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COMPATIBILITY OF SUPER-/SUB-LUMINAL SPEEDS WITH EINSTEIN'S SPECIAL RELATIVITY AXIOMS

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Abstract

In this paper, we outline the rapidly growing literature on superluminal and subluminal speeds within physical media and show that, contrary to widespread beliefs, they are compatible with Einsteins axioms of Special Relativity, provided that they are realized with a covering mathematics more appropriate for the considered broader conditions.

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1. Notes on Super/SubLuminal Speeds.

As it is well known, the advent of the Lorentz transformations [1]

$$x'^1 = x^1, \;\; x'^2 = x^2, \tag{1.1a}$$

$$x'^3 = \gamma(x^3 - \beta x^4), \;\; x'^4 = \gamma(x^4 - \beta x^3), \tag{1.1b}$$

$$\beta = \frac{v}{c}\,, \;\; \gamma = \frac{1}{\sqrt{1-\beta^2}}. \tag{1.1c}$$

and their extension by Poincaré [2] (hereon referred to as the *Lorentz-Poincaré (LP) symmetry*) leave invariant the line element in Minkowski space-time $M(x, \eta, I)$

$$x^2 = x_1^2 + x_2^2 + x_3^2 - t^2c^2 = x^\mu \eta_{\mu\nu} x^\nu, \; \mu, \nu = 1, 2, 3, 4, \tag{1.2b}$$

$$x = (x^\mu) = (x_1^\mu - x_2^\mu), \; x^4 = t, \; \eta = Diag.(1, 1, 1, -c^2), \; I = Diag.(1, 1, 1, 1), \tag{1.2a}$$

and are at the foundation of Einstein's [3] axioms of Special Relativity (SR).

As it is also well known,symmetry (1.1) identifies the *maximal causal speed* for the conditions clearly expressed by Lorentz, Poincaré and Einstein [1-3] and experimentally confirmed, namely, for *exterior dynamical problems,* consisting of *point particles and electromagnetic waves propagating in vacuum* (conceived as empty space).

In fact, the light cone, e.g., for infinitesimal displacements in (3,4)-dimensions

$$(\delta x^3)^2 - (\delta t)^2 c^2 = 0, \tag{1.3}$$

establishes the maximal causal speed in vacuum

$$\frac{\delta x^3}{\delta t} = V_{max}^{vacuum} = c. \tag{1.4}$$

For decades, *faster than light speeds* (also called "superluminal speeds") were essentially ignored because they would violate causality and other physical laws. Nevertheless, with the passing of time the study of superluminal speeds became inevitable.

Nowadays, a search on superluminal speeds at the various archives in the internet shows the existence of a large number of papers published in refereed journals, thus suggesting a study on the problem of the causal and time invariant formulation of superluminal speeds.

Under a literature on superluminal speeds of such a dimension, we regret being unable to provide a comprehensive review, and are forced to quote a few representative illustrations of the studies considered in this paper, *superluminal speeds of ordinary masses or electromagnetic waves,* by deferring the study of tachyons (see, e.g., contributions by E. Recami and his group [42,43]) to a separate paper.

To our knowledge, studies of superluminal speeds were first motivated by the Doppler interpretation of the Hubble law [4] on the cosmological redshift of light

$$z = \frac{\lambda_o}{\lambda_e} - 1 \approx Hd \equiv \frac{v}{c}. \tag{1.5}$$

where λ_o (λ_e) is the wavelength of light at the origin (that observed on Earth), d is the distance of a galaxy from Earth, and H is the Hubble constant.

In fact, following the dismissal of Zwicky [5] 1929 interpretation of the cosmological redshift as being due to loss of energy by light to intergalactic media, the expectation of galaxies at the edge of the universe with superluminal speeds $v \gg c$ became unavoidable.

As an example, in 1966, Rees [6] attempted the reconciliation of superluminal galactic speeds with SR limit (1.4) by studying the possibility that superluminal speeds are somewhat illusory. These initial studies were continued by reaching the contemporary conjecture that space itself is expanding so as to achieve compatibility of superluminal galactic speeds with SR limit (1.4) (see, e.g., Ref. [7] and counterarguments in Section 4).

Nowadays, it is well known (see, e.g., Ref. [7]) that the Doppler interpretation $z = v/c$ of the experimental data on the cosmological redshift $z = Hd$ requires the conjecture that billions of galaxies at the edge of the known universe travel at superluminal speeds, thus forcefully suggesting alternative studies due to the excessive implausibility of the conjecture with or without the expansion of space itself.[1]

[1] It should be recalled that Hubble, Fritz, de Broglie and other famous scientists died without accepting the expansion of the universe because the *acceleration* of the expansion (which is inherent in the Hubble's law $z = Hd$) occurs radially in all possible direction from Earth, thus implying a return to the Middle Ages with Earth at the center of the universe. Santilli [33,42-45] has elaborated this historical point, e.g., with Diagram 33 of Ref. [45] showing that two galaxies at double distances from Earth, thus having double speed of expansion, may have the same distance and, therefore, the same speed when seen from another galaxy.

Independently from these cosmological studies, beginning with his graduate studies in the 1960s, Santilli conducted systematic mathematical, theoretical and experimental research on *interior dynamical problems,* consisting of *extended particles and electromagnetic waves propagating within physical media* (see representative works [9-16]).

The conceptual foundations of these studies can be summarized as follows. When elementary particles move in vacuum, their only possible acceleration is that via action-at-a-distance potential interactions (more technically known as *variationally selfadjoint (SA) interactions* [15a]). In this case, it is easy to see that the achievement in vacuum of the speed of light c requires infinite energy and, therefore, the surpassing of the speed c in vacuum by ordinary masses or electromagnetic waves is excluded.

In 1981, Santilli [10] argued that the situation is substantially different when elementary particles move in interior conditions because, in this case, accelerations are the result, not only of conventional SA interactions, but also of "contact" non-potential interactions (technically known as *variationally nonselfadjoint (NSA) interactions,* [15a]) for which the notion of "potential energy" has no physical value or meaning.

It was then easy to see already in the 1980s that *under NSA interactions the local speeds of ordinary masses within physical media are unrestricted,* thus being arbitrarily bigger (or smaller) than c depending on local conditions of density, temperature, frequencies and other physical data.

The analytic background of the studies on interior conditions is given by the "true Hamilton's equations," those with external terms not derivable from a Hamiltonian

$$\frac{dr}{dt} = \frac{\partial H(r,p)}{\partial p}, \quad \frac{dp}{dt} = -\frac{\partial H(r,p)}{\partial r} + F^{NSA}(t,r,v,...), \tag{1.6}$$

and their operator counterpart, which are specifically set for the representation of open irreversible processes we cannot possibly review here [16].

The mathematical backgrounds of the studies is given by the *Lie-admissible covering of Lie's theory* since the true Hamilton's equations emerged since the 1960s [9] as admitting a Lie-admissible algebra in the brackets of their time evolution when properly written (see the more recent memoir [13] for details).

To understand the complexity of the problem, let us recall that physical theories can be claimed to have physical value if and only if they verify the *invariance over time,* namely, they predict the same numerical values under the same condition s at different times. It is easy to see that the true Hamilton's equations and their operator counterpart violate this crucial condition because they are non-canonical and non-unitary by conception.

The achievement of invariance over time for non-canonical and non-unitary theories required the construction of a new mathematics, today known as *Lie-admissible genomathematics* we cannot possibly review here (see mathematics studies [11-13] and monographs [16]).

Under these conditions, Hubble's law $z = Hd$ is evidently valid without requiring Earth at the center of the universe, but its Doppler's interpretation $z = v/c$ becomes inconsistent. Note that the far reaching conjecture of the expansion of space itself would eliminate Earth at the center of the universe only in case of a *uniform* expansion, but not under the acceleration of the expansion (see Refs. [33,42-45] for details).

A main application of these studies has been the first achievement at both nonrelativistic and relativistic levels of an *exact* representation of *all* characteristics of the neutron in its synthesis insider stars according to Rutherford's "compression of the hydrogen atom," namely, from a proton and an electron according to the known reaction (for brevity, see review [14])

$$p^+ + e^- \rightarrow n + \nu, \tag{1.7}$$

The main technical difficulty was due to the fact that the rest energy of the neutron is $0.872\ MeV\ bigger$ than the sum of the rest energies of the proton and of the electron, under which condition we would need "positive binding energies" which are anathema for quantum mechanics, since they cause the physical inconsistency of the Schrödinger equation.

Santilli's main point is that, even though there exist indeed particles with "point-like charges" (such as the electron), *there exist no "point-like wavepackets" in nature.* Therefore, Rutherford's compression of the extended wavepacket of the electron within the hyperdense medium inside the proton generates NSA interactions under which a solution of synthesis (1.7) has been indeed found [14].

The main mechanism is that contact interactions are NOSA and, therefore. they are non-unitary. The non-unitary image of Schrödinger equation achieves consistency under "positive binding energies" thanks to a new renormalization of the rest energies of the constituents (called *isorenormalization*) [14,16,34]).

The aspect important for this paper is that an apparent necessary condition for the representation of *all—* characteristics of the neutron in synthesis (1.7) is that the constituents of the neutron travel at (tangential) superluminal speeds.

Intriguingly, systematic plots of experimental data in hadron physics conducted in monograph [16d] *without* the aprioristic assumption of the Lorentz symmetry have confirmed superluminal speeds within the interior of hadrons in numerous cases, such as: phenomenological fits via gauge theories; anomalous meanlives of unstable hadrons with speed; the Bose-Einstein correlation; and other cases directly relevant for the study of superluminal speeds.

We regret not being able to review these phenomenological fits and related literature to avoid a prohibitive length of this paper. Nevertheless, their knowledge is important for a technical understanding of this paper.

We should add that, as clarified by Wall [17], the superluminal speeds of hadronic constituents here considered are not referred to tachyons as conventionally defined (see, e,g, Ref. [42]) because, as we shall see in the next sections, interior physical media cause a deformation (called "mutation") of the light cone with superluminal speeds of real valued masses. Therefore, the existence of tachyons (called in the field of this paper *isotachyons*) is shifted for speed beyond the maximal causal speed within physical media which are generally bigger than c, as shown in the next section.

More specifically, we are not excluding possible tachyonic contributions in the structure of hadrons or in other physical conditions [42]. The only point we would like to clarify is that, under the validity of isotopic theories for the hadronic structure, the speed characterizing tachyons has to be shifted beyond c (see, later on, Eq. (3.3) and related arguments).

Independently from Santilli's research, additional relevant studies on superluminal speeds are the experimental works initiated in 1992 by Enders and Nimtz [18] (see also the more recent paper [19] for additional references and paper [20]) suggesting apparent superluminal propagation of electromagnetic waves within certain physical guides.

The reconciliation of superluminal speeds with SR limit (1.4) is generally attempted by assuming that we are dealing with a "tunnel effect." However, in our view, tunnel effects generally refer to passages through a barrier, thus for distances of $1\ fm$ covered by the uncertainty principle, and not for travel over lengths of several meters, as occurring for Refs. [18-20].

Hence, it is well possible that, in reality, Refs. [18-20] deal with an interior dynamical problem in which case superluminal speeds are due to NSA interactions of electromagnetic waves with the guides (including the so-called "stray fields" that are known to be NSA) under which interactions superluminal speeds are quite natural. Additional cases of superluminal speeds of ordinary masses can also be treated as interior dynamical problems, but we regret not being able to treat them here for brevity.

Yet an additional case relevant for studies on superluminal speeds is the recent controversy at CERN on superluminal or sub-luminal neutrinos, since the original 2011 announcements [21] indicated superluminal neutrino speeds, while the subsequent paper [22] indicated subluminal speeds.

We would like to point out that the truly fundamental issue for Refs. [21,22] is the *identification of a causal and time invariant formulation of motion in interior conditions,* since for both views [21,22] neutrinos travel *underground* from CERN to the Gran Sasso Laboratory. In these conditions we have the absence of light, the absence of inertial reference frames, and other known features preventing even the correct formulation of invariance (1.1), let alone its experimental verification.

In the absence of the appropriate basic theory, the underground speed of neutrinos remains unsettled because the slightest modification of any form factor, parameter or data elaboration can produce subluminal results in Ref. [21] and superluminal results in Ref. [22], as experts in the field can verify.

Besides superluminal speeds, the study of interior dynamical problems inevitably requires the inclusion of *subluminal speeds,* namely, speeds of electromagnetic waves within physical media *smaller* than c. This is typically the case for the propagation in water of infrared or radio waves for which the reduction to photon is not possible due to the large wavelengths, yet their speed is known to be $2c/3$ [16a].

In this paper, we shall show that, when formulated with the appropriate mathematics, *Einstein's SR axioms do indeed admit fully causal and time invariant superluminal or subluminal speeds depending on the physical characteristics of the medium at hand.*

Since the Lie-admissible genomathematics is excessively advanced for the limited scope of this paper, we shall restrict our study to the particular case of interior events that are reversible over time, namely, events in which we ignore in first approximation the dispersion of light (see Figure 1). The subclass of reversible interior events can be

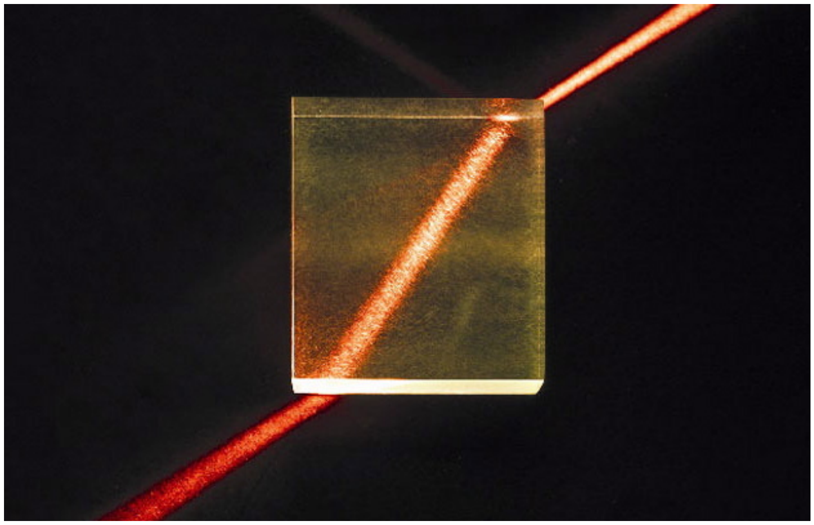

Figure 1: An illustration of the historical Lorentz problem at the foundation of this paper, the invariance of locally varying speeds of light within physical media, here illustrated with the variation of speed from air to water and then back to air. Its solution is outlined in this paper in first approximation by ignoring the dispersal of light in order to avoid the excessive complexities for the treatment of irreversible processes [13,16].

well treated via the simpler *Lie-isotopic subclass of Lie-admissible genomathematics* indicated in the next section, thus rendering this paper more accessible to the general physics audience.

Nevertheless, the reader should keep in mind that the propagation of light within physical media is an intrinsically irreversible event, as established by the simple evidence that media become warmer when traversed by light. Consequently, we should stress that the isotopic formulations presented in this paper only permit a first approximation of interior conditions, and that the proper causal and time invariant representation of irreversible interior events requires a study at the full Lie-admissible level [13,16].

2. Solution of the Historical Lorentz Problem.

As it is well known to physics historians, Lorentz first attempted the achievement of the invariance of the speed of electromagnetic waves of his time, namely, the locally *varying* speed within physical media here referred to infrared, radio and other large wavelengths not admitting a consistent reduction to photons (see Section 4 for the general case)

$$C = \frac{c}{n(t, r, v, e, \rho, \omega, \tau, ...)}, \tag{2.1}$$

where n is the familiar index of refraction with a rather complex functional dependence on local variables, such as time t, coordinates r, speeds v, energy e, density ρ, frequency ω, temperature τ and other variables.

Due to insurmountable technical difficulties, Lorentz was solely able to achieve invariance for the *constant* speed c of electromagnetic waves in vacuum, resulting in the celebrated transformations (1.1) leaving invariant line element (1.2a).

Santilli has studied for decades the solution of the *historical Lorentz problem,* namely, the achievement of the universal invariance of all possible locally varying speeds of electromagnetic waves within physical media, Eq. (2.1), which case evidently admits the constant speed c in vacuum as a particular case.

As a first step, when a member of MIT from 1974 to 1978, Santilli realized that Lorentz's inability to achieve the invariance of speeds (2.1) was due to insufficiencies of the basic theory, Lie's theory, because such a theory is strictly linear, local-differential and potential-Hamiltonian, while the invariance of speeds (2.1) is a strictly non-linear, non-local/integral and non-potential, thus non-Hamiltonian problem.

The results of these initial studies were released in monographs [15] (that originally appeared as MIT preprints to be subsequently published under affiliation to Harvard University). A main aspect of these studies is their conception as *isotopic* (intended in the Greek meaning of being "axiom preserving") lifting of the various branches of Lie's theory into such a form to admit the treatment of non-linear, non-local and non-Hamiltonian systems.

The proposal was centered in the isotopic lifting of the unit of the Lorentz symmetry, $I = Diag.(1,1,1,1)$, into a quantity $\hat{I}$ (such as a function, a matrix, an operator, etc.) with an arbitrary functional dependence on all needed local variables, under the sole condition of being positive-definite, thus invertible,

$$I = Diag.(1,1,1,1) \ \rightarrow \ \hat{I} = \hat{I}(t, r, v, e, \rho, \omega, \tau, ...) = 1/\hat{T}(t, r, v, \rho, \omega, \tau, ...) > 0. \tag{2.2}$$

which lifting remains fixed for the interior problem considered.

For consistency, the lifting of the unit required the compatible lifting of the conventional associative product between arbitrary quantities A and B of the type

$$AB \ \rightarrow \ A\hat{\times}B = A\hat{T}B, \tag{2.3}$$

under which $\hat{I}$ is indeed to the right and left unit of the theory,

$$\hat{I}\hat{\times}A = A\hat{\times}\hat{I} \equiv A, \tag{2.4}$$

for all elements A of the set considered.

Following basic assumptions (2.2)-(2.4), Santilli constructed a step by step isotopic generalization of the various branches of Lie's theory, including [15b]:

1) The isotopic lifting $\hat{\xi}(L)$ of the universal enveloping associative algebra $\xi(L)$ of a n-dimensional Lie algebra L with (Hermitean) generators $X_i, \ i = 1, 2, ..., n$, and infinite-dimensional isotopic basis (today known as the *Poincaré-Birkhoff-Witt-Santilli isotheorem* [35-42]):

$$\hat{I}, \ \ X_k, \ \ \hat{X}_i\hat{\times}\hat{X}_j, \ i \leq j, \ \ \hat{X}_i\hat{\times}\hat{X}_j\hat{\times}\hat{X}_k, \ i \leq j \leq k, \ \ldots; \tag{2.5}$$

2) The isotopic liftings of Lie algebras with closure rules (today called *Lie-Santilli isoalgebras* [*loc. cit.*]

$$[X_i \hat{,} X_j] = X_i\hat{\times}X_j - X_j\hat{\times}X_i = C_{ij}^k X'_k \tag{2.6}$$

3) The corresponding isotopic lifting of Lie's transformation groups (today called *Lie-Santilli isogroups* [*loc. cit.*]), e.g., here expressed for the time evolution

$$A(t) = U(t)A(0)U(t)^{\dagger} = [e^{H\hat{T}ti}]A(0)[e^{-it\hat{T}H}]; \tag{2.7}$$

and the isotopies of the representation theory.

The above isotopies clearly show the non-linear, non-local (integral) and non-Hamiltonian character of the isotopic theory due to the appearance of a positive-definite but otherwise arbitrary quantity $\hat{T}$ in the *exponent* of the group structure.

The representation of interior systems is then achieved via the representation of all SA interactions by means of the conventional Hamiltonian $H(r,p)$, and the representation of all NSA interactions by means of the generalized unit $\hat{I}$ (see Refs. [16,34] for concrete examples in classical and operator mechanics).

Following the construction of the isotopies of Lie's theory, Santilli introduced in letter [23] of 1983 the following isotopies of Minkowski space (1.2) (today known as the *Minkowski-Santilli isospace* [*loc. cit.*]) with the most general possible nonsingular and symmetric line element (thus including all possible Minkowskian, Riemannian, Fynslerian and other line elements in (3+1)-dimensions)

$$\hat{x}^{\hat{2}} = x^{\mu}(\hat{T}^{\rho}_{\mu}\eta_{\rho\nu}x^{\nu} = x^{\mu}\hat{\eta}_{\mu\nu}x^{\nu} = \frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2\frac{c^2}{n_4^2}, \tag{2.8a}$$

$$n_{\mu} = n_{\mu}(t, r, v, e, \rho, \omega, \tau, ...) > 0, \;\; \mu = 1, 2, 3, 4, \tag{2.9b}$$

$$\hat{T} = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2) > 0, \tag{2.8c}$$

$$\hat{I} = 1/\hat{T} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2) > 0, \tag{2.8d}$$

where: the n's are called the *characteristic quantities* of the medium considered; n_4 is the conventional index of refraction providing a geometrization of the density of the medium normalized to the value $n_4 = 1$ for the vacuum; n_1, n_2, n_3 provide a geometrization of the shape of the medium considered normalized to the values $n_1 = n_2, n_3 = 1$ for the sphere; the general *inhomogeneity* of the medium is represented by the dependence of the characteristic quantity on the local variables (e.g., the elevation for the case of our atmosphere); and the general *anisotropy* of the medium (e.g., the anisotropy of our atmosphere caused by Earth's rotation) is represented by different values of the type $n_4 \neq n_s$.[2]

Note that, for exterior dynamical problems, homogeneity and isotropy equally occur in all directions. By contrast, within a physical medium inhomogeneity and anisotropy requires the selected of a given direction in space n_s due to variations for different directions.

Following the construction of the isotopies of Lie's theory and of Minkowski spacetime, Santilli solved the historical Lorentz problem in page 551 of letter [23] via the lifting of the Lorentz symmetry characterized by the isotopic element (2.8c). This resulted in the

[2] Scientific caution is suggested before dubbing the n-characteristic quantities as "free parameters" because that would imply that, e.g., the Schwartzchild metric is characterized by four free parameters.

generalized transformations (Eqs. (15) of Ref. [23]), today known as the *Lorentz-Santilli (LS) isotransforms* [35-42] which we write in the currently used symmetrized form

$$x^{‘1} = x^1, \;\; x^{‘2} = x^2, \tag{2.9a}$$

$$x^{‘3} = \hat{\gamma}(x^3 - \hat{\beta}\, \frac{n_3}{n_4} x^4), \;\; x^{‘4} = \hat{\gamma}(x^4 - \hat{\beta}\, \frac{n_4}{n_3} x^3), \tag{2.9b}$$

$$\hat{\beta} = \frac{v_3/n_3}{c_o/n_4}, \;\; \hat{\gamma} = \frac{1}{\sqrt{1-\hat{\beta}^2}}, \tag{2.9c}$$

leaving invariant the isoline element (2.8a), thus providing indeed the invariance of the varying speeds of light (2.1) (see Ref. [34b] for the general treatment).

Jointly with the above classical formulation, Santilli constructed the corresponding operator image [24] of the above isotransformations, and then constructed the isotopies of every main aspect of the LP symmetry, including the isotopies of: the rotational symmetry [25]; the SU(2)-spin symmetry [26]; the Poincaré symmetry [27]; the spinorial covering of the Poincaré symmetry [28]; the SU(2)-isospin symmetry and local realism [29]; and the isotopies of the Minkowskian geometry [30].

The resulting isosymmetry, today known as the *Lorentz-Poincaré-Santilli (LPS) isosymmetry,* was proved in Refs. [31,32] to be "directly universal" for all infinitely possible non-singular and symmetric space-times in $(3+1)$-dimensions, thus providing the universal symmetry of all possible Riemannian, Fynslerian, and other possible line elements, with a trivial extension to arbitrary space-time dimensions, such as those for the De Sitter symmetry.

Systematic studies can be found in monographs [33.34], while independent studies can be found in monographs [35-42] and references quoted therein. A few comments are now in order to prevent possible misrepresentations that generally remain undetected by non-expert in the field.

To illustrate the universality of isoinvariant (2.8) and related isosymmetry (2.9) for all possible, symmetric and non-singular space-times in (3+1)-dimensions, let us note that they include as particular case all possible Riemannian line elements, such as the Schwartzchild's line element [27]

$$ds^2 = r^2(d\theta^2 + sin^2 d\theta^2 + d\phi^2) +$$
$$+ (1 - \frac{2 \times M}{r})^{-1} \times dr^2 - (1 - \frac{2 \times M}{r} \times dt^2 \tag{2.10a}$$

$$\hat{T}_{sch} = Diag.[1, 1, (1 - \frac{2 \times M}{r})_r^{-1}, (1 - \frac{2 \times M}{r})_t], \tag{2.10b}$$

$$\hat{I}_{sch} = Diag.[1, 1, (1 - \frac{2 \times M}{r})_r, (1 - \frac{2 \times M}{r})_t^{-1}], \tag{2.10c}$$

where one can see that the gravitational singularity is that of the isotopy, namely, the infinite value of the isotopic element and the null value of the isounit. In fact, Ref. [27]

was primarily intended to indicate the achievement of the universal symmetry for all possible (non-singular) Riemannian line elements.

A rather popular belief during the 20th century physics was that interior dynamical systems are not essential because they can be reduced to elementary particles moving in vacuum, thus recovering at the elementary level exterior conditions without non-linear, non-local and non-Hamiltonian interactions. This belief was disproved by the following:

NO REDUCTION THEOREM [13,34]: A macroscopic, nonconservative system cannot be consistently reduced to a finite number of elementary particles all in conservative conditions and, vice versa, a finite number of elementary particles all in conservative conditions cannot consistently yield a macroscopic nonconservative system under the correspondence or other principles,

Stated in different terms, the reduction of interior to exterior systems implies the belief that entropy and thermodynamical laws are "illusory" because, when interior systems are reduced to elementary particle constituents, entropy and thermodynamical laws "disappear." The above No Reduction Theorem establishes that nonconservative (thus, NSA) interactions, rather than "disappearing," originate at the most elementary level of nature.

As a concrete example, the above No Reduction Theorem is verified by a spaceship during reentry in atmosphere because its non-linear, non-local and non-Hamiltonian interactions originate at the most elementary level, that of the deep mutual penetration of the wavepackets of peripheral atomic electrons of the spaceship with the wavepackets of the electrons of atmospheric atoms.

The next aspect needed for a serious understanding of the content of this paper is the necessity to verify the time invariance indicated in the preceding section, namely, the prediction of the same numbers under the same conditions at different times.

Santilli selected a generalization of the unit for the representation of non-linear, non-local and non-Hamiltonian interactions for the intent of achieving the much needed time invariance, since the unit is the basic invariant of any theory.

However, it is easy to see that the representation of non-Hamiltonian interactions via the isounit is not sufficient *per se* to achieve the needed time invariance because isotopic theories are non-canonical at the classical level and non-unitary at the operator level by conception and construction [16,33,34]. It is then easy to see that the isounit is not preserved by the time evolution of the theory, e.g., Eq. (2.7)

$$\hat{I} \to \hat{I}' = U\, \hat{I}\, U^\dagger \neq \hat{I}, \;\; UU^\dagger \neq I \qquad 11.$$

But the isounit represent interior conditions. Therefore, its lack of conservation in time implies the transition over time from one interior system to another (e.g., from the synthesis of the neutron (1.7), for instance, to a nuclear fusion).

This occurrence is known under the name of *Theorem of Catastrophic Mathematical and Physical Inconsistencies of Non-Canonical and Non-Unitary Theories* when formulated with the mathematics of canonical and unitary theories, respectively. Regrettably, we cannot review this theorem for brevity and must refer the reader to works [13,33,34].

The resolution of the inconsistencies caused by the lack of time invariance required decades of additional research by Santilli and a number of colleagues. The solution was finally achieved following the construction of a new mathematics, today known as *Santilli isomathematics,* characterized by the isotopic lifting of the *totality* of the quantities and their operation of the mathematics used for exterior problems.

When classical non-canonical and operator non-unitary theories are elaborated with the appropriate classical and operator isomathematics, respectively, the invariance over time of numerical predictions is regained, thus offering the mathematical and physical consistency needed for applications.

3. Compatibility of Arbitrary Speeds with Einstein's SR Axioms.

The locally varying speeds of electromagnetic waves propagating within physical media left invariant by the LPS isosymmetry (2.9) are completely unrestricted and can, therefore, be smaller, equal or bigger than the speed of light in vacuum,

$$C = \frac{c}{n_4} \; <=> \; c. \tag{3,1}$$

This is due to the unrestricted functional dependence of the isotopic element (2.8c), except for the condition of being non-singular.

It is then easy to see that the *maximal causal speed* in Minkowski-Santilli isospaces is arbitrarily bigger, equal or smaller than c. In fact, the mutated light cone, called *light isocone,* in the $(s, 4)$-dimensions is given by

$$\hat{x}^{\hat{2}} = \frac{x_s^2}{n_s^2} - t^2 \frac{c^2}{n_4^2} = 0, \tag{3.2}$$

and evidently characterizes the maximal causal speed within physical media

$$V_{max,s}^{media} = c\, \frac{n_s}{n_4} \; <=> c. \tag{3.3}$$

where, as indicated in Section 1, the selection of a space direction s is necessary since physical media are generally inhomogeneous and anisotropic.

Note the need to use a covering of the speed of light for maximal causal speed under the LPS isosymmetry because interior dynamical problems are generally opaque to light. Note also that the causal character of speeds (3.3) is guaranteed by the LPS isosymmetry in exactly the same way as the LP symmetry guarantees the causal character of c.

It has been popularly believed throughout the 20th century that any deviation from the speed of electromagnetic waves in vacuum implies a "violation of Einstein's SR." In this section, we show that this belief is not technically correct, because *Einstein's SR axioms do admit arbitrary causal speeds,* provided that they are realized via the appropriate mathematics.

To begin, Lorentz transformations (1.1) provide the invariance of the "constant" speed c without any identification of its numerical value, which value is set by measurements.

Therefore, *Lorentz transformations equally apply for an arbitrary constant speed $C = c/n_4$ within physical media,* with known value in water

$$C_{water} = \frac{c}{n_4} = \frac{2}{3}c, \;\; n_4 = \frac{3}{2}. \tag{3.4}$$

in which case no violation of Einstein SR axioms can be claimed.

Additionally, the replacement in the *conventional* transforms (1.1) of the speed of light c with maximal causal speed (3.3) yields, identically, the LPS isotransformations (2.9), as one can readily verify,

$$x^{\prime 1} = x^1, \;\; x^{\prime 2} = x^2, \tag{3.5a}$$

$$x^{\prime 3} = \gamma(x^3 - \beta x^4), \tag{3.5b}$$

$$x^{\prime 4} = \gamma(x^4 - \beta x^3), \tag{3.5c}$$

$$\beta = \frac{v}{V_{max}^{media}}, \;\; \gamma = \frac{1}{\sqrt{1-\beta^2}}. \tag{3.5d}$$

Also, *the Lorentz-Santilli isosymmetry (2.9) is locally isomorphic to the conventional Lorentz symmetry by conception and construction* to the point of preserving the original structure constants [23,33,34]. Therefore, no claim that isotransforms (2.9) violate Einstein's SR axioms can be consistently voiced due to the very conception and technical realization of the isotopies of SR.

More technically, when represented on Minkowski-Santilli isospace over the isofield of real numbers [11], light isocone (3.2) becomes the perfect cone with the same maximal causal speed c as that valid in empty space [33,34].[3]

This is due to the fact that the cone axes are indeed mutated under isotopies from their original unit value to new values

$$(1_s, 1_4) \; \rightarrow \; (1/n_s^2, 1/n_4^2), \tag{3.6}$$

but, jointly, the related units are mutated by the *inverse* amounts,

$$(1_s, 1_4) \; \rightarrow \; (n_s^2, n_4^2), \tag{3.7}$$

thus preserving the original Minkowskian light cone identically.[4]

[3]The mathematically correct formulation of the Minkowski-Santilli isospace is given by the isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$ defined over the field of isoreal numbers with unit $\hat{I}$ given by Eq. (2.8c), with $\hat{x} = x\hat{I}$ as a condition to be an isonumber and $\hat{\eta} = \hat{T}\eta$ (see Ref. [30] for details).

[4]The preservation of c as the maximal causal speed in Minkowski-Santilli isospace over isofields is due to the need, for consistency, that the isoline element is an element of the real isofield, thus requiring the multiplication by the isounit (2.2), and the same should be assumed for the conventional line element. Consequently, we have the identities for constant isounits

$$x^2 = (x^\mu \eta_{\mu\nu} x^\nu) I = [x^\mu (\hat{T}\eta_{\mu\nu}) x^\nu]\hat{I} = \hat{x}^{\hat{2}}, \;\; \hat{I} = 1/\hat{T} = constant > 0,$$

and a more complex, but similar occurrence holds for the general case. the compatibility of arbitrary speeds with SR axioms is then illustrated again by the preservation of the numerical value of the line element.[16,33,34].

We finally illustrate the compatibility of arbitrary speeds with SR axioms via the following transformation of the Minkowskian coordinates

$$x^\mu \ \to \ \frac{x'^\mu}{n_\mu}, \tag{3.8}$$

under which the Minkowski line element (1.2a) is transformed into the isotopic image (2.8a), and Lorentz transformations (1.1) are turned into the LS transform by keeping in mind transformation of the speed, e.g.., along the third space direction

$$v = \frac{\delta x^3}{\delta t} \ \to \ v\frac{n_4}{n_3}. \tag{3.9}$$

By recalling that the metric of isotopic theories $\hat{\eta} = \hat{T}\eta$ includes as particular case all possible Riemannian metrics g, in this section we have attempted to indicate that *Einstein's SR axioms have a representational capability dramatically broader than that believed in the 20th century because, in addition to admitting arbitrary maximal causal speeds, they also admit interior and exterior gravitational models.*

In fact, in Ref. [3] Santilli has shown the treatment of *exterior gravitation* via SR axioms on the metric $\hat{\eta}(x) = \hat{T}(x)\eta = g(x)$, while maintaining the machinery of the Riemannian geometry (covariant derivative, Christoffel's symbols, etc.) and Einstein-Hilbert field equations under the universal LPS symmetry as a condition to achieve the above indicated invariance over time of numerical predictions. In this exterior case, the maximal causal speed is evidently that in vacuum c.

In the same Ref. [3], Santilli has shown that SR axioms equally admit *interior gravitational models,* this time, with an unrestricted functional dependence of the metric $\hat{\eta}(x, \rho,$-$\tau, \omega, ...) = \hat{T}(x, \rho, \tau, \omega, ...)\eta = g(x, \rho, \tau, \omega, ...)$ equally under the universal LPS symmetry, in which case the maximal local causal speed is arbitrarily bigger or smaller than c depending on local conditions.

Note that, under the full use of isomathematics, all the preceding enlargements of the conditions of applicability of EInstein's SR axioms can be formulated via conventional symbols as used in Eqs. (1.1)-(1.4), and merely subject them to different interpretations.

In fact, the variables x^μ can be interpreted as representing physical space-time coordinates with respect to the Lorentz unit $I = Diag.(1, 1, 1, 1)$, in which case we have the 20th century formulation of SR for exterior problems in vacuum.

Alternatively, we can consider the coordinates x^μ as being purely mathematical and assume the realization $x^\mu = x'^\mu/n_\mu$. In this case, when x'^μ are assumed as the physical coordinates and are referred to the isounit (2.8d), we realize SR axioms in such a way to have interior dynamical conditions with maximal causal speeds (3.3), exterior gravitation with the the characteristic quantities representing conventional Riemannian metrics, interior gravitation, and other possibilities.

The main point is that, at the purely abstract, realization-free level, we have one single formulation of SR, that conceived by Einstein.

3. Expected Implications.

In view of its majestic axiomatic structure and impressive experimental verifications for exterior dynamical conditions, the application of SR was systematically extended throughout the 20th century to the characterization of all possible physical conditions existing in the Universe, including those of interior dynamical problems.

Unfortunately, such an extension was generally done by *de facto* imposing the validity of the LP symmetry and by leaving essentially un-addressed a number of ensuing consistency problems.

A typical illustration studied by Santilli in various works (see, e.g. Ref. [33,34] and literature quoted therein), is the widespread claim that SR is exactly valid in water. In reality, such a claim is afflicted by a number of unresolved problems of axiomatic consistency, such as:

1) The central pillar of SR, the constancy of the speed of light, cannot be consistently formulated in water, let alone experimentally established, due to the absence in water of inertial reference frames, evidently due to the resistance experienced by any laboratory frame moving in water;

2) In water, electrons can travel faster than the local speed of light, as established by the Cherenkov light that can occur if and only if electrons travel faster than the local speed of light, thus causing evident causality problems that have not been entirely resolved in the refereed literature, to our knowledge;

3) In the event, to salvage causality, the speed of light in *vacuum* is assumed as being the maximal causal speed *in water,* we have the violation of the relativistic sum of speeds, because the sum of two speeds of light in water does not yield the speed of light in water; and other axiomatic problems.

As it is well known, the validity of SR in water is claimed via the reduction of electromagnetic waves to photons that are assumed to scatter among the water molecules, thus recovering SR at the level of photons propagating in vacuum.

It is important to indicate that attempts at recovering the validity of SR in water create additional problematic aspects, while resolving none on scientific, that is, quantitative grounds.

To understand the case, let us honor (and defend) the name of Albert Einstein by recalling that his reduction to photons was specifically referred to the *absorption* of light by the atoms of physical media, since atomic electrons can only have quantized orbits, as well known.

When the reduction of light to photons is extended to the *propagation* of light within physical media (that was never suggested by Einstein, to our knowledge), we have a number of generally unaddressed, let alone resolved problematic aspects, such as:

A) The reduction to photons is not possible for all electromagnetic waves, such as for infrared , radio and other waves with large wavelength;

B) The reduction of light to photons does not allow a consistent, quantitative representation of the angle of refraction of light in water (see Figure 1), since photons will scatter in all directions at the time of their impact with the water surface without a preferred direction of refraction;

C) The reduction of light to photons does not allow a consistent, quantitative representation of the dependence of the angle of refraction of light on the frequency of light so as to represent Newton's prism;

D) The reduction of light to photons does not allow a consistent, quantitative representation of the propagation of a light beam in water along a straight line (see also Figure 1), due to the impossibility for a large number of photons to propagate through a large number of atoms and nuclei along a straight line without appreciable scattering;

E) the reduction of light to photons does not allow a consistent quantitative representation of the *time irreversible* of the event due to the fact that light loses energy to the medium, as clearly established by the dispersion of a light beam (see also Figure 1), while 20th century treatments of photons are strictly reversible over time (this is due to the purely quadratic character of the Minkowski line element (1.2a) that, as such, admits with equal probabilities motions forward and backward in time, $t^2 = (\pm t)^2$).

All the above problematic aspects are resolved by the realization of the SR axioms permitted by Santilli isomathematics, today known as *IsoRelativity* (IR) [16,33,34,35-42].

The resolution of problematic aspects 1-3 is achieved by noting that the local speed of light within physical media is variable for isotopic axioms and that such axioms do not require inertial reference frames since, isotopic theories are non-linear, non-local and non-potential, thus non-conservative. In this way, isotopic theories avoid consistency problems in the formulation of the axioms.

Additionally, water can be safely assumed as being homogeneous and isotropic, in which case we have the values of the characteristic quantities $n_s/n_4 = 1$, and the maximal causal speed (3.3) in water assumes the value c. This prevents a violation of causality by the Cherenkov effect, while verifying the isorelativistic sum of speeds and other isotopic axioms (see again Ref. [16,33,34] for details).

The resolution of problematic aspects A-E can only be achieved, to our knowledge, via the return to the conception of light by Maxwell, Lorentz and others as being an *electromagnetic "wave" created and propagated by a universal substratum (the "ether").*[5]

In fact, such a historical conception of light with ensuing local speed (2.1) clearly resolves problematic aspects A, B and C. Problematic aspect D is resolved because the most plausible representation of the propagation of a light beam in water along a straight line is that for light being created and propagated by the universal *substratum* thus going through atoms and their nuclei without appreciable scattering.[6]

Problematic aspect E is resolved by the Minkowski-Santilli isogeometry [30] with iso-space-time (2.8a) because the isotopic element can indeed depend on time, $\hat{T} = \hat{T}(t, ...)$, thus allowing a first representation of irreversibility from primitive isotopic axioms with-

[5]It is important to note that, contrary to popular beliefs throughout the 20th century, the ether does not violate SR for the evident impossibility of identifying experimentally the "absolute reference frame," at rest with the medium.

[6]It may perhaps be interesting to note that IR allows the representation of yet another effect in water that cannot be even formulated via conventional treatments of SR, the apparent increase of dimensions of submerged objects. Such an increase is inherent in the very notion of *isocoordinates* since their numerical value must be the same as the conventional ones under isotopies, with resulting exterior and interior relation $r_{ext} \equiv \hat{r}_{int} = r_{int}\hat{I}_{int}$ for which $r_{int} > r_{ext}$ where in this case $\hat{I}_{int} < 1$.

out manipulations, while preserving Einstein's SR axioms at the abstract level.[7]

Extensive studies [9-16,23-42] have shown that problematic aspects 1-3 and A-E suffered by the application of conventional formulations of SR to water become greater when said formulations are applied to inhomogeneous and anisotropic physical media such as our atmosphere, or to the hyperdense media in the interior of hadrons, nuclei and stars.

In the author's view, the dominant aspect rendering inapplicable conventional formulations of SR for interior conditions is the linear, local and potential character of said formulations, while interior problems are structurally non-linear, non-local and non-potential, as indicated earlier.

A second important reasons is provided by the No Reduction Theorem indicated in Section 2. As an illustration, in the event light propagating within our atmosphere could be consistently reduced to photons, such a reduction would imply that the inhomogeneity and anisotropy of our atmosphere, its entropy and its thermodynamical laws and other physical features, are all "illusory" since they would all "disappear" at the level photons, while in reality these features originate at the most elementary level of nature.

A third reason is the "absence in nature of point-like wavepackets" [10] that, even though seemingly naive, implies that the interior of hadrons, nuclei and stars, as well as of high energy scattering regions, are *not* made up of isolated points as generally believed, but instead by the total mutual penetration of the wavepakects of particle constituents, since all hadrons have dimension of the order of those of their constituents (about $1\ fm$). Once the hyperdense character of the interior regions are admitted, there is the evident consequential impossibility of any consistent formulation of the LP symmetry inside hadrons, nuclei, stars and high energy scattering region.

In regard to anisotropic and inhomogeneous media, to avoid a prohibitive length we merely quote measurements [43-35] which have established the lack of exact character of the conventional 20th century formulation of SR within our atmosphere. This result was achieved via systematic measurements of *Santilli IsoRedShift,* namely, the existence of light frequency shifts toward the red without any relative motion between the source, the medium and the observer.

The same papers [43-45] have confirmed the exact validity within out atmosphere of the isotopic formulation of SR provided by IR, according to theoretical predictions dating back to the early 1980s (see paper [23] of 1983 and the systematic treatment in monographs [33] of 1991).

Ironically, Santilli conducted these measurements to initiate the study of numerical values of deviations from the Doppler shift, rather than to establish the existence of the IRS, because the mere visual observation of the redness of "direct" Sunlight at the horizon is clear evidence of a redshift beyond Doppler's law due to a clear redshift in the the absence of appreciable relative motion between Sun and Earth at Sunset and Sunrise.

In regard to the lack of exact character of the conventional realization of SR for the interior of hadrons, nuclei, stars, and high energy scattering regions, and the exact validity

[7]It should be recalled from Section 1, that the axiomatically correct representation of the irreversible propagation of light in water requires the full Lie-admissible formulations for various technical reasons, such as the necessity to have inequivalent isomodular actions to the right and left, and other reasons [13].

of the covering IR, we quote monograph [16d].

As stressed in Refs. [16,33,34], for the case of composite systems the LP symmetry was conceived and experimentally verified for *Keplerian systems,* namely for system of point particles orbiting around a *Keplerian nucleus,* as it is the case for the atomic structure.

But *hadrons, nuclei, stars and high energy scattering regions do not have nuclei.* The sole lack of existence of Keplerian nuclei is sufficient, *per se,* to prevent the exact validity of the LP symmetry in favor of the covering LPS realization [16,23-34].[8]

For a comprehensive review of the application of the isotopic formulations of 20th century theories in Newtonian mechanics, particle physics, nuclear physics, chemistry, biology, astrophysics and other fields, we suggest a consultation of monograph [42].

In summary, the mathematical, theoretical and experimental research initiated in the 1960s [9] and continued thereafter has indicated the "inapplicability" (and *not* the "violation") of the conventional 20th century realization of SR for any possible interior conditions without exception to the author's knowledge, all cases admitting exact representations by the covering isotopic formulation of SR.

In considering this scientific scenario, the reader should keep in mind that said inapplicability *not* due to insufficiencies of Einstein's SR axioms, because it is solely due to the insufficiency of the 20th century *mathematics* used for their realization.

5. Concluding Remarks

During the studies on axiom-preserving isotopies of 20th century theories, the author has stated various times that:

$>$ Rather than abusing the names of Lorentz, Poincaré, Einstein and other founders of 20th century physics by applying their theories under interior dynamical conditions they were not intended for, and cannot be properly formulated (let alone directly tested) due to the lack of inertial reference frames, the impossibility to measure the speed of light (let alone proving its constancy) , and other insufficiencies,

$>$ The best way to honor the names of Lorentz, Poincaré, Einstein and other masters is to maintain their axioms, and enlarge their conditions of exact applicability via the use of broader mathematics specifically built for broader conditions.

However, the implications in maintaining Einstein's SR axioms for interior physical conditions are so deep for quantitative sciences to imply a potential "new scientific era," as indicated in Ref. [42].

[8] It should be indicated that the representation of the synthesis of the neutron (1.7) identified already in 1978 [14] the mutation of conventional exterior masses of particles when immersed within hyperdense hadronic media. This creates unaddressed (let alone unresolved) shadows in the very numerical value of the masses of intermediate particles currently conjectured at CERN, such as the Higgs boson and others, because these masses are a direct consequence of the tacit assumption of the conventional realization of SR within hyperdense scattering region. A moment of collegial reflection appears recommendable here in view of the implied large expenditures of public funds, because the experimental confirmation of the inapplicability of the conventional formulation of SR in a medium as thin as our atmosphere renders highly unlike its exact validity within high energy scattering regions.

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