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Studies of Multi-Valued Hyperstructures for the Characterization of Matter-Antimatter Systems and their Extension

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Abstract. In this paper, we study multi-valued hyperstructures following the apparent existence in nature of a realization of two-valued hyperstructures with hyperunits characterized by matter-antimatter systems and their extensions where matter is represented with conventional mathematics and antimatter is represented with isodual mathematics.

Keywords: algebraic hyperstructure, hypergroup, hyperring, hyperfield, two-valued field, isodual spacetime, hyperunit.
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1 INTRODUCTION

As it is well known, antimatter was solely treated in the 20th century via *charge conjugation* on a Hilbert space $\mathcal{H}$ with states $\psi(x)$ over the field of complex numbers $\mathbb{C}$

$$\mathbb{C}\ \psi(x) = -\ \psi^{\dagger}(x),$$

where x is the coordinate of the representation space, such as the Minkowski spacetime.

The above approach caused a historical imbalance between matter and antimatter, because matter was treated at all known levels, from Newtonian mechanics to second quantization, while antimatter was solely treated at the level of second quantization.

The resolution of this imbalance required the construction of a new mathematics, called *Santilli isodual mathematics* [5], which is constructed via a step-by-step anti-Hermitean conjugation, denoted with the upper symbol ^{d} and called *isodual conjugation*, of each and all aspects of the 20th century mathematics used for matter. The isodual conjugation of a generic classical or operator quantity $A(x, p, \psi, \ldots)$ depending on coordinates x, momenta p, states ψ, etc. is then given by

$$A(x, p, \psi, \ldots)\ \rightarrow\ A^d(x^d, p^d, \psi^d, \ldots) = A(-x^{\dagger}, p^{\dagger}, \psi^{\dagger}, \ldots),$$

thus resulting in the new *isodual unit* $1^d = -1^{\dagger}$, *isodual real, complex or quaternionic numbers* $n^d = -n^{\dagger}$, *isodual functional analysis*, etc. [5].

The main advantage of the isodual conjugation over charge conjugation is that the former is applicable at all levels of study, thus characterizing the classical and operator isodual mechanics. The resulting *isodual theory of antimatter* has, therefore, established a complete democracy in the treatment of matter and antimatter at all levels, with intriguing implications, such as the prediction of gravitational repulsion (antigravity) for matter in the field of antimatter and vice-versa.

Despite their simplicity, the physical and mathematical differences between charge and isodual conjugations are nontrivial. From a physical viewpoint, charge conjugation solely conjugates the state, and does not conjugate the local coordinates x. This implies that, under charge conjugation, antimatter is assumed to exist in the same spacetime of matter.

By comparison, the isodual conjugation maps, for consistency, each quantity used in the representation of matter into its isodual image, thus including a necessary conjugation of spacetime with coordinates x into the novel *isodual spacetime* with isodual coordinates $x^d = -x^{\dagger}$. This conjugation implies that, under isoduality, antimatter exists in a new spacetime which is physically distinct from yet coexisting with our spacetime. In particular, their physical differences are not trivial. e.g., because the isodual conjugation of coordinates is different than inversions [5].

From a mathematical viewpoint, the co-existence of the conventional and isodual spacetimes in the same region of space creates a number of intriguing problems. At a first inspection, it is rather natural to attempt the representation of matter and antimatter via *multi-dimensional models,* e.g., via eight-dimensional mathematics essentially consisting of the Kronecker product of the four-dimensional mathematics of spacetime and its four-dimensional isodual. However, this mathematical formulation is easily seen as being unacceptable because our sensory perception deny the existence of spacetime bigger than those with four dimensions.

The compatibility of the complexities of nature with our sensory perception has motivated the construction of *multi-valued hyperstructures with hyperunits* [7]. In its most elementary possible formulation expressed via conventional operations, matter and antimatter can be represented via a two-valued hyperstructure characterized by the multiplicative hyperunit

$$E = \{1, 1^d\}$$

where $\{\ldots\}$ represents a set, with hypernumbers

$$N = \{n, n^d\}$$

and related hyperproduct

$$N \times M = \{n \times m, n^d \times^d m^d\}$$

where $\times$ is the conventional (associative) multiplication, under which E is the correct left and right hyperunit for all possible hypernumbers.

The set of hypernumbers with the indicated hyperunit and hyperproduct verifies all axioms of a numerical field, thus yielding the *two-valued hyperfield*

$$\mathcal{F} = \{F(n, \times, 1), F^d(n^d, \times^d, 1^d\}$$

from which all remaining aspects of a two-valued hypermathematics can be constructed via known procedures. Compatibility with our sensory perception is achieved by the fact that, at the abstract realization-free level, numbers and hypernumbers, spaces and hyperspaces, etc., coincide, thus avoiding the increase of dimensionality not allowed by our sensory perception.

We also indicate the possibility of extending the above two-valued example to a four-valued hyperstructure via the inclusion of *Ying's twin universes* [10], one for matter and one for antimatter, which extension has intriguing features such as characterizing a universe with identically null total physical characteristics, i.e., identically null total time, total energy, total momentum, total entropy, etc., when examined by an observer either of matter or of antimatter.

In conclusion, the resolution of the historical 20th century imbalance between matter and antimatter via the novel isodual theory of antimatter appears to produce physical evidence for the realization in nature of multi-valued hyperstructures with hyperunits, and therefore suggesting the mathematical study presented below.

2 GROUPS, RINGS AND FIELDS

In the area of algebraic structures, a mathematical entity called a group plays a key role that resonates throughout the fascinating meadows of this intriguing discipline. Still more fascinating is it that the theory of this mathematical creature, or "group theory", was thought early on by mathematicians to have only intellectual appeal. That is, nobody in his right mind thought that the group and its concomitant theoretical aspects would ever serve mankind in any way other than to stimulate his cognitive awareness. Yet as irony would prove, the mathematical group would prove to be the pathway to understanding particle physics and the subatomic entities that spin the tales of this most curious science.

Definition 2.1. Let G be a non-empty set together with a binary operation (usually called *multiplication*) that assigns to each ordered pair (a, b) of elements of G an element $a \cdot b$ in G. We say G is a *group* under this operation if the following three properties are satisfied:

(1) $a \cdot (b \cdot c) = (a \cdot b) \cdot c$, for all $a, b, c \in G$,

(2) there exists an element $e \in G$ such that $a \cdot e = e \cdot a = a$, for all $a \in G$,

(3) for every $a \in G$ there exists an element $a^{-1} \in G$ such that $a \cdot a^{-1} = a^{-1} \cdot a = e$.

We have $(a^{-1})^{-1} = a$ and $(a \cdot b)^{-1} = b^{-1} \cdot a^{-1}$, for all $a, b \in G$.

Group theory is a powerful formal method for analyzing abstract and physical systems in which symmetry is present and has surprising importance in physics, especially quantum mechanics. Various physical systems, such as crystals and the hydrogen atom, can be modeled by symmetry groups. Thus, group theory and the closely related representation theory have many applications in physics and chemistry.

In mathematics, ring theory is the study of algebraic structures in which addition and multiplication are defined and have similar properties to those familiar from the integers.

Definition 2.2. A non-empty set R is called a *ring*, if it has two binary operations called *addition* denoted by $a + b$ and multiplication denoted by $a \cdot b$ for $a, b \in R$ satisfying the following axioms:

(1) $(R, +)$ is an abelian group;

(2) multiplication is associative, i.e., $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ for all $a, b, c \in R$.

(3) distributive laws hold: $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(b + c) \cdot a = b \cdot a + c \cdot a$ for all $a, b, c \in R$.

There is a group structure with the addition operation, but not necessarily with the multiplication operation. Thus an element of a ring may or may not be invertible with respect to the multiplication operation. While the addition operation is commutative, it may or not be the case with the multiplication operation.

The philosophy of this subject is that we focus on similarities in arithmetic structure between sets (of numbers, matrices, functions or polynomials for example) which might look initially quite different but are connected by the property of being equipped with operations of addition and multiplication. The set of integers and the set of $n \times m$ matrices with real numbers as entries are examples of rings. These sets are obviously not the same, but they have some similarities, and some differences, in terms of their algebraic structure. Although people have been studying specific examples of rings for thousands of years, the emergence of ring theory as a branch of mathematics in its own right is a very recent development. Much of the activity that led to the modern formulation of ring theory took place in the first half of the 20th century. Ring theory is powerful in terms of its scope and generality, but it can be simply described as the study of systems in which addition and multiplication are possible.

Everyone is familiar with the basic operations of arithmetic, addition, subtraction, multiplication, and division. Fields are important objects of study in algebra, since they provide a useful generalization of many number systems, such as the rational numbers, real numbers, and complex numbers. In particular, the usual rules of associativity, commutativity and distributivity hold. Fields also appear in many other areas of mathematics.

Definition 2.3. Let $(F, +, \cdot)$ be a ring such that $(F - \{0\}, \cdot)$ is an abelian group. Then $(F, +, \cdot)$ is called a *field*.

A *strictly totally ordered set* consists of a set F and a binary relation $<$ which satisfies:

(1) $x \not< x$;

(2) for all x and y, exactly one of the three possibilities holds: $x < y$, $x = y$, $y < x$;

(3) if $x < y$ and $y < z$, then $x < z$.

Definition 2.4. An *ordered field* consists of a field $(F, +, \cdot)$ and a set $P \subseteq F$ of positive elements satisfying the following:

(1) if $x \in P$ and $y \in P$, then $x + y \in P$;

(2) if $x \in P$ and $y \in P$, then $x \cdot y \in P$;

(3) for each x, exactly one of the three possibilities holds: $x = 0$, $x \in P$ or $-x \in P$.

We can define a strict total ordering on an ordered field by setting $x < y$ if and only if $y - x \in P$. We may write $x \leq y$ for ($x = y$ or $x < y$).

3 HYPERGROUPS, HYPERRINGS AND HYPERFIELDS

Algebraic hyperstructures represent a natural extension of classical algebraic structures. They were introduced in 1934 by the French mathematician F. Marty [4]. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Since then, hundreds of papers and several books have been written on this topic, for example see [1, 2, 3, 9].

Let H be a non-empty set and $\times : H \times H \longrightarrow \mathcal{P}^*(H)$ be a hyperoperation, where $\mathcal{P}^*(H)$ is the set of all non-empty subsets of H. The couple $(H, \times)$ is called a *hypergroupoid*. For any two non-empty subsets A and B of H and $x \in H$, we define

$$A \times B = \bigcup_{a \in A, b \in B} a \times b, \quad A \times x = A \times \{x\} \quad \text{and} \quad x \times B = \{x\} \times B.$$

Definition 3.1. A hypergroupoid $(H, \times)$ is called a *semihypergroup* if for all a, b, c of H we have $(a \times b) \times c = a \times (b \times c)$, which means that

$$\bigcup_{u \in a \times b} u \times c = \bigcup_{v \in b \times c} a \times v.$$

A hypergroupoid $(H, \times)$ is called a *quasihypergroup* if for all a of H we have $a \times H = H \times a = H$. This condition is also called the *reproduction axiom*.

Definition 3.2. A hypergroupoid $(H, \times)$ which is both a semihypergroup and a quasihypergroup is called a *hypergroup*.

Let $(S, \cdot)$ be a semigroup and P be a non-empty subset of S. The P-hyperoperations [8] are defined as follows:

$$x \times_c y = x \cdot P \cdot y, \quad x \times_r y = x \cdot y \cdot P, \quad x \times_l y = P \cdot x \cdot y,$$

for all $x, y \in S$.

Remark that if $(S, \cdot)$ is commutative, then $\times_c = \times_r = \times_l = \times$.

Theorem 3.3. *[8] Let $(S, \cdot)$ be a semigroup and P be a non-empty subset of S. Then $(S, \times_c)$ is a semihypergroup. Moreover, $(S, \times_c)$ is a hypergroup if and only if $(S, \cdot)$ is a group.*

Example 1. The *quaternion group* is a non-abelian group of order 8. It is often denoted by Q or Q_8 and written in multiplicative form, with the following 8 elements

$$Q = \{1, -1, i, -i, j, -j, k, -k\}.$$

Here 1 is the identity element, $(-1)^2 = 1$ and $(-1)a = a(-1) = -a$ for all a in Q. The remaining multiplication rules can be obtained from the following relation:

$$i^2 = j^2 = k^2 = ijk = -1.$$

Now, suppose that $P = \{i, j, k\}$, a subset of Q_8 with three elements. Then we obtain the hypergroup $(Q_8, \times_c)$ with the following multiplication table:

$\times_c$	1	-1	i	$-i$	j	$-j$	k	$-k$
1	$\{i, j, k\}$	$\{-i, -j, -k\}$	$\{-1, -k, j\}$	$\{1, k, -j\}$	$\{k, -1, -i\}$	$\{-k, 1, i\}$	$\{-j, i, -1\}$	$\{j, -i, 1\}$
-1	$\{-i, -j, -k\}$	$\{i, j, k\}$	$\{1, k, -j\}$	$\{-1, -k, j\}$	$\{-k, 1, i\}$	$\{k, -1, -i\}$	$\{j, -i, 1\}$	$\{-j, i, -1\}$
i	$\{-1, k, -j\}$	$\{1, -k, j\}$	$\{-i, j, k\}$	$\{i, -j, -k\}$	$\{-j, -i, 1\}$	$\{j, i, -1\}$	$\{-k, -1, -i\}$	$\{k, 1, i\}$
$-i$	$\{1, -k, j\}$	$\{-1, k, -j\}$	$\{i, -j, -k\}$	$\{-i, j, k\}$	$\{j, i, -1\}$	$\{-j, -i, 1\}$	$\{k, 1, i\}$	$\{-k, -1, -i\}$
j	$\{-k, -1, i\}$	$\{k, 1, -i\}$	$\{-j, -i, -1\}$	$\{j, i, 1\}$	$\{i, -j, k\}$	$\{-i, j, -k\}$	$\{1, -k, -j\}$	$\{-1, k, j\}$
$-j$	$\{k, 1, -i\}$	$\{-k, -1, i\}$	$\{j, i, 1\}$	$\{-j, -i, -1\}$	$\{-i, j, -k\}$	$\{i, -j, k\}$	$\{-1, k, j\}$	$\{1, -k, -j\}$
k	$\{j, -i, -1\}$	$\{-j, i, 1\}$	$\{-k, 1, -i\}$	$\{k, -1, i\}$	$\{-1, -k, -j\}$	$\{1, k, j\}$	$\{i, j, -1\}$	$\{-i, -j, 1\}$
$-k$	$\{-j, i, 1\}$	$\{j, -i, -1\}$	$\{k, -1, i\}$	$\{-k, 1, -i\}$	$\{1, k, j\}$	$\{-1, -k, -j\}$	$\{-i, -j, 1\}$	$\{i, j, -1\}$

Construction 3.4. *Let $(G,\cdot)$ be an abelian group and P be any subset of G with more than one element. We define the hyperoperation $\times_P$ as follows:*

$$x \times_P y = \begin{cases} x \cdot P \cdot y = \{x \cdot h \cdot y \mid h \in P\} & \text{if } x \neq e \text{ and } y \neq e \\ x \cdot y & \text{if } x = e \text{ or } y = e. \end{cases}$$

We call this hyperoperation P_e-hyperoperation. The hyperstructure $(G, \times_P)$ is a hypergroup. Then $(G, \times_P)$ is a hypergroup.

Proof. We prove it for $P = \{s,t\}$ and the proof is analogous for any P with more elements. Let x,y,z be non unit elements of $(G,\cdot)$. Then, we have

$$\begin{aligned} x \times_P (y \times_P z) &= x \cdot P \cdot (y \cdot P \cdot z) = x \cdot P \cdot y \cdot P \cdot z, \\ (x \times_P y) \times_P z &= (x \cdot P \cdot y) \cdot P \cdot z = x \cdot P \cdot y \cdot P \cdot z. \end{aligned}$$

So $x \times_P (y \times_P z) = (x \times_P y) \times_P z$.

If one of x,y,z equals to e, say $x = e$, then we have

$$e \times_P (y \times_P z) = y \cdot P \cdot z \quad \text{and} \quad (e \times_P y) \times_P z = y \cdot P \cdot z.$$

Therefore, $\times_P$ is associative.

Now, let $x \neq e$. Then

$$x \times_P G = \{x\} \cup [x \cdot P \cdot (G - \{e\})] = \{x\} \cup [x \cdot s \cdot (G - \{e\})] \cup [x \cdot t \cdot (G - \{e\})],$$

in which we remark that the set $x \cdot s \cdot (G - \{e\})$, which contain all the elements of G except the element $x \cdot s$ and the set $x \cdot t \cdot (G - \{e\})$ contains all the elements of G except $x \cdot t$. Therefore, we have $x \times_P G = G$. The same proof for $G \times_P x = G$. Finally, the reproductivity for the unit e is obvious. Thus, $\times_P$ is reproductive. Therefore, $(G, \times_P)$ is a hypergroup. □

Remark that e is scalar unit in $(G, \times_P)$. Any element x of G has one or two inverses, the elements $(x \cdot s)^{-1}$ and $(x \cdot t)^{-1}$ when $x \cdot s \neq e$ and $x \cdot t \neq e$.

Example 2. Consider the Klein four-group $K_4 = \{e,a,b,c\}$. It is abelian, and isomorphic to the dihedral group of order 4. It is also isomorphic to the direct sum $\mathbb{Z}_2 \oplus \mathbb{Z}_2$. Multiplication table for Klein four-group is given by:

$\cdot$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Now, we consider the subgroups $P_1 = \{e, a\}$, $P_2 = \{e, b\}$ and $P_3 = \{e, c\}$. Then, according to Construction 3.4, we obtain the canonical hypergroups $(G, \times_{P_1})$, $(G, \times_{P_2})$ and $(G, \times_{P_3})$ with the following tables:

$\times_{P_1}$	e	a	b	c
e	e	a	b	c
a	a	$\{e, a\}$	$\{b, c\}$	$\{b, c\}$
b	b	$\{b, c\}$	$\{e, a\}$	$\{e, a\}$
c	c	$\{b, c\}$	$\{e, a\}$	$\{e, a\}$

$\times_{P_2}$	e	a	b	c
e	e	a	b	c
a	a	$\{e, b\}$	$\{a, c\}$	$\{e, b\}$
b	b	$\{a, c\}$	$\{e, b\}$	$\{a, c\}$
c	c	$\{e, b\}$	$\{a, c\}$	$\{e, b\}$

$\times_{P_3}$	e	a	b	c
e	e	a	b	c
a	a	$\{e, c\}$	$\{e, c\}$	$\{a, b\}$
b	b	$\{e, c\}$	$\{e, c\}$	$\{a, b\}$
c	c	$\{a, b\}$	$\{a, b\}$	$\{e, c\}$

EXAMPLE 3. Consider the group $(\mathbb{Z}_7 - \{0\}, \cdot)$, and let $P = \{2, 3\}$. Then $(\mathbb{Z}_7, \times_P)$ is a hypergroup, where the multiplication table for $\times_P$ is:

$\times_P$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	$\{1, 5\}$	$\{4, 5\}$	$\{2, 3\}$	$\{2, 6\}$	$\{1, 3\}$
3	3	$\{4, 5\}$	$\{4, 6\}$	$\{1, 3\}$	$\{2, 3\}$	$\{1, 5\}$
4	4	$\{2, 3\}$	$\{1, 3\}$	$\{4, 6\}$	$\{4, 5\}$	$\{2, 6\}$
5	5	$\{2, 6\}$	$\{2, 3\}$	$\{4, 5\}$	$\{1, 5\}$	$\{4, 6\}$
6	6	$\{1, 3\}$	$\{1, 5\}$	$\{2, 6\}$	$\{4, 6\}$	$\{2, 3\}$

Definition 3.5. A triple $(R, +, \times)$ is called a *multiplicative hyperring* if

(1) $(R, +)$ is an abelian group;

(2) $(R, \times)$ is a semihypergroup;

(3) for all $a, b, c \in R$, we have $a \times (b + c) \subseteq a \times b + a \times c$ and $(b + c) \times a \subseteq b \times a + c \times a$;

(4) for all $a, b \in R$, we have $a \times (-b) = (-a) \times b = -(a \times b)$.

If in (3) we have equalities instead of inclusions, then we say that the multiplicative hyperring is *strongly distributive*.

Definition 3.6. Let $(F, +, \times)$ be a multiplicative hyperring such that

(1) $(F - \{0\}, \times)$ is a hypergroup,

(2) $\times$ is strongly distributive with respect to $+$.

Then $(F, +, \times)$ is called a *multiplicative hyperfield.*

Proposition 3.7. *Let $(F, +, \cdot)$ be a field and P be a non-empty subset of $F - \{0\}$. Consider the P-hyperoperation defined in Theorem 3.3. Then $(F, +, \times)$ is a multiplicative hyperfield.*

Proposition 3.8. *Let $(F, +, \cdot)$ be a field and P be a non-empty subset of $(F - \{0\}, \cdot)$. Consider the hyperoperation defined in Proposition 3.4. Then $(F, +, \times)$ is a multiplicative hyperfield.*

EXAMPLE 4. Consider the finite field $(\mathbb{Z}_7, +, \cdot)$, the field of integers modulo 7, and let $H = \{1, 6\}$. Then $(\mathbb{Z}_7, +, \times)$ is a multiplicative hyperfield, where the multiplication table for $\times$ is:

$\times$	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	$\{3,4\}$	$\{1,6\}$	$\{1,6\}$	$\{3,4\}$	$\{2,5\}$
3	0	3	$\{1,6\}$	$\{2,5\}$	$\{2,5\}$	$\{1,6\}$	$\{3,4\}$
4	0	4	$\{1,6\}$	$\{2,5\}$	$\{2,5\}$	$\{1,6\}$	$\{3,4\}$
5	0	5	$\{3,4\}$	$\{1,6\}$	$\{1,6\}$	$\{3,4\}$	$\{2,5\}$
6	0	6	$\{2,5\}$	$\{3,4\}$	$\{3,4\}$	$\{2,5\}$	$\{1,6\}$

EXAMPLE 5. Consider the infinite fields $(\mathbb{Q}, +, \cdot)$, $(\mathbb{R}, +, \cdot)$ and $(\mathbb{C}, +, \cdot)$. Clearly $I = \{1, -1\}$ is a subgroup of $(\mathbb{Q} - \{0\}, \cdot)$, $(\mathbb{R} - \{0\}, \cdot)$ and $(\mathbb{C} - \{0\}, \cdot)$. Therefore, according to Proposition 3.8, $(\mathbb{Q}, +, \times)$, $(\mathbb{R}, +, \times)$ and $(\mathbb{C}, +, \times)$ are multiplicative hyperfields. For example, we have:

$$3 \times 5 = 3 \cdot I \cdot 5 = \{15, -15\},$$
$$1/2 \times 3/5 = 1/2 \cdot I \cdot 3/5 = \{3/10, -3/10\}.$$

4 HYPERUNITS

Let $(F, +, \cdot)$ be a field and $H = \widehat{I} = \{1, -1\}$. According to Proposition 3.8, $(F, +, \times)$ is a multiplicative hyperfield. We define

$$\widehat{a} = a \times \widehat{I} = \{a, -a\},$$

and we set

$$\widehat{F} = \{\widehat{a} \mid a \in F\}.$$

We define a sum and a multiplication on $\widehat{F}$ as follows:

$$\widehat{a} \oplus \widehat{b} := \widehat{a+b},$$
$$\widehat{a} \otimes \widehat{b} := \widehat{a \cdot b},$$

for all $a, b \in F$.

Theorem 4.1. $(\widehat{F}, \oplus, \odot)$ *is a field and* $\widehat{I}$ *is the unit of* $(\widehat{F}$ *(we call* $\widehat{I}$ *a hyperunit).*

5 CONCLUDING REMARKS

In this note we have indicated, in the simplest possible mathematical formulation, the apparent existence in nature of a concrete realization of multi-valued hyperstructures with a basic hyperunit given by matter-antimatter systems. We have then studied said hyperstructures in their proper mathematical formulation and provided a number of examples. By keeping in mind the complexity of nature, we hope this study may be valuable for quantitative representations of complex systems, such as biological structures ion general and the DNA code in particular, that appear to require indeed the most advanced and general possible multi-valued hyperstructures.

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