractive) and hadronic (strongly repulsive) interactions between two isoelectrons of an IE separately. Whereas the mathematical model of Santilli's earlier works seemingly treats both the interactions as a single hadronic type. Therefore, it is no wonder that the hadronic potential of Santilli's earlier works is obtained as strongly attractive one.

Acknowledgements

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Appendix

A. Potential Energy of Two Electrons at a Distance Apart

1. The potential energy of repulsion, U_e^-, of two electrons at a distance r apart is given by,

$$U_e^- = \frac{e^2}{4\pi\varepsilon_0 r} \tag{A.1}$$

where e is the charge on an electron and ε_0 is vacuum permittivity. The standard values of the involed constants are:

$$4\pi\varepsilon_0 = 1.112650\times10^{-10}\,\mathrm{J^{-1}.C^2.m^{-1}}, \quad -e = 1.602192\times10^{-19}\,\mathrm{C}$$

On substitution of the numerical value of $r = 10^{-15}m$ and using above values we obtain,

$$U_e^- = \frac{(1.602192)^2\times10^{-38}\,\mathrm{C^2}}{(1.112650\times10^{-10}\,\mathrm{J^{-1}.C^2.m^{-1}})(10^{-15}\,\mathrm{m})} = 2.307122\times10^{-13}\,\mathrm{J} = 1.44\,\mathrm{MeV} \tag{A.2}$$

2. The magnetostatic potential energy of a bar magnet of magnetic moment μ in the magnetic field intensity of $\mathbf{B}$ is given by [30],

$$U = -\,\mathbf{B}\cdot\mu \tag{A.3}$$

Hence, as above we consider that one electron is situated at a distance r in the magnetic field of the other spinning electron. Hence eq.(A.3) in this case reads as,

$$U_e^+ = -\,\mathbf{B}_e(r)\cdot\mu_e \tag{A.4}$$

However, $\mathbf{B}$ in terms of magnetic moment is given by,

$$\mathbf{B}(r) = -\,\frac{\mu_0\mu}{2\pi r^3} \tag{A.5}$$

where $\mu_0 = 4\pi\times10^{-7}\,\mathrm{J.C^{-2}.s^2.m^{-1}}$ is the vacuum permeability.

Recall that in an isoelectronium two spinning electrons have opposite spins, hence their magnetic moments are directed opposite to each other. Therefore, $\mathbf{B}_e(r)$ of eq.(A.4) is given by,

$$\mathbf{B}_e(r) = -\,\frac{\mu_0(-\mu_e)}{2\pi r^3} \tag{A.6}$$

However, it can be easily shown that

$$|\mu_e| = \mu_B = 9.274078\times10^{-24}\,\mathrm{C.s^{-1}.m^2},$$

where μ_B is the Bohr magneton.

Hence, the potential energy of the magnetostatic attraction of two spin paired electrons on using eq.(A.4) gets expressed as,

$$U_e^+ = -\frac{\left[(9.274078)^2\times10^{-48}\,\mathrm{C^2.s^{-2}.m^4}\right]\left(4\pi\times10^{-7}\,\mathrm{J.C^{-2}.s^2.m^{-1}}\right)}{2\pi\left(10^{-15}\right)^3\,\mathrm{m^3}} \tag{A.8}$$

$$= -2\times(9.274078)^2\times10^{-10}\,\mathrm{J} = -1.7201705\times10^{-8}\,\mathrm{J} = -107364.6\mathrm{MeV}$$

3. Thus from eqs.(A.2) and (A.8) we see that $U_e^+ \gg U_e^-$.

4. Which gets quantitatively supported by,

$$|U_e^+ / U_e^-| = 7.5487226\times10^4 \tag{A.9}$$

5. Whereas at $r = 1\mathrm{pm}$ we have

$U_e^+ = -1.7201705\times10^{-17}\,\mathrm{J} = -107.37\mathrm{eV}$ and

$U_e^- = 2.307122\times10^{-16}\,\mathrm{J} \approx 1440\mathrm{eV}$. Thus the repulsive potential becomes much higher than the attractive one. It may then perhaps be treated as highly loose or unstable union.

6. At $r = 10\mathrm{nm}$ we have

$U_e^+ = -1.7201705\times10^{-29}\,\mathrm{J} = -1.074\times10^{-10}\mathrm{eV}$ and

$U_e^- = 2.307122\times10^{-20}\,\mathrm{J} = 0.144\mathrm{eV}$. That is the magnetostatic attraction becomes negligible compared to electrostatic repulsion.

7. In above calculations the null potential energy is obtained at,

$$r^2 = \frac{2\times\mu_B^2\times\mu_0\times\varepsilon_0}{e^2} = 7.4559\times10^{-26}\,\mathrm{m^2} \tag{A.10}$$

$$\Rightarrow \quad r = 0.273055\mathrm{pm}$$

and the attractive and repulsive potential energies have the magnitude of $8.4493\times10^{-16}\,\mathrm{J} = 5.2736\,\mathrm{keV}$.

We present above results of calculations in Table 11.

Table 11. Electrostatic and magnetostatic potential energies of two electrons with varying distance of separation between them.

S. No.	r /m	U_e^- Repulsive (Electrostatic)	U_e^+ Attractive (Magnetostatic)
1.	10^{-15}	1.44 MeV	-1.074×10^5 MeV
2.	2.73055×10^{-13}	5.2736 keV	-5.2736 keV
3.	10^{-12}	1.44 keV	-1.0736×10^2 eV
4.	10^{-8}	0.144 eV	-1.9736×10^{-10} eV

B. The Notion of Trigger That Assist IE Formation

Let us briefly describe the concept of *trigger* that assist IE formation as elucidated by Santilli himself elsewhere [6].

1. A notion for the very existence of the IE is that of a *trigger*, namely, *external (conventional) interactions,*

$$U_e^+ = -\mu_B^2\times\frac{\mu_0}{2\pi r^3} \tag{A.7}$$

which get computed for $r = 1\,\mathrm{fm}$ as,

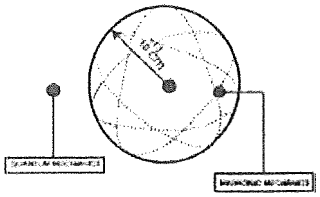

Figure 7. A schematic unit of of the hadronic horizon, namely, of the sphere of radius 1 fm *(=* 10^{-13} cm*) outside which the quantum chemistry is assumed to be exactly valid, and inside which nonlinear, nonlocal and nonpotential effects are no longer negligible, thus requiring the use of hadronic chemistry for their numerical and invarient treatment.*

which causes the identical electrons to move the one towards the other and penetrate into the hadronic horizon (Figure 7) *against their repulsive Coulombic interactions.*

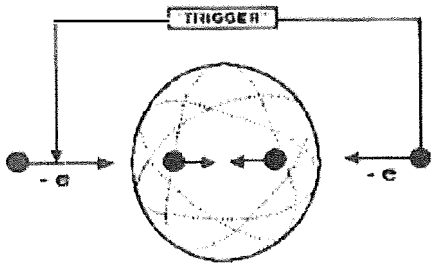

Figure 8. A schematic view of the trigger, namely, the external means suitable to force the electrons with the same charge to penetrate the hadronic barrier (c.f. Figure 7), in which the attractive hadronic forces overcome the repulsive Coulombic barrier.

Once inside the above mentioned horizon, the attractive magnetostatic force overcomes the repulsive Coulombic interaction, and at the same time a new interaction comes into play which is of hadronic origin due to the mutual overlap of wave packets of electrons, resulting in a bound state. The notion of the said *trigger* has been schematic represented in Figure 8.

2. In the case of the hadronic model of π^0 meson as a bound state of an electron and a positron at short distances, there is no need for an external trigger because the constituents naturally attract each other [26]. On the contrary, the existence of the Cooper pair does indeed require a trigger, which Santilli

and Animalu identified as being provided by the Cuprate ions [21, 22]. For the case of isolated hydrogen molecule the trigger has been identified in the two H-nuclei, which do attract the electrons [6,10] because the attraction of the electrons by the two nuclei is sufficient to cause the overlaping of the two wave packets of electrons.

C. IE as a Simple Harmonic Oscillator Within Hadronic Volume

As described in the main text and the Appendices A and B of the present paper the union of two electrons as IE is based on attractive and repulsive forces it cannot be a rigid union but would be an oscillating pair. Definitely the oscillatory motion of an IE takes place well within the hadronic volume. Therefore, an isolated harmonic oscillating IE about $\hat{r}_{12}^{0}$, to the first approximation, will have the iso-potential energy $\hat{V}(\hat{r}_{12})$ given by,

$$\hat{V}(\hat{r}_{12}) = -\,U \times \frac{1}{2} k (r_{12} - r_{12}^{0}) \times U^{\dagger} = -\left(\frac{\hat{1}}{2}\right) \hat{\times} \hat{k} \hat{\times} (\hat{r}_{12} - \hat{r}_{12}^{0}) \quad \text{(C.1)}$$

Therefore, the corresponding Schrödinger-Santilli isoequation applicable within the hadronic volume reads as,

$$\left[\frac{\hat{p}^{\hat{2}}}{\hat{2} \hat{\times} \hat{\mu}} \times \hat{I} - \left(\frac{\hat{1}}{2}\right) \hat{\times} \hat{k} \hat{\times} (\hat{r}_{12} - \hat{r}_{12}^{0}) \right] \hat{\times} |\hat{\psi}_{IE}\rangle = E_{IE} \times |\hat{\psi}_{IE}\rangle \quad \text{(C.2)}$$

where $\hat{\mu}$ is the reduced iso-mass of the IE, $\hat{k}$ is the iso-force constant of the vibration and rest of the terms have their usual meanings. The solution of eq.(C.2) gives the following expression of the vibrational energy, namely:

$$E_{\hat{v}} = \left(\hat{v} + (\hat{1}/\hat{2}) \right) \hat{\times} \hat{\nu} \qquad \text{with} \qquad \hat{v} = \hat{0},\ \hat{1},\ \hat{2}, \cdots\cdots \quad \text{(C.3)}$$

and the isonormalized isowave-function reads as,

$$\hat{\psi}_{\hat{v}}(\hat{\xi}) = \left[\hat{2}^{\hat{v}} \hat{\times} \hat{v}! \hat{\times} \hat{\pi}^{\hat{1}/\hat{2}} \right]^{-\hat{1}/\hat{2}} \hat{\times} \hat{H}_{\hat{v}}(\hat{\xi}) \hat{\times} \hat{\exp}\left(-\hat{\xi}^{\hat{2}} \hat{\div} \hat{2} \right) \quad \text{(C.4)}$$

where $\hat{\xi} = \hat{\beta}^{\hat{1}/\hat{2}} \hat{\times} \left(\hat{r}_{12} - \hat{r}_{12}^{0} \right)$ is the displacement variable in iso-Hilbert space, $\hat{\beta}$ is an isotopically lifted constant $\beta = \frac{2\pi}{h} \sqrt{m_e k}$, m_e is the mass of an electron and $\hat{H}_{\hat{v}}$ is the iso-Hermite polynomial.

Now eq.(C.3) further simplifies to,

$$E_{\hat{v}} = E_{v} = \left(v + \frac{1}{2} \right) \hat{\nu} \quad \text{(C.5)}$$

where $\hat{v}$ is the vibrational iso-quantum number. Notice that $\hat{v}$ is the isotopically lifted conventional vibrational quantum number v, which can have only the integer values and $\hat{\nu}$ is the fundamental iso-vibrational frequency within the hadronic space of IE.

Thus we see that an IE will have a zero point energy equal to $(1/2)\hat{\nu}$. Also if required we can add anharmonicity terms in eq.(C.5) as we do in the case of molecular vibrations. Indeed, the vibrational energy of IE would be relatively very small compared to its orbital energy hence the orbital energy given by eq.(30) to the first approximation measures the energy of an IE in a given orbital. To ascertain this conjecture we need to design an appropriate experiment to see if there exists a vibrational fine structure in electronic spectra of say Helium atom in singlet-singlet transition. The similar fine structure in ground singlet-first excited triplet state is also expected but less prominently because in such excitation IE breaks down.

The same would be true for the rotational fine structure of IE. More so, the rotations and vibration of IEs would be constrained by the strong electrostatic attraction between the nuclear charge and IE. This aspect needs further investigation.

As stated in the main text the IE in a heliumic system is not an isolated species but is imbedded in the electrostatic field of nuclear protons hence the simple harmonic oscillation approximation within the hadronic volume needs to be modified accordingly. For the same reason the rigid rotor treatment for the heliumic IE needs to take account of this fact. However, so far we have not attended to it.

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A Tentative Magnecular Model of Liquid Water with an Explicit Attractive Force Between Water Molecules

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Abstract: In this paper, we outline the main features of the chemical species of magnecules and their magnecular bond; we then present, apparently for the first time, experimental evidence via three different analytic methods at different laboratories on the capability by suitably polarized Hydrogen atoms to have a magnecular bond to ordinary molecules; and we submit, also apparently for the first time, a tentative model of the liquid state of water with an explicitly identified *attractive force* between the water molecules consisting of magnecular bonds between opposing polarities of the toroidal configurations of the orbits of the valence electrons of water molecules, for which the boiling temperature is the Curie temperature of the magnecular bond as established for other magnecular species. We finally point out the environmental and industrial significance of the achievement of a quantitative structure model of the water liquid state due to its extension to gasoline and other liquid fuels with ensuing possibilities to improve their combustion.

Keywords: Valence, Magnecular Bond, Magnecule, Hydrogen

1. Introduction

In preceding works (see Ref. [1] of 2005 with preceding literature quoted therein, and Refs. [2, 3] with updated literature), the author presented physical and chemical evidence suggesting the possible existence of a new chemical species consisting of *clusters of individual atoms* (H, O, C, *etc.*), *radicals* (HO, CH, *etc.*) *and ordinary molecules* (H_2, H_2O, CO, *etc.*) *bonded together by attractive forces between opposing magnetic polarities of toroidal polarizations of atomic orbitals, as well as the polarization of the magnetic moments of nuclei and electrons* (see Figure [1] for a conceptual rendering).

The new chemical species was submitted by the author under the name of *magnecules* in order to differentiate them from the conventional *molecules,* where the latter are referred to clusters of atoms solely under one or another conventional valence bond, while the former are referred to mixtures of molecular and magnecular bonds. Valence bonds are generally represented with the symbol "–", while magnecular bonds are represented with the symbol "×".

Among the various anomalous characteristics of magnecular bonds, the author tentatively identified in preceding works [1] a new feature under the name of *Hydrogen accretion,* consisting of the anomalous bond of one or more Hydrogen atoms to a conventional molecule as well as to generic clusters from 2 a.m.u. all the way to large a.m.u. We should indicate, for readers not familiar with the new Hydrogen technology here referred to, that Hydrogen accretion is definitely *impossible* for conventional Hydrogen atoms (those with a spherical distribution of their orbitals) and it is solely possible for Hydrogen atoms exposed to very large magnetic fields, such as those of the order of 10^{12} Gauss) available at atomic distances from DC electric arcs, necessary to create the toroidal polarizations (Illustrated in Figure 1).

In this paper, we present three independent experimental verifications of Hydrogen accretion obtained at three different analytic laboratories by using three different chromatographic equipment; we focus the attention in the representative values of 3, 4, 5, 6, 19, 29, and 45 a.m.u.; we tentatively interpret these species as being due to magnecular bonds of one or more Hydrogen atoms with conventional molecules; we point out expected implications for the structure of liquids; we indicate

industrial applications for new types of Hydrogen rich fossil fuels; and we offer to qualified colleagues, at no cost, samples of the anomalous gas exhibiting Hydrogen accretion for independent verifications.

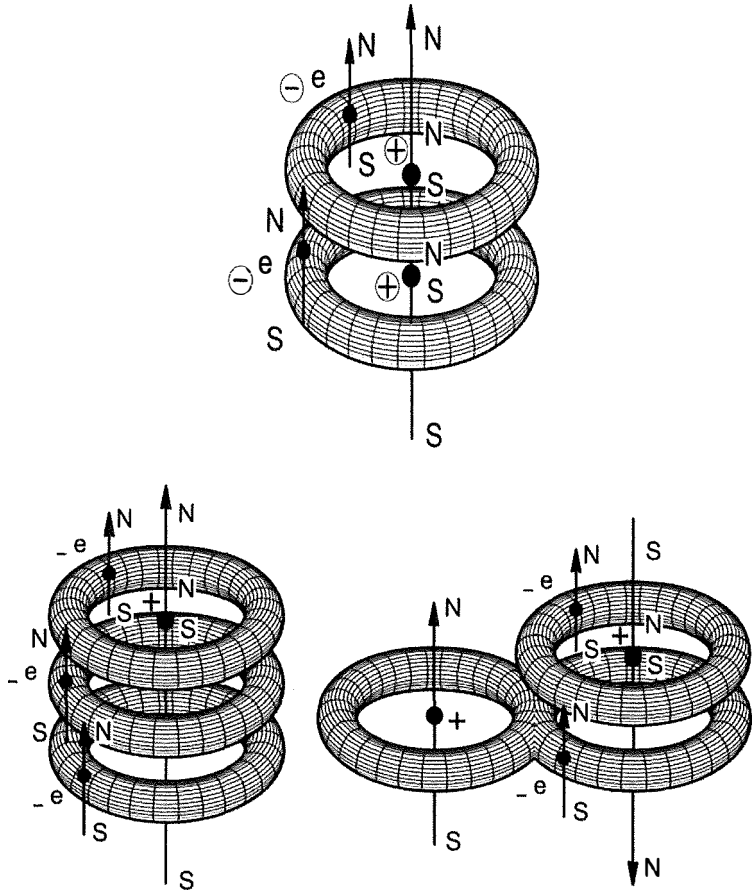


Figure 1. At the top, we show a conceptual rendering of a "two-body magnecule" at absolute zero degree temperature illustrating the dominance of the attraction due to magnetic polarizations of electron orbitals, as well as that of electrons and nuclei over repulsion forces due to opposing charges, since the atoms herein considered are assumed to have a null total charge. At the bottom, we show a conceptual rendering of the two possible configurations of a "three-body magnecule."

2. Experimental Confirmations of Hydrogen Accretion

The first independent experimental confirmation of Hydrogen accretion was obtained on November 2, 2009, at the FAI Analytic Laboratories in Atlanta, Georgia, via a GC-MS operated at 10^0 *C* column temperature, with the detection of anomalous species consisting of the increase of one value of a.m.u. from 2 to large values of a.m.u. These anomalous species were detected in a combustible gas produced and sold under the commercial name of MagneGas™ that is known to have a magnecular structure due to its origination via a submerged DC electric arc (see www.magnegas.com and Refs. [1, 2, 3]).

Among a large number of scans, we show in the top of (Figure 2) a representative scan in the rage 200 to 250 a.m.u. characterized by individual increases of 1 a.m.u. without solution of continuity, i.e.,

$$200,201,202,203,204,205,.....,250 \quad (1)$$

Following the above 2009 measurements, the author searched for years for another laboratory to perform independent verification or denial of said anomalous species. Unfortunately, no laboratory cooperated with the author's rather unusual requests in the use of a GC-MS necessary for the detection of magnecules [1-3]. For instance, analysts would insist on using the GC-MS according to procedures fully established for the detection of molecular species (e.g., by using high column temperatures, short elusion times, etc.). Practically insurmountable difficulties were encountered with recent gas chromatographic equipment using capillary feeding lines because of the general decline by analysts that the very feature to be detected, Hydrogen accretion, would clog up capillary feeding lines, thus preventing the gas sample to even enter the instrument. These and other difficulties explain the lapse of time between the preliminary identification of Hydrogen accretion and the date of this note.

Finally, in early 2010, the author became aware of the intention by HyFuels Corporation in Tarpon Springs, Florida, to have an older GC-MS/IRD restored by a specialized company in the field, and elected to wait for the availability of that instrument due to its superior capability of detecting magnecules (see Refs. [1, 2, 3] for brevity).

The desired GC-MS/IRD eventually became operational in early 2012, by comprising a HP GC model 5890, a HP MS model 5972, and a HP IRD model 5965 equipped with a HP Ultra 2 column 25m long, 0.32mm ID, and film thickness of 0.52mm, with temperatures starting at 10°C for 4 min, then incrementally raised to 250°C at 10°C/min.

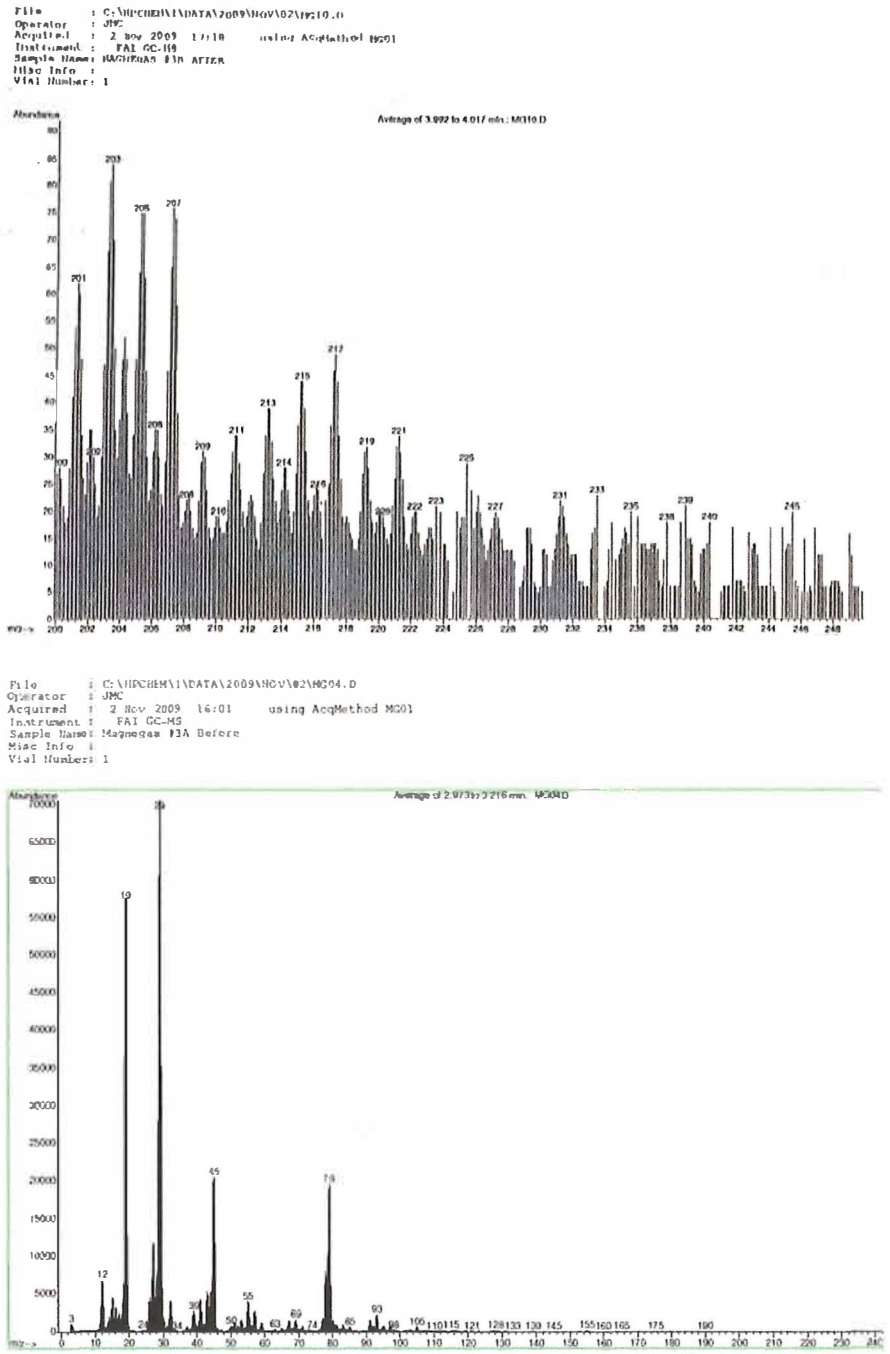


Figure 2. *The top view shows a representative scan obtained on November 2, 2009, by the FAI Analytic Laboratories of Atlanta, Georgia, on the gaseous fuel MagneGas via a contemporary GC-MS operated at low column temperature, providing experimental evidence of Hydrogen accretion from 2 to hundreds of* a.m.u. *(see the top view for a representative scan from 200 to 250* a.m.u.*). The lower view shows representative example of Hydrogen accretion with the anomalous species with 19,29,45 and other* a.m.u..

When properly operated for the detection of magnecules (e.g., by using the lowest available column temperature, the longest available elusion time, the largest available cryogenically cooled feeding line, etc.), the latter instrument did confirm the magnecular structure of MagneGas [1] as consisting of clusters fully identified in the GC-MS, but possessing no IR signature at the a.m.u. of the clusters (and not at the a.m.u. of the constituents), as shown in representative scans of (Figure 3).

In (Figure 3 and 4), we present the second experimental evidence obtained on October 9, 2012, via the above identified GC-MS/IRD confirming the existence in the MS spectrum of the anomalous species with magnecular bonds as well as species expected to be due to Hydrogen accretion.

The third experimental confirmation was obtained on November 15, 2012, at the Oneida Research Services (ORS) in Whitesboro, NY, via an IVA 110s with an accuracy of ±5% at 5000 ppm (see also Ref. [2] for details). As one can see from (Figure 5), the IVA 110s provided an accurate confirmation of the Hydrogen accretion to hundreds of a.m.u. (*only values up to* 75a.m.u. are shown in Figure 5 for brevity), including the confirmation of anomalous species with 3, 4, 5, 6, 19, 29 and 45 a.m.u.).

By recalling that Magnegas has no appreciable content of Helium, we tentatively present the following interpretation of representative new species f with 3, 4, 19, 29 and 45 a.m.u. for all possible magnecular interpretations (see also Figures 1 and 6)

$$H_3 = \{H_2 \times H, H \times H \times H \times H\}, \quad (1a)$$

$$H_4 = \{H_2 \times H_2, H_2 \times H \times H, H \times H \times H \times H \times H\}, \quad (1b)$$

$$H_3O = \{H - O - H \times H, H - (O \times H) - H\}, \quad (1c)$$

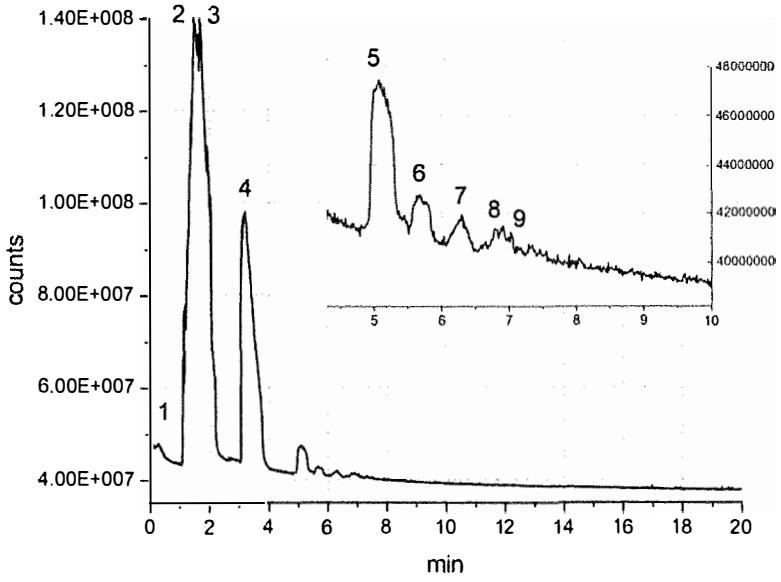
$$COH = \{C - O \times H, O - C \times H\}, \quad (1d)$$

$$CO_2H = \{C - O_2 \times H, O_2 - C \times H\}, \quad (1e)$$

with similar interpretations holding for additional anomalous species as identified, e.g., in (Figure 5).

The above magnecular interpretation is based on a number of aspects, such as:

1) The progressive Hydrogen accretion up to large a.m.u. in a gas synthesized by carbon and a liquid feedstock, such as Magnegas, whose heaviest molecule should be CO_2.
2) Known difficulties in the molecular interpretation of H_3, such as the impossibility according to quantum mechanics of bonding a third valence electron to a pair of valence electrons in singlet coupling (because the former electron has spin 1/2 while the latter electron pair has spin zero); the impossibility under a three valence electron bond of an exact representation of the binding energy of the H_2 constituent (due to the impossibility for the three valence electrons to have a continuous bond at all times); expected consequential impossibility to verify the principle of conservation of the energy at the time of the synthesis of H_3 as occurring in the species herein considered, from H_2 and H; and other insufficiencies that eventually multiply for H_4, H_5, H_6, etc.
3) The elimination of the Hydrogen accretion and of all related anomalous species when the gas is tested with a GC-TCD operated at high temperature (see Ref. [2] for details), thus confirming that species (1) have a characteristic Curie temperature at which magnecular bonds disappear, and the reduction of the gas to about 65% H_2, 30% CO plus small percentages of HO and CO_2, thus confirming that species (1) are indeed characterized by Hydrogen accretion of H_2, H_2O, CO and CO_2.



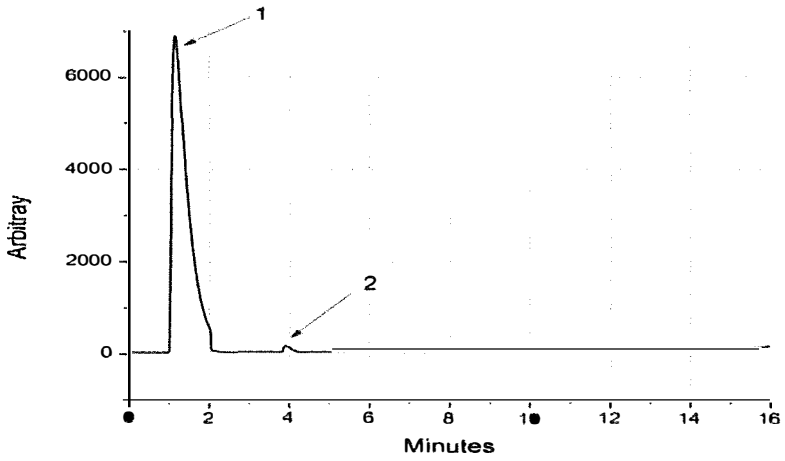
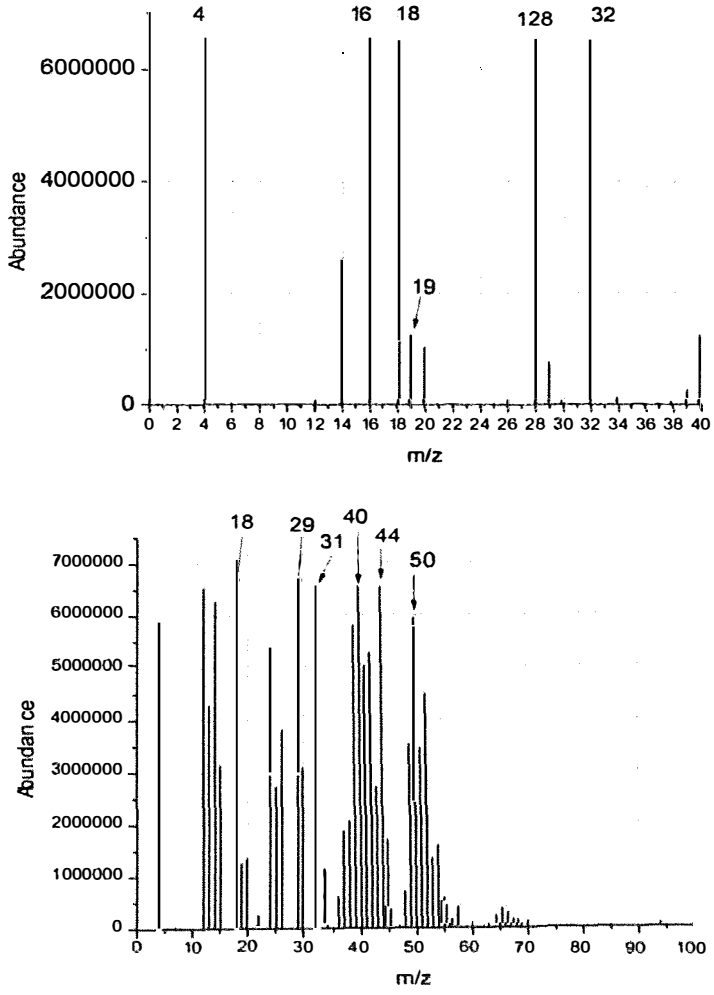


Figure 3. Representative scans achieved on October 9, 2012, via the GC-MS/IRD described in the test on MagneGas fuel. The top view presents a GC-MS scan from 2 a.m.u. *to 500* a.m.u. *with the column operated at* 10°C *and the use of 22 minutes elusion time. The bottom view presents the IRD scan of the same gas and the same injection used for the top view, that shows the existence of species well identified in the GC-MS that have no IR signature at their a.m.u., thus confirming the magnecular structure of the gas presented in Ref. [1].*



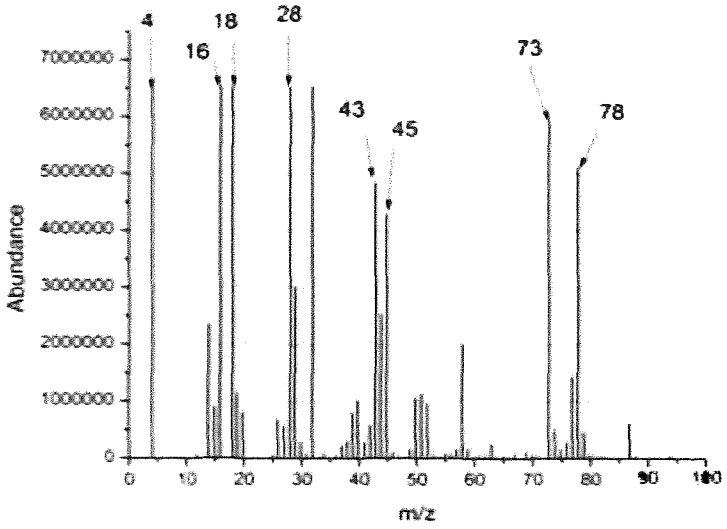


Figure 4. Representative MS scans of the magnecular clusters of (Figure 3) confirming the existence of anomalous species expected to be due to Hydrogen accretion.

ORS REPORT NO.	196809-001		
DATE TESTED	9/5/2012		
QUANTITY TESTED	2		
PACKAGE TYPE	standard		
SAMPLE	ID	EQ09012	EQ02171
Mass	2	177,131	150,391
Mass	3	93,300	77,390
Mass	4	354,930	397,630
Mass	5	373	242
Mass	6	20,779	16,402
Mass	12	1,208	753
Mass	13	73	70
Mass	14	4,517	3,051
Mass	15	641	518
Mass	16	4,393	2,411
Mass	17	9,362	3,121
Mass	18	39,386	12,362
Mass	19	5,922	2,123
Mass	20	17,950	4,954
Mass	22	178	0
Mass	24	239	0
Mass	26	1,043	236
Mass	27	466	320
Mass	28	43,690	28,234
Mass	29	1,186	881
Mass	30	1,305	382
Mass	31	228	0
Mass	32	7,328	5,828
Mass	40	469	337
Mass	42	137	0
Mass	43	459	456
Mass	44	11,546	8,718
Mass	45	399	375
Mass	46	137	0
Mass	73	320	336

Figure 5. The scan at ORS laboratories of MagneGas via IVA 110s providing the third independent confirmation of the existence of anomalous species (I) plus the identification of numerous others not considered in this note for simplicity.

3. Expected Applications of Hydrogen Accretion

We would like to close this paper with a few comments. First, it is significant for the new Hydrogen era to indicate the possible connection between Hydrogen accretion and the liquid state of water as well as of fuels such as gasoline, diesel, etc., because a deeper understanding of the latter is evidently essential for the conception and development of environmentally more acceptable fuels.

As it is well known, the liquid state is widely interpreted as being due to the bond of Hydrogen atoms belonging to different water molecules (also called "H-bridges"). It is then suggestive to assume for species (1c) the conceptual rendering of (Figure 6) since it may either represent directly said H-bonds, or provide a significant contribution for their deeper quantitative representation.

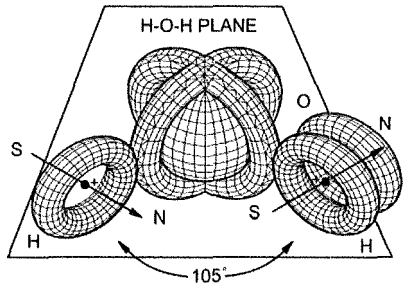


Figure 6. A conceptual rendering of anomalous species (1a) as a possible quantitative model of the H*-bonds in the liquid state of water. Note that the water molecule is represented as it occurs in nature, namely, with polarization of the orbitals perpendicular to the* H − O − H *plane, thus possessing a molecular structure particularly suited for the magnecular bond* H × H [1, 5].

In fact, the current understanding of the liquid state of

water, even though correct and valuable, is still phenomenological to a considered extent, since it misses an explicitly identified attractive force responsible for the H-bridges. By comparison, the attractive force in the magnecular bond is known theoretically and experimentally [1].

Therefore, we provide in (Figure 7) a conceptual rendering of one of the magnecular H-bonds of the liquid state of water as $H - O - H \times H - O - H$ with corresponding bonds to the left and to the Oxygen (not depicted in [Figure 7] for brevity). In particular, it was assumed in (Ref. [1]) that the water boiling temperature is the Curie temperature of magnecular bonds in the liquid state of water.

It is evident that, in the event confirmed, similar magnecular interpretations of liquid fuels, such as gasoline, diesel, etc., may allow new industrial applications of the Hydrogen accretion particularly significant from an environmental viewpoint. One such application, under full development at the industrial level, but still mostly unknown in academia, is the synthesis of new liquid fuels under the names of Hy-Gasoline™, HyDiesel™, etc. (patented and international patents pending) essentially composed by ordinary gasoline and diesel, subjected to Hydrogen accretion [1].

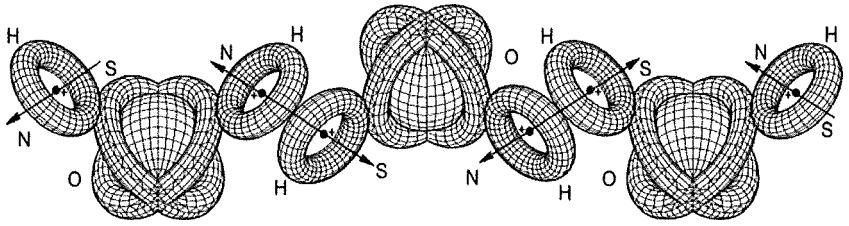


Figure 7. One of the possible magnecular bonds of H-atoms in the liquid state of water according to (Ref. [1]). The second expected magnecular H-*bond as in (Eq. 1a) is that with the* O-*atom, resulting in the typical lattice structure of the liquid state.*

In fact, Hydrogen is known to be a fuel with one of the highest flame temperatures and speed. Therefore, when Hydrogen is added to conventional fossil fuels, their Hydrogen component caused the combustion of contaminants in the exhaust, such as CO and HC in a measure proportional to the Hydrogen percentage. The importance of the industrial realization of the fuels HyGasoline, HyDiesel, etc., is that the Hydrogen component is contained in a *bonded* form which is stable at ambient temperature, rather than that of a mixture. Yet the bond is sufficiently *weaker* than the valence bond as a central condition to allow full combustion.

As indicated earlier, the gaseous fuel MagneGas is commercially produced and sold in various countries. Therefore, samples of MagneGas can be made available at no cost to qualified chemists, provided we receive assurances for the use of the same equipment and the same procedures as those described in this note and in (Refs. [1, 2, 3]).

Acknowledgements

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Magnegas - An Alternative Technology for Clean Energy

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Abstract: In the process of Hydraulic fracturing millions of gallons of water, sand and chemicals are pumped underground to break apart the rock to release the gas. In Hydraulic fracturing certain fluids and materials are used to create small fractures in order to stimulate production from new and existing oil and gas wells. This creates paths that increase the rate at which fluids can be produced from the reservoir formations, in some cases by many hundreds of percent. Although it helped in triggering this year almost 42% of decline in crude prices, on the other hand the completion of drilling process leaves behind pits with waste of the overall process. As the sludge or waste of the process is water based liquid with chemicals and hydrocarbon oil remains of the mineral stock, it is a potentially hazardous material for environment. In this view the Plasma Arc Flow Technique to convert this liquid waste into useful MAGNEGAS™ (MG) proposed by Professor Ruggero Maria Santilli is much beneficial in reducing the oil waste as well as in minimizing the environmental problems. In the present paper, origin of the concept of a new Magnecular Fuel via Hadronic Chemistry, its composition, technique, characterization and its applications in the diversified Industries are discussed.

Keywords: Magnecules, Hadronic Chemistry, Plasma Arc Flow Technique

1. Introduction

The majority of the current energy requirement of the mankind has been fulfilled by the conventional source of energy i.e. molecular combustion of fossil fuels, hydrogen or nuclear fission, etc. However, combustion of fossil fuel generates large amount of green house gas like CO_2 and hydrogen combustion depletes atmospheric O_2 by forming H_2O. Also, most of the environmental pollution caused by fossil fuel is due to chunks of un-combusted fuel that may be carcinogenic primarily because consisting of incomplete combustion of fuel. Gaseous fuels like CNG and LPG are comparatively supposed to be cleaner fuels than those of solid fossil fuels. But, majority of the Natural gas wells in United States use Hydraulic Fracturing where millions of gallons of water, sand and chemicals are pumped underground to break apart the rock and release the gas. Fracking and horizontal drilling in which wells are bored sideways through petroleum deposits have revived U.S. oil and gas output, helping to trigger this year's 42 percent decline in crude prices.

The hydraulic fracturing market is estimated to grow from $41,546.9 million in 2014 to nearly $72,629.4 million by 2019, at a CAGR of over 11.8% [1]. Hydraulic fracturing is used after the drilled hole is completed. Hydraulic fracturing is the use of fluid and material to create small fractures in a rock formation in order to stimulate production from new and existing oil and gas wells. This creates paths that increase the rate at which fluids can be produced from the reservoir formations, in some cases by many hundreds of percent. Water and sand make up 98 to 99.5 percent of the fluid used in hydraulic fracturing. In addition, chemical additives are used. The exact formulation varies depending on the well. Acid, corrosion inhibitors, biocides and many other chemicals are used to satisfy different purposes [2]. Use of biocides [3] and other toxic chemicals is raising serious concern as the sludge waste extracted from the process is posing environmental risks [4]. Scientists are worried that the chemicals used in fracturing may pose a threat either underground or when waste fluids are handled and sometimes spilled on the surface. The completion of drilling process leaves behind pits with waste of the overall process. US EPA defines a "sump" as an open pit or excavation that receives fluids such as mud, hydrocarbons, or

waste waters from oil and gas drilling and producing operations. These sumps are storing the waste generated from oil drilling process including hydraulic fracturing techniques. As the sludge or waste of the process is water based liquid with chemicals and hydrocarbon oil remains of the mineral stock, it is a potentially hazardous material for environment. As it contains the biocides and other toxic chemicals, there is a little chance of its biodegradation or bio consumption. Further, it can percolate to nearby farms and water bodies causing health issues.

Therefore, looking at the necessity and the current day demand, some new source of clean energy is required, that must be cheaper and abundant. The fuels developed should be such that can be used in existing engines without any or major modifications. These requirements have been fulfilled by changing the approach of conventional hydrocarbon fuel (in which the energy is obtained by breaking the valence bond by the process of molecular combustion) to a novel Magnecular fuel [5-8] having bonds of magnetic origin (in which one does not need significant amount of energy to break the magnecular bond and hence comparatively larger energy output can be gained without breaking of covalent bonds. The origin of this new magencular fuel lies in the first ever concept of *Santilli's Isoelectronium* [9-12] as described in the historical papers based on Hadronic Chemistry containing the isochemical models of Hydrogen and Water molecules [12, 13]. Therefore, in section 2, first, a brief description of Hadronic Chemistry, its need and its covering over conventional quantum chemistry is described. Then in section 3, a detailed description of the Magnecules, its types, classification, characteristics, technology and mechanism of Magengas production is given. Followed by, in subsection 3.1, the spectroscopic verification of the new fuel of magnecular origin is justified. Then in subsection 3.2, various applications of magnecular fuel have been elaborated. Subsection 3.3 contains the description of superiority of magnecular combustion over a molecular combustion whereas, in subsection 3.4 the use of magnegas as an additive to the conventional coal based combustion process for improved efficiency is described. Finally, in subsection 3.5, the use of Plasma Arc Flow technique in the synthesis of Magnegas from waste of Hydraulic Fracturing process is described.

2. Hadronic Chemistry

The Hadronic Chemistry is a covering of quantum chemistry where we study the addition of the effects in the electronic overlap as shown in figure 1 solely valid at distances of the order of 1 fm (only) [5, 9] (see also review [14-17] and short communications [18-20]) those are assumed to be Nonlinear - dependence of operators on powers of the wave functions greater than one, Nonlocal - dependence on integrals over the volume of wave-overlapping that, as such, cannot be reduced to a finite set of isolated points, Nonpotential - consisting of contact interactions caused by the actual physical contact of wavepackets at 1fm mutual distance with consequential zero range, for which the notion of potential energy has no mathematical or physical meaning, Non-Hamiltonian - lack of complete representability of systems via a Hamiltonian, thus requiring additional terms and, consequently, Non-Unitary - the time evolution violating the unitary condition $U \times U^{\dagger} = U^{\dagger} \times U = I$. It is named after a fundamental particle 'Hadron'. Therefore, the foundations of Hadronic Chemistry is the origin of New Magnecular Fuel called Magnecules (Magnegas) [5-8].

Santilli applied the mathematical structure of hadronic mechanics to chemical systems thereby achieving results amenable to exact representation of molecular data, precise experimental verifications and novel industrial applications.

In fact, in path breaking papers [12] and [13], Santilli and Shillady have achieved new models for the structure of Hydrogen and water molecules which, apparently for the first time,

1. exhibited a basically new, strongly attractive, non-Coulomb force among pairs of valence electrons in singlet coupling
2. explained, why these molecules have only two Hydrogen atoms,
3. achieved an exact representation of binding energy from unadulterated first axiomatic principle,
4. achieved a representation of the electric and magnetic dipoles of the water molecule which is accurate both in numerical values as well as in the sign, and
5. proved the reduction of computer time by at least a factor of 1000 folds.

For detailed historical and technical descriptions on hadronic chemistry, the reader is advised to study monograph [5] and original papers quoted therein.

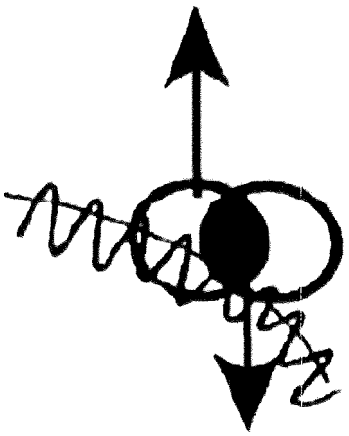

Figure 1. *A schematic view of the the deep overlapping of the wavepackets of valence electrons in singlet coupling resulting in conditions which are known to be non-linear, non-local, and non-potential (due to the zero-range, contact character of the interactions), thus not being representable via a Hamiltonian (non-Hamiltonian structure). As a result, the ultimate nature of valence bonds is outside any representational capability of quantum chemistry. Santilli has build hadronic chemistry for the specific scope of representing the conditions herein considered of the bonding of valence electrons (see the monograph Hadronic Mathematics, Mechanics and Chemistry Volume V* [5]*).*

The first breakthrough was the proposal of an isochemical model of Hydrogen molecule through the concept of a singlet

quasiparticle state of two valence electrons at a short distance of say 1 fm. The assumption was that, pairs of valence electrons from two different atoms bound themselves at short distances into a singlet (mostly, but not totally stable) quasi-particle state called an isoelectronium shown in figure 2, which describes an oo-shaped orbit around the respective two nuclei. The oo-orbital is suggested as shown in figure 2 to represent the diamagnetic character of the H-H molecule, thus being in agreement with experimental verifications.

Note that, once two valence electrons are bonded into the isoelectronium, there is no possibility for bonding additional valence electrons, that explains why Hydrogen (or water) molecule admits only two Hydrogen atoms.

Using the isomathematical lifting of the conventional Schrödinger wave equation Santilli and Shillady have shown for the first time in the history of chemistry the mathematical evidence of the existence of an attractive forces among two neutral atoms of hydrogen molecule and achieved the theoretical representation of strong valence bond namely a valence coupling between two identical electrons in singlet coupling with a strongly attractive force.

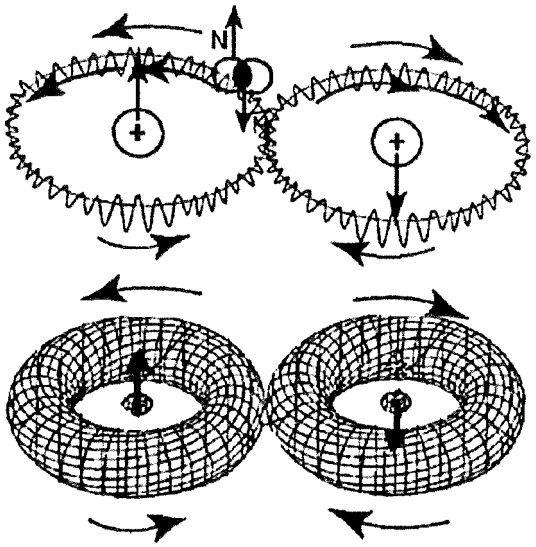

Figure 2. *A view of isochemical model of the Hydrogen molecule at absolute zero degree temperature without any rotational degrees of freedom, with the Santilli-Shillady strong valence bond between the valence electrons pair into isoelectronium quasi-particle. Note the oo-shape orbital of the isoelectronium, thus allowing a representation of the diamagnetic character of the H-molecule since, under an external strong magnetic field, the two H atoms acquire parallel but opposite magnetic polarities with null value of the total magnetic field at sufficient distances.*

Subsequent to the successful study of the isochemical molecular model of isoelectronium for Hydrogen molecules in the historical paper of (1999) [12], Santilli and Shillady proposed their second historical study [13] of hadronic chemistry for the water molecule resulting from the first axiomatic unadulterated principles of binding energy, sign and values of electric and magnetic moments and other data [13].

That constituted the first model of the water molecule as shown in figure 3, admitting the exact analytic solution from first principle in scientific history. This model exhibits a new explicitly attractive "strong" force among the atoms of the H-O diatomic radical, which is absent in conventional quantum chemistry. The equation obtained after isolifting, also explains the reason why the water molecule admits only two H-atoms. The model yields much faster convergence of series with much reduced computer times and resolves many other insufficiencies of quantum theory. Finally, the model is evidently extendable with simple adjustments to an exact solution of other dimers involving the Hydrogen, such as H-C.

Independent studies of variational calculations of isochemical models of Hydrogen and Water molecules of Santilli and Shillady in 1999 and 2000 were carried out by,

1. Aringazin and Kucherenko in their paper on Exact solution of the restricted three-body Santilli-Shillady model of H_2 molecule [23] studied the exact analytical solution of the restricted isochemical model of H_2 molecule with fixed nuclei proposed by Santilli and Shillady.
2. Aringazin independently in his paper on variational solution of the four-body Santilli-Shillady model of H_2 molecule [24] using Ritz variational approach to the four-body isochemical model of H_2 molecule suggested by Santilli and Shillady, without restriction that the isoelectronium is stable and point-like particle, the ground state energy and bond length of the H_2 molecule is calculated.
3. Pérez-Enrquez, Marn and Riera in their paper on Exact Solution of the Three-Body Santilli-Shillady Model of the Hydrogen Molecule [25] suggested new approach to the 3-body Santilli-Shillady model of the hydrogen molecule by proposing the argument that the isoelectronium follows restricted motion namely, The formation of the quasi-particle from the two electrons involves an effective mass transformation (iso-renormalization) and secondly, the isoelectronium must orbit in a spheroidal shaped region of space (isoelectronium is restricted to defined region of space).

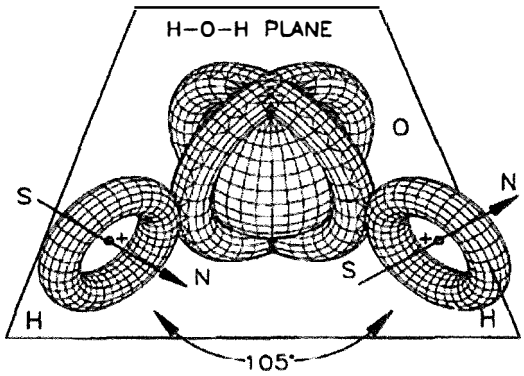

Figure 3. *A view of a water molecule H_2O at absolute zero degrees of temperature without any rotational degrees of freedom, showing the H-O-H plane, the angle 1040 5' between the H-O and O-H dimers and, above all, the natural occurrence according to which the orbital of the H atoms are not spherical, but of toroidal character for their coupling with Oxygen, thus providing direct verification of the isochemical model of the Hydrogen molecule of Figure 2.*

3. The New Chemical Species of Santilli Magnecules

Current energy requirement of mankind has been fulfilled by the conventional source of energy i.e. molecular combustion of fossil fuels, hydrogen or nuclear fission. However, combustion of fossil fuel generates large amount of green house gas like CO_2 and hydrogen combustion depletes atmospheric O_2 by forming H_2O. Also, most of the environmental pollution caused by fossil fuel is due to chunks of un-combusted fuel that may be carcinogenic primarily because consisting of incomplete combustion of fuel. Therefore the current day demand is clean energy source that is cheap and abundant.

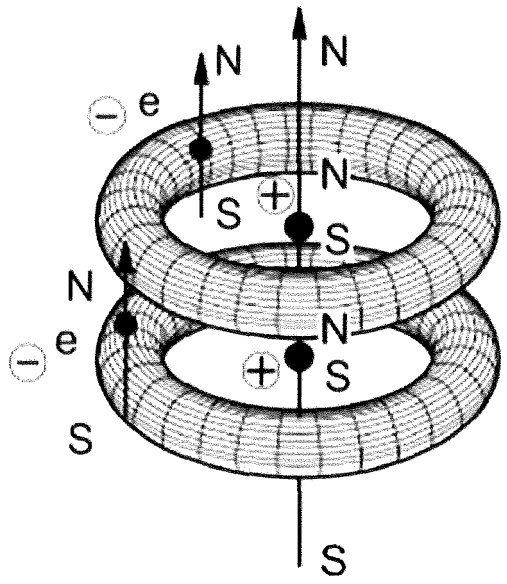

Figure 4. *A schematic view of the simplest possible di-atomic magnecule whose bond originates from opposing magnetic polarities of toroidal polarizations of the orbits of peripheral atomic electrons caused by very strong external magnetic fields whose bond is NOT that of valence.*

The fuels developed should be such that can be used in existing engines without any or major modifications. This requirement has been fulfilled by changing the approach from quantum mechanics to hadronic mechanics to hadronic chemistry. Italian-American physicist Professor R. M. Santilli [5-10] in 1998 for the first time proposed a new novel fuel characterized by hadronic mechanics/chemistry known as MAGNECULES [5-8].

Magnecules shown in figure 4, are novel chemical species having at least one magnecular bond. Principle of synthesis of magnecules is similar to the magnetization of a ferromagnet where the orbits of unbounded electrons are polarized. The atoms are held together by magnetic fields originating due to toroidal polarization of the atomic electron orbits. Rotation of the electrons within the toroid creates magnetic field which is absent for the same atom with conventional spherical distribution of electron orbitals. When two such polarized atoms are sufficiently close to each other and in north-south north-south alignment, the resulting total force between the two atoms is attractive. The polarization is brought about by high magnetic field which is obtained as in the case of high voltage DC arc as shown in left side figure 5. Thus, Santilli Magnecules in gases, liquids, and solids consist of stable clusters composed of conventional molecules, and/or individual atoms bonded together by opposing magnetic polarities of toroidal polarizations of the orbits of at least the peripheral atomic electrons when exposed to sufficiently strong external magnetic fields, as well as the polarization of intrinsic magnetic moments of nuclei and electrons as shown on right side of figure 5. A population of magnecules constitutes a chemical species when essentially pure, i.e., when molecules or other species are contained in very small percentages in a directly identifiable form.

Magnecules are called-

1. Elementary- when composed only of two molecules, e.g.: $\{H - H\} \times \{H - H\}$; and so on where '-' denotes conventional valence bond and '×' denotes magnecular bond
2. Magneplexes- when entirely composed of several identical molecules e.g.: $\{H - O - H\} \times \{H - O - H\} \times \{H - O - H\} \times \{H - O - H\} \times \{H - O - H\} \times \ldots$; and so on
3. Magneclusters- when composed of several different molecules e.g.: $\{H - H\} \times \{C - O\} \times \{O - C - O\} \times \{C = O\} \times \{H - H\} \times \cdots$; and so on

Magnecules are also classified as-

1. Isomagnecules- when having all single-valued characteristics and being reversible in time, namely, when they are characterized by isochemistry,
2. Genomagnecules- when having all single-valued characteristics and being irreversible in time, namely, when they are characterized by genochemistry; and
3. 1q`1Hypermagnecules- when having at least one multi-valued characteristic and being irreversible in time, namely, when they are characterized by hyperchemistry.

Santilli magnecules are characterized by following characteristics, namely:

1. Large atomic weights which are ten times or more than the conventional molecules.
2. Large peaks in macroscopic percentages in mass spectra, which do not belong to conventional molecules.
3. These peaks show same infra-red and ultra-violet signature as expected from the conventional molecules and/or radicals constituting the magnecule.
4. Said infrared and ultraviolet signatures are generally altered with respect to the conventional versions.
5. Magnecules have an anomalous adhesion to other substances.
6. They can break down into fragments under high energetic collisions, with subsequent recombination with other fragments and/or conventional molecules.
7. They can build up or lose individual atoms, radicals or

molecules during collision.

8. They have an anomalous penetration through other substances indicating a reduction of the average size of conventional molecules as expected under magnetic polarizations.
9. Gas magnecules show an anomalous solubility in liquids due to new magnetic bonds between gas and liquid molecules caused by magnetic induction.
10. Magnecules can be formed by molecules of immiscible liquids.
11. A gas with magnecular structure does not follow the perfect gas law.
12. Substances with magnecular structure have anomalous physical characteristics, as compared to the conventional molecules.
13. Magnecules release more energy in thermochemical reactions than that released by the same reactions among unpolarized molecular constituents.

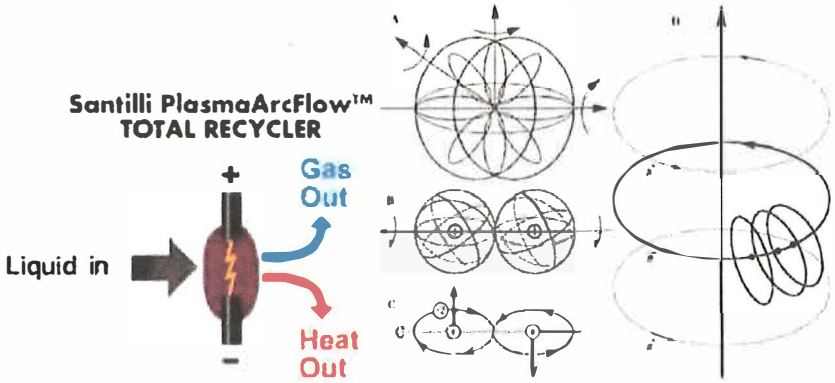

Figure 5. *Figure on the left shows the schemmatic view of the Plasma Arc Flow Reactor whereas, figure on right gives the mechanism of creation of Magnecules for the case of the Hydrogen Molecule. It consists of the use of sufficiently strong external magnetic fields which can progressively eliminate all rotations, thus reducing the hydrogen molecule to a configuration which, at absolute zero degrees temperature, can be assumed to lie in a plane. The planar configuration of the electron orbits then implies the manifestation of their magnetic moment which would be otherwise absent. The right hand side of the figure 5 outlines the geometry of the magnetic field in the immediate vicinity of an electric arc as in hadronic molecular reactors. The circular configuration of the magnetic field lines around the electric discharge, the tangential nature of the symmetry axis of the magnetic polarization of the hydrogen atoms with respect to said circular magnetic lines, and the consideration of hydrogen atoms at orbital distances from the electric arc* 10^{-8} cm, *resulting in extremely strong magnetic fields proportional to* $(10^{-8})^{-2} = 10^{16}$ Gauss, *thus being ample sufficient to create the needed polarization. The reason for these results is the intrinsic geometry of the Plasma Arc Flow.*

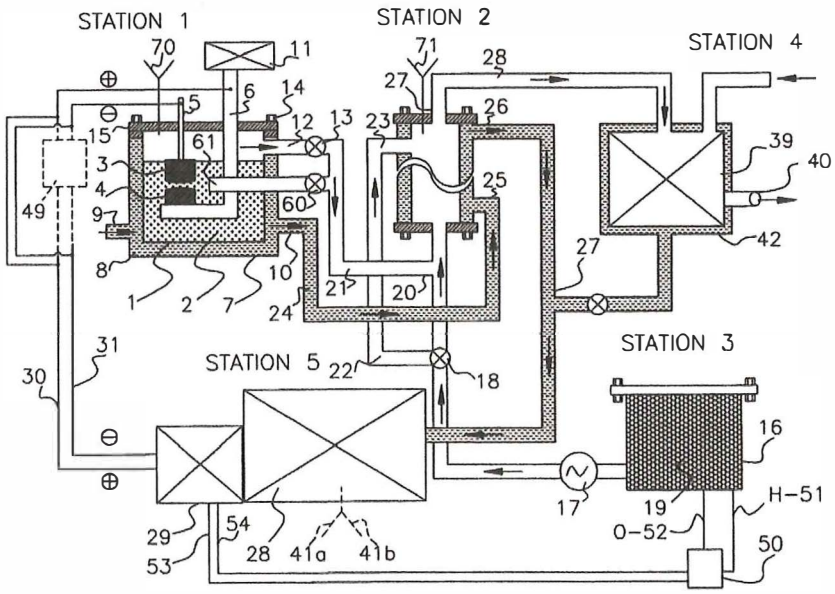

Figure 6. *Complete embodiment of the mechanism of Magnecular Fuel production comprises of the five stations as shown in figure above; 1-Pressure Vessel, 2-liquid waste as feedstock, 3,4-Carbon-base cylindrical electrodes, 5,6-copper rods, 7 -second pressure metal vessel, 8- coolant tap water or seawater, 9- Inlet, 10- Outlet, 11- Automated feeder circuit for maintaining the submerged electric arc between 3 & 4, 12- Outlet fro produced combustible gas, 13- Pressure regulator, 14- Jacket bolt, 15- fastened lid, 16 – Connecting pressure vessel, 17 – pump, 18 – flow control valve, 19- Gaseous additive from station 3, 20 to 28 – Inlet outlet Pressure pipes, 29- DC electric generator, 30, 31- connecting cables, 39- Treatment station, 40- Magnefule exit, 41a and 41b – outlets for condensed, filtered and processed drinking water and with salt respectively, 42- Heat exchanger for cooling of Magnefuel, 49- AC DC converter (rectifier), 50- Electrolytic separation equipment, 51 to 54- connecting cables, 60 and 61- outlet valves, 70- Inlet for chemical elements at station 1 and 71- at station 2.*

All the above characteristic features disappear when the magnecules are brought to a sufficiently high temperature (Curie Magnecular Temperature), which varies from species to species.

A method for the production of a clean burning liquid fuel Magnegas plus heat from a liquid feedstock requires a pressure resistant vessel containing a liquid feedstock and the vessel housing a submerged electric arc between carbon-base electrodes as shown in figure 6. By activating the submerged electric arc between said carbon base electrodes produce, by thermochemical reactions, a combustible gas which bubbles to a surface of the liquid feedstock, and transmitting said combustible gas via high pressure pipes into a tower for a catalytic liquefaction processing into a clean burning liquid fuel called Magnegas (MG) [5-8, 26]. The heat produced by the reactor is acquired by the liquid feedstock and it is used via its recirculation through external heat exchangers that can power a turbine for the production of electricity. Of course this requires an additional input to the steam to reach the supercritical temperatures.

3.1. Spectroscopic Studies on the Existence of New Type of Magnecular Bond

Current spectroscopic technologies offer variety of analytic instruments, like Gas Chromatography (GC), Liquid Chromatography (LC), Capillary Electrophoresis Chromatography (CEC), Supercritical Chromatography (SCC), Ion Chromatography (IC), Infrared Spectroscopy (IR), Raman Spectroscopy (RS), Nuclear Magnetic Resonance Spectroscopy (NMRS), X-Ray Spectroscopy (XRS), Atomic Absorption Spectroscopy (AAS), Mass Spectrometry (MS), Laser Mass Spectrometry (LMS), Flame Ionization Spectrometry (FIS), and others.

Only some of these instruments are suitable for the detection of magnecules and, when applicable, their set-up and use are considerably different than those routinely used with great success for molecules. So far, all the spectroscopic tools are devised to detect the normal distribution of electrons around nucleus. This was in accordance with all the established theories and experimental evidences for atomic status and molecular structures based on the conventional covalent and coordinate bonds. Sigma and pi bonds are also obvious parts of these theories. But, in magnecules, the toroidal polarization of the electron orbital creates a magnetic field (due to the rotation of the electrons within said toroid) which does not exist for the same atom when the electron orbital has the conventional spherical distribution. When two so polarized atoms are at a sufficiently close distance, the resulting total force between the two atoms is attractive because all acting forces are attractive except for the repulsive forces due to nuclear and electron charges. This addition of unusual property due to toroidal motion also reflects in the spectroscopic determination of magnecules. In fact, all up to date equipments are not able to interpret the magnecular bonds. Insufficiency in interpretation is due to the limitations of design of instruments.

Much of the attempts to obtain spectroscopic data using conventional tools are done to the date and the anomalies are themselves interesting and debatable. But apart from all these anomalies, basic difference between the conventional chemical bonds and magnecular bond makes everyone to confirm its existence important physical properties and industrial applications. Some of the conventional equipments with modifications to determine the magnecular species are as below: Among all available chromatographic equipment, that suitable for the detection of gas magnecules is the GC with column having ID of at least 0.32mm operated with certain conditions. By comparison, other chromatographs do not appear to permit the entrance of large magnecules. If attempted they may prove to be potentially destructive of the magnecules to be detected. Among all available spectroscopic equipment, one preferable is the IR. But the understanding should be such that an instrument is used in a negative way, that is, to verify that the magnecule considered has no IR signature. NMRS do not appear to be capable of detecting magnecules despite their magnetic nature, because NMRS are most effective for the detection of microscopic magnetic environment of H-nuclei rather than large structures. One of the prominent instruments to study magnecules is low ionization MS because other instruments seems to be destroying the magnecular structure in the time duration of detection.

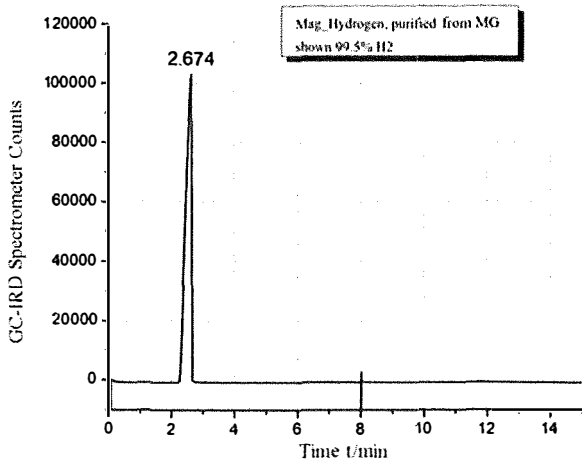

Figure 7. *A scan of Magne Hydrogen via the GC-TCD of Ref. [4] showing no appreciable difference of* MH *with pure hydrogen.*

Some of the best combination of instruments to detect magnecular species is GC coupled with MS, denoted as GC-MS and also IR coupled with this combination.

Thus best instrument for the detection of both molecules and magnecules in gases is the GC-MS equipped with the IRD denoted GC-MS/IRD. For the initial tests conducted [8] for magnecules, GC-MS scans have indicated the presence in the anomalous peaks of individual atoms of hydrogen, oxygen, and carbon evidently in addition to individual molecules. Any belief that magnegas is entirely composed by ordinary molecules, such as H_2 and CO, is disproved by experimental

evidence via GC-MS and IRD detectors. The anomalous behavior of magnecules from ordinary molecules can be understood from the observation that following the removal of magnegas from the GC-MS/IRD, the background continued to show the same anomalous peaks and reached the normal configuration only after a weekend flushing with an inert gas. When magnecule is passed through Pressure Swing Adsorption (PSA) equipment for the separation of Hydrogen from MG, a magnecules enriched with hydrogen is obtained, which is termed as Magnehydrogen [8]. Interestingly, this modified form of magnecules is energetically similar to hydrogen, but differs in physical properties to that of hydrogen. But being chemically pure, Magnehydrogen gives some readable data when studies spectroscopically as shown in figure 7. This is mainly because of the reduced complexities arising from bonding of other chemical species like CO, CO_2 etc.

3.2. Applications of Magnecules

The property of magnecules to undergo magnecular combustion with high energy output which is attributed to weak magnecular bond [8] is exploited for the industrial development of novel clean fuels such as MAGNEGAS^{TM} (See also independent review studies in [26-28] and short communications in [29, 30]). Consider the case of combustion of molecular hydrogen and oxygen to produce H_2O. The dissociation of H_2 and O_2 molecules consume 163.7 kcal/mol and thereafter the atom recombination to produce H_2O releases 221.25 kcal/mol hence the net release of energy is 57 kcal/mol. However, in case of magnecular hydrogen {H × H} and atomic oxygen O combustion (even if on considering H × H bond dissociation energy arbitrarily to be zero) the energy output is predicted to be approximately three times the value predicted by molecular structures with the same atomic constituents and combustion temperature [10].

3.2.1. Magnegas as Magnehydrogen

Recently, two different experimental confirmations have been reported of the new chemical species of Magnecules – Magne Hydrogen [8] with 99% Hydrogen content, and also having a multiple of the specific weight of conventional molecular Hydrogen. A number of features of the new species MH are pointed out therein, such as the increased energy content and the lack of seepage through the walls of a container. These features appear to be relevant for the Hydrogen industry.

The major industrial view point of this technology is Magnegas that can be obtained via combustion of carbon obtained via a submerged electric arc using Santilli's hadronic reactors of molecular type (Class III), also known as Plasma Arc Flow ^{TM} Reactor, that were first built by the R. M. Santilli in 1998 in Florida, U.S.A., and are now in regular production and sale the world over. Two of the commercially available Hadronic Reactors are shown in figures 8 and 9 respectively. Plasma Arc Flow Reactors use a submerged DC electric arc between carbon base electrodes to achieve the complete recycling of essentially any type of (nonradioactive) liquid waste into the clean burning magnegas fuel, heat usable via exchangers, and carbonaceous precipitates used for the production of electrodes. The reactors are ideally suited to recycle antifreeze waste; oil waste, sewage, and other contaminated liquids, although they can also process ordinary fresh water. The best efficiency is achieved in these reactors for the recycling of carbon-rich liquids, such as crude oil or oil waste [3, 5].

Figure 8. *Picture of a 250 kW Santilli's Hadronic Reactor [3] (also called Plasma Arc Flow Reactor) with the panels of its completely automatic and remote controls, to recycle liquid waste into magnegas usable for any fuel application, a large amount of heat and carbonaceous precipitates used to produce the electrodes. This Reactor can produce up to 5000,000 scf (140 million liters) of magnegas per week of 24 hours work per day corresponding to 3,000 gasoline gallon equivalent (11,000 gasoline liter equivalent) of magnegas per week.*

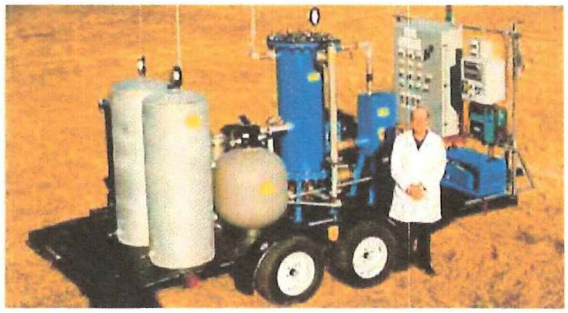

Figure 9. *Picture of a 50 kW Santilli's Hadronic Reactor (Plasma Arc Flow Recycler) mounted on a trailer for mobility to conduct test recycling where liquid wastes are located. This recycler can produce up to 84, 000 scf (up to 2.4 million liters) of magnegas per week corresponding to about 560 gasoline gallon equivalent (2, 100 gasoline liter equivalent) of magnegas per week.*

3.2.2. Magnegas as Metal Cutting Fuel

Apart from this, performance of magnecular fuel in metal cutting is more indicative of a plasma cutting feature, such as the metal cutting via plasma of ionized hydrogen atoms which recombine into H_2 when cooling in the metal surface, thus releasing the energy needed for metal cutting. The metal cutting ability is indicative of the presence of isolated atoms and dimers in the magnegas structure which recombine under combustion, to give performance similar to that of plasma cutters. Cutting and welding torch using acetylene requires a higher concentration of oxygen than is present in

air to enable the desired high temperature and facility of operation. In view of this, magnecular fuels are superior to acetylene [31]. The experimental observations for flame temperature carried out by two independent agencies [32, 33] confirmed the maximum flame temperature of magnegas to be 3670°C and 3400°C respectively. It clearly indicates its superiority to acetylene in field of cutting and welding metals.

3.2.3. Magnegas as Vehicular Fuel

Internal-combustion engines are bad polluters because they operate on fossil fuels, which contain a wide variety of components incapable of being fully combusted together within their brief residence in an engine. Combustion effluents of petroleum and even natural gas, contain undesirable hydrocarbon fragments and derivatives, often in particulate form, carbon monoxide, and gaseous oxides of nitrogen and/or of sulfur transformable to noxious droplet form (acid rain). Hydrogen was long-considered an ideal fuel because convertible completely to water via air combustion. Yet hydrogen is unsatisfactory as an internal-combustion engine fuel, because the high temperature and the rapidity of its combustion foster pre-ignition or flashback, which is greatly harmful to engine operation and structure. Also flashback is conducive to an increase of harmful Nitrogen Oxides in the atmosphere. Despite the intervening decade of research and development, no vehicle running on hydrogen as its main or sole fuel is yet commercial, notwithstanding much experimentation on fuel cell technology, which is fundamentally electrolytic and slow-generating. Polluting exhausts of fuels originating from fossil reserves as well as hydrogen are having their limitations.

Contrarily, in a paper from 2003, R. M. Santilli [34] presented theoretical and experimental evidence on the existence of a new species of Hydrogen that he called Magne Hydrogen (MH) structurally shown in figure 10, which results in clean combustion without liberation of harmful gases. Also, magnehydrogen supersedes hydrogen because of the some advantageous physical properties. It was found that the prepared gas apparently consists of 99 percent Hydrogen, although spectroscopically its specific weight (or, equivalently, molecular weight) was estimated to be 7.47 times larger than that of conventional Hydrogen. The gasification is achieved via a submerged DC electric arc between carbon electrodes that, under sufficient powers (of the order of 300kW or more) is capable of producing at atomic distances the high values of the magnetic field (estimated as being of the order of 1012 Gauss). Santilli obtained the new species MH via the use of conventional Pressure Swing Adsorption (PSA) equipment for the separation of Hydrogen from MG. From an industrial point of view, it is very important to be noted that, it is sufficient to achieve a species of MH with at least 3.3 times the specific weight of H_2 to have the same energy content of 1000 BTU/scf of Natural Gas (NG). In fact, under said conditions, MH would avoid the current needs to liquefy Hydrogen in order to achieve a sufficient range, since MH can be compressed like Natural Gas. Additionally, the magnecular structure of MH avoids the traditional seepage of Hydrogen through the walls [7, 10, 31], thus allowing long term storage that is currently prohibited by molecular Hydrogen due to current environmental laws.

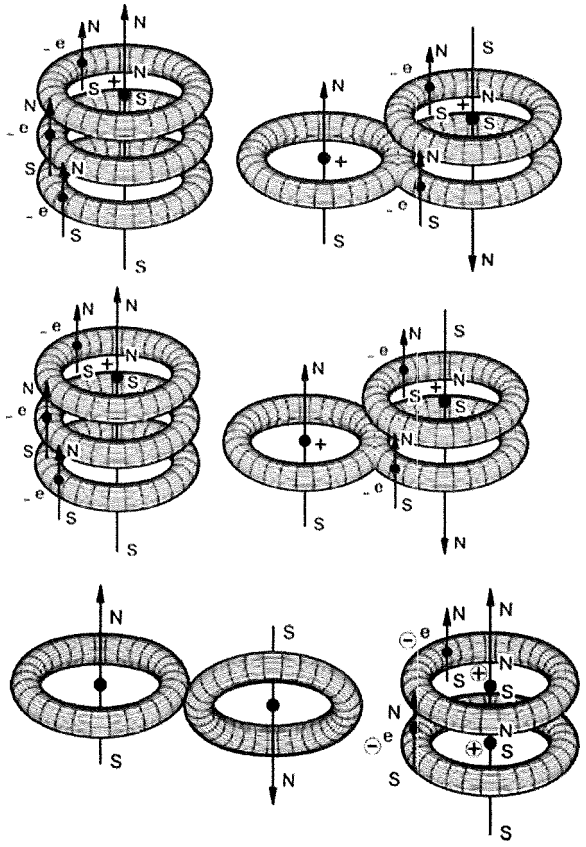

Figure 10. A conceptual rendering of the cluster Magne Hydrogen (MH) which is predicted as being composed by part of the molecular species H-H and part by the magnecular species H×H. The cluster in MH which is predicted as being composed by magnecular species H×H×H and H-H×H.

As magnecular fuel is magnetically polarized, magnegas and magnehydrogen show good binding affinity to other fuels, with which it is mixed. This gave rise to hybrid fuels like Hy-Gasoline, Hy-Diesel etc., where Hy stands for magnegas rich in hydrogen [7]. Magnegas (TM) fuel was shown by EPA accredited laboratory analysis results to be suitable for use as automotive fuel without catalytic converters while surpassing all current EPA requirements for combustion exhaust emissions, as shown in table 1 [35].

Table 1. Reproduced from the website of Magnegas technology Corp.

Element	MagneGas (MG)	Natural Gas	Gasoline	EPA Standards
Hydro-carbons	0.026 g/mi	0.380 g/mi 2460% of MG emission	0.234 g/mi 900% of MG emission	0.41 g/mi
Carbon Monoxide	0.262 g/mi	5.494 g/mi 2096% of MG emission	1.965 g/mi 750% of MG emission	3.40 g/mi
Nitrogen Oxides	0.281 g/mi	732 g/mi 260% of MG emission	0.247 g/mi 80% of MG emission	1.00 g/mi
Carbon Dioxide	235 g/mi	646.503 g/mi 275% of MG emission	458.655 g/mi 195% of MG emission	No EPA standard exists for Carbon Dioxide
Oxygen	9%-12%	0.5%-0.7% 0.04% of MG emission	0.5%-0.7% 0.04% of MG emission	No EPA standard exists for Oxygen

(Note: g/mi is the unit of measuring combustion exhaust of vehicles in grams per miles as per US Motor Vehicle Emission and Fuel Standards)

3.3. Comparison of Magnecular Combustion over Conventional Molecular Combustion

The magnecules are gaining attention of the world mainly because of the energy content it is exhibiting. The existence of dimers and atoms in magnecule were doubted due to insufficiencies and limitations of the spectroscopic equipments. But it could not divert the attention from the matter, i.e. anomalous energy content! The presence of dimers and individual atoms in magnegas can only support the quantitative interpretation of large excess of energy contained in this new fuel. This energy is accounting to be of the order of at least three times the value predicted by quantum chemistry, which is released during combustion. The admission of dimers and atoms as constituents of magnecules readily explains this anomalous energy content because said dimers and atoms are released at the time of the combustion, thus being able at that time to form molecules with exothermic reactions. In the event magnecules would not contain dimers and atoms, their only possible constituents are conventional molecules, in which case no excess energy is possible during combustion. When subjected to high voltage with a specially designed arc, atomic orbitals of conventional molecules undergo toroidal polarizarion. This toroidal polarization induces a magnetic current which is responsible for forming a magnecular bond between various neighboring toroids. This new type of attractive force also explains why molecules are in association with magnecules of individual atoms, such as H, C and O, and/or individual unpaired dimers, such as H-O and H-C. At the breakdown of the magnecules due to combustion, these individual atoms and dimers couple themselves into conventional molecules via known exothermic reactions such as [3]

$$H + H \rightarrow H_2 \quad 105\ \text{kcal/mol}$$

$$C + O \rightarrow CO \quad 225\ \text{kcal/mol}$$

$$H - O + H \rightarrow H_2O \quad 28\ \text{kcal/mol}$$

with consequential release during combustion of a large amount of energy that does not exist in fuels with a conventional molecular structure. If pure water is subjected electric arc produced with Plasma Arc Flow Reactors, quantum chemistry predicts that magnegas should be a mixture of 50% H_2 and 50% CO, with traces of O_2 and CO_2. Quantum chemistry predicts that the indicated composition consisting of 50% H_2 and 50% CO should have an energy content of about 315 BTU/cf, which is insufficient to cut metal. This prediction is also disproved by the experimental evidence that magnegas cuts metal at least 50% faster than acetylene, which has energy content of 2,300 BTU/cf.

3.4. Use of Magnegas as an Additive in Coal Combustion Process

The affinity of magnegas and magnehydrogen to fossil fuels made it best suitable to be used as additive. This is also true for its use along with coal. Increasing energy demand across the globe and dependency of major emerging economies compelled to use coal despite its dirty nature. Moreover, poor coal combustion efficiency of the old age plants is adding to the concern of insufficient combustion of coal in furnaces. This is causing serious air pollution along with huge loss of the fossil fuel due to lower than expected efficiency. Magnegas has shown some interesting results when burnt along with coal in its initial experiments. Major problem associated with coal is incomplete combustion of carbon. Addition of magnegas to coal helps in complete combustion of all combustible products in coal to the final oxidation products. Thus maximum heat is obtained and the efficiency is increased. This also reduces load on air quality as there is no incompletely burnt carbon or NO released in atmosphere. Complete combustion of carbon itself is a achievement for energy efficiency as every conversion of carbon monoxide to dioxide away from the furnace is a loss of 283.0 kJ/mol of energy. In table 2, mentioned is the tabulated representation of exhaust parameters for combustion of coal and coal along with magnegas.

Table 2. Report of exhaust gases for combustion of coal compared with coal+magnegas.

Factor	Coal	Coal + Magnegas
Oxygen	11%	13%
Carbon Dioxide(CO_2)	15%	9% (40%)
Carbonmonoxide (CO)	58 ppm	28 ppm (52%)
Nitrous oxide (NOx)	160 ppm	46 ppm(71%)
Stack temp	37100 C 7000 F	~29900 C 54000 F

The same experimental results were reconfirmed by Future Energy pvt. ltd. which is claimed in a recently filed patent in Australia. It clearly shows that the quality of exhaust is seen to be improved by addition of magnegas to coal [36]. Magnegas as additive to coal in small power plants can help to improve the combustion efficiency of these units. Also, high temperature generated in use of magnegas converts fly ash

into a glassy material and does not allow it to emit and pollute. Tremendous increase in temperature due to addition of magnegas can be trapped and converted to further generate heat in the same manner as 'waste heat recovery' boilers are working presently in many metal industries.

3.5. Use of Plasma Arc Flow Technique in Synthesis of Magnegas from Waste of Hydraulic Fracturing Process

Fracking process leaves behind a vast quantity of sludge waste containing water along with remains of hydrocarbon oil. It also contains the various chemicals added to serve variety purposes. Up to 600 chemicals are used in fracking fluid, including known carcinogens and toxins. Thus the waste generated over years becomes an issue of environmental concern.

Interestingly, this otherwise nuisance causing waste is a best candidate to be potential raw material for generation of magnegas using Plasma Arc Flow technology. For a stock of magnegas to be competent with hydrogen and acetylene in thermal parameters, it is necessary to be generated from waste containing hydrocarbon remains. Thus the waste obtained in fracking process is best for synthesis of magnegas. Thus, the waste disposal along with generation of magnegas can be achieved by reducing load on environment.

4. Conclusion

Thus, the theoretical and experimental evidences confirms Santilli's view that the chemical species of molecules, defined as stable clusters of atoms under a valance bond, does not exhaust all possible chemical species existing in nature. This conclusion is proved beyond scientific doubt, for instance, by macroscopic percentage of stable clusters, with atomic weight of several hundreds a.m.u., in light gases without an infrared signature where heaviest possible detected molecule is the CO_2 with 44 a.m.u.; the mutation of transparent oils into completely opaque substance without fluidity and other evidence provided by various scientists as well.

They also have promising application as fuel additive for improving combustion of existing fossil fuel due to their high miscibility, higher combustion/co-combustion rate and thermal output. The magnecules can also act as detonator to the conventional fuel molecules resulting in near complete combustion of the latter like the volatile matter detonating the fixed carbon of pulverized coal. Judicial use of fossil fuel can be achieved by application of clean additives like magnegas. Further, reliable and sustainable methods for generation of energy from waste are the solution for present global problems. Thus, magnegas hold promising applications as sustainable energy requirements.

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Santilli's Magnecules and Their Applications

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Abstract: Since the beginning of the industrial revolutions across the different parts of the globe, scientists are constantly trying to get access to clean; affordable and reliable energy which thus has been a cornerstone of the world's increasing prosperity and economic growth. Our use of energy in the twenty-first century must also be sustainable. Energy efficiency and conservation, as well as decarbonizing our energy sources are essential to this revolution. Reducing carbon emissions on the timescale needed to mitigate the worst risks of climate change will not be driven by our inability to find cost-effective sources of fossil fuels. Here, we are reviewing the work done by Prof. R. M. Santilli on his new chemical species of "Magnecules" which primarily consists of bonds of magnetic types enabling pollution free and environmentally benign emission of exhaust. Also, different applications have also been discussed here.

Keywords: Magnecule, Carbondioxide Emission, Global Warming, Flame

1. Introduction

All organisms modify their environment, and humans are no exception. As the human population has grown and the power of technology has expanded, the scope and nature of this modification has changed drastically. The growth of the human population, and growth in the resource base used by humanity, is maintained by a suite of human enterprises such as agriculture, industry, fishing, and international commerce. These enterprises transform the land surface (through cropping, forestry, and urbanization), alter the major biogeochemical cycles, and add or remove species and genetically distinct populations in most of Earth's ecosystems. Many of these changes are substantial and reasonably well quantified; all are ongoing. The rates, scales, kinds, and combinations of changes occurring now are fundamentally different from those at any other time in history; we are changing Earth more rapidly than we understand it. We live on a human dominated planet and the momentum of human population growth, together with the imperative for further economic development in most of the world, ensures that our dominance will increase. Finally, humanity's dominance of Earth means that we cannot escape responsibility for managing the planet [1].

At the beginning of the industrial revolution, the population of the world was 700 million. Today, the population is 7 billion and is estimated to grow to 9 billion by 2050, and about 10 billion by 2100 [2]. Most of this population growth will be in Asia and Africa, where rapidly rising economic growth will place additional demands on energy supply. The International Energy Agency (IEA) based in Paris has projected that the world's energy demand will increase from about 12 billion tone oil equivalents (t.o.e.) in 2009 to either 18 billion t.o.e. or 17 billion t.o.e. by 2035 under their 'current policies' or 'new policies' scenarios, respectively. Carbon-dioxide emissions are expected to increase from 29 gigatonnes per year (Gt.yr-1) to 43 Gt.yr-1 or 36 Gt.yr-1 under the current and new policies, respectively. The actual path we follow will depend on how efficiently and effectively we use existing and new sources of energy [3].

Liquid fuels derived from oil became the main form of energy for transportation largely because of their high energy densities (Figure 1) associated with their high energy content. However, a search for alternatives to oil for transportation energy is required to deal with the growing concerns over the rising and volatile price of oil, the vulnerability to supply disruptions, and balance-of-trade issues.

With increases over the past 30 years in the number of extreme weather events, such as temperature extremes,

floods, wildfires, droughts and storms, the overall loss trend is beginning to exceed $150 billion per year [4]. There is increasing evidence that these changes are linked with climate change [5]. As U. S. A. President Obama [6] said in his speech last month, climate change is happening -- and the effects are already being felt across the country.

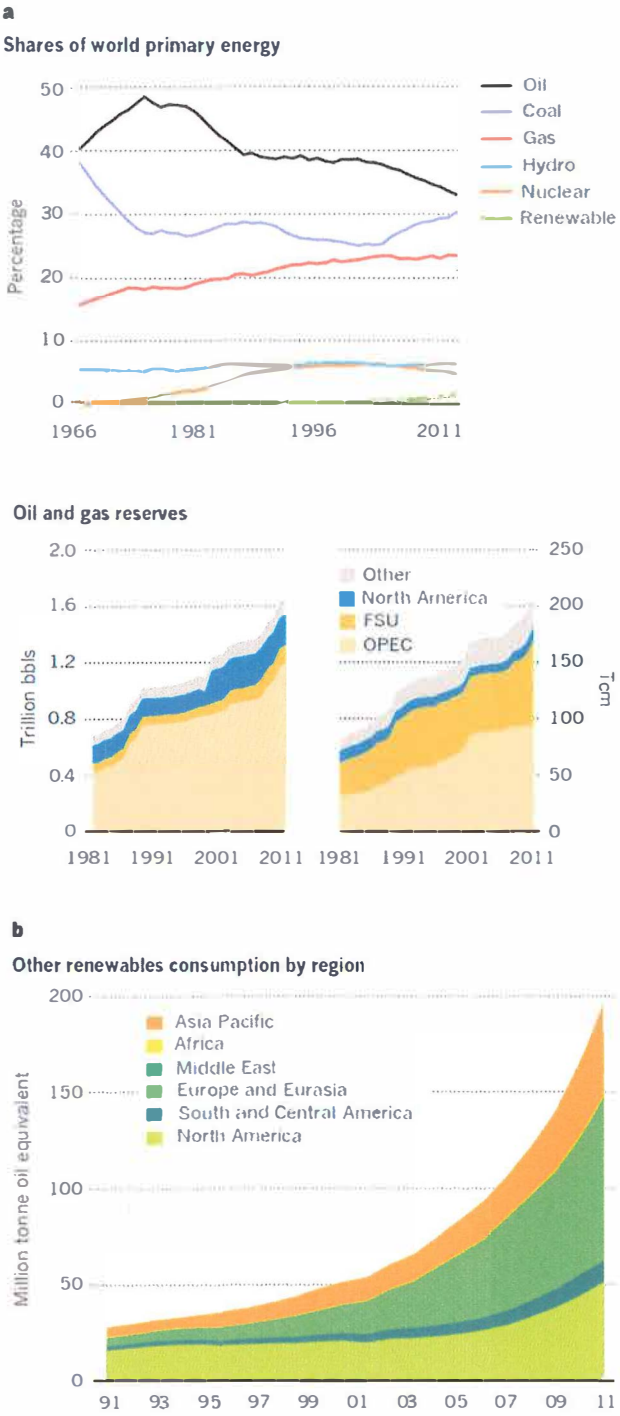

Figure 1. *Statistical review of world energy. a, Fossil energy comprises roughly 86% of the world's main energy consumption. Although the consumption of oil has increased by 31% between 1980 and 2008, the known reserves have increased comparably owing to improvements in exploration and extraction technologies. Much of the world's shale-gas reserves are not 'proven' and are not included. The fractional sum of non-carbon emitting sources of energy remained constant during the same time period. b, Growth of renewable energy was offset by the decline in nuclear power generation. Renewable energy sources in power generation grew by 17.7%. Wind generation (25.8%) accounted for more than half of renewable power generation for the first time. Renewables accounted for 3.8% of global power generation, with the highest share in Europe and Eurasia (7.1%) [3].*

2012 was the hottest year on record; the worst drought in generations covered more than half the country; record wildfires swept across western states; and an intensified Superstorm Sandy devastated communities in the East. (See The President's Plan to Fight Climate Change for more information.) U. S. Energy Sector Vulnerabilities to Climate Change and Extreme Weather examines current and potential future impacts of these climate trends on the U.S. energy sector. Researchers have identified several critical issues, including power-plant disruptions due to drought and the disruption of fuel supplies during severe storms. They've also pinpointed potential opportunities that would make our energy infrastructure more resilient to these risks.

The map above shows how the following three extreme climate trends have caused major issues to the energy sector across the country over the past ten years [6]:

1. Increasing air and water temperatures;
2. Decreasing water availability across regions and seasons; and
3. Increasing intensity and frequency of storm events, flooding and sea level rise.

The effects of these climatic changes (Table 1) are:

- Climate change has created an increased risk of shutdowns at coal, natural gas and nuclear power plants. Why? Changes in the climate mean decreased water availability -- which affects cooling at thermoelectric power plants, a requirement for operation.
- There are also higher risks to energy infrastructure located along the coasts thanks to sea level rise, the increasing intensity of storms, and higher storm surge and flooding.
- Power lines, transformers and electricity distribution systems face increasing risks of physical damage from the hurricanes, storms and wildfires that are growing more frequent and intense.

Table 1. *Relationship between climate change projections and implications for the energy sector as per US Department of Energy Report [7].*

Energy sector	Climate projection	Potential implication
Oil and gas exploration and production	• Thawing permafrost in Arctic Alaska	• Damaged infrastructure and changes to existing operations
	• Longer sea ice-free season in Arctic Alaska	• Limited use of ice-based infrastructure; Longer drilling season; new shipping routes
	• Decreasing water availability	• Impacts on drilling, production,and refining
	• Increasing intensity of storm events, sea level rise, and storm surge	• Increased risk of physical damage and disruption to offshore and coastal facilities
Fuel transport	• Reduction in river levels	• Disruption of barge transport of crude oil ,petroleum products, and coal
	• Increasing intensity and frequency of flooding	• Disruption of rail and barge transport of crude oil, petroleum products, and coal
Thermoelectric power generation (Coal, nuclear, geothermal and solar CSP)	• Increasing air temperatures	• Reduction in plant efficiencies and available generation capacity
	• Increasing water temperatures	• Reduction in plant efficiencies and available generation capacity; increased risk of exceeding thermal discharge limits
	• Decreasing water availability	• Reduction in available generation capacity; impacts on coal, natural gas, and nuclear fuel aupply chains
	• Increasing intensity of storm events, sea level rise, and storm surge	• Increased risk of physical damage and disruption to coastal facilities
	• Increasing intensity and frequency of flooding	• Increased risk of physical damage and disruption to inland facilities
Hydropower	• Increasing temperatures and evaporative losses	• Reduction in available generation capacity and changes in operations
	• Changes in precipitation and decreasing snowpack	• Reduction in available generation capacity and changes in operations
	• Increasing intensity and frequency of flooding	• Increased risk of physical damage and changes in operations
Bioenergy and biofuel production	• Increasing air temperatures	• Increased irrigation demand and risk of crop damage from extreme heat events
	• Extended growing season	• Increased production
	• Decreasing water availability	• Decreased production
	• Sea level rise and increasing intensity and frequency of flooding	• Increased risk of crop damage
Wind energy	• Variation in wind patterns	• Uncertain impact on resource potential
Solar energy	• Increasing air temperatures	• Reduction in potential generation capacity
	• Decreasing water availability	• Reduction in CSP potential generation capacity
	• Increasing air temperatures	• Reduction in transmission efficiency and available transmission capacity
Electric grid	• More frequent and severe wildfires	• Increased risk of physical damage and decreased transmission capacity
	• Increasing intensity of storm events	• Increased risk of physical damage
	• Increasing air temperatures	• Increased electricity demand for cooling; decreased fuel oil and natural gas demand for heating
Energy demand	• Increasing magnitude and frequency of extreme heat events	• Increased peak electricity demand

- Air conditioning costs will rise due to increasing temperatures and heat waves, along with the risks of blackouts and brownouts in regions throughout the country.

To overcome these environmental issues, there is urgent need of large scale production of new kind of fuels. Presently, Petroleum-derived liquid fuels are the overwhelming source of energy in the current transportation infrastructure. Now-a-days, natural gas and other bio-liquids are also used but their percentage is still very less (Figure 2). The origin of the alarming environmental problems increasingly affecting our planet are not due to fossil fuels per se, but rather to the strength of their conventional valence bond, since that strength has prohibited the achievement of full combustion during the past one hundred years of efforts.

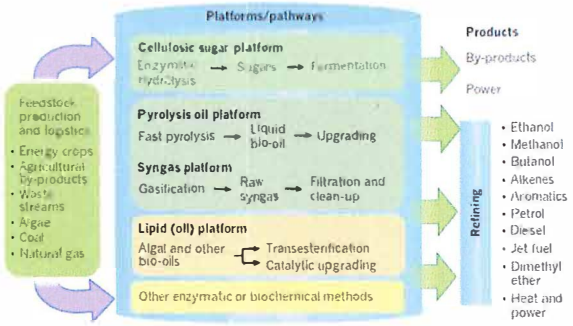

Figure 2. *Methods of producing alternative fuels from various feed stocks to products. Various feed stocks are being explored, and the pathways for producing energy or fuel investigated [3].*

2. Magnecules

Santilli magnecules in gases, liquids, and solids consist of stable clusters composed of conventional molecules, and/or diatomic radicals, and/or individual atoms bonded together by opposing magnetic polarities of toroidal polarizations of the orbits of at least the peripheral atomic electrons when exposed to sufficiently strong external magnetic fields, as well as the polarization of the intrinsic magnetic moments of nuclei and electrons (Figure 3 and 4) [7-10]. A population of magnecules constitutes a chemical species when essentially pure, i.e., when molecules or other species are contained in very small percentages in a directly identifiable form.

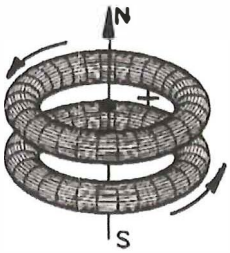

Figure 3. *A schematic view of the simplest possible diatomic magnecule whose bond originates from the toroidal polarization of the orbits of peripheral atomic electrons.*

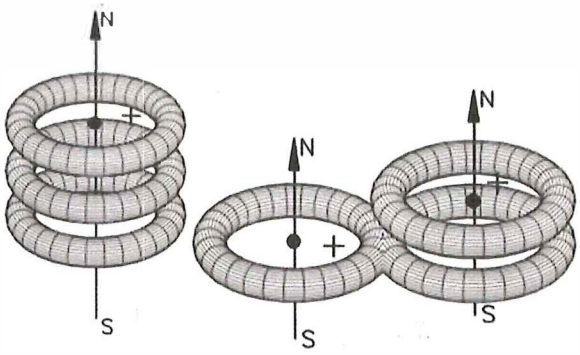

Figure 4. *A schematic view of the simplest possible multiatomic magnecular bonds. A conceptual rendering of the cluster MH_3 in MH which is predicted as being composed by magnecular species H × H × H (Top) and H - H × H (Bottom).*

Magnecules are also called:

A) *elementary* when only composed of two molecules; If the conventional valence bond is denoted with the symbol "–" and the new magnetic bond with the symbol "×", examples of elementary magnecules in gases and liquids are respectively given by

$$\{H–H\} \times \{H–H\}, \{O–O\} \times \{O–C–O\}, \text{etc.} \quad (1)$$

$$\{C_{15}–H_{20}–O\} \times \{C_{15}–H_{20}–O\}, \text{etc.} \quad (2)$$

B) *magneplexes* when entirely composed of several identical molecules; examples of magneplexes in gases and liquids are respectively given by

$$\{H–H\} \times \{H–H\} \times \{H–H\} \times., \text{etc.} \quad (3)$$

$$\{H–O–H\} \times \{H–O–H\} \times., \text{etc.} \quad (4)$$

C) *magneclusters* when composed of several different molecules and examples of magneclusters are given by

$$\{H–H\} \times \{C–O\} \times \{O–C–O\} \times \{C=O\} \times., \text{etc.} \quad (5)$$

$$\{C_{13}–H_{18}–O\} \times \{C_{14}–H_{12}–O_3\} \times \{C_{15}–H_{20}–O\} \times., \text{etc.} \quad (6)$$

A generic representation of a gas magnecules requires the presence of individual atoms and diamers (diatomic), such as:

$$\{H–H\} \times \{C–O\} \times H \times \{H–O–H\} \times C \times \{H–O\} \times., \text{etc,} \quad (7)$$

One of the most important features of magnecules is their anomalous release of energy in thermochemical reactions, in view of its evident importance for the industrial development of new clean fuels such as magnegas. This feature is crucially dependent on the existence within the magnecules of individual atoms, such as H, C and O, and/or individual diatomic free radicals, such as H–O and H–C. In fact, at the breakdown of the magnecules due to combustion, these individual atoms and diatomic free radicals couple themselves into conventional molecules via known exothermic reactions such as

$$H + H \rightarrow H_2 + 105 \text{ kcal./mol} \quad (8)$$

$$C + O \rightarrow CO + 255 \text{ kcal/mol} \quad (9)$$

$$H\text{–}O + H \rightarrow H_2O + 28 \text{ kcal/mol, etc.} \quad (10)$$

with consequential release during combustion of a large amount of energy that does not exist in fuels with a conventional molecular structure.

Finally, magnecules are called:

i) *isomagnecules* when having all single-valued characteristics and being reversible in time, namely, when they are characterized by isochemistry;
ii) *genomagnecules* when having all single-valued characteristics and being irreversible in time, namely, when they are characterized by genochemistry; and
iii) *hypermagnecules* when having at least one multi-valued characteristic and being irreversible in time, namely, when they are characterized by hyperchemistry.

3. New Molecular Internal Bonds

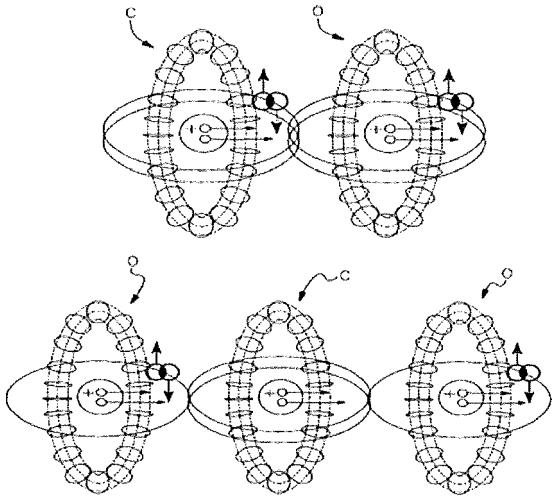

Figure 5. *A schematic view for the cases of C=O and O–C–O of the polarization of internal atomic electrons, while preserving conventional valence bonds, and the consequential creation of new bonds in conventional molecules which are not of valence type, as later on verified experimentally via IR scans.*

Magnecules can also be formed by means other than the use of external magnetic fields. For instance, magnecules can be produced by electromagnetic field with a distribution having a cylindrical symmetry; or by microwaves capable of removing the rotational degrees of freedom of molecules and atoms, resulting in magnetic polarizations. The magnetic polarization at the foundations of magnecules predicts the existence of these new internal bonds and permits their quantitative study. Recall that external magnetic fields can polarize the orbit of valence electrons, but cannot possibly break or alter valence bonds. Recall that, consequently, sufficiently strong external magnetic fields can polarize the orbits of all atomic electrons, and not only those of the valence electrons. Consider then a conventional molecule such as C=O. When exposed to the extreme magnetic fields as existing in the PlasmaArcFlow technology, the orbits of all internal electrons can be polarized, individually, for the carbon and the oxygen, in addition to the polarization of the two pairs of valence bonds. One of the various possible geometries is that in which the plane of the polarization of the internal electrons is perpendicular to that of the two pairs of valence bonds. In this case we have the birth of a new bond of magnetic origin in the interior of a conventional molecule, which is evidently given by the alignment of the two polarities North-South and North-South in the carbon and oxygen, and the consequential attraction of opposite polarities of different atoms, as illustrated in Figure 5. For the case of the O–C–O molecule we can evidently have two internal bonds of magnetic type in addition to the valence bonds, which are also given by the alignment of the magnetic polarities, resulting in one new bond, as illustrated in Figure 5.

The above new internal molecular bonds have major industrial and consumer implications, in as much as they permit the production of fuels capable of releasing under combustion anomalous amounts of energy, with consequential reduction of pollutants in the exhaust, as already proved by magnegas.

4. Main Features for the Detection of Magnecules

The experimental detection of gas magnecules requires the verification of a number of characteristic features of magnecules identified in Definition. The main features are as follows:

- Feature 1: Appearance of unexpected heavy MS peaks.
- Feature 2: Unknown character of the unexpected MS heavy peaks.
- Feature 3: Lack of IR signature of the unknown MS peaks.
- Feature 4: Mutation of IR signatures.
- Feature 5: Mutation of magnecular weights.
- Feature 6: Accretion or emission of individual atoms, diatomic radicals or ordinary molecules.
- Feature 7: Anomalous adhesion.

5. Production of Magnegas

A new technology, called Plasma Arc FlowTM, flows the waste through a submerged electric arc between conventional electrodes. Different types of Plasma Arc Flow reactors have been shown in from Figure 6 to Figure 9. The arc decomposes the molecules of the liquid into its atomic constituents; ionizes the same; and creates a plasma of mostly ionized H, C and O atoms at about 3,500 K [9]. The flow of the liquid through the arc then continuously removes the plasma from the arc following its formation. Said plasma then cools down in the surrounding liquid, and a number of chemical reactions take place resulting in the formation of magnegas which bubbles to the surface of the liquid where it

is collected for industrial or consumer use. As soon as two or more molecules near each other possessing such an extreme magnetic polarization are created, they bond to each other via opposing magnetic polarities, resulting in the elementary magnecule.

Figure 6. *50 kW Plasma Arc Flow Mobile Refinery.*

Figure 7. *100 kW Plasma Arc Flow Mobile Refinery.*

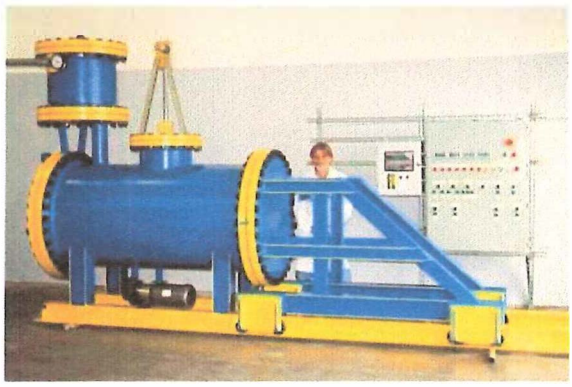

Figure 8. *Picture of a 250 kW Santilli's Hadronic Reactor (also called Plasma Arc Flow Recycler™) with the panels of its completely automatic and remote controls, to recycle liquid waste into magnegas usable for any fuel application, a large amount of heat and carbonaceous precipitates used to produce the electrodes. This Reactor can produce up to 5000, 000 scf (140 millions liters) of magnegas per week of 24 hours work per day corresponding to 3, 000 gasoline gallon equivalent (11, 000 gasoline liter equivalent) of magnegas per week computed on the basis that: 1) Gasoline contains about 110, 000 BTU/g (about 29, 000 BTU/liter); 2) Magnegas has the low energy content of 750 BTU/scf (26 BTU/liter); and 3) the gasoline gallon equivalent is given by about 150 scf of magnegas ("gasoline liter equivalent" is given by about 29 liters of magnegas). Plasma Arc Flow Recycler™ are completely self-contained. Consequently, they release no solid, liquid or gas to the environment and cause no noise or odor pollution.*

Figure 9. *Sterilization Recycler.*

When the original waste is of fossil or organic type, magnecules are essentially constituted by conventional molecules H_2, CO, CO_2, H_2O, plus individual atoms of H, O, and C, as well as radicals such as HO, CH and C-O, all these constituents being bonded together by strong magnetic fields originating from the toroidal polarization of the orbits of valence and other electrons. As a result of the expected dominance of magnetic over electric contributions, the new chemical species composing magnegas is called "magnecules" or "magneclusters". It is evident that the bonding of atoms and molecules into new clusters Constitutes new means for storing energy in a combustible gas, in addition to the conventional valence means in ordinary fuels.

Therefore, magnecules have a primary relevance for the study of the combustion of magnegas.

The MagneGas Recyclers have two operating modes:

"GASIFICATION MODE" is suitable to completely gasify target liquids for the maximum conversion of liquid to fuel and is most suitable for oily or hazardous wastes that require elimination. The waste is converted into MagneGas and carbon precipitates. In this mode the liquid is completely gasified.

"STERILIZATION MODE" is intended solely to sterilize target liquid wastes such as sewage, agricultural wastes or any effluent where eliminating bacteriological activity is beneficial to convert the waste liquid into a fertilizer and irrigation water. This results in the production of MagneGas, carbon precipitates and the same quantity of sterilized liquid. In this mode the liquid is retained but completely sterilized.

MagneGas is a cost competitive and clean burning fuel that is essentially interchangeable with Natural Gas or it can be co-combusted with existing hydrocarbon fuels and has the lowest Green House gas emissions when compared to fossil fuel. The Magnegas system occupies a small footprint and runs in a completely sealed environment. The fuel can be used for metal cutting, cooking, heating, or powering natural gas bi-fuel automobiles. MagneGas™ Recyclers shown in Figure 9 Can Process:

- Sewage
- Sludge
- Agricultural Waste

- Leachates
- Some Oil Based Liquids
- Some Industrial Waste Liquids

6. Combustion of Magnegas

It is evident that the combustion of magnegas requires the oxidation, first, of magnecules, and then that of conventional molecules H_2 and CO. To begin, the known reactions $H_2 + ½ O_2 \rightarrow H_2O$ and $CO + ½ O_2 \rightarrow CO_2$, should be replaced by the reaction:

$$\text{magnecule} + nO_2 \rightarrow mH_2O + kO_2 + lCO_2 \ldots\ldots + \Delta \text{ kcal} \quad (11)$$

which may give increased energy released per each H_2 molecule. Here, n, m, k, l, are the stoichiometric numbers, and the original magnecule is assumed to consist of both H_2 and CO molecules.

The energy balance for the combustion of magnecule is then given by:

$$E[\text{combustion}] = mE[H_2O] + (k-n)E[O_2] + lE[CO_2] - E[\text{magnecule}] \quad (12)$$

where $E[H_2O]$, $E[O_2]$, $E[CO_2]$,. are ground state energies of the molecular constituents, and E[magnecule] is ground state energy of the the original magnecule.

A way to calculate this energy balance is to use dissociation energy of the magnecule, D[magnecule]. However, we should note that D[magnecule] is different for magnecules of different mass and composition.

In general, the thermodynamic relation between the Gibbs free energy of reaction, ΔG, and the reaction equilibrium constant, K, is as follows:

$$-2.303\, RT \log K = \Delta G \quad (13)$$

where

$$\Delta G = \Delta H - T\Delta S \quad (14)$$

$R = 1{:}986$ cal.K^{-1}.mol^{-1}, T is temperature in Kelvin, ΔH is the enthalpy of reaction and ΔS is the entropy of the reaction. The latter is numerically large if the initial reactants have molecular structures more ordered than the end products, i.e. there is an increase of entropy S during the reaction.

Thus the combustion of magnegas is characterized by a very high value of the reaction constant (equilibrium) (perhaps even higher than $K = 10^{40}$ at $T = 25°C$). In fact, the combustion of magnegas is a highly exothermic reaction, and the magnecules have a structure much more ordered than the product of the combustion. Therefore, during the combustion of magnegas we have a large increase of the entropy $\Delta S > 0$. These two factors lead to very high value of the reaction equilibrium constant K for the combustion of magnegas.

It is interesting to note that the oxidation of carbon to carbon monoxide, e.g., $C + CO_2 \rightarrow 2CO$, is almost the only oxidation reaction for which ΔG decreases with the increase of the temperature. Here, the number of moles increases about twice during the reaction. As a result, the entropy greatly increases $\Delta S > 0$. Therefore, the CO molecule is more stable at high temperatures than at low temperatures (for example, it is about twice more stable at 3000°C than at 1000°C). Another ecologically very important aspect in the combustion of magnegas is therefore the reduction of CO via the oxidation of carbon atoms present in magnecules, and its subsequent dissociation as in Eq. (9) to release the oxygen needed for the burning of hydrogen. Since the stability of CO increases with the temperature, a better quality of the exhaust is reached at lower original temperatures of magnegas. This result should be compared with the opposite occurrence for natural gas and for other fuels, which are generally pre-heated prior to combustion.

Another important characteristic of a reaction is the rate of reaction. Various tests [2a] have show that the combustion of magnecules is faster than the combustion of their molecular constituents. Santilli-Shillady isochemical models of molecular structures permits the following understanding of this additional anomaly. In conventional quantum mechanical equation in relative coordinates and reduced mass for two electrons in singlet coupling we have the following Schrödinger equation:

$$\left(\frac{p^2}{m} + \frac{e^2}{r}\right)\psi(r) = E\psi(r) \quad (15)$$

where m is the electron mass. The above equation shows the repulsive Coulomb force between the point-like charges of the electrons. But the electrons have extended wavepackets of the order of 1 fm whose mutual penetration, as necessary for the valence bond, causes nonlinear, nonlocal and nonpotential interactions at the foundations of hadronic mechanics (see the preceding chapter). The only known possibility for an invariant representation of these interactions is to exit from the class of unitary equivalence of Eq. (15) via an isounitary transformation that, for simplicity, we present below in its projection into a conventional nonunitary form

$$UU^\dagger \neq I, UU^\dagger = \hat{I} = 1/\Gamma \quad (16)$$

$$U\left[\left(\frac{p^2}{m} + \frac{e^2}{r}\right)\psi(r)\right]U^\dagger$$

$$= \left[\frac{1}{m}Up^2U^\dagger + \frac{e^2}{r}UU^\dagger\right]\left(UU^\dagger\right)^{-1}\left[U\psi(r)U^\dagger\right] \quad (17)$$

$$= \left[\frac{1}{m}\hat{p}T\hat{p}T + \frac{e^2}{r}\right]\hat{\psi}(r) = E'\hat{\psi}(r)$$

where one should note the different eigen value E' from the value E of Eq.(15) (due to the general non-commutativity of the Hamiltonian and the isounit).

At this point, Santilli and Shillady introduced the following realization of the non-unitary transform, that is, of the fundamental iso-unit of hadronic chemistry, Eq.(18) of FHC,

$$UU^{\dagger} = \hat{I} = \frac{1}{T} = \exp\left\{\left[\frac{\psi(r)}{\hat{\psi}(r)}\right]\int \psi_1(r)\psi_2(r)d^3(r)\right\}$$
$$= I + \left[\frac{\psi(r)}{\hat{\psi}(r)}\right]\int \psi_1^{\dagger}(r)\psi_2(r)d(r) + \ldots\ldots \quad (18)$$

where ψ and $\hat{\psi}$ are the solutions of the unitary and nonunitary equations, and, k = 1; 2, are the conventional quantum mechanical wave functions of the two electrons.

It is evident that, as desired, the above iso-unit represents interactions that are: nonlinear, because dependent in a nonlinear way in the wave functions; nonlocal, because inclusive of a volume integral; and nonpotential, because not representable with a Hamiltonian.

Santilli and Shillady solved the above equations. First, by inserting isounit of Eq.(18) in Eq.(17), they obtained the isoequation here projected on a conventional Hilbert space

$$\left[\frac{p^2}{2m'} + \frac{e^2}{r} - V_0\frac{e^{-br}}{1-e^{-br}}\right]\hat{\psi}(r) = E'\hat{\psi}(r) \quad (19)$$

where m′ represents the iso-renormalization of the mass caused by nonpotential interactions, and one recognizes the emergence of the attractive Hulthen potential

$$V_{Hulthe'n} = V_0\frac{e^{-br}}{1-e^{-br}} \quad (20)$$

But the Hulthen potential is known to behave like the Coulomb potential at short distances and be much stronger than the latter. Therefore, Eq.(19) admits the excellent approximation

$$\left[\frac{p^2}{2m'} + V'\frac{e^{-br}}{1-e^{-br}}\right]\hat{\psi}(r) = E'\hat{\psi}(r) \quad (21)$$

where the new constants V′ reflects the "absorption" of the repulsive Coulomb potential by the much stronger attractive Hulthen potential. In this way, Santilli and Shillady achieved for the first time in the history of chemistry a valence coupling between two identical electrons in singlet coupling with a strongly attractive force, as requested by experimental evidence, which model is today known as the Santilli-Shillady strong valence bond.

In their natural conventional, and non-polarized states, H_2 and O molecules have the usual (spherical) shape due to rotations. However, an inspection of the isochemical model of the water shows that such configurations are not suited for the reaction of H and O into HO. In particular, the orbitals of H require a toroidal configuration as a condition for their bonding to the oxygen, a similar occurrence holding for the oxygen too.

It then follows that magnetically polarized molecules of hydrogen and oxygen have a bigger reaction rate than the same molecules in un-polarized conditions, since they have a distribution of the valence electrons more suitable for the reaction itself. Evidently, a bigger reaction rate implies a bigger power.

Moreover, the combustion of a magnecule consisting of H and CO, does not require the necessary previous dissociation of the $O_2 \rightarrow 2O$, 119.1 kcal), because each O-atom in a magnetically polarized oxygen molecule (oxygen molecule is ready for the combustion). Therefore, the magnecular structure acts as a kind of catalysis, in which both O-atoms of the oxygen molecule start to react with the nearest pair $H_2 \times H_2$, or $H_2 \times CO$, or $CO \times CO$ almost simultaneously; This occurrence also implies that less amounts of external energy is needed to activate the reaction, resulting, again, in an anomalous energy release in combustion. Usually, the activation energy is supplied by heat. Therefore, we can conclude by saying that the combustion of magnegas can be initiated at smaller temperature, in comparison to that of the simple mixture of hydrogen and CO gases. The reduction of pre-heating time of about one-half for magnegas as compared to that for acetylene, is in agreement with the above interpretation.

Due to the presence of magnecules viewed as heavy complexes of H and CO, one cubic foot of magnegas contains in fact larger number of H_2 and CO molecules than it is for the respective simple mixture (i.e., without the clusters) of these two gases, at the same temperature and pressure. which are interacting with each other. Thus, the larger number of H_2 and CO molecules per cubic foot of magnegas can be considered as one of the simple reasons of high energy content of magnegas which has been measured and calculated per cubic foot.

7. Applications of Magnecules

Some of the important properties of magnecules include: increased energy density; increased energy output under thermo-chemical reactions; increased adhesion with other substances; increased penetration within other substances; and other properties which are new when compared to corresponding properties of the conventional molecules constituting magnecules and any of their combination. Consequently, the new chemical species of magnecules has new industrial and consumer applications, advancement in this arena discussed brief in the section.

7.1. MagneGas

In 2001, Santilli released monograph [7] in which the first industrial realization of the new species of magnecules consisting of the gaseous fuel produced and sold worldwide under the trade name of MagneGas (MG) by the U. S. publicly traded company Magnegas Corporation (see www.magnegas.com for details) was provided in detail. Plasma Arc Flow reactor [12-19] use a submerged DC arc to achieve complete recycling of liquid waste into a clean burning combustible gas called MG, heat usable via exchangers, solid precipitates. Such kind of highly efficient and cost effective flow reactor was suitable to recycle antifreeze waste, oil waste, sewage and other contaminated liquids [12].

The first application of MG is its use as a new clean fuel for automobile (see Figure 10). When exhaust of MG was

tested, interestingly it surpasses all EPA requirement even without catalytic convertor, emits no carcinogenic, CO or other toxic chemicals, reduced carbon dioxide emission due to combustion of gasoline by about 40 %, and emits 14-20 % breathable oxygen (see Figure 11). The Plasma Arc Flow causes the removal of H and O from the arc immediately following their creation, thus preventing their recombination into H_2O, with consequential dramatic increase of the efficiency, that is, of the volume of combustible gas produced per kWh. Different studies establish the environmental superiority of magnegas over natural gas and gasoline. The following comments are now in order:

1) Magnegas does not contain (heavy) hydrocarbons since it is created at 3,500 K. Therefore, the measured hydrocarbons are expected to be due to combustion of oil, either originating from magnegas compression pumps (thus contaminating the gas), or from engine oil.
2) Carbon monoxide is fuel for magnegas (while being a combustion product for gasoline and natural gas). Therefore, any presence of CO in the exhaust is evidence of insufficient combustion.
3) Its exhaust also does not contain other pollutants such as NOx and other carcinogenic gases, yet rich in oxygen.

Figure 10. *A picture of a Ferrari 308 GTSi 1980 and two Honda Civic cars converted by the author to operate with the new clean burning magnegas without catalytic converter, yet surpassing all EPA exhaust requirements, having no carcinogenic or other toxic substance in the exhaust, reducing of about 50% the CO_2 emission due to gasoline combustion, reducing the operating temperature of about 25%, and emitting in the exhaust 10% to 14% breathable oxygen.*

Figure 11. *A picture of the readings of a 4-ways exhaust analyzer testing the exhaust of the Ferrari 308 GTSi of a preceding picture operating on magnegas "without" catalytic converter. Note: the presence of 14% breathable oxygen in the exhaust; about half the CO_2 produced by the same car when running on gasoline; the very few detected hydrocarbons originate from engine oil seeping through the piston rings because magnegas cannot contain hydrocarbons since it is synthesized at the 5, 000°C of the arc at which temperature no hydrocarbon can survive; the very small content of CO in the exhaust is due to poor combustion because CO is fuel for magnegas, while it is a byproduct of the combustion for fossil fuels, as a result of which detecting CO in the exhaust of a car running on magnegas is the same as detecting gasoline in the exhaust of a car running on gasoline.*

Figure 12. Applications of Magnegas for welding and cutting purpose.

Independent certifications by various users have established that: 1) magnegas has a pre-heat time at least half that by acetylene (which is currently used for metal cutting and has an energy content of 2,300 BTU/cf); 2) magnegas cuts metal at least 50% faster than acetylene; 3) the cut produced by magnegas is much smoother without edges as compared to that by acetylene; 4) magnegas exhaust does not contain carcinogenic or other toxic substances, while that of acetylene is perhaps the most carcinogenic and toxic of all fuels; 5) magnegas cutting does not produce the "ash-back" (local explosion of paint over metal) typical of acetylene; 6) magnegas is dramatically safer than acetylene, which is unstable and one of the most dangerous fuels currently used; and 7) magnegas cost about half that of acetylene.

In the same monograph, the second very important application of MG depicted was in the metal cutting industry, wherein it was found that MG cuts the metal much smoother, without edges and at least 50% faster than conventional acetylene (See Figure 12). Independent certifications by various users have established that:

1. magnegas has a pre-heat time at least half that by acetylene (which is currently used for metal cutting and has an energy content of2,300 BTU/cf);
2. magnegas cuts metal at least 50% faster than acetylene;
3. the cut produced by magnegas is much smoother without edges as compared to that by acetylene;
4. magnegas exhaust does not contain carcinogenic or other toxic substances, while that of acetylene is perhaps the most carcinogenic and toxic of all fuels;
5. magnegas is dramatically safer than acetylene, which is unstable and one of the most dangerous fuels currently used; and
6. magnegas cost about half that of acetylene.

Carbon dioxide (CO_2) is the primary greenhouse gas emitted through human activities. In 2012, CO_2 accounted for about 82% of all U. S. greenhouse gas emissions from human activities. Carbon dioxide is naturally present in the atmosphere as part of the Earth's carbon cycle (the natural circulation of carbon among the atmosphere, oceans, soil, plants, and animals). Human activities are altering the carbon cycle both by adding more CO_2 to the atmosphere and by influencing the ability of natural sinks, like forests, to remove CO_2 from the atmosphere. While CO_2 emissions come from a variety of natural sources, human-related emissions are responsible for the increase that has occurred in the atmosphere since the industrial revolution. The main human activity that emits CO_2 is the combustion of fossil fuels (coal, natural gas, and oil) for energy and transportation, although certain industrial processes and land-use changes also emit CO_2. The main sources of CO_2 emissions in the United States are described below.

- *Electricity.* Electricity is a significant source of energy in the United States and is used to power homes, business, and industry. The combustion of fossil fuels to generate electricity is the largest single source of CO_2 emissions in the nation, accounting for about 38% of total U.S. CO_2 emissions and 31% of total U.S. greenhouse gas emissions in 2012. The type of fossil fuel used to generate electricity will emit different amounts of CO_2. To produce a given amount of electricity, burning coal will produce more CO_2 than oil or natural gas.
- *Transportation.* The combustion of fossil fuels such as gasoline and diesel to transport people and goods is the second largest source of CO_2 emissions, accounting for about 32% of total U.S. CO_2 emissions and 27% of total U.S. greenhouse gas emissions in 2012. This category includes transportation sources such as highway vehicles, air travel, marine transportation, and rail.
- *Industry.* Many industrial processes emit CO_2 through fossil fuel combustion. Several processes also produce

CO_2 emissions through chemical reactions that do not involve combustion, for example, the production and consumption of mineral products such as cement, the production of metals such as iron and steel, and the production of chemicals. Fossil fuel combustion from various industrial processes accounted for about 14% of total U.S. CO_2 emissions and 12% of total U.S. greenhouse gas emissions in 2012. Note that many industrial processes also use electricity and therefore indirectly cause the emissions from the electricity production.

Carbon dioxide is constantly being exchanged among the atmosphere, ocean, and land surface as it is both produced and absorbed by many microorganisms, plants, and animals. However, emissions and removal of CO_2 by these natural processes tend to balance. Since the Industrial Revolution began around 1750, human activities have contributed substantially to climate change by adding CO_2 and other heat-trapping gases to the atmosphere. The most effective way to reduce carbon dioxide (CO_2) emissions is to reduce fossil fuel consumption. Many strategies for reducing CO_2 emissions from energy are cross-cutting and apply to homes, businesses, industry, and transportation. Also, it has been reported that magnegas mixed with coal, and other fossil fuels such as gasoline, diesel, etc. reduced the CO_2 emissions as well as other toxic gases emission and thus can be used in future for large scale energy formation (see Figure 13 and Figure 14).

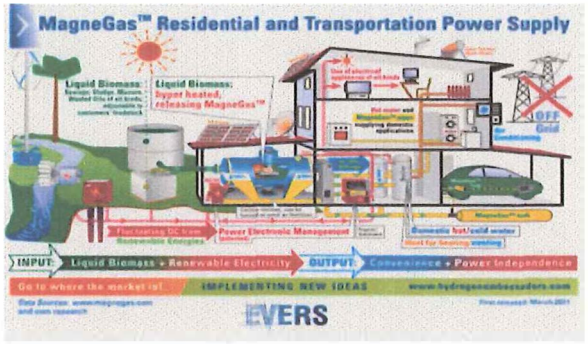

Figure 13. *Future Scope I: Applications of Magnegas for residential, transportation and industrial power supply purpose.*

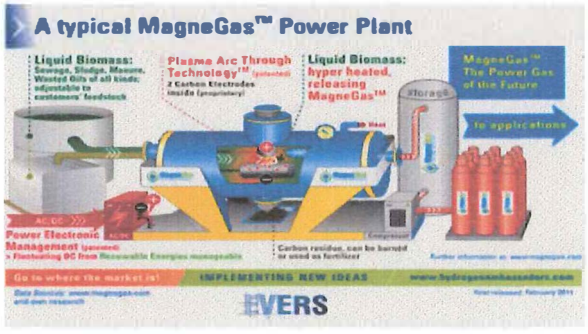

Figure 14. *Future Scope II: Applications of Magnegas in Power Plant as future scope.*

7.2. Magne Hydrogen™

In a paper from 2003, Santilli [20] presented theoretical and experimental evidence on the existence of a new species of Hydrogen.i. e. he called MagneHydrogen™ (See Figure 15). It was found that the prepared gas apparently consists of 99% Hydrogen, although spectroscopically its specific weight (or, equivalently, molecular weight) was estimated to be 7.47 times larger than that of conventional Hydrogen. This new species of MH is that originally presented by Santilli in Ref. [9], namely, a multiple of the specific weight under a high Hydrogen percentage is evidence of a new clustering of H-atoms which cannot possibly be of valence type due to the evident absence of the valence electrons necessary for a quantitative representation of the clustering of many different atoms. For the generation of MH, Santilli first developed the Plasma Arc Flow Reactor™ for the conversion of various liquids into a combustible gaseous fuel known as MagneGas™ (MG). The gasification is achieved via a submerged DC electric arc between carbon electrodes that, under sufficient powers (of the order of 300 kW or more) is capable of producing at atomic distances the high values of the magnetic field necessary for the polarization of electron orbitals into toroids (estimated as being of the order of 10^{12} Gauss. Santilli obtained the new species MH via the use of conventional Pressure Swing Adsorption (PSA) equipment for the separation of Hydrogen from MG. The purity and increased specific weight of MH depends on various factors, including the selected zeolites, the operating pressure, etc. [21].

From an industrial point of view, it is very important to be noted that, it is sufficient to achieve a species of MH with at least 3.3 the specific weight of hydrogen to have the same energy content of 1000 BTU/scf of Natural Gas (NG). In fact, under said conditions, MH would avoid the current needs to liquefy Hydrogen in order to achieve a sufficient range, since MH can be compressed like NG. Additionally, the magnecular structure of MH avoids the traditional seepage of Hydrogen through the walls, thus allowing long term storage that is currently prohibited by molecular Hydrogen due to current environmental laws. Above all, this particular type of MH has resulted to be cost competitive with respect to fossil fuels, of course, when produced in sufficiently large volumes.

1. the use of hydrogen rich wastes as liquid feedstock, such as city and farm sewage, antifreeze and or oil waste, etc., which implies an income, rather than a cost;
2. the possible utilization of steam at 400 produced by the cooling of the highly energetic processes of the reactors, which steam can be used for other income producing applications, such as desalting seawater via evaporation, production of electricity via turbines, heating of buildings, and other income producing uses; and
3. the unusually high efficiency of Santilli Hadronic Reactors of molecular types used for the process which brings the cost of electricity down to 0.005/scf.

Similar to MagneHydrogen™, Santilli have also developed magnecular fuel from oxygen gas i. e. MagneOxygen™. These gases possess specific weight and energy content greater than the corresponding values of the same gases with conventional molecular structure. It has been found that use of MagneHydrogen™ and MagneOxygen™ in fuel cells implies: 1) an increase of fuel cells voltage, power and efficiency; 2) a decrease of storage volumes; and 3) a significant decrease in operating costs. The new forms of liquid hydrogen and oxygen is expected to find for rocket propulsion with increased trust, and consequential increased payload or decreased boosters' weight with the same payload.

7.3. HHO *Gas: Aquygen™*

Santilli then provided in Ref. [11] of 2006 theoretical experimental evidence on a third industrial application of the new species of magnecules i. e. HHO gas (see Figure 16), here referred to the gas commercially produced via certain electrolyzers (see Figure 17) and essentially consisting of 2/3 Hydrogen and 1/3 Oxygen, which contains a small percentage of H and O magnecular clusters.

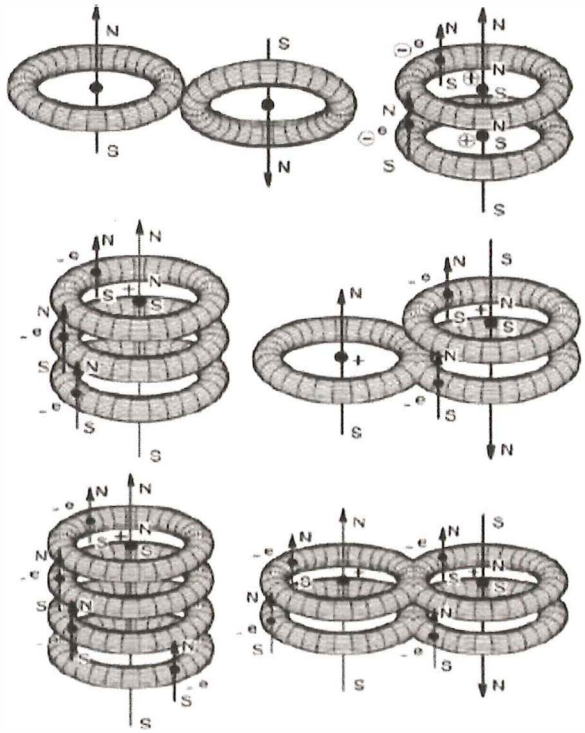

Figure 15. *A conceptual rendering of the cluster MH_2 in MH which is predicted as being composed by part of the molecular species H - H (Top, Left) and part by the magnecular species H × H (Top, Right); A conceptual rendering of the cluster MH_3 in MH which is predicted as being composed by magnecular species H × H × H (Middle, Left) and H - H × H (Middle, Right); A conceptual rendering of the cluster MH_4 in MH which is predicted as being composed by the magnecular species H × H × H × H (Bottom, Left) and H - H × H - H (Bottom, Right).*

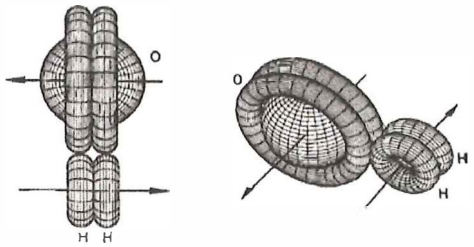

Figure 16. HHO *Gas.*

Under these conditions, Santilli suggested in Ref. [22] the name of HHO (although a similar gas produced via a different electrolyzer is known as Brown's gas). The new HHO gas is regularly produced via a new type of electrolyzer and has resulted to be distinctly different in chemical composition than the Brown gas, even though both gases share a number of common features. American company Hydrogen Technology Applications™, Inc. (HTA), which company is the owner of all intellectual rights and is currently producing and selling the HHO gas on a world wide basis under the commercial name of Aquygen™ gas.

A important feature is that the HHO gas does not require oxygen for combustion since the gas contains in its interior all oxygen needed. Another important feature of this gas is that it does not follow the *PVT* of gases with conventional molecular structure, since the gas reacquires the liquid water state at a pressure of the order of 150 psi, while conventional gases acquire the liquid state a dramatically bigger pressures. This feature suggests that the gas here considered does not possess a conventional molecular structure, namely, a structure in which the bond is of entire valence type. Another feature of the gas is its anomalous adhesion (adsorption) to gases, liquids and solids, as verified experimentally below, thus rendering its use particularly effective as an additive to improve the environmental quality of other fuels, or other applications. This

Figure 17. *The Model 1500 Aquygen Gas Generator runs on water and electricity only. It produces a stable hybrid hydrogen-oxygen gas, with many unique properties. The generator provides a superior gas for most conventional brazing, soldering and metal cutting operations. Gas Generator can be used in many exotic cutting, brazing, and fusing applications as well as glass treatment and blowing applications that cannot be performed by conventional methods and processes.*

Feature is manifestly impossible for conventional gases such as hydrogen and oxygen, thus confirming again a novel chemical structure.

AquygenTM gas supplements standard vehicle fuels such as gasoline and diesel, increasing BTUs while decreasing emissions. AquygenTM gas can also replace conventional soldering, brazing and cutting gases for use with standard equipment and techniques while producing superior results at less cost with no oxidation, no burn back, and minimal slag and, AquygenTM gas gives off no toxic fumes. AquygenTM gas has been shown to increase combustion efficiency in coal-fired furnaces leading to a dramatic decrease in fuel consumption and significant reduction in emissions, including CO_2, CO, NOx, SOx, and fly ash. Preliminary testing carried out at Western Research Institute (WRI), the leading energy research entity located in the number-one coal-producing state in the nation which has earned a name for reliable emissions testing and monitoring at power plants, showed an increase in furnace temperature upon the introduction of AquygenTM gas (A) and the ability to obtain the same furnace temperature with a decreased coal feed rate (B) which is as shown in Figure 18.

Some of the major benefits of HTAs AquygenTM Gas Enhanced Coal Combustion Process include:

1. Significant Reductions in NOx, CO, CO_2, and SO_2.
2. Increased combustion efficiency fuel savings.
3. Meets "Clean Coal Technology" requirements.
4. Reduces coal plant y ash and associated disposal costs.
5. Reduces overall emissions and produces significant revenue from the generation of carbon credits.

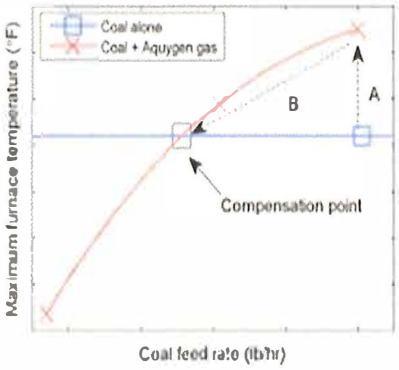

Figure 18. *Aquygen™ gas has been shown to increase combustion efficiency in coal-fired furnaces leading to a dramatic decrease in fuel consumption and significant reduction in emissions. It can be seen that there is increase in furnace temperature upon the introduction of Aquygen™ gas (A) and the ability to obtain the same furnace temperature with a decreased coal feed rate (B).*

8. Applications of Magne Gas(MG) as a Cooking or Heating Gas - Safe for Indoor Use

Carbon Monoxide (CO) is one of the most dangerous poisonous gas produced by incomplete combustion of fuels i. e. gasoline, kerosene, natural gas, propane and liquefied petroleum gas. It replaces oxygen in the blood stream causing suffocation. The results of breathing CO include loss of function, brain damage and even death. The greatest danger occurs when fossil fuels are burned indoors where they can concentrate and amplify the damage that they cause. Santilli MG is safe for indoor use. MG offer major safety advantages over fossil fuels such as natural gas and propane:

1. MG exhaust is comprised of water vapor, oxygen and carbon dioxide. Because its exhaust is free of toxic fumes and carcinogens, it is safe for indoor use.
2. Because MG exhaust has a positive oxygen balance, its does not present the dangers of oxygen depletion in closed environments that other fuels do.
3. MG is lighter than air, substantially reducing the danger of explosion compared to fossil fuels, that are heavier than air and tend to pool in and around work areas. Although it contains as much or more energy than fossil fuels, it is also less explosive.
4. Although MG is clear, it has a natural odour allowing it to be easily detected.
5. MG is magnetic, it can be easily compressed offering the same portability as propane.

9. Conclusions

In this paper, we have outlined the new chemical species of Santilli magnecules [9, 20-22] consisting of clusters of atoms, radicals and ordinary molecules whose primary characteristics are: stability at ambient temperature; a structural bond weaker than that of valence; and, consequently, an energy output bigger than that of molecular fuels with the same atomic composition.

Fuels synthesized under intense electric and magnetic fields can indeed release energy in amounts much bigger than those predicted by conventional chemical reactions. It is undeniable that the experimental confirmations of MGF exemplify a revolution in the sector of sustainable, efficient, clean fuels and over-unity power sources that do not emit harmful toxins or radioactive waste. These intriguing discoveries and experimental realizations have a significant degree of application potential in the industrial and technological sectors and, if properly implemented, may have a profoundly beneficial impact. Since, in addition, the new fuels can be produced everywhere, and have environmentally acceptable exhausts, magnegases offer serious possibilities to satisfy our ever increasing energy needs, as well as to contain the alarming environmental problems caused by fossil fuels.

Acknowledgements

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Magnecular Cleaning Coal Combustion Via Magne Gas Additive

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Abstract: Fossil fuels are contributing the largest share in meeting up energy demands of urban lifestyle across the globe. May it be gasoline or coal, all sort of fossil fuels are drilled out of earth crust to burn on the earth surface creating huge burden on air quality. Incomplete combustion of fossil fuels causes pollution of carbon monoxide and other gases. It also eats away the breathable oxygen from atmosphere. Rampant use of coal in power sector causes above problems adding to the global warming phenomenon. Magnecules and magnehydrogen are seen to be best additives to fossil fuels which can effectively enhance the combustion efficiency of fossil fuels. Present paper discusses the enhancement in combustion efficiency of fossil fuels in terms of increased utility of carbon and improved quality of emission reducing the amount of obnoxious gases.

Keywords: Coal Combnustion, Magnecules, Magnegas

1. Introduction

The new clean combustible gas, magnegas [1, 2], developed by Italian-American Scientist Ruggero Maria Santilli [1, 2], posses precisely a magnecular structure from which it has got its name. Clean water or even the sewage water can be converted to magnegas by systematic introduction to electric arc generating a high current in microseconds [3-5]. The gaseous product, so obtained, is unusual in behavior from its physical and chemical properties point of view [3]. It is a kind of short range polymeric product with two molecules of gas being bonded by a magnetic force of attraction than the conventional covalent bond. Structural detail of the properties of this gas is still a subject of discussions for about a decade. Spectroscopic results and their interpretations are of great interest of the chemists. This is mainly due to the characteristics of this new matter which are beyond the scope of interpretation of many spectroscopic and chromatographic instruments available in market. But apart from all these discussions, the most striking feature of Magnegas is that it is proving to be a promising fuel of the 21st century. So far, the success of this fuel is reported as alternative fuel in automobile sector [3-5]. Some high energy gas fuelled fields like metal cutting tools are also working successfully on this fuel.

Another potential direction of energy studies of magnegas is that the hybrid of this fuel with fossil fuels is expected to show enhanced combustion efficiency. Magnetized hydrogen from magnegas burns with elevated temperature and helps in complete combustion of carbon in coal and hydrocarbon in gasoline. Thus Hy-Diesel, Hy-Gasoline, Hy-Coal are the new age terms coined by Santilli [5].

Present paper is aimed to discuss the possibilities of a technology to use magnegas as an additive to coal in order to enhance the combustion efficiency of coal. Another equally important aspect of study is to estimate the environmental benefits of using magnegas and/or magnehydrogen as additive to reduce the emission of green house gases and limit suppress the global warming phenomenon in its long term use.

1.1. Present Global Energy Scenario

The whole world is thriving for energy in terms of electricity for industrial and agricultural development. Electricity is one of the fundamental forms of energy of

modern world. From industrial production to basic amenities and luxurious life, everything is dependent on electricity. Major part of the electricity comes from coal and nuclear power plants. But rampant use of fossil fuels like coal is a cause of worry for future. A water heater uses about two tons of coal a year. And a refrigerator consumes half-ton a year [6-8]. Even though many of us don't ever see coal, we use several tons of it every year! The rapid depleting reservoirs of coal along with dirty nature of this cheap fuel are a cause of worry for whole world due to increasing pollution and global warming from emitted gases.

Use of fossil fuels like gasoline, diesel and coal is causing a huge loss to the ecosystem across entire globe. These fuels are limited in stock and the way we are burning them, is leaving them incompletely burnt. Thus it is a dual loss in terms of energy as well environmental damage. Automobile and electricity generation are the two important sectors using large quantity of fossil fuel. Out of the two, electricity generation is at the top! Coal-fired power plants are largest emitter of majority green house gases. Switching over to natural gas is one of the possible ways for coal fired power plants, but limited supply of natural gas does not meet the requirements in developing countries. In such case, coal remains favorite fuel for generation of power in most of the largely populated countries. Coal-fired power plants currently fuel 41% of global electricity [7-10].

Coal produces 44 % of the total electricity of United States and also contributes as single largest source of pollution [7]. 68% of the total electricity of India comes from coal fired power plants [8,10]. In US, up to 353 coal-fired generators in 31 states (out of a national total of 1,169) are ripe for retirement, equal to a total of 59 gigawatt of power generating capacity. Collectively they represent as much as 18 percent of the country's coal-generating capacity and approximately six percent of the nation's power [8, 9]. In 2011, approximately 42 percent of United State's electricity was produced by burning coal [7]. But by 2012, more than three-quarters of coal-fired power plants of U.S. have outlived their 30-year life span, with 17 percent being older than half a century. Most are inefficient, operating far below both their power generation potential and the most efficient coal units on the power grid. More or less similar is the situation in all other nations obtaining large share of the electricity from coal.

1.2. Problem with Combustion of Fossil Fuels

In all, the fossil fuel combustion is a serious cause for damage of environmental conditions. Limited and steeply depleting sources of fossil fuels along with pollution due to combustion are causing great worry for the future health scenario of all living beings.

The extremely serious environmental problems caused by the disproportionate combustion of fossil fuels are well documented by Prof. R. M. Santilli as follows [4]:

(1) The combustion of fossil fuels releases in our atmosphere about sixty millions metric of tons carbon dioxide CO_2 per day that are responsible for the first large environmental problem known as "global warning" or "green house effect" [10, 11]. Of these only 30 million metric tons are estimated to be recycled by our ever decreasing forests. This implies the release in our atmosphere of about thirty million metric tons of unrecycled green house gases per day, which release is the cause of the "global warming" now visible to everybody through climactic episodes such as floods, tornadoes, hurricanes, etc. of increasing catastrophic nature.

(2) The combustion of fossil fuels causes the permanent removal from our atmosphere of about 21 million metric tons of breathable oxygen per day, a second, extremely serious environmental problem known as "oxygen depletion." [4]. Even though not disclosed by political circles and newsmedia, the very admission of an "excess" CO_2 in our atmosphere (that is, CO_2 no longer recycled by plants) is an admission of oxygen depletion because the "O_2 in the excess CO_2" was originally breathable oxygen. Hence, by recalling the atomic weight of CO_2 and O_2, we have the value $32/44 \times 30 \times 10^6 = 21.8 \times 10^8$ tons of lost oxygen per day.

(3) The combustion of fossil fuels releases in our atmosphere about fifteen million metric tons of carcinogenic and toxic substances per day [4]. This third, equally serious environmental problem is euphemistically referred to by the news media as "atmospheric pollution", while in reality it refers to the primary source of the widespread increase of cancer in our societies. For instance, it has been established by various medical studies (generally suppressed by supporters of the oil cartel) that unless of genetic origin, breast cancer is due to the inhaling of carcinogenic substances in fossil fuels exhaust. These studies have gone so far as to establish that breast cells are very receptive to a particular carcinogenic substance in fossil fuel exhaust. After all, responsible citizens should remember and propagate (rather than myopically suppress) the fact that the U. S. Environmental Protection Agency has formally admitted that diesel exhaust is carcinogenic.

1.3. Problem with Combustion of Coal for Power

Combustion of coal is associated with emission of significant quantity of volatile matter and carbon monoxide. This is a loss in terms of energy and also causing pollution due to release of green house gases (CHs and CO).

As listed in Annexure A of Kyoto Protocol, the six major greenhouse gases are Carbon dioxide (CO_2); Methane (CH_4); Nitrous oxide (N_2O); Hydro fluorocarbons (HFCs); Per-fluorocarbons (PFCs); and Sulphur hexafluoride (SF_6) [9]. Out of these six gases, three most common gases in significantly large quantity are emitted from coal fired power plants. In burning fossil fuel like coal, about 60% of the energy in the original fuel is literally thrown through the fluke, and so is the related cost, due to the notoriously poor combustion [8]. Thus, majority of the combustible and

energetic content is lost unutilized in the process.

In addition, majority of the old power plants lack essential modern pollution controls, so they damage public health. The sulfur they emit causes acid rain. The mercury they release poisons waterways and fish and causes neurological damage in children [9]. The soot they emit creates smog that causes lung disease, premature death, and triggers asthma attacks [10,11]. Burning coal demands billions of gallons of cooling water from vulnerable rivers and lakes, and leaves behind vast quantities of toxic ash residuals, while coal mining causes extensive and lasting damage both to human health and the natural environment [12].

If we look into the exact process through a deeper eye, it is seen that the coal, when heated up, gets converted into the carbon monoxide at first. Then the CO is converted to CO_2. Also, at the same time, some of the carbon combines with hydrogen to form volatile HCs. CO and HCs being gases, tend to escape out of the combustion chamber where there is a need to convert maximum gaseous content into their final oxidation products to get maximum energy. However, in reality, only part of the carbon monoxide is converted into its dioxide and a very few of the volatile matter is burnt. Thus the emission is largely containing carbon monoxide and HCs which are a waste of energy on one side and a cause of pollution and global warming on the other side.

Conversion of CO to CO_2 in atmosphere causes thermal pollution due to exothermic nature of this conversion. Depletion of breathable oxygen in the ambient air adds to the gravity of the problem. 18 to 20 ton of coal is burnt for generation of 1 MW energy. Thus a 2000 MW power plant burns about 40000 ton of coal per day and release a huge volume of gases in atmosphere.

Initial generation of heat in furnace causes formation of volatile matter. Further conversion of coal leads to formation of CO. Both of these gases are vulnerable to move upward and thus cause release along with emission. Although, black carbon is not directly emitted from the coal fired power plant in routine condition, but the possibility cannot be ruled out for smaller coal burning units like potteries and brick production.

Most of the Indian coal-fired power plants emit about 1 ton CO_2 per MW. Observed range of data obtained from Central Electricity Authority (CEA) for the year 2012-13 for all coal-fired power plants show a range of 0.9 to 2.1 ton specific emission of CO_2. Despite all the evils associated with combustion of coal, it can't be discarded altogether because of the issues of economy associated with it. Evaluation of the economic competitiveness of coal generators and comparison of the cost of electricity from individual coal-fired electricity generating units with the cost of electricity generated from an average natural gas power plant shows that cleaning the combustion of coal in coal fired power plants will be costlier. Specifically, if a coal-fired generator— after installing any needed pollution controls—would be more expensive to operate than a typical cleaner-burning and more efficient natural gas combined-cycle [6].

A graphical representation of the SPM emitted from a power plant over a period of 4 years show the exact scenario of emission.

Figure 1 and Figure 2 depict 20 readings from a period of about 4 years showing the emission exceeding prescribed limits on many occasions. Both of these units are about 30 years old.

Despite numerous problems associated with combustion of coal, still we cannot think of scrapping all coal fired power plants, because coal is the cheapest of all other energy sources. Being solid, it is easy in handling and operations as compared to natural gas which is cleaner to coal. Further, location of coal reservoirs adds a lot to the geographical restrictions of the economy of coal fired power plants. In India, there are large coal reservoirs available in central Indian states. These coal reservoirs invite coal fired power plants to these regions and no other alternate fuel can meet up this requirement in any other part of the country.

Table 1. Year wise electricity generation and CO_2 emission from coal fired power plants in India [15].

Sr. No.	Year Factor	2008-09	2009-10	2010-11	2011-12	2012-13
1	Net Generation (GWh)	467298.3811	495,343	516,877	562,991	633,793
2	Absolute Emissions (tCO_2)	510877863.2	534154395.8	554,435,897	596,550,869	666,699,576
3	Specific Emissions (t CO_2/MWh)	1.18	1.17	1.19	1.14	1.14

Thus it is more positive approach to think of a cleaner additive to coal than going for alternate fuel. Additive which is not restricted to any geographical area will be more helpful for economy and cleaner process of coal fired power plants.

It is known in chemistry that hydrogen is the best additive to improve combustion, with consequential improvement of the environmental quality of the exhaust. In fact, hydrogen has the biggest flame temperature and speed among all known fuels. Consequently, the injection of hydrogen as an additive in the flame of fossil fuels burns the uncombusted component of the exhausts in a way proportional to the used percentage of hydrogen. But, use of hydrogen as additive has several limitations and restrictions. Limitations are mainly due to cost and difficulty in handling.

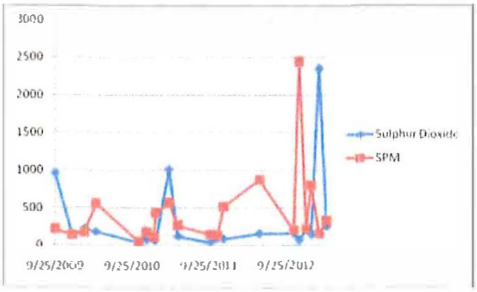

Figure 1. *Graphical representation of emission of SO_2 and SPM from a 250 MW unit-1 of Chandrapur Super Thermal Power Plant. The X-axis showing dates of collection and Y-axis represents emission in mg/Nm^3 (Year of commissioning-1983).*

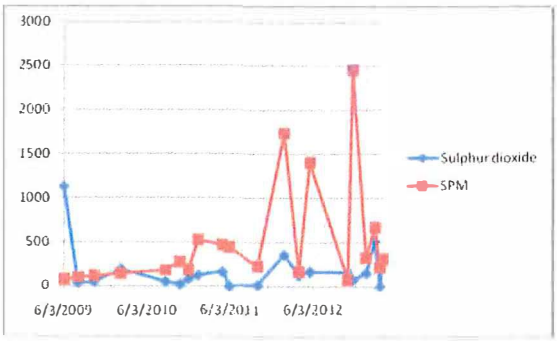

Figure 2. *Graphical representation of emission of SO_2 and SPM from a 250 MW unit-2 of Chandrapur Super Thermal Power Plant. The X-axis showing dates of collection and Y-axis represents emission in mg/Nm^3 (Year of commissioning-1984).*

2. Magnegas - the Best Option for Hydrogen as Additive at Present

Looking at the energy requirement of world and our dependency on coal for power, it seems to have no direct alternative any next morning. Continuing with the same scenario of combustion of this dirty fuel will create a miserable environmental situation in next one or two decades only.

Coal is and will be the major and cheap source of power generation in many countries. India, a developing country, and a 1/6th population of globe, is largely dependent on coal as a major source of energy for generation of electricity. Out of 1,50,000MW of total energy capacity, approximately 78,000 MW comes from coal fired thermal power plants. These power plants are mainly situated in places where there is ample coal under the earth surface. Burning coal itself is a dirty process with heavy emission of green house gases. Further coal mining has its own adverse effect on land and water in the area which ultimately ruins the agriculture and causes rehabilitation of people. One of the serious problems in burning coal for electricity is the incomplete combustion that results in loss of at least 30% fuel as uncombusted hydrocarbon, hence lesser thermal efficiency and increased cleaning cost of exhaust system and pollution control. As the data of emission shows significant quantity of carbon monoxide emitting out, it is a multi directional loss to the environment. Excavation of fossil fuel itself is against environmental balance and incomplete combustion causes decreased combustion efficiency along with heavy release of poisonous gas, carbon monoxide. Further conversion of carbon monoxide to oxide in the atmosphere is causing thermal pollution. Hence the priority need is clean combustion of the existing fossil fuel and utilizing the maximum available carbon of the fuel. This can be done by injecting magnegas [3, 4, 13] along with the conventional fossil fuel.

The property of magnecules [3, 4] to undergo magnecular combustion [1-4] with high energy output is due to weak magnecular bond. This is exploited for the industrial development of new clean fuels such as magnegas. MagneGas is a good additive for the cleaning of fossil fuel exhaust because:

1. When produced from the recycling of water-base liquid wastes, magnegas contains about 65% hydrogen, thus qualifying as an effective additive to improve fossil fuel combustion;
2. The remaining components of magnegas are rich in oxygen, thus helping to alleviate the large oxygen depletion caused by fossil fuel combustion and

The cost of magnegas is competitive over that of fossil fuel, particularly when produced by the electric power plants, because of the reduced cost of electricity and the possibility of producing magnegas from the recycling of city sewage, with a consequential income that covers most of the operating costs of PlasmaArcFlow Recyclers.

Therefore, it is a high time to come up with a realistic solution to clean the combustion of coal in power and other small coal using sectors like brick making to cement and metallurgical industries.

The only viable option is to add a suitable additive for cleaner co combustion process. Pure hydrogen gas can meet up the thermodynamic requirements but it is not viable due to safety and economical reasons. Therefore, magnegas can prove to be a better additive for co combustion with coal.

Magnegas can be synthesized on site by using any source of water and applying the famous technique pioneered by Prof. R. M. Santilli.

The magnegas can be used along with coal as additive for effective co combustion. Flame temperature of magnegas is upto 10500° F which helps in combustion of all the carbon and traces of nitrogen and sulphur to their final oxidation products in the combustion chamber itself. This ensures complete combustion of carbon to generate maximum energy. It also helps to burn all the volatile matter formed due to internal reaction of carbon and hydrogen from coal. Localized combustion of all combustible elements from coal is the main objective which can be best fulfilled by using magnegas as an additive for co combustion with coal. Some experiments are already conducted by Magnegas Corp. in collaboration with various partners across Europe, U.S. and Australia and they have reported a positive outcome of the experimental trials [3].

Magnegas achieves a flame temperature of about 3800°C. This high temperature helps in combustion of all combustible matter, thus enhancing the energy output of process. Further, it is also observed in the earlier studies that co combustion of magengas has reduced the carbon dioxide emission by about 30 to 40% and reduced emission of carbon monoxide [3].

Below mentioned is a tabulated interpretation of the results of combustion of coal in comparison with co-combustion of magnegas and coal.

Table 2. *Comparison of flue gases and stack temp in combustion of coal and coal+magnegas, Courtesy:www.magengas.com/reports/cocombustion [3].*

Factor	Coal	Coal+Magnegas
Oxygen	11%	13%
Carbon Dioxide(CO2)	15%	9%(40%)
Carbon monoxide (CO)	58ppm	28ppm (52%)
Nitrous oxide (NOx)	160ppm	46ppm (71%)
Stack temp	37100 C 700 F	~29900 C 5400 F

As magnehydrogen would be more energetic than magnegas due to resemblance of MH with hydrogen, magnehydrogen can be used in place of magnegas as a refinement to the co combustion experiment. The first independent experimental verification of the new species of Santilli MagneHydrogen was achieved in October 11, 2011, by D. Day [14-16] of the Eprida Laboratory, 3020 Canton Road Suite 104, Marietta, GA, via the use of a VSA station for the separation of MH from MG, the use of a GC-TCD for the measurement of the percentage of hydrogen in the separated gas, and the use of conventional methods for the measurement of molecular weight. In this way, Day achieved a species of MH with about 97.5% pure Hydrogen, while having 3.89 times the specific weight of H_2, and a consequential energy content of 1167 BTU/scf.

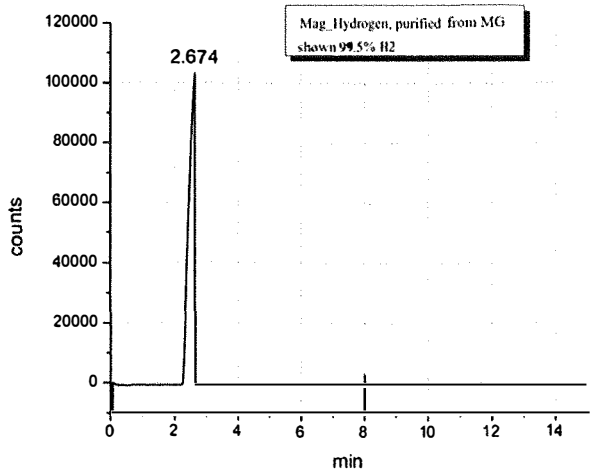

Figure 3. *GC-TCD scan of MagneHydrogen showing no appreciable difference of MH with pure hydrogen [13].*

As reported by Prof. Yang [3] from an industrial viewpoint, it is sufficient to achieve a species of MH with at least 3.3 the specific weight of H_2 to have the same energy content of 1000 BTU/scf of Natural Gas (NG). In fact, under said conditions, MH would avoid the current needs to liquefy Hydrogen in order to achieve a sufficient range, since MH can be compressed like NG. Additionally, the magnecular structure of MH avoids the traditional seepage of Hydrogen through the walls [14, 15], thus allowing long term storage that is currently prohibited by molecular Hydrogen due to current environmental laws.

Magnehydrogen is a cluster species and about 7.4 times heavier than hydrogen. These anomaly to conventional gaseous molecules help in reducing the leakage of these novel species from pipeline. Permeability issues needs to be answered while passing hydrogen through pipeline but magnehydrogen has a advantage due to its polymeric nature. Moreover, energetically it is equivalent to hydrogen and generates a high flame temperature which can prove key to cleaner combustion of coal.

Thus magnehydrogen is the closest choice to replace hydrogen as an additive and in some physical properties discussed above, it is even better to hydrogen.

Indian coal is known for its high ash content. Coal-fired power plants generate a huge quantity of ash which is an environmental problem. As per the estimates Magnegas Corporation based on studies conducted at their place, there is an elimination of particulate matter when using magnegas with coal as additive. This estimate is a source of hope for cleaning the combustion of coal-fired power plants in many states of India. Central Indian coal mines deliver a coal with ash as high as 38 to 40% and the fly ash emitting from the coal-fired power plants is a cause of land, soil and air pollution in the area.

The high temperature generated due to addition of magnegas can facilitate chemical gasification reaction occur easily in a reactor. Mixing of pulverized coal can enhance the ability of process. Even biomass and combustible garbage can be burnt with this additive. The high temperature attained inside the combustion place must be converting the ash into glass slag kind of material.

A power plant of 2340MW capacity using high ash central Indian coal consumes 40 to 45 thousand ton coal per day. The average ash content in this coal is 35%. Estimated ash per annum is ~5100000 ton/yr. Ash generation per annum as reported by this power plant ~3600000 ton/yr. Thus ~1500000 ton/yr is the fly ash generation. Assuming 98.5% efficiency of ESP, ash liberated through stack in the atmosphere is 22500ton/yr. This ash deposits in the surrounding area and pollute the ecological factors like lake, small water streams and farmland.

With addition of the magnegas, the conversion of ash into fusible mass will minimize the pollution due to fly ash.

As carbon will be converted in totality to its dioxide, the total obtainable energy will be increased. This, in fact, is the goal of overall process which will increase the energy efficiency of the coal combustion process.

As CO is highly reactive and it is difficult to measure the enthalpy of formation of CO practically by calorimeter, a simple calculative approach is described below [16].

$$C\ (graphite) + \tfrac{1}{2}\ O_2 \rightarrow CO\ (g) \quad \Delta H_1 = ?$$

$$CO\ (g) + \tfrac{1}{2}\ O_2 \rightarrow CO_2\ (g) \quad \Delta H_2 = -283.0\ kJ/mol$$

$$C\ (graphite) + O_2 \rightarrow CO_2\ (g) \quad \Delta H_3 = -393.5\ kJ/mol$$

Hess's law gives:

$$\Delta H_1 = \Delta H_3 - \Delta H_2 = -393.5 + 283.0 = -110.5\ kJ/mol$$

For every elemental carbon released in stack as carbon monoxide, we are gaining 110.5kJ/mol and loosing 283.0 kJ/mol of energy. Thus, loss amounts to 256% per mol.

This loss would be eliminated with the addition of magnegas which helps in complete combustion of carbon to CO_2.

Many metallurgical processes use heat liberated in operations to generate power. Increased waste heat due to magnegas as additive can be used effectively. An example of the availability of waste heat is given as follows [17, 18]:

In a furnace, the exhaust gases are leaving the furnace at 900°C at the rate of 2100 m^3/hour. The total heat recoverable at 180°C final exhaust can be calculated as

$$Q = V \times \rho \times C_p \times \Delta T$$

Q is the heat content in kcal

V is the flow rate of the substance in m^3/hr

ρ is density of the flue gas in kg/m^3

C_p is the specific heat of the substance in kcal/kg °C

Heat available (Q) = 2100 x 1.19 x 0.24 x (900-180) = 4,31,827 kcal/hr

Not only the metallurgical operations but the small power plants also utilise the waste heat thrown out of the stack. The term "combined cycle" describes the combination of two thermodynamic cycles, with the gas turbine (Brayton cycle) burning natural or synthetic gas from coal/residuals/oil, and its hot exhaust gas powering a small steam power plant (Rankine cycle). Combined Cycle Power Plants (CCPPs) can achieve a thermal efficiency higher than 60% today, compared to single cycle gas power plants which are limited to efficiencies of around 35 to 42% [19]. In place of natural gas, coal fired power plant using pulverized coal can also work on two cycles to increase electricity output. Also, when the total heat content in waste heat recovery is dependent on difference in temperature (ΔT), 8 to 9 times increase due to addition of magnegas will certainly increase the efficiency of waster recovery process.

In developing countries like India, there is a great deal of disparity in urban and rural life quality. In India, 42 percent of rural households use kerosene for lighting. By contrast, about 93 percent of urban households use electricity for lighting and only about 6 percent use kerosene. On average, a rural household receives six hours of electricity supply from the grid during the off-peak period (usually afternoon and night). The cost of lighting a rural household includes the cost of grid supply and the cost of kerosene. The loss of electricity in grid is also significant to be noted [20].

Distributed power generation in small capacity can serve the purpose of extending availability of electricity to rural households. This will also save losses in distribution and local employment generation is possible for rural population.

There are a few small capacity power plants established on cooperative basis in few states of India. The village community uses locally available coal, biomass and other combustible waste to generate power of few kW which is sufficient to meet their electricity requirement. These power plants are based on a crude, steam engine principle, and can't be equipped with modern pollution control facility. The overall economy does not permit use of costly ESPs.

If such small power plants are supplied with magnegas, which can be generated on site, it will help to increase the combustion efficiency of original fuel. It will also improve the quality of emission, thereby, supporting clean atmosphere.

The old age power plants of less capacity can also be run on coal + magnegas hybrid fuel to improve the efficiency of combustion. As these power plants are not suitable to be equipped with modern pollution control facility, they are emitting beyond permissible limit. Scale up of the present experimental results obtained by Magnegas Corporation can give a good solution to improve the economy of these power plants.

A bottom up model for combustion of coal with magnegas was proposed by the authors and it is still under consideration for experiments. This model study will mainly focus on Indian coal and thermal power units in India. The outcome of study can help to devise a scaled up process of co combustion of magengas and coal which can serve the purpose of all coal-firing industries in India. Apart from larger coal based power plants, there are many captive to micro power plants operated on co operative basis in villages. These power plants meet up the energy requirement of those villages. Due to lack of any sophisticated emission controlling technologies, these power plants emit a high amount of poisonous gases like CO_2, CO and NOx. Due to low stack height of these steam engine based power units, the workers are exposed to poor health and environmental conditions. The clean up aimed with co combustion of coal with magengas can provide better environmental conditions to workers in small coal fired units.

3. Conclusion

In the present paper we discussed the possibilities of using magnegas as an additive to coal in order to enhance the combustion efficiency of coal. To justify the need of novel fuel like magnegas, initially we have discussed the present scenario of energy sources and their fulfilment, problems associated with combustion of fossil fuels especially coal and their effects leading to global warming and oxygen depletion and need of mankind to find alternate fuels due to devastating effects of burning fossil fuels. Then with the short introduction to the new fuel magnegas which posses the magnetically bonded molecules and magnecules and hence justified the lower energy for its combustion called as "magnecular combustion" when used independently as well as mixed with coal as an additive for its complete and clean combustion. Hence its has been emphasized in this paper that the priority need is clean combustion of the existing fossil fuel and utilizing the maximum available carbon of the fuel which can be done by injecting magnegas along with the conventional fossil fuel.

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Experimental Confirmation of the Synthesis of Neutrons and Neutroids from a Hydrogen Gas

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Abstract: In this paper, we outline preceding mathematical, theoretical and experimental studies on the synthesis of neutrons from a hydrogen gas, and present additional systematic experimental confirmations via the use of three different neutron detectors. We also present experimental evidence of the existence of an intermediate bound state of a proton and an electron with spin zero known as the neutroid. A number of industrial applications currently under development by Thunder Energies Corporation are pointed out.

Keywords: Neutrons, Neutroids, Pseudoprotons

1. The Synthesis of the Neutron

H. Rutherford [1] suggested in 1920 that the hydrogen atom in the core of stars is "compressed" into a new neutral particle which he called the *neutron*

$$p^+ + e^- \rightarrow n. \tag{1.1}$$

The existence of the neutron was experimentally confirmed by J. Chadwick in 1932 [2]. E. Fermi [3] suggested the addition in the neutron synthesis of a massless and chargeless particle which he called *neutrino* (meaning "little neutron" in Italian), in order to conserve the angular momentum

$$p^+ + e^- \rightarrow n + \nu, \tag{1.2}$$

with spontaneous decay half-life (when the neutron is isolated) of 10 minutes,

$$n \rightarrow p^+ + e^- + \bar{\nu}, \tag{1.3}$$

where $\bar{\nu}$ is the *antineutrino*.

R. Norman and J. Dunning-Davies [4] have conducted a historical search and scientific appraisal on the neutron synthesis, by identifying an original letter in which W. Pauli suggested the addition of a new particle with spin ½ later named as the "neutrino" by Fermi, at the suggestion of Amaldi. Additionally, Ref. [4] outlines three experimental, tests on the synthesis of neutrons from a hydrogen gas, the first by Ernest J. Sternglass in 1951 (including his interesting letter to A. Einstein), the second tests were done by E. Trounson in 1952, and the third tests were done by Don Carlo Borghi and his associates in the 1960's.

None of the above tests were conducted with neutron detectors, evidently because they were not available at the time and as will become clear the reported synthesis of neutrons was in fact indirect evidence, those neutrons being secondarily created and detected via nuclear transmutations

within natural elements placed around the reactors. None of the initial three tests were published because they all implied that the neutron is in fact a conventional quantum mechanical bound state of a proton and an electron, a theoretical stance which is known to be inconsistent with accepted theory.

Beginning with the late 1970's, R. M. Santilli has conducted systematic, mathematical, theoretical, and experimental studies on the synthesis of neutrons from a hydrogen gas [6-30]. As a result of these studies, the U. S. publicly traded company *Thunder Energies Corporation* (thunder-energies.com) is now in production and sale of a *Thermal Neutron Source* (TEC-TNS) providing the controlled production of low energy neutrons from a commercially available hydrogen gas.

To briefly outline the latter developments, Santilli first identified the mathematical methods needed for the representation of the neutron synthesis known as *isomathematics* [5-9] (see also independent studies [10-15]). He then constructed the corresponding physical methods known as *isomechanics* as a branch of *hadronic mechanics* and showed its validity for the representation of the class of systems under the conditions considered [14-16]; then reached the representation of "all" characteristics of the neutron in its synthesis from the hydrogen at the non-relativistic [17, 18] and relativistic [19, 20] levels; and then conducting systematic experiments on the synthesis of neutrons from a hydrogen gas by achieving the first known direct detection of synthesized neutrons via neutron detectors [21-26]. (see also reviews [27-30], international patent pending).

The main difficulty addressed in these studies stemmed from the fact that *the rest energy of the neutron is "bigger" than the sum of the rest energies of the proton and the electron,*

$$E_p = 938.272 \text{ MeV}, \; E_e = 0.511 \text{ MeV}, \; E_n = 939.565 \text{ MeV}, \quad (1.4)$$

resulting in the *neutron mass excess*

$$E_n - (E_p + E_e) = 0.782 \text{ MeV} > 0. \quad (1.5)$$

Under the above conditions, the Schrödinger equation remains exactly valid for an electron orbiting in a vacuum at large distances around the proton (exterior dynamical problem), but it does not yield physically consistent results for the same electron when "compressed" within the hyper dense medium inside the proton (interior dynamical problem) due to numerous reasons, such as: quantum mechanics requires the necessary approximation of the proton as a massive point, in which case the electron cannot be "compressed" within its interior; the neutron synthesis would require a "positive binding energy" with ensuing "mass excess;" there is the emergence of non-linear, non-local and contact non-Hamiltonian interactions beyond any hope of treatment with the Schrödinger equation; and other reasons.

It should be indicated for the non-initiated reader that the Schrödinger equation can certainly represent particles with arbitrary *positive kinetic energies.* The indicated inconsistencies solely occur for a *positive binding energy* of a bound state of particles, since in this case the indicial equation of the Schrödinger equation fails to provide physically consistent solutions [6].

Similarly, special relativity at large, and Dirac's equation in particular, remain exactly valid for the structure of the hydrogen atom, although they become inapplicable (and certainly not violated) for the " " compression" of the same atom into the neutron, not only because of the need to achieve an " " excess mass," but also due to the inevitable emergence of non-linear, non-local and non-Hamiltonian interactions beyond any hope of consistent representation by the *mathematics* underlying relativistic quantum mechanics.

Due to the above insufficiencies, Santilli had no other choice than that of first constructing the foundations of new mathematics [5-9], consisting of a broadening of 20th century applied mathematics for the representation of *extended* particles in interior conditions. The ensuing hadronic mechanics is essentially a non-unitary "completion" of quantum mechanics much along the celebrated argument by Einstein, Podolsk and Rosen. The image of the Schrödinger equation under said transformation shows a strongly attractive Hulten potential, plus a new renormalization of the intrinsic. characteristics of particles in interior conditions achieving the crucial representation of the "mass excess" equation (1.5). The representation of all characteristics of the neutron is then consequential.

It should be finally kept in mind that isomathematics and isomechanics, as well as hadronic mechanics at large, are solely applicable under conditions which imply the overlapping of wavepackets of particles or of their charge distributions (about 1 fm), while recovering 20th century mathematics and physics uniquely and unambiguously at sufficiently large mutual distance of particles.

On experimental grounds, Refs. [21-27] have shown that the most effective way to synthesize neutrons from a hydrogen gas is the use of a gaseously submerged rapid DC discharge between a pair of tungsten electrodes with at least 3kV and 1J or, alternatively, with at least 3kW power (data hereon referred to as *threshold values*), below which neutrons are not systematically synthesized due to insufficient arc energy needed to supply the missing 0.782MeV. By recalling that, when inspected in an oscilloscope in the millisecond range, DC arcs are continuously disconnected and reconnected, properly selected steady DC arcs can also synthesize neutrons from a hydrogen gas, although in a less efficient way. Hereon, said threshold values of 3kV and 1J, Refs. [21-27] reported the lack of directly detectable neutrons, but confirmed the existence of delayed nuclear transmutations as reported in the initial tests [4] (international patent pending).

In this paper, we report recent systematic *experimental* confirmations of *industrial* synthesis of neutrons from a commercially available hydrogen gas which can be controlled via the control of the arc voltage, arc energy, pressure of the hydrogen gas, electrode gap, and other engineering means.

The non-initiated reader should know that the appraisal of our experimental results via the use of conventional doctrines is afflicted by a number of insidious inconsistencies that generally remain undetected by non-experts in the field, thus suggesting a knowledge of isomathematics and isomechanics for a technical understanding of the field.

2. The Synthesis of Neutroids

Isomathematics and isomechanics predict the existence of an intermediate state prior to the neutron synthesis called by the name *neutroid* [4, 13-15], and indicated with the symbol $\tilde{n}$., which is itself the singlet coupling of a proton and an electron at 10^{-13} cm caused by the strong Coulomb attraction (which is of the order of 10^{22} N at the indicated mutual distance) as well as by the strong attraction caused by the deep mutual penetration of the wavepackets [6,17-20]

$$p^+_\uparrow + e^-_\downarrow \rightarrow \tilde{n}. \tag{2.1}$$

The neutroid is a new particle with; null charge; null spin; null magnetic moment (in first approximation); estimated mean life of 9s; rest energy (estimated via the sole use of conventional electromagnetic interactions)

$$E_{\tilde{n}} = m_p + m_e - BE = 938.2721 + 0.511 - 0.009 \text{ MeV} =$$
$$= 938.7741 \text{ MeV} = 1.008091 \text{ amu}; \tag{2.2}$$

charge radius $R_{\tilde{n}}$ and mean life $\tau_{\tilde{n}}$

$$R_{\tilde{n}} \cong 10^{-13}\text{cm}, \quad \tau_{\tilde{n}} \approx 9\text{s}; \tag{2.3}$$

"completion" into a neutron (under the availability of the needed 0.782 MeV energy)

$$\tilde{n} \rightarrow n + \nu, \tag{2,4}$$

and decay (in the absence of said missing energy)

$$\tilde{n} \rightarrow p^+ + e^-. \tag{2.5}$$

It appears that all original tests by E. J. Sternglass, E. Trounson, Don Borghi, as well as the initial tests by R. M. Santilli below the indicated threshold values of voltage and energy [4], synthesized neutroids, rather than neutrons due to the energy of the DC arc being insufficient to supply the missing 0.782 MeV and other reasons identified later on. The above view implies that the nuclear transmutations detected by the original tests were caused by the absorption of neutroids by natural elements, their consequential instability and subsequent delayed transmutations into new elements (international patent pending).

Note that, even though neutrons and the neutroids are both neutral and have essentially the same size, *neutroids cannot be detected by available neutron detectors* because they have different spins, different rest energies, different magnetic moments, and different mean lives. Therefore, neutroids can be solely detected via the analysis of the transmutations, such as via the delayed detection of neutrons, electrons, gammas and other radiations, as detected in Refs. [4,21-26].

For a better understanding of the experiments reported in this paper, it is necessary to have a conceptual, semi-classical illustration of the various processes occurring in the synthesis of neutroids and neutrons in mind, while understanding that a technical grasp of the matter is solely possible assuming a detailed knowledge of isomathematics, isomechanics and their quantitative treatment of the synthesis.

Figure 1 depicts well known effects caused by a DC arc between tungsten electrodes submerged in a hydrogen gas, namely: the separation of the hydrogen molecule into H-atoms; the ionization of the H-atoms; the consequential creation of a plasma composed by protons and electrons; and their alignment along the tangent to a local magnetic line with opposite charges, opposing magnetic polarities and opposing spins.

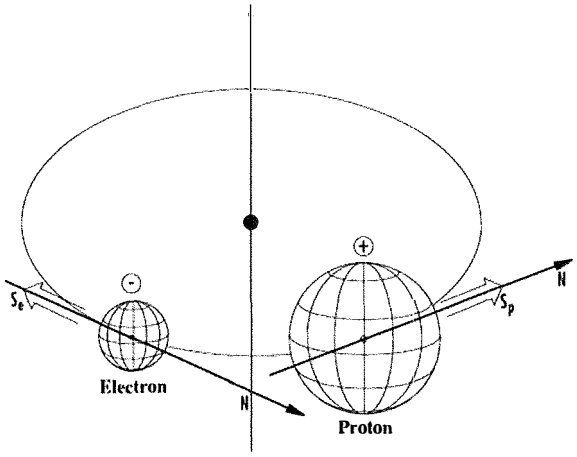


Figure 1.

Figure 2 depicts the realization of Rutherford's "compression" [1] of protons and electrons, one against the other, caused by the disconnection of a rapid DC discharge, its activation essentially creating the indicated plasma of protons and electrons. Note that, when inspected in an oscilloscope at the millisecond scale, DC discharges with constant voltage (CV) and constant current (CC) are continuously disconnected and reconnected.

Therefore, CV, CC and DC arcs submerged in hydrogen with voltage and energy values beyond their threshold do synthesize neutrons, although in a way less effective than that of a rapid DC discharge.

Figure 3 provides a conceptual illustration of the neutroid as a single bound state of a proton and an electron under partial penetration of their wavepackets, with null total charge, null spin and null magnetic moment (the latter in first approximation) since the value of the angular momentum L_e of the electron is null in the ground state. Figure 4 illustrates a novel analogy revealed by isomathematics and isomechanics, namely, the similarity between the coupling of particles with the coupling of gears [6], both requiring antiparallel angular momenta for stability due to their extended character.

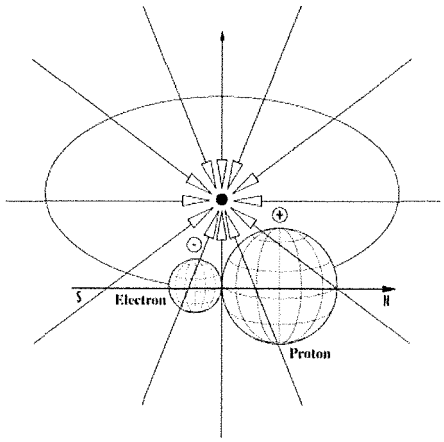


Figure 2.

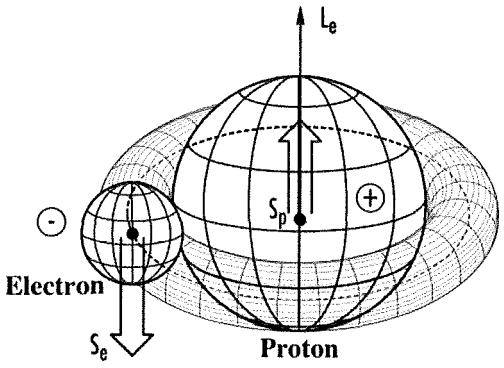


Figure 3.

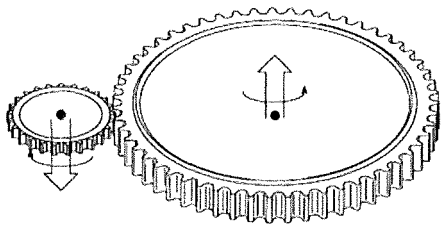


Figure 4.

It should be indicated that the structure of neutroids according to Figure 3 is impossible within conventional quantum mechanics since the sole bound states between a proton and an electron are those of the H-atoms. This is due to the point-like abstraction of the proton and the electron which is inherent in quantum mechanics without any consideration of the non-linear, non-local and non-Hamiltonian effects due to wave overlapping. When the proton is represented with its actual dimension as occurring in physical reality, said non-linear, non-local and non-Hamiltonian effects are inevitable and play a crucial role in the augmentation of basic scientific advances and knowledge founded over the past century.

To achieve a quantitative understanding of the predicted 9 s mean life of the neutroid, one should note that: the neutroid is an intermediate state prior to the full neutron synthesis; the neutron has about a 10 "minutes" mean life; and that isomathematics and isomechanics require the new isorenormalization of all characteristics of particles beginning with their rest mass, as known since 1978 (see Section 5 of Ref. [6]). Note that the neutroid has *no excited states* because following any such excitations, mutual wave overlapping of particles ceases to be appreciable, quantum mechanics returns to be exactly valid and the sole possible bound states of a proton and an electron are those of the H-atom.

Figure 5 depicts conceivable "axial couplings" of one proton and one electron, at times called *neutroids of the second (third) kind when the total angular momentum is* 1 (0). These couplings are ignored in this paper because they are predicted to have such short mean lives as to prevent industrial applications. This occurrence illustrates again the differences between quantum mechanics and isomechanics because in the former abstract particles are conceived as dimensionless points, in which case both singlet and triplet couplings are possible. By contract, the latter mechanics represents particles with their actual size, in which case a triplet coupling would be the same as coupling gears in Figure 5 with parallel spins [5].

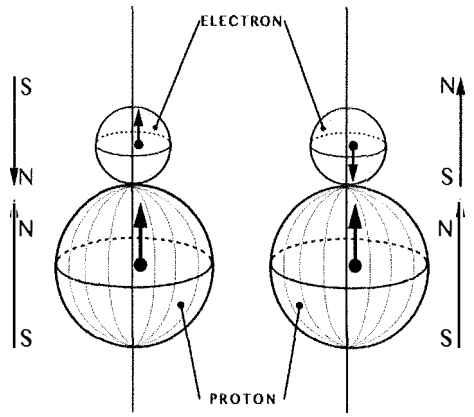


Figure 5.

Figure 6 depicts the synthesis of the neutron from a proton and an electron achieved via rapid DC discharges through tungsten electrodes submerged in a commercially available hydrogen gas with values of the voltage sufficiently bigger than 3kV and energy sufficiently bigger than 1J [5-30]. Figure 7 illustrates that the conceptual rendering via the coupling of gears requires, this time, that the smaller gear has to rotate "inside" the bigger one, besides having antiparallel angular momenta. By remembering that the electron is about 2000 times lighter than the proton, the synthesis of the neutron is essentially based on Rutherford's total "compression" of an electron within the hyper-dense medium inside the proton, in which case the electron is constrained by said medium to rotate with an angular momentum equal to the proton spin.

The total angular momentum of the electron is null and the spin of the neutron is equal to that of the proton.

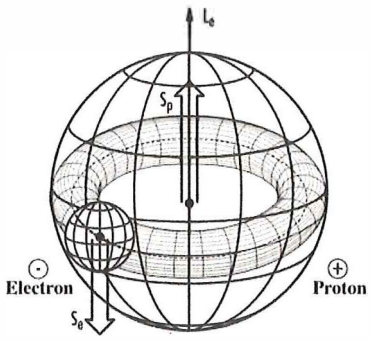


Figure 6.

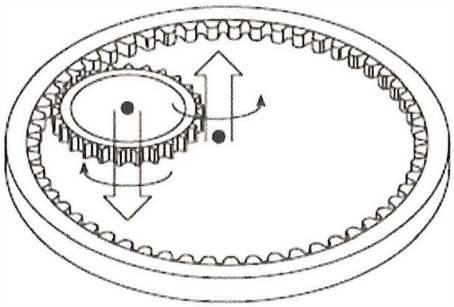


Figure 7.

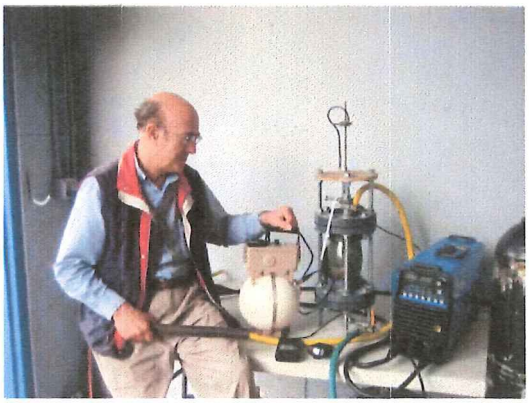
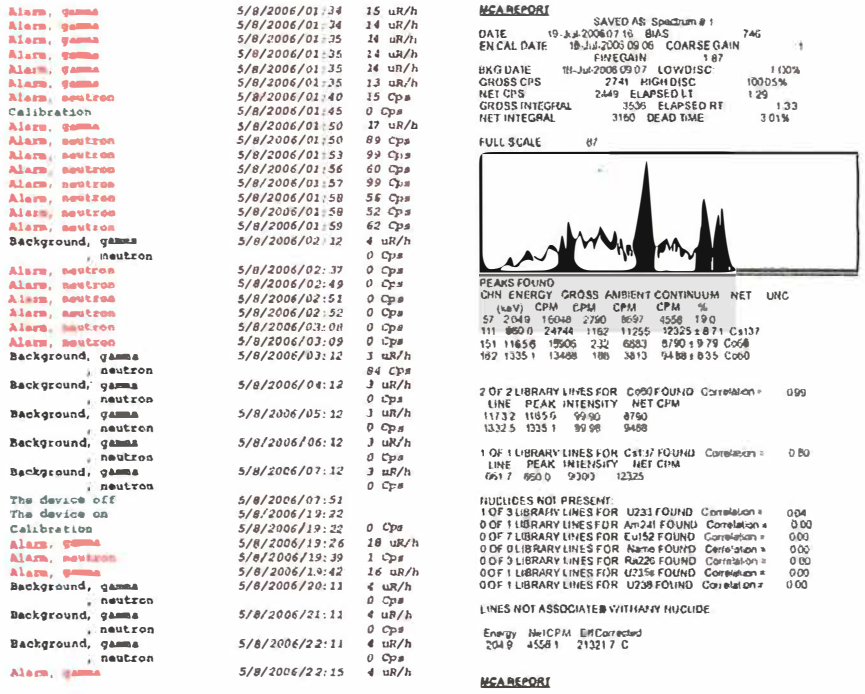


Figure 8.

Note that half-values of the angular momentum are prohibited in a vacuum, but are fully admitted within physical media due the isotopies of Lie's theory [10]. In any case, values of the angular momentum different than 1/2 would imply that the electron moves inside the proton in a direction against its hyper-dense medium, which is a physical impossibility. Note also that the above synthesis is impossible for quantum mechanics, again because the proton is dimensionless. Note finally that, the above view has permitted a numerically exact and time invariant representation of "all" characteristics of the neutron, including a representation of its anomalous magnetic moment achieved thanks to the new contribution of the rotation of the electron inside the proton different than that conceived during the time of Pauli and Fermi.



Alarm, gamma	5/8/2006/01:34	15 uR/h
Alarm, gamma	5/8/2006/01:34	14 uR/h
Alarm, gamma	5/8/2006/01:35	14 uR/h
Alarm, gamma	5/8/2006/01:35	14 uR/h
Alarm, gamma	5/8/2006/01:35	14 uR/h
Alarm, gamma	5/8/2006/01:35	13 uR/h
Alarm, neutron	5/8/2006/01:40	15 Cps
Calibration	5/8/2006/01:45	0 Cps
Alarm, gamma	5/8/2006/01:50	17 uR/h
Alarm, neutron	5/8/2006/01:50	89 Cps
Alarm, neutron	5/8/2006/01:53	99 Cps
Alarm, neutron	5/8/2006/01:56	60 Cps
Alarm, neutron	5/8/2006/01:57	99 Cps
Alarm, neutron	5/8/2006/01:58	56 Cps
Alarm, neutron	5/8/2006/01:58	52 Cps
Alarm, neutron	5/8/2006/01:59	62 Cps
Background, gamma	5/8/2006/02:12	4 uR/h
, neutron		0 Cps
Alarm, neutron	5/8/2006/02:37	0 Cps
Alarm, neutron	5/8/2006/02:49	0 Cps
Alarm, neutron	5/8/2006/02:51	0 Cps
Alarm, neutron	5/8/2006/02:52	0 Cps
Alarm, neutron	5/8/2006/03:08	0 Cps
Alarm, neutron	5/8/2006/03:09	0 Cps
Background, gamma	5/8/2006/03:12	3 uR/h
, neutron		84 Cps
Background, gamma	5/8/2006/04:12	3 uR/h
, neutron		0 Cps
Background, gamma	5/8/2006/05:12	3 uR/h
, neutron		0 Cps
Background, gamma	5/8/2006/06:12	3 uR/h
, neutron		0 Cps
Background, gamma	5/8/2006/07:12	3 uR/h
, neutron		0 Cps
The device off	5/8/2006/07:51	
The device on	5/8/2006/19:22	
Calibration	5/8/2006/19:22	0 Cps
Alarm, gamma	5/8/2006/19:26	18 uR/h
Alarm, neutron	5/8/2006/19:39	1 Cps
Alarm, gamma	5/8/2006/19:42	16 uR/h
Background, gamma	5/8/2006/20:11	4 uR/h
, neutron		0 Cps
Background, gamma	5/8/2006/21:11	4 uR/h
, neutron		0 Cps
Background, gamma	5/8/2006/22:11	4 uR/h
, neutron		0 Cps
Alarm, gamma	5/8/2006/22:15	4 uR/h

MCA REPORT

SAVED AS: Spectrum # 1

DATE 19-Jul-2006 07:16 BIAS 746
EN CAL DATE 18-Jul-2006 09:06 COARSE GAIN 1
FINE GAIN 1.87
BKG DATE 18-Jul-2006 09:07 LOW DISC 1.00%
GROSS CPS 2741 HIGH DISC 100.05%
NET CPS 2449 ELAPSED LT 1.29
GROSS INTEGRAL 3536 ELAPSED RT 1.33
NET INTEGRAL 3160 DEAD TIME 3.01%

FULL SCALE 87

PEAKS FOUND

CHN	ENERGY (keV)	GROSS CPM	AMBIENT CPM	CONTINUUM CPM	NET CPM	UNC %	
57	204.9	16048	2790	8697	4558	1.9 0	
111	350.0	24744	1162	11255	12325 ± 8.71		Cs137
151	11656	15506	232	6883	8790 ± 9.79		Co60
182	1335.1	13488	186	3813	9488 ± 8.35		Co60

2 OF 2 LIBRARY LINES FOR Co60 FOUND Correlation = 0.99

LINE	PEAK	INTENSITY	NET CPM
1173.2	1165.6	99.90	8790
1332.5	1335.1	99.98	9488

1 OF 1 LIBRARY LINES FOR Cs137 FOUND Correlation = 0.80

LINE	PEAK	INTENSITY	NET CPM
661.7	650.0	9000	12325

NUCLIDES NOT PRESENT:
1 OF 3 LIBRARY LINES FOR U231 FOUND Correlation = 0.04
0 OF 1 LIBRARY LINES FOR Am241 FOUND Correlation = 0.00
0 OF 7 LIBRARY LINES FOR Eu152 FOUND Correlation = 0.00
0 OF 0 LIBRARY LINES FOR Name FOUND Correlation = 0.00
0 OF 3 LIBRARY LINES FOR Ra226 FOUND Correlation = 0.00
0 OF 1 LIBRARY LINES FOR U235 FOUND Correlation = 0.00
0 OF 1 LIBRARY LINES FOR U238 FOUND Correlation = 0.00

LINES NOT ASSOCIATED WITH ANY NUCLIDE

Energy	Net CPM	Eff Corrected
204.9	4558.1	213217.0

MCA REPORT

Figure 9.

A technical understanding of this paper requires a knowledge of the connection between the quantitative representation of the synthesis of the neutron, and the resolution of problems in nuclear physics and chemistry that have remained fixed for one century. In fact, the resolution of all problems here referred to, is based on the use of the same methods, the novel isomathematics and isomechanics.

Recall that quantum mechanics has been unable to achieve a representation of the magnetic moment of the simplest nucleus, the deuteron (because 1% is still missing following all possible corrections) with embarrassing deviations from theoretical predictions for large nuclei such as that of plutonium. A similar situation has occurred for nuclear spins that have remained unresolved for nuclei at large. The representation permitted by isomathematics and isomechanics of protons and neutrons as *extended, and therefore, deformable charge distributions under contact non-Hamiltonian interactions when members of a nuclear structure* has permitted the achievement of the first known *numerically exact and time invariant representation of nuclear magnetic moments* [31, 32] *and spins* [33]. Recall that the so-called "hot fusions" have not achieved to date any industrially valuable results due to known extreme instabilities ultimately due to the use of excessive energies. The so-called "cold fusions" have established the existence of nuclear fusions in our Earthly environment, but they have not achieved industrially valuable results due to, at this time, energy insufficient for all necessary engineering needs. The representation of protons and neutrons as extended and deformable particle under contact non-Hamiltonian interactions when members of a nuclear structure have permitted the formulation and experimental verification of Intermediate Controlled Nuclear Fusion (CNF, also called "warm fusions") without the emission of harmful radiation and without the release of radioactive waste, which fusions occur at energies intermediate between those of the hot and cold fusions, and are controlled via a number of engineering means [34-38].

Finally, recall that, according to quantum mechanics and chemistry, identical electrons in valence couplings should *repel* each other, and certainly cannot attract each other contrary to the evidence that molecular structures are due to strongly attractive valence bonds between identical electrons. It should be indicated that non-linear, non-local and non-Hamiltonian interactions due to deep overlapping of the wavepackets of particles have been crucial for the first known articulation of an *attractive force between identical valence electrons in singlet couplings* as occurring in the reality of molecular structures [39]. Consequently, the new bond of Figures 3 and 5 are fully aligned with the bond of identical valence electrons in singlet coupling as occurring in molecular structures.

3. Theoretical Predictions

3.1. Basic Assumptions

In this section, we review the conventional activation of tabulated nuclide [40] via a neutron flux and present the corresponding activation via a flux of neutroids as is necessary for their experimental verification.

We shall represent nuclides with the known symbols (Z, A, J), where Z represents the atomic number, A represents the atomic mass, and J represents the angular momentum. The mass of nuclides are ignored since we shall be dealing with tabulated decays emitting energy [33]. Under the assumed notation, nuclides N shall be written N(Z, A, J) and the related data assumed from Table [41].

For convenience, we shall assume the same notation for neutrons and neutroids

$$n = n\,(0,\ 1,\ 1/2),\ \tilde{n} = \tilde{n}\,(0,\ 1,\ 0), \tag{3.1}$$

as well as for other particles participating in nuclear transmutations, such as

$$\beta^{-}\,(1,\ 0,\ 1/2),\ \nu\,(0,\ 0,\ 1/2),\ \gamma\,(0,\ 0,\ 1),$$

Figure 10.

$$EC = EC(-1,\ 0,\ 1/2), \tag{3.2}$$

where EC denotes Electric Capture, for which the value $Z = -1$ for the electron stands to recall that its absorption implies the decrease of the atomic number by one unity due to the neutron synthesis while the value $J = +1/2$ denotes the increase of the nuclear spin by 1/2.

In regard to emission of particles, atomic numbers, atomic masses and spins should be subtracted from the original values.

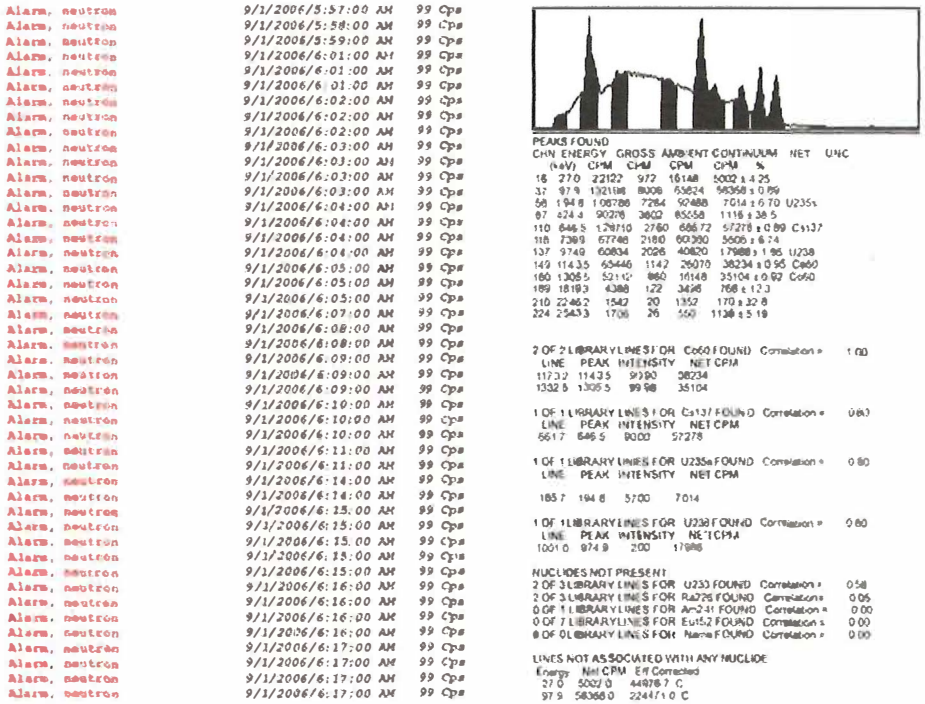


Figure 11.

3.2. Nuclides and Nucleoids

Neutron activations of tabulated nuclides N [31] follow the well-known rules (see e.g., tabulated data [41])

$$N\,(Z,\,A,\,J) + n\,(0,\,1,\,1/2) \rightarrow$$

$$\rightarrow N^0\,(Z,\,A+1,\,J+1/2), \qquad (3.3)$$

where N^0 is a known, generally unstable nuclide [31] and the possible chains of decays into a final stable isotope are equally well known [41].

Neutroid activations of a tabulated nuclide N are predicted to follow the new rules

$$N\,(Z,\,A,\,J) + \tilde{n}\,(0,\,1,\,0) \dashrightarrow$$

$$\dashrightarrow \tilde{N}\,(Z,\,A+1,\,J), \qquad (3.4)$$

where $\tilde{N}$ denotes generally unstable and untabulated nuclides first introduced in Ref. [17] under the name of *nucleoids*.

When $\tilde{N}$ is sufficiently excited to release the missing energy of 0.782MeV for synthesis (2.4), the nucleoid is predicted to have the nuclear transmutation

$$\tilde{N}\,(Z,\,A+1,\,J) \dashrightarrow N^0\,(Z,\,A+1,\,J+1/2) + \nu\,(0,\,0,\,1/2), \qquad (3.5)$$

where $N^0\,(Z,\,A+1,\,J+1/2)$ is a tabulated nuclide with known decay.

When nucleoid $\tilde{N}$ does not have the needed missing energy, it decays according to Eq. (2.5) with ensuing transmutation

$$\tilde{N}\,(Z,\,A+1,\,J) \dashrightarrow N^0\,(Z+1,\,A+1,\,J+1/2) + \beta^-\,(1,\,0,\,1/2), \qquad (3.6)$$

where $N^0\,(Z+1,\,A+1,\,J+1/2)$ is a tabulated nuclide with known decay of the type

$$N^0\,(Z+1,\,A+1,\,J+1/2) \dashrightarrow N''\,(Z+1,\,A+1,\,J) + \nu(0,\,0,\,1/2). \qquad (3.7)$$

The study of a number of additional possible transmutations and decay is left to the interested reader.

Due to the lack of existence at this writing of detectors capable of directly detecting neutroids, the sole way known to detect neutroids is given by: 1) Setting up TEC-TNS *below* the threshold values of 3kV and 1J; 2) Assuring via conventional detectors the *lack* of neutron emission; and 3) Seeking nuclear transmutations of type (3.4) - (3.7) via neutron, electron, gamma delayed emission or spectroscopic analysis of the original and of the irradiated sample.

3.3. Activations of Ag (47, 107, 1/2)

The conventional neutron activation of Ag (47, 107, 1/2) is given by [41]

$$Ag\,(47,\,107,\,1/2) + n\,(0,\,1,\,1/2) \rightarrow Ag\,(47,\,108,\,1) \rightarrow$$

$$\rightarrow Cd\,(48,\,108,\,0)[\textit{stable isotope}] + e^-\,(-1,\,0,\,1/2) + \nu(0,\,0,\,1/2), \qquad (3.8)$$

resulting in the stable nuclide Cd (48, 108, 0), as well known.

The predicted neutroid activation of Ag (47, 107, 1/2) is given by

$$Ag\,(47,\,107,\,1/2) + \tilde{n}(0,\,1,\,0) \dashrightarrow \widetilde{Ag}\,(47,\,108,\,1/2), \qquad (3.9)$$

where $\widetilde{Ag}$ (47, 108, 1/2) is a nucleoid because it is untabulated (due to the spin 1/2).

In the event the nucleoid $\widetilde{Ag}$ (47, 108, 1/2) is sufficiently excited to trigger synthesis (2.4), we have the decay

$$\widetilde{Ag}\,(47,\ 108,\ 1/2) \dashrightarrow Ag\,(47,\ 108,\ 1) + \nu(0,\ 0,\ 1/2) + \gamma(0,\ 0,\ 1), \quad (3.10)$$

where Ag (47, 108, 1) is a stable nuclide and we have used our assumption on the sign of emitted particles.

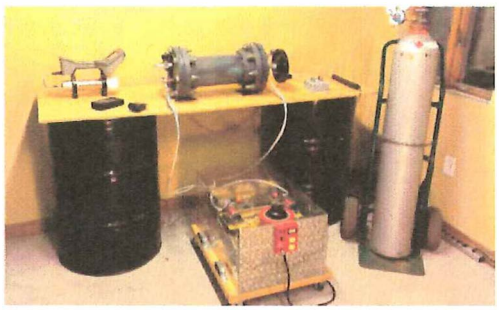


Figure 12.

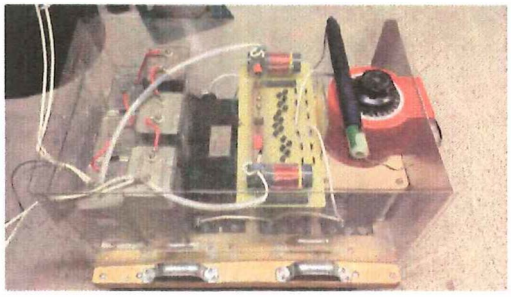


Figure 13.

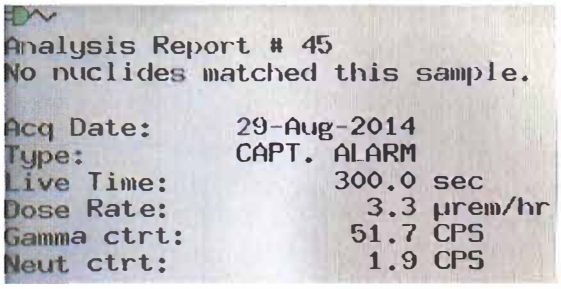


Figure 14.

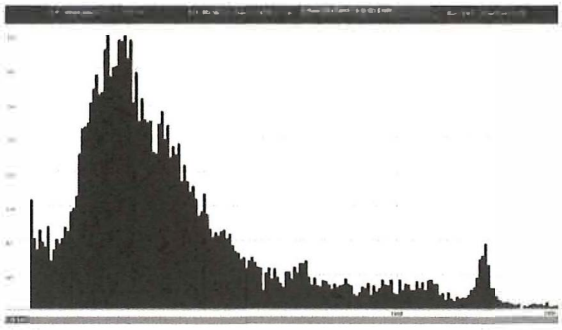


Figure 15.

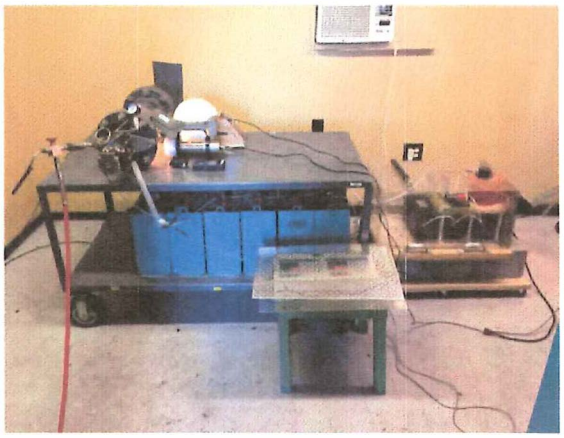


Figure 16.

Figure 17.

Figure 18.

Intriguingly, J. Sternglass suggested the apparent synthesis of neutrons from the detection of β emitted from his reactor [4]. Since his equipment did not have sufficient energy to supply the missing energy of 0.782MeV for the neutron synthesis, it appears that J. Sternglass detected a transmutation of type (3.8) caused by neutroid activation.

Under the assumption that the nucleoid is not in such an excited state to trigger synthesis (2.4), $\widetilde{Ag}$ (47, 108, 1/2) is

predicted to decay along Eq. (2.5)

$$\widetilde{Ag}(47, 108, 1/2) \rightarrow Cd\ (48, 108, 0) + \beta^{-}(1, 0, 1/2), \quad (3.11)$$

where the resulting stable nuclide Cd (48, 108, 0) is the same as for the conventional neutron irradiation, Eq. (3.8).

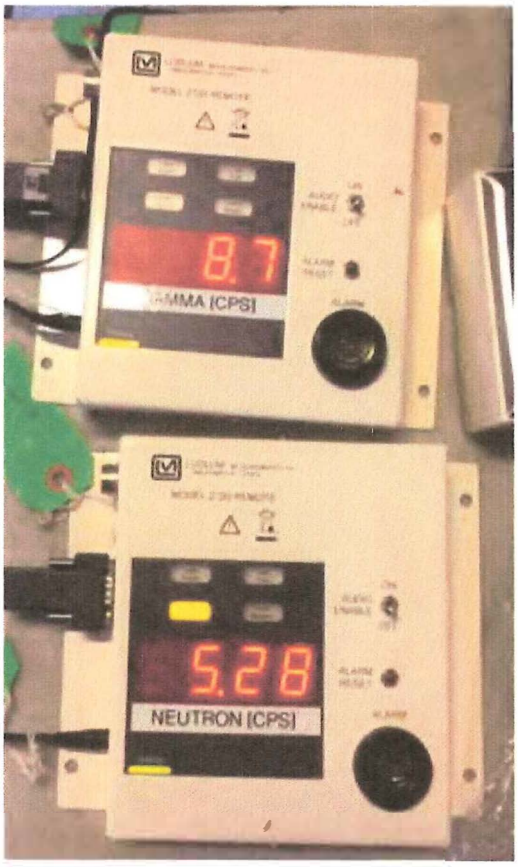
Intriguingly, E. J. Sternglass reported in his letter to A. Einstein the emission of electrons that appear to be fully in line with decay (3.11), while his experimental set up did not have sufficient energy to synthesize neutrons, thus confirming that he synthesized neutroids.

A third possible decay is given by

$$\widetilde{Ag}\ (47, 108, 1/2) + EC(-1, 0, +1/2) \rightarrow Pd\ (46, 108, 0) + \gamma(0, 0, 1), \quad (3.12)$$

where Pd (46, 108, 0) is a stable, hence tabulated isotope.

3.4. Activations of Ag (47, 109, 1/2).



Readout from Polimaster PM1704GN

12:19:20 AM	Gamma background	4 μR/h
12:19:20 AM	Neutron background	0.02 cps
1:19:20 AM	Gamma background	4 μR/h
1:19:20 AM	Neutron background	0.02 cps
2:19:20 AM	Gamma background	4 μR/h
2:19:20 AM	Neutron background	0.02 cps
3:19:20 AM	Gamma background	4 μR/h
3:19:20 AM	Neutron background	0.02 cps
4:19:20 AM	Gamma background	4 μR/h
4:19:20 AM	Neutron background	0.02 cps
5:19:20 AM	Gamma background	4 μR/h
5:19:20 AM	Neutron background	0.02 cps
6:19:20 AM	Gamma background	4 μR/h
6:19:20 AM	Neutron background	0.02 cps
7:19:20 AM	Gamma background	5 μR/h
7:19:20 AM	Neutron background	0.02 cps
7:46:20 AM	**Neutron alarm**	**0.02 cps**
8:19:20 AM	Gamma background	4 μR/h
8:19:20 AM	Neutron background	0.02 cps
9:19:20 AM	Gamma background	4 μR/h
9:19:20 AM	Neutron background	0.02 cps
10:17:58 AM	**Neutron alarm**	**0.02 cps**
10:19:20 AM	Gamma background	5 μR/h
10:19:20 AM	Neutron background	0.02 cps
11:19:20 AM	Gamma background	4 μR/h
11:19:20 AM	Neutron background	0.02 cps
11:47:20 AM	**Neutron alarm**	**0.02 cps**
12:19:20 PM	Gamma background	3 μR/h
12:19:20 PM	Neutron background	0.02 cps
1:19:20 PM	Gamma background	3 μR/h
1:19:20 PM	Neutron background	0.01 cps

Figure 19.

The conventional neutron activation is given by

$$Ag\ (47, 109, 1/2) + n(0, 1, 1/2) \rightarrow Ag\ (47, 110, 1) \rightarrow$$

$$\rightarrow Cd\ (48, 110, 0) + \beta^{-}(1, 0, 1/2) + \nu\ (0, 0, 1/2), \quad (3.13)$$

resulting in the stable nuclide Cd (48, 110, 0), as well known. The predicted neutroid activation is given by

$$Ag\ (47, 109, 1/2) + \tilde{n}(0, 1, 0) \rightarrow \widetilde{Ag}\ (47, 110, 1/2), \quad (3.14)$$

where $\widetilde{Ag}$ (47, 110, 1/2) is an un-tabulated nucleoid with: first possible decay under synthesis (2.4)

$$\widetilde{Ag}\ (47, 108, 1/2) + EC\ (-1, 0, +1/2) \rightarrow Pd\ (46, 110, 0) + \gamma\ (0, 0, 1), \quad (3.15)$$

or alternative decay according to neutroid decay (2.5)

$$\widetilde{Ag}\ (47, 110, 1/2) \rightarrow Cd\ (48, 110, 0) + \beta^{-}(-1, 0, 1/2), \quad (3.16)$$

and other decays whose study is left to the interested reader.

3.5. Activations of Au (79, 197, 3/2)

The conventional neutron activation is given by

$$\text{Au}(79, 197, 3/2) + n(0, 1, 1/2) \rightarrow \text{Au}(79, 198, 2) \rightarrow$$
$$\rightarrow \text{Hg}(80, 198, 0) + \beta^{-}(1, 0, 1/2) + \nu(0, 0, 1/2) + \gamma(0, 0, 1), \tag{3.17}$$

resulting in the stable nuclide Hg (80, 198, 0), as well known. The predicted neutroid activation is given by

$$\text{Au}(79, 197, 3/2) + \tilde{n}(0, 1, 0) \rightarrow \widetilde{\text{Au}}(79, 198, 3/2), \tag{3.18}$$

where $\widetilde{\text{Au}}(79, 198, 3/2)$ is a nucleoid with decay predicted along synthesis (2.4) although without the emission of a neutrino

$$\widetilde{\text{Au}}(79, 198, 3/2) \rightarrow \text{Pt}(78, 198, 0) + \beta^{-}(1, 0, 1/2) + \gamma(0, 0, 1), \tag{3.19}$$

because the emission of a neutrino as requested by synthesis (2.4) would yield the inconsistent decay

$$\widetilde{\text{Au}}(79, 198, 3/2) \rightarrow \text{Hg}(80, 198, 0) + \nu(0, 0, 1/2) + \beta^{-}(1, 0, 1/2) + \nu(0, 0, 1/2) + \gamma(0, 0, 1).$$

An alternative decay is given by

$$\widetilde{\text{Au}}(79, 198, 3/2) \rightarrow \text{Hg}(80, 198, 0) + \beta^{-}(1, 0, 1/2) + \gamma(0, 0, 1), \tag{3.20}$$

resulting again in the same stable nuclide Hg (80, 198, 0) as that released by neutron activation, although without the possibility of consistently admitting the emission of a neutrino, thus casting doubts on basic syntheses (1.2) and (2.4)

The study of additional, virtually endless neutroid activations is left to the interested reader.

4. Experimental Verifications

4.1. Basic Information

The first direct detection of the synthesis of neutrons from a hydrogen gas was achieved by R. M. Santilli in 2006 [21, 22] the complete documentation being available in Ref. [23] also of 2006.

Tests [21-23] were first done with the experimental set up of Figure 8 comprising: a vertical reactor fabricated from a translucent PVC pipe of 3" diameter and 12" length filled up with a commercially available hydrogen gas at a maximum of 10psi; a 5kW DC welder produced by Miller Electric delivering an electric arc between submerged tungsten electrodes of 1/3" diameter with 20V and 200A; and various neutron and gamma detectors, by Ludlum, Berkeley Nucleonics and Polimaster with internal recording of data. This first experimental set up *did not* produce neutrons, but produced neutroids due to the steady character of the DC arc. In fact, tests [21, 22] essentially produced transmutations of natural elements surrounding the reactors, at times delayed up to 15 minutes.

Tests [21-23] were also done with the equipment of Figure 10 comprising: a high-pressure metal reactor of 1° diameter and 2° length containing hydrogen gas up to 100psi; a DC welder produced by Miller Electric with up to 50kW used to deliver an arc between carbon electrodes (because tungsten electrodes would instantly melt at the power here considered) of about 30V and 1,000A corresponding to 30KVA; and the same neutron detectors of the preceding tests. The tests produced such neutron alarms, particularly at the activation and disconnection of the arc, to mandate the rapid evacuation of the laboratory, as illustrated by the data of Figure 11 (see Ref. [23] for comprehensive data). In view of this occurrence, no additional tests have been done since 2006 with the hydrogen gas at 100psi for obvious reasons connected to operator safety.

The third tests were done in 2014 [24, 25] with the setup of Figure 12 comprising: a reactor fabricated from a translucent PVC pipe of 6": diameter and 24" length containing a commercially available hydrogen gas at a maximum of 30psi; a high voltage 3kW power source supplied by Information Unlimited of New Hampshire, depicted in Figure 13, delivering a sequence of rapid DC arcs between submerged, 1" diameter carbon electrodes with up to 15kV and up to 2,500J when connected to a 20 μF capacitor; and various neutron-gamma detectors with internal recording of all detections, including a SAM 935 by Berkeley Nucleonics, a PM1703 by Polimaster, and other detectors. The reactor was fabricated in translucent PVC so as to allow visual inspection of the working setup filled with air, then filling up of the reactor with hydrogen, activating and disconnecting the reactor at a distance, and downloading data on neutrons and gamma CPS from the memory banks of the detectors. Tests [24, 25] established the controlled synthesis of neutrons from a hydrogen gas with an industrially valuable flux with up to 1.9 neutron CPS corresponding to about 4,500CPS when pro-rated to the sphere centered in the arc. Tests [24, 25] also established the thermal character of the synthesized neutrons due to lack of energy from the plasma surrounding the arc which was insufficient to create high-energy motion perpendicular to the arc. In this paper, we report the following tests:

Figure 20.

TESTS 1: Done with the experimental set up of Figure 16 comprising: the same power source as that of tests [24, 25] (Figure 13) charging six capacitors for a total of 10μF; the Ludlum 375 and SAM 940 neutron and gamma detector; the reactor depicted in Figure 17 showing the remote controlled stepper motor (on the left), the internal electrodes and various valves; and the remote control station depicted in Figure 18 comprising from the right, the control of the gap between the electrodes, the main switch to activate the power source the neutron and gamma remote Ludlum sensors and the Polimaster PM1703 to verify the lack of harmful radiation at the remote control station for the safety of the operator. Some of the neutron CPS detected by the Ludlum 375 are depicted in the left of Figure 19, and the confirmation of neutron detected by the SAM 940 is depicted in the right (see also Ref. [42] for a short movie on these tests).

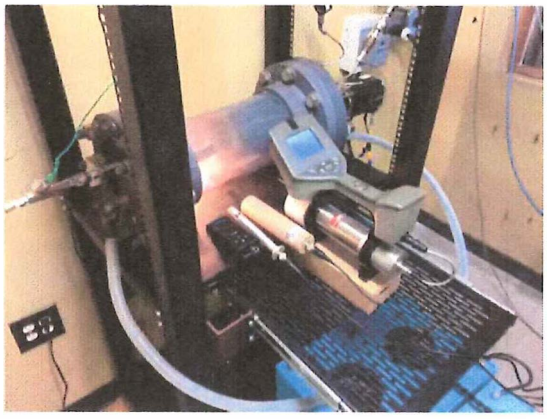


Figure 21.

TESTS 2: Done with the experimental set up of Figure 20 known as the Thermal Neutron Source (TNS) 20 comprising: the same reactor of Figure 17 and the same high voltage power source of Figure 13 used in preceding tests, the latter being connected of rapid discharge capacitors with 20 μF supplied by General Atomic of San Diego, California; the Ludlum 375, SAM 940 and Polimaster PM1704 neutron and gamma detectors with position of their sensors illustrated in Figure 21; the remote control station illustrated in Figure 22 comprising from the right and the remote control of the stepper motor to achieve the desired gap between the carbon electrodes, the main switch to activate the power source; the Ludlum remote sensors for neutron and gamma CPS; and the remote control of the power used in the power source. A number of neutron CPS detected during these tests are presented in Section 4.2 and 4.3.

Figure 22.

TESTS 3: The same experimental set up as that of Tests 2 with the high voltage power source of up to 20kV and 5kW delivering a rapid DC discharge between the submerged electrodes of up to 3,000J following the charge of capacitors with 20μF (see Picture 23 showing in front-right the 5kW high voltage power source).

TESTS 4 the same experimental set up of Tests 3 with the Magna-Power source, model XR10000-0.80/480POS+LXI with 8kW, 10kV and 0.8A. During the tests, the source was set at 8kV and 0.5A, but operated with voltage oscillating between 2.5kV and 0.5A, with a rapid rate of charging the capacitors at about 3 discharges per second, that resulted in an essentially steady production of 4 to 6 neutron CPS (see Picture 23 showing in front-left the 8kW high voltage Magna-Power source).

TESTS 5: The same experimental set up of Tests 3 but with the power source supplied by Information Unlimited directly connected to the arc without capacitors (the MagnaPower source could not be used because of its automatic disconnect when used to deliver DC arcs).

The various tests were done according to the following written routine: verification that all switches are off and the capacitors are discharged; flushing the reactor with air from a compressor to remove hydrogen residues; selection of the desired power; consequential selection of the gap between the electrodes; remote activation of the power unit; verification of the existence of acceptable arc in air; disconnection of all switches and verification that the capacitors are discharged; flushing the reactor with hydrogen;

setting up the desired hydrogen pressure in the reactor; activate cameras filming the reactor as well as the remote sensors; activate remotely the TNS; conduct the test for the desired duration of time; disconnect all switches; move the electrodes into a short to discharge residual charges; verification that the capacitors are indeed discharged; and collect neutron and gamma CPS from the Ludlum 375, the SAM 940 and the Polimaster PM1704.

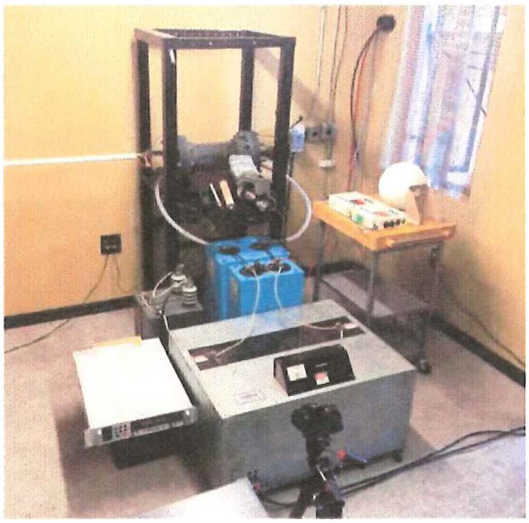


Figure 23.

4.2. Experimental Confirmation of the Synthesis of Neutroids

The reactor (TNS) is working with low power, as we don't want to synthesize neutrons, but only neutroids which require lower energies. No neutron detection should occur during the test, but readings are expected sometime after the arc has been turned off, due to the decay of the nuclei made unstable by the neutroids themselves. These neutron emissions are expected to be very directional and random, so the detectors are not likely to see them at the same time.

Some of the experimental results obtained are reported in Table 1.

Table 1.

Summary of neutroids tests	
Test 1. Reactor filled with H_2 *at ambient pressure, power unit set at* 0.75 kW *power:*	
Start Time	11:23 am
End Time	11:27 am
Ludlum Neutron Detections during test	None
Ludlum Gamma Detections during test (CPS)	11.6, 8.8, 10.7, 12.9, 13.8, 14.1 (coherent with background)
Ludlum neutron detections after the end	1 CPS 4 minutes later
SAM Neutron Detections	0 CPS, 0 counts
SAM Gamma Detections	43.9633 CPS
Test 2 - Same configuration as previous test	
Start Time	1:40 pm
End Time	1:44 pm
Summary of neutroids tests	
Ludlum Neutron Detections during test	None
Ludlum Gamma Detections during test	Coherent with background
Ludlum neutron detections after the end	None
SAM Neutron Detections	0.0033 CPS, 1 counts
SAM Gamma Detections	44.3733 CPS
Test 3 - Same configuration as previous test	
Start Time	1:50 pm
End Time	1:55 pm
Ludlum Neutron Detections during test	None
Ludlum Gamma Detections during test	Coherent with background
Ludlum Neutron detections after the end	1 CPS 30 sec later
SAM Neutron Detections	0.0017 CPS, 1 counts
SAM Gamma Detections	44.995 CPS
Test 4 - Same configuration as previous test	
Start Time	2:04 pm
End Time	2:10 pm
Ludlum Neutron Detections during test	None
Ludlum Gamma Detections during test	Coherent with background
Ludlum neutron detections after the end	None
SAM Neutron Detections	0 CPS, 0 counts
SAM Gamma Detections	47.7433 CPS
Test 5 - Reactor with H_2 *at* 10 psi, *power unit set at* 0.75 kW *power*	
Start Time	2:25 pm
End Time	2:30 pm
Ludlum Neutron Detections during test	None
Ludlum Gamma Detections during test	Coherent with background
Ludlum neutron detections after the end	1 CPS 1 min later
SAM Neutron Detections	0.005 CPS, 3 counts
SAM Gamma Detections	44.6533 CPS

4.3. Experimental Confirmation of the Synthesis of Neutrons

The reactor (TNS) is working with different power units and various levels of power. Two power units designed ad hoc by Information Unlimited and one commercially available by MagnaPower have been used. The pressure of hydrogen inside the reactor has been used as a variable as well, changing it in a controlled way to verify the effects on neutron CPS.

It should be noted that the Ludlum detector provides a real-time record of the neutron and gamma counts, while the SAM940 detector only records an average neutron and gamma CPS for the entire duration of the test. The neutron CPS values reported for the Ludlum in the tests with the two Information Unlimited power units (see Tables 2-3) are just the peak values corresponding to the capacitors discharge, which typically happen every 5 seconds. The typical shape of neutron emission during test time can be seen in Figure 24, together with the trend of power consumption from the unit. During the tests with the MagnaPower unit (see Table 4), however, on the other end, the discharges were much quicker, resulting in a continuous neutron production, as can be seen in Figure 25, so the values reported in the table do represent the actual neutron CPS variations recorded by Ludlum detector.

Table 2.

Summary of neutron tests with Informatiou Unlimited power unit 1	
Test 1 - Reactor filled with H_2 *at* 5 psi, *power unit set at* 2 kW, *electrodes gap at* 2.2 mm	
Ludlum Neutron Detections (CPS)	4.06, 4.06, 4.06, 3.06, 3.05, 3.05, 4.05, 4.05, 4.06
Ludlum Gamma Detections (CPS)	15.6, 11.2, 8.6, 9.0, 10.9, 9.8 (coherent with background)
SAM Neutron Detections (CPS)	0.5444
SAM Gamma Detections (CPS)	46.2611 (coherent with background)
Polimaster Detection	Neutron Alarm
Test 2 - Reactor filled with H_2 *at* 20 psi, *power unit set at* 2 kW, *electrodes gap at* 2 mm	
Ludlum Neutron Detections (CPS)	4.00, 6.00, 5.00, 5.00, 4.00, 5.00, 4.00, 4.00, 4.00
Ludlum Gamma Detections (CPS)	coherent with background
Test 3 - Reactor filled with H_2 *at* 20 psi, *power unit set at* 2 kW, *electrodes gap at* 2.2 mm	
Ludlum Neutron Detections (CPS)	4.00, 4.00, 2.00, 5.00, 4.00, 4.00, 4.00, 4.00, 5.00
Ludlum Gamma Detections (CPS)	coherent with background
Test 4 - Reactor filled with H_2 *at* 27 psi, *power unit set at* 2 kW, *electrodes gap at* 1.9 mm	
Ludlum Neutron Detections (CPS)	4.00, 4.00, 4.00, 4.00, 4.00, 5.00, 5.00, 4.5, 4.00
Ludlum Gamma Detections (CPS)	coherent with background
Test 5 - Reactor filled with H_2 *at 20 psi, power unit set at 2 kW, Ludlum detector at 2 cm from the reactor wall, electrodes gap at 2 mm.*	
Ludlum Neutron Detections (CPS)	6.00, 5.00, 5.00, 5.00, 5.00, 5.00, 5.00, 5.00, 5.00
Ludlum Gamma Detections (CPS)	coherent with background
Test 6 - Reactor filled with H_2 *at* 20 psi, *power unit set at* 2 kW, *Ludlum at* 1.5 m *from the reactor wall, electrodes gap at* 2 mm	
Ludlum Neutron Detections (CPS)	1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00
Ludlum Gamma Detections (CPS)	coherent with background
Test 7 - Reactor filled with H_2 *at* 15 psi, *power unit set at* 2 kW, *Ludlum again at* 2 cm *from the wall, electrodes gap at* 2 mm	
Ludlum Neutron Detections (CPS)	4.00, 3.00, 4.00, 4.00, 4.00, 4.00, 4.00
Ludlum Gamma Detections (CPS)	coherent with background
Test 8 - Reactor filled with H_2 *at* 15 psi, *power unit set at* 2 kW, *electrodes gap at* 2 mm	
Ludlum Neutron Detections (CPS)	3.00, 3.00, 2.76, 2.58, 2.64, 3.33, 3.41, 3.41, 4.20
Ludlum Gamma Detections (CPS)	coherent with background
Test 9 - Reactor filled with H_2 *at* 20 psi, *power unit set at* 2 kW, *electrodes gap at* 2 mm	
Ludlum Neutron Detections (CPS)	4.00, 5.00, 5.00, 4.00, 5.00, 4.00, 5.00
Ludlum Gamma Detections (CPS)	coherent with background
Test 10 - Reactor filled with H_2 *at* 25 psi, *power unit set at* 2kW, *electrodes gap at* 1.9 mm	
Ludlum Neutron Detections (CPS)	5.00, 6.00, 5.00, 5.00, 5.00, 4.00, 5.00, 6.00, 4.00
Ludlum Gamma Detections (CPS)	coherent with background
Test 11 - Reactor filled with H_2 *at* 30 psi, *power unit set at* 2kW, *electrodes gap at* 1.9 mm	
Ludlum Neutron Detections (CPS)	4.00, 6.00, 5.00, 6.00, 4.00, 4.00, 5.00, 5.00, 5.00
Ludlum Gamma Detections (CPS)	coherent with background
Test 12 - Reactor filled with H_2 *at* 30 psi, *power unit set at* 2kW, *electrodes gap at* 1.8 mm	
Ludlum Neutron Detections (CPS)	4.00, 4.00, 4.00, 5.00, 4.02, 4.00, 5.00, 4.00, 5.00
Ludlum Gamma Detections (CPS)	coherent with background
Test 13 - Reactor filled with H_2 *at* 30 psi, *power unit set at* 2 kW, *electrodes gap at* 1.7 mm	
Ludlum Neutron Detections (CPS)	2.28, 2.28, 2.28, 2.53, 3.32, 3.41, 2.42, 2.42, 2.30
Ludlum Gamma Detections (CPS)	coherent with background
Test 14 - Reactor filled with H_2 *at* 30 psi, *power unit set at* 2 kW, *electrodes gap at* 1.6 mm	
Ludlum Neutron Detections (CPS)	1.73, 1.42, 1.71, 1.42, 1.71, 1.85, 1.46, 1.36, 1.68
Ludlum Gamma Detections (CPS)	coherent with background

Table 3.

Summary of neutron tests with Information Unlimited power unit 2	
Test 1 - Reactor filled with H_2 *at* 10 psi, *power unit set at* 3.5 kW, *electrodes gap at* 2.0 mm, *then at* 4.5 kW *with electrodes gap at* 2.2 mm	
Ludlum Neutron Detections (CPS), 3.5 kW	3.00, 3.05, 3.05, 3.02, 3.02, 3.05, 3.05, 3.02, 3.02
Ludlum Neutron Detections (CPS), 4.5 kW	4.09, 4.05, 3.05, 3.05, 4.05, 3.05, 3.05, 4.02, 3.09
Ludlum Gamma Detections (CPS)	coherent with background
SAM Neutron Detections (CPS)	0.372222
SAM Gamma Detections (CPS)	45.4222
PoliMaster Detection	Neutron Alarm
Test 2 - Reactor filled with H_2 *at* 5 psi, *power unit set at* 2.5 kW, *electrodes gap at* 2.0 mm	
Ludlum Neutron Detections (CPS)	4.00, 4.00, 4.00, 4.00, 4.00, 4.00, 4.00, 4.00, 4.00
Ludlum Gamma Detections (CPS)	coherent with background
Test 3 - Reactor filled with H_2 *at* 10 psi, *power unit set at* 2.5 kW, *electrodes gap at* 2.0 mm	
Ludlum Neutron Detections (CPS)	3.01, 3.02, 5.00, 4.00, 4.00, 4.00, 4.00, 5.00, 4.00
Ludlum Gamma Detections (CPS)	coherent with background
Test 4 - Reactor filled with H_2 *at* 20 psi, *power unit set at* 3.5 kW, *electrodes gap at* 2.0 mm	

Summary of neutron tests with Information Unlimited power unit 2	
Ludlum Neutron Detections (CPS)	5.00, 4.00, 4.00, 4.00, 4.00, 5.00, 5.00, 5.00, 5.00
Ludlum Gamma Detections (CPS)	coherent with background
Test 5 - Reactor filled with H_2 *at* 30 psi, *power unit set at* 3.5 kW, *electrodes gap at* 2.0 mm	
Ludlum Neutron Detections (CPS)	4.02, 4.01, 5.03, 4.02, 4.01, 4.03, 4.03, 4.03, 4.03
Ludlum Gamma Detections (CPS)	coherent with background

Table 4.

Summary of neutron tests with Magna Power	
Test 24 - Reactor filled with H_2 *at* 10 psi, *power unit operating at* 1.5 kV *and* 0.5 mA, *electrodes gap at* 2.0 mm	
Ludlum Neutron Detections (CPS)	3.50, 3.75, 3.94, 3.98, 4.00, 4.00, 4.00, 6.00, 6.01
Ludlum Gamma Detections (CPS)	coherent with background
SAM Neutron Detections (CPS)	0.222222
SAM Gamma Detections (CPS)	47.4778
PoliMaster Detection	Neutron Alarm
Test 25 - Reactor filled with H_2 *at atmospheric pressure, power unit operating at* 1.5 kV *and* 0.5 mA, *electrodes gap at* 2.0 mm	
Ludlum Neutron Detections (CPS)	5.12, 6.25, 4.78, 3.39, 3.69, 4.78, 3.92, 2.96, 3.48
Ludlum Gamma Detections (CPS)	coherent with background
SAM Neutron Detections (CPS)	0.366667
SAM Gamma Detections (CPS)	61.1444
Test 26 - Reactor filled with H_2 *at* 10 psi, *power unit power unit operating at* 1.5 kV *and* 0.5 mA, *electrodes gap at* 2.1 mm	
Ludlum Neutron Detections (CPS)	8.27, 9.08, 12.07, 10.03, 8.01, 12.07, 7.50, 6.25, 9.37
Ludlum Gamma Detections (CPS)	coherent with background
SAM Neutron Detections (CPS)	0.688889
SAM Gamma Detections (CPS)	44.5889
Test 27 - Reactor filled with H_2 *at* 20 psi, *power unit power unit operating at* 1.5 kV *and* 0.5 mA, *electrodes gap at* 1.9 mm	
Ludlum Neutron Detections (CPS)	4.25, 4.12, 4.06, 4.25, 6.01, 5.01, 4.50, 6.01, 4.12
Ludlum Gamma Detections (CPS)	coherent with background
SAM Neutron Detections (CPS)	N/A
SAM Gamma Detections (CPS)	N/A
Test 28 - Reactor filled with H_2 *at* 30 psi, *power unit power unit operating at* 1.5 kV *and* 0.5 mA, *electrodes gap at* 1.8 mm	
Ludlum Neutron Detections (CPS)	4.58, 5.29, 4.32, 5.16, 4.58, 5.29, 4.64, 5.32, 4.66
Ludlum Gamma Detections (CPS)	coherent with background
SAM Neutron Detections (CPS)	N/A
SAM Gamma Detections (CPS)	N/A

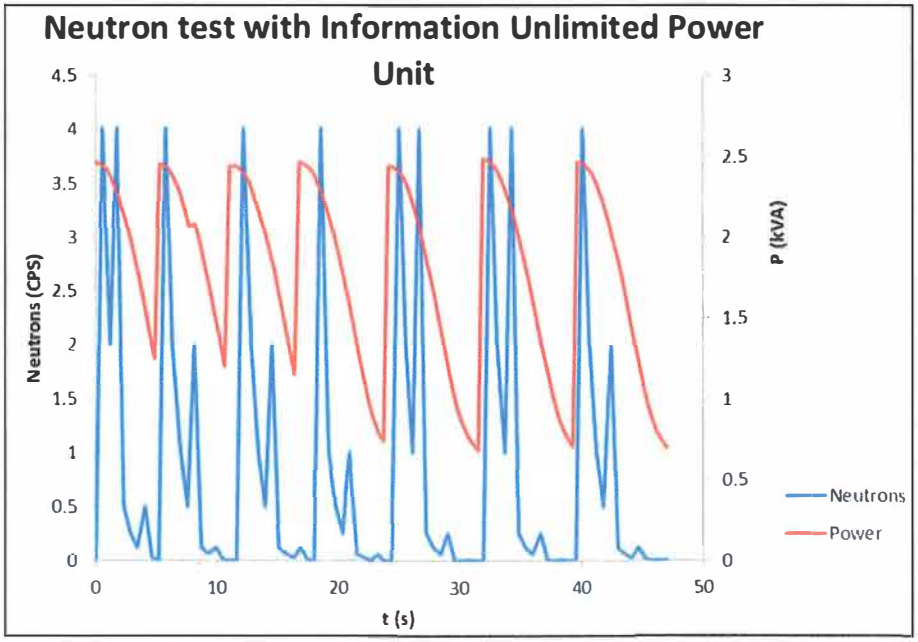


Figure 24.

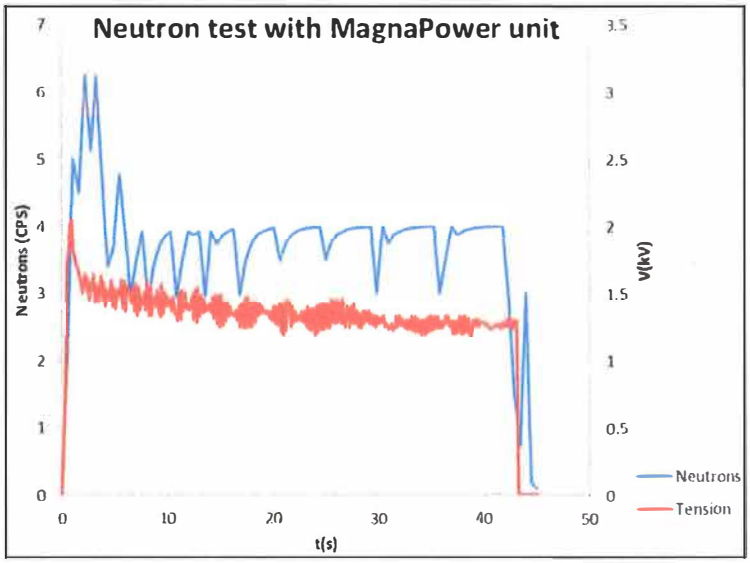


Figure 25.

5. Concluding Remarks

As reviewed in Section 1, H. Rutherford [1] conceived the neutron in 1920 as a "compressed hydrogen atom" in the core of stars. The existence of the neutron was experimentally verified by J. Chadwick [2] in 1932. The complexity of the problems created by the neutron synthesis emerged soon thereafter with the exchange in the 1940's between W. Pauli and E. Fermi [4] that resulted in the hypothesis of the emission of the massless and chargeless neutrino.

The first experimental tests on the laboratory synthesis of the neutron from the hydrogen were done in the 1950's by E. J. Sternglass, were repeated soon thereafter by E. Trounson, and then conducted by Don Carlo Borghi and his associates in the 1960's [4]. All of these initial tests presented serious evidence of delayed nuclear transmutations that could solely be due to a flux of neutrons synthesized by the reactors, although none of them directly detected neutrons. Also, these initial tests implied that the neutron is a conventional, quantum mechanical bound state of a proton and an electron under strong Coulomb attractions, which view was known since the early 20th century to be impossible for various reasons [5, 6, 15]. Consequently, none of the initial tests were published in refereed scientific journals.

The lack of direct detection of neutrons while detecting delayed nuclear transmutations suggested to Don Borghi the hypothesis of the existence of an intermediate state with spin zero called the *neutroid* [4]. Nuclear transmutations were then assumed to be due to the absorption of neutroids by nuclei of natural elements, with completion of the neutron synthesis inside nuclei, resulting in the transmutation of natural elements.

In the late 1970's, while being in the faculty of Harvard University under DOE support, R. M. Santilli [5-7] initiated systematic studies on the neutron synthesis beginning with the identification of the insufficiency of the *mathematics* underlying 20th century science, due to their local-differential character with ensuing abstraction of particles as being point-like. Santilli's argument is that such an abstraction is certainly acceptable for particles at large mutual distances (exterior dynamical conditions), but it becomes insufficient when extended particles are in conditions of mutual penetration (interior dynamical conditions) due to the emergence of non-linear, non-local and contact non-Hamiltonian interactions simply beyond any hope of quantitative treatment with 20th century mathematics, let alone physical theories. In order to achieve the needed representation, Santilli worked out the foundations of the new mathematics, today known as *isomathematics* and related *isomechanics* in which the proton is represented with its actual, experimentally measured dimension and density [5-12].

Thanks to the broader mathematical and physical methods, Santilli achieved the first and only known exact representation of *all* characteristics of the neutron in its synthesis from a proton and electron at the nonrelativistic [17, 18] and relativistic [18, 19] levels (see also reviews [28-30]).

Based on the understanding of the neutron synthesis achieved via these mathematical and theoretical studies, Santilli conducted systematic experimental confirmations of the synthesis of both neutrons and neutroids from a hydrogen gas [21-27]. Subsequently, the U. S. publicly traded company *Thunder Energies Corporation* (TEC, thunder-energies.com) began production and sale of the *Thermal Neutron Sources* (TNS) used in our tests, which source can provide on demand the desired flux of low energy neutrons from a commercially available hydrogen gas via the control of the electric power, hydrogen pressure, electrode gap and other engineering means (patent pending).

In this paper, we have presented systematic experimental confirmation of the syntheses of both neutrons and neutroids from a hydrogen gas (hereon referred to as the neutron

synthesis). A few comments are recommendable on the implications of these experimental confirmations

To begin, particularly significant are the scientific and industrial applications of the novel TNS which are under development at Thunder Energies Corporation, including: 1) The detection of fissionable material that can be smuggled in containers or suitcases via the detection of the radiation emitted by their decay when irradiated by thermal neutrons; 2) The recycling of nuclear waste via their stimulated decay and reduction of their mean lives when activated by a sufficiently strong flux of sufficiently energetic neutrons and other particles; 3) The detection and concentrations in mines of precious minerals and other elements via the detection of the sharp gammas emitted under their irradiation by thermal neutrons irradiation; 4) The treatment of cancerous cells when irradiated by a thin beam of very low energy neutrons; 5) The study of esoenergetic nuclear transmutations without the emission of harmful radiations and without the release of radioactive waste as illustrated by Eqs. (3.8)-(3.10); 6) The test of welds in naval constructions; and other applications open for collaboration to qualified scholars.

A broad implication of our experimental confirmation is an expected revision of 20th century views on the structure, rather than the classification of particles because *the neutron results to be a generalized bound state of one proton and one electron under the laws of isomathematics and isomechanics* thus being isoparticles, rather than conventional particles, intended as isorepresentations of the Lorentz-Poincaré-Santilli isosymmetry [15]

In particular, the neutron cannot be a bound state of quarks because such an assumption would imply the manifestly untenable consequences that, at the time of the neutron synthesis, the permanently stable proton and electron literally "disappear" from the universe to be mysteriously transformed into the hypothetical quarks and, at the time of the neutron spontaneous decay, quarks would "disappear" from the universe and the permanently stable proton and electron reappear because of "fiat."

Independently from these insufficiencies, the assumption of quarks as the ultimate constituents of matter is a theoretical proposition afflicted by numerous consistency problems that have generally remained unaddressed by experts in the field (see e.g., the 1981 Ref. [44] and references quoted therein), such as: the lack of detection of quarks at the extremely high energies currently available at CERN; the impossibility of defining quarks as physical particles in our spacetime since they are not representable with the Poincaré symmetry; the impossibility of achieving a serious quark confinement, one with an identically null probability of tunnel effects into free particles due to the uncertainty principle, and other problems.

The return to actual physical particles as the ultimate constituents of matter raises the still broader impossibility of using one single model, the standard model, for both the classification of particles into family and the structure of individual particles of a given family, contrary to historical teaching, such as that for atoms that required a classification model and a different, yet compatible, model for the structure of individual atoms. The implausibility of a joint classification and structure by the same model is ultimately established by the inapplicability for the structure problem of the *mathematics,* let alone the physics, which is so effective for the classification problem.

The view here adopted is that advocated in Refs. [5, 6] (see also Refs. [13-15]) consisting in the assumption of the standard model as the final classification of particles into families, while the problems of the structure of individual particles of a given family, is open for study by interested physicists (see Ref. [6], Section 5, for a structure model of mesons with physical constituents, Ref. [27, 44] for a corresponding structure model of baryons, and Ref. [30] for a review).

A serious understanding of the implications of the synthesis of the neutron from a proton and an electron requires the additional awareness that the inability to achieve new environmentally acceptable nuclear energies is apparently due to the same insufficiencies of 20th century sciences for the neutron synthesis, namely, the abstraction of protons and neutrons as massive points. In fact, their representation as extended, and therefore deformable particles has permitted the first and only known numerically exact and time invariant representation of nuclear magnetic moments [31, 32] and spins [33]. The same representation has permitted the initiation (now under development at Thunder Energies Corporation) of the *Intermediate Controlled Nuclear Syntheses* (also called "warm syntheses without the emission of harmful radiation and without the release of radioactive nuclear waste ") [34-38].

An additional implication of the neutron synthesis is given by the characterization of new composite, thus unstable particles with actual physical constituents and mean lives of the order of seconds, thus having industrial, let alone scientific relevance, with the clear understanding that these new particles are impossible for the point-like abstraction of their constituents under quantum mechanical laws, while they are fully consistent under the representation of their constituents as extended particles verifying covering laws (isoparticles).

The first particle of this new class is the neutroid treated in Sections 2–4 whose industrial relevance is set by its predicted mean life of 9 seconds, thus being suitable for new activations.

A second new particle stems from the fact that the experimentally established finite probability for an electron to be "compressed" inside the proton implies the consequential existence of a smaller, yet finite probability to compress an electron, this time, inside a neutron, resulting in the new *pseudoproton* denoted with the symbol (in the notations of Section 3.1) $p^-(-1, 1, 1/2)$ [46] (see also Refs. [45, 47]) with reaction

$$n + e^- \to p^-(-1, 1, 1/2). \tag{5.1}$$

We also have the intermediate state with spin zero called the *pseudoprotoid* [48]

$$n_{\uparrow} + e^{-}_{\downarrow} \rightarrow \tilde{p}^{-}(-1, 1, 0). \quad (5.2)$$

The relevance of these new particles is evident since they are *negatively charged,* thus being *attracted* by nuclei and resolving the central problem of nuclear fusions at large, the Coulomb barrier [26], as illustrated by the esoenergetic nuclear transmutations *without the emission of neutrons* in lieu of Eqs. (3.8)-(3.12) [46] (international patent pending).

$$\text{Ag}\,(47, 107, 1/2) + \tilde{p}^{-}\,(-1, 1, 1/2) \dashrightarrow \widetilde{\text{Ag}}\,(46, 108, 1) \dashrightarrow$$

$$\dashrightarrow \text{Pd}\,(46, 108, 0)[\textit{stable isotope}] - \gamma\,(0, 0, 1), \quad (5.3a)$$

$$\text{Ag}\,(47\ 107, 1/2) + \tilde{p}^{-}\,(-1, 1, 0) \rightarrow \widetilde{\text{Ag}}^{0}(46, 108, 1/2) \dashrightarrow$$

$$\rightarrow \text{Pd}\,(46, 108, 0)[\textit{stable isotope}] - \nu\,(0, 0, 1/2). \quad (5.3b)$$

Thanks to the use of the novel technologies based on isomathematics and isomechanics, there is the prediction of additional, heavier composite particles such as the *deuteroid and triturid* [48].

Additionally, we should mention that the use of isomathematcs has permitted the construction of the novel *isochemistry* that has achieved the first and only known attractive force between identical valence electrons in singlet coupling, resulting in new models of the Hydrogen and water molecules [49] (see also the reviews [30, 50]), as well as a new two-body model of the helium [51] permitted by Santilli strong valence bond [49].

In the authors' view, the most important implication of the synthesis of neutrons from a hydrogen gas is the apparent lack of existence of the neutrino in favor of the *etherino* [52, 26] (see also a review with comments in Ref. [4]), indicated with the symbol a from the Latin *aether,* which is intended as a longitudinal "impulse" (or a quasi-particle) supplying the missing 0.782 MeV from the ether conceived as a universal substratum with extremely high energy density for the characterization and propagation of electromagnetic waves and elementary particles according to the reaction in lieu of (1.2)

$$p^{+} + a + e^{-} \rightarrow n. \quad (5.4)$$

A first reason for the external origin of the missing energy is that the relative kinetic energy between the proton and the electron cannot supply the missing energy since their cross section at about 1MeV is essentially null. A second reason is that, when written in the l.h.s., as needed for any synthesis, the antineutrino is expected to have negative energy and, in any case, to have a virtually null cross section with the proton and the electron. A third reason is that, in the event stars provide the missing energy, they would never produce light due to the cooling caused by the delivery of 10^{50} MeV per seconds representing the number of synthesized neutrons per second in a star.

It should be indicated that, contrary to a rather popular belief, the admission of a universal substratum *does not* violate special relativity in vacuum for the evident impossibility for us to identify a reference frame at rest with the ether [52]

The above aspects are complemented by from a number of insufficiencies of the neutrino hypothesis that have remained generally unaddressed, let alone unresolved for close to one century, including [17-30]:

1) The inability of representing the *spin* of the neutrino whenever the proton is represented as an extended particle due to the constrained orbital motion of the electron inside the proton (Figure 6) which motion accounts for both the neutron spin as well as its anomalous magnetic moment;

2) The inability of admitting the *energy* of the neutrino in reaction (1.2), since the energy is already missing for the neutron, and the request for additional energy for the neutrino implies inconsistencies in the representation of all characteristics of the neutron during its synthesis from the hydrogen;

3) The inability to represent with energy the bell-shaped energy distribution in nuclear beta decay, since said shape is due to the dependence of the energy on the angle of the beta emission caused by the Coulomb interactions between the electron and the nucleus, with maximal value for a radial emission, and smaller energy values for smaller angles of emission;

4) The excessive paucity of the experimental evidence used to claim the existence of the neutrino, compared to the extremely large number of events, which disparity prevents the existence of the neutrino to be finally sealed in history. As an example, the flux of solar neutrinos reaching Earth is of the order of trillions of neutrinos per square centimeter per second while, by comparison, their experimental detection has been so limited that the Italian Government terminated the operation of the neutrino tests at the Italian Gran Sasso Laboratory. Independently from such an insufficiency, the paucity of neutrino detection justifies alternative interpretation of the experimental data, such as that via the etherino.

5) The progressive decrease of the credibility of the neutrino hypothesis caused by the standard model because of the proliferation of the original hypothesis into electron, muon, and tau neutrinos without directly verifiable physical differences; then their interchange without a clear physical process; then the assumption that neutrinos have masses; and expected new conjectures. The decrease of credibility of the Pauli-Fermi hypothesis is due to implausibility that massive particles can cross entire planets and stars without appreciable scatterings. By comparison, the etherino hypothesis is more plausible since the etherino propagates through the universal *substratum* underlying nuclei and all matter.

We should point out that the etherino can indeed represent the experimental data obtained so far for the neutrino hypothesis, to such an extent that the etherino interpretation would require a change of names jointly with the reformulation of decay or scattering processes via isomathematics and isomechanics. The reader should also note the full compatibility of the etherino hypothesis with the above adopted duality of using the standard model for the classification of particles into families and a different model

for the structure of individual particles of a given family.

We should also indicate that the etherino hypothesis has no known consistent formulation within the context of quantum mechanics due to its *external* nature with respect to the Hilbert space and other reasons. By contrast, the covering isomathematics and isomechanics have been formulated to achieve a generalization of the Hilbert space such to include external sources such as the etherino.

In regard to nuclear transmutations, the etherino can be characterized with the symbol along the assumptions of Section 2.1 $a = a(0,0,0)$ where one should note the zero value of the spin due to its redundancy, since the spin $1/2$ of the neutron is represented by the novel orbital contribution in the neutron synthesis (Figure 6 and related comments in Section 2 [17-26]).

Consequently, nuclear transmutations implying the conjecture of the emission of a neutrino as in, Eq. (3.7) become [52] are reinterpreted in the form

$$\mathrm{N}(Z,A,J) + \tilde{n}(0,1,0) \to \tilde{\mathrm{N}}(Z,A+1,J) + a(0,0,0) \to$$

$$\mathrm{N}'(Z+1,A+1,J+1/2) \to \mathrm{N}''(Z+1,A+1,J), \quad (5.5)$$

where N′ and N" are tabulated nuclides.

As a final comment, interested readers should meditate a moment on the cosmological implications of the etherino hypothesis, since it implies a return to the historical view on the continuous creation of matter in the universe due to the extremely large transfer of energy from the ether to neutrons synthesized in stars, supernova and other astrophysical processes [52].

All in all, we can safely conclude with the view, evident to all "True Researchers" in the sense of Albert Einstein, that, despite historical advances, our current mathematical, physical and chemical knowledge is extremely limited and so much remains to be discovered, provided that the emphasis is placed in *the pursuit of new knowledge*, rather than in "the preservation of old doctrines" for evident political gains.

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Note Added in Proof

Following the completion of this paper, the authors have been informed of the important historical contribution in the field by E. Recami which is here added as Ref. [53] with a link for its access.

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A Possible Angular Quantization as a Complement to the Conventional Radial Quantization in the Hydrogen Atom and Aqueous Systems

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Abstract: In this work we propose, apparently for the first time, a possible *angular* quantization as a complement for the conventional *radial* quantization with the intent of initiating quantitative studies regarding the capability of liquid water to acquire and propagate information. We articulate the proposed angular quantization via the absorption of thermal energy by the hydrogen atom in the ground state at absolute zero degree temperature prior to the transition to the first excited state. We extend the proposed angular quantization to the hydrogen and water molecules; and conclude that if our model of angular quantization is confirmed, the liquid state of water has the capability of acquiring and propagating a truly vast quantity of information, explaining demonstrated chemo-analogous biological effects apart from chemical exposure.

Keywords: Hydrogen Atom, Quantization, Energy Absorption, Aqueous System, Information

1. Experimental Relevance

This work represents a first step towards the specific explanation of informational effects upon biological systems mediated through aqueous systems as defined through the rigorous formalism of hadronic mechanics. As is well known, quantum chemistry, although a laudable and accurate theory, is unable to usefully describe aqueous dynamics beyond clear limitations.

> "Recall that quantum chemistry was unable to achieve an exact and invariant representation of the main characteristics of the water molecule from unadulterated first principles despite efforts over the past century. In fact, a historical 2% has been missing in the representation of the water binding energy, while the representation of its electric and magnetic moments was embarrassingly wrong even in the signs." (Santilli 2005, p. 142).

Naturally, any proposed new formalism which aims to describe a complex system from first principles must begin with the most abundant component piece of the aqueous assembly: the hydrogen atom. In this way the many experimental effects we have demonstrated (Norman et al. 2016; Norman and Dunning-Davies, 2017) upon biological morphology and proliferation through chemo-molecularly derived biologically efficacious *information alone apart from molecular exposure of cells and tissues* can be explained and tested. Why have we been able to demonstrate in many experiments, that encoded *information alone* once mediated through aqueous systems affects diseases such as MRSA and the morphology of cancerous cells, in a way similar to the source drug or agent molecule from which the information was derived? This formalism will begin to explain at a causal level how the aqueous system holds information thereby affecting biological outcomes by way of hadronic mechanics and the resultant chemistry of the magnecular model (Santilli, 2001, 2005, 2017; Norman and Dunning-Davies, 2017).

2. Theory

In this note, we hope to initiate quantitative studies regarding the capacity of liquid water to acquire and propagate a great variety of information at large distances, as is made evident by sharks which sense blood and prey at such distances as to exclude, in our opinion, a sensible quantitative interpretation solely via dilution, which would indicate in some species, sensitivity of scent distinction of one part in ten billion, or one drop in an Olympic-size swimming pool (web ref. [11, 12]).

Our study is based on the principles of quantum mechanics according to which the energy absorption by the hydrogen atom cannot be continuous and, therefore, must be discrete.

To begin, we consider the ground state of the hydrogen atom at absolute zero degree temperature. We assume in first approximation that such a ground state is entirely contained in a plane, thus ignoring fluctuations due to the uncertainty principle and other contributions because of expected second order effects.

The conventional quantization of the hydrogen atom shall be called hereon *radial quantization,* (and be denoted with the superfix "*r*") to indicate its emphasis on a toroidal distribution of quantized orbits as per the magnecular model.

We will study the absorption by the hydrogen atom of thermal energy (e.g., due to infrared waves) in cases where energetic quanta are assumed to be *smaller* than the energy needed for the transition from the ground state to the first excited state.

Our main assumption is that, in order to verify quantum mechanics, the indicated absorption cannot be continuous. Consequently, the said absorption is here assumed, apparently for the first time, to be quantized. Alternative views would imply either deviations from quantum mechanics or the inability by the hydrogen atom to absorb heat.

Since, by assumption, the absorbed energy does not allow the transition to the first excited state, the most plausible transition of the hydrogen atom is that from the planar distribution to a distribution in three dimensions, thus implying the appearance of an *angular component* in the shape of the electron orbits.

The quantization of the absorbed thermal energy by the hydrogen atom when insufficient for the transition to an excited state is hereon called *angular quantization* (and denoted with the superfix "*a*") to indicate its emphasis on angular distributions over the plane of the ground state at 0° *K*.

The next issue is the identification of the *shape* of the orbit of the electron for the indicated energy absorption. As the considered excitation cannot lead to the first excited state, since the electron orbit is expected to remain near that of the ground state, and since the excited orbit under consideration cannot be radial to avoid violation of quantum mechanics, the most plausible shape is that of a *toroid distribution* much along the lines of that introduced in (Santilli, 2001) at the emergence of the new chemical species of magnecules.

The next issue is the shape of the electron orbit inside the indicated toroid. Since the said trajectory cannot be radial for the reason indicated above, the most plausible trajectory in angular quantization is that of a helix entirely contained in the indicated toroids, as illustrated in Figure 1.

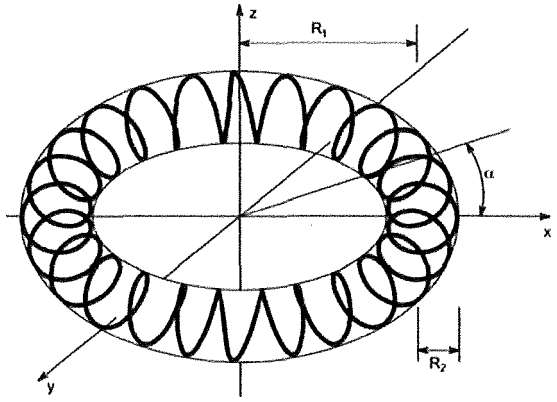

Figure 1. *Above.*

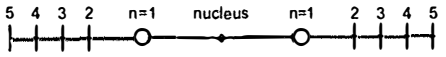

FIG. 2A

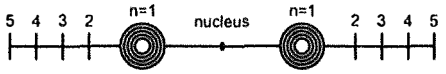

FIG. 2B

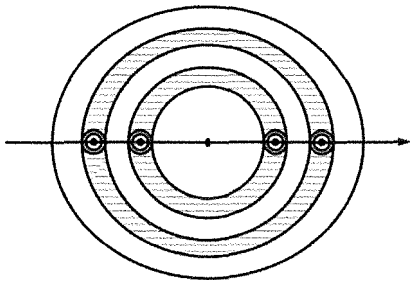

FIG. 2C

Figure 2. *Above.*

In order to appraise the plausibility of orbital quantization, we believe that the first quantitative treatment should be done in the simplest possible, semi-classical Bohr approximation, and then pass to full treatments in first and second quantization.

Therefore, we consider the wavelength of the electron of the hydrogen atom for the ground state and the first excited state in the Bohr approximation

$$\lambda_1^r = 2\pi r_1 = 6.28b, \lambda_2^r = \pi r_2 = 12.57b. \quad (1)$$

with related difference

$$\Delta\lambda^r = \lambda_2^r - \lambda_1^r = 6.29\ b \quad (2)$$

where b = 0.053 × 10−8 cm is the well known Bohr radius.

The thermal energy absorption here considered should verify the condition

$$\Delta\lambda^a \leq 6.29\ b. \quad (3)$$

We consider now the corresponding (negative) values of the energy

$$E_1^r = 1.36\ eV,\ E_2^r = {}^{1.36}/_{2^2}\ eV = 0.34eV \quad (4)$$

Consequently, the thermal energy absorbed by the hydrogen atom E^a here under consideration is restricted by the condition

$$E^a \leq 1.02\ eV. \quad (5)$$

We consider now the quantization of the helical trajectory of the electron inside the toroid as illustrated in Figure 1. As one can see, the said trajectory is characterized by two lengths: r_1, which is the original radius of the ground state; and r_2, which is the radius of the toroid; plus the number of turns p of the helix in the said toroid.

The quantization connected to r_1 is conventional and, therefore, it is indicated with the traditional quantum number $n = 1,2,3,...$; the quantization connected to r_2 will be indicated with the new quantum number $m = 1,2,3,...$; finally, the number of turns cannot be fractional for stability, thus constituting a third quantum number $p = 1,2,3,....$

Therefore, the wavelength created by thermal energy λ^a has the following functional dependence

$$\lambda^a = \lambda^a(r_1, r_2, n, m, p) \quad (6)$$

under conditions (3) and (5).

By keeping in mind the limitations of our model that are unavoidable for first studies, the following ideas are important. In order to initiate a quantitative treatment, we approximate the toroidal helix of Figure 1 with a cylindrical helix with length $2\pi r_1$, cross-sectional area with radius r_2 and the same number of turns p, in which case the length of the helix and, therefore, the wavelength electron in orbital quantization, is given by the known expression

$$\lambda^a_{n=m=p=1} = \sqrt{(2\pi r_1/np)^2 + (2\pi r_2 p/m)^2} \quad (7)$$

where one recognizes the conventional quantization for radius r_1, and two new quantizations - that for the radius r_2 and that for the turns.

The first important implication of our model is the introduction, apparently for the first time since the initiation of studies on the hydrogen atom, of a basically new quantization that is geometrically based on the angle α of Figure 1, although mathematically treated through the radius r_2.

It should be stressed that representation (7) is only an *approximation* of the expected full angular quantization of the electron trajectory inside the toroid which is currently under study. In fact, expression (7) is a mere "linearization" of the toroid. However, to be valid, the said linearization should occur for the helix in the toroid without solution of continuity, that is, without beginning and end. The verification of the latter condition is done via the condition that the number of turns p be positive integers.

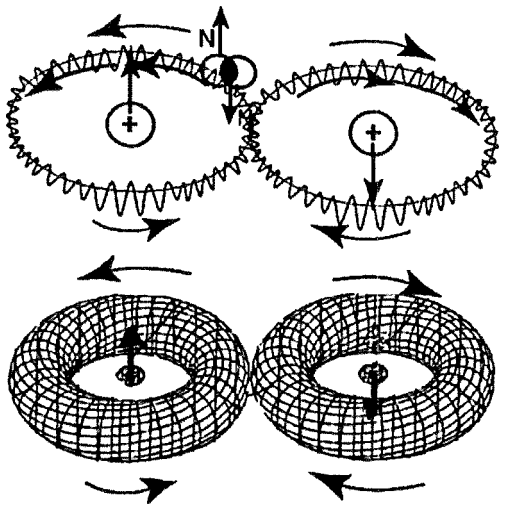

Figure 3. *Above.*

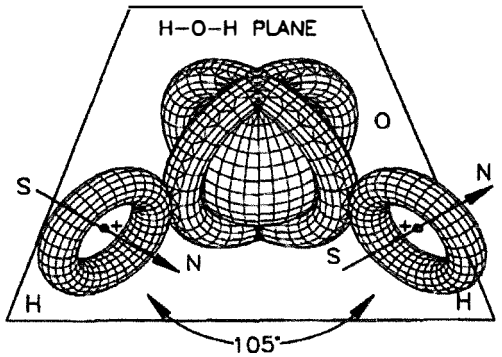

Figure 4. *Above.*

The following comments are in order:

1. When the radius of the toroid is null, $r_2 = 0$ (in which case $p = 1$), the absorption of thermal energy is null, and Eq. (7) recovers Bohr's quantization uniquely and identically,

$$\lambda^a \equiv \lambda^r = 2\pi r_2/n \quad (8)$$

as a result of which our model can be considered as being a "completion" of the Bohr model with quantized angular contributions.

2. The smallest possible quantized absorption of thermal energy by the ground state occurs for $n = m = p = 1$, with wavelength

$$\lambda^{a}_{n=m=p=1} = \sqrt{(2\pi r_1)^2 + (2\pi r_2)^2} \qquad (9)$$

namely, the wavelength is given by Bohr's wavelength for the ground state plus a correction term.

3. When the absorbed thermal energy is that with the equality in value (5), there is the transition to Bohr's first excited state with no toroids. The same occurs for subsequent excited states.
4. It is evident that, for large values of the quantum numbers *n, m* at constant *p*, the absorption of thermal energy tends to zero, and so does the associated wavelength

$$Lim\lambda^{a}_{n,m\to\infty} = 0 \qquad (10)$$

By keeping in mind the limitations of our model that are unavoidable for first studies, the following comments are in order also:

i. In addition to the right and left degrees of freedom of the helix for a given *n*, there exists a large number of values of *m* and *p* verifying conditions (3) and (5), as necessitated for reasons indicated below. This occurrence is conceptually illustrated in the sectional view of Part B of Figure 2 with several concentric toroids associated with the ground state without encompassing the first excited state.

ii. Until now we have considered the absorption of thermal energy (e.g., due to infrared waves) by the *structure* of the hydrogen atom. In addition, we have the thermal energy due to collisions, which causes known rotations of the hydrogen atom conceptually illustrated in part C of Figure 2. The latter absorption occurs for *the hydrogen atom as a whole* and, as such, cannot be quantized.

iii. The application of a sufficiently strong external magnetic field to the hydrogen atom (such as that caused by a high current DC discharge in a hydrogen gas) eliminates the rotation, therefore exposing the quantized toroids of the new species of magnecules (Santilli, 2001) as experimentally confirmed by Day and Yang et al (Day, 2004; Yang et al. 2013, 2013*a*). A second important implication of our model presented in this note for the first time, is that the toroidal distributions at the foundation of the new chemical species of magnecules have a quantized structure in full agreement with quantum mechanics.

Needless to say, in order to allow, in due time, experimental verifications, the proposed angular quantization must be extended to the hydrogen molecule. For this purpose, we adopt the *Santilli-Shillady isochemical model of the hydrogen molecule* (Santilli, 2001) which is based on an actual *attractive force* between the identical valence electrons in singlet couplings; that attractive force being of sufficient strength to bond the electron pair into a quasiparticle called *isoelectronium.*

With reference to Figure 3, the application of our angular quantization to the hydrogen molecule is then given by the extension of the initial helical trajectory of one single electron (Figure 1) to the helical expression of two bonded electrons represented in the isoelectronium in the oo-shaped toroid of Figure 3. Therefore, the extension of our model to the isoelectronium of the hydrogen molecule appears indeed to allow the extension of the angular quantization from the hydrogen atom to the hydrogen molecule.

Verification and other issues:

The experimental verification of the proposed angular quantization is rather difficult because it requires:

1) the use of hydrogen at absolute zero degree temperature;

2) the irradiation of such a liquid state with infrared radiation with energy insufficient to cause the transition to the first excited state;

and

3) the accurate measurement of the temperature increase of the liquid. The possible detection of a discrete increase of temperature would then confirm our angular quantization.

A number of additional experiments based on spectral emissions is conceivable but would require detailed specific studies beyond the introductory level of this note.

However, second order demonstration and partial confirmation of the model is possible. In (Chen et al., 2017) (pp. 3-4) we read: "At water-water distances of R < 1 nm, water molecules experience changes in their orientational distribution on the order of 10° to 20° per molecule." It is possible to encode a 7Hz carrier frequency modulated at 3 KHz (Norman et al. 2016; Norman and Dunning-Davies, 2017) with the information associated with salt water. Perturbations of the collective H bond network causing ionic signature angular variance can be predicted. One would compare demonstrated angular perturbations of the pure water aqueous system exposed to the carrier frequency with no encoding, to that with encoding. We can predict the effect of the encoded information will be to create a similar perturbation in angular orientational distributions as that demonstrated by exposure to a chemical ionic constituent. If we apply the mathematics of our model and do indeed gain increased perturbation effects and improved biological informational response with predicted alterations in encoding methodologies, we may at least partially confirm the factual utility of the model from those second order effects. If so, we may also then speculate those long range perturbations, perhaps in the context of extended coherence implied via the Schumann fundamental (roughly analogous to our carrier frequency), might account for the shark's ability to sense blood and prey at extreme distance (web ref. [11, 12]), and demonstrate informational capacity in water.

In order to flesh out preliminary considerations on the possible application of angular quantization to the liquid state of water, we consider the conventional knowledge of the water molecule depicted in Figure 4, see e.g., (Eisenberg and Kauznann, 1969) in which the two hydrogen atoms do not have a spherical distribution, but their electrons are distributed in toroids as a condition for establishing a proper bond with the corresponding valence electron of the oxygen. The same toroidal distribution is then necessary for the two valence electrons of the oxygen.

Our angular quantization implies that the isoelectronia of the two $H - O$ dimers of the water molecule describe a

helical trajectory within the oo-shaped toroids of Figure 4. Consequently, the water molecule can indeed acquire a substantial amount of data expressed precisely by the angular quantization described above, as a function of the descriptively encoded multiplicity of quantum numbers associated with the evolving toroids, the angular interactivities demonstrated by individual water molecules and their collective orientation in the H-bond network. Future work will spell out the toroid as a proposed new sort of harmonic oscillator, within the context of the magnecular H bond model.

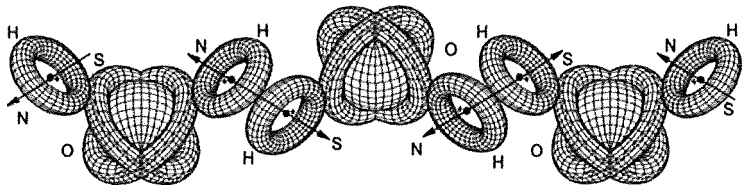

Figure 5. *Above.*

In order to initiate a quantitative study pertaining to informational propagation in aqueous systems, it is necessary, in our view, to use a model of the liquid state with an explicitly articulated *attractive force* between the water molecules, such as the magnecular model of Santilli (Santilli, 2017). In fact, only as a function of this attractive force can the information acquired by one hydrogen atom of the water molecule be propagated to the corresponding hydrogen atom of the next water molecule, evidently with progressively decreasing intensity.

3. Conclusion

In conclusion, it appears that should it be confirmed, our model of angular quantization can indeed allow a quantitative understanding of functional informational capacity associated with the water molecule permitting acquisition of a large volume of information propagated at considerable distance across liquid aqueous systems.

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