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Exact and Invariant Representation of Nuclear Magnetic Moments and Spins According to Hadronic Mechanics

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Abstract: In order to render this paper minimally self-sufficient, we review and specialize the main structure of the isomathematics to nuclear constituents as extended and deformable charge distributions under linear and non-linear, local and non-local and Hamiltonian as well non-Hamiltonian interactions; we then review and specialize for the nuclear structure the main laws of the isotopic branch of hadronic mechanics known as isomechanics; we review and specialize the method for turning quantum mechanical nuclear models for point-like nucleons into covering isomechanical models for extended and deformable constituents under the most general known realization of strong interactions; we then review and specialize to nuclear structures the consequential notion of isoparticles; we then review the ensuing, first known, numerically exact and time invariant representation of the magnetic moments of stable nuclides; we then review the structure of the neutron as a bound state according to isomechanics of an isoproton and an isoelectron; and we finally review the ensuing three-body structure of the Deuteron. Via the use of the preceding advances. We then present, apparently for the first time, a numerically exact and time invariant representation of the spin of stable nuclides, firstly, via their approximation as isotopic bound states of isodeuterons, isoneutron and isoprotons, and secondly, via their reduction to isobound states of isoprotons and isoelectrons. Some observations on the nuclear configurations so obtained have also been presented in the case of the first model and in view of the second option we have identified in isoelectrons the nuclear glue which tightly holds isonucleons of stable nuclide in the atomic nucleus in the preferred orientation of their intrinsic spins. In Appendix A, we provide a technical review specialized for the first time to nuclear physics of the Lie-Santilli theory and its main application to the notion of isoparticles as isoirreducible isounitary isorepresentations of the Lorentz-Poincaré-Santilli isosymmetry.

Keywords: Hadronic Mechanics, Nuclear Magnetic Moments, Nuclear Spins

PACS: 21.10. Hw; 21.30.-x; 21.60. De; 21.30. Fe

This paper is dedicated to the memory of Enrico Fermi who: expressed doubts as to whether conventional geometries apply to the structure of particles; supported the introduction of the size of nucleons for basic advances in nuclear physics; and suggested that the anomalous magnetic moment of nuclei may be due to the deformation of their charge distributions under the strong nuclear forces [1], *all visions that are quantitatively studies in this paper.*

1. Introduction

In the authors view, quantum mechanics is exactly valid for the atomic structure, but it is only approximately valid for the nuclear structure because quantum mechanics achieved a very accurate representation of atomic data, compared to the known inability by quantum mechanics to achieve an accurate representation of nuclear data, thus supporting the historical argument by Einstein, Podolsky and Rosen according to which quantum mechanics is “incomplete" [2].

A first reason for the above dichotomy is the fact that the mathematics underlying quantum mechanics (including the local-differential calculus, functional analysis, Hilbert spaces, Lie algebras, etc.) can only represent a finite number of isolated point-particles moving in vacuum, which conditions are known as characterizing *exterior dynamical problems*. The abstraction of particles into dimensionless points is evidently effective for the atomic structure due to the large mutual

distances of the atomic constituents, but the same abstraction is ineffective for the nuclear structure because nuclear constituents consist of extended charge distributions in conditions of partial mutual penetration, which conditions are known as characterizing broader interior dynamical problems (Figure 1).

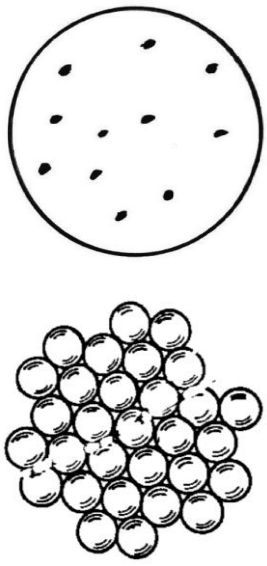

Figure 1. *A conceptual rendering (top view) of the abstraction of the nuclear structure as a sphere with isolated point-particles in its interior which is necessary for the applicability of the mathematics underlying quantum mechanics, namely, the local-differential calculus with related Hilbert spaces and Lie's theory. By contrast (bottom view), the nuclear structure consists of hyperdense, extended charge distributions in conditions of partial mutual penetration, as one can verify by the fact that nuclear volumes are generally smaller than the sum of the volumes of the constituent protons and neutrons. In the authors view, the inability by quantum mechanics to achieve an exact representation of nuclear data is due to the evident insufficiency of the abstraction of the nuclear structure depicted in the top when compared to the physical reality of the bottom view. This first insufficiency establishes the fundamental need of a basically new mathematics for the representation of extended charge distributions as they occur in the nuclear reality (Section 2).*

A second reason for the above dichotomy is that the fundamental symmetries of non-relativistic and relativistic quantum mechanics, the Galileo and Poincaré symmetries respectively, are solely valid for a *Keplerian system*, namely, for a system of particles orbiting around a heavier center, as it is the case indeed for atomic structures. By contrast, as stressed in the recent literature, *nuclei do not have nuclei* and, therefore, the symmetries valid for systems of point particles with a Keplerian nucleus cannot possibly be exactly valid for structurally different systems of extended particles without a Keplerian nucleus (Figure 2).

A third reason for the above dichotomy is that none of the 20th century sciences, including quantum mechanics and special relativity, can represent Fermi's historical hypothesis that the deviations of the values of nuclear magnetic moments from the predictions of relativistic quantum mechanics are due to deformations of the charge distributions of protons and neutrons (nucleons) when under the strong interactions of a nuclear structure, with consequential alteration (called in this paper *mutation*) of their intrinsic magnetic moments (Figure 3). This insufficiency is evidently due to the fact that *dimensionless points cannot experience deformations.* Therefore, a mathematics which can solely represent dimensionless points is structurally unable to represent the deformation of extended charge distributions as they occur in the nuclear reality.

A fourth reason for the above dichotomy is the fact that *dimensionless points can only experience interactions at a distance, thus derivable from a potential* (interactions technically known as *variationally self-adjoint* [3a]). In view of this basic feature, recent representations of the strong nuclear force have reached un-reassuring limits, such as a Hamiltonian with forty or so potentials, without the desired achievement of an exact representation of nuclear data. In the authors view, it is necessary to complement these conventional studies with the admission that the interactions between extended charge distributions under conditions of partial mutual penetrations are of *contact* type, thus not being derivable from a potential (interactions technically known as *variationally non-selfadjoint* [3a]). Consequently, it is recommendable to ascertain whether some of the potential components of nuclear Hamiltonians should be replaced with non-Hamiltonian representations.

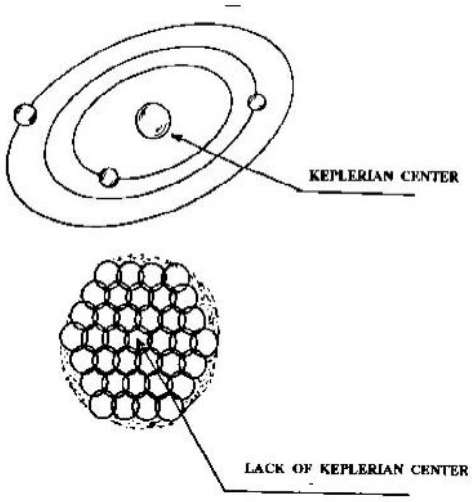

Figure 2. *A conceptual rendering of the second impossibility for quantum mechanics to be exactly valid for the nuclear structure, which is given by the fact that the basic symmetries of non-relativistic and relativistic quantum mechanics, the Galileo and Poincaré symmetries respectively, only apply for Keplerian systems of particles orbiting around a heavier nucleus. By contrast, R. M. Santilli has stated several times in his writings that "nuclei do not have nuclei," thus implying a necessary breaking of said fundamental symmetries, with consequential lack of exact character of non-relativistic and relativistic quantum mechanics for the nuclear structure. The same breaking is confirmed by numerous additional evidences, such as the fact that the partial mutual penetration of nucleons in a nuclear structure implies the presence of contact interactions not representable with a Hamiltonian, thus implying the inapplicability of the entire Lie theory, let alone of Lie's symmetries, due to its strictly Hamiltonian character. This second insufficiency establishes the need for a covering of Lie's theory for the construction of the symmetries of systems of extended particles without Keplerian nuclei under Hamiltonian as well as non-Hamiltonian internal forces (Section 2 and Appendix A).*

A fifth reason for the above dichotomy is that quantum mechanics is certainly effective for the description of nuclear *fissions* due to the effective representation of the fission debris as point particles, but quantum mechanics has proved to be ineffective for the achievement of nuclear *fusions* for all the above indicated reasons, plus the fact that *nuclear fusions are structurally irreversible over time while quantum mechanics is structurally reversible,* hence the need for a covering of quantum mechanics that can represent extended charge distributions with Hamiltonian and non-Hamiltonian interactions in generally irreversible conditions.

In this paper, we shall briefly outline decades of research by one of us (R. M. Santilli) [3-33] for: the construction of a generalization of 20^{th} century mathematics suitable to represent extended particles (Figure 1); the generalization of Lie's theory for the construction of symmetries of systems of extended particles without Keplerian center under Hamiltonian and non-Hamiltonian internal forces (Figure 2); the representation of Fermi's historical hypothesis on the deformability of nucleons; and the consequential, first known, exact and time invariant representation of nuclear magnetic moments (Figure 3).

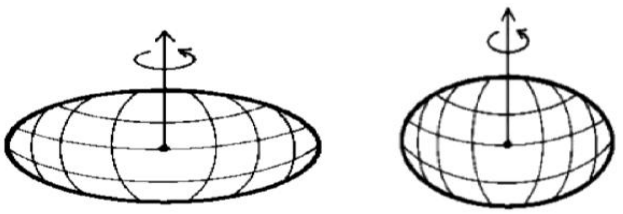

Figure 3. *A third insufficiency of quantum mechanics for nuclear structures is given by the historical prediction by Enrico Fermi [1] that the anomalous values of nuclear magnetic moments is due to deformations of the charge distribution of protons and neutrons when under the strong interactions of the nuclear structure, with consequential alteration of their conventional magnetic moments. In fact, a quantitative treatment of Fermi's teaching requires the use of the deformation theory which is known to be incompatible with quantum mechanics. This third insufficiency establishes the need that the novel mathematics and Lie's theory for extended charge distributions should be constructed in such a way to be compatible with the deformation theory "ab initio" (Section 2 and Appendix A).*

Since the advances considered here [3-33] are only known to a restricted number of experts, and they are generally unknown to the nuclear physics community, in order to render minimally understandable the advances presented in this paper, it has been necessary to: outline in Section 2 the novel mathematics (known as *isomathematcs* for the reversible case and *genomathematics* for the irreversible form); outline in Section 3 the corresponding invariant branches of hadronic mechanics (known as *isomechanics* and *genomechanics* respectively); outline in Section 3.1 the non-relativistic nuclear isomechanics; outline is Section 3.2 the relativistic nuclear isomechanics; outline in Section 4 a simple construction of iso- and gene-mechanics; outline in subsequent sections the exact and time invariant representation of nuclear magnetic moments (Section 5), the test of spinorial symmetry by neutron interferometry (Section 6), and then outline the emerging new structure of the neutron (Section 7), deuteron and nuclei at large (Section 8). Above all, it has emerged as recommendable to formulate advances [3-33] in a form directly applicable to nuclear physics, rather than leaving such an adaptable to the imagination of non-initiated readers.

We shall then present, apparently for the first time, the achievement of an exact and time invariant representation of the spin of stable nuclide which, thanks to the above advances, is compatible with the mutation of the intrinsic magnetic moments of nucleons, and then indicate the implications of these advances in nuclear physics for basically new, environmentally acceptable forms of nuclear energies. For the sake of self sufficiency of this presentation we start with a very brief description of stable and unstable nuclides in Section 9, a brief description of new and old vistas of nuclear forces with the earlier conjectural assertions of the stability of nucleons in Section 10. In Section 11 we have developed notations to represent isoneutrons and isodeuterons. We have presented in Section 12 two models of nuclear configuration, namely (i) considering isodeuterons, isoneutrons and isoprotons as isonucleons (Section 12.1) and (ii) isoprotons and isoelectrons as isonucleons (Section 12.2). These nuclear configurations were written down in a way to be commensurate with the experimental nuclear spins and tabulated in Section 13 for both the nuclear models stated above. We have also presented our observations in Section 14 on these nuclear configurations with an idea to provide the facts about the isonucleons within the nuclides that would help in developing corresponding theories of nuclear stability and generate new explanations of other nuclear properties (Sections 14.1 and 14.2). In the second model of nuclear configuration arrived at in this paper we propose, apparently for the first time, that the isoelectrons serve as the nuclear glue that tightly holds the nuclear isoprotons together in the atomic nucleus (Section 14.2). For the sake of ready reference we have also presented the Lorentz-Poincaré-Santilli Isosymmetry and its characterization of isoparticles in Appendix A.

2. Elements of Iso-Mathematics and Geno-Mathematics

The first known time-invariant representation of extended and deformable charge distributions in interior dynamical conditions was proposed by Santilli in the early 1980s [3b] via the *isotopic* (in the sense of being axiom-reserving) lifting of the associative product AB between generic quantities (numbers, functions, matrices, operators, etc.) into the form, todays known as *Santilli isoproduct,*

$$AB = AB \to A\hat{T}B = A\hat{\times}B \tag{1}$$

where $\hat{T}$ is solely restricted to be invertible, but otherwise possesses an arbitrary dependence on local variables such as:

time t, coordinates r, velocities v, density μ, temperature τ, index of refraction ρ, frequency ω, wave functions ψ, etc., $\hat{T} = \hat{T}(t, r, v, \mu, \tau, \rho, \omega, \psi, ...)$.

When $\hat{T}$ is positive-definite and invariant under time-reversal $t \to -t$, it is called isotopic element, and when it is positive-definite (or merely Hermitean) but non-invariant under time reversal, it is called the genotopic element.

The representation of extended and deformable charge distributions is then immediately achieved via realizations of $\hat{T}$ of the type [3]

$$\hat{T} = Diag.\left(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, \frac{1}{n_4^2}\right) e^{-\Gamma(\psi,...)\int \psi^\dagger \psi dr^3} \tag{2}$$

where: $n_k = n_k(t, r, v, \mu, \tau, \delta, \omega, \psi, ...), k = 1, 2, 3,$ represent, in this simple case, the deformable semi-axes of a nucleon assumed for simplicity to be an ellipsoid; $n_4 = \rho$ characterizes the density of the nucleon considered; all quantities $n_\mu, \mu = 1, 2, 3, 4$, called characteristic quantities of the nucleon considered, are normalized to the value $n_\mu = 1$ for exterior conditions in vacuum; $\Gamma(\psi, ...)$ is a positive definite function or operator characterizes all non-linear interactions not representable with the conventional Hamiltonian; and the integral in the exponent of Eq. (2) tends to zero at mutual distances of particles much bigger than their charge radius (about $1\,\text{fm} = 10^{-13}$ cm), thus implying the limit

$$\lim_{r \gg 1\,\text{fm}} \hat{T} = 1, \tag{3}$$

for which

$$\lim_{r \gg 1\,\text{fm}} \left(A \hat{\times} B\right) = AB. \tag{4}$$

When $\hat{T}$ verifies the conditions

$$\hat{T}(t, ...) = \hat{T}^\dagger(t, ...) = \hat{T}(-t, ...) = \hat{T}^\dagger(-t, ...) \tag{5}$$

it is called the *isotopic element*, while under the verification of the conditions

$$\hat{T}(t, ...) = \hat{T}^\dagger(t, ...) \neq \hat{T}(-t, ...) = \hat{T}^\dagger(-t, ...) \tag{6}$$

$\hat{T}$ is called the genotopic element. Conditions (5) characterize the use of isomathematics, while conditions (6) characterize the use of the broader genomathematics. The most important mathematical difference is that the conventional Lie theory with historical product between Hermitean operators

$$[A, B] = AB - BA, \tag{7}$$

at the foundations of quantum mechanics is lifted in the former case into *Santilli Lie-isotopic theory* with basic product

$$[A, \hat{\times}, B] = A \hat{\times} B - B \hat{\times} A = A\hat{T}B - B\hat{T}A, \tag{8}$$

while in the latter case Lie's theory is lifted into the broader *Santilli Lie-admissible theory* with covering product

$$(A, \hat{\times}, B) = A\hat{T}(-t, ...)B - B\hat{T}(t, , ,)A = A\hat{R}B - B\hat{S}A, \tag{9}$$

$$R = R^\dagger, \; S = S^\dagger, \; R \neq S, \tag{10}$$

according to conceptions, formulations and terminologies first introduced by Santilli in Ref. [3b].

It should be indicated from the upset the importance of conditions (5) and (6) for nuclear physics. In fact, conditions (5) characterize a *stable nuclide* composed by extended nucleons when isolated from the rest of the universe, thus being reversible over time. By contrast, conditions (6) characterize *irreversible nuclear reactions*, such as nuclear syntheses.

In fact, as it is well known, the time reversibility of quantum mechanics is ultimately due to the *invariance of the Lie product under anti-Hermiticity* (for hermitean operators A and B)

$$[A, B] \equiv -[A, B]^\dagger. \tag{11}$$

It is then easy to see that isomathematics and its ensuing physical formulations are also time reversal invariant due to the *invariance of the Lie-Santilli isoproduct under anti Hermiticity,*

$$[A, \hat{\times}, B] \equiv -[A, \hat{\times}, B]^\dagger. \tag{12}$$

By contrast, genomathematics and its related physical formulations are irreversible over time precisely because *Santilli's Lie-admissible product violates, by central conception, the invariance under anti-Hermiticity*

$$(A, \hat{\times}, B) \neq -(A, \hat{\times}, B)^\dagger. \tag{13}$$

Monograph [3b] presented the lifting of most 20th century applied mathematics via the systematic lifting of all products into the isotopic form (1), although all liftings were formulated on conventional numeric fields.

Since this paper deals with magnetic moments and spins of stable, thus reversible nuclides, we shall mainly use isomathematics under basic conditions (5). However, it is recommendable for the non-initiated reader to know that that the extension to irreversible nuclear processes is immediate, thus being recommendable when applicable.

Subsequently, Santilli discovered that the emerging formulations were not invariant over time, that is, they failed to predict the same numerical values under the same conditions at different times. In order to resolve this basic insufficiency, Santilli re-examined in 1993 [4] conventional numeric fields $F(n, \times, 1)$ with classification of numbers n into real, complex or quaternionic numbers n, conventional associative product $nm = n \times m \in F$ and basic multiplicative unit 1, $1 \times n = n \times 1 = n \quad \forall n \in F$.

In this way, Santilli [4] discovered that the axioms of numeric field also admit solution with an arbitrary basic unit $\hat{I}$, under the conditions that: 1) all numbers are lifted in the isonumbers

$$n \to \hat{n} = n\hat{I}; \tag{14}$$

2) all products are lifted into the *isoproduct* (1),

$$nm \to n \hat{\times} m = n\hat{T}m; \tag{15}$$

and 3) the conventional unit 1 of 20th century numeric field is lifted into the isounit under the sole conditions of being positive-definite and being the inverse of isotopic element $\hat{T}$,

$$\hat{I} = \frac{1}{\hat{T}} \tag{16}$$

Under these conditions all axioms of a numeric field are verified and $\hat{I}$ is the correct left and right multiplicative unit,

$$\hat{I} \hat{\times} \hat{n} = \hat{n} \hat{\times} \hat{I} = \hat{n} \quad \forall \hat{n} \in \hat{F}. \tag{17}$$

Under conditions (4), $\hat{I}$ is called Santilli isounit, while under broader conditions (5) it is called Santilli genounit [4].

This lead to the discovery of new numeric fields $\hat{F}(\hat{n}, \hat{\times}, \hat{I})$ called isofields under conditions (4) and genofields under conditions (5) with corresponding novel isoreal, isocomplex and isoquaternionic numbers and general, genocomplex and genoquarternionic genonumbers $\hat{n} = n\hat{I}$.

Following the discovery of isonumbers and genonumbers, all theories originally formulated on conventional fields [3] where lifted into formulations defined over isofields and genofields [5, 6], but the crucial time invariance of the numeric predictions was still missing.

In order to resolve this impasse, Santilli reinspected in 1995 the Newton-Leibnitz differential calculus and discovered that, contrary to popular beliefs in mathematics and physics for centuries, the Newton-Leibnitz differential calculus depends on the assumed basic multiplicative unit because, in the event said unit has a functional dependence on the differentiation variable, the ordinary differential dr must be generalized into the form first introduced in memoir [7]

$$\hat{d}\hat{r} = \hat{T}d[r\hat{I}(\hat{r},...)] = dr + r\hat{T}d\hat{I}(\hat{r},...), \tag{18}$$

and called *isodifferential* under conditions (4) and *genodifferential* under conditions (5), with corresponding *isoderivatives* (and *genoderivatives*) [7]

$$\frac{\hat{\partial}\hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \frac{\partial \hat{f}(\hat{r})}{\partial \hat{r}} + \hat{f}(\hat{r})\hat{T}\frac{\partial \hat{I}(\hat{r},...)}{\partial \hat{r}}, \tag{19}$$

where, for consistency, coordinates and functions must be isoscalars, that is, have values in $\hat{F}$ with structures

$$\hat{r} = r\hat{I}(\hat{r},...), \quad \hat{f}(\hat{r},...) = f(\hat{r},...)\hat{I}(\hat{r},...). \tag{20}$$

It should be stressed that the representation of nuclear magnetic moments and spin presented in this paper depends crucially on a non-potential component of the nuclear force due to partial mutual penetration of the charge distribution of nucleons, which non-potential components is represented precisely via the isodifferential calculus and, therefore, with the novel additional terms in the r.h.s. of Eqs. (18) and (19).

In memoir [7] Santilli introduced a third broader mathematics under the name of *hypermathematics* which is given by a covering of genomathematics when the genounit is multi-valued (rather than multi-dimensional), e.g. of the ordered type $\hat{I} = \{\hat{I}_1, \hat{I}_2, ..., \hat{I}_n\}$ w here n can assume an arbitrarily larger values such as $n = 10^{50}$ as needed for biological structures.

The discovery of the generalized differential calculus signed the achievement in memoir [7] of mathematical maturity in the generalizations of 20th century applied mathematics at large, that stimulated seminal advances in mathematics (see representative monographs [8-11]) as well as generalized physical and chemical theories, including novel industrial applications indicated below.

The above studies lead to the following chain of generalized mathematics:

1. *IsoMathematics*, which is used for the representation of stable and isolated, thus time-reversible nuclei composed by extended nucleons in conditions of partial mutual penetration and is characterized by the lifting of the totality of 20th century applied mathematics in such a way to admit a positive-definite and time-reversal invariant isounit (5) at all levels of treatment.
2. GenoMathematics, which is used for the representation of time-irreversible nuclear reactions and it is characterized by a dual lifting of the totality of 20th century mathematics in such a way to admit a positive-definite time-noninvariant genounit (6) at all levels of treatment, one genounit, $\hat{I}(t,...) = 1/\hat{T}(t,...)$ characterizes motion forward in time, and its time reversal image $\hat{I}(-t,...) = 1/\hat{T}(-t,...)$ characterizes motion backward in time, irreversibility over time being assured by inequivalent forward and backward genounits $\hat{I}(t,...) \neq \hat{I}(-t,...)$.

The knowledge of the above distinct mathematics is important for researchers to prevent the use of time non-invariant isounits that may eventually imply irreversible contributions for the structure of isolated and stable nuclei, with evident inconsistencies.

Important independent contributions on the foundations of isomathematics and genomathematics can be found in monographs [8-11] and in their bibliographies.

The main methodological problems for the representation of nuclear magnetic moments and spins are the following:

2.I: The representation of the deformation of the charge distribution of protons and neutrons when members of a nuclear structure and the ensuing mutation of their intrinsic magnetic moments according to Fermi's historical hypothesis [2]. This first central problem was solved by Santilli via the use of the isotopies of the rotational symmetry [12], as reviewed in the

next section and in Appendix A.

2.II: The representation of the mutation of the intrinsic magnetic moments of nuclear constituents in a way compatible with the conventional ten conservation laws of total physical quantities (the conservation of the total angular momentum, total linear momentum, the center of motion, and the total energy), which must hold for all isolated bound states of particles. This problem was solved by Santilli by showing the isotopies of the Lorentz and of the Poincaré symmetry [15-17] do verify indeed said conventional total conservation laws because in the lifting of Lie's theory into the Lie-Santilli isotheory the generators of Lie algebras (that represent said conservation laws) remain unchanged, and only their products lifted for the representation of extended shapes and non-Hamiltonian interactions.

2.III: The representation of the spin of stable nuclides in a way compatible with mutation of the magnetic moments of protons and neutrons under strong nuclear interactions. This problem will be solved, apparently for the first time in this paper, by showing that the isotopies of the $SU(2)$-spin symmetry do indeed admit a "hidden" degree of freedom directly connected to the mutation of spin.

The central physical notion used in this paper for the solution of the above problems and for the characterization of extended-deformable nuclear constituents in conditions of partial mutual penetration is that of *isoparticle,* specialized to *isoprotons, isoneutron and isoelectrons.*

The understanding of the notion of isoparticle and, therefore, of this paper, requires at least some knowledge of the central branch of isomathematics used for the derivation of the new notion of isoparticle, which is given by the isotopies of Lie's theory, originally proposed by Santilli in monograph [3b], including the isotopies of universal enveloping associative algebras, Lie's theorem and Lie's transformation groups.

Among a rather large literature in the field, Santilli's papers specifically devoted to the notions of isoparticle are given by the isotopies of: the rotational symmetry $O(3)$ [12]; the $SU(2)$ spin symmetry [13, 14]; the Lorentz symmetry $O(3.1)$ in classical [15] and operator [16] forms; the isotopies of the Poincaré symmetry $P(3.1)$ [17]; the spinorial covering of the Poincaré symmetry [18]; and the isotopies of the Minkowskian geometry [19]. The notion of isoparticle was then studied in details in Refs. [20-23].

In view of these advances, the isotopies of Lie's theory are today called the *Lie-Santilli isotheory* (see independent studies [24-33]).

Due to its fundamental character for the exact and time invariant representation of magnetic moments and spins, the notion of isoparticle will be reviewed in detail in Appendix A.

3. Elements of Nuclear IsoMechanics and GenoMechanics

The non-unitary covering of quantum mechanics was proposed under the name of *hadronic mechanics* by R. M. Santilli in monograph [3b] of 1981 (see page 112 for the proposal of the name of the new mechanics.) The original proposal comprised two branches, the isotopic branch with Lie-isotopic structure (8) and in the genotopic branch with Lie-admissible structure (9).

A fundamental contribution to hadronic mechanics (which is fully valid nowadays) was provided in paper [34] of 1982 by the mathematician (late) H. C. Myung and R. M. Santilli via the isotopies and genotopies of the Hilbert space (today known as the *Hilbert-Myung-Santilli isospace and genospace* respectively) and the indication that hadronic mechanics removes the divergencies of quantum mechanics via the isotopies of Dirac Delta "distribution" (today known as the *Dirac-Myung-Santilli isodelta "function"* and the fast convergence of isotopic series (see, e.g., Ref. [35]).

These initial studies were formulated on a conventional field and elaborated via the conventional differential calculus. Hadronic mechanics achieved a mature formulation only following the discovery of the novel isonumbers and genonumbers [4] in 1993 and of the isodifferential and genodifferential calculus [7] in 1996 (see monographs [22] for a general presentation of hadronic mechanics, including the fundamental notion of iso- and geno-particles).

With the passing of time, the above indicated two branches of hadronic mechanics acquired the names of *isomechanics* and *genomechanics*, respectively. Since these names have received a rather wide acceptance by the physics community, they have been adopted in this paper.

The reader should be aware that hadronic mechanics has a variety of applications in disparate fields all dealing with interior dynamical problems. The main reference for the specialization of isomechanics to nuclear physics is given by memoir [26] of 1998, while the main reference for genomechanics is given by memoir [37] of 2006.

Since hadronic mechanics at large, as well as isotopic and genotopic branches are essentially unknown to the nuclear physics community, it appears recommendable to provide in this section an elementary review specialized to nuclear physics sufficient for the understanding of the derivation of exact and invariant magnetic moments and spins, with the understanding that an in depth study of memoirs [35, 36] is essential for serious knowledge.

3.1. Elements of Non-relativistic Nuclear IsoMechanics

Non-relativistic nuclear isomechanics is characterized by the lifting of Planck's constant $\hbar$ into a 3×3-dimensional, positive-definite *space isounit* [4, 7]

$$\hbar \to \hat{I}_{\hat{r}} = 1/\hat{T}_{\hat{r}} = Diag.(n_1^2, n_2^2, n_3^2) > 0 \tag{21}$$

where the quantities $n_k^2, k = 1,2,3$: represents ab initio the semiaxes of the extended-deformable shape of nucleons when members of a nuclear structure (see the l.h.s. of Figure 3); are

normalized to the perfect sphere $n_k^2 = 1$ in empty space; are restricted to be positive-definite and time-reversal invariant; and possess an unrestricted functional dependence on all needed local variables (see Section 1) $n_k^2 = n_k^2(\hat{t}, \hat{r}, \hat{v}, \hat{\mu}, \hat{\tau}, \hat{\rho}, \omega, \hat{\psi}, ...) > 0$, where the "hat" denotes the referral to *internal* variables, while variables without a "hat" refer to those of the *external* observer.

Note that we have ignored in Eq.(21) for simplicity the multiplicative exponential term representing internal non-linear, non-local and non-Hamiltonian interactions as in Eq. (2) since this term can be embedded into the n_k^2 via their simple redefinition.

Assumption (21) implies that all possible products AB of conventional nuclear formulations (including the product of numbers, functions, matrices, etc.) have to be lifted to the isoproduct [4],

$$AB \to A \hat{\times} B = A\hat{T}_{\hat{r}} B. \tag{22}$$

with Lie-isotopic structure (8) [4].

Assumptions (21) and (22) also implies that conventional numeric fields $F(n, \times, I)$ are lifted into isofields $\hat{F}(\hat{n}, \hat{\times}, \hat{I}_{\hat{r}})$ with isonumbers $\hat{n} = n\hat{I}_{\hat{r}}$ [4].

Note that, in the event the characteristic quantities $n's$ depend on time in a way not invariant under time-reversal, instead of the single unit (21) and product (22) for action to the right and to the left, we would have the genoproduct and genounit for motion forward in time [36]

$$AB \to A > B = AT(t,...)B, \ \hat{I}^{>} = 1/T(t,...), \tag{23}$$

and the genoproduct and genounit for motion backward in time

$$AB \to A < B = AT(-t,...)B, \ T(t,...) \neq T(-t), \ ^{<}\hat{I} = 1/T(-t,...) \tag{24}$$

with Lie-admissible structure (9).

Therefore, the use of time-reversal non-invariant quantities $n_k, k = 1,2,3$ for the study of a stable, time-reversal invariant nuclear structure would imply the inclusion of un-warranted irreversible contributions that should solely be admitted for irreversible nuclear reactions [36].

Nuclear isomechanics is additionally characterized by the lifting of time t into the isotime

$$t = t_{ext} \to \hat{t} = t_{int}\hat{I}_{\hat{t}} \tag{25}$$

where t_{ext} is the time of the external observer, t_{int} is the intrinsic time in the interior of nuclei, and $\hat{I}_t$ is different than $\hat{I}_r$, both dimensionally and numerically.

The representation of nuclear magnetic moments and spins has been achieved in the above stated paper via the simpler case in which $\hat{I}_{\hat{t}} = 1$ and the sole use of the time of the external observer $t = t_{ext}$. Consequently, isotime will be ignored for simplicity.

Nevertheless, the non-initiated reader should be aware that, on strict technical grounds, isomechanics implies that the time in the interior of nuclei is generally different than the external time [22].

The carrier isospace of isocoordinates $\hat{r} = r\hat{I}_{\hat{r}}$ is given by the Euclid-Santilli isospace [7] $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$ with isometric $\hat{\delta} = \hat{T}_{\hat{r}}\delta$ where $\delta = Diag.(1,1,1)$ the conventional Euclidean metric, with isoline element

$$\hat{r}^{\hat{2}} = \hat{r}^i \hat{\times} \hat{\delta}_{ij} \hat{\times} \hat{r}^j == (r^i \hat{T}_i^k \delta_{kj} r^j)\hat{I}_{\hat{r}} =$$

$$= \left(\frac{r_1^2}{n_1^2} + \frac{r_2^2}{n_2^2} + \frac{r_3^2}{n_3^2} \right) \hat{I}_{\hat{r}} \tag{26}$$

where one should keep in mind that the elements of the isometric must be isonumbers as a condition for the isoline element to be an isoscalar with value in the isoreal isofield R.

The understanding of nuclear isomechanics requires the knowledge that the isotopies map ellipsoids on conventional Euclidean space into the perfect sphere in the Euclid-Santilli isospace. This is due to the fact that the deformation of the semiaxes $1_k \to 1/n_k^2$ is done with the joint inverse deformation of the isounit $1_k \to n_k^2$, by therefore yielding the original value of the perfect sphere 1_k in isospace.

The reconstruction of the perfect sphere in isospace is essential for the isomorphism of the Lie-Santilli isorotations $\hat{O}(3)$ with the conventional rotations $O(3)$ under the central condition of including the deformation theory (Appendix A).

The isooperator isospace is given by the Hilbert-Myung-Santilli isospace [34] H defined on isofields of isocomplex isonumbers C with isounit (21), isostates $|\hat{\psi}(\hat{t}, \hat{r})\rangle$ and isoinner isoproduct

$$\langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle \hat{I}_{\hat{r}} = \langle \hat{\psi} | \hat{T}_r | \hat{\psi} \rangle \hat{I}_{\hat{r}}, \tag{27}$$

isonormalization

$$\langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle I_{\hat{r}} = \hat{I}_{\hat{r}} \tag{28}$$

and isoexpectation values for an iso-Hermitean isooperator, $\hat{Q}$,

$$\langle \hat{Q} \rangle = \langle \hat{\psi} | \hat{\times} \hat{Q} \hat{\times} | \hat{\psi} \rangle \hat{I}_{\hat{r}} = \langle \hat{\psi} | \hat{T}_{\hat{r}} \hat{Q} \hat{T}_{\hat{r}} | \hat{\psi} \rangle \hat{I}_{\hat{r}} \tag{29}$$

with particular properties

$$\hat{I}_{\hat{r}} \hat{\times} | \hat{\psi} \rangle = | \hat{\psi} \rangle, \ \ \langle \hat{\psi} | \hat{\times} \hat{I}_{\hat{r}} \hat{\times} | \hat{\psi} \rangle \hat{I}_{\hat{r}} = \hat{I}_{\hat{r}}. \tag{30}$$

confirming that $\hat{I}_r$ is the correct isounit of the theory.

The dynamical equations of non-relativistic nuclear isomechanics are given by the Schrödinger-Santilli isoequation on H over C [3, 7]

$$-\hat{i} \hat{\times} \hat{\partial}_{\hat{t}} | \hat{\psi} \rangle = \hat{H} \hat{\times} | \hat{\psi} \rangle = \hat{H}(\hat{r}, \hat{p}) \hat{T}_{\hat{r}} (\hat{\psi}, \hat{\partial}\hat{\psi}, ...) | \hat{\psi} \rangle =$$

$$= \hat{E}\hat{\times}|\hat{\psi}\rangle = E|\hat{\psi}\rangle \tag{31}$$

the *isolinear isomomentum*, introduced for the first time in memoir [7] following the discovery of the isodifferential calculus

$$\hat{p}\hat{\times}|\hat{\psi}\rangle = -\hat{i}\hat{\times}\hat{\partial}_{\hat{r}}|\hat{\psi}\rangle = -i\hat{I}_{\hat{r}}\partial_{\hat{r}}|\hat{\psi}\rangle, \tag{32}$$

the *Heisenberg-Santilli IsoEquation* in the infinitesimal version [3, 7]

$$\hat{i}\hat{\times}\frac{\hat{d}\hat{Q}}{\hat{d}\hat{t}} = [\hat{Q}\hat{,}\hat{H}] = \hat{Q}\hat{\times}\hat{H} - \hat{H}\hat{\times}\hat{Q} = \hat{Q}\hat{T}_{\hat{r}}\hat{H} - \hat{H}\hat{T}_{\hat{r}}\hat{Q} \tag{33}$$

the integrated version to a finite transform (see Refs [22, 36] for the correct formulation in isomechanics)

$$\hat{Q}(\hat{t}) = UQ(0)U^{\dagger} = e^{\hat{H}\hat{T}\hat{t}\hat{i}}Q(0)e^{-i\hat{t}\hat{T}\hat{H}}, \tag{34}$$

$$UU^{\dagger} \neq I, \tag{35}$$

and the isocommutation rules

$$[\hat{r}_i\hat{,}\hat{p}_j] = \delta_{ij}\hat{I}_{\hat{r}}, \quad [\hat{p}_i\hat{,}\hat{p}_j] = [\hat{r}_i\hat{,}\hat{r}_j] = 0. \tag{36}$$

where the "hat" on operators denotes their definition on $\hat{\mathrm{H}}$ over $\hat{\mathrm{C}}$

3.2. Elements of Relativistic Nuclear Isomechanics

Relativistic nuclear isomechanics is characterized by the lifting of Planck's constant $\hbar$ into a 4×4 -dimensional, positive-definite, thus diagonalizable isounit (see Refs. [22] and the memoir [44])

$$\hbar \to \hat{I} = 1/\hat{T} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2) > 0 \tag{37}$$

where: the $n_k^2, k = 1,2,3$ continue to represent deformed nucleons; n_4 is a geometrization of the hyperdense medium inside nucleons' the characteristics quantities $n_\mu, \mu = 1,2,3,4$ are subjected to the normalization for the vacuum $n_\mu^2 = 1$; the multiplicative exponential term as in Eq. (1) is absorbed by the n_μ which have an arbitrary functional dependence on local internal variables solely subjected to be invariant under time reversal.

Assumption (35) implies that the totality of all products AB of relativistic nuclear isoformulations are lifted into the isoproducts $A\hat{\times}B = ATB$ defined on isofields $\hat{I}(\hat{n},\hat{\times},\hat{I})$.

Again, care must be exercised in the study of stable nuclei in order to prevent the transition from isomechanics to genomechanics that occurs whenever the characteristic quantities n_μ are not invariant under time reversal.

Let $M(x,\eta,I)$ be the conventional Minkowski space with coordinates $x = (x^1, x^2, x^3, x^4 = t)$, metric $\eta = Diag.(1,1,1,-c^2)$ and unit $I = Diag.(1,1,1,1)$. Then, the relativistic isospace of the isocoordinates $\hat{x} = x\hat{I}$ is given by the Minkowski-Santilli isospace $\hat{M}(\hat{x},\hat{\eta},\hat{I})$ [15, 19] over the isofield of isoreal isonumbers R with isometric

$$\hat{\eta} = \hat{T}\eta = Diag.\left(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, -\frac{c^2}{n_4^2}\right)\hat{I}, \tag{38}$$

where the multiplication by $\hat{I}$ is necessary for the elements of the isometric to be isoscalars, with isoinvariant

$$\hat{x}^{\hat{2}} = \hat{x}^{\mu}\hat{\times}\hat{\eta}_{\mu\nu}\hat{\times}\hat{x}^{\nu} = (x^{\mu}\hat{\eta}_{\mu\nu}x^{\nu})\hat{I} =$$

$$= \left(\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \times \frac{c^2}{n_4^2}\right)\hat{I}, \tag{39}$$

It is evident that, according to then original conception [15], the isotopies of the Minkowski space represent locally varying speeds of light $C = c/n_4$, with consequent mutation of the conventional light cone, which features have been shown in memoir [38] to be compatible with the abstract axions of special relativity.

However, non-initiated readers should be aware that the isotopies reconstruct the perfect light cone in isospace $\hat{M}$ including c as the maximal causal speed. This is due to the fact that the speed of light is mutated in the value $c^2 \to c^2/n_4^2$, while the corresponding unit is mutated by the inverse amount $1_4 \to n_4^2$, thus preserving the maximal causal speed c in isospace $\hat{M}$ over the isofield $\hat{\mathrm{R}}$.

By linearizing the second order isoinvariant of the Poincaré-Santilli isosymmetry $P(3.1)$ as in the conventional case (see Appendix A), one reaches the fundamental equations of relativistic nuclear isomechanics which is given by the Dirac-Santilli isoequation [18]

$$(\hat{\eta}^{\mu\nu}\hat{\gamma}_{\mu}\hat{p}_{\nu} + \hat{i}\hat{\times}\hat{m}\hat{\times}\hat{C})\hat{\times}|\hat{\psi}(\hat{x})\rangle =$$

$$= (-i\hat{I}\hat{\eta}^{\mu\nu}\hat{\gamma}_{\mu}\hat{\partial}_{\nu} + imC)|\hat{\psi}(\hat{x})\rangle = 0. \tag{40}$$

which clearly illustrate the lifting of Plank's constant (35) when compared to the conventional equation, where the *Dirac-Santilli isogamma matrices* have a structure

$$\hat{\gamma}_k = \frac{1}{n_k}\begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \ \hat{\gamma}_4 = i\frac{1}{n_4}\begin{pmatrix} I_{2\times2} & 0 \\ 0 & -I_{2\times2} \end{pmatrix}, \tag{41}$$

with anti-isocommutation rules

$$\{\hat{\gamma}_{\mu}\hat{,}\hat{\gamma}_{\nu}\} = \hat{\gamma}_{\mu}\hat{T}\hat{\gamma}_{\nu} + \hat{\gamma}_{\nu}\hat{T}\hat{\gamma}_{\mu} = 2\hat{\eta}_{\mu\nu}, \tag{42}$$

Note in Eq. (38) the replacement of the speed of light c with the isospeed $C = c/n_4$. This is necessary because c is no longer invariant under the Poincaré-Santilli isosymmetry, while C is indeed invariant (Appendix A).

It should be indicated that the above formulation of the Dirac-santilli isoequation is solely based on the isotopies of spacetime without the isotopies of the spin of nucleons, since such an isotopy is sufficient for the derivation of nuclear magnetic moments and spins.

For a general study of the Dirac-Santilli isoequation, including the mutation of spacetime and spins as well as in regular and irregular realizations, we refer the interested reader to memoir [36].

The following comments are now in order:

3.1. By conception and construction, nuclear isomechanics is solely valid within regions of space of the nuclear radius (the order of 1 fm), because at larger distances the isounit recovers the convectional Planck's constant and, consequently, isomechanics recovers quantum mechanics uniquely and unambiguously (see Figure 4).

3.2. Also by conception and construction, nuclear isomechanics preserves the axioms of quantum mechanics and merely realize them via a broader mathematics. In fact, isomechanics and quantum mechanics coincide at the abstract realization-free level, to such an extent that they be expressed via the same equations only subjected to different realizations.

3.3. The name "hadronic mechanics" was suggested by Santilli [3b] for the representation of "hadrons" at large, thus including the representation of protons and neutrons. Consequently, nuclear isomechanics has been specifically constructed for the study of the nuclear stricture, while its covering genomechanics has been constructed to study nuclear reactions.

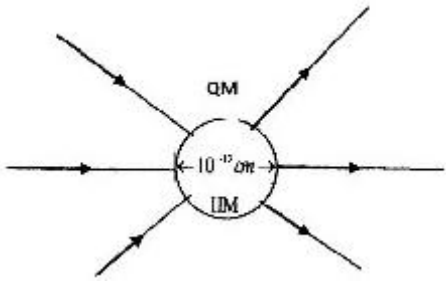

Figure 4. *A central feature of hadronic mechanics verified by all its branches is that the new mechanics is solely valid at distances of the order of* $1\ fm = 10^{-13}\,cm$ *because at larger distances it recovers quantum mechanics uniquely and unambiguously since at larger distances the isounit recovers Planck's constant.*

3.4. As it is well known, non-linear interactions (here referred to nonlinearity in the wave-functions) cannot be consistently represented via quantum mechanics since, in this case, they can be solely represented with a Hamiltonian $H(r, p, \psi)$, with the ensuing violation of the superposition and other laws. Consequently, quantum mechanics cannot consistently define nuclear constituents under non-linear terms of the nuclear force. By contrast, nuclear isomechanics can consistently represent non-linear terms in the nuclear force because all non-linear contributions are embedded in the isounit (or the isotopic element), by therefore maintaining the superposition principle on isospace over isofields. Additionally, nuclear isomechanics reconstructs linearity on isospaces over isofields with evident computational advantages.

3.5. The elementary review of this section has been necessarily incomplete to avoid excessive length. Therefore, interested readers are suggested to study memoir [36] for more technical details and monographs [22] for a comprehensive presentation. Particularly important is the acquisition of technical knowledge on properties such as: iso-Hermiticity coincides with conventional Hermiticity, as a result of which all observables of quantum mechanics remain observable for nuclear isomechanics; nuclear isomechanics eliminates the divergencies of quantum mechanics because all products of divergent series are lifted into the form given in Eq. (22) where the absolute value of the isotopic element $\hat{T}_{\hat{I}}$ is very small (see the negative sign of the exponent of Eq. (1); nuclear isomechanics is a "completion" of quantum mechanics according to the Einstein-Podolsky-Rosen argument, thus providing a concrete and explicit realization of "hidden variables" λ via the isotopic element $\hat{T}_{\hat{I}}$; and other important properties [22, 36].

3.6. The replacement of Planck's constant $\hbar$ into the integro-differential operator $\hat{I}$ is a representation of the expectation that, when nucleons are represented as expended charge distributions in conditions of partial mutual penetration, the energy exchange is at least in part continuous. However, the deviations from discrete energy exchanges in nuclear is very small due to the very small absolute value of the isotopic element 2. By contrast, the deviation of quantized energy exchanges for a proton in the core of a star are expected to be finite due to its total immersion with a hyperdense hadronic media for which quantized energy exchanges cannot be even defined.

4. Simple Construction of Isomechanics and Genomechanics

For the benefit of experimental nuclear physicists, it is important to note that any given quantum mechanical nuclear model can be lifted via an elementary procedure into the corresponding isomechanical form, by therefore performing the transition from the point-like abstraction of nucleons, to extended-deformable nucleons under potential as well as contact non-potential interactions.

Isomechanics is a structurally *non-unitary* theory when formulated on a conventional Hilbert space over a conventional numeric field, Eq. (35), while quantum mechanics is unitary. Therefore, the novel isomechanical contributions due to the extended-deformable character of nucleons as well as to the non-potential component of the nuclear force can be represented, from Eq. (21), with a non-unitary transform of the type

$$UU^{\dagger} = \hat{I} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2) \times e^{\Gamma(\psi,...)} \int \psi^{\dagger} \psi dr^3 \qquad (43)$$

It is then easy to see that the application of the above non-unitary transform to the "totality" of the formalist of a quantum nuclear model characterizes its isomechanical formulation in its entirety

$$I \to \hat{I} = U \times I \times U^{\dagger} = 1/\hat{T}, \tag{44a}$$

$$n \to \hat{n} = U \times n \times U^{\dagger} = n \times U \times U^{\dagger} = \\ = n \times \hat{I} \in \hat{F},, n \in F, \tag{44b}$$

$$e^{A} \to U \times e^{A} \times U^{\dagger} = \hat{I} \times e^{\hat{T} \times \hat{A}} = (e^{\hat{A} \times \hat{T}}) \times \hat{I}, \tag{44c}$$

$$A \times B \to U \times (A \times B) \times U^{\dagger} = \\ = (U \times A \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times B \times U^{\dagger}) = \hat{A} \hat{\times} \hat{B}, \tag{44d}$$

$$[X_i, X_j] \to U \times [X_i X_j] \times U^{\dagger} = \\ = [\hat{X}_i \hat{,} \hat{X}_j] = U \times (C_{ij}^{k} \times X_k) \times U^{\dagger} = \hat{C}_{ij}^{k} \hat{\times} \hat{X}_k = \\ = C_{ij}^{k} \times \hat{X}_k, \tag{44e}$$

$$\langle \psi | \times | \psi \rangle \to U \times \langle \psi | \times | \psi \rangle \times U^{\dagger} = \\ = \langle \psi | \times U^{\dagger} \times (U \times U^{\dagger})^{-1} \times U \times | \psi \rangle \times (U \times U^{\dagger}) = \\ = \langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle \times \hat{I}, \tag{44f}$$

$$H \times | \psi \rangle \to U \times (H \times | \psi \rangle) = \\ = (U \times H \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times | \psi \rangle) = \\ = \hat{H} \hat{\times} | \hat{\psi} \rangle, etc. \tag{44g}$$

It is easy to see that the application of an additional non-unitary transform causes the lack of time invariance of the isounit

$$W \times W^{\dagger} \neq I, \tag{45a}$$

$$\hat{I} \to \hat{I}' = W \times \hat{I} \times W^{\dagger} \neq \hat{I}, \tag{45b}$$

with consequential lack of invariance of the numeric predictions, with activation of the catastrophic inconsistencies [37], as well as the loss of the represented system.

However, any given nonn-unitary transform can be identically rewritten in the isounitary form on $\hat{\mathrm{H}}$ over $\hat{\mathrm{C}}$

$$W \times W^{\dagger} = \hat{I}, \quad W = \hat{W} \times \hat{T}^{1/2}, \tag{46}$$

$$W \times W^{\dagger} = \hat{W} \hat{\times} \hat{W}^{\dagger} = \hat{W}^{\dagger} \hat{\times} \hat{W} = \hat{I}, \tag{47}$$

under which we have the invariance of the isounit and isoproduct [22, 36, 37]

$$\hat{I} \to \hat{I}' = \hat{W} \hat{\times} \hat{I} \hat{\times} \hat{W}^{\dagger} = \hat{I}, \tag{48a}$$

$$\hat{A} \hat{\times} \hat{B} \to \hat{W} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{W}^{\dagger} = \\ = (\hat{W} \times \hat{T} \times \hat{A} \times \hat{T} \times \hat{W}^{\dagger}) \times (\hat{T} \times \hat{W}^{\dagger})^{-1} \times \hat{T} \times (\hat{W} \times \\ \times \hat{T})^{-1} \times (\hat{W} \times \hat{T} \times \hat{B} \times \hat{T} \times \hat{W}^{\dagger}) = \\ = \hat{A}' \times (\hat{W}^{\dagger} \times \hat{T} \times \hat{W})^{-1} \times \hat{B}' = \hat{A}' \times \hat{T} \times \hat{B}' = \hat{A}' \hat{\times} \hat{B}', etc. \tag{48b}$$

from which the invariance of the entire isotopic formalism follows.

Note that the invariance is ensured by the *numerically invariant values of the isounit and of the isotopic element under isounitary transforms,*

$$\hat{I} \to \hat{I}' \equiv \hat{I}, \tag{49a}$$

$$A \hat{\times} B \to A' \hat{\times}' B' \equiv A' \hat{\times} B', \tag{49b}$$

in a way fully equivalent to the invariance of quantum mechanics, as expected to be necessarily the case due to the preservation of the abstract axioms under isotopies. The resolution of the inconsistencies for non-invariant theories is then consequential.

It should be indicated that the above lifting of quantum into isomechanical models solely apply for the so-called *regular representations of the Lie-Santilli isotheory* (see Appendix A), that can be essentially expressed as representations preserving the conventional value of the spin, thus being sufficient for nuclear constituents.

Howsoever, the reader should be aware of the existence of *irregular representations of the Lie-Santilli isotheory* (see also Appendix A), which can be indicated as realizations of the axioms causing anomalous values of the spin, as expected for a proton when in the core of a star subjected to enormous pressures under which the very definition of conventional spin is technically flawed.

The lifting of a quantum mechanical nuclear model into the covering genomechanical version can be equally done via an elementary procedure, by performing the transition from time-reversible description to an irreversible one when applicable, e.g., for nuclear reactions.

Recall that genomathematics represent irreversibility by embedding the direction of time in the most ultimate quantities, the unit and related product. Therefore, the creation of a time ordering requires *two* different non-unitary transforms

$$UU^{\dagger} \neq I, \; WW^{\dagger} \neq I, \; UW^{\dagger} \neq I, \tag{50}$$

Then Planck's constant can be lifted in the form applicable for motion forward in time

$$\hbar = I \to \hat{I}^{>} = UIW^{\dagger} = 1/\hat{T}^{>} > 0, \tag{51}$$

with corresponding lifting of all products AB into the ordered genoproduct to the right

$$AB \to A > B = A\hat{T}^{>}B, \tag{52}$$

and lifting of $\hbar$ for motion backward in time

$$\hbar = I \to^{<} \hat{I} = WIU^{\dagger} = 1/^{<} \hat{T} > 0, \tag{53}$$

and corresponding lifting of all quantum products into the form ordered to the left

$$AB \to A < B = A^{<}\hat{T}B. \tag{54}$$

The irreversible character of the representation is then assured by the different values of the forward and backward genounits, with consequential incoherence of the related geno-Hilbert spaces (see memoir [37] for details).

5. Exact and Invariant Representation of Nuclear Magnetic Moments

Following the preparatory advances outlined in the preceding sections [3-37], the representation of Fermi's historical hypothesis on the representation of nuclear magnetic moments via the deformation of the charge distribution of nucleons (Section 1), becomes direct and immediate.

The first exact and time invariant representation of the anomalous magnetic moment of the Deuteron (where the term "anomalous" refers to deviations from quantum predictions) was achieved by R. M. Santilli in 1993 while visiting the JINRT in Dubna, Russia, and was presented at the local *International Symposium Deuteron-1993* [39]. The results were then extended to the representation of the anomalous magnetic moments of all stable nuclides in memoir [36] of 1998.

Let us recall from Refs. [2] that the magnetic moment of nucleons can be expressed in terms of their spin

$$\mu = g^S S + g^L L \tag{55}$$

where the g's are the spin and orbital gyro-magnetic factors with values in unit of nuclear magnetons for protons and neutrons

$$g_p^S = 5.585\,\text{nm}, \quad g_n^S = -3.826\,\text{nm}, \tag{56}$$

$$g_p^L = 1, \; g_n^L = 0. \tag{57}$$

By assuming that $L = 0$ for the ground state, the quantum mechanical (qm) prediction of the magnetic moment of the Deuteron is given by

$$\mu_D^{qm} = g_p^S S + g_n^S S = 0.879, \tag{58}$$

while the experimental value is given by '

$$\mu_D^{exp} = 0.857, \tag{59}$$

thus implying a deviation of 0.02 nm *in excess* between the prediction of quantum mechanics from experimental values.

It should be stressed that the "small" character of the deviation 0.02 nm may be misleading because it refers to the smallest nucleus, with increasingly embarrassing deviations for heavier nuclei, thus establishing the need for the exact and invariant representation of all nuclear magnetic moments, and not only that for the deuteron (Figure 5).

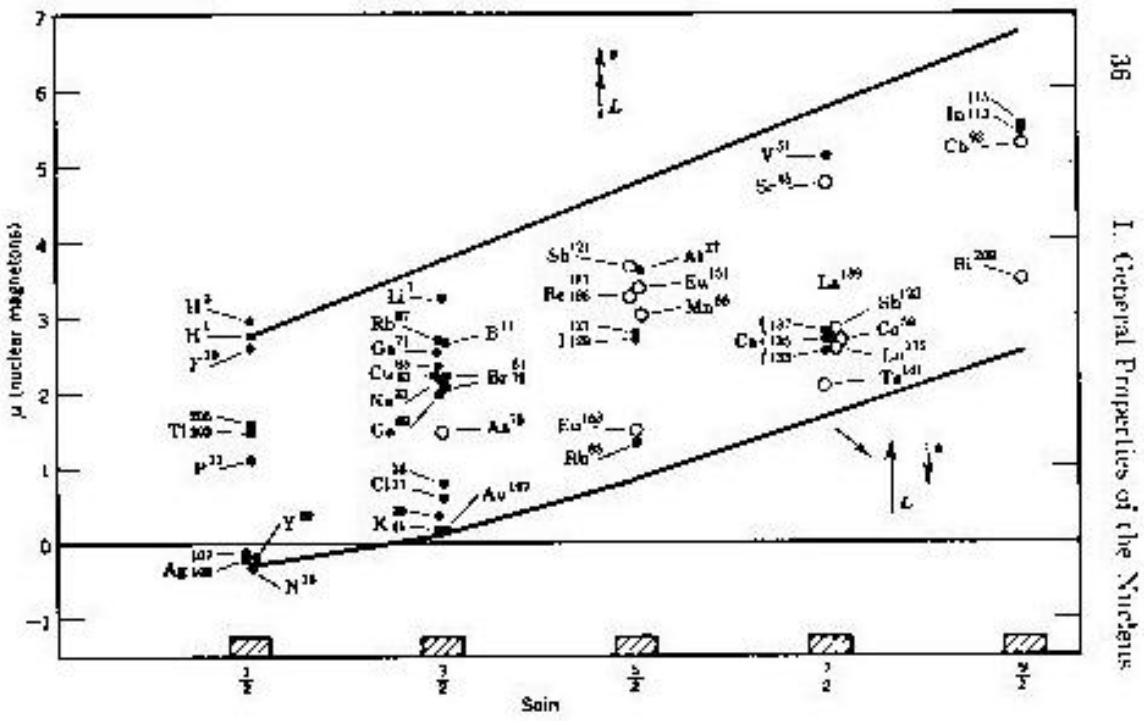

Figure 5. *On rigorous scientific grounds, a theory can be considered as being "exactly valid" for given physical conditions when it represents the entirety of the experimental data from unadulterated first principles. In this figure we reproduce the so-called "Schmidt limits" representing minimal and maximal values of nuclear magnetic moments. In the authors view, the Schmidt limits are a direct representation of the "deviations" of quantum mechanics from nuclear experimental data because they represent the deviation from quantum predictions for the simplest possible nucleus, the Deuteron, with increasingly embarrassing deviations for heavier nuclei. The achievement of an exact and invariant representation of nuclear magnetic moments according to Fermi's teaching (Section 1) has been a main motivation for the construction of the new isomathematics and isomechanics, as shown in Section 5.*

Attempts at the achievement of an exact representation of the anomalous magnetic moment of the Deuteron have been attempted for about one century via the use of quantum mechanics without any result that will resist the test of time.

The first attempts have been done by using an *ad hoc* combination of orbital angular momenta $L \neq 0$ of the proton and the neutron. However, the assumption $L \neq 0$ is in contradiction with the experimental evidence that the isolated Deuteron is in its ground state and, therefore, the orbital angular momenta of its constituents must be $L = 0$.

Numerous additional attempts have been done via relativistic corrections and relativistic field theory, by achieving the needed exact representation of the Deuteron magnetic moment with the introduction of arbitrary parameters or special form factors, thus, without deriving the needed value from first adulterated principles.

Additionally, it should be indicated that the reduction of protons and neutrons to the hypothetical quarks creates additional problems and solves none, because the hypothetical orbits of the hypothetical quarks inside nucleons are too small to admit a hypothetical polarization suitable for the representation of the Deuteron magnetic moment.

In conclusion, in 1993 the exact representation of the magnetic moment of the simplest nucleus, the Deuteron, let alone those of heavier nuclei (see Figure 5) had remained elusive because the proposed representations have contradictions or manipulations that will not resist the test of time.

In this way, Fermi's historical hypothesis acquires its full light when represented via isomathematics and isomechanics. The central conceptual and technical notion of nuclear isomechanics is that *the constituents of nuclei are "isoparticles"* (Ref. [40] and Appendix A), namely, *ordinary particles experience a mutation of their "intrinsic" characteristics when in conditions of partial penetration of their charge distributions (and/or wavepacket) as occurring in the nuclear structure, with ensuing exposure to the strong nuclear force.*[1]

The first intrinsic characteristics of particles experiencing a mutation under nuclear conditions is that of their intrinsic magnetic moments. Its explicit expression can be easily derived from the Dirac-Santilli isoequation (40) by repeating the corresponding procedure for the quantum case, yield the following *mutation of the intrinsic magnetic moment in the transition from particles to isoparticles* (see Ref. [39] as well as, for more details, Ref. [18])

$$\tilde{\mu}^{is} = \frac{n_4}{n_3}\mu, \tag{60}$$

where: (is) stands for isomechanics; we consider the magnetic moment along its symmetry axis, as usual; n_3 is the deformed semiaxis in the third direction; and n_4 a geometrization of the hyperdense medium inside nucleons.

We should note the use the upper symbol $\tilde{\mu}$, rather than $\hat{\mu}$, since the latter indicates elements of isofields because the use of the symbol $\hat{\mu}$ would indicate the transition from a scalar to an isoscalar (Section 2), which is merely given by the multiplication of the conventional magnetic moment and the isounit,

$$\hat{\tilde{\mu}} = \tilde{\mu}\hat{I}, \tag{61}$$

Due to the lack of impact to numerical values, the above isoscalar extension will be ignored hereon for simplicity.

From Eq. (60), we have the following mutation of the quantum mechanical magnetic moment (μ)

$$\tilde{\mu}_D^{is} = \tilde{g}_p^S S + \tilde{g}_n^S S = \frac{n_4}{n_3}\left(g_p^S S_p + g_n^S S\right) \tag{62}$$

where we have assumed for simplicity that the mutations of the charge distributions of the proton and the neutron are the same, since they have essentially the same volume and the same hyperdensity.

The numeric value of n_4 has been the subject of extensive phenomenological and experimental studies via the Bose-Einstein correlation and other particle experiments, resulting in the value (see Refs. [41-44] and Eqs. (6.1.101), page 864, Vol. IV of monographs [23])

$$n_4 = 0.654,\ n_4^2 = 0.355, \tag{63}$$

Consequently, under the numeric value of the third semiaxes

$$n_3 = 0.670,\ n_3^2 = 0.449, \tag{64}$$

we reach the following numerically exact and time invariant representation of the anomalous magnetic moment of the Deuteron according to isomathematics and isomechanics

$$\tilde{\mu}_D^{is} = \tilde{g}_p^S S + \tilde{g}_n^S S = 0.857\text{nm} \tag{65}$$

where we should note the use of slightly different numerical values than those used in the original derivation [39] due to advances occurred since 1993.

As one can see, the proton and the neutron are mutated from perfect spherical shapes when in vacuum under sole action-a-distance electromagnetic interactions, to a *oblate* charge distributions when constituents of the Deuteron, as expected in view of their high rotational conditions.

Note that *the deformability of nucleons under strong interactions does not imply the alteration of their volume due to their hyperdense character.* By assuming that the original semiaxes are normalized to 1, we then have the restriction on the numeric value of the remaining semiaxes

[1] The condition of partial mutual penetration of the charge distributions of protons and neutrons when nuclear constituents, can be easily derived by comparing the experimental values of nuclear volumes with those of protons and neutrons.

$$\frac{1}{n_1^2}+\frac{1}{n_2^2}+\frac{1}{n_3^2}=3, \tag{66}$$

under which we obtain the value of the remaining semiaxes of the oblate spheroid under the evident identification

$$n_1 = n_2 = 1.635,\ n_1^2 = n_2^2 = 2.574 \tag{67}$$

Even though the above values are certainly not suggested to be final, we can state that isomathematics and isomechanics provides the first known numerical values of the semiaxes of the proton and the neutron when constituents of the Deuteron, in a way compatible with the numerically exact and time invariant representation of the anomalous magnetic moment of the Deuteron (Figure 6).

As indicated earlier, the extension of the above representation for the Deuteron was extended to all stable nuclei in memoir [36] via the general *isomechanics representation of nuclear magnetic moments* [36].

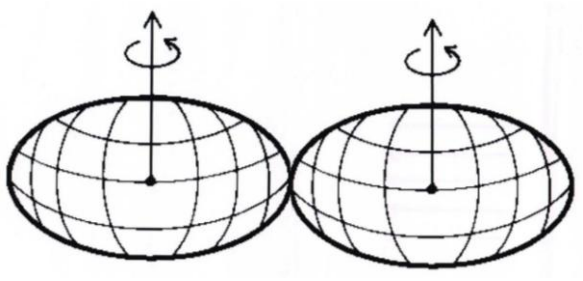

Figure 6. *A conceptual rendering of the oblate shape of the proton and the neutron when constituents of the Deuteron in its ground state achieved, for the first time to our knowledge, by nuclear isomechanics from the anomalous magnetic moment of the Deuteron, with values of the semiaxes* $n_3^2 = 0.449,\ n_1^2 = n_2^2 = 2.574$ *(see Section 5). It should be stressed that the construction has been done along the conventional conception of the Deuteron, namely, with the proton and the neutron with parallel spins in order to represent the spin* $_1$ *of the Deuteron. However, quantum mechanical axioms predicts that the sole stable bound state between two particles with spin 1/2 is the singlet with antiparallel spins, since couplings with parallel spins are predicted to be highly unstable due to strong "repulsive" forces. Therefore, in Section 8 we shall first review the structure of the Deuteron according to nuclear isomechanics with a representation of the spin 1 without structural inconsistencies, and then provide a more accurate representation of the magnetic moment of the Deuteron.*

$$\tilde{\mu}_N^{is} = \sum_{k=1,...,Z}\left[\frac{n_4}{n_{3kp}}\left(g_{pk}^S S + g_{pk}^L L\right)\right] + \sum_{k=1,...,A-Z}\left[\frac{n_4}{n_{3kn}}\left(g_{nk}^S S + g_{nk}^L L\right)\right], \tag{68}$$

where we have assumed that: all nucleons as nuclear constituents nuclei have the same hyper density geometrized by n_4; the deformation of the charge distributions may vary with the increase of the constituents; and anomalous orbital contributions may eventually emerge for heavier nuclei.

The verification that Eq. (68) provides indeed a representation of the magnetic moment of all stable nuclei will be shown in a subsequent paper. At this moment we merely limit ourselves with the representation later on of the magnetic moment light stable nuclei. The following comments are now in order:

5.1. Representation (65) is invariant over time because the mutation of intrinsic magnetic moments, Eq. (61) is a consequence of the Dirac-Santilli isoequation (which is invariant under the Poincaré-Santilli isosymmetry (Refs. [15-18] and Appendix A).

5.2. As one can see, representation (65) does not require the mutation of the spin of nuclear constituents because the sole mutation of the Minkowski spacetime into isospace (39) underlying the Dirac-Santilli isoequaiton (40) has been sufficient. This does not exclude extreme physical conditions, such as those at the core of stars that may require the mutation also of the spin.

5.3. As clearly shown by Eq. (62), the mutation of the intrinsic magnetic moment of nucleons under their conventional value of the spin creates the problem of the intrinsic compatibility of the approach. This problem is solved in Appendix A, where we show that the degree of freedom of regular isotopies of the SU(2) spin identified in Refs. [13, 14] can represent indeed the mutation of spin, thus achieving full compatibility under isomathematics and isomechanics between mutation of intrinsic magnetic moments and conventional values of spins.

5.4. We should indicate that value (63) for the geometrization of the hyperdense medium inside nucleons has been derived via experimental data on different events, such as the fireball of proton-Antiproton annihilation in the Bose-Einstein correlation, while direct experimental data for nucleons are not available at this writing. Therefore, it is possible that value (63) and, consequently, value (64), may need revisions following direct test on the density of the medium inside nucleons.

6. Test of the Spinorial Symmetry Via Neutron Interferometry

It is evident that the deformability of protons and neutrons under sufficient external forces requires a direct experimental verification. The ideally suited test is the so-called 4π neutron interferometric experiment which consists of (see Figure 7): a thermal neutron beam which is first coherently split into two beams by a perfect crystal; one of the two split beams passes through the gap of an electromagnet with the magnetic field calibrated to such the value $7,496$ G causing two complete spin flips (720° from which the name 4π) of the neutron on account of its intrinsic magnetic moment $-1.913148 \pm 0.000066 \mu_N$. The two beams are then coherently recombined. Various analysis are then conducted between the original beam and the recombined one.

When electromagnet gap is empty and, therefore, the split neutron beam travels in empty space, all known tests confirm

the achievement of two complete spin flips in full agreement with quantum mechanics. However, in order to avoid stray fields, the electromagnet gap is filled up with Mu-metal or other heavy metal sheets. In the latter cased, the test essentially provides a *test of the spinorial symmetry of neutrons under the intense electric and magnetic fields in the vicinity of Mu-metal nuclei, without any appreciable contributions from the strong interactions of Mu- metal nuclei.*

The rather bizarre history of this fundamental test can be summarized as follows. The Austrian neutron interferometric experimentalist H. Rauch and his Austrian associate A. Zeilinger participated to the 1981 *Third Workshop on Lie-Admissible Formulations*, and presented preliminary results of a 4π neutron interferometric experiment that was going on via thermal neutron beams available at the nuclear facilities in Grenoble, France.

In particular, Rauch and Zeilinger reported at said 1981 meeting that they were *not* measuring 720° rotations, by rather the following values of minimal and maximal rotations [45]

$$\theta_{min} = 715.87^{o},\ \theta_{max} = 719.67^{o},\ \theta_{aver} = 712.07^{o} \qquad (69)$$

which evidently *do not* contain 720°. In particular, Rauch and Zeilinger reported an angle of rotation systematically *smaller* then that expected, a feature referred to as the *angle slow-down effect* and expected to be due to a *decrease* of the intrinsic magnetic moment of the neutron under the strong fields of the Mu-metal nuclei.

The Austrian theoretical physicist G. Eder [46] who also attended the indicated 1981 workshop by presenting a Lie-admissible mutation of the rotational symmetry representing the decrease of the intrinsic magnetic moment of the neutron under the considered conditions in agreement with data (69).

Based on these studies, Santilli [47] presented at the same 1981 workshop the notion of *Lie-admissible mutation of elementary particles* (also called *genoparticles* under strong nuclear interactions considered as external (which is a condition to sue the irreversible Lie-admissible formulations). It should be noted that the Lie-isotopic notion of isoparticle presented in this paper is an evident particular case of the notion of genoparticles presented in 1981.

Immediately following the announcement of the above studies, H. Feshback, then chairman of the Department of Physics at MIT, strenuously opposed the completion of the 4π neutron interferometric experiment by Rauch and Zeilinger. the opposition, first by Feshback and then by his world wide collective was such that Rauch was prohibited the access at his own laboratory in Grenoble and was, therefore, prohibited its completion (see Refs. [48] and their three volumes of documentations).

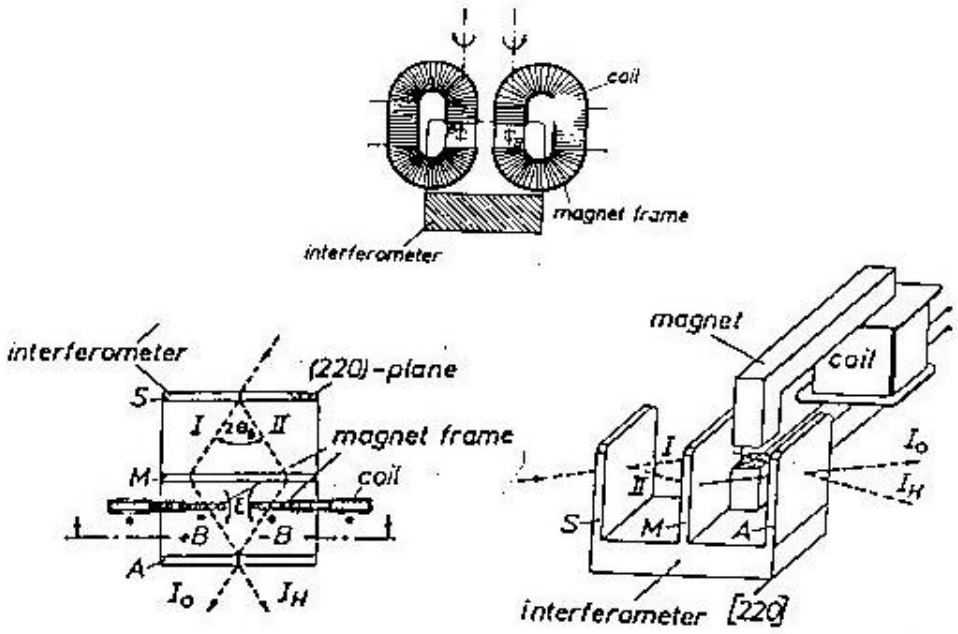

Figure 7. *A schematic view of one of the most fundamental experiments for the past half a century, the 4π neutron interferometric spinorial symmetry experiments described in Section 6 [45-51]. The lack of resolution of this experiments due to political obstructions [48] is one of the reasons fueling the growing view according to which we are currently eyewitnessing one of the biggest scientific obscurantism in the history of mankind.*

Subsequently, Rauch was offered the position of Director of the Atominstitut in Wien, Austria, while Zeilinger was invited for a one year stay at MIT after which he received a chair in physics at an Austrian university.

Following the above events in the early 1980s, the 4π neutron interferometric experiment was occasionally repeated, but either without heavy metal sheets in the electromagnet gap, by splitting the gap into two opposite contributions or in other versions essentially assuring the verification of the exact spinorial symmetry.

To our best knowledge, the current situation (October 2015) is the following. On one side, Rauch and Zeilinger have dismissed measurements (69) and claim the exact validity of the 4π symmetry (without any systematic experimental resolution on record), as reported by D. Kendellen [49] (see also book [50]).

By contrast, Santilli [51] claims that; 1) an accurate and unbiassed comparative analysis of the original and the recombined neutron beams show clear deviations from 4π rotations of at least 1%, even though the neutrons are solely exposed to *electromagnetic* interactions, thus expecting bigger deviations under nuclear strong interactions; 2) the deformability of the neutron is such a fundamental physical problem to require a systematic repetition of the 4π tests; and 3) Nowadays, the experiment can be repeated for a multiple of two complete rotations, with ensuing resolutory results (see Ref. [51] for details).

In the authors opinion, a reason for the incredible c hostility by the nuclear physics community against this fundamental experiment is the lack of technical knowledge of the Lie-Santilli isotheory according to which their fear of the violation of the "spinorial" symmetry in the 4π tests has no technical foundations because the experiment here considered deals with the *deformation of the charge distribution of the neutron while fully preserving its spin* $1/2$. In fact, the authors believe that the very name "spinorial" symmetry experiment is erroneous and misleading, since the Fermi-Dirac character of the neutrons remains fully valid under a deformation of their charge distribution (Appendix A).

In the final analysis, the serious scientist should keep in mind that perfectly rigid bodies solely exist in academic environments but they do not exist in nature. Therefore, the serious scientific issue is the measurement of the deformation of the charge distribution of neutrons for given sufficiently strong external forces, with the understanding that the deformability itself should be outside credible doubts.

7. The Synthesis of the Neutron from the Hydrogen

As it is well known, stars initiate their life as an aggregate of Hydrogen. The first nuclear synthesis in the core of a star is that of the neutron from the Hydrogen atom according to the historical reaction [2]

$$p^+ + e^- \to n + \nu \tag{70}$$

Deuterium, Tritium and other nuclei are synthesized only following the synthesis of the neutron. It is then evident that the understanding of the first and most basic synthesis of the neutron is crucial for a deeper understanding of the subsequent nuclear syntheses.

Unfortunately, the synthesis of the neutron is vastly ignored even at the most important Ph. D. courses in nuclear physics because it is *incompatible with quantum mechanics and special relativity.* This is due to the fact that *the rest energy of the neutron is bigger than the sum of the rest energies of the proton and of the electron,* as established by the known data

$$E_p = 938.272\,\text{MeV},\ E_e = 0.511\,\text{MeV},\ E_n = 939.565\,\text{MeV}, \tag{71a}$$

$$E_n - (E_p + E_e) = 0.782\,\text{MeV} > 0, \tag{71b}$$

Under these conditions, the Schrödinger equation does not yield physically consistent results due to the need for a “positive binding energy" resulting in a “mass excess" that are beyond any descriptive capacity of non-relativistic quantum mechanics.

Synthesis (70) is also incompatible with special relativity and relativistic quantum mechanics because the conventional Dirac equation, which is so effective for the description of the electron orbiting around the proton in the Hydrogen atom, becomes completely ineffective for the description of the same electron when “compressed" inside the proton in the core of a star according to Rutherford.

The proposal to build a non-unitary covering of quantum mechanics under the name of *hadronic mechanics*, including its isotopic and genotopic branches, was submitted in monograph [3b] precisely for the achievement of a quantitative representation of the synthesis of the neutron from the Hydrogen, and then apply the results to other nuclear syntheses.

Following decades of preparatory research [3-51], a numerically exact and time invariant representation of *all* characteristics of the neutron in its synthesis form the Hydrogen atom was achieved at the non-relativistic level via the Schrödinger-Santilli isoequation (31) in Refs. [52-54], and at the relativistic level via the Dirac-Santilli isoequations (40) in Refs. [18, 54].

The first laboratory synthesis of the neutron from a Hydrogen gas was done by the Italian priest-physoicist Don Carlo Borghi and his associates in the mid 1960s [55]. Santilli conducted comprehensive tests for the laboratory synthesis of the neutron from the Hydrogen reported in Refs. [56-60]. The above body of scientific knowledge is now used by the U. S. publicly traded company *Thunder Energies Corporation* for the industrial production of a *Thermal Neutron Source* (see the, e.g., Ref.[61] video [62]. Excellent reviews of the mathematical, theoretical and experimental aspects for the synthesis of the neutron from the Hydrogen are available in Refs.[63, 64].

The following comments are in order:

7.1. *Refs. [52-64] imply that the proton and the electron are actual physical constituents of the neutron, although in their mutated form known as “isoproton" and “isoelectron"* [40] (see Appendix A). In fact, one of the necessary condition to achieve a numerical representation of all characteristics of the neutron in its synthesis from the Hydrogen is that the electron rest energy is mutated according to a mechanism today known as *isorenormalization*.

It should be indicated that these results turn the conjecture of undetectable and unconfinable “point-like" quarks to a

mathematical abstraction of the structure of hadrons because the proton and the electron are the only massive permanently stable particles detected to date. As such, they cannot "disappear" (sic) at the time of the neutron synthesis to be replaced by the hypothetical quarks. Additionally, at the time of the neutron decay, quarks cannot "disappear" (sic) while the emitted proton and electron "reappear"(sic).

The name "hadronic mechanics" was suggested in Ref. [3b] precisely to permit a basically new structure model of all unstable particles with actual physical constituents, generally given by massive physical particles produced in their decay with the lowest mode. Advances along these lines have been reported in memoir [43].

It should be stressed that this new structure model of hadrons *is not* in conflict with the standard model of elementary particles because quarks remain necessary for its elaboration, although in their true scientific meaning of being purely mathematical representations of a purely mathematical internal symmetry formulated in a purely mathematical complex-valued unitary space.

We merely return to the teaching of all classifications that have historically required *two* different but compatible models, one model for the *classification* into families, and a different model for the *structure* of each element of a given family. The same historical teaching is confirmed by the fact that, in the transition from the classification to the structure of atoms there was the need for a new mathematical and physical theories. Similarly, in the transition from the classification of hadrons to their structure there is also the need for new mathematics and physical theories for the reasons indicated in Sections 1 - 3.

As a final comment, the serious scholar should be made aware of potentially large environmental and societal implications in abandoning the conjecture of the hypothetical and unconfinable quarks as actual physical constituents of hadrons in favor of physical particles in their isotopic form. For instance, the admission of the isoelectron as a physical constituent of the neutron allows the conception and experimental study of a number of basically new clean nuclear energies, originally proposed in Refs. [65] and currently under study at *Thunder Energies Corporation* as well as at other companies. By contract, the admission of the hypothetical quarks as the physical constituents of hadrons prohibits such possible environmentally large advances.

7.2. *Refs. [52-64] imply that the neutrino does not appear to exist as physical particles, thus creating the intriguing problem of seeking alternative conceptions.*

In his studies of synthesis (70), Enrico Fermi [1] had no other choice than that of representing the proton as a dimensionless point, resulting in the consequentially *necessary* hypothesis of the "neutrino" (meaning "little neutron" in Italian).

Thanks to the availability of the novel isomathematics (Section 2), in Refs. [52-64] we were able to represent the proton in its actual shape and dimension. This permitted the discovery of a new angular motion and related magnetic moment for the constrained rotation of the isoelectron when compressed in the hyperdense medium inside the proton (Figure 8), which new angular momentum is completely absent when the proton is abstracted as a dimensionless particle.

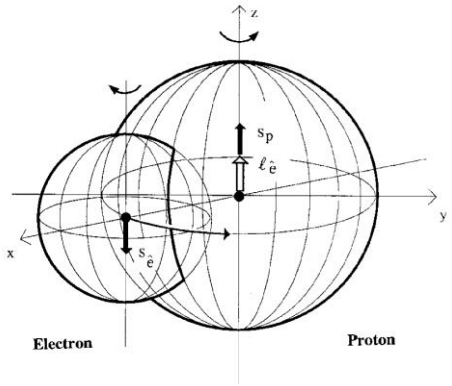

Figure 8. *A basic novelty in Santilli's synthesis of the neutron from the Hydrogen atom is the appearance of a constrained angular motion of the electron when totally immersed within the hyperdense proton. This orbital motion eliminates the need for the emission of the hypothetical neutrino; is solely permitted by the representation of the proton as extended according to hadronic mechanics; and did not exist during Fermi's time since quantum mechanics can solely represents the proton as a massive point [52-63].*

In turn, the constrained orbital motion of the isoelectron inside the proton must be equal to the proton spin (evidently to prevent that the extended wave-packet of the isoelectron moves within and against the hyperdense medium inside the problem), resulting in a null total angular momentum of the isoelectron in synthesis (70) as a result of which the spin of the neutron coincides with the spin of the proton.[2]

The conclusion is that *studies* [52-64] *eliminate any possibility for the production of a neutrino in synthesis* (70). In fact, the emission of a neutrino would *violate*, rather than verify, the conservation of the total angular momentum since the spin $1/2$ of the neutrino is represented by the constrained orbital angular moment of the isoelectron inside the proton. Additionally, reaction (70) already misses 0.782 MeV for the synthesis of the neutron. Any need for the additional energy to produce the hypothetical neutrino would cause catastrophic inconsistencies.

In a nutshell, Enrico Fermi did salvage the conservation of the angular momentum in the synthesis of the neutron with the hypothesis of the neutrino, but he did not salvage quantum mechanics and special relativity in the same synthesis.

7.3. *Refs.* [52-64] *have the intriguing implications of implying the apparent return to the "continuous creation" in the universe as the most plausible way at this moment to explain the missing* 0.782 MeV *for the synthesis of the neutron from the Hydrogen atom.*

[2] It should be recalled that half-odd-integer angular momenta are prohibited in quantum mechanics because they violate the unitarity of the theory, but they are fully allowed for the covering isomechanics precisely in view of its non-unitary structure (see Refs. [18, 22, 52-54] and Appendix A).

One of the biggest mysteries in the synthesis of the neutron from the Hydrogen is the origin of the missing 0.782 MeV (assuming that the neutrino does not exist, otherwise the missing energy would be much bigger). This energy cannot be provided by the relative kinetic energy between the proton and the electron because at that energy value their cross section is virtually null, thus prohibiting any synthesis.

Additionally, the missing energy of 0.782 MeV cannot be provided by the star because, at the initiation of nuclear syntheses, stars synthesize up to 10^{50} neutrons per second. The assumption that the missing energy is provided by the star would then imply that the star *loses* about 10^{50} MeV per second, under which conditions a star would never initiate the majestic event of producing light.

In an attempt to initiate the solution of this mystery, Santilli has suggested that the missing energy of 0.782 MeV is provided by space conceived as a universal substratum with a very high energy density. via a "longitudinal impulse" (rather than a particle) submitted under the name of "etherino" with the symbol " a " (from the Latin aether), thus implying the replacement of the quantum mechanical reaction (70) with the isomechanical reaction [66]

$$p^+ + a + e^- \to n, \tag{72}$$

where one should note the need for the energy carrying impulse to be in the left (rather than the right) of the reaction, and that the use in the left of the antineutrino would increase the missing energy due to its negative energy state [*loc. cit.*].

It should be noted that the historical hypothesis of the neutrino was essentially dismissed by the lack of detection of the "solar neutrinos" (namely, neutrinos emitted by the Sun during its synthesis of the neutron), according to which our particle laboratories should be traversed by an extremely large flux of neutrinos none of which has been detected with such evidence to be acceptable by the scientific community at large.

The advent of the standard model has produced additional reasons for the dismissal of neutrinos since the standard model requires a variety of different neutrinos without clear physical differences, all neutrinos being assumed to have a mass. It is now widely accepted that particles with mass simply cannot traverse nuclei, planets and stars with a very small of no scattering, thus mandating a basically new interpretation of physical reality.

The hypothesis of the etherino has been submitted because of a possible resolution of these insufficiencies via a more realistic interrelation of experimental data. In fact, the traversing to nuclei, planets and stars without appreciable scattering is more plausibly interpreted by the etherino rather than by the neutrino, since the former refers to a longitudinal impulse propagating through the universal substratum, while the latter is assumed to be a massive particle that should traverse without appreciable scattering hyperdense media inside nuclei, planets and stars.

We should also clarify that a number of claimed "experimental verifications" of the neutrino do not refer to the direct detection of the neutrino which is impossible, but refer to the detection of ordinary particles predicted as being emitted under the neutrino hypothesis. The point is that the emission of exactly the same particles is predicted by the etherino and perhaps other hypotheses. Finally, we should indicate that the claimed "experimental verifications" of the neutrino hypothesis are based on very few events out of billions of events, thus lacking the credibility needed to resist the test of time.

In summary, the lack of existence of the neutrino as a physical particle emitted in the synthesis of the neutron creates one of the most fascinating scientific problems in history, that of the possible continuous creation in the universe (see, e.g., the historical paper [67]), since the missing energy for the neutron synthesis is "created" in the core of stars in the sense that it is acquired from the universal substratum. In turn such a fascinating problem has implications for virtually all quantitative sciences, including lack of expansion of the universe due to loss of energy by galactic light to the intergalactic medium [68], possible future interstellar travel at arbitrary speeds whose energy source would be permitted by a universal substratum with very high energy density [38], and other intriguing open problems.

8. Three-Body Structure of the Deuteron According to IsoMechanics

There comes a moment in the life of a serious scientist at which physical realities have to be admitted, no matter how against preferred doctrines, as a condition not to exit from the boundaries of science.

The physical reality here referred to is that *despite more than half a century of attempts, quantum mechanics has failed to achieve a constant representation of the structure of the simplest nucleus, the Deuteron, with embarrassing deviations for heavier nuclei,* in view of the following insufficiencies [69]:

8.1. *Quantum mechanics has been unable to represent the stability of the Deuteron.* As it is well known, the neutron is naturally unstable when isolated. Therefore, quantum mechanics has failed to explain how the neutron becomes permanently stable when bonded to the proton in the structure of the Deuteron.

8.2. *Quantum mechanics has been unable to achieve a consistent representation of the spin* 1 *of the ground state of the Deuteron.* The basic axioms of quantum mechanics require that the stable bound state of one proton and one neutron is the singlet with total spin zero, while the spin of the Deuteron is 1. For the intent of maintaining quantum mechanics, 20^{th} century nuclear physics has assumed a combination of orbital states requiring excited conditions which are in direct contradiction with the physical evidence that the spin 1 occurs for the Deuteron in its "ground" state.

8.3. *Quantum mechanics has been unable to identify the physical origin of the attractive force binding the proton and the neutron in the Deuteron.* Since the neutron is neutral, there is no

known electrostatic origin of the attractive force needed for the existence of the Deuteron, while their magnetostatic force is "repulsive" in their triplet coupling. As a result of these occurrences, a "strong" force was conjectured for the bond of nuclear constituents [2] and its existence was subsequently confirmed. Nevertheless, the physical origin of the strong nuclear force has remained unidentified by quantum mechanics to this writing.

8.4. *Quantum mechanics has been unable to achieve a consistent representation of the Deuteron space parity.* According to experimental evidence, of the space parity is positive for the deuteron in its ground state because the angular momentum is null, while the quantum mechanical representation of the spin 1 of the Deuteron requires excited orbital states, resulting in an additional direct conflict between quantum predictions and experimental realities.

8.5. *Quantum mechanics has been unable to reach an exact representation of the magnetic moment of the Deuteron,* as discussed in Section 5.

Following the achievement of the non-relativistic and relativistic presentation of the structure of the neutron as a bound state of one isoproton and one isoelectron (Refs. [51-54] and Section 7), Santilli proposed in Part V of monograph [69] the *structure of the deuteron according to isomechanics as a three body bound state of two isoprotons in triplet coupling and one isoelectrons withy null total angular momentum which is exchanged in between the two isoprotons as a kind of isogluon, hereon referred to as the "iso-Deuteron"* (see Figure 9).

The new three-body structure model of the Deuteron achieves a numerically exact and time invariant representation of *all* characteristics of the Deuteron, including its binding energy, charge radius, stability, spin, parity, etc., which representation is here assumed as known for brevity from Ref. [69] (see also the excellent reviews [33, 70]).

The conceptual and, therefore, the most important reasons for the proposal of the iso-Deuteron were several [69]. The first origination is that *the reduction of the Deuteron to protons and electrons (although in a mutated form) sets clear foundations for stability* since the proton and the electron are the only stable massive particle known to mankind.

The second origination of the iso-Deuteron is that *the spin* 1 *of the Deuteron is direct evidence that it is a "three-body," rather than a two-body state,* because the configuration of two nucleons in triplet coupling, which is necessary for the representation of the spin 1 in the ground state, can only be achieved in a consistent way via the addition of a third particle with null total angular momenta as in Figure 9.

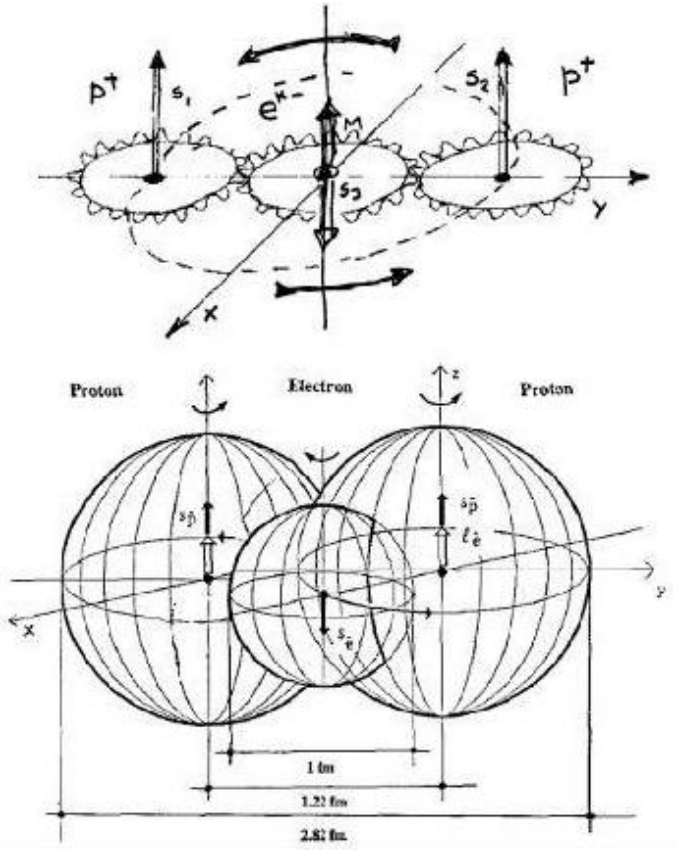

Figure 9. *A schematic view from Part V of Ref. [69] on the structure of the Deuteron following the reduction of the neutron to a hadronic bound state of an isoproton and an isoelectron. Note from the top view that the two isoprotons are in triplet coupling, while the isoelectron with null total angular momentum is exchanged between them, thus allowing the first known representation of the spin 1 of the Deuteron in its true ground state.*

The third origination of the iso-Deuteron is that *isomathematics and isomechanics are the only known methods achieving an explicit and concrete strongly "attractive" force in the Deuteron structure.* In the transition from quantum mechanical to isomechanical nuclear models via the non-unitary transform of Section 4, and realization of the isotopic element of type (1), there is the emergence of a strongly attractive Hulthén potential (see Ref. [69] for details) originating from the partial mutual penetration of the deformed charge distributions of the constituents. Note that in the structure of the iso-Deuteron there is *no* repulsive electrostatic force due to the continuous exchange of the isoelectron between the two isoprotons.

Particularly significant for this paper is the deeper representation of the anomalous magnetic moment of the Deuteron which is permitted by its three-body isotopic structure. In Section 5, we presented a first representation of the magnetic moment of the Deuteron based on its representation as a bound state of an isoproton and an isoneutron in triplet coupling to represent the spin 1 (see Figure 9).

However, as also indicated in Section 5, this representation is basically insufficient because the triplet coupling of Figure 3 generates strongly *repulsive* forces under which no stable bound state is possible. Santilli's three-body model of the iso-Deuteron allows an exact and time invariant representation of the magnetic moment without any known inconsistencies, which is essentially given by the muted magnetic moments of the two isoproton, plus a contributions from the isoelectron (see Ref. [69] for details).

This section concludes the review of past advances in nuclear physics permitted by isomathematics and isomechanics that are necessary for an understanding of the numerically exact and time invariant representation of the spin of stable nuclides presented in the following sections.

9. Stable and Unstable Nuclides

Notice that deuteron is the simplest neuclide having one proton and one neutron and is stable. However, we see that it, in fact, is an *isonuclide*. When we survey the elements of the periodic table we find that out of 289 primordial nuclides 254 are stable ones. The stability of nuclides depends also on evenness or oddness of its atomic number Z, neutron number N and, consequently, of their sum, the mass number A. Oddness of both Z and N tends to lower the nuclear binding energy, making odd nuclei, generally, less stable. This fact we have depicted [71] in Table 1.

However, in this paper, we are presenting, apparently for the first time, a structure model of stable nuclides of the first three rows of the periodic table, hereon called *stable isonuclides*, as bound states of extended, thus deformable isoprotons and isoelectrons according to the laws of hadronic mechanics, under the condition of recovering in first approximation the conventional structure model of nuclides as quantum mechanical bound states of point-like protons and neutrons.

We shall then show, also apparently for the first time, that the reduction of nuclides to isoprotons and isoelectrons allows the first known achievement of an exact representation of the spin of all stable nuclides.

Table 1. *Even and odd nucleon numbers. A is the atomic mass number, Z is the atomic number, N is the number of neutrons in the nucleus, EE is the even-even proton-neutron combination, OO is the odd-odd proton-neutron combination, EO is the even-odd proton-neutron combination and OE is the odd-even proton-neutron combination.*

A	Even		Odd		Total
Z,N	EE	OO	EO	OE	
Stable	148	5	53	48	254
	153		101		
Long-lived	22	4	4	5	35
	26		9		
All primordial	170	9	57	53	289
	179		110		

Next we will indicate without treatment that the reduction of nuclides to isoprotons and isoelectrons puts the foundations for an exact representation of the magnetic moment of all nuclides for studies to be presented in a subsequent paper. We shall also indicate, for studies in a subsequent paper, that the transition of the nuclear structure from that in terms of point-like protons and neutrons to that in terms of is extended, thus deformable isoprotons and isoelectrons offers realistic possibilities for studying basically new forms of clear nuclear energies.

10. Old and New Vistas in Nuclear Forces

For the semi-quantitative discussion conventionally one uses the following expression of nuclear binding energy, namely:

$$\frac{BE}{\text{MeV}} = 931.4\left(Z\times m_{\text{H}} + (A-Z)\times m_n - M\right) \qquad (73)$$

where m_H and m_n are the masses on amu scale of hydrogen and neutron respectively and M is atomic mass on amu scale of the given element. Notice that the mass of electrons has not been included separately in the above expression because it remains included in m_H. The standard plot of binding energies of all nuclides is shown in Figure 10.

Glasstone [72] further asserts that the nuclear binding energy is the result of ($n-n$), ($n-p^+$) and (p^+-p^+) forces operating within the nucleus. The experimental data on the nuclear scattering and correspondence of binding energies of the identically same mass number elements (isobars) it was concluded that the magnitudes of ($n-n$), ($n-p^+$) and (p^+-p^+) forces of attraction are almost equal [72].

In view of the above assertion it was expected that the diproton and the dineutron nuclei should be stable as deuteron is a stable nucleus (which consists of one proton and one neutron).

But so far neither of the former two particles have been detected as stable particles. However, in proton-proton chain reaction it is suspected that a di-proton is formed in the first step which immediately disintegrates into two protons (> 99.99 %) and to deuteron plus β^+ (< 0.01 %) (however the measurement of corresponding half-lives could not succeed) [73]. Of course, it is certain that there is no nucleus made up only of two neutrons because it doesn't constitute a chemical element.

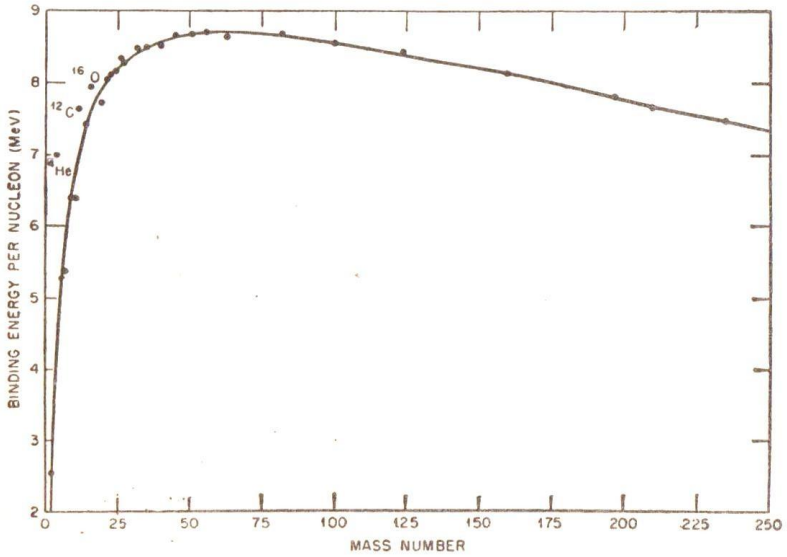

Figure 10. *Binding Energy per nucleon as a function of mass number of stable nuclides.*

On the other hand, the existence of a strong attraction between the pair ($n - p^+$) is exemplified by the stability of deuteron and the stable nuclides He-4, Li-6, B-10, C-12, N-14, O-16, Ne-20, Mg-24, Si-28, S-32, Ar-36 and Ca-40, they all have equal number of protons and neutrons. Besides these nuclides in other stable nuclides we do have neutrons and none of the neutrons disintegrates. The stabilization of neutrons in a nucleus is a subject matter of nuclear physics and the 20th century attempts to explain the said stability are based on quantum mechanics but a satisfactory quantum mechanical description still eludes [18, 22, 23, 33, 36].

The reader may very well notice that the structures of neutron and deuteron that Santilli had proposed, which we have described in brief in Sections 7 and 8 respectively, in fact, are in the form of isoneutron and isodeuteron respectively. These structures involve the mutual deep but partial penetration of the wave packets of electron and proton(s) (c.f. Figures 8 and 9). Thus the quantum mechanical perception of these particles as the point particles has been replaced by the respective tiny but finite size particles all well within the hadronic horizon. However, when we go beyond deuteron the size of the nucleus grows but less significantly (the standard cube root formula [74] estimates the nuclear radius of 1.25 fm for hydrogen nucleus to 4.275 fm for calcium-40 nucleus. Thus the nucleons have increased 40 times but the nuclear radius has increased only 3.4 times).

Therefore, on the lines of the structure of a neutron and a deuteron proposed by Santilli we hereby, apparently for the first time, propose that,

1. an atomic nucleus is composed of nucleons as particles of tiny volume of hadronic dimensions,
2. the wave packets of nucleons penetrate mutually but partially that produces strong nuclear force and
3. the said mutual penetration of wave packets between heteronucleons perhaps produces very strong attractive force compared to that between homonucleons.

This is what has been indicated in Section 1 and shown in Figure 1, that is — the nucleons within a nucleus are in a state of mutual but partial penetration of their wave packets. Thus all nucleons in a nucleus, in fact, are the isonucleons, namely isoneutrons, isodeuterons, isoelectrons and isoprotons.

Of course, one needs to investigate and evaluate quantitatively the magnitude of nuclear forces so generated via the methods of hadronic mechanics but at this juncture we consider that it would be profitable first to generate nuclear configuration of stable nuclides as if the nucleus of all stable nuclides are composed of isonucleons, which is likely to present enough ground for carrying out the detailed investigation of the corresponding quantitative haronic physics. Indeed, we have presented in Section 3 a brief description of Santilli's initial work on nuclear isomechanics and genomechanics.

In the following Section 11 we will see that there are two options for developing nuclear configuration. The first one, the model-I, is through the isodeuterons, isoneutrons and isoprotons as the building nucleons and the second option, the model-II, is through the isoprotons and isoelectrons as the building nucleons, both of them are easily interconvertible. We will also discuss the advantages and limitations of each.

11. Notations for Representation of IsoNeutronand IsoDeuteron

In order to develop the nuclear configuration of nuclides the first logical option is offered by the fact that the deuteron is a stable nuclide similar to a proton. In Section 8 we have described that the deuteron is a hadronic bound state of an isoneutron and an isoproton. But as described in Section 7 the neutron is indeed an isoneutron, which is a hadronic bound state of one isoproton and one isoelectron. However, the isoneutron is an unstable nuclide, which decays radiatively by β^- emission with half-life of 614.6 s [75] (In 1967 experiment the half-life of free neutron was recorded as 10.8 min [76]). But when it makes a union with an isoproton its instability vanishes altogether. Hence in this hadronic choice we have developed nuclear configuration of stable nuclides commensurate with the observed nuclear spin using isodeuterons, isoneutrons and if required used isoprotons. However, recall that each isodeuteron is made up of 2 isoprotons of parallel spin and one isoelectron of zero spin, and the isoneutron consists of one isoproton of half spin and one isoelectron of zero spin hence it is easy to convert the nuclear configuration of the first choice into the one in terms of isoprotons of 1/2 spin, isoprotons of -1/2 spin and isoelectrons of zero spin, that is our second choice. However, we can directly write the nuclear configuration in the second choice just by choosing correct number of isoprotons with 1/2 and -1/2 spin commensurate with the experimental nuclear spin, because isoelectron doesn't contribute to the nuclear spin.

A simple notation to represent Santilli's isoneutron, $\hat{n}$, is as given below as a compressed hydrogen atom, namely:

$$ha = (p^+, e^-)_{qm} \to \left(\hat{p}^+\left(\uparrow\right), \hat{e}^-(J=0)\right)_{hm} \equiv \hat{n}\left(\uparrow\right) \qquad (74)$$

where ha denotes the hydrogen atom; qm denotes quantum mechanics; p^+ denotes the conventional proton; e^- denotes the conventional electron; hm denotes hadronic mechanics; $\hat{p}^+$ denotes the isoproton; $\hat{e}^-$ denotes an isoelectron; J is the spin and $\uparrow$ denotes spin 1/2. The total angular momentum of the isoelectron is null because the particle is constrained to rotate within the hyperdense proton in singlet coupling, thus acquiring a value of the orbital angular momentum equal but opposite to its spin (Figure 8).

Similarly, the notation of an isodeuteron, $\hat{d}$, is obtained as given below, namely:

$$d(J=?) = \left(p^+\left(\uparrow\right), n\left(\downarrow\right)\right)_{qm} \to \left(\hat{p}^+\left(\uparrow\right), \hat{e}^-(J=0), \hat{p}^+\left(\uparrow\right)\right)_{hm}$$
$$\equiv \hat{d}(J=1) = \hat{d}\left(\uparrow\uparrow\right) \qquad (75)$$

where $\downarrow$ denotes the spin -1/2. The spin 1 of the isodeuteron is because of two up spins, $\uparrow\uparrow$, of two isoprotons.

The stability of deuteron gets excellently explained by the Santilli iso-deuteron model, Eq. (75). Namely, as the structure $\left(\hat{p}^+\left(\uparrow\right), \hat{e}^-(J=0)\right)_{hm}$ is unstable, there is a natural tendency of the bound electron in $\left(\hat{p}^+\left(\uparrow\right), \hat{e}^-(J=0), \hat{p}^+\left(\uparrow\right)\right)_{hm}$ to get released from the grip of its isoproton to which it is bound at the given instant of time, but no sooner it succeeds in getting released it immediately gets trapped into the hyper-dense medium of the other very closely placed proton. This is how isodeuteron enjoys its stability against radioactivity. This interpretation of nuclear stability and instability reasonably good.

In the next Section 12 we consider only the stable nuclides of periodic table up to the atomic number 82.

12. Proposed Nuclear Configuration of Stable IsoNuclides

We adopt ${}^{A}_{Z}\mathrm{X}_{N}(J) = \mathrm{X}(A,Z,J)$ to represent nuclides, where X represents the symbol of the chemical element, A is the mass number i.e. the total number of protons and neutrons, Z is the atomic number i.e. the total number of protrons, N is the total number of neutrons, and J is the nuclear spin. Obviously $(A-Z)$ is the total number of neutrons, N, in the nucleus. Notice that we have incorporated nuclear spin, J, in the conventional representation of nuclide.

In this paper we propose, apparently for the first time, the extension of Santilli isodeuteron to all stable nuclides under the proposed name of IsoNuclides with the symbol ${}^{A}_{Z}\hat{\mathrm{X}}_{N}(J)$. Notice that in this notation we have still retained the symbols A,N,Z because it would be easy to correlate with conventional description.

Now as stated in preceding sections there are two options for developing nuclear configuration of nuclides.

In the *model-I* the adopted working rule is that we are bound by the requirement of producing that nuclear configuration which predicts correctly the experimental nuclear spin. Our method is further based on the observed stability of an isodeuteron that indicates that the isonucleons of a nuclide first prefer to adopt the isodeuteron structures and in this way the unaccounted neutrons and protons stay in the nucleus as isoneutrons and isoprotons with appropriate spin orientation.

In the *model-II* we fix the number of isoelectrons equal to the number of neutrons (because in a nucleus an isoelectron with null spin is carried into through the neutron as isoneutron) and obviously the number of isoprotons of a nucleus equals to the mass number, A, of the nuclide. Thus our method is then to choose the number of isoprotons with spin 1/2 and -1/2 that correctly predicts the experimental nuclear spin of the nuclide.

12.1. Isodeuteron, Isoneutron and Isoproton as Constituents of Atomic Nuclei. Model-I

In this first option with the guidelines described above in this

Section 12 we note that ${}^{3}_{2}\hat{\mathrm{He}}_1(0)$ can be readily interpreted as a hadronic bound state of an isodeuteron and an isoproton in singlet coupling (perhaps necessary for stability). Accordingly it gets represented as under,

$$\begin{aligned} {}^{3}_{2}\hat{\mathrm{He}}_1(1/2) &= \left(\hat{d}\left(\uparrow\uparrow\right),\ \hat{p}^{+}\left(\downarrow\right)\right)_{hm} \\ &\equiv \left({}^{2}_{1}\hat{d}_1(1),\ \hat{p}^{+}(-1/2)\right)_{hm} \end{aligned} \tag{76}$$

Notice that in this isonuclide there we have one separate isoproton and two mutated protons as isoprotons in the form of $\hat{d}\left(\uparrow\uparrow\right)$.

Similarly, ${}^{4}_{2}\hat{\mathrm{He}}_2(0)$ can be readily interpreted as a hadronic bound state of two isodeuterons in singlet coupling, namely,

$$\begin{aligned} {}^{4}_{2}\hat{\mathrm{He}}_2(0) &= \left(\hat{d}\left(\uparrow\uparrow\right),\hat{d}\left(\downarrow\downarrow\right)\right)_{hm} \\ &\equiv \left({}^{2}_{1}\hat{d}_1(1),{}^{2}_{1}\hat{d}_1(-1)\right)_{hm}. \end{aligned} \tag{77}$$

Along the same linens, ${}^{6}_{3}\hat{\mathrm{Li}}_3(1)$ can be readily interpreted as a hadronic bound state of ${}^{4}_{2}\hat{\mathrm{He}}_2(0)$ and one isodeuteron ${}^{2}_{1}\hat{d}_1(1)$

$${}^{6}_{3}\hat{\mathrm{Li}}_3(1) = \left({}^{4}_{2}\hat{\mathrm{He}}_2(0),{}^{2}_{1}\hat{d}_1(1)\right)_{hm} \tag{78}$$

and similarly for the remaining stable nuclides (see Table 2).

For the isotopic structure model of ${}^{7}_{3}\hat{\mathrm{Li}}_4(3/2)$ we have the more complex model

$${}^{7}_{3}\hat{\mathrm{Li}}_4(3/2) = \left({}^{4}_{2}\hat{\mathrm{He}}_2(0),\ {}^{2}_{1}\hat{d}_1(1),\ \hat{n}(1/2)\right)_{hm} \tag{79}$$

Therefore, we can symbolically write the *nuclear configuration of stable isonuclies* as under,

$$\begin{aligned} {}^{A}_{Z}\hat{\mathrm{X}}_N(J) = [&x_1({}^{2}_{1}\hat{d}_1(1)),\ x_2({}^{2}_{1}\hat{d}_1(-1)), \\ &x_3(\hat{n}(1/2)),\ x_4(\hat{n}(-1/2)), \\ &x_5(\hat{p}^{+}(1/2)),\ x_6(\hat{p}^{+}(-1/2))] \end{aligned} \tag{80}$$

where $\hat{\mathrm{X}}$ denotes the isonuclide, x_i's are the number of the isonuclear or nuclear species depicted in the braces next to them. Notice that in this model-I in any nucleus the isoprotons would be in the form of isoneutrons, isodeuterons and remaining as separate isoprotons hence if the atomic number of a nuclide demands more protons than those accounted by isodeuterons and isoneutrons (the striking example is that of He-3, c.f. Eq. (76) they will be separate mutated proptons (i.e. the isoprotons). In view of this in above expression (80) the last two terms on the right hand side account for the separate isoprotons that are demanded by its atomic number, *Z*.

In this way the expression of the atomic mass number, A, is obtained as,

$$A = 2x_1 + 2x_2 + x_3 + x_4 + x_5 + x_6 \tag{81}$$

the atomic number, Z, is given by,

$$Z = x_1 + x_2 + x_5 + x_6 \tag{82}$$

Therefore, obviously the number of nuclear neutrons, N, is given by,

$$N = A - Z = x_1 + x_2 + x_3 + x_4. \tag{83}$$

Whereas, the total number of isoprotons $\mathbb{P}_{\hat{p}^+}$, get computed as,

$$\mathbb{P}_{\hat{p}^+} = 2x_1 + 2x_2 + x_3 + x_4 + x_5 + x_6 \tag{84}$$

and the total number of isoelectrons, $\mathbb{E}_{\hat{e}^-}$, get computed as,

$$\mathbb{E}_{\hat{e}^-} = x_1 + x_2 + x_3 + x_4 \tag{85}$$

It is no wonder that $N = \mathbb{E}_{\hat{e}^-}$ because with each isoneutron there is associated one isoelectron. Moreover, the nuclear spin J gets computed as,

$$J = x_1 - x_2 + \frac{1}{2}x_3 - \frac{1}{2}x_4 + \frac{1}{2}x_5 - \frac{1}{2}x_6 \tag{86}$$

Therefore, the isonuclide, ${}^{A}_{Z}\hat{\mathrm{X}}_N(J)$, gets reduced to isoprotons, $\hat{p}^{+}$ and isoelectrons, $\hat{e}^{-}$, that gets expressed as,

$${}^{A}_{Z}\hat{\mathrm{X}}_N(J) = \left(\mathbb{P}_{\hat{p}^+},\ \mathbb{E}_{\hat{e}^-}\right) \tag{87}$$

12.2. Isoprotons and Isonelectrons as Constituents of Atomic Nuclei. Model-II

Recall that all nuclear protons are indistinguishable whereas the isoprotons of the nuclear isoneutrons too remain indistinguishable because the isoneutrons have a natural tendency to get converted to protons. Therefore, we cannot label which proton out of the available nuclear protons at a given instant of time is actually bound to an isoelectron. In this way there must be on an average at a given moment of time a fixed number of isoprotons and the same number of isoelectrons, and remaining number of nucleons are the protons and are equal to the atomic number of the chemical element. However, in view of the housing of all protons and neutrons in extremely small nuclear volume (see also Section 10) there must be at least partial mutual penetration of wave packets of protons besides in addition to that with the wave packets of electrons that describe the isoneutron and isodeuteron. Hence all nuclear protons and neutrons taken together need to be treated as an assemblage of

isoprotons and isoelectrons. Of course, the mutual penetration of wave packets of protons and the mutual penetration of wave packets of electrons and protons would definitely produce different hadronic effects hence needs to be quantitatively investigated by the tools of hadronic mechanics. Therefore, in this *model-II* we treat every nucleon of a nucleus an isonucleon.

Thus the counter part of Eq. (80) in this case would read as,

$$\begin{aligned} {}^{A}_{Z}\hat{\mathrm{X}}_{N}(J) = \big[& 2x_1\left(\hat{p}^{+}(1/2)\right), 2x_2\left(\hat{p}^{+}(-1/2)\right), \\ & x_3\left(\hat{p}^{+}(1/2)\right), x_4\left(\hat{p}^{+}(-1/2)\right), \\ & x_5\left(\hat{p}^{+}(1/2)\right), x_6\left(\hat{p}^{+}(-1/2)\right), \\ & (x_1+x_2+x_3+x_4)\left(\hat{e}^{-}(0)\right)\big] \end{aligned} \tag{88}$$

which gets simplified to,

$$\begin{aligned} {}^{A}_{Z}\hat{\mathrm{X}}_{N}(J) = \big[& (2x_1+x_3+x_5)\left(\hat{p}^{+}(1/2)\right), \\ & (2x_2+x_4+x_6)\left(\hat{p}^{+}(-1/2)\right), \\ & (x_1+x_2+x_3+x_4)\left(\hat{e}^{-}(0)\right)\big] \end{aligned} \tag{89}$$

where the number of isoprotons with spin 1/2, $\mathbb{P}(1/2)$, is given by,

$$\mathbb{P}(1/2) = 2x_1 + x_3 + x_5, \tag{90}$$

number of the isoprotons with spin -1/2, $\mathbb{P}(-1/2)$, is given by,

$$\mathbb{P}(-1/2) = 2x_2 + x_4 + x_6, \tag{91}$$

and number of the isoelectrons with spin 0, $\mathbb{E}(0) = N$, is given by,

$$\mathbb{E}(0) = x_1 + x_2 + x_3 + x_4. \tag{92}$$

Alternatively, we can directly express ${}^{A}_{Z}\mathrm{X}_{N}(J)$ as follows,

$${}^{A}_{Z}\hat{\mathrm{X}}_{N}(J) = \left[\mathbb{P}(1/2), \mathbb{P}(-1/2), \mathbb{E}(0)\right] \tag{93}$$

where $\mathbb{P}(1/2)+\mathbb{P}(-1/2) = A$ and $Z = \mathbb{P}(1/2)+\mathbb{P}(-1/2)-N$. Since, all nuclear spins are null or positive numbers we have $\mathbb{P}(1/2) > \mathbb{P}(-1/2)$.

We would like to stress that the methods of writing nuclear configuration described above are entirely general that make no distinction between stable and unstable nuclides. However, with the above adopted notations we are now well equipped to build the nuclear configuration of stable nuclides as isonuclides ${}^{A}_{Z}\hat{\mathrm{X}}_{N}(J)$, that we present in the next Section 13.

13. Hadronic Mechanics Based Configuration of Stable Nuclides

In this paper we are primarily presenting the nuclear configuration of the stable nuclides. The nuclides of atomic number higher than 82 are all radioactive therefore we have developed the nuclear configuration up to the chemical element Pb. Now onwards we will use the short hand notation of an isoneutron and an isodeuteron given in the extreme right hand side of Eqs. (74) and (75), namely $\hat{n}\left(\uparrow\right)$ and $\hat{d}\left(\uparrow\uparrow\right)$ respectively. Moreover, henceforth all nuclear protons would be treated as isoprotons whether the wave packet of any one of them penetrates with that of an isoelectron or not. This is so because as discussed in Section 10, in view of the extremely small size of atomic nuclei, all nuclear protons indeed get transformed to isoprotons.

13.1. Nuclear Configuration of Stable Isotopes as Isonuclides. Model-I

The observed stability of deuteron does indicate that the stable nuclides first prefer to have the isodeuteron structure from the available number of neutrons and protons. Whereas the remaining unaccounted neutrons and protons stay in the nucleus as isoneutrons and isoprotons.

Thus we have followed a nuclear version of the Aufbau type principle with the requirement that the resulting nuclear configuration should correctly predict the observed nuclear spin of each isotope of the elements. We are presenting in column 3 of Table 2 the so arrived at nuclear configuration of the stable isonuclides up to the element Pb of the periodic table along with the observed nuclear spin (in colum 5) against each isonuclide for the ready reference. All the nuclear spins reported now onwards are taken from the Ref. [80] unless otherwise other sources are cited.

13.2. Nuclear Configuration of Stable Isotopes as Isonuclides. Model-II

The nuclear configuration in terms of isoprotons and isoelectrons that replicate the observed nuclear spin is easy to write. We first write number of isoelectrons equal to the number of neutrons, N, in the nucleus and then write the number of isoprotons equal to the mass number, A, of the nuclide, which then is distributed in up and down spin isoprotons so that the net spin of the combination equals the experimental nuclear spin.

Equivalently, on realizing that each isodeuteron has two isoprotons of same spin and one isoelectron of null spin, and the isoproton of each isoneutron has the same spin as that of the latter. The total number of nuclear isoelectrons is given by the sum of the number of isodeuterons and isoneutrons in a given isonuclide. The nuclear configuration of the *model-II* has been listed in the column 4 of Table 2. Notice that the nuclear configuration in this option of all nuclei correctly predicts the respective observed nuclear spin.

Table 2. *Nuclear configuration of stable, primordial and very long lived isonuclides for nuclear model-I and model-II*

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
1	${}_{1}^{1}H_{0}(1/2)$ Proton (not an isonuclide)	$p^{+}(\uparrow)$	$p^{+}(\uparrow)$	1/2
	${}_{1}^{2}\hat{H}_{1}(1)$ (isodeuteron)	$\hat{d}(\uparrow\uparrow)$	$2\hat{p}^{+}(\uparrow),\hat{e}^{-}(\uparrow\downarrow)$	1
2	${}_{2}^{3}\hat{He}_{1}(1/2)$	$\hat{d}(\uparrow\uparrow),\ \hat{p}^{+}(\downarrow)$	$2\hat{p}^{+}(\uparrow),\hat{p}^{+}(\downarrow),$ $\hat{e}^{-}(\uparrow\downarrow)$	1/2
	${}_{2}^{4}\hat{He}_{2}(0)$	$\hat{d}(\uparrow\uparrow),\ \hat{d}(\downarrow\downarrow)\ \equiv\ \left[{}_{2}^{4}\hat{He}_{2}(0)\right]$	$2\hat{p}^{+}(\uparrow),2\hat{p}^{+}(\downarrow),$ $2\hat{e}^{-}(\uparrow\downarrow)$	0
3	${}_{3}^{6}\hat{Li}_{3}(1)$	$\left[{}_{2}^{4}\hat{He}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow)$	$4\hat{p}^{+}(\uparrow),2\hat{p}^{+}(\downarrow),$ $3\hat{e}^{-}(\uparrow\downarrow)$	1
	${}_{3}^{7}\hat{Li}_{4}(3/2)$	$\left[{}_{2}^{4}\hat{He}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\hat{n}(\uparrow)$	$5\hat{p}^{+}(\uparrow),2\hat{p}^{+}(\downarrow),$ $4\hat{e}^{-}(\uparrow\downarrow)$	3/2
4	${}_{4}^{9}\hat{Be}_{5}(3/2)$	$\left[{}_{2}^{4}\hat{He}_{2}(0)\right],2\,\hat{d}(\uparrow\uparrow),\ \hat{n}(\downarrow)$	$6\hat{p}^{+}(\uparrow),3\hat{p}^{+}(\downarrow),$ $5\hat{e}^{-}(\uparrow\downarrow)$	3/2
5	${}_{5}^{10}\hat{B}_{5}(3)$	$\left[{}_{2}^{4}\hat{He}_{2}(0)\right],3\,\hat{d}(\uparrow\uparrow)$	$8\hat{p}^{+}(\uparrow),2\hat{p}^{+}(\downarrow),$ $5\hat{e}^{-}(\uparrow\downarrow)$	3
	${}_{5}^{11}\hat{B}_{6}(3/2)$	$2\left[{}_{2}^{4}\hat{He}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\hat{n}(\uparrow)$	$7\hat{p}^{+}(\uparrow),4\hat{p}^{+}(\downarrow),$ $6\hat{e}^{-}(\uparrow\downarrow)$	3/2
6	${}_{6}^{12}\hat{C}_{6}(0)$	$2\left[{}_{2}^{4}\hat{He}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{d}(\downarrow\downarrow)$ $\equiv\ 3\left[{}_{2}^{4}\hat{He}_{2}(0)\right]$	$6\hat{p}^{+}(\uparrow),6\hat{p}^{+}(\downarrow),$ $6\hat{e}^{-}(\uparrow\downarrow)$	0
7	${}_{6}^{13}\hat{C}_{7}(1/2)$	$3\left[{}_{2}^{4}\hat{He}_{2}(0)\right],\ \hat{n}(\uparrow)$	$7\hat{p}^{+}(\uparrow),6\hat{p}^{+}(\downarrow),$ $7\hat{e}^{-}(\uparrow\downarrow)$	1/2
	${}_{7}^{14}\hat{N}_{7}(1)$	$3\left[{}_{2}^{4}\hat{He}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow)$	$8\hat{p}^{+}(\uparrow),6\hat{p}^{+}(\downarrow),$ $7\hat{e}^{-}(\uparrow\downarrow)$	1
	${}_{7}^{15}\hat{N}_{8}(1/2)$	$3\left[{}_{2}^{4}\hat{He}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{n}(\downarrow)$	$8\hat{p}^{+}(\uparrow),7\hat{p}^{+}(\downarrow),$ $8\hat{e}^{-}(\uparrow\downarrow)$	1/2
8	${}_{8}^{16}\hat{O}_{8}(0)$	$3\left[{}_{2}^{4}\hat{He}_{2}(0)\right]\hat{d}(\uparrow\uparrow),\ \hat{d}(\downarrow\downarrow)$ $\equiv\ 4\left[{}_{2}^{4}\hat{He}_{2}(0)\right]$	$8\hat{p}^{+}(\uparrow),8\hat{p}^{+}(\downarrow),$ $8\hat{e}^{-}(\uparrow\downarrow)$	0
	${}_{8}^{17}\hat{O}_{9}(5/2)$	$3\left[{}_{2}^{4}\hat{He}_{2}(0)\right],2\,\hat{d}(\uparrow\uparrow),\ \hat{n}(\uparrow)$	$11\hat{p}^{+}(\uparrow),6\hat{p}^{+}(\downarrow),$ $9\hat{e}^{-}(\uparrow\downarrow)$	5/2
	${}_{8}^{18}\hat{O}_{10}(0)$	$4\left[{}_{2}^{4}\hat{He}_{2}(0)\right],\ \hat{n}(\uparrow),\ \hat{n}(\downarrow)$	$9\hat{p}^{+}(\uparrow),9\hat{p}^{+}(\downarrow),$ $10\hat{e}^{-}(\uparrow\downarrow)$	0
9	${}_{9}^{19}\hat{F}_{10}(1/2)$	$4\left[{}_{2}^{4}\hat{He}_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{n}(\downarrow)$	$10\hat{p}^{+}(\uparrow),9\hat{p}^{+}(\downarrow),$ $10\hat{e}^{-}(\uparrow\downarrow)$	1/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
10	$^{20}_{10}\hat{N}e_{10}(0)$	$4\left[{}^{4}_{2}\hat{H}e_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{d}(\downarrow\downarrow)\ \equiv\ 5\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$	$10\hat{p}^{+}(\uparrow),10\hat{p}^{+}(\downarrow),\ 10\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{21}_{10}\hat{N}e_{11}(3/2)$	$4\left[{}^{4}_{2}\hat{H}e_{2}(0)\right],2\hat{d}(\uparrow\uparrow),\ \hat{n}(\downarrow)$	$12\hat{p}^{+}(\uparrow),9\hat{p}^{+}(\downarrow),\ 11\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{22}_{10}\hat{N}e_{12}(0)$	$5\left[{}^{4}_{2}\hat{H}e_{2}(0)\right],\ \hat{n}(\uparrow),\ \hat{n}(\downarrow)$	$11\hat{p}^{+}(\uparrow),11\hat{p}^{+}(\downarrow),\ 12\hat{e}^{-}(\uparrow\downarrow)$	0
11	$^{23}_{11}\hat{N}a_{12}(3/2)$	$5\left[{}^{4}_{2}\hat{H}e_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\hat{n}(\uparrow)$	$13\hat{p}^{+}(\uparrow),10\hat{p}^{+}(\downarrow),\ 12\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{24}_{12}\hat{M}g_{12}(0)$	$5\left[{}^{4}_{2}\hat{H}e_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{d}(\downarrow\downarrow)\ \equiv\ 6\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$	$12\hat{p}^{+}(\uparrow),12\hat{p}^{+}(\downarrow),\ 12\hat{e}^{-}(\uparrow\downarrow)$	0
12	$^{25}_{12}\hat{M}g_{13}(5/2)$	$5\left[{}^{4}_{2}\hat{H}e_{2}(0)\right],2\hat{d}(\uparrow\uparrow),\ \hat{n}(\uparrow)$	$15\hat{p}^{+}(\uparrow),10\hat{p}^{+}(\downarrow),\ 13\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{26}_{12}\hat{M}g_{14}(0)$	$6\left[{}^{4}_{2}\hat{H}e_{2}(0)\right],\ \hat{n}(\uparrow),\ \hat{n}(\downarrow)$	$13\hat{p}^{+}(\uparrow),13\hat{p}^{+}(\downarrow),\ 14\hat{e}^{-}(\uparrow\downarrow)$	0
13	$^{27}_{13}\hat{A}l_{14}(5/2)$	$5\left[{}^{4}_{2}\hat{H}e_{2}(0)\right],3\hat{d}(\uparrow\uparrow),\ \hat{n}(\downarrow)$	$16\hat{p}^{+}(\uparrow),11\hat{p}^{+}(\downarrow),\ 14\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{28}_{14}\hat{S}i_{14}(0)$	$6\left[{}^{4}_{2}\hat{H}e_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{d}(\downarrow\downarrow)\ \equiv\ 7\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$	$14\hat{p}^{+}(\uparrow),14\hat{p}^{+}(\downarrow),\ 14\hat{e}^{-}(\uparrow\downarrow)$	0
14	$^{29}_{14}\hat{S}i_{15}(1/2)$	$7\left[{}^{4}_{2}\hat{H}e_{2}(0)\right],\ \hat{n}(\uparrow)$	$15\hat{p}^{+}(\uparrow),14\hat{p}^{+}(\downarrow),\ 15\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{30}_{14}\hat{S}i_{16}(0)$	$7\left[{}^{4}_{2}\hat{H}e_{2}(0)\right],\ \hat{n}(\uparrow),\ \hat{n}(\downarrow)$	$15\hat{p}^{+}(\uparrow),15\hat{p}^{+}(\downarrow),\ 16\hat{e}^{-}(\uparrow\downarrow)$	0
15	$^{31}_{15}\hat{P}_{16}(0)$	$7\left[{}^{4}_{2}\hat{H}e_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{n}(\downarrow)$	$16\hat{p}^{+}(\uparrow),15\hat{p}^{+}(\downarrow),\ 16\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{32}_{16}\hat{S}_{16}(0)$	$7\left[{}^{4}_{2}\hat{H}e_{2}(0)\right],\ \hat{d}(\uparrow\uparrow),\ \hat{d}(\downarrow\downarrow)\ \equiv\ 8\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$	$16\hat{p}^{+}(\uparrow),16\hat{p}^{+}(\downarrow),\ 16\hat{e}^{-}(\uparrow\downarrow)$	0
16	$^{33}_{16}\hat{S}_{17}(3/2)$	$7\left[{}^{4}_{2}\hat{H}e_{2}(0)\right],2\hat{d}(\uparrow\uparrow),\ \hat{n}(\downarrow)$	$18\hat{p}^{+}(\uparrow),15\hat{p}^{+}(\downarrow),\ 17\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{34}_{16}\hat{S}_{18}(3/2)$	$8\left[{}^{4}_{2}\hat{H}e_{2}(0)\right],\ \hat{n}(\uparrow),\ \hat{n}(\downarrow)$	$17\hat{p}^{+}(\uparrow),17\hat{p}^{+}(\downarrow),\ 18\hat{e}^{-}(\uparrow\downarrow)$	0

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17	$^{36}_{16}\hat{S}_{20}$ (3/2)	$8\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 2\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$18\hat{p}^{+}(\uparrow), 18\hat{p}^{+}(\downarrow), 20\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{35}_{17}\hat{Cl}_{18}$(3/2)	$8\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), \hat{n}(\uparrow)$	$19\hat{p}^{+}(\uparrow), 16\hat{p}^{+}(\downarrow), 18\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{37}_{17}\hat{Cl}_{20}$(3/2)	$8\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 2\hat{n}(\uparrow), \hat{n}(\downarrow)$	$20\hat{p}^{+}(\uparrow), 17\hat{p}^{+}(\downarrow), 20\hat{e}^{-}(\uparrow\downarrow)$	3/2
18	$^{36}_{18}\hat{Ar}_{18}(0)$	$8\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow) \equiv 9\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$	$18\hat{p}^{+}(\uparrow), 18\hat{p}^{+}(\downarrow), 18\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{38}_{18}\hat{Ar}_{20}(0)$	$9\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$19\hat{p}^{+}(\uparrow), 19\hat{p}^{+}(\downarrow), 20\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{40}_{18}\hat{Ar}_{22}(0)$	$9\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 2\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$20\hat{p}^{+}(\uparrow), 20\hat{p}^{+}(\downarrow), 22\hat{e}^{-}(\uparrow\downarrow)$	0
19	$^{39}_{19}\hat{K}_{20}$(3/2)	$9\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), \hat{n}(\uparrow)$	$21\hat{p}^{+}(\uparrow), 18\hat{p}^{+}(\downarrow), 20\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{40}_{19}\hat{K}_{21}$(4)	$8\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 3\hat{d}(\uparrow\uparrow), 2\hat{n}(\uparrow)$	$24\hat{p}^{+}(\uparrow), 16\hat{p}^{+}(\downarrow), 21\hat{e}^{-}(\uparrow\downarrow)$	4
	$^{41}_{19}\hat{K}_{22}$(3/2)	$9\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 2\hat{n}(\uparrow), \hat{n}(\downarrow)$	$22\hat{p}^{+}(\uparrow), 19\hat{p}^{+}(\downarrow), 22\hat{e}^{-}(\uparrow\downarrow)$	3/2
20	$^{40}_{20}\hat{Ca}_{20}(0)$	$9\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow) \equiv 10\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$	$20\hat{p}^{+}(\uparrow), 20\hat{p}^{+}(\downarrow), 20\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{42}_{20}\hat{Ca}_{22}(0)$	$10\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$21\hat{p}^{+}(\uparrow), 21\hat{p}^{+}(\downarrow), 22\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{43}_{20}\hat{Ca}_{23}$(7/2)	$9\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 2\hat{d}(\uparrow\uparrow), 3\hat{n}(\uparrow)$	$25\hat{p}^{+}(\uparrow), 18\hat{p}^{+}(\downarrow), 23\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{44}_{20}\hat{Ca}_{24}(0)$	$10\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 2\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$22\hat{p}^{+}(\uparrow), 22\hat{p}^{+}(\downarrow), 24\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{46}_{20}\hat{Ca}_{26}(0)$	$10\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 3\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$23\hat{p}^{+}(\uparrow), 23\hat{p}^{+}(\downarrow), 26\hat{e}^{-}(\uparrow\downarrow)$	0

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21	$^{48}_{20}\hat{Ca}_{28}(0)$	$10\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 4\,\hat{n}(\uparrow), 4\,\hat{n}(\downarrow)$	$23\hat{p}^{+}(\uparrow), 23\hat{p}^{+}(\downarrow), 26\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{45}_{21}\hat{Sc}_{24}(7/2)$	$9\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 3\,\hat{d}(\uparrow\uparrow), 2\,\hat{n}(\uparrow), \hat{n}(\downarrow)$	$26\hat{p}^{+}(\uparrow), 19\hat{p}^{+}(\downarrow), 24\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{46}_{22}\hat{Ti}_{24}(0)$	$11\left[{}^{4}_{2}\hat{He}_{2}(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$23\hat{p}^{+}(\uparrow), 23\hat{p}^{+}(\downarrow), 24\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{47}_{22}\hat{Ti}_{25}(5/2)$	$10\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 2\,\hat{d}(\uparrow\uparrow), 2\,\hat{n}(\uparrow), \hat{n}(\downarrow)$	$26\hat{p}^{+}(\uparrow), 21\hat{p}^{+}(\downarrow), 25\hat{e}^{-}(\uparrow\downarrow)$	5/2
22	$^{48}_{22}\hat{Ti}_{26}(0)$	$11\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 2\,\hat{n}(\uparrow), 2\,\hat{n}(\downarrow)$	$24\hat{p}^{+}(\uparrow), 24\hat{p}^{+}(\downarrow), 26\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{49}_{22}\hat{Ti}_{27}(7/2)$	$10\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 2\,\hat{d}(\uparrow\uparrow), 4\,\hat{n}(\uparrow), \hat{n}(\downarrow)$	$28\hat{p}^{+}(\uparrow), 21\hat{p}^{+}(\downarrow), 27\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{50}_{22}\hat{Ti}_{28}(0)$	$11\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 3\,\hat{n}(\uparrow), 3\,\hat{n}(\downarrow)$	$25\hat{p}^{+}(\uparrow), 25\hat{p}^{+}(\downarrow), 28\hat{e}^{-}(\uparrow\downarrow)$	0
23	$^{50}_{23}\hat{V}_{27}(6)$	$9\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 5\,\hat{d}(\uparrow\uparrow), 3\,\hat{n}(\uparrow), \hat{n}(\downarrow)$	$31\hat{p}^{+}(\uparrow), 19\hat{p}^{+}(\downarrow), 27\hat{e}^{-}(\uparrow\downarrow)$	6
	$^{51}_{23}\hat{V}_{28}(7/2)$	$11\left[{}^{4}_{2}\hat{He}_{2}(0)\right], \hat{d}(\uparrow\uparrow), 5\,\hat{n}(\uparrow)$	$29\hat{p}^{+}(\uparrow), 22\hat{p}^{+}(\downarrow), 28\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{50}_{24}\hat{Cr}_{26}(0)$	$12\left[{}^{4}_{2}\hat{He}_{2}(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$25\hat{p}^{+}(\uparrow), 25\hat{p}^{+}(\downarrow), 26\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{52}_{24}\hat{Cr}_{28}(0)$	$12\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 2\,\hat{n}(\uparrow), 2\,\hat{n}(\downarrow)$	$26\hat{p}^{+}(\uparrow), 26\hat{p}^{+}(\downarrow), 28\hat{e}^{-}(\uparrow\downarrow)$	0
24	$^{53}_{24}\hat{Cr}_{29}(3/2)$	$12\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 4\,\hat{n}(\uparrow), \hat{n}(\downarrow)$	$28\hat{p}^{+}(\uparrow), 25\hat{p}^{+}(\downarrow), 29\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{54}_{24}\hat{Cr}_{30}(0)$	$12\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 3\,\hat{n}(\uparrow), 3\,\hat{n}(\downarrow)$	$27\hat{p}^{+}(\uparrow), 27\hat{p}^{+}(\downarrow), 30\hat{e}^{-}(\uparrow\downarrow)$	0
25	$^{55}_{25}\hat{Mn}_{30}(5/2)$	$12\left[{}^{4}_{2}\hat{He}_{2}(0)\right], \hat{d}(\uparrow\uparrow), 4\,\hat{n}(\uparrow), \hat{n}(\downarrow)$	$30\hat{p}^{+}(\uparrow), 25\hat{p}^{+}(\downarrow), 30\hat{e}^{-}(\uparrow\downarrow)$	5/2
26	$^{54}_{26}\hat{Fe}_{28}(0)$	$13\left[{}^{4}_{2}\hat{He}_{2}(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$27\hat{p}^{+}(\uparrow), 27\hat{p}^{+}(\downarrow), 28\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{56}_{26}\hat{Fe}_{30}(0)$	$13\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 2\,\hat{n}(\uparrow), 2\,\hat{n}(\downarrow)$	$28\hat{p}^{+}(\uparrow), 28\hat{p}^{+}(\downarrow), 30\hat{e}^{-}(\uparrow\downarrow)$	0

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27	$^{57}_{26}\hat{F}e_{31}$ (1/2)	$13\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 3\,\hat{n}(\uparrow), 2\,\hat{n}(\downarrow)$	$29\hat{p}^{+}(\uparrow), 28\hat{p}^{+}(\downarrow), 31\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{58}_{26}\hat{F}e_{32}$ (0)	$13\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 3\,\hat{n}(\uparrow), 3\,\hat{n}(\downarrow)$	$29\hat{p}^{+}(\uparrow), 29\hat{p}^{+}(\downarrow), 32\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{59}_{27}\hat{C}o_{32}$ (7/2)	$13\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 5\,\hat{n}(\uparrow)$	$33\hat{p}^{+}(\uparrow), 26\hat{p}^{+}(\downarrow), 32\hat{e}^{-}(\uparrow\downarrow)$	7/2
28	$^{58}_{28}\hat{N}i_{30}$ (0)	$14\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$29\hat{p}^{+}(\uparrow), 29\hat{p}^{+}(\downarrow), 30\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{60}_{28}\hat{N}i_{32}$ (0)	$14\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 2\,\hat{n}(\uparrow), 2\,\hat{n}(\downarrow)$	$30\hat{p}^{+}(\uparrow), 30\hat{p}^{+}(\downarrow), 32\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{61}_{28}\hat{N}i_{33}$ (3/2)	$14\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 4\,\hat{n}(\uparrow), \hat{n}(\downarrow)$	$32\hat{p}^{+}(\uparrow), 29\hat{p}^{+}(\downarrow), 33\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{62}_{28}\hat{N}i_{34}$ (0)	$14\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 3\,\hat{n}(\uparrow), 3\,\hat{n}(\downarrow)$	$31\hat{p}^{+}(\uparrow), 31\hat{p}^{+}(\downarrow), 34\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{64}_{28}\hat{N}i_{36}$ (0)	$14\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 4\,\hat{n}(\uparrow), 4\,\hat{n}(\downarrow)$	$32\hat{p}^{+}(\uparrow), 32\hat{p}^{+}(\downarrow), 36\hat{e}^{-}(\uparrow\downarrow)$	0
29	$^{63}_{29}\hat{C}u_{34}$ (3/2)	$14\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 3\,\hat{n}(\uparrow), 2\,\hat{n}(\downarrow)$	$33\hat{p}^{+}(\uparrow), 30\hat{p}^{+}(\downarrow), 34\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{65}_{29}\hat{C}u_{36}$ (3/2)	$14\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 4\,\hat{n}(\uparrow), 3\,\hat{n}(\downarrow)$	$34\hat{p}^{+}(\uparrow), 31\hat{p}^{+}(\downarrow), 36\hat{e}^{-}(\uparrow\downarrow)$	3/2
30	$^{64}_{30}\hat{Z}n_{34}$ (0)	$15\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 2\,\hat{n}(\uparrow), 2\,\hat{n}(\downarrow)$	$32\hat{p}^{+}(\uparrow), 32\hat{p}^{+}(\downarrow), 34\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{66}_{30}\hat{Z}n_{36}$ (0)	$15\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 3\,\hat{n}(\uparrow), 3\,\hat{n}(\downarrow)$	$33\hat{p}^{+}(\uparrow), 33\hat{p}^{+}(\downarrow), 36\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{67}_{30}\hat{Z}n_{37}$ (5/2)	$15\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 6\,\hat{n}(\uparrow), \hat{n}(\downarrow)$	$36\hat{p}^{+}(\uparrow), 31\hat{p}^{+}(\downarrow), 37\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{68}_{30}\hat{Z}n_{38}$ (0)	$15\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 4\,\hat{n}(\uparrow), 4\,\hat{n}(\downarrow)$	$34\hat{p}^{+}(\uparrow), 34\hat{p}^{+}(\downarrow), 38\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{68}_{30}\hat{Z}n_{38}$ (0)	$15\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 5\,\hat{n}(\uparrow), 5\,\hat{n}(\downarrow)$	$35\hat{p}^{+}(\uparrow), 35\hat{p}^{+}(\downarrow), 40\hat{e}^{-}(\uparrow\downarrow)$	0
31	$^{69}_{31}\hat{G}a_{38}$ (3/2)	$15\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 4\,\hat{n}(\uparrow), 3\,\hat{n}(\downarrow)$	$36\hat{p}^{+}(\uparrow), 33\hat{p}^{+}(\downarrow), 38\hat{e}^{-}(\uparrow\downarrow)$	3/2

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32	$^{71}_{31}\hat{Ga}_{40}$ (3/2)	$15\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $5\,\hat{n}(\uparrow)$, $4\,\hat{n}(\downarrow)$	$37\hat{p}^{+}(\uparrow), 34\hat{p}^{+}(\downarrow)$, $40\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{70}_{32}\hat{Ge}_{38}$ (0)	$16\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $3\,\hat{n}(\uparrow)$, $3\,\hat{n}(\downarrow)$	$35\hat{p}^{+}(\uparrow), 35\hat{p}^{+}(\downarrow)$, $38\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{72}_{32}\hat{Ge}_{40}$ (0)	$16\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $4\,\hat{n}(\uparrow)$, $4\,\hat{n}(\downarrow)$	$36\hat{p}^{+}(\uparrow), 36\hat{p}^{+}(\downarrow)$, $40\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{73}_{32}\hat{Ge}_{41}$ (9/2)	$16\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $9\,\hat{n}(\uparrow)$	$41\hat{p}^{+}(\uparrow), 32\hat{p}^{+}(\downarrow)$, $41\hat{e}^{-}(\uparrow\downarrow)$	9/2
	$^{74}_{32}\hat{Ge}_{42}$ (0)	$16\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $5\,\hat{n}(\uparrow)$, $5\,\hat{n}(\downarrow)$	$37\hat{p}^{+}(\uparrow), 37\hat{p}^{+}(\downarrow)$, $42\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{76}_{32}\hat{Ge}_{44}$ (0)	$16\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $6\,\hat{n}(\uparrow)$, $6\,\hat{n}(\downarrow)$	$38\hat{p}^{+}(\uparrow), 38\hat{p}^{+}(\downarrow)$, $44\hat{e}^{-}(\uparrow\downarrow)$	0
33	$^{75}_{33}\hat{As}_{42}$ (3/2)	$16\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $5\,\hat{n}(\uparrow)$, $4\,\hat{n}(\downarrow)$	$39\hat{p}^{+}(\uparrow), 36\hat{p}^{+}(\downarrow)$, $42\hat{e}^{-}(\uparrow\downarrow)$	3/2
34	$^{74}_{34}\hat{Se}_{40}$ (0)	$17\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $3\,\hat{n}(\uparrow)$, $3\,\hat{n}(\downarrow)$	$37\hat{p}^{+}(\uparrow), 37\hat{p}^{+}(\downarrow)$, $40\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{76}_{34}\hat{Se}_{42}$ (0)	$17\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $4\,\hat{n}(\uparrow)$, $4\,\hat{n}(\downarrow)$	$38\hat{p}^{+}(\uparrow), 38\hat{p}^{+}(\downarrow)$, $42\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{77}_{34}\hat{Se}_{43}$ (0)	$17\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $5\,\hat{n}(\uparrow)$, $4\,\hat{n}(\downarrow)$	$39\hat{p}^{+}(\uparrow), 38\hat{p}^{+}(\downarrow)$, $43\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{78}_{34}\hat{Se}_{44}$ (0)	$17\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $5\,\hat{n}(\uparrow)$, $5\,\hat{n}(\downarrow)$	$39\hat{p}^{+}(\uparrow), 39\hat{p}^{+}(\downarrow)$, $44\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{80}_{34}\hat{Se}_{46}$ (0)	$17\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $6\,\hat{n}(\uparrow)$, $6\,\hat{n}(\downarrow)$	$40\hat{p}^{+}(\uparrow), 40\hat{p}^{+}(\downarrow)$, $46\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{82}_{34}\hat{Se}_{48}$ (0)	$17\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $7\,\hat{n}(\uparrow)$, $7\,\hat{n}(\downarrow)$	$41\hat{p}^{+}(\uparrow), 41\hat{p}^{+}(\downarrow)$, $48\hat{e}^{-}(\uparrow\downarrow)$	0
35	$^{79}_{35}\hat{Br}_{44}$ (3/2)	$17\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $5\,\hat{n}(\uparrow)$, $4\,\hat{n}(\downarrow)$	$41\hat{p}^{+}(\uparrow), 38\hat{p}^{+}(\downarrow)$, $44\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{81}_{35}\hat{Br}_{46}$ (3/2)	$17\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $6\,\hat{n}(\uparrow)$, $5\,\hat{n}(\downarrow)$	$42\hat{p}^{+}(\uparrow), 39\hat{p}^{+}(\downarrow)$, $46\hat{e}^{-}(\uparrow\downarrow)$	3/2
36	$^{78}_{36}\hat{Kr}_{42}$ (0)	$18\left[{}^{4}_{2}\hat{He}_{2}(0)\right]$, $3\,\hat{n}(\uparrow)$, $3\,\hat{n}(\downarrow)$	$39\hat{p}^{+}(\uparrow), 39\hat{p}^{+}(\downarrow)$, $42\hat{e}^{-}(\uparrow\downarrow)$	0

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37	$^{80}_{36}\hat{K}r_{44}$ (0)	$18\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 4\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$40\hat{p}^{+}(\uparrow), 40\hat{p}^{+}(\downarrow), 44\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{82}_{36}\hat{K}r_{46}$ (0)	$18\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 5\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$41\hat{p}^{+}(\uparrow), 41\hat{p}^{+}(\downarrow), 46\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{83}_{36}\hat{K}r_{47}$ (9/2)	$18\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 10\hat{n}(\uparrow), \hat{n}(\downarrow)$	$46\hat{p}^{+}(\uparrow), 37\hat{p}^{+}(\downarrow), 47\hat{e}^{-}(\uparrow\downarrow)$	9/2
	$^{84}_{36}\hat{K}r_{48}$ (0)	$18\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$42\hat{p}^{+}(\uparrow), 42\hat{p}^{+}(\downarrow), 48\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{86}_{36}\hat{K}r_{50}$ (0)	$18\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 7\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$43\hat{p}^{+}(\uparrow), 43\hat{p}^{+}(\downarrow), 50\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{85}_{37}\hat{R}b_{48}$ (5/2)	$18\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 7\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$45\hat{p}^{+}(\uparrow), 40\hat{p}^{+}(\downarrow), 48\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{87}_{37}\hat{R}b_{50}$ (3/2)	$18\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 7\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$45\hat{p}^{+}(\uparrow), 42\hat{p}^{+}(\downarrow), 50\hat{e}^{-}(\uparrow\downarrow)$	3/2
38	$^{84}_{38}\hat{S}r_{46}$ (0)	$19\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 4\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$42\hat{p}^{+}(\uparrow), 42\hat{p}^{+}(\downarrow), 46\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{86}_{38}\hat{S}r_{48}$ (0)	$19\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 5\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$43\hat{p}^{+}(\uparrow), 43\hat{p}^{+}(\downarrow), 48\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{87}_{38}\hat{S}r_{49}$ (9/2)	$19\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 10\hat{n}(\uparrow), \hat{n}(\downarrow)$	$48\hat{p}^{+}(\uparrow), 39\hat{p}^{+}(\downarrow), 49\hat{e}^{-}(\uparrow\downarrow)$	9/2
	$^{88}_{38}\hat{S}r_{50}$ (0)	$19\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$44\hat{p}^{+}(\uparrow), 44\hat{p}^{+}(\downarrow), 50\hat{e}^{-}(\uparrow\downarrow)$	0
39	$^{89}_{39}\hat{Y}_{50}$ (1/2)	$19\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 5\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$45\hat{p}^{+}(\uparrow), 44\hat{p}^{+}(\downarrow), 50\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{90}_{40}\hat{Z}r_{50}$ (0)	$20\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 5\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$45\hat{p}^{+}(\uparrow), 45\hat{p}^{+}(\downarrow), 50\hat{e}^{-}(\uparrow\downarrow)$	0
40	$^{91}_{40}\hat{Z}r_{51}$ (5/2)	$20\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 8\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$48\hat{p}^{+}(\uparrow), 43\hat{p}^{+}(\downarrow), 51\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{92}_{40}\hat{Z}r_{52}$ (0)	$20\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$46\hat{p}^{+}(\uparrow), 46\hat{p}^{+}(\downarrow), 52\hat{e}^{-}(\uparrow\downarrow)$	0

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41	$^{94}_{40}\hat{Zr}_{54}$ (0)	$20\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 7\,\hat{n}(\uparrow), 7\,\hat{n}(\downarrow)$	$47\hat{p}^{+}(\uparrow), 47\hat{p}^{+}(\downarrow), 54\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{96}_{40}\hat{Zr}_{56}$ (0)	$20\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 8\,\hat{n}(\uparrow), 8\,\hat{n}(\downarrow)$	$48\hat{p}^{+}(\uparrow), 48\hat{p}^{+}(\downarrow), 56\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{93}_{41}\hat{Nb}_{52}$ (9/2)	$20\left[{}^{4}_{2}\hat{He}_{2}(0)\right], \hat{d}(\uparrow\uparrow), 9\,\hat{n}(\uparrow), 2\,\hat{n}(\downarrow)$	$51\hat{p}^{+}(\uparrow), 42\hat{p}^{+}(\downarrow), 52\hat{e}^{-}(\uparrow\downarrow)$	9/2
	$^{92}_{42}\hat{Mo}_{50}$ (0)	$21\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 4\,\hat{n}(\uparrow), 4\,\hat{n}(\downarrow)$	$46\hat{p}^{+}(\uparrow), 46\hat{p}^{+}(\downarrow), 50\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{94}_{42}\hat{Mo}_{52}$ (0)	$21\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 5\,\hat{n}(\uparrow), 5\,\hat{n}(\downarrow)$	$47\hat{p}^{+}(\uparrow), 47\hat{p}^{+}(\downarrow), 52\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{95}_{42}\hat{Mo}_{53}$ (5/2)	$21\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 8\,\hat{n}(\uparrow), 3\,\hat{n}(\downarrow)$	$50\hat{p}^{+}(\uparrow), 45\hat{p}^{+}(\downarrow), 53\hat{e}^{-}(\uparrow\downarrow)$	5/2
42	$^{96}_{42}\hat{Mo}_{54}$ (0)	$21\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 6\,\hat{n}(\uparrow), 6\,\hat{n}(\downarrow)$	$48\hat{p}^{+}(\uparrow), 48\hat{p}^{+}(\downarrow), 54\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{97}_{42}\hat{Mo}_{55}$ (5/2)	$21\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 9\,\hat{n}(\uparrow), 4\,\hat{n}(\downarrow)$	$51\hat{p}^{+}(\uparrow), 46\hat{p}^{+}(\downarrow), 55\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{98}_{42}\hat{Mo}_{56}$ (0)	$21\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 7\,\hat{n}(\uparrow), 7\,\hat{n}(\downarrow)$	$49\hat{p}^{+}(\uparrow), 49\hat{p}^{+}(\downarrow), 56\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{100}_{42}\hat{Mo}_{58}$ (0)	$21\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 8\,\hat{n}(\uparrow), 8\,\hat{n}(\downarrow)$	$50\hat{p}^{+}(\uparrow), 50\hat{p}^{+}(\downarrow), 58\hat{e}^{-}(\uparrow\downarrow)$	0
43	$^{xx}_{43}\hat{Tc}_{yy}$ (?)	No Stable Nuclide	No Stable Nuclide	
44	$^{96}_{44}\hat{Ru}_{52}$ (0)	$22\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 4\,\hat{n}(\uparrow), 4\,\hat{n}(\downarrow)$	$48\hat{p}^{+}(\uparrow), 48\hat{p}^{+}(\downarrow), 52\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{98}_{44}\hat{Ru}_{54}$ (0)	$22\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 5\,\hat{n}(\uparrow), 5\,\hat{n}(\downarrow)$	$49\hat{p}^{+}(\uparrow), 49\hat{p}^{+}(\downarrow), 54\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{99}_{44}\hat{Ru}_{55}$ (5/2)	$22\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 8\,\hat{n}(\uparrow), 3\,\hat{n}(\downarrow)$	$52\hat{p}^{+}(\uparrow), 47\hat{p}^{+}(\downarrow), 55\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{100}_{44}\hat{Ru}_{56}$ (0)	$22\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 6\,\hat{n}(\uparrow), 6\,\hat{n}(\downarrow)$	$50\hat{p}^{+}(\uparrow), 50\hat{p}^{+}(\downarrow), 56\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{101}_{44}\hat{Ru}_{57}$ (5/2)	$22\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 9\,\hat{n}(\uparrow), 4\,\hat{n}(\downarrow)$	$53\hat{p}^{+}(\uparrow), 48\hat{p}^{+}(\downarrow), 57\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{102}_{44}\hat{Ru}_{58}$ (0)	$22\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 7\,\hat{n}(\uparrow), 7\,\hat{n}(\downarrow)$	$51\hat{p}^{+}(\uparrow), 51\hat{p}^{+}(\downarrow), 58\hat{e}^{-}(\uparrow\downarrow)$	0

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45	$^{104}_{44}\hat{\mathrm{Ru}}_{60}$ (0)	$22\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$52\hat{p}^{+}(\uparrow), 52\hat{p}^{+}(\downarrow), 60\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{103}_{45}\hat{\mathrm{Rh}}_{58}$ (1/2)	$22\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], \hat{d}(\uparrow\uparrow), 6\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$52\hat{p}^{+}(\uparrow), 51\hat{p}^{+}(\downarrow), 58\hat{e}^{-}(\uparrow\downarrow)$	1/2
46	$^{102}_{46}\hat{\mathrm{Pd}}_{56}$ (0)	$23\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 5\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$51\hat{p}^{+}(\uparrow), 51\hat{p}^{+}(\downarrow), 56\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{104}_{46}\hat{\mathrm{Pd}}_{58}$ (0)	$23\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$52\hat{p}^{+}(\uparrow), 52\hat{p}^{+}(\downarrow), 58\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{105}_{46}\hat{\mathrm{Pd}}_{59}$ (5/2)	$23\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 9\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$55\hat{p}^{+}(\uparrow), 50\hat{p}^{+}(\downarrow), 59\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{106}_{46}\hat{\mathrm{Pd}}_{60}$ (0)	$23\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 7\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$53\hat{p}^{+}(\uparrow), 53\hat{p}^{+}(\downarrow), 60\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{108}_{46}\hat{\mathrm{Pd}}_{62}$ (0)	$23\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 8\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$54\hat{p}^{+}(\uparrow), 54\hat{p}^{+}(\downarrow), 62\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{110}_{46}\hat{\mathrm{Pd}}_{64}$ (0)	$23\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 9\hat{n}(\uparrow), 9\hat{n}(\downarrow)$	$55\hat{p}^{+}(\uparrow), 55\hat{p}^{+}(\downarrow), 64\hat{e}^{-}(\uparrow\downarrow)$	0
47	$^{107}_{47}\hat{\mathrm{Ag}}_{60}$ (1/2)	$23\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], \hat{d}(\uparrow\uparrow), 6\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$54\hat{p}^{+}(\uparrow), 53\hat{p}^{+}(\downarrow), 60\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{109}_{47}\hat{\mathrm{Ag}}_{62}$ (1/2)	$23\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], \hat{d}(\uparrow\uparrow), 7\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$55\hat{p}^{+}(\uparrow), 54\hat{p}^{+}(\downarrow), 62\hat{e}^{-}(\uparrow\downarrow)$	1/2
48	$^{106}_{48}\hat{\mathrm{Cd}}_{58}$ (0)	$24\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 5\hat{n}(\uparrow), 5\hat{n}(\downarrow)$	$53\hat{p}^{+}(\uparrow), 53\hat{p}^{+}(\downarrow), 58\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{108}_{48}\hat{\mathrm{Cd}}_{60}$ (0)	$24\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 6\hat{n}(\uparrow), 6\hat{n}(\downarrow)$	$54\hat{p}^{+}(\uparrow), 54\hat{p}^{+}(\downarrow), 60\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{110}_{48}\hat{\mathrm{Cd}}_{62}$ (0)	$24\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 7\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$55\hat{p}^{+}(\uparrow), 55\hat{p}^{+}(\downarrow), 62\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{111}_{48}\hat{\mathrm{Cd}}_{63}$ (1/2)	$24\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 8\hat{n}(\uparrow), 7\hat{n}(\downarrow)$	$56\hat{p}^{+}(\uparrow), 55\hat{p}^{+}(\downarrow), 63\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{112}_{48}\hat{\mathrm{Cd}}_{64}$ (0)	$24\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 8\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$56\hat{p}^{+}(\uparrow), 56\hat{p}^{+}(\downarrow), 64\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{113}_{48}\hat{\mathrm{Cd}}_{65}$ (1/2)	$24\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 9\hat{n}(\uparrow), 8\hat{n}(\downarrow)$	$57\hat{p}^{+}(\uparrow), 56\hat{p}^{+}(\downarrow), 65\hat{e}^{-}(\uparrow\downarrow)$	1/2

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49	$^{116}_{48}\hat{\mathrm{Cd}}_{68}$ (0)	$24\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 10\,\hat{n}(\uparrow), 10\,\hat{n}(\downarrow)$	$58\hat{p}^{+}(\uparrow), 58\hat{p}^{+}(\downarrow), 68\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{113}_{49}\hat{\mathrm{In}}_{64}$ (9/2)	$24\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], \hat{d}(\uparrow\uparrow), 11\,\hat{n}(\uparrow), 4\,\hat{n}(\downarrow)$	$61\hat{p}^{+}(\uparrow), 52\hat{p}^{+}(\downarrow), 64\hat{e}^{-}(\uparrow\downarrow)$	9/2
	$^{115}_{49}\hat{\mathrm{In}}_{66}$ (9/2)	$24\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], \hat{d}(\uparrow\uparrow), 12\,\hat{n}(\uparrow), 5\,\hat{n}(\downarrow)$	$62\hat{p}^{+}(\uparrow), 53\hat{p}^{+}(\downarrow), 66\hat{e}^{-}(\uparrow\downarrow)$	9/2
50	$^{112}_{50}\hat{\mathrm{Sn}}_{62}$ (0)	$25\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 6\,\hat{n}(\uparrow), 6\,\hat{n}(\downarrow)$	$56\hat{p}^{+}(\uparrow), 56\hat{p}^{+}(\downarrow), 62\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{114}_{50}\hat{\mathrm{Sn}}_{64}$ (0)	$25\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 7\,\hat{n}(\uparrow), 7\,\hat{n}(\downarrow)$	$57\hat{p}^{+}(\uparrow), 57\hat{p}^{+}(\downarrow), 64\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{115}_{50}\hat{\mathrm{Sn}}_{65}$ (1/2)	$25\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 8\,\hat{n}(\uparrow), 7\,\hat{n}(\downarrow)$	$58\hat{p}^{+}(\uparrow), 57\hat{p}^{+}(\downarrow), 65\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{116}_{50}\hat{\mathrm{Sn}}_{66}$ (0)	$25\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 8\,\hat{n}(\uparrow), 8\,\hat{n}(\downarrow)$	$58\hat{p}^{+}(\uparrow), 58\hat{p}^{+}(\downarrow), 66\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{117}_{50}\hat{\mathrm{Sn}}_{67}$ (1/2)	$25\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 9\,\hat{n}(\uparrow), 8\,\hat{n}(\downarrow)$	$59\hat{p}^{+}(\uparrow), 58\hat{p}^{+}(\downarrow), 67\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{118}_{50}\hat{\mathrm{Sn}}_{68}$ (0)	$25\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 9\,\hat{n}(\uparrow), 9\,\hat{n}(\downarrow)$	$59\hat{p}^{+}(\uparrow), 59\hat{p}^{+}(\downarrow), 68\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{119}_{50}\hat{\mathrm{Sn}}_{69}$ (1/2)	$25\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 10\,\hat{n}(\uparrow), 9\,\hat{n}(\downarrow)$	$60\hat{p}^{+}(\uparrow), 59\hat{p}^{+}(\downarrow), 69\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{120}_{50}\hat{\mathrm{Sn}}_{70}$ (0)	$25\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 10\,\hat{n}(\uparrow), 10\,\hat{n}(\downarrow)$	$60\hat{p}^{+}(\uparrow), 60\hat{p}^{+}(\downarrow), 70\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{122}_{50}\hat{\mathrm{Sn}}_{72}$ (0)	$25\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 11\,\hat{n}(\uparrow), 11\,\hat{n}(\downarrow)$	$61\hat{p}^{+}(\uparrow), 61\hat{p}^{+}(\downarrow), 72\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{124}_{50}\hat{\mathrm{Sn}}_{74}$ (0)	$25\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], 12\,\hat{n}(\uparrow), 12\,\hat{n}(\downarrow)$	$62\hat{p}^{+}(\uparrow), 62\hat{p}^{+}(\downarrow), 74\hat{e}^{-}(\uparrow\downarrow)$	0
51	$^{121}_{51}\hat{\mathrm{Sb}}_{70}$ (5/2)	$25\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right], \hat{d}(\uparrow\uparrow), 11\,\hat{n}(\uparrow), 8\,\hat{n}(\downarrow)$	$63\hat{p}^{+}(\uparrow), 58\hat{p}^{+}(\downarrow), 70\hat{e}^{-}(\uparrow\downarrow)$	5/2

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52	$^{123}_{51}\hat{S}b_{72}$ (7/2)	$25\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $13\,\hat{n}(\uparrow)$, $8\,\hat{n}(\downarrow)$	$65\hat{p}^{+}(\uparrow), 58\hat{p}^{+}(\downarrow)$, $72\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{120}_{52}\hat{T}e_{68}$ (0)	$26\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $8\,\hat{n}(\uparrow)$, $8\,\hat{n}(\downarrow)$	$60\hat{p}^{+}(\uparrow), 60\hat{p}^{+}(\downarrow)$, $68\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{122}_{52}\hat{T}e_{70}$ (0)	$26\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $9\,\hat{n}(\uparrow)$, $9\,\hat{n}(\downarrow)$	$61\hat{p}^{+}(\uparrow), 61\hat{p}^{+}(\downarrow)$, $70\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{123}_{52}\hat{T}e_{71}$ (1/2)	$26\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $10\,\hat{n}(\uparrow)$, $9\,\hat{n}(\downarrow)$	$62\hat{p}^{+}(\uparrow), 61\hat{p}^{+}(\downarrow)$, $71\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{124}_{52}\hat{T}e_{72}$ (0)	$26\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $10\,\hat{n}(\uparrow)$, $10\,\hat{n}(\downarrow)$	$62\hat{p}^{+}(\uparrow), 62\hat{p}^{+}(\downarrow)$, $72\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{125}_{52}\hat{T}e_{73}$ (1/2)	$26\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $11\,\hat{n}(\uparrow)$, $10\,\hat{n}(\downarrow)$	$63\hat{p}^{+}(\uparrow), 62\hat{p}^{+}(\downarrow)$, $73\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{126}_{52}\hat{T}e_{74}$ (0)	$26\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $11\,\hat{n}(\uparrow)$, $11\,\hat{n}(\downarrow)$	$63\hat{p}^{+}(\uparrow), 63\hat{p}^{+}(\downarrow)$, $74\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{128}_{52}\hat{T}e_{76}$ (0)	$26\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $12\,\hat{n}(\uparrow)$, $12\,\hat{n}(\downarrow)$	$64\hat{p}^{+}(\uparrow), 64\hat{p}^{+}(\downarrow)$, $76\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{130}_{52}\hat{T}e_{78}$ (0)	$26\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $13\,\hat{n}(\uparrow)$, $13\,\hat{n}(\downarrow)$	$63\hat{p}^{+}(\uparrow), 63\hat{p}^{+}(\downarrow)$, $74\hat{e}^{-}(\uparrow\downarrow)$	0
53	$^{127}_{53}\hat{I}_{74}$ (5/2)	$26\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $12\,\hat{n}(\uparrow)$, $9\,\hat{n}(\downarrow)$	$66\hat{p}^{+}(\uparrow), 61\hat{p}^{+}(\downarrow)$, $74\hat{e}^{-}(\uparrow\downarrow)$	5/2
54	$^{124}_{54}\hat{X}e_{70}$ (0)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $8\,\hat{n}(\uparrow)$, $8\,\hat{n}(\downarrow)$	$62\hat{p}^{+}(\uparrow), 62\hat{p}^{+}(\downarrow)$, $70\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{126}_{54}\hat{X}e_{72}$ (0)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $9\,\hat{n}(\uparrow)$, $9\,\hat{n}(\downarrow)$	$63\hat{p}^{+}(\uparrow), 63\hat{p}^{+}(\downarrow)$, $72\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{128}_{54}\hat{X}e_{74}$ (0)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $10\,\hat{n}(\uparrow)$, $10\,\hat{n}(\downarrow)$	$64\hat{p}^{+}(\uparrow), 64\hat{p}^{+}(\downarrow)$, $74\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{129}_{54}\hat{X}e_{75}$ (1/2)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $11\,\hat{n}(\uparrow)$,	$65\hat{p}^{+}(\uparrow), 64\hat{p}^{+}(\downarrow)$, $75\hat{e}^{-}(\uparrow\downarrow)$	1/2

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55	$^{130}_{54}\hat{X}e_{76}$ (0)	$10\,\hat{n}(\downarrow)$ $27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $11\,\hat{n}(\uparrow)$, $11\,\hat{n}(\downarrow)$	$65\hat{p}^{+}(\uparrow), 65\hat{p}^{+}(\downarrow)$, $76\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{131}_{54}\hat{X}e_{77}$ (3/2)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $13\,\hat{n}(\uparrow)$, $10\,\hat{n}(\downarrow)$	$67\hat{p}^{+}(\uparrow), 64\hat{p}^{+}(\downarrow)$, $77\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{132}_{54}\hat{X}e_{78}$ (0)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $12\,\hat{n}(\uparrow)$, $12\,\hat{n}(\downarrow)$	$66\hat{p}^{+}(\uparrow), 66\hat{p}^{+}(\downarrow)$, $78\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{134}_{54}\hat{X}e_{80}$ (0)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $13\,\hat{n}(\uparrow)$, $13\,\hat{n}(\downarrow)$	$67\hat{p}^{+}(\uparrow), 67\hat{p}^{+}(\downarrow)$, $80\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{136}_{54}\hat{X}e_{82}$ (0)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $14\,\hat{n}(\uparrow)$, $14\,\hat{n}(\downarrow)$	$68\hat{p}^{+}(\uparrow), 68\hat{p}^{+}(\downarrow)$, $82\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{133}_{55}\hat{C}s_{78}$ (7/2)	$27\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $14\,\hat{n}(\uparrow)$, $9\,\hat{n}(\downarrow)$	$70\hat{p}^{+}(\uparrow), 63\hat{p}^{+}(\downarrow)$, $78\hat{e}^{-}(\uparrow\downarrow)$	7/2
56	$^{130}_{56}\hat{B}a_{74}$ (0)	$28\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $9\,\hat{n}(\uparrow)$, $9\,\hat{n}(\downarrow)$	$65\hat{p}^{+}(\uparrow), 65\hat{p}^{+}(\downarrow)$, $74\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{132}_{56}\hat{B}a_{76}$ (0)	$28\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $10\,\hat{n}(\uparrow)$, $10\,\hat{n}(\downarrow)$	$66\hat{p}^{+}(\uparrow), 66\hat{p}^{+}(\downarrow)$, $76\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{134}_{56}\hat{B}a_{78}$ (0)	$28\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $11\,\hat{n}(\uparrow)$, $11\,\hat{n}(\downarrow)$	$67\hat{p}^{+}(\uparrow), 67\hat{p}^{+}(\downarrow)$, $78\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{135}_{56}\hat{B}a_{79}$ (3/2)	$28\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $13\,\hat{n}(\uparrow)$, $10\,\hat{n}(\downarrow)$	$69\hat{p}^{+}(\uparrow), 66\hat{p}^{+}(\downarrow)$, $79\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{136}_{56}\hat{B}a_{80}$ (0)	$28\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $12\,\hat{n}(\uparrow)$, $12\,\hat{n}(\downarrow)$	$68\hat{p}^{+}(\uparrow), 68\hat{p}^{+}(\downarrow)$, $80\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{137}_{56}\hat{B}a_{81}$ (3/2)	$28\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $14\,\hat{n}(\uparrow)$, $11\,\hat{n}(\downarrow)$	$70\hat{p}^{+}(\uparrow), 67\hat{p}^{+}(\downarrow)$, $81\hat{e}^{-}(\uparrow\downarrow)$	3/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
57	$^{138}_{56}\hat{Ba}_{82}$ (0)	$28\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 13\,\hat{n}(\uparrow), 13\,\hat{n}(\downarrow)$	$69\hat{p}^{+}(\uparrow), 69\hat{p}^{+}(\downarrow), 82\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{138}_{57}\hat{La}_{81}$ (5)	$28\left[{}^{4}_{2}\hat{He}_{2}(0)\right], \hat{d}(\uparrow\uparrow), 16\,\hat{n}(\uparrow), 8\,\hat{n}(\downarrow)$	$74\hat{p}^{+}(\uparrow), 64\hat{p}^{+}(\downarrow), 81\hat{e}^{-}(\uparrow\downarrow)$	5
	$^{139}_{57}\hat{La}_{82}$ (7/2)	$28\left[{}^{4}_{2}\hat{He}_{2}(0)\right], \hat{d}(\uparrow\uparrow), 15\,\hat{n}(\uparrow), 10\,\hat{n}(\downarrow)$	$73\hat{p}^{+}(\uparrow), 66\hat{p}^{+}(\downarrow), 82\hat{e}^{-}(\uparrow\downarrow)$	7/2
58	$^{136}_{58}\hat{Ce}_{78}$ (0)	$29\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 10\,\hat{n}(\uparrow), 10\,\hat{n}(\downarrow)$	$68\hat{p}^{+}(\uparrow), 68\hat{p}^{+}(\downarrow), 78\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{138}_{58}\hat{Ce}_{80}$ (0)	$29\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 11\,\hat{n}(\uparrow), 11\,\hat{n}(\downarrow)$	$69\hat{p}^{+}(\uparrow), 69\hat{p}^{+}(\downarrow), 80\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{140}_{58}\hat{Ce}_{82}$ (0)	$29\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 12\,\hat{n}(\uparrow), 12\,\hat{n}(\downarrow)$	$70\hat{p}^{+}(\uparrow), 70\hat{p}^{+}(\downarrow), 82\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{142}_{58}\hat{Ce}_{84}$ (0)	$29\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 13\,\hat{n}(\uparrow), 13\,\hat{n}(\downarrow)$	$71\hat{p}^{+}(\uparrow), 71\hat{p}^{+}(\downarrow), 84\hat{e}^{-}(\uparrow\downarrow)$	0
59	$^{141}_{59}\hat{Pr}_{82}$ (5/2)	$29\left[{}^{4}_{2}\hat{He}_{2}(0)\right], \hat{d}(\uparrow\uparrow), 13\,\hat{n}(\uparrow), 10\,\hat{n}(\downarrow)$	$73\hat{p}^{+}(\uparrow), 68\hat{p}^{+}(\downarrow), 82\hat{e}^{-}(\uparrow\downarrow)$	5/2
60	$^{142}_{60}\hat{Nd}_{82}$ (0)	$30\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 11\,\hat{n}(\uparrow), 11\,\hat{n}(\downarrow)$	$71\hat{p}^{+}(\uparrow), 71\hat{p}^{+}(\downarrow), 82\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{143}_{60}\hat{Nd}_{83}$ (7/2)	$30\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 15\,\hat{n}(\uparrow), 8\,\hat{n}(\downarrow)$	$75\hat{p}^{+}(\uparrow), 68\hat{p}^{+}(\downarrow), 83\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{144}_{60}\hat{Nd}_{84}$ (0)	$30\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 12\,\hat{n}(\uparrow), 12\,\hat{n}(\downarrow)$	$72\hat{p}^{+}(\uparrow), 72\hat{p}^{+}(\downarrow), 84\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{145}_{60}\hat{Nd}_{85}$ (7/2)	$30\left[{}^{4}_{2}\hat{He}_{2}(0)\right], 16\,\hat{n}(\uparrow), 9\,\hat{n}(\downarrow)$	$76\hat{p}^{+}(\uparrow), 69\hat{p}^{+}(\downarrow), 85\hat{e}^{-}(\uparrow\downarrow)$	7/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
	$^{146}_{60}\hat{N}d_{86}$ (0)	$30\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 13\,\hat{n}(\uparrow), 13\,\hat{n}(\downarrow)$	$73\hat{p}^{+}(\uparrow), 73\hat{p}^{+}(\downarrow), 86\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{148}_{60}\hat{N}d_{88}$ (0)	$30\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 14\,\hat{n}(\uparrow), 14\,\hat{n}(\downarrow)$	$74\hat{p}^{+}(\uparrow), 74\hat{p}^{+}(\downarrow), 88\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{150}_{60}\hat{N}d_{90}$ (0)	$30\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 15\,\hat{n}(\uparrow), 15\,\hat{n}(\downarrow)$	$75\hat{p}^{+}(\uparrow), 75\hat{p}^{+}(\downarrow), 90\hat{e}^{-}(\uparrow\downarrow)$	0
61	$^{xxx}_{61}\hat{P}m_{yy}$ (?)	No Stable Nuclide	No Stable Nuclide	
	$^{144}_{62}\hat{S}m_{82}$ (0)	$31\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 10\,\hat{n}(\uparrow), 10\,\hat{n}(\downarrow)$	$72\hat{p}^{+}(\uparrow), 72\hat{p}^{+}(\downarrow), 82\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{147}_{62}\hat{S}m_{85}$ (7/2)	$31\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 15\,\hat{n}(\uparrow), 8\,\hat{n}(\downarrow)$	$77\hat{p}^{+}(\uparrow), 70\hat{p}^{+}(\downarrow), 85\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{148}_{62}\hat{S}m_{86}$ (0)	$31\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 12\,\hat{n}(\uparrow), 12\,\hat{n}(\downarrow)$	$74\hat{p}^{+}(\uparrow), 74\hat{p}^{+}(\downarrow), 86\hat{e}^{-}(\uparrow\downarrow)$	0
62	$^{149}_{62}\hat{S}m_{87}$ (7/2)	$31\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 16\,\hat{n}(\uparrow), 9\,\hat{n}(\downarrow)$	$78\hat{p}^{+}(\uparrow), 71\hat{p}^{+}(\downarrow), 87\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{150}_{62}\hat{S}m_{88}$ (0)	$31\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 13\,\hat{n}(\uparrow), 13\,\hat{n}(\downarrow)$	$75\hat{p}^{+}(\uparrow), 75\hat{p}^{+}(\downarrow), 88\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{152}_{62}\hat{S}m_{90}$ (0)	$31\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 14\,\hat{n}(\uparrow), 14\,\hat{n}(\downarrow)$	$76\hat{p}^{+}(\uparrow), 76\hat{p}^{+}(\downarrow), 90\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{154}_{62}\hat{S}m_{92}$ (0)	$31\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 15\,\hat{n}(\uparrow), 15\,\hat{n}(\downarrow)$	$77\hat{p}^{+}(\uparrow), 77\hat{p}^{+}(\downarrow), 92\hat{e}^{-}(\uparrow\downarrow)$	0
63	$^{151}_{63}\hat{E}u_{88}$ (5/2)	$31\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 14\,\hat{n}(\uparrow), 11\,\hat{n}(\downarrow)$	$78\hat{p}^{+}(\uparrow), 73\hat{p}^{+}(\downarrow), 88\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{153}_{63}\hat{E}u_{90}$ (5/2)	$31\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 15\,\hat{n}(\uparrow), 12\,\hat{n}(\downarrow)$	$79\hat{p}^{+}(\uparrow), 74\hat{p}^{+}(\downarrow), 90\hat{e}^{-}(\uparrow\downarrow)$	5/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
64	$^{152}_{64}\hat{G}d_{88}$ (0)	$32\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 12\,\hat{n}(\uparrow), 12\,\hat{n}(\downarrow)$	$76\hat{p}^{+}(\uparrow), 76\hat{p}^{+}(\downarrow), 88\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{154}_{64}\hat{G}d_{90}$ (0)	$32\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 13\,\hat{n}(\uparrow), 13\,\hat{n}(\downarrow)$	$77\hat{p}^{+}(\uparrow), 77\hat{p}^{+}(\downarrow), 90\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{155}_{64}\hat{G}d_{91}$ (3/2)	$32\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 15\,\hat{n}(\uparrow), 12\,\hat{n}(\downarrow)$	$79\hat{p}^{+}(\uparrow), 76\hat{p}^{+}(\downarrow), 91\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{156}_{64}\hat{G}d_{92}$ (0)	$32\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 14\,\hat{n}(\uparrow), 14\,\hat{n}(\downarrow)$	$78\hat{p}^{+}(\uparrow), 78\hat{p}^{+}(\downarrow), 92\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{157}_{64}\hat{G}d_{93}$ (3/2)	$32\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 16\,\hat{n}(\uparrow), 13\,\hat{n}(\downarrow)$	$80\hat{p}^{+}(\uparrow), 77\hat{p}^{+}(\downarrow), 93\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{158}_{64}\hat{G}d_{94}$ (0)	$32\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 15\,\hat{n}(\uparrow), 15\,\hat{n}(\downarrow)$	$79\hat{p}^{+}(\uparrow), 79\hat{p}^{+}(\downarrow), 94\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{160}_{64}\hat{G}d_{96}$ (0)	$32\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 16\,\hat{n}(\uparrow), 16\,\hat{n}(\downarrow)$	$80\hat{p}^{+}(\uparrow), 80\hat{p}^{+}(\downarrow), 96\hat{e}^{-}(\uparrow\downarrow)$	0
65	$^{159}_{65}\hat{T}b_{94}$ (3/2)	$32\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 15\,\hat{n}(\uparrow), 14\,\hat{n}(\downarrow)$	$81\hat{p}^{+}(\uparrow), 78\hat{p}^{+}(\downarrow), 94\hat{e}^{-}(\uparrow\downarrow)$	3/2
66	$^{156}_{66}\hat{D}y_{90}$ (0)	$33\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 12\,\hat{n}(\uparrow), 12\,\hat{n}(\downarrow)$	$78\hat{p}^{+}(\uparrow), 78\hat{p}^{+}(\downarrow), 90\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{158}_{66}\hat{D}y_{92}$ (0)	$33\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 13\,\hat{n}(\uparrow), 13\,\hat{n}(\downarrow)$	$79\hat{p}^{+}(\uparrow), 79\hat{p}^{+}(\downarrow), 92\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{160}_{66}\hat{D}y_{94}$ (0)	$33\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 14\,\hat{n}(\uparrow), 14\,\hat{n}(\downarrow)$	$80\hat{p}^{+}(\uparrow), 80\hat{p}^{+}(\downarrow), 94\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{161}_{66}\hat{D}y_{95}$ (5/2)	$33\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 17\,\hat{n}(\uparrow), 12\,\hat{n}(\downarrow)$	$83\hat{p}^{+}(\uparrow), 78\hat{p}^{+}(\downarrow), 95\hat{e}^{-}(\uparrow\downarrow)$	5/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
	$^{162}_{66}\hat{D}y_{96}$ (0)	$33\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 15\,\hat{n}(\uparrow), 15\,\hat{n}(\downarrow)$	$81\hat{p}^{+}(\uparrow), 81\hat{p}^{+}(\downarrow), 96\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{163}_{66}\hat{D}y_{97}$ (5/2)	$33\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 18\,\hat{n}(\uparrow), 13\,\hat{n}(\downarrow)$	$84\hat{p}^{+}(\uparrow), 79\hat{p}^{+}(\downarrow), 97\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{164}_{66}\hat{D}y_{98}$ (0)	$33\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 16\,\hat{n}(\uparrow), 16\,\hat{n}(\downarrow)$	$82\hat{p}^{+}(\uparrow), 82\hat{p}^{+}(\downarrow), 98\hat{e}^{-}(\uparrow\downarrow)$	0
67	$^{165}_{67}\hat{H}o_{98}$ (7/2)	$33\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 18\,\hat{n}(\uparrow), 13\,\hat{n}(\downarrow)$	$86\hat{p}^{+}(\uparrow), 79\hat{p}^{+}(\downarrow), 98\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{162}_{68}\hat{E}r_{94}$ (0)	$34\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 13\,\hat{n}(\uparrow), 13\,\hat{n}(\downarrow)$	$81\hat{p}^{+}(\uparrow), 81\hat{p}^{+}(\downarrow), 94\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{164}_{68}\hat{E}r_{96}$ (0)	$34\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 14\,\hat{n}(\uparrow), 14\,\hat{n}(\downarrow)$	$82\hat{p}^{+}(\uparrow), 82\hat{p}^{+}(\downarrow), 96\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{166}_{68}\hat{E}r_{98}$ (0)	$34\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 15\,\hat{n}(\uparrow), 15\,\hat{n}(\downarrow)$	$83\hat{p}^{+}(\uparrow), 83\hat{p}^{+}(\downarrow), 98\hat{e}^{-}(\uparrow\downarrow)$	0
68	$^{167}_{68}\hat{E}r_{99}$ (7/2)	$34\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 19\,\hat{n}(\uparrow), 12\,\hat{n}(\downarrow)$	$87\hat{p}^{+}(\uparrow), 80\hat{p}^{+}(\downarrow), 99\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{168}_{68}\hat{E}r_{100}$ (0)	$34\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 16\,\hat{n}(\uparrow), 16\,\hat{n}(\downarrow)$	$84\hat{p}^{+}(\uparrow), 84\hat{p}^{+}(\downarrow), 100\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{170}_{68}\hat{E}r_{102}$ (0)	$34\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 17\,\hat{n}(\uparrow), 17\,\hat{n}(\downarrow)$	$86\hat{p}^{+}(\uparrow), 86\hat{p}^{+}(\downarrow), 102\hat{e}^{-}(\uparrow\downarrow)$	0
69	$^{169}_{69}\hat{T}m_{100}$ (1/2)	$34\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 16\,\hat{n}(\uparrow), 15\,\hat{n}(\downarrow)$	$85\hat{p}^{+}(\uparrow), 84\hat{p}^{+}(\downarrow), 100\hat{e}^{-}(\uparrow\downarrow)$	1/2
70	$^{168}_{70}\hat{Y}b_{98}$ (0)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 14\,\hat{n}(\uparrow), 14\,\hat{n}(\downarrow)$	$84\hat{p}^{+}(\uparrow), 84\hat{p}^{+}(\downarrow), 98\hat{e}^{-}(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
71	$^{170}_{70}\hat{Y}b_{100}$ (0)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 15\,\hat{n}(\uparrow), 15\,\hat{n}(\downarrow)$	$85\hat{p}^{+}(\uparrow), 85\hat{p}^{+}(\downarrow), 100\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{171}_{70}\hat{Y}b_{101}$ (1/2)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 16\,\hat{n}(\uparrow), 15\,\hat{n}(\downarrow)$	$86\hat{p}^{+}(\uparrow), 85\hat{p}^{+}(\downarrow), 101\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{172}_{70}\hat{Y}b_{102}$ (0)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 16\,\hat{n}(\uparrow), 16\,\hat{n}(\downarrow)$	$86\hat{p}^{+}(\uparrow), 86\hat{p}^{+}(\downarrow), 102\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{173}_{70}\hat{Y}b_{103}$ (5/2)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 19\,\hat{n}(\uparrow), 14\,\hat{n}(\downarrow)$	$89\hat{p}^{+}(\uparrow), 84\hat{p}^{+}(\downarrow), 103\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{174}_{70}\hat{Y}b_{104}$ (0)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 17\,\hat{n}(\uparrow), 17\,\hat{n}(\downarrow)$	$87\hat{p}^{+}(\uparrow), 87\hat{p}^{+}(\downarrow), 104\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{176}_{70}\hat{Y}b_{106}$ (0)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 18\,\hat{n}(\uparrow), 18\,\hat{n}(\downarrow)$	$88\hat{p}^{+}(\uparrow), 88\hat{p}^{+}(\downarrow), 106\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{175}_{71}\hat{L}u_{104}$ (7/2)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 19\,\hat{n}(\uparrow), 14\,\hat{n}(\downarrow)$	$91\hat{p}^{+}(\uparrow), 84\hat{p}^{+}(\downarrow), 104\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{176}_{71}\hat{L}u_{105}$ (7)	$35\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 23\,\hat{n}(\uparrow), 11\,\hat{n}(\downarrow)$	$95\hat{p}^{+}(\uparrow), 81\hat{p}^{+}(\downarrow), 105\hat{e}^{-}(\uparrow\downarrow)$	7
72	$^{174}_{72}\hat{H}f_{102}$ (0)	$36\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 15\,\hat{n}(\uparrow), 15\,\hat{n}(\downarrow)$	$87\hat{p}^{+}(\uparrow), 87\hat{p}^{+}(\downarrow), 102\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{176}_{72}\hat{H}f_{104}$ (0)	$36\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 16\,\hat{n}(\uparrow), 16\,\hat{n}(\downarrow)$	$88\hat{p}^{+}(\uparrow), 88\hat{p}^{+}(\downarrow), 104\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{177}_{72}\hat{H}f_{105}$ (7/2)	$36\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 20\,\hat{n}(\uparrow), 13\,\hat{n}(\downarrow)$	$92\hat{p}^{+}(\uparrow), 85\hat{p}^{+}(\downarrow), 105\hat{e}^{-}(\uparrow\downarrow)$	7/2
	$^{178}_{72}\hat{H}f_{106}$ (0)	$36\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 17\,\hat{n}(\uparrow), 17\,\hat{n}(\downarrow)$	$89\hat{p}^{+}(\uparrow), 89\hat{p}^{+}(\downarrow), 106\hat{e}^{-}(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
73	$^{179}_{72}\hat{H}f_{107}$ (9/2)	$36\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 22\,\hat{n}(\uparrow), 13\,\hat{n}(\downarrow)$	$94\hat{p}^{+}(\uparrow), 85\hat{p}^{+}(\downarrow), 107\hat{e}^{-}(\uparrow\downarrow)$	9/2
	$^{180}_{72}\hat{H}f_{108}$ (0)	$36\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 18\,\hat{n}(\uparrow), 18\,\hat{n}(\downarrow)$	$90\hat{p}^{+}(\uparrow), 90\hat{p}^{+}(\downarrow), 108\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{181}_{73}\hat{T}a_{108}$ (7/2)	$36\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 20\,\hat{n}(\uparrow), 15\,\hat{n}(\downarrow)$	$94\hat{p}^{+}(\uparrow), 87\hat{p}^{+}(\downarrow), 108\hat{e}^{-}(\uparrow\downarrow)$	7/2
74	$^{180}_{74}\hat{W}_{106}$ (0)	$37\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 16\,\hat{n}(\uparrow), 16\,\hat{n}(\downarrow)$	$90\hat{p}^{+}(\uparrow), 90\hat{p}^{+}(\downarrow), 106\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{182}_{74}\hat{W}_{108}$ (0)	$37\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 17\,\hat{n}(\uparrow), 17\,\hat{n}(\downarrow)$	$91\hat{p}^{+}(\uparrow), 91\hat{p}^{+}(\downarrow), 108\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{183}_{74}\hat{W}_{109}$ (1/2)	$37\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 18\,\hat{n}(\uparrow), 17\,\hat{n}(\downarrow)$	$92\hat{p}^{+}(\uparrow), 91\hat{p}^{+}(\downarrow), 109\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{184}_{74}\hat{W}_{110}$ (0)	$37\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 18\,\hat{n}(\uparrow), 18\,\hat{n}(\downarrow)$	$92\hat{p}^{+}(\uparrow), 92\hat{p}^{+}(\downarrow), 110\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{186}_{74}\hat{W}_{112}$ (0)	$37\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 19\,\hat{n}(\uparrow), 19\,\hat{n}(\downarrow)$	$93\hat{p}^{+}(\uparrow), 93\hat{p}^{+}(\downarrow), 112\hat{e}^{-}(\uparrow\downarrow)$	0
75	$^{185}_{75}\hat{R}e_{110}$ (5/2)	$37\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 19\,\hat{n}(\uparrow), 16\,\hat{n}(\downarrow)$	$95\hat{p}^{+}(\uparrow), 90\hat{p}^{+}(\downarrow), 110\hat{e}^{-}(\uparrow\downarrow)$	5/2
	$^{187}_{75}\hat{R}e_{112}$ (5/2)	$37\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 20\,\hat{n}(\uparrow), 17\,\hat{n}(\downarrow)$	$96\hat{p}^{+}(\uparrow), 91\hat{p}^{+}(\downarrow), 112\hat{e}^{-}(\uparrow\downarrow)$	5/2
76	$^{184}_{76}\hat{O}s_{108}$ (0)	$38\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 16\,\hat{n}(\uparrow), 16\,\hat{n}(\downarrow)$	$92\hat{p}^{+}(\uparrow), 92\hat{p}^{+}(\downarrow), 108\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{186}_{76}\hat{O}s_{110}$ (0)	$38\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 17\,\hat{n}(\uparrow), 17\,\hat{n}(\downarrow)$	$93\hat{p}^{+}(\uparrow), 93\hat{p}^{+}(\downarrow), 110\hat{e}^{-}(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
77	$^{187}_{76}\hat{O}s_{111}$ (1/2)	$38\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 18\,\hat{n}(\uparrow), 17\,\hat{n}(\downarrow)$	$94\hat{p}^{+}(\uparrow), 93\hat{p}^{+}(\downarrow), 111\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{188}_{76}\hat{O}s_{112}$ (0)	$38\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 18\,\hat{n}(\uparrow), 18\,\hat{n}(\downarrow)$	$94\hat{p}^{+}(\uparrow), 94\hat{p}^{+}(\downarrow), 112\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{189}_{76}\hat{O}s_{113}$ (3/2)	$38\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 20\,\hat{n}(\uparrow), 17\,\hat{n}(\downarrow)$	$96\hat{p}^{+}(\uparrow), 93\hat{p}^{+}(\downarrow), 113\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{190}_{76}\hat{O}s_{114}$ (0)	$38\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 19\,\hat{n}(\uparrow), 19\,\hat{n}(\downarrow)$	$95\hat{p}^{+}(\uparrow), 95\hat{p}^{+}(\downarrow), 114\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{192}_{76}\hat{O}s_{116}$ (0)	$38\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 20\,\hat{n}(\uparrow), 20\,\hat{n}(\downarrow)$	$96\hat{p}^{+}(\uparrow), 96\hat{p}^{+}(\downarrow), 116\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{191}_{77}\hat{I}r_{114}$ (3/2)	$38\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 19\,\hat{n}(\uparrow), 18\,\hat{n}(\downarrow)$	$97\hat{p}^{+}(\uparrow), 94\hat{p}^{+}(\downarrow), 114\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{193}_{77}\hat{I}r_{116}$ (3/2)	$38\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], \hat{d}(\uparrow\uparrow), 20\,\hat{n}(\uparrow), 19\,\hat{n}(\downarrow)$	$98\hat{p}^{+}(\uparrow), 95\hat{p}^{+}(\downarrow), 116\hat{e}^{-}(\uparrow\downarrow)$	3/2
78	$^{192}_{78}\hat{P}t_{114}$ (0)	$39\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 18\,\hat{n}(\uparrow), 18\,\hat{n}(\downarrow)$	$96\hat{p}^{+}(\uparrow), 96\hat{p}^{+}(\downarrow), 114\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{194}_{78}\hat{P}t_{116}$ (0)	$39\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 19\,\hat{n}(\uparrow), 19\,\hat{n}(\downarrow)$	$97\hat{p}^{+}(\uparrow), 97\hat{p}^{+}(\downarrow), 116\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{195}_{78}\hat{P}t_{117}$ (1/2)	$39\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 20\,\hat{n}(\uparrow), 19\,\hat{n}(\downarrow)$	$98\hat{p}^{+}(\uparrow), 97\hat{p}^{+}(\downarrow), 117\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{196}_{78}\hat{P}t_{118}$ (0)	$39\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 20\,\hat{n}(\uparrow), 20\,\hat{n}(\downarrow)$	$98\hat{p}^{+}(\uparrow), 98\hat{p}^{+}(\downarrow), 118\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{198}_{78}\hat{P}t_{120}$ (0)	$39\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 21\,\hat{n}(\uparrow), 21\,\hat{n}(\downarrow)$	$99\hat{p}^{+}(\uparrow), 99\hat{p}^{+}(\downarrow), 120\hat{e}^{-}(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
79	$^{197}_{79}\hat{A}u_{118}$ (3/2)	$39\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $20\,\hat{n}(\uparrow)$, $19\,\hat{n}(\downarrow)$	$100\hat{p}^{+}(\uparrow), 97\hat{p}^{+}(\downarrow)$, $118\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{196}_{80}\hat{H}g_{116}$ (0)	$40\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $18\,\hat{n}(\uparrow)$, $18\,\hat{n}(\downarrow)$	$98\hat{p}^{+}(\uparrow), 98\hat{p}^{+}(\downarrow)$, $116\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{198}_{80}\hat{H}g_{118}$ (0)	$40\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $19\,\hat{n}(\uparrow)$, $19\,\hat{n}(\downarrow)$	$99\hat{p}^{+}(\uparrow), 99\hat{p}^{+}(\downarrow)$, $118\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{199}_{80}\hat{H}g_{119}$ (1/2)	$40\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $20\,\hat{n}(\uparrow)$, $19\,\hat{n}(\downarrow)$	$100\hat{p}^{+}(\uparrow), 99\hat{p}^{+}(\downarrow)$, $119\hat{e}^{-}(\uparrow\downarrow)$	1/2
80	$^{200}_{80}\hat{H}g_{120}$ (0)	$40\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $20\,\hat{n}(\uparrow)$, $20\,\hat{n}(\downarrow)$	$100\hat{p}^{+}(\uparrow), 100\hat{p}^{+}(\downarrow)$, $120\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{201}_{80}\hat{H}g_{121}$ (3/2)	$40\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $22\,\hat{n}(\uparrow)$, $19\,\hat{n}(\downarrow)$	$102\hat{p}^{+}(\uparrow), 99\hat{p}^{+}(\downarrow)$, $121\hat{e}^{-}(\uparrow\downarrow)$	3/2
	$^{202}_{80}\hat{H}g_{122}$ (0)	$40\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $21\,\hat{n}(\uparrow)$, $21\,\hat{n}(\downarrow)$	$101\hat{p}^{+}(\uparrow), 101\hat{p}^{+}(\downarrow)$, $122\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{204}_{80}\hat{H}g_{124}$ (0)	$40\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $22\,\hat{n}(\uparrow)$, $22\,\hat{n}(\downarrow)$	$102\hat{p}^{+}(\uparrow), 102\hat{p}^{+}(\downarrow)$, $124\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{203}_{81}\hat{T}l_{122}$ (1/2)	$40\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $20\,\hat{n}(\uparrow)$, $21\,\hat{n}(\downarrow)$	$102\hat{p}^{+}(\uparrow), 101\hat{p}^{+}(\downarrow)$, $122\hat{e}^{-}(\uparrow\downarrow)$	1/2
81	$^{205}_{81}\hat{T}l_{124}$ (1/2)	$40\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $\hat{d}(\uparrow\uparrow)$, $21\,\hat{n}(\uparrow)$, $22\,\hat{n}(\downarrow)$	$103\hat{p}^{+}(\uparrow), 102\hat{p}^{+}(\downarrow)$, $124\hat{e}^{-}(\uparrow\downarrow)$	1/2
	$^{204}_{82}\hat{P}b_{122}$ (0)	$41\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $20\,\hat{n}(\uparrow)$, $20\,\hat{n}(\downarrow)$	$102\hat{p}^{+}(\uparrow), 102\hat{p}^{+}(\downarrow)$, $122\hat{e}^{-}(\uparrow\downarrow)$	0
82	$^{206}_{82}\hat{P}b_{124}$ (0)	$41\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $21\,\hat{n}(\uparrow)$, $21\,\hat{n}(\downarrow)$	$103\hat{p}^{+}(\uparrow), 103\hat{p}^{+}(\downarrow)$, $124\hat{e}^{-}(\uparrow\downarrow)$	0
	$^{207}_{82}\hat{P}b_{125}$ (1/2)	$41\left[{}^{4}_{2}\hat{H}e_{2}(0)\right]$, $22\,\hat{n}(\uparrow)$, $21\,\hat{n}(\downarrow)$	$104\hat{p}^{+}(\uparrow), 103\hat{p}^{+}(\downarrow)$, $125\hat{e}^{-}(\uparrow\downarrow)$	1/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
	$^{208}_{82}\hat{P}b_{126}(0)$	$41\left[{}^{4}_{2}\hat{H}e_{2}(0)\right], 22\,\hat{n}(\uparrow),$ $22\,\hat{n}(\downarrow)$	$104\hat{p}^{+}(\uparrow), 104\hat{p}^{+}(\downarrow),$ $126\hat{e}^{-}(\uparrow\downarrow)$	0

14. Some Observations

The purpose of presenting the nuclear configuration in terms of isonucleons in this paper is to make available adequate ground so that one can attempt to (i) develop a theory of nuclear stability and (ii) acquire understanding of other nuclear properties of the stable nuclides. In Table 2 we have proton as the first entry and 278 stable, primordial and very long lived isonuclides up to and including atomic number 82 of the periodic table. Beyond Pb all elements are radioactive. There are two elements namely Tc ($Z = 43$) and Pm ($Z = 61$) having no stable isotopes, that has also been mentioned in Table 2. The columns 3, 4 and 5 of Table 2 depict respectively the hadronic mechanics based nuclear configuration of models I and II, and experimental nuclear spin of the isonuclide. In the present Section 14 we summarize our observations on them.

14.1. Nuclear Configuration of Model-I

14.1.1. Proton

In Table 2 there are two entries corresponding to $Z = 1$. They are the isotopes of hydrogen, the first element of the periodic table.

Thus, ${}^{1}_{1}H_{0}$, in fact, is the proton, the fundamental particle, which is a stable particle. For its description no hadronic mechanics is required, hence it is not an isonuclide.

14.1.2. Isodeuteron

Hydrogen of mass number 2 is conventionally termed as deuterium. Its nucleus, indeed, is an isonucleus hence it is termed as isodeuteron that gets represented as ${}^{2}_{1}H_{1}$. We represent this system in our proposed notation as,

$$\begin{aligned} &{}^{2}_{1}H_{1} : n = 1,\ p^{+} = 1\} \Rightarrow \\ &\left[\left(\hat{p}^{+}(\uparrow), \hat{e}^{-}(J = 0), \hat{p}^{+}(\uparrow)\right)\right]_{hm} \\ &\equiv {}^{2}_{1}H_{1}(1) \equiv \hat{d}(\uparrow\uparrow),\ \text{stable},\ J = 1 \end{aligned} \qquad (94)$$

How the nuclear spin of value 1 for isodeuteron originates gets easily understood from Figure 9.

14.1.3. Other Stable Isonuclides of Table 2

All the stable isonuclides of Table 2 beyond hydrogen are the combination of isodeuterons and isoneutrons except He-3 which consists of an isodeuteron and an isoproton.

I. Stable Isonuclides with Null Nuclear Spin

Out of total of 278 isonuclides of Table 2 there we have 163 isonuclides having nuclear spin of 0. From this group 9 isonuclides consists only of all spin paired isodeuterons, and they can be considered as possessing 1, 3, 4, 5, 6, 7, 8, 9 and 10 ${}^{4}_{2}\hat{B}e_{2}(0)$ centers. Notice that the number 2 is notoriously missing in this list. That corresponds to the isonuclide ${}^{8}_{4}\hat{B}e_{4}(0)$ that we know is unstable and instantaneously disintegrates to α-particles. The same observation in terms of isodeuterons speaks as follows. Recall that the isodeuteron is a stable combination of isonucleons. However, two isodeuterons in the singlet coupling are also stable, which actually is the α-particle. Next on addition of one isodeuteron to it there we form an isonuclide of Li-6, which also is a stable isonuclide. But on further adding one more isodeuteron with total nuclear spin zero we obtain Be-8 isonuclide which is unstable. Thus we see that three isodeuteron in low spin state is stable but the four isodeuteron in zero spin state is unstable (But notice that in the case of stable Be-9 there we have two parallel spin isodeuterons coupled with one isoneutron of opposite spin. It means that the addition of one isoneutron to Be-8 forces one spin paired isodeuterons to assume parallel spin and itself combines to them with opposite spin that imparts stability to Be-9 with net nuclear spin of 3/2.). However, the next stable isonuclide is B-10 consisting of 5 isodeuterons. But in this case there we have two spin paired isodeuterons and three unpaired ones, ironically which is not a combination of 2α-particles and one isodeuteron similar to Li-6. The next stable isonuclide is C-12 that consists of 6 isodeuterons in the spin paired state, which is equivalent to strongly bound combination of 3α-particles. It is surprising that the combination of 2α-particles is unstable but the combination of 3α-particles is stable one. Here onwards 4 to 10α-particles combination are all stable ones.

The remaining 154 isonuclides with null nuclear spin consist of even number of isodeuterons and even number of isoneutrons and they are all spin paired. Notice that not only the isoneutrons get stabilized but also the zero spin di-isoneutrons are getting stabilized in the environment of zero spin isodeuterons. Amongst them from Ca-42 and onwards we have the combination of di-isoneutrons and isodeuterons. The number of spin paired di-isoneuterons continuously increases and rises ultimately to 22 in number in the case of Pb-208 that consists of 82 spin paired isodeuterons. Recall that a dineutron is not a stable entity but 22 spin paired di-isoneutrons of Pb-208 in the presence of 41 α-particles are stable. We need to investigate further what interactions are responsible for this extraordinary stability. However, we also need to take into account the nuclear configuration of adjacent unstable isonuclide. For example, Ca-40 and Ca-42 are both stable but Ca-39 and Ca-41 are unstable nuclides and Ca-43 is stable one. The nuclear configuration commensurate with the observed nuclear spin of

Ca-39 and Ca-41 respectively are:

$$9\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right],\hat{p}^{+}(\uparrow),\hat{d}(\uparrow\uparrow)$$

and

$$8\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right],4\hat{d}(\uparrow\uparrow),\hat{n}(\downarrow)$$

They both are unstable nuclear configurations. Thus in going from Ca-39 to Ca-40 the nuclear configuration transforms as

$$9\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right],\hat{p}^{+}(\uparrow),\hat{d}(\uparrow\uparrow)$$
$$\downarrow$$
$$9\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right],\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\equiv 10\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right]$$

That is, in Ca-39 last pair of an isodeuteron and a proton are in high spin state but on the availability of one more isoneutron in case of Ca-40 not only one additional isodeuteron gets formed but also both the isodeuterons get spin paired, imparting stability.

Whereas in going from Ca-40 to Ca-41 the nuclear configuration transforms as

$$10\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right]\rightarrow 8\left[{}^{4}_{2}\hat{\mathrm{He}}_{2}(0)\right],4\hat{d}(\uparrow\uparrow),\hat{n}(\downarrow)$$

That is, 2α-particles out of 10 in Ca-40 become 4 spin unpaired isodeuterons on the availability of an additional isoneutron in the case of Ca-41 and at the same time the additional isoneutron orients its spin opposite to that of the isodeuterons and the result is nuclear instability of Ca-41. Next we observe that when we add one isodeuteron to Ca-41 to form Ca-42 strikingly the four spin unpaired isodeuterons of the former isonuclide as well as the two isoneutrons get spin paired to form Ca-42, a stable isonuclide.

Further, on going from Ca-42 to Ca-43 we see from Table 2 that one α-particle out of 10 of Ca-42 gets spin unpaired providing two isodeuterons and simultaneously the spin paired isoneutrons of Ca-42 become spin unpaired to form three spin unpaired isoneutrons of Ca-43. Still Ca-43 is a stable isonuclide. Moreover, the reason of the nuclear stability of Ca-44 seems to be the same as that of Ca-42 because in the former we have one spin paired di-isoneutron whereas in the latter case we have two spin paired di-isoneutrons.

It seems from the above observations that the spin pairing of the isonucleons is not the only parameter that determines the nuclear stability. Other factors need to be identified. This would get further substantiated by considering the stable isonuclides with non-zero nuclear spin in next subsection.

II. Stable Isonuclides with Non-zero Nuclear Spin

Moreover, there are 104 stable isonuclides (in addition to isodeuteron) in Table 2 with non-zero nuclear spin. All have the combination of isodeuterons and isoneutrons except He-3, which consists of one isodeuteron with both its spins up and an isoproton with spin down.

1. Notice that in Table 2 there we have highest nuclear spin of 7 (Lu-176). There also we have isonuclides with nuclear spins 6 (V-50), 5 (La-138) and 9/2 (Ge-73, Kr-83, Sr-87, Nb-93, In-113, In-115 and Hf-179). That is even though the spins are parallel the isonuclides are stable.
2. Also we notice that three parallel spin isodeuterons in the environment of α-particles are also stable they are B-10, K-40 (it also has 2 parallel spin isoneutrons) and Sc-45 (it also consists of one parallel spin isoneutron and one spin zero di-isoneutron).
3. In addition to these parallel spin high spin states there we have various combination of parallel spin isodeuterons combined with parallel or opposite spin isoneutrons resulting in the intermediate nuclear spins from 1/2 to 7/2.
4. As we know that an isolated single isoneutron is unstable but it gets stabilized in the form of an isodeuteron on the one hand but on the other hand it also gets stabilized in the environment of spin paired isodeuterons. This is the case of the nuclear spin of 1/2 of the isonuclides due only to a single isoneutron. From Table 2 we find that-

a) in the cases of C-13 and Si-29 we have a single isoneutron in the environment of 3 and 7α-particles and both the isonuclides are stable.
b) Another set of stable isonuclides with a single isoneutron consist of Fe-57, Se-77, Sn-115, Sn-117, Sn-119, Te-125, Xe-129, Xe-131, Yb-171, W-183, Os-187, Hg-199 and Pb-207. These isonuclides offer the environment of spin paired isodeuterons along with the spin paired isoneutrons to the last isoneutron resulting in the stability of the last isoneutron.

5. Another set of stabilized single isoneutron is in combination with high spin state of isodeuterons in the environment of α-particles. We list them as follows.

(a). The cases of a single isoneutron in the environment of α-particles along with a single isodeuteron are of two types. The high spin (that is the net nuclear spin of 3/2) states are Li-7, B-11, Na-23, Cl-35 and K-39. The low spin (that is the net nuclear spin of 1/2) stable isonuclides are N-15, F-19 and P-31.

(b). The cases of a single isoneutron in the environment of α-particles along with two parallel spin isodeuterons are also of two types. The high spin (that is the net nuclear spin of 5/2) stable states are O-17 and Mg-25. The low spin (that is the net nuclear spin of 3/2) stable states are Be-9, Ne-21 and S-33.

(c). The cases of a single isoneutron in the environment of α-particles along with three parallel spin isodeuterons are two in number. The high spin (that is net nuclear spin of 7/2) state is Sc-45 and the low spin (that is net nuclear spin of 5/2) state is Al-27.

(d). We have already seen in Section 14.1.3.1 that spin paired isoneutrons get stabilized in the environment of α-particles. Now we find that the combination of one isodeuteron and

one isoneutron also get stabilized in the environment of α-particles when accompanied by the zero spin di-isoneutrons. The corresponding isotopes with high spin (the net nuclear spin of 3/2) are Cl-37, K-41, Cu-63, Cu-65, Ga-71, As-75, Br-79, Br-81, Tb-159, Ir-191, Ir-193 and Au-197. Whereas the low spin (the net nuclear spin of 1/2) isonuclides are Y-89, Rh-103, Ag-107, Ag-109, Tm-169, Tl-203 and Tl-205.

(e). We have seen above that K-40 is a stable isonuclide. Herein di-isoneutron of spin 1 is getting stabilized in the environment offered by α-particles and three parallel spin isodeuterons. Three parallel spin isoneutrons get stabilized in Ca-43 that offers the environment of α -particles and two parallel spin isodeuterons.

(f). We also see that up to the net 9 parallel spin isoneutrons get stabilized in the environment of 36 α-particles and 13 di-isoneutrons in the case of Hf-179 whereas in the case of Ge-73 the 9 parallel spin isoneutrons get stabilized in the environment of 16 α-particles, no isodeuterons are required for this stabilization.

We have described above certain representative observations but on closer scrutiny of Table 2 we can spell out many more observations. However, the main task of presenting the nuclear configurations of Table 2 has been to provide ample facts that would provide base to evolve a comprehensive theory of nuclear stability against radioactivity and find out the factors that lead to nuclear instability.

While attempting to explain the nuclear stability we definitely need to consider unstable isonuclides in the immediate vicinity of the stable isonuclides along with their nuclear configurations commensurate with their observed spins. For example let us consider the cases of stable Nb-93 and In-113. We know that Nb-92 and Nb-94 are unstable isotopes and their experimentally observed nuclear spins are 7 and 6 respectively whereas that of Nb-93 it is 9/2. That is Nb-93 lies in between the higher nuclear spin isotopes. The nuclear configuration of Nb-92 is

$$19\left[{}^{4}_{2}\hat{\mathrm{He}}_2(0)\right],3\left[\hat{d}(\uparrow\uparrow)\right],\left[\hat{n}(\uparrow)\hat{n}(\downarrow)\right],8\left[\hat{n}(\uparrow)\right]$$

that on adding one isoneutron changes to

$$20\left[{}^{4}_{2}\hat{\mathrm{He}}_2(0)\right],\left[\hat{d}(\uparrow\uparrow)\right],2\left[\hat{n}(\uparrow)\hat{n}(\downarrow)\right],7\left[\hat{n}(\uparrow)\right]$$

That is the addition of one isoneutron forces two isodeuterons out of three parallel spin isodeuterons to get spin paired and simultaneously itself gets spin paired with one isoneutron leaving 7 parallel spin isoneutrons. The outcome is the stable Nb-93. Now to this stable isotope on adding one isoneutron it forces one pair of spin paired isoneutrons to become spin unpaired resulting in total number of 10 parallel spin isoneutrons. The resultant nuclear configuration obtained is

$$20\left[{}^{4}_{2}\hat{\mathrm{He}}_2(0)\right],\left[\hat{d}(\uparrow\uparrow)\right],\left[\hat{n}(\uparrow)\hat{n}(\downarrow)\right],10\left[\hat{n}(\uparrow)\right]$$

which is unstable Nb-94.

Similarly, in the sequence In-112, In-113 and In-114 the nuclear configuration transforms as

$$24\left[{}^{4}_{2}\hat{\mathrm{He}}_2(0)\right],\left[\hat{d}(\uparrow\uparrow)\right],7\left[\hat{n}(\uparrow)\hat{n}(\downarrow)\right]$$

$$\downarrow$$

$$24\left[{}^{4}_{2}\hat{\mathrm{He}}_2(0)\right],\left[\hat{d}(\uparrow\uparrow)\right],4\left[\hat{n}(\uparrow)\hat{n}(\downarrow)\right],7\left[\hat{n}(\uparrow)\right]$$

$$\downarrow$$

$$24\left[{}^{4}_{2}\hat{\mathrm{He}}_2(0)\right],\left[\hat{d}(\uparrow\uparrow)\right],8\left[\hat{n}(\uparrow)\hat{n}(\downarrow)\right]$$

Notice that in this sequence spin 1 states are unstable and 9/2 spin state is a stable one.

The above described are a few representative examples but they adequately pose the kind of challenge we need to undertake in order to explain nuclear stability/instability. One may think that an answer may be found through developing corresponding shell model and corresponding magic numbers.

In order to check if magic numbers play any role in nuclear configuration through isoneucleons we have also compiled the nuclear configuration in terms of isodeuterons as the only constituent and depicted in Table 3.

Table 3. *Isonuclides composed only of isodeuterons*

Nuclear Configuration	Isonuclide $^{A}_{Z}\hat{\mathrm{X}}_{N}(J)$	Isodeuterons and Nuclear Stability / Instability	Nuclear Magnetic dipole Moment μ / μ_N	Nuclear Electric Quadrupole Moment Q/eb
$\hat{d}(\uparrow\uparrow)$	Isodeuteron	1 (odd) stable	0.85743823	+0.00286
$\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$	${}^{4}_{2}\hat{\mathrm{He}}_2(0)$	2 (even) stable	0	N/A
$\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right],\ \hat{d}(\uparrow\uparrow)$	${}^{6}_{3}\hat{\mathrm{Li}}_3(1)$	2, 1 (odd) stable	0.8220473	-0.00083
$2\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$	${}^{8}_{4}\hat{\mathrm{Be}}_4(0) \equiv 2\left[{}^{4}_{2}\hat{\mathrm{He}}_2(0)\right]$	2, 2 (even) unstable	0	N/A

Nuclear Configuration	Isonuclide $\hat{{}^{A}_{Z}X}_{N}(J)$	Isodeuterons and Nuclear Stability / Instability	Nuclear Magnetic dipole Moment μ / μ_N	Nuclear Electric Quadrupole Moment Q/eb
$\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$, $\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow)$	${}^{10}_{5}\hat{B}_{5}(3)$	2, 3 (odd) stable	1.8006448	0.08472
$3\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$	${}^{12}_{6}\hat{C}_{6}(0)$	2, 4 (even) stable	0	N/A
$3\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$, $\hat{d}(\uparrow\uparrow)$	${}^{14}_{7}\hat{N}_{7}(1)$	2, 5 (odd) stable	0.403761	0.0193
$4\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$	${}^{16}_{8}\hat{O}_{8}(0)$	2, 6 (even) stable	0	N/A
$4\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$ $\hat{d}(\uparrow\uparrow)$	${}^{18}_{9}\hat{F}_{9}(0)$	2, 6, 1 (odd) unstable	N/A	N/A
$5\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$	${}^{20}_{10}\hat{Ne}_{10}(0)$	2, 6, 2 (even) stable	0	N/A
$4\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$, $\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow)$	${}^{22}_{11}\hat{Na}_{11}(3)$	2, 6, 3 (odd) unstable	1.746	N/A
$6\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$	${}^{24}_{12}\hat{Mg}_{12}(0)$	2, 6, 4 (even) stable	0	N/A
$4\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$, $\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow)$, $\hat{d}(\uparrow\uparrow)$	${}^{26}_{13}\hat{Al}_{13}(5)$	2, 6, 5 (odd) unstable	N/A	N/A
$7\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$	${}^{28}_{14}\hat{Si}_{14}(0)$	2, 6, 6 (even) stable	0	N/A
$7\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$, $\hat{d}(\uparrow\uparrow)$	${}^{30}_{15}\hat{P}_{15}(1)$	2, 6, 6, 1 (odd) unstable	N/A	N/A
$8\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$	${}^{32}_{16}\hat{S}_{16}(0)$	2, 6, 6, 2 (even) stable	0	N/A
$8\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$, $\hat{n}(\uparrow),\hat{p}(\downarrow)$	${}^{34}_{17}\hat{Cl}_{17}(0)$	2, 6, 6, 2 (even) unstable	0	N/A
$9\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$	${}^{36}_{18}\hat{Ar}_{18}(0)$	2, 6, 6, 4 (even) stable	0	N/A
$8\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$, $\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow),\hat{d}(\uparrow\uparrow)$	${}^{38}_{19}\hat{K}_{19}(3)$	2, 6, 6, 5 (odd) unstable	1.371	N/A
$10\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$	${}^{40}_{20}\hat{Ca}_{20}(0)$	2, 6, 6, 6 (even) stable	0	N/A
$10\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$, $\hat{n}(\uparrow),\hat{p}(\downarrow)$	${}^{42}_{21}\hat{Sc}_{21}(0)$	2, 6, 6, 6 (even) unstable	0	N/A
$11\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$	${}^{44}_{22}\hat{Ti}_{22}(0)$	2, 6, 6, 8 (even) unstable	0	N/A
$11\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$,	${}^{46}_{23}\hat{V}_{23}(0)$	2, 6, 6, 8 (even) unstable	0	N/A

Nuclear Configuration	Isonuclide $\overset{A}{Z}\hat{\mathrm{X}}_N(J)$	Isodeuterons and Nuclear Stability / Instability	Nuclear Magnetic dipole Moment μ/μ_N	Nuclear Electric Quadrupole Moment Q/eb
$\hat{n}(\uparrow)$, $\hat{p}(\downarrow)$				
$12\left[\hat{d}(\uparrow\uparrow),\hat{d}(\downarrow\downarrow)\right]$	$\overset{48}{24}\hat{\mathrm{Cr}}_{24}(0)$	2, 6, 6, 8, 2 (even) unstable	0	N/A

From the column 3 of Table 3 we learn that the stable nuclides consists of odd number i.e. 1, 3, 5, 7 and 9, and even number i.e. 2, 6, 8, 10, 12, 14, 16, 18 and 20 isodeuterons. We also notice that there are no nuclides, stable or unstable, with 17, 21 and 23 isodeuterons this we have depicted in Table 3 by including Cl-34 (consisting of 16 spin paired isodeuterons and spin paired one isoneutron and one isoproton), Sc-42 (consisting of 20 spin paired isodeuterons and spin paired one isoneutron and one isoproton) and V-46 (consisting of 22 spin paired isodeuterons and spin paired one isoneutron and one isoproton). All of them are unstable isonuclides. The nearest stable isonuclides are Cl-35 (consisting of 16 spin paired isodeuterons, and one each isodeuteron and isoneutron with parallel spins), Sc-45 (consisting of 18 spin paired isodeuterons, two spin paired isoneutrons and parallel spin 3 isodeuterons and one isoneutron) and V-51 (consisting of 22 spin paired isodeuterons, and parallel spin one isodeuteron and 5 isoneutrons) respectively (can be seen in Table 2).

Thus, the column 3 of Table 3 doesn't appear to point out the existence of a system of magic numbers for isodeuterons when housed in a nucleus. Moreover, in Table 3 we have also depicted the nuclear magnetic dipole and electric quadrupole moments [77, 78] in columns 4 and 5 respectively. That reveals the nonspherical nuclear charge distribution in the case of non-zero nuclear quadrupole moments.

14.2. Nuclear Configuration of Model-II

In this model we treat atomic nuclei as constituted of up-spin isoprotons, down-spin isoprotons and null-spin isoelectrons. The column 4 of Table 2 list nuclear configuration of all stable nuclides in terms of these isonucleons. The striking feature of these nuclear configuration is that the number of isoelectrons, $\mathbb{E}(0)$, is equal to the number of neutrons (c.f. Eqs. (84) and (91)) and in this sense we may say that the isoelectrons have *replaced* neutrons.

Traditionally the nuclear stability is described by the ratio N/Z (number of neutrons to atomic number) and it is argued that this ratio increases from value 1 at lower atomic numbers to 1.537 in the case of Pb-208 (the last stable nuclide) because with the increase of atomic number the nuclear charge increases, which results in tremendous increase in repulsive force amongst nuclear protons. This repulsion gets minimized by the presence of neutrons. At higher atomic number the neutrons in a nuclide need to out number protons to attain nuclear stability, hence the said ratio increases up to 1.537 for Pb-208. Of course, there is a limitation on the effectiveness of neutrons to overcome the nuclear repulsive force otherwise by mere increase of number of neutrons all nuclide could have been stabilized. Hence, other explanation of nuclear stability were looked for. That resulted in postulation of a host of new subatomic particles. In layman's language scientists were looking for a *nuclear glue* which is responsible for tightly holding nucleons together within an atomic nucleus.

With this background we now interpret the stability of an isodeuteron in terms of the ability of an isoelectron to effectively hold two isoprotons together. Hence, we are in a position to say that the isoelectron acts in this case as an effective nuclear glue that holds tightly two isoprotons together. In view of this interpretation we now, apparently for the first time, hypothesize that *in all stable nuclides their isoelectrons act as effective glue that tightly hold their isoprotons together in the nucleus, of course, with appropriate distribution of up and down spins amongst isoprotons*. We caution the reader that the isoelectrons as nuclear glue has entirely different base than what the nuclear glue is described in the conventional nuclear physics. In the present model we have not postulated any new subatomic particle. Our proposal is that the conventional electrons and protons get transformed respectively to isoelectrons and isoprotons by way of mutual partial penetration of their wave packets in view of their very close proximity and that acts as the nuclear glue.

In view of the rôle of the nuclear glue played by isoelectrons we hereby propose that instead of N/Z ratio it would be more appropriate to use the ratio $\mathbb{E}(0)/Z$ to qualitatively describe nuclear stability.

Of course, the proposal of nuclear configuration in terms of isoprotons and isoelectrons with latter as the nuclear glue, opens up new vistas for further investigations on the topics of nuclear stability and understanding of all other nuclear properties.

The nuclear configuration of nuclides of *model-II* are listed in column 4 of Table 2. The observations and analysis of these nuclear configurations are described in Section 14.2.1.

14.2.1. Observations and Analysis of Nuclear Configuration of Model-II

In the *model-II we view the nucleus as a pool of isoprotons with the isoelectrons immersed in it*. In light of this we are presenting our preliminary visualization of only a few nuclides of lower atomic numbers and for the time being we are postponing our analysis of higher atomic number nuclides.

1. In the case of *isodeuteron* there we have one isoelectron and two isoprotons (both with up spin). Hence the isoelectron acts as a solitary nuclear glue that tightly holds both the up spin isoprotons. The most obvious geometry of these three

isonucleons is linear that perfectly matches with the structure proposed by Santilli (see Figure 9). Thus oblate elliptical shape of isodeuteron described in Figure 6 perfectly matches with the present description. It seems that the zero spin isodeuteron is energetically unstable hence even if it is formed in some nuclear transmutations that gets quickly converted to the spin 1 isodeuteron.

2. The next entry in Table 2 is **He-3** with the nuclear spin 1/2. It is the case of a pool of 3 isoprotons and in that one isoelectron is immersed. The obvious minimum energy geometry is the one in which the isoelectron is at the center of an equilateral triangle and the three isoprotons situated at the vertices of it. Again in this case too the shape would be elliptical due to its overall spinning motion. He-3 with nuclear spin of 3/2 has not been observed so far, which must be energetically unstable nuclide. Therefore, even if it is formed in some nuclear transmutations it gets quickly transformed to He-3 of 1/2 spin. Thus we learn that the low nuclear spin state He-3 is the preferred one. Moreover, when we add one down spin isoproton to an isodeuteron nucleus we get He-3 nucleus but we see that this addition does not disturbed the nuclear stability of, though the geometry changes from linear to planner.
3. Just for comparison with He-3 nuclide let us consider **H-3** *(triton)* nuclide. The latter nucleus too possesses 3 isoprotons with the net spin of 1/2 but consists of 2 isoelectrons. The minimum energy arrangement would be trigonal-bipyramidal in that the two isoelectrons occupy axial positions above and below the horizontal plane of symmetry. However, in this arrangement the penetration of wave packets of isoelectrons into those of isoprotons would not be as deep as one isoelectron in He-3 achieves. This perhaps leads to instability. It decays with β^- emission to the stable He-3 nuclide and its half life is 12.329 y that is it is not highly unstable nuclide. This is understandable because by emission of one isoelectron a stable He-3 geometrical arrangement is achived.
4. The **He-4** nuclide is the case of a pool of 4 isoprotons and immersed in it are two isoelectrons. The minimum energy shape in this case would be that of an octahedron in which isoelectrons occupy the two diagrammatically opposite axial positions and 4 isoprotons occupy the remaining 4 vertices. The spins of isoprotons would be alternately up and down so that the net nuclear spin is null. The charge distribution would be spherically symmetric in view of the repulsion between axial isoelectrons.
5. The **Li-6** nuclide is a case of a pool of 6 isoprotons and 3 isoelectrons immersed in it. The minimum energy shape seems to be the two trigonal-pyramids in a staggered geometry with 6 isoprotons at the vertices. All the 3 isoelectrons occupy axial positions and out of them one is at the center holding tightly both the trigonal pyramids. The observed spin 1 originate from the one up spin isoproton on each side of the central isoelectron. If one isoelectron is added to Li-6 arrangement described herein then we will have to house 2 isoelectrons at the center of the axial position. An equally probable geometry could be one He-4 arrangement and one isodeuteron moiety oriented above one of the axial isoelectrons such that the isodeuteron moeity and the axial isoelectrons of He-4 moiety form a straight line. Such an arrangement would not be stable because of the strong electrostatic repulsion between two central isoelectrons. The resultant nuclide would be He-6. However, it has two decay paths with half life of 806.7 ms. One is the obvious decay to Li-6 just by getting rid of the extra electron and in the second path simultaneously an α -particle is emitted, the daughter nuclides are a deuteron and He-4 nuclides.
6. The **Li-7** nuclide is a case of a pool of 7 isoprotons and 3 isoelectrons immersed in it. The obvious minimum energy geometry would be having two H-3 trigonal-bipyramids fused by one isoproton at the center such that its wave packet simultaneously allows penetration of wave packets of two adjacent central isoelectrons on its left and right hand sides. The observed spin 3/2 is because of the two up spin isoprotons on each trigonal plane and one up spin isoproton of the fusing isoproton. If we add one isoelectron to Li-7 the resultant nuclide would be He-7, which decays to He-6 by neutron emission which in turn decays by two simultaneous paths to Li-6 and He-4 along with a deuteron by β^- emission.
7. The case of **Be-8** is unique. It has a pool of 8 isoprotons and 4 isoelectrons immersed in it. The minimum energy shape would be two compressed octahedrons in the staggered orientation one above the other. Thus the four vertices of each octahedron would be alternately occupied by up and down spin isoprotons and the two axial vertices of each octahedron are occupied by one isoelectrons each. However, in this way middle two isoelectrons would come close to each other hence this arrangement cannot sustain itself. As a result of it the two octahedrons get separated. This is the reason why Be-8 is not a stable nuclide disintegrating to α-particles. If we add 1 isoelectron to Be-8 the resultant nuclide would be Li-8 which in turn disintegrates to Be-8 by β^- emission with half life of 840.3 ms.
8. The **Be-9** nuclide is a case of a pool of 9 isoprotons and 5 isoelectrons immersed in it. The obvious minimum energy geometrical arrangement of isonucleons consist of 2 H-3 trigonal-bipyramids fused by the triagonal planar geometry of He-3 in a staggered orientation with respect to both the trigonal-bipyramids. The observed spin of 3/2 is due to the spin 1/2 of one He-3 and two H-3 geometries. Notice that the wave packet of the isoelectron of the central He-3 geometry will be effectively shielded by the wave packets of its three isoprotons hence the wave packets of the isoelectrons of both the H-3 geometries oriented towards the central H-3 geometry wold penetrate into the wave packets of the central H-3 isoprotons. This seems to impart stability to Be-9. If we add 1 isoelectron to Be-9 nuclide the resultant nuclide would be Li-9 which disintegrates by two paths to Be-9 and Be-8 along with

a neutron with the half life of 178.3 ms.

9. The **B-10** is the case of a pool of 10 isoprotons and 5 isoelectrons immersed in it. The minimum energy packing of isonucleons would be two He-4 type octahedrons in the staggered orientation one above the other and one isodeuteron fusing them so that 2 central isoprotons and five axial isoelectrons are in a straight line. The spin 3 of B-10 originates from 8 up spin isoprotons 3 in each octahedron plus two in the fusing isodeuteron leaving two down spin isoprotons one each in the octahedron geometry. If we add one isoelectron to B-10 nuclide the resultant nuclide would be Be-10 nuclide which disintegrates back to B-10 by β^- emission with half life of 1.39×10^6 y.
10. The **B-11** is the case of a pool of 11 isoprotons and 6 isoelectrons immersed in it. The minimum energy packing of the isonucleons would be 2 He-4 structures in staggered orientation and the remaining 3 isoprotons and 2 isoelectrons linearly and alternately coupled acts as the fusing chain of the two octahedral structures. The nuclear spin of 3/2 is due to 3 up spin central isoprotons. If we add one isoelectron to the B-11 nuclide the resultant nuclide would be Be-11 which partly decays back to B-11 and partly to Li-7 and α-particle by β^- emission with half life of 13.81 s.

The above presented visualization of isonucleons in nuclides appears to be satisfactorily reasonable. We would extend the work on the same lines for all stable and unstable nuclides.

15. Concluding Remarks

In this paper, we have reviewed the numerous insufficiencies of quantum mechanics for the representation of the structure of stable nuclides, and the ensuing greater insufficiencies for the representation of the structure of unstable nuclides and nuclear reactions at large due to their structural irreversibility over time compared to the strict reversibility of quantum mechanical axions.

We have pointed out that the origin of the insufficiencies rests primarily in the *mathematics* of quantum mechanics, rather than in its axioms, due to its local-differential character with consequential abstraction of nuclear constituents as being point-like particles, compared to the evident need for the nuclear structure to represent nucleons as they are in the nuclear reality: extended charge distributions.

We have then reviewed the rudiments of the novel *isomathematics* which has been constructed precisely for the representation of nuclei as being composed by extended constituents in conditions of partial mutual penetration, thus resulting in the most general known interactions of linear and non-linear, local and non-local as well as Hamiltonian and non-Hamiltonian type.

We have then reviewed the rudiments of the covering of quantum mechanics known as *isomechanics* specifically formulated for the nuclear structure, by stressing that it essentially consists in an axiom-preserving "completion" of quantum mechanics along the historical argument by Einstein, Podolsky and Rosen, which is solely valid at one fermi distances while recovering quantum mechanics uniquely and identically for bigger distances.

We have then reviewed the use of the above new formulations for the first and only achievement on scientific records of an exact and time invariant representation of the magnetic moments of stable nuclei via the implementation of Fermi's historical hypothesis that the charge distributions of protons and neutrons is deformed when they are members of a nuclear structure, with a consequential deformation of their intrinsic magnetic moments (see Figures 8 and 9 for neutron as isoneutron and deuteron as isodeuteron respectively).

The conceptual and technically most dominant aspect of the above advances is that the admission of contact, non-linear, non-local and non-Hamiltonian interactions causes alterations of the *intrinsic* characteristics of particles called *isorenormalizations* that are simply beyond any possible quantitative treatment via 20th century knowledge.

Consequently, we reviewed in the Appendix A the rudiments of the covering of Lie's theory known as the *Lie-Santilli isotheory* which has been specifically constructed for the invariant treatment of systems with extended-deformable constituents with the most general known interactions.

The most prominent salient part of the Appendix A is the review of the *Lorentz-Poincaré-Santilli isosymmetry* and its characterization of isoparticles, with particular emphasis in the characterization of nuclear constituents as extended-deformable isoparticles.

We finally review the use of all the above knowledge for the first and only known numerically exact and time invariant representation of *all* characteristics of the neutron in its synthesis from the hydrogen atom as being composed by one isoproton and one isoelectron, with the consequential representation of *all* characteristics of the deuteron as being composed by two isoprotons and one isoelectron.

By using the above advances, we then present, apparently for the first time, two exact and invariant representations of the nuclear spin of the stable nuclides. The model-I is based on nuclear structures composed by isoprotons, isoneutrons and isodeuterons as isonucleons and the model-II is based on the final reduction of nuclides to isomechanical bound states of the respective isoprotons and isoelectrons.

In the *former model* we have considered that with the available neutrons and protons of the nuclide they first prefer to have the stable isodeuteron structure and the remaining nucleons stay as isoneutrons and isoprotons in the nucleus. In doing so the rule followed is that the so generated nuclear configuration should correctly reproduce the experimental nuclear spin of the given nuclide. Thus in Table 2 we have listed nuclear configuration of all stable nuclides up to the atomic number 82, that is up to Pb-208. Then we have analyzed these nuclear configurations and presented our observations in terms of the number of isodeuterons (both their low spin and high spin

combinations) and their rôle in stabilization of various combination of spin paired and/or parallel spin isoneutrons. We have tried to look if these nuclear configurations indicate corresponding magic numbers but the data in Table 3 fail to provide any indication. However, it seems that unless we systematically compare the nuclear configuration based on model-I of neighbouring unstable nuclides about the stable nuclides considered in Table 2 we may not be able to throw much light on the factors responsible for nuclear stability/instability. Indeed, our data of Tables 2 and 3 of model-I has opened up an entirely a new line of research in the fields of nuclear stability/instability and nuclear magnetic moments including nuclear electric quadrupole moments.

Whereas in arriving at the *model-II* we have first reinterpreted the stable structure of an isodeuteron in the sense that the isoelectron of it acts as a nuclear glue that tightly holds its two isoprotons. This proposal of isoelectrons acting as the nuclear glue we have, perhaps for the first time, extended to all stable nuclides. There we have assumed that a given nuclide consists of a pool of isoprotons and the isoelectrons are immersed in it, which in essence is the model-II of this paper. The working rule is that the number of isoelectrons is equal to the number of neutrons in the nuclide and the number of isoprotons is equal to the mass number of the nuclide. Next these isprotons are distributed in two groups of up and down spins in such a way to correctly predict the experimental nuclear spin of the given nuclide. The resulting nuclear configurations of all stable nuclides are listed in the column 4 of Table 2. Herein we have presented our preliminary observations on the so developed nuclear configuration. Of course, we have so far analyzed only a very few light nuclides in terms of geometrical arrangements of isoprotons and isoelectrons of H-2, H-3 (unstable), He-3, He-4, Li-6, Li-7, Be-8 (unstable), Be-9, B-10 and B-11 nuclides. Our assigned geometrical arrangements of isonucleons seem to provide reasonably satisfactory rational behind nuclear stability/instability. Particularly the reason of instability of H-3 and Be-8 so obtained seems to be rationally correct and encouraging.

The remarkable feature of both the models of nuclear configuration presented in this paper is that we need not to invent nuclear particles other than the basic subatomic particles, namely electrons, protons and neutrons.

Moreover, as stated in the main text of this paper the methods of writing nuclear configuration of a nuclide in both the models are equally applicable to unstable nuclides too hence while dealing with the nuclear stability/instability one can easily write down nuclear configurations of neighbouring unstable nuclides about a given stable one with identically the same rules as those we have followed in the case of stable nuclides and then attempt to rationalize nuclear stability/instability meaningfully.

Both the models promise new vistas of nuclear physics that lays a foundation of carrying out further investigations based on hadronic mechanics to strengthen our knowledge of nuclear physics.

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Appendix A

The Lorentz-Poincaré-Santilli IsoSymmetry and its Characterization of IsoParticles

A.1 Definition of IsoParticles

The 20th century definition of *particles* is that of unitary irreducible representation of the Lorentz-Poincaré (LP) symmetry on a Hilbert space over the field of complex numbers. This definition implies that all interactions are derivable from a potential and representable with a Hamiltonian as a central condition for the very applicability of Lie's theory at large, and that of the LP symmetry in particular. In turn, the local-differential mathematics underlying Lie's theory implies that the particles are abstracted as being point-like, as it is evident from the restriction of the interactions to actions-at-a-distance.

A central aim of this paper is the representation of nuclear constituents as they are in the physical reality, namely, extended, non-spherical and deformable charge distributions according to the representation of Eq. (1) which is structurally non-Hamiltonian, in the sense that it cannot be represented with a Hamiltonian, thus requiring a new quantity other than the Hamiltonian. In order to achieve a time invariant representation, isomathematics selects Santilli isounit Eq. (37), $\hat{I} = 1/\hat{T} > 0$ for the representation of the new interactions.

Additionally, the comparison of experimental data on nuclear volumes with those on the volume of protons and neutrons, establishes that, when they are members of a nuclear structure, protons and neutrons are in conditions of partial mutual penetration of their charge distributions.

These data imply the emergence of new nuclear interactions that are non-existence in the 20th century notion of particles, which are given by non-linear (in the wave functions), non-local (of integral and other type) and variationally non-selfadjoint [3a]. The latter interactions are also not representable with a Hamiltonian and can be invariantly represented with the exponent in the isotopic element, Eq. (1), or in the isounit.

The above basic assumptions imply the applicability of the Lie-Santilli isotheory [3b, 7, 22, 24-33] at large that was constructed precisely for the representation of non-Hamiltonian systems under the most general known linear and non-linear, local and non-local and Hamiltonian as well as non-Hamiltonian interactions,

Finally, the above basic assumptions imply that the universal symmetry for the non-relativistic treatment of isolated and

stable nuclei is the *Galileo-Santilli isosymmetry* [21, 22], while that for the relativistic treatment is the *Lorentz-Poincaré-Santilli isosymmetry* [12-23]. We can, therefore, introduce the following:

DEFINITION A.1 [18, 21, 22]: A non-relativistic (relativistic) isoparticle is an isounitary, isoirreducible isorepresentation of the Galileo-Santilli (Lorentz-Poincaré-Santilli) isosymmetry on a Hilbert-Myung-Santilli isospace over a Santilli isofields.

Within the context of this paper, whenever nuclear constituents are called "protons" and "neutrons" we refer to their quantum mechanical characterization as *point-like particles* under sole action-at-a-distance, potential-Hamiltonian interactions. Nuclear constituents according to this paper must necessarily be isoparticles at large, and isoprotons, isoneutron and isoelectrons in particular. In this Appendix we provide a summary characterization of relativistic isoparticles, while the particular case of non-relativistic isoparticles is referred to Refs. [18, 21] for brevity.

It should be stressed that a technical knowledge of the notion of isoparticle can solely be acquired from the study of Refs. [18, 21, 22]. In particular, a necessary pre-requisite for a technical characterization is the knowledge of Kadeisvili isofunctional analysis [22] we cannot possibly review to prevent excessive length.

A.2 The Lie-Santilli IsoTheory

The main branches of the Lie-Santilli isotheory can be outlined as follows (see the original proposal [3a] for the isotopies of enveloping algebras, Lie algebras and Lie group; Ref. [7] for their upgrading in terms of the isodifferential calculus over isofields; the final formulation in Ref. [22]; and Refs. [24-33] for independent studies):

Universal Enveloping Isoassociative Algebras

Let $E = E(L)$ be the universal enveloping associative algebra of an N-dimensional Lie algebra L with ordered (Hermitean) generators $X_k, k = 1,2,\ldots,N$, and attached antisymmetric algebra isomorphic to the Lie algebra, $[E(L)]^- \approx L$ over a field F (of characteristic zero), and let the infinite-dimensional basis I, X_k, $X_i \times X_j$, $i \leq j,\ldots$ of $E(L)$ be characterized by the Poincaré-Birkhoff-Witt theorem. We then have the following:

THEOREM A.1 [3b, 7]: (Poincaré-Birkhoff-Witt-Santilli theorem): The isocosets of the isounit and of the standard isomonomials

$$\hat{I},\ X_k,\ \hat{X}_i \hat{\times} \hat{X}_j, i \leq j;\ \ \hat{X}_i \hat{\times} \hat{X}_j \hat{\times} \hat{X}_k; i \leq j \leq k,\ldots, \quad \text{(A.1)}$$

form an infinite dimensional basis of the universal enveloping isoassociative algebra $\hat{E}(\hat{L})$ (also called isoenvelope for short) of a Lie-Santilli isoalgebra $\hat{L}$.

The first application of the above theorem, also formulated in Ref. [3b] and then reexamined by various authors, is a rigorous characterization of the *isoexponentiation*, i.e.,

$$\hat{e}^{\hat{I}\hat{\times}\hat{w}\hat{\times}\hat{X}} =$$
$$= \hat{I} + \hat{i} \hat{\times} \hat{w} \hat{\times} \hat{X} \hat{/} \hat{1}! + (\hat{i} \hat{\times} \hat{w} \hat{\times} \hat{X}) \hat{\times} (\hat{i} \hat{\times} \hat{w} \hat{\times} \hat{X}) \hat{/} \hat{2}! + \ldots =$$
$$= \hat{I} \times (e^{i \times w \times T \times X}) = (e^{i \times w \times X \times T}) \times \hat{I}, \quad \text{(A.2a)}$$

$$\hat{i} = i \times \hat{I}, \hat{w} = w \times \hat{I} \in \hat{F}. \quad \text{(A.2b)}$$

where quantities with a "hat" are formulated on isospaces over isofields and those without are their projection on conventional spaces over conventional fields.

The non-triviality of the Lie-Santilli isotheory is established by the emergence of the isotopic element T directly in the exponent, thus ensuring the desired generalization, thus establishing "ab initio" that while Lies theory can solely characterize linear, local-dofferential and Hamiltonian systems, the covering Lie-Santilli isotheory characterize the most general known non-linear, non-local and non-canonical or non-unitary systems.

LIE-SANTILLI ISOALGEBRAS.

As it is well known, Lie algebras are the antisymmetric algebras $L \approx [\xi(L)]^-$ attached to the universal enveloping algebras $\xi(L)$. This main characteristic is preserved although enlarged under isotopies as expressed by the following:

THEOREM A.2 [3b, 7] (Lie-Santilli Second theorem): The antisymmetric isoalgebras $\hat{L}$ attached to the isoenveloping algebras $\hat{E}(\hat{L})$ verify the isocommutation rules.

$$[\hat{X}_i \hat{,} \hat{X}_j] = \hat{X}_i \hat{\times} \hat{X}_j - \hat{X}_j \hat{\times} \hat{X}_i =$$
$$= X_i \times T(x,v,\xi,\omega,\psi,\partial\psi,\ldots) \times X_j - X_j \times T(x,v,\xi,\omega,\psi,\partial\psi,\ldots) \times X_i =$$
$$= \hat{C}_{ij}^k(x,v,\xi,\omega,\psi,\partial\psi,\ldots) \hat{\times} \hat{X}_k = C_{ij}^k(x,v,\xi,\omega,\psi,\partial\psi,\ldots) \times X_k,, \quad \text{(A.3)}$$

where T is the projection of the isotopic element $\hat{T}$ on a conventional space over a conventional field, and the C's, called the "structure isofunctions" of $\hat{L}$, generally have an explicit dependence on local variables, and are restricted by the conditions (Lie-Santilli Third Theorem)

$$[X_i \hat{,} X_j] + [X_j \hat{,} X_i] = 0, \quad \text{(A.4a)}$$

$$[[X_i \hat{,} X_j] \hat{,} X_k] + [[X_j \hat{,} X_k] \hat{,} X_i] + [[X_k \hat{,} X_i] \hat{,} X_j] = 0. \quad \text{(A.4b)}$$

It was stated in the original proposal [3b, 7] that all isoalgebras $\hat{L}$ are isomorphic to the original algebra L for all

positive-definite isotopic elements. In other words, the isotopies cannot characterize any new Lie algebras because all possible Lie algebras are known from Cartan classification. Therefore, Lie-Santilli isoalgebras merely provide new non-linear, non=local and non=canonical or non-unitary realizations of existing Lie algebras.

LIE-SANTILLI ISOGROUPS.

Under certain integrability and smoothness conditions hereon assumed, Lie algebras L can be "exponentiated" to their corresponding Lie transformation groups G and, vice-versa, Lie transformation groups G admit corresponding Lie algebras L when computed in the neighborhood of the unit I.

These basic properties are preserved under isotopies although broadened to the most general possible, axiom-preserving nonlinear, nonlocal and noncanonical transformations groups according to the following:

THEOREM A.3 [3b, 7] (Lie-Santilli isogroups): The isogroup characterized by finite (integrated) form $\hat{G}$ of isocommutation rules (1.12) on an isospace $\hat{S}(\hat{x},\hat{F})$ over an isofield $\hat{F}$ with common isounit $\hat{I}=1/\hat{T}>0$ is a group mapping each element $\hat{x}\in\hat{S}$ into a new element $\hat{x}'\in\hat{S}$ via the isotransformations

$$\hat{x}'=\hat{g}(\hat{w})\hat{\times}\hat{x},\ \ \hat{x},\hat{x}'\in\hat{S},\ \ \hat{w}\in\hat{F}, \tag{A.5}$$

with the following isomodular action to the right:

1) The map $\hat{g}\hat{\times}\hat{S}$ into $\hat{S}$ is isodifferentiable $\forall\hat{g}\in\hat{G}$;

2) $\hat{I}$ is the left and right unit

$$\hat{I}\hat{\times}\hat{g}=\hat{g}\hat{\times}\hat{I}\equiv\hat{g},\ \ \forall\hat{g}\in\hat{G}; \tag{A.6}$$

3) the isomodular action is isoassociative, i.e.,

$$\hat{g}_1\hat{\times}(\hat{g}_2\hat{\times}\hat{x})=(\hat{g}_1\hat{\times}\hat{g}_2)\hat{\times}\hat{x},\ \ \forall\hat{g}_1,\hat{g}_2\in\hat{G}; \tag{A.7}$$

4) in correspondence with every element $\hat{g}(\hat{w})\in\hat{G}$ there is the inverse element $\hat{g}^{-\hat{I}}=\hat{g}(-\hat{w})$ such that

$$\hat{g}(\hat{0})=\hat{g}(\hat{w})\hat{\times}\hat{g}(-\hat{w})=\hat{I}; \tag{A.8}$$

5) the following composition laws are verified

$$\hat{g}(\hat{w})\hat{\times}\hat{g}(\hat{w}')=\hat{g}(\hat{w}')\hat{\times}\hat{g}(\hat{w})=\hat{g}(\hat{w}+\hat{w}'),\forall\hat{g}\in\hat{G},\ \hat{w}\in\hat{F}; \tag{A.9}$$

with corresponding isomodular action to the left, and general expression

$$\hat{g}(\hat{w})=\prod_k\hat{e}^{\hat{I}\hat{\times}\hat{w}_k\hat{X}_k}\ \hat{\times}\ \hat{g}(0)\hat{\times}\prod_k\hat{e}^{-\hat{I}\hat{\times}\hat{w}_k\hat{X}_k}, \tag{A.10}$$

Another important property is that conventional group composition laws admit a consistent isotopic lifting, resulting in the following

THEOREM A.4 [3b, 7] (Baker-Campbell-Hausdorff-Santilli theorem):

$$(\hat{e}^{\hat{X}_1})\hat{\times}(\hat{e}^{\hat{X}_2})=\hat{e}^{\hat{X}_3}, \tag{A.11a}$$

$$\hat{X}_3=\hat{X}_1+\hat{X}_2+[\hat{X}_1\hat{,}\hat{X}_2]\hat{/}\hat{2}+[(\hat{X}_1-\hat{X}_2)\hat{,}[\hat{X}_1\hat{,}\hat{X}_2]]\hat{/}\hat{12}+\ldots. \tag{A.11b}$$

Let $\hat{G}_1$ and $\hat{G}_2$ be two isogroups with respective isounits $\hat{I}_1$ and $\hat{I}_2$. The direct isoproduct $\hat{G}_1\hat{\times}\hat{G}_2$ is the isogroup of all ordered pairs

$$(\hat{g}_1,\hat{g}_2),\ \ \hat{g}_1\in\hat{G}_1,\hat{g}_2\in\hat{G}_2, \tag{A.12}$$

with isomultiplication

$$(\hat{g}_1,\hat{g}_2)\hat{\times}(\hat{g}_1',\hat{g}_2')=(\hat{g}_1\hat{\times}\hat{g}_1',\hat{g}_2\hat{\times}\hat{g}_2'), \tag{A.13}$$

total isounit $(\hat{I}_1,\hat{I}_2)$ and inverse $(\hat{g}_1^{-\hat{I}_1},\hat{g}_2^{-\hat{A}.13})$.

The following particular case is important for the isotopies of inhomogeneous groups. Let $\hat{G}$ be an isogroup with isounit $\hat{I}$ and $\hat{G}_{\hat{a}}$ the group of all its inner automorphisms. Let $\hat{G}_{\hat{a}}^{o}$ be a subgroup of $\hat{G}_{\hat{a}}$ with isounit $\hat{I}^o$, and let $\Lambda(\hat{g})$ be the image of $\hat{g}\in\hat{G}$ under $\hat{G}_{\hat{a}}$. The semi-direct isoproduct $\hat{G}\hat{\times}\hat{G}_{\hat{a}}^{o}$ is the isogroup of all ordered pairs $(\hat{g},\hat{\Lambda})\hat{\times}(\hat{g}^o,\hat{\Lambda}^o)$ with total isounit

$$I_{tot}=\hat{I}\times\hat{I}^o. \tag{A.14}$$

The studies of the isotopies of the remaining aspects of the structure of Lie groups is then consequential. It is hoped the reader can see from the above elements that the entire conventional Lie theory does indeed admit a consistent and nontrivial lifting into the covering Lie-Santilli formulation.

A.3 Classification of Lie-Santilli IsoTheories

The Lie-Santilli isotheories are classified into [7]:

3.1) *Regular isotheories* when the C's of rules (A.3) are constant; and

3.2) *Irregular isotheories* when the C's of rules (A.3) are functions of local variables.

We should recall for the benefit of concrete applications in nuclear physics that *all regular Lie-Santilli isotheories can be constructed via the application of a non-canonical or non-unitary transformation to the totality of the conventional formulation of Lie's theory,* according to the rule of Section 4.

From now on, except for an illustration in Section 16.13, we should solely consider regular realizations of the Lie-santilli isotheories because amply sufficient for nuclear applications, although the use of irregular realizations appear to be necessary for astrophysical applications.

We should also recall that "structure functions" are impossible for Lie's theory, and they are solely possible for the covering Lie-Santilli isotheory, by therefore establishing the non-trivial character of Santilli isotopies.

A.4 The Fundamental Theorem on IsoSymmetries

As recalled in Section 16.1, the fundamental symmetries of the 20th century physics characterize point-like abstractions of particles in vacuum under linear, local and potential interactions, and are given by the *Galilei symmetry* $G(3.1)$ for non-relativistic treatment, the *Lorentz-Poincaré symmetry* $P(3.1)$ or relativistic formulations, the rotational symmetry $O(3)$, the $SU(2)$ symmetries and others.

A central objective of hadronic mechanics is the broadening of these fundamental symmetries to represent extended, non-spherical and deformable particles under linear and non-linear, local and non-local and potential as well as non-potential interactions *in such a way to preserve the original symmetries at the abstract level* as a necessary condition to maintain the conventional total conservation laws for isolated stable systems.

This central objective is achieved by the following property first proved by Santilli in Ref. [22b]:

THEOREM A.5: Let G *be an N-dimensional Lie symmetry of a K-dimensional metric or pseudo-metric space* $S(x,m,F)$ *over a field* F,

$$G:\ x' = \Lambda(w)\times x,\ \ y' = \Lambda(w)\times y,\ \ x,y\in \hat{S}, \tag{A.15a}$$

$$(x'-y')^\dagger \times \Lambda^\dagger \times m\times \Lambda \times (x-y) \equiv (x-y)^\dagger \times m\times (x-y), \tag{A.15b}$$

$$\Lambda^\dagger(w)\times m\times \Lambda(w) \equiv m. \tag{A.15c}$$

Then, all infinitely possible isotopies $\hat{G}$ of G acting on the isospace $\hat{S}(\hat{x},\hat{M},\hat{F})$, $\hat{M} = \hat{m}\times \hat{I} = (\hat{T}_i^k \times m_{kj})\times \hat{I}$ characterized by the same generators and parameters of G and the infinitely possible, common isounits $\hat{I} = 1/\hat{T} > 0$ leave invariant the isocomposition

$$\hat{G}:\ x' = \hat{\Lambda}(w)\times x,\ \ y' = \hat{\Lambda}(w)\times y,\ \ x,y\in \hat{S}, \tag{A.16a}$$

$$(x'-y')^\dagger \times \hat{\Lambda}^\dagger \times \hat{m}\times \hat{\Lambda} \times (x-y) \equiv (x-y)^\dagger \times \hat{m}\times (x-y) \tag{A.16b}$$

$$\hat{\Lambda}^\dagger(\hat{w})\times \hat{m}\times \hat{\Lambda}(\hat{w}) \equiv \hat{m}. \tag{A.16c}$$

and all infinitely possible so constructed isosymmetries $\hat{G}$ are locally isomorphic to the original symmetry G.

For a proof of the above theorem, one may inspect Section 1.2, Vol. II of Ref. [22].

To achieve a technical understanding of the Lie-Santilli isotheory and its applications in nuclear physics, the reader should note that, while a given Lie symmetry G is unique as well known, there can be an infinite number of covering isosymmetries $\hat{G}$ with generally different explicit forms of the transformations due to the infinite number of possible isotopic elements.

In fact, systems are characterized by the Hamiltonian H in the conventional scattering theory with trivial unit $I = Diag.(1,1,...,1)$. In this case, changing the Hamiltonian implies the referral to a different system, but the symmetry transformations remain the same. In the isoscattering theory, systems are characterized by the Hamiltonian H plus the isotopic element T. In this case, changing the isotopic element implies the referral to a different system as well as the characterization of generally different transformations due to the appearance of the isotopic element in the very structure of the isosymmetry.

Note also that all possible isosymmetries can be explicitly and uniquely constructed via the sole knowledge of the conventional symmetry and the isotopic element (1). in fact, as implied by Theorem A.5, the existence of the original symmetry plus the condition $\hat{I} > 0$ ensure verification of the integrability conditions for the existence of finite transformations, a property hereon tacitly implied.

Recall that all quantities that are Hermitean in quantum mechanics are iso-Hermitean in hadronic mechanics as one can verify via Eq. (29), to such as extent that Hermiticity and iso-Hermiticity coincide at the abstract realization-free level,

$$\hat{X}^{\hat{\dagger}} \equiv \hat{X}^\dagger. \tag{A.17}$$

The following property is then crucial for the physical consistency of the nuclear applications of hadronic mechanics, particularly the isomechanical models of closed-isolated stable nuclei:

THEOREM A.6 [22]: Physical quantities that are Hermitean and conserved in quantum mechanics remain iso-Hermitean and iso-conserved in isomechanics.

The proof of the theorem can be easily done via the local isomorphism of conventional Lie algebras L and their isotopic covering $\hat{L}$ since isotopies do not change the generators, and merely generalize their associative products.

Recall that the basic space time symmetries, the Galileo and the Lorentz-Poincaré symmetries, characterize ten total conservation laws for the total linear momentum P, the total angular momentum J, the tonal energy H, the uniform motion of the center of mass M.

Theorem A.6 then assures that all total-external quantities that are conserved for quantum mechanical models remain conserved for their covering isomechanical form achieved via the rules of Section 4.

A.5 The Minkowski-Santilli IsoGeometry

Let $M(x,\eta,I)$ be the conventional Minkowski space over the field of real numbers R, with coordinates $x = (x^\mu) = (x^1,x^2,x^3,x^4 = t),\ \mu = 1,2,3,4$, metric $\eta = Diag.(1,1,1,-c^2)$, unit $I = Diag.(1,1,1,1)$ and line element

$$(x-y)^2 \to (\hat{x}-\hat{y})^{\hat{2}} =$$
$$= [(x_1-y_1)^2 + (x_2-y_2)^2 + (x_3-y_3)^2 - (t_1-t_2)^2 c^2]I, \tag{A.18}$$

As it is well known, the Lorentz-Poincaré symmetry, hereon denoted $P(3.1)$, leaves invariant the above line element and constitutes the ultimate structural foundations of special

relativity because it permits the unique and unambiguous characterization of its basic axioms and physical laws for exterior problems of point-particles moving in vacuum.

The fundamental isospace of relativistic isomechanics is the Minkowski-Santilli isospace [15] $\hat{M}(\hat{x},\hat{\eta},\hat{I})$ over the isoreals $\hat{R}$, with isocoordinates $\hat{x} = x\hat{I}$, isometric from Eq. (37) is

$$\hat{\eta} = \hat{T}\eta,\ \hat{T} = Diag\left(\frac{1}{n_1^2},\frac{1}{n_2^2},\frac{1}{n_3^2},-\frac{c^2}{n_4^2}\right), \qquad (A.19)$$

isounit $\hat{I} = 1/\hat{T} > 0$, and isoline element

$$(\hat{x}-\hat{y})^{\hat{2}} = \left[(\hat{x}-\hat{y})^{\mu}\hat{\times}\hat{\eta}_{\mu\nu}\hat{\times}(\hat{x}-\hat{y})^{\nu}\right] = \left[\frac{(x_1-y_1)^2}{n_1^2}+\frac{(x_2-y_2)^2}{n_2^2}+\frac{(x_3-y_3)^2}{n_3^2}-\frac{(t_1-t_2)^2c^2}{n_4^2}\right]\hat{I}, \qquad (A.20)$$

where the isometric characteristic quantities n_μ are positive-definite but have otherwise an unrestricted functional dependence on all needed quantities, such as space-time coordinates x, velocities v, accelerations a, energy E, distance d, frequencies ω, temperature τ, wavefunction ψ, their derivatives $\partial\psi$, etc.

$$n_\mu = n_\mu(x,v,a,E,d,\omega,\tau,\psi,\partial\psi,...) > 0,\ \mu = 1,2,3,4, \qquad (A.21)$$

Isoprotons, isoneutrons and isoelectrons are defined on isospace $\hat{M}(\hat{x},\hat{\eta},\hat{I})$ over the isoreals. As one can see, isometric (A.19) is the most general possible metric with signature $(+,+,+,-)$, thus including as particular case the Riemannian, Fynslerian, Minkowskian and other possible metric.

The Minkowski-Santilli isogeometry [19] is the geometry of isospace $\hat{M}(\hat{x},\hat{\eta},\hat{I})$ and can be conceptually identified as the Riemannian geometry reformulated with respect to the isofields of isoreals because the isometric is indeed dependent on local coordinates, thus requiring the machinery of the Riemannian geometry, such as Christoffel symbols, covariant derivatives, etc., although reformulated with respect to isomathematics.

The intriguing part of the Minkowski-Santilli isogeometry is that it has zero curvature as necessary from the local isomorphism of isospace $\hat{M}(\hat{x},\hat{\eta},\hat{I})$ with the conventional space $M(x,\eta,I)$. It should be stressed that the lack of curvature was a necessary prerequisite for the construction of the symmetry of isoinvariant (A.20) (see Refs. [22] for details).

A.6 The Lorentz-Poncaré-Santilli IsoSymmetry

Following the prior construction of the isotopies of Lie's theory [3b], the universal isosymmetry of all infinitely possible isoline elements (A.19) was first identified by Santilli in 1983 [15], subjected to systematic studies in Refs. [15-19], and presented in a systematicw ay in monographs [21, 22], resulting in a new isosymmetry today known as the Lorentz-Poincaré-Santilli isosymmetry (LPD) and denoted with the symbol $\hat{P}(3.1)$.

The isosymmetry $\hat{P}(3.1)$ can be defined as the isotransformations on Minkowski-Santilli isospaces over isoreals

$$\hat{x}' = \hat{\Lambda}(\hat{w})\hat{\times}\hat{x},\ \hat{x}' = \hat{x}+\hat{A}(\hat{x},...), \qquad (A.22a)$$

$$\hat{\Lambda}^{\dagger}\hat{\times}\hat{\eta}\hat{\times}\hat{\Lambda} = \Lambda\times\hat{\eta}\times\Lambda^{\dagger} = \hat{I}\times\hat{\eta}\times\hat{I}, \qquad (A.22b)$$

where we shall preserve the symbol $\times$ of ordinary multiplications hereon, under the *isomodularity condition*

$$\hat{Det}\,(\hat{\Lambda}) = \pm\hat{I}, \qquad (A.23)$$

where the quantity $\hat{A}$ is identified below and $\hat{w} = w\times\hat{I}$ represents isoparameters.

The regular isoconnected component of the LPS isosymmetry $\hat{P}^0(3.1)$ is characterized by the condition

$$\hat{Det}\,\hat{\Lambda} = +\hat{I}, \qquad (A.24)$$

and can be written

$$\hat{P}^0(3.1) = \hat{SO}(3.)\times\hat{T}(3.1)\times\hat{D}, \qquad (A.25)$$

where D is the 11th dimensionality of the LPS isosymmetry identified below.

By expanding the preceding finite isotransforms (A.22) in terms of the isounit, the *regular LPS isoalgebra* is characterized by the conventional generators of the LP algebra and the isocommutation rules [21, 22, 25]

$$[J_{\mu\nu}\hat{,}J_{\alpha\beta}] = i(\hat{\eta}_{\nu\alpha}J_{\beta\mu}-\hat{\eta}_{\mu\alpha}J_{\beta\nu}-\hat{\eta}_{\nu\beta}J_{\alpha\mu}+\hat{\eta}_{\mu\beta}J_{\alpha\nu}), \qquad (A.26a)$$

$$[J_{\mu\nu}\hat{,}P_\alpha] = i\times(\hat{\eta}_{\mu\alpha}\times P_\nu-\hat{\eta}_{\nu\alpha}\times P_\mu), \qquad (A.26b)$$

$$[P_\mu\hat{,}P_\nu] = 0. \qquad (A.26c)$$

The iso-Casimir isoinvariants of $\hat{P}(3.1)$ are given by [loc. cit]

$$\hat{C}_1 = \hat{I}(x,...), \qquad (A.27a)$$

$$\hat{C}_2 = P^{\hat{2}} = P_\mu\hat{\times}P^\mu = P^\mu\times\hat{\eta}_{\mu\nu}\times P^\nu = P_k\times g_{kk}\times P_k - p_4\times g_{44}\times P_4, \qquad (A.27b)$$

$$\hat{C}_3 = W^{\hat{2}} = W_\mu\hat{\times}W^\mu,\ W_\mu = \hat{\varepsilon}_{\mu\alpha\beta\rho}\hat{\times}J^{\alpha\beta}\hat{\times}P^\rho, \qquad (A.27c)$$

and they are at the foundation of classical and operator

isorelativistic kinematics [43].

It is easy to prove that *the LPS isosymmetry is locally isomorphic to the conventional LP symmetry.* It then follows that the isotopies increase significantly the arena of applicability of the LP (as well as any Lie symmetry) by lifting the Minkowskian spacetime (A.18) to all infinitely possible isospacetime (A.20).

Note that *isolinear isomomenta isocommute, Eqs. (A.26c), that is, they commute in isospace over isoreals, but they do not generally commute when projected in the ordinary Minkowski space.* This occurrence is a clear confirmation of a nonlinear structure of isorelativity with rather deep gravitational implications not considered in this paper.

Yet, this property is significant because it appears, for the first time to our knowledge, the possibility of identifying a possible gravitational component in the structure of nuclei, as studied preliminarily in Refs. [79].

The *isoirregular LPS isoalgebra* is characterized by *structure functions*, thus no longer being locally isomorphic to the conventional LP symmetry. The study of the irregular realization is left to the interested reader for brevity.

By using the original generators of the LP symmetry, the isotopic element (37) and Lie-Santilli isotheory, regular LPS isotransformations can be easily identified as outlined below.

A.7 Regular IsoRotations

The regular isorotations, first presented in Ref. [12], and then treated in details in Refs. [22] via isofunctional analysis in general, and isotrgonometric functions in particular. Since the isounit $\hat{I} = Diag(n_1^2, n_2^2, n_3^2)$ is positive-definite, the isosymmetry $\hat{SO}(3)$ is locally isomorphic to the conventional rotational symmetry $O(3)$ (Figure 11).

Isorotations provide the technical characterization of the deformation of protons and neutrons when members of a nuclear structure under strong interactions. In their projection on an ordinary Euclidean space, isorotations can be written in the (1-2)-plane (see Ref. [22] for the general case).

$$x^{1'} = x^1 \cos[[\theta(n_1 n_2)^{-1}] - \\ -x^2 \frac{n_1^2}{n_2^2} \sin[\theta(n_1 \times n_2)^{-1}], \tag{A.28a}$$

$$x^{2'} = x^1 \frac{n_2^2}{n_1^2} \sin[\theta(n_1 n_2)^{-1}] + \\ +x^2 \cos[\theta(n_1 n_2)^{-1}], \tag{A.28b}$$

The isomorphism of $\hat{SO}(3) \approx O(3)$ is due to the fact that ellipsoid deformations of the semiaxes of the perfect sphere are compensated on isospaces over isofields by the inverse deformation of the related unit

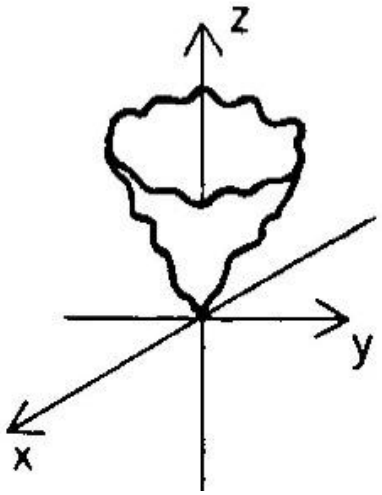

Figure 11. *It was popularly believed in the 20th century physics that the Lorentz symmetry is broken for locally varying speeds of light within physical media, here represented with a wiggly light cone. the Lie-Santilli isosymmetries have restored the exact validity of the Lorentz symmetry for all possible subluminal and superluminal speeds, thus confirming the preservation of the abstract axioms of special relativity for interior dynamical problems [15, 22].*

$$Radius \quad 1_k \to 1/n_k^2, \; Unit \quad 1_k \to n_k^2. \tag{A.29a}$$

$$\hat{r}^{\hat{2}} = \hat{r}_1^{\hat{2}} + \hat{r}_2^{\hat{2}} + \hat{r}_3^{\hat{2}}. \tag{A.29b}$$

resulting in the reconstruction of the perfect sphere on isospace called the *isosphere*, (A.29b), with consequential reconstruction of the exact rotational symmetry.

A.8 Regular Lorentz-Santilli IsoTransformations

The regular Lorentz-Santilli (LS) isotransforms were first identified in Ref. [15] and then studied in details in monographs [22]. Their elaboration also requires the use of the isofunctional analysis we cannot possibly review in this paper for brevity. It is easy to prove from the positive-definite character of the isounit $\hat{I} = Diag.\left(n_1^2, n_2^2, n_3^2, n_4^2\right)$ that the Lorentz-Santilli isosymmetry $\hat{SO}(3.1)$ is locally isomorphic to the conventional symmetry $SO(3.1)$ (Figure 11).

The LS isotransformations are at the foundations of the relativistic results of this paper as well as of their invariance over time. They were first derived in Ref. [15] of 1983 and can be presented projected in the conventional Minkowski (3-4)-plane (see monograph [22b] for the general case)

$$x^{1'} = x^1, \; x^{2'} = x^2, \tag{A.30a}$$

$$x^{3'} = \hat{\gamma}\left(x^3 - \hat{\beta} \times \frac{n_3}{n_4} x^4\right), \tag{A.30b}$$

$$x^{4'} = \hat{\gamma}\left(x^4 - \hat{\beta} \times \frac{n_4}{n_3} x^3\right), \tag{A.30c}$$

where

$$\hat{\beta} = \frac{v_3 / n_3}{c_o / n_4}, \tag{A.31a}$$

$$\hat{\gamma} = \frac{1}{\sqrt{1-\hat{\beta}^2}}. \tag{A.31b}$$

It should be indicated that the main aim of Ref. [15] was the solution of the historical Lorentz probem, namely, the achievement of the universal symmetry for locally varying speeds of light within physical media $C = c / n_4$. Since this problem is highly non-linear, its solution could not be derived via the conventional Lie's theory. For this reason, Santilli conducted decades of studies for the generalization of Lie's theory into a form valid for nonlinear systems, first presented in monograph [3b], as a prerequisite for the solution of Lorentz's historical problem.

The isomorphism $\hat{SO}(3.1) \approx SO(3.1)$ is due to the reconstruction of the exact light cone on isospace over isofields called the light isocone. In fact, jointly with the deformation of the light cone

$$x^2 = x_3^2 - t^2 c^2 = 0 \to \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n^4} = 0, \tag{A.32}$$

we have the corresponding *inverse* deformations of the units,

$$d_3 = 1 \to \hat{I}_3 = n_3^3 \;\; d_4 = 1 \to \hat{I}_4 = n_4^2, \tag{A.33}$$

thus reconstructing the original light cone on isospaces over isofields.

The reader should be aware that the above reconstruction includes the preservation on isospace over isofields of the original characteristic angle of the conventional light cone, namely, *the maximal causal speed on isospace over isofields is the conventional speed of light* c *in vacuum* [22].

A.9 Regular IsoTranslations

The regular isotranslations $T(4)$ were first studied in Ref. [16] and then studied in details in monographs [22]. and can be expressed in their projection in the conventional Minkowski space with the following lifting of the conventional translations $x^{\mu'} = x^\mu + a^\mu, \mu = 1,2,3,4,$ and a^μ constants,

$$x^{\mu'} = x^\mu + A^\mu(a,\ldots), \tag{A.34}$$

where

$$A^\mu = a^\mu(n_\mu^{-2} + a^\alpha[n_\mu^{-2}; P_\alpha]/1! + \ldots), \tag{A.35}$$

and there is no summation on the μ indices.

Note the high nonlinearity of the isotranslations. This is due to the fact that the above expressions are the projection in the conventional spacetime since, when written on a Minkowski-Santilli isospace over isofields, isotransformations coincide with conventional translations.

A.10 Regular IsoDilations and IsoContractions

The *regular isodilations and isoContractions* $D(1)$ were first identified in Ref. [16] and then studied in details in monographs [22]. They constitute a *basically new spacetime symmetry* with vast implications, e.g., for grand unified theories [71], and can be expressed via the transformation

$$\hat{\eta} \to \hat{\eta}' = w^{-1}\hat{\eta}, \hat{I} \to \hat{I}' = w\hat{I}, \tag{A.36}$$

with ensuing invariance

$$(x^\mu \hat{\eta}_{\mu\nu} x^\nu)\hat{I} \equiv \left[x^\mu (w^{-1}\hat{\eta}_{\mu\nu}) x^\nu\right](w \times \hat{I}) = \\ = (x^\mu \hat{\eta}_{\mu\nu} x^\nu) \times \hat{I}', w \in R. \tag{A.37}$$

It was popularly believed in the 20th century that the LP symmetry was 10 -dimensional. The above invariance establishes that, instead, *the LPS isosymmetry as well as the LP conventional symmetry are* 11*-dimensional.*

A.11 Regular IsoInversions

The regular isoinversions are given by [22b]

$$\hat{\pi} \hat{\times} x = \pi x = (-r, tc), \tag{A.38a}$$

$$\hat{\tau} \hat{\times} x = \tau x = (r, -tc). \tag{A.38b}$$

where π and τ are the conventional space and time inversion operators.

A.12 Regular $\hat{SU}(2)$ *IsoSymmetry*

In this section we provide the solution, apparently for the first time, of a central problem for the consistent and time invariant representation of nuclear magnetic moments via the deformations of the charge distributions of nucleons with consequential mutation of their intrinsic magnetic moment, under the conservation of conventional, values of the spins.

By remembering the lack of uniqueness of the isounits and related isotopic element, the simplest regular two-dimensional irreducible isorepresentations of $\hat{SU}(2)$ are characterized by the lifting of the two-dimensional complex-valued unitary space with metric $\delta = Diag.(1,1)$ into the isotopic image [12, 15, 22]

$$\hat{I} = Diag.(n_1^2, n_2^2), \;\; \hat{T} = Diag.(1/n_1^2, 1/n_2^2), \tag{A.39a}$$

$$\hat{\delta} = \hat{T} \times \delta = Diag.(1/n_1^2, 1/n_2^2), \tag{A.39b}$$

$$Det\, \hat{\delta} = (n_1 n_2)^{-2} = 1, \tag{A.39c}$$

The basic non-unitary transform (43) of Section 4 us then given by

$$U\times U^{\dagger}=\hat{I}=\begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix},\quad T=\begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix} \tag{A.40a}$$

$$U=\begin{pmatrix} i\times n_1 & 0 \\ 0 & i\times n_2 \end{pmatrix},\quad U^{\dagger}=\begin{pmatrix} -i\times n_1 & 0 \\ 0 & -i\times n_2 \end{pmatrix}, \tag{A.40b}$$

here the n's are well behaved nowhere null functions, resulting in the *regular Pauli-Santilli isomatrices* [*loc. cit*]

The related lifting of Pauli's matrices are then given by the *Regular Paili-Santilli isomatrices* [13, 14]

$$\sigma_k\to\hat{\sigma}_k=U\times\sigma_k\times U^{\dagger}, \tag{A.41a}$$

$$\hat{\sigma}_1=\begin{pmatrix} 0 & n_1^2 \\ n_2^2 & 0 \end{pmatrix},\quad \hat{\sigma}_2=\begin{pmatrix} 0 & -in_1^2 \\ in_2^2 & 0 \end{pmatrix},$$

$$\hat{\sigma}_3=\begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}. \tag{A.41b}$$

Another realization of the regular hadronic spin 1/2 is given by *non diagonal* nonunitary transforms [*loc. cit.*].

$$\begin{aligned} U&=\begin{pmatrix} 0 & n_1 \\ n_2 & 0 \end{pmatrix},\quad U^{\dagger}=\begin{pmatrix} 0 & n_2 \\ n_1 & 0 \end{pmatrix},\\ \hat{I}&=\begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix},\quad \hat{T}=\begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix}, \end{aligned} \tag{A.42}$$

with corresponding alternative version of the regular Pauli-Santilli isomatrices,

$$\hat{\sigma}_1=\begin{pmatrix} 0 & n_1n_2 \\ n_1n_2 & 0 \end{pmatrix},\quad \hat{\sigma}_2=\begin{pmatrix} 0 & -in_1n_2 \\ i\times n_1n_2 & 0 \end{pmatrix},$$

$$\hat{\sigma}_3=\begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, \tag{A.43}$$

or by more general realizations with Hermitean non diagonal isounits $\hat{I}$ [15b].

All regular Pauli-Santilli isomatrices verify the following *isocommutation rules and isoeigenvalue equations* on H over C

$$\begin{aligned} &[\hat{\sigma}_i\,\hat{,}\,\hat{\sigma}_j]=\\ &=\hat{\sigma}_i\hat{T}\hat{\sigma}_j-\hat{\sigma}_j\hat{T}\hat{\sigma}_i=2i\varepsilon_{ijk}\hat{\sigma}_k, \end{aligned} \tag{A.44a}$$

$$\begin{aligned} &\hat{\sigma}^{\hat{2}}\hat{\times}|\hat{\psi}\rangle=\\ &(\hat{\sigma}_1T\times\hat{\sigma}_1+\hat{\sigma}_2T\times\hat{\sigma}_2+\hat{\sigma}_3T\times\hat{\sigma}_3)T|\hat{\psi}\rangle=3\times|\hat{\psi}\rangle, \end{aligned} \tag{A.44b}$$

$$\hat{\sigma}_3\hat{\times}|\hat{\psi}\rangle=\hat{\sigma}_3\times T\times|\hat{\psi}\rangle=\pm1\times|\hat{\psi}\rangle, \tag{A.44c}$$

thus preserving conventional structure constants and eigenvalues for spin $1/2$ under non-Hamiltonian/nonunitary interaction, while adding the degree of freedom

$$n_1^2=\lambda,\ n_2^2=\lambda^{-1}, \tag{A.45}$$

That indeed is fully compatible with the mutation of intrinsic magnetic moments of spin $1/2$ particles, Eq. (60).

Additionally, *the regular Pauli-Santilli isomatrices provide an explicit and concrete realization of hidden variables*, with intriguing implications for local realism studied in detail in ref. [14]. In turn, the above aspect confirm the origination of isomechanics as a concrete and explicit realization of the "incompleteness" of quantum mechanics according to Einstein, Podolsky and Rosen [1].

A.13 Irregular $\hat{S}U(2)$ IsoSymmetry

As indicated throughout this paper, there appears to be no need for a mutation of the spin of nuclear constituents to achieve an exact representation of nuclear magnetic moments and spins.

Nevertheless, the issue persists as to whiter a proton in the core of a star should have the same spin when member of a nuclear structure. Santilli has introduced the irregular isotopies of the $SU(2)$ spin precisely for future studies of this important problem for the structure of stars.

One illustrative example of *irregular Pauli-Santilli isomatrices* is given by [12-14]

$$\tilde{\sigma}_1=\begin{pmatrix} 0 & n_1^2 \\ n_2^2 & 0 \end{pmatrix},\quad \tilde{\sigma}_2=\begin{pmatrix} 0 & -in_1^2 \\ in_2^2 & 0 \end{pmatrix},$$

$$\tilde{\sigma}_3=\begin{pmatrix} wn_1^2 & 0 \\ 0 & wn_2^2 \end{pmatrix}. \tag{A.46}$$

where w is the *mutation parameter,* with isocommutation rules and eigenvalue equations

$$\begin{aligned} &[\tilde{\sigma}_1,\tilde{\sigma}_2]=iw^{-1}\tilde{\sigma}_3,\ [\tilde{\sigma}_2,\tilde{\sigma}_3]=iws\tilde{\sigma}_1,\\ &[\tilde{\sigma}_3,\tilde{\sigma}_2]=isw\tilde{\sigma}_1, \end{aligned} \tag{A.47a}$$

$$\begin{aligned} &\tilde{\sigma}^{\hat{2}}\hat{\times}|\hat{\psi}\rangle=\\ &(\tilde{\sigma}_1T\tilde{\sigma}_1+\tilde{\sigma}_2T\tilde{\sigma}_2+\tilde{\sigma}_3T\tilde{\sigma}_3)T|\hat{\psi}\rangle=(2+w^2)\times|\hat{\psi}\rangle, \end{aligned} \tag{A.47b}$$

$$\tilde{\sigma}_3\hat{\times}|\hat{\psi}\rangle=\tilde{\sigma}_3T|\hat{\psi}\rangle=\pm w|\hat{\psi}\rangle,\ w\neq1, \tag{A.47c}$$

Additional examples of irregular Pauli-Santilli isomatrices can be found in Refs. [12-14].

The assumption of a mutated spin in hyperdense interior conditions evidently implies the inapplicability (rather than the violation) of the Fermi-Dirac statistics, Pauli's exclusion principle and other quantum mechanical laws, with the understanding that, by central assumption, non-Hamiltonian bound states of particles as a whole must have conventional total quantum values. Therefore, *we are here referring to possible internal exchanges of angular momentum and spin*

always in such a way as to cancel out and yield total conventional values.

A.14 IsoRelativity IsoAxioms

As shown in this paper, a numerically exact and time invariant representation of nuclear magnetic moments and spins has required the isotopies of 20th century mathematics, with ensuing isotopies of quantum mechanics into isomechanics.

Interested readers should be aware that the above isotopies imply the inapplicability of special relativity for the nuclear structure in favor of a covering relativity known as isorelativity [15, 21-23]. The central aim of special relativity is the invariance of the speed of light in vacuum. A central aim of isorelativity is the invariance of local varying speeds of light $C = c / n_4$ within physical media as shown in Appendix A.8.

A rudimentary knowledge of the covering relativity is important to prevent major misrepresentations of the results of this paper as well as in possible further advances because the the appraisals of the new nuclear structure provided by isomechanics via special relativity would be equivalent to the appraisal of the results by special relativity via Newtonian mechanics.

The isotopies of the axioms of special relativity, today known as *IsoAxioms*, were initiated by Santilli in paper [15] of 1983; they received a first systematic formulation by Santilli in monographs [21] of 1991; they were finalized in monographs [22] of 1995 jointly with the discovery of the isodifferential calculus; and their experimental verifications were presented ion Refs. [23].

In this paper we specialize, apparently for the first time, the isoaxioms for the isomechanical structure of stable and isolated nuclei whose constituents are isoparticles. The gravitational formulation of the isoaxioms of Reg. [71] should be kep in mind because it offers, also apparently for the first time, the possibility of addressing the origin of gravitational field in the structure of nuclei.

The first implication of the isotopies of special relativity is the *abandonment of the speed of light in vacuum as the maximal causal speed in favor of a covering geometrization of physical media.* This occurrence is easily seen by specializing the isoline element (27) to the *isolight isocone* [23, 37]

$$\hat{x}^{\hat{2}} = \left(\frac{x_k^2}{n_k^2} - t^2 \frac{c^2}{n_4^2} \right) = 0, \tag{A.48}$$

thus leading to the Maximal Causal Speed V_{max} of IsoAxiom 5.1 below.

The remaining isoaxioms can be uniquely and unambiguously identified via a procedure parallel to the construction of the axioms of special relativity from the Lorentz-Poincaré symmetry.

Another departure from 20th century views is that isoaxioms refer to generally inhomogeneous and anisotropic physical media, as it os typically the case of the medium within spinning charge distributions., Therefore, the isoaxioms are formulated below for a generic space direction k ,

ISOAXIOM A. I: The maximal causal speed in a given space direction k in the interior of nuclei is given by

$$V_{max,k} = c \frac{n_k}{n_4}, \tag{A.49}$$

ISOAXIOM A. II: The local isospeed of light is given by

$$\hat{c} = \frac{c}{n_4} \tag{A.50}$$

where c is the speed of light in vacuum.

ISOAXIOM A. III: The addition of isospeeds in the k -direction follows the isotopic law

$$V_{tot,k} = \frac{v_{1,k} / n_k + v_{2,k/n_k}}{1 + \dfrac{v_{1,k} v_{2,k}}{c^2} \dfrac{n_4^2}{n_k^2}}. \tag{A.51}$$

ISOAXIOM A. IV: The isodilatation of isotime, the isocontraction of isolengths, the variation of mass with isospeed, and the mass-energy isoequivalence principle follow the isotopic laws

$$\Delta t' = \hat{\gamma}_k \, \Delta t, \tag{A.52a}$$

$$\Delta \ell' = \hat{\gamma}_k^{-1} \, \Delta \ell, \tag{A.52b}$$

$$m' = \hat{\gamma}_k \, m, \tag{A.52c}$$

$$E = m V_{max}^2 = m c^3 \frac{n_k^2}{n_4^2} \tag{A.52d}$$

where $\hat{\gamma}$ and $\hat{\beta}$ have values (32).

ISOAXIOM A. V: The frequency isoshift of light propagating within a nucleus in the k -direction follows the Doppler-Santilli isotopic law

$$\omega_e = \omega_o \hat{\gamma}_k \left[1 \pm \frac{v / n_k}{c / n_4} \cos \alpha \right] \tag{A.53}$$

where ω_e is the frequency experimentally measured in the outside, ω_o is the frequency at the origin inside a nucleus, and we have ignored for simplicity the isotopies of trigonometry (see Refs. [23] for brevity).

It should be stressed that in the above formulations as well as in the next section we present the isoaxioms in their *projection on the conventional Minkowski space.* while their technical treatment requires the full use of the various branches of isomathematcs, including the formulation of the isoaxioms on a Minkowski-Santilli isos; ace over an isofield.

A main feature is that, when the isoaxioms are represented on

isospace over isofields, they coincide with the conventional axioms of special relativity by conception and technical realization. In particular, the maximal causal speed $V_{max} \neq c$ solely occurs in the projection of the isoaxioms on Minkowski space because, at the isotopic level, the maximal causal speed is c for all possible isogravitational problems.

A.15 Predicted Implications of the IsoAxioms for the Nuclear Structure

In this final section, we identify the most important predictions of isorelativity [15, 21, 22] emerging as a consequence of our exact and invariant representation of nuclear magnetic moments and spins, and present their preliminary appraisals by soliciting comments from interested colleagues.

Isoaxioms clearly imply *two* different representations of the nuclear structure, the first is the representation of nuclear characteristics as measured from outside observer here indicated with the subindex "ext," and the second representation is that in the interior of nuclei here indicated with the subindex "int."

These two representations are necessary for the evident reason that the exterior observer is assumed as being in vacuum thus obeying conventional relativity axioms while the second representation occurs within hyperdense physical media, here assumed as obeying the covering isorelativity axioms.

A first implications of isorelativity is that *the time of the exterior observer is not necessarily the same as that in the interior of nuclei.* In fact, by recalling the isodilation and isocontraction of Appendix A.10, we can write the identity

$$t_{ext} 1 = t_{int} \hat{I}_t . \tag{A.54}$$

Since for the nuclear structures considered in this paper $\hat{I}_t = n_4^2 < 1$ as in Eq. (63), one can see that the interior time evolution of nucleons is p*redicted to be "faster" than that of an outsider observer.*

Note that at the abstract realization-free level there is no distinction between interior and exterior times as typical for all isotopies [22] since Eq. (A.54) can be written

$$t \equiv \hat{t} \tag{A.55}$$

where t is an ordinary scalar, while $\hat{t}$ is an isoscalar (Section 2). Therefore, t_{ext} and t_{int} are the projection of Eq. (A.55) in our spacetime.

For the case of distances, we can write the corresponding differentiations between external and internal distances according to the isotopic law

$$r_{ext} 1 = r_{int,k} \hat{I}_k . \tag{A.56}$$

Since the space isounits are generally smaller than one from Eqs. (64), one can see again that space distances perceived in the outside observer are predicted to be bigger than the actual distances in the interior.

Intriguingly, isolaw (A.56) is verified in ordinary water where, as we all know, dimensions perceived from the outsider are bigger than those actually occurring within water (Figure 12). Therefore, our argument is that, since isolaw (A.55) is verified in a medium with relatively big density such as water, the possibility of a similar occurrence in much denser media such as nuclei deserves due scientific process.

Figure 12. *This picture illustrates the representation by isorelativity of the known effect that dimensions in water appears as being bigger then their actual dimensions when seen fro an outside observer, thus warranting the study of the corresponding effect within nuclei.*

Next, the speed of light in vacuum c has no mathematical or physical meaning for isorelativity and, in particular, it is not invariant under the time evolution. The sole mathematically and physically accepted quantity is Lorentz locally varying speed $C = c / n_4$.

In fact, the relativistic sum of two ordinary speeds of light does not yield the speed of light within physical media such as water and the same is expected within nuclei. By contrast, the isorelativistic sum of two locally varying speeds of light does indeed yield the local speed of light according to isoaxioms III,

$$V_{tot} = \frac{c / n_4 + c / n_4}{1 + \dfrac{c^2 / n_4^2}{c^2 / n_4^2}} = \frac{c}{n_4}. \tag{A.57}$$

In particular, one should note that a necessary condition for the isorepresentation of nuclear magnetic moments is that the local speed of light in the interior of nuclei is bigger than that in vacuum, see Eq. (64). This is a confirmation of the similar condition for $C > c$ which is necessary for the synthesis of the neutron from bthe hydrogen (Section 7).

Yet another prediction of isorelativity according to isoaxioms A. IV is that *the energy isoequivalent according to isoaction (A.52d) is "bigger" than that described from the outside.* This is a typical occurrence for all structure models of hadrons, nuclei and stars according to iksomathematics, and it is nowadays known as *isorenormalization.*

Consequently, in considering the structure model of nuclei as

isobound states of isoprotons and isoelectrons, the reader should be aware that the rest energy of the isoelectron is isorenormalized to a minimum value of 1.293 MeV in the first approximation of ignoring Coulomb interactions, with bigger predicted values of the rest energy of the isoelectrons when including Coulomb interactions (due to the Coulomb attraction between isoprotons and isoelectrons).

As an illustration, a necessary condition for the achievement of an exact representation of the synthesis of the neutron from the hydrogen is that (by ignoring coulomb interactions) the isoremnormalized rest energy of the electron is 1.293 MeV.

Finally, we mention the prediction of isorelativity according to which *the frequency of the photons emitted by nuclei and measured in the outside is bigger than that at the point of emission in the interior ofg nuclei.* This additional effect is due to the *isoblueshift*, namely, the acquisition of energy by photons from hot environments without any relative motion, which was predicted by Santilli in 1992 [21], and experimentally verified in hot gases in 2010 [80] (see Refs. [68] for a comprehensive bibliography).

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