Apparent Resolution of the Coulomb Barrier for Nuclear Fusions Via the Irreversible Lie-admissible Branch of Hadronic Mechanics

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In this paper, we report decades of mathematical, theoretical, experimental and industrial studies aiming at the resolution of the Coulomb barrier for nuclear fusions, here referred to the extremely big *repulsive* Coulomb force between natural nuclei that has prevented the achievement of controlled nuclear fusion to date. The studies have been done via the Lie-isotopic completion (for reversible processes) and Lie-admissible completion (for irreversible processes) of quantum mechanics into the various branches of hadronic mechanics. We first outline the prior representations via hadronic mechanics of: the synthesis of the neutron from the Hydrogen in the core of stars; the experimental data of the Deuteron in its true ground state (that with null orbital contributions); the stability of the neutron under strong nuclear forces; and the nuclear stability despite strongly repulsive protonic forces. Thanks to these preceding studies, we present apparently for the first time: 1) The prediction by hadronic mechanics of the existence of new, negatively charged, unstable nuclei, called *pseudo-nuclei* and denoted with the symbol $\tilde{N}$ (patent pending), which are characterized by a strongly attractive hadronic bond of electrons and natural nuclei, by therefore resolving the Coulomb barrier since pseudo-nuclei would be *attracted* (rather than repelled) by natural nuclei, with ensuing new conception of nuclear fusions here called *hyperfusions;* 2) The identification of engineering means for the synthesis of pseudo-nuclei which is given by the hadronic reactors for the synthesis of the neutron from the proton and the electron; 3) Laboratory evidence according to which the synthesis of pseudo-nuclei and related hyperfusions appear to be the origin of the limited, yet sustained and controlled excess energy achieved by the *Intermediate Controlled Nuclear Fusions.*

1 Introduction

As it is well known, nuclear fusions have indeed been achieved at various energies, but none of them has achieved to date the *sustainability and controllability* necessary for industrial usages, such as the production of electricity, due to a number of yet unresolved theoretical and engineering problems, such as:

Problem 1: Means to resolve the *repulsion* between natural, positively charged nuclei, called the *Coulomb barrier,* which reaches very big repulsive values of the macroscopic order of Newtons at the mutual distances of about 1 fm necessary to activate attractive strong nuclear forces,

$$\begin{aligned} F &= Z\,\frac{e^2}{r^2} = \\ &= Z\,(8.99\times 10^9)\,\frac{(1.60\times x10^{-19})^2}{(10^{-15})^2} = Z\times 230\,\mathrm{N}\,, \end{aligned} \tag{1}$$

where Z represents the number of proton-proton pairs.

Problem 2: Means to control the anti-parallel coupling of nuclear spins, in which absence there would be a violation of the angular momentum conservation law with nuclear fusions solely possible at random.

Problem 3: Means to achieve "clean" nuclear fusions, ideally referring to those without the emission of harmful radiations and without the release of radioactive waste.

In this paper, we study, apparently for the first time, the possibility of synthesizing new, negatively charged, unstable nuclei, hereon called *pseudo-nuclei,* which are characterized by a strongly attractive bond between negatively charged electrons and positively charged natural nuclei.

In the event the synthesized nuclei have a sufficient mean life, pseudo-nuclei would bypass the Coulomb barrier (Problem 1) because they would be *attracted* (rather than repelled) by natural, positively charged nuclei all the way to mutual distances 10^{-13} cm = 1 fm needed to activate strong nuclear interactions.

Pseudo-nuclei also offer realistic possibilities for a resolution of Problem 2, because, in view of their opposite charges and magnetic moments, pseudo-nuclei would couple automatically with natural nuclei in anti-parallel spin alignment.

Engineering tests are expected to initiate with the synthesis of *light* pseudo-nuclei, whose fusion with natural nuclei would be the best arena for the possible resolution of Problem 3.

We shall hereon identify generic nuclei N with the familiar expression $N(Z, A, J, u)$ where Z represents the total number of protons, A represents the total number of protons and neutrons, J represents the nuclear spin, and u represents the mass in Atomic Mass Units, also denoted amu. We shall also use tabulated symbols for individual nuclei, such as: H for Hydrogen, D for Deuteron, C for Carbon, *etc*. Measurements of nuclear data used in this paper are available from [1–4].

Our feasibility study shall initiate with the synthesis of the smallest possible pseudo-nucleus, here called the *pseudo-Deuteron-2e* and denoted with the symbol $\tilde{D}_{2e}$ (Figs. 1, 2, 3), according to the reaction

$$(\beta^-_\uparrow, \beta^-_\downarrow) + D(1,2,1) \rightarrow \tilde{D}_{2e}(1,2,1)\,, \tag{2}$$

where: the subindex $2e$ represents the total number of bonded electrons; the total charge of the pseudo-nucleus is *negative,* $Z^{tot} = -1$ (since it is the result of one positive + and two negative − elementary charges); and the electron pair with antiparallel coupling is an expected image at short distances of Pauli's exclusion principle in molecular structures.

The existence of synthesis (2) would evidently imply the existence of the *pseudo-Deuteron-3e* with structure

$$(\beta^-_\uparrow, \beta^-_\downarrow) + \beta^-_\downarrow + D(1,2,1_\uparrow) \rightarrow \tilde{D}_{3e}(1,2,1/2_\uparrow)\,, \tag{3}$$

with intriguing characteristics due to the very big magnetic moment of the third electron for nuclear standards, and ensuing possibility of resolving Problem 2.

Following a quantitative representation of pseudo-Deuterons, in this paper we shall study the possible fusion of pseudo-nuclei and natural nuclei, called *hyperfusion* and here referred to nuclear fusions without the Coulomb barrier and with natural antiparallel spin alignments, including the following possible hyperfusion

$$\begin{aligned}&\tilde{D}_{2e}(1,2,1) + C(6,12,0) \rightarrow \rightarrow N(7,14,1) + \beta^-_\uparrow + \beta^-_\downarrow\\ &\Delta_N = 10.272\,\mathrm{MeV}\,,\end{aligned} \tag{4}$$

and others that apparently occurred in recently measured excess heat in nuclear fusions to be reviewed in Sect. 4.

In order to conduct the indicated feasibility study, in this paper we shall adopt:

1) The 1935 historical argument by A. Einstein, B. Podolsky and N. Rosen that *Quantum mechanics is not a complete theory* [5];

2) The historical verifications of the EPR argument by W. Heisenberg [6], L. de Broglie [7] and D. Bohm [8], as well as the recent verifications by R. M. Santilli [9–11];

3) The experiments establishing deviations of quantum mechanical predictions from physical reality in various fields, including: nuclear physics [12]; electrodynamics [13–15]; condensed matter physics [16]; heavy ion physics [17]; time dilation for composite particles [18, 19]; Bose-Einstein correlation [20, 21]; cosmology [22, 23]; and various epistemological arguments [24–26].

4) The *approximate validity* of quantum mechanics in nuclear physics due to its inability over one century of achieving [27–29]: a quantitative representation of the fundamental synthesis of the neutron from a proton and an electron in the core of stars; an exact representation of nuclear magnetic moments; an exact representation of the spin of nuclei in their true ground state (that without the usual orbital excitations); a quantitative representation of the stability of neutrons when members of a nuclear structure; a quantitative representation of the stability of nuclei despite the huge Coulomb repulsion between nuclear protons; and other insufficiencies.

5) The completion of quantum mechanics into hadronic mechanics [30–32] (see [33] for an outline and [27–29] for a review); and the studies conducted during the *2020 International Teleconference on the EPR Argument* [34] (see its overviews [36, 37] and monographs [38]–[47] for independent studies).

By using a language specifically intended for nuclear physicists, in Sect. 2 we review the branches of hadronic mechanics used in our study [48]–[76]; in Sect. 3, we show that hadronic mechanics allows a quantitative representation of the synthesis of the pseudo-nuclides; in Sect. 4, we show that the synthesis of pseudo-Deuterons appears to be verified by the sustainable and controllable excess energy produced by the *Intermediate Controlled Nuclear Fusions* (ICNF) [77]–[146]; and in Sect. 5 we summarize the results.

For the self-sufficiency of this presentation, in the appendices we outline preceding studies playing a crucial role for the consistent derivation and application of pseudo-nuclei. In Appendix A, we study the possible resolution of Problem 3 (nuclear fusions without harmful radiations); in Appendix B, we outline the representation via hadronic mechanics of the synthesis of the neutron from a proton and an electron (which is fundamental for the synthesis of pseudo-nuclei); and in Appendix C we present, apparently for the first time, the representation of nuclear stability permitted by hadronic mechanics despite the natural instability of the neutron and despite the strongly repulsive protonic forces in which absence no resolution of the Coulomb barrier for nuclear fusions appears to be plausible.

2 Selection of the basic methods

2.1 Basic notions of hadronic mechanics

Recall that, according to Einstein, Podolsky and Rosen [5], the primary limitation of quantum mechanics in nuclear physics is its *locality,* namely, the representation of protons and neutrons as *massive points*. Therefore, the foundations of hadronic mechanics were built in the late 1970's by R. M. Santilli at Harvard University under DOE support [48, 49] for the primary purpose of representing the actual dimension, shape and density of protons and neutrons in a form invariant over time.

Recall also that quantum mechanical point-particles can solely admit linear, local and potential interactions, hereon called *Hamiltonian interactions* (and technically identified as *variationally self-adjoint (SA) interactions* [48]).

By contrast, clear nuclear data establish that nuclear volumes are generally *smaller* than the sum of the volumes of the constituent protons and neutrons. Consequently, when they are members of a nuclear structure, protons and neutrons are generally in conditions of partial mutual penetration of their dense charge distributions, resulting in additional interactions that are: *non-linear* in the wave function, as first studied by W. Heisenberg [6]; *non-local* in the sense of occurring over volumes, as first studied by L. de Broglie [7]; and of contact, thus zero-range type, *not derivable from a potential,* as first studied by R. M. Santilli [49]. The latter interactions are hereon called *non-Hamiltonian interactions* (and are technically identified as *variationally non-self-adjoint (NSA) interactions* [48]).

2.2 Lie-isotopic branch of hadronic mechanics

In Sect. 3, we shall study the representation of *stable,* thus time reversible nuclei. Their possible bonds with electrons are also reversible over time since the decay of pseudo-nuclei reproduce the original, permanently stable constituents.

In this section, we outline the branch of hadronic mechanics suggested for the consistent representation of time reversible systems, which is known as the *isotopic branch* and comprises the novel *iso-mathematics* [30] (see also [43, 47]) and *iso-mechanics* [31] (see also [38] and [45]), where the prefix "iso" is intended in the Greek meaning of denoting *preservation of the original axioms.*

By recalling that quantum mechanics is based on Lie algebras, the above methods are also known as *Lie-isotopic formulations* to indicate that they are based on the isotopies of Lie algebras not treated here for brevity [49] (see also [39]).

Recall that quantum mechanics is characterized by a universal enveloping algebra of Hermitean operators A, B, with conventional associative product $A \times B = AB$ on a Hilbert space $\mathcal{H}$ with states $|\psi\rangle$ and normalization $\langle\psi|\psi\rangle = 1$ over the field of complex numbers C, Schrödinger equation $H(r, p)|\psi\rangle = E|\psi\rangle$, canonical commutation rules and the familiar quantum mechanical methods used in nuclear physics over the past century.

Santilli achieved the first known representation of non-Hamiltonian/NSA interactions in a time-reversible way via a *new operator* called the *isotopic element* and indicated with the symbol $\hat{T}$, which is sandwiched in between all possible quantum products AB, resulting in the new, associativity-preserving product called *iso-product* (Eq. (5), p. 71 of [49])

$$A \star B = A\hat{T}B, \quad \hat{T} = \hat{T}(\hat{\psi}, ...) > 0\,, \tag{5}$$

with ensuing generalized multiplicative unit, called *iso-unit* and related identity axion

$$\begin{aligned} &\hat{I}(\hat{\psi}, ...) = 1/\hat{T}(\hat{\psi}, ...) > 0\,, \\ &I \star A = A \star I = A\,, \end{aligned} \tag{6}$$

where the dependence on $\hat{\psi}$ represents non-linearity in the appropriate iso-space of iso-mechanics.

For the case of the Deuteron as a two-body bound state according to hadronic mechanics, the isotopic element has a realization of the type [29, 36]

$$\begin{aligned} &\hat{T}(\hat{\psi}, ...) = 1/\hat{I}(\hat{\psi}, ...) = \\ &\quad = \prod_{\alpha=1,2} \text{Diag}\left(\frac{1}{n^2_{1,\alpha}}, \frac{1}{n^2_{2,\alpha}}, \frac{1}{n^2_{3,\alpha}}, \frac{1}{n^2_{4,\alpha}}\right) e^{-\Gamma(\hat{\psi},...)}\,, \\ &\quad n_{\mu,\alpha} > 0, \quad \Gamma > 0, \;\; \mu = 1, 2, 3, 4, \;\; \alpha = 1, 2\,, \end{aligned} \tag{7}$$

by therefore characterizing:

1) The dimension and shape of the proton and neutron via semi-axes $n^2_{k,\alpha}$, $k = 1, 2, 3$ (with n_3 parallel to the spin);

2) The density $n^2_{4,\alpha}$ of the proton and of the neutron with normalizations for the vacuum to the value $n^2_{\mu,\alpha} = 1$.

3) Non-Hamiltonian/NSA interactions between the proton and the neutron caused by the mutual penetration of their dense charge distribution, which interactions are represented via the exponential term $e^{\Gamma(\hat{\psi},...)} > 0$, where Γ is positive-definite but possesses otherwise an unrestricted functional dependence on all needed local variables.

Despite their simplicity, isotopies (5)-(6) requested the step-by-step, completion of all aspects of quantum mechanics into iso-mechanics, as illustrated by the basic *Schrödinger-Santilli iso-equation* (Ch. 5, p. 182 on, [31])

$$H \star |\hat{\psi}\rangle = H(r, p)\,\hat{T}(\hat{\psi}, ...)\,|\hat{\psi}\rangle = E\,|\hat{\psi}\rangle\,, \tag{8}$$

as well as the *Heisenberg-Santilli iso-equation* for an observable A

$$\begin{aligned} i\,\frac{dA}{dt} &= [A, H]^\star = \\ &= A \star H - H \star S = A\hat{T}H - H\hat{T}A\,, \end{aligned} \tag{9}$$

whose time-reversibility is assured by the conservation of the total energy,

$$i\,\frac{dH}{dt} = [H, H]^\star \equiv 0\,, \tag{10}$$

as well as the invariance of (9) under anti-Hermiticity,

$$[A, H]^\star \equiv -[A, H]^{\star\dagger}\,. \tag{11}$$

As clearly illustrated by iso-equation (8)-(9), the representation of stable nuclei via iso-mechanics requires *two operators,* the conventional Hamiltonian H for the representation of Hamiltonian/SA interactions and the isotopic element $\hat{T}$ for the representation of the dimension, shape, density and non-Hamiltonian/NSA interactions of protons and neutrons in a nuclear structure.

To reach a preliminary understanding of the subsequent sections, interested readers should be aware that, despite their simplicity, Eqs. (5)-(6) require a step-by-step completion of *all* aspects of 20^{th} century applied mathematics into the novel iso-mathematics, with no exception known to the author, including the new: *iso-numbers* [50] (see also [42])

$$\hat{n} = n\hat{I}\,; \tag{12}$$

iso-functions [49] (see also [38]) $\hat{f}(\hat{r}) = [f(r\hat{I})]\hat{I}$; and iso-differential calculus [51] (see also [46])

$$\hat{d}\hat{r} = \hat{T}d(r\hat{I}) = dr + r\hat{T}d\hat{I},$$
$$\frac{\hat{\partial}\hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \hat{I}\,\frac{\partial\hat{f}(\hat{r})}{\partial\hat{r}}\,, \tag{13}$$

that allowed the completion of the iso-Schrödinger and iso-Heisenberg representations

$$\hat{p}\star|\hat{\psi}(\hat{r})\rangle = -\hat{i}\star\hat{\partial}_{\hat{r}}|\hat{\psi}(\hat{r})\rangle = -i\,\hat{I}\partial_{\hat{r}}|\hat{\psi}(\hat{r})\rangle\,,$$
$$\left[\hat{r}_i,\hat{p}_j\right]^\star\star|\hat{\psi}(\hat{r})\rangle = -i\,\hat{\delta}_{ij}|\hat{\psi}(\hat{r})\rangle = -i\,\hat{I}\delta_{ij}|\hat{\psi}(\hat{r})\rangle\,, \tag{14}$$
$$\left[\hat{r}_i,\hat{r}_j\right]^\star\star|\hat{\psi}(\hat{r})\rangle = [\hat{p}_i,\hat{p}_j]^\star\star|\hat{\psi}(\hat{r})\rangle = 0\,,$$

as well as the completion of the Heisenberg uncertainties for point particles under electromagnetic interactions into the *Heisenberg-Santilli iso-uncertainties* for extended hadrons under strong interactions [9–12]

$$\Delta r\,\Delta p = \frac{1}{2}|\langle\hat{\psi}|\star[\hat{r},\hat{p}]^*\star|\hat{\psi}\rangle| \approx \frac{1}{2}\hat{T} \ll 1\,. \tag{15}$$

It should be finally noted that all aspects of iso-mathematics and iso-mechanics can be constructed very simply via a systematic non-unitary transformation of *all* the corresponding 20th century formulations [52], e.g.,

$$\begin{aligned}
&UU^\dagger = \hat{I}(\hat{\psi},...) = 1/\hat{T} > 0\,,\\
&\hbar = 1 \;\rightarrow\; U\hbar U^\dagger = \hat{I}\,,\\
&r \;\rightarrow UrU^\dagger = \hat{r}\,,\\
&p \;\rightarrow\; UpU^\dagger = \hat{p}\,,\\
&U(AB)U^\dagger = \hat{A}\hat{T}\hat{B}\,,\\
&U(H|\psi\rangle U^\dagger = \hat{H}\star|\hat{\psi}\rangle =\\
&\quad = \left[\frac{1}{\hat{2}\star\hat{m}}\sum_{k=1,2,3}\hat{p}_k\star\hat{p}_k + \hat{V}(\hat{r})\right]\star|\hat{\psi}\rangle =\\
&\quad = \hat{E}\star|\hat{\psi}\rangle = E|\hat{\psi}\rangle\,.
\end{aligned} \tag{16}$$

The invariance over time of the numeric values of the isotopic element and of the iso-unit is finally assured by the reformulation of conventional non-unitary transformations (15) into the *iso-unitary iso-transformations* of hadronic mechanics [52]

$$\begin{aligned}
&WW^\dagger = \hat{I}, \quad W = \hat{W}\hat{T}^{1/2}\,,\\
&WW^\dagger = \hat{W}\star\hat{W}^\dagger = \hat{W}^\dagger\star\hat{W} = \hat{I}\,,
\end{aligned} \tag{17}$$

under which

$$\begin{aligned}
&\hat{I}\rightarrow\hat{I}' = \hat{W}\star\hat{I}\star\hat{W}^\dagger \equiv \hat{I}\,,\\
&\hat{A}\star\hat{B}\rightarrow\hat{W}\star(\hat{A}\star\hat{B})\star\hat{W}^\dagger =\\
&\qquad = \hat{A}'\star\hat{B}' = \hat{A}'\hat{T}\hat{B}'\,,\\
&\hat{A}' = \hat{W}\star A\star\hat{W}^\dagger),\ \hat{B}' = \hat{W}\star\hat{B}\star\hat{W}^\dagger\,,\\
&\hat{T} = (W^\dagger\star\hat{W})^{-1}\,.
\end{aligned} \tag{18}$$

The invariance of isotopic formulation then follows (see [29] for a technical review via *iso-symmetries,* namely, the isotopic completion of 20th century space-time symmetries).

2.3 Lie-admissible branch of hadronic mechanics

In Sect. 4, we shall study apparent nuclear fusions that are permitted by pseudo-Deuterons without Coulomb barrier and with a natural antiparallel alignment of nuclear spins. The primary difference between stable nuclei and nuclear fusions is that the former constitute time reversible systems, thus allowing their treatment via time reversible isotopic methods, while the latter are *irreversible over time* by therefore requiring for their consistent treatment the *irreversible branch of hadronic mechanics* known as *Lie-admissible or genotopic formulations* [53]–[70] (see [30–32] for a general treatment), where the prefix "geno" is intended this time in the Greek sense of *inducing new axioms.*

In the author's view, an important problem of nuclear fusions, that has remained essentially unaddressed for about one century, is that *the representation of nuclear fusions via quantum mechanics generally violates causality,* because the same Schrödinger equation applies for both, the fusion process as well as its time reversal image which requires the spontaneous disintegration of the synthesized nucleus, resulting in solutions that generally admit effects preceding their cause.

The primary objective of Santilli's research in the late 1970's at Harvard University under DOE support was the construction of the EPR completion of time reversible quantum mechanics into an irreversible form representing nuclear fusions without causality problems. The study was essentially along the Ph. D. thesis at the University of Torino, Italy, on the time irreversible, Lie-admissible generalization of quantum mechanics [53, 54, 56].

The need for new clean nuclear energies to contain the deterioration of our environment (that was already visible in the late 1970's), joint with the lack of controlled nuclear fusions, stimulated a considerable volume of research in the period 1977–1985 under DOE support, including papers [57]–[61] five *Workshops on Lie-admissible formulations* [61], the *First International Conference on Nonpotential Interactions and their Lie-Admissible Treatment* [62], the first *Workshops on Hadronic Mechanics* [63, 64], and various reprint volumes, such as [65]. The post-1985 references on Lie-admissible mathematics and mechanics are too numerous for comprehensive quotations. We here merely quote *The third international conference on the Lie-admissible treatment of non-potential interactions* [66] and special contributions [67]–[70].

As a main aspect in the representation of nuclear fusions via irreversible genotopic methods let us recall that quantum mechanics is a time reversible theory beginning with its axiomatic structure. In particular, the right modular action of the Hermitean Hamiltonian on a Hilbert state, $H|\psi\rangle = E|\psi\rangle$, is equivalent to the corresponding left modular action, $\langle\psi|H = -\langle\psi|E'$, $E' \equiv E$, and the same holds for isotopic methods in view of the Hermiticity of the isotopic element $\hat{T} = \hat{T}^\dagger$.

Following extensive studies, the foundations of irreversible formulations were achieved in the 1979 Harvard University paper [57] via the following inequivalent right and left modular actions of a Hamiltonian on a Hilbert state. The right modular action (indicated with the symbol >) is assumed to represent *motion forward in time,* while the left modular action (indicated with the symbol <) is assumed to represent *motion backward in time,* with forward ("for") and backward ("bac") geno-Schödinger equations

$$\begin{aligned} &\hat{H} \equiv \hat{H}^\dagger, \\ &\hat{H} > |\psi\rangle = \hat{H}\hat{R}|\psi\rangle = E^{for}|\psi\rangle, \\ &\langle\psi| < \hat{H} = \langle\psi|\hat{S}\hat{H} = \langle\psi|E^{bac}, \end{aligned} \tag{19}$$

which assure irreversibility whenever the *genotopic operators* $\hat{R}, \hat{S}$ are different

$$\hat{R} \neq \hat{S}, \quad E^{for} \neq E^{bac}, \tag{20}$$

isotopic formulations being a particular case for $\hat{R} = \hat{S} = \hat{T}$.

Note that *genotopic formulations maintain the observability of the total energy* [31], by therefore avoiding the use of complex-valued Hamiltonians to represent irreversibility with the consequential loss of observability.

According to the above assumptions, geno-mathematics (geno-mechanics) essentially consists of *two* inequivalent isomathematics (iso-mechanics), one with *all* products ordered to the right and the other ordered to the left.

By using (19), the *genotopic time evolution* (for the simple case $\hat{t} = t$) is given by (Eqs. (19), p. 153 of [49])

$$\begin{aligned} \hat{A}(t) &= e_>^{Hti} > \hat{A}(0) < e_< e^{-itH} = \\ &= e^{H\hat{S}ti}\hat{A}(0)\, e^{-it\hat{S}H}, \end{aligned} \tag{21}$$

with infinitesimal form

$$\begin{aligned} &i\,\frac{\hat{d}\hat{A}}{\hat{d}t} = (\hat{A}, \hat{H}) = \\ &\quad = \hat{A} < \hat{H} - \hat{H} > \hat{A} = \hat{A}\hat{S}\hat{H} - \hat{H}\hat{R}\hat{A} = \\ &\quad = (\hat{A}\hat{T}\hat{H} - \hat{H}\hat{T}\hat{A}) + (\hat{A}\hat{J}\hat{H} - \hat{H}\hat{J}\hat{A}), \\ &\hat{S} = \hat{T} + \hat{J}, \quad \hat{R} = -\hat{T} + \hat{J}. \end{aligned} \tag{22}$$

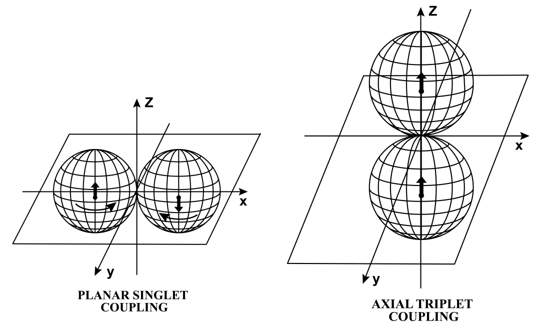

Fig. 1: In this figure, we illustrate the two stable bound states of particles with spin predicted by hadronic mechanics, which are given by the "planar singlet coupling" on the left and the "axiom triplet coupling" on the right.

The important methodological, as well as historical feature of genotopic formulations is that their brackets (A, H) are *jointly Lie-admissible and Jordan-admissible* according to the American mathematician A. A. Albert [71], in the sense that the antisymmetric brackets $[A, B]^\star$ verify the Lie algebra axioms, while the symmetric brackets $\{A, B\}^\star$ verify the Jordan axioms.

Intriguingly, the symmetric term of brackets (21) provides a representation of the *external terms* $\hat{F}^{NSA}$ of Lagrange's and Hamilton's equations as one can see for the particular case

$$\begin{aligned} &\hat{S} = 1, \quad \hat{R} = -1 + \frac{\hat{F}}{\hat{H}}, \\ &i\,\frac{\hat{d}\hat{A}}{\hat{d}t} = (\hat{A}, \hat{H}) = \hat{A}\hat{H} - \hat{H}\hat{A} + \hat{A}\hat{F}, \end{aligned} \tag{23}$$

by therefore realizing Jordan's wish that his symmetric algebra may, one day, see physical applications (for details, see Sect. 2 of [27]).

Recall that iso-mathematics and iso-mechanics can be constructed with the sole use of *one*, single, non-unitary transformation of the conventional applied mathematics and quantum mechanics, Eqs. (16). Similarly, geno-mathematics and geno-mechanics can be constructed, this time, via *two* different non-unitary transformations of conventional applied mathematics and quantum mechanics, and this includes the lifting of quantum mechanical nuclear models with sole potential interactions into their covering hadronic models with potential, as well as contact, non-potential interactions (see [52] for brevity).

3 Negatively charged pseudo-Deuterons

3.1 Basic assumptions

As indicated in Sect. 1, no study of pseudo-nuclei, with ensuing resolution of the Coulomb barrier for nuclear fusions, appears to be plausible without the prior resolution of a number of basic problems in nuclear physics, beginning with the resolution of the *locality* of quantum mechanics via the invariant representation of the dimension, shape and density of protons and neutrons outlined in Sect. 2.

In this section, we outline the second necessary requirement for the indicated task, the numerically exact and time invariant representation of the experimental data of the Deuteron in its true ground state (that with null orbital contributions) under the assumption that the neutron is an extended structureless neutral particle with spin 1/2.

The indicated task additionally requires the representation according to hadronic mechanics (outlined in Appendix B) of the synthesis of the neutron from a proton and an electron in the core of stars. In fact, the neutron synthesis is prohibited by quantum mechanics for numerous technical reasons, despite the huge proton-electron Coulomb *attraction,* with ensuing expectation that pseudo-nuclei are not possible because not allowed by quantum mechanics.

The latter view is quickly dispelled by the century old evidence that the neutron is indeed synthesized in the core of stars from a proton and an electron, by therefore confirming the Einstein-Podolsky-Rosen argument that *Quantum mechanics is not a complete theory* [5].

In turn, the representations of *all* characteristics of the neutron during its synthesis from the proton and the electron have allowed the resolution of the last nuclear problems needed for the study of pseudo-nuclei, which are given by the understanding of nuclear stability despite the neutron natural instability and despite the huge repulsive protonic forces. The latter resolutions are presented apparently for the first time in Appendix C.

3.2 Representation of the Deuteron experimental data

As it is well known, the only stable bound state between a proton and a neutron predicted by quantum mechanics (qm) is the singlet coupling

$$D = (p_\uparrow, n_\downarrow)_{qm}\,, \tag{24}$$

for which the total spin would be zero, $J_D = 0$, contrary to clear experimental evidence for which the spin of the Deuteron is $J_D = 1$.

For the intent of maintaining quantum mechanics as an exact discipline in nuclear physics, the spin of the Deuteron is generally associated to a collection of *orbital states* $L_D = 1$ (see e.g. [75]), which association is however contrary to the experimental evidence for which *the spin of the Deuteron has the value* $J_D = 1$ *in the true ground state,* namely, a state for which all excited orbital contributions are null.

Following the non-relativistic and relativistic representations of all characteristics of the neutron in its synthesis from the proton and the electron [84]–[103], the numerically exact and time invariant representation of all the experimental data of the Deuteron in its true ground state has been achieved by R. M. Santilli [29, 77, 78, 83] (see also [79–81]).

Under the assumption that the neutron is an extended structureless particle, the representation of the spin $J_D = 1$ was achieved via the notion of *hadronic spin* (first introduced in Sect. 6.8, p. 250 of [31] and [10]) which is given by iso-unitary, iso-irreducible iso-representations of the Lie-Santilli isoalgebra $\widehat{\mathrm{SU}}(2)$ whose iso-fundamental iso-representation can be constructed quite easily via the following *non-unitary* transformation of Pauli's matrices

$$UU^\dagger = \hat{I} = \mathrm{Diag}\,(\lambda^{-1}, \lambda)\,, \quad \hat{T} = \mathrm{Diag}\,(\lambda, \lambda^{-1})\,, \tag{25}$$

including an explicit and concrete realization of Bohm's *hidden variables* λ [8], first introduced in Eqs. (6.8.20), p. 254 of [31], and resulting in the *iso-Pauli matrices* generally called *Pauli-Santilli iso-matrices*

$$\begin{aligned} &\hat{\Sigma}_k = U\Sigma_k U^\dagger, \;\; \Sigma_k = \sigma_k \hat{I}\,, \\ &\hat{\sigma}_1 = \begin{pmatrix} 0 & \lambda \\ \lambda^{-1} & 0 \end{pmatrix}, \;\; \hat{\sigma}_2 = \begin{pmatrix} 0 & -i\lambda \\ i\lambda^{-1} & 0 \end{pmatrix}, \\ &\hat{\sigma}_3 = \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & -\lambda \end{pmatrix}, \end{aligned} \tag{26}$$

and then used in various works (see e.g. [10]).

As one can see, the iso-Pauli matrices verify the iso-commutation rules

$$\begin{aligned} [\hat{\sigma}_i, \hat{\sigma}_j]^* &= \hat{\sigma}_i \star \hat{\sigma}_j - \hat{\sigma}_j \star \hat{\sigma}_i = \\ &= \hat{\sigma}_i \hat{T} \hat{\sigma}_j - \hat{\sigma}_j \hat{T} \hat{\sigma}_i = i2\epsilon_{ijk}\hat{\sigma}_k\,, \end{aligned} \tag{27}$$

showing the clear iso-morphism $\widehat{\mathrm{SU}}(2) \approx \mathrm{SU}(2)$, as well as the iso-eigenvalue equations on an iso-state $|\hat{b}\rangle$ of the *Hilbert-Myung-Santilli iso-space* $\hat{\mathcal{H}}$ [76] over the iso-field of iso-complex iso-numbers $\hat{C}$ [50]

$$\begin{aligned} &\hat{S}_k = \frac{\hat{1}}{2} \star \hat{\sigma}_k = \frac{1}{2}\hat{\sigma}_k\,, \\ &\hat{\sigma}_3 \star |\,\hat{b}\,\rangle = \hat{\sigma}_3 \hat{T}\,|\,\hat{b}\,\rangle = \pm\,|\,\hat{b}\,\rangle\,, \\ &\hat{\sigma}^{\hat{2}} \star |\,\hat{b}\,\rangle = (\hat{\sigma}_1\hat{T}\hat{\sigma}_1 + \hat{\sigma}_2\hat{T}\hat{\sigma}_2 + \hat{\sigma}_3\hat{T}\hat{\sigma}_3)\hat{T}\,|\,\hat{b}\,\rangle = 3\,|\,\hat{b}\,\rangle. \end{aligned} \tag{28}$$

The addition of hadronic spins (Sect. 6.11, p. 265 of [31]) allowed the identification of two stable couplings of spin 1/2 *extended* particles called *planar singlet coupling* and *axial triplet coupling* which are illustrated in Fig. 1.

The configuration of the Deuteron allowing the representation of the spin $J_D = 1$ in its true ground state is evidently the axial triplet coupling, first identified in Fig. 13, p 91 of [36] (Fig. 2)

$$\tilde{D} = \begin{pmatrix} \hat{p}_\uparrow \\ \star \\ \hat{n}_\uparrow \end{pmatrix}. \tag{29}$$

Two complementary, numerically exact and time invariant representations of the Deuteron magnetic moment

$$\mu_D^{ex} = 0.85647\,\mu_{\mathrm{N}}\,, \tag{30}$$

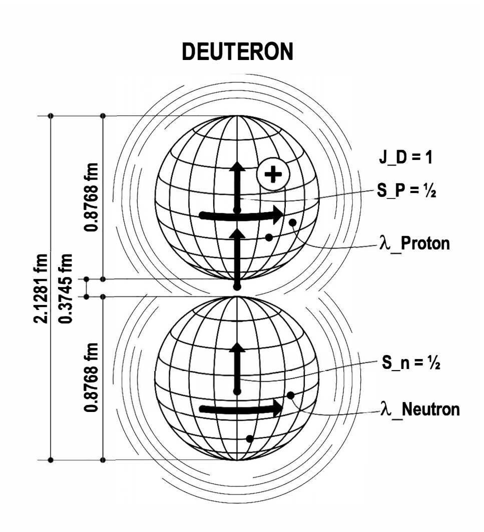

Fig. 2: In this figure, we reproduce known experimental data on the dimensions of the Deuteron [2] and its constituent proton and neutron [4], as well as their interpretation as a hadronic bound state in axial triplet coupling (Fig. 1), thus representing for the first time the spin of the Deuteron $S_D = 1$ in its ground state, that with null angular contributions $L_D = 0$ [83].

were achieved via hadronic mechanics. The first representation was reached in Eq. (3.6), p. 124 of the 1994 paper [77] (see also the 1998 monograph [78]) via the following numeric values of the characteristic quantities of isotopic element (26)

$$b_1 = \frac{1}{n_1} = b_2 = \frac{1}{n_2} = 1.0028\,, \qquad b_3 = \frac{1}{n_3} = 1.662, \;\; b_4 = \frac{1}{n_4} = 1.653\,. \tag{31}$$

The second representation of the magnetic moment of the Deuteron (35) was reached in the recent paper [83] via the realization of Bohm's hidden variable λ

$$\lambda = e^{\phi} \geq 0\,, \tag{32}$$

and the factorization (from Eq. (6.8.18), p. 254 of [31]),

$$\hat{\sigma}_3|\,\hat{b}\,\rangle = \sigma_3 \hat{I}|\,\hat{b}\,\rangle = \sigma_3 e^{\phi\sigma_3}|\,\hat{b}\,\rangle\,, \tag{33}$$

resulting in the relation

$$\mu_{hm}|\,\hat{b}\,\rangle = e^{\phi\sigma_3}\mu_{qm}|\,\hat{b}\,\rangle = e^{\phi\sigma_3} gS\,|\,\hat{b}\,\rangle\,, \tag{34}$$

from which the magnetic moment (35) is exactly represented via the following numeric value of Bohm's hidden variable λ [83]

$$\lambda = e^{\phi} = e^{0.97666} = 2.65557\,. \tag{35}$$

The invariance over time of the representations follows from the derivation of iso-Pauli matrices (31) from the isotopies of the Poincaré symmetry (see the general review [29] for brevity).

The representation of the rest energy and charge radius of the Deuteron (Fig. 2) were first achieved via the iso-Schrödinger equation of hadronic mechanics for a two-body, proton-neutron system (Fig. 12) [78] and then extended to a *restricted three-body system* comprising two protons and an electron (Fig. 13 and reviews [79–81]). The stability of the Deuteron despite the natural instability of the neutron is studied in Appendix C.

3.3 Predicted characteristics of the pseudo-Deuteron-2e

In this section, we study the possible bound state (2) of an electron pair and the Deuteron into a negatively charged unstable nucleus called *pseudo-Deuteron-2e* and denoted with the symbol $\tilde{D}_{2e}(1,2,1)$ (Sect. 1), conceptually proposed in Sect. 8.2.8, p. 96 of [36], and here studied at the non-relativistic level with the structure according to hadronic mechanics (hm)

$$[(\beta^-_{\uparrow}, \beta^-_{\downarrow})_{hm} + D(1,2,1)]_{hm} = \tilde{D}_{2e}(1,2,1)\,, \tag{36}$$

where:

3.1.1) The bond between the electron pair and the Deuteron is primarily due to their very big *attractive* Coulomb force of 460 N at the mutual distance of 10^{-13} cm, Eq. (1), as well as non-Hamiltonian/NSA interactions caused by the motion of the electron pair within the wave packet of the Deuteron here presented as an example of the *Einstein-Podolsky-Rosen (EPR) entanglement* [12].

3.1.2) The electron pair in synthesis (36) is the valence electron bond represented via hadronic chemistry under the name of *isoelectronium* (see Chapter 4 on, [72] and applications [73, 74]) which is the sole valence electron pair with an *attractive force* known to this author despite their equal charges.

3.1.3) The electron pair and the Deuteron are assumed, for simplicity, to constitute single bodies in structure equations.

3.1.4) Synthesis (36) is assumed in first approximation to be reversible over time with spontaneous decay

$$\tilde{D}_{2e}(1,2,1) \;\rightarrow\; D(1,2,1) + \beta^-_{\uparrow} + \beta^-_{\downarrow}\,. \tag{37}$$

3.1.5) Synthesis (36) is studied via the iso-mathematics and iso-mechanics of hadronic mechanics outlined in Sect. 2.2 under the sole assumption of the following non-relativistic form of isotopic element (7) [29, 36]

$$\begin{aligned} &\hat{T}(\hat{\psi}, ...) = 1/\hat{I}(\hat{\psi}, ...) = \\ &= \prod_{\alpha=1,2} \mathrm{Diag}\left(\frac{1}{n^2_{1,\alpha}}, \frac{1}{n^2_{2,\alpha}}, \frac{1}{n^2_{3,\alpha}}\right) e^{-\Gamma(\hat{\psi},...)}, \\ &n_{\mu,\alpha} > 0, \quad \Gamma > 0, \;\; \mu = 1,2,3 \;\; \alpha = 1,2\,. \end{aligned} \tag{38}$$

3.1.6) We assume that both the electron pair and the Deuteron are spherical with characteristic quantities $n_\mu = 1$, $\mu = 1, 2, 3$, by therefore reducing isotopic element (36) to its exponential term

$$\hat{T}(\hat{\psi}, ...) = 1/\hat{I}(\hat{\psi}, ...) = e^{-\Gamma(\hat{\psi},...)}. \tag{39}$$

3.1.7) To avoid insidious instabilities, the orbit of the electron pair around the Deuteron is assumed to be in a plane and a perfect circle on iso-spaces over iso- fields.

Following the study of synthesis (36) with two electrons, we shall study the synthesis with a bigger number of electrons, such as the *pseudo-Deuteron-3e* (Fig. 4).

A generic hyperfusion between a pseudo-nucleus $\tilde{N}_{ke}(Z_1, A_1, J_1)$ with k bonded electrons and a natural nucleus $N(Z_2, A_2, J_2)$ will be denoted

$$\begin{aligned} &\tilde{N}_{ke}(Z_1, A_1, J_1) + N(Z_2, A_2, J_2) \to \\ &\to N(Z_1 + Z_2, A_1 + A_2, J_1 + J_2) + k\beta^- . \end{aligned} \tag{40}$$

With reference to Fig. 3, we consider now the quantum mechanical Schrödinger equation for the bond of an electron pair with rest energy $M_{2e} = 1.022$ MeV to the Deuteron with rest energy $M_D = 1875.6129$ MeV

$$\left[-\frac{1}{2m} \sum_{k=1,2,3} p_k p_k + V_c(r) \right] |\, \psi(r)\, \rangle = E\, |\, \psi(r)\, \rangle\,, \tag{41}$$

where m is the reduced mass

$$m = \frac{M_D \times 2m_e}{M_d + 2M_e} \approx M_{2e} = 1.022 \text{ MeV}\,, \tag{42}$$

and the attraction is that of the Coulomb force between the electron pair and the proton

$$V_c = \frac{(+e) \times (-2e)}{r} = -2\, \frac{e^2}{r}\,. \tag{43}$$

We now assume that the considered bond is characterized by a second interaction due to the overlapping of the wave packets of the electrons with that of the Deuteron (illustrated with the dashed area of Fig. 2), resulting in a deep EPR entanglement (Sect. 3 of [15]) with ensuing contact, non-Hamiltonian/NSA interactions represented by isotopic element (27).

In order to achieve an interaction in the iso-Schrödinger equation which is *additive* to the Coulomb interaction, we select the following simplified form of the isotopic element that has produced various numerically exact representations of experimental data [29]

$$\hat{T} = 1/\hat{I} = e^{+V_h(\hat{r})/V_c(\hat{r})}\,, \tag{44}$$

where $V_h(\hat{r})$ is the *Hulten potential* in the hadronic system of iso-coordinates v$\hat{r} = r\hat{I}$

$$V_h(\hat{r}) = -K_h\, \frac{e^{b\hat{r}}}{1 - e^{b\hat{r}}}\,, \tag{45}$$

b represents the charge radius of the pseudo-Deuteron here assumed to be of the order of 2 fm,

$$R_{\tilde{D}} = b \approx 2\,\text{fm} = 2 \times 10^{-13}\,\text{cm}, \tag{46}$$

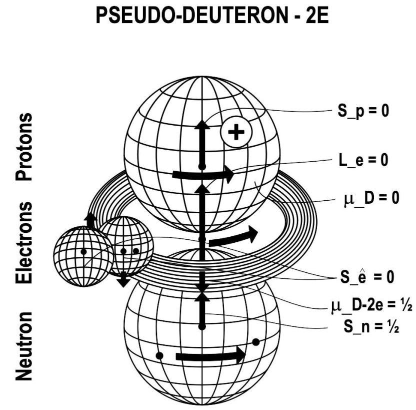

Fig. 3: In this figure, we illustrate the structure of the pseudo-Deuteron-2e predicted by hadronic mechanics as a bound state of an electron pair and a Deuteron (Sect. 3).

and K_h is the Hulten constant.

Under the above assumptions, the iso-Schrödinger equation in the structure of the pseudo-Deuteron is uniquely characterized by the following *non-unitary* transformation of the quantum mechanical description

$$\begin{aligned} &UU^\dagger = \hat{I} = 1/\hat{T} = e^{[-V_h(\hat{r})]/[-V_c(\hat{r})]} \approx 1 + \tfrac{\hat{V}_h(\hat{r})}{V_c(\hat{r})} + ... \\ &(UU^\dagger)^{-1} = \hat{T} = e^{V_h(\hat{r})/V_c(\hat{r})} \approx 1 - \tfrac{V_h(\hat{r})}{V_c(\hat{r})} + ... \end{aligned} \tag{47}$$

when applied to (41) (first studied in Sect. 5.1, p. 827 on, of the 1978 Harvard University memoir [82] and upgraded in Sect. 2.7.2 of [29]) with final result

$$\left[-\frac{1}{2m} \hat{\Delta}_{\hat{r}} - V_c(\hat{r}) - V_h(\hat{r}) \right] |\, \hat{\psi}(\hat{r})\, \rangle = E_h\, |\, \hat{\psi}(\hat{r})\, \rangle\,. \tag{48}$$

Recall that the Hulten potential behaves like the Coulomb potential at short distances (see Eq. (5.1.15), p. 885 of [82]),

$$V_h(\hat{r}) \approx \frac{k_h}{b\hat{r}}\,. \tag{49}$$

Consequently, the strongly attractive Hulten potential *absorbs* the attractive Coulomb potential with a mere redefinition K'_h, of the constant K_h, resulting in the iso-Schrödinger equation

$$\left[-\frac{1}{2m_e} \hat{\Delta}_{\hat{r}} - K'_h\, \frac{e^{b\hat{r}}}{1 - e^{b\hat{r}}} \right] |\, \hat{\psi}(\hat{r})\, \rangle = E_{be}\, |\, \hat{\psi}(\hat{r})\, \rangle, \tag{50}$$

where E_{be} is the binding energy of the Hulten potential and $\bar{m}_e$ is the iso-renormalized mass of the electron, that is, the renormalization of the mass caused by non-Hamiltonian interactions.

For our initial feasibility study, we assume that the pseudo-Deuteron has the mass $m_{\tilde{D}} \approx 2m_e + m_D = 1.876\,\text{MeV} - E_{be}$, the mean life of $\tau_{\tilde{D}} \approx 1$ s and the charge radius $R_{\tilde{D}} = b^{-1} = 2 \times 10^{-13}$ cm.

By following the structure model of pions as hadronic bound states of electrons and positrons of Eqs. (5.1.14), p. 836, [82], we reach the following *non-relativistic structure equations of the pseudo-Deuteron-2e*

$$\begin{aligned} &\left[\frac{1}{r^2}\left(\frac{d}{dr}r^2\frac{d}{dr}\right) + \tilde{2}\bar{m}_e\left(E_{be} + K'_h\frac{e^{-br}}{1-e^{-br}}\right)\right] = 0\,, \\ &m_{\tilde{D}} = 2m_e + m_D - E_{be}\,, \\ &\tau_{\tilde{D}}^{-1} = 2\pi\lambda^2\,|\hat{\psi}(0)|^2\,\frac{\alpha^2 E_1}{\hbar} = 1\,\text{s}\,, \\ &R_{\tilde{D}} = b^{-1} = 2\times 10^{-13}\,\text{cm}\,. \end{aligned} \tag{51}$$

The solution of the above equations was reduced (see Eqs. (5.1.32a) and (5.1.32b), p. 840 of [82]) to the numeric values of two parameters denoted k_1 and k_2 that, in our case, become

$$k_1[1-(k_2-1)2] = \frac{1}{2\hbar c}\left(m_{\tilde{D}}b^{-1}\right) = 2.5\times 10^{-2} m_{\tilde{D}}\,, \tag{52}$$

$$\frac{(k_2-1)^3}{k_1} = 2.9\times 10^{-6}\left(\tau_{\tilde{D}}^{-1}b^{-1}\right) = 1.45\times 10^{-19}\,, \tag{53}$$

whose numeric solutions are given by

$$k_2 \approx 1\,, \quad k_1 \approx 1.45\,. \tag{54}$$

As it is well known, the binding energy is represented by the familiar *finite* spectrum of the Hulten potential (Eq. (5.1.20), p. 837, [82]) that in our case has the null value

$$\begin{aligned} &E_{be} = -\frac{1}{4K_h k_2}\left(\frac{k_2}{N} - N\right)^2 = 0\,, \\ &k_2 = K_h\,\frac{m_{\tilde{D}}}{\hbar^2 b^2} = 1\,, \end{aligned} \tag{55}$$

suggesting the existence of *one and only one energy value* that with $N = 1$ and $E_{be} = 0$ as expected because contact interactions have no potential.

In conclusion, the use of non-Hamiltonian/NSA interactions yields structure model (54) of the pseudo-Deuteron-2e predicting the following rest energy

$$m_{\tilde{D}} \approx 2m_e + m_D = 1.876\,\text{MeV}, \tag{56}$$

with the evident understanding that the above value needs a correction via hadronic mechanics of the Coulomb binding energy which is currently under study.

3.4 Spin of the pseudo-Deuteron-2e

Evidently, the total spin of the electron pair in structure (36) is identically null, while the Deuteron-2e is represented in its ground state, thus implying that the orbital angular momentum of the electron pair has the value $L_{2e} = 0$. Consequently, the total angular momentum of the electron pair is null and *the spin of the pseudo-Deuteron-2e coincides with that of the conventional Deuteron.*

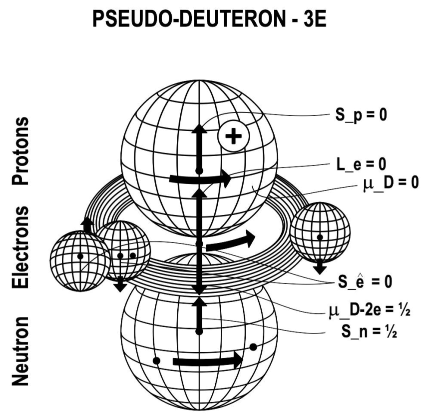

Fig. 4: In this figure, we illustrate the structure of the pseudo-Deuteron-3e predicted by hadronic mechanics as a bound state of an electron and a pseudo-Deuteron-2e (Sect. 3).

3.5 Magnetic moment of the pseudo-Deuteron-2e

Evidently, the magnetic moment of the electron pair in structure (36) is identically null. However, the *rotation* of the two elementary charges in the ground state creates a rather big magnetic moment (per nuclear standards) in the direction opposite that of the Deuteron magnetic moment.

In fact, the magnetic moment of the electron is given by

$$\begin{aligned} \mu_e^{spin} &= -9.284764\times 10^{-24}\,\text{J/T} = \\ &= 1838.2851\,\mu_{\text{N}}, \end{aligned} \tag{57}$$

(where J/T stands for Joules per Tesla and μ_{N} is the nuclear magnetron) thus being 2, 162-times *bigger* than the magnetic moment of the Deuteron.

Direct calculations of the magnetic moment of elementary charges rotating within a dense hadronic medium are unknown at this writing. To have an order of magnitude of the magnetic moment of the pseudo-Deuteron-2e, we use the orbital magnetic moment of the electron in the synthesis of the neutrons from the Hydrogen in the core of star done in [84]–[97] (see also reviews [98]–[103]) which, in order to counter magnetic moment (57) to reach the neutron magnetic moment of $-1.9130\,\mu_{\text{N}}$, is given from (84) by $\mu_e = 1833.5801\,\mu_{\text{N}}$, resulting in the tentative prediction of the *magnetic moment of the pseudo-Deuteron-2e*

$$\mu_{\tilde{D}-2e} = -3.666\,\mu_{\text{N}}. \tag{58}$$

Evidently, a much bigger magnetic moment is predicted for the pseudo-Deuteron-3e.

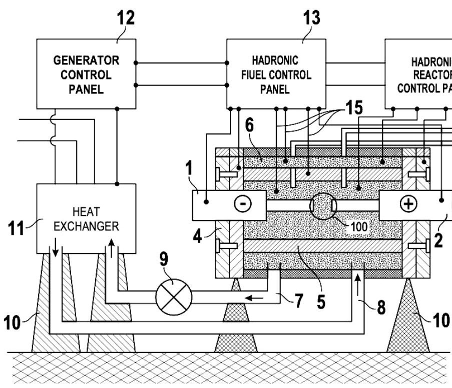

Fig. 5: In this figure, we provide a conceptual rendering of the main features of the hadronic reactor for the engineering realization of Intermediate Controlled Nuclear Fusions described in Sect. 4.3 according to Laws I-V of hadronic mechanics, including: 1. Stationary cathode; 2. Controllable anode; 3. Servomotor for the remote control of the electrode gap; 4. High pressure metal vessel; 5. External flanges; 6. Gaseous hadronic fuel, pump and tank; 7. Liquid coolant; 8. Outlet and inlet ports for the liquid coolant; 9. Liquid coolant pump; 10. Heat exchanger; 11. Electric power released to the grid; 12-13. Separate stands for the hadronic reactor and for the heat exchanger; 14. Electric power in; 15. DC power cables; 16. DC generator; 17. Variety of detectors for temperature, pressure, radiation, *etc*.; 18-20. Integrated remote and automatic control panels for the electric generator (19), hadronic fuel (20) and hadronic reactor (21), with automatic disconnect for any pre-set values for: outlet power; hadronic fuel temperature, pressure and flow; liquid coolant temperature, pressure and flow; *etc*; 21. Area in which the nuclear fusion occurs (for technical details see U. S. patents [129–131]).

4 Hyperfusion

4.1 Basic assumptions

In this section, we show that, according to our best understanding and documentation, the *Intermediate Controlled Nuclear Fusions* (ICNF) tested from 2005 to 2016 [78] [104]–[128]: 1) Produce fully controlled energy without harmful radiations in excess of the used energy; 2) The production of excess energy via ICNF hadronic reactors has been limited to a few minutes for safety reasons, but it is expected to be continuous under sufficient funding and engineering; 3) The primary origin of the sustainable and controlled production of clean excess energy, here studied for the first time, appears to be primarily due to the capability by the ICNF technology of turning the nuclei of at least one of the two hadronic fuels into pseudo-nuclei (Sect. 3).

As done in the ICNF tests here considered, we assume that the hadronic fuels are *light, natural and stable elements* in their solid, liquid or gaseous form. The selection of their form is made following the engineering realization of the hadronic laws for nuclear fusions reviewed below.

Since ICNF are irreversible over time, in order to avoid the causality problems in the use of quantum mechanics or iso-mechanics identified earlier, all elaborations of ICNF are tacitly assumed to be done via the Lie-admissible geno-mathematics and geno-mechanics of Sect. 2.3.

4.2 Physical laws of controlled nuclear fusions according to hadronic mechanics

Following the quantitative representation of the neutron synthesis and its use for the exact representation of deuteron data, the physical laws of new clean nuclear energies predicted by hadronic mechanics have been presented for the first time in the 1998 monograph [78], specialized in the 2007 paper [104] and then developed at the scientific and industrial levels in subsequent years [105]–[128] according to the following classification:

Class I: Clean nuclear energies predicted via stimulated nuclear transmutations (Sect. III-4, p. 127 of [78]);

Class II: Clean nuclear energies predicted via controlled nuclear fusions (Sect. IV-3, p. 183 of [78]);

Class III: Clean energies predicted at the atomic-molecular level via contributions from energies of Class I and II (Sect. V-4, p. 287 of [78]).

In this section, we adopt the physical laws of Class II presented in Sect. 8, p. 149 of [104] and here specialized for the engineering realization of ICNF:

HADRONIC LAW I: Hadronic fusion reactors should have means for the systematic and controlled exposure of nuclei out of their electronic clouds. In the absence of such engineering means, it is assumed that nuclear fusions may indeed occur, but only at random.

HADRONIC LAW II: Whenever the nuclei of hadronic fuels have non-null spins, hadronic fusion reactors should have means for the systematic and controlled coupling of nuclear spins either in planar singlet or in axial triplet coupling (Fig. 1). In the absence of said engineering means, it is assumed that nuclear fusions may occur, but again, only at random.

HADRONIC LAW III: Hadronic fusion reactors should have means for the systematic and controlled transmutation of the nuclei of at least one of the two hadronic fuels into pseudo-nuclei (Sect. 3). In the absence of said engineering means, nuclear fusions remain possible but at a smaller efficiency rate.

HADRONIC LAW IV: The search for ICNF without the emission of harmful radiation or the release of radioactive waste should use light, natural and stable elements as hadronic fuels. Hadronic mechanics predicts that the use of heavy natural elements as hadronic fuels creates such instantaneous energy surges to trigger processes that may inevitably emit neutrons (see Appendix A for details).

HADRONIC LAW V: The energy used by hadronic reactors to achieve a desired energy output should be the minimal possible for the operation of all engineering components of the

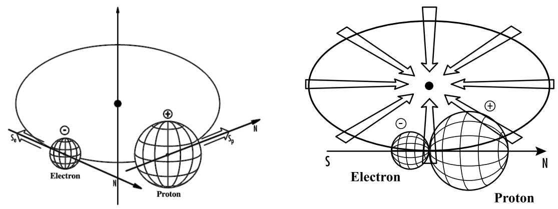

Fig. 6: In this figure, we illustrate the action of an electric DC arc between carbon electrodes submerged within a Hydrogen gas. The left view illustrates the ionization of the gas and the orientation of the proton and electron along a magnetic line, which occurs during the activation of the arc. The right view illustrates the compression of the plasma surrounding the arc in all radial directions toward its symmetry axis, which occurs during the disconnection of the arc [104].

reactors. The Lie-admissible branch of hadronic mechanics predicts that any energy in excess of the indicated minimum creates instabilities with ensuing decrease of efficiency.

4.3 Engineering realization and test of hadronic reactors for ICNF

The main principle for the engineering realization of ICNF according to Laws I-V suggested by decades of tests is the use of *DC arcs between carbon electrodes submerged within a gas.* This assumption implies that the first recommendable hadronic fuel is given by *Carbon* $C(6, 12, 0)$, while the second hadronic fuel is a properly selected, commercially available *gaseous fuel* flown through the arc to control its temperature and maximize efficiency (see U. S. patents [129–131]). Note that *Carbon nuclei have spin zero,* by therefore avoiding the need for the engineering realization of Hadronic Law II.

The action of submerged DC arcs on the gaseous hadronic fuel is the following:

1) Following their activation, DC arcs consume the point of the carbon electrode where they occur, with consequential disconnection and reconnection between points with the shortest distance. Hence, when activated, DC arcs consist of a continuous sequence of connection and disconnections generally occurring in [ms].

2) During their activation and under sufficient DC power (generally of a minimum of 40 kW), DC arcs ionize the gas, by creating a plasma in their surroundings comprising electrons, nuclei and atoms (left view of Fig. 6).

3) During their reconnection, DC arcs have been proved by the technology for the neutron synthesis (Fig. 11, 12 and Appendix B) to *compress* the surrounding plasma in all radial directions toward its symmetry axis (right view of Fig. 6).

Consequently, to the author's best knowledge, submerged DC arcs provide the best known means for the verification of Hadronic Laws I-V, with particular reference to the synthesis of pseudo-nuclei (Hadronic Law III).

Fig. 7: In the top figure, we show the main components of the Nitrogen hadronic reactor [105]; in the bottom left picture, we show the team of experimentalists from Princeton Gamma Spectroscopy Corporation [118]–[121] headed by L. Ying, President, who confirmed all results of [105]; in the bottom right picture, we show the confirmation of the lack of neutron or other harmful radiations by R. Brenna [110].

Hadronic reactors for the engineering realization of ICNF consist of: a metal vessel containing a gaseous hadronic fuel at pressure traversed by internal electrodes with remote means for the monitoring and control of the arc power, arc gap, gas pressure, gas temperature, vessel temperatures, gas flow through the arc, heat exchanger; a variety of neutrons and other detectors; interconnected, remote, monitoring and control panels of the various functions with automatic disconnect of all systems in the event of any deviation of the data from pre-set values (for details, see Fig. 5 and U. S. patents [129–131]).

In regards to manufacturing data, tests [104]–[109] were done via hadronic reactors comprising: cylindrical metal vessels with outside diameters ranging from 1 foot to 2 feet and length ranging from 2 feet to 6 feet, said vessels being certified to withstand internal pressures at least up to three times the expected operating pressure; electrodes fabricated form cylindrical graphite rods ranging from 1 to 2 inches diameter with the non-consuming anode generally being 4 inches long and the consuming cathode generally being a minimum 6 inches long; a metal jacket surrounding said vessel containing a coolant (such as water) which is recirculated through a heat exchanger; specially designed control panels for the various functions; and other engineering means (Fig. 5).

Fig. 8: In this figure, we show the Oxygen hadronic reactor for the ICNF of Helium and Carbon into Oxygen (Sect. 4.5) with the structure outlined in Fig. 5, including: in the top view, the engineer Chris Lynch, the reactor, its control panel and the power unit; in the bottom left view, the scorching of the cathode despite its continuous cooling by the flow of the Helium; in the bottom right view, the production of steam operated by the author.

All tests [104]–[109] were done via a 50 kW Miller Electric Dimension 1000, AC-DC converter operating at 40 kW by therefore supplying 0.866 kWh per minute. The reader should be aware that the use (in lieu of a commercially available AC-DC converter) of the special DC power unit for the neutron synthesis (Appendix B) provides a significant increase of the energy output due to special features of the DC arc not outlined here for brevity. All ICNF tests conducted from 2005 to 2019 by the U. S. publicly traded company *Thunder Energies Corporation,* now the private company *Hadronic Technologies Corporation* which owns all intellectual rights on ICNF. All ICNF tests were done from 2005 to 2016 with *private funds.*

Samples of the solid and gaseous hadronic fuels where taken for all ICNF tests outlined in Sects. 4.4, 4.5, 4.6, under the Trail of Custody by the technician Jim Alban before and after the tests and sent out for analysis by independent companies.

More specifically, laboratory bottles were filled up with the gaseous hadronic fuel at ambient temperature of about 80^{o} F = 26^{o} C before and after the activation of the reactor. Said samples were individually marked and shipped to *Oneida Research Services* (ORS) of Whitebnodo, New York, for the *two* different analyses of reports [118]–[124], the first for *molecular* counts and the second for *nuclear* counts with Atomic Mass Units from 2 u to 400 u, which were done via an Internal Vapor Analyzer, model 110-s operated with SOP MEL-1070. The generally low pressure of the various laboratory bottles were also marked and are reported in the individual ORS reports.

The following information is important for the proper interpretation of ORS reports: 1) The molecular and nuclear counts are inequivalent due to molecular anomalies caused by the DC arc discussed in Sect. 4.7; 2) The total number of nuclear counts before and after the tests are not the same because the Avogadro number is not conserved under nuclear fusions; 3) Reported nuclear counts generally refer to primary counts out of possible counts from 2 u to 400 u, thus implying that the sum of all counts per given sample does not necessarily add to 100; 4) The type of gas was not generally disclosed to ORS, thus implying that the used gaseous hadronic fuel is generally identified in a given report by the biggest number of reported counts; 5) The approximate character of the analyses is unquestionable, yet sufficient to establish the *existence* of an excess clean energy produced by ICNF, with the understanding that readers interested in the utmost possible accuracy should wait for proper funding.

Also under the Trail of Custody by Jim Alban, samples of the graphite used for the electrodes were taken before and after each test, marked and shipped to *Constellation Technology* of Largo, Florida, for analyses available in reports [125]–[128]. It should be noted that the latter analyses are for *solid* traces of new elements in the electrodes following tests, that confirm *some* of the nuclear fusions detected by the ORS analyses, but not all, since the primary nuclear fusions occur at the gaseous level.

4.4 ICNF with the Nitrogen hadronic reactor [105]

The *Nitrogen hadronic reactor* ([105]–[107] and Fig. 7) was built according to the specifications described in Sect. 4.3 and in Fig. 5. In particular, the metal vessel was built out of Schedule 40 steel tube 1 foot × 2 feet with 1/2 inch thickness weighting 325 lbs plus side flanges weighting 125 lbs each for a total of 575 lbs certified to withstand 300 psi. All tests were done with gaseous hadronic fuels at 100 psi and for a maximum of *two minutes* due to the rapidity of the temperature surge.

Among the variety of tests with the Nitrogen hadronic reactor from 2005 to 2016, we outline below the following tests with the understanding that, to avoid an excessive length, all technical details are referred to [105]–[107]:

4.4.1 ICNF with Deuterium and Carbon

The Nitrogen hadronic reactor was filled up with a commercial grade Deuterium gas at 100 psi pressure under 40 kW DC power. Following two minutes of operation, the external temperature of the reactor went from 26^{o} C to 150^{o} C. Two laboratory bottles before and after the activation of the reactor were filled up with the gas at the pressure indicated in the reports, market HCN1 and HCN2, respectively and shipped to ORS. The results are available in [118] and will be analyzed for nuclear fusions in Sect. 4.7.

4.4.2 ICNF with Hydrogen and Carbon

The preceding results were confirmed by tests in the Nitrogen hadronic reactor with a commercial grade Hydrogen at 100 psi pressure and 40 kW DC power, as reported in Sect. 6 of [105] and in ORS report [120] for bottles market HC1 and HC2 (see Sect. 4.7 for their study). An important result of this test is that, under the same conditions of pressure, power, electrodes, *etc*. of the preceding test with Deuterium and Carbon, the operation with a hydrogen gas produced an energy excess bigger than that with Deuterium gas, since in two minutes of operation the temperature of the exterior wall of the reactor went from 26° C to 254° C with about 1.72% *increase* of the temperature compared to the test with Deuterium.

4.4.3 ICNF with Magnegas and Carbon

The most successful tests with the Nitrogen hadronic reactor occurred with the use as hadronic fuel of *magnegas,* the gaseous fuel with the new magnecular structure [72, 131] (Fig. 10). The results of the ORS analyses are reported in [120] for bottles marked MG1 and MG2. A main result of various tests is that the Nitrogen hadronic reactor operating with magnegas at 100 psi pressure under 40 kW power went from 26° C to 254° C in *one minute,* rather than the two minutes as for then Hydrogen-Carbon tests, thus implying a 3.44 increase of efficiency of the Deuteron-Carbon tests.

We should indicate the conduction of additional tests with the Nitrogen hadronic reactor by using various gaseous fuels whose analyses are available from [119].

4.5 ICNF with the Oxygen hadronic reactor

The Oxygen hadronic reactor (see [106]–[107], independent studies in [113], ORS reports [121, 122] and Fig. 8) was built in 2010 for testing the ICNF of Helium and Carbon into Oxygen, by therefore using Helium as the gaseous hadronic fuel.

The hadronic reactor comprised: a vertical 1 foot × 4 feet Schedule 40 steel cylinder certified to withstand 500 psi; a chamber surrounding said vessel for flowing water as coolant; the flow of the gaseous hadronic fuel through the electrodes for its cooling; and the remaining engineering component illustrated in Fig. 5.

The reactor was additionally built to test the feasibility of the new principle of combustion subsequently released in 2018 under the name of *HyperCombustion* [145] which is intended to achieve the full combustion of fossil fuels via a combination of a conventional combustion plus ICNF in Parts Per Million by Volume, ppmv.

For the test done in April 2010, the laboratory bottles prior and after the test were filled up with the internal gas at the pressure indicated in the reports, market HT1 and HT2 and shipped to ORS for analysis whose results are available from [121]. The tests were repeated in February 2011, market HE1 and HE2 and sent to ORS for analyses whose results are available from [122] (see Sect. 4.7 for their analysis).

The primary result of the tests was the proof under various eyewitnesses that, when filled up with Helium at 100 psi and operated with a 40 kW AC-DC converter, the Oxygen hadronic reactor did indeed produce a steam sustainable for two minutes after which the cooling system was insufficient to maintain the reactor at a constant temperature.

During additional tests done on May 15, 2011, with the Helium hadronic fuel at 150 psi, in two minutes of operation the mixture of Helium and synthesized Oxygen sent the internal temperature gauge off the 10 000° C limit and melted the top Helium recirculation port, with an impressive release of the incandescent interior gas after which all tests with the Oxygen hadronic reactor were terminated for safety. The technicians (who eyewitnessed the discharge at a distance) nicknamed *Dragons* the hadronic reactors (Dragons I, II and III for the Nitrogen, Oxygen and Silicon reactors, respectively).

4.6 ICNF with the Silicon hadronic reactor

The Silicon hadronic reactor ([106]–[108], video [109] and Fig. 9) was built to test the ICNF of Oxygen and Carbon into Silicon via the use of *air* as hadronic fuel, instead of pure Oxygen, because the natural mixture of 78% Nitrogen and 21% Oxygen is known to quench the Oxygen reactance experienced in Section 4.5.

The hadronic reactor consisted of a Schedule 40 steel tube with 1 foot diameter and 6 feet long certified to withstand a 5 000 psi pressure. Air was continuously pumped through the reactor at 1 000 psi. The arc was powered by a 50 kW AC-DC converter. The reactor was surrounded by a jacket as in Fig. 5 in which water was continuously pumped at ambient pressure. The superheated air and cooling water from the reactor were mixed to power an electricity producing turbine whose data are analyzed in Sect. 4.7. To avoid an excessive length, we suggest interested readers to view video [109] for a detailed description of this third hadronic reactor including the identification of the various members of the experimental team.

The analyses for the gaseous part of the test are available from ORS [123, 124], and the analyses for the solid part are available from Constellation Technology [125]–[128]. Independent studies are available from [110]–[117].

The main result of the tests with the Silicon hadronic reactor is that, under various eyewitness (see Fig. 9 and [109]), *the Silicon hadronic reactor did prove the capability by ICNF to produce clean excess energy for* 15 min *(fifteen minutes),*

Fig. 9: In this figure, we illustrate the hadronic reactor for the ICNF of Oxygen and Carbon into Silicon (Sect. 4.6), with a structure outlined in Fig. 5 and described in detail in the video [109], including: in the top row, a front and rear view; in the middle view, the reactor and the touch screen for remote control; in the bottom view, one of control panels and some of the technicians (from the left) Chrys Lynch, Jim Alban and Michael Rodriguez eye-witnessing the sustainable and controllable production of steam from the turbine for the duration of 15 min.

after which the remote monitoring and control panels automatically disconnected the operation for the inability of the cooling system to maintain the reactor temperature within a pre-set safety value. No additional tests were done with the Silicon hadronic reactor due to lack of funds for the construction of a properly engineered *prototype hadronic power plant.*

4.7 Representation of ICNF excess energy via hyperfusions

The ICNF tests conducted from 2015 to 2016 via the Nitrogen, Oxygen and Silicon hadronic reactors [104]–[109] were conceived, conducted and reported under the assumption that the Hadronic Laws for nuclear fusions available at that time [78] were verified by the processing, via a submerged DC arc, of the gaseous hadronic fuel into the new chemical species of *magnecules* [72, 131] (see the MG1 counts in [120] for a sample of its anomalous chemical structure), due to the verification by magnecules such as $C \times D$ (Fig. 10) of Hadronic Laws I and II. The fusion $C \times D \rightarrow N$ was then supposed to be permitted by the compression of the magnecules during the disconnection of the DC arc (Fig. 6).

Subsequent studies have revealed that the sole creation of

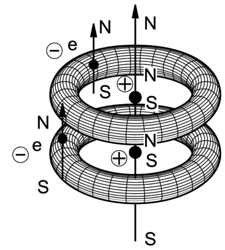

Fig. 10: In this figure, we illustrate the simplest possible case of the new chemical species of magnecules [72–74, 131], which is characterized by two atoms with a toroid polarization of their orbits caused by a DC arc which atoms are bonded together according to an axial triplet coupling thanks to their newly acquired magnetic field which does not exist in natural atomic configurations.

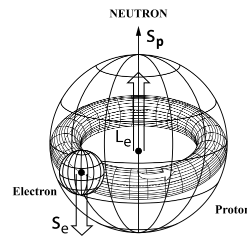

Fig. 11: In this figure, we illustrate the synthesis of the neutron from a proton and an electron in the core of stars according to hadronic mechanics reviewed in Appendix B [84]–[103].

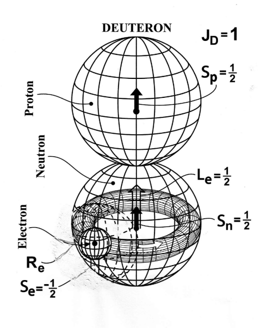

Fig. 12: A view of the structure of the Deuteron according to the synthesis of the neutron from a proton and an electron in the core of Stars ([83] and Appendix B).

a magnecular structure for gaseous hadronic fuels is insufficient for a consistent representation of the nuclear fusions reported in the ORS nuclear counts for various reasons, including :

1) As shown below, the representation of ORS data on the fusion of Oxygen, Silicon and higher nuclei requires the prior fusion of two Deuterons into the Helium, which has not been achieved to date in a sustainable form via conventional

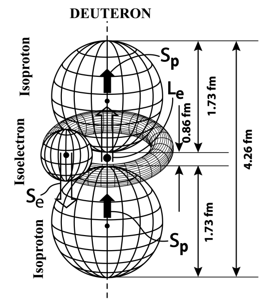

Fig. 13: In this figures, we illustrate the representation by hadronic mechanics of the stability of natural nuclei despite the natural instability of their neutrons. Said representation is intrinsic in the hadronic synthesis of the neutron from a proton and an electron (Appendix B), and it is given by the decoupling of the electron to assure a symmetric position between the two attracting protons (Appendix C). Note that the model implies the reduction of all matter in the universe to protons and electrons.

engineering means;

2) Deuterium atoms are ionized by the DC current of the ICNF tests, by therefore resulting in Deuterons and electrons;

3) Explicit calculations done via the Ampere law have shown that the compression caused by the disconnection of the DC arc is unable to overcome the *extremely big Coulomb repulsion* between two Deuterons at 1 fm mutual distance, Eq. (1), by therefore rendering impossible their fusions into the Helium.

To the author's knowledge, the best possibility for two Deuterons to fuse into the Helium is that one of them acquires the form of pseudo-Deuteron (Sect. 3) since in that case all Hadronic Laws I-V are verified due to opposite charges and magnetic moments, including the planar spin coupling, under an extremely big Coulomb *attraction,* and the consequential inevitable activation of strong nuclear forces under which the fusion is inevitable. In this section, we study the excess clean energy produced by the ICNF [104]–[109] to illustrate the plausibility of their being in reality hyperfusions.

The excess energy produced by the Nitrogen hadronic reactor (Sect. 4.4.1) over the used energy in two minutes of 1.333 kWh was tentatively appraised in Sect. 4. Eq. (4.3) of [105] resulting in the value

$$\Delta E = E^{out} - E^{in} = \\ = 2.203 - 1.333\,\text{kWh} = 0.87\,\text{kWh}\,. \tag{59}$$

These preliminary appraisals were confirmed by the independent analysis [113]. By using a different method, Sect. 3.3 of the independent study [110] reached the value

$$\Delta E = 2.88\,\text{MJ} = 0.138\,\text{kWh}\,. \tag{60}$$

The tests of Sect. 4.4.2 then imply a clean excess energy

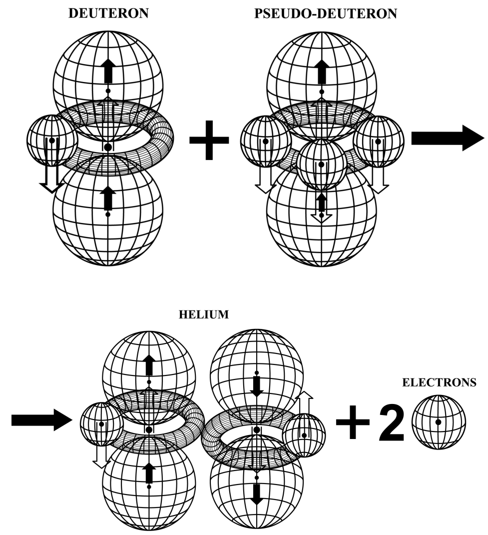

Fig. 14: In this figure, we illustrate the hyperfusion of a natural Deuteron and a pseudo-Deuteron-2e into the Helium plus the emission of an electron pair (Sects. 3 and 4).

of 1.73% bigger then that of (59), i.e.,

$$\Delta E = E^{out} - E^{in} = \\ = 2.823 - 1.333\,\text{kWh} = 1.49\,\text{kWh}\,. \tag{61}$$

For the case of magnegas as hadronic fuel (Sect. 4.4.3) we would then have an excess clean energy 3.44 times bigger that that of (59) in only one minute,

$$\Delta E = E^{out} - E^{in} = \\ = 23.656 - 0.666\,\text{kWh} = 2.99\,\text{kWh}\,. \tag{62}$$

It should be noted that the above preliminary appraisals are significantly *below* the excess energy actually produced by the hadronic reactors because said appraisals used the *external temperature* of the hadronic reactors, rather than the actual *internal temperature*. As an example, calculations done for the tests indicated in Section 4.1.2 with the Helium as hadronic fuel at 150 psi and the temperature of the internal gas in excess of 10 000$^{\text{o}}$ C in two minutes of operation we would have a multiple of value (62). The same large thermal values can be obtained from the tests of Section 4.6 with the Silicon hadronic reactor operating at 1 000 psi.

In view of the indicated insufficiencies of thermal calculations of excess clean energy produced by the hadronic reactors, in this section we present, apparently for the first time, an alternative approximate calculation of excess energy output based on the energy produced by the primary nuclear fusions reported in the ORS counts. Along these lines, [118] reports the following primary increased counts Δu among numerous other counts that are omitted in this first study for brevity,

$$\begin{aligned}
&(a)\ \ \Delta 2u : 18,550,801 - 16,075,402 = \\
&\qquad = 2,475,399\,\text{ppmv}\,, \\
&(b)\ \ \Delta 3u : 41,165 - 30,269 = 10,896\,\text{ppmv}\,, \\
&(c)\ \ \Delta 4u : 76 - 0 = 76\,\text{ppmv}\,, \\
&(d)\ \ \Delta 14u : 3,555 - 2,841 = 714\,\text{ppmv}\,, \\
&(e)\ \ \Delta 16u : 3,010 - 1,205 = 1,805\,\text{ppmv}\,, \\
&(f)\ \ \Delta 18u : 2,949 - 2,718 = 231\,\text{ppmv}\,, \\
&(g)\ \ \Delta 28u : 30,171 - 24,684 = 3,687\,\text{ppmv}\,.
\end{aligned} \tag{63}$$

Recall the following tabulated values (see e.g. [1])

$$\begin{aligned}
&E_e = 0.511\,\text{MeV}\,, \\
&E_p = 938.272\,\text{MeV}\,, \quad E_n = 939.565\,\text{MeV}\,, \\
&E_D = 2.014102\,\text{u}\,, \quad E_{He} = 4.002603\,\text{u}\,, \\
&E_C = 12.00000\,\text{u}\,, \quad E_N = 14.003074\,\text{u}\,, \\
&E_O = 15.994915\,\text{u}\,, \quad E_{Si} = 27.976927\,\text{u}\,, \\
&1\ u = 931.5\,\text{MeV}\,.
\end{aligned} \tag{64}$$

Recall also that the commercial grade Deuterium gas used for the tests contained a considerable percentage of Hydrogen. From our studies on the neutron synthesis (Appendix B), we expect that the reactor first synthesized the neutron according to the *endothermic* reaction (82)

$$\hat{e}^- + \hat{a} + p^+ \to n\,, \quad \Delta E_n = -0.782\,\text{MeV}\,. \tag{65}$$

Immediately following their synthesis, neutrons are compressed by the arc against the protons, by therefore synthesizing the Deuterium. This would explain the considerable increase of counts (a) in (63)

$$n + p \to D(1,2,1)\,, \quad \Delta E_D = 1.7045\,\text{MeV}\,. \tag{66}$$

Count (b) of (63) is interpreted as the synthesis of the Tritium with an energy balance here assumed, for simplicity, to be similar to that for the Deuterium. Count (c) of (63) is evidently the synthesis of the Helium from two Deuterons which, according to our view, can be best interpreted as the hyperfusion between a pseudo-Deuteron-2e and a natural Deuteron according to the general rule of (28) (Fig. 14),

$$\begin{aligned}
&\tilde{D}_{2e}(1,2,1_\uparrow) + D(1,2,1_\downarrow) \to He(2,4,0) + \beta^-_\uparrow + \beta^-_\downarrow\,, \\
&\Delta E_{4u} = 23.8473315\,\text{MeV}\,.
\end{aligned} \tag{67}$$

Count (d) of Eq. (63) is interpreted via the synthesis of the Nitrogen,

$$\begin{aligned}
&\tilde{D}_{2e}(1,2,1) + C(6,12,0) \to N(7,14,1) + 2\beta^-\,, \\
&\Delta E_{14u} = 10.272582\,\text{MeV}\,.
\end{aligned} \tag{68}$$

Count (e) is interpreted as due to the synthesis of the Oxygen

$$\begin{aligned}
&\tilde{H}e_{ke}(2,4,0) + C(6,12,0) \to O(8,16,0) + k\beta^-\,, \\
&\Delta E_{16u} = 7.161372\,\text{MeV}\,.
\end{aligned} \tag{69}$$

Count (f) of (63) is evidently due to the synthesis of the Silicon

$$\begin{aligned}
&\tilde{O}_{ke}(8,16,0) + C(6,12,0) \to Si(14,28,0) + k\beta^-\,, \\
&DeltaE_{15u} = 16.755822\,\text{MeV}\,.
\end{aligned} \tag{70}$$

Even though incomplete, the above ICNF are sufficient to illustrate the sustainable and controllable production of clean energy by ICNF.

By using the data from (63) to (70), we have the following energy output for the counts of (63):
Deuterium synthesis: $2,283,555$ MeV in ppmv which is given by the energy released by the Deuterium synthesis 2,475,399 ppmv $\times$ 1.7045 MeV = $4,219,317$ MeV in ppmv *less* the energy needed for the neutron synthesis $2,475,399 \times 0.782 = 1,935,762$ MeV;
Helium synthesis: 76 ppmv $\times$ 23.8 MeV = $1,808.8$ MeV in ppmv;
Nitrogen synthesis: 714 ppmv $\times$ 10.3 MeV = $7,354.2$ MeV in ppmv;
Oxygen synthesis: $1,805 \times 7.2$ MeV = $12,996$ MeV in ppmv;
Silicon synthesis: $3,687$ ppmv $\times$ 16.7 MeV = $61,573$ MeV;
resulting in the total energy output of $54,546,518$ MeV in ppmv corresponding to the total energy output of

$$54,546,518\,\text{MeV} = 2.43 \times 10^{-12}\,\text{kWh}\,. \tag{71}$$

By assuming that the gas in the reactor is a perfect gas, by assuming the related law

$$PV = nRT\,, \tag{72}$$

and by recalling that one mole contains 6.02×10^{23} particles, that is, 6.02×10^{17} *millions* of particles, the total energy output for data (63) is given by

$$\begin{aligned}
&\Delta E = E^{out} - E^{in} = \\
&= 2.43 \times 10{-}12 \times n \times 6.02 \times 10^{17} - 1.333\,\text{kwh} \approx \\
&\approx n \times 14.628 \times 10^5\,\text{kwh}\,.
\end{aligned} \tag{73}$$

For a first approximation of the number of moles n, we assume the conversions for: pressure 100 psi = 6.8 atm; length 1 foot = 2.54 cm; radius of the cylinder r = 1 foot = 13.97 cm; height of the cylinder h = 2 ft = 60.85 cm; and volume of the gas $V = \pi \times r^2 h = 3.14 \times 196 \times 60.85 = 37,455\,\text{cm}^3 = 37.45$ L. Consequently, $PV = 6.8 \times 37.45 = 254.66$. Since the gas constant is given (in our units) by $R = 0.0821$, we have $PV/R = 254.66/0.0821 = 3,101.827$.

Consequently, an approximate value of the total number of moles under the indicated assumptions is given by

$$n = 3 \times 10^3 \times \frac{1}{T}. \tag{74}$$

We now assume that the internal gas is Hydrogen and that its temperature varies from the expected 1 000 000° C in the small area of the nuclear fusions all the way to temperatures of the order of 1 000° C in the back of the reactor wall, resulting in an average temperature of the order of 10^5 C. A similar temperature value is reached via calculations based on the transmission of from 26° C to 150° C through a 1/2 inch steel wall within a period of time of the order of sixty seconds via a gas, such as Hydrogen, with the smallest possible density.

By using the equivalency 0° C ≡ 273.15 K, we have 150° C ≡ 423.15 K, our approximate value of the number of moles is given by

$$\begin{aligned} n = 3 \times 10^3 \times \frac{1}{T} = 3 \times 10^3 \times \frac{1}{423} \times 10^{-5} = \\ = 0.00704 \times 10^{-2} = 7.04 \times 10^{-4}. \end{aligned} \tag{75}$$

Corrections of the above value for the total number of moles of a Deuterium gas reduce the above value to

$$n \approx 7 \times 10^{-5}. \tag{76}$$

The approximate total output of controlled clean energy of the considered ICNF is then given by

$$\begin{aligned} \Delta E = E^{out} - E^{in} = \\ = 7 \times 10^{-5} \times 15 \times 10^5 - 1.333\,\text{kWh} \approx 100\,\text{kWh}. \end{aligned} \tag{77}$$

It is easy to see that, for the case of the tests of the Silicon hadronic reactor (Section 4.6) done at 1,000 psi of the gaseous hadronic fuel, the repetition of the above analysis yields a total sustainable and controllable, clean energy output of the order of 1,000 kWh, out of which the surplus electric energy released by a turbine operated electric generator is expected to be of the order of 100 kWh.

The author has no words to indicate again the approximate character of the above appraisal. More accurate calculations are planned for the forthcoming paper [146].

5 Concluding remarks

In this paper, we have recalled the generally forgotten insufficiencies of quantum mechanics in nuclear physics in view of its inability in about one century of achieving: A) A quantitative representation of the fundamental synthesis of the neutron from a proton and an electron in the core of stars; B) An exact representation of nuclear magnetic moments; C) An exact representation of the spin of nuclei in their true ground state (that without the usual orbital excitations); D) A quantitative representation of the stability of neutrons when members of a nuclear structure; E) A quantitative representation of the stability of nuclei despite the huge Coulomb repulsion between positive nuclear charges; and other insufficiencies.

We then recalled the largely forgotten experiments establishing deviations of quantum mechanical predictions from physical reality in various fields, including: electrodynamics; condensed matter physics; heavy ion physics; time dilation for composite particles; Bose-Einstein correlation; propagation of light within physical media; and in other fields.

We additionally recalled that *quantum mechanics is reversible over time* due to the invariance of Heisenberg's equation under anti-Hermiticity and for other reasons. Consequently, quantum mechanics cannot provide a consistent representation of energy-releasing processes such as nuclear fusion due to their *irreversibility over time.* In particular, we have shown that the treatment of nuclear fusions via quantum mechanics may violate causality laws (e.g., because of solutions in which effects precede the cause), because the same Schrödinger equation applies for nuclear fusions forward as well as backward in time.

We then recalled that the axiomatic origin of the above insufficiencies of quantum mechanics has been first identified in 1935 by A. Einstein, B. Podolsky and N. Rosen and rests in the *locality* of the theory (EPR argument) [5], beginning at the level of the Newton-Leibnitz calculus, due to the sole possibility of characterizing particles and nuclei as massive points, thus creating conceptual and technical difficulties in fusing two points into a third point.

We then briefly reviewed the EPR completion of quantum mechanics into hadronic mechanics for the characterization of particles and nuclei as *extended, thus deformable and hyperdense* under conventional, Hamiltonian interactions plus contact, thus zero-range, non-Hamiltonian interactions caused by mutual penetrations, with an elementary review of:

i) The Lie-isotopic (i.e. axiom-preserving) branch of hadronic mechanics including *iso-mathematics and iso-mechanics* (Sect. 2.2) for the representation of extended particles and their non-Hamiltonian interactions via the isotopic element $\hat{T} = \hat{T}^\dagger > 0$ of the universal enveloping iso-associative algebra of Hermitean operators with product $A \star B = A\hat{T}B$ and ensuing iso-Schrödinger equation $H \star |\psi\rangle = H\hat{T}|\psi\rangle = E|\psi\rangle$ with apparent resolution of quantum mechanical insufficiencies for *stable* nuclei.

ii) The Lie-admissible branch of hadronic mechanics, also called genotopic branch, including *geno-mathematics and geno-mechanics* (Sect. 2.3) based on forward enveloping algebra with ordered products to the right $A > b = ARB$, $R > 0$ representing motion forward in time, and backward enveloping algebra with ordered products to the left $A < B = ASB, S > 0$ representing motion backward in time, with the corresponding forward and backward geno-Schrödinger equations $H > |\psi^{for}\rangle = HR|\psi^{for}\rangle = E^{for}|\psi^{for}\rangle$, $\langle\psi^{bac}| < H = \langle\psi^{bac}|S\,H = E^{bac}\langle\psi^{bac}|$, and axiomatically consistent resolution of the quantum mechanical causality problems for irreversible processes whenever $R \neq S$.

Thanks to half a century preparatory studies on the above issues, in this paper we have presented apparently for the first time:

1) The prediction by hadronic mechanics of the existence of new, negatively charged, unstable nuclei, called *pseudo-nuclei,* which are characterized by a hadronic bond of negatively charged electrons and positively charged natural nuclei, by therefore resolving the Coulomb barrier since pseudo-nuclei would be *attracted* (rather than repelled) by natural nuclei, with ensuing new conception of nuclear fusions between pseudo-nuclei and natural nuclei, here called *hyperfusions* (Section 3);

2) The identification of engineering means for the synthesis of pseudo-nuclei which is given by the hadronic reactors for the synthesis of the neutron from the proton and the electron (Sects. 4.1-4.6 and Appendix B);

3) Laboratory evidence according to which the synthesis of pseudo-nuclei and related hyperfusions appear to be the origin of the limited, yet sustained and controlled excess energies achieved by the *Intermediate Controlled Nuclear Fusions* (Section 4.7).

In view of the inability by quantum mechanics in about one century under large public funds to achieve industrially applicable nuclear fusions, and the consequential, rapidly increasing deterioration of our environment, the author hopes that appropriate academic and governmental entities initiate the implementation of a true scientific democracy for qualified inquiries, which requires the continuation of the search for clean nuclear energies along quantum mechanical lines, *jointly* with the search based on new vistas, such as the forgotten EPR argument.

Appendices

A Is neutron radiation truly necessary for nuclear fusions?

As it is well known, it has been generally assumed for about one century that the emission of harmful neutrons is necessary for nuclear fusions (see e.g. [132]), as it is the case for the *Tokamak nuclear fusion* of Deuterium and Tritium into Helium plus neutron [133, 134]

$$\begin{aligned} &D(1,2,1) + D(1,3,1/2) \rightarrow He(2,4,0) + n\,, \\ &\Delta E = +17.6\,\mathrm{MeV}\,. \end{aligned} \tag{78}$$

The author respectfully suggests the conduction of *experimental verifications* of the need for the emission of neutrons in Deuteron-type fusions prior to its systematic use under public support, in view of the following opposing evidence:

A.1. The need for the emission of neutrons in nuclear fusions was historically established for the fusions of *heavy nuclei* but, to the author's best knowledge, no quantitative study is currently available on a similar need for the fusion of *light nuclei,* such as the Deuterium and the Tritium.

A.2. It is known that, in the core of stars, Deuterons fuse into the Helium without neutron emission,

$$\begin{aligned} &D(1,2,1_\uparrow) + D(1,2,1_\downarrow) \rightarrow He(2,4,0)\,, \\ &\Delta E = +29.523\,\mathrm{MeV}\,, \end{aligned} \tag{79}$$

since the Coulomb barrier is overcome by the extreme local pressures, while collective fusions leading to the explosion of the star are prohibited by the random spin alignment of fusion (79).

A.3. There exists valid evidence of excess heat creation in condensed matter due to nuclear fusions of light nuclei without the emission of neutrons [135–137].

A.4. The assumption of the necessary emission of neutrons in the fusion of light nuclei is based on a theory, quantum mechanics, which is only *approximately valid* in nuclear physics due to its inability in one century of achieving exact representations of basic nuclear data (Sect. 1).

A.5. Clear experimental evidence achieved in major physics laboratories has established the existence of deviations of quantum mechanical predictions from physical reality in various fields [12]–[26].

A.6. The unverified assumption of the necessary emission of neutrons in nuclear fusion is made in oblivion of the Einstein-Podolsky-Rosen argument that *Quantum mechanics is not a complete theory* [5].

A.7. The totality of the *intermediate Controlled Nuclear Fusions* occurred with the independently certified absence of any neutron emission [104]–[128].

Since sustainability has not been achieved in about one century for nuclear fusions with neutron emission, nuclear fusions without neutron emission should deserve the same scientific process.

B The synthesis of the neutron in a star

In 1920, E. Rutherford [138] suggested that the hydrogen atom in the core of stars is "compressed" into a new particle that he called the *neutron*

$$e^- + p^+ \;\rightarrow\; n\,. \tag{80}$$

In 1932, J. Chadwick [139] provided an experimental confirmation of the existence of the neutron.

In 1933, W. Pauli [140] pointed out that synthesis (80) violates the conservation of angular momentum.

In 1935, E. Fermi [141] submitted the hypothesis that the synthesis of the neutron occurs with the joint *emission* of a neutral and massless particle ν with spin 1/2 that he called the *neutrino* (meaning "little neutron" in Italian)

$$e^- + p^+ \;\rightarrow\; n + \nu\,. \tag{81}$$

In 1978, R. M. Santilli [82] (see also the 2021 update [29]) identified various arguments according to which quantum mechanics is *inapplicable* to (rather than violated by) the neutron synthesis, beginning with the fact that the rest energy of the neutron is *bigger* than the sum of the rest energies of the proton and the electron,

$$\begin{aligned} &E_p = 938.272\,\text{MeV}\,, \quad E_e = 0.511\,\text{MeV}\,, \\ &E_n = 939.565\,\text{MeV}\,, \\ &\Delta E = E_n - (E_p + E_e) = 0.782\,\text{MeV} > 0\,, \end{aligned} \tag{82}$$

by therefore requiring a *positive binding energy* and resulting in a *mass excess* for which the Schrödinger and Dirac equations admit no physically meaningful solutions.

Consequently, when he was at Harvard University under support of the U.S. Department of Energy, R. M. Santilli proposed [49] the construction of the non-unitary Einstein-Podolski-Rosen completion of quantum mechanics into a new mechanics called *hadronic mechanics* (Sect. 2).

Following the achievement of mathematical and physical maturity [51] (see [30, 31] for detailed treatment), the Lie-isotopic branch of hadronic mechanics allowed the representation of *all* characteristics of the neutron at the non-relativistic and relativistic levels via a structure model of the neutron consisting of an electron e^- totally compressed inside the extended and dense proton p^+ in singlet coupling [84]–[89] (Figs. 11, 12 and 13).

At the non-relativistic level, the exact representation of the **mass, mean life and charge radius of the neutron** were achieved via structure equations of type (51) [84].

The exact representation of the **spin of the neutron** was achieved thanks to the appearance of the *internal orbital motion* of the electron within the extended and dense proton with angular momentum $L_e = 1/2$ (which is necessary to avoid major resistive forces), resulting in the following realization of Rutherford's original conception of the neutron, Eq. (80),

$$\begin{aligned} &\hat{e}^- + \hat{p}^+ \rightarrow \hat{e}^-_{spin} + \hat{e}^-_{orb} + \hat{p}^+ \rightarrow n\,, \\ &S^{spin}_{\hat{e}} + S^{orb}_{\hat{e}} + S^{spin}_{\hat{p}} = -\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{1}{2}\,, \end{aligned} \tag{83}$$

(where the "hat" denotes treatment via hadronic mechanics) according to which *the spin of the neutron coincides with that of the proton,* as expected since the proton is assumed to be at rest in synthesis (80) and its mass is about 1 800 times that of the electron. Note that the internal orbital motion of the electron is impossible for quantum mechanics due to the representation of the proton as a point.

Recall that, in Rutherford's synthesis (80), we have the following tabulated magnetic moments of the electron, the proton and the neutron all considered in nuclear magnetrons

$$\begin{aligned} &\mu^{spin}_{\hat{e}} = +1838.285\,\mu_{\text{N}}, \quad \mu^{spin}_{\hat{p}} = +2.7/92\,\mu_{\text{N}}\,, \\ &\mu^{spin}_{n} = -1.913\,\mu_{\text{N}}\,, \end{aligned} \tag{84}$$

where one should note that the direction of the magnetic moment of the electron is the same as that of the proton because of the double inversion of the spin and of the charge. One should also note the very big value of the intrinsic magnetic moment of the electron for nuclear standards which is intrinsic in the synthesis of the neutron from the Hydrogen.

The exact representation of the **anomalous magnetic moment of the neutron** was achieved thanks to the indicated internal orbital motion of the electron with the value

$$\mu^{orb}_{\hat{e}} = -1842.990\,\mu_{\text{N}}\,, \tag{85}$$

by keeping in mind that the orbital magnetic moment of the electron is opposite that of the proton due to opposite charges. The exact representation of the anomalous magnetic moment of the neutron was then reached via the sum [84]

$$\begin{aligned} &\mu^{spin}_{e} + \mu^{orb}_{e} + \mu^{spin}_{p} = \\ &= +1838.285 - 1842.990 + +2.792\,\mu_{\text{N}} = \\ &= -1.913\,\mu_{\text{N}}\,. \end{aligned} \tag{86}$$

The above representation is considered to be a confirmation of the internal orbital motion of the electron in synthesis (80) because of the representation of the *negative* value of the neutron magnetic moment.

The **relativistic representation** of all characteristics of the neutron in synthesis (80) was reached in the 1995 paper [89] (see review [28]) via the isotopies $\hat{\mathcal{P}}(3.1)$ of the spinorial covering of the Lorentz-Poincaré symmetry and cannot be reviewed here for brevity.

Following the mathematical and physical understanding of the neutron synthesis in the core of stars, Santilli and his associates conducted systematic experimental and industrial tests on the laboratory synthesis of the neutron from a commercial grade Hydrogen gas [90]–[97] (see also independent studies [98]–[103]). These tests eventually lead to the production and sale by the U.S. publicly traded company *Thunder Energies Corporation* (now the private company *Hadronic Technologies Corporation* //www.hadronictechnologies.com) of the *Directional Neutron Source* (DNS) producing on demand a flux of low energy neutrons in the desired direction (Fig. 15).

In regard to the mass excess of synthesis (80), we should recall that the missing energy of 0.782 MeV cannot be provided by the relative kinetic energy between the electron and the proton because, at that energy, the electron-proton cross section is essentially null, thus prohibiting any synthesis.

Similarly, said missing energy cannot be provided by a star such as our Sun, because the Sun synthesizes about 10^{38} – 10^{39} neutrons *per second,* that would require such a big amount of energy (about 10^{38} MeV/s) to prevent a star from producing light due to insufficient internal temperature.

For these and other reasons, Santilli [142] proposed in 2007 the hypothesis that *the missing energy in the neutron synthesis is provided by space as a universal substratum with*

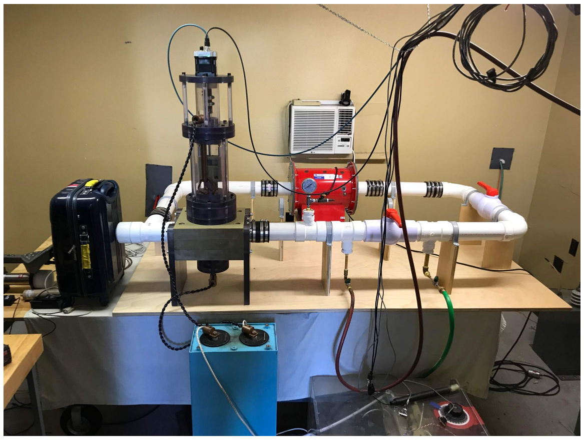

Fig. 15: In this figure, we illustrate the Directional Neutron Source (DNS) produced and sold by the U. S. publicly traded company Thunder Energies Corporation (now www.hadronictechnologies.com) which produces on demand a flux of low energy, directional neutrons synthesized from a commercial-grade Hydrogen gas contained in the loop of the hadronic reactor.

an extremely big energy density needed for the characterization and propagation of electromagnetic waves and elementary particles. The missing energy is transferred from space to the neutron by a massless, chargeless and spinless longitudinal impulse called *etherino* and denoted with the letter $\hat{a}$ (from the Latin *aether*), according to the synthesis

$$\hat{e}^- + \hat{a} + \hat{p}^+ \rightarrow n\,, \tag{87}$$

where one should note that, contrary to the case of the neutrino in synthesis (81), the etherino is on the *left* of the synthesis as a condition to supply the missing energy.

Independently from the above studies, the Sun releases into light 2.3×10^{38} MeV/s [143], corresponding to about 4.3×10^6 t/s. Since in a Gregorian year there are 10^7 seconds, the loss of mass by the Sun ΔM_S per year due to light emission is given by

$$\Delta M_S = 10^{23} \text{ metric tons per year}\,. \tag{88}$$

This loss of mass is of such a size to cause a decrease of planetary orbits detectable in astrophysical laboratories, contrary to centuries of measurements on the stability of planetary orbits.

Therefore, Santilli proposed the etherino hypothesis [142] for the intent of representing the gravitational stability of the Sun via a return to the historical cosmological model based on the continuous creation of matter in the universe. In fact, the energy needed for the star to synthesize neutrons 10^{38} MeV/s is essentially equal to the energy needed for the neutron synthesis, by therefore representing the stability of the star with intriguing cosmological implications [22, 23], e.g. for supernova explosions and neutron stars.

Note that the *permanently stable* protons and electrons cannot possibly disappear from the universe during the neutron synthesis to be replaced by the hypothetical quarks. Note also that the neutron is naturally unstable (when isolated) and decays into the original stable constituents. The above features imply new recycling nuclear waste via their stimulated decay caused by photon irradiation with a suitable resonating frequency $\gamma^{res} = 1.293$ MeV and transmutations of the type [78, 95]

$$M\gamma^{res} + N(Z, A, J) \rightarrow N'(Z + M, A, J + K) + M\beta^-\,, \tag{89}$$

(where K is the spin corrections due to the emission of electrons) under which transmutations the long mean lives of nuclear waste can be reduced in a way proportional to the intensity of the gamma irradiation.

Additionally, we should mention that, following the compression of the electron within the proton, hadronic mechanics predicts the subsequent compression (evidently with a smaller probability) of an electron, this time, within a neutron, resulting in a *new negatively charged particle* called *pseudo-proton* $\tilde{p}^-$ [144] with an additional possibility of recycling nuclear waste via pseudo-proton irradiation and ensuing transmutations

$$M\tilde{p}^- + N(Z, A, J) \rightarrow N'(Z - M, A, J + K)\,, \tag{90}$$

under which long mean lives (calculated via hadronic mechanics) can be reduced to seconds.

C Representation of nuclear stability

We close this paper with an outline of the representation of nuclear stability according to hadronic mechanics.

C.1 Representation of nuclear stability despite the instability of the neutron

According to quantum mechanics, no stable nuclei should exist in nature because nuclei are assumed to be quantum mechanical bound states of protons, which are permanently stable, and neutrons which are naturally unstable, with a mean life of 879.6 ± 0.8 s and spontaneous decay [1]

$$n \rightarrow p^+ + e^- + \bar{\nu}\,. \tag{91}$$

Following decades of study of the problem, this author was unable to *formulate,* let alone solve, the above problem due to the lack of any possible quantum mechanical representation of the synthesis of the neutron in a star.

Following a conceptual suggestion in [36], we here indicate, apparently for the first time, that the quantitative representation of nuclear stability despite the natural instability of the neutron is an intrinsic feature of the hadronic synthesis of the neutron from the proton and the electron because, starting from the hadronic structure of the neutron (Fig. 11 and Appendix B) with ensuing hadronic structure of the Deuteron as a two-body bound state of a proton and a neutron in axial triplet coupling (Fig. 12 and Sect. 3), the electron naturally decouples from the proton to acquire a position intermediate between the two protons (Fig. 13). Consequently, the decoupled electron assumes an intermediate position between the two attracting protons by occupying the distance between their charge distributions of 0.3745 fm, which would otherwise be empty according to available experimental data (Fig. 2, [2]). Intriguingly, the suggested decoupling allows a novel representation of the known *exchange forces* in nuclear physics because the transition of the decoupled electron from one proton to the other evidently implies a proton-neutron exchange.

Note that the indicated decoupling of the neutron is impossible for 20$^{\text{th}}$ century physics. Note also that, in the preceding Deuteron model based on planar singlet couplings of Sect. IV-2.5, p. 171 of [78], the decoupling here considered is impossible while, for the axial triplet coupling the decoupled electron remains with null total angular momentum due to its motion within the nuclear medium, by therefore confirming the uniqueness and importance of the axial triplet coupling (Fig. 1). Note finally that the decoupling of nuclear neutrons into protons and electrons implies the reduction of all matter in the universe to protons and electrons.

C.2 Representation of nuclear stability despite repulsive protonic forces

Additionally, stable nuclei should not exist in nature according to quantum mechanics because equal charge nuclear protons *repel* each other with an extremely big Coulomb force of the order of hundreds of Newtons Eq. (1).

We here indicate, also apparently for the first time, that hadronic mechanics can indeed represent this second problem of nuclear stability via a mechanism similar to the achievement by quantum chemistry of a strongly *attractive* force between the *identical* electrons of valence couplings [72–74].

Let us consider the nucleus with the minimal number of proton pairs, which is evidently given by the Helium $He(2,4,0)$ [1]. Various measurements [3] have established that the Helium has a charge radius of 1.678 fm, against the radius of two protons and two neutrons each having the value of 0.841 fm [4] with the total radius of 1.678 fm .

The above measurements confirm the primary assumption of hadronic mechanics, according to which nuclei are composed by extended protons and neutrons in conditions of partial mutual penetration of their dense charge distributions with ensuing non-Hamiltonian interactions (Sect. 2).

Let us assume in first approximation that the Helium is a quantum mechanical (qm) bound state of two Deuterons with anti-parallel spins

$$He(2,4,0) = [\, D(1,2,1_\uparrow), D(1,2,1_\downarrow)\,]_{qm} \,. \tag{92}$$

We assume the representation of the non-Hamiltonian interactions via non-unitary transform (47) of the quantum model (92) thus yielding the expression

$$\begin{aligned} & U\left[\frac{1}{2m}\,\delta^{ij}p_ip_j + V_c(r)\right]|\,\psi(r)\,\rangle\, U^\dagger = \\ & = \left\{-\frac{1}{m}\,\hat\partial_{\hat r}\hat\partial_{\hat r} + V_c(\hat r)\left[1-\frac{V_h(\hat r)}{V_c(\hat r)}\right]\right\}\,|\,\hat\psi(\hat r)\,\rangle = \\ & = \left[-\frac{1}{m}\,\hat\Delta_{\hat r} - K_h'\,\frac{e^{b\hat r}}{1-e^{b\hat r}}\right]|\,\hat\psi(\hat r)\,\rangle = E_h\,|\,\hat\psi(\hat r)\,\rangle\,, \end{aligned} \tag{93}$$

where the last expression has been reached by "absorbing" the Coulomb potential into the Hulten potential as in (50).

Consequently, the above analysis confirms that non-linear, non-local and non-potential nuclear interactions due to the mutual penetration of nucleons can be so strongly attractive to overcome repulsive Coulomb force between protons.

It is easy to see that the hadronic conversion of the repulsive Coulomb into a strongly attractive Hulten-type or other potentials also applies for other nuclear potentials, such as the *Yukawa potential* [147], the *Woods-Saxon potential* [148] and other potentials or their combination [149].

C.3 Representation of the Helium data

We now show that, following the overcoming of the repulsive ptotonic forces, non-Hamiltonian interactions remain so strong to represent the characteristics of Helium.

Note that the representation of the spin and magnetic moment of Helium follows from the antiparallel Deuteron spins of model (92). The representation of the rest energy, mean life and charge radius of Helium can be done via the hadronic structure model of the pion (Sect. 5.1, p. 827 on, [82]) and of the Deuteron (Sect. IV-2.5, p. 171 on, [78])

$$\begin{aligned} & \left[\frac{1}{r^2}\left(\frac{d}{dr}r^2\frac{d}{dr}\right) + \tilde m_d\left(E + K_h'\frac{e^{-br}}{1-e^{-br}}\right)\right] = 0\,, \\ & E_{he} = 2E_d - E_{be} = 3.7284\times 10^3\ \text{MeV}\,, \\ & \tau^{-1} = 2\pi\lambda^2\,|\hat\psi(0)|^2\,\frac{\alpha^2 E_1}{\hbar} = \infty\,, \\ & R = b^{-1} = 1\ \text{fm}, \end{aligned} \tag{94}$$

where $\hat m = m/\rho$ is the *iso-renormalized mass* of the Deuteron, that is, the mass renormalized from non-Hamiltonian interactions (Eq. (5.1.7b), p. 833, of [82]) here adjusted for Helium

$$\begin{aligned} & \tilde m = \tilde E_d = \frac{E_d}{\rho} = \frac{E_{he}}{2} = 1.8542\times 10^3\ \text{MeV}\,, \\ & \rho = 1.0188 > 1\,, \end{aligned} \tag{95}$$

The solution of Eqs. (94) was studied in all details in Sect. 5.1, p. 836 on, [82] (see also the recent review in [29]) and reduced to the numeric values of two parameters denoted k_1 and k_2, Eqs. (5.1.32a) and (5.1.32b), p. 840 [82], that become in our case

$$\tau = \frac{48\times(137)^2}{4\pi bc}\,\frac{k_1}{(k_2-1)^3} = \infty\,, \tag{96}$$

$$E_{he} = k_1[1-(k_2-1)2]\,\frac{2\bar{h}c}{b} = 3.7284\times10^3\,\mathrm{MeV}\,, \qquad (97)$$

with numeric solutions

$$k_2 = 1\,, \quad k_1 = 4.9\,, \qquad (98)$$

that should be compared with the numeric solutions for the meson octet of [29, 82].

Intriguingly, the known finite spectrum of the Hulten potential (see Eq. (5.1.20), p. 837, [82])

$$BE_h = -\frac{1}{4K_h k_2}\left(\frac{k_2}{n}-n\right)^2\,, \quad k_2 = K_h\,\frac{\tilde{E}_d}{\hbar^2 b^2} = 1\,, \qquad (99)$$

admits *only one value, Helium*, for $n = 1$, with null value of the binding energy, $BE_h = 0$, as expected for the sole *non-potential* interactions of model (94), since the representation of the Helium binding energy requires the addition of a *potential* force here left to interested readers.

In conclusion, the above model confirms that nuclear forces are some of the most complex forces in nature, since they include a linear, local and potential component represented by the Hamiltonian which is responsible for nuclear binding energies, plus a non-linear, non-local and non-potential component represented by the isotopic element which is responsible for the nuclear stability.

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