

ISSN: 2326-8867

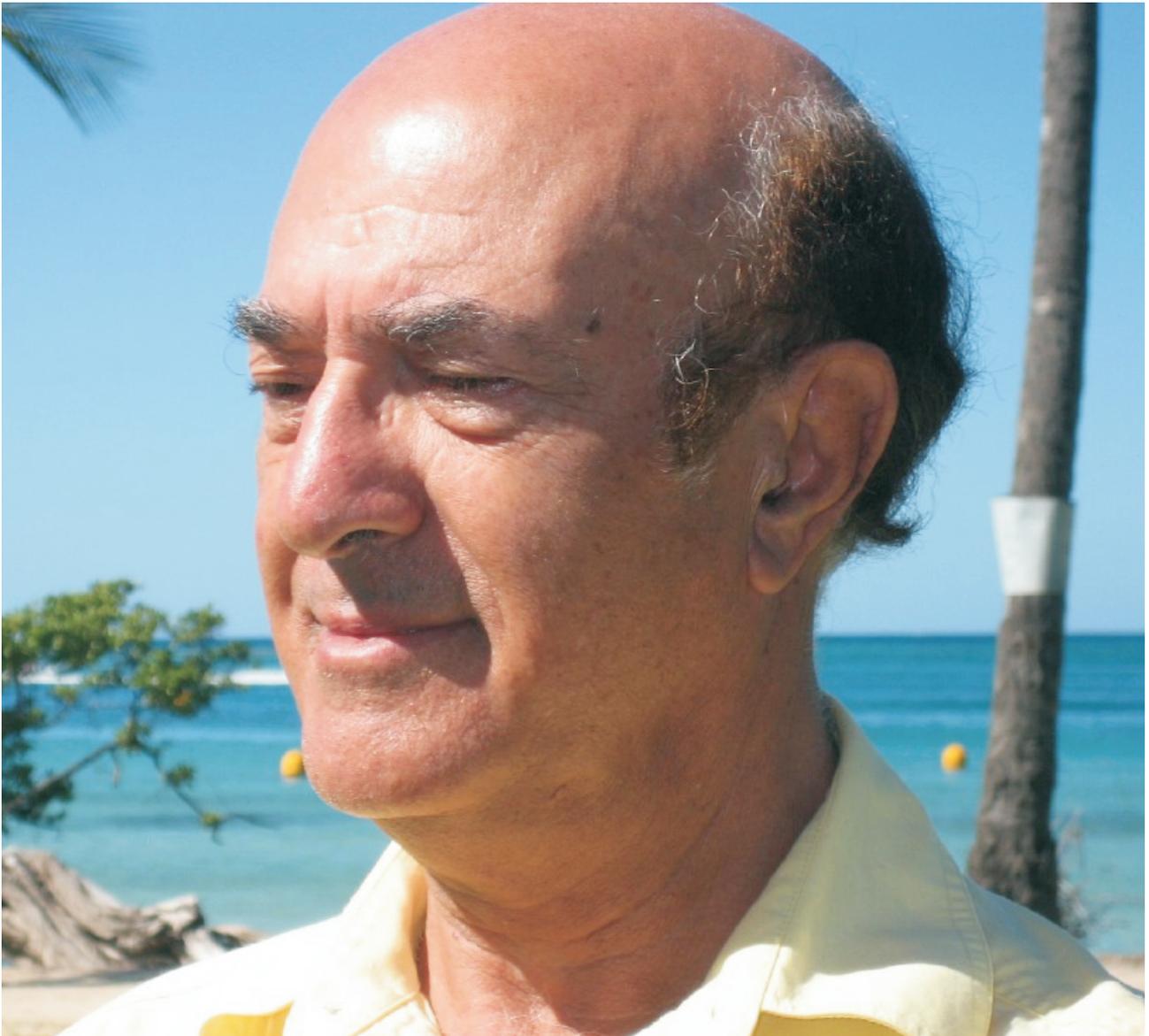
Volume 5 Number 2-1 March 2016

<http://www.sciencepublishinggroup.com/specialissue/122014>

Special Issue

Issue II: Foundations of Hadronic Mechanics

American Journal of Modern Physics



Special Issue:

Issue II: Foundations of Hadronic Mechanics

Lead Guest Editor

Richard Anderson

Board of Trustees, the R. M. Santilli Foundation, Palm Harbor, Florida, USA

Introduction

In continuation of the new mathematics discussed in the preceding special issue entitled Foundations of Hadronic Mathematics, we recall that the Italian-American scientist R. M. Santilli proposed in 1978 the construction of a covering of quantum mechanics called hadronic mechanics, which is solely valid at mutual distances of one Fermi while recovering quantum mechanics identically and uniquely at larger mutual distances. By using the novel iso-, geno- and hyper-mathematics and their isoduals the Hadronic mechanics is divided into isomechanics, genomechanics and hypermechanics for the representation of single-valued, reversible, single-valued irreversible and multi-valued irreversible matter-system or reactions, respectively, with corresponding isodual for antimatter composite systems or reactions.

Thanks to the collaboration of numerous physicists, hadronic mechanics has now received applications and experimental verifications in classical mechanics, particle physics, nuclear physics, astrophysics, cosmology and other fields. The special issue of the AJMP entitled the Foundations of Hadronic Mechanics shall review some of these applications and present new advances that can potentially stimulate the birth of new technologies. It should be indicated that novel technologies solely predicted by hadronic mechanics have reached industrial applications, such as Thunder Energy Corporation, a U. S. publicly traded company with stock symbol TNRG, that has developed the first laboratory synthesis of neutrons from a hydrogen gas and is now entering into production and sale of the related equipment.

For more information about the Special Issue, please pay a visit to the following website:

<http://www.sciencepublishinggroup.com/specialissue/122014>

CONTENTS

Vol. 5 No. 2-1

March 2016

Outline of Hadronic Mathematics, Mechanics and Chemistry as Conceived by R. M. Santilli	
Richard Anderson	1
Studies on Santilli's Isonumber Theory	
Arun S. Muktibodh	17
Santilli Synthesis of the Neutron According to Hadronic Mechanics	
Chandrakant S. Burande	37
Studies on Santilli Three-Body Model of the Deuteron According to Hadronic Mechanics	
Sudhakar S. Dhondge	46
Exact and Invariant Representation of Nuclear Magnetic Moments and Spins According to Hadronic Mechanics	
Anil A. Bhalekar, Ruggero Maria Santilli	56
Hadronic Nuclear Energy: An Approach Towards Green Energy	
Indrani B. Das Sarma	119
Possibilities for the Detection of Santilli Neutroids and Pseudo-protons	
Victor de Haan	131
Study of Bose-Einstein Correlation Within the Framework of Hadronic Mechanics	
Chandrakant S. Burande	137
Compatibility of Arbitrary Speeds with Special Relativity Axioms for Interior Dynamical Problems	
Ruggero Maria Santilli	143
Santilli's Isodual Mathematics and Physics for Antimatter	
P. M. Bhujbal	161
Possible Role of Antimatter Galaxies for the Stability of the Universe	
S. Beghella-Bartoli, R. M. Santilli	185

American Journal of Modern Physics (AJMP)

JOURNAL INFORMATION

American Journal of Modern Physics (AJMP) is published bimonthly by Science Publishing Group (SciencePG), online at <http://www.sciencepublishinggroup.com/ajmp>

SUBMISSION

To submit, please visit: <http://www.sciencepublishinggroup.com/login>
Any questions, just contact: journal@sciencepublishinggroup.com

REPRINTS

Reprints of the published journal are issued by SciencePG.
Cost: 50USD/one hard copy.
E-mail: journal@sciencepublishinggroup.com

ADVANTAGES

Peer Review

All papers submitted to journals in SciencePG are peer-reviewed.

Fast Publication

Papers of good quality can be published within two months after being submitted to SciencePG.

High Visibility

Each journal in SciencePG uses the OpenURL standard, making it easy for libraries to link users as directly as possible from citation to the full text of the article.

Open Access

Open Access makes knowledge available to all, without the barrier of affordability and without restrictions on using it to develop related areas of research.

Abstracting and Indexing

WorldCat, CrossRef, JournalSeek, Academickeys, ResearchBib, Directory of Research Journals Indexing (DRJI) and others.

COPYRIGHT

The authors' publications in SciencePG are distributed under Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>). The license was developed to facilitate open access, namely, free immediate access to and unrestricted reuse of original works of all types.

Under this license, authors retain ownership of the copyright for their publications, but grant SciencePG a non-exclusive license to publish the work in paper form and allow anyone to reuse, distribute and reproduce the content as long as the original work is properly cited.

Appropriate attribution can be provided by simply citing the original work. No permission is required from the authors or the publishers. For any reuse or distribution of a work, users must also make clear the license terms under which the work was published.

The standard license will be applied to the authors' publications, which ensures the publications freely and openly available in perpetuity.

For more information contact us by

E-mail: service@sciencepublishinggroup.com



Outline of Hadronic Mathematics, Mechanics and Chemistry as Conceived by R. M. Santilli

Richard Anderson

The R. M. Santilli Foundation, Palm Harbor, Florida, U.S.A.

Email address:

board@santilli-foundation.org

To cite this article:

Richard Anderson. Outline of Hadronic Mathematics, Mechanics and Chemistry as Conceived by R. M. Santilli. *American Journal of Modern Physics*. Special Issue: Issue II: Foundations of Hadronic Mechanics. Vol. 5, No. 2-1, 2016, pp. 1-16.

doi: 10.11648/j.ajmp.2016050201.11

Received: August 11, 2015; **Accepted:** August 24, 2015; **Published:** May 18, 2016

Abstract: In this paper, we outline the various branches of hadronic mathematics and their applications to corresponding branches of hadronic mechanics and chemistry as conceived by the Italian-American scientist Ruggero Maria Santilli. According to said conception, hadronic mathematics comprises the following branches for the treatment of matter in conditions of increasing complexity: 1) 20th century mathematics based on Lie's theory; 2) IsoMathematics based on Santilli's isotopies of Lie's theory; 3) GenoMathematics based on Santilli's formulation of Albert's Lie-admissibility; 4) HyperMathematics based on a multi-valued realization of genomathematics with classical operations; and 5) HyperMathematics based on Vougiouklis H_v hyperstructures expressed in terms of hyperoperations. Additionally, hadronic mathematics comprises the anti-Hermitian images (called isoduals) of the five preceding mathematics for the description of antimatter also in conditions of increasing complexity. The outline presented in this paper includes the identification of represented physical or chemical systems, the main mathematical structure, and the main dynamical equations per each branch. We also show the axiomatic consistency of various branches of hadronic mathematics as sequential coverings of 20th century mathematics; and indicate a number of open mathematical problems. Novel physical and chemical applications permitted by hadronic mathematics are presented in subsequent collections.

Keywords: Santilli isomathematics, Genomathematics, Hypermathematics

1. 20th Century Mathematics, Mechanics and Chemistry

$$F(n, \times, 1), n \tag{2}$$

Basic Associative product

$$nm = n \times m, 1 \times n = n \times 1 = n \forall n \in F \tag{3}$$

1.1. Represented Systems

Single-valued, closed-isolated, time-reversible systems of point-like particles moving in vacuum solely under action at a distance Hamiltonian interactions, such as the structure of atoms and molecules.

Measurement units of time, energy, etc. all positive
 Ordinary functional analysis $f(r) \in F$,
 Ordinary differential calculus
 Conventional Lie theory

1.2. Main Mathematical Structure

$$[X_i, X_j] = X_i \times X_j - X_j \times X_i == C_{ij}^k \times X_k, \tag{4}$$

Basic unit

$$A(w) = e^{X \times w \times i} \times A(0) \times e^{-i \times w \times X}. \tag{5}$$

$$I = +1 \tag{1}$$

Euclidean geometry and topology

Basic numeric fields n = real, complex, quaternionic numbers

$$E(r, \delta, 1), r = (r^k), k = 1, 2, 3, \delta = \text{Diag.}(1, 1, 1), \tag{6}$$

$$r^2 = r^i \times \delta_{ij} \times r^j = r_1^2 + r_2^2 + r_3^2 \in F, \tag{7}$$

Minkowskian geometry

$$M(x, \eta, I): x = (x^\mu), \mu = 1,2,3,4, x^4 = t, \quad (8)$$

$$\eta = \text{Diag.}(+1, +1, +1, -c^2), \quad (9)$$

$$x^2 = x^\mu \times \eta_{\mu\nu} \times x^\nu = x_1^2 + x_2^2 + x_3^2 - t^2 c^2 \in F, \quad (10)$$

Riemannian geometry

$$R(x, g(x), I): x = (x^\mu), \mu = 1,2,3,4, x^4 = t, \quad (11)$$

$$x^2 = x^\mu \times g(x)_{\mu\nu} \times x^\nu \in F \quad (11)$$

$$x^2 = x^\mu \times g(x)_{\mu\nu} \times x^\nu \in F \quad (12)$$

Symplectic geometry.

$$\omega = dr^k \wedge dp_k \quad (13)$$

1.3. Dynamical Equations

Newton equation

$$m \times \frac{dv}{dt} - F^{SA}(t, r, v,) = 0, \quad (14)$$

Variational principle

$$\delta A = \delta \int (p_k \times dr^k - H \times dt) = 0. \quad (15)$$

Hamilton's equations without external terms

$$\frac{dr^k}{dt} = \frac{\partial H(r,p)}{\partial p_k}, \quad \frac{dp_k}{dt} = -\frac{\partial H(r,p)}{\partial r^k}, \quad (16)$$

Hilbert space H over C with states $|\psi\rangle$ over (C)
Expectation value of a Hermitean operator A

$$\langle A \rangle = \langle \psi | \times A \times | \psi \rangle \in C, \quad (17)$$

Heisenberg equation

$$i \times \frac{dA}{dt} = [A, H] = A \times H - H \times A, \quad (18)$$

Schrödinger equations

$$H \times |\psi\rangle = E \times |\psi\rangle \quad (19)$$

$$p \times |\psi\rangle = -i \times \partial_r |\psi\rangle \quad (20)$$

Dirac equation

$$(\eta^{\mu\nu} \times \gamma_\mu \times p_\nu - i \times m \times c) \times |\psi\rangle = 0. \quad (21)$$

$$\{\gamma_\mu, \gamma_\nu\} = \gamma_\mu \times \gamma_\nu + \gamma_\nu \times \gamma_\mu = 2 \times \eta_{\mu\nu}, \quad (22)$$

Comments and References

The literature on 20th century mathematics, mechanics and chemistry is so vast and so easily identifiable to discourage discriminatory partial listings.

2. Isomathematics, Isomechanics and Isochemistry

2.1. Represented Systems [1-5]

Single-value, closed-isolated, time-reversible system of extended-deformable particles with action at a distance Hamiltonian and contact non-Hamiltonian interactions, such as the structure of hadrons, nuclei and stars, in the valence electron bonds and other systems.

2.2. Main Mathematical Structures [1-5]

Santilli IsoUnit \hat{I} and isotopic element \hat{T}^1

$$\hat{I} = \hat{I}(r, p, a, \psi, \dots) = 1/\hat{T}(r, p, a, \psi, \dots) > 0, \quad (23)$$

Santilli IsoFields

$$\hat{F}(\hat{n}, \hat{\otimes}, \hat{I}), \hat{n} = n \times \hat{I}, \quad (24)$$

Santilli isoproduct

$$\hat{n} \hat{\otimes} \hat{m} = \hat{n} \times \hat{T} \times \hat{m} \in \hat{F}, \quad (25)$$

$$\hat{I} \hat{\otimes} \hat{n} = \hat{n} \hat{\otimes} \hat{I} = \hat{n} \forall \hat{n} \in \hat{F}, \quad (26)$$

Representation via the isotopic element of extended-deformable particles under non-Hamiltonian interactions

$$\hat{T} = \text{Diag.} \left(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2} \right) \times e^{\Gamma(r,p,\psi,\partial\psi,\dots)} \quad (27)$$

IsoCoordinates $\hat{r} = r \times \hat{I} \in \hat{F}$,

IsoFunctional analysis $\hat{f}(\hat{r}) = f(\hat{r}) \times \hat{I} \in \hat{F}$,

IsoDifferential Calculus

$$d\hat{r} = dr + r \times \hat{T} \times d\hat{I}, \quad (28)$$

$$\frac{\partial \hat{f}(\hat{r})}{\partial \hat{r}} = \hat{I} \times \frac{\partial f(\hat{r})}{\partial r}, \quad (29)$$

Santilli Lie-Isotopic Theory

$$[X_i, \hat{X}_j] = X_i \hat{\otimes} X_j - X_j \hat{\otimes} X_i = C_{ij}^k(r, p, \dots) \times X_k, \quad (30)$$

$$A(w) = \hat{e}^{X \times w \times X} \hat{\otimes} A(0) \hat{\otimes} \hat{e}^{-i \times w \times X}. \quad (31)$$

Santilli Iso-Euclidean Geometry

$$\hat{E}(\hat{r}, \hat{\delta}, \hat{I}), \hat{\delta}(r, p, z, \psi, \dots) = \hat{T}(r, p, z, \psi, \dots) \times \delta, \quad (32)$$

$$\hat{T} = \text{Diag.} (1/n_1^2, 1/n_2^2, 1/n_3^2), \quad (33)$$

$$\hat{r}^2 = \hat{r}^i \hat{\otimes} \hat{\delta}_{ij} \hat{\otimes} \hat{r}^j = \left(\frac{r_1^2}{n_1^2} + \frac{r_2^2}{n_2^2} + \frac{r_3^2}{n_3^2} \right) \times \hat{I} \in \hat{F}, \quad (34)$$

¹See Santilli's curriculum

<http://www.world-lecture-series.org/santilli-cv>

Prizes and Nominations

<http://www.santilli-foundation.org/santilli-nobel-nominations.html>

and scientific archive

<http://www.santilli-foundation.org/news.html>

Santilli Iso-Minkowskian Geometry

$$\widehat{M}(\widehat{x}, \widehat{\eta}, \widehat{I}): \widehat{x} = (\widehat{x}^\mu), \mu = 1,2,3,4, x_4 = t, \quad (35)$$

$$\widehat{\eta}(x, \psi, \dots) = \widehat{T}(x, \psi, \dots) \times \eta, \quad (36)$$

$$\widehat{T} = \text{Diag.} (1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2), \quad (37)$$

$$\widehat{x}^2 = \widehat{x}^\mu \widehat{\eta}_{\mu\nu} \widehat{x}^\nu = \left(\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2}\right) \times \widehat{I} \in \widehat{F}, \quad (38)$$

Santilli Iso-Riemannian Geometry

$$\widehat{R}(\widehat{x}, \widehat{g}, \widehat{I}): \widehat{g} = \widehat{T}(x, v, \dots) \times g(x), \quad (39)$$

$$\widehat{x}^2 = \left(\frac{g_{11}}{n_1^2} + \frac{g_{22}}{n_2^2} + \frac{g_{33}}{n_3^2} - \frac{g_{44}}{n_4^2}\right) \times \widehat{I} \in \widehat{F}, \quad (40)$$

Santilli Iso-Symplectic Geometry

$$\widehat{\omega} = \widehat{d}\widehat{r}^k \widehat{\wedge} \widehat{d}\widehat{p}_k \quad (41)$$

2.3. IsoDynamical IsoEquations s [1-5]

Newton-Santilli IsoEquation

$$\widehat{m} \widehat{\otimes} \frac{d\widehat{v}}{d\widehat{t}} - F^{SA}(t, r, p) = m \times \frac{dv}{dt} - F^{SA}(t, r, p) - F^{NSA}(t, r, p, \dots) = 0, \quad (42)$$

IsoVariational principle

$$\widehat{\delta}\widehat{A} = \widehat{\delta} \widehat{\int} (\widehat{p}_k \widehat{\otimes} \widehat{d}\widehat{r}^k - \widehat{H} \widehat{\otimes} \widehat{d}\widehat{t}) = 0. \quad (43)$$

Hamilton-Santilli IsoEquations

$$\frac{d\widehat{r}^k}{d\widehat{t}} = \frac{\partial \widehat{H}(\widehat{r}, \widehat{p})}{\partial \widehat{p}_k}, \quad \frac{d\widehat{p}_k}{d\widehat{t}} = -\frac{\partial \widehat{H}(\widehat{r}, \widehat{p})}{\partial \widehat{r}^k}, \quad (44)$$

Iso-Hilbert space \widehat{H} over C with states $|\widehat{\psi}\rangle$ over the isofield \widehat{C}^2

IsoExpectation value of a Hermitean operator \widehat{A} on \widehat{H}

$$\langle \widehat{A} \rangle = \langle \widehat{\psi} | \widehat{A} | \widehat{\psi} \rangle \in \widehat{C} \quad (45)$$

Heisenberg-Santilli IsoEquation

$$\widehat{i} \widehat{\otimes} \frac{d\widehat{A}}{d\widehat{t}} = [\widehat{A}; \widehat{H}] = \widehat{A} \widehat{\otimes} \widehat{H} - \widehat{H} \widehat{\otimes} \widehat{A} = \widehat{A} \times \widehat{T}(\widehat{\psi}, \dots) \times \widehat{H}(\widehat{r}, \widehat{p}) - \widehat{H}(\widehat{r}, \widehat{p}) \times \widehat{T}(\widehat{\psi}, \dots) \times \widehat{A} \quad (46)$$

Schrödinger-Santilli IsoEquation

$$\widehat{H} \widehat{\otimes} |\widehat{\psi}\rangle = \widehat{H}(\widehat{r}, \widehat{p}) \times \widehat{T}(\widehat{\psi}, \widehat{\delta}\widehat{\psi}, \dots) \times |\widehat{\psi}\rangle = \widehat{E} \widehat{\otimes} |\widehat{\psi}\rangle = E \times |\widehat{\psi}\rangle, \quad (47)$$

$$\widehat{p} \widehat{\otimes} |\widehat{\psi}\rangle = -\widehat{i} \widehat{\otimes} \widehat{\partial}_{\widehat{r}} |\widehat{\psi}\rangle = -i \times \widehat{I} \times \partial_{\widehat{r}} |\widehat{\psi}\rangle, \quad (48)$$

Dirac-Santilli IsoEquation

$$(\widehat{\eta}^{\mu\nu} \widehat{\otimes} \widehat{\gamma}_\mu \widehat{\otimes} \widehat{p}_\nu - \widehat{i} \widehat{\otimes} \widehat{m} \widehat{\otimes} \widehat{c}) \widehat{\otimes} |\widehat{psi}\rangle = 0. \quad (49)$$

$$\{\widehat{\gamma}_\mu, \widehat{\gamma}_\nu\} = \widehat{\gamma}_\mu \widehat{\otimes} \widehat{\gamma}_\nu + \widehat{\gamma}_\nu \widehat{\otimes} \widehat{\gamma}_\mu = \widehat{2} \widehat{\otimes} \widehat{\eta}_{\mu\nu} = 2 \times \widehat{\eta}_{\mu\nu}, \quad (50)$$

2.4. Comments and References

As it is well known, the local-differential calculus of 20th century mathematics can solely represent a finite set of isolated dimensionless points. In view of this structural feature, Newton formulated his celebrated equations (14) for massive points, resulted in a conception of nature that was adopted by Galileo and Einstein, became the dominant notion of 20th century sciences, and was proved to be valid for classical or quantum particles moving in vacuum at large mutual distances, such as for our planetary system or the atomic structure.

However, when bodies move within physical media, such as for a spaceship during re-entry in our atmosphere or for a proton in the core of a star, point-like abstractions of particles became excessive, e.g., because a macroscopic collection of point-particles cannot have entropy (since all known Hamiltonian interactions are invariant under time reversal), with consequential violation of thermodynamical laws and other insufficiencies.

Besides the clear identification of these insufficiencies, the first historical contribution by the Italian-American scientist Ruggero Maria Santilli (see Footnote 1) has been the generalization of 20th century mathematics into such a form to admit a time invariant representation of extended, and therefore deformable particles under conventional Hamiltonian as well as contact non-Hamiltonian interactions, with implications for all quantitative sciences.

The above central objective was achieved in monographs [1] originally written by Santilli during his stay at MIT from 1974 to 1977 (where they appeared as MIT preprints). Monographs [1] were then completed by Santilli during his stay at Harvard University from 1977 to 1982 under DOE support, and released for publication only following the delivery at Harvard of a post Ph. D. seminar Course in the field.

The representation of extended-deformable bodies moving within physical media was achieved via an axiom-preserving lifting, called isotopy, of the conventional associative product $AB = A \times B$ between generic quantities A, B (such as numbers, functions, matrices, operators, etc.) into the form $A \widehat{\otimes} B = A \times \widehat{T} \times B$, Eq. (25). Conventional interactions are represented via conventional Hamiltonian, while actual shape and non-Hamiltonian interactions are represented via realization of the quantity \widehat{T} , called isotopic element, of the

²As shown in the seminal paper [6] of 1982, but vastly ignored for the past four decades, isomechanics formulated on iso-Hilbert spaces over isofields eliminates the divergencies of quantum mechanics and related scattering theories. This important feature is primarily due to the fact that, for all physical and chemical applications worked out to date, the isounit $\widehat{I} = 1/\widehat{T} > 0$ must have a large value of the exponential type (27) and, consequently, the isotopic element \widehat{T} must have a very small value. This occurrence eliminates the singularity of the Dirac delta "distribution" when lifted to the Dirac-Myung-Santilli delta "isofunction" as shown by the realization of the type

$$\widehat{\delta}(r - r_0) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ik\widehat{T}(r-r_0)} dk,$$

with $\widehat{T} = \frac{N}{r-r_0}$, $N \ll 1$. Similarly, perturbative and other series with Hermitean operators that are divergent or slowly convergent in quantum mechanics can be lifted into isoserries of the type

$$A(w) = \widehat{I} + \frac{w(A\widehat{T}H - H\widehat{T}A)}{1!} + \dots$$

that are manifestly convergent for $w > 1$ but $\widehat{T} \ll w$. As shown by A. O. E. Animalu and R. M. Santilli in five papers published proceedings [25], the above lack of divergences carries over to the covering of the scattering theory known as isoscattering theory, by therefore achieving numerical results without the use of infinities for the renormalization of divergent series.

type (27).

Santilli then achieved in monographs [1] the axiom-preserving isotopies of the various branches of Lie's theory, e.g., Eqs. (30), (31,) including their elaboration via the initiation of the isotopies of functional analysis. In particular, Santilli showed that the isotopies of the rotational symmetry $SO(3)$ characterized by isotopic element (27) do represent extended, generally non-spherical and deformable bodies. Finally, Santilli proved in Vol. II of Ref. [1] the significance of his Lie-isotopic theory by showing that it characterizes the Birkhoffian covering of classical Hamiltonian mechanics and its "direct universality" for the representation of all possible, non-singular, generally non-Hamiltonian Newtonian systems in the frame of the experimenter, which direct universality was subsequently proved to hold also for isotopic operator theories. The above advances were formulated on an ordinary numeric field.

Subsequently, Santilli discovered in 1993 [2] that the axioms of numeric fields with characteristic zero do not necessarily require that the basic multiplicative unit is the trivial number +1, since said axioms admit arbitrary generalized units, today called *Santilli isounits*, provided that they are positive-definite and are the inverse of the isotopic element, $\hat{I} = 1/\hat{T} > 0$. This second historical discovery identified new numbers today known as *Santilli isoreal, isocomplex and isoquaternionic numbers* of the First (Second) kind when the isounit is outside (an element of) the original field. This discovery prompted a flurry of reformulation over Santilli isofields of all preceding isotopies, including most importantly the reformulation of Santilli's Lie-isotopic theory.

Despite the above momentous advances, Santilli remained dissatisfied because the isotopic formulations of the early 1990s were not invariant under their time evolution, thus being unable to predict the same numerical values under the same conditions at different times. Since the entire 20th century mathematics had been isotopically lifted by the early 1990s, Santilli was left with no other choice than that of reinspecting the Newton-Leibnitz differential calculus by discovering that, contrary to a popular belief in mathematics and physics for some four centuries, the differential calculus is indeed dependent on the basic multiplicative unit. In this way, Santilli achieved in memoir [3] of 1996 the third historical discovery according to which the ordinary differential calculus needs generalizations of the type (28), (29) whenever the isounit depends on the local variable of differentiation. This discovery signaled the achievement of mathematical maturity of isomathematics that permitted numerous advances in physics and chemistry as well as novel industrial applications.

All in all, Santilli has written about 150 papers on the isotopies of all various aspects of 20th century mathematics. These contributions are reported in monographs [4] of 1995 that remain to this day the most comprehensive presentation on isotopies. In the subsequent series of monographs [5] of 2008, Santilli introduces the names of *Hadronic Mathematics, Mechanics and Chemistry* which have been adopted for this review due to their wide acceptance.

Numerous authors have made important contributions in Santilli isomathematics, among whom we quote: the mathematician H. C. Myung who initiated (with R. M. Santilli) [6] the isotopies of Hilbert Spaces, including the momentous elimination of the divergencies of quantum mechanics under sufficiently small values of the isotopic element \hat{T} ; the mathematicians D. S. Sourlas and G. T. Tsagas [7] who conducted in 1993 the first comprehensive study of the Lie-Santilli isotheory; the theoretician J. V. Kadeisvili [8] who presented systematic studies of Santilli's isotopies of 20th century geometries and relativities; the mathematician Chun-Xuan Jiang [9] who conducted in 2001 systematic studies of Santilli IsoNumber Theory; the mathematicians R. M. Falcon Ganfornina and J. Nunez Valdes who wrote in 2001 the now historical, first mathematically rigorous treatment of Santilli isotopies [10], and the historical achieved isotopology [11] which provides the ultimate mathematical structure of the Newton-Santilli isoequations (42) for extended-deformable particles under Hamiltonian and non-hamiltonian interactions achieved in memoir [3]; the mathematician S. Georgiev who wrote one of the most monumental and important mathematical works in scientific history [12], by showing that Santilli's IsoDifferential Calculus implies a variety of fully consistent coverings of 20th century mathematics; the mathematician A. S. Muktibodh [13] who presented the first known generalization of Santilli isonumber theory for the case of characteristic $p \neq 0$; the physicists I. Gandzha and J. Kadeisvili who presented in 2011 [14] a comprehensive review of Santilli isomathematics and its applications in physics and chemistry; plus additional seminal advances presented in the subsequent papers of this collection.

3. Genomathematics, Genomechanics and Genochemistry

3.1. Represented Systems s [1-5]

Single-valued, time-irreversible system of extended-deformable particles under action at a distance Hamiltonian and contact non-Hamiltonian interactions, as occurring in nuclear reactions, biological structures and chemical reactions.

3.2. Main Mathematical Structure s [1-5]

Santilli Forward GenoUnit

$$\hat{I}^> = \hat{I}^>(t^>r^>, p^>, a^>, \psi^>, \partial^>\psi^>, \dots) = 1/\hat{T}^> > 0, \quad (51)$$

Santilli Backward GenoUnit

$$\hat{I}^< = \hat{I}^<(<r^<, <p^<, <a^<, <\psi^<, \partial^<\psi^<, \dots) = 1/\hat{T}^< > 0, \quad (52)$$

Condition for time-irreversibility

$$\hat{I}^> \neq \hat{I}^< \quad (53)$$

Forward GenoFields

$$\hat{F}^>(\hat{n}^>, >, \hat{I}^>), \hat{n}^> = n \times \hat{I}^> \quad (54)$$

Backward GenoFields

$$\langle \hat{F}^>(\langle \hat{n}^>, \langle \hat{I}^>), \langle \hat{n}^> = \langle \hat{I}^> \times n, \quad (55)$$

Forward GenoProduct

$$\hat{n}^> \times \hat{m}^> = \hat{n}^> \times \hat{I}^> \times \hat{m}^> \in \hat{F}^>, \quad (56)$$

$$\hat{I}^> > \hat{n}^> = \hat{n}^> > \hat{I}^> = \hat{n}^> \vee \hat{n}^> \in \hat{F}^> \quad (57)$$

Backward Genoproduct

$$\langle \hat{n}^> \langle \langle \hat{m}^> = \langle \hat{n}^> \times \langle \hat{I}^> \times \langle \hat{m}^> \in \langle \hat{F}^>, \quad (58)$$

$$\langle \hat{I}^> \langle \langle \hat{n}^> = \langle \hat{n}^> \langle \langle \hat{I}^> = \langle \hat{n}^> \vee \langle \hat{n}^> \in \langle \hat{F}^>, \quad (59)$$

Representation of forward extended-deformable particles under non-Hamiltonian interactions

$$\hat{T}^> = \text{Diag.} \left(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2} \right) \times e^{\Gamma(t,r,p,\psi,\partial\psi,\dots)} \quad (60)$$

Forward GenoCoordinates

$$\hat{r}^> = r \times \hat{I}^> \in \hat{F}^>, \quad (61)$$

Backward GenoCoordinates

$$\langle \hat{r}^> = \langle \hat{I}^> \times r \in \langle \hat{F}^>, \quad (62)$$

Forward GenoFunctional analysis

$$\hat{f}^>(\hat{r}^>) = f(\hat{r}^>) \times \hat{I}^> \in \hat{F}^>, \quad (63)$$

Backward GenoFunctional analysis

$$\langle \hat{f}^>(\langle \hat{r}^>) = f(\langle \hat{r}^>) \times \langle \hat{I}^> \in \langle \hat{F}^>, \quad (64)$$

Forward GenoDifferential Calculus

$$\hat{d}^>\hat{r}^> = dr + r \times \hat{T}^> \times d\hat{I}^>, \quad (65)$$

$$\frac{\partial^>\hat{f}^>(\hat{r}^>)}{\partial^>\hat{r}^>} = \hat{I}^> \times \frac{\partial \hat{f}^>(\hat{r}^>)}{\partial \hat{r}^>}, \quad (66)$$

Backward GenoDifferential Calculus

$$\langle \hat{d}^>\langle \hat{r}^> = dr + r \times \langle \hat{T}^> \times \langle d\hat{I}^>, \quad (67)$$

$$\frac{\langle \partial \langle \hat{f}^>(\langle \hat{r}^>)}{\langle \partial \langle \hat{r}^>} = \langle \hat{I}^> \times \frac{\partial \langle \hat{f}^>(\langle \hat{r}^>)}{\partial \langle \hat{r}^>}, \quad (68)$$

Santilli Lie-Admissible Theory

$$(X_i \hat{X}_j) = X_i \langle X_j - X_j \rangle X_i = C_{ij}^k(t, r, p, \psi, \dots) \times X_k, \quad (69)$$

$$A(w) = \hat{e}_>^{X \times w \times X} > A(0) \ll \hat{e}^{-i \times w \times X}. \quad (70)$$

Santilli Forward Geno-Euclidean Geometry

$$\hat{E}^>(\hat{r}^>, \hat{\delta}^>, \hat{I}^>), \hat{\delta}^>(t, r, p, \psi, \dots) = \hat{T}^>(t, r, p, \psi, \dots) \times \delta, \quad (71)$$

$$\hat{r}^>^2 = \hat{r}^>^i > \hat{\delta}_{ij}^> > \hat{r}^>^j \in \hat{F}^>, \quad (72)$$

$$\hat{\delta}^> \neq \hat{\delta}^> \text{transp} \quad (73)$$

Santilli Backward Geno-Euclidean Geometry

$$\langle \hat{E}^>(\langle \hat{r}^>, \langle \hat{\delta}^>, \langle \hat{I}^>), \langle \hat{\delta}^>(t, r, p, \psi, \dots) = \langle \hat{T}^>(t, r, p, \psi, \dots) \times \delta, \quad (74)$$

$$\langle \hat{r}^>^2 = \langle \hat{r}^>^i \langle \langle \hat{\delta}_{ij}^> \langle \langle \hat{r}^>^j \in \langle \hat{F}^>, \quad (75)$$

$$\langle \hat{\delta}^> \neq \langle \text{transp} \hat{\delta}^> \quad (76)$$

Santilli Forward Geno-Minkowskian Geometry ($\mu = 1,2,3,4$)

$$\hat{M}^>(\hat{x}^>, \hat{\eta}^>, \hat{I}^>): \hat{x}^> = (\hat{x}^>^\mu), x_4^> = t^>, \quad (77)$$

$$\hat{\eta}^>(x, \psi, \dots) = \hat{T}^>(x, \psi, \dots) \times \eta, \quad (78)$$

$$\hat{x}^>^2 = \hat{x}^>^\mu > \hat{\eta}_{\mu\nu}^> > \hat{x}^> \in \hat{F}^>, \quad (79)$$

$$\hat{\eta}^> \neq \hat{\eta}^> \text{transp} \quad (80)$$

Santilli Backward Geno-Minkowskian Geometry ($\mu = 1,2,3,4$)

$$\langle \hat{M}^>(\langle \hat{x}^>, \langle \hat{\eta}^>, \langle \hat{I}^>): \langle \hat{x}^> = (\hat{x}^>^\mu), \langle x_4^> = \langle t^>, \quad (81)$$

$$\langle \hat{\eta}^>(x, v, \dots) = \langle \hat{T}^>(x, v, \dots) \times \eta, \quad (82)$$

$$\langle \hat{x}^>^2 = \langle \hat{x}^>^\mu \langle \langle \hat{\eta}_{\mu\nu}^> \langle \langle v \hat{x}^> \in \langle \hat{F}^>, \quad (83)$$

$$\langle \hat{\eta}^> \neq \langle \text{transp} \hat{\eta}^> \quad (84)$$

Santilli Forward Geno-Riemannian Geometry

$$\hat{R}^>(\hat{x}^>, \hat{g}^>, \hat{I}^>): \hat{g}^> = \hat{T}^>(x, v, \dots) \times g(x), \quad (85)$$

$$\hat{x}^>^2 = \hat{x}^>^\mu > \hat{g}_{\mu\nu}^> > \hat{x}^> \in \hat{F}^>, \quad (86)$$

$$\hat{g}^> \neq \hat{g}^> \text{transp} \quad (87)$$

Santilli Backward Geno-Riemannian Geometry

$$\langle \hat{R}^>(\langle \hat{x}^>, \langle \hat{g}^>, \langle \hat{I}^>): \langle \hat{g}^> = \langle \hat{T}^>(x, v, \dots) \times g(x), \quad (88)$$

$$\langle \hat{x}^>^2 = \langle \hat{x}^>^\mu \langle \langle \hat{g}_{\mu\nu}^> \langle \langle v \hat{x}^> \in \langle \hat{F}^>, \quad (89)$$

$$\langle \hat{g}^> \neq \langle \text{transp} \hat{g}^> \quad (90)$$

Santilli Forward Geno-Symplectic Geometry

$$\hat{\omega}^> = \hat{d}^>\hat{r}^>^k \hat{\lambda}^> \hat{d}^>\hat{p}_k^> \quad (91)$$

Santilli Backward Geno-Symplectic Geometry

$$\langle \hat{\omega}^> = \langle \hat{d}^>\langle \hat{r}^>^k \langle \hat{\lambda}^> \langle \hat{d}^>\langle \hat{p}_k^> \quad (92)$$

3.3. Genodynamical Genoequations [1-5]

Newton-Santilli Forward GenoEquation

$$\hat{m}^> > \frac{\hat{d}^>\hat{v}^>}{\hat{d}^>\hat{t}^>} - F^{>SA}(t, r, p) = [m \times \frac{dv}{dt}]^> - F^{SA}>(t, r, p) - F^{NSA}>(t, r, p, \dots) = 0, \quad (93)$$

Newton-Santilli Backward GenoEquation

$$\langle \hat{m}^> \langle \frac{\langle \hat{d}^>\langle \hat{v}^>}{\langle \hat{d}^>\langle \hat{t}^>} - \langle F^{SA}>(t, r, p) =$$

$$=^< [m \times \frac{dv}{dt}] -^<SA F(t, r, p) -^<NSA F(t, r, p, \dots) = 0, \quad (94)$$

Forward GenoVariational principle

$$\delta^> \hat{A}^> = \delta^> \int^> (\hat{p}_k^> \hat{r}^> - \hat{H}^>) dt = 0. \quad (95)$$

Backward GenoVariational principle

$$^<\delta \hat{A} = ^<\delta \int^< (\hat{p}_k \hat{r}^k - \hat{H}) dt = 0. \quad (96)$$

Forward Hamilton-Santilli GenoEquations

$$[\frac{d\hat{r}^k}{dt}]^> = [\frac{\partial \hat{H}(\hat{r}, \hat{p})}{\partial \hat{p}_k}]^>, \quad [\frac{d\hat{p}_k}{dt}]^> = -[\frac{\partial \hat{H}(\hat{r}, \hat{p})}{\partial \hat{r}^k}]^>, \quad (97)$$

Backward Hamilton-Santilli GenoEquations

$$^<[\frac{d\hat{r}^k}{dt}] = \frac{\partial \hat{H}(\hat{r}, \hat{p})}{\partial \hat{p}_k}, \quad ^<[\frac{d\hat{p}_k}{dt}] = -^<[\frac{\partial \hat{H}(\hat{r}, \hat{p})}{\partial \hat{r}^k}], \quad (98)$$

Forward Geno-Hilbert space $\hat{H}^>$ with states $|\hat{\psi}^>$ over the isofield $\hat{C}^>$

GenoExpectation value of a Hermitean operator \hat{A} on $\hat{H}^>$

$$^<\hat{A}^> = ^<\langle \hat{\psi} | \hat{A} | \hat{\psi} \rangle \in \hat{C} \quad (99)$$

Heisenberg-Santilli GenoEquation³

$$\hat{i} \hat{\times} \frac{d\hat{A}}{dt} = (\hat{A}, \hat{H}) = \hat{A} \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{A} = \hat{A} \hat{\times} T(\hat{\psi}, \dots) \times \hat{H}(\hat{r}, \hat{p}) - \hat{H}(\hat{r}, \hat{p}) \times \hat{T}(\hat{\psi}, \dots) \times \hat{A} \quad (100)$$

Forward Schrödinger-Santilli GenoEquation

$$\hat{H}^> |\hat{\psi}^> = \hat{H}^>(\hat{r}, \hat{p}) \times \hat{T}^>(\hat{\psi}, \hat{\partial}\hat{\psi}, \dots) \times |\hat{\psi}^> = \hat{E}^> |\hat{\psi}^> \quad (101)$$

$$\hat{p}^> |\hat{\psi}^> = -\hat{i}^> \hat{\partial}_{\hat{r}} |\hat{\psi}^> = -i \times \hat{I} \times \partial_{\hat{r}} |\hat{\psi}^>, \quad (102)$$

Backward Schrödinger-Santilli GenoEquation

$$^<\langle \hat{\psi} | ^<\hat{H} = ^<\langle \hat{\psi} | \hat{\times} \hat{T}(\hat{\psi}, \hat{\partial}\hat{\psi}, \dots) \hat{\times} \hat{H}(\hat{r}, \hat{p}) = ^<\langle \hat{\psi} | ^<\hat{E} = ^<\langle \hat{\psi} | \hat{\times} \hat{E}, \quad (103)$$

$$^<\langle \hat{\psi} | ^<\hat{p} = -^<\langle \hat{\psi} | ^<\hat{i} \hat{\times} \hat{\partial} = -i \times ^<\langle \hat{\psi} | \hat{\times} \hat{I} \quad (104)$$

Forward Dirac-Santilli IsoEquation

$$(\hat{\eta}^{\mu\nu} \hat{r}^{\mu} \hat{p}^{\nu} - \hat{i}^> \hat{m}^>) \hat{r}^{\mu} \hat{p}^{\nu} = 0. \quad (105)$$

$$\{\hat{r}^{\mu}, \hat{r}^{\nu}\}^> = [\hat{r}^{\mu} \hat{\times} \hat{r}^{\nu} + \hat{r}^{\nu} \hat{\times} \hat{r}^{\mu}]^> = \hat{2}^> \hat{\eta}^{\mu\nu}, \quad (106)$$

Backward Dirac-Santilli GenoEquation

$$^<\langle \hat{\psi} | ^<(\hat{p}_\nu \hat{r}^\mu - \hat{\eta}^{\mu\nu}) = 0. \quad (107)$$

$$\{\hat{r}^{\mu}, \hat{r}^{\nu}\} = [\hat{r}^{\mu} \hat{\times} \hat{r}^{\nu} + \hat{r}^{\nu} \hat{\times} \hat{r}^{\mu}] = \hat{2} \hat{\eta}^{\mu\nu} = 2 \hat{\eta}^{\mu\nu}, \quad (108)$$

3.4. Comments and References

As it is also well known, all 20th century mathematical, physical or chemical formulations are reversible over time. Following research over half a century initiated during his Ph. D. studies at the University of Torino, Italy, in the mid 1960s [15, 17-23,4,5], R. M., Santilli has made the additional historical discovery of the first and only known, axiomatically consistent, generalization of 20th century mathematics as well as of its covering isomathematics into a form embedding irreversibility over time in ordered forward and backward units, in corresponding ordered forward and backward products and, consequently, in all subsequent mathematical structures, resulting in the new mathematics nowadays known as *Santilli forward and backward genomathematics* with corresponding physical and chemical theories for the representation of irreversible processes.

Since the reversibility over time of 20th century theories can be reduced to the invariance under anti-Hermiticity of the Lie product between Hermitean operators, $[a, b] = ab - ba = -[a, b]^{\dagger}$, Santilli presented in 1967 [15] the first known (p, q)-deformation of the Lie product $(a, b) = pab - qba$, where p, q are scalars and the product ab is generally non-associative. Following an intense search in European mathematical libraries, Santilli discovered that the new product verifies the axiom of *Lie-admissibility* by the American mathematician A. A. Albert [16] in the sense that the attached anti-symmetric product $[a, b] = (a, b) - (b, a)$ verifies the axioms of a Lie algebra.

Since spaceship during re-entry are notoriously irreversible over time, Santilli was invited by the Center for Theoretical Physics of the University of Miami, Florida, under NASA support, where he moved with his wife Carla and newly born daughter Luisa in August 1967, and published a number of additional works in Lie-admissibility, including the first known Lie-admissible generalization of Hamilton and Heisenberg equations [17,18], nowadays considered at the foundation of hadronic mechanics and chemistry, as well as the first and only known Lie-admissible formulation of dissipative plasmas surrounding spaceships during reentry [19].

Santilli then spent seven years, from 1968 to 1974, at the Department of Physics of Boston University, and then three years, from 1974 to 1977, at MIT, during which time he wrote, in his words, *Phys.. Rev of career-oriented papers nobody reads*. In September 1977, Santilli joined Harvard University and was invited by the DOE to study irreversible processes because all energy releasing processes are irreversible over time. In April 1978, Santilli published under his DOE support his most important mathematical contribution [20] (see also monographs [21]) in which he achieved a Lie-admissible covering of the various branches of Lie's theory, Eqs. (69), (70), including the most general known time evolution whose brackets characterize an algebra, Eqs. (1000). It should be indicated that the isotopies of Lie's theory outlined in the preceding section were derived by Santilli as a particular case of the broader Lie-admissible theory of Ref. [20], and then

³By including the multi-valued (Section 4) and hyperstructural formulations (Section 5), Lie-admissible equations (100) are so broad that it will take centuries for their generalizations. For this reason, Santilli has requested in his will that his tombstone should have the engraving

$$i\hat{A} = A < H - H > A$$

below his name.

published in monographs [1].

Subsequently, Santilli discovered in paper [2] of 1993 that the axiom of a numeric field, besides admitting a generalization of the multiplicative unit, also admit the restriction of the associative product to an ordered form to the right and, separately, to the left. In this way, Santilli discovered two additional classes of new numbers, today known as *Santilli forward and backward genoreal, genocomplex and genoquaternionic numbers*. In the seminal memoir [3] of 1996 Santilli discovered two additional coverings of the ordinary differential calculus and of its isotopic covering, today known as *Santilli forward and backward genodifferential calculi*, Eqs. (65) to (68). Santilli called a *genotopy* [20] the lifting of isomathematics into ordered formulations to the right and to the left in the Greek sense of inducing a covering of Lie's axioms, Eqs. (69), (70).

As it is well known, thousands of papers have been published beginning from the late 1980s on the so-called q-deformations of Lie algebras with product $(a, b) = ab - qba$ which are an evident particular case of Santilli Lie-admissible product [15]. What is lesser known, or not admitted, all q-deformations did not achieve invariance over time, thus being afflicted by serious inconsistencies, since they consisted of non-unitary theories formulated via the mathematics of unitary theories. Santilli solved this problem in 1997 by achieving the first and only known invariant formulation of q- as well as of (p, q)-deformations [22].

We should indicate that Santilli's conception of a genotopic lifting of his preceding isomathematics (indicated in Section 2 by "hat" on symbols plus the "arrow of time") is necessary to achieve a consistent representation of irreversibility because point-like particles can only experience action-at-a-distance interactions that are reversible over time. Therefore, a simple genotopy of 20th century mathematics based on the conventional associative product would be axiomatically inconsistent. Consequently, to represent irreversibility it is first necessary to lift 20th century mathematics into isomathematics, with consequential representation of extended-deformable particles via realizations of type (27) so that extended particles can experience non-Hamiltonian interactions needed for irreversibility. It is then necessary to add irreversibility via the ordering of all products. It should also be indicated that, when formulated via time-dependent isounits, isomathematics can become genomathematics via the identifications $\hat{I}(t, \dots) = \hat{I}^\dagger(t, \dots) = \hat{I}^\triangleright, \hat{I}(-t, \dots) = \hat{I}^\triangleleft(-t, \dots) = \hat{I}, \hat{I}(t, \dots) \neq \hat{I}(-t)$, and the judicious addition of ordered products.

Systematic studies on the Lie-Admissible treatment of irreversible systems were presented in memoir [3] and monographs [4]. Santilli's subsequent memoir [23] of 2006 remains to this day the most comprehensive presentation of Lie-admissible treatments of irreversibility at the classical and operator levels. Monographs [5] of 2008 presented an update. Paper collection [24] presents all available independent contributions in Lie-admissibility up to [1984]. The Proceedings of the *Third International Conference on Lie-admissible Treatment of Irreversible Systems* [25] present numerous additional independent contributions as well as references for the five *Workshops on Lie-0admissible*

Algebras organized by Santilli at Harvard University, and for the preceding two international conference in Lie-admissibility, the first at the Université d'Orleans, France, in 1981 and the second at the Castle Prince Pignatelli, Italy, in 1995 (see also the general review [14] and large literature quoted therein).

As it is well known, there exists a large number of papers on Lie-admissible algebras within the context of non-associative algebras (see Tomber's Bibliography [26] listing all significant papers in the field up to 1986). It should be indicated that, regrettably, these studies have no connection with Santilli genomathematics since the latter deals with the irreversible generalizations of all aspects of 20th century mathematics.

4. Classical Hypermathematics, Hypermechanics and Hyperchemistry

4.1. Represented Systems s [1-5]

Multi-valued, time-irreversible systems of extended-deformable particles or constituents under the most general known Hamiltonian and non-Hamiltonian interaction, as occurring for multi-valued universes or the structure of the DNA.

4.2. Main Mathematical Structures [1-5]

Basic HyperUnits and HyperProducts

$$\hat{I}^\triangleright = \{\hat{I}_1^\triangleright, \hat{I}_2^\triangleright, \hat{I}_3^\triangleright, \dots\} = 1/\hat{S}, \quad (109)$$

$$\hat{I}^\triangleleft = \{\hat{I}_1^\triangleleft, \hat{I}_2^\triangleleft, \hat{I}_3^\triangleleft, \dots\} = \frac{1}{\hat{R}}, \quad (110)$$

Forward and Backward HyperProducts

$$A > B = \{A \times \hat{S}_1 \times B, A \times \hat{S}_2 \times B, A \times \hat{S}_3 \times B, \dots\}, \hat{I}^\triangleright > \\ A = A > \hat{I}^\triangleright = A \times I, \quad (111)$$

$$A < B = \{A \times \hat{R}_1 \times B, A \times \hat{R}_2 \times B, A \times \hat{R}_3 \times B, \dots\}, \hat{I}^\triangleleft < \\ A = -A < \hat{I}^\triangleleft = I \times A, \quad (112)$$

$$A = A^\dagger, B = B^\dagger, \hat{R} = \hat{S}^\dagger. \quad (113)$$

Classical hypermathematics then follow as for genomathematics with multi-valued units, quantities and operations.

4.3. Classical Hyper-Dynamical Equations s [1-5]

The same as those for genomathematics, but with multi-valued hyperunits, quantities and operations.

Comments and References

The multi-valued three-dimensional (rather than multi-dimensional) realization of genomathematics outlined in Section 4 emerged from specific biological needs. The Australian biologist C. Illert [27] confirmed that the *shape* of seashells can indeed be represented in a three-dimensional Euclidean space as known since Fourier's time, but proved that the *growth in time* of a seashell cannot any longer be

consistently represented in a conventional, three-dimensional Euclidean space, and achieved a consistent representation via the doubling of the three reference axis.

Santilli [27,28] confirmed Illert's findings because the conventional Euclidean geometry has no time arrow and, consequently, cannot consistently represent a strictly irreversible system, such as the growth of seashells. Additionally, Santilli proved that this geno-Euclidean geometry, Eqs. (71) to (73), is equally unable to represent the growth in time of seashells despite its irreversible structure, however, an axiomatically consistent and exact representation of the growth of seashells was possible via the multi-valued realization of the forward geno-Euclidean geometry, thus beginning to illustrate the complexity of biological structures.

The multi-valued, rather than multi-dimensional character of classical hypermathematics is indicated by Santilli as follows [28] *We perceive the growth of a seashell specifically in three dimensions from our Eustachian lobes. Therefore, an irreversible mathematics suitable to represent the growth of sea shells must be perceived by us as being in three dimensions. However, Illert has shown the need to double the three Cartesian axis. Classical hypermathematics has been conceived and structured in such a way that the increase of the reference axes is complemented by a corresponding multi-valued hyperunit in such a way that a classical hyper-Euclidean geometry, when seen at the abstract level, remains indeed three-dimensional as necessary to achieve representation of biological structures compatible with our sensory perception.*

5. Hope Hypermathematics, Hypermechanics and Hyperchemistry

5.1. Represented Systems

The most complex known multi-valued, time-irreversible requiring extremely large number of data, such as the DNA code [31-35].

5.2. Comments and References

Despite the preceding structural generalization of 20th century mathematics, Santilli remained dissatisfied in view of the complexity of nature, particularly of biological entities because advances in the *structure* of the DNA are indeed possible via classical hypermathematics, as we shall see in the third collection of this series dedicated to chemistry (e.g., via Santilli hypermagnecules), but any attempt at representing the DNA *code* via any of the preceding mathematics can be proved to be excessively restrictive due to the volume, complexity, diversification and coordination of the information.

Therefore, Santilli approved one of the most important mathematicians in hyperstructures, T. Vougiouklis from Greece, and asked for his assistance in further generalizing the preceding mathematics via hyperstructures defined on hyperfields, as necessary for applications implying measurements, and formulated via hyperoperations (called

"hope") permitting the needed broadening of the representational capability.

The above contact lead to the hypermathematics indicated in this section as presented in Refs. [29-33] which is based on Vougiouklis H_v hyperaxioms and which mathematics, in Santilli's words, *constitutes the most general mathematics that can be conceived nowadays by the human mind.*

6. Isodual Mathematics, Mechanics and Chemistry

6.1. Represented Systems

Single-valued, closed-isolated, time-reversible systems of classical and operatorpoint-like antiparticles moving in vacuum solely under action at a distance Hamiltonian interactions, such as the structure of antimatter atoms and antimatter molecules [2,36-43].

6.2. Main Mathematical Structure [2, 36-43]

Basic isodual unit

$$1^d = -1^\dagger = -1, \quad (114)$$

Isodual numeric fields

$$F^d(n^d, \times^d, 1^d), n^d = n \times 1^d, n^d \times^d m^d = n^d \times (1^d)_{-1} \times m^d \in F^d,$$

$$n^d = \text{isodual! real, complex, quatern. ! numbers}, \quad (115)$$

Isodual functional analysis

$$f^d(r^d) = f(r^d) \times 1^d \in F^d \quad (116)$$

Isodual differential calculus

$$d^d r^d = (1)^{-1} \times dr^d = dr, \quad (117)$$

$$\frac{\partial^d f^d(r^d)}{\partial^d r^d} = 1^d \times \frac{\partial f^d(r^d)}{\partial r^d}, \quad (118)$$

Santilli Isodual Lie theory

$$[X_i, X_j]^d = (X_i \times X_j - X_j \times X_i)^d = -C_{ij}^k \times X_k, \quad (119)$$

$$A^d(w^d) = e_d^{x \times w \times i} \times^d A^d(0) \times^d e_d^{-i \times w \times x}. \quad (120)$$

Santilli isodual Euclidean geometry

$$E^d(d, \delta^d, 1^d), r^d = (r^{dk}), k = 1,2,3,$$

$$\delta^d = \text{Diag.}(-1, -1, -1), \quad (121)$$

$$r^{d2d} = r^{di} \times \delta_{ij} \times^d r^{dj} = (r_1^2 + r_2^2 + r_3^2) \times 1^d \in F^d, \quad (122)$$

Santilli Isodual Minkowskian geometry ($\mu = 1,2,3,4$)

$$M^d(x^d, \eta^d, I^d): x^d = (x^{d\mu}), x^{d4} = t^d = t \times 1^d = -t, \quad (123)$$

$$\eta^d = \text{Diag.}(-1, -1, -1, +c^{d2d}), \quad (124)$$

$$x^{d2d} = (x^\mu \times \eta_{\mu\nu} \times x^\nu)^d$$

$$= (x_1^2 + x_2^2 + x_3^2 - t^2 c^2) \times 1^d \in F^d, \quad (125)$$

Isodual Riemannian geometry, Santilli Isodual Symplectic Geometry.

6.3. Isodual Dynamical Equations [2, 36-43]

Newton-Santilli Isodual Equation

$$m^d \times^d \frac{d^d v^d}{d^d t^d} - F^{dSA}(t^d, r^d, v^d) = 0, \quad (126)$$

Isodual Variational Principle

$$\delta^d A^d = \delta^d \int^d (p_k^d \times^d d^d r^{dk} - H^d \times^d d^d t^d) = 0. \quad (127)$$

Hamilton-Santilli Isodual Equations without external terms

$$\frac{d^d r^{dk}}{d^d t^d} = \frac{\partial^d H^d(r^d, p^d)}{\partial^d p_k^d}, \quad \frac{d^d p_k^d}{d^d t^d} = -\frac{\partial^d H^d(r^d, p^d)}{\partial^d r^{dk}}, \quad (128)$$

Isodual Hilbert space H^d over C with states $|\psi^d \rangle = -\langle \psi|$ over C^d

Expectation value of a Hermitean operator A

$$\langle A^d \rangle = \langle \psi| \times A^d \times |\psi \rangle \in C^d m \quad (129)$$

Heisenberg-Santilli Isodual Equations

$$i^d \times^d \frac{d^d A^d}{d^d t^d} = [A, H]^d = (A \times H - H \times A)^d, \quad (130)$$

Schrödinger-Santilli Isodual Equations

$$H^d \times^d |\psi^d \rangle = E^d \times^d |\psi^d \rangle = -E \times |\psi \rangle \quad (131)$$

$$p^d \times^d |\psi^d \rangle = +i^d \times^d \partial_{r^d}^d |\psi^d \rangle \quad (132)$$

Dirac-Santilli Isodual Equation

$$(\eta^{d\mu\nu} \times^d \gamma_\mu^d \times^d p_\nu^d + i^d \times^d m^d \times^d c^d) \times |\psi \rangle = 0. \quad (133)$$

$$\{\gamma_\mu, \gamma_\nu\}^d = (\gamma_\mu \times \gamma_\nu + \gamma_\nu \times \gamma_\mu)^d = 2^d \times^d \eta_{\mu\nu}^d, \quad (134)$$

Comments and References

In addition to the the study of irreversible processes and the representation of extended-deformable particles, during his Ph. D. studies of the md 1960s Santilli was interested to ascertain whether a far away galaxy is made up of matter or of antimatter. He soon discovered that none of the mathematics and physics he had learned during his graduate studies was applicable for a quantitative study of the problem considered since, at that time, antimatter was solely represented in second quantization, while the study of far away antimatter galaxies requested their representation at the purely *classical and neutral* level. In this way, Santilli initiated a solitary scientific journey that lasted for half a century.

This occurrence created one of the biggest imbalances in scientific history because matter was treated at all possible levels, from Newtonian mechanics to second quantization, while antimatter was solely treated in second quantization. The imbalance originated from the fact that special and

general relativities had been conceived decades before the discovery of antimatter and, therefore, they had no possibility of representing antimatter at the classical and neutral (as well as charged) level.

It should be stressed that the ongoing trend to extend the application of special and general relativities to the classical treatment of antimatter is afflicted by a number of serious inconsistencies, such as the impossibility to achieve a consistent representation of neutral antimatter, the impossibility to reach a consistent representation of matter-antimatter annihilation (evidently due to the lack of a suitable conjugation from matter to antimatter), violation of the PCT theorem and other inconsistencies that remain generally ignored.

Being an applied mathematician by instinct and training, Santilli knew that the imbalance was the result of a purely mathematical insufficiency because the transition from matter to antimatter is an anti-homomorphism. Consequently, the description of antimatter required a mathematics which is anti-homomorphic to conventional mathematics.

Santilli dedicated a decade to the search of the needed mathematics for antimatter. Following an additional extended search done at the Department of Mathematics of Harvard University under DOE support in the early 1980s, *Santilli concluded that a mathematics suitable for the joint classical and operator treatment of antimatter did not exist and had to be constructed.*

In the early 1980s, Since he had introduced the isoproduct $A \times B = A \times \hat{T}B, \hat{T} > 0$, Eq. (25). Consequently, it was natural to introduce its *negative-definite* counterpart which he called *isodual* and denoted with the upper index d , namely $A \times^d B = A \times \hat{T}^d B, \hat{T}^d = (\hat{T}^d)^\dagger < 0$. While constructing the isotopies of 20th century mathematics presented in Section 2, Santilli initiated the construction of their isodual image but published no paper in the new mathematics for over a decade.

This caution was due to the fact that, despite the lack of any visible mathematical inconsistency, Santilli remained skeptical on a mathematics based on a negative-definite product is afflicted by known physical inconsistencies, such as the violation of causality for negative time, energies and other physical quantities.

A breakthrough occurred in paper [2] of 1993. During the achievement of the broadest possible realizations of the abstract axioms of a numeric field (of characteristic zero), Santilli discovered that realizations with negative-definite units were simply unavoidable. This lead to the discovery of additional new numbers, today known as *Santilli isodual real, isodual complex and isodual quaternionic numbers* occurring for $I^d = -1$, Eq. (14), with isodual products (5), which are at the foundation of the isodual mathematics of this section and the additional numbers known as *Santilli isodual iso- and isodual geno-real, complex and quaternionic numbers* which are at the foundation of the isodual isomathematics and isodual genomathematics of Sections 7 and 8m respectively [2].

The discovery of isodual numbers is truly historical in our view due to its far reaching implications. In fact, the discovery

established the existence of the desired *isodual mathematics* as an anti-isomorphic image of 20th century mathematics for the representation of antimatter. Additionally, the discovery permitted the resolution of the problems of causality for negative values of physical quantities.

To avoid insidious inconsistencies generally not seen by non-experts in the field, the isodual map must be applied for consistency to the *totality* of quantities and their operations. This led to Santilli's conception of antimatter as possessing its negative-definite physical quantities for time, energy, momentum, frequency, etc, but such negative values are referred to *negative units* of measurements. Consequential a theory with negative time referred to negative units of time is as causal as our reality with a positive time referred to positive units, and the same holds for all other physical quantities.

Following the resolution of these basic issues, Santilli published in 1994 his first paper [36] specifically devoted to the isodual representation of antimatter. In mathematical memoir [3] of 1996, Santilli achieved the first isodual mathematical and physical representation of antimatter. In paper [37] of 1998, Santilli achieved his first goal of the early 1960s, namely, a consistent classical representation of neutral (as well as charged) antimatter.

By the early 1990s, Santilli had shown that *isodual mathematics represents all available experimental, data on antimatter at the classical and operator level*. Hence, he initiated the second phase of his studies, namely, the identification of new predictions for subsequent experimental verification.

A breakthrough occurred at the 1996 *First International Conference on Antimatter* held in Sepino, Italy [38]. By that time, Santilli had shown that the only conceivable representation of *neutral* antimatter required the conjugation of the sign of all physical quantities (jointly with the corresponding conjugation of their units of measurements). Since photons are neutral, the application of the same principle to light implies light emitted by antimatter, that Santilli called *isodual light*, is physically different than light emitted by matter in an experimentally verifiable way, e.g., because antimatter light is predicted to be *repelled* by a matter gravitational field.

Santilli then passed to a deeper geometric study of the gravitational field of antimatter. As indicated earlier, general relativity was formulated decades before the discovery of antimatter and, therefore, had no clue for the representation of the gravitational field of antimatter bodies. In Ref.[39] of 1998, Santilli conducted an in depth geometric study of antimatter, and in monograph [40] of 2006, Santilli completed the gravitational study of antimatter via the isodual Riemannian geometry.

All these studies concluded with the prediction of *gravitational repulsion* (antigravity) between matter and antimatter at all levels of analysis, from the isodual Newton-Santilli equations (26) to isodual second quantization. These aspects will be studied in the second collection of this series dedicated to hadronic mechanics.

Thanks to all the above advances, Santilli was finally in a

position to address his original main aim of the 1960s, namely, ascertain whether a far away galaxy is made up of matter or of antimatter. The preceding studies had established that the light emitted by antimatter must have a *negative index of refraction* that, as such, require *concave* lenses for its focusing. Consequently, Santilli secured the construction of a revolutionary telescope with concave lenses. About fifty years following his original aim, Santilli finally published in 2013 [41] measurements of the night sky with his new telescope showing images that can be solely due to light with a negative index of refraction which light, in turn, can solely originate from far away antimatter stars or galaxies (see also the two independent confirmations [42,43]).

An intriguing aspect that should be of interest to pure mathematicians is the conclusion of these studies illustrating the power of new mathematics, to the effect that none of the large numbers of telescopes available nowadays can detect antimatter stars or galaxies since they all have *convex* lenses. Similarly, as humans evolved in a matter world, we will never be able to see antimatter with our eyes since our cornea is convex and, as such, it will disperse antimatter light all over the retina.

Needless to say, isodual mathematics and its application to antimatter have implications so intriguing that are stimulating the participation of a large number of scientists as we shall report in the second collection of this series

7. Isodual Isomathematics, Isodual Isomechanics and Isodual Isochemistry

7.1. Represented Systems [2, 36-43]

Single-value, closed-isolated, time-reversible system of classical or operator extended-deformable antiparticles with action at a distance Hamiltonian and contact non-Hamiltonian interactions, such as the structure of antimatter hadrons, nuclei and stars, in the antimatter valence electron bonds and other antimatter systems.

7.2. Main Mathematical Structure[2, 36-43]

Basic Isodual IsoUnit

$$\hat{I}^d = \hat{I}^d(r^d, p^d, a^d, \psi^d, \partial^d \psi^d, \dots) = 1^d / \hat{T}^d < 0, \quad (135)$$

Basic Isodual IsoFields

$$\hat{F}^d(\hat{n}^d, \hat{\alpha}^d, \hat{I}^d), \hat{n}^d = n \times \hat{I}^d, \hat{n}^d \hat{\alpha}^d \hat{m}^d = \hat{n}^d \times \hat{T}^d \times \hat{m}^d \in \hat{F}^d, \quad (136)$$

Isodual IsoCoordinates $\hat{r}^d = r \times \hat{I}^d \in \hat{F}^d$,

Isodual IsoFunctional analysis $\hat{f}^d(\hat{r}^d) = f(\hat{r}^d) \times \hat{I}^d \in \hat{F}^d$,

Isodual IsoDifferential Calculus

$$\hat{d}^d \hat{r}^d = dr - r^d \times \hat{T}^d \times d\hat{I}^d, \quad (137)$$

$$\frac{\partial^d \hat{f}^d(\hat{r}^d)}{\partial^d \hat{r}^d} = \hat{I}^d \times \frac{\partial \hat{f}^d(\hat{r}^d)}{\partial \hat{r}^d}, \quad (138)$$

Santilli Isodual Lie-Isotopic Theory

$$[X_i, X_j]^d = X_i \hat{\times} X_j - X_j \hat{\times} X_i)^d == -C_{ij}^k(r, p, \dots) \times X_k, \quad (139)$$

$$A^d(w^d) = \hat{e}_d^{x^d \times w^d \times i^d} \hat{\times}^d A^d(0^d) \hat{\times}^d \hat{e}_d^{-i^d \times w^d \times x^d}. \quad (140)$$

Santilli Isodual Iso-Euclidean Geometry

$$\hat{E}^d(\hat{r}^d, \hat{\delta}^d, \hat{I}^d), \hat{\delta}^d(r^d, p^d, a^d, \psi, \dots) = \hat{T}^d(r^d, p^d, a^d, \psi^d, \dots) \times \delta, \quad (141)$$

$$\hat{T}^d = \text{Diag.} (1/n_1^2, 1/n_2^2, 1/n_3^2)^d, \quad (142)$$

$$\hat{r}^{d2d} = (\hat{r}^i \hat{\times} \hat{\delta}_{ij} \hat{\times} \hat{r}^j)^d = (\frac{r_1^2}{n_1^2} + \frac{r_2^2}{n_2^2} + \frac{r_3^2}{n_3^2})^d \times \hat{I}^d \in \hat{F}^d, \quad (143)$$

Santilli Isodual Iso-Minkowskian Geometry ($\mu = 1, 2, 3, 4$)

$$\hat{M}^d(\hat{x}^d, \hat{\eta}^d, \hat{I}^d): \hat{x}^d = (\hat{x}^{d\mu}), \hat{x}_4^d = \hat{t}^d = t \times \hat{I}^d, \quad (144)$$

$$\hat{\eta}^d(x^d, \psi^d, \dots) = \hat{T}^d(x^d, \psi^d, \dots) \times \eta, \quad (145)$$

$$\hat{T}^d = \text{Diag.} (1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2)^d, \quad (146)$$

$$\hat{x}^{d2d} = (\hat{x}^\mu \hat{\times} \hat{\eta}_{\mu\nu} \hat{\times} \hat{x}^\nu)^d == (\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2})^d \times \hat{I}^d \in \hat{F}^d, \quad (147)$$

Santilli Isodual Iso-Riemannian Geometry

$$\hat{R}^d(\hat{x}^d, \hat{g}^d, \hat{I}^d): \hat{g}^d = \hat{T}^d(x^d, v^d, \dots) \times g(x), \quad (148)$$

$$\hat{x}^{d2d} = (\frac{g_{11}}{n_1^2} + \frac{g_{22}}{n_2^2} + \frac{g_{33}}{n_3^2} - \frac{g_{44}}{n_4^2})^d \times \hat{I}^d \in \hat{F}^d, \quad (149)$$

Santilli Isodual Iso-Symplectic Geometry

$$\hat{\omega}^d = \hat{d}\hat{r}^{dk} \hat{\wedge}^d \hat{d}\hat{p}_k^d \quad (150)$$

7.3. Isodual IsoDynamical IsoEquation[2,36-43

Newton-Santilli Isodual IsoEquation

$$\hat{m}^d \hat{\times}^d \frac{\hat{d}^d \hat{v}^d}{\hat{d}^d \hat{t}^d} - F^{dSA}(r^d, p^d) == (m \times \frac{dv}{dt})^d - F^{dSA}(r^d, p^d) - F^{dNSA}(r^d, p^d, \dots) = 0^d = 0, \quad (151)$$

Isodual IsoVariational principle

$$\delta^d \hat{A}^d = \delta^d \int \hat{p}_k^d \hat{\times}^d \hat{d}^d \text{hatr}^{dk} - \hat{H}^d \hat{\times}^d \hat{d}^d \hat{t}^d = 0^d = 0. \quad (152)$$

Hamilton-Santilli Isodual IsoEquations

$$\frac{\hat{d}^d \hat{r}^{dk}}{\hat{d}^d \hat{t}^d} = \frac{\hat{\partial}^d \hat{H}^d(\hat{r}^d, \hat{p}^d)}{\hat{\partial}^d \hat{p}_k^d}, \quad \frac{\hat{d}^d \hat{p}_k}{\hat{d}^d \hat{t}^d} = + \frac{\hat{\partial}^d \hat{H}^d(\hat{r}^d, \hat{p}^d)}{\hat{\partial}^d \hat{r}^{dk}}, \quad (153)$$

Isodual iso-Hilbert space \hat{H}^d over C with states $|\hat{\psi}^d\rangle = \langle \hat{\psi} |$ over \hat{C}^d

Expectation value of a Hermitean operator A

$$\langle A^d \rangle = \langle \hat{\psi} | \hat{\times}^d A^d \hat{\times}^d | \hat{\psi} \rangle \in C^d \quad (154)$$

Heisenberg-Santilli Isodual IsoEquation

$$\hat{I}^d \hat{\times}^d \hat{d}^d \text{hatA}^d \text{over} \hat{d}^d \hat{t}^d = [\hat{A}, \hat{H}]^d = (\hat{A} \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{A})^d = (\hat{A} \times \hat{T}(\hat{\psi}, \hat{\delta}\hat{\psi}, \dots) \times \hat{H}(\hat{r}, \hat{p}) - \hat{H}(\hat{r}, \hat{p}) \times \hat{T}(\hat{\psi}, \hat{\delta}\hat{\psi}, \dots) \times \hat{A})^d, \quad (155)$$

Schrödinger-Santilli Isodual IsoEquation

$$(\hat{H} \hat{\times} | \hat{\psi} \rangle)^d = \langle \hat{\psi}^d | \hat{\times}^d \hat{H}^d = (\hat{H}(\hat{r}, \hat{p}) \times \hat{T}(\hat{\psi}, \hat{\delta}\hat{\psi}, \dots) \times | \hat{\psi} \rangle)^d = - \langle \hat{\psi}^d | \hat{\times}^d \hat{E}^d = - \langle \hat{\psi}^d | \times \hat{E}^d, \quad (156)$$

$$(\hat{p} \hat{\times} | \hat{\psi} \rangle)^d = \langle \hat{\psi}^d | \hat{\times}^d \hat{\partial}_{r^d} = -i \times \langle \hat{\psi}^d | \hat{\times}^d \hat{\partial}_{r^d}, \quad (157)$$

Dirac-Santilli Isodual IsoEquation

$$[(\hat{\eta}^{\mu\nu} \hat{\times} \hat{\gamma}_\mu \hat{\times} \hat{p}_\nu - \hat{I} \hat{\times} \hat{m} \hat{\times} \hat{c}) \hat{\times} | \hat{psi} \rangle]^d = 0. \quad (158)$$

$$\{\hat{\gamma}_\mu, \hat{\gamma}_\nu\}^d = (\hat{\gamma}_\mu \hat{\times} \hat{\gamma}_\nu + \hat{\gamma}_\nu \hat{\times} \hat{\gamma}_\mu)^d = \hat{2}^d \hat{\times}^d \hat{\eta}_{\mu\nu}^d, \quad (159)$$

Comments and References

See monograph [40] with particular reference to the use of the isodual isomathematics for the achievement of a grand unification of electroweak and gravitational interactions inclusive of matter and antimatter.

8. Isodual Genomathematics, Isodual Genomechanics and Isodual Genochemistry

8.1. Represented Systems [2, 36-43]

Single-valued, time-irreversible system of extended-deformable antiparticles under action at a distance Hamiltonian and contact non-Hamiltonian interactions, as occurring in antimatter nuclear reactions, antimatter biological structures and antimatter chemical reactions.

8.2. Main Mathematical Structure [2, 36-43]

Backward Isodual GenoUnit

$$\hat{I}^{>d} = \hat{I}^{>d}(t^{>r}, p^{>d}, a^{>d}, \psi^{>d}, \partial^{>d} \psi^{>d}, \dots) = 1/\hat{T}^{>d} > 0, \quad (160)$$

Forward Isodual GenoUnit

$$\langle^d \hat{I} = \langle^d \hat{I}(\langle^d r, \langle^d p, \langle^d a, \langle^d \psi, \langle^d \partial \langle^d \psi, \dots) = 1/\langle^d \hat{T} > 0, \quad (161)$$

Condition for time-irreversibility

$$\hat{I}^{>d} \neq \langle^d \hat{I} \quad (162)$$

Backward Isodual GenoFields

$$\hat{F}^{>d}(\hat{n}^{>d}, \dots, \hat{I}^{>d}), \hat{n}^{>d} = n \times \hat{I}^{>d}, \hat{n}^{>d} >^d \hat{m}^{>d} = \hat{n}^{>d} \times \hat{I}^{>d} \times \hat{m}^{>d} \in \hat{F}^{>d}, \quad (163)$$

Forward Isodual GenoFields

$$\langle^d \hat{F}(\langle^d \hat{n}, \langle^d \hat{I}), \langle^d \hat{n} = \langle^d \hat{I} \times n, \langle^d \hat{n} \langle^d \langle^d \hat{m} = \langle^d \hat{n} \times \langle^d \hat{T} \times \langle^d \hat{m} \in \langle^d \hat{F}, \quad (164)$$

Backward Isodual GenoCoordinates

$$\hat{r}^{>d} = r \times \hat{I}^{>d} \in \hat{F}^{>d}, \quad (165)$$

Forward Isodual GenoCoordinates

$$\langle^d \hat{r} = \langle^d \hat{I} \times r \in \langle^d \hat{F}, \quad (166)$$

Backward Isodual GenoFunctional analysis

$$\hat{f}^{>d}(\hat{r}^{>d}) = f(\hat{r}^{>d}) \times \hat{I}^{>d} \in \hat{F}^{>d}, \quad (167)$$

Forward Isodual GenoFunctional analysis

$$\langle^d \hat{f}(\langle^d \hat{r}) = f(\langle^d \hat{r}) \times \langle^d \hat{I} \in \langle^d \hat{F} \quad (168)$$

Backward Isodual GenoDifferential Calculus

$$\hat{d}^{>d} \hat{r}^{>d} = dr + r \times \hat{T}^{>d} \times d\hat{I}^{>d}, \quad (169)$$

$$\frac{\hat{\partial}^{>d} \hat{f}^{>d}(\hat{r}^{>d})}{\hat{\partial}^{>d} \hat{r}^{>d}} = \hat{I}^{>d} \times \frac{\partial \hat{f}^{>d}(\hat{r}^{>d})}{\partial \hat{r}^{>d}}, \quad (170)$$

Forward Isodual GenoDifferential Calculus

$$\langle^d \hat{d} \langle^d \hat{r} = dr + r \times \langle^d \hat{T} \times d \langle^d \hat{I}, \quad (171)$$

$$\frac{\langle^d \hat{\partial} \langle^d \hat{f}(\langle^d \hat{r})}{\langle^d \hat{\partial} \langle^d \hat{r}} = \langle^d \hat{I} \times \frac{\partial \langle^d \hat{f}(\langle^d \hat{r})}{\partial \langle^d \hat{r}}, \quad (172)$$

Santilli Isodual Lie-Admissible Theory

$$(X_i \hat{\times} X_j)^d = (X_i \langle X_j - X_j \rangle X_i)^d = -C_{ij}^{dk}(t^d, r^d, p^d, \psi^d, \dots) \times X_k, \quad (173)$$

$$A^d(w^d) = \hat{e}_{>d}^{X \times w \times i} \rangle^d A(0) \langle^d \hat{e}^{-i \times w \times X}. \quad (174)$$

Santilli Backward Geno-Euclidean Geometry

$$\hat{E}^{>d}(\hat{r}^{>d}, \hat{\delta}^{>d}, \hat{I}^{>d}), \hat{\delta}^{>d}(t, r, p, \psi, \dots) = \hat{T}^{>d}(t, r, p, \psi, \dots) \times \delta, \quad (175)$$

$$\hat{r}^{>d \hat{2}d} = (\hat{r}^{>di} \rangle^d \hat{\delta}_{ij}^{>d} \rangle^d \hat{r}^{>dj} \in F^{>d}, \quad (176)$$

$$\hat{T}^{>d} \neq \hat{T}^{>d \text{ transp}} \quad (177)$$

Santilli Forward Isodual Geno-Euclidean Geometry

$$\langle^d \hat{E}(\langle^d \hat{r}, \langle^d \hat{\delta}, \langle^d \hat{I}), \langle^d \hat{\delta}(t, r, p, \psi, \dots) = \langle^d \hat{T}(t, r, p, \psi, \dots) \times \delta, \quad (178)$$

$$\langle^d \hat{2}d \hat{r} = \langle^d \hat{r} \langle^d \hat{\delta}_{ij} \langle^d \langle^d \hat{r} \in \langle^d F, \quad (179)$$

$$\langle^d \hat{T} \neq \langle^d \hat{T} \text{ transp} \quad (180)$$

Santilli Backward Isodual Geno-Minkowskian Geometry ($\mu = 1, 2, 3, 4$)

$$\hat{M}^{>d}(\hat{x}^{>d}, \hat{\eta}^{>d}, \hat{I}^{>d}): \hat{x}^{>d} = (\hat{x}^{>d\mu}), x_4^{>d} = t^{>d}, \quad (181)$$

$$\hat{\eta}^{>d}(x, \psi, \dots) = \hat{T}^{>d}(x, \psi, \dots) \times \eta, \quad (182)$$

$$\hat{x}^{>d2d} = \hat{x}^{>d\mu} \rangle^d \hat{\eta}_{\mu\nu}^{>d} \rangle^d \hat{x}^{>d\nu} \in \hat{F}^{>d}, \quad (183)$$

$$\hat{\eta}^{>d} \neq \hat{\eta}^{>d \text{ transp}} \quad (184)$$

Santilli Forward Isodual Geno-Minkowskian Geometry ($\mu = 1, 2, 3, 4$)

$$\langle^d \hat{M}(\langle^d \hat{x}, \langle^d \hat{\eta}, \langle^d \hat{I}): \langle^d \hat{x} = (\hat{x}^\mu), \langle^d x_4 = \langle^d t, \quad (185)$$

$$\langle^d \hat{\eta}(x, v, \dots) = \langle^d \hat{T}(x, v, \dots) \times \eta, \quad (186)$$

$$\langle^d \hat{x} \langle^d 2d = \langle^d \mu \hat{x} \langle^d \langle^d \hat{\eta}_{\mu\nu} \langle^d \langle^d v \hat{x} \in \langle^d \hat{F}, \quad (187)$$

$$\langle^d \hat{\eta} \neq \langle^d \hat{\eta} \text{ transp} \quad (188)$$

Santilli Backward Isodual Geno-Riemannian Geometry

$$\hat{R}^{>d}(\hat{x}^{>d}, \hat{g}^{>d}, \hat{I}^{>d}): \hat{g}^{>d} = \hat{T}^{>d}(x, v, \dots) \times g(x), \quad (189)$$

$$\hat{x}^{>d2d} = x^{>d\mu} \cdot \hat{g}_{\mu\nu}^{>d} \rangle^d x^{>d\nu} \in \hat{F}^{>d} \quad (190)$$

$$\hat{T}^{>d} \neq \hat{T}^{>d \text{ transp}} \quad (191)$$

Santilli Forward Isodual Geno-Riemannian Geometry

$$\langle^d \hat{R}(\langle^d \hat{x}, \langle^d \hat{g}, \langle^d \hat{I}): \langle^d \hat{g} = \langle^d \hat{T}(x, v, \dots) \times g(x), \quad (192)$$

$$\langle^d \hat{x} \langle^d 2d = \langle^d \mu \hat{x} \langle^d \langle^d \hat{g}_{\mu\nu} \langle^d \langle^d v \hat{x} \in \langle^d \hat{F}, \quad (193)$$

$$\langle^d \hat{g} \neq \langle^d \hat{g} \text{ transp} \quad (194)$$

Santilli Backward Isodual Geno-Symplectic Geometry

$$\hat{\omega}^{>d} = \hat{d}^{>d} \hat{r}^{>d} \hat{\times} \hat{\lambda}^{>d} \hat{d}^{>d} \hat{p}_k^{>d} \quad (195)$$

Santilli Forward Isodual Geno-Symplectic Geometry

$$\langle^d \hat{\omega} = \langle^d \hat{d} \langle^d \hat{r} \langle^d \hat{\lambda} \langle^d \hat{d} \langle^d \hat{p}_k \quad (196)$$

8.3. Isodual Genodynamical GenoEquations [2, 36-43]

Newton-Santilli Backward Isodual GenoEquation

$$\hat{m}^{>d} \rangle^d \frac{\hat{d}^{>d} \hat{p}^{>d}}{\hat{d}^{>d} \hat{t}^{>d}} - F^{>dSA}(t, r, p) = [m \times \frac{dv}{dt}]^{>d} - F^{SA>d}(t, r, p) - F^{NSA>d}(t, r, p, \dots) = 0, \quad (197)$$

Newton-Santilli Forward Isodual GenoEquation

$$\langle^d \hat{m} \langle^d \frac{\langle^d \hat{d} \langle^d \hat{p}}{\langle^d \hat{d} \langle^d \hat{t}} - \langle^d dSA F(t, r, p) = \langle^d [m \times \frac{dv}{dt}] - \langle^d dSA F(t, r, p) - \langle^d dNSA F(t, r, p, \dots) = 0, \quad (198)$$

Backward Isodual GenoVariational principle

$$\hat{\delta}^{>d} \hat{A}^{>d} = \hat{\delta}^{>d} \int^{>d} (\hat{p}_k^{>d} \rangle^d \hat{d}^{>d} \hat{r}^{>d} \hat{\times} \hat{d}^{>d} \hat{p}_k - \hat{H}^{>d} \rangle^d \hat{d}^{>d} \hat{t}^{>d}) = 0. \quad (199)$$

Forward Isodual GenoVariational principle

$$\langle^d \hat{\delta} \langle^d \hat{A} = \langle^d \hat{\delta} \langle^d \int^{>d} (\langle^d \hat{p}_k \langle^d \hat{d} \langle^d \hat{r} \langle^d \hat{d} \langle^d \hat{H} \langle^d \hat{d} \langle^d \hat{t}) = 0. \quad (200)$$

Backward Isodual Hamilton-Santilli GenoEquations

$$\hat{d}^{>d} \hat{r}^{>d} \hat{d} \hat{t} = [\frac{\hat{\partial} \hat{H}(r, p)}{\hat{\partial} \hat{p}_k}]^{>d}, [\frac{\hat{d}^{>d} \hat{p}_k^{>d}}{\hat{d}^{>d} \hat{t}^{>d}}] = -[\frac{\hat{\partial} \hat{H}(r, p)}{\hat{\partial} \hat{r}^k}]^{>d}, \quad (201)$$

Forward isodual Hamilton-Santilli GenoEquations

$$\frac{\langle^d \hat{a} \langle^d \hat{r}^k}{\langle^d \hat{a} \langle^d \hat{t}} = \langle^d \frac{\partial \hat{H}(\hat{r}, \hat{p})}{\partial \hat{p}_k} \rangle, \quad \frac{\langle^d \hat{a} \langle^d \hat{p}_k}{\langle^d \hat{a} \langle^d \hat{t}} = - \langle^d \left[\frac{\partial \hat{H}(\hat{r}, \hat{p})}{\partial \hat{r}^k} \right] \rangle, \quad (202)$$

Heisenberg-Santilli IsoDual GenoEquations

$$\hat{i} \hat{\times} \frac{d\hat{A}}{d\hat{t}} = (\hat{A}, \hat{H}) = \hat{A} \langle \hat{H} - \hat{H} \rangle \hat{A} = A \times^< T(\hat{\psi}, \hat{\partial}\hat{\psi}, \dots) \times \hat{H}(\hat{r}, \hat{p}) - \hat{H}(\hat{r}, \hat{p}) \times \hat{T}^>(\hat{\psi}, \hat{\partial}\hat{\psi}, \dots) \times \hat{A} \quad (203)$$

Schrödinger-Santilli Backward Isodual GenoEquations

$$\hat{H}^>d \rangle^d |\hat{\psi}^>d \rangle = \hat{H}^>d(\hat{r}, \hat{p}) \times \hat{T}^>d(\hat{\psi}, \hat{\partial}\hat{\psi}, \dots) \times |\hat{\psi}^>d \rangle = \hat{E}^>d \rangle^d |\hat{\psi}^>d \rangle = E^>d \times |\hat{\psi}^>d \rangle, \quad (204)$$

$$\hat{p}^>d \rangle |\hat{\psi}^>d \rangle = -i^>d \rangle \hat{\partial}_{\hat{r}}^>d |\hat{\psi}^>d \rangle = -i \times \hat{I}^> \times \partial_{\hat{r}} |\hat{\psi}^>d \rangle, \quad (205)$$

Schrödinger-Santilli Forward Isodual GenoEquations

$$\langle^d \langle^d \hat{\psi} | \langle^d \hat{H} = \langle^d \langle^d \hat{\psi} | \times^d \hat{T}(\hat{\psi}, \hat{\partial}\hat{\psi}, \dots) \times^d \hat{H}(\hat{r}, \hat{p}) = \langle^d \langle^d \hat{\psi} | \times^d \hat{E} = \langle^d \langle^d \hat{\psi} | \times^d E, \quad (206)$$

$$\langle^d \langle^d \hat{\psi} | \langle^d \hat{p} = - \langle^d \langle^d \hat{\psi} | \langle^d \hat{i} \langle^d \hat{\partial} = -i \times \langle^d \hat{\psi} | \langle^d \partial \times^d \hat{I} \quad (207)$$

Dirac-Santilli Backward Isodual IsoEquation

$$(\hat{\eta}^>d \mu \nu \rangle^d \hat{\gamma}_{\mu}^>d \rangle^d \hat{p}_{\nu}^>d - i^>d \rangle \hat{m}^>d \rangle^d \hat{c}^>d) \rangle |\hat{p}si^>d \rangle = 0. \quad (208)$$

$$\{\hat{\gamma}_{\mu} \hat{\gamma}_{\nu}\}^>d = [\hat{\gamma}_{\mu} \hat{\times} \hat{\gamma}_{\nu} + \hat{\gamma}_{\nu} \hat{\times} \hat{\gamma}_{\mu}]^>d = \hat{2}^>d \rangle \hat{\eta}_{\mu\nu}^>d, \quad (209)$$

Dirac-Santilli Forward Isodual GenoEquation

$$\langle^d \langle^d \hat{p}_{\nu} \langle^d \hat{\gamma}_{\mu} \langle^d \hat{c}^{\mu\nu} \hat{\eta} - \langle^d \hat{i} \langle^d \hat{m} \langle^d \hat{c} = 0. \quad (210)$$

$$\langle^d \{\hat{\gamma}_{\mu} \hat{\gamma}_{\nu}\} \langle^d = \langle^d [\hat{\gamma}_{\mu} \hat{\times} \hat{\gamma}_{\nu} + \hat{\gamma}_{\nu} \hat{\times} \hat{\gamma}_{\mu}] \langle^d \hat{2} \langle^d \hat{\eta}_{\mu\nu} = 2 \times^d \hat{\eta}_{\mu\nu}, \quad (211)$$

Comments and References

See memoir [20] which constitutes the most comprehensive study of antimatter in irreducible conditions available at this writing.

9. Isodual Classical and Hope Isodual Hypermathematics

Isodual Hyper-Formulations are generally considered to be part of the Hyper-Formulations of Section 4 and 5 because the classification of ordered sets of hyperunits includes isodual realizations, as illustrated in the paper [44] and references quoted therein.

10. Simple Method for the Construction of Regular Hadronic Mathematics

10.1. Introduction [4.5]

Hadronic formulations are called *regular* when the

structure quantities C_{ij}^k of Santilli's Lie-Isotopic algebras, Eqs. (3), Lie-admissible algebras, Eqs. (69) (zzz) and their isoduals, Eqs. (119-, (139), are *constant*. When the structure quantities are *functions of the local variables* $C_{ij}^k(t, r, p, \psi, \partial\psi, \dots)$, hadronic formulations are called *irregular*.

In this section, we shall review a very simple method for the construction of regular hadronic formulations via the mere use of non-unitary transformations of the corresponding conventional formulations. We shall then review the axiomatic consistency of hadronic formulations by showing that Santilli iso-, geno-, hyper-units and their isoduals are invariant under the transformations, thus implying the crucial invariance over time of extended-deformable shapes and their non-Hamiltonian interactions that are invariantly represented precisely nwith such generalized units.

No method exists to our knowledge at this writing (June 2015) for the construction of irregular hadronic formulations via maps of conventional formulations and, therefore, irregular hadronic formulations characterize a new axiomatic structure still mostly unexplored.

10.2. Simple Construction of Regular Iso-Formulations [4.5]

A simple method has been identified in Refs. [4,5] for the construction of the Lie-Santilli isothory, all its underlying isomathematics and all physical methods This method is important because it permits a simple implementation of conventional models into their isotopic covering without the need for advanced mathematics. The method consists in:

(i) Representing all conventional potential interactions with a Hamiltonian $H(r, p)$ and all extended-deformable shapes and their non-Hamiltonian interactions and effects with Santilli's isounit $\hat{I}(r, p, \psi, \partial\psi, \dots)$;

(ii) Identifying the latter interactions with a nonunitary transform

$$U \times U^{\dagger} = \hat{I} \neq I \quad (212)$$

and

(iii) Subjecting the *totality* of conventional mathematical and physical quantities and all their operations to the above nonunitary transform, resulting in expressions of the type

$$I \rightarrow \hat{I} = U \times I \times U^{\dagger} = 1/\hat{I}, \quad (213)$$

$$a \rightarrow \hat{a} = U \times a \times U^{\dagger} = a \times U \times U^{\dagger} = a \times \hat{I}, a \in F, \quad (214)$$

$$e^A \rightarrow U \times e^A \times U^{\dagger} = \hat{I} \times e^{\hat{T} \times \hat{A}} = (e^{\hat{A} \times \hat{T}}) \times \hat{I}, \quad (215)$$

$$A \times B \rightarrow U \times (A \times B) \times U^{\dagger} = (U \times A \times U^{\dagger}) \times (U \times B \times U^{\dagger}) = \hat{A} \hat{\times} \hat{B}, \quad (216)$$

$$[X_i, X_j] \rightarrow U \times [X_i X_j] \times U^{\dagger} = [\hat{X}_i, \hat{X}_j] = U \times (C_{ij}^k \times X_k) \times U^{\dagger} = \hat{C}_{ij}^k \hat{\times} \hat{X}_k = C_{ij}^k \times \hat{X}_k, \quad (217)$$

$$\langle \psi | \times | \psi \rangle \rightarrow U \times \langle \psi | \times | \psi \rangle \times U^{\dagger} = \langle \psi | \times U^{\dagger} \times (U \times | \psi \rangle \times U^{\dagger}) = \langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle \times \hat{I}, \quad (218)$$

$$H \times | \psi \rangle \rightarrow U \times (H \times | \psi \rangle) = (U \times H \times U^{\dagger}) \times (U \times$$

$$U^{\dagger})^{-1} \times (U \times |\psi \rangle) = \hat{H} \hat{\times} |\hat{\psi} \rangle, \text{etc.} \quad (219)$$

Note that serious inconsistencies emerge in the event even 'one' single quantity or operation is not subjected to the above non-unitary map. In the absence of comprehensive liftings, we would have a situation equivalent to the elaboration of quantum spectral data of the hydrogen atom with isomathematics, resulting in large deviations from reality.

The construction of isodual iso-formulations is simply done via Santilli's isodual map, namely, via the simple anti-hermitean image of the above isotopic formulations.

10.3. Axiomatic Consistency of Iso-Formulation [4.5]

Let us recall that Santilli's central assumption is the representation of extended-deformable shapes and their non-Hamiltonian interactions via the isounit. Therefore, any change of the numerical value of the isounit implies the inability to represent the same system over time, besides activating the *Theorem of Catastrophic Mathematical and Physical Inconsistencies of Non-Canonical and Non-Unitary Theories* when formulated via the mathematics of conventional canonical and unitary theories, respectively [23].

It is easy to see that the application of an additional nonunitary transform

$$W \times W^{\dagger} \neq I, \quad (220)$$

to the preceding expressions causes their *lack of invariance*, with consequential activation of the theorem of catastrophic inconsistencies. This is due to the *change of the value of the basic isounit* under additional non-unitary transformations

$$\hat{I} \rightarrow \hat{I}' = W \times \hat{I} \times W^{\dagger} \neq \hat{I}, \quad (221)$$

However, any given nonunitary transform can be identically rewritten in the isounitary form [3]

$$W \times W^{\dagger} = \hat{I}, \quad W = \hat{W} \times \hat{T}^{1/2}, \quad (222)$$

$$W \times W^{\dagger} = \hat{W} \hat{\times} \hat{W}^{\dagger} = \hat{W}^{\dagger} \hat{\times} \hat{W} = \hat{I}, \quad (223)$$

under which we have the invariance of the isounit and isoproduct [7]

$$\hat{I} \rightarrow \hat{I}' = \hat{W} \hat{\times} \hat{I} \hat{\times} \hat{W}^{\dagger} = \hat{I}, \quad (224)$$

$$\begin{aligned} \hat{A} \hat{\times} \hat{B} \rightarrow \hat{W} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{W}^{\dagger} &= (\hat{W} \times \hat{T} \times \hat{A} \times \hat{T} \times \hat{W}^{\dagger}) \times \\ &(\hat{T} \times \hat{W}^{\dagger})^{-1} \times \hat{T} \times (\hat{W} \times \hat{T})^{-1} \times (\hat{W} \times \hat{T} \times \hat{B} \times \hat{T} \times \hat{W}^{\dagger}) = \\ \hat{A}' \times (\hat{W}^{\dagger} \times \hat{T} \times \hat{W})^{-1} \times \hat{B}' &= \hat{A}' \times \hat{T} \times \hat{B}' = \hat{A}' \hat{\times} \hat{B}', \text{etc.} \end{aligned} \quad (225)$$

from which the invariance of the entire isotopic formalism follows.

Note that the invariance is ensured by the *numerically invariant values of the isounit and of the isotopic element under non-unitary-isounitary transformations*,

$$\hat{I} \rightarrow \hat{I}' \equiv \hat{I}, \quad (226)$$

$$A \hat{\times} B \rightarrow A' \hat{\times} B' \equiv A' \hat{\times} B', \quad (227)$$

in a way fully equivalent to the invariance of Lie's theory and quantum mechanics, as expected to be necessarily the case due to the preservation of the abstract axioms under isotopies. The resolution of the inconsistencies for non-invariant theories is then consequential.

The proof of the invariance of Santilli isodual iso-formulations is an interesting exercise for non-initiated readers.

10.4. Simple Construction of Regular GenoMathematics and its IsoDual [4.5]

An important feature of the Lie-Santilli genotheory is its *form invariance* under the appropriate geno-transformations in a way fully similar to the invariance of the mathematical and physical structures of quantum mechanics under unitary transformations.

This feature can be shown via a *pair* of non-unitary transformations

$$V \times V^{\dagger} \neq I, W \times W^{\dagger} \neq I, V \times W^{\dagger} \neq I, W \times V^{\dagger} \neq I, \quad (228)$$

under which we have the characterization of the forward and backward genounits and related genoproduct

$$I \rightarrow V \times I \times W^{\dagger} = \hat{I}^{\triangleright}, \text{eqno} \quad (229)$$

$$A \times B \rightarrow V \times (A \times B) \times W^{\dagger} = A^{\triangleright} \triangleright B^{\triangleright} \quad (230)$$

$$I \rightarrow W \times I \times V = {}^{\triangleleft} \hat{I}, \quad (231)$$

$$A \times B \rightarrow W \times (A \times B) \times V = {}^{\triangleleft} A \triangleleft {}^{\triangleleft} B / \quad (232)$$

10.5. Axiomatic Consistency of GenoMathematics and its Isodual [4.5]

It is easy to see that the above dual non-unitary transformations can always be identically rewritten as the *geno-unitary transforms* on geno-Hilbert spaces over complex genofields,

$$V \times V^{\dagger} \neq 1, V = {}^{\triangleleft} \hat{V} \times \hat{R}^{1/2}, V \times V^{\dagger} = {}^{\triangleleft} \hat{V} \triangleleft {}^{\triangleleft} \hat{V}^{\dagger} = {}^{\triangleleft} \hat{V}^{\dagger} \triangleleft {}^{\triangleleft} \hat{V} = {}^{\triangleleft} \hat{I}, \quad (233)$$

$$W \times W^{\dagger} \neq 1, W = \hat{W}^{\triangleright} \times \hat{S}^{1/2}, W \times W^{\dagger} = \hat{W}^{\triangleright} \triangleright \hat{W}^{\triangleright \dagger} = \hat{W}^{\triangleright \dagger} \triangleright \hat{W}^{\triangleright} = \hat{I}^{\triangleright}, \quad (234)$$

under which we have indeed the following forward geno-invariance laws [3j]

$$\hat{I}^{\triangleright} \rightarrow \hat{I}'^{\triangleright} = \hat{W}^{\triangleright} \triangleright \hat{I}^{\triangleright} \triangleright \hat{W}^{\triangleright \dagger} = \hat{I}^{\triangleright}, \quad (235)$$

$$\hat{A} \triangleright \hat{B} \rightarrow \hat{W}^{\triangleright} \triangleright (\hat{A} \triangleright \hat{B}) \triangleright \hat{W}^{\triangleright \dagger} = \hat{A}' \triangleright \hat{B}', \quad (236)$$

$$\begin{aligned} \hat{H}^{\triangleright} \triangleright | \triangleright = \hat{E}^{\triangleright} \triangleright | \triangleright = E \times | \triangleright \rightarrow \hat{W}^{\triangleright} \triangleright \hat{H}^{\triangleright} \triangleright | \triangleright = \hat{H}'^{\triangleright} \triangleright | \triangleright = \\ | \triangleright = \hat{W}^{\triangleright} \triangleright \hat{E}^{\triangleright} \triangleright | \triangleright = E \times | \triangleright, \end{aligned} \quad (237)$$

with corresponding rules for the backward and classical counterparts.

The above rules confirm the achievement of the *invariance of the numerical values of genounits, geno-products and geno-eigenvalues*, thus permitting physically consistent

applications.

The invariance of the isodual geno-formulations can then be proved via the isodual map applied to the above procedure.

11. Open Mathematical Problems

Among a predictable large number of basic open problems, we list for the interested readers the following ones:

Study methods to transform nonlinear models on conventional spaces into isolinear models on isospaces over isofields;

See whether simple solutions of isolinear equations on isospaces over isofields provide at least a solution of their nonlinear projection on conventional spaces over conventional fields;

Study the removal of divergencies in quantum mechanics and scattering theories (Footnote 2) by isomechanics on an iso-Hilbert space over an isofield.

Study the regular and irregular isorepresentations of the Lie-Santilli isothory;

Study Santilli isoMinkowskian geometry via the machinery of the Riemannian geometry, yet lack of curvature [39];

Study the Lie-admissible theory in Santilli's sense, that is, as a generalization of Lie's theory elaborated via genomathematics;

Study Santilli geno-Euclidean, geno-Minkowskian and geno-Riemannian geometries where irreversibility is embedded in the non symmetric character of the metric [23];

extend the Tsagas, Ganformina-Nunez isotopology to the genotopic form and their isoduals.

Research funds are available from the R. M. Santilli Foundation for partial support of research in the above listed and related open problems in hadronic mathematics.

Acknowledgments

Special thanks are due to Prof. R. M. Santilli for allowing the free use of his fifty years of scientific works, for endless consultations and for encouragement without which this paper could not have been written. Additional special thanks are due to the Trustees and various affiliates of the R. M. Santilli Foundation for their extended consultations to verify the accuracy of the main contributions. Final thanks are due to Mrs. Sherri Stone for an accurate editorial control of the manuscript. Needless to say, I am solely responsible for the content of this paper also in view of numerous last minutes technical, bibliographical and linguistic changes.

References

- [1] R. M. Santilli, *Foundations of Theoretical Mechanics*, Volumes I (1978) and II (1982) Springer-Verlag, Heidelberg, Germany (1983).
<http://www.santilli-foundation.org/docs/Santilli-209.pdf>,
<http://www.santilli-foundation.org/docs/santilli-69.pdf>
- [2] R. M. Santilli, Isonumbers and genonumbers of dimension 1, 2, 4, 8, their isoduals and pseudoisoduals, and "hidden numbers" of dimension 3, 5, 6, 7, Algebras, Groups and Geometries (1993), 273-322. <http://www.santilli-foundation.org/docs/Santilli-34.pdf>
- [3] R. M. Santilli, Nonlocal-integral isotopies of differential calculus, geometries and mechanics, Rendiconti Circolo Matematico di Palermo, Supplemento, in press, 1996, <http://www.santilli-foundation.org/docs/Santilli-37.pdf>
- [4] R. M. Santilli, *Elements of Hadronic Mechanics*, Vol. I (1995), Vol. II (1995), Academy of Sciences, Kiev, <http://www.santilli-foundation.org/docs/Santilli-300.pdf>,
<http://www.santilli-foundation.org/docs/Santilli-301.pdf>
- [5] R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry*, Volumes I, II, III, IV and V, International Academic Press (2008), <http://www.i-b-r.org/Hadronic-Mechanics.htm>
- [6] H. C. Myung and R. M. Santilli, "Modular-isotopic Hilbert space formulation of the exterior strong problem," *Hadronic Journal* , 1277-1366 (1982), <http://www.santilli-foundation.org/docs/Santilli-201.pdf>.
- [7] D. S. Sourlas and G. T. Tsagas, *Mathematical Foundation of the Lie-Santilli Theory*, Ukraine Academy of Sciences (1993), available as free download from <http://www.santilli-foundation.org/docs/santilli-70.pdf>
- [8] J. V. Kadeisvili, *Santilli's Isotopies of Contemporary Algebras, Geometries and Relativities*, Ukraine Academy of Sciences, Second edition (1997), available as free download from <http://www.santilli-foundation.org/docs/Santilli-60.pdf>
- [9] Chun-Xuan Jiang, *Foundations of Santilli Isonumber Theory*, International Academic Press (2001), <http://www.i-b-r.org/docs/jiang.pdf>
- [10] Raul M. Falcon Ganformina and Juan Nunez Valdes, *Fundamentos de la Isotopia de Santilli*, International Academic Press (2001), <http://www.i-b-r.org/docs/spanish.pdf>, English translations Algebras, Groups and Geometries Vol. 32, pages 135-308 (2015), <http://www.i-b-r.org/docs/Aversa-translation.pdf>
- [11] Raul M. Falcon Ganformina and Juan Nunez Valdes, "Studies on the Tsagas-Sourlas-Santilli Isotopology," *Algebras, Groups and Geometries* Vol. 20, 1 (2003), <http://www.santilli-foundation.org/docs/isotopologia.pdf>
- [12] S. Georgiev, *Foundations of the IsoDifferential Calculus*, Volumes, I, II, III, IV and V, Nova Scientific Publisher (2015 on).
- [13] A. S. Muktibodh, "Iso-Galois Fields," *Hadronic Journal* , 13-29 (2015), <http://www.santilli-foundation.org/docs/Iso-Galoisfields.pdf>
- [14] I. Gandzha and J. Kadeisvili, *New Sciences for a New Era" Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli*, Sankata Printing Press, Nepal (2011), available in free pdf download from the link, <http://www.santilli-foundation.org/santilli-scientific-discoveries.html>
- [15] R. M. Santilli, "Embedding of Lie algebras in Lie-admissible algebras. *Nuovo Cimento* , 570 (1967), <http://www.santilli-foundation.org/docs/Santilli-54.pdf>
- [16] A. A. Albert, *Trans. Amer. Math. Soc.* , 552 (1948).
- [17] R. M. Santilli, "An introduction to Lie-admissible algebras," *Suppl. Nuovo Cimento* , 1225 (1968).
- [18] R. M. Santilli, "Dissipativity and Lie-admissible algebras," *Meccanica* , 3 (1969).

- [19] P. Roman and R. M. Santilli, "A Lie-admissible model for dissipative plasma," *Lettere Nuovo Cimento* Vol. 2, 449-455 (1969)
- [20] R. M. Santilli, On a possible Lie-admissible covering of the Galilei relativity in Newtonian mechanics for nonconservative and Galilei non-invariant systems, *Hadronic J* [1978], 223-423; Addendum, *ibid.* 1 (1978), 1279-1342. <http://www.santilli-foundation.org/docs/Santilli-58.pdf>
- [21] R. M. Santilli, *Lie-admissible Approach to the Hadronic Structure*, Volumes I (1978) and II (1981), Hadronic Press (1981), <http://www.santilli-foundation.org/docs/santilli-71.pdf>, <http://www.santilli-foundation.org/docs/santilli-72.pdf>
- [22] R. M. Santilli, "Invariant Lie-admissible formulation of quantum deformations," *Found. Phys.* , 1159- 1177 (1997), <http://www.santilli-foundation.org/docs/Santilli-06.pdf>
- [23] R. M. Santilli, "Lie-admissible invariant representation of irreversibility for matter and antimatter at the classical and operator levels," *Nuovo Cimento B* 121, 443 (2006), <http://www.santilli-foundation.org/docs/Lie-admiss-NCB-I.pdf>
- [24] A. Schoeber, Editor, *Irreversibility and Non-potentiality in Statistical Mechanics*, Hadronic Press (1984), <http://www.santilli-foundation.org/docs/Santilli-110.pdf>
- [25] C. Corda, Editor, *Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes*, Kathmandu University (2011), <http://www.santilli-foundation.org/docs/Nepal-2011.pdf>
- [26] M. Tomber, *Bibliography and Index in non-Associative Algebras*, Volumes I, II, III, Hadronic Press (1986), <http://www.santilli-foundation.org/docs/Tomber.pdf>
- [27] C. R. Illert and R. M. Santilli, *Foundations of Theoretical Conciology*, Hadronic Press (1995), <http://www.santilli-foundation.org/docs/santilli-109.pdf>
- [28] R. M. Santilli, *Isotopic, Genotopic and Hyperstructural Methods in Theoretical Biology*, Ukraine Academy of Sciences, Kiev (1994), <http://www.santilli-foundation.org/docs/santilli-67.pdf>
- [29] T. Vougiouklis, "The Santilli theory 'invasion' in hyperstructures," *Algebras, Groups and Geometries* Vol. 28, pages 83-104 (2011), <http://www.santilli-foundation.org/docs/santilli-invasion.pdf>
- [30] Pipina Nikolaidou, Thomas Vougiouklis, "The Lie-Santilli admissible hyperalgebras of type A_n ," *Ratio Matematica* Vol. 26, pages 113-128 (2014), *American Journal of Modern Physics* Vol. 4, pages 5-9 (2015), <http://www.santilli-foundation.org/docs/115-RM-2677.pdf>
- [31] T. Vougiouklis, "On the Iso- H_v numbers," in press, <http://www.santilli-foundation.org/docs/isoHv-numbers.pdf>
- [32] R. M. Santilli, "Representation of antiparticles via isodual numbers, spaces and geometries," *Comm. Theor. Phys.* vol. 3, 1994, pp. 153-181, <http://www.santillifoundation.org/docs/Santilli-112.pdf>
- [33] R. M. Santilli, "Does antimatter emit a new light?" Invited paper for the proceedings of the International Conference on Antimatter, held in Sepino, Italy, on May 1996, invited publication in *Hyperfine Interactions* vol. 109, 1997, pp. 63-81, <http://www.santilli-foundation.org/docs/Santilli-28.pdf>
- [34] R.M. Santilli and T. Vougiouklis, "Lie-admissible hyperalgebras," *Italian Journal of Pure and Applied mathematics* Vol. 31, pages 239-254 (2013), http://www.santilli-foundation.org/docs/111_santilli-vougiouklis.pdf
- [35] B. Davvaz, *Hyperings Theory and Applications*, International Academic Press (2007), <http://www.santilli-foundation.org/docs/Davvaz.pdf>
- [36] R. M. Santilli, "Representation of antiparticles via isodual numbers, spaces and geometries," *Comm. Theor. Phys.* vol. 3, 1994, pp. 153-181, <http://www.santillifoundation.org/docs/Santilli-112.pdf>
- [37] Santilli, R. M. "Classical isodual theory of antimatter and its prediction of antigravity," *Intern. J. Modern Phys.* 1999, A 14, 2205-2238, <http://www.santilli-foundation.org/docs/Santilli-09.pdf>
- [38] R. M. Santilli, "Does antimatter emit a new light?" Invited paper for the proceedings of the International Conference on Antimatter, held in Sepino, Italy, on May 1996, invited publication in *Hyperfine Interactions* vol. 109, 1997, pp. 63-81, <http://www.santilli-foundation.org/docs/Santilli-28.pdf>
- [39] Santilli, R. M. "Isominkowskian Geometry for the Gravitational Treatment of Matter and its Isodual for Antimatter," *Intern. J. Modern Phys.* 1998, D 7, 351, <http://www.santilli-foundation.org/docs/Santilli-35.pdf>
- [40] R. M. Santilli, *Isodual Theory of Antimatter with Applications to Antigravity, Grand Unifications and Cosmology*, Springer (2006).
- [41] R. M. Santilli, "Apparent detection of antimatter galaxies via a telescope with convex lenses," *Clifford Analysis, Clifford Algebras and their Applications* vol. 3, 2014, pages 1-26 (Cambridge, UK), <http://www.santilli-foundation.org/docs/Antimatter-telescope-2013-final.pdf>
- [42] P. Bhujbal, J. V. Kadeisvili, A. Nas, S Randall, and T. R. Shelke, "Preliminary confirmation of antimatter detection via Santilli telescope with concave lenses," *Clifford Analysis, Clifford Algebras and their Applications* Vol. 3, pages 27-39, 2014 (Cambridge, UK), www.santilli-foundation.org/docs/Con-Ant-Tel-2013.pdf
- [43] S. Beghella-Bartoli, Prashant M. Bhujbal, Alex Nas, "Confirmation of antimatter detection via Santilli telescope with concave lenses," *American Journal of Modern Physics* Vol. 4, pages 34-41 (2015), <http://www.santilli-foundation.org/docs/antimatter-detect-2014.pdf>
- [44] B. Davvaz, R. M. Santilli and T. Vougiouklis, *Studies Of Multi-Valued Hyperstructures For The characterization Of Matter-Antimatter Systems And Their Extension* Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes, C. Corda, Editor, Kathmandu University (2011), pages 45-57, <http://www.santilli-foundation.org/docs/Davvaz-Sant-Vou.pdf>
- [45] Scientific archives from 1967 to 2007, <http://www.santilli-foundation.org/docs/Santilli-64.pdf>, Scientific Archives from 2007 to 2015, <http://www.santilli-foundation.org/news.html>

Studies on Santilli's Isonumber Theory

Arun S. Muktibodh

Mohota College of Science, Nagpur, India

Email address:

amukti2000@yahoo.com

To cite this article:

Arun S. Muktibodh. Studies on Santilli's Isonumber Theory. *American Journal of Modern Physics*. Special Issue: Issue II: Foundations of Hadronic Mechanics. Vol. 5, No. 2-1, 2016, pp. 17-36. doi: 10.11648/j.ajmp.2016050201.12

Received: August 10, 2015; **Accepted:** August 11, 2015; **Published:** May 18, 2016

Abstract: Beginning with studies in the 1980s at the Department of Mathematics of Harvard University, the Italian-American scientist R. M. Santilli discovered new realizations of the abstract axioms of numeric fields with characteristic zero, based on an axiom-preserving generalization of conventional associative product and consequential positive-definite generalization of the multiplicative unit, today known as Santilli *isonumbers* [1], and the resulting novel numeric fields are known as Santilli *isofields*. By remembering that 20th century mathematics was formulated on numeric fields, their generalization into isofields stimulated a corresponding generalization of all of 20th century mathematics and its application to mechanics, today known as Santilli *isomathematics* and *isomechanics*, respectively, which is used for the representation of extended-deformable particles moving within physical media under Hamiltonian as well as contact non-Hamiltonian interactions. Additionally, Santilli discovered a second realization of the abstract axioms of a numeric field, this time with arbitrary (non-singular) negative definite generalized unit and related multiplication, today known as Santilli *isodual isonumber* [1] that have stimulated a second covering of 20th century mathematics and mechanics known as Santilli *isodual isomathematics* and *isodual isomechanics*. The latter methods are used for the classical as well as operator form of antimatter in full democracy with the study of matter. In this paper, we present a comprehensive study of Santilli's epoch making discoveries of isonumbers and their isoduals along with their application to isomechanics and its isodual for matter and antimatter, respectively.

Keywords: Isonumber, Isodual Number, Isodual-Isonumber, Genonumber

1. Introduction

As it is well known, modern mathematics has a strong foundation on number theory, algebraic structures such as groups, rings, algebra, vector spaces and related methods have found vast applications in all quantitative sciences. More general structures like groupoids, semigroups, monoids, quasigroups and loops were also being studied in 20th century, although their applications in quantitative sciences are under development. The detailed consolidated account of these generalized structures is found in Survey of Binary Systems by R.H.Bruck [2].

While the scientific discoveries and mathematical knowledge were moving hand in hand, towards the end of 20th century there were few mathematically unexplained physical phenomena in Quantum Physics and Quantum Chemistry. These new physical situations could not be faithfully described by the existing mathematical structures and called for more generalized mathematical structures.

It was Enrico Fermi, [3] beginning of chapter VI, p.111 said "..... there are some doubts as to whether the usual concepts of geometry hold for such small region of space." His inspiring doubts on the exact validity of quantum mechanics for the nuclear structure led to the genesis of the whole new kind of generalized mathematics, called isomathematics and generalized mechanics, called as Hadronic mechanics.

In fact, the prevailing Newtonian and Einsteinian 'Dynamical systems' called as 'Exterior Dynamical systems' which are characterized as 'local', 'linear' 'Lagrangian' and 'Hamiltonian' could not accommodate these obscure situations. Thus it was the pressing demand of time to formulate new mathematical theory which could deal with the obscure phenomena and develop a new physical theory. This stupendous task was taken up by the Italian-American theoretical physicist Ruggero Maria Santilli, President of Institute for Basic Research, Palm Harbor, Florida, USA and did the pioneering work by defining axiom-preserving, nonlinear, nonlocal and noncanonical isotopies of conventional mathematical structures, including units, fields,

vector spaces, transformation theory, algebras, groups, geometries, Hilbert spaces etc. while at Department of Mathematics of Harvard University in the early 80's. Prof. Santilli has rightly said;

“There can not be really new physical theories without really new mathematics, and there can not be really new mathematics without new numbers”.

The founders of analytic mechanics, such as Lagrange, Hamilton [4] and others classified dynamical systems in to two kinds. First one is the ‘Exterior Dynamical system’ and the second one is the more complex but generalized ‘Interior Dynamical system’.

However, over a period of time the the above distinction was abandoned preventing the identification of limitations of the prevailing mathematical and physical theories. One can easily notice that *Lie's Theory* is exactly applicable to the exterior dynamical systems. It was Prof. Santilli who at the Department of Mathematics of Harvard University, for the first time, drew the attention of the scientific community towards the crucial distinction between exterior and interior dynamical systems and presented insufficiencies of prevailing mathematical and physical theories by submitting the so-called *axiom-preserving, nonlinear, nonlocal, and noncanonical isotopies of Lie's theory* [5] under the name *Lie Isotopic theory*. Further generalization as *Lie-admissible theory* [6,7] was also achieved by him.

Exterior Dynamical Systems: In this system Point-like particles are moving in a homogeneous and isotropic vacuum with local-differential and potential-canonical equations of motion. These are linear, local, Newtonian Lagrangian and Hamiltonian. Conventional Mathematical structures such as Algebras, Geometries, Analytical Mechanics, Lie Theory can faithfully represent these systems.

Interior Dynamical Systems: In this system we consider extended non-spherical deformable particles moving within non-homogeneous anisotropic physical medium. These are non-linear, non-local, non-Newtonian, non-Lagrangian and non-Hamiltonian. The mathematical structures needed to describe these systems are most general possible which are axiom preserving; non-linear and non-local formulations of current mathematical structures.

During a talk at the conference *Differential Geometric Methods in Mathematical Physics* held in Clausthal, Germany, in 1980, Ruggero Maria Santilli submitted new numbers based on certain axiom preserving generalization of the multiplication, today known as *isotopic numbers* or *isonumbers*[1] in short. This generalization induced the so-called isotopies of the conventional multiplication with consequential generalization of the multiplicative unit, where the Greek word “isotopy” from the Greek word “ ἰσότης τοῦ τοῦ ” implied the meaning “same topology” [8,9]. Subsequently, Ruggero Maria Santilli submitted a **new conjugation**, under the name *isoduality* which yields an additional class of numbers, today known as **isodual isonumbers** [1].

The discovery of isonumbers was made with the specific need of quantitative representation of the transition from Exterior Dynamical Systems to Interior Dynamical System.

It should be quite clear that there can not be new numbers without new fields. This led Santilli to define ‘Isofield’ which is the first new algebraic structure defined by him. This concept of ‘Isofield’ further led to a plethora of new isoalgebraic structures and a whole new ‘Isomathematics’ which is a step further in Modern Mathematics. Subsequently, ‘Isomathematics’ has grown in to a huge tree with various branches like ‘Isofunctional Analysis’, ‘Isocalculus’, ‘Isoalgebra’, isocryptography etc.

Prof. Santilli attracted great attention from academic community at Chinese Academy of Sciences during a workshop in China on August 23, 1997. Since then Prof Santilli and his associates in various countries around the world have produced numerous papers, monographs, conference proceedings which cover approximately 10,000 pages of research work.

Today Number theory has advanced as an important branch of axiomatized mathematics with highly sophisticated applications in the Modern world of computer science and information technology. After some advances in 19th century due to Gauss [10], Abel [11], Hamilton [4], Cayley [12], Galois [13] and others, major important advances were made during 20th century which included axiomatic formulation, the algebraic number theory [14].

The classification of all normed algebras with identity, over reals, in view of the previous studies by Hurwitz[15], Albert [16], and N.Jacobson [17] may be expressed in the following important Theorem.

Theorem 1.1. All possible normed algebras with multiplicative unit over the field of real numbers are given by algebras of dimension 1 (real numbers), 2 (complex numbers), 4 (quaternions), and 8 (octonions).

In this comprehensive presentation of the development of ‘Isonumber theory’ we cover the following important aspects of fundamental importance as formulated by Prof. R. M. Santilli [18], [1].

Starting with the brief background of the origin of ‘isonumit’ and isofield, we present the theory of isonumbers, pseudoisonumbers, “hidden numbers” and their isoduals. Genonumbers, pseudogenonumbers and their isoduals are also of fundamental importance. We will study the isotopies and isodualities of the notions of numbers, fields and normed algebras with unit ref.[1]. In short, in this paper we are going to study the properties of isonumbers and their isoduals [1].

In his study Santilli has taken into account the four normed algebras over reals as given in the above theorem. The isotopic lifting of these algebras give rise to isotopies of normed algebras with multiplicative unit of dimension 1,2,4 and 8 which includes realization of ‘isoreal numbers’, ‘isocomplex numbers’, ‘isoquaternions’ and ‘isooctonions’. Isodualities of these structures give isodual isonumbers.

The mathematical non-triviality of these structures is evident due to lack of unitary equivalence of isotopic and genotopic theories to conventional ones, non-applicability of

trigonometry and some other aspects. On the other hand, the physical non-triviality of these structures emerges from the fact that this theory of isonumbers is at the foundations of Li-isotopic theory used successfully to study nonlinear, nonlocal, and nonhamiltonian dynamical systems. The more general Lie-admissible theory emerges from the more general genonumbers.

In a nutshell, the theory of isonumbers is at the foundation of current studies of nonlinear-nonlocal-nonhamiltonian systems in nuclear, particle and statistical physics, superconductivity and other fields.

1.1. Origin of Isonumbers

The concept of ‘Isotopy’ plays a vital role in the development of this new age mathematics ref. R. H. Bruck [2] and [19].

The first and foremost algebraic structure defined by Santilli is ‘isofield’. Elements of an isofield are called as ‘isonumbers’. The conversion of unit 1 to the isounit $\hat{1}$ is of paramount importance for further development of ‘Isomathematics’.

The reader should be aware that there are various definitions of “fields” in the mathematical literature [20], [21], [22] and [14] with stronger or weaker conditions depending on the given situation. Often “fields” are assumed to be associative under the multiplication.

i.e.

$$a \times (b \times c) = (a \times b) \times c \quad \forall a, b, c \in F$$

We formally define an isofield [23], [24] as follows.

Definition 1.1 Given a “field” F , here defined as a ring with with elements a, b, c, \dots , sum $a + b$, multiplication ab , which is commutative and associative under the operation of conventional addition $+$ and (generally nonassociative but) alternative under the operation of conventional multiplication \times and respective units 0 and 1 , “Santilli’s isofields” are rings of elements $\hat{a} = a\hat{1}$ where a are elements of F and $\hat{1} = T^{-1}$ is a positive-definite $n \times n$ matrix generally outside F equipped with the same sum $\hat{a} + \hat{b}$ of F with related additive unit $\hat{0} = 0$ and a new multiplication $\hat{a} * \hat{b} = \hat{a}T\hat{b}$, under which $\hat{1} = T^{-1}$ is the new left and right unit of F in which case \hat{F} satisfies all axioms of the original field.

T is called the isoelement. In the above definitions we have assumed “fields” to be alternative, i.e.

$$a \times (b \times b) = (a \times b) \times b, \quad (a \times a) \times b = a \times (a \times b) \quad \forall a, b \in F$$

Thus, “isofields” as per above definition are not in general isoassociative, i.e. they generally violate the isoassociative law of the multiplication, i.e.

$$\hat{a} \hat{\times} (\hat{b} \hat{\times} \hat{c}) = (\hat{a} \hat{\times} \hat{b}) \hat{\times} \hat{c}, \quad \forall \hat{a}, \hat{b}, \hat{c} \in \hat{F}$$

but rather satisfy isoalternative laws.

The specific need to generalize the definition of “number” to ‘real numbers’, complex numbers, ‘quaternions’ and ‘octonians’ suggested the above definition. The resulting new numbers are ‘isoreal numbers’, isocomplex numbers, ‘isoquaternions’ and ‘isooctonians’ respectively, where ‘isooctonians’ are alternative but not associative.

The ‘isofields’ $\hat{F} = \hat{F}(\hat{a}, +, \hat{\times})$ are given by elements $\hat{a}, \hat{b}, \hat{c}, \dots$ characterized by one-to-one and invertible maps $a \rightarrow \hat{a}$ of the original element $a \in F$ equipped with two operations $(+, \hat{\times})$, the conventional addition $+$ of F and a new multiplication $\hat{\times}$ called “isomultiplication” with corresponding conventional additive unit 0 and a generalized multiplicative unit $\hat{1}$, called “multiplicative isounit” under which all the axioms of the original field F are preserved.

Santilli has shown that the transition from exterior dynamical system to interior dynamical system can be effectively represented via the isotopy of conventional multiplication of numbers a and b from its simple possible associative form $a \times b$ in to the isotopic multiplication, or isomultiplication for short, as introduced in [8].

The lifting of the product $ab = a \times b$ of conventional numbers in to the form

$$a \hat{\times} b := a \times T \times b \tag{1}$$

denoted by $\hat{\times} = \times T \times$, where T is a fixed invertible quantity for all possible a, b called isotopic element.

This isomultiplication then lifts the conventional unit 1 defined by $1 \times a = a \times 1 = a$ to the multiplicative isounit $\hat{1}$ defined by

$$\hat{1} \hat{\times} a = a \hat{\times} \hat{1} = a, \text{ where } \hat{1} = T^{-1} \tag{2}$$

Under the condition that $\hat{1}$ preserves all the axioms of 1 the lifting $1 \rightarrow \hat{1}$ is an isotopy, i.e. the conventional unit 1 and the iso unit $\hat{1}$ (as well as the conventional product $a \times b$ and its isotopic form $a \hat{\times} b$) have the same basic axioms and coincide at the abstract level by conception. The isounit $\hat{1}$ is so chosen that it follows the axioms of the unit 1 namely; boundedness, smoothness, nowhere degeneracy, hermiticity and positive-definiteness. This ensures that the lifting $1 \rightarrow \hat{1}$ is an isotopy and conventional unit 1 and the isounit $\hat{1}$ coincide at the abstract level of conception.

Thus, the isonumbers are the generalization of the conventional numbers characterized by the isounit and the isoproduct as defined above.

The liftings $a \rightarrow \hat{a}$, and $\times \rightarrow \hat{\times}$ can be used jointly or individually.

It is important to note that unlike isotopy of multiplication $\times \rightarrow \hat{\times}$, the lifting of the addition $+\rightarrow \hat{+}$ implies general loss of left and right distributive laws. Hence the study of such a lifting is the question of independent mathematical investigation.

The first generalization was introduced by Prof. Santilli when he generalized the real, complex and quaternion numbers [23], [24] based on the lifting of the unit 1 into isounit $\hat{1}$ as defined above. Resulting numbers are called isorealnumbers, isocomplex numbers and isoquaternion numbers.

In fact, this lifting leads to a variety of algebraic structures which are often used in physics. The following flowchart is self explanatory.

Isonumbers \rightarrow Isofields \rightarrow Isospaces \rightarrow Isotransformations \rightarrow Isoalgebras \rightarrow Isogroups \rightarrow Isosymmetries \rightarrow Isorepresentations \rightarrow Isogeometries etc.

The isounit is generally assumed to be outside the original field with all the possible compatible conditions imposed on it. For rudiments of isomathematics reader can refer to [1, 6, 7, 25].

The lifting of unit I to isounit \hat{I} may be represented as, $I \rightarrow \hat{I}(t, r, \dot{r}, p, T, \psi, \psi^\dagger, \partial\psi, \partial\psi^\dagger, \dots)$. where t is time, r is the position vector, p is the momentum vector, ψ is the wave function and ψ^\dagger are the corresponding partial differentials. The positive definiteness of the isounit \hat{I} is assured by, $\hat{I}(t, r, \dot{r}, p, T, \psi, \psi^\dagger, \partial\psi, \partial\psi^\dagger, \dots) = \frac{1}{T} > 0$ where T is called the isotopic element, a positive definite quantity.

The isonumbers are generated as, $\hat{n} = n \times \hat{I}$, $n = 0, 1, 2, 3, \dots$.

Isofields are of two types, isofield of first kind; wherein the isounit does not belong to the original field, and isofield of second kind; wherein the isounit belongs to the original field. The elements of the isofield are called as isonumbers. This leads to number of new terms and parallel developments of conventional mathematics.

2. Isounits and Their Isoduals

As stated earlier, the isonumbers and their product can first be introduced as the generalization of conventional numbers by equations (1) and (2) as above.

Prof. Santilli further, introduced *isodual isonumbers* [26, 27, 28] by lifting the isounit into the form

$$\hat{1}^d \hat{\times}^d a = a \hat{\times}^d \hat{1}^d = a, \text{ where } \hat{1}^d := -\hat{1} \quad (3)$$

called the *isodual isounit* following lifting of iso multiplication defined in (1) into the *isodual multiplication* called *isoduality* as

$$a \hat{\times} b \rightarrow a \hat{\times}^d b := a \times T^d \times b = -a \times T \times b = -a \hat{\times} b \text{ where } T^d = -T \quad (4)$$

The *isodual isonumbers* were first conceived as characterized by isodual multiplication (4) with respect to the multiplicative *isodual isounit* $\hat{1}^d = -\hat{1}$.

The significance of isonumbers and isodual isonumbers lies in fulfilling the specific physical needs refs [18, 29, 30, 31] as given below;

- In the exterior dynamical system ordinary particles moving in the vacuum are characterized by conventional numbers.
- In the interior dynamical system ordinary particles moving in the physical medium are characterized by isonumbers.
- In the exterior dynamical system ordinary antiparticles moving in vacuum are characterized by isodual numbers.

In the interior dynamical system the antiparticles moving in the physical medium are characterized by isodual isonumbers.

Interpretation of customary characterization of antiparticles via negative-energy solutions of Dirac's equations behave in an un-physical way when interpreted with respect to the same numbers and unit 1 of particles, forcing various hypothetical assumptions and postulates, where as, reinterpretation of antiparticles with same negative energy solutions when interpreted as belonging to the field of isodual numbers behave in a fully physical way ref [1]. This treatment of antiparticles with isodual numbers also leads to intriguing geometrical implications which predict another universe, called as *isodual universe*, interconnected to our universe via isoduality and identified by the isodualities of Riemannian geometry and their isoduals refs.[31, 24, 32]. Thus, the isodual theory emerged from the identification of negative units in the antiparticle component of the conventional Dirac equation and the reconstruction of the theory with respect to this new negative unit. Hence isoduality provides a mere reinterpretation of Dirac's original notion of antiparticle leaving all numerical predictions electro-weak interactions essentially unchanged.

In view of the definition of an isofield [1], we can say that an isofield is an additive abelian group equipped with a new unit (called isounit) and isomultiplication defined appropriately so that the resulting structure becomes a field.

If the original field is alternative then the isofield also satisfies weaker isoalternative laws as follows.

$$\hat{a} \hat{\times} (\hat{b} \hat{\times} \hat{b}) = (\hat{a} \hat{\times} \hat{b}) \hat{\times} \hat{b} \text{ and } \hat{a} \hat{\times} (\hat{a} \hat{\times} \hat{b}) = (\hat{a} \hat{\times} \hat{a}) \hat{\times} \hat{b}.$$

We mention two important propositions by Santilli.

Proposition 2.1. The necessary and sufficient condition for the lifting (where the multiplication is lifted but elements not the elements) $F(a, +, \times) \rightarrow \hat{F}(a, +, \hat{\times}), \hat{\times} = \times T \times, \hat{1} = T^{-1}$

to be an isotopy (that is for \hat{F} to verify all axioms of the original field F) is that T is a non-null element of the original field F .

Proposition 2.2. The lifting (where both the multiplication and the elements are lifted)

$$F(a, +, \times) \rightarrow \hat{F}(\hat{a}, +, \hat{\times}), \hat{a} = a \times \hat{1}, \hat{\times} = \times T \times, \hat{1} = T^{-1}$$

constitutes an isotopy even when the multiplicative isounit $\hat{1}$ is not an element of the original field.

The above proposition guarantees the physically fundamental capability of generating Plank's unit \mathcal{V} of quantum mechanics into an integro-differential operator $\hat{1}$ for quantitative treatment of nonlocal interactions [33].

As the first application of the isotopies of numbers Santilli

considers the set $S = \langle in \rangle$, the set of all purely imaginary numbers. This set is not closed ($i^2 = -1 \notin S$). On the other hand, the same set S represented as $\hat{S}(\hat{n}, +, \hat{\times})$ with $\hat{n} = in$ constitutes an isofield. i.e. it verifies all the axioms of a field including closure under isomultiplication because $T = i^{-1}$ and $\hat{n} \hat{\times} \hat{m} = in \hat{\times} im = inm \in \hat{S}$.

This illustrates an important fact that, even when a given set does not constitute a field, there may exist an isotopy under which it verifies the axioms of a field.

As stated earlier the lifting of $+$ to $\hat{+}$ does not necessarily produce an isotopy of a given field. This lifting does not preserve the distributivity in the resulting set as stated in the following proposition 2.3.

Proposition 2.3 The lifting $F(a, +, \times) \rightarrow \hat{F}(\hat{a}, \hat{+}, \hat{\times})$ where

$$\hat{a} = a \times 1, \quad \hat{+} = +K +, \quad \hat{0} = -K = -K \times \hat{1},$$

$$\hat{\times} = \times T \times, \quad \hat{1} = T^{-1} \quad \text{where } K \text{ is}$$

an element of the original field F and T is an arbitrary invertible quantity, is not an isotopy for all nontrivial values of the quantity $K \neq 0$, because it preserves all the axioms of proposition 2.1 except the distributive law.

Based on the failure of distributivity Santilli defines "pseudoisofields" as follows.

Definition 2.1 Let $\hat{F}(\hat{a}, +, \hat{\times})$ be an isofield as defined above. Then the "pseudoisofields" $\hat{\hat{F}}(\hat{\hat{a}}, \hat{\hat{+}}, \hat{\hat{\times}})$ are given by the images of $\hat{F}(\hat{a}, +, \hat{\times})$ under all possible liftings of the addition $+ \rightarrow \hat{\hat{+}} = +K +$, with additive isounit $\hat{\hat{0}} = -\hat{K} = -K \times \hat{1}, K \neq 0$ in which case the elements $\hat{\hat{a}}$ are called the "pseudoisounumbers".

For the algebra of isonumbers and isodual numbers readers are advised to refer [1, 34].

Images of field, isofield and pseudoisofield under the change of sign of the isounit $\hat{1} \rightarrow \hat{1}^d = -1$ is called the *Isotopic conjugation* or *isoduality* ref. [28, 29, 30].

Definition 2.2 Let $F(a, +, \times)$ be a field as per definition 1.1. Then the isodual field $F^d(a^d, +, \times^d)$ is constituted by the elements called "isodual numbers"

$$a^d := a \times 1^d = -a \tag{5}$$

defined with respect to the "isodual multiplication" and related "isodual unit"

$$\times^d := \times 1^d \times = -\times, \quad 1^d = -1. \tag{6}$$

Definition 2.3 Let $F(\hat{a}, +, \hat{\times})$ be an isofield as per definition 1.1. Then the isodual isofield $F^d(\hat{a}^d, +, \hat{\times}^d)$ is constituted by the elements called "isodual isonumbers"

$$\hat{a}^d := a^c \times 1^d = -a^c \times 1 \tag{7}$$

where a^c is the conventional conjugation of F (e.g. complex conjugation) defined in terms of the "isodual isomultiplication"

$$\hat{\times}^d := \times T^d \times = -\hat{\times}, \quad T^d = -T. \tag{8}$$

Definition 2.4 Let $F(\hat{a}, \hat{+}, \hat{\times})$ be a pseudofield $\hat{\hat{F}}(\hat{\hat{a}}, \hat{\hat{+}}, \hat{\hat{\times}})$ as per definition 2.1. Then the "isodual pseudofield" $F(\hat{a}^d, \hat{+}^d, \hat{\times}^d)$ is given by the image of the original isofield under isodualities (6) and (7) plus the additional isoduality

$$\hat{0} \rightarrow \hat{0}^d = -0 \tag{9}$$

and its elements \hat{a}^d are called "isodual pseudonumbers."

2.1. Classes of Isofields

Kadeisvilli [35] classified isounits into five primary classes according to their usefulness.

- **CLASS I: Isounits**:- These are the isounits when they are sufficiently smooth, bounded, nowhere singular, Hermitian and positive-definite. This class is of primary use in physics for characterization of ordinary particles moving in interior physical conditions. This class represents the isotopy of the conventional unit.
- **CLASS II: Isodual Isounits**:- They are same as isounits except that they are negative-definite. Isodual isounits are used in physics to characterize antiparticles via reinterpretation of the negative energy solutions of Dirac's equation [31, 36]. They represent *isodual isotopy* according to *isodual conjugation*.
- **CLASS III: Singular Isounits**:- These occur when isounits are considered as a divergent limit, $1 \Rightarrow \pm\infty$. These are used to represent gravitational collapse into a singularity and other limit conditions ref.[37, 23].
- **CLASS IV: Indefinite Isounits** :- This class represents isounits which are sufficiently smooth, bounded, nowhere singular, Hermitian and can smoothly interconnect positive definite with negative definite values. These are particularly used in mathematics.
- **CLASS V: General Isounits**, when they are solely Hermitian:- This is the most general class which includes preceding ones and permits a large variety of additional realizations including those in terms of discrete structures, discontinuous functions, distributions etc.

Isofields can be classified according to the isounits as defined above. They are;

1. Isofields.
2. Isodual isofields.
3. Singular isofields.
4. Indefinite isofields.
5. General isofields.

The following four fundamental numbers are generated depending upon the isofield we consider;

1. (a) Ordinary numbers: real numbers $R(n, +, \times)$,

complex numbers $C(c, +, \times)$, quaternions $Q(q, +, \times)$ and octonians $O(o, +, \times)$ which are used in the characterization of particles in vacuum.

(b) Isonumbers: isoreal numbers $\hat{R}(\hat{n}, +, \hat{\times})$, isocomplex numbers $\hat{C}(\hat{c}, +, \hat{\times})$, isoquaternions $\hat{Q}(\hat{q}, +, \hat{\times})$ and isoocionians $\hat{O}(\hat{o}, +, \hat{\times})$ which are used for the characterization of particles within the physical media.

(c) Isodual numbers: isodual real numbers $R^d(n^d, +, \times^d)$, isodual complex numbers $C^d(c^d, +, \times^d)$, isodual quaternions $Q^d(q^d, +, \times^d)$ and isodual octonians $O^d(o^d, +, \times^d)$ which are used in the characterization of antiparticles in vacuum.

(d) Isodual isonumbers: isodual isoreal numbers $\hat{R}^d(\hat{n}^d, +, \hat{\times}^d)$, isodual isocomplex numbers $\hat{C}^d(\hat{c}^d, +, \hat{\times}^d)$, isodual isoquaternions $\hat{Q}^d(\hat{q}^d, +, \hat{\times}^d)$ and isodual isoocionians $\hat{O}^d(\hat{o}^d, +, \hat{\times}^d)$ which are used for the characterization of particles within the physical media.

2. Genofield is the generalization of isofield with the selection of an ordering of the multiplication to the left or to the right and applied for the more general Lie-admissible branch of hadronic mechanics.
3. Pseudofields, and
4. Pseudogenofields are the further generalization based on lifting of addition which relaxes at least one axiom of conventional fields, and which do have applications in other fields.
5. Hyper numbers can be constructed from hyperstructures ref.[35].

2.2. Isospaces

Let $S(x, g, R(n, +, \times))$ be a metric (or pseudo metric) n -dimensional space with local coordinates x and (Hermitean) metric $g = g^\dagger$ over the field of reals $R(n, +, \times)$. Then the *isospace* $\hat{S}(x, \hat{g}, \hat{R}(n, +, \hat{\times}))$ first introduced in [38] is characterized by;

$$\begin{aligned} \hat{S}(x, \hat{g}, \hat{R}(n, +, \hat{\times})): \quad \hat{g} &= T \times g, \\ \hat{\times} &= \times T, \quad \hat{1} = T^{-1}. \end{aligned} \quad (10)$$

Also the *isodual isospace* [28] is given by;

$$\begin{aligned} \hat{S}^d(x, \hat{g}^d, \hat{R}^d(n^d, +, \hat{\times}^d)): \quad \hat{g}^d &= T^d \times g = -T \times g, \\ \hat{\times}^d &= \times T^d \times = -\times T \times, \quad \hat{1}^d = -\hat{1}. \end{aligned} \quad (11)$$

Note that isospaces $\hat{S}(x, \hat{g}, \hat{R}(n, +, \hat{\times}))$ coincide with spaces $S(x, g, R(n, +, \times))$ at the abstract level of conception. Spaces have the most general known curvature and integral character owing to the arbitrariness in the isotopic

element T . The isometries $\hat{g} = T \times g$ have the most general possible, nonlinear, nonlocal, noncanonical dependence in all variables,

$$g = g(x) \rightarrow \hat{g} = T(t, x, \dot{x}, \ddot{x}, \dots) \times g(x) = \hat{g}(t, x, \dot{x}, \ddot{x}, \dots). \quad (12)$$

The isospaces which are most important for physical and mathematical applications are *isoeuclidean spaces* $\hat{E}(x, \hat{\sigma}, \hat{R})$, *isominkowski spaces* $\hat{M}(x, \hat{\eta}, \hat{R})$ and *isoriemanian spaces* $\hat{R}(x, \hat{g}, \hat{R})$. These are the foundations of the representation of nonlinear, nonlocal, and noncanonical interior systems in nonrelativistic and gravitational interior problems [31, 23].

Also, *pseudoisospaces* can be defined as the images $\hat{S}(x, \hat{g}, \hat{R}(n, \hat{\times}))$ of the original space characterized by further lifting $+ \rightarrow \hat{+} = +K+$, $0 \rightarrow \hat{0} = -K$. Subsequently, *isodual pseudoisospaces* are also defined.

2.4. Isoalgebras

The concept of isoalgebra was fundamental in the correct description of interior dynamical systems. As conventional numbers constitute normed algebras with unit, isoalgebras were defined to represent isonumbers ref. [21, 8, 39]. An isovector space \hat{U} with elements $A, B, C \dots$ and isomultiplication \hat{e} over an isofield $\hat{F}(a, +, \hat{\times})$ with elements a, b, c and isomultiplication $a \hat{\times} b$ with multiplicative isounit $\hat{1} = T^{-1}$ is called (associative or nonassociative) *isoalgebra* when it satisfies right and left scalar and distributive laws;

$$(a \hat{\times} A) \hat{\circ} B = A \hat{\circ} (a \hat{\times} B) = a \hat{\times} (A \hat{\circ} B). \quad (15)$$

$$(A \hat{\times} a) \hat{\circ} B = A \hat{\circ} (B \hat{\times} a) = (A \hat{\circ}) B \hat{\times} a \quad (16)$$

$$\begin{aligned} A \hat{\circ} (B + C) &= A \hat{\circ} B + A \hat{\circ} C, \\ (B + C) \hat{\circ} A &= B \hat{\circ} A + C \hat{\circ} A \end{aligned} \quad (17)$$

for all the elements $A, B, C \in \hat{U}$ and $a, b, c \in \hat{F}$.

Note that the isoalgebra \hat{U} may contain the matrices where as the iso field \hat{F} can contain ordinary numbers.

The isoalgebra \hat{U} is an *isodivision algebra* if the equation $A \hat{\times} x = B$ always admits a solution in \hat{U} , for nonzero A .

Isonorm can be defined in the following manner;

Let \hat{e}_k be an "isobasis" of \hat{U} over the isofield $\hat{F}(a, +, \hat{\times})$. Then the generic element $A \in \hat{U}$ can be written as $A = \sum_{k=1, \dots, m} n_k \hat{\times} \hat{e}_k$, with $n_k \in \hat{F}$ and $\hat{e}^2 = \sum_k \hat{e}_k \hat{\circ} \hat{e}_k = 1$. The *isonorm* of \hat{U} in the isobasis considered, is then given by;

$$A_s = \left(\sum_{k=1, \dots, m} n_k^2 \right)^{\frac{1}{2}} \times \hat{1} = \left(\sum_{k=1, \dots, m} n_k \hat{\times} n_k \right)^{\frac{1}{2}} \times \hat{1} \in \hat{F} \quad (18)$$

The isoalgebra \hat{U} is said to be isoassociative if;

$$A \hat{\odot} (B \hat{\odot} C) = (A \hat{\odot} B) \hat{\odot} C, \forall A, B, C \in \hat{U} \quad (19)$$

(isoassociative law)

and

$$A^2 \hat{\odot} B = A \hat{\odot} (A \hat{\odot} B), A \hat{\odot} B^2 = (A \hat{\odot} B) \hat{\odot} B \quad (20)$$

(isoalternative laws)

The isoalgebra \hat{U} is said to be *Lie-isotopic* when the isoproduct \hat{e} satisfies Lie-algebra axioms (anticommutativity and Jacobi laws) in the following form;

$$A \hat{\odot} B = ATB - BTA, \quad A, T, B, \text{ etc.} = \text{assoc.} \quad (21)$$

It is said to be *lie-admissible* when the antisymmetric bracket product is;

$$[A, B] := A \hat{\odot} B - B \hat{\odot} A \quad (22)$$

and is *Lie-isotopic* as in the realization;

$$A \hat{\odot} B = ARB - BSA. \quad (23)$$

We shall be mainly interested in the isoassociative isonormed algebras with isounit $\hat{1}$ which can be extended to isoalternative algebras in order to include isoocionians.

Extension of U and \hat{U} under the pseudofield $\hat{F}(a, \hat{+}, \hat{\times})$ implies loss of distributive laws and hence do not remain algebras in the real sense, however, we call them *pseudoisoalgebras* ref.[39].

2.5. Isoreal Numbers and Their Isoduals

2.5.1. Real Numbers

Real numbers constitute a one-dimensional normed associative and commutative algebra $U(1)$ ref.[1].

Real numbers are realized ref.[8] as a one-dimensional real Euclidean space $E_1(x, R(n, +, \times))$ which represents a straight line with origin at 0 , local coordinates x , metric $\mathcal{D} = 1$, additive unit 0 and multiplicative unit 1 . Another characterization of real numbers is defined by the isomorphism of the reals $R(n, +, \times)$ into the commutative one-dimensional multiplicative group of dilations $G(1)$ defined by;

$$x' = n \times x, \quad n \in R(n, +, \times), \quad x, x' \in E_1(x, \mathcal{D}, R). \quad (24)$$

The basis is given by

$$e = 1 \quad (25)$$

with the norm defined by

$$|n| = (n \times n)^{\frac{1}{2}} > 0 \quad (26)$$

and

$$|n \times n'| = |n| \times |n'|. \quad (27)$$

2.5.2. Isodual Real Numbers

Isodual Real numbers constitute a one-dimensional isodual associative and commutative normed algebra $U^d(1)$ which is anti-isomorphic to $U(1)$ ref.[1].

Isodual real numbers are the conventional numbers n defined with respect to the isodual unit $1^d = -1$. The isodual conjugation of real numbers is then written as

$$n = n \times 1 \rightarrow n^d = n \times 1^d = -n. \quad (28)$$

Note that, such a sign inversion occurs when the isodual real numbers are projected in the field of conventional real numbers. As a result, all the numerical values change sign under isoduality.

The one-dimensional real isodual Euclidean space $E^d(x, \mathcal{D}^d, R^d(n^d, +, \times^d))$ is a straight line, with conventional additive unit 0 , and isodual multiplicative unit $1^d = -1$. The $R^d(n^d, +, \times^d)$ represents the Euclidean space $E^d(x, \mathcal{D}^d, R^d(n^d, +, \times^d))$. Also, the isodual dilations are defined by

$$x' = n^d \times^d x = n \times x \quad (29)$$

This establishes an isomorphism between $R^d(n^d, +, \times^d)$ and the isodual group of dilations $G^d(1)$ (the conventional group reformulated according to the multiplicative unit 1^d). Santilli points out that $E_1(x, \mathcal{D}, R)$ and $E_1^d(x, \mathcal{D}^d, R^d)$ are antiisomorphic and the same property holds for $G(1)$ and $G^d(1)$. Also, the isodual dilations coincide with dilations as defined above. Santilli further says that "this could be the a reason for the lack of detection of isodual numbers until then." ref.s [38, 27, 28].

In the isodual case, the isodual basis is given by

$$e^d = 1^d \quad (30)$$

with isodual norm

$$|n| = (n \times n)^{\frac{1}{2}} \times 1^d = |n| \times 1^d = -|n| < 0 \quad (31)$$

satisfying the axioms

$$|n^d \times n'^d|^d = |n^d|^d \times^d |n'^d|^d. \quad (32)$$

2.5.3. Isoreal Numbers

Isoreal numbers constitute a one-dimensional, isonormed isoassociative and isocommutative isoalgebra $\hat{U}(1) \approx U(1)$ ref.[1].

Isoreal numbers are the numbers $\hat{n} = n \times \hat{1}$ of an isofield of Class I, with isomultiplication defined by $\hat{\times} = \times T \times$ and isounit $\hat{1} = T^{-1} > 0$, generally outside the original field $R(n, +, \times)$. These can be represented as the isoeuclidean spaces $\hat{E}_{1,1}(x, \hat{\delta}, \hat{R}(\hat{n}, +, \hat{\times}))$ with $\hat{\delta} = T\delta$, over $\hat{R}(\hat{n}, +, \hat{\times})$ the isotopes of conventional one-dimensional Euclidean spaces $E_1(x, \delta, R)$.

Some of the important remarks are as follows.

- The conventional Euclidean space $E_1(x, \delta, R)$ and its isotopic covering $\hat{E}_{1,1}(x, \hat{\delta}, \hat{R})$ are locally isomorphic due to the joint liftings $\delta \rightarrow \hat{\delta} = T \times \delta$ and $1 \rightarrow \hat{1} = T^{-1}$.
- $\hat{E}_{1,1}(x, \hat{\delta}, \hat{R})$ is not a Riemannian space because of the intrinsic dependence of the isometric $\hat{\delta}$ on the derivatives \dot{x}, \ddot{x}, \dots as well as the fact that the basic unit is not the conventional quantity 1.
- However, $\hat{E}_{1,1}(x, \hat{\delta}, \hat{R})$ is a simple, yet bona-fide isoriemannian space [24], because $\hat{\delta} = T \times \delta = \hat{\delta}(t, x, \dot{x}, \ddot{x}, \dots)$, where the local dependence is generally nonlinear, nonlocal and noncanonical in all variables.

In fact, the *one-dimensional isospace* $\hat{E}_{1,1}(x, \hat{\delta}, \hat{R})$ represents a one-dimensional generalization of conventional straight line, called as *isoline*. This is because of its intrinsically nonlinear, nonlocal and noncanonical metric $\hat{\delta}(t, x, \dot{x}, \ddot{x}, \dots)$ with multiplicative isounit $\hat{1} = \hat{1}(t, x, \dot{x}, \ddot{x}, \dots)$. Then $\hat{R}_1(\hat{n}, \hat{+}, \hat{\times})$ can be realized via *isodilations* on $\hat{E}_1(x, \hat{\delta}, \hat{R})$ as;

$$x' = \hat{n} \hat{\times} x = n \times x, \quad (33)$$

which is isodual dilation and represents one-dimensional isogroup of isodilations $\hat{G}(1)$ same as the group $G(1)$ realized with respect to isounit $\hat{1}$.

Again, the *isobasis* is given by

$$\hat{e} = \hat{1} \quad (34)$$

with *isonorm* defined as;

$$\|\hat{n}\| := (n \times n)^{\frac{1}{2}} \times \hat{1} = |n| \times \hat{1} \quad (35)$$

which is the conventional norm only rescaled to the new unit $\hat{1}$. We then also have

$$\|\hat{n} \hat{\times} \hat{n}'\| = \|\hat{n}\| \hat{\times} \|\hat{n}'\|. \quad (36)$$

2.5.4. Isodual Isoreal Numbers

The isodual isoreal numbers are the realization of the one-dimensional isodual, isonormed, isoassociative and isocommutative isoalgebra $\hat{U}^d(1) \approx U^d(1)$ ref.[1].

These are the isodual numbers

$$\hat{n}^d = n \times \hat{1}^d, \quad \hat{1}^d = -\hat{1} \quad (37)$$

in the isodual isofield $\hat{R}_{11}^d(\hat{n}^d, +, \times^d)$. These correspond to $\hat{E}_{11}^d(x, \hat{\delta}^d, \hat{R}^d)$ the isoeuclidean space of Class II $\hat{E}_{11}^d(x, \hat{\delta}^d, \hat{R}^d)$ of dimension one with *isodual isodilations*

$$x' = \hat{n}^d \hat{\times}^d x \quad (38)$$

coinciding with dilations (24). This also characterizes an isomorphism isodual isoreal numbers with the one-dimensional isodual isogroup $\hat{G}^d(1)$. The underlying isomorphism

$$E_1^d(x, \delta^d, R^d(n^d, +, \times^d)) \approx$$

$$E_{11,1}^d(x, \hat{\delta}^d, \hat{R}^d(\hat{n}^d, +, \hat{\times}^d))$$

implies the $\hat{G}^d(1) \approx G^d(1)$.

The *isodual isobasis* is defined by

$$\hat{e}^d = \hat{1}^d \quad (39)$$

The isodual isonorm

$$\|\hat{n}^d\|^d := (n \times n)^{\frac{1}{2}} \times \hat{1}^d = -\|\hat{n}\| \quad (40)$$

verifies the axioms

$$\|\hat{n}^d \hat{\times}^d \hat{n}'^d\|^d = \|\hat{n}^d\|^d \hat{\times}^d \|\hat{n}'^d\|^d \quad (41)$$

2.6. Isocomplex Numbers and Their Isoduals

2.6.1. Complex Numbers

Complex numbers constitute a two-dimensional, normed associative and commutative algebra $U(2)$ ref.[1].

Complex numbers $c = n_0 + n_1 i$ where n_0 and n_1 are real numbers and i is an imaginary unit, are represented in a Gauss plane which is a realization of two-dimensional Euclidean space $E_2(x, \delta, R(n, +, \times))$ satisfying

$$x^2 = x^t \delta_{ij} x_j = x_1^2 + x_2^2 \in R(n, +, \times) \quad (42)$$

whose group of isometries is one dimensional Lie Group $O(2)$, the invariance of the circle. Hence, complex numbers can be represented via fundamental representation of $O(2)$ as follows.

A one-to-one correspondence between complex numbers and points in the Gauss plane can be obtained by following *dilative rotations*

$$z' = (x_1 + x_2 i)' = c \circ z = (n_0 + n_1 i) \circ (x_1 + x_2 i) \quad (43)$$

and multiplication

$$c \circ z = (n_0, n_1) \circ (x_1, x_2) = (n_0 x_1 - n_1 x_2, n_0 x_2 + n_1 x_1) \quad (44)$$

which preserve all the properties of a field.

Representation of a complex number via matrices has the following form

$$c := n_0 \times I_0 + n_1 \times I_1 = \begin{pmatrix} n_0 & n_1 \times i \\ n_1 \times i & n_0 \end{pmatrix} \quad (45)$$

where

$$-\bar{c}^d = c, i^d = i \quad (46)$$

which are well known as the identity and fundamental representation of $O(2)$.

Norm can also be defined as

$$|c| = |n_0 + n_1 \times i| := (\text{Det}c)^{\frac{1}{2}} = (n_0^2 + n_1^2)^{\frac{1}{2}} \quad (47)$$

Also, the identification of basis in terms of matrices is $e_1 = I_0$ and $e_2 = I_1$.

2.6.2. Isodual Complex Numbers

Isodual complex numbers constitute a two-dimensional isodual, normed, associative and commutative algebra $U^d(2)$ anti-isomorphic to $U(2)$ ref.[1].

Isodual complex numbers are given by

$$C^d = \{(c^d, +, \times^d) | \times^d = -\times, i^d = -i; c^d = \bar{c}, \bar{c}^d = -\bar{c}, \bar{c} \in \bar{C}\} \quad (48)$$

where \bar{c} is the complex conjugation. Thus, given a complex number $c = n_0 + n_1 \times i$, its isodual is given by

$$c^d = -\bar{c} = n_0^d + n_1^d \times i = -n_0 - n_1 \times i = -n_0 + n_1 \times i \in C^d. \quad (49)$$

Considering the group of isometries, the one-dimensional isodual Lie group $O^d(2)$ i.e. the image of O_2 under the lifting $I = \text{diag.}(1, 1) \rightarrow I^d = \text{diag.}(-1, -1)$ of the two-dimensional isodual Euclidean space

$E_2^d(x, \delta^d, R^d(n^d, +, \times^d))$ with basic invariant

$$\begin{aligned} x^{2d} &= x^t \delta^d x = x_i \delta_{ij}^d x_j = x_1^{2d} + x_2^{2d} = \\ x_1 \times^d x_1 + x_2 \times^d x_2 &= -x_1^2 - x_2^2 \in R^d(n^d, +, \times^d) \end{aligned} \quad (50)$$

isodual complex numbers can be characterized by the isorepresentation of $O^d(2)$.

Now, the image of the conventional plane under isoduality is the isodual Gauss plane. Also, a one-to-one correspondence between the points $P = (x_1, x_2)$ and complex numbers can be defined by *isodual dilative rotations* as

$$z' = (x_1 + x_2 \times i)' = c^d \circ^d z = (-n_0 + n_1 \times i) \circ^d (x_1 + x_2 \times i) \quad (51)$$

following the multiplication rules

$$\begin{aligned} c^d \circ^d z &= (-n_0, n_1) \circ^d (x_1, x_2) = \\ (-n_0 \times x_1 \times n_1 \times x_2, -n_0 \times x_2 + n_1 \times x_1) \end{aligned} \quad (52)$$

which preserve all the properties of a field.

Isodual transformations form an isodual group $G^d(2)$ antiisomorphic to $G(2)$. Even the one-to-one correspondence between complex numbers and Gauss plane continues under isoduality.

Matrix representation of isodual complex numbers can be defined as

$$c^d := n_0^d \times i_0^d + n_1^d \times i_1^d = \begin{pmatrix} -n_0 & n_1 \times i \\ n_1 \times i & -n_0 \end{pmatrix}, \quad (53)$$

$$i_0^d = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, i_1^d = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \quad (54)$$

with the isodual unit and isodual representations of $O^d(2)$ respectively.

The *isodual norm* can be defined as $|c^d|^d = |-n_0 + n_1 \times i|^d :=$

$$[\text{Det}_R(c^d \times T^d)]^{\frac{1}{2}} \times i_0^d = (c^{-d} \times^d c^d)^{\frac{1}{2}} \times i_0^d$$

which may be written as

$$|c^d|^d = (c \times \bar{c}) \times i_0^d = (n_0^2 + n_1^2) \times i_0^d \quad (55)$$

and verifies the axioms

$$|c^d \circ c'^d|^d = |c^d|^d \times^d |c'^d|^d \in R^d, \quad c^d, c'^d \in C^d. \quad (56)$$

The *isodual basis* in terms of matrices is given by

$$e_i^d = i_0^d, \quad e_2^d = i_1^d. \quad (57)$$

2.6.3. Isocomplex Numbers

Isocomplex numbers constitute a two-dimensional, isonormed, isoassociative and isocommutative isoalgebras over the isoreals $\hat{U}(2) \approx U(2)$ ref.[1].

In this case we consider the isofield of isocomplex numbers

$$\hat{C} = \{(\hat{c}, +, \hat{\times}) \mid \hat{\times} = \times T \times, \hat{1} = T^{-1}, \hat{c} = c \times \hat{1}, c \in C(c, +, \times)\} \quad (58)$$

with the generic element $\hat{c} = \hat{n}_0 + \hat{n}_1 \times i$. Here we need the two-dimensional isoeuclidean space of class I, $\hat{E}_{1,2}(x, \hat{\delta}, \hat{R}(\hat{n}, +, \hat{\times}))$. The most important realization used in the physical literature has the diagonalized and positive-definite isotopic element and isounit

$$T = \text{diag.}(b_1^2, b_2^2), \hat{1} = \text{diag.}(b_1^{-1}, b_2^{-2}), b_k > 0, k = 1, 2. \quad (59)$$

with basic isoseparation

$$\begin{aligned} \hat{e}_1 &= \hat{I}_0, & \hat{e}_{k+1} \\ &= \hat{i}_k, & k = 1, 2, 3. \end{aligned} \quad (60)$$

The group of isometries of this space is the Lie group $\hat{O}(2) \approx O(2)$, the group constructed with respect to the multiplicative isounit $\hat{1} = \text{diag.}(b_1^{-2}, b_2^{-2})$ which provides the invariance of all possible ellipses with semiaxes $a = b_1^{-2}, b = b_2^{-2}$ as the infinitely possible deformation of the circle $x^2 = x_1^2 + x_2^2 \in R(n, +, \times)$. Thus, isocomplex numbers are characterizable via fundamental representation of $\hat{O}(2)$.

Isocomplex numbers $\hat{c} = (\hat{n}_0, \hat{n}_1)$ can also be characterized to be the set of points $P = (\hat{x}_1, \hat{x}_2)$ on the *isogauss plane* on $\hat{E}_{1,2}(x, \hat{\delta}, \hat{R}(\hat{n}, +, \hat{\times}))$.

In fact, a one-to one correspondence between isocomplex numbers $\hat{C}(\hat{c}, +, \hat{\times})$ and the points on the isogauss plane can be defined via following *isodilative isorotations*

$$z' = (x_1 + x_2 \times i)' = \hat{c} \hat{\circ} z \quad (61)$$

characterized by the isomultiplication defined as

$$\hat{c} \hat{\circ} z = (\hat{n}_0, \hat{n}_1) \hat{\circ} (x_1, x_2) =$$

$$= ([(n_0 \times x_0) \times \hat{1} - \Delta^{\frac{1}{2}} \times (n_1 \times n_2) \times \hat{1}], [(n_0 \times x_2) \times \hat{1} + (n_1 \times x_1) \times \hat{1}]),$$

with

$$\Delta = \text{Det}T = b_1^2 \times b_2^2 \quad (62)$$

Isocomplex numbers also admit following two-by-two matrix representation.

$$\hat{c} = \hat{n}_0 \times \hat{i}_0 + \hat{n}_1 \hat{i}_1 = \begin{pmatrix} n_0 \times b_1^{-2} & i \times n_1 \times b_1^2 \times \Delta^{-\frac{1}{2}} \\ i \times n_1 \times b_2^2 \times \Delta^{-\frac{1}{2}} & n_0 \times b_1^{-2} \end{pmatrix} \quad (63)$$

where

$$\hat{1} = \hat{I}_0 = \begin{pmatrix} b_1^{-2} & 0 \\ 0 & b_2^{-2} \end{pmatrix}, \hat{I}_1 = \Delta^{-\frac{1}{2}} \begin{pmatrix} 0 & i \times b_1^2 \\ i \times b_2^2 & 0 \end{pmatrix} \quad (64)$$

and

$$\Delta = \text{Det}T = b_1^2 b_2^2 \quad (65)$$

which characterize the isounit and the fundamental (adjoint)representation of $\hat{O}(2)$ respectively.

The set of matrices (63) is closed under addition and isomultiplication. Also, each element possesses the isoinverse

$$\hat{c}^{-1} = \hat{c}^{-1} \times \hat{1} \quad (66)$$

where \hat{c}^{-1} is the ordinary inverse. As a result, $\hat{S}(\hat{c}, +, \hat{\times})$ is an isofield with the local isomorphism $\hat{S}(\hat{c}, +, \hat{\times}) \approx \hat{C}(\hat{c}, +, \hat{\times})$. We note that the one-to-one correspondence between complex numbers and Gauss plane is preserved under isotopy. It is important know that the realization of complex numbers as matrices is not unique.

The *isonorm* is defined as

$$\|\hat{c}\| = [\text{Det}_R(\hat{c} \times T)]^{\frac{1}{2}} \times \hat{I}_0 = (n_0^2 + \Delta n_1^2)^{\frac{1}{2}} \times \hat{I}_0 \quad (67)$$

which readily verifies the axiom

$$\|\hat{c} \hat{\circ} \hat{c}'\| = \|\hat{c}\| \hat{\times} \|\hat{c}'\| \in \hat{R}, \hat{c}, \hat{c}' \in \hat{C}. \quad (68)$$

The *isobasis* is given by

$$\hat{e}_1 = \hat{I}_0, \hat{e}_2 = \hat{1}. \quad (69)$$

2.6.4. Isodual Isocomplex Numbers

The isodual isocomplex numbers constitute a two-dimensional, isodual, isonormed, isoassociative and isocommutative isoalgebras over the isodual isoreals $\hat{U}^d(2) \approx U^d(2)$ ref.[1].

Now the isodual isocomplex numbers are defined as

$$\begin{aligned} \hat{C}^d &= \{(\hat{c}^d, +, \hat{\times}^d) \mid \hat{c} = -\bar{c} \hat{1}^d, \hat{\times}^d = \times T^d \times, T^d \\ &= -T, \hat{1}^d = T^{-1d}, c \in C(c, +, \times)\} \end{aligned} \quad (70)$$

with generic element $\hat{c}^d = \hat{n}^d + \hat{n}_1^d \times i^d = -\hat{n}_0 + \hat{n}_1 \times i$. Here we need a two-dimensional isodual isoeuclidean space $E_{II,2}^d(x, \delta^d, \hat{R}^d(n^d, +, \times^d))$ with the realization

$$T^d = \text{diag}(-b_1^2, -b_2^2), \hat{1}^d = \text{diag}(-b_1^{-2}, -b_2^{-2}), b_k > 0, k = 1, 2, \tag{71}$$

with basic isoseparation

$$x^{2d} = (x^t \delta^d x) \times \hat{1}^d = (x_i \delta_{ij}^d x_j) \times \hat{1}^d = (-x_1 b_1^2 x_1 - x_2 b_2^2 x_2) \times \hat{1}^d \in \hat{R}^d(\hat{n}^d, +, \times^d), \tag{72}$$

whose group of isosymmetries is the isodual isorthogonal group $\hat{O}^d(2) \approx O^d(2)$.

The *isodual isogauss plane* is defined as the set of points $P = (\hat{x}_1, x_2)$ on $\hat{E}_{1,2}^d(x, \delta^d, \hat{R}^d(\hat{n}^d, +, \times^d))$ which characterize the isocomplex numbers $\hat{c} = (-\hat{n}_0, \hat{n}_1)$.

The correspondence between the isodual isocomplex numbers $\hat{C}^d(\hat{c}^d, +, \times^d)$ and the isodual gauss plane can be made one-to-one by the isodual isodilative isorotations

$$z' = (x_1 + x_2 \times i)' = \hat{c}^d \circ^d z \tag{73}$$

having rule for multiplication as

$$\hat{c}^d \circ^d z = (\hat{n}_0, \hat{n}_1) \circ^d (x_1, x_2) = [(-n_0 \times x_0) \times \hat{1} + \Delta^{\frac{1}{2}} \times (n_1 \times x_2) \times \hat{1}], [(-n_0 \times x_2) \times \hat{1} + (n_1 \times x_1) \times \hat{1}]. \tag{74}$$

Isodual isogauss planes characterizes isodual isofield. Also the isodual isotransformations forms an isodual isogroup $\hat{G}^d(2) \approx G^d(2)$.

Isodual isocomplex numbers also admit the following two-by-two matrix representation.

$$\hat{c}^d = \hat{n}_0^d \times^d \hat{I}_0^d + n_1^d \times^d \hat{I}^d = \begin{pmatrix} -n_0 \times b_1^{-2} & i \times n_1 \times b_1^2 \times \Delta^{\frac{1}{2}} \\ i \times n_1 \times b_2^2 \times \Delta^{\frac{1}{2}} & -n_0 \times b_2^{-2} \end{pmatrix} \tag{75}$$

where

$$\hat{1}^d = \hat{I}_0^d = \begin{pmatrix} -b_1^{-2} & 0 \\ 0 & -b_2^{-2} \end{pmatrix}, \hat{I}^d = \begin{pmatrix} 0 & -i \times b_1^2 \times \Delta^{\frac{1}{2}} \\ -i \times b_2^2 \times \Delta^{\frac{1}{2}} & 0 \end{pmatrix}. \tag{76}$$

This satisfies isomultiplication rule (74) characterizing the isodual isounit and fundamental representation of $\hat{O}^d(2)$.

The set of matrices representing isodual complex numbers $\hat{S}^d(\hat{c}^d, +, \times^d)$, is closed under addition and isomultiplication. Each element possesses the isodual isoinverse

$$(\hat{c}^{-1})^d = (\hat{c}^d)^{-1} \times \hat{I}^d. \tag{77}$$

As a result we get a local isomorphism $\hat{S}^d(\hat{c}^d, +, \times^d) \approx \hat{C}^d(\hat{c}^d, +, \times^d)$.

Now, the *isodual isonorm* can be defined as

$$\|\hat{c}\|^d = [Det_R(\hat{c}^d \times T^d)]^{\frac{1}{2}} \times \hat{I}_0^d = (n_0^2 + \Delta n_1^2)^{\frac{1}{2}} \times \hat{I}^d, \tag{78}$$

which verifies

$$\|\hat{c}^d \circ^d \hat{c}'^d\|^d = \|\hat{c}^d\|^d \times^d \|\hat{c}'^d\|^d \in \hat{R}^d, \hat{c}^d, \hat{c}'^d \in \hat{C}^d. \tag{79}$$

The isodual isobasis is given by

$$\hat{e}_1^d = \hat{I}_0^d, \hat{e}_2^d = \hat{I}^d. \tag{80}$$

2.7. Isoquaternions and Their Isoduals

2.7.1. Quaternions

Quaternions constitute a normed, associative, non-commutative algebra of dimension 4 over reals $U(4)$ ref.[1].

Quaternions $q \in Q(q, +, \times)$ admit a realization in the complex Hermitean plane $E_2(z, \delta, C)$ with separation

$$E_2(z, \delta, C): z \circ z, z^{-i} \delta_{ij} z^j = z^{-1} z^1 + z^{-2} z^2, \delta \circ \delta \equiv \delta \tag{81}$$

with basic (unimodular) invariant $SU(2)$. Hence quaternions have a fundamental representation $SU(2)$ by *Pauli's matrices*.

Quaternions Q can be realized as the pairs of complex numbers, $q = (c_1, c_2)$, $q \in Q$ and $c_1, c_2 \in C$ with multiplication \circ . Hermitean dilative rotation on $E_2(z, \delta, C)$ which leaves $z^\dagger z$ invariant is given by

$$z'^1 = c_1 \circ z^1 + c_2 \circ z^2, z'^2 = -\bar{c}_2 \circ z^1 + \bar{c}_1 \circ z^2, \tag{82}$$

where the dilation is represented by $\bar{c}_1 \circ c_1 + \bar{c}_2 \circ c_2 \neq 1$.

These transformations form a group $G(4)$. This group is associative but noncommutative resulting into a one-to-one correspondence with quaternions.

Quaternions can be represented via matrices over the field of complex numbers $C(c, +, \times)$ as

$$q = \begin{pmatrix} c_1 & c_2 \\ -\bar{c}_2 & \bar{c}_1 \end{pmatrix} \quad (83)$$

with

$$c_1 = n_0 + n_3 \times i, \quad c_2 = n_1 + n_2 \times i \quad (84)$$

The matrix q admits the representation

$$q = n_0 \times I_0 + n_1 \times i_1 + n_2 \times i_2 + n_3 \times i_3 \quad (85)$$

where I_0, i_1, i_2, i_3 are the Pauli's matrices

$$I_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad i_1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad (86)$$

$$i_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad i_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

with fundamental relations

$$i_n \times i_m = -\mathcal{E}_{nmk} \times i_k, \quad n \neq m, \quad n, m = 1, 2, 3. \quad (87)$$

where \mathcal{E}_{nmk} is the tensor of rank three. The norm of the quaternion can be defined as

$$|q| = (q^\dagger q)^{\frac{1}{2}} = \left(\sum_{k=1,2,3} n_k^2 \right)^{\frac{1}{2}}, \quad (88)$$

satisfying

$$|q \circ q'| = |q| \times |q'| \in R, \quad q, q' \in Q \quad (89)$$

The basis is defined by

$$e_1 = I_0, \quad e_{k+1} = i_k, \quad k = 1, 2, 3. \quad (90)$$

2.7.2. Isodual Quaternions

Isodual quaternions constitute an isodual four-dimensional, normed associative and noncommutative algebra over the isodual reals $U^d(4)$ which is anti-isomorphic to $U(4)$ ref.[1].

Isodual quaternions $q^d \in Q^d(q^d, +, \times^d)$ can be represented via the isodual Hermitean Euclidean space

$$E_2^d(z^d, \delta^d, C^d(c^d, +, \times^d)) : (z^{-id} \delta_{ij}^d z^{jd}) \times I^d \quad (91)$$

$$= (-z^{-1} z^1 - z^{-2} z^2) \times I^d \in R^d.$$

Isodual complex numbers can also be realized via pairs of isodual complex numbers as

$$q^d = (c^d, c^{2d}), \quad q^d \in Q^d, \quad c_1^d, c_2^d \in C^d.$$

Also, the isodual Hermitean dilative rotation on

$E_2^d(z^d, \delta^d, C^d(c^d, +, \times^d))$ leaving invariant $z^d \dagger \delta^d z^d$ is given by

$$z'^{1d} = c_1^d \circ^d z^{1d} - \bar{c}_2^d \circ^d z^{2d}, \quad (92)$$

$$z'^{2d} = c_2^d \circ^d z^{1d} + \bar{c}_1^d \circ^d z^{2d}$$

where the dilation is represented by the value $\bar{c}_1^d \circ^d c_1^d + \bar{c}_2^d \circ^d c_2^d \neq -1$.

These transformations form an associative but noncommutative isodual group $G^d(4)$ which is in one-to-one correspondence with isodual quaternions $Q^d(q^d, +, \times^d)$.

As a result there is a matrix representation of isodual complex numbers over the field of isodual complex numbers $C^d(c^d, +, \times^d)$ as

$$q^d = \begin{pmatrix} c_1^d & -\bar{c}_2^d \\ c_2^d & \bar{c}_1^d \end{pmatrix} \quad (93)$$

under the condition

$$c_1^d = -n_0 + n_3 \times i, \quad c_2^d = -n_1 + n_2 \times i \quad (94)$$

where $-\bar{c}^d = c$, $i^d = i$.

We can represent q^d as

$$q^d = n_0^d \times^d I_0^d + n_1^d \times^d i_1^d + n_2^d \times^d i_2^d + n_3^d \times^d i_3^d =$$

$$= -n_0 \times I_0 + n_1 \times i_1 + n_2 \times i_2 + n_3 \times i_3 \quad (95)$$

where i 's are the Pauli's matrices. Note that Pauli's matrices change sign under isoduality although their product with isodual numbers is isoselfdual.

Isodual norm is then defined as

$$|q^d| = [Det_C(q^d \times T^d)] \times I^d = \left(- \sum_{k=0,1,2,3} n_k^2 \right)^{\frac{1}{2}} \times I^d \quad (96)$$

satisfying

$$|q^d \circ^d q'^d|^d = |q^d|^d \times^d |q'^d|^d \in R^d, \quad (97)$$

$$q^d, q'^d \in Q^d.$$

The isodual basis is defined as

$$e_1^d = I_0^d, \quad e_{k+1}^d = i_k, \quad k = 1, 2, 3. \quad (98)$$

2.7.3. Isoquaternions

Isoquaternions constitute a four-dimensional, isonormed, isoassociative, non-isocommutative isoalgebra over the

isoreals $\hat{U}(4) \approx U(4)$, ref.[1].

Isoquaternions $\hat{q} \in \hat{Q}(\hat{q}, +, \hat{\times})$ can be represented using two-dimensional, complex Hermitean isoeuclidean space of class I, $\hat{E}_{I,2}(\hat{z}, \hat{\delta}, \hat{C})$, $\hat{z}^k = z^k$, $\hat{z}_k = \hat{\delta}_{ki} \hat{z}^i$, $\hat{\delta} = \hat{T} \hat{\delta} = \hat{\delta}_{ij}$ on the isofield $\hat{C}(\hat{c}, +, \hat{\times})$ with real separation given by

$$\hat{z} \hat{\dagger} \hat{\delta} \hat{z} = \bar{z}^1 b_1^2 z^1 + \bar{z}^2 b_2^2 z^2, \quad \hat{\delta} \hat{\dagger} \hat{\delta} = \delta > 0, \quad (99)$$

with basic isotopic element and isounit

$$T = \text{Diag.}(b_1^2, b_2^2), \quad \hat{1} = \text{Diag.}(b_1^{-2}, b_2^{-2}), b_k > 0, \quad (100)$$

The (unimodular) invariance group of this space is the Lie-isotopic group $S\hat{U}(2)$. Isoquaternions can also be characterized by fundamental representation of $S\hat{U}(2)$ algebra. A Hermitean isodilative isorotation on $\hat{E}_{I,2}(\hat{z}, \hat{\delta}, \hat{C}(\hat{c}, +, \hat{\times}))$ is given by

$$\hat{z}'^1 = \hat{c}_1 \hat{\circ} \hat{z}^1 + \hat{c}_2 \hat{\circ} \hat{z}^2, \quad \hat{z}'^2 = -\hat{c}_2 \hat{\circ} \hat{z}^1 + \hat{c}_1 \hat{\circ} \hat{z}^2, \quad (101)$$

where the dilation is represented by the value $\hat{c}_1 \hat{\circ} \hat{c}_1 + \hat{c}_2 \hat{\circ} \hat{c}_2 \neq \hat{1}$. Representation of isoquaternions into two-by-two matrices on $\hat{C}(\hat{c}, +, \hat{\times})$ is characterized by the isorepresentations of the Lie-isotopic algebra $S\hat{U}(2)$ ref. [40, 41, 42]. These can be expressed in terms of the basic isounit

$$\hat{I} = \hat{1}_0 = \begin{pmatrix} b_1^{-2} & 0 \\ 0 & b_2^{-2} \end{pmatrix} \quad (102)$$

and fundamental representation of $S\hat{U}(2)$ as

$$\begin{aligned} \hat{i}_1 &= \Delta^{-\frac{1}{2}} \begin{pmatrix} 0 & ib_1^2 \\ ib_2^2 & 0 \end{pmatrix}, \quad \hat{i}_2 = \Delta^{-\frac{1}{2}} \begin{pmatrix} 0 & b_1^2 \\ -b_2^2 & 0 \end{pmatrix}, \\ \hat{i}_3 &= \Delta^{-\frac{1}{2}} \begin{pmatrix} ib_2^2 & 0 \\ 0 & -ib_1^2 \end{pmatrix} \end{aligned} \quad (103)$$

Note that the matrices above satisfy the properties of isotopic image

$$\begin{aligned} \hat{i}_n \hat{\circ} \hat{i}_m &= \Delta^{-\frac{1}{2}} \epsilon_{nmk} \hat{1}_k, \quad n \neq m, \\ n, m &= 1, 2, 3, \quad \Delta = b_1^2 b_2^2, \end{aligned} \quad (104)$$

and hence are closed under commutators, which is a necessary condition for the existence of an isotopy. This results into a Lie-isotopic $S\hat{U}(2)$ algebra

$$[\hat{i}_n, \hat{i}_m] := \hat{i}_n \hat{\circ} \hat{i}_m - \hat{i}_m \hat{\circ} \hat{i}_n = -2\Delta^{-\frac{1}{2}} \epsilon_{nmk} \hat{i}_k. \quad (105)$$

Isoquaternions can be represented in the form

$$\hat{q} = n_0 I_0 + n_1 \hat{i}_1 + n_2 \hat{i}_2 + n_3 \hat{i}_3 =$$

$$\begin{pmatrix} (n_0 b_1^{-2} + \Delta^{-\frac{1}{2}} i n_3 b_2^2) & \Delta^{-\frac{1}{2}} (-n_2 + i n_1) b_1^2 \\ \Delta^{-\frac{1}{2}} (n_2 + i n_1) b_2^2 & (n_0 b_2^{-2} - \Delta^{-\frac{1}{2}} i n_3 b_1^2) \end{pmatrix}. \quad (106)$$

Note that the set $\hat{S}(\hat{q}, +, \hat{\times})$ is a four dimensional vector space over the isoreals $\hat{R}(\hat{n}, +, \hat{\times})$ which is closed under the operation of conventional addition and isomultiplication and hence, is an isofield. Thus, $\hat{S}(\hat{q}, +, \hat{\times}) \approx \hat{Q}(\hat{q}, +, \hat{\times})$.

The *isornorm* of the isoquaternions is defined as follows

$$\|\hat{q}\| = [\text{Det}_R(\hat{q}T)]^{\frac{1}{2}} \hat{1}_0, \quad (107)$$

and may be written as

$$\|\hat{q}\| = [n_0^2 + \Delta(n_1^2 + n_2^2 + n_3^2)] \hat{1}_0, \quad (108)$$

and then

$$\|\hat{q} \hat{\circ} \hat{q}'\| = \|\hat{q}\| \hat{\times} \|\hat{q}'\| \in \hat{R}, \quad \hat{q}, \hat{q}', \hat{\circ} \in \hat{Q} \quad (109)$$

The *isobasis* is defined as

$$\hat{e}_1 = \hat{I}_0, \quad \hat{e}_{k+1} = \hat{i}_k, \quad k = 1, 2, 3. \quad (110)$$

2.7.4. Isodual Isoquaternions

The isodual isoquaternions constitute a four-dimensional, isodual, isonormed, isoassociative, non-isocommutative isoalgebra over the isodual isoreals $\hat{U}(4) \approx U^d(4)$ ref. [1].

The *isodual isoquaternions* $\hat{q}^d \in \hat{Q}^d(\hat{q}^d, +, \hat{\circ}^d)$ by a two-dimensional isodual complex Hermitean isoeuclidean space of class II over the isodual isocomplex field as

$$\begin{aligned} E_{II,2}^d(\hat{z}^d, \hat{\delta}^d, \hat{C}^d(c^d, +, \hat{\times}^d)) : \hat{z}^d \hat{\gamma} \hat{\delta}^d z^d \\ z^{-1d} \hat{\times}^d z^{1d} + z^{-2d} \hat{\times}^d z^{2d} = -z^{-1} b_1^2 z^1 - z^{-2} b_2^2 z^2. \end{aligned} \quad (111)$$

having basic isodual isotopic element and *isodual isounit*

$$T^d = \text{Diag.}(-b_1^2, -b_2^2), \hat{1}^d = \text{Diag.}(-b_1^{-2}, -b_2^{-2}) \quad (112)$$

having invariance as the isodual Lie-isotopic group $S\hat{U}^d$. An *isodual Hermitean isodilative isorotation* on $E_{II,2}^d(\hat{z}^d, \hat{\delta}^d, \hat{C}^d(c^d, +, \hat{\times}^d))$ is given by

$$\begin{aligned} \hat{z}'^{1d} &= \hat{c}_1^d \hat{\circ}^d \hat{z}^{1d} - \bar{c}_2^d \hat{\circ}^d \hat{z}^{2d}, \\ \hat{z}'^{2d} &= \hat{c}_2^d \hat{\circ}^d \hat{z}^{1d} + \bar{c}_1^d \hat{\circ}^d \hat{z}^{2d}, \end{aligned} \quad (113)$$

where dilation is represented by

$$\hat{c}_1^d \circ^d \hat{c}_1^d + \bar{c}_2^d \circ^d \hat{c}_2^d \neq \hat{1}^d.$$

Isodual Isoquaternions can also be realized as the isodual isorepresentation of $S\hat{U}^d(2)$ and can be written as

$$\begin{aligned} \hat{q}^d &= \hat{n}_0^d + \hat{n}_1^d \hat{\times}^d \hat{i}_1^d + \hat{n}_2^d \hat{\times}^d \hat{i}_2^d + \hat{n}_3^d \hat{\times}^d \hat{i}_3^d = \\ &= -\hat{n}_0 + \hat{n}_1 \hat{i}_1 + \hat{n}_2 \hat{i}_2 + \hat{n}_3 \hat{i}_3 = \\ &= \begin{pmatrix} (-n_0 b_1^{-2} + \Delta \frac{1}{2} i n_3 b_2^2) & \Delta \frac{1}{2} (-n_2 + i n_1) b_1^2 \\ \Delta \frac{1}{2} (n_2 + i n_1) b_2^2 & (-n_0 b_2^{-2} - \Delta \frac{1}{2} i n_3 b_1^2) \end{pmatrix} \end{aligned} \quad (114)$$

Note that the set of all the matrices ${}^d(\hat{q}^d, +, \hat{\times}^d)$ is an isofield and hence ${}^d(\hat{q}^d, +, \hat{\times}^d) \approx \hat{Q}^d(\hat{q}^d, +, \hat{\times}^d)$.

The *isodual isonorm* is defined as

$$\begin{aligned} \hat{q}^d, {}^d &= [Det_R(\hat{q}^d T^d)]^{\frac{1}{2}} \hat{I}_0^d = \\ &= [-n_0^2 - \Delta(n_1^2 + n_2^2 + n_3^2)] \hat{I}_0^d, \end{aligned} \quad (115)$$

$$\begin{aligned} \|\hat{q}^d \circ^d \hat{q}'^d\|^d &= \|\hat{q}^d\|^d \hat{\times}^d \|\hat{q}'^d\|^d \in \hat{R}^d, \\ \hat{q}^d, \hat{q}'^d, \circ^d &\in \hat{Q}^d \end{aligned} \quad (116)$$

The *isodual isobasis* is defined as

$$\hat{e}_1^d = \hat{I}_0^d \quad \hat{e}_{k+1}^d = \hat{i}_k^d, \quad k=1,2,3. \quad (117)$$

2.8. Isooctonians and Their Isoduals

2.8.1. Octonians

Octonians constitute and eight-dimensional normed, non-associative and non-commutative, alternative algebra $U(8)$ over the field of reals $R(n, +, \times)$. ref.[20, 21].

Octonians $o \in O(o, +, \times)$ can be realized as two-dimensional quaternions $o = (q_1, q_2)$ with multiplication rules

$$\begin{aligned} o \circ o' &= (q_1, q_2) \circ (q'_1, q'_2) = \\ &= (q_1 \circ q'_1 + q_1 \circ q'_2, -\bar{q}_1 \circ q'_2 + \bar{q}_1 \circ q_2). \end{aligned} \quad (118)$$

The *antiautomorphic conjugation* of an octonian is defined as

$$\bar{o} = (\bar{q}_1, -q_2). \quad (119)$$

The *norm* of an octonian is defined as

$$|o| := (\bar{o} \circ o)^{\frac{1}{2}} = |q_1| + |q_2|, \quad (120)$$

with the basic axioms

$$|o \circ o'| = |o| \times |o'| \in R, \quad o, o' \in O. \quad (121)$$

It is important to note that Octonions do not constitute a realization of the abstract axioms of a numeric field and, therefore, they do not constitute numbers as conventionally known in mathematics due to the non-associative character of their multiplication (see ref. [1]).

2.8.2. Isodual Octonians

The isodual octonians constitute an eight-dimensional isodual, normed, non-associative, and non-commutative algebra $U^d(8)$ over the isodual real numbers $R^d(n^d, +, \times^d)$ ref. [1].

Isodual octonians are defined as

$$o^d = (q_1^d, q_2^d) \quad (122)$$

over the isodual reals $R^d(n^d, +, \times^d)$. The isodual multiplication of isodual octonians is defined by

$$\begin{aligned} o^d \circ^d o'^d &= (q_1^d, q_2^d) \circ^d (q_1'^d, q_2'^d) = \\ &= (q_1^d \circ^d q_1'^d - \bar{q}_1^d \circ^d q_2'^d, q_1^d \circ^d q_2'^d + \bar{q}_1^d \circ^d q_2^d). \end{aligned} \quad (123)$$

The isodual antiautomorphic conjugation of an octonian is defined as

$$\bar{o}^d = (\bar{q}_1^d, -q_2^d). \quad (124)$$

The *isodual norm* of an octonian is defined as

$$|o^d| := (\bar{o}^d \circ o^d)^{\frac{1}{2}} = |q_1^d| + |q_2^d|, \quad (125)$$

with the basic axioms

$$|o^d \circ^d o'^d| = |o^d|^d \times^d |o'^d|^d \in R^d, \quad o^d, o'^d \in O. \quad (126)$$

2.8.4. Isodual Isooctonians

Isodual isooctonians form an eight-dimensional isodual, isonormed, non-isoassociative, non-isocommutative, but isoalternative isoalgebra $\hat{U}^d(8) \approx U^d(8)$ over the isodual isofield $\hat{R}^d(\hat{n}^d, +, \hat{\times}^d)$, ref. [43].

Isodual isooctonians $\hat{o}^d \in \hat{O}^d(\hat{o}^d, +, \hat{\times}^d)$ can be defined as the pair of isoquaternions $\hat{o}^d = (\hat{q}_1^d, \hat{q}_2^d)$ over the isodual isoreals $\hat{R}^d(\hat{n}^d, \hat{\times}^d)$ with the multiplication rule

$$\begin{aligned} \hat{o}^d \circ^d \hat{o}'^d &= (\hat{q}_1^d, \hat{q}_2^d) \circ^d (\hat{q}_1'^d, \hat{q}_2'^d) = \\ &= (\hat{q}_1^d \circ^d \hat{q}_1'^d - \bar{\hat{q}}_1^d \circ^d \hat{q}_2'^d, \hat{q}_1^d \circ^d \hat{q}_2'^d + \bar{\hat{q}}_1^d \circ^d \hat{q}_2^d) \end{aligned} \quad (131)$$

The isodual isoantiautomorphism is defined as

$$\tilde{o}^d = (\tilde{q}^d, -\hat{q}_2^d) \quad (132)$$

The *isodual isonorm* is defined as

$$\|\hat{\sigma}^d\|^d := (\tilde{\sigma}^d \circ^d \hat{\sigma}^d)^{\frac{1}{2}} \times \hat{1}^d = \|\hat{q}_1^d\|^d + \|\hat{q}_2^d\|^d \quad (133)$$

which readily verifies

$$\|\hat{\sigma}^d \circ^d \hat{\sigma}'^d\| = \|\hat{\sigma}^d\|^d \hat{\times}^d \|\hat{\sigma}'^d\|^d \in \hat{R}^d, \quad \hat{\sigma}^d, \hat{\sigma}'^d \in \hat{O}^d. \quad (134)$$

Again it is important to note that Isodual iso-octonians do not constitute a realization of the abstract axioms of a numeric field and, therefore, they do not constitute numbers as conventionally known in mathematics due to the non-associative character of their multiplication (see Ref. [1]).

3. Grand Unification of Numeric Fields

Isotopic generalization has brought about a grand unification of the conventional numbers into one single, abstract notion of isonumber. It is important to note that the unification of all numbers was conjectured by Prof. Santilli in numerous publications through out his research for many years. Finally it was proved by Kadeisville, Kamiya and Santilli ref.[40]. The following theorem is the main result in this regard.

Theorem 3.1. *Let $F(a, +, \times)$ be the fields of real numbers, complex numbers and quaternions, respectively, $F^d(a^d, +, \times^d)$ the isodual fields, $a^d := a \times 1^d = -a$ the isofields, and $\times^d := \times 1^d \times = -\times$, $1^d = -1$. the isodual isofields as defined in the preceding section. Then all these fields can be constructed with the same methods for the construction of $F(\hat{a}, +, \hat{\times})$ from $F^d(\hat{a}^d, +, \hat{\times}^d)$, under the relaxation of the condition of positive-definiteness of the isounit, thus achieving a unification of all the fields, isofields and their isoduals into the single, abstract isofield of Class III, denoted by R .*

3.1. Hidden Numbers of Dimension 3, 5, 6, 7

Based on the historical problem ‘The four and eight square problem and division algebras’ ref.[21], Prof. Santilli conjectured the possibility of ‘Hidden numbers’ of dimension 3, 5, 6 and 7. The numbers studied by Santilli, namely, reals, complex, quaternions and octonians are the solution of the following problem.

$$(a_1^2 + a_2^2 + \dots + a_n^2) \times (b_1^2 + b_2^2 + \dots + b_n^2) = A_1^2 + A_2^2 + \dots + A_n^2$$

with

$$A_k = \sum_{r,s} c_{krs} \times a_r \times b_s \quad (135)$$

where all the a 's, b 's and c 's are elements of a field $F(a, +, \times)$ with conventional operations $+$ and \times . It is

well known that the only possible solutions of the problem are of dimension 1, 2, 4 and 8. These facts are incorporated in the theorem 1.1, restated here

Theorem 3.2 All possible normed algebras with multiplicative unit over the field of real numbers are given by algebras of dimension 1 (real numbers), 2 (complex numbers), 4 (quaternions), and 8 (octonians).

The question posed by Santilli: Is ‘Does the classification according to above theorem persist under isotopies, pseudoisotopies and their isodualities?’ or ‘Is it incomplete?’ First, we investigate this problem for isotopies of the multiplication. The above problem, equation (135) is reformulated under the isotopies of the multiplication as follows.

The isotopic lifting of the multiplication

$$\times \rightarrow \hat{\times} = \times T \times, 1 \rightarrow \hat{1} = T^{-1} \quad (136)$$

transforms the problem (135) in to

$$(a_1^{\hat{2}} + a_2^{\hat{2}} + \dots + a_n^{\hat{2}}) \hat{\times} (b_1^{\hat{2}} + b_2^{\hat{2}} + \dots + b_n^{\hat{2}}) = A_1^{\hat{2}} + A_2^{\hat{2}} + \dots + A_n^{\hat{2}} \quad (137)$$

with

$$A_k = \sum_{r,s} c_{krs} \hat{\times} a_r \hat{\times} b_s \quad (138)$$

where all the a 's, b 's and c 's are elements of an isofield $\hat{F}(a, +, \hat{\times})$ in which $\hat{1}$ is an element of the original field, can be simplified to the conventional operations as

$$(a_1^2 + a_2^2 + \dots + a_n^2) \times (b_1^2 + b_2^2 + \dots + b_n^2) = T^{-2} \times (A_1^2 + A_2^2 + \dots + A_n^2) \quad (139)$$

with

$$A_k = T^2 \times \sum_{r,s} c_{krs} \times a_r \times b_s. \quad (140)$$

Comparing the original problem and its isotopic conversion as formulated above, we observe that the reformulation of the problem is same as the original problem and hence the isotopic lifting and isoduality of the field $F(a, +, \times) \rightarrow \hat{F}(\hat{a}, +, \hat{\times})$ does not change the solution of the problem. As the result we get the following theorem.

Theorem 3.3. *All possible isonormed isoalgebras with multiplicative isounit over the field of the isoreals are the isoalgebras of dimension 1 (isoreals), 2 (isocomplex), 4 (isoquaternions), and 8 (isooctonians) and the classification persists under isoduality.*

Further, lifting of addition gives the third formulation which is pseudoisotopic type

$$+ \rightarrow \hat{+} = +\hat{K}, \quad 0 \rightarrow \hat{0} = -\hat{K}, \quad \hat{K} = K \times \hat{1} \quad (141)$$

under which (137), (138) can be written over the pseudoisofield $\hat{F}(\hat{a}, \hat{+}, \hat{\times})$ as

$$(\hat{a}_1^2 \hat{+} \hat{a}_2^2 \hat{+} \dots \hat{+} \hat{a}_n^2) \hat{\times} (\hat{b}_1^2 \hat{+} \hat{b}_2^2 \hat{+} \dots \hat{+} \hat{b}_n^2) = \hat{A}_1^2 \hat{+} \hat{A}_2^2 \hat{+} \dots \hat{+} \hat{A}_n^2 \quad (142)$$

with

$$\hat{A}_k = \sum_{r,s} \hat{c}_{krs} \hat{\times} \hat{a}_r \hat{\times} \hat{b}_s = (\sum_{r,s} c_{krs} a_r b_s) \hat{1} = A_k \times \hat{1} \quad (143)$$

This can be written in the conventional operations as

$$\begin{aligned} & [(a_1^2 + a_2^2 + \dots + a_n^2)\hat{1} + (n-1)K\hat{1}]T \\ & [(b_1^2 + b_2^2 + \dots + b_n^2)\hat{1} + (n-1)K\hat{1}] = \\ & = (A_1^2 + A_2^2 + \dots + A_n^2)\hat{1} + (n-1)K\hat{1}, \quad \hat{A}_k = A_k \hat{1} \quad (144) \end{aligned}$$

The solution to (144) of dimension other than 1,2,4,8 under the pseudoisofield $\hat{F}(\hat{a}, \hat{+}, \hat{\times})$ was envisaged by prof.Santilli as a conjecture under the loss of the needed axioms of a field, such as distributive laws.

It was found that the solution do exist, but under the loss of number of axioms of the original field, in addition to the loss of distributivity. We consider a representative example of "Hidden numbers" of dimension 3 as follows

$$(\hat{1}^2 + \hat{2}^2 + \hat{3}^2) * (\hat{5}^2 + \hat{6}^2 + \hat{7}^2) = \hat{1}2^2 + \hat{2}4^2 + \hat{3}0^2 \quad (145)$$

Note that also the condition on \hat{A}_k is true, that the elements in the r.h.s can be written as the combinations of the elements on the l.h.s as

$$12 = 2 \times 6, \quad 24 = 2 \times 5 + 2 \times 7, \quad 30 = 3 \times 3 + 3 \times 7. \quad (146)$$

Hence we can rewrite the problem as

$$\begin{aligned} & [(1^2 + 2^2 + 3^2)\hat{1} + 2K]T[(5^2 + 6^2 + 7^2)\hat{1} + 2K\hat{1}] \\ & = (12^2 + 24^2 + 30^2)\hat{1} + 2K\hat{1} \quad (147) \end{aligned}$$

which on simplification gives a quadratic equation in K as

$$4K^2 + 246K - 80 = 0 \quad (148)$$

with solution

$$K = 0.325... \quad (149)$$

Thus the solution exists, but is not an integer. This implies the loss of closure under isoaddition for the case of integers. However, the closure can be regained if the original field is enlarged to include all real numbers. The issue whether such solutions do indeed form a pseudoisofield is open for the mathematicians.

As algebras of dimensions higher than 8 are not alternative [21], also, as this property persists under isotopies and

pseudoisotopies, leads to the fact that formulations (137) and (142) are restricted to dimensions $n \leq 8$.

Prof. Santilli ref.[1] identified following open problems with regards to the notion of isofields.

- Investigative study of "number with singular unit", i.e. isofields of class IV which are at the foundations of the isotopic studies of gravitational collapse.
- The study of isofields of characteristic $p \neq 0$, to see whether new fields and therefore new Lie-algebras are permitted by isotopies.

Author of this article has defined 'Iso-Galois fields' ref.[44] which are basically finite isofields essentially of nonzero characteristic. As predicted by Santilli these isofields have important applications in Cryptography, Genetics, Fractal geometry etc.

- The study of the integro-differential topology characterized by isofields with local differential structure and integral isounits.

3.2. Genonumbers and Their Isoduals

We have seen that the two degrees of freedom due to isotopic lifting of *addition* and *multiplication* give rise to isofields and pseudoisofields respectively. These fields are at the foundation of the *Lie-isotopic theory* [8, 9, 45].

Also, there exists a third degree of freedom caused by the *ordering* of the above operations which leads to further generalization of a field which is at the foundation of *Lie-admissible algebras* [8, 9, 18].

Given a field $F(a, +, \times)$ of ordinary numbers with generic elements $a, b, c \dots$, with addition $a + b = b + a$ and multiplication $a \times b$, we can define the following.

Genoaddition: Addition of a to b from the left, denoted by $a +^> b$ and addition of b to a from the right denoted by $a < + b$ are called *genoadditions*.

Genomultiplication: Multiplication of a times b from the left denoted by $a \times^> b$, and multiplication b times a from the right denoted by $a < \times b$ are called *genomultiplications*.

It is worthwhile to note that ordering of multiplication is fully compatible with its basic axioms, such as commutativity for real and complex numbers, associativity for quaternions, and alternativity for the octonions. In the case of real and complex numbers we will have

$$a \times^> b \equiv b \times^> a, \quad a < \times b \equiv b < \times a \quad (150)$$

The identity of multiplication from left and right can be different and hence two genomultiplications can very well be different i.e.

$$a \times^> b \neq a < \times b \quad (151)$$

with realization,

$$a \times^> b := aRb, \quad a \times^< b := aSb, \quad R \neq S, \quad (152)$$

where R and S are fixed isotopic elements, called the genotopic elements. These are sufficiently smooth, bounded and nowhere singular (not necessarily Hermitean) outside the original field.

The left and right generalized *genounits* can be defined in the following manner

$$a \hat{\times} b \quad (153)$$

$$\hat{1} = T^{-1} \quad (154)$$

Note that all the axioms and properties of the original field are preserved under the mentioned left or right multiplication and multiplicative units under the appropriate ordering for all the dimensions 1,2,4,8. This procedure leads to new fields called as *genofield* denoted by $\hat{F}^>(\hat{a}^>, +, \hat{\times}^>)$ (right genofield) or $\hat{F}^<(\hat{a}^<, +, \hat{\times}^<)$ (left genofield) or $\hat{F}^>(\hat{a}^>, +, \hat{\times}^>)$. Also, *isodual genofields* are defined by the antiautomorphic conjugations

$$R \rightarrow R^d = -R, \quad S \rightarrow S^d = -S \quad (155)$$

denoted by $\hat{F}^{>d}(\hat{a}^{>d}, +, \hat{\times}^{>d})$.

Note that isofields are the particular case of genofields where the genotopic elements coincide. i.e.

$$\hat{F}^{>d}(\hat{a}^{>d}, +, \hat{\times}^{>d})_{|R=S=T} = F(\hat{a}, +, \hat{\times}). \quad (156)$$

R-S mutation of the Lie product: is defined as

$$(A, B) = ARB - BSA \quad (157)$$

which is Lie-admissible via the attached antisymmetric product

$$[A, \hat{B}] = (A, B) - (B, A) = ATB - BTA, T = R - S \quad (158)$$

which is Lie-isotopic.

The lifting $[A, B] \rightarrow [A, \hat{B}]$ is called an *isotopy*. The lifting $[A, B] \rightarrow (A, B)$ is called a *genotopy*, ref. [8, 1].

The Lie-isotopic algebras are defined by one single isotopy of the enveloping associative algebra and related unit

$$AB = A \times B \rightarrow A \hat{\times} B = ATB, \quad 1 \rightarrow \hat{1} = T^{-1}. \quad (159)$$

For the consistent formulation of Lie-isotopic algebras they must be defined over an isofield $\hat{F}(\hat{a}, +, \hat{\times})$ with isounit $\hat{1} = T^{-1}$.

Note that for the conventional multiplication \times there is no ordering as $1^> =^< 1 = 1$. The above ordering can be defined for isomultiplication $\hat{\times}$ wherein we can have different isounits.

The Lie-admissible algebras can be generated by two different isotopies of the original associative algebra using left and right isounits with corresponding isotopies as

$$AB \rightarrow ARB := A \times^> B, \quad 1 \rightarrow \hat{1}^> = R^{-1}, \quad (160)$$

$$BA \rightarrow BSA := B \times^< A, \quad 1 \rightarrow \hat{1}^< = S^{-1}. \quad (161)$$

which must be defined over the genofields $\hat{F}^>(\hat{a}^>, +, \hat{\times}^>)$ with isounits $\hat{1}^>$. Here, the isounits related with the left and right isomultiplication are disjoint and can indeed be Hermitean and real-valued, which admit Kadeisville classification into classes I, II, III, IV and V.

However, in physics the isounits (left and right) used have a real physical significance when they are inter-related by a Hermitean conjugation as

$$\hat{1}^> = (\hat{1}^<)^{\dagger} \quad (162)$$

This representation of the genounits (and hence genofields) provides approximation of irreversibility ref.[18].

It is important to note that conventional addition admits no meaningful ordering as $0^> =^< 0 \equiv 0$. However, the ordering exists for the isoaddition $\hat{+} = +K +$ as $\hat{+}^> \neq^< \hat{+}$ with $K^> \neq^< K$. But there is loss of distributive law for the resulting genofield under genoadditions $\hat{+}^>$.

All the above discussion leads to a broadest generalization of the existing theory of numbers through

1. *pseudogenofields* $\hat{F}^>(\hat{a}^>, \hat{+}^>, \hat{\times}^>)$ defined via genotopies of all aspects of conventional fields $F(a, +, \times)$ and
2. *isodual pseudogenofields* $\hat{F}^{>d}(\hat{a}^{>d}, \hat{+}^{>d}, \hat{\times}^{>d})$ defined via isoduality of pseudogenofields.

This new generalization of the conventional numbers leads to the following categorization of numbers:

- Conventional numbers of dimension 1,2,4,8 and their isoduals;
- Isonumbers of the same dimension and their isoduals;
- Genonumbers of the same dimensions and their isoduals;
- Pseudoisonumbers of the same dimension and their isoduals;
- Pseudogenonumbers of the same dimension and their isoduals;
- "Hidden pseudoisonumbers" of dimension 3,4, 5,7 and their osoduals;
- "Hidden pseudogenonumbers" of dimension 3,4,5,7 and their isoduals.

Note that each of these can be defined for the fields of characteristic 0 or for $p \neq 0$.

In addition to above generalization, we can have an ordered set of values for the multiplicative unit such as

$1^> = \{2, \frac{4}{3}, 6, \dots\}$ defined as applicable to the right or left.

This possibility leads to the new numbers called as *hyper-Santillian numbers*. These include hyper-real, hyper complex, hyper-quaternion numbers which have vast applications in biological sciences.

In the further generalization, the multiplicative unit can very well have non-zero negative values. This leads to a new class of numbers called *iso-dual Santillian numbers*. This further leads to a new kinds of conventional iso-dual numbers called as *iso-topic isodual numbers*, *geno-topic iso-dual numbers* and *hyper-structural isodual*. These numbers have applications for antimatter.

The above generalization of the conventional numbers gives us, in all, eleven classes of new numbers namely, *the iso-topic numbers*, *genotopic to the right and left*, *right and left hyper-structural numbers*, *iso-dual conventional numbers*, *iso-dual iso-topic numbers*, *iso-dual geno-topic to the right and left numbers* and *hyper-structural iso-dual to the right and left numbers*. Each class is applicable to the real, complex and quaternion numbers where each of the applications have infinite number of possible units.

4. Applications and Advances

Quantum mechanics was sufficient to deal with 'Exterior Dynamical systems' which are liner, local, lagrangian and hamiltonian. The main purpose of formulating the new generalized mathematics was to deal with the insufficiencies in the modern mathematics to describe 'Interior Dynamical systems' which are intrinsically non-linear, non-local, non-hamiltonian and non-lagrangian. The axiom-preserving generalization of quantum mechanics which can also deal with non-linear, non-local non-hamiltonian and non-lagrangian systems is called the *Hadronic mechanics*. The mechanics; built specifically to deal with 'hadrons' (strongly interacting particles) ref. [18]. Prof. Santilli, in 1978 when at Harvard University, proposed 'Hadronic mechanics' under the support from U. S. Department of Energy, which was subsequently studied by number of mathematicians, theoreticians and experimentalists. Hadronic mechanics is directly universal; that is, capable of representing all possible nonlinear, nonlocal, nonhamiltonian, continuous or discrete, inhomogeneous and anisotropic systems (universality), directly in the frame of the experimenter (direct universality). In particular the hadronic mechanics has shown that quantum mechanics is completely inapplicable to the synthesis of neutron [46], as mass of the neutron is greater than the sum of the masses of proton and electron (called "mass defect") of which it is made. In this case quantum equations are completely inconsistent. Hadronic mechanics has achieved numerically exact results in the cases in which quantum mechanics results are not valid. For further details of isonumber theory we recommend refs. [47, 1, 48, 46, 49].

As far as mathematics is concerned, one of the major applications of isonumber theory is in Cryptography, ref. [50]. Cryptograms can be lifted to iso-cryptograms which render highest security for a given crypto-system. Isonumbers,

hypernumbers and their pseudo-formulations can be used effectively for the tightest security via new disciplines, *isocryptology*, *genocryptology*, *hypercryptology*, *pseudocryptology* etc. More complex cryptograms can be achieved using *pseudocryptograms* in which we have the additional hidden selection of addition and multiplication to the left and those to the right whose results are generally different among themselves. Yet more complex pseudocryptograms can be achieved in which the result of each individual operations of addition and multiplication is given by a set of numbers [50]. *Santillian iso-crypto systems* have maximum security due to a large variety of isounits which can be changed automatically and continuously, achieving maximum possible security needed for the modern age banking and other systems related with information technology.

Reformulations of conventional numbers to the most generalized isonumbers and subsequently to genonumbers and hypernumbers led to a vast variety of parallel developments in the conventional mathematics including hyperstructures [51] and its various branches such as 'iso-functional analysis' ref [35], iso-calculus ref [52], iso-cryptography [50] etc.

Iso-Galois fields [53], Iso-permutation groups [54, 53] have been defined by this author, which can play an important role in cryptography and other branches of mathematics where finite fields are used. Investigations are underway.

Isomathematics can also explain complex biological structures and hence has applications in Fractal geometry. Further applications in Neuroscience and Genetics can provide new insight in these disciplines.

References

- [1] R. M. Santilli, isonumbers and genonumbers of dimension 1,2,4,8, their isoduals and pseudoduals, and "hidden numbers" of dimension 3,5,6,7, Algebras, Groups and Geometries 10, 273-322 (1993).
- [2] R. H. Bruck, A Survey of Binary Systems, Springer Verlag, (1958).
- [3] E.Fermi, Nuclear Physics, Univ. of Chicago Press (1950).
- [4] W. R. Hamilton, Lectures on quaternions, Dublin, Ireland (1853).
- [5] R. M. Santilli, Hadronic J.1, 223(1978), Addendum 1, 1343 (1978).
- [6] R. M. Santilli, Lie-Admissible Approach to Hadronic Structure, Vol. I: NonApplicability of the Galilei and Einstein Relativities ?, Hadronic Press, Tarpon Springs, FL (1978)
- [7] R. M. Santilli, Lie-Admissible Approach to Hadronic Structure, Vol. II: Coverings of the Galilei and Einstein Relativities ?, (1982) and Vol. III: Identification of the Hadronic constituents with Physical Partcles ?, Hadronic Press, Tarpon Springs, FL.
- [8] R. M. Santilli, On a possible Lie-Admissible covering of the Galilei relativity in Newtonian mechanics fo conservative and Galilei form-noninvariant systems, Hadronic J. 1. 223-423 (1978) and Addendum, Hadronic J. I. 1279-1342 (1978).

- [9] R. M. Santilli, Foundations of Theoretical Mechanics, Vol.II: Birkhoffian Generalization of Hamiltonian Mechanics, Springer-Verlag, Heidelberg, New York (1982).
- [10] C. F. Gauss, Die 4 Beweise der Zeriengung ganzer algebraischer Funktionen (1799), Edited by E. Netto, Ostwalds Klassiker der exkten Wissenschaften n. 144.
- [11] N. H. Abel, Overes Completes, Christiania Publisher, (1881).
- [12] A. Cayley, Phyl. Magaz. and J. of Science 3, 210 (1845).
- [13] E. Galois, Overers Mathematiques, Gautier-Villars, Paris, (1897).
- [14] O. Haupt and Sengenhorst, Algebraic extension of a field, in Fundamentals of Mathematics, Vol.I, Edited by H. Behnke et al., MIT Press, Cambridge MA (1974).
- [15] A. Hurwitz, Uber die Composition der quadratischen Formen vonbeliebigvielen Variablen, Nach.kon.Gesell.der Wiss. Gottingen, 309-316 (1898).
- [16] A. A. Albert, Ann.Math. 43, 161, (1942).
- [17] N. Jacobson, Composition Algebras and their automorphism, Rendiconti Circolo Matematico Palermo 7, series II, 55 (1958), Modern Algebra, edited by A. A. Albert, Prentice Hall, New York (1963).
- [18] R. M. Santilli, Elements of Hadronic Mechanics, Vol. I and II, Ukraine Academy of Sciences, Kiev, second edition 1995 <http://www.santilli-foundation.org/docs/Santilli-300.pdf> <http://www.santilli-foundation.org/docs/Santilli-301.pdf>.
- [19] A. S. Muktibodh, Foundations of Isomathematics, American Institute of Physics, AIP Conference Proceedings, Vol. 1558, 707, (2013).
- [20] G. Pickert and H. Steiner, Complex numbers and quaternions, in Fundamentals of Mathematics, edited by Behnek et al., MIT Press (1974).
- [21] C. H. Curtis, The Four and Eight Square Problem and Division Algebras, in Studies in Modern Algebras, Edited by A. A. Albert, Prentice Hall (1963).
- [22] E. Kleinfeld, A characterization of the Cayley numbers, in Studies in Mathematics, Edited by A. A. Albert, Prentice Hall, New York (1963).
- [23] R. M. Santilli, Isotopies of contemporary mathematical structures, I; Isotopies of fields, vector spaces, transformation theory, Lie Algebras, analytic mechanics and space-time symmetries, Algebras, groups and Geometries 8, 169-266 (1991).
- [24] R. M. Santilli, Isotopies of contemporary mathematical structures, II; Isotopies of symplectic geometry, affine geometry, Riemannian geometry and Einstein gravitation, Algebras, Groups and Geometries, 8, 275-390 (1991).
- [25] R. M. Santilli, Hadronic Mathematics, Mechanics and Chemistry. Vol.III: Iso-, Geno-, Hyper-, Formulations for Matter and their isoduals for Antimatter. Palm Harbor, FL34682, U.S.A.; International Academic Press, (2008).
- [26] R. M. Santilli, Isodual Theory of Antimatter with Application to Antigravity, Grand Unification and Cosmology, Vol.151 of Fundamental Theories of Physics. 3300 A A Dordrecht, The Netherlands: Springer, (2006).
- [27] R. M. Santilli, Lie-Isotopic liftings of Lie symmetries. I. General Considerations, Hadronic J., Vol. 8, pp. 25-35, January, 1985.
- [28] R. M. Santilli, Lie-Isotopic liftings of Lie symmetries. II., Lifting of Rotations, Hadronic J. Vol.8, pp. 36-51, January,(1985).
- [29] R. M. Santilli, Foundations of Hadronic Mechanics, Hadronic J. Suppl. 4B,1-25(1989).
- [30] R. M. Santilli, Isodual spaces and antiparticles, Comm. Theor. Phys. Vol. 3 (1994).
- [31] R. M. Santilli, Isotopic Generalization of Galilei's and Einstein's Relativities, Vol. II, Classical isotopies, 1-st edition Hadronic Press, Palm Harbor, FL(1991): 2-nd edition Ukraine Academy of Sciences, Kiev.
- [32] J. V. Kadeisvili, Santilli's Lie-isotopic generalizations of Contemporary Algebras, Geometries and Relativities, 1-st edition (1992), Hadronic Press, Palm Harbor, FL, 2-nd Edition, Ukraine Academy of Sciences, Kiev.
- [33] R. M. Santilli, Non-local Formulation of the Bose-Einstein correlation within the context of hadronic mechanics, Hadronic J. 15, 1-99, (1992).
- [34] A. A. Bhalekar, Santilli's New Mathematics for Chemists and Biologists. An Introductory Account Hadronic J. (2013).
- [35] J. V. Kadeisvili, Elements of Functional isoanalysis, Algebras Groups and Geometries, 9, 283-318,(1992).
- [36] A. K. Aringazin, A. Jannussis, D.F. Lopez, M. Nishioka and B. Veljanoski, Santilli's Lie-isotopic Generalization of Galilei's and Einstein's Relativities, Kostarakis Publisher, Athens, Greece (1991).
- [37] R. M. Santilli, Nonlinear, nonlocal, noncanonical axiom-preserving isotopies, Q-operator deformations of Lie symmetries, Proceedings of third international Wigner Symposium, Oxford University, Sept. (1993).
- [38] R. M. Santilli, Lie-isotopic lifting of the special relativity for extended-deformable particles, Lettere Nuovo Cimento 37, 545-555 (1983).
- [39] R. M. Santilli, Isotopic Generalization of Galilei's and Einstein's Relativities, Vol. I, Mathematical Foundations, 1-st edition Hadronic Press, Palm Harbor, FL (1991): 2-nd edition Ukraine Academy of Sciences, Kiev.
- [40] J. V. Kadeisvili, M. Kamiya and R. M. Santilli, A characterization of isofields and their isoduals, Algebras Groups and Geometries, Vol 10, 168-185(1993).
- [41] R. M. Santilli, Contributed paper to the proceedings, Third International Wigner Symposium, Oxford University(1993), and JINR Rapid Comm. 6, 24 (1993).
- [42] Klimyk, R. M. Santilli, Algebras, Groups and Geometries 10, 323 (1993).
- [43] R. M. Santilli, Algebras, Groups and Geometries, 10, 273 (1993).
- [44] A. S. Muktibodh, Iso-Galois fields, Hadronic J., Vol. 36 (2), 225 (2013).
- [45] D. S. Surlas and G. T. Tsagas, Mathematical foundations of Lie-Santilli theory, Ukraine Academy of Sciences, Kiev.

- [46] R. M. Santilli, *Hadronic Mathematics, Physics and Chemistry*, Vols. I [4a], II[4b], III[4c], IV[4d] and V[4c], New York: International Academic Press.
- [47] H. Davenport, *Aritmetica superiore, Un'introduzione alla teoria dei numeri*, Bologna: Zanichelli, (1994).
- [48] R. M. Santilli, *Circolo matematico di Palermo*, (Special issue on Santilli isotopies) Supp., Vol. 42 (1996).
- [49] C. X. Jiang, *Fundamentals of the Theory of Santillian numbers*, International Academic Press, America-Europe-Asia (2002).
- [50] D. E. R. Denning, *Cryptography and Data Security*, Addison Wesley, Reading M A (1983).
- [51] T. Vougiouklis, *Hyperstructures and their Representations*, Hadronic Press, Palm Harbor, FL (1994).
- [52] S. Georiev, J. Kadeisville, *Foundations of Iso-Differential Calculus*, Vol.I and Vol.2 (2014).
- [53] A. S. Muktibodh, *Iso-Galois Field and Isopermutation Group*, *Hadronic J.* 13, Vol. 38, I (2015).
- [54] A. S. Muktibodh, *Isopermutation Group, Clifford Analysis Clifford Algebra and Applications*, Vol 3, No. 3 (2014).

Santilli Synthesis of the Neutron According to Hadronic Mechanics

Chandrakant S. Burande

Vilasrao Deshmukh College Engineering and Technology, Mouda, Dist. Nagpur, India

Email address:

csburande@gmail.com

To cite this article:

Chandrakant S. Burande. Santilli Synthesis of the Neutron According to Hadronic Mechanics. *American Journal of Modern Physics*. Special Issue: Issue II: Foundations of Hadronic Mechanics. Vol. 5, No. 2-1, 2016, pp. 37-45. doi: 10.11648/j.ajmp.2016050201.13

Received: July 17, 2016; **Accepted:** July 18, 2016; **Published:** May 18, 2016

Abstract: In 1920 H. Rutherford conjectured that neutron is a compressed hydrogen atom in the core of the stars. W. Pauli noted that such synthesis of neutron violates the conservation of the angular momentum. Therefore, E. Fermi proposed emission of massless particle, called "neutrino". However, R.M. Santilli more recently noted that, even though the angular angular momentum would be conserved, the neutrino hypothesis does not allow non-relativistic quantum mechanics to be valid because the rest energy of the neutron is bigger than the sum of the rest energies of the proton and electron, under these conditions Schrodinger equation becomes inconsistent. Similarly, Santilli showed that relativistic quantum mechanics is also inapplicable (rather than violated) because, even though exactly valid for the electron at large distance from the proton in the hydrogen atom, the celebrated Dirac's equation is clearly inapplicable for the representation of electron when immersed inside the proton. In this paper, we study Santilli's decades of mathematical, theoretical and experimental research, first for the construction of the covering hadronic mechanics, and then the resulting numerically exact and time invariant representation at the non-relativistic and relativistic levels of "all" characteristics of the neutron in its synthesis from a proton and an electron. In particular, we show that, within said covering context, the representation of proton as an extended particle implies the existence of an orbital angular momentum of the electron within the hyperdense proton which is totally non-existence for quantum mechanics, under which the total angular momentum is conserved without any need for the conjectural neutrino. We finally study Santilli's suggestive hypothesis of the "etherino" as a longitudinal impulse (rather than particle) from the ether as a universal substratum that delivers missing energy for the synthesis of the neutron.

Keywords: Neutron, Binding Energy, Isoelectron, Hulthen Potential, Lie-Santilli Isoalgebras

1. Introduction

In 1920, Rutherford [1] submitted the hypothesis that hydrogen atoms in the core of stars are compressed into new neutral particles having the size of the proton that he called neutrons (**Figure 1**), according to the synthesis

$$p^+ + e^- \rightarrow n. \tag{1}$$

The existence of the neutron was confirmed in 1932 by Chadwick [2]. However, Pauli [3] noted that the spin 1/2 of the neutron cannot be represented via a quantum state of proton and electron, each having spin 1/2. Fermi [4] adopted Pauli's objection and, he then developed the theory of weak interactions according to which the synthesis of the neutron is characterized by either the emission of a neutral and

massless particle, named neutrino (ν) or absorption of antineutrino ($\bar{\nu}$). The particle reactions as per proposed theory of weak interaction are given by

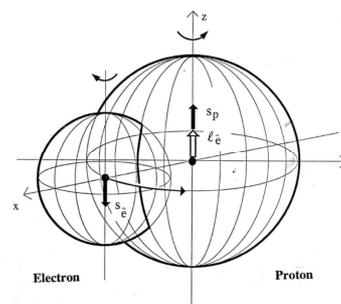


Figure 1. A schematic view of the the deep overlapping of the wavepackets of electrons into hyperdense medium of proton with singlet coupling.

$$p^+ + e^- \rightarrow n + \nu, \quad \text{or} \quad (2)$$

$$p^+ + e^- + \bar{\nu} \rightarrow n. \quad (3)$$

However, Santilli [5-7] has dismissed the Fermi's version of synthesis of neutron on following grounds:

1. the sum of the rest energies of the proton and of the electron,

$$\begin{aligned} m_p + m_e &= 938.272 \text{ MeV} + 0.511 \text{ MeV} \\ &= 938.783 \text{ MeV} \end{aligned} \quad (4)$$

is smaller than the rest energy of the neutron,

$$m_n = 939.565 \text{ MeV} \quad (5)$$

with positive energy (binding energy) difference of 0.78 MeV,

2. Schrödinger equation does not admit positive binding energy for quantum bound states when electron totally immersed within the hyper-dense medium inside the proton structure,
3. classical theory of antimatter requires that the anti-neutrino has a negative energy, although, eq.(3) is needed positive energy to supply the missing energy, 0.78 MeV,
4. neither, antineutrino can deliver the 0.78 MeV needed for the neutron synthesis because the cross section of former with electron or proton is null, and
5. the proton and the electron are the only experimentally discovered stable massive particles. Hence, emission of neutrino in neutron formation does not have any relevance. Moreover, it cannot be directly detected.

2. Hadronic Energy

The only bound state of a proton and an electron predicted by quantum mechanics is the hydrogen atom, with smallest orbit (Bohr's orbit) of the order of 10^{-8} cm. Santilli's hadronic mechanics has identified the existence of an additional bound state when the electron orbits within the proton structure at distances of the order of 10^{-13} cm or less. The difference between these two bound states is depicted in "Figure 2".

Remarkably, Santilli has proved that the hadronic state is one and only one, the neutron, and its first excited state is the Hydrogen atom which is formed when the electron leaves the proton structure, thus recovering all conventional quantum states. In this sense, the energy levels of the hydrogen atom are the excited states of the neutron.

The mutual overlapping of the charge distribution or wavepackets of electron and proton leads to new interactions of contact type. However, it is not possible via conventional quantum mechanics to represent these new interactions for various reasons, such as:

1. quantum mechanics can only represent particles as dimensionless point masses; quantum mechanics has a local-differential structure ruling out any consistent

2. quantum mechanics can only represent interactions derivable from a local potential, while contact interactions of the type required to be considered herein can be represented with anything except a potential or a Hamiltonian.

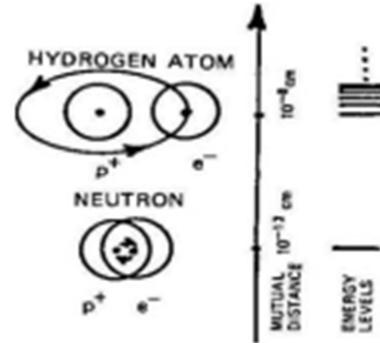


Figure 2. A schematic comparison of Bohr's orbit and hadronic structure identified by Santilli.

In this event, Santilli's isomechanics is ideally suited for a quantitative study of the neutron synthesis because, in addition to all interactions characterizing the hydrogen atom, it allows the new interactions caused by deep mutual penetration of the constituents. This method has been used by Santilli in numerous applications. Santilli [5-8] obtained an isoequation for the neutron by isotopically lifting of Schrödinger equation introducing additional potential term of Coulomb nature that reads as,

$$\begin{aligned} &\left[\frac{1}{m} \hat{p} \times \hat{T} \times \hat{p} \times \hat{T} - \frac{ze^2}{r} + \frac{e^2}{r} \hat{T} \right] \times \hat{\psi}(r) \\ &= E \times \hat{\psi}(r) \end{aligned} \quad (6)$$

with isounit,

$$\hat{I} = U \times I \times U^\dagger = 1 / \hat{T} > 0. \quad (7)$$

The suitable isounit to represent the two particle penetration (now termed as an isoelectron), is defined as, with isounit,

$$\begin{aligned} \hat{I} &= \text{Diag} \left(n_1^2(1), n_2^2(1), n_3^2(1), n_4^2(1) \right) \\ &\times \text{Diag} \left(n_1^2(2), n_2^2(2), n_3^2(2), n_4^2(2) \right) \\ &\times \exp \left((\psi / \hat{\psi}) \times \int \hat{\psi}(r)_{1\downarrow}^2 \times \hat{\psi}(r)_{2\uparrow} dr \right) \end{aligned} \quad (8)$$

where the two diagonal matrices represent the shapes (assumed to be spheroids) and the densities of the particles considered, while the last term represents the non-Hamiltonian interactions. For spherical point-like charge particle, such as electrons, the diagonal matrices get reduced to 1. Next, the evaluation of the volume integral into a constant,

$$N = \int \hat{\psi}(r)_{1\downarrow}^2 \times \hat{\psi}(r)_{2\uparrow} dr, \quad (9)$$

and the expansion of the isoexponent up to the second term, yields,

$$\hat{I} \approx e^{N \times \psi / \hat{\psi}} \approx 1 + N \times \psi / \hat{\psi} \quad (10)$$

$$\hat{T} \approx e^{-N \times \psi / \hat{\psi}} \approx 1 - N \times \psi / \hat{\psi} \quad (11)$$

$$|\hat{I}| \gg 1, \quad |\hat{T}| \ll 1, \quad \lim_{r \rightarrow 1, fm} \hat{I} = 1. \quad (12)$$

In above equations ψ and $\hat{\psi}$ behave respectively as

$$\psi \approx P \times e^{-br} \quad (13)$$

$$\hat{\psi} \approx Q \times (1 - e^{-br}) / r \quad (14)$$

where P and Q are constants and b is inverse of hadronic horizon, r_h . Using eqs.(13) and (14), the isotopic element depicted in eq.(11) reads as

$$\hat{T} \approx 1 - N \times \psi / \hat{\psi} = 1 - r \times V_0 \frac{e^{-br}}{(1 - e^{-br})}, \quad (15)$$

where $V_0 = NP / Q$. Now, by introducing Hulthen potential,

$$V_{Hulthen} = V_0 \frac{e^{-br}}{1 - e^{-br}} \quad (16)$$

where V_0 is the Hulthen's constant, the isotopic element can be written as

$$\hat{T} \approx 1 - N \times \psi / \hat{\psi} = 1 - r \times V_{Hulthen}. \quad (17)$$

Further, at small distances, the Hulthen potential behaves like Coulomb potential,

$$V_{Hulthen} \approx \frac{V_0}{b \times r}. \quad (18)$$

which is very strong as the quantity b in the denominator is of the order of 10^{-13} cm, thus resulting the multiplicative factor of the order of 10^{13} . As a result, inside the hadronic horizon, the Coulomb potential is absorbed by the Hulthen potential, thus we can write

$$\begin{aligned} & \frac{e^2}{r} \times \hat{T} - \frac{ze^2}{r} \\ & \approx \frac{e^2}{r} \times (1 - r \times V_{Hulthen}) - \frac{ze^2}{r} \\ & = -V \times \frac{e^{-br}}{(1 - e^{-br})} \end{aligned} \quad (19)$$

where $z = 1$ and $V = e^2 V_0$.

Using eqs.(6) and (19), Santilli obtained the nonrelativistic radial isoequation of the hadronic two-body structure model that reads as

$$\begin{aligned} & \left[\frac{1}{r^2} \left(\frac{d}{dr} r^2 \frac{d}{dr} \right) \right] \times \hat{\psi}(r) \\ & + \left[\frac{m}{\rho^2 \hbar^2} \left(E_{hb} + V \frac{e^{-br}}{1 - e^{-br}} \right) \right] \times \hat{\psi}(r) = 0 \end{aligned} \quad (20)$$

where E_{hb} is hadronic binding energy. Assuming the change of variable, $x = 1 - e^{-br}$, eq.(20) can be written as

$$\begin{aligned} & \left[x(1-x) \frac{d^2}{dx^2} \right] S(x) \\ & - \left[(2|A|^{1/2} + 1) \frac{d}{dx} + \beta^2 \right] S(x) = 0 \end{aligned} \quad (21)$$

where

$$\begin{aligned} k_2 &= \frac{mV_0}{\hbar^2 \rho^2 b^2}, \\ A &= \frac{m}{\hbar^2 \rho^2 b^2} E_b < 0, \end{aligned} \quad (22)$$

The solution of eq.(21) is then given by

$$\begin{aligned} G_n(x) &= \sum_{k=1}^{k=n} \binom{n-1}{k-1} \binom{n+k+2|A|^{1/2}-1}{k} x^k \end{aligned} \quad (23)$$

with isonormalized isoeigenfunction

$$\begin{aligned} \hat{\psi}(r) &= \left[\frac{\Gamma(2|A|^{1/2} + 3)}{\Gamma(3)\Gamma(2|A|^{1/2})} \right]^{1/2} \\ & \times \frac{1 - e^{-br}}{r} e^{-|A|^{1/2} br} \end{aligned} \quad (24)$$

the expression for hadronic binding energy is then obtained as

$$|E_{hb}| = E^{Bind} = \frac{V_0}{4k_2} \left[\frac{k_2}{n} - n \right]^2. \quad (25)$$

The boundary conditions demand that $k_2 > n$. This indicates the finite value of eigenvalues for Hulthen potential. This is in concurrence with the hadronic bound state. Further, for an isoparticle to be bounded inside the hadronic horizon b^{-1} , its wavelength, λ must be proportional to the horizon itself, and we shall write

$$\lambda = \frac{1}{2\pi k_1 b} \quad (26)$$

where k_1 is a positive quantity that must be constant for a stationary state. Next the hadronic kinetic energy E_{hk} is given by

$$E_{hk} = \frac{\hat{p}^2}{2m} \approx \frac{\hbar^2 \rho^2 b^2}{2m}. \quad (27)$$

Now, from eqs.(22) and (27), we have

$$V_0 = 2k_2 E_{hk}. \quad (28)$$

Hence, the hadronic total energy of the hadronic bound state is given by

$$E_{ht} = 2E_{hr} + 2E_{hk} - E_{hb} \\ \approx 2k_1 \left[1 - (k_2 - 1)^2 \right] \hbar b c_0. \quad (29)$$

where c_0 is the speed of light in vacuum, and note that the last approximation holds for hadronic bound states where the rest energy is insignificant with respect to the kinetic energy. Thus, at this point we obtained the expression for the total energy of the two-body hadronic bound state which is depend on two unknown quantities, k_1 and k_2 . To achieve a numerical solution, we now introduce second expression, the meanlife, τ of the unstable hadron

$$\tau^{-1} = \lambda^2 \left| \hat{\psi}(0) \right|^2 \frac{\alpha^2 E_{hk}}{\pi \hbar}, \quad (30)$$

where α is the fine structure constant. By using the above expressions, we can write

$$\hat{\psi}(0) \left[\frac{1}{2} \frac{(k_2 - 1) \Gamma \left[\frac{1}{2} (k_2 - 1) + 2 \right]}{3! \Gamma \left[\frac{1}{2} (k_2 - 1) + 2 \right]} \right]^{1/2} \\ \times b = \frac{(k_2 - 1)^{3/2}}{(48)^{1/2}} b \quad (31)$$

The meanlife of the hadronic bound state then becomes

$$\tau^{-1} = \frac{4\pi}{48(137)^2} \frac{(k_2 - 1)^3}{k_1} b c_0 \quad (32)$$

Thus, we obtained a system of two equations in terms of two unknown quantities k_1 and k_2 , total rest energy, E_{tot} , the meanlife, τ and the charge radius, R_c of the two-body hadronic bound state, that it is reproduced identically below:

$$k_1 \left[1 - (k_2 - 1)^2 \right] = \frac{E_{ht}}{2\hbar b c_0} \quad (33)$$

$$\frac{(k_2 - 1)^3}{k_1} = \frac{48(137)^2}{4\pi b c_0} \tau^{-1}. \quad (34)$$

On substituting $b = 10^{-13}$, $\tau^{-1} = 10^{-3}$ and $E_{ht} = 939$ in eqs.(33) and (34), we extract

$$k_1 = 2.6, \quad k_2 = 1 + 0.81 \times 10^{-8} \approx 1. \quad (35)$$

For admissible state, $n = 1$, we further have

$$\frac{k_2}{n} - n \approx 0, \quad (36)$$

$$E_{hb} = -\frac{V_0}{4k_2} \times \left(\frac{k_2}{n} - n \right) \approx 0 \quad (37)$$

Thus, this proves that the in nonrelativistic approximation the hadronic binding energy is insignificant. Further, the numerical value of the hadronic kinetic energy is obtained as

$$E_{hk} \approx k_1 \hbar b c_0 = 6.63 \times 10^{-23} \text{ MeV} \approx 0 \quad (38)$$

which is also insignificant. The reason for being very small hadronic binding energy and ignorable in first approximation is due to the fact that contact resistive forces have no potential energy. The main physical origin of hadronic structure is the contact, zero-range, interaction due to the complete immersion of one wavepacket within the other.

Finally, Santilli arrives at the following result namely the total hadronic energy of the neutron is primarily characterized by the rest energy of the proton and the isonormalized rest energy of the isoelectron,

$$E_n \approx E_p + E_{hr,e} = E_p + \frac{m_e c_0^2}{\rho^2} \\ = 938.272 + 1.293 = 939.565 \text{ MeV} \quad (39)$$

where $\rho^2 = 0.3952$ is a geometrization of the departure of the interior of hadrons from our space-time. Since the proton is not mutated in this first approximation as per our assumption, have

$$b_1 = b_2 = b_3 = 1, \quad (40)$$

$$\rho^2 = n_4^2 = b_4^{-2} = \frac{0.511}{1.293} = 0.3952 \quad (41)$$

$$\rho = n_4 = b_4^{-1} = 0.6286. \quad (42)$$

Notice that the above value for the characterization of the density of the neutron coincides with the experimental value of the density of the fireball of the Bose-Einstein correlation.

3. The Neutron Spin

The conceptual interpretation of the observed spin $1/2$ of the neutron, for the first, was successfully explained by Santilli as follows. Considering the initiation of Rutherford's process of compression of the isoelectron within the proton in singlet coupling, it is evident that, as soon as the penetration begins, the isoelectron is trapped inside the hyperdense medium inside the proton, thus resulting in a constrained orbiting motion of the isoelectron that must superpose on the proton spin (FIGURE 2). Santilli stresses that the proton is not mutated because it is 2000 times heavier than the electron, and that the coupling must be in singlet for stability. This implies

that, for the case of the neutron structure, the spin of the electron is also not mutated. However, the angular momentum of electron is mutated inside the hadronic sphere. The needed mutation of the quantum into the hadronic angular momentum is trivially given by the nonunitary transforms

$$U \times U^\dagger = \hat{I} = \frac{1}{2}, \quad \hat{T} = 2, \quad (43)$$

The mutation is supported by the isotopic invariance of the Hilbert space. Nonunitary lifting of angular momentum, in this case, reads

$$\begin{aligned} &\langle l, m | \times L_3 \times | l, m \rangle \times I \\ &\rightarrow U \times [\langle l, m | \times L_3 \times | l, m \rangle] \times U^\dagger \\ &= \langle \hat{l}, \hat{m} | 2 \times \hat{L}_3 \times 2 | \hat{l}, \hat{m} \rangle \times \frac{1}{2}, \end{aligned} \quad (44)$$

In order to represent the spin of neutron Santilli (1990) used irregular isorepresentations of Lie-Santilli isoalgebras [9-11], namely, isorepresentations characterized by nonunitary isounitary transforms for the generators different than those for the product. This difference is rather natural for the structure of the neutron, since the basic nonunitary transform for the rest energy has already been selected for calculation of binding energy. This irregular isopresentation of $\hat{SO}(3)$ based on the the isodifferential calculus and isolinear momentum is given below [9-11]:

$$[\hat{r}_i, \hat{r}_j] = [\hat{p}_i, \hat{p}_j] = 0, \quad (45)$$

$$[\hat{r}_i, \hat{p}_j] = \hat{\delta}_{ij} = \hat{I} \times \delta_{ij} = \rho \delta_{ij} \quad (46)$$

$$\hat{L}^2 \times \hat{Y}_{\hat{l}\hat{m}}(\hat{\theta}, \hat{\phi}) = \rho^2 \times \hat{l}(\hat{l}+1) \hat{Y}_{\hat{l}\hat{m}}(\hat{\theta}, \hat{\phi}), \quad (47)$$

$$\hat{L}_3 \times \hat{Y}_{\hat{l}\hat{m}}(\hat{\theta}, \hat{\phi}) = \rho \times \hat{m} \times \hat{Y}_{\hat{l}\hat{m}}(\hat{\theta}, \hat{\phi}), \quad (48)$$

$$\hat{l} = 1, 2, 3, \dots \quad \hat{m} = \hat{l}, \hat{l}-1, \dots, -\hat{l}. \quad (49)$$

Notice that the isotopic lifting of the integer value of the angular momentum, $l = 1, 2, 3, \dots$ into the value $\rho \times \hat{l}$, where, again, $\hat{l} = 1, 2, 3, \dots$, the value $\hat{l} = 0$ being excluded by boundary conditions, ρ being a variable depending on the local conditions. For the study of the neutron spin on the line of hadronic mechanics, Santilli selected the following two-dimensional irregular isorepresentation of $\widehat{SU}(2)$:

$$\hat{I} = \begin{pmatrix} g_{11} & 0 \\ 0 & g_{22} \end{pmatrix}, \quad \hat{T} = \begin{pmatrix} g_{11}^{-1} & 0 \\ 0 & g_{22}^{-1} \end{pmatrix} \quad (50)$$

$$\begin{aligned} \hat{J}_1 &= \frac{1}{2} \begin{pmatrix} 0 & g_{11}^{-1/2} \\ g_{22}^{-1/2} & 0 \end{pmatrix}, \\ \hat{J}_2 &= \frac{1}{2} \begin{pmatrix} 0 & -ig_{11}^{-1/2} \\ ig_{22}^{-1/2} & 0 \end{pmatrix} \end{aligned} \quad (51)$$

$$\begin{aligned} [\hat{J}_1, \hat{J}_2] &= i\hat{J}_3 \\ &= \frac{1}{2} \times \frac{\Delta^{1/2}}{2} \begin{pmatrix} g_{11}^{-1} & 0 \\ 0 & g_{22}^{-1} \end{pmatrix} \end{aligned} \quad (52)$$

$$\hat{J}_3 \times | \hat{j}, \hat{s} \rangle = \hat{J}_3 \times \hat{T} | \hat{j}, \hat{s} \rangle = \pm \frac{\Delta}{2} | \hat{j}, \hat{s} \rangle \quad (53)$$

In this case, Santilli [9-11] has selected the two-dimensional irregular isorepresentation of $\widehat{SU}(2)$ and then computed the total angular momentum of the neutron model, $n = (p^+, \hat{e}^-)_{hm}$ as,

$$J_n = J_p + \hat{L}_e^{orb} + \hat{J}_e^{int} = \frac{1}{2} + \rho - \frac{\Delta}{2} \quad (54)$$

resulting in the values anticipated above, namely:

$$\rho = \frac{1}{2}, \quad \Delta = 1. \quad (55)$$

It shows that the spin of the isoelectron is not mutated and the angular momentum is mutated in such a way that the isoelectron is merely carried out by the proton spin.

4. The Neutron Magnetic Moment

In view of the hadronic orbiting motion of isoelectron, the magnetic moment of the neutron was generated by Santilli by considering the following three contributions,

$$\mu_n = \mu_p^{int} - \mu_e^{orb} + \mu_e^{int} \quad (56)$$

The observed values of magnetic moment of neutron and proton are respectively,

$$\mu_n = -1.9 \times \frac{e}{2m_p c_0}, \quad \mu_p = 2.7 \times \frac{e}{2m_p c_0}. \quad (57)$$

Now, on rearranging magnetic moment of neutron as

$$\begin{aligned} \mu_n &= -1.9 \times \frac{e}{2m_p c_0} \\ &= 2.7 \times \frac{e}{2m_p c_0} - 4.6 \times \frac{e}{2m_p c_0}, \end{aligned} \quad (58)$$

and comparing with eq.(56), we obtain following identity:

$$-\mu_e^{orbital} + \mu_e^{intrinsic} = -4.6 \times \frac{e}{2m_p c_0}. \quad (59)$$

This is equivalent to

$$\begin{aligned} -\mu_e^{orbital} + \mu_e^{intrinsic} &= \frac{-2.5 \times 10^{-3} e}{2m_e c_0} \\ &= -2.5 \times 10^{-3} \mu_e. \end{aligned} \quad (60)$$

From eq.(60), Santilli derived the desired value of, $\mu_e^{orbital}$, that is

$$\mu_e^{orbital} = (1 + 2.5 \times 10^{-3}) \times \mu_e. \quad (61)$$

The small value of the total magnetic moment of the isoelectron is fully compatible with the null value of its total angular momentum.

5. Santilli Aetherino Hypothesis

Santilli replaces the neutrino as a physical particle in our space-time with a longitudinal impulse originated by the ether as a universal substratum that he calls "etherino" [12]. In this view, all physical quantities missing in the neutron synthesis, such as energy and spin, are delivered by said impulse.

A particular motivation for the etherino hypothesis is due to the evident difficulties in accepting that neutrino now believed to have mass could traverse entire planets and stars without appreciable scattering. By contrast, this difficulty is resolved by the propagation of a longitudinal impulse in the universal substratum because it would underlie matter.

Additionally, the replacement of the neutrino with the etherino appears to preserve the experimental evidence in the field because what is today detected and interpreted as a "neutrino scattering" could in reality be due to the scattering of the longitudinal impulse with targets.

Hence, the etherino hypothesis appears to resolve some of the insufficiencies of the neutrino conjecture, may eventually resulting to be fully compatible with available experimental data, and is already stimulating rather intriguing research on superluminal communications, that are the only possible for interstellar contact [12] due to evident insufficiencies of electromagnetic waves for galactic distances.

6. Don Borghi Experiment on the Synthesis of Neutrons

The first experiment on the synthesis of neutrons from protons and electrons was conducted by Carlo Borghi, C. Giori and A. Dall'Olio in the 1960 at the CEN Laboratories in Recife, Brazil [13], [14]. Hydrogen gas at fraction of 1 bar pressure was obtained from the electrolytical separation of water and was placed in the interior of a cylindrical metal chamber (called klystron) and kept mostly ionized by an electric arc with about 500 V and 10 mA. Additionally, the gas was traversed by microwaves with 10s frequency. Suitable materials which are vulnerable to nuclear transmutation when exposed to a neutron flux, were placed exterior of the chamber. Following exposures of the order of days or weeks, the

experimentalists reported nuclear transmutations that were based on the observed neutron counts of up to 104 cps. Don Borghi experiment has been strongly criticized by academia on pure theoretical grounds without the actual repetition of the tests. Note that experiment makes no claim of direct detection of neutrons, and only claims the detection of clear nuclear transmutations.

To verify the claim of Don Borghi's experiment, Santilli repeated this experiment in large number of laboratories and institutions the world over.

7. Santilli Experiment on the Synthesis of Neutrons

Santilli conceived his experiment [15], [16] as being solely based on the use of an electric arc within a cold (i.e., at atmospheric temperature) hydrogen gas without any use of microwave at all. Three different klystrons were manufactured, tested and used for the measurements. The specifications of detectors were used for measurements are given below:

1. A detector model PM1703GN manufactured by Polimaster, Inc., with sonic and vibration alarms as well as memory for printouts, with the photon channel activated by CsI and the neutron channel activated by LiI.
2. A photon-neutron detector SAM 935 manufactured by Berkeley Nucleonics, Inc., with the photon channel activated by NaI and the neutron channel activated by He-3 also equipped with sonic alarm and memory for printouts of all counts. This detector was used to verify the counts from the preceding one.
3. A BF3 activated neutron detector model 12-4 manufactured by Ludlum Measurements, Inc., without counts memory for printouts. This detector was used to verify the counts by the preceding two detectors.

Electric arcs were powered by welders manufactured by Miller Electric, Inc., including a Syncrowave 300, a Dynasty 200, and a Dynasty 700 capable of delivering an arc in DC or AC mode, the latter having frequencies variable from 20 to 400 Hz.

Klystron-I was cylindrical and sealed, of about 6" outside diameter and 12" height, made of commercially available, transparent, PolyVinyl Chloride (PVC) housing along its symmetry axis a pair of tungsten electrodes. The electrodes gap was controllable by sliding the top conducting rod through the seal of the flange. The klystron cylindrical wall was transparent so as to allow a visual detection of arc. After initiation of DC arc there was no detection for hours. However, shaking of klystron the neutrons were detected in a systematic and repetitive way. The detection was triggered by a neutron-type particle, excluding contributions from photons. However, these detections were anomalous, that is, they did not appear to be due to a flux of actual neutrons originating from the klystron. This anomaly is established by the repeated "delayed detections," that is, exposure of the detector to the

klystron with no counts of any type, moving the detector away from the klystron (at times for miles), then seeing the detectors enter into off-scale vibrations and sonic alarms with zero photon counts.

Klystron-II was a rectangular, transparent, made up of PVC of dimension. This klystron was small in size than earlier one to avoid implosion caused by combustion with atmospheric oxygen. This test was conducted only once because of instantaneous off-scale detection of neutrons by all detectors which led to evacuation of the laboratory. Hence, this test was not repeated for safety.

Klystron-III was cylindrical made up of carbon steel pipe with 12" outer diameter, 0.5" wall thickness, 24" length and 3" thick end flanges to sustain hydrogen pressure up to 500 psi with the internal arc between throated tungsten electrodes controlled by outside mechanisms. This test was conceived for the conduction of the test at bigger hydrogen pressure compared to that of Klystron I. The test was conducted only once at 300 psi hydrogen pressures because of instantaneous, off-scale, neutron detections such to cause another evacuation of the laboratory.

The main purpose of Santilli's of conducting these tests was to establish the production of neutron-type particles via a DC arc within a hydrogen gas. He has experimented identical tests with other gases, but no meaningful counts were detected other than hydrogen. No neutron, photon or other radiation was measured from electric arcs submerged within liquids. Hence, the reported findings appear to be specific for electric arcs within a hydrogen gas under the conditions stated above.

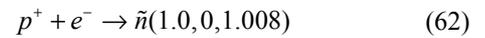
8. The Don Borghi-Santilli Neutroid

Santilli [5,15] excludes that the entities produced in the tests with Klystron I are true neutrons for various reasons, such as:

1. The anomalous behavior of the detector, in the case of the 15 minute delay, namely the self-activated detection indicates first the absorption of "entities" producing nuclear transmutations that, in turn release ordinary neutrons.
2. The environment inside stars can indeed provide the missing energy of 0.78 MeV for the neutron synthesis, but the environment inside Klystron-I cannot do the same due to the very low density of the hydrogen gas.
3. The physical laws of hadronic mechanics do not allow the synthesis of the neutron under the conditions of Klystron-I because of the need of the trigger, namely, an external event permitting the transition from quantum to hadronic conditions. In fact, the tests with Klystrons-II and III do admit the trigger required by hadronic mechanics. However, Santilli did not discard that the "entities" produced in the tests with Klystrons-II and III are indeed actual neutrons, due to the instantaneous, off-scale nature of the neutron alarms in clear absence of photon or vibrations.

In view of above reasons, Don Borghi [13], [14] submitted the hypothesis that the "entities" are neutron-type particles

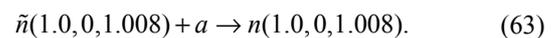
called "neutroids". Santilli adopted this hypothesis and presented the first technical characterization of neutroids with the symbol, \tilde{n} and the characteristics in conventional nuclear units, $A = 1, Z = 0, J = 0, amu = 0.008$. Hence, Santilli assumed that in Klystron-I, he produced the following reaction precisely along Rutherford's original conception



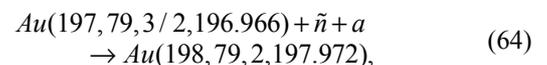
where the value $J = 0$ is used for the primary purpose of avoiding the spin anomaly in the neutron synthesis as indicated above and the rest energy of the neutroids is assumed as being that of the hydrogen atom.

9. Interpretation of Don Borghi and Santilli Experiments

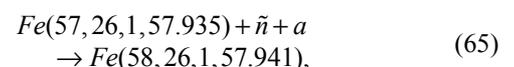
In Don Borghi's and Santilli's experiments the various substances placed in the exterior of the klystrons did indeed experience nuclear transmutations. If we discard the Don Borghi's klystron and Santilli's Klystron-I to produce actual neutrons, then the main question arises from where the neutrons originated and detected. Evidently, only two possibilities remain, namely, that the detected neutrons were actually synthesized in the walls of the klystrons, or by the activated substances themselves following the absorption of the neutroids produced by the klystrons. Considering the neutrino hypothesis has no sense for the neutron synthesis for various reasons, Santilli [5, 15] assumes that the energy, spin and magnetic anomalies in the neutron synthesis are accounted for by their transfer either from nuclei or from the aether via his etherino hypothesis



Assuming the binding energy of a neutroid is similar to that of an ordinary nucleon (since neutroids are assumed to be converted into neutrons when inside nuclei, or to decompose into protons and electrons, thus recovering again the nucleon binding energy), Santilli indicates the following possible nuclear reaction for one of the activated substances in Don Borghi's tests



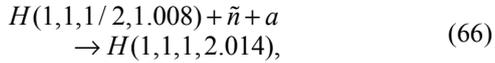
produces known nuclide, hence it indicates that neutrons were synthesized by the activating substances themselves on absorption of neutroid. The nuclear reaction with steel wall of the klystron,



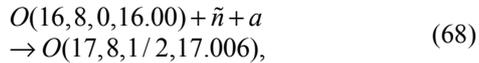
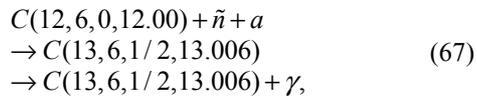
yields an unknown nuclide, $Fe(58, 26, 1, 57.941)$ because the known nuclide is $Fe(58, 26, 0, 57.933)$. This indicates that the neutrons in Don Borghi experiment were not

synthesized in the walls of his klystron. Eq.(2) also allow an interpretation of some of Santilli detections [5], [15], with the understanding that the anomalous behavior of the detectors, such as the delayed neutron counts, requires special studies and perhaps the existence of some additional event not clearly manifested in Don Borghi's tests.

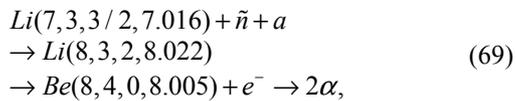
To initiate the study, Santilli considered the first possible reaction inside the klystron



delivers ordinary deuteron on coupling of hydrogen atom and neutroid. This indicates neutrons cannot be originated inside the klystron-I. Next, Santilli considered following nuclear reactions with the polycarbonate of Klystron-I wall containing about 75 percent carbon and 18.9 percent oxygen



do not give conventional activation processes. Thus, in Santilli's experiment too, it does not appear that the detected neutrons are synthesized by the walls of klystron. The above analysis leads us to the only remaining possibility that in Santilli tests, the neutrons are synthesized by the detectors themselves. To study this possibility, Santilli considered the reaction using Li-activated detectors,



that behaves fully equivalent to detection of neutrinos or neutrons. This indicated that neutrons detected in Santilli experiment were synthesized by the substance used for detection after absorption of neutrinos.

10. Concluding Remarks

It is observed that Santilli's discovery of hadronic mechanics appropriately represents, at both non-relativistic and relativistic levels, "all" characteristics of neutron according to Rutherford's conjecture of its synthesis from hydrogen atom in the core of a star. A first implication of the studies is that the orbital motion of the electron within the hyperdense proton allows the conservation of the total angular momentum without any need for the conjecture of the hypothetical neutrino. Another important implication is the dismissal of quarks as the actual physical constituents of the neutron since the proton and the electron cannot "disappear" at the time of the neutron synthesis to be replaced by the hypothetical quarks, and then "reappear" at the time of the neutron decay. We show that, besides the above mathematical and theoretical studies, Santilli has provided numerous

experimental verification of the laboratory synthesis of the neutron from a hydrogen gas in support of Rutherford's historical hypothesis.

Acknowledgments

The author is highly grateful to Professor R. M. Santilli, his team and Professor Anderson for valuable guidance in preparing and for suggestions to improve this paper. The financial support from the R. M. Santilli Foundation is gratefully acknowledged. The author is also highly indebted to Professor A. A. Bhalekar for suggesting me this topic and helping me in writing this paper by his valuable advices and suggestions in its overall improvement.

References

- [1] H. Rutherford, *Proc. Roy. Soc. A*, 97, 374 (1920).
- [2] J. Chadwick, *Proc. Roy. Soc. A*, 136, 692 (1932).
- [3] W. Pauli, *Handbuch der Physik*, 24, Berlin, Springer Verlag, 1933.
- [4] E. Fermi, *Nuclear Physics*, Chicago, University of Chicago Press, 1949.
- [5] R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry*, Vol. I-V, International Academic Press, Palm Harbor, U.S.A., 2008.
- [6] R. M. Santilli, "Apparent Consistency of Rutherford's Hypothesis on Neutron as a Compressed Hydrogen Atom", *Hadronic J.* 13, 513-531(1900).
- [7] R. M. Santilli, "Apparent Consistency of Rutherford's Hypothesis on Neutron Structure via Hadronic Generalization of Quantum Mechanics-I: Nonrelativistic Treatment", in *ICTP-1991*, International Academic Press, Palm Harbor, Florida, 1992.
- [8] I. Gandzha and J. Kadeisvily, *New Science for A New Era*, Sankata Printing Press, Kathmandu, Nepal, 2001. <http://www.santilli-foundation.org/docs/RMS>
- [9] R. M. Santilli, *Lie-admissible Approach to the Hadronic Structure, Volume I: Non-applicability of the Galilei and Einstein Relativities* in the series Monographs in Theoretical Physics, Hadronic Press, Palm Harbor, Florida, 1978.
- [10] R. M. Santilli, *Lie-admissible Approach to the Hadronic Structure, Volume II: Coverings of the Galilei and Einstein Relativities* in the series Monographs in Theoretical Physics, Hadronic Press, Palm Harbor, Florida, 1981.
- [11] R. M. Santilli, "Nonlocal-Integral Isotopies of Differential Calculus, Mechanics and Geometries," in *Isotopies of Contemporary Mathematical Structures, Rendiconti Circolo Matematico Palermo*, Suppl. Vol. 42, 7-82 (1996), <http://www.santilli-foundation.org/docs/Santilli-37.pdf>
- [12] R. M. Santilli, "The etherino and/or the neutrino hypothesis", *Foundation of Physics*, 37, 670 (2007).
- [13] C. Borghi, C. Giori C. and A. Dall'Olio, *Communications of CENUFPE*, 8(1969) and 25 (1971).

- [14] C. Borghi, C. Giori C. and A. Dall'Olio, (in Russian), *Phys. Atomic Nuclei*, 56, 205 (1993).
- [15] R. M. Santilli, "Apparent Confirmation of Don Borghi's Experiment toward the Synthesis of Neutron from Protons and Electrons", *Hadronic Journal*, textbf30, 709 (1997).
- [16] R. M. Santilli, *The Synthesis of the Neutron*, <http://www.neutronstructure.org/neutronsynthesis.htm>.

Studies on Santilli Three-Body Model of the Deuteron According to Hadronic Mechanicsⁱ

Sudhakar S. Dhondge

Department of Chemistry, S. K. Porwal College, Nagpur, India

Email address:

s_dhondge@hotmail.com

To cite this article:

Sudhakar S. Dhondge. Studies on Santilli Three-Body Model of the Deuteron According to Hadronic Mechanics. *American Journal of Modern Physics*. Special Issue: Issue II: Foundations of Hadronic Mechanics. Vol. 5, No. 2-1, 2016, pp. 46-55.

doi: 10.11648/j.ajmp.2016050201.14

Received: July 8, 2016; **Accepted:** July 9, 2016; **Published:** May 18, 2016

Abstract: In this paper, we outline the inapplicability (rather than the violation) of quantum mechanics for the representation of the synthesis of the neutron from the Hydrogen atom in the core of a star, and we outline the corresponding inability of quantum mechanics for a consistent representation of all characteristics of the deuteron as a two-body state of one proton and one neutron in its ground state. We then outline the first representation of all characteristics of the neutron achieved by R. M. Santilli via a generalized two-body bound state of one proton and one electron in conditions of total mutual penetration according to the laws of hadronic mechanics, thus implying the mutation of particles into isoparticles under the Lorentz-Santilli isosymmetry. We then outline the first representation of all characteristics of the deuteron also achieved by R. M. Santilli via a generalized three-body bound state of two isoprotons and one isoelectron, including the first known exact and time invariant representation of the deuteron spin, magnetic moment, binding energy, stability, charge radius, dipole moment, etc. We finally study further advances of Santilli three-body model of the deuteron in preparation of its extension to all nuclei, such as: the admission of exact analytic solution for the structure of the deuteron as a restricted three-body system; the validity in first approximation of the structure of the deuteron as a two-body system of one isoproton and one iso neutron; the importance for the representation of experimental data of the deformability of the charge distribution of the proton and the neutron which is prohibited by quantum mechanics but readily permitted by hadronic mechanics in the notion of isoparticle; and other aspects.

Keywords: Neutron, Deuteron, Hadronic Mechanics

1. Introduction

The nucleus of deuterium is called a deuteron and it contains one proton and one neutron, whereas the far more common hydrogen nucleus contains no neutron. The isotope name is formed from the Greek deuterons meaning "second", to denote the two particles composing the nucleus. Thus Deuteron is normally considered as the combination of proton and neutron and thus it is considered as a two body system by quantum mechanical bound state. It is the simplest bound state of nucleons and therefore gives us an ideal system for studying the nucleon-nucleon interaction. In analogy with the ground state of the hydrogen atom, it is reasonable to assume that the ground state of the deuteron also has zero orbital angular momentum $L = 0$. However the measured total angular momentum is $J = 1$ (one unit of $\hbar/2\pi$)

thus it obviously follows that the proton and neutron spins are parallel: $s_n + s_p = 1/2 + 1/2 = 1$. On the other hand, its high stability is to the tune of 2.2 MeV. The stability of deuteron plays a very important part of the existence of the universe.

The structure of deuteron and its physical properties were first proposed by Santilli [1, 2]. Although Deuteron is a simple molecule, quantum mechanics has been unable to explain its different properties like the spin, magnetic moment, binding energy, stability, charge radius, dipole moment, etc. The magnetic moment of deuteron was for the first time represented exactly by Santilli [3]. Also for the first time the notion of isoproton and isoelectron was introduced by Santilli [4, 5], which was further elaborated by him [6, 7]. He made Rutherford's conjecture of neutron a quantitative description based on his Hadronic Mechanics [8-10]. Santilli

under the covering laws of Hadronic Mechanics has demonstrated and established that all nuclei and therefore all the matter at large are supposed to be composed of protons and electrons in their isoprotons and isoelectrons realization characterized by Lorentz-Santilli isosymmetry [4, 5, 8]. The conception of nuclei as quantum mechanical bound states of proton and neutron remains valid but only as a first approximation. Thus, Santilli's reduction of the neutron to a hadronic bound state of a proton and an electron suggests the reduction of all nuclei and, therefore, all matter in the universe, to protons and electrons. However, on technical grounds, the constituents of nuclei are given by protons and electron in their form mutated by contact non-Hamiltonian, thus nonunitary interactions called isoprotons and isoelectrons [5, 11] (for further details see [6, 7] and technically defined as isounitary irreducible representations of the Lorentz-Poincaré-Santilli isosymmetry.

Hadronic mechanics not only allows the reduction of a nuclei into (iso) protons and (iso) electrons, but also achieves, for the first time, a numerically exact and invariant representation of various nuclear data beyond any dream of representation via quantum mechanics.

For the sake of some sort of continuity we start in the next Section with a very brief description of neutron structure based on Santilli hadronic mechanics and then would devote all succeeding Sections to hadronic mechanics of deuteron as developed by Santilli.

2. A Brief Review of Neutron Structure Based on Santilli's Hadronic Mechanics

In the history of science Santilli for the first time quantified the Rutherford conjecture that a neutron is indeed a compressed hydrogen atom using his hadronic mechanics. The main motivation to develop corresponding hadronic mechanics has been the inadequacy of quantum mechanics to arrive at experimentally established properties of neutron e.g. its spin, magnetic moment, its stability within nucleus (an isolated neutron is unstable having half life of about 10 min), etc. For the details of all these aspects can be found in [8-10]. However, herein we recall only the main features of Santilli's quantification of neutron structure and synthesis to illustrate the continuity of nuclear structure from neutron to deuteron according to hadronic mechanics.

In order to make Rutherford's conjecture a quantitative one he proposed a model in which the wave packets of an electron and a proton mutually overlap to form a dynamic union such that electron revolves around proton as shown in Figure 1.

In other words, the proton and the electron are actual physical constituents of the neutron in our space-time, not in their conventional quantum mechanical states, but in generalized states due to the total penetration of the wave packet of the electron within the hyperdense proton, for which Santilli has suggested the names of "isoproton," "here

denoted \hat{p}^+ , and "isoelectron," here denoted \hat{e}^- , these new states are technically realized as irreducible isorepresentation of the Lorentz-Poincaré-Santilli isosymmetry. In this way he studied the representation of "Rutherford's compression" of the Hydrogen atom into a neutron inside a star via a non-unitary transform of the conventional structure of the Hydrogen atom (HA).

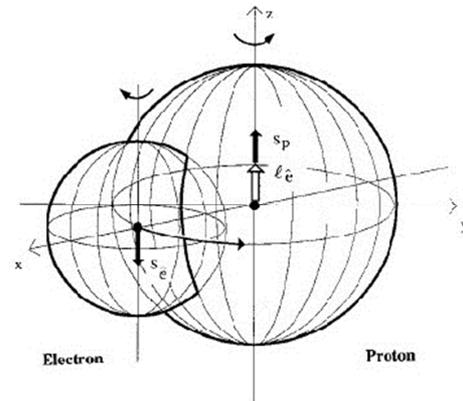


Figure 1. A conceptual view of Rutherford's compression of the electron inside the hyperdense proton in singlet coupling (necessary for stability), resulting in the constrained orbital angular momentum of the electron under which the total angular momentum of the electron is zero and the spin of the neutron coincides with that of proton.

Thus the mutated electron and proton as shown in Figure 1 are termed as *isoelectron* and *isoproton* respectively. The iso-prefix stems from the need of Santilli isomathematics [12] to describe the process of the said mutation. The said mutation gets mathematically expressed as,

$$HA \equiv (p^+, e^-)_{QM} \rightarrow n = (\hat{p}^+, \hat{e}^-)_{HM} \quad (1)$$

where subscripts QM and HM stands for the horizons of quantum mechanics and hadronic mechanics respectively. From the model of Figure 1 it is evident that the dimensions of interaction between isoelectron and isoproton are of 1 fm or less. But to maintain an electron within such a short nuclear volume very strong attractive force is needed because the conventional electrostatic attraction at such a short distances turns out to be grossly inadequate. This then indicated that an external trigger is operating that forces an electron to penetrate within the hyperdense medium of a proton. This in hadronic mechanics has been quantified through corresponding Hulthén potential, which produces very large attractive force compared to the conventional electrostatic force.

The reader is advised to refer to the references cited herein for the details of the Rutherford-Santilli model of neutron and its synthesis both in Stars and in laboratory.

3. Santilli's Structured Model of Deuteron as a Hadronic Bound State of Two Protons and One Electron

Santilli considered deuteron as a hadronic bound state of

two protons and one electron verifying the laws and symmetries of hadronic mechanics. According to him:

1. The deuteron is a stable light, natural isotope that, as such, is reversible over time.
2. Thus Santilli assumes the quantum mechanical structure less of the deuteron (denoted as “d”)

$$d \approx (p^+, n)_{QM} \tag{2}$$

as valid in first approximation, and reduces the deuteron to two protons and one electron according to the structure:

$$d = (\hat{p}^+, \hat{e}^-, \hat{p}^+)_{HM} \tag{3}$$

In the above equation all the constituents are isoparticles, namely, two iso- protons and one isoelectron. Their iso-character has been depicted by (^) over the symbols.

3. Contrary to expectations, contact interactions generate a special version of restricted three body system that admits an exact analytic solution.

In this communication we intend to review the insufficiencies of quantum mechanics for a quantitative representation of experiential data on the deuteron and then review their exact and invariant representation via Santilli’s isomechanics and underlying isomathematics.

3.1. Insufficiencies of Quantum Mechanics to Adequately Describe the Structure of Deuteron

3.1.1. Quantum Mechanics has been Unable to Represent or Explain the Stability of the Deuteron

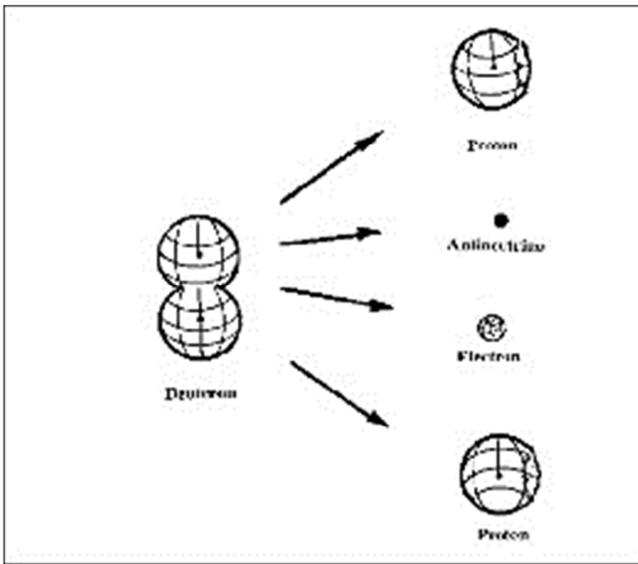


Figure 2. Three body model of the deuteron.

This problem might be also due to unavailability of the technical literature of quantitative numerical proofs that, when bonded to a proton, the neutron cannot decay, as an evident condition for stability. Thus the stability of the deuteron has been left fundamentally unexplained by quantum mechanics till date. Santilli illustrated the inability by quantum mechanics to represent the stability of the

deuteron, since the neutron is naturally unstable and, therefore, the deuteron should decay into two protons, an electron and the hypothetical antineutrino. Even today, no reason is known that why neutron should become stable when coupled to a proton. Santilli represented three body model of the deuteron and its stability as shown in Figure 2.

3.1.2. Quantum Mechanics has been Unable to Represent the Spin 1 of the Ground State of the Deuteron

According to quantum mechanics the most stable bound state of two particles is with the opposite spins and hence should have SPIN ZERO. No such state has been detected in the deuteron. Thus quantum mechanics has been unable to represent the spin 1 of the ground state of the deuteron. This is illustrated in Figure 3.

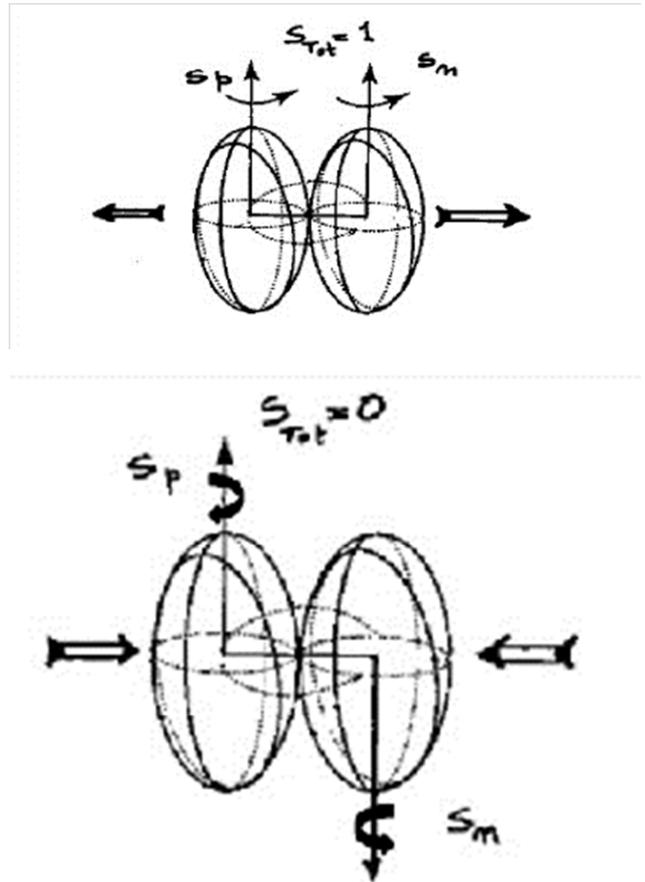


Figure 3. Figures above represent the impossibility of quantum mechanics to represent the spin 1 of the deuteron in a way compatible with its size. First figure explains how spin 1 can solely be achieved with a triplet coupling in which case no stable nucleus is conceivable due to very strong repulsive forces at the distance of nuclear forces. Thus only stable state is the singlet but in this case the total angular momentum is zero, in disagreement with experimental evidence.

3.1.3. Quantum Mechanics has been Unable to Reach an exact Representation of the Magnetic Moment of the Deuteron

It has been observed that non-relativistic quantum mechanics misses 0.022 Bohr units corresponding to 2.6% of the experimental value. Relativistic corrections reduce the error down to about 1% but under highly questionable

theoretical assumptions, such as the use for ground state of a mixture of different energy levels that are assumed to exist without any emission or absorption of quanta as expected by quantum mechanics. The situation becomes worst for the magnetic moments of heavier nuclei.

3.1.4. *Quantum Mechanics has been Unable to Identify the Physical Origin of the Attractive Force that Binds Together the Proton and the Neutron in the Deuteron*

Since the neutron is neutral, there is no known electrostatic origin of the attractive force needed for the existence of the deuteron. The only Coulomb force for the proton-neutron system is that of the magnetic moments, which force is REPULSIVE for the case of spin 1 with parallel spin. Therefore, a "strong" force was conjectured and its existence was subsequently proved to be true.

3.1.5. *Quantum Mechanics has also been Unable to Treat the Deuteron Space Parity in a Way Consistent with the Rest of the Theory*

The experimental value of the space parity of the deuteron is positive for the ground state, because the angular momentum L is null. However, nuclear physicists assume for the calculation of the magnetic moment of deuteron that the ground state is a mixture of the lowest state with $L = 0$ with other states in which the angular momentum is not null. This produces incompatibility of these calculations with the positive parity of the ground state.

3.2. Inferences

Thus from above discussion we can infer that, after about one century of research, quantum mechanics has left unresolved fundamental problems even for the case of the smallest possible nucleus, the deuteron, with progressively increasing unresolved problems for heavier nuclei. Following these insufficiencies, any additional belief on the final character of quantum mechanics in nuclear physics is a sheer political posture in disrespect of the societal need to search for a more adequate mechanics.

Not only quantum mechanics is not exactly valid in nuclear physics, but the very assumption of neutrons as nuclear constituents is approximately valid since neutrons are composite particles. Therefore, the main objective of this chapter is the identification of stable, massive physical constituents of nuclei and their theoretical treatment that admits in first approximation the proton-neutron model, while permitting deeper advances.

The replacement of protons and neutrons with the hypothetical quark is mathematically significant, with the clarification that, in Santilli's view, *quarks cannot be physical particles* because, as stresses several times by Santilli, *quarks are purely mathematical representations of a purely mathematical symmetry realized in a purely mathematical internal unitary space without any possible formulation in our spacetime* (because of the O'Rafearthaigh's theorem).

Consequently, *quark masses are purely mathematical*

parameters and cannot be physical inertial masses. As also stressed several times, on true scientific grounds, inertial masses can only be defined as the eigenvalues of the second order Casimir invariant of the Lorentz-Poincaré symmetry. But this basic symmetry is notoriously inapplicable for the representation of quarks because of their particular features. Therefore, quark "masses" cannot have inertia. Additionally, Santilli points out that the hypothetical orbits of the hypothetical quarks are excessively small to allow an exact representation of nuclear magnetic moments via their polarization. In fact, various attempts have been made in representing magnetic moments when reducing nuclei to quarks with the result of bigger deviations from experimental data than those for the proton-neutron structure. Similar increases of the problematic aspects occur for all other insufficiencies of quantum mechanics in nuclear physics. Consequently, the reduction of nuclei to quarks will be ignored hereon because of its excessive deviation from solid physical foundations as well as experimental data.

In conclusion, quarks can indeed be considered as replacements of protons and neutrons, with the understanding that nuclei made up of *quarks cannot have any weight*, since, according to Albert Einstein, gravity can solely be defined for bodies existing in our spacetime.

4. Deuteron and Hadronic Mechanics

It is evident from the above facts that quantum mechanics has been unable to treat the deuteron space parity, in a way consistent with the rest of the theory [1, 8, 10]. Thus quantum mechanics has not been able to solve fundamental problems even for the case of the smallest possible nucleus, the deuteron, with progressively increasing unresolved problems for heavier nuclei.

4.1. Deuteron Structure

The nuclear force solely applies up to the distance of 10^{-13} cm, which distance coincides with the charge radius of the proton as well as the electron wavepacket, and that the sole stable orbit for the two protons under contact strong interactions is the circle. The size of the deuteron then forces the charge distribution of two protons as essentially being in contact with each other. It can be said that the electron is totally immersed within a proton, expectedly exchanging its penetration from one proton to the other.

Now the spin of the deuteron in its ground state is 1; the spin of the protons is 1/2; the spin of the isoelectron is 1/2; and that the mutated angular momentum of the isoelectron is -1/2. So Santilli assumed the structure of the deuteron as being composed of two un-mutated protons with parallel spins rotating around the central isoelectron to allow the triplet coupling of protons, and then the two coupled particles in line have an orbital motion around the isoelectron at the center, resulting in the first approximation in the following hadronic structure model of the deuteron [2].

$$d = (p_{\uparrow}^+, \hat{e}_{\downarrow}^-, p_{\uparrow}^+)_{HM} \quad (4)$$

Thus, proton is the only stable particle and neutron is unstable, comprising of proton and electron. Santilli assumed that nuclei are a collection of protons and neutrons, in first approximation, while at a deeper level a collection of mutated protons and electrons. It has been proved that a three-body structure provides the only known consistent representation of all characteristics of the deuteron, first achieved by R. M. Santilli. Thus Coulomb and contact attractive forces in pair-wise singlet couplings proton-isoelectron are so strong to overcome Coulomb repulsion among the two protons and form a bound state that is permanently stable when isolated, as already established for the valence bond and Cooper pairs of identical electrons.

Volodymyr Krasnoholovets has tried to resolve the above anomalies in his recent paper [13]. He analyzed the problem of the deuteron from the viewpoint of the constitution of the real space that he developed. He concluded that the nucleus does not hold the electrons in the orbital position and polarized inertons [14-16] of atomic electrons directly interact with the nucleus. He also analyzed the problem of the motion of nucleons in the deuteron, which takes into account their interaction with the space and concluded that nucleons in the deuteron oscillate along the polar axis and also undergo rotational oscillations. In other words, the nucleons execute radial and rotationally oscillatory motions. Trying to account for the reasons for nuclear forces, he has analyzed major views available in the literature including quantum field theories, hadronic mechanics, and even the Vedic literature.

R. M. Santilli in 1998 provided the consistent representation of all the characteristics of the deuteron using its three body model [2] that involves isomathematics based methods of hadronic mechanics. His hadronic mechanics method explains the strong attraction between protons and neutrons via the Hulthén potential concept [17]. Thus the hadronic mechanics:

1. could successfully explain the experimental value of spin 1 of the deuteron;
2. offered the exact and invariant representation of the total magnetic moment of the deuteron;
3. provided a physical insight into the deuteron size and charge.

4.2. Size of Deuteron

It has been observed experimentally that the proton has the following values for the charge radius and diameter (size) $R_p = 0.8 \times 10^{-13} = 0.8 \text{ fm}$; $D_p = 1.6 \text{ fm}$. Whereas, the value of the size of the deuteron given in literature is: $D_d = 4.31 \text{ fm}$.

Structure model represented by equation 4 does indeed fully justifies the above data in accordance with Figure 4. In fact, the above data indicate that the charge radii of the two protons are separated by approximately 1.1 fm, namely, an amount that is fully sufficient, on one side, to allow the triplet alignment of the two protons as in the upper part of Figure 4 and, on the other side, to generate contact nonlocal effects from the penetration of the wave packet (here referred to the square of the probability amplitude) of the central

spinning electron within the two peripheral protons.

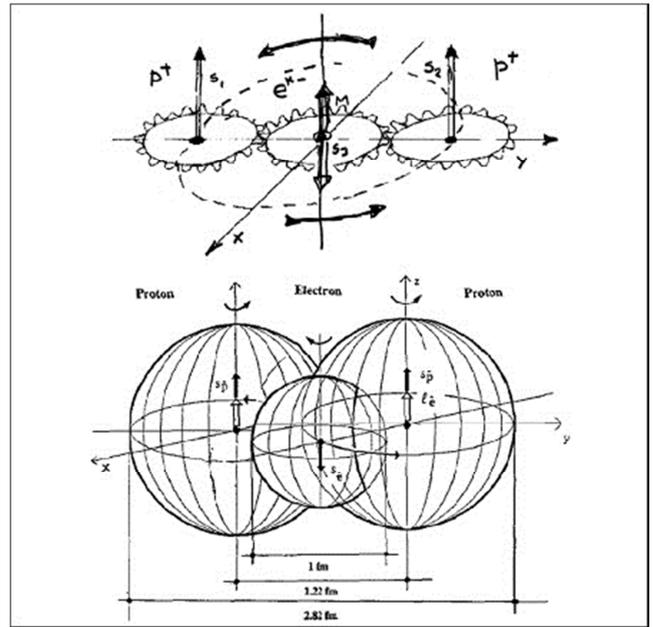


Figure 4. Represents the structure of the deuteron as a restricted three body of two un-mutated protons (due to their weight) and one mutated electron. The top view uses the very effective “gear model” to avoid the highly repulsive triplet couplings, while the bottom view is the same as the top view, the particles being represented with overlapping spheres.

4.3. Representation of the Stability of the Deuteron

As indicated earlier, the lack of a quantitative representation of the stability of the deuteron when composed by the stable proton and the unstable neutron has been one of the fundamental problems left unsolved by quantum mechanics in about one century of research.

By comparison, protons and electrons are permanently stable particles. Therefore, structure model equation (4) resolves the problem of the stability of the deuteron in a simple, direct, and visible way. The deuteron has no unstable particle in its structure and, consequently it is stable due to the strength of the nuclear force.

In fact, as shown below, the Coulomb and contact attractive forces in pair-wise singlet couplings proton-isoelectron are so “strong” to overcome Coulomb repulsion among the two protons and form a bound state that is permanently stable when isolated, as already established for the valence bond and Cooper pairs of identical electrons.

4.4. Deuteron Charge

Model given by equation 4 represents the deuteron positive charge +e. This is due to the fact that hadronic mechanics generally implies the mutation of all characteristics of particles, thus including the mutation of conventional charges Q , and so that mutated charge of the deuteron constituents

$$\hat{Q}_{p1} = ae, \quad \hat{Q}_e = be, \quad \hat{Q}_{p2} = ce \quad (5)$$

where a, b, c are positive-definite parameters, and e is the

elementary charge. These mutations are necessary for consistency with other aspects, such as the reconstruction of the exact isospin symmetry in nuclear physics. However, these mutations are only internal, under the condition of recovering the conventional total charge +e for the system as a whole, as it is the case for closed non-Hamiltonian systems. Consequently, the charge mutations are subject to cancelation in such a way to yield the total charge +e, i.e.,

$$Q_d = (a + b + c)e = e; a + b + c = 1 \quad (6)$$

However, the mutations of the charge is expected to be quite small in value as being a second order effect ignorable at a first approximation, the deuteron structure does not require the mutual penetration of the charge distribution of protons.

4.5. Representation of the Deuteron Spin

According to quantum mechanics the most stable state between two particles with spin 1/2 is the singlet, for which the total spin is zero. Thus for the ground state of the deuteron as a bound state of a proton and a neutron should have spin zero. This is exactly contrary to the experimental value of spin 1. When the deuteron is assumed to be a three-body bound state of two protons with an intermediate electron, hadronic mechanics achieves the exact and invariant representation of the spin 1 of model represented by equation 4.

It can be seen that the electron is trapped inside one of the two protons, thus being constrained to have an angular momentum equal to the spin of the proton itself. In this case, with reference to Figure 4 the total angular momentum of the isoelectron is null. Thus the ground state has null angular momentum, the total angular momentum of the deuteron is given by the sum of the spin 1/2 of the two isoprotons.

According to quantum mechanics fractional angular momenta are prohibited because they violate the crucial condition of unitarity, with consequential violation of causality, probability laws, and other basic physical axioms.

For hadronic mechanics, the isotopic lifting and of the spin S and angular momentum L of the electron when immersed within a hyperdense hadronic medium are characterized by

$$\hat{S}^2 \hat{T} | \hat{s} \rangle = (PS)(PS+1) | \hat{s} \rangle \quad (7)$$

$$\hat{S}_3 \hat{T} | \hat{s} \rangle = \pm(PS) | \hat{s} \rangle \quad (8)$$

$$\hat{L}^2 \hat{T} | \hat{a} \rangle = (QL)(QL+1) | \hat{a} \rangle \quad (9)$$

$$\hat{Q}_3 \hat{T} | \hat{a} \rangle = \pm(QL) | \hat{a} \rangle \quad (10)$$

where $S=1/2$ $L=0,1,2,\dots$, where P and Q are arbitrary (non-null) positive parameters and isotopically lifted S and L are \hat{S} and \hat{L} respectively.

Santilli introduced the above isotopy of SU(2)-spin to prevent the belief of the perpetual motion that is inherent when the applicability of quantum mechanics is extended in

the core of a star.

In fact, quantum mechanics predicts that an electron moves in the core of a star with an angular momentum that is conserved in exactly the same manner as when the same electron orbits around proton in vacuum, thus an electron in the core of a star can only have a locally varying angular momentum and spin as represented by Eqs. 7 - 10.

In case of the isoelectron in the deuteron, we have the constraint that the orbital angular momentum must be equal but opposite to that of the spin:

$$\hat{S} = (P) \frac{1}{2} = -\hat{L} = Q, \quad Q = -\frac{P}{2}, \quad \hat{J}_{tot} = 0 \quad (11)$$

The exact and invariant representation of the spin 1 of the ground state of the deuteron then follows according to the rule

$$J_d = S_{p1} + S_{p2} = 1 \quad (12)$$

Now suppose that the quantum mechanical angular momentum operator L has expectation value 1, then

$$\langle a | L | a \rangle = 1 \quad (13)$$

Under isotopic lifting the above expression easily acquires the value 1/2 for $\hat{T} = 1/2$, $\hat{L} = 2$.

$$\langle \hat{a} | \hat{T} \hat{L} \hat{T} | \hat{a} \rangle = 1/2 \quad (14)$$

However, in this case the isounit is given by $\hat{I} = 1/\hat{T} = 2$. Therefore, when the isoeigenvalue of the angular momentum is properly represented as an isonumber (an ordinary number multiplied by the isounit), one recovers the original value 1.

$$\langle \hat{a} | \hat{T} \hat{L} \hat{T} | \hat{a} \rangle \hat{I} = 1 \quad (15)$$

thus recovering causality and other laws.

It should be noted that there is no violation of Pauli's exclusion principle in this case since that principle only applies to "identical" particles and does not apply to protons and neutrons, as well known (more explicitly, one of the two protons of Eq. 4 is in actuality the neutron since it has embedded in its interior, the isoelectron).

4.6. Magnetic Moment of Deuteron

The experimental values of magnetic moment of deuteron and its constituents are:

$$\mu_d = \frac{0.8754eh}{2\pi M_p c}; \quad \mu_p = \frac{2.795782eh}{4\pi M_p c} \quad (16)$$

and

$$\mu_e = \frac{eh}{4\pi M_e c} = \frac{eh}{4\pi M_p c} \cdot \frac{M_p}{M_e} = \frac{938.272}{0.511} \cdot \frac{eh}{4\pi M_p c}$$

$$= 1.836 \times 10^3 \cdot \frac{eh}{4\pi M_p c} \quad (17)$$

We know that deuteron is in its ground state with null angular momentum and there is no orbital contribution to the total magnetic moment from the two protons. Thus the exact and invariant representation of the total magnetic moment of the deuteron is then given by:

$$\begin{aligned} \mu_d &= 2\mu_p + \mu_{\text{tot},e} = 2 \times 2.792 \frac{eh}{4\pi M_p c} + \mu_{\text{tot},e} \\ &= 0.8754 \frac{eh}{4\pi M_p c} \end{aligned} \quad (18)$$

$$\begin{aligned} \mu_{\text{tot},e} &= 0.8754 \frac{eh}{4\pi M_p c} - 5.584 \frac{eh}{4\pi M_p c} \\ &= -4.709 \frac{eh}{4\pi M_p c} = -4.709 \frac{eh}{4\pi M_e c} \cdot \frac{M_e}{M_p} \\ &= -8.621 \times 10^{-4} \frac{eh}{4\pi M_e c} = \mu_{e,\text{orb}} - \mu_{\dot{e},\text{spin}} \end{aligned} \quad (19)$$

In the above equation, missing contribution is provided by the total magnetic moment of the isoelectron. The latter numerical value is given by the difference between the orbital and the intrinsic magnetic moment that is very small (per electron's standard) since the total angular momentum of the isoelectron is indeed small. Also note the correct value of the sign because the isoelectron has the orbital motion in the direction of the proton spin. But the charge is of opposite sign.

Thus the direction of the orbital magnetic moment of the isoelectron is opposite to that of the proton, as represented in equation 4. The small value of the total magnetic moment of the isoelectron for the case of the deuteron is close to the corresponding value for the neutron.

4.7. Deuteron Force

The assumption that the deuteron is a bound state of a proton and a neutron does not provide any explanation for physical origin of the nuclear forces. Quantum mechanics provides mathematical description of the attractive force via number of potentials, although none of them admits a clear physical explanation of the strong attraction between protons and neutrons. Santilli has always tried to generalize quantum mechanics for nuclear physics by providing fundamentally different notions and representations by using hadronic mechanics principles.

We have seen that Model represented by equation 4 permits a clear resolution of this additional insufficiency of quantum mechanics via the precise identification of two types of nuclear forces, the first derivable from a Coulomb potential and the second of contact type represented with the

isounit. On the inspection of Figure 4 we see that the constituents of deuteron are in specific configuration such that there we have short range pair-wise opposite signs of charges and magnetic moments with long range identical signs of charges and magnetic moments. Thus it implies that the net attractive Coulomb force in the deuteron is determined by the following expression of potential:

$$V_d = \frac{e^2}{0.6 \text{ fm}} + \frac{e^2}{1.2 \text{ fm}} - \frac{\mu_p \cdot \mu_e}{0.6 \text{ fm}} + \frac{\mu_p \cdot \mu_e}{1.2 \text{ fm}} \quad (20)$$

In addition, the constituents admit an attractive force not derivable from a potential due to the deep penetration of their wavepackets in singlet pair-wise couplings, which force is the same as that of the two identical electrons in the Cooper and valence pairs, the structure of mesons, the structure of the neutron, and can be represented via the isounit:

$$\hat{I} = \exp\left(F(r) \int \psi_{\downarrow}^{\dagger}(r) \times \psi_{\uparrow}^{\dagger}(r) d^3r\right) \quad (21)$$

The projection of the above force characterizes a strongly attractive Hulthen potential, that behaves at short distances like the Coulomb potential, thereby absorbing the latter and resulting in a single, dominating, attractive Hulthen well with great simplification of the calculations. Thus it can be seen that besides the above potential and contact force, no additional nuclear force is needed for an exact and invariant representation of the remaining characteristics of the deuteron, such as binding and total energies. It can be proved that the isoelectron is not restricted to exist within one of the two protons, because there lies a 50% isoprobability of moving from the interior of one proton to that of the other proton. Therefore, the proton-neutron exchange is confirmed by model given by equation 4.

4.8. Deuteron Binding Energy

We know that quantum mechanics is a purely Hamiltonian theory in the sense that the sole admitted forcings are those derivable from a potential. So direct and immediate consequence is the impossibility of quantitative representation of the deuteron binding energy. The experimental binding energy of deuteron is

$$E_d = -2.26 \text{ MeV} \quad (22)$$

that is, a representation via equations, rather than via the existing epistemological arguments. Thus the mathematics underlying quantum mechanics, being local differential, can only represent the proton and the neutron of model as being point-like particles. As a result of this fact quantum mechanics admits no binding energy at all for the Deuteron, including the absence of binding energy of Coulomb type, because the neutron is abstracted as a neutral massive point. The lack of a quantum mechanical binding energy for the Deuteron persists even under the assumption that the Deuteron is composed of six hypothetical quarks because attractive and repulsive contributions between the

hypothetical quarks of the proton and those of the neutron cancel out, resulting in no force acting at all between the proton and the neutron, irrespective of whether attractive or repulsive.

Model given by equation 4, under the covering laws of hadronic mechanics has permitted the achievement of the first quantitative representation of the binding as well as the total energy of the Deuteron in scientific history, thus illustrating the validity of Santilli's original proposal of 1978 [18] to build the covering hadronic mechanics.

According to hadronic mechanics, the binding energy is mainly characterized by forces derivable from a potential since the contact forces due to mutual wave-overlapping of wave packets have no potential energy. Hence, the binding energy of the deuteron is due to the potential component of the deuteron binding force given by equation 20. This can be verified by using known values of charges and magnetic moments for the two electron-proton pairs of the deuteron and their mutual distances.

Now, Hadronic mechanics also permits the exact and invariant representation of the total energy of the deuteron, that is direct verification of model given by equation 4.

Now 1 amu = 941.49432 MeV gives,

$$M_p = \frac{938.265 \text{ MeV}}{c^2} = 1.00727663 \text{ amu}$$

$$M_e = \frac{0.511 \text{ MeV}}{c^2} = 5.48597 \times 10^{-4} \text{ amu}$$

The mass of a nucleus with A nucleons and Z protons without the peripheral atomic electrons is characterized by

$$M_{\text{nucleus}} = M_{\text{amu}} - Z \times M_e + 15.73 \times Z^{-1/3} \times 10^{-6} \text{ amu} \quad (23)$$

and thus for deuteron

$$M_d = 2.1035 \text{ amu} = 1875.563 \text{ MeV} \quad (24)$$

The iso-Schrödinger equation for model given by equation 4 can be reduced to that of the neutron, under the assumption that the isoelectron spends 50% of the time within one proton and 50% within the other, thus reducing model (equation 4) in first approximation to a two-body system of two identical particles with un-isorenormalized mass given by

$$\hat{M} = 937.782 \text{ amu} \quad (25)$$

The main differences are given by different numerical values for the energy, meanlife and charge radius. Thus Santilli derived the structured equation of the deuteron as a two-body nonrelativistic approximation

$$d = (\hat{p}_\uparrow, \hat{p}_\uparrow)_{\text{hm}} \quad (26)$$

$$\left(-\frac{\hbar^2}{2M_p} \nabla^2 - V \times \frac{\exp(-r/R)}{1 - \exp(-r/R)} \right) |\hat{p}\rangle = E |\hat{p}\rangle \quad (27)$$

$$E_d = 2E_p - |E| = 1875 \text{ MeV} \quad (28)$$

$$\tau_d^{-1} = 2\lambda^2 |\hat{e}(0)|^2 \alpha^2 E_e / \hbar = \infty \quad (29)$$

$$R_d = 4.32 \times 10^{-13} \text{ cm} \quad (30)$$

The above equations admit a consistent solution reducible to the algebraic expressions as for the case of Rutherford-Santilli neutron,

$$k_2 = 1, \quad k_1 = 2.5 \quad (31)$$

It is worth noting that, in the above model, the deuteron binding energy is zero,

$$E = -V \left(\frac{k_2 - 1}{4k_2} \right)^2 \approx 0 \quad (32)$$

because all potential contributions have been included in the structure of \hat{P} and, for the binding of the two \hat{P} all potential forces have been absorbed by the nonlocal forces and k_2 has now reached the limit value of 1 (while being close to but bigger than 1). It has been observed that a more accurate description can be obtained via the restricted three-body configuration of Figure 4. This model gives an exact solution. The model can be constructed via a nonunitary transform of the conventional restricted three-body Schrödinger equation for two protons with parallel spin 1/2 and one isoelectron with null total angular momentum as per Figure 4 with conventional Hamiltonian $H = T + V_{\text{Coul}}$, where V_{Coul} is given by equation 20. The nonunitary transforms then produces an additional strong Hulthén potential that can absorb the Coulomb potential resulting in a solvable equation.

4.9. Electric Dipole Moment and Parity of Deuteron

It is well known that the electric dipole moment of the proton, neutron and Deuteron are null. The preservation of these values by hadronic mechanics is assured by the general property that axiom-preserving lifting preserves the original numerical values, and the same holds for parity. The positive parity of the deuteron is represented by hadronic mechanics via the expression

$$\text{Isoparity} = (-1)^L \quad (33)$$

The value for unperturbed deuteron in its ground state $\hat{L} = L = 0$. It should be noted that on one hand, the parity of the deuteron is positive ($L=0$), while on the other hand, in order to attempt a recombination of deuteron magnetic moments and spin, the unperturbed deuteron is assumed as being a mixture of different levels, some of which have non-null values of L , thus implying the impossibility of a positive parity.

Thus Santilli has shown that the isotopic branch of nonrelativistic hadronic mechanics permits the exact and invariant representation of "all" the characteristics of the deuteron composed of two isoprotons and one isoelectron, at

the same time resolving all quantum insufficiencies spelled out in the main text above.

4.10. Reduction of Matter to Isoproton and Isoelectrons

It is evident that, following the reduction of the neutron to a proton and an electron and the reduction of the deuteron to two protons and one electron, Santilli has indeed achieved the important reduction of all matter to protons and electrons, since the reduction of the remaining nuclei to protons and electron is consequential, e.g., as a hadronic bound state of two mutated deuterons represents Helium nucleus.

We would like to close our discussion by indicating Santilli's additional astro-physical contribution given by the fact that the so-called "neutron stars" are in reality an extremely high density and high temperature fluid composed by the original constituents of the star, protons and electrons in their isoprotons and isoelectrons realization, in conditions of deep mutual penetration under the laws of hadronic mechanics.

5. Conclusion

As it is well known, the local-differential structure of quantum mechanics solely permits the representation of particles as being massive points. This abstraction has been proved to be effective for the representation of the structure of atoms, since the atomic constituents are at very large mutual distances compared to the size of charge distributions or wave packets of particles.

As shown by R. M. Santilli in mathematical and physical details, the insufficiency of quantum mechanics to represent the characteristics of the neutron in its synthesis from the hydrogen atom in the core of a star are due precisely to the insufficiency of the representation of the proton and electron as massive points.

In fact, the representation of the proton as an extended charge distribution of 1 fm radius has permitted the representation of all characteristics of the neutron as a compressed hydrogen atom in the core of stars [8]. As an illustration, the anomalous magnetic moments of the neutron is readily represented by a contribution which is impossible for quantum mechanics, but intrinsic in the very conception of hadronic mechanics, namely, the contribution from the orbital motion of the electron when totally compressed inside the proton.

The same advances have shown that the characteristics of the electron change in the transition from isolated conditions in vacuum to the condition of total penetration within the hyperdense proton.

This difference has been quantitatively and invariantly represented by Santilli via, firstly, the transition from Lie' theory to the covering lie-Santilli isothory, and, secondly, via the transition from particles to isoparticles, namely, the transition from irreducible unitary representations from the conventional Lorentz symmetry to those of the covering Lorentz-Santilli isosymmetry. An exact and time invariant representation of all characteristic of the neutron as a

generalized bound state of one isoproton and one isoelectron then follow.

Following, and only following the achievement of a constant, exact and invariant representation of the structure of the neutron Santilli has applied the results to the structure of the deuteron conceived as a three-body generalized bound state of two isoprotons and one isoelectron [2].

This has permitted the exact and invariant representation of all characteristics of the deuteron, with intriguing implications, such as the reduction of all matter in the universe, to protons and electrons in various dynamical conditions.

As an illustration, Santilli's astrophysical contributions finds their root in the fact that the so-called "neutron stars" are in reality an extremely high density and high temperature fluid composed by the original constituents of the star, protons and electrons, in conditions of deep mutual penetration under the laws of hadronic mechanics.

Needless to say, a virtually endless list of intriguing open problems have emerged from the above new vistas in nuclear physics, among which we mention: the need to reexamine from its foundation the notion of nuclear force due to the emergence of a component not derivable from a potential whose control may lead to new clean nuclear energies; the implications of Santilli's deuteron structure on the natural radioactivity elsewhere; the exact and invariant representation of the spin and magnetic moments of all nuclei; and others.

Acknowledgments

The author would like to thank Prof. R. M. Santilli and Prof., A. A. Bhalekar for invaluable assistance in the preparation of this paper.

References

- [1] R. M. Santilli, *J. New Energy*, Vol. 1, 1, (1999). <http://www.santilli-foundation.org/docs/Santilli-114.pdf>
- [2] R. M. Santilli, *Intern. J. Phys.*, Vol. 4, 1, (1998). <http://www.santilli-foundation.org/docs/Santilli-07.pdf>
- [3] R. M. Santilli, "A quantitative isotopic representation of the deuteron magnetic moment," in *Proceedings of the International Symposium, "DUBNA DEUTERON-93"*, Joint Institute for Nuclear Research, Dubna, Russia (1994) <http://www.santilli-foundation.org/docs/Santilli-134.pdf>
- [4] R. M. Santilli, "Theory of mutation of elementary particles and its application to Rauch's experiment on the spinorial symmetry," ICTP preprint # IC/91/265 (1991) <http://www.santilli-foundation.org/docs/Santilli-141.pdf>
- [5] R. M. Santilli, "The notion of nonrelativistic isoparticle," ICTP preprint # IC/91/265 (1991) <http://www.santilli-foundation.org/docs/Santilli-145.pdf>
- [6] R. M. Santilli, *Elements of Hadronic Mechanics*, vol. I and II, Naukova Dumka Publishers, Kiev, second edition, 1995. <http://www.santilli-foundation.org/docs/santilli-300.pdf>

- [7] R. M. Santilli, Hadronic Mathematics, Mechanics and Chemistry, Vol. I [a], II [b], III [c], IV [d] and V [e], International Academic Press, Palm Harbor, Florida, U.S.A. <http://www.i-b-r.org/Hadronic-Mechanics.htm>
- [8] I. Gandzha and J. Kadeisvily, New Sciences for a New Era. Mathematical, Physical Discoveries of Ruggero Maria Santilli, Sankata Printing Press, Kathmandu, Nepal, 2011.
- [9] C. S. Burande, Numerical Analysis and Applied Mathematics ICNAAM 2013 AIP Conf. Proc. 1558, 693 (2013). <http://dx.doi.org/10.1063/1.4825586> <http://www.santilli-foundation.org/docs/burande.pdf>
- [10] R. M. Santilli, The Structure of the neutron as predicted by Hadronic Mechanics, <http://www.neutronstructure.org>.
- [11] R. M. Santilli, The Structure of the neutron as predicted by Hadronic Mechanics, <http://www.neutronstructure.org>.
- [12] R. M. Santilli, Elements of Hadronic Mechanics, vol. I and II, Naukova Dumka Publishers, Kiev, second edition, 1995. <http://www.santilli-foundation.org/docs/santilli-300.pdf> <http://www.santilli-foundation.org/docs/santilli-301.pdf>
- [13] V. Krasnoholovets, Scientific Inquiry, Vol. 7, no. 1, 25, June 30, (2006).
- [14] V. Krasnoholovets and D. Ivanovsky, "Motion of a particle and the vacuum", Physics Essays Vol. 6, no. 4, 554-263 (1993) (arXiv.org e-print archive; <http://arXiv.org/abs/quant-ph/9910023>).
- [15] V. Krasnoholovets, "Motion of a relativistic particle and the vacuum". Physics Essays Vol. 10, no. 3, 407-416 (1997) (arXiv.org e-print archive [quant-ph/9903077](http://arXiv.org/abs/quant-ph/9903077)).
- [16] V. Krasnoholovets, "On the nature of spin, inertia and gravity of a moving canonical particle". Indian J. Theor. Phys., Vol. 48, no. 2, 97-132 (2000b) (also arXiv.org e-print archive; <http://arXiv.org/abs/quant-ph/0103110>).
- [17] L. Hulthén, Ark. Mat. Astron. Fys., 28A, 5 (1942).
- [18] R. M. Santilli, Hadronic J. Vol. 1, 574-901 (1978). <http://www.santilli-foundation.org/docs/santilli-73.pdf>

¹This work was partly presented at the International Conference of Numerical Analysis and Applied Mathematics (ICNAAM) - 2014, at Rhodes, Greece, September 22-27, 2014.

Exact and Invariant Representation of Nuclear Magnetic Moments and Spins According to Hadronic Mechanics

Anil A. Bhalekar^{1,2}, Ruggero Maria Santilli²

¹Department of Chemistry, R. T. M. Nagpur University, Amravati Road Campus, Nagpur, India

²Thunder Energies Corporation, Tarpon Springs, Florida, U. S. A.

Email address:

anabha@hotmail.com (A. A. Bhalekar), research@thunder-energies.com (R. M. Santilli)

To cite this article:

Anil A. Bhalekar, Ruggero Maria Santilli. Exact and Invariant Representation of Nuclear Magnetic Moments and Spins According to Hadronic Mechanics. *American Journal of Modern Physics*. Special Issue: Issue II: Foundations of Hadronic Mechanics.

Vol. 5, No. 2-1, 2016, pp. 56-118. doi: 10.11648/j.ajmp.2016050201.15

Received: January 13, 2016; **Accepted:** January 14, 2016; **Published:** June 1, 2016

Abstract: In order to render this paper minimally self-sufficient, we review and specialize the main structure of the isomathematics to nuclear constituents as extended and deformable charge distributions under linear and non-linear, local and non-local and Hamiltonian as well non-Hamiltonian interactions; we then review and specialize for the nuclear structure the main laws of the isotopic branch of hadronic mechanics known as isomechanics; we review and specialize the method for turning quantum mechanical nuclear models for point-like nucleons into covering isomechanical models for extended and deformable constituents under the most general known realization of strong interactions; we then review and specialize to nuclear structures the consequential notion of isoparticles; we then review the ensuing, first known, numerically exact and time invariant representation of the magnetic moments of stable nuclides; we then review the structure of the neutron as a bound state according to isomechanics of an isoproton and an isoelectron; and we finally review the ensuing three-body structure of the Deuteron. Via the use of the preceding advances. We then present, apparently for the first time, a numerically exact and time invariant representation of the spin of stable nuclides, firstly, via their approximation as isotopic bound states of isodeuterons, isoneutron and isoprotons, and secondly, via their reduction to isobound states of isoprotons and isoelectrons. Some observations on the nuclear configurations so obtained have also been presented in the case of the first model and in view of the second option we have identified in isoelectrons the nuclear glue which tightly holds isonucleons of stable nuclide in the atomic nucleus in the preferred orientation of their intrinsic spins. In Appendix A, we provide a technical review specialized for the first time to nuclear physics of the Lie-Santilli theory and its main application to the notion of isoparticles as iso-irreducible isounitary isorepresentations of the Lorentz-Poincaré-Santilli isosymmetry.

Keywords: Hadronic Mechanics, Nuclear Magnetic Moments, Nuclear Spins

PACS: 21.10. Hw; 21.30.-x; 21.60. De; 21.30. Fe

This paper is dedicated to the memory of Enrico Fermi who: expressed doubts as to whether conventional geometries apply to the structure of particles; supported the introduction of the size of nucleons for basic advances in nuclear physics; and suggested that the anomalous magnetic moment of nuclei may be due to the deformation of their charge distributions under the strong nuclear forces [1], all visions that are quantitatively studied in this paper.

1. Introduction

In the authors view, quantum mechanics is exactly valid for the atomic structure, but it is only approximately valid for the

nuclear structure because quantum mechanics achieved a very accurate representation of atomic data, compared to the known inability by quantum mechanics to achieve an accurate representation of nuclear data, thus supporting the historical argument by Einstein, Podolsky and Rosen according to which quantum mechanics is "incomplete" [2].

A first reason for the above dichotomy is the fact that the mathematics underlying quantum mechanics (including the local-differential calculus, functional analysis, Hilbert spaces, Lie algebras, etc.) can only represent a finite number of isolated point-particles moving in vacuum, which conditions are known as characterizing *exterior dynamical problems*. The abstraction of particles into dimensionless points is evidently effective for the atomic structure due to the large mutual

distances of the atomic constituents, but the same abstraction is ineffective for the nuclear structure because nuclear constituents consist of extended charge distributions in conditions of partial mutual penetration, which conditions are known as characterizing broader interior dynamical problems (Figure 1).

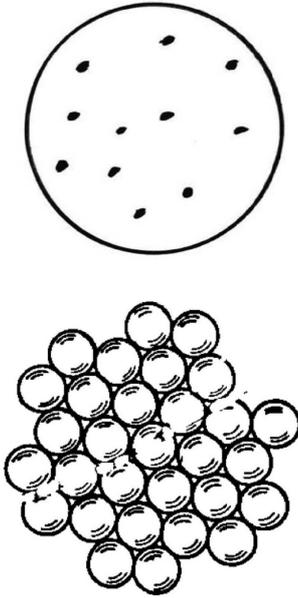


Figure 1. A conceptual rendering (top view) of the abstraction of the nuclear structure as a sphere with isolated point-particles in its interior which is necessary for the applicability of the mathematics underlying quantum mechanics, namely, the local-differential calculus with related Hilbert spaces and Lie's theory. By contrast (bottom view), the nuclear structure consists of hyperdense, extended charge distributions in conditions of partial mutual penetration, as one can verify by the fact that nuclear volumes are generally smaller than the sum of the volumes of the constituent protons and neutrons. In the authors view, the inability by quantum mechanics to achieve an exact representation of nuclear data is due to the evident insufficiency of the abstraction of the nuclear structure depicted in the top when compared to the physical reality of the bottom view. This first insufficiency establishes the fundamental need of a basically new mathematics for the representation of extended charge distributions as they occur in the nuclear reality (Section 2).

A second reason for the above dichotomy is that the fundamental symmetries of non-relativistic and relativistic quantum mechanics, the Galileo and Poincaré symmetries respectively, are solely valid for a *Keplerian system*, namely, for a system of particles orbiting around a heavier center, as it is the case indeed for atomic structures. By contrast, as stressed in the recent literature, *nuclei do not have nuclei* and, therefore, the symmetries valid for systems of point particles with a Keplerian nucleus cannot possibly be exactly valid for structurally different systems of extended particles without a Keplerian nucleus (Figure 2).

A third reason for the above dichotomy is that none of the 20th century sciences, including quantum mechanics and special relativity, can represent Fermi's historical hypothesis that the deviations of the values of nuclear magnetic moments from the predictions of relativistic quantum mechanics are due to deformations of the charge distributions of protons and neutrons (nucleons) when under the strong interactions of a

nuclear structure, with consequential alteration (called in this paper *mutation*) of their intrinsic magnetic moments (Figure 3). This insufficiency is evidently due to the fact that *dimensionless points cannot experience deformations*. Therefore, a mathematics which can solely represent dimensionless points is structurally unable to represent the deformation of extended charge distributions as they occur in the nuclear reality.

A fourth reason for the above dichotomy is the fact that *dimensionless points can only experience interactions at a distance, thus derivable from a potential* (interactions technically known as *variationally self-adjoint* [3a]). In view of this basic feature, recent representations of the strong nuclear force have reached un-reassuring limits, such as a Hamiltonian with forty or so potentials, without the desired achievement of an exact representation of nuclear data. In the authors view, it is necessary to complement these conventional studies with the admission that the interactions between extended charge distributions under conditions of partial mutual penetrations are of *contact* type, thus not being derivable from a potential (interactions technically known as *variationally non-selfadjoint* [3a]). Consequently, it is recommendable to ascertain whether some of the potential components of nuclear Hamiltonians should be replaced with non-Hamiltonian representations.

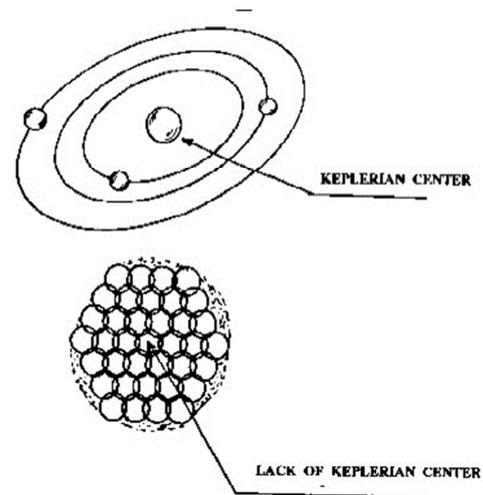


Figure 2. A conceptual rendering of the second impossibility for quantum mechanics to be exactly valid for the nuclear structure, which is given by the fact that the basic symmetries of non-relativistic and relativistic quantum mechanics, the Galileo and Poincaré symmetries respectively, only apply for Keplerian systems of particles orbiting around a heavier nucleus. By contrast, R. M. Santilli has stated several times in his writings that "nuclei do not have nuclei," thus implying a necessary breaking of said fundamental symmetries, with consequential lack of exact character of non-relativistic and relativistic quantum mechanics for the nuclear structure. The same breaking is confirmed by numerous additional evidences, such as the fact that the partial mutual penetration of nucleons in a nuclear structure implies the presence of contact interactions not representable with a Hamiltonian, thus implying the inapplicability of the entire Lie theory, let alone of Lie's symmetries, due to its strictly Hamiltonian character. This second insufficiency establishes the need for a covering of Lie's theory for the construction of the symmetries of systems of extended particles without Keplerian nuclei under Hamiltonian as well as non-Hamiltonian internal forces (Section 2 and Appendix A).

A fifth reason for the above dichotomy is that quantum mechanics is certainly effective for the description of nuclear *fissions* due to the effective representation of the fission debris as point particles, but quantum mechanics has proved to be ineffective for the achievement of nuclear *fusions* for all the above indicated reasons, plus the fact that *nuclear fusions are structurally irreversible over time while quantum mechanics is structurally reversible*, hence the need for a covering of quantum mechanics that can represent extended charge distributions with Hamiltonian and non-Hamiltonian interactions in generally irreversible conditions.

In this paper, we shall briefly outline decades of research by one of us (R. M. Santilli) [3-33] for: the construction of a generalization of 20th century mathematics suitable to represent extended particles (Figure 1); the generalization of Lie's theory for the construction of symmetries of systems of extended particles without Keplerian center under Hamiltonian and non-Hamiltonian internal forces (Figure 2); the representation of Fermi's historical hypothesis on the deformability of nucleons; and the consequential, first known, exact and time invariant representation of nuclear magnetic moments (Figure 3).

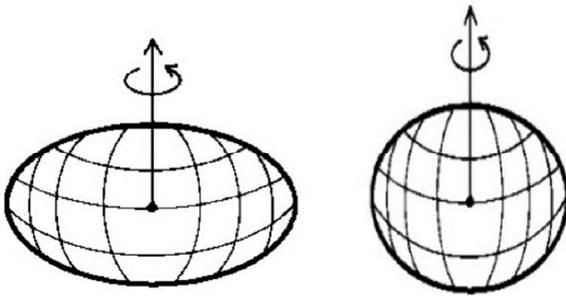


Figure 3. A third insufficiency of quantum mechanics for nuclear structures is given by the historical prediction by Enrico Fermi [1] that the anomalous values of nuclear magnetic moments is due to deformations of the charge distribution of protons and neutrons when under the strong interactions of the nuclear structure, with consequential alteration of their conventional magnetic moments. In fact, a quantitative treatment of Fermi's teaching requires the use of the deformation theory which is known to be incompatible with quantum mechanics. This third insufficiency establishes the need that the novel mathematics and Lie's theory for extended charge distributions should be constructed in such a way to be compatible with the deformation theory "ab initio" (Section 2 and Appendix A).

Since the advances considered here [3-33] are only known to a restricted number of experts, and they are generally unknown to the nuclear physics community, in order to render minimally understandable the advances presented in this paper, it has been necessary to: outline in Section 2 the novel mathematics (known as *isomathematics* for the reversible case and *genomathematics* for the irreversible form); outline in Section 3 the corresponding invariant branches of hadronic mechanics (known as *isomechanics* and *genomechanics* respectively); outline in Section 3.1 the non-relativistic nuclear isomechanics; outline in Section 3.2 the relativistic nuclear isomechanics; outline in Section 4 a simple construction of iso- and gene-mechanics; outline in subsequent sections the exact and time invariant

representation of nuclear magnetic moments (Section 5), the test of spinorial symmetry by neutron interferometry (Section 6), and then outline the emerging new structure of the neutron (Section 7), deuteron and nuclei at large (Section 8). Above all, it has emerged as recommendable to formulate advances [3-33] in a form directly applicable to nuclear physics, rather than leaving such an adaptable to the imagination of non-initiated readers.

We shall then present, apparently for the first time, the achievement of an exact and time invariant representation of the spin of stable nuclide which, thanks to the above advances, is compatible with the mutation of the intrinsic magnetic moments of nucleons, and then indicate the implications of these advances in nuclear physics for basically new, environmentally acceptable forms of nuclear energies. For the sake of self sufficiency of this presentation we start with a very brief description of stable and unstable nuclides in Section 9, a brief description of new and old vistas of nuclear forces with the earlier conjectural assertions of the stability of nucleons in Section 10. In Section 11 we have developed notations to represent isoneutrons and isodeuterons. We have presented in Section 12 two models of nuclear configuration, namely (i) considering isodeuterons, isoneutrons and isoprotons as isonucleons (Section 12.1) and (ii) isoprotons and isoelectrons as isonucleons (Section 12.2). These nuclear configurations were written down in a way to be commensurate with the experimental nuclear spins and tabulated in Section 13 for both the nuclear models stated above. We have also presented our observations in Section 14 on these nuclear configurations with an idea to provide the facts about the isonucleons within the nuclides that would help in developing corresponding theories of nuclear stability and generate new explanations of other nuclear properties (Sections 14.1 and 14.2). In the second model of nuclear configuration arrived at in this paper we propose, apparently for the first time, that the isoelectrons serve as the nuclear glue that tightly holds the nuclear isoprotons together in the atomic nucleus (Section 14.2). For the sake of ready reference we have also presented the Lorentz-Poincaré-Santilli Isosymmetry and its characterization of isoparticles in Appendix A.

2. Elements of Iso-Mathematics and Geno-Mathematics

The first known time-invariant representation of extended and deformable charge distributions in interior dynamical conditions was proposed by Santilli in the early 1980s [3b] via the *isotopic* (in the sense of being axiom-reserving) lifting of the associative product AB between generic quantities (numbers, functions, matrices, operators, etc.) into the form, today known as *Santilli isoproduct*,

$$AB = AB \rightarrow A\hat{T}B = A \times B \tag{1}$$

where \hat{T} is solely restricted to be invertible, but otherwise possesses an arbitrary dependence on local variables such as:

time t , coordinates r , velocities v , density μ , temperature τ , index of refraction ρ , frequency ω , wave functions Ψ , etc., $\hat{T} = \hat{T}(t, r, v, \mu, \tau, \rho, \omega, \Psi, \dots)$.

When \hat{T} is positive-definite and invariant under time-reversal $t \rightarrow -t$, it is called isotopic element, and when it is positive-definite (or merely Hermitean) but non-invariant under time reversal, it is called the genotopic element.

The representation of extended and deformable charge distributions is then immediately achieved via realizations of \hat{T} of the type [3]

$$\hat{T} = \text{Diag} \left(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, \frac{1}{n_4^2} \right) e^{-\Gamma(\psi, \dots)} \int \psi^\dagger \psi dr^3 \quad (2)$$

where: $n_k = n_k(t, r, v, \mu, \tau, \delta, \omega, \psi, \dots)$, $k = 1, 2, 3$, represent, in this simple case, the deformable semi-axes of a nucleon assumed for simplicity to be an ellipsoid; $n_4 = \rho$ characterizes the density of the nucleon considered; all quantities $n_\mu, \mu = 1, 2, 3, 4$, called characteristic quantities of the nucleon considered, are normalized to the value $n_\mu = 1$ for exterior conditions in vacuum; $\Gamma(\psi, \dots)$ is a positive definite function or operator characterizes all non-linear interactions not representable with the conventional Hamiltonian; and the integral in the exponent of Eq. (2) tends to zero at mutual distances of particles much bigger than their charge radius (about $1 \text{ fm} = 10^{-13} \text{ cm}$), thus implying the limit

$$\lim_{r \gg 1 \text{ fm}} \hat{T} = 1, \quad (3)$$

for which

$$\lim_{r \gg 1 \text{ fm}} (A \hat{\times} B) = AB. \quad (4)$$

When \hat{T} verifies the conditions

$$\hat{T}(t, \dots) = \hat{T}^\dagger(t, \dots) = \hat{T}(-t, \dots) = \hat{T}^\dagger(-t, \dots) \quad (5)$$

it is called the *isotopic element*, while under the verification of the conditions

$$\hat{T}(t, \dots) = \hat{T}^\dagger(t, \dots) \neq \hat{T}(-t, \dots) = \hat{T}^\dagger(-t, \dots) \quad (6)$$

\hat{T} is called the *genotopic element*. Conditions (5) characterize the use of isomathematics, while conditions (6) characterize the use of the broader genomathematics. The most important mathematical difference is that the conventional Lie theory with historical product between Hermitean operators

$$[A, B] = AB - BA, \quad (7)$$

at the foundations of quantum mechanics is lifted in the former case into *Santilli Lie-isotopic theory* with basic product

$$[A \hat{\times}, B] = A \hat{\times} B - B \hat{\times} A = A \hat{T} B - B \hat{T} A, \quad (8)$$

while in the latter case Lie's theory is lifted into the broader *Santilli Lie-admissible theory* with covering product

$$(A \hat{\times}, B) = A \hat{T}(-t, \dots) B - B \hat{T}(t, \dots) A = A \hat{R} B - B \hat{S} A, \quad (9)$$

$$R = R^\dagger, S = S^\dagger, R \neq S, \quad (10)$$

according to conceptions, formulations and terminologies first introduced by Santilli in Ref. [3b].

It should be indicated from the upset the importance of conditions (5) and (6) for nuclear physics. In fact, conditions (5) characterize a *stable nuclide* composed by extended nucleons when isolated from the rest of the universe, thus being reversible over time. By contrast, conditions (6) characterize *irreversible nuclear reactions*, such as nuclear syntheses.

In fact, as it is well known, the time reversibility of quantum mechanics is ultimately due to the *invariance of the Lie product under anti-Hermiticity* (for hermitean operators A and B)

$$[A, B] \equiv -[A, B]^\dagger. \quad (11)$$

It is then easy to see that isomathematics and its ensuing physical formulations are also time reversal invariant due to the *invariance of the Lie-Santilli isoproduct under anti-Hermiticity*,

$$[A \hat{\times}, B] \equiv -[A \hat{\times}, B]^\dagger. \quad (12)$$

By contrast, genomathematics and its related physical formulations are irreversible over time precisely because *Santilli's Lie-admissible product violates, by central conception, the invariance under anti-Hermiticity*

$$(A \hat{\times}, B) \neq - (A \hat{\times}, B)^\dagger. \quad (13)$$

Monograph [3b] presented the lifting of most 20th century applied mathematics via the systematic lifting of all products into the isotopic form (1), although all liftings were formulated on conventional numeric fields.

Since this paper deals with magnetic moments and spins of stable, thus reversible nuclides, we shall mainly use isomathematics under basic conditions (5). However, it is recommendable for the non-initiated reader to know that that the extension to irreversible nuclear processes is immediate, thus being recommendable when applicable.

Subsequently, Santilli discovered that the emerging formulations were not invariant over time, that is, they failed to predict the same numerical values under the same conditions at different times. In order to resolve this basic insufficiency, Santilli re-examined in 1993 [4] conventional numeric fields $F(n, \times, 1)$ with classification of numbers n into real, complex or quaternionic numbers n , conventional associative product $nm = n \times m \in F$ and basic multiplicative unit 1, $1 \times n = n \times 1 = n \quad \forall n \in F$.

In this way, Santilli [4] discovered that the axioms of

numeric field also admit solution with an arbitrary basic unit \hat{I} , under the conditions that: 1) all numbers are lifted in the isonumbers

$$n \rightarrow \hat{n} = n\hat{I}; \tag{14}$$

2) all products are lifted into the *isoproduct* (1),

$$nm \rightarrow n\hat{\times}m = n\hat{T}m; \tag{15}$$

and 3) the conventional unit 1 of 20th century numeric field is lifted into the isounit under the sole conditions of being positive-definite and being the inverse of isotopic element \hat{T} ,

$$\hat{I} = \frac{1}{\hat{T}} \tag{16}$$

Under these conditions all axioms of a numeric field are verified and \hat{I} is the correct left and right multiplicative unit,

$$\hat{I}\hat{\times}\hat{n} = \hat{n}\hat{\times}\hat{I} = \hat{n} \quad \forall \hat{n} \in \hat{F}. \tag{17}$$

Under conditions (4), \hat{I} is called Santilli isounit, while under broader conditions (5) it is called Santilli genounit [4].

This lead to the discovery of new numeric fields $\hat{F}(\hat{n}, \hat{\times}, \hat{I})$ called isofields under conditions (4) and genofields under conditions (5) with corresponding novel isoreal, isocomplex and isoquaternionic numbers and general, genocomplex and genoquaternionic genonumbers $\hat{n} = n\hat{I}$.

Following the discovery of isonumbers and genonumbers, all theories originally formulated on conventional fields [3] where lifted into formulations defined over isofields and genofields [5, 6], but the crucial time invariance of the numeric predictions was still missing.

In order to resolve this impasse, Santilli reinspected in 1995 the Newton-Leibnitz differential calculus and discovered that, contrary to popular beliefs in mathematics and physics for centuries, the Newton-Leibnitz differential calculus depends on the assumed basic multiplicative unit because, in the event said unit has a functional dependence on the differentiation variable, the ordinary differential dr must be generalized into the form first introduced in memoir [7]

$$\hat{d}\hat{r} = \hat{T}d[r\hat{I}(\hat{r}, \dots)] = dr + r\hat{T}d\hat{I}(\hat{r}, \dots), \tag{18}$$

and called *isodifferential* under conditions (4) and *genodifferential* under conditions (5), with corresponding *isoderivatives* (and *genoderivatives*) [7]

$$\frac{\hat{\partial}\hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \frac{\partial\hat{f}(\hat{r})}{\partial\hat{r}} + \hat{f}(\hat{r})\hat{T} \frac{\partial\hat{I}(\hat{r}, \dots)}{\partial\hat{r}}, \tag{19}$$

where, for consistency, coordinates and functions must be isoscalars, that is, have values in \hat{F} with structures

$$\hat{r} = r\hat{I}(\hat{r}, \dots), \quad \hat{f}(\hat{r}, \dots) = f(\hat{r}, \dots)\hat{I}(\hat{r}, \dots). \tag{20}$$

It should be stressed that the representation of nuclear

magnetic moments and spin presented in this paper depends crucially on a non-potential component of the nuclear force due to partial mutual penetration of the charge distribution of nucleons, which non-potential components is represented precisely via the isodifferential calculus and, therefore, with the novel additional terms in the r.h.s. of Eqs. (18) and (19).

In memoir [7] Santilli introduced a third broader mathematics under the name of *hypermathematics* which is given by a covering of genomathematics when the genounit is multi-valued (rather than multi-dimensional), e.g. of the ordered type $\hat{I} = \{\hat{I}_1, \hat{I}_2, \dots, \hat{I}_n\}$ where n can assume an arbitrarily larger values such as $n=10^{50}$ as needed for biological structures.

The discovery of the generalized differential calculus signed the achievement in memoir [7] of mathematical maturity in the generalizations of 20th century applied mathematics at large, that stimulated seminal advances in mathematics (see representative monographs [8-11]) as well as generalized physical and chemical theories, including novel industrial applications indicated below.

The above studies lead to the following chain of generalized mathematics:

1. *IsoMathematics*, which is used for the representation of stable and isolated, thus time-reversible nuclei composed by extended nucleons in conditions of partial mutual penetration and is characterized by the lifting of the totality of 20th century applied mathematics in such a way to admit a positive-definite and time-reversal invariant isounit (5) at all levels of treatment.
2. *GenoMathematics*, which is used for the representation of time-irreversible nuclear reactions and it is characterized by a dual lifting of the totality of 20th century mathematics in such a way to admit a positive-definite time-noninvariant genounit (6) at all levels of treatment, one genounit, $\hat{I}(t, \dots) = 1/\hat{T}(t, \dots)$ characterizes motion forward in time, and its time reversal image $\hat{I}(-t, \dots) = 1/\hat{T}(-t, \dots)$ characterizes motion backward in time, irreversibility over time being assured by inequivalent forward and backward genounits $\hat{I}(t, \dots) \neq \hat{I}(-t, \dots)$.

The knowledge of the above distinct mathematics is important for researchers to prevent the use of time non-invariant isounits that may eventually imply irreversible contributions for the structure of isolated and stable nuclei, with evident inconsistencies.

Important independent contributions on the foundations of isomathematics and genomathematics can be found in monographs [8-11] and in their bibliographies.

The main methodological problems for the representation of nuclear magnetic moments and spins are the following:

- 2.1: The representation of the deformation of the charge distribution of protons and neutrons when members of a nuclear structure and the ensuing mutation of their intrinsic magnetic moments according to Fermi's historical hypothesis [2]. This first central problem was solved by Santilli via the use of the isotopies of the rotational symmetry [12], as

reviewed in the next section and in Appendix A.

2.II: The representation of the mutation of the intrinsic magnetic moments of nuclear constituents in a way compatible with the conventional ten conservation laws of total physical quantities (the conservation of the total angular momentum, total linear momentum, the center of motion, and the total energy), which must hold for all isolated bound states of particles. This problem was solved by Santilli by showing the isotopies of the Lorentz and of the Poincaré symmetry [15-17] do verify indeed said conventional total conservation laws because in the lifting of Lie's theory into the Lie-Santilli isothory the generators of Lie algebras (that represent said conservation laws) remain unchanged, and only their products lifted for the representation of extended shapes and non-Hamiltonian interactions.

2.III: The representation of the spin of stable nuclides in a way compatible with mutation of the magnetic moments of protons and neutrons under strong nuclear interactions. This problem will be solved, apparently for the first time in this paper, by showing that the isotopies of the $SU(2)$ -spin symmetry do indeed admit a "hidden" degree of freedom directly connected to the mutation of spin.

The central physical notion used in this paper for the solution of the above problems and for the characterization of extended-deformable nuclear constituents in conditions of partial mutual penetration is that of *isoparticle*, specialized to *isoprotons*, *isoneutrons* and *isoelectrons*.

The understanding of the notion of isoparticle and, therefore, of this paper, requires at least some knowledge of the central branch of isomathematics used for the derivation of the new notion of isoparticle, which is given by the isotopies of Lie's theory, originally proposed by Santilli in monograph [3b], including the isotopies of universal enveloping associative algebras, Lie's theorem and Lie's transformation groups.

Among a rather large literature in the field, Santilli's papers specifically devoted to the notions of isoparticle are given by the isotopies of: the rotational symmetry $O(3)$ [12]; the $SU(2)$ spin symmetry [13, 14]; the Lorentz symmetry $O(3,1)$ in classical [15] and operator [16] forms; the isotopies of the Poincaré symmetry $P(3,1)$ [17]; the spinorial covering of the Poincaré symmetry [18]; and the isotopies of the Minkowskian geometry [19]. The notion of isoparticle was then studied in details in Refs. [20-23].

In view of these advances, the isotopies of Lie's theory are today called the *Lie-Santilli isothory* (see independent studies [24-33]).

Due to its fundamental character for the exact and time invariant representation of magnetic moments and spins, the notion of isoparticle will be reviewed in detail in Appendix A.

3. Elements of Nuclear IsoMechanics and GenoMechanics

The non-unitary covering of quantum mechanics was proposed under the name of *hadronic mechanics* by R. M.

Santilli in monograph [3b] of 1981 (see page 112 for the proposal of the name of the new mechanics.) The original proposal comprised two branches, the isotopic branch with Lie-isotopic structure (8) and in the genotopic branch with Lie-admissible structure (9).

A fundamental contribution to hadronic mechanics (which is fully valid nowadays) was provided in paper [34] of 1982 by the mathematician (late) H. C. Myung and R. M. Santilli via the isotopies and genotopies of the Hilbert space (today known as the *Hilbert-Myung-Santilli isospace and genospace* respectively) and the indication that hadronic mechanics removes the divergencies of quantum mechanics via the isotopies of Dirac Delta "distribution" (today known as the *Dirac-Myung-Santilli isodelta "function"* and the fast convergence of isotopic series (see, e.g., Ref. [35]).

These initial studies were formulated on a conventional field and elaborated via the conventional differential calculus. Hadronic mechanics achieved a mature formulation only following the discovery of the novel isonumbers and genonumbers [4] in 1993 and of the isodifferential and genodifferential calculus [7] in 1996 (see monographs [22] for a general presentation of hadronic mechanics, including the fundamental notion of iso- and geno-particles).

With the passing of time, the above indicated two branches of hadronic mechanics acquired the names of *isomechanics* and *genomechanics*, respectively. Since these names have received a rather wide acceptance by the physics community, they have been adopted in this paper.

The reader should be aware that hadronic mechanics has a variety of applications in disparate fields all dealing with interior dynamical problems. The main reference for the specialization of isomechanics to nuclear physics is given by memoir [26] of 1998, while the main reference for genomechanics is given by memoir [37] of 2006.

Since hadronic mechanics at large, as well as isotopic and genotopic branches are essentially unknown to the nuclear physics community, it appears recommendable to provide in this section an elementary review specialized to nuclear physics sufficient for the understanding of the derivation of exact and invariant magnetic moments and spins, with the understanding that an in depth study of memoirs [35, 36] is essential for serious knowledge.

3.1. Elements of Non-relativistic Nuclear IsoMechanics

Non-relativistic nuclear isomechanics is characterized by the lifting of Planck's constant \hbar into a 3×3 -dimensional, positive-definite *space isounit* [4, 7]

$$\hbar \rightarrow \hat{I}_f = 1 / \hat{T}_f = \text{Diag.}(n_1^2, n_2^2, n_3^2) > 0 \quad (21)$$

where the quantities $n_k^2, k = 1, 2, 3$: represents ab initio the semiaxes of the extended-deformable shape of nucleons when members of a nuclear structure (see the l.h.s. of Figure 3); are normalized to the perfect sphere $n_k^2 = 1$ in empty space; are restricted to be positive-definite and time-reversal invariant; and possess an unrestricted functional dependence on all needed local variables (see Section 1)

$n_k^2 = n_k^2(\hat{t}, \hat{r}, \hat{v}, \hat{\mu}, \hat{t}, \hat{\rho}, \omega, \hat{\psi}, \dots) > 0$, where the "hat" denotes the referral to *internal* variables, while variables without a "hat" refer to those of the *external* observer.

Note that we have ignored in Eq.(21) for simplicity the multiplicative exponential term representing internal non-linear, non-local and non-Hamiltonian interactions as in Eq. (2) since this term can be embedded into the n_k^2 via their simple redefinition.

Assumption (21) implies that all possible products AB of conventional nuclear formulations (including the product of numbers, functions, matrices, etc.) have to be lifted to the isoproduct [4],

$$AB \rightarrow A \hat{\times} B = A \hat{T}_r B. \quad (22)$$

with Lie-isotopic structure (8) [4].

Assumptions (21) and (22) also implies that conventional numeric fields $F(n, \times, I)$ are lifted into isofields $\hat{F}(\hat{n}, \hat{\times}, \hat{I}_r)$ with isonumbers $\hat{n} = n \hat{I}_r$ [4].

Note that, in the event the characteristic quantities n 's depend on time in a way not invariant under time-reversal, instead of the single unit (21) and product (22) for action to the right and to the left, we would have the genoproduct and genounit for motion forward in time [36]

$$AB \rightarrow A > B = AT(t, \dots)B, \hat{I}^> = 1/T(t, \dots), \quad (23)$$

and the genoproduct and genounit for motion backward in time

$$AB \rightarrow A < B = AT(-t, \dots)B, T(t, \dots) \neq T(-t), \hat{I}^< = 1/T(-t, \dots) \quad (24)$$

with Lie-admissible structure (9).

Therefore, the use of time-reversal non-invariant quantities $n_k, k=1,2,3$ for the study of a stable, time-reversal invariant nuclear structure would imply the inclusion of un-warranted irreversible contributions that should solely be admitted for irreversible nuclear reactions [36].

Nuclear isomechanics is additionally characterized by the lifting of time t into the isotime

$$t = t_{ext} \rightarrow \hat{t} = t_{int} \hat{I}_r \quad (25)$$

where t_{ext} is the time of the external observer, t_{int} is the intrinsic time in the interior of nuclei, and \hat{I}_r is different than \hat{I}_r , both dimensionally and numerically.

The representation of nuclear magnetic moments and spins has been achieved in the above stated paper via the simpler case in which $\hat{I}_r = 1$ and the sole use of the time of the external observer $t = t_{ext}$. Consequently, isotime will be ignored for simplicity.

Nevertheless, the non-initiated reader should be aware that, on strict technical grounds, isomechanics implies that the time in the interior of nuclei is generally different than the external time [22].

The carrier isospace of isocoordinates $\hat{r} = r \hat{I}_r$ is given by

the Euclid-Santilli isospace [7] $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$ with isometric $\hat{\delta} = \hat{I}_r \delta$ where $\delta = \text{Diag.}(1,1,1)$ the conventional Euclidean metric, with isoline element

$$\begin{aligned} \hat{r}^2 &= \hat{r}^i \hat{\times} \hat{\delta}_{ij} \hat{\times} \hat{r}^j = (r^i \hat{T}_i^k \delta_{kj} r^j) \hat{I}_r = \\ &= \left(\frac{r_1^2}{n_1^2} + \frac{r_2^2}{n_2^2} + \frac{r_3^2}{n_3^2} \right) \hat{I}_r \end{aligned} \quad (26)$$

where one should keep in mind that the elements of the isometric must be isonumbers as a condition for the isoline element to be an isoscalar with value in the isoreal isofield R .

The understanding of nuclear isomechanics requires the knowledge that the isotopies map ellipsoids on conventional Euclidean space into the perfect sphere in the Euclid-Santilli isospace. This is due to the fact that the deformation of the semiaxes $1_k \rightarrow 1/n_k^2$ is done with the joint inverse deformation of the isounit $1_k \rightarrow n_k^2$, by therefore yielding the original value of the perfect sphere 1_k in isospace.

The reconstruction of the perfect sphere in isospace is essential for the isomorphism of the Lie-Santilli isorotations $\hat{O}(3)$ with the conventional rotations $O(3)$ under the central condition of including the deformation theory (Appendix A).

The isooperator isospace is given by the Hilbert-Myung-Santilli isospace [34] H defined on isofields of isocomplex isonumbers C with isounit (21), isostates $|\hat{\psi}(\hat{t}, \hat{r})\rangle$ and isoinner isoproduct

$$\langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle \hat{I}_r = \langle \hat{\psi} | \hat{T}_r | \hat{\psi} \rangle \hat{I}_r, \quad (27)$$

isonormalization

$$\langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle \hat{I}_r = \hat{I}_r \quad (28)$$

and isoexpectation values for an iso-Hermitean isooperator, \hat{Q} ,

$$\langle \hat{Q} \rangle = \langle \hat{\psi} | \hat{\times} \hat{Q} \hat{\times} | \hat{\psi} \rangle \hat{I}_r = \langle \hat{\psi} | \hat{T}_r \hat{Q} \hat{T}_r | \hat{\psi} \rangle \hat{I}_r \quad (29)$$

with particular properties

$$\hat{I}_r \hat{\times} | \hat{\psi} \rangle = | \hat{\psi} \rangle, \langle \hat{\psi} | \hat{\times} \hat{I}_r \hat{\times} | \hat{\psi} \rangle \hat{I}_r = \hat{I}_r. \quad (30)$$

confirming that \hat{I}_r is the correct isounit of the theory.

The dynamical equations of non-relativistic nuclear isomechanics are given by the Schrödinger-Santilli isoequation on H over C [3, 7]

$$\begin{aligned} -\hat{i} \hat{\times} \hat{\partial}_t | \hat{\psi} \rangle &= \hat{H} \hat{\times} | \hat{\psi} \rangle = \hat{H}(\hat{r}, \hat{p}) \hat{T}_r (\hat{\psi}, \hat{\partial} \hat{\psi}, \dots) | \hat{\psi} \rangle = \\ &= \hat{E} \hat{\times} | \hat{\psi} \rangle = E | \hat{\psi} \rangle \end{aligned} \quad (31)$$

the *isolinear isomomentum*, introduced for the first time in memoir [7] following the discovery of the isodifferential

calculus

$$\hat{p} \hat{\times} |\hat{\psi}\rangle = -\hat{i} \hat{\times} \hat{\partial}_r |\hat{\psi}\rangle = -\hat{i} \hat{I}_r \partial_r |\hat{\psi}\rangle, \quad (32)$$

the Heisenberg-Santilli IsoEquation in the infinitesimal version [3, 7]

$$\hat{i} \hat{\times} \frac{d\hat{Q}}{dt} = [\hat{Q}, \hat{H}] = \hat{Q} \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{Q} = \hat{Q} \hat{T}_r \hat{H} - \hat{H} \hat{T}_r \hat{Q} \quad (33)$$

the integrated version to a finite transform (see Refs [22, 36] for the correct formulation in isomechanics)

$$\hat{Q}(\hat{t}) = UQ(0)U^\dagger = e^{\hat{H}\hat{T}\hat{t}} Q(0) e^{-\hat{t}\hat{T}\hat{H}}, \quad (34)$$

$$UU^\dagger \neq I, \quad (35)$$

and the isocommutation rules

$$[\hat{r}_i \hat{\times} \hat{p}_j] = \delta_{ij} \hat{I}_r, \quad [\hat{p}_i \hat{\times} \hat{p}_j] = [\hat{r}_i \hat{\times} \hat{r}_j] = 0. \quad (36)$$

where the "hat" on operators denotes their definition on \hat{H} over \hat{C}

3.2. Elements of Relativistic Nuclear Isomechanics

Relativistic nuclear isomechanics is characterized by the lifting of Planck's constant \hbar into a 4×4 -dimensional, positive-definite, thus diagonalizable isounit (see Refs. [22] and the memoir [44])

$$\hbar \rightarrow \hat{I} = 1/\hat{T} = \text{Diag.}(n_1^2, n_2^2, n_3^2, n_4^2) > 0 \quad (37)$$

where: the $n_k^2, k=1,2,3$ continue to represent deformed nucleons; n_4 is a geometrization of the hyperdense medium inside nucleons' the characteristics quantities $n_\mu, \mu=1,2,3,4$ are subjected to the normalization for the vacuum $n_\mu^2 = 1$; the multiplicative exponential term as in Eq. (1) is absorbed by the n_μ which have an arbitrary functional dependence on local internal variables solely subjected to be invariant under time reversal.

Assumption (35) implies that the totality of all products AB of relativistic nuclear isoformulations are lifted into the isoproducts $A \hat{\times} B = ATB$ defined on isofields $\hat{I}(\hat{n}, \hat{\times}, \hat{I})$.

Again, care must be exercised in the study of stable nuclei in order to prevent the transition from isomechanics to genomechanics that occurs whenever the characteristic quantities n_μ are not invariant under time reversal.

Let $M(x, \eta, I)$ be the conventional Minkowski space with coordinates $x = (x^1, x^2, x^3, x^4 = t)$, metric $\eta = \text{Diag.}(1, 1, 1, -c^2)$ and unit $I = \text{Diag.}(1, 1, 1, 1)$. Then, the relativistic isospace of the isocoordinates $\hat{x} = \hat{x}\hat{I}$ is given by the Minkowski-Santilli isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$ [15, 19] over the isofield of isoreal isonumbers R with isometric

$$\hat{\eta} = \hat{T}\eta = \text{Diag.}\left(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, -\frac{c^2}{n_4^2}\right)\hat{I}, \quad (38)$$

where the multiplication by \hat{I} is necessary for the elements of the isometric to be isoscalars, with isoinvariant

$$\begin{aligned} \hat{x}^{\hat{2}} &= \hat{x}^\mu \hat{\times} \hat{\eta}_{\mu\nu} \hat{\times} \hat{x}^\nu = (x^\mu \hat{\eta}_{\mu\nu} x^\nu) \hat{I} = \\ &= \left(\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \times \frac{c^2}{n_4^2}\right) \hat{I}, \end{aligned} \quad (39)$$

It is evident that, according to then original conception [15], the isotopies of the Minkowski space represent locally varying speeds of light $C = c/n_4$, with consequent mutation of the conventional light cone, which features have been shown in memoir [38] to be compatible with the abstract axioms of special relativity.

However, non-initiated readers should be aware that the isotopies reconstruct the perfect light cone in isospace \hat{M} including c as the maximal causal speed. This is due to the fact that the speed of light is mutated in the value $c^2 \rightarrow c^2/n_4^2$, while the corresponding unit is mutated by the inverse amount $1_4 \rightarrow n_4^2$, thus preserving the maximal causal speed c in isospace \hat{M} over the isofield \hat{R} .

By linearizing the second order isoinvariant of the Poincaré-Santilli isosymmetry $P(3.1)$ as in the conventional case (see Appendix A), one reaches the fundamental equations of relativistic nuclear isomechanics which is given by the Dirac-Santilli isoequation [18]

$$\begin{aligned} &(\hat{\eta}^{\mu\nu} \hat{\gamma}_\mu \hat{p}_\nu + \hat{i} \hat{\times} \hat{m} \hat{\times} \hat{C}) \hat{\times} |\hat{\psi}(\hat{x})\rangle = \\ &= (-i\hat{I} \hat{\eta}^{\mu\nu} \hat{\gamma}_\mu \partial_\nu + imC) |\hat{\psi}(\hat{x})\rangle = 0. \end{aligned} \quad (40)$$

which clearly illustrate the lifting of Plank's constant (35) when compared to the conventional equation, where the Dirac-Santilli isogamma matrices have a structure

$$\hat{\gamma}_k = \frac{1}{n_k} \begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \quad \hat{\gamma}_4 = i \frac{1}{n_4} \begin{pmatrix} I_{2 \times 2} & 0 \\ 0 & -I_{2 \times 2} \end{pmatrix}, \quad (41)$$

with anti-isocommutation rules

$$\{\hat{\gamma}_\mu \hat{\times} \hat{\gamma}_\nu\} = \hat{\gamma}_\mu \hat{T} \hat{\gamma}_\nu + \hat{\gamma}_\nu \hat{T} \hat{\gamma}_\mu = 2\hat{\eta}_{\mu\nu}, \quad (42)$$

Note in Eq. (38) the replacement of the speed of light c with the isospeed $C = c/n_4$. This is necessary because c is no longer invariant under the Poincaré-Santilli isosymmetry, while C is indeed invariant (Appendix A).

It should be indicated that the above formulation of the Dirac-santilli isoequation is solely based on the isotopies of spacetime without the isotopies of the spin of nucleons, since such an isotopy is sufficient for the derivation of nuclear magnetic moments and spins.

For a general study of the Dirac-Santilli isoequation,

including the mutation of spacetime and spins as well as in regular and irregular realizations, we refer the interested reader to memoir [36].

The following comments are now in order:

3.1. By conception and construction, nuclear isomechanics is solely valid within regions of space of the nuclear radius (the order of 1 fm), because at larger distances the isounit recovers the conventional Planck’s constant and, consequently, isomechanics recovers quantum mechanics uniquely and unambiguously (see Figure 4).

3.2. Also by conception and construction, nuclear isomechanics preserves the axioms of quantum mechanics and merely realize them via a broader mathematics. In fact, isomechanics and quantum mechanics coincide at the abstract realization-free level, to such an extent that they be expressed via the same equations only subjected to different realizations.

3.3. The name "hadronic mechanics" was suggested by Santilli [3b] for the representation of "hadrons" at large, thus including the representation of protons and neutrons. Consequently, nuclear isomechanics has been specifically constructed for the study of the nuclear structure, while its covering genomechanics has been constructed to study nuclear reactions.

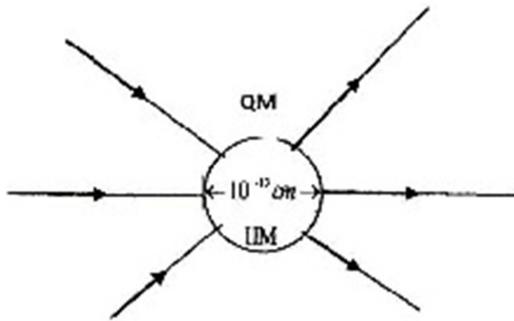


Figure 4. A central feature of hadronic mechanics verified by all its branches is that the new mechanics is solely valid at distances of the order of 1 fm = 10⁻¹³ cm because at larger distances it recovers quantum mechanics uniquely and unambiguously since at larger distances the isounit recovers Planck’s constant.

3.4. As it is well known, non-linear interactions (here referred to nonlinearity in the wave-functions) cannot be consistently represented via quantum mechanics since, in this case, they can be solely represented with a Hamiltonian $H(r, p, \psi)$, with the ensuing violation of the superposition and other laws. Consequently, quantum mechanics cannot consistently define nuclear constituents under non-linear terms of the nuclear force. By contrast, nuclear isomechanics can consistently represent non-linear terms in the nuclear force because all non-linear contributions are embedded in the isounit (or the isotopic element), by therefore maintaining the superposition principle on isospace over isofields. Additionally, nuclear isomechanics reconstructs linearity on isospaces over isofields with evident computational advantages.

3.5. The elementary review of this section has been necessarily incomplete to avoid excessive length. Therefore, interested readers are suggested to study memoir [36] for more

technical details and monographs [22] for a comprehensive presentation. Particularly important is the acquisition of technical knowledge on properties such as: iso-Hermiticity coincides with conventional Hermiticity, as a result of which all observables of quantum mechanics remain observable for nuclear isomechanics; nuclear isomechanics eliminates the divergencies of quantum mechanics because all products of divergent series are lifted into the form given in Eq. (22) where the absolute value of the isotopic element \hat{T}_f is very small (see the negative sign of the exponent of Eq. (1); nuclear isomechanics is a "completion" of quantum mechanics according to the Einstein-Podolsky-Rosen argument, thus providing a concrete and explicit realization of "hidden variables" λ via the isotopic element \hat{T}_f ; and other important properties [22, 36].

3.6. The replacement of Planck’s constant \hbar into the integro-differential operator \hat{I} is a representation of the expectation that, when nucleons are represented as expended charge distributions in conditions of partial mutual penetration, the energy exchange is at least in part continuous. However, the deviations from discrete energy exchanges in nuclear is very small due to the very small absolute value of the isotopic element 2. By contrast, the deviation of quantized energy exchanges for a proton in the core of a star are expected to be finite due to its total immersion with a hyperdense hadronic media for which quantized energy exchanges cannot be even defined.

4. Simple Construction of Isomechanics and Genomechanics

For the benefit of experimental nuclear physicists, it is important to note that any given quantum mechanical nuclear model can be lifted via an elementary procedure into the corresponding isomechanical form, by therefore performing the transition from the point-like abstraction of nucleons, to extended-deformable nucleons under potential as well as contact non-potential interactions.

Isomechanics is a structurally *non-unitary* theory when formulated on a conventional Hilbert space over a conventional numeric field, Eq. (35), while quantum mechanics is unitary. Therefore, the novel isomechanical contributions due to the extended-deformable character of nucleons as well as to the non-potential component of the nuclear force can be represented, from Eq. (21), with a non-unitary transform of the type

$$UU^\dagger = \hat{I} = \text{Diag.}(n_1, n_2^2, n_3^2, n_4^2) \times e^{\Gamma(\psi, \dots)} \int \psi^\dagger \psi dr^3 \quad (43)$$

It is then easy to see that the application of the above non-unitary transform to the "totality" of the formalist of a quantum nuclear model characterizes its isomechanical formulation in its entirety

$$I \rightarrow \hat{I} = U \times I \times U^\dagger = I / \hat{T}, \quad (44a)$$

$$n \rightarrow \hat{n} = U \times n \times U^\dagger = n \times U \times U^\dagger = n \times \hat{I} \in \hat{F}, n \in F, \quad (44b)$$

$$e^A \rightarrow U \times e^A \times U^\dagger = \hat{I} \times e^{\hat{I} \times A} = (e^{\hat{A} \times \hat{T}}) \times \hat{I}, \quad (44c)$$

$$A \times B \rightarrow U \times (A \times B) \times U^\dagger = (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times B \times U^\dagger) = \hat{A} \times \hat{B}, \quad (44d)$$

$$[X_i, X_j] \rightarrow U \times [X_i, X_j] \times U^\dagger = [\hat{X}_i, \hat{X}_j] = U \times (C_{ij}^k \times X_k) \times U^\dagger = \hat{C}_{ij}^k \times \hat{X}_k = C_{ij}^k \times \hat{X}_k, \quad (44e)$$

$$\langle \psi | \times | \psi \rangle \rightarrow U \times \langle \psi | \times | \psi \rangle \times U^\dagger = \langle \psi | \times U^\dagger \times (U \times U^\dagger)^{-1} \times U \times | \psi \rangle \times (U \times U^\dagger) = \langle \hat{\psi} | \times | \hat{\psi} \rangle \times \hat{I}, \quad (44f)$$

$$H \times | \psi \rangle \rightarrow U \times (H \times | \psi \rangle) = (U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times | \psi \rangle) = \hat{H} \times | \hat{\psi} \rangle, etc. \quad (44g)$$

It is easy to see that the application of an additional non-unitary transform causes the lack of time invariance of the isounit

$$W \times W^\dagger \neq I, \quad (45a)$$

$$\hat{I} \rightarrow \hat{I}' = W \times \hat{I} \times W^\dagger \neq \hat{I}, \quad (45b)$$

with consequential lack of invariance of the numeric predictions, with activation of the catastrophic inconsistencies [37], as well as the loss of the represented system.

However, any given non-unitary transform can be identically rewritten in the isounitary form on \hat{H} over \hat{C}

$$W \times W^\dagger = \hat{I}, \quad W = \hat{W} \times \hat{T}^{1/2}, \quad (46)$$

$$W \times W^\dagger = \hat{W} \times \hat{W}^\dagger = \hat{W}^\dagger \times \hat{W} = \hat{I}, \quad (47)$$

under which we have the invariance of the isounit and isoproduct [22, 36, 37]

$$\hat{I} \rightarrow \hat{I}' = \hat{W} \times \hat{I} \times \hat{W}^\dagger = \hat{I}, \quad (48a)$$

$$\begin{aligned} \hat{A} \times \hat{B} &\rightarrow \hat{W} \times (\hat{A} \times \hat{B}) \times \hat{W}^\dagger = \\ &= (\hat{W} \times \hat{T} \times \hat{A} \times \hat{T} \times \hat{W}^\dagger) \times (\hat{T} \times \hat{W}^\dagger)^{-1} \times \hat{T} \times (\hat{W} \times \\ &\quad \times \hat{T})^{-1} \times (\hat{W} \times \hat{T} \times \hat{B} \times \hat{T} \times \hat{W}^\dagger) = \\ &= \hat{A}' \times (\hat{W}^\dagger \times \hat{T} \times \hat{W})^{-1} \times \hat{B}' = \hat{A}' \times \hat{T} \times \hat{B}' = \hat{A}' \times \hat{B}', etc. \end{aligned} \quad (48b)$$

from which the invariance of the entire isotopic formalism follows.

Note that the invariance is ensured by the numerically invariant values of the isounit and of the isotopic element

under isounitary transforms,

$$\hat{I} \rightarrow \hat{I}' \equiv \hat{I}, \quad (49a)$$

$$A \times B \rightarrow A' \times B' \equiv A' \times B', \quad (49b)$$

in a way fully equivalent to the invariance of quantum mechanics, as expected to be necessarily the case due to the preservation of the abstract axioms under isotopies. The resolution of the inconsistencies for non-invariant theories is then consequential.

It should be indicated that the above lifting of quantum into isomechanical models solely apply for the so-called *regular representations of the Lie-Santilli isothery* (see Appendix A), that can be essentially expressed as representations preserving the conventional value of the spin, thus being sufficient for nuclear constituents.

Howsoever, the reader should be aware of the existence of *irregular representations of the Lie-Santilli isothery* (see also Appendix A), which can be indicated as realizations of the axioms causing anomalous values of the spin, as expected for a proton when in the core of a star subjected to enormous pressures under which the very definition of conventional spin is technically flawed.

The lifting of a quantum mechanical nuclear model into the covering genomechanical version can be equally done via an elementary procedure, by performing the transition from time-reversible description to an irreversible one when applicable, e.g., for nuclear reactions.

Recall that genomathematics represent irreversibility by embedding the direction of time in the most ultimate quantities, the unit and related product. Therefore, the creation of a time ordering requires *two* different non-unitary transforms

$$UU^\dagger \neq I, WW^\dagger \neq I, UW^\dagger \neq I, \quad (50)$$

Then Planck's constant can be lifted in the form applicable for motion forward in time

$$\hbar = I \rightarrow \hat{I}^> = UIW^\dagger = 1/\hat{T} > 0, \quad (51)$$

with corresponding lifting of all products AB into the ordered genoproduct to the right

$$AB \rightarrow A > B = A \hat{T}^\dagger B, \quad (52)$$

and lifting of \hbar for motion backward in time

$$\hbar = I \rightarrow \hat{I}^< = WIU^\dagger = 1/\hat{T} < 0, \quad (53)$$

and corresponding lifting of all quantum products into the form ordered to the left

$$AB \rightarrow A < B = A^\dagger \hat{T} B. \quad (54)$$

The irreversible character of the representation is then assured by the different values of the forward and backward genounits, with consequential incoherence of the related

Attempts at the achievement of an exact representation of the anomalous magnetic moment of the Deuteron have been attempted for about one century via the use of quantum mechanics without any result that will resist the test of time.

The first attempts have been done by using an *ad hoc* combination of orbital angular momenta $L \neq 0$ of the proton and the neutron. However, the assumption $L \neq 0$ is in contradiction with the experimental evidence that the isolated Deuteron is in its ground state and, therefore, the orbital angular momenta of its constituents must be $L = 0$.

Numerous additional attempts have been done via relativistic corrections and relativistic field theory, by achieving the needed exact representation of the Deuteron magnetic moment with the introduction of arbitrary parameters or special form factors, thus, without deriving the needed value from first adulterated principles.

Additionally, it should be indicated that the reduction of protons and neutrons to the hypothetical quarks creates additional problems and solves none, because the hypothetical orbits of the hypothetical quarks inside nucleons are too small to admit a hypothetical polarization suitable for the representation of the Deuteron magnetic moment.

In conclusion, in 1993 the exact representation of the magnetic moment of the simplest nucleus, the Deuteron, let alone those of heavier nuclei (see Figure 5) had remained elusive because the proposed representations have contradictions or manipulations that will not resist the test of time.

In this way, Fermi's historical hypothesis acquires its full light when represented via isomathematics and isomechanics. The central conceptual and technical notion of nuclear isomechanics is that *the constituents of nuclei are "isoparticles"* (Ref. [40] and Appendix A), namely, *ordinary particles experience a mutation of their "intrinsic" characteristics when in conditions of partial penetration of their charge distributions (and/or wavepacket) as occurring in the nuclear structure, with ensuing exposure to the strong nuclear force.*¹

The first intrinsic characteristics of particles experiencing a mutation under nuclear conditions is that of their intrinsic magnetic moments. Its explicit expression can be easily derived from the Dirac-Santilli isoequation (40) by repeating the corresponding procedure for the quantum case, yield the following *mutation of the intrinsic magnetic moment in the transition from particles to isoparticles* (see Ref. [39] as well as, for more details, Ref. [18])

$$\tilde{\mu}^{is} = \frac{n_4}{n_3} \mu, \quad (60)$$

where: (is) stands for isomechanics; we consider the magnetic moment along its symmetry axis, as usual; n_3 is the

deformed semiaxis in the third direction; and n_4 a geometrization of the hyperdense medium inside nucleons.

We should note the use the upper symbol $\tilde{\mu}$, rather than $\hat{\mu}$, since the latter indicates elements of isofields because the use of the symbol $\hat{\mu}$ would indicate the transition from a scalar to an isoscalar (Section 2), which is merely given by the multiplication of the conventional magnetic moment and the isounit,

$$\hat{\mu} = \tilde{\mu} \hat{I}, \quad (61)$$

Due to the lack of impact to numerical values, the above isoscalar extension will be ignored hereon for simplicity.

From Eq. (60), we have the following mutation of the quantum mechanical magnetic moment (μ)

$$\tilde{\mu}_D^{is} = \tilde{g}_p^S S + \tilde{g}_n^S S = \frac{n_4}{n_3} (g_p^S S_p + g_n^S S) \quad (62)$$

where we have assumed for simplicity that the mutations of the charge distributions of the proton and the neutron are the same, since they have essentially the same volume and the same hyperdensity.

The numeric value of n_4 has been the subject of extensive phenomenological and experimental studies via the Bose-Einstein correlation and other particle experiments, resulting in the value (see Refs. [41-44] and Eqs. (6.1.101), page 864, Vol. IV of monographs [23])

$$n_4 = 0.654, \quad n_4^2 = 0.355, \quad (63)$$

Consequently, under the numeric value of the third semiaxes

$$n_3 = 0.670, \quad n_3^2 = 0.449, \quad (64)$$

we reach the following numerically exact and time invariant representation of the anomalous magnetic moment of the Deuteron according to isomathematics and isomechanics

$$\tilde{\mu}_D^{is} = \tilde{g}_p^S S + \tilde{g}_n^S S = 0.857 \text{ nm} \quad (65)$$

where we should note the use of slightly different numerical values than those used in the original derivation [39] due to advances occurred since 1993.

As one can see, the proton and the neutron are mutated from perfect spherical shapes when in vacuum under sole action-a-distance electromagnetic interactions, to a *oblate* charge distributions when constituents of the Deuteron, as expected in view of their high rotational conditions.

Note that *the deformability of nucleons under strong interactions does not imply the alteration of their volume due to their hyperdense character*. By assuming that the original semiaxes are normalized to 1, we then have the restriction on the numeric value of the remaining semiaxes

¹ The condition of partial mutual penetration of the charge distributions of protons and neutrons when nuclear constituents, can be easily derived by comparing the experimental values of nuclear volumes with those of protons and neutrons.

$$\frac{1}{n_1^2} + \frac{1}{n_2^2} + \frac{1}{n_3^2} = 3, \tag{66}$$

under which we obtain the value of the remaining semiaxes of the oblate spheroid under the evident identification

$$n_1 = n_2 = 1.635, n_3^2 = n_2^2 = 2.574 \tag{67}$$

Even though the above values are certainly not suggested to be final, we can state that isomathematics and isomechanics provides the first known numerical values of the semiaxes of the proton and the neutron when constituents of the Deuteron, in a way compatible with the numerically exact and time invariant representation of the anomalous magnetic moment of the Deuteron (Figure 6).

As indicated earlier, the extension of the above representation for the Deuteron was extended to all stable nuclei in memoir [36] via the general *isomechanics representation of nuclear magnetic moments* [36].

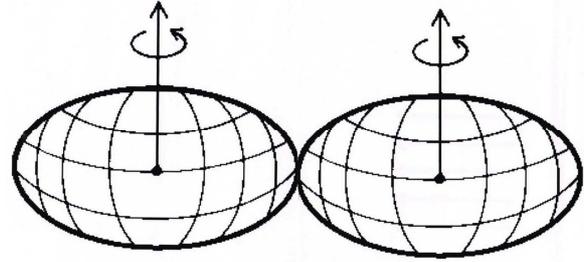


Figure 6. A conceptual rendering of the oblate shape of the proton and the neutron when constituents of the Deuteron in its ground state achieved, for the first time to our knowledge, by nuclear isomechanics from the anomalous magnetic moment of the Deuteron, with values of the semiaxes $n_3 = 0.449, n_1 = n_2 = 2.574$ (see Section 5). It should be stressed that the construction has been done along the conventional conception of the Deuteron, namely, with the proton and the neutron with parallel spins in order to represent the spin 1 of the Deuteron. However, quantum mechanical axioms predicts that the sole stable bound state between two particles with spin 1/2 is the singlet with antiparallel spins, since couplings with parallel spins are predicted to be highly unstable due to strong “repulsive” forces. Therefore, in Section 8 we shall first review the structure of the Deuteron according to nuclear isomechanics with a representation of the spin 1 without structural inconsistencies, and then provide a more accurate representation of the magnetic moment of the Deuteron.

$$\tilde{\mu}_N^{is} = \sum_{k=1, \dots, Z} \left[\frac{n_4}{n_{3kp}} (g_{pk}^S S + g_{pk}^L L) \right] + \sum_{k=1, \dots, A-Z} \left[\frac{n_4}{n_{3kn}} (g_{nk}^S S + g_{nk}^L L) \right], \tag{68}$$

where we have assumed that: all nucleons as nuclear constituents nuclei have the same hyper density geometrized by n_4 ; the deformation of the charge distributions may vary with the increase of the constituents; and anomalous orbital contributions may eventually emerge for heavier nuclei.

The verification that Eq. (68) provides indeed a representation of the magnetic moment of all stable nuclei will be shown in a subsequent paper. At this moment we merely limit ourselves with the representation later on of the magnetic moment light stable nuclei. The following comments are now in order:

5.1. Representation (65) is invariant over time because the mutation of intrinsic magnetic moments, Eq. (61) is a consequence of the Dirac-Santilli isoequation (which is invariant under the Poincaré-Santilli isosymmetry (Refs. [15-18] and Appendix A).

5.2. As one can see, representation (65) does not require the mutation of the spin of nuclear constituents because the sole mutation of the Minkowski spacetime into isospace (39) underlying the Dirac-Santilli isoequation (40) has been sufficient. This does not exclude extreme physical conditions, such as those at the core of stars that may require the mutation also of the spin.

5.3. As clearly shown by Eq. (62), the mutation of the intrinsic magnetic moment of nucleons under their conventional value of the spin creates the problem of the intrinsic compatibility of the approach. This problem is solved in Appendix A, where we show that the degree of freedom of regular isotopies of the SU(2) spin identified in Refs. [13, 14] can represent indeed the mutation of spin, thus achieving full compatibility under isomathematics and isomechanics

between mutation of intrinsic magnetic moments and conventional values of spins.

5.4. We should indicate that value (63) for the geometrization of the hyperdense medium inside nucleons has been derived via experimental data on different events, such as the fireball of proton-Antiproton annihilation in the Bose-Einstein correlation, while direct experimental data for nucleons are not available at this writing. Therefore, it is possible that value (63) and, consequently, value (64), may need revisions following direct test on the density of the medium inside nucleons.

6. Test of the Spinorial Symmetry Via Neutron Interferometry

It is evident that the deformability of protons and neutrons under sufficient external forces requires a direct experimental verification. The ideally suited test is the so-called 4π neutron interferometric experiment which consists of (see Figure 7): a thermal neutron beam which is first coherently split into two beams by a perfect crystal; one of the two split beams passes through the gap of an electromagnet with the magnetic field calibrated to such the value 7,496 G causing two complete spin flips (720° from which the name 4π) of the neutron on account of its intrinsic magnetic moment $-1.913148 \pm 0.000066 \mu_N$. The two beams are then coherently recombined. Various analysis are then conducted between the original beam and the recombined one.

When electromagnet gap is empty and, therefore, the split neutron beam travels in empty space, all known tests confirm

the achievement of two complete spin flips in full agreement with quantum mechanics. However, in order to avoid stray fields, the electromagnet gap is filled up with Mu-metal or other heavy metal sheets. In the latter case, the test essentially provides a *test of the spinorial symmetry of neutrons under the intense electric and magnetic fields in the vicinity of Mu-metal nuclei, without any appreciable contributions from the strong interactions of Mu-metal nuclei.*

The rather bizarre history of this fundamental test can be summarized as follows. The Austrian neutron interferometric experimentalist H. Rauch and his Austrian associate A. Zeilinger participated to the 1981 *Third Workshop on Lie-Admissible Formulations*, and presented preliminary results of a 4π neutron interferometric experiment that was going on via thermal neutron beams available at the nuclear facilities in Grenoble, France.

In particular, Rauch and Zeilinger reported at said 1981 meeting that they were *not* measuring 720° rotations, by rather the following values of minimal and maximal rotations [45]

$$\theta_{min} = 715.87^\circ, \theta_{max} = 719.67^\circ, \theta_{aver} = 712.07^\circ \quad (69)$$

which evidently *do not* contain 720° . In particular, Rauch and Zeilinger reported an angle of rotation systematically *smaller* than that expected, a feature referred to as the *angle slow-down effect* and expected to be due to a *decrease* of the

intrinsic magnetic moment of the neutron under the strong fields of the Mu-metal nuclei.

The Austrian theoretical physicist G. Eder [46] who also attended the indicated 1981 workshop by presenting a Lie-admissible mutation of the rotational symmetry representing the decrease of the intrinsic magnetic moment of the neutron under the considered conditions in agreement with data (69).

Based on these studies, Santilli [47] presented at the same 1981 workshop the notion of *Lie-admissible mutation of elementary particles* (also called *genoparticles* under strong nuclear interactions considered as external (which is a condition to sue the irreversible Lie-admissible formulations). It should be noted that the Lie-isotopic notion of isoparticle presented in this paper is an evident particular case of the notion of genoparticles presented in 1981.

Immediately following the announcement of the above studies, H. Feshbach, then chairman of the Department of Physics at MIT, strenuously opposed the completion of the 4π neutron interferometric experiment by Rauch and Zeilinger. the opposition, first by Feshbach and then by his world wide collective was such that Rauch was prohibited the access at his own laboratory in Grenoble and was, therefore, prohibited its completion (see Refs. [48] and their three volumes of documentations).

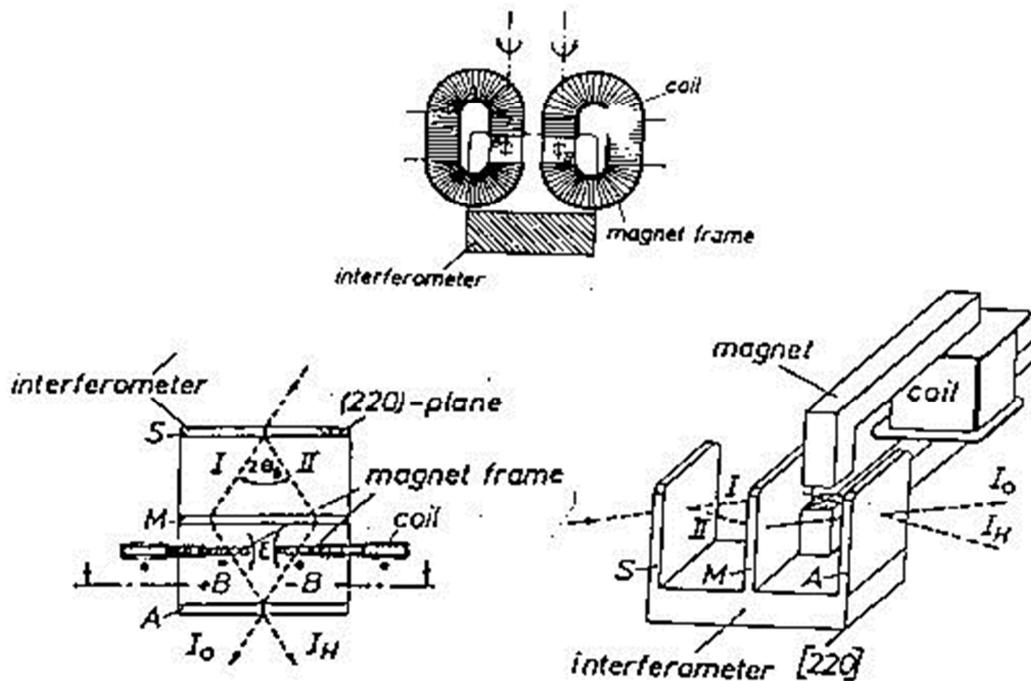


Figure 7. A schematic view of one of the most fundamental experiments for the past half a century, the 4π neutron interferometric spinorial symmetry experiments described in Section 6 [45-51]. The lack of resolution of this experiments due to political obstructions [48] is one of the reasons fueling the growing view according to which we are currently eyewitnessing one of the biggest scientific obscurantism in the history of mankind.

Subsequently, Rauch was offered the position of Director of the Atominstitut in Wien, Austria, while Zeilinger was invited for a one year stay at MIT after which he received a chair in physics at an Austrian university.

Following the above events in the early 1980s, the 4π

neutron interferometric experiment was occasionally repeated, but either without heavy metal sheets in the electromagnet gap, by splitting the gap into two opposite contributions or in other versions essentially assuring the verification of the exact spinorial symmetry.

To our best knowledge, the current situation (October 2015) is the following. On one side, Rauch and Zeilinger have dismissed measurements (69) and claim the exact validity of the 4π symmetry (without any systematic experimental resolution on record), as reported by D. Kendellen [49] (see also book [50]).

By contrast, Santilli [51] claims that; 1) an accurate and unbiased comparative analysis of the original and the recombined neutron beams show clear deviations from 4π rotations of at least 1%, even though the neutrons are solely exposed to *electromagnetic* interactions, thus expecting bigger deviations under nuclear strong interactions; 2) the deformability of the neutron is such a fundamental physical problem to require a systematic repetition of the 4π tests; and 3) Nowadays, the experiment can be repeated for a multiple of two complete rotations, with ensuing resolutory results (see Ref. [51] for details).

In the authors opinion, a reason for the incredible hostility by the nuclear physics community against this fundamental experiment is the lack of technical knowledge of the Lie-Santilli isothory according to which their fear of the violation of the "spinorial" symmetry in the 4π tests has no technical foundations because the experiment here considered deals with the *deformation of the charge distribution of the neutron while fully preserving its spin 1/2*. In fact, the authors believe that the very name "spinorial" symmetry experiment is erroneous and misleading, since the Fermi-Dirac character of the neutrons remains fully valid under a deformation of their charge distribution (Appendix A).

In the final analysis, the serious scientist should keep in mind that perfectly rigid bodies solely exist in academic environments but they do not exist in nature. Therefore, the serious scientific issue is the measurement of the deformation of the charge distribution of neutrons for given sufficiently strong external forces, with the understanding that the deformability itself should be outside credible doubts.

7. The Synthesis of the Neutron from the Hydrogen

As it is well known, stars initiate their life as an aggregate of Hydrogen. The first nuclear synthesis in the core of a star is that of the neutron from the Hydrogen atom according to the historical reaction [2]



Deuterium, Tritium and other nuclei are synthesized only following the synthesis of the neutron. It is then evident that the understanding of the first and most basic synthesis of the neutron is crucial for a deeper understanding of the subsequent nuclear syntheses.

Unfortunately, the synthesis of the neutron is vastly ignored even at the most important Ph. D. courses in nuclear physics because it is *incompatible with quantum mechanics and special relativity*. This is due to the fact that *the rest energy of the neutron is bigger than the sum of the rest energies of the*

proton and of the electron, as established by the known data

$$E_p = 938.272 \text{ MeV}, E_e = 0.511 \text{ MeV}, E_n = 939.565 \text{ MeV}, \tag{71a}$$

$$E_n - (E_p + E_e) = 0.782 \text{ MeV} > 0, \tag{71b}$$

Under these conditions, the Schrödinger equation does not yield physically consistent results due to the need for a "positive binding energy" resulting in a "mass excess" that are beyond any descriptive capacity of non-relativistic quantum mechanics.

Synthesis (70) is also incompatible with special relativity and relativistic quantum mechanics because the conventional Dirac equation, which is so effective for the description of the electron orbiting around the proton in the Hydrogen atom, becomes completely ineffective for the description of the same electron when "compressed" inside the proton in the core of a star according to Rutherford.

The proposal to build a non-unitary covering of quantum mechanics under the name of *hadronic mechanics*, including its isotopic and genotopic branches, was submitted in monograph [3b] precisely for the achievement of a quantitative representation of the synthesis of the neutron from the Hydrogen, and then apply the results to other nuclear syntheses.

Following decades of preparatory research [3-51], a numerically exact and time invariant representation of *all* characteristics of the neutron in its synthesis from the Hydrogen atom was achieved at the non-relativistic level via the Schrödinger-Santilli isoequation (31) in Refs. [52-54], and at the relativistic level via the Dirac-Santilli isoequations (40) in Refs. [18, 54].

The first laboratory synthesis of the neutron from a Hydrogen gas was done by the Italian priest-physicist Don Carlo Borghi and his associates in the mid 1960s [55]. Santilli conducted comprehensive tests for the laboratory synthesis of the neutron from the Hydrogen reported in Refs. [56-60]. The above body of scientific knowledge is now used by the U. S. publicly traded company *Thunder Energies Corporation* for the industrial production of a *Thermal Neutron Source* (see the, e.g., Ref.[61] video [62]. Excellent reviews of the mathematical, theoretical and experimental aspects for the synthesis of the neutron from the Hydrogen are available in Refs.[63, 64].

The following comments are in order:

7.1. Refs. [52-64] imply that the proton and the electron are actual physical constituents of the neutron, although in their mutated form known as "isoproton" and "isoelectron" [40] (see Appendix A). In fact, one of the necessary condition to achieve a numerical representation of all characteristics of the neutron in its synthesis from the Hydrogen is that the electron rest energy is mutated according to a mechanism today known as *isorenormalization*.

It should be indicated that these results turn the conjecture of undetectable and unconfined "point-like" quarks to a mathematical abstraction of the structure of hadrons because the proton and the electron are the only massive permanently

stable particles detected to date. As such, they cannot “disappear” (sic) at the time of the neutron synthesis to be replaced by the hypothetical quarks. Additionally, at the time of the neutron decay, quarks cannot “disappear” (sic) while the emitted proton and electron “reappear” (sic).

The name “hadronic mechanics” was suggested in Ref. [3b] precisely to permit a basically new structure model of all unstable particles with actual physical constituents, generally given by massive physical particles produced in their decay with the lowest mode. Advances along these lines have been reported in memoir [43].

It should be stressed that this new structure model of hadrons is *not* in conflict with the standard model of elementary particles because quarks remain necessary for its elaboration, although in their true scientific meaning of being purely mathematical representations of a purely mathematical internal symmetry formulated in a purely mathematical complex-valued unitary space.

We merely return to the teaching of all classifications that have historically required *two* different but compatible models, one model for the *classification* into families, and a different model for the *structure* of each element of a given family. The same historical teaching is confirmed by the fact that, in the transition from the classification to the structure of atoms there was the need for a new mathematical and physical theories. Similarly, in the transition from the classification of hadrons to their structure there is also the need for new mathematics and physical theories for the reasons indicated in Sections 1 - 3.

As a final comment, the serious scholar should be made aware of potentially large environmental and societal implications in abandoning the conjecture of the hypothetical and unconfined quarks as actual physical constituents of hadrons in favor of physical particles in their isotopic form. For instance, the admission of the isoelectron as a physical constituent of the neutron allows the conception and experimental study of a number of basically new clean nuclear energies, originally proposed in Refs. [65] and currently under study at *Thunder Energies Corporation* as well as at other companies. By contrast, the admission of the hypothetical quarks as the physical constituents of hadrons prohibits such possible environmentally large advances.

7.2. Refs. [52-64] imply that the neutrino does not appear to exist as physical particles, thus creating the intriguing problem of seeking alternative conceptions.

In his studies of synthesis (70), Enrico Fermi [1] had no other choice than that of representing the proton as a dimensionless point, resulting in the consequentially necessary hypothesis of the “neutrino” (meaning “little neutron” in Italian).

Thanks to the availability of the novel isomathematics (Section 2), in Refs. [52-64] we were able to represent the proton in its actual shape and dimension. This permitted the discovery of a new angular motion and related magnetic moment for the constrained rotation of the isoelectron when compressed in the hyperdense medium inside the proton (Figure 8), which new angular momentum is completely

absent when the proton is abstracted as a dimensionless particle.

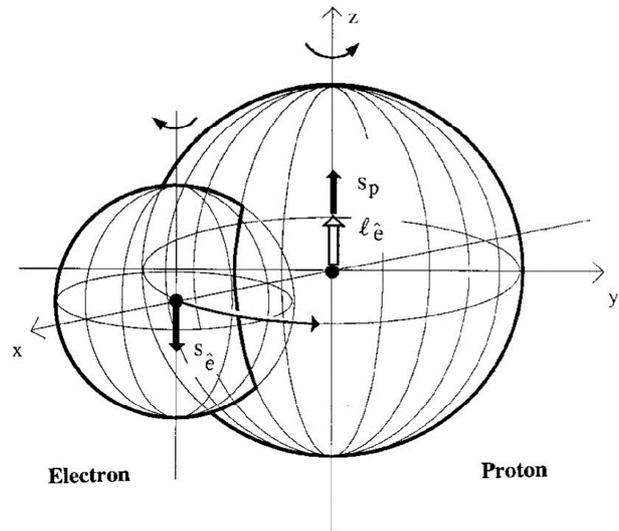


Figure 8. A basic novelty in Santilli's synthesis of the neutron from the Hydrogen atom is the appearance of a constrained angular motion of the electron when totally immersed within the hyperdense proton. This orbital motion eliminates the need for the emission of the hypothetical neutrino; is solely permitted by the representation of the proton as extended according to hadronic mechanics; and did not exist during Fermi's time since quantum mechanics can solely represent the proton as a massive point [52-63].

In turn, the constrained orbital motion of the isoelectron inside the proton must be equal to the proton spin (evidently to prevent that the extended wave-packet of the isoelectron moves within and against the hyperdense medium inside the problem), resulting in a null total angular momentum of the isoelectron in synthesis (70) as a result of which the spin of the neutron coincides with the spin of the proton.²

The conclusion is that studies [52-64] eliminate any possibility for the production of a neutrino in synthesis (70). In fact, the emission of a neutrino would *violate*, rather than verify, the conservation of the total angular momentum since the spin $1/2$ of the neutrino is represented by the constrained orbital angular moment of the isoelectron inside the proton. Additionally, reaction (70) already misses 0.782 MeV for the synthesis of the neutron. Any need for the additional energy to produce the hypothetical neutrino would cause catastrophic inconsistencies.

In a nutshell, Enrico Fermi did salvage the conservation of the angular momentum in the synthesis of the neutron with the hypothesis of the neutrino, but he did not salvage quantum mechanics and special relativity in the same synthesis.

7.3. Refs. [52-64] have the intriguing implications of implying the apparent return to the “continuous creation” in the universe as the most plausible way at this moment to explain the missing 0.782 MeV for the synthesis of the

² It should be recalled that half-odd-integer angular momenta are prohibited in quantum mechanics because they violate the unitarity of the theory, but they are fully allowed for the covering isomechanics precisely in view of its non-unitary structure (see Refs. [18, 22, 52-54] and Appendix A).

neutron from the Hydrogen atom.

One of the biggest mysteries in the synthesis of the neutron from the Hydrogen is the origin of the missing 0.782 MeV (assuming that the neutrino does not exist, otherwise the missing energy would be much bigger). This energy cannot be provided by the relative kinetic energy between the proton and the electron because at that energy value their cross section is virtually null, thus prohibiting any synthesis.

Additionally, the missing energy of 0.782 MeV cannot be provided by the star because, at the initiation of nuclear syntheses, stars synthesize up to 10^{50} neutrons per second. The assumption that the missing energy is provided by the star would then imply that the star *loses* about 10^{50} MeV per second, under which conditions a star would never initiate the majestic event of producing light.

In an attempt to initiate the solution of this mystery, Santilli has suggested that the missing energy of 0.782 MeV is provided by space conceived as a universal substratum with a very high energy density. via a "longitudinal impulse" (rather than a particle) submitted under the name of "etherino" with the symbol "*a*" (from the Latin aether), thus implying the replacement of the quantum mechanical reaction (70) with the isomechanical reaction [66]



where one should note the need for the energy carrying impulse to be in the left (rather than the right) of the reaction, and that the use in the left of the antineutrino would increase the missing energy due to its negative energy state [*loc. cit.*].

It should be noted that the historical hypothesis of the neutrino was essentially dismissed by the lack of detection of the "solar neutrinos" (namely, neutrinos emitted by the Sun during its synthesis of the neutron), according to which our particle laboratories should be traversed by an extremely large flux of neutrinos none of which has been detected with such evidence to be acceptable by the scientific community at large.

The advent of the standard model has produced additional reasons for the dismissal of neutrinos since the standard model requires a variety of different neutrinos without clear physical differences, all neutrinos being assumed to have a mass. It is now widely accepted that particles with mass simply cannot traverse nuclei, planets and stars with a very small or no scattering, thus mandating a basically new interpretation of physical reality.

The hypothesis of the etherino has been submitted because of a possible resolution of these insufficiencies via a more realistic interrelation of experimental data. In fact, the traversing to nuclei, planets and stars without appreciable scattering is more plausibly interpreted by the etherino rather than by the neutrino, since the former refers to a longitudinal impulse propagating through the universal substratum, while the latter is assumed to be a massive particle that should traverse without appreciable scattering hyperdense media inside nuclei, planets and stars.

We should also clarify that a number of claimed "experimental verifications" of the neutrino do not refer to the

direct detection of the neutrino which is impossible, but refer to the detection of ordinary particles predicted as being emitted under the neutrino hypothesis. The point is that the emission of exactly the same particles is predicted by the etherino and perhaps other hypotheses. Finally, we should indicate that the claimed "experimental verifications" of the neutrino hypothesis are based on very few events out of billions of events, thus lacking the credibility needed to resist the test of time.

In summary, the lack of existence of the neutrino as a physical particle emitted in the synthesis of the neutron creates one of the most fascinating scientific problems in history, that of the possible continuous creation in the universe (see, e.g., the historical paper [67]), since the missing energy for the neutron synthesis is "created" in the core of stars in the sense that it is acquired from the universal substratum. In turn such a fascinating problem has implications for virtually all quantitative sciences, including lack of expansion of the universe due to loss of energy by galactic light to the intergalactic medium [68], possible future interstellar travel at arbitrary speeds whose energy source would be permitted by a universal substratum with very high energy density [38], and other intriguing open problems.

8. Three-Body Structure of the Deuteron According to IsoMechanics

There comes a moment in the life of a serious scientist at which physical realities have to be admitted, no matter how against preferred doctrines, as a condition not to exit from the boundaries of science.

The physical reality here referred to is that *despite more than half a century of attempts, quantum mechanics has failed to achieve a constant representation of the structure of the simplest nucleus, the Deuteron, with embarrassing deviations for heavier nuclei*, in view of the following insufficiencies [69]:

8.1. *Quantum mechanics has been unable to represent the stability of the Deuteron.* As it is well known, the neutron is naturally unstable when isolated. Therefore, quantum mechanics has failed to explain how the neutron becomes permanently stable when bonded to the proton in the structure of the Deuteron.

8.2. *Quantum mechanics has been unable to achieve a consistent representation of the spin 1 of the ground state of the Deuteron.* The basic axioms of quantum mechanics require that the stable bound state of one proton and one neutron is the singlet with total spin zero, while the spin of the Deuteron is 1. For the intent of maintaining quantum mechanics, 20th century nuclear physics has assumed a combination of orbital states requiring excited conditions which are in direct contradiction with the physical evidence that the spin 1 occurs for the Deuteron in its "ground" state.

8.3. *Quantum mechanics has been unable to identify the physical origin of the attractive force binding the proton and the neutron in the Deuteron.* Since the neutron is neutral, there

is no known electrostatic origin of the attractive force needed for the existence of the Deuteron, while their magnetostatic force is “repulsive” in their triplet coupling. As a result of these occurrences, a “strong” force was conjectured for the bond of nuclear constituents [2] and its existence was subsequently confirmed. Nevertheless, the physical origin of the strong nuclear force has remained unidentified by quantum mechanics to this writing.

8.4. *Quantum mechanics has been unable to achieve a consistent representation of the Deuteron space parity.* According to experimental evidence, of the space parity is positive for the deuteron in its ground state because the angular momentum is null, while the quantum mechanical representation of the spin 1 of the Deuteron requires excited orbital states, resulting in an additional direct conflict between quantum predictions and experimental realities.

8.5. *Quantum mechanics has been unable to reach an exact representation of the magnetic moment of the Deuteron,* as discussed in Section 5.

Following the achievement of the non-relativistic and relativistic presentation of the structure of the neutron as a bound state of one isoproton and one isoelectron (Refs. [51-54] and Section 7), Santilli proposed in Part V of monograph [69] the structure of the deuteron according to isomechanics as a

three body bound state of two isoprotons in triplet coupling and one isoelectrons with null total angular momentum which is exchanged in between the two isoprotons as a kind of isogluon, hereon referred to as the “iso-Deuteron” (see Figure 9).

The new three-body structure model of the Deuteron achieves a numerically exact and time invariant representation of *all* characteristics of the Deuteron, including its binding energy, charge radius, stability, spin, parity, etc., which representation is here assumed as known for brevity from Ref. [69] (see also the excellent reviews [33, 70]).

The conceptual and, therefore, the most important reasons for the proposal of the iso-Deuteron were several [69]. The first origination is that *the reduction of the Deuteron to protons and electrons (although in a mutated form) sets clear foundations for stability* since the proton and the electron are the only stable massive particle known to mankind.

The second origination of the iso-Deuteron is that *the spin 1 of the Deuteron is direct evidence that it is a “three-body,” rather than a two-body state*, because the configuration of two nucleons in triplet coupling, which is necessary for the representation of the spin 1 in the ground state, can only be achieved in a consistent way via the addition of a third particle with null total angular momenta as in Figure 9.

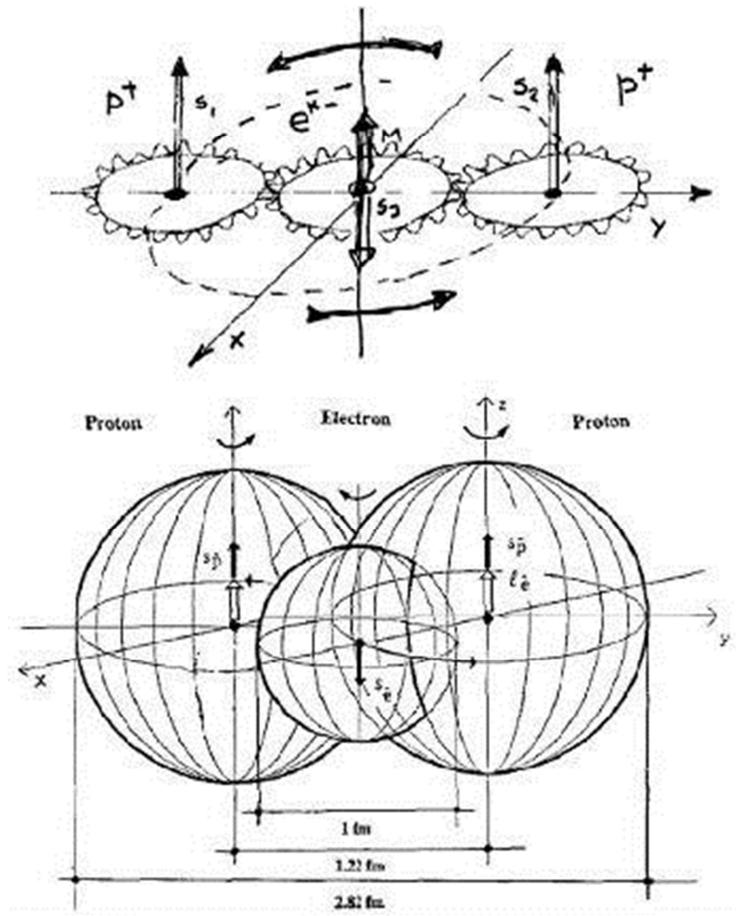


Figure 9. A schematic view from Part V of Ref. [69] on the structure of the Deuteron following the reduction of the neutron to a hadronic bound state of an isoproton and an isoelectron. Note from the top view that the two isoprotons are in triplet coupling, while the isoelectron with null total angular momentum is exchanged in between them, thus allowing the first known representation of the spin 1 of the Deuteron in its true ground state.

The third origination of the iso-Deuteron is that *isomathematics and isomechanics are the only known methods achieving an explicit and concrete strongly "attractive" force in the Deuteron structure.* In the transition from quantum mechanical to isomechanical nuclear models via the non-unitary transform of Section 4, and realization of the isotopic element of type (1), there is the emergence of a strongly attractive Hulthén potential (see Ref. [69] for details) originating from the partial mutual penetration of the deformed charge distributions of the constituents. Note that in the structure of the iso-Deuteron there is *no* repulsive electrostatic force due to the continuous exchange of the isoelectron between the two isoprotons.

Particularly significant for this paper is the deeper representation of the anomalous magnetic moment of the Deuteron which is permitted by its three-body isotopic structure. In Section 5, we presented a first representation of the magnetic moment of the Deuteron based on its representation as a bound state of an isoproton and an isoneutron in triplet coupling to represent the spin 1 (see Figure 9).

However, as also indicated in Section 5, this representation is basically insufficient because the triplet coupling of Figure 3 generates strongly *repulsive* forces under which no stable bound state is possible. Santilli's three-body model of the iso-Deuteron allows an exact and time invariant representation of the magnetic moment without any known inconsistencies, which is essentially given by the muted magnetic moments of the two isoproton, plus a contributions from the isoelectron (see Ref. [69] for details).

This section concludes the review of past advances in nuclear physics permitted by isomathematics and isomechanics that are necessary for an understanding of the numerically exact and time invariant representation of the spin of stable nuclides presented in the following sections.

9. Stable and Unstable Nuclides

Notice that deuteron is the simplest nuclide having one proton and one neutron and is stable. However, we see that it, in fact, is an *isonuclide*. When we survey the elements of the periodic table we find that out of 289 primordial nuclides 254 are stable ones. The stability of nuclides depends also on evenness or oddness of its atomic number Z , neutron number N and, consequently, of their sum, the mass number A . Oddness of both Z and N tends to lower the nuclear binding energy, making odd nuclei, generally, less stable. This fact we have depicted [71] in Table 1.

However, in this paper, we are presenting, apparently for the first time, a structure model of stable nuclides of the first three rows of the periodic table, hereon called *stable isonuclides*, as bound states of extended, thus deformable isoprotons and isoelectrons according to the laws of hadronic mechanics, under the condition of recovering in first approximation the conventional structure model of nuclides as quantum mechanical bound states of point-like protons and

neutrons.

We shall then show, also apparently for the first time, that the reduction of nuclides to isoprotons and isoelectrons allows the first known achievement of an exact representation of the spin of all stable nuclides.

Table 1. Even and odd nucleon numbers. A is the atomic mass number, Z is the atomic number, N is the number of neutrons in the nucleus, EE is the even-even proton-neutron combination, OO is the odd-odd proton-neutron combination, EO is the even-odd proton-neutron combination and OE is the odd-even proton-neutron combination.

A Z, N	Even		Odd		Total
	EE	OO	EO	OE	
Stable	148	5	53	48	254
	153		101		
Long-lived	22	4	4	5	35
	26		9		
All primordial	170	9	57	53	289
	179		110		

Next we will indicate without treatment that the reduction of nuclides to isoprotons and isoelectrons puts the foundations for an exact representation of the magnetic moment of all nuclides for studies to be presented in a subsequent paper. We shall also indicate, for studies in a subsequent paper, that the transition of the nuclear structure from that in terms of point-like protons and neutrons to that in terms of is extended, thus deformable isoprotons and isoelectrons offers realistic possibilities for studying basically new forms of clear nuclear energies.

10. Old and New Vistas in Nuclear Forces

For the semi-quantitative discussion conventionally one uses the following expression of nuclear binding energy, namely:

$$\frac{BE}{\text{MeV}} = 931.4(Z \times m_H + (A - Z) \times m_n - M) \quad (73)$$

where m_H and m_n are the masses on amu scale of hydrogen and neutron respectively and M is atomic mass on amu scale of the given element. Notice that the mass of electrons has not been included separately in the above expression because it remains included in m_H . The standard plot of binding energies of all nuclides is shown in Figure 10.

Glasstone [72] further asserts that the nuclear binding energy is the result of $(n - n)$, $(n - p^+)$ and $(p^+ - p^+)$ forces operating within the nucleus. The experimental data on the nuclear scattering and correspondence of binding energies of the identically same mass number elements (isobars) it was concluded that the magnitudes of $(n - n)$, $(n - p^+)$ and $(p^+ - p^+)$ forces of attraction are almost equal [72].

In view of the above assertion it was expected that the diproton and the dineutron nuclei should be stable as deuteron is a stable nucleus (which consists of one proton and one neutron). But so far neither of the former two particles have

been detected as stable particles. However, in proton-proton chain reaction it is suspected that a di-proton is formed in the first step which immediately disintegrates into two protons ($> 99.99\%$) and to deuteron plus β^+ ($< 0.01\%$) (however

the measurement of corresponding half-lives could not succeed) [73]. Of course, it is certain that there is no nucleus made up only of two neutrons because it doesn't constitute a chemical element.

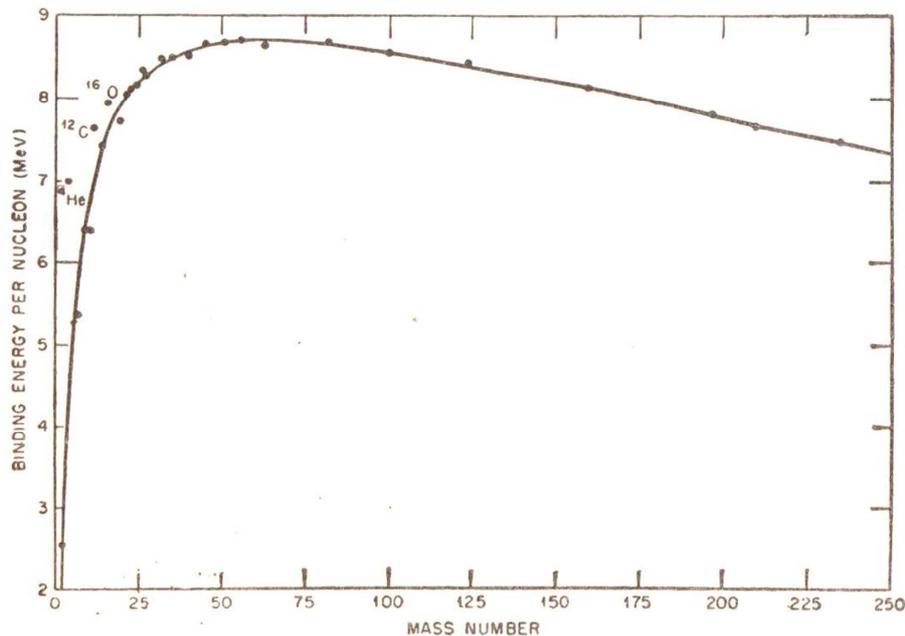


Figure 10. Binding Energy per nucleon as a function of mass number of stable nuclides.

On the other hand, the existence of a strong attraction between the pair ($n-p^+$) is exemplified by the stability of deuteron and the stable nuclides He-4, Li-6, B-10, C-12, N-14, O-16, Ne-20, Mg-24, Si-28, S-32, Ar-36 and Ca-40, they all have equal number of protons and neutrons. Besides these nuclides in other stable nuclides we do have neutrons and none of the neutrons disintegrates. The stabilization of neutrons in a nucleus is a subject matter of nuclear physics and the 20th century attempts to explain the said stability are based on quantum mechanics but a satisfactory quantum mechanical description still eludes [18, 22, 23, 33, 36].

The reader may very well notice that the structures of neutron and deuteron that Santilli had proposed, which we have described in brief in Sections 7 and 8 respectively, in fact, are in the form of isoneutron and isodeuteron respectively. These structures involve the mutual deep but partial penetration of the wave packets of electron and proton(s) (c.f. Figures 8 and 9). Thus the quantum mechanical perception of these particles as the point particles has been replaced by the respective tiny but finite size particles all well within the hadronic horizon. However, when we go beyond deuteron the size of the nucleus grows but less significantly (the standard cube root formula [74] estimates the nuclear radius of 1.25 fm for hydrogen nucleus to 4.275 fm for calcium-40 nucleus. Thus the nucleons have increased 40 times but the nuclear radius has increased only 3.4 times).

Therefore, on the lines of the structure of a neutron and a deuteron proposed by Santilli we hereby, apparently for the first time, propose that,

1. an atomic nucleus is composed of nucleons as particles

of tiny volume of hadronic dimensions,

2. the wave packets of nucleons penetrate mutually but partially that produces strong nuclear force and
3. the said mutual penetration of wave packets between heteronucleons perhaps produces very strong attractive force compared to that between homonucleons.

This is what has been indicated in Section 1 and shown in Figure 1, that is — the nucleons within a nucleus are in a state of mutual but partial penetration of their wave packets. Thus all nucleons in a nucleus, in fact, are the isonucleons, namely isoneutrons, isodeuterons, isoelectrons and isoprotons.

Of course, one needs to investigate and evaluate quantitatively the magnitude of nuclear forces so generated via the methods of hadronic mechanics but at this juncture we consider that it would be profitable first to generate nuclear configuration of stable nuclides as if the nucleus of all stable nuclides are composed of isonucleons, which is likely to present enough ground for carrying out the detailed investigation of the corresponding quantitative hadronic physics. Indeed, we have presented in Section 3 a brief description of Santilli's initial work on nuclear isomechanics and genomechanics.

In the following Section 11 we will see that there are two options for developing nuclear configuration. The first one, the model-I, is through the isodeuterons, isoneutrons and isoprotons as the building nucleons and the second option, the model-II, is through the isoprotons and isoelectrons as the building nucleons, both of them are easily interconvertible. We will also discuss the advantages and limitations of each.

11. Notations for Representation of IsoNeutron and IsoDeuteron

In order to develop the nuclear configuration of nuclides the first logical option is offered by the fact that the deuteron is a stable nuclide similar to a proton. In Section 8 we have described that the deuteron is a hadronic bound state of an isoneutron and an isoproton. But as described in Section 7 the neutron is indeed an isoneutron, which is a hadronic bound state of one isoproton and one isoelectron. However, the isoneutron is an unstable nuclide, which decays radiatively by β^- emission with half-life of 614.6 s [75] (In 1967 experiment the half-life of free neutron was recorded as 10.8 min [76]). But when it makes a union with an isoproton its instability vanishes altogether. Hence in this hadronic choice we have developed nuclear configuration of stable nuclides commensurate with the observed nuclear spin using isodeuterons, isoneutrons and if required used isoprotons. However, recall that each isodeuteron is made up of 2 isoprotons of parallel spin and one isoelectron of zero spin, and the isoneutron consists of one isoproton of half spin and one isoelectron of zero spin hence it is easy to convert the nuclear configuration of the first choice into the one in terms of isoprotons of 1/2 spin, isoprotons of -1/2 spin and isoelectrons of zero spin, that is our second choice. However, we can directly write the nuclear configuration in the second choice just by choosing correct number of isoprotons with 1/2 and -1/2 spin commensurate with the experimental nuclear spin, because isoelectron doesn't contribute to the nuclear spin.

A simple notation to represent Santilli's isoneutron, \hat{n} , is as given below as a compressed hydrogen atom, namely:

$$ha = (p^+, e^-)_{qm} \rightarrow (\hat{p}^+(\uparrow), \hat{e}^-(J=0))_{hm} \equiv \hat{n}(\uparrow) \quad (74)$$

where ha denotes the hydrogen atom; qm denotes quantum mechanics; p^+ denotes the conventional proton; e^- denotes the conventional electron; hm denotes hadronic mechanics; \hat{p}^+ denotes the isoproton; \hat{e}^- denotes an isoelectron; J is the spin and \uparrow denotes spin 1/2. The total angular momentum of the isoelectron is null because the particle is constrained to rotate within the hyperdense proton in singlet coupling, thus acquiring a value of the orbital angular momentum equal but opposite to its spin (Figure 8).

Similarly, the notation of an isodeuteron, \hat{d} , is obtained as given below, namely:

$$\begin{aligned} d(J=?) &= (p^+(\uparrow), n(\downarrow))_{qm} \rightarrow (\hat{p}^+(\uparrow), \hat{e}^-(J=0), \hat{p}^+(\uparrow))_{hm} \\ &\equiv \hat{d}(J=1) = \hat{d}(\uparrow\uparrow) \end{aligned} \quad (75)$$

where \downarrow denotes the spin -1/2. The spin 1 of the isodeuteron is because of two up spins, $\uparrow\uparrow$, of two isoprotons.

The stability of deuteron gets excellently explained by the Santilli iso-deuteron model, Eq. (75). Namely, as the structure $(\hat{p}^+(\uparrow), \hat{e}^-(J=0))_{hm}$ is unstable, there is a natural tendency

of the bound electron in $(\hat{p}^+(\uparrow), \hat{e}^-(J=0), \hat{p}^+(\uparrow))_{hm}$ to get released from the grip of its isoproton to which it is bound at the given instant of time, but no sooner it succeeds in getting released it immediately gets trapped into the hyper-dense medium of the other very closely placed proton. This is how isodeuteron enjoys its stability against radioactivity. This interpretation of nuclear stability and instability reasonably good.

In the next Section 12 we consider only the stable nuclides of periodic table up to the atomic number 82.

12. Proposed Nuclear Configuration of Stable IsoNuclides

We adopt ${}^A_Z X_N(J) = X(A, Z, J)$ to represent nuclides, where X represents the symbol of the chemical element, A is the mass number i.e. the total number of protons and neutrons, Z is the atomic number i.e. the total number of protons, N is the total number of neutrons, and J is the nuclear spin. Obviously $(A-Z)$ is the total number of neutrons, N , in the nucleus. Notice that we have incorporated nuclear spin, J , in the conventional representation of nuclide.

In this paper we propose, apparently for the first time, the extension of Santilli isodeuteron to all stable nuclides under the proposed name of IsoNuclides with the symbol ${}^A_Z \hat{X}_N(J)$. Notice that in this notation we have still retained the symbols A, N, Z because it would be easy to correlate with conventional description.

Now as stated in preceding sections there are two options for developing nuclear configuration of nuclides.

In the *model-I* the adopted working rule is that we are bound by the requirement of producing that nuclear configuration which predicts correctly the experimental nuclear spin. Our method is further based on the observed stability of an isodeuteron that indicates that the isonucleons of a nuclide first prefer to adopt the isodeuteron structures and in this way the unaccounted neutrons and protons stay in the nucleus as isoneutrons and isoprotons with appropriate spin orientation.

In the *model-II* we fix the number of isoelectrons equal to the number of neutrons (because in a nucleus an isoelectron with null spin is carried into through the neutron as isoneutron) and obviously the number of isoprotons of a nucleus equals to the mass number, A , of the nuclide. Thus our method is then to choose the number of isoprotons with spin 1/2 and -1/2 that correctly predicts the experimental nuclear spin of the nuclide.

12.1. Isodeuteron, Isoneutron and Isoproton as Constituents of Atomic Nuclei. Model-I

In this first option with the guidelines described above in this Section 12 we note that ${}^3_2 \hat{\text{He}}_1(0)$ can be readily interpreted as a hadronic bound state of an isodeuteron and an isoproton in singlet coupling (perhaps necessary for stability). Accordingly it gets represented as under,

$$\begin{aligned}
 {}^3_2\hat{\text{He}}_1(1/2) &= (\hat{d}(\uparrow\uparrow), \hat{p}^+(\downarrow))_{hm} \\
 &\equiv ({}^2_1\hat{d}_1(1), \hat{p}^+(-1/2))_{hm} \quad (76)
 \end{aligned}$$

Notice that in this isonuclide there we have one separate isoproton and two mutated protons as isoprotons in the form of $\hat{d}(\uparrow\uparrow)$.

Similarly, ${}^4_2\hat{\text{He}}_2(0)$ can be readily interpreted as a hadronic bound state of two isodeuterons in singlet coupling, namely,

$$\begin{aligned}
 {}^4_2\hat{\text{He}}_2(0) &= (\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow))_{hm} \\
 &\equiv ({}^2_1\hat{d}_1(1), {}^2_1\hat{d}_1(-1))_{hm} \quad (77)
 \end{aligned}$$

Along the same lines, ${}^6_3\hat{\text{Li}}_3(1)$ can be readily interpreted as a hadronic bound state of ${}^4_2\hat{\text{He}}_2(0)$ and one isodeuteron ${}^2_1\hat{d}_1(1)$

$${}^6_3\hat{\text{Li}}_3(1) = \left({}^4_2\hat{\text{He}}_2(0), {}^2_1\hat{d}_1(1) \right)_{hm} \quad (78)$$

and similarly for the remaining stable nuclides (see Table 2).

For the isotopic structure model of ${}^7_3\hat{\text{Li}}_4(3/2)$ we have the more complex model

$${}^7_3\hat{\text{Li}}_4(3/2) = \left({}^4_2\hat{\text{He}}_2(0), {}^2_1\hat{d}_1(1), \hat{n}(1/2) \right)_{hm} \quad (79)$$

Therefore, we can symbolically write the *nuclear configuration of stable isonuclides* as under,

$$\begin{aligned}
 {}^A_Z\hat{\text{X}}_N(J) &= [x_1({}^2_1\hat{d}_1(1)), x_2({}^2_1\hat{d}_1(-1)), \\
 &x_3(\hat{n}(1/2)), x_4(\hat{n}(-1/2)), \\
 &x_5(\hat{p}^+(1/2)), x_6(\hat{p}^+(-1/2))] \quad (80)
 \end{aligned}$$

where $\hat{\text{X}}$ denotes the isonuclide, x_i 's are the number of the isonuclear or nuclear species depicted in the braces next to them. Notice that in this model-I in any nucleus the isoprotons would be in the form of isoneutrons, isodeuterons and remaining as separate isoprotons hence if the atomic number of a nuclide demands more protons than those accounted by isodeuterons and isoneutrons (the striking example is that of He-3, c.f. Eq. (76) they will be separate mutated protons (i.e. the isoprotons). In view of this in above expression (80) the last two terms on the right hand side account for the separate isoprotons that are demanded by its atomic number, Z .

In this way the expression of the atomic mass number, A , is obtained as,

$$A = 2x_1 + 2x_2 + x_3 + x_4 + x_5 + x_6 \quad (81)$$

the atomic number, Z , is given by,

$$Z = x_1 + x_2 + x_5 + x_6 \quad (82)$$

Therefore, obviously the number of nuclear neutrons, N , is given by,

$$N = A - Z = x_1 + x_2 + x_3 + x_4 \quad (83)$$

Whereas, the total number of isoprotons $\mathbb{P}_{\hat{p}^+}$, get computed as,

$$\mathbb{P}_{\hat{p}^+} = 2x_1 + 2x_2 + x_3 + x_4 + x_5 + x_6 \quad (84)$$

and the total number of isoelectrons, $\mathbb{E}_{\hat{e}^-}$, get computed as,

$$\mathbb{E}_{\hat{e}^-} = x_1 + x_2 + x_3 + x_4 \quad (85)$$

It is no wonder that $N = \mathbb{E}_{\hat{e}^-}$ because with each isoneutron there is associated one isoelectron. Moreover, the nuclear spin J gets computed as,

$$J = x_1 - x_2 + \frac{1}{2}x_3 - \frac{1}{2}x_4 + \frac{1}{2}x_5 - \frac{1}{2}x_6 \quad (86)$$

Therefore, the isonuclide, ${}^A_Z\hat{\text{X}}_N(J)$, gets reduced to isoprotons, \hat{p}^+ and isoelectrons, \hat{e}^- , that gets expressed as,

$${}^A_Z\hat{\text{X}}_N(J) = \left(\mathbb{P}_{\hat{p}^+}, \mathbb{E}_{\hat{e}^-} \right) \quad (87)$$

12.2. Isoprotons and Isolelectrons as Constituents of Atomic Nuclei. Model-II

Recall that all nuclear protons are indistinguishable whereas the isoprotons of the nuclear isoneutrons too remain indistinguishable because the isoneutrons have a natural tendency to get converted to protons. Therefore, we cannot label which proton out of the available nuclear protons at a given instant of time is actually bound to an isoelectron. In this way there must be on an average at a given moment of time a fixed number of isoprotons and the same number of isoelectrons, and remaining number of nucleons are the protons and are equal to the atomic number of the chemical element. However, in view of the housing of all protons and neutrons in extremely small nuclear volume (see also Section 10) there must be at least partial mutual penetration of wave packets of protons besides in addition to that with the wave packets of electrons that describe the isoneutron and isodeuteron. Hence all nuclear protons and neutrons taken together need to be treated as an assemblage of isoprotons and isoelectrons. Of course, the mutual penetration of wave packets of protons and the mutual penetration of wave packets of electrons and protons would definitely produce different hadronic effects hence needs to be quantitatively investigated

by the tools of hadronic mechanics. Therefore, in this *model-II* we treat every nucleon of a nucleus an isonucleon.

Thus the counter part of Eq. (80) in this case would read as,

$$\begin{aligned}
 {}^A_Z\hat{X}_N(J) = & [2x_1(\hat{p}^+(1/2)), 2x_2(\hat{p}^+(-1/2)), \\
 & x_3(\hat{p}^+(1/2)), x_4(\hat{p}^+(-1/2)), \\
 & x_5(\hat{p}^+(1/2)), x_6(\hat{p}^+(-1/2)), \\
 & (x_1 + x_2 + x_3 + x_4)(\hat{e}^-(0))] \quad (88)
 \end{aligned}$$

which gets simplified to,

$$\begin{aligned}
 {}^A_Z\hat{X}_N(J) = & [(2x_1 + x_3 + x_5)(\hat{p}^+(1/2)), \\
 & (2x_2 + x_4 + x_6)(\hat{p}^+(-1/2)), \\
 & (x_1 + x_2 + x_3 + x_4)(\hat{e}^-(0))] \quad (89)
 \end{aligned}$$

where the number of isoprotons with spin 1/2, $\mathbb{P}(1/2)$, is given by,

$$\mathbb{P}(1/2) = 2x_1 + x_3 + x_5, \quad (90)$$

number of the isoprotons with spin -1/2, $\mathbb{P}(-1/2)$, is given by,

$$\mathbb{P}(-1/2) = 2x_2 + x_4 + x_6, \quad (91)$$

and number of the isoelectrons with spin 0, $\mathbb{E}(0) = N$, is given by,

$$\mathbb{E}(0) = x_1 + x_2 + x_3 + x_4. \quad (92)$$

Alternatively, we can directly express ${}^A_Z\hat{X}_N(J)$ as follows,

$${}^A_Z\hat{X}_N(J) = [\mathbb{P}(1/2), \mathbb{P}(-1/2), \mathbb{E}(0)] \quad (93)$$

where $\mathbb{P}(1/2) + \mathbb{P}(-1/2) = A$ and $Z = \mathbb{P}(1/2) + \mathbb{P}(-1/2) - N$. Since, all nuclear spins are null or positive numbers we have $\mathbb{P}(1/2) > \mathbb{P}(-1/2)$.

We would like to stress that the methods of writing nuclear configuration described above are entirely general that make no distinction between stable and unstable nuclides. However, with the above adopted notations we are now well equipped to build the nuclear configuration of stable nuclides as isonuclides ${}^A_Z\hat{X}_N(J)$, that we present in the next Section 13.

13. Hadronic Mechanics Based Configuration of Stable Nuclides

In this paper we are primarily presenting the nuclear configuration of the stable nuclides. The nuclides of atomic

number higher than 82 are all radioactive therefore we have developed the nuclear configuration up to the chemical element Pb. Now onwards we will use the short hand notation of an isoneutron and an isodeuteron given in the extreme right hand side of Eqs. (74) and (75), namely $\hat{n}(\uparrow)$ and $\hat{d}(\uparrow\uparrow)$ respectively. Moreover, henceforth all nuclear protons would be treated as isoprotons whether the wave packet of any one of them penetrates with that of an isoelectron or not. This is so because as discussed in Section 10, in view of the extremely small size of atomic nuclei, all nuclear protons indeed get transformed to isoprotons.

13.1. Nuclear Configuration of Stable Isotopes as Isonuclides. Model-I

The observed stability of deuteron does indicate that the stable nuclides first prefer to have the isodeuteron structure from the available number of neutrons and protons. Whereas the remaining unaccounted neutrons and protons stay in the nucleus as isoneutrons and isoprotons.

Thus we have followed a nuclear version of the Aufbau type principle with the requirement that the resulting nuclear configuration should correctly predict the observed nuclear spin of each isotope of the elements. We are presenting in column 3 of Table 2 the so arrived at nuclear configuration of the stable isonuclides up to the element Pb of the periodic table along with the observed nuclear spin (in column 5) against each isonuclide for the ready reference. All the nuclear spins reported now onwards are taken from the Ref. [80] unless otherwise other sources are cited.

13.2. Nuclear Configuration of Stable Isotopes as Isonuclides. Model-II

The nuclear configuration in terms of isoprotons and isoelectrons that replicate the observed nuclear spin is easy to write. We first write number of isoelectrons equal to the number of neutrons, N , in the nucleus and then write the number of isoprotons equal to the mass number, A , of the nuclide, which then is distributed in up and down spin isoprotons so that the net spin of the combination equals the experimental nuclear spin.

Equivalently, on realizing that each isodeuteron has two isoprotons of same spin and one isoelectron of null spin, and the isoproton of each isoneutron has the same spin as that of the latter. The total number of nuclear isoelectrons is given by the sum of the number of isodeuterons and isoneutrons in a given isonuclide. The nuclear configuration of the *model-II* has been listed in the column 4 of Table 2. Notice that the nuclear configuration in this option of all nuclei correctly predicts the respective observed nuclear spin.

Table 2. Nuclear configuration of stable, primordial and very long lived isonuclides for nuclear model-I and model-II

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
1	${}^1_1\text{H}_0(1/2)$ Proton (not an isonuclide)	$p^+(\uparrow)$	$p^+(\uparrow)$	1/2
	${}^2_1\hat{\text{H}}_1(1)$ (isodeuteron)	$\hat{d}(\uparrow\uparrow)$	$2\hat{p}^+(\uparrow), \hat{e}^-(\uparrow\downarrow)$	1
2	${}^3_2\hat{\text{He}}_1(1/2)$	$\hat{d}(\uparrow\uparrow), \hat{p}^+(\downarrow)$	$2\hat{p}^+(\uparrow), \hat{p}^+(\downarrow), \hat{e}^-(\uparrow\downarrow)$	1/2
	${}^4_2\hat{\text{He}}_2(0)$	$\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow) \equiv \left[{}^4_2\hat{\text{He}}_2(0) \right]$	$2\hat{p}^+(\uparrow), 2\hat{p}^+(\downarrow), 2\hat{e}^-(\uparrow\downarrow)$	0
3	${}^6_3\hat{\text{Li}}_3(1)$	$\left[{}^4_2\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow)$	$4\hat{p}^+(\uparrow), 2\hat{p}^+(\downarrow), 3\hat{e}^-(\uparrow\downarrow)$	1
	${}^7_3\hat{\text{Li}}_4(3/2)$	$\left[{}^4_2\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow), \hat{n}(\uparrow)$	$5\hat{p}^+(\uparrow), 2\hat{p}^+(\downarrow), 4\hat{e}^-(\uparrow\downarrow)$	3/2
4	${}^9_4\hat{\text{Be}}_5(3/2)$	$\left[{}^4_2\hat{\text{He}}_2(0) \right], 2\hat{d}(\uparrow\uparrow), \hat{n}(\downarrow)$	$6\hat{p}^+(\uparrow), 3\hat{p}^+(\downarrow), 5\hat{e}^-(\uparrow\downarrow)$	3/2
5	${}^{10}_5\hat{\text{B}}_5(3)$	$\left[{}^4_2\hat{\text{He}}_2(0) \right], 3\hat{d}(\uparrow\uparrow)$	$8\hat{p}^+(\uparrow), 2\hat{p}^+(\downarrow), 5\hat{e}^-(\uparrow\downarrow)$	3
	${}^{11}_5\hat{\text{B}}_5(3/2)$	$2\left[{}^4_2\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow), \hat{n}(\uparrow)$	$7\hat{p}^+(\uparrow), 4\hat{p}^+(\downarrow), 6\hat{e}^-(\uparrow\downarrow)$	3/2
6	${}^{12}_6\hat{\text{C}}_6(0)$	$2\left[{}^4_2\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow) \equiv 3\left[{}^4_2\hat{\text{He}}_2(0) \right]$	$6\hat{p}^+(\uparrow), 6\hat{p}^+(\downarrow), 6\hat{e}^-(\uparrow\downarrow)$	0
	${}^{13}_6\hat{\text{C}}_7(1/2)$	$3\left[{}^4_2\hat{\text{He}}_2(0) \right], \hat{n}(\uparrow)$	$7\hat{p}^+(\uparrow), 6\hat{p}^+(\downarrow), 7\hat{e}^-(\uparrow\downarrow)$	1/2
7	${}^{14}_7\hat{\text{N}}_7(1)$	$3\left[{}^4_2\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow)$	$8\hat{p}^+(\uparrow), 6\hat{p}^+(\downarrow), 7\hat{e}^-(\uparrow\downarrow)$	1
	${}^{15}_7\hat{\text{N}}_7(1/2)$	$3\left[{}^4_2\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow), \hat{n}(\downarrow)$	$8\hat{p}^+(\uparrow), 7\hat{p}^+(\downarrow), 8\hat{e}^-(\uparrow\downarrow)$	1/2
8	${}^{16}_8\hat{\text{O}}_8(0)$	$3\left[{}^4_2\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow) \equiv 4\left[{}^4_2\hat{\text{He}}_2(0) \right]$	$8\hat{p}^+(\uparrow), 8\hat{p}^+(\downarrow), 8\hat{e}^-(\uparrow\downarrow)$	0
	${}^{17}_8\hat{\text{O}}_9(5/2)$	$3\left[{}^4_2\hat{\text{He}}_2(0) \right], 2\hat{d}(\uparrow\uparrow), \hat{n}(\uparrow)$	$11\hat{p}^+(\uparrow), 6\hat{p}^+(\downarrow), 9\hat{e}^-(\uparrow\downarrow)$	5/2
9	${}^{18}_8\hat{\text{O}}_{10}(0)$	$4\left[{}^4_2\hat{\text{He}}_2(0) \right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$9\hat{p}^+(\uparrow), 9\hat{p}^+(\downarrow), 10\hat{e}^-(\uparrow\downarrow)$	0
	${}^{19}_9\hat{\text{F}}_{10}(1/2)$	$4\left[{}^4_2\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow), \hat{n}(\downarrow)$	$10\hat{p}^+(\uparrow), 9\hat{p}^+(\downarrow), 10\hat{e}^-(\uparrow\downarrow)$	1/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
10	${}_{10}^{20}\hat{N}e_{10}(0)$	$4\left[{}^4_2\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)$ $\equiv 5\left[{}^4_2\hat{H}e_2(0)\right]$	$10\hat{p}^+(\uparrow), 10\hat{p}^+(\downarrow),$ $10\hat{e}^-(\uparrow\downarrow)$	0
	${}_{10}^{21}\hat{N}e_{11}(3/2)$	$4\left[{}^4_2\hat{H}e_2(0)\right], 2\hat{d}(\uparrow\uparrow), \hat{n}(\downarrow)$	$12\hat{p}^+(\uparrow), 9\hat{p}^+(\downarrow),$ $11\hat{e}^-(\uparrow\downarrow)$	3/2
	${}_{10}^{22}\hat{N}e_{12}(0)$	$5\left[{}^4_2\hat{H}e_2(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$11\hat{p}^+(\uparrow), 11\hat{p}^+(\downarrow),$ $12\hat{e}^-(\uparrow\downarrow)$	0
11	${}_{11}^{23}\hat{N}a_{12}(3/2)$	$5\left[{}^4_2\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), \hat{n}(\uparrow)$	$13\hat{p}^+(\uparrow), 10\hat{p}^+(\downarrow),$ $12\hat{e}^-(\uparrow\downarrow)$	3/2
	${}_{12}^{24}\hat{M}g_{12}(0)$	$5\left[{}^4_2\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)$ $\equiv 6\left[{}^4_2\hat{H}e_2(0)\right]$	$12\hat{p}^+(\uparrow), 12\hat{p}^+(\downarrow),$ $12\hat{e}^-(\uparrow\downarrow)$	0
12	${}_{12}^{25}\hat{M}g_{13}(5/2)$	$5\left[{}^4_2\hat{H}e_2(0)\right], 2\hat{d}(\uparrow\uparrow), \hat{n}(\uparrow)$	$15\hat{p}^+(\uparrow), 10\hat{p}^+(\downarrow),$ $13\hat{e}^-(\uparrow\downarrow)$	5/2
	${}_{12}^{26}\hat{M}g_{14}(0)$	$6\left[{}^4_2\hat{H}e_2(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$13\hat{p}^+(\uparrow), 13\hat{p}^+(\downarrow),$ $14\hat{e}^-(\uparrow\downarrow)$	0
13	${}_{13}^{27}\hat{A}l_{14}(5/2)$	$5\left[{}^4_2\hat{H}e_2(0)\right], 3\hat{d}(\uparrow\uparrow), \hat{n}(\downarrow)$	$16\hat{p}^+(\uparrow), 11\hat{p}^+(\downarrow),$ $14\hat{e}^-(\uparrow\downarrow)$	5/2
	${}_{14}^{28}\hat{S}i_{14}(0)$	$6\left[{}^4_2\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)$ $\equiv 7\left[{}^4_2\hat{H}e_2(0)\right]$	$14\hat{p}^+(\uparrow), 14\hat{p}^+(\downarrow),$ $14\hat{e}^-(\uparrow\downarrow)$	0
14	${}_{14}^{29}\hat{S}i_{15}(1/2)$	$7\left[{}^4_2\hat{H}e_2(0)\right], \hat{n}(\uparrow)$	$15\hat{p}^+(\uparrow), 14\hat{p}^+(\downarrow),$ $15\hat{e}^-(\uparrow\downarrow)$	1/2
	${}_{14}^{30}\hat{S}i_{16}(0)$	$7\left[{}^4_2\hat{H}e_2(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$15\hat{p}^+(\uparrow), 15\hat{p}^+(\downarrow),$ $16\hat{e}^-(\uparrow\downarrow)$	0
15	${}_{15}^{31}\hat{P}_{16}(0)$	$7\left[{}^4_2\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), \hat{n}(\downarrow)$	$16\hat{p}^+(\uparrow), 15\hat{p}^+(\downarrow),$ $16\hat{e}^-(\uparrow\downarrow)$	1/2
	${}_{16}^{32}\hat{S}_{16}(0)$	$7\left[{}^4_2\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)$ $\equiv 8\left[{}^4_2\hat{H}e_2(0)\right]$	$16\hat{p}^+(\uparrow), 16\hat{p}^+(\downarrow),$ $16\hat{e}^-(\uparrow\downarrow)$	0
16	${}_{16}^{33}\hat{S}_{17}(3/2)$	$7\left[{}^4_2\hat{H}e_2(0)\right], 2\hat{d}(\uparrow\uparrow), \hat{n}(\downarrow)$	$18\hat{p}^+(\uparrow), 15\hat{p}^+(\downarrow),$ $17\hat{e}^-(\uparrow\downarrow)$	3/2
	${}_{16}^{34}\hat{S}_{18}(3/2)$	$8\left[{}^4_2\hat{H}e_2(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$17\hat{p}^+(\uparrow), 17\hat{p}^+(\downarrow),$ $18\hat{e}^-(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
17	${}_{16}^{36}\hat{S}_{20}(3/2)$	$8\left[{}^4_2\hat{H}e_2(0)\right], 2\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$18\hat{p}^+(\uparrow), 18\hat{p}^+(\downarrow), 20\hat{e}^-(\uparrow\downarrow)$	0
	${}_{17}^{35}\hat{Cl}_{18}(3/2)$	$8\left[{}^4_2\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), \hat{n}(\uparrow)$	$19\hat{p}^+(\uparrow), 16\hat{p}^+(\downarrow), 18\hat{e}^-(\uparrow\downarrow)$	3/2
	${}_{17}^{37}\hat{Cl}_{20}(3/2)$	$8\left[{}^4_2\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 2\hat{n}(\uparrow), \hat{n}(\downarrow)$	$20\hat{p}^+(\uparrow), 17\hat{p}^+(\downarrow), 20\hat{e}^-(\uparrow\downarrow)$	3/2
18	${}_{18}^{36}\hat{Ar}_{18}(0)$	$8\left[{}^4_2\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)$ $\equiv 9\left[{}^4_2\hat{H}e_2(0)\right]$	$18\hat{p}^+(\uparrow), 18\hat{p}^+(\downarrow), 18\hat{e}^-(\uparrow\downarrow)$	0
	${}_{18}^{38}\hat{Ar}_{20}(0)$	$9\left[{}^4_2\hat{H}e_2(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$19\hat{p}^+(\uparrow), 19\hat{p}^+(\downarrow), 20\hat{e}^-(\uparrow\downarrow)$	0
	${}_{18}^{40}\hat{Ar}_{22}(0)$	$9\left[{}^4_2\hat{H}e_2(0)\right], 2\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$20\hat{p}^+(\uparrow), 20\hat{p}^+(\downarrow), 22\hat{e}^-(\uparrow\downarrow)$	0
19	${}_{19}^{39}\hat{K}_{20}(3/2)$	$9\left[{}^4_2\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), \hat{n}(\uparrow)$	$21\hat{p}^+(\uparrow), 18\hat{p}^+(\downarrow), 20\hat{e}^-(\uparrow\downarrow)$	3/2
	${}_{19}^{40}\hat{K}_{21}(4)$	$8\left[{}^4_2\hat{H}e_2(0)\right], 3\hat{d}(\uparrow\uparrow), 2\hat{n}(\uparrow)$	$24\hat{p}^+(\uparrow), 16\hat{p}^+(\downarrow), 21\hat{e}^-(\uparrow\downarrow)$	4
	${}_{19}^{41}\hat{K}_{22}(3/2)$	$9\left[{}^4_2\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), 2\hat{n}(\uparrow), \hat{n}(\downarrow)$	$22\hat{p}^+(\uparrow), 19\hat{p}^+(\downarrow), 22\hat{e}^-(\uparrow\downarrow)$	3/2
20	${}_{20}^{40}\hat{Ca}_{20}(0)$	$9\left[{}^4_2\hat{H}e_2(0)\right], \hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)$ $\equiv 10\left[{}^4_2\hat{H}e_2(0)\right]$	$20\hat{p}^+(\uparrow), 20\hat{p}^+(\downarrow), 20\hat{e}^-(\uparrow\downarrow)$	0
	${}_{20}^{42}\hat{Ca}_{22}(0)$	$10\left[{}^4_2\hat{H}e_2(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$21\hat{p}^+(\uparrow), 21\hat{p}^+(\downarrow), 22\hat{e}^-(\uparrow\downarrow)$	0
	${}_{20}^{43}\hat{Ca}_{23}(7/2)$	$9\left[{}^4_2\hat{H}e_2(0)\right], 2\hat{d}(\uparrow\uparrow), 3\hat{n}(\uparrow)$	$25\hat{p}^+(\uparrow), 18\hat{p}^+(\downarrow), 23\hat{e}^-(\uparrow\downarrow)$	7/2
	${}_{20}^{44}\hat{Ca}_{24}(0)$	$10\left[{}^4_2\hat{H}e_2(0)\right], 2\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$22\hat{p}^+(\uparrow), 22\hat{p}^+(\downarrow), 24\hat{e}^-(\uparrow\downarrow)$	0
	${}_{20}^{46}\hat{Ca}_{26}(0)$	$10\left[{}^4_2\hat{H}e_2(0)\right], 3\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$23\hat{p}^+(\uparrow), 23\hat{p}^+(\downarrow), 26\hat{e}^-(\uparrow\downarrow)$	0
	${}_{20}^{48}\hat{Ca}_{28}(0)$	$10\left[{}^4_2\hat{H}e_2(0)\right], 4\hat{n}(\uparrow), 4\hat{n}(\downarrow)$	$23\hat{p}^+(\uparrow), 23\hat{p}^+(\downarrow), 26\hat{e}^-(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
21	${}_{21}^{45}\hat{\text{Sc}}_{24}(7/2)$	$9\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], 3\hat{d}(\uparrow\uparrow),$ $2\hat{n}(\uparrow), \hat{n}(\downarrow)$	$26\hat{p}^+(\uparrow), 19\hat{p}^+(\downarrow),$ $24\hat{e}^-(\uparrow\downarrow)$	7/2
	${}_{22}^{46}\hat{\text{Ti}}_{24}(0)$	$11\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$23\hat{p}^+(\uparrow), 23\hat{p}^+(\downarrow),$ $24\hat{e}^-(\uparrow\downarrow)$	0
	${}_{22}^{47}\hat{\text{Ti}}_{25}(5/2)$	$10\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], 2\hat{d}(\uparrow\uparrow),$ $2\hat{n}(\uparrow), \hat{n}(\downarrow)$	$26\hat{p}^+(\uparrow), 21\hat{p}^+(\downarrow),$ $25\hat{e}^-(\uparrow\downarrow)$	5/2
22	${}_{22}^{48}\hat{\text{Ti}}_{26}(0)$	$11\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], 2\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$24\hat{p}^+(\uparrow), 24\hat{p}^+(\downarrow),$ $26\hat{e}^-(\uparrow\downarrow)$	0
	${}_{22}^{49}\hat{\text{Ti}}_{27}(7/2)$	$10\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], 2\hat{d}(\uparrow\uparrow),$ $4\hat{n}(\uparrow), \hat{n}(\downarrow)$	$28\hat{p}^+(\uparrow), 21\hat{p}^+(\downarrow),$ $27\hat{e}^-(\uparrow\downarrow)$	7/2
	${}_{22}^{50}\hat{\text{Ti}}_{28}(0)$	$11\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], 3\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$25\hat{p}^+(\uparrow), 25\hat{p}^+(\downarrow),$ $28\hat{e}^-(\uparrow\downarrow)$	0
23	${}_{23}^{50}\hat{\text{V}}_{27}(6)$	$9\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], 5\hat{d}(\uparrow\uparrow),$ $3\hat{n}(\uparrow), \hat{n}(\downarrow)$	$31\hat{p}^+(\uparrow), 19\hat{p}^+(\downarrow),$ $27\hat{e}^-(\uparrow\downarrow)$	6
	${}_{23}^{51}\hat{\text{V}}_{28}(7/2)$	$11\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], \hat{d}(\uparrow\uparrow), 5\hat{n}(\uparrow)$	$29\hat{p}^+(\uparrow), 22\hat{p}^+(\downarrow),$ $28\hat{e}^-(\uparrow\downarrow)$	7/2
24	${}_{24}^{50}\hat{\text{Cr}}_{26}(0)$	$12\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$25\hat{p}^+(\uparrow), 25\hat{p}^+(\downarrow),$ $26\hat{e}^-(\uparrow\downarrow)$	0
	${}_{24}^{52}\hat{\text{Cr}}_{28}(0)$	$12\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], 2\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$26\hat{p}^+(\uparrow), 26\hat{p}^+(\downarrow),$ $28\hat{e}^-(\uparrow\downarrow)$	0
	${}_{24}^{53}\hat{\text{Cr}}_{29}(3/2)$	$12\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], 4\hat{n}(\uparrow), \hat{n}(\downarrow)$	$28\hat{p}^+(\uparrow), 25\hat{p}^+(\downarrow),$ $29\hat{e}^-(\uparrow\downarrow)$	3/2
	${}_{24}^{54}\hat{\text{Cr}}_{30}(0)$	$12\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], 3\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$27\hat{p}^+(\uparrow), 27\hat{p}^+(\downarrow),$ $30\hat{e}^-(\uparrow\downarrow)$	0
	${}_{25}^{55}\hat{\text{Mn}}_{30}(5/2)$	$12\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], \hat{d}(\uparrow\uparrow),$ $4\hat{n}(\uparrow), \hat{n}(\downarrow)$	$30\hat{p}^+(\uparrow), 25\hat{p}^+(\downarrow),$ $30\hat{e}^-(\uparrow\downarrow)$	5/2
26	${}_{26}^{54}\hat{\text{Fe}}_{28}(0)$	$13\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$27\hat{p}^+(\uparrow), 27\hat{p}^+(\downarrow),$ $28\hat{e}^-(\uparrow\downarrow)$	0
	${}_{26}^{56}\hat{\text{Fe}}_{30}(0)$	$13\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], 2\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$28\hat{p}^+(\uparrow), 28\hat{p}^+(\downarrow),$ $30\hat{e}^-(\uparrow\downarrow)$	0
	${}_{26}^{57}\hat{\text{Fe}}_{31}(1/2)$	$13\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], 3\hat{n}(\uparrow), 2\hat{n}(\downarrow)$	$29\hat{p}^+(\uparrow), 28\hat{p}^+(\downarrow),$ $31\hat{e}^-(\uparrow\downarrow)$	1/2
	${}_{26}^{58}\hat{\text{Fe}}_{32}(0)$	$13\left[{}_{2}^{4}\hat{\text{He}}_2(0)\right], 3\hat{n}(\uparrow), 3\hat{n}(\downarrow)$	$29\hat{p}^+(\uparrow), 29\hat{p}^+(\downarrow),$ $32\hat{e}^-(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
27	${}_{27}^{59}\hat{\text{Co}}_{32}(7/2)$	$13 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $5 \hat{n}(\uparrow)$	$33\hat{p}^+(\uparrow), 26\hat{p}^+(\downarrow),$ $32\hat{e}^-(\uparrow\downarrow)$	7/2
	${}_{28}^{58}\hat{\text{Ni}}_{30}(0)$	$14 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \hat{n}(\uparrow), \hat{n}(\downarrow)$	$29\hat{p}^+(\uparrow), 29\hat{p}^+(\downarrow),$ $30\hat{e}^-(\uparrow\downarrow)$	0
	${}_{28}^{60}\hat{\text{Ni}}_{32}(0)$	$14 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 2 \hat{n}(\uparrow), 2 \hat{n}(\downarrow)$	$30\hat{p}^+(\uparrow), 30\hat{p}^+(\downarrow),$ $32\hat{e}^-(\uparrow\downarrow)$	0
28	${}_{28}^{61}\hat{\text{Ni}}_{33}(3/2)$	$14 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 4 \hat{n}(\uparrow), \hat{n}(\downarrow)$	$32\hat{p}^+(\uparrow), 29\hat{p}^+(\downarrow),$ $33\hat{e}^-(\uparrow\downarrow)$	3/2
	${}_{28}^{62}\hat{\text{Ni}}_{34}(0)$	$14 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 3 \hat{n}(\uparrow), 3 \hat{n}(\downarrow)$	$31\hat{p}^+(\uparrow), 31\hat{p}^+(\downarrow),$ $34\hat{e}^-(\uparrow\downarrow)$	0
	${}_{28}^{64}\hat{\text{Ni}}_{36}(0)$	$14 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 4 \hat{n}(\uparrow), 4 \hat{n}(\downarrow)$	$32\hat{p}^+(\uparrow), 32\hat{p}^+(\downarrow),$ $36\hat{e}^-(\uparrow\downarrow)$	0
29	${}_{29}^{63}\hat{\text{Cu}}_{34}(3/2)$	$14 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $3 \hat{n}(\uparrow), 2 \hat{n}(\downarrow)$	$33\hat{p}^+(\uparrow), 30\hat{p}^+(\downarrow),$ $34\hat{e}^-(\uparrow\downarrow)$	3/2
	${}_{29}^{65}\hat{\text{Cu}}_{36}(3/2)$	$14 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $4 \hat{n}(\uparrow), 3 \hat{n}(\downarrow)$	$34\hat{p}^+(\uparrow), 31\hat{p}^+(\downarrow),$ $36\hat{e}^-(\uparrow\downarrow)$	3/2
	${}_{30}^{64}\hat{\text{Zn}}_{34}(0)$	$15 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 2 \hat{n}(\uparrow), 2 \hat{n}(\downarrow)$	$32\hat{p}^+(\uparrow), 32\hat{p}^+(\downarrow),$ $34\hat{e}^-(\uparrow\downarrow)$	0
	${}_{30}^{66}\hat{\text{Zn}}_{36}(0)$	$15 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 3 \hat{n}(\uparrow), 3 \hat{n}(\downarrow)$	$33\hat{p}^+(\uparrow), 33\hat{p}^+(\downarrow),$ $36\hat{e}^-(\uparrow\downarrow)$	0
30	${}_{30}^{67}\hat{\text{Zn}}_{37}(5/2)$	$15 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 6 \hat{n}(\uparrow), \hat{n}(\downarrow)$	$36\hat{p}^+(\uparrow), 31\hat{p}^+(\downarrow),$ $37\hat{e}^-(\uparrow\downarrow)$	5/2
	${}_{30}^{68}\hat{\text{Zn}}_{38}(0)$	$15 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 4 \hat{n}(\uparrow), 4 \hat{n}(\downarrow)$	$34\hat{p}^+(\uparrow), 34\hat{p}^+(\downarrow),$ $38\hat{e}^-(\uparrow\downarrow)$	0
	${}_{30}^{68}\hat{\text{Zn}}_{38}(0)$	$15 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 5 \hat{n}(\uparrow), 5 \hat{n}(\downarrow)$	$35\hat{p}^+(\uparrow), 35\hat{p}^+(\downarrow),$ $40\hat{e}^-(\uparrow\downarrow)$	0
31	${}_{31}^{69}\hat{\text{Ga}}_{38}(3/2)$	$15 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $4 \hat{n}(\uparrow), 3 \hat{n}(\downarrow)$	$36\hat{p}^+(\uparrow), 33\hat{p}^+(\downarrow),$ $38\hat{e}^-(\uparrow\downarrow)$	3/2
	${}_{31}^{71}\hat{\text{Ga}}_{40}(3/2)$	$15 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $5 \hat{n}(\uparrow), 4 \hat{n}(\downarrow)$	$37\hat{p}^+(\uparrow), 34\hat{p}^+(\downarrow),$ $40\hat{e}^-(\uparrow\downarrow)$	3/2
	${}_{32}^{70}\hat{\text{Ge}}_{38}(0)$	$16 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 3 \hat{n}(\uparrow), 3 \hat{n}(\downarrow)$	$35\hat{p}^+(\uparrow), 35\hat{p}^+(\downarrow),$ $38\hat{e}^-(\uparrow\downarrow)$	0
32	${}_{32}^{72}\hat{\text{Ge}}_{40}(0)$	$16 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 4 \hat{n}(\uparrow), 4 \hat{n}(\downarrow)$	$36\hat{p}^+(\uparrow), 36\hat{p}^+(\downarrow),$ $40\hat{e}^-(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
33	${}_{32}^{73}\hat{\text{Ge}}_{41} (9/2)$	$16 \left[{}_2^4\hat{\text{He}}_2(0) \right], 9 \hat{n}(\uparrow)$	$41\hat{p}^+(\uparrow), 32\hat{p}^+(\downarrow),$ $41\hat{e}^-(\uparrow\downarrow)$	9/2
	${}_{32}^{74}\hat{\text{Ge}}_{42} (0)$	$16 \left[{}_2^4\hat{\text{He}}_2(0) \right], 5 \hat{n}(\uparrow), 5 \hat{n}(\downarrow)$	$37\hat{p}^+(\uparrow), 37\hat{p}^+(\downarrow),$ $42\hat{e}^-(\uparrow\downarrow)$	0
	${}_{32}^{76}\hat{\text{Ge}}_{44} (0)$	$16 \left[{}_2^4\hat{\text{He}}_2(0) \right], 6 \hat{n}(\uparrow), 6 \hat{n}(\downarrow)$	$38\hat{p}^+(\uparrow), 38\hat{p}^+(\downarrow),$ $44\hat{e}^-(\uparrow\downarrow)$	0
	${}_{33}^{75}\hat{\text{As}}_{42} (3/2)$	$16 \left[{}_2^4\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $5 \hat{n}(\uparrow), 4 \hat{n}(\downarrow)$	$39\hat{p}^+(\uparrow), 36\hat{p}^+(\downarrow),$ $42\hat{e}^-(\uparrow\downarrow)$	3/2
34	${}_{34}^{74}\hat{\text{Se}}_{40} (0)$	$17 \left[{}_2^4\hat{\text{He}}_2(0) \right], 3 \hat{n}(\uparrow), 3 \hat{n}(\downarrow)$	$37\hat{p}^+(\uparrow), 37\hat{p}^+(\downarrow),$ $40\hat{e}^-(\uparrow\downarrow)$	0
	${}_{34}^{76}\hat{\text{Se}}_{42} (0)$	$17 \left[{}_2^4\hat{\text{He}}_2(0) \right], 4 \hat{n}(\uparrow), 4 \hat{n}(\downarrow)$	$38\hat{p}^+(\uparrow), 38\hat{p}^+(\downarrow),$ $42\hat{e}^-(\uparrow\downarrow)$	0
	${}_{34}^{77}\hat{\text{Se}}_{43} (0)$	$17 \left[{}_2^4\hat{\text{He}}_2(0) \right], 5 \hat{n}(\uparrow), 4 \hat{n}(\downarrow)$	$39\hat{p}^+(\uparrow), 38\hat{p}^+(\downarrow),$ $43\hat{e}^-(\uparrow\downarrow)$	1/2
	${}_{34}^{78}\hat{\text{Se}}_{44} (0)$	$17 \left[{}_2^4\hat{\text{He}}_2(0) \right], 5 \hat{n}(\uparrow), 5 \hat{n}(\downarrow)$	$39\hat{p}^+(\uparrow), 39\hat{p}^+(\downarrow),$ $44\hat{e}^-(\uparrow\downarrow)$	0
	${}_{34}^{80}\hat{\text{Se}}_{46} (0)$	$17 \left[{}_2^4\hat{\text{He}}_2(0) \right], 6 \hat{n}(\uparrow), 6 \hat{n}(\downarrow)$	$40\hat{p}^+(\uparrow), 40\hat{p}^+(\downarrow),$ $46\hat{e}^-(\uparrow\downarrow)$	0
	${}_{34}^{82}\hat{\text{Se}}_{48} (0)$	$17 \left[{}_2^4\hat{\text{He}}_2(0) \right], 7 \hat{n}(\uparrow), 7 \hat{n}(\downarrow)$	$41\hat{p}^+(\uparrow), 41\hat{p}^+(\downarrow),$ $48\hat{e}^-(\uparrow\downarrow)$	0
35	${}_{35}^{79}\hat{\text{Br}}_{44} (3/2)$	$17 \left[{}_2^4\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $5 \hat{n}(\uparrow), 4 \hat{n}(\downarrow)$	$41\hat{p}^+(\uparrow), 38\hat{p}^+(\downarrow),$ $44\hat{e}^-(\uparrow\downarrow)$	3/2
	${}_{35}^{81}\hat{\text{Br}}_{46} (3/2)$	$17 \left[{}_2^4\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $6 \hat{n}(\uparrow), 5 \hat{n}(\downarrow)$	$42\hat{p}^+(\uparrow), 39\hat{p}^+(\downarrow),$ $46\hat{e}^-(\uparrow\downarrow)$	3/2
36	${}_{36}^{78}\hat{\text{Kr}}_{42} (0)$	$18 \left[{}_2^4\hat{\text{He}}_2(0) \right], 3 \hat{n}(\uparrow), 3$ $\hat{n}(\downarrow)$	$39\hat{p}^+(\uparrow), 39\hat{p}^+(\downarrow),$ $42\hat{e}^-(\uparrow\downarrow)$	0
	${}_{36}^{80}\hat{\text{Kr}}_{44} (0)$	$18 \left[{}_2^4\hat{\text{He}}_2(0) \right], 4 \hat{n}(\uparrow), 4$ $\hat{n}(\downarrow)$	$40\hat{p}^+(\uparrow), 40\hat{p}^+(\downarrow),$ $44\hat{e}^-(\uparrow\downarrow)$	0
	${}_{36}^{82}\hat{\text{Kr}}_{46} (0)$	$18 \left[{}_2^4\hat{\text{He}}_2(0) \right], 5 \hat{n}(\uparrow), 5$ $\hat{n}(\downarrow)$	$41\hat{p}^+(\uparrow), 41\hat{p}^+(\downarrow),$ $46\hat{e}^-(\uparrow\downarrow)$	0
	${}_{36}^{83}\hat{\text{Kr}}_{47} (9/2)$	$18 \left[{}_2^4\hat{\text{He}}_2(0) \right], 10 \hat{n}(\uparrow),$ $\hat{n}(\downarrow)$	$46\hat{p}^+(\uparrow), 37\hat{p}^+(\downarrow),$ $47\hat{e}^-(\uparrow\downarrow)$	9/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
37	${}_{36}^{84}\hat{\text{K}}\text{r}_{48} (0)$	$18 \left[{}_2^4\hat{\text{H}}\text{e}_2(0) \right], 6 \hat{n}(\uparrow), 6 \hat{n}(\downarrow)$	$42\hat{p}^+(\uparrow), 42\hat{p}^+(\downarrow), 48\hat{e}^-(\uparrow\downarrow)$	0
	${}_{36}^{86}\hat{\text{K}}\text{r}_{50} (0)$	$18 \left[{}_2^4\hat{\text{H}}\text{e}_2(0) \right], 7 \hat{n}(\uparrow), 7 \hat{n}(\downarrow)$	$43\hat{p}^+(\uparrow), 43\hat{p}^+(\downarrow), 50\hat{e}^-(\uparrow\downarrow)$	0
	${}_{37}^{85}\hat{\text{R}}\text{b}_{48} (5/2)$	$18 \left[{}_2^4\hat{\text{H}}\text{e}_2(0) \right], \hat{d}(\uparrow\uparrow), 7 \hat{n}(\uparrow), 4 \hat{n}(\downarrow)$	$45\hat{p}^+(\uparrow), 40\hat{p}^+(\downarrow), 48\hat{e}^-(\uparrow\downarrow)$	5/2
	${}_{37}^{87}\hat{\text{R}}\text{b}_{50} (3/2)$	$18 \left[{}_2^4\hat{\text{H}}\text{e}_2(0) \right], \hat{d}(\uparrow\uparrow), 7 \hat{n}(\uparrow), 6 \hat{n}(\downarrow)$	$45\hat{p}^+(\uparrow), 42\hat{p}^+(\downarrow), 50\hat{e}^-(\uparrow\downarrow)$	3/2
	${}_{38}^{84}\hat{\text{S}}\text{r}_{46} (0)$	$19 \left[{}_2^4\hat{\text{H}}\text{e}_2(0) \right], 4 \hat{n}(\uparrow), 4 \hat{n}(\downarrow)$	$42\hat{p}^+(\uparrow), 42\hat{p}^+(\downarrow), 46\hat{e}^-(\uparrow\downarrow)$	0
	${}_{38}^{86}\hat{\text{S}}\text{r}_{48} (0)$	$19 \left[{}_2^4\hat{\text{H}}\text{e}_2(0) \right], 5 \hat{n}(\uparrow), 5 \hat{n}(\downarrow)$	$43\hat{p}^+(\uparrow), 43\hat{p}^+(\downarrow), 48\hat{e}^-(\uparrow\downarrow)$	0
38	${}_{38}^{87}\hat{\text{S}}\text{r}_{49} (9/2)$	$19 \left[{}_2^4\hat{\text{H}}\text{e}_2(0) \right], 10 \hat{n}(\uparrow), \hat{n}(\downarrow)$	$48\hat{p}^+(\uparrow), 39\hat{p}^+(\downarrow), 49\hat{e}^-(\uparrow\downarrow)$	9/2
	${}_{38}^{88}\hat{\text{S}}\text{r}_{50} (0)$	$19 \left[{}_2^4\hat{\text{H}}\text{e}_2(0) \right], 6 \hat{n}(\uparrow), 6 \hat{n}(\downarrow)$	$44\hat{p}^+(\uparrow), 44\hat{p}^+(\downarrow), 50\hat{e}^-(\uparrow\downarrow)$	0
39	${}_{39}^{89}\hat{\text{Y}}_{50} (1/2)$	$19 \left[{}_2^4\hat{\text{H}}\text{e}_2(0) \right], \hat{d}(\uparrow\uparrow), 5 \hat{n}(\uparrow), 6 \hat{n}(\downarrow)$	$45\hat{p}^+(\uparrow), 44\hat{p}^+(\downarrow), 50\hat{e}^-(\uparrow\downarrow)$	1/2
	${}_{40}^{90}\hat{\text{Z}}\text{r}_{50} (0)$	$20 \left[{}_2^4\hat{\text{H}}\text{e}_2(0) \right], 5 \hat{n}(\uparrow), 5 \hat{n}(\downarrow)$	$45\hat{p}^+(\uparrow), 45\hat{p}^+(\downarrow), 50\hat{e}^-(\uparrow\downarrow)$	0
40	${}_{40}^{91}\hat{\text{Z}}\text{r}_{51} (5/2)$	$20 \left[{}_2^4\hat{\text{H}}\text{e}_2(0) \right], 8 \hat{n}(\uparrow), 3 \hat{n}(\downarrow)$	$48\hat{p}^+(\uparrow), 43\hat{p}^+(\downarrow), 51\hat{e}^-(\uparrow\downarrow)$	5/2
	${}_{40}^{92}\hat{\text{Z}}\text{r}_{52} (0)$	$20 \left[{}_2^4\hat{\text{H}}\text{e}_2(0) \right], 6 \hat{n}(\uparrow), 6 \hat{n}(\downarrow)$	$46\hat{p}^+(\uparrow), 46\hat{p}^+(\downarrow), 52\hat{e}^-(\uparrow\downarrow)$	0
	${}_{40}^{94}\hat{\text{Z}}\text{r}_{54} (0)$	$20 \left[{}_2^4\hat{\text{H}}\text{e}_2(0) \right], 7 \hat{n}(\uparrow), 7 \hat{n}(\downarrow)$	$47\hat{p}^+(\uparrow), 47\hat{p}^+(\downarrow), 54\hat{e}^-(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
41	${}_{40}^{96}\hat{Zr}_{56} (0)$	$20 \left[{}_2^4\hat{H}e_2(0) \right], 8 \hat{n}(\uparrow), 8 \hat{n}(\downarrow)$	$48\hat{p}^+(\uparrow), 48\hat{p}^+(\downarrow), 56\hat{e}^-(\uparrow\downarrow)$	0
	${}_{41}^{93}\hat{Nb}_{52} (9/2)$	$20 \left[{}_2^4\hat{H}e_2(0) \right], \hat{d}(\uparrow\uparrow), 9 \hat{n}(\uparrow), 2 \hat{n}(\downarrow)$	$51\hat{p}^+(\uparrow), 42\hat{p}^+(\downarrow), 52\hat{e}^-(\uparrow\downarrow)$	9/2
42	${}_{42}^{92}\hat{Mo}_{50} (0)$	$21 \left[{}_2^4\hat{H}e_2(0) \right], 4 \hat{n}(\uparrow), 4 \hat{n}(\downarrow)$	$46\hat{p}^+(\uparrow), 46\hat{p}^+(\downarrow), 50\hat{e}^-(\uparrow\downarrow)$	0
	${}_{42}^{94}\hat{Mo}_{52} (0)$	$21 \left[{}_2^4\hat{H}e_2(0) \right], 5 \hat{n}(\uparrow), 5 \hat{n}(\downarrow)$	$47\hat{p}^+(\uparrow), 47\hat{p}^+(\downarrow), 52\hat{e}^-(\uparrow\downarrow)$	0
	${}_{42}^{95}\hat{Mo}_{53} (5/2)$	$21 \left[{}_2^4\hat{H}e_2(0) \right], 8 \hat{n}(\uparrow), 3 \hat{n}(\downarrow)$	$50\hat{p}^+(\uparrow), 45\hat{p}^+(\downarrow), 53\hat{e}^-(\uparrow\downarrow)$	5/2
	${}_{42}^{96}\hat{Mo}_{54} (0)$	$21 \left[{}_2^4\hat{H}e_2(0) \right], 6 \hat{n}(\uparrow), 6 \hat{n}(\downarrow)$	$48\hat{p}^+(\uparrow), 48\hat{p}^+(\downarrow), 54\hat{e}^-(\uparrow\downarrow)$	0
42	${}_{42}^{97}\hat{Mo}_{55} (5/2)$	$21 \left[{}_2^4\hat{H}e_2(0) \right], 9 \hat{n}(\uparrow), 4 \hat{n}(\downarrow)$	$51\hat{p}^+(\uparrow), 46\hat{p}^+(\downarrow), 55\hat{e}^-(\uparrow\downarrow)$	5/2
	${}_{42}^{98}\hat{Mo}_{56} (0)$	$21 \left[{}_2^4\hat{H}e_2(0) \right], 7 \hat{n}(\uparrow), 7 \hat{n}(\downarrow)$	$49\hat{p}^+(\uparrow), 49\hat{p}^+(\downarrow), 56\hat{e}^-(\uparrow\downarrow)$	0
	${}_{42}^{100}\hat{Mo}_{58} (0)$	$21 \left[{}_2^4\hat{H}e_2(0) \right], 8 \hat{n}(\uparrow), 8 \hat{n}(\downarrow)$	$50\hat{p}^+(\uparrow), 50\hat{p}^+(\downarrow), 58\hat{e}^-(\uparrow\downarrow)$	0
	43	${}_{43}^{xx}\hat{Tc}_{yy} (?)$	No Stable Nuclide	No Stable Nuclide
44	${}_{44}^{96}\hat{Ru}_{52} (0)$	$22 \left[{}_2^4\hat{H}e_2(0) \right], 4 \hat{n}(\uparrow), 4 \hat{n}(\downarrow)$	$48\hat{p}^+(\uparrow), 48\hat{p}^+(\downarrow), 52\hat{e}^-(\uparrow\downarrow)$	0
	${}_{44}^{98}\hat{Ru}_{54} (0)$	$22 \left[{}_2^4\hat{H}e_2(0) \right], 5 \hat{n}(\uparrow), 5 \hat{n}(\downarrow)$	$49\hat{p}^+(\uparrow), 49\hat{p}^+(\downarrow), 54\hat{e}^-(\uparrow\downarrow)$	0
	${}_{44}^{99}\hat{Ru}_{55} (5/2)$	$22 \left[{}_2^4\hat{H}e_2(0) \right], 8 \hat{n}(\uparrow), 3 \hat{n}(\downarrow)$	$52\hat{p}^+(\uparrow), 47\hat{p}^+(\downarrow), 55\hat{e}^-(\uparrow\downarrow)$	5/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
45	$^{100}_{44}\hat{R}u_{56} (0)$	$22 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{n}(\downarrow) \end{matrix} \right], 6 \hat{n}(\uparrow), 6$	$50\hat{p}^+(\uparrow), 50\hat{p}^+(\downarrow),$ $56\hat{e}^-(\uparrow\downarrow)$	0
	$^{101}_{44}\hat{R}u_{57} (5/2)$	$22 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{n}(\downarrow) \end{matrix} \right], 9 \hat{n}(\uparrow), 4$	$53\hat{p}^+(\uparrow), 48\hat{p}^+(\downarrow),$ $57\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{102}_{44}\hat{R}u_{58} (0)$	$22 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{n}(\downarrow) \end{matrix} \right], 7 \hat{n}(\uparrow), 7$	$51\hat{p}^+(\uparrow), 51\hat{p}^+(\downarrow),$ $58\hat{e}^-(\uparrow\downarrow)$	0
	$^{104}_{44}\hat{R}u_{60} (0)$	$22 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{n}(\downarrow) \end{matrix} \right], 6 \hat{n}(\uparrow), 6$	$52\hat{p}^+(\uparrow), 52\hat{p}^+(\downarrow),$ $60\hat{e}^-(\uparrow\downarrow)$	0
	$^{103}_{45}\hat{R}h_{58} (1/2)$	$22 \left[\begin{matrix} \hat{H}e_2(0) \\ 6 \hat{n}(\uparrow), 7 \hat{n}(\downarrow) \end{matrix} \right], \hat{d}(\uparrow\uparrow),$	$52\hat{p}^+(\uparrow), 51\hat{p}^+(\downarrow),$ $58\hat{e}^-(\uparrow\downarrow)$	1/2
46	$^{102}_{46}\hat{P}d_{56} (0)$	$23 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{n}(\downarrow) \end{matrix} \right], 5 \hat{n}(\uparrow), 5$	$51\hat{p}^+(\uparrow), 51\hat{p}^+(\downarrow),$ $56\hat{e}^-(\uparrow\downarrow)$	0
	$^{104}_{46}\hat{P}d_{58} (0)$	$23 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{n}(\downarrow) \end{matrix} \right], 6 \hat{n}(\uparrow), 6$	$52\hat{p}^+(\uparrow), 52\hat{p}^+(\downarrow),$ $58\hat{e}^-(\uparrow\downarrow)$	0
	$^{105}_{46}\hat{P}d_{59} (5/2)$	$23 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{n}(\downarrow) \end{matrix} \right], 9 \hat{n}(\uparrow), 4$	$55\hat{p}^+(\uparrow), 50\hat{p}^+(\downarrow),$ $59\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{106}_{46}\hat{P}d_{60} (0)$	$23 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{n}(\downarrow) \end{matrix} \right], 7 \hat{n}(\uparrow), 7$	$53\hat{p}^+(\uparrow), 53\hat{p}^+(\downarrow),$ $60\hat{e}^-(\uparrow\downarrow)$	0
	$^{108}_{46}\hat{P}d_{62} (0)$	$23 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{n}(\downarrow) \end{matrix} \right], 8 \hat{n}(\uparrow), 8$	$54\hat{p}^+(\uparrow), 54\hat{p}^+(\downarrow),$ $62\hat{e}^-(\uparrow\downarrow)$	0
47	$^{110}_{46}\hat{P}d_{64} (0)$	$23 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{n}(\downarrow) \end{matrix} \right], 9 \hat{n}(\uparrow), 9$	$55\hat{p}^+(\uparrow), 55\hat{p}^+(\downarrow),$ $64\hat{e}^-(\uparrow\downarrow)$	0
	$^{107}_{47}\hat{A}g_{60} (1/2)$	$23 \left[\begin{matrix} \hat{H}e_2(0) \\ 6 \hat{n}(\uparrow), 7 \hat{n}(\downarrow) \end{matrix} \right], \hat{d}(\uparrow\uparrow),$	$54\hat{p}^+(\uparrow), 53\hat{p}^+(\downarrow),$ $60\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{109}_{47}\hat{A}g_{62} (1/2)$	$23 \left[\begin{matrix} \hat{H}e_2(0) \\ 7 \hat{n}(\uparrow), 8 \hat{n}(\downarrow) \end{matrix} \right], \hat{d}(\uparrow\uparrow),$	$55\hat{p}^+(\uparrow), 54\hat{p}^+(\downarrow),$ $62\hat{e}^-(\uparrow\downarrow)$	1/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
48	$^{106}_{48}\hat{\text{C}}\text{d}_{58} (0)$	$24 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{H}}\text{e}_2(0) \right], 5 \hat{n}(\uparrow), 5 \hat{n}(\downarrow)$	$53\hat{p}^+(\uparrow), 53\hat{p}^+(\downarrow), 58\hat{e}^-(\uparrow\downarrow)$	0
	$^{108}_{48}\hat{\text{C}}\text{d}_{60} (0)$	$24 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{H}}\text{e}_2(0) \right], 6 \hat{n}(\uparrow), 6 \hat{n}(\downarrow)$	$54\hat{p}^+(\uparrow), 54\hat{p}^+(\downarrow), 60\hat{e}^-(\uparrow\downarrow)$	0
	$^{110}_{48}\hat{\text{C}}\text{d}_{62} (0)$	$24 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{H}}\text{e}_2(0) \right], 7 \hat{n}(\uparrow), 7 \hat{n}(\downarrow)$	$55\hat{p}^+(\uparrow), 55\hat{p}^+(\downarrow), 62\hat{e}^-(\uparrow\downarrow)$	0
	$^{111}_{48}\hat{\text{C}}\text{d}_{63} (1/2)$	$24 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{H}}\text{e}_2(0) \right], 8 \hat{n}(\uparrow), 7 \hat{n}(\downarrow)$	$56\hat{p}^+(\uparrow), 55\hat{p}^+(\downarrow), 63\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{112}_{48}\hat{\text{C}}\text{d}_{64} (0)$	$24 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{H}}\text{e}_2(0) \right], 8 \hat{n}(\uparrow), 8 \hat{n}(\downarrow)$	$56\hat{p}^+(\uparrow), 56\hat{p}^+(\downarrow), 64\hat{e}^-(\uparrow\downarrow)$	0
	$^{113}_{48}\hat{\text{C}}\text{d}_{65} (1/2)$	$24 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{H}}\text{e}_2(0) \right], 9 \hat{n}(\uparrow), 8 \hat{n}(\downarrow)$	$57\hat{p}^+(\uparrow), 56\hat{p}^+(\downarrow), 65\hat{e}^-(\uparrow\downarrow)$	1/2
49	$^{116}_{48}\hat{\text{C}}\text{d}_{68} (0)$	$24 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{H}}\text{e}_2(0) \right], 10 \hat{n}(\uparrow), 10 \hat{n}(\downarrow)$	$58\hat{p}^+(\uparrow), 58\hat{p}^+(\downarrow), 68\hat{e}^-(\uparrow\downarrow)$	0
	$^{113}_{49}\hat{\text{I}}\text{n}_{64} (9/2)$	$24 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{H}}\text{e}_2(0) \right], \hat{d}(\uparrow\uparrow), 11 \hat{n}(\uparrow), 4 \hat{n}(\downarrow)$	$61\hat{p}^+(\uparrow), 52\hat{p}^+(\downarrow), 64\hat{e}^-(\uparrow\downarrow)$	9/2
	$^{115}_{49}\hat{\text{I}}\text{n}_{66} (9/2)$	$24 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{H}}\text{e}_2(0) \right], \hat{d}(\uparrow\uparrow), 12 \hat{n}(\uparrow), 5 \hat{n}(\downarrow)$	$62\hat{p}^+(\uparrow), 53\hat{p}^+(\downarrow), 66\hat{e}^-(\uparrow\downarrow)$	9/2
	$^{112}_{50}\hat{\text{S}}\text{n}_{62} (0)$	$25 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{H}}\text{e}_2(0) \right], 6 \hat{n}(\uparrow), 6 \hat{n}(\downarrow)$	$56\hat{p}^+(\uparrow), 56\hat{p}^+(\downarrow), 62\hat{e}^-(\uparrow\downarrow)$	0
50	$^{114}_{50}\hat{\text{S}}\text{n}_{64} (0)$	$25 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{H}}\text{e}_2(0) \right], 7 \hat{n}(\uparrow), 7 \hat{n}(\downarrow)$	$57\hat{p}^+(\uparrow), 57\hat{p}^+(\downarrow), 64\hat{e}^-(\uparrow\downarrow)$	0
	$^{115}_{50}\hat{\text{S}}\text{n}_{65} (1/2)$	$25 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{H}}\text{e}_2(0) \right], 8 \hat{n}(\uparrow), 7 \hat{n}(\downarrow)$	$58\hat{p}^+(\uparrow), 57\hat{p}^+(\downarrow), 65\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{116}_{50}\hat{\text{S}}\text{n}_{66} (0)$	$25 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{H}}\text{e}_2(0) \right], 8 \hat{n}(\uparrow), 8 \hat{n}(\downarrow)$	$58\hat{p}^+(\uparrow), 58\hat{p}^+(\downarrow), 66\hat{e}^-(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J	
51	$^{117}_{50}\hat{\text{Sn}}_{67} (1/2)$	$25 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{He}}_2(0) \right], 9 \hat{n}(\uparrow), 8 \hat{n}(\downarrow)$	$59\hat{p}^+(\uparrow), 58\hat{p}^+(\downarrow), 67\hat{e}^-(\uparrow\downarrow)$	1/2	
	$^{118}_{50}\hat{\text{Sn}}_{68} (0)$	$25 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{He}}_2(0) \right], 9 \hat{n}(\uparrow), 9 \hat{n}(\downarrow)$	$59\hat{p}^+(\uparrow), 59\hat{p}^+(\downarrow), 68\hat{e}^-(\uparrow\downarrow)$	0	
	$^{119}_{50}\hat{\text{Sn}}_{69} (1/2)$	$25 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{He}}_2(0) \right], 10 \hat{n}(\uparrow), 9 \hat{n}(\downarrow)$	$60\hat{p}^+(\uparrow), 59\hat{p}^+(\downarrow), 69\hat{e}^-(\uparrow\downarrow)$	1/2	
	$^{120}_{50}\hat{\text{Sn}}_{70} (0)$	$25 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{He}}_2(0) \right], 10 \hat{n}(\uparrow), 10 \hat{n}(\downarrow)$	$60\hat{p}^+(\uparrow), 60\hat{p}^+(\downarrow), 70\hat{e}^-(\uparrow\downarrow)$	0	
	$^{122}_{50}\hat{\text{Sn}}_{72} (0)$	$25 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{He}}_2(0) \right], 11 \hat{n}(\uparrow), 11 \hat{n}(\downarrow)$	$61\hat{p}^+(\uparrow), 61\hat{p}^+(\downarrow), 72\hat{e}^-(\uparrow\downarrow)$	0	
	$^{124}_{50}\hat{\text{Sn}}_{74} (0)$	$25 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{He}}_2(0) \right], 12 \hat{n}(\uparrow), 12 \hat{n}(\downarrow)$	$62\hat{p}^+(\uparrow), 62\hat{p}^+(\downarrow), 74\hat{e}^-(\uparrow\downarrow)$	0	
	$^{121}_{51}\hat{\text{Sb}}_{70} (5/2)$	$25 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow), 11 \hat{n}(\uparrow), 8 \hat{n}(\downarrow)$	$63\hat{p}^+(\uparrow), 58\hat{p}^+(\downarrow), 70\hat{e}^-(\uparrow\downarrow)$	5/2	
	$^{123}_{51}\hat{\text{Sb}}_{72} (7/2)$	$25 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow), 13 \hat{n}(\uparrow), 8 \hat{n}(\downarrow)$	$65\hat{p}^+(\uparrow), 58\hat{p}^+(\downarrow), 72\hat{e}^-(\uparrow\downarrow)$	7/2	
	$^{120}_{52}\hat{\text{Te}}_{68} (0)$	$26 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{He}}_2(0) \right], 8 \hat{n}(\uparrow), 8 \hat{n}(\downarrow)$	$60\hat{p}^+(\uparrow), 60\hat{p}^+(\downarrow), 68\hat{e}^-(\uparrow\downarrow)$	0	
	$^{122}_{52}\hat{\text{Te}}_{70} (0)$	$26 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{He}}_2(0) \right], 9 \hat{n}(\uparrow), 9 \hat{n}(\downarrow)$	$61\hat{p}^+(\uparrow), 61\hat{p}^+(\downarrow), 70\hat{e}^-(\uparrow\downarrow)$	0	
	52	$^{123}_{52}\hat{\text{Te}}_{71} (1/2)$	$26 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{He}}_2(0) \right], 10 \hat{n}(\uparrow), 9 \hat{n}(\downarrow)$	$62\hat{p}^+(\uparrow), 61\hat{p}^+(\downarrow), 71\hat{e}^-(\uparrow\downarrow)$	1/2
		$^{124}_{52}\hat{\text{Te}}_{72} (0)$	$26 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{He}}_2(0) \right], 10 \hat{n}(\uparrow), 10 \hat{n}(\downarrow)$	$62\hat{p}^+(\uparrow), 62\hat{p}^+(\downarrow), 72\hat{e}^-(\uparrow\downarrow)$	0
$^{125}_{52}\hat{\text{Te}}_{73} (1/2)$		$26 \left[\begin{matrix} \hat{4} \\ \hat{2} \end{matrix} \hat{\text{He}}_2(0) \right], 11 \hat{n}(\uparrow), 10 \hat{n}(\downarrow)$	$63\hat{p}^+(\uparrow), 62\hat{p}^+(\downarrow), 73\hat{e}^-(\uparrow\downarrow)$	1/2	

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
53	$^{126}_{52}\hat{\text{T}}\text{e}_{74} (0)$	$26 \left[\begin{smallmatrix} \hat{\text{H}}\text{e}_2(0) \\ 2 \end{smallmatrix} \right], 11 \hat{n}(\uparrow),$ $11 \hat{n}(\downarrow)$	$63\hat{p}^+(\uparrow), 63\hat{p}^+(\downarrow),$ $74\hat{e}^-(\uparrow\downarrow)$	0
	$^{128}_{52}\hat{\text{T}}\text{e}_{76} (0)$	$26 \left[\begin{smallmatrix} \hat{\text{H}}\text{e}_2(0) \\ 2 \end{smallmatrix} \right], 12 \hat{n}(\uparrow),$ $12 \hat{n}(\downarrow)$	$64\hat{p}^+(\uparrow), 64\hat{p}^+(\downarrow),$ $76\hat{e}^-(\uparrow\downarrow)$	0
	$^{130}_{52}\hat{\text{T}}\text{e}_{78} (0)$	$26 \left[\begin{smallmatrix} \hat{\text{H}}\text{e}_2(0) \\ 2 \end{smallmatrix} \right], 13 \hat{n}(\uparrow),$ $13 \hat{n}(\downarrow)$	$63\hat{p}^+(\uparrow), 63\hat{p}^+(\downarrow),$ $74\hat{e}^-(\uparrow\downarrow)$	0
	$^{127}_{53}\hat{\text{I}}_{74} (5/2)$	$26 \left[\begin{smallmatrix} \hat{\text{H}}\text{e}_2(0) \\ 2 \end{smallmatrix} \right], \hat{d}(\uparrow\uparrow),$ $12 \hat{n}(\uparrow), 9 \hat{n}(\downarrow)$	$66\hat{p}^+(\uparrow), 61\hat{p}^+(\downarrow),$ $74\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{124}_{54}\hat{\text{X}}\text{e}_{70} (0)$	$27 \left[\begin{smallmatrix} \hat{\text{H}}\text{e}_2(0) \\ 2 \end{smallmatrix} \right], 8 \hat{n}(\uparrow), 8$ $\hat{n}(\downarrow)$	$62\hat{p}^+(\uparrow), 62\hat{p}^+(\downarrow),$ $70\hat{e}^-(\uparrow\downarrow)$	0
	$^{126}_{54}\hat{\text{X}}\text{e}_{72} (0)$	$27 \left[\begin{smallmatrix} \hat{\text{H}}\text{e}_2(0) \\ 2 \end{smallmatrix} \right], 9 \hat{n}(\uparrow), 9$ $\hat{n}(\downarrow)$	$63\hat{p}^+(\uparrow), 63\hat{p}^+(\downarrow),$ $72\hat{e}^-(\uparrow\downarrow)$	0
	$^{128}_{54}\hat{\text{X}}\text{e}_{74} (0)$	$27 \left[\begin{smallmatrix} \hat{\text{H}}\text{e}_2(0) \\ 2 \end{smallmatrix} \right], 10 \hat{n}(\uparrow),$ $10 \hat{n}(\downarrow)$	$64\hat{p}^+(\uparrow), 64\hat{p}^+(\downarrow),$ $74\hat{e}^-(\uparrow\downarrow)$	0
	$^{129}_{54}\hat{\text{X}}\text{e}_{75} (1/2)$	$27 \left[\begin{smallmatrix} \hat{\text{H}}\text{e}_2(0) \\ 2 \end{smallmatrix} \right], 11 \hat{n}(\uparrow),$ $10 \hat{n}(\downarrow)$	$65\hat{p}^+(\uparrow), 64\hat{p}^+(\downarrow),$ $75\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{130}_{54}\hat{\text{X}}\text{e}_{76} (0)$	$27 \left[\begin{smallmatrix} \hat{\text{H}}\text{e}_2(0) \\ 2 \end{smallmatrix} \right], 11 \hat{n}(\uparrow),$ $11 \hat{n}(\downarrow)$	$65\hat{p}^+(\uparrow), 65\hat{p}^+(\downarrow),$ $76\hat{e}^-(\uparrow\downarrow)$	0
	$^{131}_{54}\hat{\text{X}}\text{e}_{77} (3/2)$	$27 \left[\begin{smallmatrix} \hat{\text{H}}\text{e}_2(0) \\ 2 \end{smallmatrix} \right], 13 \hat{n}(\uparrow),$ $10 \hat{n}(\downarrow)$	$67\hat{p}^+(\uparrow), 64\hat{p}^+(\downarrow),$ $77\hat{e}^-(\uparrow\downarrow)$	3/2
54	$^{132}_{54}\hat{\text{X}}\text{e}_{78} (0)$	$27 \left[\begin{smallmatrix} \hat{\text{H}}\text{e}_2(0) \\ 2 \end{smallmatrix} \right], 12 \hat{n}(\uparrow),$ $12 \hat{n}(\downarrow)$	$66\hat{p}^+(\uparrow), 66\hat{p}^+(\downarrow),$ $78\hat{e}^-(\uparrow\downarrow)$	0
	$^{134}_{54}\hat{\text{X}}\text{e}_{80} (0)$	$27 \left[\begin{smallmatrix} \hat{\text{H}}\text{e}_2(0) \\ 2 \end{smallmatrix} \right], 13 \hat{n}(\uparrow),$ $13 \hat{n}(\downarrow)$	$67\hat{p}^+(\uparrow), 67\hat{p}^+(\downarrow),$ $80\hat{e}^-(\uparrow\downarrow)$	0
	$^{136}_{54}\hat{\text{X}}\text{e}_{82} (0)$	$27 \left[\begin{smallmatrix} \hat{\text{H}}\text{e}_2(0) \\ 2 \end{smallmatrix} \right], 14 \hat{n}(\uparrow),$ $14 \hat{n}(\downarrow)$	$68\hat{p}^+(\uparrow), 68\hat{p}^+(\downarrow),$ $82\hat{e}^-(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
55	$^{133}_{55}\hat{C}_{s_{78}} (7/2)$	$27 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], \hat{d}(\uparrow\uparrow),$ $14 \hat{n}(\uparrow), 9 \hat{n}(\downarrow)$	$70\hat{p}^+(\uparrow), 63\hat{p}^+(\downarrow),$ $78\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{130}_{56}\hat{B}a_{74} (0)$	$28 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 9 \hat{n}(\uparrow), 9$ $\hat{n}(\downarrow)$	$65\hat{p}^+(\uparrow), 65\hat{p}^+(\downarrow),$ $74\hat{e}^-(\uparrow\downarrow)$	0
	$^{132}_{56}\hat{B}a_{76} (0)$	$28 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 10 \hat{n}(\uparrow),$ $10 \hat{n}(\downarrow)$	$66\hat{p}^+(\uparrow), 66\hat{p}^+(\downarrow),$ $76\hat{e}^-(\uparrow\downarrow)$	0
	$^{134}_{56}\hat{B}a_{78} (0)$	$28 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 11 \hat{n}(\uparrow),$ $11 \hat{n}(\downarrow)$	$67\hat{p}^+(\uparrow), 67\hat{p}^+(\downarrow),$ $78\hat{e}^-(\uparrow\downarrow)$	0
56	$^{135}_{56}\hat{B}a_{79} (3/2)$	$28 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 13 \hat{n}(\uparrow),$ $10 \hat{n}(\downarrow)$	$69\hat{p}^+(\uparrow), 66\hat{p}^+(\downarrow),$ $79\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{136}_{56}\hat{B}a_{80} (0)$	$28 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 12 \hat{n}(\uparrow),$ $12 \hat{n}(\downarrow)$	$68\hat{p}^+(\uparrow), 68\hat{p}^+(\downarrow),$ $80\hat{e}^-(\uparrow\downarrow)$	0
	$^{137}_{56}\hat{B}a_{81} (3/2)$	$28 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 14 \hat{n}(\uparrow),$ $11 \hat{n}(\downarrow)$	$70\hat{p}^+(\uparrow), 67\hat{p}^+(\downarrow),$ $81\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{138}_{56}\hat{B}a_{82} (0)$	$28 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 13 \hat{n}(\uparrow),$ $13 \hat{n}(\downarrow)$	$69\hat{p}^+(\uparrow), 69\hat{p}^+(\downarrow),$ $82\hat{e}^-(\uparrow\downarrow)$	0
57	$^{138}_{57}\hat{L}a_{81} (5)$	$28 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], \hat{d}(\uparrow\uparrow),$ $16 \hat{n}(\uparrow), 8 \hat{n}(\downarrow)$	$74\hat{p}^+(\uparrow), 64\hat{p}^+(\downarrow),$ $81\hat{e}^-(\uparrow\downarrow)$	5
	$^{139}_{57}\hat{L}a_{82} (7/2)$	$28 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], \hat{d}(\uparrow\uparrow),$ $15 \hat{n}(\uparrow), 10 \hat{n}(\downarrow)$	$73\hat{p}^+(\uparrow), 66\hat{p}^+(\downarrow),$ $82\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{136}_{58}\hat{C}e_{78} (0)$	$29 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 10 \hat{n}(\uparrow),$ $10 \hat{n}(\downarrow)$	$68\hat{p}^+(\uparrow), 68\hat{p}^+(\downarrow),$ $78\hat{e}^-(\uparrow\downarrow)$	0
58	$^{138}_{58}\hat{C}e_{80} (0)$	$29 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 11 \hat{n}(\uparrow),$ $11 \hat{n}(\downarrow)$	$69\hat{p}^+(\uparrow), 69\hat{p}^+(\downarrow),$ $80\hat{e}^-(\uparrow\downarrow)$	0
	$^{140}_{58}\hat{C}e_{82} (0)$	$29 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 12 \hat{n}(\uparrow),$ $12 \hat{n}(\downarrow)$	$70\hat{p}^+(\uparrow), 70\hat{p}^+(\downarrow),$ $82\hat{e}^-(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
59	$^{142}_{58}\hat{C}e_{84} (0)$	$29 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{H}e_2(0) \end{matrix} \right], 13 \hat{n}(\uparrow),$ $13 \hat{n}(\downarrow)$	$71\hat{p}^+(\uparrow), 71\hat{p}^+(\downarrow),$ $84\hat{e}^-(\uparrow\downarrow)$	0
	$^{141}_{59}\hat{P}r_{82} (5/2)$	$29 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{H}e_2(0) \end{matrix} \right], \hat{d}(\uparrow\uparrow),$ $13 \hat{n}(\uparrow), 10 \hat{n}(\downarrow)$	$73\hat{p}^+(\uparrow), 68\hat{p}^+(\downarrow),$ $82\hat{e}^-(\uparrow\downarrow)$	5/2
60	$^{142}_{60}\hat{N}d_{82} (0)$	$30 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{H}e_2(0) \end{matrix} \right], 11 \hat{n}(\uparrow),$ $11 \hat{n}(\downarrow)$	$71\hat{p}^+(\uparrow), 71\hat{p}^+(\downarrow),$ $82\hat{e}^-(\uparrow\downarrow)$	0
	$^{143}_{60}\hat{N}d_{83} (7/2)$	$30 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{H}e_2(0) \end{matrix} \right], 15 \hat{n}(\uparrow),$ $8 \hat{n}(\downarrow)$	$75\hat{p}^+(\uparrow), 68\hat{p}^+(\downarrow),$ $83\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{144}_{60}\hat{N}d_{84} (0)$	$30 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{H}e_2(0) \end{matrix} \right], 12 \hat{n}(\uparrow),$ $12 \hat{n}(\downarrow)$	$72\hat{p}^+(\uparrow), 72\hat{p}^+(\downarrow),$ $84\hat{e}^-(\uparrow\downarrow)$	0
	$^{145}_{60}\hat{N}d_{85} (7/2)$	$30 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{H}e_2(0) \end{matrix} \right], 16 \hat{n}(\uparrow),$ $9 \hat{n}(\downarrow)$	$76\hat{p}^+(\uparrow), 69\hat{p}^+(\downarrow),$ $85\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{146}_{60}\hat{N}d_{86} (0)$	$30 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{H}e_2(0) \end{matrix} \right], 13 \hat{n}(\uparrow),$ $13 \hat{n}(\downarrow)$	$73\hat{p}^+(\uparrow), 73\hat{p}^+(\downarrow),$ $86\hat{e}^-(\uparrow\downarrow)$	0
	$^{148}_{60}\hat{N}d_{88} (0)$	$30 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{H}e_2(0) \end{matrix} \right], 14 \hat{n}(\uparrow),$ $14 \hat{n}(\downarrow)$	$74\hat{p}^+(\uparrow), 74\hat{p}^+(\downarrow),$ $88\hat{e}^-(\uparrow\downarrow)$	0
61	$^{xxx}_{61}\hat{P}m_{yy} (?)$	No Stable Nuclide	No Stable Nuclide	
	$^{144}_{62}\hat{S}m_{82} (0)$	$31 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{H}e_2(0) \end{matrix} \right], 10 \hat{n}(\uparrow),$ $10 \hat{n}(\downarrow)$	$72\hat{p}^+(\uparrow), 72\hat{p}^+(\downarrow),$ $82\hat{e}^-(\uparrow\downarrow)$	0
	$^{147}_{62}\hat{S}m_{85} (7/2)$	$31 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{H}e_2(0) \end{matrix} \right], 15 \hat{n}(\uparrow),$ $8 \hat{n}(\downarrow)$	$77\hat{p}^+(\uparrow), 70\hat{p}^+(\downarrow),$ $85\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{148}_{62}\hat{S}m_{86} (0)$	$31 \left[\begin{matrix} \hat{H}e_2(0) \\ \hat{H}e_2(0) \end{matrix} \right], 12 \hat{n}(\uparrow),$ $12 \hat{n}(\downarrow)$	$74\hat{p}^+(\uparrow), 74\hat{p}^+(\downarrow),$ $86\hat{e}^-(\uparrow\downarrow)$	0
62				

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
63	$^{149}_{62}\hat{\text{Sm}}_{87} (7/2)$	$31 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 16 \hat{n}(\uparrow),$ $9 \hat{n}(\downarrow)$	$78\hat{p}^+(\uparrow), 71\hat{p}^+(\downarrow),$ $87\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{150}_{62}\hat{\text{Sm}}_{88} (0)$	$31 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 13 \hat{n}(\uparrow),$ $13 \hat{n}(\downarrow)$	$75\hat{p}^+(\uparrow), 75\hat{p}^+(\downarrow),$ $88\hat{e}^-(\uparrow\downarrow)$	0
	$^{152}_{62}\hat{\text{Sm}}_{90} (0)$	$31 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 14 \hat{n}(\uparrow),$ $14 \hat{n}(\downarrow)$	$76\hat{p}^+(\uparrow), 76\hat{p}^+(\downarrow),$ $90\hat{e}^-(\uparrow\downarrow)$	0
	$^{154}_{62}\hat{\text{Sm}}_{92} (0)$	$31 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 15 \hat{n}(\uparrow),$ $15 \hat{n}(\downarrow)$	$77\hat{p}^+(\uparrow), 77\hat{p}^+(\downarrow),$ $92\hat{e}^-(\uparrow\downarrow)$	0
	$^{151}_{63}\hat{\text{Eu}}_{88} (5/2)$	$31 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $14 \hat{n}(\uparrow), 11 \hat{n}(\downarrow)$	$78\hat{p}^+(\uparrow), 73\hat{p}^+(\downarrow),$ $88\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{153}_{63}\hat{\text{Eu}}_{90} (5/2)$	$31 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $15 \hat{n}(\uparrow), 12 \hat{n}(\downarrow)$	$79\hat{p}^+(\uparrow), 74\hat{p}^+(\downarrow),$ $90\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{152}_{64}\hat{\text{Gd}}_{88} (0)$	$32 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 12 \hat{n}(\uparrow),$ $12 \hat{n}(\downarrow)$	$76\hat{p}^+(\uparrow), 76\hat{p}^+(\downarrow),$ $88\hat{e}^-(\uparrow\downarrow)$	0
	$^{154}_{64}\hat{\text{Gd}}_{90} (0)$	$32 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 13 \hat{n}(\uparrow),$ $13 \hat{n}(\downarrow)$	$77\hat{p}^+(\uparrow), 77\hat{p}^+(\downarrow),$ $90\hat{e}^-(\uparrow\downarrow)$	0
	$^{155}_{64}\hat{\text{Gd}}_{91} (3/2)$	$32 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 15 \hat{n}(\uparrow),$ $12 \hat{n}(\downarrow)$	$79\hat{p}^+(\uparrow), 76\hat{p}^+(\downarrow),$ $91\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{156}_{64}\hat{\text{Gd}}_{92} (0)$	$32 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 14 \hat{n}(\uparrow),$ $14 \hat{n}(\downarrow)$	$78\hat{p}^+(\uparrow), 78\hat{p}^+(\downarrow),$ $92\hat{e}^-(\uparrow\downarrow)$	0
64	$^{157}_{64}\hat{\text{Gd}}_{93} (3/2)$	$32 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 16 \hat{n}(\uparrow),$ $13 \hat{n}(\downarrow)$	$80\hat{p}^+(\uparrow), 77\hat{p}^+(\downarrow),$ $93\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{158}_{64}\hat{\text{Gd}}_{94} (0)$	$32 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 15 \hat{n}(\uparrow),$ $15 \hat{n}(\downarrow)$	$79\hat{p}^+(\uparrow), 79\hat{p}^+(\downarrow),$ $94\hat{e}^-(\uparrow\downarrow)$	0
	$^{160}_{64}\hat{\text{Gd}}_{96} (0)$	$32 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 16 \hat{n}(\uparrow),$ $16 \hat{n}(\downarrow)$	$80\hat{p}^+(\uparrow), 80\hat{p}^+(\downarrow),$ $96\hat{e}^-(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
65	$^{159}_{65}\hat{\text{Tb}}_{94} (3/2)$	$32 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $15 \hat{n}(\uparrow), 14 \hat{n}(\downarrow)$	$81\hat{p}^+(\uparrow), 78\hat{p}^+(\downarrow),$ $94\hat{e}^-(\uparrow\downarrow)$	3/2
	$^{156}_{66}\hat{\text{Dy}}_{90} (0)$	$33 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 12 \hat{n}(\uparrow),$ $12 \hat{n}(\downarrow)$	$78\hat{p}^+(\uparrow), 78\hat{p}^+(\downarrow),$ $90\hat{e}^-(\uparrow\downarrow)$	0
	$^{158}_{66}\hat{\text{Dy}}_{92} (0)$	$33 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 13 \hat{n}(\uparrow),$ $13 \hat{n}(\downarrow)$	$79\hat{p}^+(\uparrow), 79\hat{p}^+(\downarrow),$ $92\hat{e}^-(\uparrow\downarrow)$	0
	$^{160}_{66}\hat{\text{Dy}}_{94} (0)$	$33 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 14 \hat{n}(\uparrow),$ $14 \hat{n}(\downarrow)$	$80\hat{p}^+(\uparrow), 80\hat{p}^+(\downarrow),$ $94\hat{e}^-(\uparrow\downarrow)$	0
66	$^{161}_{66}\hat{\text{Dy}}_{95} (5/2)$	$33 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 17 \hat{n}(\uparrow),$ $12 \hat{n}(\downarrow)$	$83\hat{p}^+(\uparrow), 78\hat{p}^+(\downarrow),$ $95\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{162}_{66}\hat{\text{Dy}}_{96} (0)$	$33 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 15 \hat{n}(\uparrow),$ $15 \hat{n}(\downarrow)$	$81\hat{p}^+(\uparrow), 81\hat{p}^+(\downarrow),$ $96\hat{e}^-(\uparrow\downarrow)$	0
	$^{163}_{66}\hat{\text{Dy}}_{97} (5/2)$	$33 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 18 \hat{n}(\uparrow),$ $13 \hat{n}(\downarrow)$	$84\hat{p}^+(\uparrow), 79\hat{p}^+(\downarrow),$ $97\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{164}_{66}\hat{\text{Dy}}_{98} (0)$	$33 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 16 \hat{n}(\uparrow),$ $16 \hat{n}(\downarrow)$	$82\hat{p}^+(\uparrow), 82\hat{p}^+(\downarrow),$ $98\hat{e}^-(\uparrow\downarrow)$	0
67	$^{165}_{67}\hat{\text{Ho}}_{98} (7/2)$	$33 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $18 \hat{n}(\uparrow), 13 \hat{n}(\downarrow)$	$86\hat{p}^+(\uparrow), 79\hat{p}^+(\downarrow),$ $98\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{162}_{68}\hat{\text{Er}}_{94} (0)$	$34 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 13 \hat{n}(\uparrow),$ $13 \hat{n}(\downarrow)$	$81\hat{p}^+(\uparrow), 81\hat{p}^+(\downarrow),$ $94\hat{e}^-(\uparrow\downarrow)$	0
	$^{164}_{68}\hat{\text{Er}}_{96} (0)$	$34 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 14 \hat{n}(\uparrow),$ $14 \hat{n}(\downarrow)$	$82\hat{p}^+(\uparrow), 82\hat{p}^+(\downarrow),$ $96\hat{e}^-(\uparrow\downarrow)$	0
68	$^{166}_{68}\hat{\text{Er}}_{98} (0)$	$34 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 15 \hat{n}(\uparrow),$ $15 \hat{n}(\downarrow)$	$83\hat{p}^+(\uparrow), 83\hat{p}^+(\downarrow),$ $98\hat{e}^-(\uparrow\downarrow)$	0
	$^{167}_{68}\hat{\text{Er}}_{99} (7/2)$	$34 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 19 \hat{n}(\uparrow),$ $12 \hat{n}(\downarrow)$	$87\hat{p}^+(\uparrow), 80\hat{p}^+(\downarrow),$ $99\hat{e}^-(\uparrow\downarrow)$	7/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
69	$^{168}_{68}\hat{\text{E}}\text{r}_{100} (0)$	$34 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{H}}\text{e}_2(0) \right], 16 \hat{n}(\uparrow),$ $16 \hat{n}(\downarrow)$	$84\hat{p}^+(\uparrow), 84\hat{p}^+(\downarrow),$ $100\hat{e}^-(\uparrow\downarrow)$	0
	$^{170}_{68}\hat{\text{E}}\text{r}_{102} (0)$	$34 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{H}}\text{e}_2(0) \right], 17 \hat{n}(\uparrow),$ $17 \hat{n}(\downarrow)$	$86\hat{p}^+(\uparrow), 86\hat{p}^+(\downarrow),$ $102\hat{e}^-(\uparrow\downarrow)$	0
	$^{169}_{69}\hat{\text{T}}\text{m}_{100} (1/2)$	$34 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{H}}\text{e}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $16 \hat{n}(\uparrow), 15 \hat{n}(\downarrow)$	$85\hat{p}^+(\uparrow), 84\hat{p}^+(\downarrow),$ $100\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{168}_{70}\hat{\text{Y}}\text{b}_{98} (0)$	$35 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{H}}\text{e}_2(0) \right], 14 \hat{n}(\uparrow),$ $14 \hat{n}(\downarrow)$	$84\hat{p}^+(\uparrow), 84\hat{p}^+(\downarrow),$ $98\hat{e}^-(\uparrow\downarrow)$	0
70	$^{170}_{70}\hat{\text{Y}}\text{b}_{100} (0)$	$35 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{H}}\text{e}_2(0) \right], 15 \hat{n}(\uparrow),$ $15 \hat{n}(\downarrow)$	$85\hat{p}^+(\uparrow), 85\hat{p}^+(\downarrow),$ $100\hat{e}^-(\uparrow\downarrow)$	0
	$^{171}_{70}\hat{\text{Y}}\text{b}_{101} (1/2)$	$35 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{H}}\text{e}_2(0) \right], 16 \hat{n}(\uparrow),$ $15 \hat{n}(\downarrow)$	$86\hat{p}^+(\uparrow), 85\hat{p}^+(\downarrow),$ $101\hat{e}^-(\uparrow\downarrow)$	1/2
	$^{172}_{70}\hat{\text{Y}}\text{b}_{102} (0)$	$35 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{H}}\text{e}_2(0) \right], 16 \hat{n}(\uparrow),$ $16 \hat{n}(\downarrow)$	$86\hat{p}^+(\uparrow), 86\hat{p}^+(\downarrow),$ $102\hat{e}^-(\uparrow\downarrow)$	0
	$^{173}_{70}\hat{\text{Y}}\text{b}_{103} (5/2)$	$35 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{H}}\text{e}_2(0) \right], 19 \hat{n}(\uparrow),$ $14 \hat{n}(\downarrow)$	$89\hat{p}^+(\uparrow), 84\hat{p}^+(\downarrow),$ $103\hat{e}^-(\uparrow\downarrow)$	5/2
	$^{174}_{70}\hat{\text{Y}}\text{b}_{104} (0)$	$35 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{H}}\text{e}_2(0) \right], 17 \hat{n}(\uparrow),$ $17 \hat{n}(\downarrow)$	$87\hat{p}^+(\uparrow), 87\hat{p}^+(\downarrow),$ $104\hat{e}^-(\uparrow\downarrow)$	0
	$^{176}_{70}\hat{\text{Y}}\text{b}_{106} (0)$	$35 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{H}}\text{e}_2(0) \right], 18 \hat{n}(\uparrow),$ $18 \hat{n}(\downarrow)$	$88\hat{p}^+(\uparrow), 88\hat{p}^+(\downarrow),$ $106\hat{e}^-(\uparrow\downarrow)$	0
71	$^{175}_{71}\hat{\text{L}}\text{u}_{104} (7/2)$	$35 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{H}}\text{e}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $19 \hat{n}(\uparrow), 14 \hat{n}(\downarrow)$	$91\hat{p}^+(\uparrow), 84\hat{p}^+(\downarrow),$ $104\hat{e}^-(\uparrow\downarrow)$	7/2
	$^{176}_{71}\hat{\text{L}}\text{u}_{105} (7)$	$35 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{H}}\text{e}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $23 \hat{n}(\uparrow), 11 \hat{n}(\downarrow)$	$95\hat{p}^+(\uparrow), 81\hat{p}^+(\downarrow),$ $105\hat{e}^-(\uparrow\downarrow)$	7
72	$^{174}_{72}\hat{\text{H}}\text{f}_{102} (0)$	$36 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{H}}\text{e}_2(0) \right], 15 \hat{n}(\uparrow),$ $15 \hat{n}(\downarrow)$	$87\hat{p}^+(\uparrow), 87\hat{p}^+(\downarrow),$ $102\hat{e}^-(\uparrow\downarrow)$	0

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
	${}_{72}^{176}\hat{\text{Hf}}_{104} (0)$	$36 \left[{}_2^4\hat{\text{He}}_2(0) \right], 16 \hat{n}(\uparrow),$ $16 \hat{n}(\downarrow)$	$88\hat{p}^+(\uparrow), 88\hat{p}^+(\downarrow),$ $104\hat{e}^-(\uparrow\downarrow)$	0
	${}_{72}^{177}\hat{\text{Hf}}_{105} (7/2)$	$36 \left[{}_2^4\hat{\text{He}}_2(0) \right], 20 \hat{n}(\uparrow),$ $13 \hat{n}(\downarrow)$	$92\hat{p}^+(\uparrow), 85\hat{p}^+(\downarrow),$ $105\hat{e}^-(\uparrow\downarrow)$	7/2
	${}_{72}^{178}\hat{\text{Hf}}_{106} (0)$	$36 \left[{}_2^4\hat{\text{He}}_2(0) \right], 17 \hat{n}(\uparrow),$ $17 \hat{n}(\downarrow)$	$89\hat{p}^+(\uparrow), 89\hat{p}^+(\downarrow),$ $106\hat{e}^-(\uparrow\downarrow)$	0
	${}_{72}^{179}\hat{\text{Hf}}_{107} (9/2)$	$36 \left[{}_2^4\hat{\text{He}}_2(0) \right], 22 \hat{n}(\uparrow),$ $13 \hat{n}(\downarrow)$	$94\hat{p}^+(\uparrow), 85\hat{p}^+(\downarrow),$ $107\hat{e}^-(\uparrow\downarrow)$	9/2
	${}_{72}^{180}\hat{\text{Hf}}_{108} (0)$	$36 \left[{}_2^4\hat{\text{He}}_2(0) \right], 18 \hat{n}(\uparrow),$ $18 \hat{n}(\downarrow)$	$90\hat{p}^+(\uparrow), 90\hat{p}^+(\downarrow),$ $108\hat{e}^-(\uparrow\downarrow)$	0
73	${}_{73}^{181}\hat{\text{Ta}}_{108} (7/2)$	$36 \left[{}_2^4\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $20 \hat{n}(\uparrow), 15 \hat{n}(\downarrow)$	$94\hat{p}^+(\uparrow), 87\hat{p}^+(\downarrow),$ $108\hat{e}^-(\uparrow\downarrow)$	7/2
	${}_{74}^{180}\hat{\text{W}}_{106} (0)$	$37 \left[{}_2^4\hat{\text{He}}_2(0) \right], 16 \hat{n}(\uparrow),$ $16 \hat{n}(\downarrow)$	$90\hat{p}^+(\uparrow), 90\hat{p}^+(\downarrow),$ $106\hat{e}^-(\uparrow\downarrow)$	0
	${}_{74}^{182}\hat{\text{W}}_{108} (0)$	$37 \left[{}_2^4\hat{\text{He}}_2(0) \right], 17 \hat{n}(\uparrow),$ $17 \hat{n}(\downarrow)$	$91\hat{p}^+(\uparrow), 91\hat{p}^+(\downarrow),$ $108\hat{e}^-(\uparrow\downarrow)$	0
74	${}_{74}^{183}\hat{\text{W}}_{109} (1/2)$	$37 \left[{}_2^4\hat{\text{He}}_2(0) \right], 18 \hat{n}(\uparrow),$ $17 \hat{n}(\downarrow)$	$92\hat{p}^+(\uparrow), 91\hat{p}^+(\downarrow),$ $109\hat{e}^-(\uparrow\downarrow)$	1/2
	${}_{74}^{184}\hat{\text{W}}_{110} (0)$	$37 \left[{}_2^4\hat{\text{He}}_2(0) \right], 18 \hat{n}(\uparrow),$ $18 \hat{n}(\downarrow)$	$92\hat{p}^+(\uparrow), 92\hat{p}^+(\downarrow),$ $110\hat{e}^-(\uparrow\downarrow)$	0
	${}_{74}^{186}\hat{\text{W}}_{112} (0)$	$37 \left[{}_2^4\hat{\text{He}}_2(0) \right], 19 \hat{n}(\uparrow),$ $19 \hat{n}(\downarrow)$	$93\hat{p}^+(\uparrow), 93\hat{p}^+(\downarrow),$ $112\hat{e}^-(\uparrow\downarrow)$	0
75	${}_{75}^{185}\hat{\text{Re}}_{110} (5/2)$	$37 \left[{}_2^4\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $19 \hat{n}(\uparrow), 16 \hat{n}(\downarrow)$	$95\hat{p}^+(\uparrow), 90\hat{p}^+(\downarrow),$ $110\hat{e}^-(\uparrow\downarrow)$	5/2
	${}_{75}^{187}\hat{\text{Re}}_{112} (5/2)$	$37 \left[{}_2^4\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $20 \hat{n}(\uparrow), 17 \hat{n}(\downarrow)$	$96\hat{p}^+(\uparrow), 91\hat{p}^+(\downarrow),$ $112\hat{e}^-(\uparrow\downarrow)$	5/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J	
76	$^{184}_{76}\hat{O}S_{108} (0)$	$38 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 16 \hat{n}(\uparrow),$ $16 \hat{n}(\downarrow)$	$92\hat{p}^+(\uparrow), 92\hat{p}^+(\downarrow),$ $108\hat{e}^-(\uparrow\downarrow)$	0	
	$^{186}_{76}\hat{O}S_{110} (0)$	$38 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 17 \hat{n}(\uparrow),$ $17 \hat{n}(\downarrow)$	$93\hat{p}^+(\uparrow), 93\hat{p}^+(\downarrow),$ $110\hat{e}^-(\uparrow\downarrow)$	0	
	$^{187}_{76}\hat{O}S_{111} (1/2)$	$38 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 18 \hat{n}(\uparrow),$ $17 \hat{n}(\downarrow)$	$94\hat{p}^+(\uparrow), 93\hat{p}^+(\downarrow),$ $111\hat{e}^-(\uparrow\downarrow)$	1/2	
	$^{188}_{76}\hat{O}S_{112} (0)$	$38 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 18 \hat{n}(\uparrow),$ $18 \hat{n}(\downarrow)$	$94\hat{p}^+(\uparrow), 94\hat{p}^+(\downarrow),$ $112\hat{e}^-(\uparrow\downarrow)$	0	
	$^{189}_{76}\hat{O}S_{113} (3/2)$	$38 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 20 \hat{n}(\uparrow),$ $17 \hat{n}(\downarrow)$	$96\hat{p}^+(\uparrow), 93\hat{p}^+(\downarrow),$ $113\hat{e}^-(\uparrow\downarrow)$	3/2	
	$^{190}_{76}\hat{O}S_{114} (0)$	$38 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 19 \hat{n}(\uparrow),$ $19 \hat{n}(\downarrow)$	$95\hat{p}^+(\uparrow), 95\hat{p}^+(\downarrow),$ $114\hat{e}^-(\uparrow\downarrow)$	0	
	$^{192}_{76}\hat{O}S_{116} (0)$	$38 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 20 \hat{n}(\uparrow),$ $20 \hat{n}(\downarrow)$	$96\hat{p}^+(\uparrow), 96\hat{p}^+(\downarrow),$ $116\hat{e}^-(\uparrow\downarrow)$	0	
	77	$^{191}_{77}\hat{I}r_{114} (3/2)$	$38 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], \hat{d}(\uparrow\uparrow),$ $19 \hat{n}(\uparrow), 18 \hat{n}(\downarrow)$	$97\hat{p}^+(\uparrow), 94\hat{p}^+(\downarrow),$ $114\hat{e}^-(\uparrow\downarrow)$	3/2
		$^{193}_{77}\hat{I}r_{116} (3/2)$	$38 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], \hat{d}(\uparrow\uparrow),$ $20 \hat{n}(\uparrow), 19 \hat{n}(\downarrow)$	$98\hat{p}^+(\uparrow), 95\hat{p}^+(\downarrow),$ $116\hat{e}^-(\uparrow\downarrow)$	3/2
	78	$^{192}_{78}\hat{P}t_{114} (0)$	$39 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 18 \hat{n}(\uparrow),$ $18 \hat{n}(\downarrow)$	$96\hat{p}^+(\uparrow), 96\hat{p}^+(\downarrow),$ $114\hat{e}^-(\uparrow\downarrow)$	0
$^{194}_{78}\hat{P}t_{116} (0)$		$39 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 19 \hat{n}(\uparrow),$ $19 \hat{n}(\downarrow)$	$97\hat{p}^+(\uparrow), 97\hat{p}^+(\downarrow),$ $116\hat{e}^-(\uparrow\downarrow)$	0	
$^{195}_{78}\hat{P}t_{117} (1/2)$		$39 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 20 \hat{n}(\uparrow),$ $19 \hat{n}(\downarrow)$	$98\hat{p}^+(\uparrow), 97\hat{p}^+(\downarrow),$ $117\hat{e}^-(\uparrow\downarrow)$	1/2	
$^{196}_{78}\hat{P}t_{118} (0)$		$39 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{H}e_2(0) \right], 20 \hat{n}(\uparrow),$ $20 \hat{n}(\downarrow)$	$98\hat{p}^+(\uparrow), 98\hat{p}^+(\downarrow),$ $118\hat{e}^-(\uparrow\downarrow)$	0	

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
79	${}_{78}^{198}\hat{\text{Pt}}_{120} (0)$	$39 \left[{}_2^4\hat{\text{He}}_2(0) \right], 21 \hat{n}(\uparrow),$ $21 \hat{n}(\downarrow)$	$99\hat{p}^+(\uparrow), 99\hat{p}^+(\downarrow),$ $120\hat{e}^-(\uparrow\downarrow)$	0
	${}_{79}^{197}\hat{\text{Au}}_{118} (3/2)$	$39 \left[{}_2^4\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $20 \hat{n}(\uparrow), 19 \hat{n}(\downarrow)$	$100\hat{p}^+(\uparrow), 97\hat{p}^+(\downarrow),$ $118\hat{e}^-(\uparrow\downarrow)$	3/2
80	${}_{80}^{196}\hat{\text{Hg}}_{116} (0)$	$40 \left[{}_2^4\hat{\text{He}}_2(0) \right], 18 \hat{n}(\uparrow),$ $18 \hat{n}(\downarrow)$	$98\hat{p}^+(\uparrow), 98\hat{p}^+(\downarrow),$ $116\hat{e}^-(\uparrow\downarrow)$	0
	${}_{80}^{198}\hat{\text{Hg}}_{118} (0)$	$40 \left[{}_2^4\hat{\text{He}}_2(0) \right], 19 \hat{n}(\uparrow),$ $19 \hat{n}(\downarrow)$	$99\hat{p}^+(\uparrow), 99\hat{p}^+(\downarrow),$ $118\hat{e}^-(\uparrow\downarrow)$	0
	${}_{80}^{199}\hat{\text{Hg}}_{119} (1/2)$	$40 \left[{}_2^4\hat{\text{He}}_2(0) \right], 20 \hat{n}(\uparrow),$ $19 \hat{n}(\downarrow)$	$100\hat{p}^+(\uparrow), 99\hat{p}^+(\downarrow),$ $119\hat{e}^-(\uparrow\downarrow)$	1/2
	${}_{80}^{200}\hat{\text{Hg}}_{120} (0)$	$40 \left[{}_2^4\hat{\text{He}}_2(0) \right], 20 \hat{n}(\uparrow),$ $20 \hat{n}(\downarrow)$	$100\hat{p}^+(\uparrow), 100\hat{p}^+(\downarrow),$ $120\hat{e}^-(\uparrow\downarrow)$	0
	${}_{80}^{201}\hat{\text{Hg}}_{121} (3/2)$	$40 \left[{}_2^4\hat{\text{He}}_2(0) \right], 22 \hat{n}(\uparrow),$ $19 \hat{n}(\downarrow)$	$102\hat{p}^+(\uparrow), 99\hat{p}^+(\downarrow),$ $121\hat{e}^-(\uparrow\downarrow)$	3/2
	${}_{80}^{202}\hat{\text{Hg}}_{122} (0)$	$40 \left[{}_2^4\hat{\text{He}}_2(0) \right], 21 \hat{n}(\uparrow),$ $21 \hat{n}(\downarrow)$	$101\hat{p}^+(\uparrow), 101\hat{p}^+(\downarrow),$ $122\hat{e}^-(\uparrow\downarrow)$	0
81	${}_{80}^{204}\hat{\text{Hg}}_{124} (0)$	$40 \left[{}_2^4\hat{\text{He}}_2(0) \right], 22 \hat{n}(\uparrow),$ $22 \hat{n}(\downarrow)$	$102\hat{p}^+(\uparrow), 102\hat{p}^+(\downarrow),$ $124\hat{e}^-(\uparrow\downarrow)$	0
	${}_{81}^{203}\hat{\text{Tl}}_{122} (1/2)$	$40 \left[{}_2^4\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $20 \hat{n}(\uparrow), 21 \hat{n}(\downarrow)$	$102\hat{p}^+(\uparrow), 101\hat{p}^+(\downarrow),$ $122\hat{e}^-(\uparrow\downarrow)$	1/2
	${}_{81}^{205}\hat{\text{Tl}}_{124} (1/2)$	$40 \left[{}_2^4\hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow),$ $21 \hat{n}(\uparrow), 22 \hat{n}(\downarrow)$	$103\hat{p}^+(\uparrow), 102\hat{p}^+(\downarrow),$ $124\hat{e}^-(\uparrow\downarrow)$	1/2
82	${}_{82}^{204}\hat{\text{Pb}}_{122} (0)$	$41 \left[{}_2^4\hat{\text{He}}_2(0) \right], 20 \hat{n}(\uparrow),$ $20 \hat{n}(\downarrow)$	$102\hat{p}^+(\uparrow), 102\hat{p}^+(\downarrow),$ $122\hat{e}^-(\uparrow\downarrow)$	0
	${}_{82}^{206}\hat{\text{Pb}}_{124} (0)$	$41 \left[{}_2^4\hat{\text{He}}_2(0) \right], 21 \hat{n}(\uparrow),$ $21 \hat{n}(\downarrow)$	$103\hat{p}^+(\uparrow), 103\hat{p}^+(\downarrow),$ $124\hat{e}^-(\uparrow\downarrow)$	0
	${}_{82}^{207}\hat{\text{Pb}}_{125} (1/2)$	$41 \left[{}_2^4\hat{\text{He}}_2(0) \right], 22 \hat{n}(\uparrow),$ $21 \hat{n}(\downarrow)$	$104\hat{p}^+(\uparrow), 103\hat{p}^+(\downarrow),$ $125\hat{e}^-(\uparrow\downarrow)$	1/2

Atomic Number, Z	Isonuclides of Chemical Elements	Nuclear Configuration Model-I	Nuclear Configuration Model-II	Nuclear Spin, J
	${}^{208}_{82}\hat{\text{Pb}}_{126}(0)$	$41 \left[{}^4_2\hat{\text{He}}_2(0) \right], 22 \hat{n}(\uparrow),$ $22 \hat{n}(\downarrow)$	$104 \hat{p}^+(\uparrow), 104 \hat{p}^+(\downarrow),$ $126 \hat{e}^-(\uparrow\downarrow)$	0

14. Some Observations

The purpose of presenting the nuclear configuration in terms of isonucleons in this paper is to make available adequate ground so that one can attempt to (i) develop a theory of nuclear stability and (ii) acquire understanding of other nuclear properties of the stable nuclides. In Table 2 we have proton as the first entry and 278 stable, primordial and very long lived isonuclides up to and including atomic number 82 of the periodic table. Beyond Pb all elements are radioactive. There are two elements namely Tc ($Z=43$) and Pm ($Z=61$) having no stable isotopes, that has also been mentioned in Table 2. The columns 3, 4 and 5 of Table 2 depict respectively the hadronic mechanics based nuclear configuration of models I and II, and experimental nuclear spin of the isonuclide. In the present Section 14 we summarize our observations on them.

14.1. Nuclear Configuration of Model-I

14.1.1. Proton

In Table 2 there are two entries corresponding to $Z=1$. They are the isotopes of hydrogen, the first element of the periodic table.

Thus, ${}^1_1\text{H}_0$, in fact, is the proton, the fundamental particle, which is a stable particle. For its description no hadronic mechanics is required, hence it is not an isonuclide.

14.1.2. Isodeuteron

Hydrogen of mass number 2 is conventionally termed as deuterium. Its nucleus, indeed, is an isonucleus hence it is termed as isodeuteron that gets represented as ${}^2_1\hat{\text{H}}_1$. We represent this system in our proposed notation as,

$${}^2_1\text{H}_1 : n = 1, p^+ = 1 \} \Rightarrow$$

$$\left[\left(\hat{p}^+(\uparrow), \hat{e}^-(J=0), \hat{p}^+(\uparrow) \right) \right]_{hm}$$

$$\equiv {}^2_1\text{H}_1(1) \equiv \hat{d}(\uparrow\uparrow), \text{ stable, } J=1 \quad (94)$$

How the nuclear spin of value 1 for isodeuteron originates gets easily understood from Figure 9.

14.1.3. Other Stable Isonuclides of Table 2

All the stable isonuclides of Table 2 beyond hydrogen are the combination of isodeuterons and isoneutrons except He-3 which consists of an isodeuteron and an isoproton.

I. Stable Isonuclides with Null Nuclear Spin

Out of total of 278 isonuclides of Table 2 there we have 163 isonuclides having nuclear spin of 0. From this group 9 isonuclides consists only of all spin paired isodeuterons, and

they can be considered as possessing 1, 3, 4, 5, 6, 7, 8, 9 and 10 ${}^4_2\hat{\text{Be}}_2(0)$ centers. Notice that the number 2 is notoriously

missing in this list. That corresponds to the isonuclide ${}^8_4\hat{\text{Be}}_4(0)$ that we know is unstable and instantaneously disintegrates to α -particles. The same observation in terms of isodeuterons speaks as follows. Recall that the isodeuteron is a stable combination of isonucleons. However, two isodeuterons in the singlet coupling are also stable, which actually is the α -particle. Next on addition of one isodeuteron to it there we form an isonuclide of Li-6, which also is a stable isonuclide. But on further adding one more isodeuteron with total nuclear spin zero we obtain Be-8 isonuclide which is unstable. Thus we see that three isodeuteron in low spin state is stable but the four isodeuteron in zero spin state is unstable (But notice that in the case of stable Be-9 there we have two parallel spin isodeuterons coupled with one isoneutron of opposite spin. It means that the addition of one isoneutron to Be-8 forces one spin paired isodeuterons to assume parallel spin and itself combines to them with opposite spin that imparts stability to Be-9 with net nuclear spin of 3/2.). However, the next stable isonuclide is B-10 consisting of 5 isodeuterons. But in this case there we have two spin paired isodeuterons and three unpaired ones, ironically which is not a combination of 2α -particles and one isodeuteron similar to Li-6. The next stable isonuclide is C-12 that consists of 6 isodeuterons in the spin paired state, which is equivalent to strongly bound combination of 3α -particles. It is surprising that the combination of 2α -particles is unstable but the combination of 3α -particles is stable one. Here onwards 4 to 10α -particles combination are all stable ones.

The remaining 154 isonuclides with null nuclear spin consist of even number of isodeuterons and even number of isoneutrons and they are all spin paired. Notice that not only the isoneutrons get stabilized but also the zero spin di-isonutrons are getting stabilized in the environment of zero spin isodeuterons. Amongst them from Ca-42 and onwards we have the combination of di-isonutrons and isodeuterons. The number of spin paired di-isonutrons continuously increases and rises ultimately to 22 in number in the case of Pb-208 that consists of 82 spin paired isodeuterons. Recall that a dineutron is not a stable entity but 22 spin paired di-isonutrons of Pb-208 in the presence of 41 α -particles are stable. We need to investigate further what interactions are responsible for this extraordinary stability. However, we also need to take into account the nuclear configuration of adjacent unstable isonuclide. For example, Ca-40 and Ca-42 are both stable but Ca-39 and Ca-41 are unstable nuclides and Ca-43 is stable one. The nuclear configuration commensurate with the observed nuclear spin of Ca-39 and Ca-41 respectively are:

$$9 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \hat{p}^+(\uparrow), \hat{d}(\uparrow\uparrow)$$

and

$$8 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 4\hat{d}(\uparrow\uparrow), \hat{n}(\downarrow)$$

They both are unstable nuclear configurations. Thus in going from Ca-39 to Ca-40 the nuclear configuration transforms as

$$\begin{array}{c} 9 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \hat{p}^+(\uparrow), \hat{d}(\uparrow\uparrow) \\ \downarrow \\ 9 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow) \equiv 10 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right] \end{array}$$

That is, in Ca-39 last pair of an isodeuteron and a proton are in high spin state but on the availability of one more isoneutron in case of Ca-40 not only one additional isodeuteron gets formed but also both the isodeuterons get spin paired, imparting stability.

Whereas in going from Ca-40 to Ca-41 the nuclear configuration transforms as

$$10 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right] \rightarrow 8 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 4\hat{d}(\uparrow\uparrow), \hat{n}(\downarrow)$$

That is, 2α -particles out of 10 in Ca-40 become 4 spin unpaired isodeuterons on the availability of an additional isoneutron in the case of Ca-41 and at the same time the additional isoneutron orients its spin opposite to that of the isodeuterons and the result is nuclear instability of Ca-41. Next we observe that when we add one isodeuteron to Ca-41 to form Ca-42 strikingly the four spin unpaired isodeuterons of the former isonuclide as well as the two isoneutrons get spin paired to form Ca-42, a stable isonuclide.

Further, on going from Ca-42 to Ca-43 we see from Table 2 that one α -particle out of 10 of Ca-42 gets spin unpaired providing two isodeuterons and simultaneously the spin paired isoneutrons of Ca-42 become spin unpaired to form three spin unpaired isoneutrons of Ca-43. Still Ca-43 is a stable isonuclide. Moreover, the reason of the nuclear stability of Ca-44 seems to be the same as that of Ca-42 because in the former we have one spin paired di-isoneutron whereas in the latter case we have two spin paired di-isoneutrons.

It seems from the above observations that the spin pairing of the isonucleons is not the only parameter that determines the nuclear stability. Other factors need to be identified. This would get further substantiated by considering the stable isonuclides with non-zero nuclear spin in next subsection.

II. Stable Isonuclides with Non-zero Nuclear Spin

Moreover, there are 104 stable isonuclides (in addition to isodeuteron) in Table 2 with non-zero nuclear spin. All have the combination of isodeuterons and isoneutrons except He-3, which consists of one isodeuteron with both its spins up and an

isoproton with spin down.

1. Notice that in Table 2 there we have highest nuclear spin of 7 (Lu-176). There also we have isonuclides with nuclear spins 6 (V-50), 5 (La-138) and 9/2 (Ge-73, Kr-83, Sr-87, Nb-93, In-113, In-115 and Hf-179). That is even though the spins are parallel the isonuclides are stable.
2. Also we notice that three parallel spin isodeuterons in the environment of α -particles are also stable they are B-10, K-40 (it also has 2 parallel spin isoneutrons) and Sc-45 (it also consists of one parallel spin isoneutron and one spin zero di-isoneutron).
3. In addition to these parallel spin high spin states there we have various combination of parallel spin isodeuterons combined with parallel or opposite spin isoneutrons resulting in the intermediate nuclear spins from 1/2 to 7/2.
4. As we know that an isolated single isoneutron is unstable but it gets stabilized in the form of an isodeuteron on the one hand but on the other hand it also gets stabilized in the environment of spin paired isodeuterons. This is the case of the nuclear spin of 1/2 of the isonuclides due only to a single isoneutron. From Table 2 we find that-
 - a) in the cases of C-13 and Si-29 we have a single isoneutron in the environment of 3 and 7 α -particles and both the isonuclides are stable.
 - b) Another set of stable isonuclides with a single isoneutron consist of Fe-57, Se-77, Sn-115, Sn-117, Sn-119, Te-125, Xe-129, Xe-131, Yb-171, W-183, Os-187, Hg-199 and Pb-207. These isonuclides offer the environment of spin paired isodeuterons along with the spin paired isoneutrons to the last isoneutron resulting in the stability of the last isoneutron.
5. Another set of stabilized single isoneutron is in combination with high spin state of isodeuterons in the environment of α -particles. We list them as follows.
 - (a). The cases of a single isoneutron in the environment of α -particles along with a single isodeuteron are of two types. The high spin (that is the net nuclear spin of 3/2) states are Li-7, B-11, Na-23, Cl-35 and K-39. The low spin (that is the net nuclear spin of 1/2) stable isonuclides are N-15, F-19 and P-31.
 - (b). The cases of a single isoneutron in the environment of α -particles along with two parallel spin isodeuterons are also of two types. The high spin (that is the net nuclear spin of 5/2) stable states are O-17 and Mg-25. The low spin (that is the net nuclear spin of 3/2) stable states are Be-9, Ne-21 and S-33.
 - (c). The cases of a single isoneutron in the environment of α -particles along with three parallel spin isodeuterons are two in number. The high spin (that is net nuclear spin of 7/2) state is Sc-45 and the low spin (that is net nuclear spin of 5/2) state is Al-27.
 - (d). We have already seen in Section 14.1.3.1 that spin paired isoneutrons get stabilized in the environment of α -particles. Now we find that the combination of one isodeuteron and one isoneutron also get stabilized in the environment of α -particles when accompanied by

the zero spin di-isoneutrons. The corresponding isotopes with high spin (the net nuclear spin of 3/2) are Cl-37, K-41, Cu-63, Cu-65, Ga-71, As-75, Br-79, Br-81, Tb-159, Ir-191, Ir-193 and Au-197. Whereas the low spin (the net nuclear spin of 1/2) isonuclides are Y-89, Rh-103, Ag-107, Ag-109, Tm-169, Tl-203 and Tl-205.

- (e). We have seen above that K-40 is a stable isonuclide. Herein di-isoneutron of spin 1 is getting stabilized in the environment offered by α -particles and three parallel spin isodeuterons. Three parallel spin isoneutrons get stabilized in Ca-43 that offers the environment of α -particles and two parallel spin isodeuterons.
- (f). We also see that up to the net 9 parallel spin isoneutrons get stabilized in the environment of 36 α -particles and 13 di-isoneutrons in the case of Hf-179 whereas in the case of Ge-73 the 9 parallel spin isoneutrons get stabilized in the environment of 16 α -particles, no isodeuterons are required for this stabilization.

We have described above certain representative observations but on closer scrutiny of Table 2 we can spell out many more observations. However, the main task of presenting the nuclear configurations of Table 2 has been to provide ample facts that would provide base to evolve a comprehensive theory of nuclear stability against radioactivity and find out the factors that lead to nuclear instability.

While attempting to explain the nuclear stability we definitely need to consider unstable isonuclides in the immediate vicinity of the stable isonuclides along with their nuclear configurations commensurate with their observed spins. For example let us consider the cases of stable Nb-93 and In-113. We know that Nb-92 and Nb-94 are unstable isotopes and their experimentally observed nuclear spins are 7 and 6 respectively whereas that of Nb-93 it is 9/2. That is Nb-93 lies in between the higher nuclear spin isotopes. The nuclear configuration of Nb-92 is

$$19 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], 3 \left[\hat{d}(\uparrow\uparrow) \right], \left[\hat{n}(\uparrow)\hat{n}(\downarrow) \right], 8 \left[\hat{n}(\uparrow) \right]$$

that on adding one isoneutron changes to

$$20 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \left[\hat{d}(\uparrow\uparrow) \right], 2 \left[\hat{n}(\uparrow)\hat{n}(\downarrow) \right], 7 \left[\hat{n}(\uparrow) \right]$$

That is the addition of one isoneutron forces two isodeuterons out of three parallel spin isodeuterons to get spin paired and simultaneously itself gets spin paired with one isoneutron leaving 7 parallel spin isoneutrons. The outcome is the stable Nb-93. Now to this stable isotope on adding one isoneutron it forces one pair of spin paired isoneutrons to become spin unpaired resulting in total number of 10 parallel spin isoneutrons. The resultant nuclear configuration obtained is

$$20 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \left[\hat{d}(\uparrow\uparrow) \right], \left[\hat{n}(\uparrow)\hat{n}(\downarrow) \right], 10 \left[\hat{n}(\uparrow) \right]$$

which is unstable Nb-94.

Similarly, in the sequence In-112, In-113 and In-114 the nuclear configuration transforms as

$$24 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \left[\hat{d}(\uparrow\uparrow) \right], 7 \left[\hat{n}(\uparrow)\hat{n}(\downarrow) \right]$$

↓

$$24 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \left[\hat{d}(\uparrow\uparrow) \right], 4 \left[\hat{n}(\uparrow)\hat{n}(\downarrow) \right], 7 \left[\hat{n}(\uparrow) \right]$$

↓

$$24 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right], \left[\hat{d}(\uparrow\uparrow) \right], 8 \left[\hat{n}(\uparrow)\hat{n}(\downarrow) \right]$$

Notice that in this sequence spin 1 states are unstable and 9/2 spin state is a stable one.

The above described are a few representative examples but they adequately pose the kind of challenge we need to undertake in order to explain nuclear stability/instability. One may think that an answer may be found through developing corresponding shell model and corresponding magic numbers.

In order to check if magic numbers play any role in nuclear configuration through isoneucleons we have also compiled the nuclear configuration in terms of isodeuterons as the only constituent and depicted in Table 3.

Table 3. Isonuclides composed only of isodeuterons

Nuclear Configuration	Isonuclide $\sum \hat{X}_N(J)$	Isideuterons and Nuclear Stability / Instability	Nuclear Magnetic dipole Moment μ / μ_N	Nuclear Electric Quadrupole Moment Q/e ^b
$\hat{d}(\uparrow\uparrow)$	Isodeuteron	1 (odd) stable	0.85743823	+0.00286
$\left[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow) \right]$	$\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0)$	2 (even) stable	0	N/A
$\left[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow) \right], \hat{d}(\uparrow\uparrow)$	$\begin{smallmatrix} 6 \\ 3 \end{smallmatrix} \hat{\text{Li}}_3(1)$	2, 1 (odd) stable	0.8220473	-0.00083
$2 \left[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow) \right]$	$\begin{smallmatrix} 8 \\ 4 \end{smallmatrix} \hat{\text{Be}}_4(0)$ $\equiv 2 \left[\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \hat{\text{He}}_2(0) \right]$	2, 2 (even) unstable	0	N/A

Nuclear Configuration	Isonuclide $\frac{A}{Z}\hat{X}_N(J)$	Isodeuteron and Nuclear Stability / Instability	Nuclear Magnetic dipole Moment μ / μ_N	Nuclear Electric Quadrupole Moment Q/eb
$[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)],$ $\hat{d}(\uparrow\uparrow), \hat{d}(\uparrow\uparrow), \hat{d}(\uparrow\uparrow)$	$^{10}_5\hat{B}_5(3)$	2, 3 (odd) stable	1.8006448	0.08472
$3[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)]$	$^{12}_6\hat{C}_6(0)$	2, 4 (even) stable	0	N/A
$3[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)], \hat{d}(\uparrow\uparrow)$	$^{14}_7\hat{N}_7(1)$	2, 5 (odd) stable	0.403761	0.0193
$4[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)]$	$^{16}_8\hat{O}_8(0)$	2, 6 (even) stable	0	N/A
$4[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)]$ $\hat{d}(\uparrow\uparrow)$	$^{18}_9\hat{F}_9(0)$	2, 6, 1 (odd) unstable	N/A	N/A
$5[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)]$	$^{20}_{10}\hat{Ne}_{10}(0)$	2, 6, 2 (even) stable	0	N/A
$4[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)],$ $\hat{d}(\uparrow\uparrow), \hat{d}(\uparrow\uparrow), \hat{d}(\uparrow\uparrow)$	$^{22}_{11}\hat{Na}_{11}(3)$	2, 6, 3 (odd) unstable	1.746	N/A
$6[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)]$	$^{24}_{12}\hat{Mg}_{12}(0)$	2, 6, 4 (even) stable	0	N/A
$4[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)],$ $\hat{d}(\uparrow\uparrow), \hat{d}(\uparrow\uparrow), \hat{d}(\uparrow\uparrow),$ $\hat{d}(\uparrow\uparrow), \hat{d}(\uparrow\uparrow)$	$^{26}_{13}\hat{Al}_{13}(5)$	2, 6, 5 (odd) unstable	N/A	N/A
$7[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)]$	$^{28}_{14}\hat{Si}_{14}(0)$	2, 6, 6 (even) stable	0	N/A
$7[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)], \hat{d}(\uparrow\uparrow)$	$^{30}_{15}\hat{P}_{15}(1)$	2, 6, 6, 1 (odd) unstable	N/A	N/A
$8[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)]$	$^{32}_{16}\hat{S}_{16}(0)$	2, 6, 6, 2 (even) stable	0	N/A
$8[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)],$ $\hat{n}(\uparrow), \hat{p}(\downarrow)$	$^{34}_{17}\hat{Cl}_{17}(0)$	2, 6, 6, 2 (even) unstable	0	N/A
$9[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)]$	$^{36}_{18}\hat{Ar}_{18}(0)$	2, 6, 6, 4 (even) stable	0	N/A
$8[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)],$ $\hat{d}(\uparrow\uparrow), \hat{d}(\uparrow\uparrow), \hat{d}(\uparrow\uparrow)$	$^{38}_{19}\hat{K}_{19}(3)$	2, 6, 6, 5 (odd) unstable	1.371	N/A
$10[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)]$	$^{40}_{20}\hat{Ca}_{20}(0)$	2, 6, 6, 6 (even) stable	0	N/A
$10[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)],$ $\hat{n}(\uparrow), \hat{p}(\downarrow)$	$^{42}_{21}\hat{Sc}_{21}(0)$	2, 6, 6, 6 (even) unstable	0	N/A
$11[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)]$	$^{44}_{22}\hat{Ti}_{22}(0)$	2, 6, 6, 8 (even) unstable	0	N/A
$11[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)],$ $\hat{n}(\uparrow), \hat{p}(\downarrow)$	$^{46}_{23}\hat{V}_{23}(0)$	2, 6, 6, 8 (even) unstable	0	N/A
$12[\hat{d}(\uparrow\uparrow), \hat{d}(\downarrow\downarrow)]$	$^{48}_{24}\hat{Cr}_{24}(0)$	2, 6, 6, 8, 2 (even) unstable	0	N/A

From the column 3 of Table 3 we learn that the stable nuclides consists of odd number i.e. 1, 3, 5, 7 and 9, and even number i.e. 2, 6, 8, 10, 12, 14, 16, 18 and 20 isodeuterons. We also notice that there are no nuclides, stable or unstable, with 17, 21 and 23 isodeuterons this we have depicted in Table 3 by including Cl-34 (consisting of 16 spin paired isodeuterons and spin paired one isoneutron and one isoproton), Sc-42 (consisting of 20 spin paired isodeuterons and spin paired one isoneutron and one isoproton) and V-46 (consisting of 22 spin paired isodeuterons and spin paired one isoneutron and one isoproton). All of them are unstable isonuclides. The nearest stable isonuclides are Cl-35 (consisting of 16 spin paired isodeuterons, and one each isodeuteron and isoneutron with parallel spins), Sc-45 (consisting of 18 spin paired isodeuterons, two spin paired isoneutrons and parallel spin 3 isodeuterons and one isoneutron) and V-51 (consisting of 22 spin paired isodeuterons, and parallel spin one isodeuteron and 5 isoneutrons) respectively (can be seen in Table 2).

Thus, the column 3 of Table 3 doesn't appear to point out the existence of a system of magic numbers for isodeuterons when housed in a nucleus. Moreover, in Table 3 we have also depicted the nuclear magnetic dipole and electric quadrupole moments [77, 78] in columns 4 and 5 respectively. That reveals the nonspherical nuclear charge distribution in the case of non-zero nuclear quadrupole moments.

14.2. Nuclear Configuration of Model-II

In this model we treat atomic nuclei as constituted of up-spin isoprotons, down-spin isoprotons and null-spin isoelectrons. The column 4 of Table 2 list nuclear configuration of all stable nuclides in terms of these isonucleons. The striking feature of these nuclear configuration is that the number of isoelectrons, $\mathbb{E}(0)$, is equal to the number of neutrons (c.f. Eqs. (84) and (91)) and in this sense we may say that the isoelectrons have *replaced* neutrons.

Traditionally the nuclear stability is described by the ratio N/Z (number of neutrons to atomic number) and it is argued that this ratio increases from value 1 at lower atomic numbers to 1.537 in the case of Pb-208 (the last stable nuclide) because with the increase of atomic number the nuclear charge increases, which results in tremendous increase in repulsive force amongst nuclear protons. This repulsion gets minimized by the presence of neutrons. At higher atomic number the neutrons in a nuclide need to out number protons to attain nuclear stability, hence the said ratio increases up to 1.537 for Pb-208. Of course, there is a limitation on the effectiveness of neutrons to overcome the nuclear repulsive force otherwise by mere increase of number of neutrons all nuclide could have been stabilized. Hence, other explanation of nuclear stability were looked for. That resulted in postulation of a host of new subatomic particles. In layman's language scientists were looking for a *nuclear glue* which is responsible for tightly holding nucleons together within an atomic nucleus.

With this background we now interpret the stability of an isodeuteron in terms of the ability of an isoelectron to

effectively hold two isoprotons together. Hence, we are in a position to say that the isoelectron acts in this case as an effective nuclear glue that holds tightly two isoprotons together. In view of this interpretation we now, apparently for the first time, hypothesize that *in all stable nuclides their isoelectrons act as effective glue that tightly hold their isoprotons together in the nucleus, of course, with appropriate distribution of up and down spins amongst isoprotons*. We caution the reader that the isoelectrons as nuclear glue has entirely different base than what the nuclear glue is described in the conventional nuclear physics. In the present model we have not postulated any new subatomic particle. Our proposal is that the conventional electrons and protons get transformed respectively to isoelectrons and isoprotons by way of mutual partial penetration of their wave packets in view of their very close proximity and that acts as the nuclear glue.

In view of the rôle of the nuclear glue played by isoelectrons we hereby propose that instead of N/Z ratio it would be more appropriate to use the ratio $\mathbb{E}(0)/Z$ to qualitatively describe nuclear stability.

Of course, the proposal of nuclear configuration in terms of isoprotons and isoelectrons with latter as the nuclear glue, opens up new vistas for further investigations on the topics of nuclear stability and understanding of all other nuclear properties.

The nuclear configuration of nuclides of *model-II* are listed in column 4 of Table 2. The observations and analysis of these nuclear configurations are described in Section 14.2.1.

14.2.1. Observations and Analysis of Nuclear Configuration of Model-II

In the *model-II* we view the nucleus as a pool of isoprotons with the isoelectrons immersed in it. In light of this we are presenting our preliminary visualization of only a few nuclides of lower atomic numbers and for the time being we are postponing our analysis of higher atomic number nuclides.

1. In the case of *isodeuteron* there we have one isoelectron and two isoprotons (both with up spin). Hence the isoelectron acts as a solitary nuclear glue that tightly holds both the up spin isoprotons. The most obvious geometry of these three isonucleons is linear that perfectly matches with the structure proposed by Santilli (see Figure 9). Thus oblate elliptical shape of isodeuteron described in Figure 6 perfectly matches with the present description. It seems that the zero spin isodeuteron is energetically unstable hence even if it is formed in some nuclear transmutations that gets quickly converted to the spin 1 isodeuteron.
2. The next entry in Table 2 is **He-3** with the nuclear spin 1/2. It is the case of a pool of 3 isoprotons and in that one isoelectron is immersed. The obvious minimum energy geometry is the one in which the isoelectron is at the center of an equilateral triangle and the three isoprotons situated at the vertices of it. Again in this case too the shape would be elliptical due to its overall

spinning motion. He-3 with nuclear spin of 3/2 has not been observed so far, which must be energetically unstable nuclide. Therefore, even if it is formed in some nuclear transmutations it gets quickly transformed to He-3 of 1/2 spin. Thus we learn that the low nuclear spin state He-3 is the preferred one. Moreover, when we add one down spin isoproton to an isodeuteron nucleus we get He-3 nucleus but we see that this addition does not disturb the nuclear stability of, though the geometry changes from linear to planar.

3. Just for comparison with He-3 nuclide let us consider **H-3** (*triton*) nuclide. The latter nucleus too possesses 3 isoprotons with the net spin of 1/2 but consists of 2 isoelectrons. The minimum energy arrangement would be trigonal-bipyramidal in that the two isoelectrons occupy axial positions above and below the horizontal plane of symmetry. However, in this arrangement the penetration of wave packets of isoelectrons into those of isoprotons would not be as deep as one isoelectron in He-3 achieves. This perhaps leads to instability. It decays with β^- emission to the stable He-3 nuclide and its half life is 12.329 y that is it is not highly unstable nuclide. This is understandable because by emission of one isoelectron a stable He-3 geometrical arrangement is achieved.
4. The **He-4** nuclide is the case of a pool of 4 isoprotons and immersed in it are two isoelectrons. The minimum energy shape in this case would be that of an octahedron in which isoelectrons occupy the two diagrammatically opposite axial positions and 4 isoprotons occupy the remaining 4 vertices. The spins of isoprotons would be alternately up and down so that the net nuclear spin is null. The charge distribution would be spherically symmetric in view of the repulsion between axial isoelectrons.
5. The **Li-6** nuclide is a case of a pool of 6 isoprotons and 3 isoelectrons immersed in it. The minimum energy shape seems to be the two trigonal-pyramids in a staggered geometry with 6 isoprotons at the vertices. All the 3 isoelectrons occupy axial positions and out of them one is at the center holding tightly both the trigonal pyramids. The observed spin 1 originates from the one up spin isoproton on each side of the central isoelectron. If one isoelectron is added to Li-6 arrangement described herein then we will have to house 2 isoelectrons at the center of the axial position. An equally probable geometry could be one He-4 arrangement and one isodeuteron moiety oriented above one of the axial isoelectrons such that the isodeuteron moiety and the axial isoelectrons of He-4 moiety form a straight line. Such an arrangement would not be stable because of the strong electrostatic repulsion between two central isoelectrons. The resultant nuclide would be He-6. However, it has two decay paths with half life of 806.7 ms. One is the obvious decay to Li-6 just by getting rid of the extra electron and in the second path simultaneously an α -particle is emitted, the daughter nuclides are a deuteron and He-4 nuclides.
6. The **Li-7** nuclide is a case of a pool of 7 isoprotons and 3 isoelectrons immersed in it. The obvious minimum energy geometry would be having two H-3 trigonal-bipyramids fused by one isoproton at the center such that its wave packet simultaneously allows penetration of wave packets of two adjacent central isoelectrons on its left and right hand sides. The observed spin 3/2 is because of the two up spin isoprotons on each trigonal plane and one up spin isoproton of the fusing isoproton. If we add one isoelectron to Li-7 the resultant nuclide would be He-7, which decays to He-6 by neutron emission which in turn decays by two simultaneous paths to Li-6 and He-4 along with a deuteron by β^- emission.
7. The case of **Be-8** is unique. It has a pool of 8 isoprotons and 4 isoelectrons immersed in it. The minimum energy shape would be two compressed octahedrons in the staggered orientation one above the other. Thus the four vertices of each octahedron would be alternately occupied by up and down spin isoprotons and the two axial vertices of each octahedron are occupied by one isoelectrons each. However, in this way middle two isoelectrons would come close to each other hence this arrangement cannot sustain itself. As a result of it the two octahedrons get separated. This is the reason why Be-8 is not a stable nuclide disintegrating to α -particles. If we add 1 isoelectron to Be-8 the resultant nuclide would be Li-8 which in turn disintegrates to Be-8 by β^- emission with half life of 840.3 ms.
8. The **Be-9** nuclide is a case of a pool of 9 isoprotons and 5 isoelectrons immersed in it. The obvious minimum energy geometrical arrangement of isonucleons consist of 2 H-3 trigonal-bipyramids fused by the triangular planar geometry of He-3 in a staggered orientation with respect to both the trigonal-bipyramids. The observed spin of 3/2 is due to the spin 1/2 of one He-3 and two H-3 geometries. Notice that the wave packet of the isoelectron of the central He-3 geometry will be effectively shielded by the wave packets of its three isoprotons hence the wave packets of the isoelectrons of both the H-3 geometries oriented towards the central H-3 geometry would penetrate into the wave packets of the central H-3 isoprotons. This seems to impart stability to Be-9. If we add 1 isoelectron to Be-9 nuclide the resultant nuclide would be Li-9 which disintegrates by two paths to Be-9 and Be-8 along with a neutron with the half life of 178.3 ms.
9. The **B-10** is the case of a pool of 10 isoprotons and 5 isoelectrons immersed in it. The minimum energy packing of isonucleons would be two He-4 type

octahedrons in the staggered orientation one above the other and one isodeuteron fusing them so that 2 central isoprotons and five axial isoelectrons are in a straight line. The spin 3 of B-10 originates from 8 up spin isoprotons 3 in each octahedron plus two in the fusing isodeuteron leaving two down spin isoprotons one each in the octahedron geometry. If we add one isoelectron to B-10 nuclide the resultant nuclide would be Be-10 nuclide which disintegrates back to B-10 by β^- emission with half life of 1.39×10^6 y.

10. The **B-11** is the case of a pool of 11 isoprotons and 6 isoelectrons immersed in it. The minimum energy packing of the isonucleons would be 2 He-4 structures in staggered orientation and the remaining 3 isoprotons and 2 isoelectrons linearly and alternately coupled acts as the fusing chain of the two octahedral structures. The nuclear spin of 3/2 is due to 3 up spin central isoprotons. If we add one isoelectron to the B-11 nuclide the resultant nuclide would be Be-11 which partly decays back to B-11 and partly to Li-7 and α -particle by β^- emission with half life of 13.81 s.

The above presented visualization of isonucleons in nuclides appears to be satisfactorily reasonable. We would extend the work on the same lines for all stable and unstable nuclides.

15. Concluding Remarks

In this paper, we have reviewed the numerous insufficiencies of quantum mechanics for the representation of the structure of stable nuclides, and the ensuing greater insufficiencies for the representation of the structure of unstable nuclides and nuclear reactions at large due to their structural irreversibility over time compared to the strict reversibility of quantum mechanical axioms.

We have pointed out that the origin of the insufficiencies rests primarily in the *mathematics* of quantum mechanics, rather than in its axioms, due to its local-differential character with consequential abstraction of nuclear constituents as being point-like particles, compared to the evident need for the nuclear structure to represent nucleons as they are in the nuclear reality: extended charge distributions.

We have then reviewed the rudiments of the novel *isomathematics* which has been constructed precisely for the representation of nuclei as being composed by extended constituents in conditions of partial mutual penetration, thus resulting in the most general known interactions of linear and non-linear, local and non-local as well as Hamiltonian and non-Hamiltonian type.

We have then reviewed the rudiments of the covering of quantum mechanics known as *isomechanics* specifically formulated for the nuclear structure, by stressing that it essentially consists in an axiom-preserving "completion" of quantum mechanics along the historical argument by Einstein, Podolsky and Rosen, which is solely valid at one fermi distances while recovering quantum mechanics uniquely and

identically for bigger distances.

We have then reviewed the use of the above new formulations for the first and only achievement on scientific records of an exact and time invariant representation of the magnetic moments of stable nuclei via the implementation of Fermi's historical hypothesis that the charge distributions of protons and neutrons is deformed when they are members of a nuclear structure, with a consequential deformation of their intrinsic magnetic moments (see Figures 8 and 9 for neutron as isoneutron and deuteron as isodeuteron respectively).

The conceptual and technically most dominant aspect of the above advances is that the admission of contact, non-linear, non-local and non-Hamiltonian interactions causes alterations of the *intrinsic* characteristics of particles called *isorenormalizations* that are simply beyond any possible quantitative treatment via 20th century knowledge.

Consequently, we reviewed in the Appendix A the rudiments of the covering of Lie's theory known as the *Lie-Santilli isothory* which has been specifically constructed for the invariant treatment of systems with extended-deformable constituents with the most general known interactions.

The most prominent salient part of the Appendix A is the review of the *Lorentz-Poincaré-Santilli isosymmetry* and its characterization of isoparticles, with particular emphasis in the characterization of nuclear constituents as extended-deformable isoparticles.

We finally review the use of all the above knowledge for the first and only known numerically exact and time invariant representation of *all* characteristics of the neutron in its synthesis from the hydrogen atom as being composed by one isoproton and one isoelectron, with the consequential representation of *all* characteristics of the deuteron as being composed by two isoprotons and one isoelectron.

By using the above advances, we then present, apparently for the first time, two exact and invariant representations of the nuclear spin of the stable nuclides. The model-I is based on nuclear structures composed by isoprotons, isoneutrons and isodeuterons as isonucleons and the model-II is based on the final reduction of nuclides to isomechanical bound states of the respective isoprotons and isoelectrons.

In the *former model* we have considered that with the available neutrons and protons of the nuclide they first prefer to have the stable isodeuteron structure and the remaining nucleons stay as isoneutrons and isoprotons in the nucleus. In doing so the rule followed is that the so generated nuclear configuration should correctly reproduce the experimental nuclear spin of the given nuclide. Thus in Table 2 we have listed nuclear configuration of all stable nuclides up to the atomic number 82, that is up to Pb-208. Then we have analyzed these nuclear configurations and presented our observations in terms of the number of isodeuterons (both their low spin and high spin combinations) and their rôle in stabilization of various combination of spin paired and/or parallel spin isoneutrons. We have tried to look if these nuclear configurations indicate corresponding magic numbers but the data in Table 3 fail to provide any indication. However, it

seems that unless we systematically compare the nuclear configuration based on model-I of neighbouring unstable nuclides about the stable nuclides considered in Table 2 we may not be able to throw much light on the factors responsible for nuclear stability/instability. Indeed, our data of Tables 2 and 3 of model-I has opened up an entirely a new line of research in the fields of nuclear stability/instability and nuclear magnetic moments including nuclear electric quadrupole moments.

Whereas in arriving at the *model-II* we have first reinterpreted the stable structure of an isodeuteron in the sense that the isoelectron of it acts as a nuclear glue that tightly holds its two isoprotons. This proposal of isoelectrons acting as the nuclear glue we have, perhaps for the first time, extended to all stable nuclides. There we have assumed that a given nuclide consists of a pool of isoprotons and the isoelectrons are immersed in it, which in essence is the model-II of this paper. The working rule is that the number of isoelectrons is equal to the number of neutrons in the nuclide and the number of isoprotons is equal to the mass number of the nuclide. Next these isoprotons are distributed in two groups of up and down spins in such a way to correctly predict the experimental nuclear spin of the given nuclide. The resulting nuclear configurations of all stable nuclides are listed in the column 4 of Table 2. Herein we have presented our preliminary observations on the so developed nuclear configuration. Of course, we have so far analyzed only a very few light nuclides in terms of geometrical arrangements of isoprotons and isoelectrons of H-2, H-3 (unstable), He-3, He-4, Li-6, Li-7, Be-8 (unstable), Be-9, B-10 and B-11 nuclides. Our assigned geometrical arrangements of isonucleons seem to provide reasonably satisfactory rational behind nuclear stability/instability. Particularly the reason of instability of H-3 and Be-8 so obtained seems to be rationally correct and encouraging.

The remarkable feature of both the models of nuclear configuration presented in this paper is that we need not to invent nuclear particles other than the basic subatomic particles, namely electrons, protons and neutrons.

Moreover, as stated in the main text of this paper the methods of writing nuclear configuration of a nuclide in both the models are equally applicable to unstable nuclides too hence while dealing with the nuclear stability/instability one can easily write down nuclear configurations of neighbouring unstable nuclides about a given stable one with identically the same rules as those we have followed in the case of stable nuclides and then attempt to rationalize nuclear stability/instability meaningfully.

Both the models promise new vistas of nuclear physics that lays a foundation of carrying out further investigations based on hadronic mechanics to strengthen our knowledge of nuclear physics.

Acknowledgments

The present work is supported by R. M. Santilli Foundation, Florida, USA. The authors acknowledge the kind assistance of

V. M. Tangde in the conversion of this paper from LaTeX to word format and particularly for afresh compiling in word all Tables of this paper. Thanks are finally due to Sherry Stone for a detailed linguistic control of the manuscript.

Appendix A

The Lorentz-Poincaré-Santilli IsoSymmetry and its Characterization of IsoParticles

A.1 Definition of IsoParticles

The 20th century definition of *particles* is that of unitary irreducible representation of the Lorentz-Poincaré (LP) symmetry on a Hilbert space over the field of complex numbers. This definition implies that all interactions are derivable from a potential and representable with a Hamiltonian as a central condition for the very applicability of Lie's theory at large, and that of the LP symmetry in particular. In turn, the local-differential mathematics underlying Lie's theory implies that the particles are abstracted as being point-like, as it is evident from the restriction of the interactions to actions-at-a-distance.

A central aim of this paper is the representation of nuclear constituents as they are in the physical reality, namely, extended, non-spherical and deformable charge distributions according to the representation of Eq. (1) which is structurally non-Hamiltonian, in the sense that it cannot be represented with a Hamiltonian, thus requiring a new quantity other than the Hamiltonian. In order to achieve a time invariant representation, isomathematics selects Santilli isounit Eq. (37), $\hat{I} = 1/\hat{T} > 0$ for the representation of the new interactions.

Additionally, the comparison of experimental data on nuclear volumes with those on the volume of protons and neutrons, establishes that, when they are members of a nuclear structure, protons and neutrons are in conditions of partial mutual penetration of their charge distributions.

These data imply the emergence of new nuclear interactions that are non-existence in the 20th century notion of particles, which are given by non-linear (in the wave functions), non-local (of integral and other type) and variationally non-selfadjoint [3a]. The latter interactions are also not representable with a Hamiltonian and can be invariantly represented with the exponent in the isotopic element, Eq. (1), or in the isounit.

The above basic assumptions imply the applicability of the Lie-Santilli isotheory [3b, 7, 22, 24-33] at large that was constructed precisely for the representation of non-Hamiltonian systems under the most general known linear and non-linear, local and non-local and Hamiltonian as well as non-Hamiltonian interactions,

Finally, the above basic assumptions imply that the universal symmetry for the non-relativistic treatment of isolated and stable nuclei is the *Galileo-Santilli isosymmetry* [21, 22], while that for the relativistic treatment is the *Lorentz-Poincaré-Santilli isosymmetry* [12-23]. We can, therefore, introduce the following:

DEFINITION A.1 [18, 21, 22]: *A non-relativistic*

(relativistic) isoparticle is an isounitary, isoirreducible isorepresentation of the Galileo-Santilli (Lorentz-Poincaré-Santilli) isosymmetry on a Hilbert-Myung-Santilli isospace over a Santilli isofields.

Within the context of this paper, whenever nuclear constituents are called “protons” and “neutrons” we refer to their quantum mechanical characterization as *point-like particles* under sole action-at-a-distance, potential-Hamiltonian interactions. Nuclear constituents according to this paper must necessarily be isoparticles at large, and isoprotons, isoneutron and isoelectrons in particular. In this Appendix we provide a summary characterization of relativistic isoparticles, while the particular case of non-relativistic isoparticles is referred to Refs. [18, 21] for brevity.

It should be stressed that a technical knowledge of the notion of isoparticle can solely be acquired from the study of Refs. [18, 21, 22]. In particular, a necessary pre-requisite for a technical characterization is the knowledge of Kadeisvili isofunctional analysis [22] we cannot possibly review to prevent excessive length.

A.2 The Lie-Santilli IsoTheory

The main branches of the Lie-Santilli isothory can be outlined as follows (see the original proposal [3a] for the isotopies of enveloping algebras, Lie algebras and Lie group; Ref. [7] for their upgrading in terms of the isodifferential calculus over isofields; the final formulation in Ref. [22]; and Refs. [24-33] for independent studies):

Universal Enveloping Isoassociative Algebras

Let $E = E(L)$ be the universal enveloping associative algebra of an N -dimensional Lie algebra L with ordered (Hermitian) generators $X_k, k = 1, 2, \dots, N$, and attached antisymmetric algebra isomorphic to the Lie algebra, $[E(L)]^- \approx L$ over a field F (of characteristic zero), and let the infinite-dimensional basis $I, X_k, X_i \times X_j, i \leq j, \dots$ of $E(L)$ be characterized by the Poincaré-Birkhoff-Witt theorem. We then have the following:

THEOREM A.1 [3b, 7]: (Poincaré-Birkhoff-Witt-Santilli

$$\begin{aligned}
 [\hat{X}_i \hat{\wedge} \hat{X}_j] &= \hat{X}_i \hat{\times} \hat{X}_j - \hat{X}_j \hat{\times} \hat{X}_i = \\
 &= X_i \times T(x, v, \xi, \omega, \psi, \partial \psi, \dots) \times X_j - X_j \times T(x, v, \xi, \omega, \psi, \partial \psi, \dots) \times X_i = \\
 &= \hat{C}_{ij}^k(x, v, \xi, \omega, \psi, \partial \psi, \dots) \hat{\times} \hat{X}_k = C_{ij}^k(x, v, \xi, \omega, \psi, \partial \psi, \dots) \times X_k,
 \end{aligned}
 \tag{A.3}$$

where T is the projection of the isotopic element \hat{T} on a conventional space over a conventional field, and the C 's, called the “structure isofunctions” of \hat{L} , generally have an explicit dependence on local variables, and are restricted by the conditions (Lie-Santilli Third Theorem)

$$[X_i \hat{\wedge} X_j] + [X_j \hat{\wedge} X_i] = 0, \tag{A.4a}$$

$$[[X_i \hat{\wedge} X_j] \hat{\wedge} X_k] + [[X_j \hat{\wedge} X_k] \hat{\wedge} X_i] + [[X_k \hat{\wedge} X_i] \hat{\wedge} X_j] = 0. \tag{A.4b}$$

theorem): The isocosets of the isounit and of the standard isomonomials

$$\hat{I}, X_k, \hat{X}_i \hat{\times} \hat{X}_j, i \leq j; \hat{X}_i \hat{\times} \hat{X}_j \hat{\times} \hat{X}_k; i \leq j \leq k, \dots, \tag{A.1}$$

form an infinite dimensional basis of the universal enveloping isoassociative algebra $\hat{E}(\hat{L})$ (also called isoenvelope for short) of a Lie-Santilli isoalgebra \hat{L} .

The first application of the above theorem, also formulated in Ref. [3b] and then reexamined by various authors, is a rigorous characterization of the *isoexponentiation*, i.e.,

$$\begin{aligned}
 \hat{e}^{\hat{i} \hat{\times} \hat{w} \hat{\times} \hat{X}} &= \\
 &= \hat{I} + \hat{i} \hat{\times} \hat{w} \hat{\times} \hat{X} / \hat{1}! + (\hat{i} \hat{\times} \hat{w} \hat{\times} \hat{X}) \hat{\times} (\hat{i} \hat{\times} \hat{w} \hat{\times} \hat{X}) / \hat{2}! + \dots = \\
 &= \hat{I} \times (e^{i \times w \times X}) = (e^{i \times w \times X \times T}) \times \hat{I},
 \end{aligned}
 \tag{A.2a}$$

$$\hat{i} = i \times \hat{I}, \hat{w} = w \times \hat{I} \in \hat{F}. \tag{A.2b}$$

where quantities with a “hat” are formulated on isospaces over isofields and those without are their projection on conventional spaces over conventional fields.

The non-triviality of the Lie-Santilli isothory is established by the emergence of the isotopic element T directly in the exponent, thus ensuring the desired generalization, thus establishing “ab initio” that while Lie theory can solely characterize linear, local-differential and Hamiltonian systems, the covering Lie-Santilli isothory characterize the most general known non-linear, non-local and non-canonical or non-unitary systems.

LIE-SANTILLI ISOALGEBRAS.

As it is well known, Lie algebras are the antisymmetric algebras $L \approx [\xi(L)]^-$ attached to the universal enveloping algebras $\xi(L)$. This main characteristic is preserved although enlarged under isotopies as expressed by the following:

THEOREM A.2 [3b, 7] (Lie-Santilli Second theorem): The antisymmetric isoalgebras \hat{L} attached to the isoenveloping algebras $\hat{E}(\hat{L})$ verify the isocommutation rules.

It was stated in the original proposal [3b, 7] that all isoalgebras \hat{L} are isomorphic to the original algebra L for all positive-definite isotopic elements. In other words, the isotopies cannot characterize any new Lie algebras because all possible Lie algebras are known from Cartan classification. Therefore, Lie-Santilli isoalgebras merely provide new non-linear, non-local and non-canonical or non-unitary realizations of existing Lie algebras.

LIE-SANTILLI ISOGROUPS.

Under certain integrability and smoothness conditions

hereon assumed, Lie algebras L can be "exponentiated" to their corresponding Lie transformation groups G and, vice-versa, Lie transformation groups G admit corresponding Lie algebras L when computed in the neighborhood of the unit I .

These basic properties are preserved under isotopies although broadened to the most general possible, axiom-preserving nonlinear, nonlocal and noncanonical transformations groups according to the following:

THEOREM A.3 [3b, 7] (Lie-Santilli isogroups): The isogroup characterized by finite (integrated) form \hat{G} of isocommutation rules (1.12) on an isospace $\hat{S}(\hat{x}, \hat{F})$ over an isofield \hat{F} with common isounit $\hat{I} = 1/\hat{T} > 0$ is a group mapping each element $\hat{x} \in \hat{S}$ into a new element $\hat{x}' \in \hat{S}$ via the isotransformations

$$\hat{x}' = \hat{g}(\hat{w}) \hat{\times} \hat{x}, \quad \hat{x}, \hat{x}' \in \hat{S}, \quad \hat{w} \in \hat{F}, \quad (\text{A.5})$$

with the following isomodular action to the right:

- 1) The map $\hat{g} \hat{\times} \hat{S}$ into \hat{S} is isodifferentiable $\forall \hat{g} \in \hat{G}$;
- 2) \hat{I} is the left and right unit

$$\hat{I} \hat{\times} \hat{g} = \hat{g} \hat{\times} \hat{I} \equiv \hat{g}, \quad \forall \hat{g} \in \hat{G}; \quad (\text{A.6})$$

- 3) the isomodular action is isoassociative, i.e.,

$$\hat{g}_1 \hat{\times} (\hat{g}_2 \hat{\times} \hat{x}) = (\hat{g}_1 \hat{\times} \hat{g}_2) \hat{\times} \hat{x}, \quad \forall \hat{g}_1, \hat{g}_2 \in \hat{G}; \quad (\text{A.7})$$

- 4) in correspondence with every element $\hat{g}(\hat{w}) \in \hat{G}$ there is the inverse element $\hat{g}^{-1} = \hat{g}(-\hat{w})$ such that

$$\hat{g}(\hat{0}) = \hat{g}(\hat{w}) \hat{\times} \hat{g}(-\hat{w}) = \hat{I}; \quad (\text{A.8})$$

- 5) the following composition laws are verified

$$\hat{g}(\hat{w}) \hat{\times} \hat{g}(\hat{w}') = \hat{g}(\hat{w}') \hat{\times} \hat{g}(\hat{w}) = \hat{g}(\hat{w} + \hat{w}'), \quad \forall \hat{g} \in \hat{G}, \quad \hat{w} \in \hat{F}; \quad (\text{A.9})$$

with corresponding isomodular action to the left, and general expression

$$\hat{g}(\hat{w}) = \prod_k \hat{e}^{i \hat{\times} \hat{w}_k \hat{\times} \hat{X}_k} \hat{\times} \hat{g}(0) \hat{\times} \prod_k \hat{e}^{-i \hat{\times} \hat{w}_k \hat{\times} \hat{X}_k}, \quad (\text{A.10})$$

Another important property is that conventional group composition laws admit a consistent isotopic lifting, resulting in the following

THEOREM A.4 [3b, 7] (Baker-Campbell-Hausdorff-Santilli theorem):

$$(\hat{e}^{\hat{X}_1}) \hat{\times} (\hat{e}^{\hat{X}_2}) = \hat{e}^{\hat{X}_3}, \quad (\text{A.11a})$$

$$\hat{X}_3 = \hat{X}_1 + \hat{X}_2 + [\hat{X}_1, \hat{X}_2] \hat{\wedge} 2 + [(\hat{X}_1 - \hat{X}_2), [\hat{X}_1, \hat{X}_2]] \hat{\wedge} 2 + \dots \quad (\text{A.11b})$$

Let \hat{G}_1 and \hat{G}_2 be two isogroups with respective isounits \hat{I}_1 and \hat{I}_2 . The direct isoproduct $\hat{G}_1 \hat{\times} \hat{G}_2$ is the isogroup of

all ordered pairs

$$(\hat{g}_1, \hat{g}_2), \quad \hat{g}_1 \in \hat{G}_1, \hat{g}_2 \in \hat{G}_2, \quad (\text{A.12})$$

with isomultiplication

$$(\hat{g}_1, \hat{g}_2) \hat{\times} (\hat{g}'_1, \hat{g}'_2) = (\hat{g}_1 \hat{\times} \hat{g}'_1, \hat{g}_2 \hat{\times} \hat{g}'_2), \quad (\text{A.13})$$

total isounit (\hat{I}_1, \hat{I}_2) and inverse $(\hat{g}_1^{-1}, \hat{g}_2^{-1})$.

The following particular case is important for the isotopies of inhomogeneous groups. Let \hat{G} be an isogroup with isounit \hat{I} and \hat{G}_a the group of all its inner automorphisms. Let \hat{G}_a^o be a subgroup of \hat{G}_a with isounit \hat{I}^o , and let $\Lambda(\hat{g})$ be the image of $\hat{g} \in \hat{G}$ under \hat{G}_a^o . The semi-direct isoproduct $\hat{G} \hat{\times} \hat{G}_a^o$ is the isogroup of all ordered pairs $(\hat{g}, \hat{\Lambda}) \hat{\times} (\hat{g}^o, \hat{\Lambda}^o)$ with total isounit

$$I_{tot} = \hat{I} \times \hat{I}^o. \quad (\text{A.14})$$

The studies of the isotopies of the remaining aspects of the structure of Lie groups is then consequential. It is hoped the reader can see from the above elements that the entire conventional Lie theory does indeed admit a consistent and nontrivial lifting into the covering Lie-Santilli formulation.

A.3 Classification of Lie-Santilli IsoTheories

The Lie-Santilli isotheories are classified into [7]:

3.1) *Regular isotheories* when the C 's of rules (A.3) are constant; and

3.2) *Irregular isotheories* when the C 's of rules (A.3) are functions of local variables.

We should recall for the benefit of concrete applications in nuclear physics that *all regular Lie-Santilli isotheories can be constructed via the application of a non-canonical or non-unitary transformation to the totality of the conventional formulation of Lie's theory*, according to the rule of Section 4.

From now on, except for an illustration in Section 16.13, we should solely consider regular realizations of the Lie-santilli isotheories because amply sufficient for nuclear applications, although the use of irregular realizations appear to be necessary for astrophysical applications.

We should also recall that "structure functions" are impossible for Lie's theory, and they are solely possible for the covering Lie-Santilli isotheory, by therefore establishing the non-trivial character of Santilli isotopies.

A.4 The Fundamental Theorem on IsoSymmetries

As recalled in Section 16.1, the fundamental symmetries of the 20th century physics characterize point-like abstractions of particles in vacuum under linear, local and potential interactions, and are given by the *Galilei symmetry* $G(3.1)$ for non-relativistic treatment, the *Lorentz-Poincaré symmetry* $P(3.1)$ or relativistic formulations, the rotational symmetry $O(3)$, the $SU(2)$ symmetries and others.

A central objective of hadronic mechanics is the broadening of these fundamental symmetries to represent extended, non-spherical and deformable particles under linear and non-linear, local and non-local and potential as well as non-potential interactions *in such a way to preserve the original symmetries at the abstract level* as a necessary condition to maintain the conventional total conservation laws for isolated stable systems.

This central objective is achieved by the following property first proved by Santilli in Ref. [22b]:

THEOREM A.5: *Let G be an N -dimensional Lie symmetry of a K -dimensional metric or pseudo-metric space $S(x,m,F)$ over a field F ,*

$$G: x' = \Lambda(w) \times x, y' = \Lambda(w) \times y, x, y \in \hat{S}, \quad (A.15a)$$

$$(x' - y')^\dagger \times \Lambda^\dagger \times m \times \Lambda \times (x - y) \equiv (x - y)^\dagger \times m \times (x - y), \quad (A.15b)$$

$$\Lambda^\dagger(w) \times m \times \Lambda(w) \equiv m. \quad (A.15c)$$

Then, all infinitely possible isotopies \hat{G} of G acting on the isospace $\hat{S}(\hat{x}, \hat{M}, \hat{F})$, $\hat{M} = \hat{m} \times \hat{I} = (\hat{T}_i^k \times m_{ij}) \times \hat{I}$ characterized by the same generators and parameters of G and the infinitely possible, common isounits $\hat{I} = 1/\hat{T} > 0$ leave invariant the isocomposition

$$\hat{G}: x' = \hat{\Lambda}(w) \times x, y' = \hat{\Lambda}(w) \times y, x, y \in \hat{S}, \quad (A.16a)$$

$$(x' - y')^\dagger \times \hat{\Lambda}^\dagger \times \hat{m} \times \hat{\Lambda} \times (x - y) \equiv (x - y)^\dagger \times \hat{m} \times (x - y) \quad (A.16b)$$

$$\hat{\Lambda}^\dagger(\hat{w}) \times \hat{m} \times \hat{\Lambda}(\hat{w}) \equiv \hat{m}. \quad (A.16c)$$

and all infinitely possible so constructed isosymmetries \hat{G} are locally isomorphic to the original symmetry G .

For a proof of the above theorem, one may inspect Section 1.2, Vol. II of Ref. [22].

To achieve a technical understanding of the Lie-Santilli isothory and its applications in nuclear physics, the reader should note that, while a given Lie symmetry G is unique as well known, there can be an infinite number of covering isosymmetries \hat{G} with generally different explicit forms of the transformations due to the infinite number of possible isotopic elements.

In fact, systems are characterized by the Hamiltonian H in the conventional scattering theory with trivial unit $I = \text{Diag.}(1,1,\dots,1)$. In this case, changing the Hamiltonian implies the referral to a different system, but the symmetry transformations remain the same. In the isoscattering theory, systems are characterized by the Hamiltonian H plus the isotopic element T . In this case, changing the isotopic element implies the referral to a different system as well as the characterization of generally different transformations due to the appearance of the isotopic element in the very structure of the isosymmetry.

Note also that all possible isosymmetries can be explicitly and uniquely constructed via the sole knowledge of the

conventional symmetry and the isotopic element (1). in fact, as implied by Theorem A.5, the existence of the original symmetry plus the condition $\hat{I} > 0$ ensure verification of the integrability conditions for the existence of finite transformations, a property hereon tacitly implied.

Recall that all quantities that are Hermitean in quantum mechanics are iso-Hermitean in hadronic mechanics as one can verify via Eq. (29), to such an extent that Hermiticity and iso-Hermiticity coincide at the abstract realization-free level,

$$\hat{X}^\dagger \equiv \hat{X}^\dagger. \quad (A.17)$$

The following property is then crucial for the physical consistency of the nuclear applications of hadronic mechanics, particularly the isomechanical models of closed-isolated stable nuclei:

THEOREM A.6 [22]: *Physical quantities that are Hermitean and conserved in quantum mechanics remain iso-Hermitean and iso-conserved in isomechanics.*

The proof of the theorem can be easily done via the local isomorphism of conventional Lie algebras L and their isotopic covering \hat{L} since isotopies do not change the generators, and merely generalize their associative products.

Recall that the basic space time symmetries, the Galileo and the Lorentz-Poincaré symmetries, characterize ten total conservation laws for the total linear momentum P , the total angular momentum J , the total energy H , the uniform motion of the center of mass M .

Theorem A.6 then assures that all total-external quantities that are conserved for quantum mechanical models remain conserved for their covering isomechanical form achieved via the rules of Section 4.

A.5 The Minkowski-Santilli IsoGeometry

Let $M(x,\eta,I)$ be the conventional Minkowski space over the field of real numbers R , with coordinates $x = (x^\mu) = (x^1, x^2, x^3, x^4 = t), \mu = 1, 2, 3, 4$, metric $\eta = \text{Diag.}(1,1,1,-c^2)$, unit $I = \text{Diag.}(1,1,1,1)$ and line element

$$(x - y)^2 \rightarrow (\hat{x} - \hat{y})^2 = [(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2 - (t_1 - t_2)^2 c^2] I, \quad (A.18)$$

As it is well known, the Lorentz-Poincaré symmetry, hereon denoted $P(3.1)$, leaves invariant the above line element and constitutes the ultimate structural foundations of special relativity because it permits the unique and unambiguous characterization of its basic axioms and physical laws for exterior problems of point-particles moving in vacuum.

The fundamental isospace of relativistic isomechanics is the Minkowski-Santilli isospace [15] $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$ over the isoreals R , with isocoordinates $\hat{x} = x\hat{I}$, isometric from Eq. (37) is

$$\hat{\eta} = \hat{T}\hat{\eta}, \quad \hat{T} = \text{Diag}\left(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, -\frac{c^2}{n_4^2}\right), \quad (\text{A.19})$$

isounit $\hat{I} = 1/\hat{T} > 0$, and isoline element

$$\begin{aligned} & (\hat{x} - \hat{y})^{\hat{2}} = \\ & [(\hat{x} - \hat{y})^{\mu} \hat{\eta}_{\mu\nu} \hat{\eta}_{\nu\lambda} (\hat{x} - \hat{y})^{\lambda}] = \\ & = \left[\frac{(x_1 - y_1)^2}{n_1^2} + \frac{(x_2 - y_2)^2}{n_2^2} + \frac{(x_3 - y_3)^2}{n_3^2} - \frac{(t_1 - t_2)^2 c^2}{n_4^2} \right] \hat{I}, \quad (\text{A.20}) \end{aligned}$$

where the isometric characteristic quantities n_{μ} are positive-definite but have otherwise an unrestricted functional dependence on all needed quantities, such as space-time coordinates x , velocities v , accelerations a , energy E , distance d , frequencies ω , temperature τ , wavefunction ψ , their derivatives $\partial\psi$, etc.

$$n_{\mu} = n_{\mu}(x, v, a, E, d, \omega, \tau, \psi, \partial\psi, \dots) > 0, \quad \mu = 1, 2, 3, 4, \quad (\text{A.21})$$

Isoprotons, isoneutrons and isoelectrons are defined on isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$ over the isoreals. As one can see, isometric (A.19) is the most general possible metric with signature $(+, +, +, -)$, thus including as particular case the Riemannian, Fynslerian, Minkowskian and other possible metric.

The Minkowski-Santilli isogeometry [19] is the geometry of isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$ and can be conceptually identified as the Riemannian geometry reformulated with respect to the isofields of isoreals because the isometric is indeed dependent on local coordinates, thus requiring the machinery of the Riemannian geometry, such as Christoffel symbols, covariant derivatives, etc., although reformulated with respect to isomathematics.

The intriguing part of the Minkowski-Santilli isogeometry is that it has zero curvature as necessary from the local isomorphism of isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$ with the conventional space $M(x, \eta, I)$. It should be stressed that the lack of curvature was a necessary prerequisite for the construction of the symmetry of isoinvariant (A.20) (see Refs. [22] for details).

A.6 The Lorentz-Poncaré-Santilli IsoSymmetry

Following the prior construction of the isotopies of Lie's theory [3b], the universal isosymmetry of all infinitely possible isoline elements (A.19) was first identified by Santilli in 1983 [15], subjected to systematic studies in Refs. [15-19], and presented in a systematic way in monographs [21, 22], resulting in a new isosymmetry today known as the Lorentz-Poincaré-Santilli isosymmetry (LPD) and denoted with the symbol $\hat{P}(3.1)$.

The isosymmetry $\hat{P}(3.1)$ can be defined as the isotransformations on Minkowski-Santilli isospaces over isoreals

$$\hat{x}' = \hat{\Lambda}(\hat{w}) \hat{\times} \hat{x}, \quad \hat{x}' = \hat{x} + \hat{A}(\hat{x}, \dots), \quad (\text{A.22a})$$

$$\hat{\Lambda}^{\dagger} \hat{\times} \hat{\eta} \hat{\times} \hat{\Lambda} = \Lambda \times \hat{\eta} \times \Lambda^{\dagger} = \hat{I} \times \hat{\eta} \times \hat{I}, \quad (\text{A.22b})$$

where we shall preserve the symbol \times of ordinary multiplications hereon, under the *isomodularity condition*

$$\hat{D}et(\hat{\Lambda}) = \pm \hat{I}, \quad (\text{A.23})$$

where the quantity \hat{A} is identified below and $\hat{w} = w \times \hat{I}$ represents isoparameters.

The regular isoconnected component of the LPS isosymmetry $\hat{P}^0(3.1)$ is characterized by the condition

$$\hat{D}et \hat{\Lambda} = +\hat{I}, \quad (\text{A.24})$$

and can be written

$$\hat{P}^0(3.1) = \hat{S}O(3.) \times \hat{T}(3.1) \times \hat{D}, \quad (\text{A.25})$$

where D is the 11th dimensionality of the LPS isosymmetry identified below.

By expanding the preceding finite isotransforms (A.22) in terms of the isounit, the *regular LPS isoalgebra* is characterized by the conventional generators of the LP algebra and the isocommutation rules [21, 22, 25]

$$\begin{aligned} & [J_{\mu\nu}, \hat{\times} J_{\alpha\beta}] = \\ & = i(\hat{\eta}_{\nu\alpha} J_{\beta\mu} - \hat{\eta}_{\mu\alpha} J_{\beta\nu} - \hat{\eta}_{\nu\beta} J_{\alpha\mu} + \hat{\eta}_{\mu\beta} J_{\alpha\nu}), \quad (\text{A.26a}) \end{aligned}$$

$$[J_{\mu\nu}, \hat{\times} P_{\alpha}] = i(\hat{\eta}_{\mu\alpha} \times P_{\nu} - \hat{\eta}_{\nu\alpha} \times P_{\mu}), \quad (\text{A.26b})$$

$$[P_{\mu}, \hat{\times} P_{\nu}] = 0. \quad (\text{A.26c})$$

The iso-Casimir isoinvariants of $\hat{P}(3.1)$ are given by [loc. cit]

$$\hat{C}_1 = \hat{I}(x, \dots), \quad (\text{A.27a})$$

$$\begin{aligned} \hat{C}_2 = P^{\hat{2}} & = P_{\mu} \hat{\times} P^{\mu} = P^{\mu} \hat{\times} \hat{\eta}_{\mu\nu} \times P^{\nu} = \\ & = P_k \times g_{kk} \times P_k - p_4 \times g_{44} \times P_4, \quad (\text{A.27b}) \end{aligned}$$

$$\hat{C}_3 = W^{\hat{2}} = W_{\mu} \hat{\times} W^{\mu}, \quad W_{\mu} = \hat{\epsilon}_{\mu\alpha\beta\rho} \hat{\times} J^{\alpha\beta} \hat{\times} P^{\rho}, \quad (\text{A.27c})$$

and they are at the foundation of classical and operator *isorelativistic kinematics* [43].

It is easy to prove that *the LPS isosymmetry is locally isomorphic to the conventional LP symmetry*. It then follows that the isotopies increase significantly the arena of applicability of the LP (as well as any Lie symmetry) by lifting the Minkowskian spacetime (A.18) to all infinitely possible isospacetime (A.20).

Note that *isolinear isomomenta isocommute*, Eqs. (A.26c), that is, *they commute in isospace over isoreals, but they do not*

generally commute when projected in the ordinary Minkowski space. This occurrence is a clear confirmation of a nonlinear structure of isorelativity with rather deep gravitational implications not considered in this paper.

Yet, this property is significant because it appears, for the first time to our knowledge, the possibility of identifying a possible gravitational component in the structure of nuclei, as studied preliminarily in Refs. [79].

The *isoirregular LPS isoalgebra* is characterized by *structure functions*, thus no longer being locally isomorphic to the conventional LP symmetry. The study of the irregular realization is left to the interested reader for brevity.

By using the original generators of the LP symmetry, the isotopic element (37) and Lie-Santilli isothory, regular LPS isotransformations can be easily identified as outlined below.

A.7 Regular IsoRotations

The regular isorotations, first presented in Ref. [12], and then treated in details in Refs. [22] via isofunctional analysis in general, and isotrigonometric functions in particular. Since the isounit $\hat{I} = \text{Diag}(n_1^2, n_2^2, n_3^2)$ is positive-definite, the isosymmetry $\hat{SO}(3)$ is locally isomorphic to the conventional rotational symmetry $O(3)$ (Figure 11).

Isorotations provide the technical characterization of the deformation of protons and neutrons when members of a nuclear structure under strong interactions. In their projection on an ordinary Euclidean space, isorotations can be written in the (1-2)-plane (see Ref. [22] for the general case).

$$x^{1'} = x^1 \cos[\theta(n_1 n_2)^{-1}] - x^2 \frac{n_1^2}{n_2^2} \sin[\theta(n_1 \times n_2)^{-1}], \tag{A.28a}$$

$$x^{2'} = x^1 \frac{n_2^2}{n_1^2} \sin[\theta(n_1 n_2)^{-1}] + x^2 \cos[\theta(n_1 n_2)^{-1}], \tag{A.28b}$$

The isomorphism of $\hat{SO}(3) \approx O(3)$ is due to the fact that ellipsoid deformations of the semiaxes of the perfect sphere are compensated on isospaces over isofields by the inverse deformation of the related unit

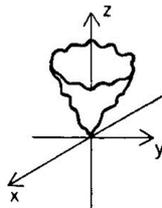


Figure 11. It was popularly believed in the 20th century physics that the Lorentz symmetry is broken for locally varying speeds of light within physical media, here represented with a wiggly light cone. the Lie-Santilli isosymmetries have restored the exact validity of the Lorentz symmetry for all possible subluminal and superluminal speeds, thus confirming the preservation of the abstract axioms of special relativity for interior dynamical problems [15, 22].

$$\text{Radius } 1_k \rightarrow 1/n_k^2, \text{ Unit } 1_k \rightarrow n_k^2. \tag{A.29a}$$

$$\hat{r}^2 = \hat{r}_1^2 + \hat{r}_2^2 + \hat{r}_3^2. \tag{A.29b}$$

resulting in the reconstruction of the perfect sphere on isospace called the *isosphere*, (A.29b), with consequential reconstruction of the exact rotational symmetry.

A.8 Regular Lorentz-Santilli IsoTransformations

The regular Lorentz-Santilli (LS) isotransforms were first identified in Ref. [15] and then studied in details in monographs [22]. Their elaboration also requires the use of the isofunctional analysis we cannot possibly review in this paper for brevity. It is easy to prove from the positive-definite character of the isounit $\hat{I} = \text{Diag}(n_1^2, n_2^2, n_3^2, n_4^2)$ that the Lorentz-Santilli isosymmetry $\hat{SO}(3.1)$ is locally isomorphic to the conventional symmetry $SO(3.1)$ (Figure 11).

The LS isotransformations are at the foundations of the relativistic results of this paper as well as of their invariance over time. They were first derived in Ref. [15] of 1983 and can be presented projected in the conventional Minkowski (3-4)-plane (see monograph [22b] for the general case)

$$x^{1'} = x^1, \quad x^{2'} = x^2, \tag{A.30a}$$

$$x^{3'} = \hat{\gamma} \left(x^3 - \hat{\beta} \times \frac{n_3}{n_4} x^4 \right), \tag{A.30b}$$

$$x^{4'} = \hat{\gamma} \left(x^4 - \hat{\beta} \times \frac{n_4}{n_3} x^3 \right), \tag{A.30c}$$

where

$$\hat{\beta} = \frac{v_3 / n_3}{c_o / n_4}, \tag{A.31a}$$

$$\hat{\gamma} = \frac{1}{\sqrt{1 - \hat{\beta}^2}}. \tag{A.31b}$$

It should be indicated that the main aim of Ref. [15] was the solution of the historical Lorentz problem, namely, the achievement of the universal symmetry for locally varying speeds of light within physical media $C = c/n_4$. Since this problem is highly non-linear, its solution could not be derived via the conventional Lie's theory. For this reason, Santilli conducted decades of studies for the generalization of Lie's theory into a form valid for nonlinear systems, first presented in monograph [3b], as a prerequisite for the solution of Lorentz's historical problem.

The isomorphism $\hat{SO}(3.1) \approx SO(3.1)$ is due to the reconstruction of the exact light cone on isospace over isofields called the light isocone. In fact, jointly with the deformation of the light cone

$$x^2 = x_3^2 - t^2 c^2 = 0 \rightarrow \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2} = 0, \quad (\text{A.32})$$

we have the corresponding *inverse* deformations of the units,

$$d_3 = 1 \rightarrow \hat{I}_3 = n_3^3 \quad d_4 = 1 \rightarrow \hat{I}_4 = n_4^2, \quad (\text{A.33})$$

thus reconstructing the original light cone on isospaces over isofields.

The reader should be aware that the above reconstruction includes the preservation on isospace over isofields of the original characteristic angle of the conventional light cone, namely, *the maximal causal speed on isospace over isofields is the conventional speed of light C in vacuum* [22].

A.9 Regular IsoTranslations

The regular isotranslations $T(4)$ were first studied in Ref. [16] and then studied in details in monographs [22]. and can be expressed in their projection in the conventional Minkowski space with the following lifting of the conventional translations $x^{\mu'} = x^\mu + a^\mu$, $\mu = 1, 2, 3, 4$, and a^μ constants,

$$x^{\mu'} = x^\mu + A^\mu(a, \dots), \quad (\text{A.34})$$

where

$$A^\mu = a^\mu (n_\mu^{-2} + a^\alpha [n_\mu^{-2}; P_\alpha] / 1! + \dots), \quad (\text{A.35})$$

and there is no summation on the μ indices.

Note the high nonlinearity of the isotranslations. This is due to the fact that the above expressions are the projection in the conventional spacetime since, when written on a Minkowski-Santilli isospace over isofields, isotransformations coincide with conventional translations.

A.10 Regular IsoDilations and IsoContractions

The *regular isodilations and isoContractions* $D(1)$ were first identified in Ref. [16] and then studied in details in monographs [22]. They constitute a *basically new spacetime symmetry* with vast implications, e.g., for grand unified theories [71], and can be expressed via the transformation

$$\hat{\eta} \rightarrow \hat{\eta}' = w^{-1} \hat{\eta}, \quad \hat{I} \rightarrow \hat{I}' = w \hat{I}, \quad (\text{A.36})$$

with ensuing invariance

$$\begin{aligned} (x^\mu \hat{\eta}_{\mu\nu} x^\nu) \hat{I} &\equiv [x^\mu (w^{-1} \hat{\eta}_{\mu\nu}) x^\nu] (w \times \hat{I}) = \\ &= (x^\mu \hat{\eta}_{\mu\nu} x^\nu) \times \hat{I}', \quad w \in R. \end{aligned} \quad (\text{A.37})$$

It was popularly believed in the 20th century that the LP symmetry was 10 -dimensional. The above invariance establishes that, instead, *the LPS isosymmetry as well as the LP conventional symmetry are 11 -dimensional*.

A.11 Regular IsoInversions

The regular isoinversions are given by [22b]

$$\hat{\pi} \hat{\times} x = \pi x = (-r, tc), \quad (\text{A.38a})$$

$$\hat{\tau} \hat{\times} x = \tau x = (r, -tc). \quad (\text{A.38b})$$

where π and τ are the conventional space and time inversion operators.

A.12 Regular $\hat{S}U(2)$ IsoSymmetry

In this section we provide the solution, apparently for the first time, of a central problem for the consistent and time invariant representation of nuclear magnetic moments via the deformations of the charge distributions of nucleons with consequential mutation of their intrinsic magnetic moment, under the conservation of conventional, values of the spins.

By remembering the lack of uniqueness of the isounits and related isotopic element, the simplest regular two-dimensional irreducible isorepresentations of $\hat{S}U(2)$ are characterized by the lifting of the two-dimensional complex-valued unitary space with metric $\delta = \text{Diag.}(1, 1)$ into the isotopic image [12, 15, 22]

$$\hat{I} = \text{Diag.}(n_1^2, n_2^2), \quad \hat{T} = \text{Diag.}(1/n_1^2, 1/n_2^2), \quad (\text{A.39a})$$

$$\hat{\delta} = \hat{T} \times \delta = \text{Diag.}(1/n_1^2, 1/n_2^2), \quad (\text{A.39b})$$

$$\text{Det } \hat{\delta} = (n_1 n_2)^{-2} = 1, \quad (\text{A.39c})$$

The basic non-unitary transform (43) of Section 4 us then given by

$$U \times U^\dagger = \hat{I} = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, \quad T = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix} \quad (\text{A.40a})$$

$$U = \begin{pmatrix} i \times n_1 & 0 \\ 0 & i \times n_2 \end{pmatrix}, \quad U^\dagger = \begin{pmatrix} -i \times n_1 & 0 \\ 0 & -i \times n_2 \end{pmatrix}, \quad (\text{A.40b})$$

here the n 's are well behaved nowhere null functions, resulting in the *regular Pauli-Santilli isomatrices* [loc. cit]

The related lifting of Pauli's matrices are then given by the *Regular Paili-Santilli isomatrices* [13, 14]

$$\sigma_k \rightarrow \hat{\sigma}_k = U \times \sigma_k \times U^\dagger, \quad (\text{A.41a})$$

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1^2 \\ n_2^2 & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -in_1^2 \\ in_2^2 & 0 \end{pmatrix},$$

$$\hat{\sigma}_3 = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}. \quad (\text{A.41b})$$

Another realization of the regular hadronic spin 1/2 is given by *non diagonal* nonunitary transforms [loc. cit.].

$$\begin{aligned}
 U &= \begin{pmatrix} 0 & n_1 \\ n_2 & 0 \end{pmatrix}, & U^\dagger &= \begin{pmatrix} 0 & n_2 \\ n_1 & 0 \end{pmatrix}, \\
 \hat{I} &= \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, & \hat{T} &= \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix},
 \end{aligned}
 \tag{A.42}$$

with corresponding alternative version of the regular Pauli-Santilli isomatrices,

$$\begin{aligned}
 \hat{\sigma}_1 &= \begin{pmatrix} 0 & n_1 n_2 \\ n_1 n_2 & 0 \end{pmatrix}, & \hat{\sigma}_2 &= \begin{pmatrix} 0 & -i n_1 n_2 \\ i n_1 n_2 & 0 \end{pmatrix}, \\
 \hat{\sigma}_3 &= \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix},
 \end{aligned}
 \tag{A.43}$$

or by more general realizations with Hermitean non diagonal isounits \hat{I} [15b].

All regular Pauli-Santilli isomatrices verify the following *isocommutation rules and iso-eigenvalue equations* on H over C

$$\begin{aligned}
 [\hat{\sigma}_i, \hat{\sigma}_j] &= \\
 &= \hat{\sigma}_i \hat{T} \hat{\sigma}_j - \hat{\sigma}_j \hat{T} \hat{\sigma}_i = 2i \varepsilon_{ijk} \hat{\sigma}_k,
 \end{aligned}
 \tag{A.44a}$$

$$(\hat{\sigma}_1 T \times \hat{\sigma}_1 + \hat{\sigma}_2 T \times \hat{\sigma}_2 + \hat{\sigma}_3 T \times \hat{\sigma}_3) T |\hat{\psi}\rangle = 3 \times |\hat{\psi}\rangle,
 \tag{A.44b}$$

$$\hat{\sigma}_3 \hat{\times} |\hat{\psi}\rangle = \hat{\sigma}_3 \times T \times |\hat{\psi}\rangle = \pm 1 \times |\hat{\psi}\rangle,
 \tag{A.44c}$$

thus preserving conventional structure constants and eigenvalues for spin 1/2 under non-Hamiltonian/nonunitary interaction, while adding the degree of freedom

$$n_1^2 = \lambda, \quad n_2^2 = \lambda^{-1},
 \tag{A.45}$$

That indeed is fully compatible with the mutation of intrinsic magnetic moments of spin 1/2 particles, Eq. (60).

Additionally, *the regular Pauli-Santilli isomatrices provide an explicit and concrete realization of hidden variables*, with intriguing implications for local realism studied in detail in ref. [14]. In turn, the above aspect confirm the origination of isomechanics as a concrete and explicit realization of the "incompleteness" of quantum mechanics according to Einstein, Podolsky and Rosen [1].

A.13 Irregular $\hat{S}U(2)$ IsoSymmetry

As indicated throughout this paper, there appears to be no need for a mutation of the spin of nuclear constituents to achieve an exact representation of nuclear magnetic moments and spins.

Nevertheless, the issue persists as to whether a proton in the core of a star should have the same spin when member of a nuclear structure. Santilli has introduced the irregular isotopies of the $SU(2)$ spin precisely for future studies of this important problem for the structure of stars.

One illustrative example of *irregular Pauli-Santilli isomatrices* is given by [12-14]

$$\begin{aligned}
 \tilde{\sigma}_1 &= \begin{pmatrix} 0 & n_1^2 \\ n_2^2 & 0 \end{pmatrix}, & \tilde{\sigma}_2 &= \begin{pmatrix} 0 & -i n_1^2 \\ i n_2^2 & 0 \end{pmatrix}, \\
 \tilde{\sigma}_3 &= \begin{pmatrix} w n_1^2 & 0 \\ 0 & w n_2^2 \end{pmatrix}.
 \end{aligned}
 \tag{A.46}$$

where w is the *mutation parameter*, with isocommutation rules and eigenvalue equations

$$\begin{aligned}
 [\tilde{\sigma}_1, \tilde{\sigma}_2] &= i w^{-1} \tilde{\sigma}_3, & [\tilde{\sigma}_2, \tilde{\sigma}_3] &= i w s \tilde{\sigma}_1, \\
 [\tilde{\sigma}_3, \tilde{\sigma}_2] &= i s w \tilde{\sigma}_1,
 \end{aligned}
 \tag{A.47a}$$

$$\begin{aligned}
 \tilde{\sigma}^2 \hat{\times} |\hat{\psi}\rangle &= \\
 (\tilde{\sigma}_1 T \tilde{\sigma}_1 + \tilde{\sigma}_2 T \tilde{\sigma}_2 + \tilde{\sigma}_3 T \tilde{\sigma}_3) T |\hat{\psi}\rangle &= (2 + w^2) \times |\hat{\psi}\rangle,
 \end{aligned}
 \tag{A.47b}$$

$$\tilde{\sigma}_3 \hat{\times} |\hat{\psi}\rangle = \tilde{\sigma}_3 T |\hat{\psi}\rangle = \pm w \times |\hat{\psi}\rangle, \quad w \neq 1,
 \tag{A.47c}$$

Additional examples of irregular Pauli-Santilli isomatrices can be found in Refs. [12-14].

The assumption of a mutated spin in hyperdense interior conditions evidently implies the inapplicability (rather than the violation) of the Fermi-Dirac statistics, Pauli's exclusion principle and other quantum mechanical laws, with the understanding that, by central assumption, non-Hamiltonian bound states of particles as a whole must have conventional total quantum values. Therefore, *we are here referring to possible internal exchanges of angular momentum and spin always in such a way as to cancel out and yield total conventional values.*

A.14 IsoRelativity IsoAxioms

As shown in this paper, a numerically exact and time invariant representation of nuclear magnetic moments and spins has required the isotopies of 20th century mathematics, with ensuing isotopies of quantum mechanics into isomechanics.

Interested readers should be aware that the above isotopies imply the inapplicability of special relativity for the nuclear structure in favor of a covering relativity known as isorelativity [15, 21-23]. The central aim of special relativity is the invariance of the speed of light in vacuum. A central aim of isorelativity is the invariance of local varying speeds of light $C = c/n_4$ within physical media as shown in Appendix A.8.

A rudimentary knowledge of the covering relativity is important to prevent major misrepresentations of the results of this paper as well as in possible further advances because the the appraisals of the new nuclear structure provided by isomechanics via special relativity would be equivalent to the appraisal of the results by special relativity via Newtonian mechanics.

The isotopies of the axioms of special relativity, today known as *IsoAxioms*, were initiated by Santilli in paper [15] of

1983; they received a first systematic formulation by Santilli in monographs [21] of 1991; they were finalized in monographs [22] of 1995 jointly with the discovery of the isodifferential calculus; and their experimental verifications were presented in Refs. [23].

In this paper we specialize, apparently for the first time, the isoaxioms for the isomechanical structure of stable and isolated nuclei whose constituents are isoparticles. The gravitational formulation of the isoaxioms of Reg. [71] should be kept in mind because it offers, also apparently for the first time, the possibility of addressing the origin of gravitational field in the structure of nuclei.

The first implication of the isotopies of special relativity is the *abandonment of the speed of light in vacuum as the maximal causal speed in favor of a covering geometrization of physical media*. This occurrence is easily seen by specializing the isoline element (27) to the *isolight isocone* [23, 37]

$$\hat{x}^2 = \left(\frac{x_k^2}{n_k^2} - t^2 \frac{c^2}{n_4^2} \right) = 0, \quad (\text{A.48})$$

thus leading to the Maximal Causal Speed V_{max} of IsoAxiom 5.1 below.

The remaining isoaxioms can be uniquely and unambiguously identified via a procedure parallel to the construction of the axioms of special relativity from the Lorentz-Poincaré symmetry.

Another departure from 20th century views is that isoaxioms refer to generally inhomogeneous and anisotropic physical media, as it is typically the case of the medium within spinning charge distributions. Therefore, the isoaxioms are formulated below for a generic space direction k ,

ISOAXIOM A. I: The maximal causal speed in a given space direction k in the interior of nuclei is given by

$$V_{max,k} = c \frac{n_k}{n_4}, \quad (\text{A.49})$$

ISOAXIOM A. II: The local isospeed of light is given by

$$\hat{c} = \frac{c}{n_4} \quad (\text{A.50})$$

where c is the speed of light in vacuum.

ISOAXIOM A. III: The addition of isospeeds in the k -direction follows the isotopic law

$$V_{tot,k} = \frac{v_{1,k}/n_k + v_{2,k}/n_k}{1 + \frac{v_{1,k}v_{2,k}}{c^2} \frac{n_4^2}{n_k^2}}. \quad (\text{A.51})$$

ISOAXIOM A. IV: The isodilatation of isotime, the isocontraction of isolengths, the variation of mass with isospeed, and the mass-energy isoequivalence principle follow the isotopic laws

$$\Delta t' = \hat{\gamma}_k \Delta t, \quad (\text{A.52a})$$

$$\Delta \ell' = \hat{\gamma}_k^{-1} \Delta \ell, \quad (\text{A.52b})$$

$$m' = \hat{\gamma}_k m, \quad (\text{A.52c})$$

$$E = mV_{max}^2 = mc^3 \frac{n_k^2}{n_4^2} \quad (\text{A.52d})$$

where $\hat{\gamma}$ and $\hat{\beta}$ have values (32).

ISOAXIOM A. V: The frequency isoshift of light propagating within a nucleus in the k -direction follows the Doppler-Santilli isotopic law

$$\omega_e = \omega_o \hat{\gamma}_k \left[1 \pm \frac{v/n_k}{c/n_4} \cos \alpha \right] \quad (\text{A.53})$$

where ω_e is the frequency experimentally measured in the outside, ω_o is the frequency at the origin inside a nucleus, and we have ignored for simplicity the isotopies of trigonometry (see Refs. [23] for brevity).

It should be stressed that in the above formulations as well as in the next section we present the isoaxioms in their *projection on the conventional Minkowski space*. While their technical treatment requires the full use of the various branches of isomathematics, including the formulation of the isoaxioms on a Minkowski-Santilli isospace over an isofield.

A main feature is that, when the isoaxioms are represented on isospace over isofields, they coincide with the conventional axioms of special relativity by conception and technical realization. In particular, the maximal causal speed $V_{max} \neq c$ solely occurs in the projection of the isoaxioms on Minkowski space because, at the isotopic level, the maximal causal speed is c for all possible isogravitational problems.

A.15 Predicted Implications of the IsoAxioms for the Nuclear Structure

In this final section, we identify the most important predictions of isorelativity [15, 21, 22] emerging as a consequence of our exact and invariant representation of nuclear magnetic moments and spins, and present their preliminary appraisals by soliciting comments from interested colleagues.

Isoaxioms clearly imply *two* different representations of the nuclear structure, the first is the representation of nuclear characteristics as measured from outside observer here indicated with the subindex "ext," and the second representation is that in the interior of nuclei here indicated with the subindex "int."

These two representations are necessary for the evident reason that the exterior observer is assumed as being in vacuum thus obeying conventional relativity axioms while the second representation occurs within hyperdense physical media, here assumed as obeying the covering isorelativity axioms.

A first implication of isorelativity is that *the time of the*

exterior observer is not necessarily the same as that in the interior of nuclei. In fact, by recalling the isodilation and isocontraction of Appendix A.10, we can write the identity

$$t_{ext}1 = t_{int}\hat{I}_t. \quad (\text{A.54})$$

Since for the nuclear structures considered in this paper $\hat{I}_t = n_4^2 < 1$ as in Eq. (63), one can see that the interior time evolution of nucleons is predicted to be "faster" than that of an outsider observer.

Note that at the abstract realization-free level there is no distinction between interior and exterior times as typical for all isotopies [22] since Eq. (A.54) can be written

$$t \equiv \hat{t} \quad (\text{A.55})$$

where t is an ordinary scalar, while \hat{t} is an isoscalar (Section 2). Therefore, t_{ext} and t_{int} are the projection of Eq. (A.55) in our spacetime.

For the case of distances, we can write the corresponding differentiations between external and internal distances according to the isotopic law

$$r_{ext}1 = r_{int,k}\hat{I}_k. \quad (\text{A.56})$$

Since the space isounits are generally smaller than one from Eqs. (64), one can see again that space distances perceived in the outside observer are predicted to be bigger than the actual distances in the interior.

Intriguingly, isolaw (A.56) is verified in ordinary water where, as we all know, dimensions perceived from the outsider are bigger than those actually occurring within water (Figure 12). Therefore, our argument is that, since isolaw (A.55) is verified in a medium with relatively big density such as water, the possibility of a similar occurrence in much denser media such as nuclei deserves due scientific process.



Figure 12. This picture illustrates the representation by isorelativity of the known effect that dimensions in water appears as being bigger than their actual dimensions when seen from an outside observer; thus warranting the study of the corresponding effect within nuclei.

Next, the speed of light in vacuum C has no mathematical or physical meaning for isorelativity and, in particular, it is not invariant under the time evolution. The sole mathematically and physically accepted quantity is Lorentz locally varying speed $C = c/n_4$.

In fact, the relativistic sum of two ordinary speeds of light

does not yield the speed of light within physical media such as water and the same is expected within nuclei. By contrast, the isorelativistic sum of two locally varying speeds of light does indeed yield the local speed of light according to isoaxioms III,

$$V_{tot} = \frac{c/n_4 + c/n_4}{1 + \frac{c^2/n_4^2}{c^2/n_4^2}} = \frac{c}{n_4}. \quad (\text{A.57})$$

In particular, one should note that a necessary condition for the isorepresentation of nuclear magnetic moments is that the local speed of light in the interior of nuclei is bigger than that in vacuum, see Eq. (64). This is a confirmation of the similar condition for $C > c$ which is necessary for the synthesis of the neutron from bthe hydrogen (Section 7).

Yet another prediction of isorelativity according to isoaxioms A. IV is that *the energy isoequivalent according to isoaction (A.52d) is "bigger" than that described from the outside*. This is a typical occurrence for all structure models of hadrons, nuclei and stars according to iksomathematics, and it is nowadays known as *isorenormalization*.

Consequently, in considering the structure model of nuclei as isobound states of isoprotons and isoelectrons, the reader should be aware that the rest energy of the isoelectron is isorenormalized to a minimum value of 1.293 MeV in the first approximation of ignoring Coulomb interactions, with bigger predicted values of the rest energy of the isoelectrons when including Coulomb interactions (due to the Coulomb attraction between isoprotons and isoelectrons).

As an illustration, a necessary condition for the achievement of an exact representation of the synthesis of the neutron from the hydrogen is that (by ignoring coulomb interactions) the isoremnormalized rest energy of the electron is 1.293 MeV.

Finally, we mention the prediction of isorelativity according to which *the frequency of the photons emitted by nuclei and measured in the outside is bigger than that at the point of emission in the interior ofg nuclei*. This additional effect is due to the *isoblueshift*, namely, the acquisition of energy by photons from hot environments without any relative motion, which was predicted by Santilli in 1992 [21], and experimentally verified in hot gases in 2010 [80] (see Refs. [68] for a comprehensive bibliography).

References

- [1] E. Fermi, *Nuclear Physics*, Chicago University Press (1948). See also, E. Fermi, *Elementary Particles*, Yale University Press, 2012 edition, Foreword by T. Appelquist.
- [2] A. Einstein, B. Podolsky, N. Rosen - Physical review, 1935.
- [3] R. M. Santilli, *Foundation of Theoretical Mechanics*, Volumes I (1978) [3a], and Volume II (1982) [3b], Springer-Verlag, Heidelberg, Germany, <http://www.santilli-foundation.org/docs/Santilli-209.pdf> <http://www.santilli-foundation.org/docs/santilli-69.pdf>.

- [4] R. M. Santilli, "Isonumbers and Genonumbers of Dimensions 1, 2, 4, 8, their Isoduals and Pseudoduals, and "Hidden Numbers" of Dimension 3, 5, 6, 7," *Algebras, Groups and Geometries* Vol. 10, 273 (1993).
<http://www.santilli-foundation.org/docs/Santilli-34.pdf>.
- [5] R. M. Santilli, *Isotopies of contemporary mathematical structures, I: Isotopies of fields, vector spaces, transformation theory, Lie Algebras, analytic mechanics and space-time symmetries, Algebras, Groups and Geometries* 8 (1991), 266.
- [6] R. M. Santilli, *Isotopies of contemporary mathematical structures, II: Isotopies of symplectic geometry, affine geometry, Riemannian geometry and Einstein gravitation, Algebras, Groups and Geometries* 8 (1991), 275-390.
- [7] R. M. Santilli, "Nonlocal-Integral Isotopies of Differential Calculus, Mechanics and Geometries," in *Isotopies of Contemporary Mathematical Structures*, P. Vetro Editor, *Rendiconti Circolo Matematico Palermo, Suppl. Vol. 42*, 7-82 (1996).
<http://www.santilli-foundation.org/docs/Santilli-37.pdf>.
- [8] Chun-Xuan Jiang, *Foundations of Santilli Isonumber Theory*, International Academic Press (2001),
<http://www.i-b-r.org/docs/jiang.pdf>.
- [9] Raul M. Falcon Ganfornina and Juan Nunez Valdes, *Fundamentos de la Isdotopia de Santilli*, International Academic Press (2001), <http://www.i-b-r.org/docs/spanish.pdf>.
English translations *Algebras, Groups and Geometries* Vol. 32, pages 135-308 (2015),
<http://www.i-b-r.org/docs/Aversa-translation.pdf>.
- [10] Raul M. Falcón Ganfornina and Juan Núñez Valdes, "Studies on the Tsagas-Sourlas-Santilli Isotopology," *Algebras, Groups and Geometries* Vol. 20, 1 (2003),
<http://www.santilli-foundation.org/docs/isotopologia.pdf>.
- [11] S. Georgiev, *Foundations of the IsoDifferential Calculus*, Volumes, I, II, III, IV and V, Nova Scientific Publisher (2015 on).
- [12] R. M. Santilli, "Isotopies of Lie Symmetries," I (basic theory) and II (isotopies of the rotational symmetry), *Hadronic J.* Vol. 8, 36 and 85 (1985),
<http://www.santilli-foundation.org/docs/santilli-65.pdf>.
- [13] R. M. Santilli, "Isotopic Lifting of the SU(2) Symmetry with Applications to Nuclear Physics," *JINR rapid Comm.* Vol. 6, 24-38 (1993),
<http://www.santilli-foundation.org/docs/Santilli-19.pdf>.
- [14] R. M. Santilli, "Isorepresentation of the Lie-isotopic SU(2) Algebra with Application to Nuclear Physics and Local Realism," *Acta Applicandae Mathematicae* Vol. 50, 177 (1998),
<http://www.santilli-foundation.org/docs/Santilli-27.pdf>.
- [15] R. M. Santilli, "Lie-isotopic Lifting of Special Relativity for Extended Deformable Particles," *Lettere Nuovo Cimento* 37, 545 (1983),
<http://www.santilli-foundation.org/docs/Santilli-50.pdf>.
- [16] R. M. Santilli, "Lie-isotopic Lifting of Unitary Symmetries and of Wigner's Theorem for Extended and Deformable Particles," *Lettere Nuovo Cimento* Vol. 38, 509 (1983),
<http://www.santilli-foundation.org/docs/Santilli-51.pdf>.
- [17] R. M. Santilli, "Nonlinear, Nonlocal and Noncanonical Isotopies of the Poincaré Symmetry," *Moscow Phys. Soc.* Vol. 3, 255 (1993),
<http://www.santilli-foundation.org/docs/Santilli-40.pdf>.
- [18] R. M. Santilli, "Recent theoretical and experimental evidence on the synthesis of the neutron," *Communication of the JINR, Dubna, Russia, No. E4-93-252* (1993), published in the *Chinese J. System Eng. and Electr.* Vol. 6, 177 (1995),
<http://www.santilli-foundation.org/docs/Santilli-18.pdf>.
- [19] R. M. Santilli, "Isominkowskian Geometry for the Gravitational Treatment of Matter and its Isodual for Antimatter," *Intern. J. Modern Phys. D* 7, 351 (1998),
<http://www.santilli-foundation.org/docs/Santilli-35.pdf>.
- [20] R. M. Santilli, "The notion of nonrelativistic isoparticle," *ICTP preprint IC/91/265* (1991)
<http://www.santilli-foundation.org/docs/Santilli-145.pdf>.
- [21] R. M. Santilli, *Isotopic Generalizations of Galilei and Einstein Relativities*, Vols. I [14a] and II [14b] (1991), International Academic Press.
<http://www.santilli-foundation.org/docs/Santilli-01.pdf>.
<http://www.santilli-foundation.org/docs/Santilli-61.pdf>.
- [22] R. M. Santilli, *Elements of Hadronic Mechanics*, Vol. I and Vol. II (1995) [15b], Academy of Sciences, Kiev,
<http://www.santilli-foundation.org/docs/Santilli-300.pdf>.
<http://www.santilli-foundation.org/docs/Santilli-301.pdf>.
- [23] R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry*, Volumes I to V, International Academic Press, (2008), <http://www.i-b-r.org/Hadronic-Mechanics.htm>.
- [24] D. S. Sourlas and G. T. Tsagas, *Mathematical Foundation of the Lie-Santilli Theory*, Ukraine Academy of Sciences (1993),
<http://www.santilli-foundation.org/docs/santilli-70.pdf>.
- [25] J. V. Kadeisvili, "An introduction to the Lie-Santilli isotopic theory," *Mathematical Methods in Applied Sciences* 19, 1349 (1996),
<http://www.santilli-foundation.org/docs/Santilli-30.pdf>.
- [26] S. Georgiev, "Elements of Santilli's Lie - Isotopic Time Evolution Theory," in press at *Applied Physics and Mathematics* (2015).
<http://www.santilli-foundation.org/docs/IsoLie1.pdf>.
- [27] R. Anderson, "Comments on the Regular and Irregular IsoRepresentations of the Lie-Santilli IsoAlgebras," invited paper, in press *American Journal of Modern Physics* (2015).
<http://www.santilli-foundation.org/docs/Lie-Santilli-IsoTheory-2015.pdf>.
- [28] J. V. Kadeisvili, *Santilli's Isotopies of Contemporary Algebras, Geometries and Relativities*, Ukraine Academy of Sciences, Second edition (1997),
<http://www.santilli-foundation.org/docs/Santilli-60.pdf>.
- [29] R. Anderson, *Outline of Hadronic Mathematics, Mechanics and Chemistry as Conceived by R. M. Santilli* In press at a refereed journal (2015).
<http://www.santilli-foundation.org/docs/hadronic-math-mec-chem.pdf>.
- [30] A. K. Aringazin, "Studies on the Lie-Santilli IsoTheory with Unit of general Form," *Algebras, Groups and Geometries* 28, 299 (2011),
<http://www.santilli-foundation.org/docs/Aringazin-2012.pdf>.
- [31] J. V. Kadeisvili, *Advances in the Lie-Santilli IsoTheory*, invited keynote lecture.
<http://www.world-lecture-series.org/invited-2014-keynote-lectures>.

- [32] A. K. Aringazin and K. M. Aringazin, "Universality of Santilli's iso-Minkowskian geometry" in *Frontiers of Fundamental Physics*, M. Barone and F. Selleri, Editors, Plenum (1995), <http://www.santilli-foundation.org/docs/Santilli-29.pdf>.
- [33] I. Gandzha and J. Kadeisvili, *New Sciences for a New Era: Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli*, Sankata Printing Press, Nepal (2011), <http://www.santilli-foundation.org/docs/RMS.pdf>
- [34] H. C. Myung and R. M. Santilli, "Modular-isotopic Hilbert space formulation of the exterior strong problem," *Hadronic Journal* 5, 1277-1366 (1982). <http://www.santilli-foundation.org/docs/Santilli-201.pdf>.
- [35] M. Nishioka, "Expansion of the Dirac-Myungf-Santilli Delta Function to Field Theory," *Lettere Nuovo Cimento* Vol. 39, pages 369-372 (1984), <http://www.santilli-foundation.org/docs/Santilli-202.pdf>.
- [36] R. M. Santilli, "Nuclear realization of hadronic mechanics and the exact representation of nuclear magnetic moments," *R. M. Santilli, Intern. J. of Phys.* Vol. 4, 1-70 (1998). <http://www.santilli-foundation.org/docs/Santilli-07.pdf>.
- [37] R. M. Santilli, "Lie-admissible invariant representation of irreversibility for matter and antimatter at the classical and operator levels," *Nuovo Cimento B* 121, 443 (2006), <http://www.santilli-foundation.org/docs/Lie-admiss-NCB-I.pdf>.
- [38] R. M. Santilli, "Compatibility of Arbitrary Speeds with Special Relativity Axioms for Interior Dynamical Problems." *American Journal of Modern Physics*, in press (2015). <http://www.santilli-foundation.org/docs/superluminal-speeds.pdf>.
- [39] R. M. Santilli, "A quantitative isotopic representation of the deuteron magnetic moment," in *Proceedings of the International Symposium 'Dubna Deuteron-93*, Joint Institute for Nuclear Research, Dubna, Russia (1994), <http://www.santilli-foundation.org/docs/Santilli-134.pdf>.
- [40] R. M. Santilli, "The notion of non-relativistic isoparticle," *ICTP preprint IC/91/265* (1991) <http://www.santilli-foundation.org/docs/Santilli-145.pdf>.
- [41] R. M. Santilli, "Nonlocal formulation of the Bose-Einstein correlation within the context of hadronic mechanics," *Hadronic J.* 15, 1-50 and 81-133 (1992), <http://www.santilli-foundation.org/docs/Santilli-116.pdf>.
- [42] F. Cardone and R. Mignani, "Nonlocal approach to the Bose-Einstein correlation, *Euop. Phys. J. C* 4, 705 (1998). see also "Metric description of hadronic interactions from the Bose-Einstein correlation," *JETP* Vol. 83, p.435 (1996). <http://www.santilli-foundation.org/docs/Santilli-130.pdf>.
- [43] R. M. Santilli, "Relativistic hadronic mechanics: non-unitary axiom-preserving completion of quantum mechanics" *Foundations of Physics* Vol. 27, p.625-739 (1997) <http://www.santilli-foundation.org/docs/Santilli-15.pdf>.
- [44] C. S. Burande, "Study of Bose-Einstein Correlation Within the Framework of Hadronic Mechanics," *International Journal of Modern Physics*, in press (2015), [http://www.santilli-foundation.org/docs/122014002\(3\).pdf](http://www.santilli-foundation.org/docs/122014002(3).pdf).
- [45] H. Rauch et A. Zeilinger, "Demonstration of SU(2) symmetry by neutron interferometers," in the *Proceedings of the 1981 Third Workshop on Lie-Admissible Formulations*, *Hadronic J.*, 4, 1280 (1981).
- [46] G. Eder, *Hadronic J.* 4, 634 (1981).
- [47] R. M. Santilli, "Experimental, theoretical and mathematical elements for a possible Lie-admissible generalization of the notion of particle under strong interactions," *Hadronic J.* 4, p. 1166-1258 (1981).
- [48] R. M. Santilli, *Ethical Probe of Einstein Followers in the U. S. A.*, Alpha Publishing, Nonantum, MA (1984). <http://www.scientificethics.org/IlGrandeGrido.htm>.
- [49] D. Kendellen, "Neutron Interferometry and 4π Spinor Symmetry Experiment, Preprint of the North Carolina State University, April 21 (2008). http://wikis.lib.ncsu.edu/images/0/06/Neutron_Interferometry.pdf.
- [50] H. Rauch H. and S. A. Werner, *Neutron Interferometry*, Oxford University Press, New York, 2000).
- [51] R. M. Santilli, "Theory of mutation of elementary particles and its application to Rauch's experiment on the spinorial symmetry," *ICTP preprint IC/91/46* (1991) <http://www.santilli-foundation.org/docs/Santilli-141.pdf>.
- [52] R. M. Santilli, "Apparent consistency of Rutherford's hypothesis on the neutron as a compressed hydrogen atom," *Hadronic J.* 13, 513 (1990). <http://www.santilli-foundation.org/docs/Santilli-21.pdf>.
- [53] R. M. Santilli, "Apparent consistency of Rutherford's hypothesis on the neutron structure via the hadronic generalization of quantum mechanics - I: Nonrelativistic treatment", *ICTP communication IC/91/47* (1992). <http://www.santilli-foundation.org/docs/Santilli-150.pdf>.
- [54] R. M. Santilli, "Representation of the synthesis of the neutron inside stars from the Hydrogen atom," *Communication of the Joint Institute for Nuclear Research, Dubna, Russia, number JINR-E4-93-352* (1993).
- [55] C. Borghi, C. Giori C. and A. Dall'Olio, *Communications of CENUFPE*, Number 8 (1969) and 25 (1971), reprinted via Santilli's proposal in the (*Russian Phys. Atomic Nuclei*, Vol. 56, p. 205 (1993).
- [56] R. M. Santilli, Confirmation of Don Borghi's experiment on the synthesis of neutrons, *arXiv publication*, August 15, 2006. <http://arxiv.org/pdf/physics/0608229v1.pdf>.
- [57] R. M. Santilli, The synthesis of the neutron according to hadronic mechanics and chemistry," *Journal Applied Sciences* 5, 32 (2006).
- [58] R. M. Santilli, "Apparent confirmation of Don Borghi's experiment on the laboratory synthesis of neutrons from protons and electrons," *Hadronic J.* 30, 29 (2007). <http://www.i-b-r.org/NeutronSynthesis.pdf>.
- [59] R. M. Santilli, "Documentation of scans from the Polimaster and SAM 935 detectors during tests [11-13]. <http://www.neutronstructure.org/neutron-synthesis-3.htm>.
- [60] R. M. Santilli, "The structure of the neutron," website www.neutronstructure.org/neutron-synthesis-2.htm
- [61] R. M. Santilli and A. Nas, "Confirmation of the Laboratory Synthesis of Neutrons from a Hydrogen Gas," *Journal of Computational Methods in Sciences and Eng, Journal of Computational Methods in Sciences and Engineering* Vol. 14, pp 405-415 (2014). www.thunder-energies.com/docs/neutron-synthesis-2014.pdf.

- [62] R. M. Santilli and A. Nas, 12 minutes Film on the Laboratory Synthesis of Neutrons from the Hydrogen Gas". www.world-lecture-series.org/newtron-synthesis-08-14.
- [63] J. V. Kadeisvili, "The Rutherford-Santilli Neutron," *Hadronic J.* Vol. 31, pages 1-125, 2008, pdf version of the published paper. <http://www.i-b-r.org/Rutherford-Santilli-II.pdf> also available in html format at. <http://www.i-b-r.org/Rutherford-Santilli-neutron.htm>.
- [64] C. S. Burande, "Santilli Synthesis of the Neutron According to Hadronic Mechanics," *American Journal of Modern Physics*, in press (2015). <http://www.santilli-foundation.org/docs/>.
- [65] R. M. Santilli, "Hadronic energies," *Hadronic Journal*, Vol. 17, 311 (1994). <http://www.santilli-foundation.org/docs/Santilli->
- [66] R. M. Santilli, "The etherino and/or the Neutrino Hypothesis?" *Found. Phys.* 37, p. 670 (2007). www.santilli-foundation.org/docs/neutron-synthesis-2015.pdf
- [67] H. Bondi and T. Gold, "The Steady-State Theory of the Universe," *Notices of the Royal Astronomical Society*, Vol. 108, p.252 (MNRAS Homepage).
- [68] P. Fleming, "Collected papers, interviews, seminars and international press releases on the lack of expansion of the universe," <http://www.santilli-foundation.org/docs/No-universe-expans.pdf>.
- [69] R. M. Santilli, *The Physics of New Clean Energies and Fuels According to Hadronic Mechanics*, Special issue of the *Journal of New Energy*, 318 pages (1998). <http://www.santilli-foundation.org/docs/Santilli-114.pdf>
- [70] S. S. Dhondge, "Studies on Santilli Three-Body Model of the Deuteron According to Hadronic Mechanics," *American Journal of Modern Physics*, in press (2015). <http://www.santilli-foundation.org/docs/deuteron-review-2015.pdf>.
- [71] <https://en.wikipedia.org/wiki/Nuclide>.
- [72] S. Glasstone, *Sourcebook on Atomic Energy*. Affiliated East-West Press Pvt. Ltd., New Delhi: D. Van Nostrand Company, INC., Princeton, New Jersey, U. S. A., Third ed., 1967, 1969 (Student Edition).
- [73] https://en.wikipedia.org/wiki/Isotopes_of_helium.
- [74] https://en.wikipedia.org/wiki/Atomic_nucleus.
- [75] R. B. Firestone and L. P. Ekström, "LBNL Isotopes Project - LUNDS Universitet. WWW Table of Radioactive Isotopes." <http://ie.lbl.gov/toi/index.asp>, January 2004.
- [76] C. Christensen, A. Nielsen, A. Bahnsen, W. Brown, and B. Rustad, "The half-life of the free neutron," *Physics Letters B*, vol. 26, pp. 11-13, December 1967.
- [77] T. Gray, M. Whitby, and N. Mann, "PeriodicTable." <http://www.periodictable.com/Isotopes/001.1/index.full.html>, September 2013.
- [78] N. J. Stone, *Atomic Data and Nuclear Data Tables*, vol. 90, pp. 75 - 176, 2005.
- [79] R. M. Santilli, "Rudiments of isogravitation for matter and its isodual for antimatter," *American Journal of Modern Physics* Vol. 4(5), pages 59-75 (2015). <http://www.santilli-foundation.org/docs/isogravitation.pdf>.
- [80] R. M. Santilli, "Experimental verifications of isoredshift with possible absence of universe expansion, big bang, dark matter and dark energy," *The Open Astronomy Journal* 3, 124 (2010), available as free download from <http://www.santilli-foundation.org/docs/Santilli-isoredshift.pdf>.

Hadronic Nuclear Energy: An Approach Towards Green Energy

Indrani B. Das Sarma

Department of Applied Chemistry, Jhulelal Institute of Technology, Lonara, Nagpur, India

Email address:

dassarmaindrani@rediffmail.com

To cite this article:

Indrani B. Das Sarma. Hadronic Nuclear Energy: An Approach Towards Green Energy. *American Journal of Modern Physics*. Special Issue: Issue II: Foundations of Hadronic Mechanics. Vol. 5, No. 2-1, 2016, pp. 119-130. doi: 10.11648/j.ajmp.2016050201.16

Received: August 11, 2015; **Accepted:** August 12, 2015; **Published:** June 1, 2016

Abstract: Nuclear energy is undoubtedly the largest energy source capable of meeting the total energy requirements to a large extent in long terms. However the conventional nuclear energy involves production of high level radioactive wastes which possesses threat, both to the environment and mankind. The modern day demand of clean, cheap and abundant energy gets fulfilled by the novel fuels that have been developed through hadronic mechanics / chemistry. In the present paper, a review of Prof. R. M. Santilli's Hadronic nuclear energy by intermediate controlled nuclear synthesis and particle type like stimulated neutron decay and double beta decay has been presented.

Keywords: Intermediate Controlled Nuclear Synthesis, Stimulated Neutron Decay

1. Introduction

The ever increasing demand for good quality of livelihood has ultimately culminated in increasing global energy demands. The demand can be met conventionally by either molecular combustion or nuclear fission. The former is achieved by combustion of fossil fuel or hydrogen which produces large amount of green house gases as well as depletes breathable oxygen from the environment. The latter does not generate green house gases or depletes breathable oxygen but creates large amount of radioactive wastes. Moreover, the shielding from the high energy ionizing and non-ionizing radiations is cumbersome and expensive. The handling of the highly radioactive wastes poses environmental as well as security threat. Thus, handling of these wastes requires great deal of safety requirements. There are several ways that are used to curb the either menace such as using better furnace design, improvising fuels and additives for molecular combustion or improvising fuel geometry and reactor design for efficient nuclear fission. In either case the perilous waste products are not completely eliminated. Although there are energy sources that have zero emissions like the energy harnessed from renewable sources like solar, wind, tidal, geo-thermal, wave, ocean-thermal and so on but are mainly time and location dependent. Hence cannot be universally employed for harnessing energy or power generation.

On the other hand, the nucleus of an atom has always been considered to be the source of unlimited energy since its discovery in 1911 by Ernest Rutherford [1]. The basic nuclear processes are of two types viz., fission and fusion. Both these processes generate large amount of energy which can be conveniently harnessed for useful work. The fission reaction is exoergic and criticality can be attained easily but fusion is endoergic and achieving criticality is comparatively difficult. Hence fission has been extensively explored for destructive as well as constructive work.

The unlimited source of the atomic nucleus due to fission process was initially exclusively exploited for destructive purpose. However, post World War II the focus shifted more towards constructive work. Attention was turned to the peaceful and directly beneficial application of nuclear energy. In the course of developing nuclear weapons the Soviet Union and rest of the Western world had discovered range of new technologies. Scientists also realized that the tremendous heat produced in the process could be tapped either for direct use or for generating electricity. It was also clear that this new form of energy had tremendous potential for the development of compact long-lasting power sources which could have various applications.

The world's first artificial nuclear reactor was Chicago Pile-1. It was a research reactor. Its construction was a part of the Manhattan Project. It was carried out by the Metallurgical

Laboratory, University of Chicago under the supervision of Enrico Fermi, alongwith Leó Szilárd (discoverer of the chain reaction), Martin Whittaker, Walter Zinn and George Weil [2, 3]. The first man-made self-sustaining nuclear chain reaction was initiated in Chicago Pile-1 on December 2, 1942. The apparatus was described as a *crude pile of black bricks and wooden timbers* by Fermi. The pile contained large amount of graphite (771,000 lbs) and uranium (80,590 lbs of uranium oxide and 12,400 lbs of uranium metal). The pile was in the form of flattened ellipsoid measuring 25 feet wide and 20 feet high. The neutron producing uranium pellets were separated from one another by graphite blocks in the pile. Some of the free neutrons produced by the natural decay of uranium were absorbed by other uranium atoms, causing nuclear fission of those atoms and the release of additional free neutrons. The graphite between the uranium pellets was neutron moderator that thermalized neutrons, increasing fission cross-section. The control rods were of cadmium, indium (for preventing uncontrolled chain reaction) and silver (measuring the flux). Unlike the modern reactors, it lacked radiation shield or cooling system.

The first nuclear reactor to produce electricity by fission was the Experimental Breeder Reactor-I or Chicago Pile-4 designed and operated by Argonne National Laboratory, Idaho, USA under the supervision of Walter Zinn. This LMFBR went critical in December 1951. It produced 0.8 kW in a test run on December 20, 1951[4] and 100 kW of electrical power the following day, [5] having a design output of 200 kW of electrical power.

With advancement of technologies, the modern fission reactors have high energy output but have disadvantages such as enrichment of fuel and / or moderator; disposal of high energy radioactive waste and cumbersome shielding from high energy ionizing radiations. Thus, the energy harnessed is not completely green. On the other hand fusion process does not generate large amount of nuclear waste and if nuclei combine at threshold energy then the chances of crossing the fission barrier and emission of ionizing radiation are reduced considerably.

2. Nuclear Fusion

The nuclear fusion has always been considered the holy grail of unlimited clean energy. The reason for this is probably the thermonuclear reactions taking place in the sun and other stars [6]. In this case, nuclear fusion is achieved by using extremely high temperatures. The average kinetic energy of the combining nuclei increases proportionately with temperature. The temperature is determined by Lawson criteria [7] as given by expression 1.

$$n\tau_E \geq L \equiv \frac{12 k_B T}{E_{ch} \langle \sigma v \rangle} \quad (1)$$

where, n , τ_E , E_{ch} and k_B are particle density, confinement time, energy of charged fusion product and Boltzmann constant respectively.

The quantity $T/\langle \sigma v \rangle$ is a function of temperature with an

absolute minimum. Replacing the function with its minimum value provides an absolute lower limit for the product $n\tau_E$. This is the Lawson criterion. At the temperature predicted by the Lawson criterion the energy of the colliding particles confined within the plasma are high enough to overcome the Coulomb barrier and chances of fusion increases. The colliding nuclei are confined within the plasma by gravitational or magnetic or inertial confinement. The controlled thermonuclear fusion reactions take place in an environment allowing some of the resulting energy to be harnessed for constructive purposes. Since this reaction takes place at very high temperature, so is popularly known as Hot Fusion. The major drawback is that it is not self sustaining and compound nucleus undergoes fission leading to formation of radioactive wastes. This is because the atomic electron clouds are completely stripped off. Kinetic energies of the colliding nuclei are increased to overcome the coulombic barrier and the energy attained by the compound nucleus is generally higher than the fission barrier which results in fission reaction or nuclear decay as prominent exit channels.

Fleischmann, Pons and Hawkins [8] in the year 1989 reported their historic but the most debatable findings. They observed unusual excess heat in the electrolysis of heavy water using deuterium loaded palladium electrodes. This they presumed to be due nuclear fusion reaction. Since the reaction taking place is at low temperature, they termed it as cold fusion on similar terms as hot fusion. Cold fusion or low energy nuclear fusion is known to occur under certain conditions in metal hydrides. It produces excess heat and nuclear ash such as helium, charged particles and sometimes very low level of neutrons. In certain cases the host metal has been found to be transmuted into other elements. The cold fusion reaction has been reported with palladium, titanium, nickel, some superconducting ceramics and so on. It has been observed due to varied triggers like ultrasonic waves, laser beam, electrical current. The major explanation for this phenomenon is reported to be the induction of electrostatic pressure to the reacting nuclei within the lattice of the metal. This environment is difficult to achieve and hence the phenomenon is non-reproducible. This could be due to insufficient energy required to expose the atomic nuclei from within the covering atomic electron cloud.

2.1. Nuclear Processes and Quantum Mechanics

Quantum mechanics is based on Galilei and Poincaré symmetries [9]. They are applicable only for Keplerian systems, where the various particles orbit around a centrally located nucleus, such as planets around central star / sun or electrons around nucleus. However, quantum mechanics is not applicable in understanding interaction between those particles which lack such symmetries like interaction between two electrons in a sigma bond or lateral overlap as in π -bonds. The Hamiltonian nature of quantum mechanics restricts the understanding of nuclear forces. Hence, to represent the nuclear force with a potential up to 35 different potentials have been added without achieving the required exact representation. The linear, local and Hamiltonian character of quantum mechanics is effective for the classification of

hadrons under their point-like approximation, but is inadequate for structure related problems due to expected nonlinear, nonlocal and non-Hamiltonian effects occurring within the hyper dense media inside hadrons.

Thus, Prof. Santilli [10] states: *According to the standard model, at the time of the neutron synthesis from protons and electrons inside a star, the permanently stable protons and electrons simply disappear from the universe to be replaced by conjectural quarks, and then the proton and the electron simply reappear at the time of the neutron decay. These beliefs are simply repugnant to me because excessively irrational, thus showing the conduction of particle physics via academic authority, rather than scientific veritas.*

The quantum theory fails to explain the following even for the simplest nucleus of deuterium [9, 10]-

1. The spin 1 of deuterium since quantum axioms require that the single stable bound state of two particles with spin 1/2, (proton and neutron) must be the singlet state with spin zero.
2. To represent the magnetic moment of deuterium.
3. The stability of unstable neutron when coupled to proton in a nucleus (e.g. deuterium). $T_{1/2}$ of neutron \cong 15 minutes.
4. Quantum Mechanics is inapplicable for explaining the synthesis of neutron from a proton and an electron as occurring in stars because; in this case the Schrödinger equation becomes inconsistent.

It is unsuitable for all processes that are irreversible over time, like nuclear fusions, because quantum mechanics is reversible over time, thus admitting the time reversal event which violates energy conservation, causality and other basic laws.

2.2. Hadronic Mechanics

Quantum mechanics was conceived for the study of interactions among particles at large mutual distances which is represented with differential equations defined over a finite set of isolated points. It does not have the scope for the study of the additional nonlocal-integral interactions due to mutual wave overlapping. These interactions are defined over an entire volume and cannot be effectively approximated by their abstraction into finite number of isolated points. Thus, the same cannot be derived from a Hamiltonian or their derivatives [9].

Thus, Prof. Santilli has founded more fundamental theory

of the universe, named after the composite nuclear particle *hadron* as *Hadronic Mechanics*. Hadronic mechanics was formulated as an extension of quantum mechanics, encompassing its insufficiencies for the study of the additional nonlocal-integral interactions due to mutual wave overlapping. Thus the range of hadronic, quantum and classical mechanics can be depicted as in Figure 1.

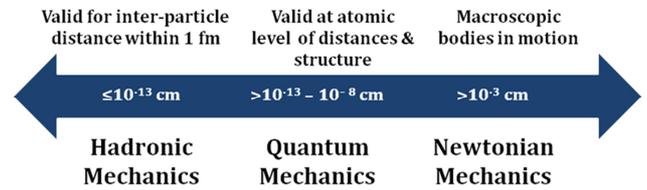


Figure 1. Various range of validity for Hadronic, Quantum and Newtonian Mechanics.

The emergence of strongly attractive force for deeply overlapping particles is one of the fundamental contributions of hadronic mechanics. There are varied instances where hadronic mechanics could satisfactorily explain the interactions such as quantitative treatment of neutron synthesis from protons and electrons (as occurring in stars), nuclear fusion, explanation of nuclear structure, strong nuclear binding energy, strong interaction between two electrons in a sigma bond, formation of magnecular bonds, formation of cooper pair in superconductors, and so on. Thus, hadronic mechanics could provide a quantitative treatment for the possible utilization of inextinguishable energy contained inside the neutron and formation of light nuclei. In other words, the study of new clean energies and fuels that cannot even be conceived with the 20th century doctrines and other basic advances can be done with the new sciences. So, hadronic mechanics is rightly called as new sciences for new era [10].

The modern day demand of clean, cheap and abundant energy source can be fulfilled by changing the approach from quantum mechanics to hadronic mechanics to hadronic chemistry. In view of this, Prof. Ruggero Maria Santilli proposed various types of new non-nuclear as well as nuclear fuels. Non-nuclear fuels are basically magnecules that show magnecular combustion similar to conventional molecular combustion albeit cleaner, greener and with higher calorific values probably due to stored magnetostatic energy within the magnecules [12, 13]. The hadronic fuels are summarized in Figure 2.

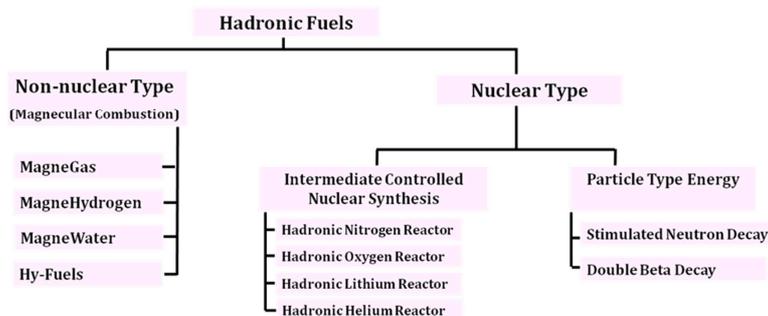


Figure 2. The classification of various hadronic fuels.

2.3. Nuclear Type Hadronic Fuels

The nuclear fuels proposed by Prof. Santilli under hadronic mechanics are controlled nuclear reactions (fusion as well as fission) without ionizing radiations and radioactive waste. The nuclear fission could be adequately explained by quantum mechanics by considering the fragments as point mass. However, the same theory fails to explain nuclear fusion because considering the reacting nuclei as point mass is impractical [10]. Nucleus is a hyper dense medium containing protons and neutrons. Since neutrons are made up of protons and electron, hence Prof. Santilli projects nucleus of an element as collection of mutated protons and electrons. The basic assumptions [11,12] proposed by Prof. Santilli are-

1. *Nuclear force:* Nuclear force was initially considered to be derived completely from a potential. So it was represented with a Hamiltonian. However, Prof. Santilli assumed that nuclear force is partly of action-at-a-distance, potential type that can be represented with a Hamiltonian and partly is of contact, non-potential type that cannot be represented with a Hamiltonian. This assumption implies that the time evolution of nuclear structure and processes is essentially of non-unitary type. So the use of hadronic mechanics is mandatory as it is the only known axiomatically consistent and time invariant non-unitary formulations of nuclear structures and their processes.
2. *Stable nuclei:* Nuclei have no nuclei of their own and are composed of particles in contact with each other having mutual penetration of about 10^{-3} of their charge distributions shown in Figure 3. So, the nuclear force is expected to be partially of potential and partially of non-potential type, with ensuing non-unitary character of the theory, and related applicability of hadronic mechanics.

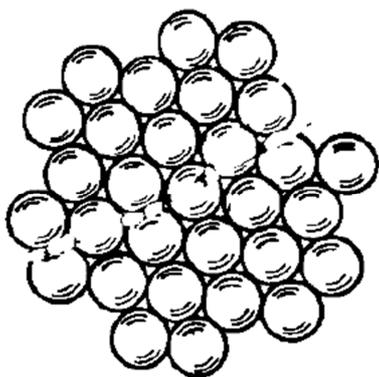


Figure 3. Schematic representation used by Prof. Santilli to illustrate that nuclei have no nuclei of their own and are composed of particles in contact with each other.

Let A be the time evolution of a Hermitean operator in the infinitesimal and finite forms derived from Heisenberg-Santilli Lie-isotopic equations proposed in 1978 by Prof. Santilli for stable, reversible, interior dynamical problems.

$$i \frac{dA}{dt} = [A, H] = ATH - HTA \tag{2}$$

$$A(t) = \exp(HTit)A(0)\exp(-itTH)$$

where the Hermitean Hamiltonian

$$H = \frac{p^2}{2m} + V(r)$$

represents all possible nuclear forces truly derived from potential V (r);

Isotopic element T represents all contact non-potential interactions allowing the nuclear structure with all constituents in actual contact with each other, and the simplest possible realizations of type

$$T = \exp\left(-F(r) \int \psi^\dagger(r)\psi(r)d^3r\right) > 0 \tag{3}$$

which recovers quantum mechanics when there is no appreciable overlapping of the wavefunctions ψ of nuclear constituents; and the inverse of isotopic element

$$I = \frac{1}{T} > 0 \tag{4}$$

represents the basic, right and left unit of the theory at all levels, non-zero values of T depicts non-Hamiltonian interactions (presence of contact).

The stability of the nucleus (reversibility over time) is represented by the identity of the basic iso-unit to the right and to the left, namely, for motions forward and backward in time.

3. *Unstable nuclei and nuclear fusion:* According to the Heisenberg-Santilli Lie-admissible equations for the time evolution of Hermitean operator A in their infinitesimal and finite forms

$$i \frac{dA}{dt} = (A; H) = ARH - HSA \tag{5}$$

where H is Hermitean representing the non-conserved total energy
genotopic elements R and S represents the non-potential interactions.

Irreversibility is depicted by the different values of the genounit for forward (f) and backward (b) motions in time

$$I = \frac{1}{R} \neq I = \frac{1}{S} \tag{6}$$

Here, the Lie-admissible branch of hadronic mechanics is ideally suited to represent the decay of unstable nuclei as well as nuclear fusions, since both are irreversible over time.

4. *Neutron synthesis:* Rutherford's conjecture on neutron as a compressed hydrogen atom was experimentally verified later by Don Borghi's experiment. It is also well-known that synthesis of neutron from the compressed hydrogen gas is precursor to synthesis of all natural elements in a star. So, the synthesis of the neutron is the most fundamental nuclear synthesis. As shown in Figure 4 (a) the original drawing used by Prof. Santilli to illustrate the physical difference between the hydrogen atom and neutron synthesis from proton and

electron. The figure 4 (b) depicts the additional non-linear, non-local and non-potential interactions due to deep wave overlapping of proton and electron in

neutron which is otherwise absent in hydrogen atom. This non-Hamiltonian character requires a non-unitary extension of quantum mechanics.

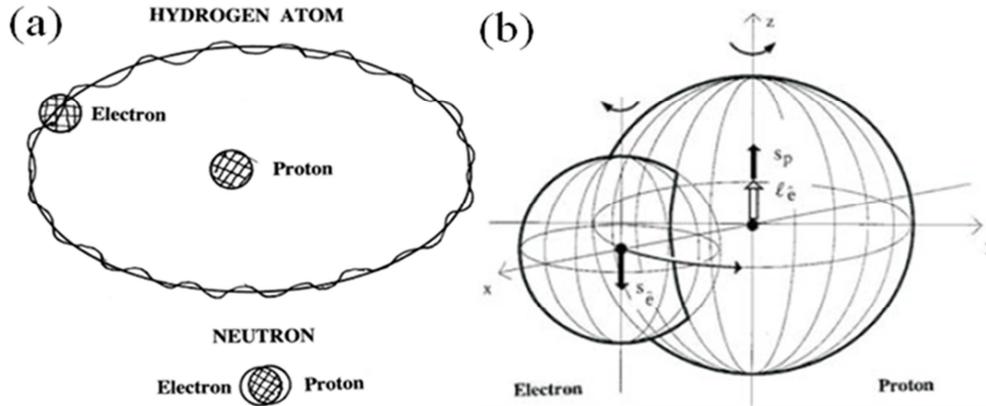


Figure 4. (a) Original drawing used by Prof. Santilli to illustrate the physical difference between the hydrogen atom and neutron. (b) additional non-linear, non-local and non-potential interactions due to deep wave overlapping of proton and electron in neutron.

Consequently, Prof. Santilli quantified neutron synthesis using hadronic mechanics as

$$p^+ + a + e^- \rightarrow n \tag{7}$$

where ‘a’ is Santilli’s etherino which is a conventional Hilbert space the transfer of 0.782 MeV and spin 1/2 missing in the neutron synthesis from the environment to the neutron structure.

The etherino disappears at the covering level of hadronic mechanics and the neutron synthesis on a iso-Hilbert space over an iso-fields. Finally, the missing 0.782 MeV energy required for the synthesis of the neutron is provided by the environment. For instance, a star would not start producing light due to huge amount of energy needed for the synthesis of large number of neutrons. Thus, it was concluded that for continuous creation of neutron in the universe the missing energy is provided by the ether as a universal substratum.

5. *Nuclear Structure:* Prof. Santilli assumes the unitary classification of baryons as valid, but introduces new structure models of each member of the baryonic family with physical constituents that can be produced free, thus being detected in the spacetime. Resolution of historical objections is merely achieved by assuming that, when in interior conditions (only), barionic constituents obey hadronic mechanics and symmetries with subsequent mutations (denoted by hat) of their intrinsic characteristics. Proton is assumed to be an elementary stable particle without known structure and neutron to be an unstable particle comprising of a proton \hat{p}^+ and an electron \hat{e}^- in mutated conditions due to their total mutual immersion and resulting synthesis

$$n = (\hat{p}^+, \hat{e}^-)_{hm} \tag{8}$$

As a result, it is assumed that nucleus is a collection of protons and neutrons in first approximation, while being at a

deeper level it is a collection of mutated protons and electrons.

2.3.1. Controlled Nuclear Synthesis (CNS)

The hot fusion or cold fusion reactions are difficult to achieve. The high temperature required for hot fusion and random occurrence of cold fusion limits their use for economic energy output. One of the major successes of hadronic mechanics and iso sciences is their ability to obtain industrial realization of fusion reactions without any ionizing radiations. These reactions are controlled as well as have intermediate energy requirements than hot or cold fusions hence are called as controlled nuclear synthesis (CNS) or intermediate controlled nuclear synthesis (ICNS). *Controlled Nuclear Synthesis (CNS) are given by systematic energy releasing nuclear fusions whose rate of synthesis (or of energy output) is controllable via one or more mechanisms capable of performing the engineering optimization of the applicable laws* [11, 12].

There are various physical laws which are to be obeyed by all nuclear fusions to occur in a systematic way rather than in a random way. The CNS is governed by Santilli's laws for controlled nuclear synthesis [11, 12]:

1. *The orbitals of peripheral atomic electrons are controlled such that nuclei are systematically exposed.* Nuclei are shielded by the electron cloud. It is obvious that nuclear synthesis between two atoms is impossible at low energies because the electron cloud restricts approachability of the interacting nuclei. This law explains the inability of the cold fusions to achieve energy output of industrial significance because in this case the energy necessary for systematic exposure of nucleus from electron cloud is low. This law also emphasises the need for the proposed intermediate synthesis in which the first energy requirement is precisely the control of atomic clouds.
2. *CNS occurs when nuclei spins are either in singlet planar coupling or triplet axial coupling.* This law shows the structural difference between quantum and

hadronic mechanics. The constituents of a bound state of two quantum particles must be point-like to avoid structural inconsistencies such as local-differential topology. As a result, as per quantum mechanics singlet and triplet couplings are equally possible. However, when the actual extended character of the constituents is taken into account, it is clear that triplet planar couplings of extended particles at short distances are strongly repulsive, while singlet planar couplings are strongly attractive. Planar means that the two nuclei have a common median plane and axial means a common axial symmetry as shown in figure 5. This law was the basis to build hadronic mechanics via gear model. In fact, the coupling of gears in triplet (parallel spins) causes extreme repulsion, while the only possible coupling of gears is in singlet (antiparallel spins). The emergence of strongly attractive force for the singlet planar or triplet axial couplings is one of the fundamental contributions of hadronic mechanics to fusion processes since such a force is totally absent for quantum mechanics, while it appears naturally in all spinning and deeply overlapping particles.

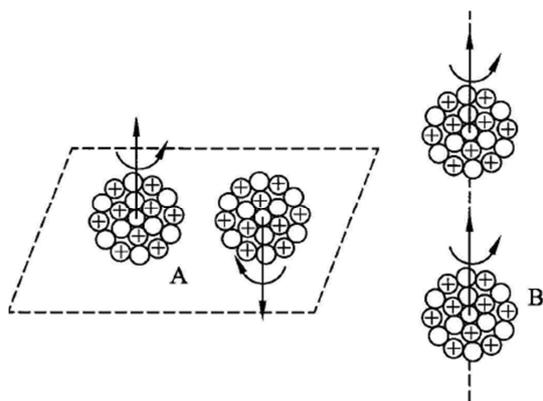


Figure 5. Schematic representations of the only two stable couplings permitted by hadronic mechanics for nuclear synthesis; the singlet planar coupling (A) and the triplet axial coupling (B). All other spin configurations have been proved to produce strongly repulsive forces under which no CNS is possible.

3. The most probable CNS are those occurring at threshold energies and without the release of massive particles or ionizing radiations. In other words, CNS occurring at threshold energies are green in nature as they do not emit ionizing radiations or ejectiles. The threshold energy mostly hinders fusion reaction. If the energy is lower than the threshold energy then industrially meaningful nuclear syntheses is not possible as in case for cold fusions, although random synthesis may occur due to tunnelling effect. On the other hand, if the energy of the interacting nuclei is higher than the threshold energy then the excess energy is reflected as excitation energy of the resulting compound nucleus. Thus, excitation energy of the compound nucleus is directly proportional to the energy of the interacting nuclei. The excitation energy is dissipated by emission of gamma

photon or particles or fission of the resulting compound nucleus as shown in figure 6.

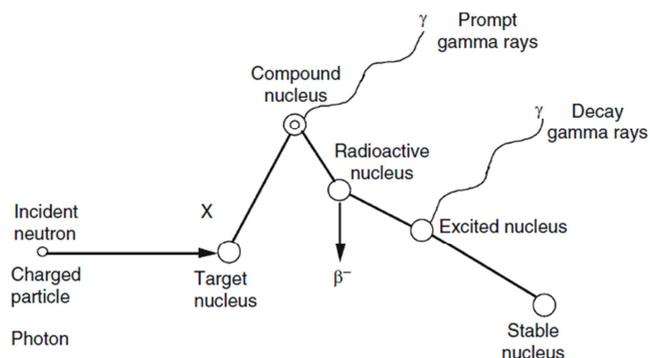


Figure 6. Formation of compound nucleus having high excitation energy.

The calculations based on hadronic mechanics indicate that the probability of a nuclear synthesis with the release of neutrons is much smaller than that of synthesis without the emission of massive particles. This law has been verified by ICNS data and also it appears to be verified by nuclear syntheses spontaneously occurring in nature. However, this does not mean that CNS with secondary emission is impossible. This only suggests that the nuclear synthesis could be green which was earlier unimaginable.

4. *CNS requires trigger, an external mechanism that forces exposed nuclei to come in fm range (hadronic horizon).* All nuclei are positively charged, thus repel each other at distances bigger than one Fermi. Nuclear synthesis is impossible without overcoming the coulombic repulsion that brings nuclei inside the hadronic horizon. Inside the hadronic horizon, the preceding laws are verified (particularly second law on spin couplings). The synthesis is inevitable due to the activation of the strongly attractive hadronic forces (typical non-potential interaction) that overcome the repulsive Coulomb force.

Considering the Fleishmann-Pons electrolytic cell in purview of Santilli's Law of CNS, it is clear that this cell does verify the conservation of the energy, angular momentum and has a trigger characterized by the electrostatic pressure compressing deuterium inside the palladium. However, Fleishmann-Pons electrolytic cell does not verify first law (control of atomic clouds to expose nuclei) and second law (control of spin couplings). Here the nuclear spin couplings occur at random; there is lack of identified mechanism for systematic exposure of the interacting nuclei and optimization of the verified laws. Consequently, nuclear syntheses occur at random, preventing economic values of the energy output. Thus, it is evident that for nuclear synthesis of economic value to occur all the above laws should be verified.

2.3.2. *Magnecules: A Precursor for Nuclear Synthesis*

Magnecules proposed by Prof. R. M. Santilli is a novel chemical species that have at least one magnecular bond between two atoms or radicals or molecules. The atoms are

held together by magnetic fields originating due to toroidal polarization of the atomic electron orbits [13, 14]. The rotation of the electrons within the toroid creates the fifth field force, the magnetic field, which cannot originate for the same atom if the conventional spherical electron distribution in orbitals is a physical reality. When two such polarized atoms are sufficiently close to each other and in north-south north-south alignment, the resulting total force between the two atoms is attractive as shown in Fig.7.

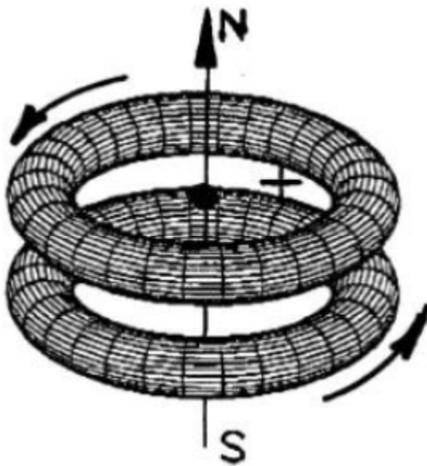


Figure 7. Conceptual depiction of an elementary magnecule comprising two identical or different atoms whose bond is entirely of magnecular character, namely, originating from opposing polarities North-South-North-South of the toroidal distributions of orbitals, as well as the polarization of nuclear and electron magnetic moments.

Thus, the simple principle of synthesizing magnecules is similar to the magnetization of a ferromagnet where the orbits of unbounded electrons are polarized. The added beauty of magnecules is that the nucleus is systematically exposed and the two nuclei can approach each other without appreciable columbic barrier. The internuclear distance is governed by the bond energy of the magnecular bond. The nuclei of the interacting atoms approach more closely than in case of conventional molecular bond allowing the required singlet planar or triplet axial coupling for nuclear synthesis. Thus, when a trigger brings the two nuclei within 1 fm range (hadronic horizon) the fusion becomes inevitable and a new nucleus is formed. Since the parent nuclei are not having high energy the resulting daughter nuclei also does not possess high excitation energy, consequently there is no nuclear emission. Thus the process is green.

Thus, the ICNS proposed by Santilli are of the generic type [11, 12]

$$\begin{aligned}
 &N_1(A_1, Z_1, J_1^P, u_1) + N_2(A_2, Z_2, J_2^P, u_2) + TR \\
 &\quad \rightarrow N_3(A_3, Z_3, J_3^P, u_3) + \text{Heat} \tag{9} \\
 &\text{where } A_1 + A_2 = A_3, \quad Z_1 + Z_2 = Z_3, \\
 &J_1 + J_2 = J_3, \quad P_1 + P_2 = P_3
 \end{aligned}$$

A is the atomic number
 Z is the nuclear charge
 J^P is the nuclear angular momentum with parity

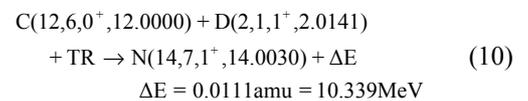
u is the nuclear energy in amu units

TR is trigger mechanism (high voltage DC arc in hadronic reactor) and mass defect is observed in form of heat

Nuclear synthesis via green mechanism is known to occur silently in nature [11]. This can be verified from the chemical analyses of about one hundred million years old amber sample. The trapped air bubbles showed 40% nitrogen, whereas the current percentage of nitrogen in atmosphere is approximately 80%. Other chemical analyses verify the above analysis that the increase of nitrogen in our atmosphere has been gradual.

According to Prof. Santilli, these data indicate *the natural synthesis of nitrogen from lighter elements*. The most probable mode of nitrogen synthesis in nature seems to be initiated by *lightning* as quantitative explanation of *thunder* is impossible by conventional chemical reactions, thus requiring nuclear syntheses. Numerical explanation of thunder requires energy equivalent to hundreds of tons of explosives that simply cannot be explained via conventional processes due to the very small cylindrical volume of air affected by lightning and its extremely short duration of the order of nanoseconds. However, the nitrogen syntheses by lightning provide numerical explanation of thunder as well as the gradual increase of nitrogen in the atmosphere. Among all possible syntheses, the most probable one results in being the *synthesis of nitrogen from carbon and deuterium*.

According to Prof. Santilli, these data indicate *the natural synthesis of nitrogen from lighter elements*. The most probable mode of nitrogen synthesis in nature seems to be initiated by *lightning* as quantitative explanation of *thunder* is impossible by conventional chemical reactions, thus requiring nuclear syntheses. Numerical explanation of thunder requires energy equivalent to hundreds of tons of explosives that simply cannot be explained via conventional processes due to the very small cylindrical volume of air affected by lightning and its extremely short duration of the order of nanoseconds. However, the nitrogen syntheses by lightning provide numerical explanation of thunder as well as the gradual increase of nitrogen in the atmosphere. Among all possible syntheses, the most probable one results in being the *synthesis of nitrogen from carbon and deuterium*.



However, the amount of deuterium present in the atmosphere is negligible to justify thunder quantitatively. Here, Prof. Santilli emphasizes the synthesis of neutrons by lightning from protons and electrons.

The neutron synthesis is expected to be a pre-requisite for the synthesis of deuterium in atmosphere which in turn synthesizes nitrogen which justifies energy of the thunder quantitatively. The same synthesis has been reproduced in laboratory quantitatively by Prof. Santilli using hadronic reactor.

2.4. Synthesis of Nitrogen from Carbon and Deuterium by ICNS

The fusion reaction/nuclear synthesis taking place in hadronic reactor using deuterium as fuel and carbon electrode have shown to yield clean energy without formation of any radioactive species or ionizing radiations [15, 16]. This synthesis is of industrial importance because it yields 10^{10} BTU of energy per hour which is equivalent to 10^{30} ICNS per hour. The electric arc of the hadronic reactor polarizes carbon and hydrogen atoms by forming $C \times H \times H$ magneucle, having triplet axial spin coupling. Under a suitable trigger (either high DC voltage or any other suitable means) the magneucle $C \times H \times H$ yield a nucleus with $A=14$, $Z=8$, $J^P=1^+$. However, this is impossible as O (14, 8) has spin $J=0$ and any other nucleus of the above mentioned type does not exist.

So, Prof. Santilli postulated that the nature synthesizes a neutron from proton, electron and etherino as

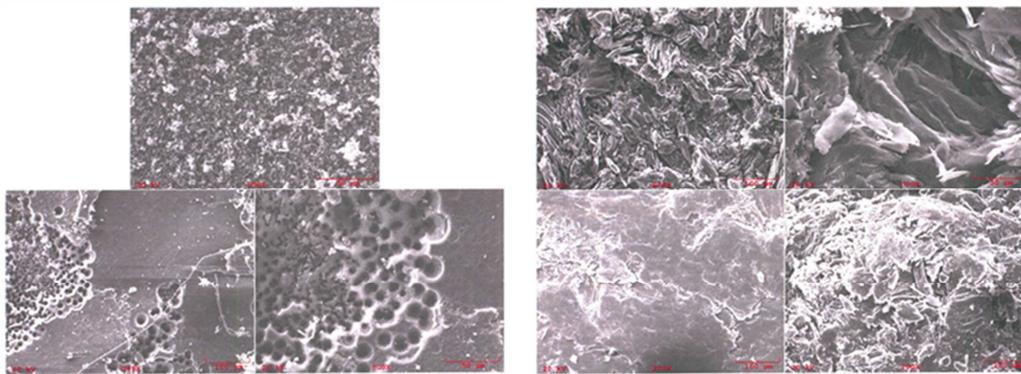
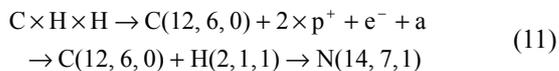
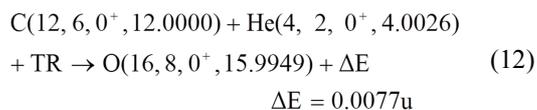


Figure 8. Semi-quantitative chemical compositions of the used electrodes were obtained using an Amray Scanning Electron Microscope equipped with an iXRF energy dispersive spectrometer (SEM-EDS) by Constellation Technology on 15th October 2013. The elements detected were aluminium, silicon, calcium, chlorine and iron which were initially absent or if present were in trace amount.

(ii). Synthesis of Oxygen from Carbon and Helium

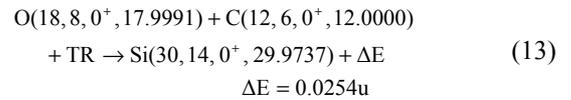


This nucleosynthetic reaction also verifies all conservation laws. Here, the interior of the reactor was cleaned, and various components replaced. A vacuum was pulled out of the inner chamber and the reactor was filled up with commercial grade helium at 100 psi. It was found that oxygen content decreased to a non-detectable amount but the CO increased from a non-detectable amount to 4.24%.

The formation of CO depicts synthesis of oxygen at the tip of the DC arc hitting the carbon in the cathode surface. The resulting large local heat rapidly expels the synthesized oxygen from the DC arc, preventing any additional nuclear synthesis. However, high affinity of carbon and oxygen results in formation of carbon monoxide.

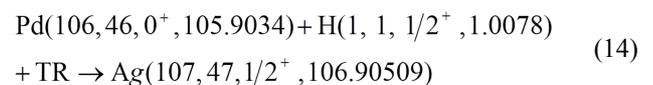
Other Examples of ICNS

(i). Synthesis of Silicon from Oxygen and Carbon



This nucleosynthetic reaction verifies all conservation laws. The controlled fusion of oxygen and carbon into silica was done using CO₂ (green house gas) as hadronic fuel for the production of clean energy [11]. The whitish powder formed on the edge of carbon electrodes of the hadronic reactor suggests synthesis of silica. Hadronic reactor was filled up with CO₂ at pressure. The DC arc efficiently separates it into O₂ and C. O₂ and C burns to produce CO which in the presence of oxygen and an arc, reproduces CO₂. Thus recovering the energy used for the separation of CO₂. However, along with the conventional combustion, the hadronic reactor produces a net positive energy output due to the fusion of oxygen and carbon into silica [17].

(iii). Synthesis Silver from Palladium and Hydrogen



This nucleosynthetic reaction depicts the basic difference between pre-existing studies on cold fusion and the proposed ICNS. In this reaction palladium 106 is used as cathode and reactor is filled with hydrogen at a certain pressure.

If cold fusion occurs, then fusion reactions should take place inside the palladium cathode. However, the engineering implementation of the new CNS laws inside the palladium electrodes is virtually impossible, thus explaining the reason for the lack of its consideration in the industrial research. According to Prof. Santilli the nuclear fusions may occur in such conditions at random, thus preventing the controlled energy output necessary of industrial relevance.

Thus, albeit this reaction verifies conventional nuclear conservation laws but is not of industrial relevance owing to

its random nature.

2.5. Santilli Hadronic Reactors

Hadronic reactors are the upgraded hadronic refineries originally designed by Prof. Santilli [11]. They use magnecular fuels for production of heat that can be used for power generation. The reactors house trigger mechanisms like high voltage DC arc or pressure impulse, etc as shown in Fig. 9 to facilitate controlled nuclear synthesis. Hadronic reactors can withstand higher pressure as compared to the hadronic refineries.

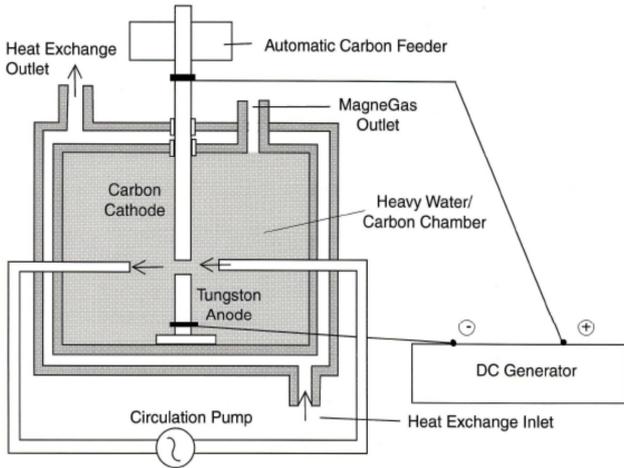
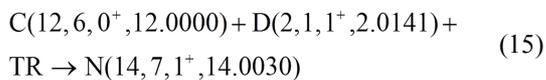


Figure 9. Schematic diagram of hadronic reactor based on an upgradation of the hadronic refineries showing emphasis on the production and use of a magnecular fuel in the latter, to the production and use of heat in the former.

These reactors are named based on the product or the fuel used. Hadronic nitrogen reactor is the most primitive type of hadronic reactor.

2.5.1. Hadronic Nitrogen Reactor

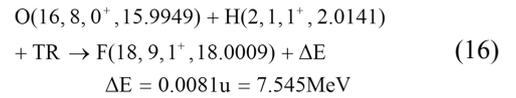
The reactor is filled with D₂ gas at 3,000 psi and is re-circulated through graphite electrodes. The trigger mechanism is by pulse DC arc of 100,000 V, 5 mA and other means. The heat is dissipated by the external heat exchanger. The heat is due to the nucleosynthetic reaction between deuterium and carbon occurring in reactor



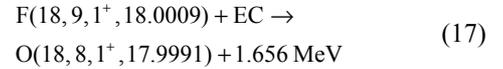
2.5.2. Hadronic Oxygen Reactor

It is one of the simplest reactors as the reaction does not require spin polarizations for conservation of the angular momentum. So the reactor is similar to the one shown in Fig. 5 housing carbon electrodes. The vessel is filled up with a 50-50 mixture of oxygen 16 and helium at 3,000 psi, which is re-circulated through 50 kW electric arc to create magnecules of the type O He. The trigger is DC pulses of 100,000 V and 5 mA, or by impulse pressures or other mechanisms. The heat produced is dissipated by the external heat exchangers.

Nucleosynthetic reaction occurring in the reactor



The instability of F(18, 9, 1⁺, 18.0009) results in secondary process



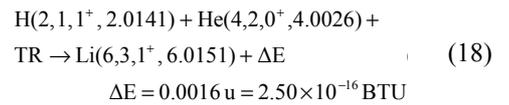
Thus the total energy output per synthesis is equivalent to 9.201 MeV 1.30x10⁻¹⁵ BTU.

If 10³⁰ syntheses occur per hour then amount of green energy yielded would be substantial.

2.5.3. First Hadronic Lithium Reactor

First lithium reactor is the same as that of the oxygen reactor. The only difference is that the vessel is filled with 50-50 mixture of hydrogen and helium gases at 3,000 psi. The mixture is also recirculated through a 50 kW electric arc that creates magnecules H He. The trigger is given by a high voltage pulse DC current or impulse pressure or other mechanism.

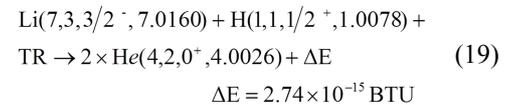
Nucleosynthetic reaction occurring in the reactor is



2.5.4. Second Hadronic Lithium Reactor

It is more complex than the first hadronic lithium reactor because of the need of lithium nuclei and a beam of protons with opposite polarization to avoid random reactions. The current technology allows a variety of engineering realizations of the needed polarization where a proton beam with down polarization enters a chamber of lithium with up polarization. Both polarizations are achieved via magnetic fields. The efficiency of the hadronic reactor depends on the geometry of the proton beams, the lithium chamber as well as required trigger.

Nucleosynthetic reaction occurring in the reactor



Assuming efficiency of 10¹⁶ per minute one mole of lithium would produce energy equivalent to 1.7 x 10⁶ J hour⁻¹.

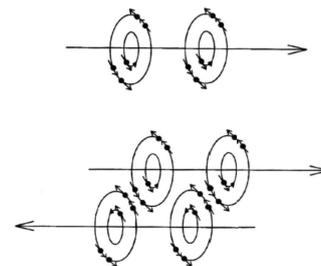


Figure 10. Schematic view of singlet (antiparallel) spin coupling required to synthesize helium from deuterium.

2.5.5. Hadronic Helium Reactor

It is one of the most difficult as it requires the application of a trigger to two different beams of deuterium gas with opposite spin polarizations as depicted in Fig. 10. The reactor as shown in Fig. 11 is a metal vessel that houses two parallel but separate electric arcs with opposing polarities so as to produce opposite polarizations of the deuterium gas. The flow of the gas through said arcs from opposite directions creates the superposition of the beams in the area located between said arcs with spin couplings as shown in Fig. 11. The trigger seems to be the impulse pressure.

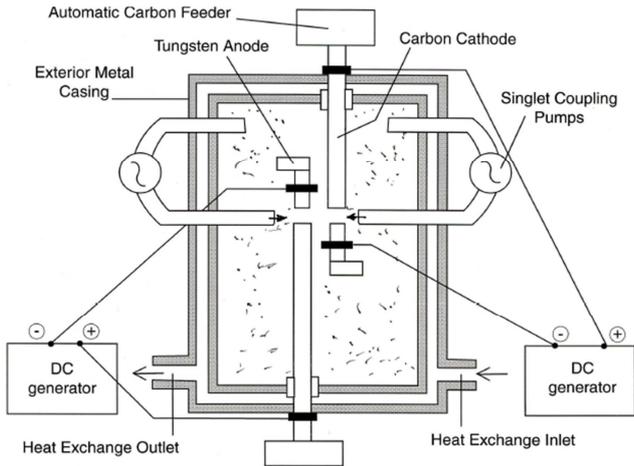
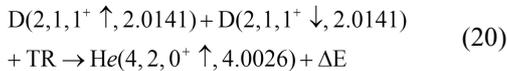


Figure 11. Schematic representation of hadronic reactor for nucleosynthesis of helium.

The nucleosynthetic reaction is-



2.6. Particle Type Hadronic Energy: Stimulated Decay of Energy

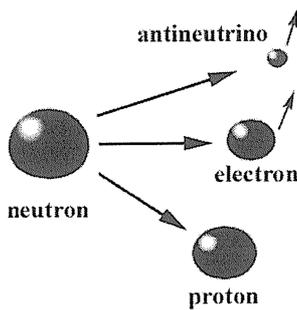


Figure 12. Schematic representation of stimulated neutron decay.

Hadronic nuclear energy can also be obtained by fission reactions or decay of stable nuclei. Theoretically any stable nuclei can be disintegrated into its nuclear constituents by photons having higher energy than the binding energy of the nuclei to be disintegrated. With higher stable nuclei the energy of the photons required to disintegrate also increases. The low binding nuclei like 2_1H and 9_4Be are well-known to undergo photo-disintegration with 2.22 MeV and 2.62 MeV photons respectively [1]. Similarly, stimulated decay of

neutrons as represented in Fig. 12 is also a well-known phenomenon. The prediction and its quantitative treatment can be done by hadronic mechanics.

According to Prof. Santilli, neutron is an unlimited source of energy because it decays releasing highly energetic electron and neutrino that can be easily trapped with a metal shield. It is known that an isolated neutron is highly unstable and has half life of approximately 15 minutes. However, as a constituent of nuclei, it shows high stability which has been attributed to a strong nuclear force of attraction. The neutron shows stimulated decay as



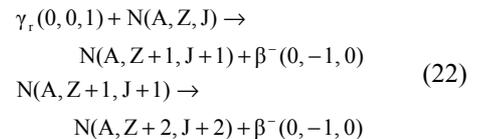
where β^- has spin zero for the conservation law of the angular momentum. β^- can also be considered either as an electron and a neutrino or as an electron and an antietherino with opposing spin $1/2$. However, this difference is irrelevant for the stimulated decay of the neutron.

When a resonating photon hits a nucleus, it excites the isoelectron inside a neutron irrespective of whether the photon penetrates or not inside the neutron. The excited isoelectron leaves the neutron structure, thus causing its stimulated decay. This is due to the fact that hadronic mechanics predicts only one energy level for the proton and the electron in conditions of total mutual immersion (as in case of neutron). Range of hadronic mechanics is given by the radius of neutron that is 1 fm. Thus, the excited isoelectron excites the proton and reassumes its conventional quantum features when moving in vacuum.

Numerous additional triggers are predicted by hadronic mechanics such as photons with a wavelength equal to the neutron size. Here, the whole neutron is excited, rather than the isoelectron in its interior, but the result is always the stimulated decay.

Double Beta Decay

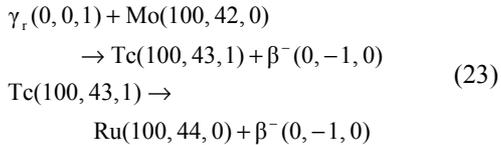
In this typical example of double decay first reaction is stimulated and the second is spontaneous [11].



The original isotope should admit stimulated decay of at least one of its peripheral neutrons via one photon with a resonating frequency verifying all conservation laws of the energy, angular momentum, etc. The new nucleus formed should undergo spontaneous beta decay so that with one resonating photon there is production of two electrons whose kinetic energy is trapped with a metal shield to produce heat. The original isotope is metallic so that, following the emission of two electrons, it acquires an electric charge suitable for the production of a DC current between the metallic isotope and the metallic shield. The energy balance is positive. The initial and final isotopes are light, natural and stable elements so that the new energy is *clean* (since the electrons can be easily

trapped with a thin metal shield), and produce non-radioactive waste.

E.g. double beta decay of the Mo(100, 42, 0)



Mo(100, 42, 0) is naturally stable with mass 99.9074771 amu. Tc(100, 43) has mass 99.9076576 amu and is naturally unstable with spontaneous decay into Ru (100, 44, 0) and half life of 15.8 s. Ru (100, 44) is naturally stable with mass 99.9042197 amu. Although the mass of Mo(100, 42, 0) is smaller than that of Tc(100, 43, 1), yet the conservation of energy can be verified with a resonating frequency of 0.16803 MeV (obtained for $n=1/7$) where n is normalization constant.

But the mass of the original isotope is bigger than that of the final isotope for a value much bigger than that of the resonating photon, with usable hadronic energy (HE) power nuclear reaction

$$\begin{aligned} \text{HE} &= M(100, 42) - M(100, 44) - \\ & \quad E(\gamma) - 2 \times E(e) \\ &= 3.034 - 0.184 - 1.022 \text{ MeV} \\ &= 1.828 \text{ MeV} \end{aligned} \quad (24)$$

where Santilli subtracts the conventional rest energy of the two electrons because it is not usable as a source of energy in this case.

Under the assumptions of using a coherent beam with resonating photons hitting a sufficient mass of Mo(100, 42, 0) suitable to produce 10^{20} stimulated nuclear transmutations per hour, we have the following:

Hadronic production of heat :

$$2 \times 10^{20} \text{ MeV/h} = 3 \times 10^4 \text{ BTU/h}, \quad (25)$$

Hadronic production of electricity :

$$2 \times 10^{20} \text{ e/h} = 200 \text{ C/h} = 55 \text{ mA}. \quad (26)$$

3. Applications of Hadronic Nuclear Energy

3.1. Intermediate Controlled Nuclear Synthesis

1. Green power generation source

ICNS can be industrially exploited for power generation. Since there are no ionizing radiations or particular emissions, it is green and can be used for sustainable development.

2. Synthesis of heavy and super heavy elements

Synthesis of heavy elements particularly of the seventh period is conventionally done by bombarding two heavy nuclei. The reacting rather bombarding nuclei have high energy to overcome the coulombic barrier. This results in high excitation energy of the resulting daughter

nuclei which is often higher than the fission barrier. Thus, fission is one of the pre-dominant exit channel.

However, if ICNS is used then the nuclei of the participating atoms can be exposed in controlled manner as well as can be brought near each other to a considerable extent without initiating coulombic repulsion. This is due to magnetic bond in the participating nuclei which in this case are magnecules rather than mere atoms or ions. The trigger mechanism then pushes the participating nuclei within hadronic radius where fusion is inevitable. Consequently formation of heavy nucleus takes place. Converting heavy nucleus into respective magnecules requires high magnetic field and would be a costly affair. However, the daughter heavy nuclei produced would be considerable stable making the synthesis green and viable.

Stable heavy daughter nuclei formed would allow study of its actual chemical characteristic instead of predicting on the basis of periodic table and spectroscopic studies.

3.2. Particulate Type Nuclear Energy

1. Green power generation source

Stimulated decay of neutron and double beta decay can be used for power generation. The by product is electron which can be stopped with a metal sheet. This results in a clean and green power source.

2. Recycling of nuclear waste

It may also be used for recycling of nuclear waste generated due to existing conventional nuclear energy facility by stimulated neutron decay using photon with resonating frequency (or energy) of 1.294 MeV.

4. Conclusion

ICNS seems to be more promising than hot or cold fusion in terms of reproducibility and energy input to output ratio. The successful achievement of ICNS with industrial relevance depends on the proper selection of the *hadronic fuel*. The hadronic fuel is mainly due to-

- The original and final nuclides are light, natural and stable isotope.
- The nuclear syntheses cause no emission of ionizing radiations.
- The energy produced ΔE is much bigger than the total energy used by the equipment for its production.

Stimulated beta decay and double beta decay also seems to be promising prospect for green power generation. Apart from power generation, ICNS and stimulated decay holds promising prospect for synthesis of heavy elements and recycling of nuclear wastes respectively.

Acknowledgement

The financial support for this work from The R. M. Santilli Foundation is gratefully acknowledged. The author expresses her deep gratitude to Professor R. M. Santilli, Professor R.

Anderson and Professor A. A. Bhalekar for encouragements and valuable guidance in preparing this paper.

References

- [1] D. D. Sood, A. V. R. Reddy and N. Ramamoorthy; Fundamentals of Radiochemistry; Indian Association of Nuclear Chemists and Allied Scientists; 3rd Edition; (2007).
- [2] <http://webapps1.cityofchicago.org/landmarkweb/web/landmarkdetails.htm?lanId=1383> downloaded on 25-05-2015.
- [3] Frontiers Research Highlights 1946-1996. p. 11. published by the Office of Public Affairs, Argonne National Laboratory, Argonne, IL.
- [4] Experimental Breeder Reactor-I, U.S. Department of Energy National Laboratory
- [5] Rich Michal, Fifty years ago in December: Atomic reactor EBR-I produced first electricity, American Nuclear Society, Nuclear News. November 2001.
- [6] H. A. Bethe; Energy Production in Stars; Nobel Lectures, Physics 1963-1970, Elsevier Publishing Company, Amsterdam, (1972).
- [7] J. D. Lawson, Some Criteria for a Power producing thermonuclear reactor, Proceedings of the Physical Society. Section B Volume 70 Number 1, December 1955.
- [8] Fleischmann, M., S. Pons, and M. Hawkins, Electrochemically induced nuclear fusion of deuterium. *J. Electroanal. Chem.*, 1989. 261: p. 301.
- [9] R. M. Santilli; Foundations of Hadronic Chemistry with applications to New Clean Energies and Fuels; Kluwer Academic Publishers, Netherlands, 2001
- [10] A. K. Aringazin and M. G. Kucherenko; Exact variational solution of the restricted three-body Santilli-Shillady model of the hydrogen molecule; *Hadronic Journal*, 23, 1-56, (2000). (physics/0001056).
- [11] I. Gandzha and J. Kadeisvily; New Sciences for a New Era: Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli; San-Marino draft dated June 28, 2011.
- [12] R. M. Santilli; The novel "Controlled Intermediate Nuclear Fusion" and a report on its industrial realization as predicted by Hadronic Mechanics; *Hadronic Journal*, 31, 115-200, (2008).
- [13] M. G. Kucherenko and A. K. Aringazin; Estimate of the polarized magnetic moment of the isoelectronium in the hydrogen molecule; *Hadronic Journal*, 21, 895-902, (1998).
- [14] A. K. Aringazin; Toroidal configuration of the orbit of the electron of the hydrogen atom under strong external magnetic fields; *Hadronic Journal*, 24, 395-434, (2001).
- [15] R. M. Santilli; Experimental confirmation of Nitrogen synthesis from deuterium and carbon without harmful radiations; *New Advances in Physics*, 5, 29-36, (2011).
- [16] R. M. Santilli; Additional Confirmation of the "Intermediate Controlled Nuclear Fusions" without harmful radiation or waste; Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes, Kathmandu University; Nepal, pp. 163-177, (2011).
- [17] J. V. Kadeisvili, C. Lynch and Y. Yang; Confirmations of Santilli's Intermediate Controlled Nuclear Fusion of Deuterium and Carbon into Nitrogen without harmful radiations; *The Open Physical Chemistry Journal*, 5, 17-27, (2013).

Possibilities for the Detection of Santilli Neutroids and Pseudo-protons

Victor de Haan

Bon Physics B.V., Laan Van Heemstede, Puttershoek, The Netherlands

Email address:

victor@bonphysics.nl

To cite this article:

Victor de Haan. Possibilities for the Detection of Santilli Neutroids and Pseudo-protons. *American Journal of Modern Physics*. Special Issue: Issue II: Foundations of Hadronic Mechanics. Vol. 5, No. 2-1, 2016, pp. 131-136. doi: 10.11648/j.ajmp.2016050201.17

Received: July 8, 2015; **Accepted:** July 9, 2015; **Published:** June 1, 2016

Abstract: Following systematic mathematical, theoretical and experimental studies on the synthesis of the neutron from hydrogen, R. M. Santilli noted delayed neutron detections following the termination of tests, and attempted to represent them with the hypothesis of a new state of the hydrogen with spin zero called *neutroid* consisting of a proton and an electron at 1 fm mutual distance in singlet coupling. More recently, Santilli predicted the possible existence of a second new particle called *pseudo-proton* characterized by the synthesis of the electron with the neutron, therefore resulting in a negatively charged unstable particle with a mean life expected in the range of that of the neutron and a mass of the order of the hydride ion. Subsequently, Santilli has indicated that, in the event confirmed, the pseudo-proton could eliminate the Coulomb barrier for nuclear syntheses and trigger nuclear transmutations with large release of heat without neutron emission, thus identifying a possible novel use of hydrogen for the industrial production of a basically new clean nuclear energy. In view of the latter possibility, in this paper specific experiments are proposed for the verification or denial of the existence of Santilli neutroid and pseudo-proton and, in case of confirmation, accurate measurements of their characteristics and production in numbers sufficient for industrial application.

Keywords: Pseudo-proton, Low Energy Nuclear Transmutation, Hydrogen Energy Source, Neutron, Pseudo-protonium, Detection, Spectroscopy

1. Introduction

Recently, Santilli et al. called for the experimental verification of the so-called *pseudo-proton* [1]. The pseudo-proton is a particle predicted by Santilli [2] based on the hadronic mechanics he has derived since the 1970's (for an overview see [3,4]). The pseudo-proton is expected to play an important role in the upcoming field of low energy nuclear transmutations due to its ability to penetrate the atomic core with relative ease due to its negative charge [5]. It is expected that it plays a crucial role in the synthesis of hydrogen ions [6], [7]. Hence the experimental detection of the pseudo proton is important for future development of hydrogen energy.

2. Synthesis of Neutron from Hydrogen

The synthesis of the neutron from the hydrogen inside a star was proposed in 1920 by H. Rutherford [8],

experimentally established in 1932 by J. Chadwick [9] and formalized by E. Fermi [10] via the familiar reaction



The properties of proton, electron and neutron are well known and a summary is given in table 1. The rest mass of the neutron, n is 0.782 MeV larger than the sum of the rest energies of both proton and electron, so that this reaction can only occur in sufficiently strong fields, for instance in the inside of stars or in the arc of a very strong discharge [1]. In 1978, R. M. Santilli [11] pointed out that quantum mechanics is not applicable to the above neutron synthesis because the rest energy of the neutron is 0.782 MeV bigger than the sum of the rest energies of the proton and the electron, as shown by the energy of the rest mass for proton, electron and neutron respectively $E_p = 938.272$ MeV, $E_e = 0.511$ MeV and

$$E_n = 939.565 \text{ MeV},$$

$$E_n - (E_p + E_e) = 0.782 \text{ MeV} > 0 \quad (2)$$

thus requiring a 'positive binding energy' with a resulting 'mass excess' under which the Schrödinger equation no longer admits physically consistent solutions.

In order to explain neutron synthesis, Santilli [11] proposed the construction of a non-unitary covering of quantum mechanics under the name of *Hadronic Mechanics*. However, non-unitary theories on a conventional Hilbert space over a conventional numeric field are known to violate causality. Only following the achievement of sufficient mathematical and theoretical maturity (for more details see [5]), while visiting in 1991 the ICTP in Trieste, Italy, Santilli [12] achieved the first known, non-relativistic, exact representation of *all* characteristics of the neutron in its synthesis from the hydrogen in the core of a star. A relativistic representation was achieved in 1992 when visiting the JINR in Dubna, Russia [13].

Laboratory synthesis of the neutron from a hydrogen gas was achieved in the 1960s by the Italian priest-physicist Don Carlo Borghi and his associates [14] via a metal chamber containing hydrogen at pressure traversed by microwaves and an electric discharge used to keep hydrogen ionized.

Santilli could not account for the synthesis of the neutron via microwaves. However, the same formulations predicted the laboratory synthesis of the neutron under an electric discharge suitable to force the penetration of the electron within the proton following their strong Coulomb attraction (according to Rutherford [8]) and provide 0.782 MeV needed for the synthesis according to equation (2). Extensive tests apparently confirmed the laboratory synthesis of the neutron according to the predictions of Santilli's work [15]. Thereafter, Santilli continued systematic experimental research on the laboratory synthesis of the neutron from a hydrogen gas at the laboratory of the Institute for Basic Research, Palm Harbor, Florida (see the extensive data in Ref. [16]).

Systematic experimentations and industrial realizations are now conducted by the U. S. publicly traded company Thunder Energies Corporation (TEC) [17], which is currently organizing the production and sale of a *Thermal Neutron Source* (TEC domain names and patents pending) essentially consisting of a pressure vessel containing commercially available hydrogen traversed by a suitable discharge and control of hydrogen pressure, electric power and other characteristics to obtain the desired neutron flux output (see corporate paper [18] and movie [19] on a pre-production prototype.)

A recent technical review of the synthesis of the neutron has been provided by U. Abundo [20]; a review recommended to the general physics audience was provided in 2006 by the late J. V. Kadeisvili [21]; while a general review of the various mathematical, theoretical and experimental aspects has been provided in 2011 by I. Gandzha and the late J. V. Kadeisvili [22].

3. Neutroid and Pseudo-proton

Essential the bases for the prediction of the pseudo-proton is the natural assumption that mathematical point particles do not exist in nature. In real life particles will have a finite size and there is a finite probability that particles will overlap. When particles overlap, the intrinsic properties of the particles can change and can recover when they are released again in vacuum. A more elaborate overview is given in [22]. Hence, when a hydrogen atom consisting of a proton, p^+ and an electron, e^- is sufficiently compressed (so that its radius is of the order of 1 fm) they can form a neutron via the above described procedure.

Table 1. Properties for several particles. For the pseudo-proton the range in which the property is expected is indicated. ¹ in units of $1.602177 \cdot 10^{19} \text{ C}$.

Name	Sym	Description
Electron	e^-	Elementary particle
Proton	p^+	Elementary particle
Neutron	n	Elementary particle (standard model) or bound proton and electron (Santilli)
Hydrogen atom	H	Proton + orbit electron
Hydrogen ion	H_2^+	Ionized hydrogen molecule
Pseudo-proton	\tilde{p}^-	Synthesis of neutron and electron (Santilli)
Hydride ion	H^-	Proton + 2 electrons in orbit
Anti-proton	\bar{p}^-	Anti particle of proton

	Charge ¹	Mass 10^{-27} kg	MeV	Magnetic mom. 10^{-26} J/T
e^-	-1	0.00091	0.511	928.477
p^+	1	1.67262	938.272	1.410608
n	0	1.67493	939.565	0.966237
H	0	1.67353	938.783	
H_2^+	-1	3.34615	1877.055	
\tilde{p}^-	-1	1.672...1.677	937...941	≈ 1
H^-	-1	1.67444	939.294	
\bar{p}^-	-1	1.67262	938.272	1.410608

Both tests by Don Borghi [14] and Santilli [15,16] indicated the existence of nuclear transmutations that could solely be explained via the absorption of neutrons or 'neutron-type' particles. Santilli conducted extensive studies of this occurrence based on the detection of neutron counts that were delayed for up to 15 minutes following the termination of the tests.

These delayed neutron counts occurred under certain characteristics of the reactor, such as power, hydrogen pressure and others, insufficient to provide 0.782 MeV needed for the synthesis of the neutron according to equation (2). By contrast, neutrons were detected under sufficient power, hydrogen density and other characteristics, to such an extend to require various evacuations of the laboratory.

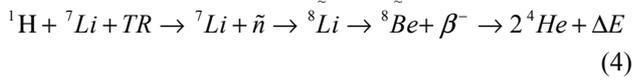
In this way, Santilli proposed the possible existence of a new particle called *neutroid* [16] with essentially the characteristics of the neutron except having spin zero which is created by the reaction



under insufficient conditions to achieve the full reaction according to equation (1). A number of nuclear

transmutations caused by the absorption of neutroids could then allow a quantitative representation of the delayed detection of neutrons (see Ref. [16] for brevity).

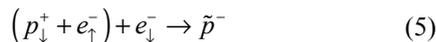
Abundo [20] noted that, in the event confirmed, Santilli neutroids would not experience the notorious Coulomb barrier opposing nuclear syntheses and can, therefore, trigger nuclear transmutations with large release of heat without the emission of neutrons, such as



where $\Delta E > \approx 15.3$ MeV per ${}^7\text{Li}$ nucleus and TR represents Santilli's trigger, namely, a mechanisms triggering the transmutation, such as instantaneous increase of power or pressure. Note that the symbol ${}^1\text{H}$ in the above reaction represents the hydrogen atom. Therefore, to our best knowledge, Abundo's transmutation [20] is the first in scientific records in which *the hydrogen atom, rather than the proton, triggers a nuclear transmutation, thus stimulating potential new technologies centrally based on hydrogen.*

To understand the occurrence, one should note that Santilli neutroid is essentially a new (unstable) form of the hydrogen atom with the proton and electron in singlet coupling kept together by the very strong Coulomb attraction at 1 fm mutual distance. Santilli's trigger is, therefore, a mechanism essentially triggering the transition from the hydrogen to the neutroid that, as such, has a number of possible engineering realizations.

Following the systematic studies on the synthesis of the neutron from hydrogen, Santilli predicted in the Appendix of Ref. [2] the possible existence of a second new particle under the name of *pseudo-proton* with symbol \tilde{p}^- , generated by the synthesis of the electron, this time, with the hydrogen atom, according to the reaction

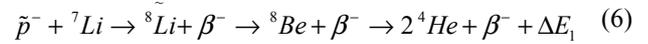


Note that, in comparison with Fermi's reaction (equation (1)), Santilli's reaction does not require the emission of a neutrino. The pseudo-proton will have a spin 1/2, a negative unit charge, essentially the same charge radius and rest energy of the proton, and a mean life (when isolated) of the order of that of the neutron. The properties of the pseudo-proton are summarized in table 1. It is expected that the magnetic moment is of the same order as that of the proton and neutron as the charge radius and rest energy are also of the same order. It will be determined by the precise shape and velocity of the constituting charge distribution. The charge and mass of the pseudo-proton will be similar to that of the anti-proton or hydride ion.

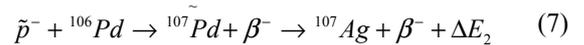
More recently, Santilli [5] indicated that, in the event confirmed, *the pseudo-proton would not only eliminate the Coulomb barrier for nuclear syntheses, but could actually be attracted by nuclei, thus triggering esoenergetic nuclear transmutations without neutron emission and significantly increase the possibilities for the industrial development of a*

basically new and clean nuclear energy [5] (TEC domain names and patents pending).

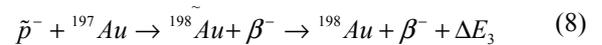
In order to illustrate the significance of the possibility, Santilli indicated the possible triggering by pseudo-protons of nuclear transmutations, such as [5] the transmutation of Li-7



with $\Delta E_1 = 2.887 \cdot 10^{-12}$ J, or Pd-106



or Au-197



where Au-198 is unstable and decays in 2.69 days into betas and Hg-198 with the release of 1.372 MeV, and others, all releasing a rather significant amount of heat, none of them emitting neutrons, while beta emission being easily trapped by a metal shield.

In summary, the above studies indicate possible, basically new and clean nuclear energies centered on the use of hydrogen, and its processing via suitable reactors first into neutroids and/or pseudo-protons and then their use to trigger esoenergetic nuclear transmutations. Due to the evident significance of these studies, both per se, as well as with respect to the emerging new hydrogen era, in this paper experiments are studied aimed at the confirmation or denial of the existence of Santilli neutroids and pseudo-protons and, in case affirmative, reach accurate measurements of their characteristics and their production in number sufficient for industrial applications. It should be noted that in this paper the possible connection between Santilli pseudo-proton and the anti-protons produced at various laboratories is ignored, since such a topic has been preliminarily studied in Refs. [2,5], although the technologies for the production of the two particles can benefit each other, thus warranting additional studies.

From the above it is clear that it is expected that neutrons and pseudo-protons will be produced in a sufficiently strong discharge in hydrogen gas as has been performed by Santilli [1]. That neutrons can be produced in hydrogen gas is well known and experimentally verified by many experiments [23-26] and developed for industrial applications [27]. In general it is assumed that neutrons are produced as a by-product of the fusion of two protons, deuterons, tritons or their combinations resulting in the production of neutrons with high kinetic energies, so-called fast neutrons. The discharge acts as a means to accelerate the hydrogen ions to such high energies that the Coulomb repulsion can be overcome so that the fusion reaction can occur. However, the energy needed to overcome the Coulomb repulsion so that two ions can come as close to each other as several femtometer (the size of the proton, or range of the strong

force) is of the order of 1.4 MeV which is much larger than the kinetic energy available. As a recourse for this problem it is assumed that quantum tunneling exists. In that case the ion nuclei need to overlap and the tunneling probability will be large enough to give a reasonable yield. In such a case also the possibility for the above mentioned reactions exists and neutron or pseudo-proton production can occur.

4. Detection Possibilities

In the above it has been made clear that it is possible that neutrons or pseudo-protons can be created in a hydrogen gas discharge. In the following it is discussed how these particle might be detected.

4.1. Neutrons

Neutrons are neutral and have limited interaction with most materials. One should differentiate between fast neutrons that are created during fusion of fission reactions and thermal neutrons that are created by the Santilli synthesis or by moderation (i.e. slowing down) of fast neutrons. The kinetic energy of fast neutrons is of the order of MeV while the kinetic energy of thermal neutrons is of the order of several tens of meV. When fast neutrons have collisions with cores of the material constituting atoms or molecules, part of their energy will be released to the core and the neutrons energy decreases. After many collisions the fast neutron is slowed down to a speed comparable to the speed of the atoms or molecules of the material. At room temperature this corresponds to an average kinetic energy of 25 meV. Only after the neutrons are slowed down their energy is low enough to be absorbed during the interaction with the nucleus.

Thermal neutrons are detected by a nuclear absorption process inside the detector. The nuclear absorption creates a charged particle pair that is detected by means of an appropriate mechanism. This can be an avalanche detector in case of a gas-filled detector like a ^3He or BF_3 detector. This can also be a photo-multiplier in case of a scintillator detector like a LiF crystal or Li-glass. The key point is that one can discriminate between fast and slow neutrons by means of a time-of-flight technique or by means of an appropriate thermal neutron absorbing material or fast neutron moderator.

The time-of-flight technique is based on the measurement of the arrival time of the neutron in the detector after it has been created. During this time the neutron has to travel the known distance from source to detector. As the travel time and travel distance are measured, the velocity of the neutrons can be determined. Slow neutrons will travel with speeds of the order of 1 km/s while fast neutrons will travel with speed of at least 10000 km/s. To determine the travel time it is needed to know when the neutrons are emitted from the source. One method is by using a pulsed source so that the time interval in which the neutrons are created is known.

When the source is surrounded by a thermal neutron absorber (like for instance Cadmium, Gadolinium, Lithium or Boron) no thermal neutrons can escape the source. Then

any excess neutron detected by the detector after turning on the source is due to the moderation of fast neutrons created in the source. When the source is also surrounded by a fast neutron moderator the number of excess neutrons in the detector will increase as more neutrons will be moderated. With this scheme it is possible to determine a) whether or not neutrons are created and b) whether fast neutrons or slow neutrons are created and c) when slow neutrons are created a indication of their energy distribution can be measured. The cost for a time-of-flight detection including the needed materials for absorption and moderation is estimated between 20 and 50 kUSD.

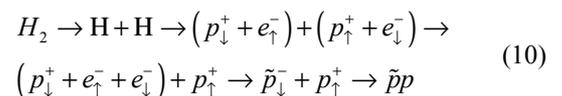
4.2. Pseudo-protons

Pseudo protons are negatively charged particle with a mass comparable with that of the hydrogen atom or proton. It can be discriminated from an electron in a mass spectrometer due to the large mass difference and from a proton because of its reversed charge. In a hydrogen discharge also hydride ions will be produced. The mass and charge of the hydride ion is comparable to that of the pseudo-proton. Hence, it is possible that already in experiments that have been performed in the past the pseudo-proton was detected, but interpreted as hydride ion [25-28]. The mass difference between the hydride ion and the pseudo-proton is expected to be very small of the order of 0.1 % (see table 1). Hence, when using a mass spectrometer to determine what kind of particle is detected, the accuracy should be high enough to measure this small difference. An additional test might be done by measuring the magnetic moment of the detected negatively charged particle by means of an appropriate device as for example a penning trap. Mass spectrometers can be bought from the shelves, but one with an accuracy of 0.1 % for a mass of approximately 1 amu is not readily available and must be developed. In such a case it can be combined with the magnetic moment measurements. Estimated costs for such a development are between 500 kUSD and 1 MUSD.

4.3. Pseudo-protonium Atom

The pseudo-proton can quickly capture a proton to form a *pseudo-protonium* atom, $\tilde{p}p$ where the proton and pseudo-proton circle around each other like in a 2-body system.

Hence, one can imagine that in a strong electrical discharge in hydrogen gas the following reaction might occur



Hence, in the event confirmed, pseudo-protons can form pseudo-protonium atoms during the discharge process. These atoms have energy levels determined by the energy levels of a two body system. The reduced mass of this system is about 1836 times larger than that of the hydrogen atom, comparable to those of protonium [32], resulting in energy levels in the keV range. These levels can be detected by means of X-ray spectrometry. The Lyman [33] (transitions from level $n > 1$

to 1) and Balmer [33] series (transitions from level $n > 2$ to 2) for this atom are shown in table 2.

The intensity of the spectral lines depends on the number density of the pseudo-protonium atoms and hence will depend strongly on the discharge parameters and the pressure of the hydrogen gas. If these series can be found in the spectra of hydrogen gas discharges with sufficient accuracy, then also the reduced mass of the pseudo-protonium atom and hence the mass of the pseudo-proton can be measured. It should be noted that in some Energy-dispersive X-ray spectroscopy experiments peaks are recorded that are close to the estimated value giving clues under what conditions pseudo-protonium might be formed [34]. The estimated costs of such an experiment are between 100 and 200 kUSD.

Table 2. The energy differences and wavelength in nm of the Lyman and Balmer series for pseudo-protonium atom.

Lyman Transition of n	2→1	3→1	4→1	5→1	∞→1
Energy difference in keV	9.367	11.102	11.71	11.99	12.49
Wavelength in nm	0.1318	0.1112	0.1054	0.1030	0.0989

Balmer Transition of n	3→2	4→2	5→2	6→2	7→2	∞→2
Energy difference in keV	1.735	2.342	2.623	2.776	2.868	3.122
Wavelength in nm	0.712	0.527	0.471	0.445	0.431	0.395

5. Conclusions

Following lifelong mathematical, theoretical, and experimental studies on the synthesis of the neutron from hydrogen [19] Santilli proposed the possible existence of two new unstable particles: the neutroid [16], which is essentially a new, spin zero bound state of a proton and an electron in singlet coupling at 1 fm mutual distance, and the pseudo-proton [2,5], which is characterized by the 'compression' of two electrons with opposite spins inside the proton, rather than a single 'compression' according to Rutherford [8].

By remembering that the synthesis of the neutron from the hydrogen exists in nature, and that the probability for the synthesis of the pseudo-proton is smaller yet close to that of the neutron, Santilli neutroids and/or pseudo-protons may generate basically new applications of hydrogen for industrially meaningful, and environmentally acceptable new energies. Since these possibilities are already under industrial development [34], it appears advisable to conduct joint scientific studies.

In this paper experimental means to verify or deny the existence of Santilli neutroids and pseudo-protons are described. In the affirmative case, these means can achieve accurate measurements of the particles characteristics and provide information to produce them in sufficient number to have industrial significance. Since the charge and mass of the pseudo-proton are comparable to those of the hydride ion, it is possible that in previous experiments the pseudo-proton has already been measured but not identified.

For the identification, sufficiently accurate measurements of its mass are needed (accuracy at least 0.1 %) or a

measurement of the magnetic moment by means of a penning trap should be performed. It is also possible that another kind of hydrogen molecule, called a pseudo-protonium atom, is formed when a pseudo-proton and proton are in orbit around each other. When a sufficient density of pseudo-protonium is realized, it is possible to measure its Lyman and Balmer series. When this property can be measured with high accuracy, it can be used to determine the reduced mass of the pseudo-protonium atom and hence the mass of the pseudo-proton.

Acknowledgments

The author would like to thank R.M. Santilli for valuable comments and Thunder Energy Cooperation for financial support for writing this paper.

References

- [1] R. Santilli, A. Nas, *Confirmation of the laboratory synthesis of neutrons from a hydrogen gas*, Journal of Computational Methods in Sciences and Engineering 14 (6) (2014) 405–414. URL <http://www.thunderenergies.com/docs/neutron-synthesis-2014.pdf>
- [2] R. Santilli, *Apparent detection of antimatter galaxies via a refractive telescope with concave lenses*, Clifford Analysis, Clifford Algebras and their Applications 3 (2014) 1–26.
- [3] R. Santilli, *Elements of Hadronic Mechanics*, Volumes I, 1st Edition, Ukraine Academy of Sciences, Kiev, 1995. URL <http://www.santilli-foundation.org/docs/Santilli-300.pdf>
- [4] R. Santilli, *Elements of Hadronic Mechanics*, Volumes II, 1st Edition, Ukraine Academy of Sciences, Kiev, 1995. URL <http://www.santilli-foundation.org/docs/Santilli-301.pdf>
- [5] R. Santilli, *Apparent nuclear transmutations without neutron emission triggered by pseudoprotons*, American Journal of Modern Physics 4 (2015) 15–18. URL <http://www.thunderenergies.com/docs/pseudoproton-2014.pdf>
- [6] M. D. Giulio, E. Filippo, D. Manno, V. Nassisi, *Analysis of nuclear transmutations observed in d- and h-loaded pd films*, International Journal of Hydrogen Energy 27 (5) (2002) 527–531.
- [7] T. Ohmori, T. Mizuno, H. Minagawa, M. Enyo, *Low temperature nuclear transmutation forming iron on/in gold electrode during light water electrolysis*, International Journal of Hydrogen Energy 22 (5) (1997) 459 – 463.
- [8] H. Rutherford, Bakerian lecture: *Nuclear constitution of atoms*, Proceedings of the Royal Society A 97 (1920) 374.
- [9] J. Chadwick, *The existence of a neutron*, Proceedings of the Royal Society 320 A 136 (1932) 692.
- [10] E. Fermi, *Nuclear Physics*, University of Chicago Press, 1949.
- [11] R. Santilli, *Foundation of Theoretical Mechanics*, Volumes I (1978) and II (1982), Springer-Verlag, Heidelberg, Germany, 1978. URL <http://www.santilli-foundation.org/docs/Santilli-209.pdf> and <http://www.santilli-foundation.org/docs/santilli-69.pdf>

- [12] R. Santilli, *Apparent consistency of rutherford's hypothesis on the neutron structure via the hadronic generalization of quantum mechanics - i: Nonrelativistic treatment*, ICTP communication IC/91/47. URL <http://www.santilli-foundation.org/docs/Santilli-150.pdf>
- [13] R. Santilli, *Representation of the synthesis of the neutron inside stars from the hydrogen atom. communication of the joint institute for nuclear research, dubna, russia, number jinr-e4-93-352* (1993), Chinese J. System Engineering and Electronics 6 (1995) 177–199. URL: <http://www.santilli-foundation.org/docs/Santilli-18.pdf>
- [14] C. Borghi, C. Giori, A. Dalloio, *Evidence on the emission of neu-trons from a cold hydrogen plasma*, Communications of CENUFPE Number 8 (1969) and 25 (1971), reprinted via Santilli's proposal in the (Russian) Phys. Atomic Nuclei, 56, 205 (1993).
- [15] R. Santilli, *Confirmation of don borghi's experiment on the synthesis of neutrons from protons and electrons*, ArXiv Physics e-prints arXiv: physics/0608229. URL <http://arxiv.org/pdf/physics/0608229v1.pdf>
- [16] R. Santilli, *Apparent confirmation of don borghi's experiment on the laboratory synthesis of neutrons from protons and electrons*, Hadronic Journal 30 (2007) 709–729. URL <http://www.i-b-r.org/NeutronSynthesis.pdf> <http://www.neutronstructure.org/neutron-synthesis-3.htm> <http://www.neutronstructure.org/neutron-synthesis-2.htm>
- [17] P. Fleming, Thunder energies corporation, website. URL <http://www.thunder-energies.com>
- [18] R. Santilli, A. Nas, *Confirmation of the laboratory synthesis of neutrons from a hydrogen gas*, Thunder Energies Corporation, 1444 Tarpon Springs, FL 34689, U.S.A.
- [19] R. Santilli, A. Nas, *12 minutes film on the laboratory synthesis of neutrons from a hydrogen gas*, WORLD LECTURE SERIES ON HADRONIC MATHEMATICS, MECHANICS AND CHEMISTRY. URL <http://www.world-lecture-series.org/newtron-synthesis-08-14>
- [20] U. Abundo, *Interpretation and enhancement of the excess energy of rossi's reactor via santilli neutroids and nucleoids*, Hadronic Journal 37 (2014) 697 – 737. URL <http://www.thunder-fusion.com/docs/abundo-paper-2014.pdf> (106MB)
- [21] J. Kadeisvili, *The rutherford-santilli neutron*, Hadronic Journal 31 (2008) 1. URL <http://www.i-b-r.org/Rutherford-Santilli-neutron.htm> URL: <http://www.thunder-energies.com/docs/pseudoproton-2014.pdf>
- [22] I. Gandzha, J. Kadeisvili, *New Sciences for a New Era: Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli*, Sankata Printing Press, Nepal, 2011. URL <http://www.santilli-foundation.org/docs/RMS.pdf>
- [23] J. Verbeke, K. Leung, J. Vujic, *Development of a sealed-accelerator-tube neutron generator*, Applied Radiation and Isotopes 53 (45) (2000) 801 – 809.
- [24] M. Saltmarsh, *Warm fusion*, Nature 434 (45) (2005) 1077–1078.
- [25] B. Naranjo, J. Gimzewski, S. Putterman, *Observation of nuclear fusion driven by a pyroelectric crystal*, Nature 434 (45) (2005) 1115–1117.
- [26] J. Reijonen, F. Gicquel, S. Hahto, M. King, T. Lou, K. Leung, *Dd neutron generator development at LBNL*, Applied Radiation and Isotopes 63 (56) (2005) 757 – 763, 8th International Conference on Applications of Nuclear Techniques.
- [27] J. Sved, *Applications for gas-plasma target neutron generators*, Nuclear Research Applications and Utilization of Accelerators, Vienna, Austria, 48 May 2009.
- [28] K. Ehlers, K. Leung, *Multicusp negative ion source*, Review of Scientific Instruments 51 (6) (1980) 721–727.
- [29] M. Bacal, A. Hatayama, I. Matsumiya, M. Hamabe, T. Kuroda, Y. Oka, *Extraction physics in volume h-ion sources*, Review of Scientific Instruments 77 (3) (2006) 03A502.
- [30] M. Bacal, *Negative hydrogen ion production in fusion dedicated ion sources*, Chemical Physics 398 (0) (2012) 3 – 6, chemical Physics of Low-Temperature Plasmas (in honour of Prof Mario Capitelli).
- [31] P. Franzen, D. Wnderlich, U. Fantz, the NNBI Team, *On the electron extraction in a large rf-driven negative hydrogen ion source for the iter nbi system*, Plasma Physics and Controlled Fusion 56 (2) (2014) 025007.
- [32] C. Batty, *Antiprotonic-hydrogen atoms*, Reports on Progress in Physics 52 (1989) 1165 – 1216.
- [33] F. Jenkins, H. White, *Fundamentals of Optics*, 11th Edition, McGraw-Hill, 1982.
- [34] E. Storms, *The explanation of low energy nuclear reaction, an explanation of the relationship between observation and explanation*, 1st Edition, Infinite Energy Press, Concord, NH, USA, 2014.

Study of Bose-Einstein Correlation Within the Framework of Hadronic Mechanics

Chandrakant S. Burande

Vilasrao Deshmukh College Engineering and Technology, Mouda, Nagpur, India

Email address:

csburande@gmail.com

To cite this article:

Chandrakant S. Burande. Study of Bose-Einstein Correlation Within the Framework of Hadronic Mechanics. *American Journal of Modern Physics*. Special Issue: Issue II: Foundations of Hadronic Mechanics. Vol. 5, No. 2-1, 2016, pp. 137-142. doi: 10.11648/j.ajmp.2016050201.18

Received: July 8, 2015; **Accepted:** July 9, 2015; **Published:** June 1, 2016

Abstract: The Bose-Einstein correlation is the phenomenon in which protons and antiprotons collide at extremely high energies; coalesce one into the other resulting into the fireball of finite dimension. They annihilate each other and produces large number of mesons that remain correlated at distances very large compared to the size of the fireball. It was believed that special relativity and relativistic quantum mechanics are the valid frameworks to represent this phenomenon. Although, R.M. Santilli showed that the Bose-Einstein correlation requires four arbitrary parameters (chaoticity parameters) to fit the experimental data which parameters are prohibited by the basic axioms of relativistic quantum mechanics, such as that for the vacuum expectation values. Moreover, Santilli showed that correlated mesons can not be treated as a finite set of isolated point-like particles as required for the exact validity of the Lorentz and Poincare's symmetries, because the event is non-local due to overlapping of wavepackets and consequential non-Hamiltonian effects. Therefore, the Bose-Einstein correlation is incompatible with the axiom of expectation values of quantum mechanics. In this paper, we study Santilli's exact and invariant representation of the Bose-Einstein correlation via relativistic hadronic mechanics including the exact representation of experimental data from the first axiomatic principles without adulterations, and consequential exact validity of the Lorentz-Santilli and Poincare-Santilli isosymmetries under non-local and non-Hamiltonian internal effect. We finally study the confirmation of Santilli's representation of the Bose-Einstein correlation by F. Cardone and R. Mignani.

PACS: 13.38Dg, 03.30Tp, 10.30Cp, 02.20.Sv

Keywords: Bose-Einstein Correlation, Special Relativity, Lorentz-Santilli Isosymmetry

1. Introduction

The main ingredient of hadronic mechanics [1], [2] is that strong interactions have a nonlocal component of contact, due to deep wave-overlappings at mutual distances of 1 Fermi. This nonlocal component can not be represented by the conventional quantum mechanics. However, novel hadronic mechanics encompass entire local and nonlocal effects with remarkable experimental evidences. Thus, the most fundamental experimental verifications of hadronic mechanics are, those which manifested the expected nonlocality of the strong interactions. Among them, the most important tests are those on the Bose-Einstein correlation [3], [4], [5], [6], [7], in which protons and antiprotons are made to collide at very big or very small energies and annihilate each other in a region called the fireball. The annihilation produces

various unstable hadrons whose final states are given by correlated mesons which are "in phase" with each other despite large mutual distances compared to the size of the fireball. Correlated mesons can not be treated as a finite set of isolated point-like particles. It is non-local event due overlapping of wavepackets. There are several nonlocal theories which attempted to reduce nonlocal event into a finite set of isolated points distributed over the finite volume of the fireball. However, these theories are discarded by Santilli for the fact that the Bose-Einstein correlation is incompatible with the axiom of expectation values of quantum mechanics. It is purely manipulated nonlocal interaction to verify the quantum laws.

The first exact and invariant formulation of the

Bose-Einstein correlation via relativistic hadronic mechanics was done by R. M. Santilli [8] in 1962. F. Cardone and R. Mignani [9], [10] was the first to verify Santilli's theoretical isorelativistic calculation with experimental data (Figure 1) and they published their result in 1996.

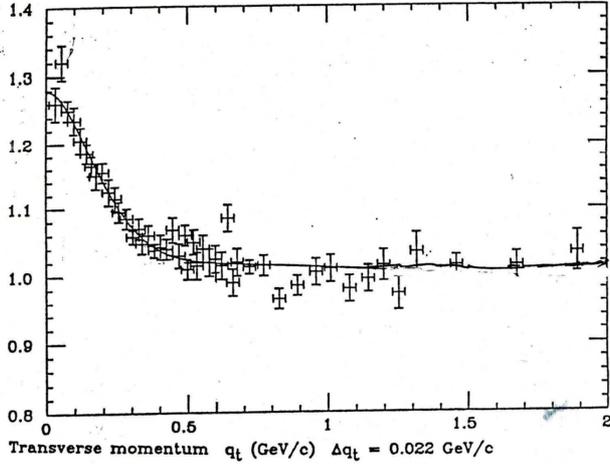


Figure 1. The exact fit of Santilli's two-point Bose-Einstein isocorrelation function at high energy.

2. Conventional Treatment of the Bose-Einstein Correlation

Consider a quantum system in 2-dimensions represented on a Hilbert space H with initial and final states $|a_k\rangle, |b_k\rangle, k=1,2$. The vacuum expectation values of an operator A are given by [3]

$$\langle A \rangle = \langle a_k | \times A \times | b_k \rangle = \sum_{k=1,2} a_k \times A_{kk} \times b_k \quad (1)$$

which is necessarily diagonal, to fulfill the condition that operator corresponds to observable quantity must be Hermitian. The two-points correlation function of the Bose-Einstein correlation is defined by

$$C_2 = \frac{P(p_1, p_2)}{P(p_1) \times P(p_2)} \quad (2)$$

where $P(p_1, p_2)$ is the two particles probability density subjected to Bose-Einstein symmetrization, and $P(p_k), k=1,2$ is the corresponding quantity for the k^{th} particle with 4-momentum, p_k . The two-particles density is computed via the vacuum expectation value

$$P(p_1, p_2) = \int \psi_{12}^\dagger(x_1, x_2; r_1, r_2) \psi_{12}(x_1, x_2; r_1, r_2) \times F(r_1) F(r_2) \times d_1^4 d_2^4 \quad (3)$$

where ψ_{12} is the probability amplitude to produce two bosons at r_1 and r_2 that are detected at x_1 and x_2 , given by

$$\psi_{12} = \frac{1}{\sqrt{2}} \left[e^{ip_1(x_1-r_1)+ip_2(x_2-r_2)} \right] + \frac{1}{\sqrt{2}} \left[e^{ip_1(x_1-r_2)+ip_2(x_2-r_1)} \right] \quad (4)$$

With the use of above equations, we obtain the final expression for the two-point correlation function

$$C_2 = 1 + e^{-Q_{12}^2 R^2}, \quad (5)$$

where $Q_{12} = p_1 - p_2$ is the momentum transfer, where R is the Gaussian width and r is generally assumed to be the radius of the fireball.

3. Incompatibility of the Bose-Einstein Correlation with Relativistic Quantum Mechanics

The Bose-Einstein correlation given by eq.(5) derived from conventional quantum mechanics, deviates from experimental results. This tempt to the introduction of a first, completely unknown parameter λ , called "chaoticity parameter", namely;

$$C_2 = 1 + \lambda e^{-Q_{12}^2 R^2}. \quad (6)$$

Note that the introduction of chaoticity parameter is quite arbitrary and it is impossible to derive the above parameter from any axiom of relativistic quantum mechanics. Hence, the chaoticity parameter λ introduced in eq.(6) is the first direct evidence of the incompatibility of the Bose-Einstein correlation with quantum axioms. Although, the modified eq.(6) too deviates dramatically from experimental data. In order to fit the desired experimental data eq.(6) was further modified by introducing an increasing number of completely unknown and arbitrary parameter, namely,

$$C_2 = 1 + \lambda_1 e^{-Q_{12}^2 R^2} + \lambda_2 e^{-Q_{12}^2 R^2} + \lambda_3 e^{-Q_{12}^2 R^2} + \lambda_4 e^{-Q_{12}^2 R^2} \quad (7)$$

which is strongly objected by Santilli because the only scientific route of achieving the addition terms in Bose-Einstein correlation function is by a formulation of nondiagonal elements of the expectation values. However, latter are prohibited by relativistic quantum mechanics for observable quantity (Hermitian operator). Thus, in the context of study of Bose-Einstein correlation, relativistic quantum mechanics has the following insufficiencies:

1. The theory can only represent the proton and the antiprotons as dimensionless points. Hence, the particle correlation and then the existence of the fireball is impossible with assumption of dimensionless points;

2. The point-like abstraction of particles has a number of technical consequences, such as the independent terms of the densities in eq.(3). That directly prohibit the boson correlation;

and

3. Relativistic quantum mechanics must insist the fireball to be necessarily spherical, so as to prevent the loss of the rotational symmetry to protect from the violation of spacetime symmetries.

$$\begin{aligned} b_\mu &= b_\mu(t, x, p, E, \dots) > 0, \\ n_\mu &= n_\mu(t, x, p, E, \dots) > 0, \end{aligned} \tag{13}$$

$$\begin{aligned} \hat{T} &= \hat{T}(t, x, p, E, \dots), \\ \hat{I} &= \hat{I}(t, x, p, E, \dots) = \hat{T}^{-1}. \end{aligned} \tag{14}$$

4. Representation of the Bose-Einstein Correlation with Relativistic Hadronic Mechanics

In regard of derivation of Bose-Einstein correlation function and its experimental validity, Santilli observed that:

1. The Bose-Einstein correlation is incompatible with the axioms of relativistic quantum mechanics because of the impossibility to admit off-diagonal terms in the two-point correlation function from unadulterated first principles, and other reasons; and

2. The Bose-Einstein correlation is directly compatible with the axioms of the covering relativistic hadronic mechanics because of the admission of nonlocal non-Hamiltonian interactions and the appearance of off-diagonal terms from first principles.

The axiom of isoexpectation value for relativistic hadronic mechanics [8] is given by

$$\begin{aligned} \langle \hat{A} | \hat{a}_k | \hat{T} \times \hat{A} \times \hat{T} \times | \hat{b}_k \rangle \\ = \sum_{ijk} \hat{a}_i \times \hat{T}_i^j \times \hat{A}_{ji} \times \hat{T}_j^k \times \hat{b}_k \end{aligned} \tag{8}$$

where \hat{T} is the isotopic element, and the "hat" denotes quantities defined on isospaces over isofields. The main new feature is that the operator \hat{A} must be Hermitian, thus diagonal, to be observable, but the isotopic element does not need to be diagonal. Santilli isorelativity with Minkowski-Santilli isospace $\hat{M}(x, \hat{\eta}, \hat{R})$, isoinvariant, isometric, isotopic element and isounit given respectively by [8]

$$\begin{aligned} \hat{x}^2 &= (\hat{x}^\mu \times \hat{\eta}_{\mu\nu} \times \hat{x}^\nu) \times \hat{I} \\ &= [x^\mu \times (\hat{T}_\mu^\nu \times \eta_{\nu\rho}) \times x^\rho] \times \hat{I} \in \hat{R}, \end{aligned} \tag{9}$$

$$\begin{aligned} \hat{\eta} &= \text{Diag}(b_1^2, b_2^2, b_3^2, -b_4^2) \times \Gamma \\ &= \text{Diag}(1/n_1^2, 1/n_2^2, 1/n_3^2, -1/n_4^2) \times \Gamma, \end{aligned} \tag{10}$$

$$\begin{aligned} \hat{T} &= \text{Diag}(b_1^2, b_2^2, b_3^2, b_4^2) \times \Gamma \\ &= \text{Diag}(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2) \times \Gamma, \end{aligned} \tag{11}$$

$$\begin{aligned} \hat{I} &= \text{Diag}(1/b_1^2, 1/b_2^2, 1/b_3^2, 1/b_4^2) \times \Gamma^{-1} \\ &= \text{Diag}(n_1^2, n_2^2, n_3^2, n_4^2) \times \Gamma^{-1}, \end{aligned} \tag{12}$$

It observed that unlike chaoticity parameter, the characteristic quantities must represent physically measurable quantities, namely, $1/b_k^2 = n_k^2$, $k = 1, 2, 3$ must characterize the semiaxes of the Bose-Einstein fireball according to a proper normalization and $1/b_4^2 = n_4^2$ must characterize the density of the fireball in a way compatible with other experiments.

Note that the value of the type $b_1^2 = b_2^2 = b_3^2$ emerging from isorepresentation would be inconsistent because the Bose-Einstein fireball cannot possibly be a sphere due to the extreme energies of the collision. The fireball must be a very elongated spheroidal ellipsoid, for instance, of the type $b_3^2 \gg b_1^2 = b_2^2$. Moreover, the numerical value of the density $b_4^2 = 1/n_4^2$ must be compatible with numerical values from different experiments on comparable densities, such as we observed in case of protons and neutrons.

The correlation function on an iso-Hilbert space \hat{H} with initial and final isostates $|\hat{a}_k\rangle, |\hat{b}_k\rangle$; $k = 1, 2$ and the non-diagonal isotopic element in the explicit form is given by

$$\begin{aligned} \hat{T} &= \text{Diag}(b_1^2, b_2^2, b_3^2, b_4^2) \times \Gamma \\ &= \text{Diag}(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2) \times \Gamma, \end{aligned} \tag{15}$$

$$\Gamma = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \tag{16}$$

where

$$B = B' \left| 1 - \exp\left(\int \psi_{b_2}^2 \psi_{a_1} dx\right) \right|$$

$$C = C' \left| 1 - \exp\left(\int \psi_{a_2}^2 \psi_{b_1} dx\right) \right|$$

and values of constants, A, B, C and D are determined by the condition of normalization, that is

$$\text{Det}(\Gamma) = 1. \tag{17}$$

Notice that isoexpectation value of eq.(11), isotopic element

1. Allows indeed off-diagonal terms in the isoexpectation values;
2. Represents the overlapping of the wavepackets of particles via the integrals in the exponents of Γ ; and
3. Eliminates all correlations when said overlapping is null.

Next, the isorepresentation is given by a trivial isotopy of the conventional treatment, with the use now of the nontrivial

isoexpectation values of eq.(8). We then have the two-points isocorrelation function

$$\hat{C}_2 = \frac{\hat{P}(p_1, p_2)}{\hat{P}(p_1) \times \hat{P}(p_2)}, \quad (18)$$

where $\hat{P}(p_1, p_2)$ is the two-particle isoprobability density subjected to proper symmetrization, $\hat{P}(p_1)$ and $\hat{P}(p_2)$ are the corresponding quantity for the k particle with 4-momentum. The two-particles isoprobability density is now given by the isoeigenvalue expression

$$\hat{P}(p_k) = \int \hat{\Psi}_{12}^\dagger(x_k; r_k) \hat{\Psi}_{12}(x_k; r_k) \times F(r_k) \times d_r^4 d_r^4, \quad k=1,2 \quad (19)$$

Note that the crucial difference between eqs.(3) and (19) given by the isotopic lifting of all quantities and their operations and the appearance in the former of the isotopic element allowing the mixing of nondiagonal terms. Another major difference between conventional and isotopic treatments is that the probability densities for particles 1 and 2 are factorized in the conventional treatment, eq.(3), while they cannot be factorized in the isotopic treatment. This is due to the fact that protons, antiprotons, and all produced mesons are pointlike for relativistic quantum mechanics, while they are extended in case Santilli's treatment. Hence, the separation of the densities would be equivalent to ceasing all correlations. The isotopy of of the conventional treatment referred to isoexpectation values of eq.(11), including the symmetrization of the isotopic element and isowavefunctions for all possible directions, plus the assumed normalizations then leads to isodensity, that is

$$\hat{F}(r_1, r_2) = \sum_\mu \hat{\eta}_{\mu\mu} \frac{b_{\mu\mu}}{4\pi^2} e^{-\frac{r^2 b_\mu^2}{2}} \quad (20)$$

where r is the radius of the sphere in which the correlated mesons are detected.

The continuation of calculations via a simple isotopy of the conventional treatment, the final expression of the two-points isocorrelation function, derived for the first time by Santilli is given by [8]

$$\begin{aligned} \hat{C}_2 &= 1 + \frac{1}{3} \sum_\mu b_\mu^2 \times e^{-q_r^2 K^2 / b_\mu^2} \\ &= 1 + \frac{1}{3} b_1^2 \times e^{-q_r^2 K^2 / b_1^2} + \frac{1}{3} b_2^2 \times e^{-q_r^2 K^2 / b_2^2} \\ &\quad + \frac{1}{3} b_3^2 \times e^{-q_r^2 K^2 / b_3^2} - \frac{1}{3} b_4^2 \times e^{-q_r^2 K^2 / b_4^2}, \\ K^2 &= b_1^2 + b_2^2 + b_3^2 = 3. \end{aligned} \quad (21)$$

In the above isorepresentations, all operations are now conventional. Hence, the above expressions are the

projections in our spacetime of the isocorrelation functions on isospace.

5. Exact Poincare Symmetry under Nonlocal and Non-Hamiltonian Interaction

As indicated earlier, a crucial insufficiency of the conventional treatment of the Bose-Einstein correlation, is the inability to provide an invariant representation of the fireball, due to its prolate character under which the conventional rotational symmetry no longer applies. The Bose-Einstein correlation creates a fireball characterized by a spheroid prolated in the direction of the proton-antiproton flight. Following its creation, the fireball expands rapidly, resulting in the correlated mesons. Consequently, the original characteristic quantities, here denoted $b_k^2 = 1/n_k^2$, have an explicit dependence on time. By assuming that the prolateness is along the third axis, we have

$$\begin{aligned} K^2(t) &= b_1^2(t) + b_2^2(t) + b_3^2(t) \\ &\neq \text{const}, \quad b_3^2(t) \gg b_1^2(t) = b_2^2(t). \end{aligned} \quad (23)$$

However, the fireball must preserve its shape during its expansion when considered as isolated from the rest of the universe. This implies that all characteristic quantities have the same factorizable time dependence. In conclusion, the fireball can be studied at the time of its formation with constant characteristic quantities $b_k^2 = 1/n_k^2$ and the following isoinvariant formulated on the Euclidean-Santilli isospace [11], [12] with isounit

$$\begin{aligned} \hat{R}^2 &= (x_1^2 b_1^2 + x_2^2 b_2^2 + x_3^2 b_3^2) \\ &\times \hat{I} = \left(\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} \right) \times \hat{I}, \end{aligned} \quad (24)$$

$$\begin{aligned} \hat{I} &= \text{Diag}(1/b_1^2, 1/b_2^2, 1/b_3^2) \\ &= \text{Diag}(n_1^2, n_2^2, n_3^2). \end{aligned} \quad (25)$$

The reconstruction of the exact Lorentz symmetry $\hat{O}(3)$ [11], [12] for the Bose-Einstein correlation follows the same lines. Since the speed of light is assumed to be locally varying, we have mutated light cones of the type,

$$\hat{n}^2 = (x_3^2 \times b_3^2 - x_4^2 \times b_4^2) \times \hat{I} = \frac{x_3^2}{n_3^2} - \frac{x_4^2}{n_4^2}, \quad (26)$$

$$\hat{I} = \text{Diag}(1/b_3^2, 1/b_4^2) = \text{Diag}(n_3^2, n_4^2). \quad (27)$$

It is again easy to see that the mutated light cone in our spacetime is the perfect light cone in isospace, called light isocone, because, again, the mutation of each axis is complemented by the inverse mutation of the corresponding

unit. Recall that isorelativity and special relativity coincide at the abstract, realization free level, as confirmed by the speed of light in vacuum to be the constant maximal causal speed in isospace. Consequently, the understanding of the isorepresentation of the Bose-Einstein correlation requires the knowledge that, rather than "violating" special relativity as at times perceived, in reality allows the maximal possible enlargement of the arena of applicability of Einsteinian axioms.

6. Theoretical Prediction

It is important now to identify the theoretical prediction of isorepresentation so that we can compare them below with experimental data.

Prediction 1: The minimum value of the two-points isocorrelation function, first identified by Santilli, evidently holding for infinite momentum transfer.

$$\hat{C}_2^{Min} = 1 \tag{28}$$

Prediction 2: The maximal value is predicted to be evidently holding for null momentum transfer. The isorepresentation to be valid if all data must remain between the above minimum and maximum values.

$$\hat{C}_2^{Max} = 1 + \frac{1}{3} + \frac{1}{3} = 1.67 \tag{29}$$

Prediction 3: Isorepresentation also predicts the maximum value of the isodensity, occurring for \hat{C}_2^{Max} . In fact, for $q_i = 0$ we have no correlations, in which case we have

$$b_k^2 = 1, \quad k = 1, 2, 3, \tag{30}$$

$$K^2 = b_1^2 + b_2^2 + b_3^2 = 3,$$

$$\hat{C}_2^{Max} = 1 + \frac{K^4}{3} - \frac{K^2 b_4^2}{3} = 1.67, \tag{31}$$

$$b_4^2 = 2.33, \quad n_4^2 = 0.429, \tag{32}$$

$$b_4 = 1.526, \quad n_4 = 0.654.$$

Prediction 4: By assuming that $K^2 = 3$ and that the fireball is very prolate, with $b_3^2 = 30b_1^2 = 30b_2^2$, we obtain the following prediction on the remaining characteristic quantities. From the isoaxioms, Santilli also have the following additional predictions:

$$b_1^2 = b_2^2 = 0.043, \quad b_3^2 = 2.816, \tag{33}$$

$$b_1^2 = n_1^2 = n_2^2 = 10.666, \quad n_3^2 = 0.355. \tag{34}$$

Prediction 5: The maximal causal speed within the fireball is bigger than that in vacuum,

$$V_{max} = c_0 \left(\frac{b_4}{b_3} \right) > c_0. \tag{35}$$

Prediction 6: Time t within the fireball flows faster than time predicted by special relativity,

$$t = \hat{\gamma} \times t_0 > \gamma \times t_0. \tag{36}$$

Prediction 7: Lengths l inside the fireball are smaller than lengths predicted by special relativity,

$$l = \hat{\gamma}^{-1} \times l_0 < \gamma^{-1} \times l_0. \tag{37}$$

Prediction 8: Mass behavior with speed is bigger than that predicted by special relativity,

$$m = \hat{\gamma} \times m_0 > \gamma \times m_0. \tag{38}$$

Prediction 9: The energy equivalence of the fireball is bigger than that predicted by special relativity or, equivalently, for a given energy, the mass is smaller,

$$E = m \times V_{max} > E_0 = m \times c_0^2. \tag{39}$$

Prediction 10 : Frequencies of light emitted inside the fireball, exist the same isoblueshifted, namely, with an increase of frequency as compared to the corresponding behavior predicted by special relativity,

$$\omega = \hat{\gamma} \times \omega_0. \tag{40}$$

Prediction 11: The speed of light within the fireball is bigger than that in vacuum, $c = c_0 > b_4 > c_4$, by smaller than the maximal causal speed,

$$c = c_0 \times b_4 < V_{max} = c_0 \left(\frac{b_4}{b_3} \right). \tag{41}$$

The isoblueshift of light is nothing mysterious because it is a mere manifestation of the high energy density of the medium in which light propagates. Isoblueshift, as the increase of frequencies as predicted by special relativity in vacuum, is then a mere consequence of the medium transfer energy to light. A similar situation occurs for all other predictions.

7. Experimental Verification

F. Cardone and R. Mignani [9], [10] in 1992 had contested the eq.(15) for actual experimental data. The Bose-Einstein two-point correlation function derived by Santilli is perfectly matched with experimental results at high energy. The numerical values of the characteristic functions for the fireball of the Bose-Einstein correlation resulting from this exercise are

$$b_1 = 0.267 \pm 0.054, \quad b_2 = 0.437 \pm 0.035,$$

$$b_3 = 1.661 \pm 0.013, \quad b_4 = 1.653 \pm 0.015. \quad (42)$$

A most important feature of the above data is that they characterize the medium inside the fireball as being iso-Minkowskian of Group III, Type 9, thus confirming that all hadrons heavier than kaons have the same iso-Minkowskian features. The fit of FIGURE 1 and the above values provide the following experimental verifications:

1. The experimental data do indeed lie between the theoretically minimum and maximal value;
2. The experimental data confirm all eleven theoretical predictions;
3. The experimental proof confirms the reconstruction of the exact character of the Poincare symmetry for the Bose-Einstein correlation.

F. Cardone and R. Mignani investigation provides remarkable experimental verification of Santilli isorelativity and relativistic hadronic mechanics. This experimental verification on Bose-Einstein Correlation reveals the nonlocality of strong interactions of correlated mesons.

8. Concluding Remarks

Santilli's thorough investigation found that special relativity, the Lorentz and Poincare's symmetries, and relativistic quantum mechanics are not exactly valid to represent the Bose-Einstein correlation because their predictions from a large deviation from experimental data. Moreover, the resulting representations fail to tender non-explanation for the introduction of chaoticity parameters which are needed to fit the experimental data. Furthermore, it is observed that there are flaws in treating correlated mesons as a finite set of isolated point-like particles, since the Bose-Einstein correlation is purely non-local event with deep overlapping of wavepackets that cannot be treated by conventional quantum mechanics.

We have shown in this paper that Santilli's representation of the Bose-Einstein correlation via relativistic hadronic mechanics has indeed achieved a numerically exact and time invariant representation of experimental data from unadulterated first axioms, with consequential exact validity of the Lorentz-Santilli and Poincare-Santilli isosymmetry that are known to be locally isomorphic to the corresponding conventional symmetry. Hence, relativistic hadronic mechanics restores indeed the validity of the conventional spacetime symmetries, although at the covering isotopic level.

We conclude the paper with a study of the confirmation of all results by Santilli independently provided by F. Cardone and R. Mignani, including the all important exact and invariant representation experimental data.

Acknowledgments

The author is highly grateful to Professor R. M. Santilli, his team and Professor Anderson for valuable guidance in preparing and for suggestions to improve this paper. The author is also highly indebted to Professor A. A. Bhalekar for suggesting me this topic and helping me in writing this paper by his valuable advices and suggestions in its overall improvement.

References

- [1] R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry*, Vol. I-V, International Academic Press, Palm Harbor, U.S.A., 2008.
- [2] I. Gandzha and J. Kadeisvily, *New Sciences for a New Era: Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli*, The Institute for Basic Research, Palm Harbor, Florida, U.S.A., 2011.
- [3] B. Lorstad, *Int. J. Mod. Phys. A* 4, 2861 (1989).
- [4] D. H. Boal et al. *Rev. Mod. Phys.* 62, 553 (1990).
- [5] R. Adler et al., *Z. Phys. C* 63, 541 (1994).
- [6] M. Gaspero et al., *Phys. Lett. B* 58, 861 (1995).
- [7] G. Goldhaber, S. Goldhaber, W. Lee, and A. Pais, *Phys. Rev.* 120, 300, (1960).
- [8] R. M. Santilli, *Hadronic J.* 15, 1 and 81 (1992).
- [9] F. Cardone and R. Mignani, *JETP*, 83, 435 (1996).
- [10] F. Cardone, M. Gasperini and R. Mignani *Euop. Phys. J. C* 4, 705 (1998).
- [11] R. M. Santilli, *Lie-admissible Approach to the Hadronic Structure, Volume I: Non-applicability of the Galilei and Einstein Relativities* in the series Monographs in Theoretical Physics, Hadronic Press, Palm Harbor, Florida, 1978.
- [12] R. M. Santilli, *Lie-admissible Approach to the Hadronic Structure, Volume II: Coverings of the Galilei and Einstein Relativities* in the series Monographs in Theoretical Physics, Hadronic Press, Palm Harbor, Florida, 1981.

Compatibility of Arbitrary Speeds with Special Relativity Axioms for Interior Dynamical Problems

Ruggero Maria Santilli

Thunder Energies Corporation, Tarpon Springs, Florida, U.S.A.

Email address:

research@i-b-r.org

To cite this article:

Ruggero Maria Santilli. Compatibility of Arbitrary Speeds with Special Relativity for Interior Dynamical Problems. *American Journal of Modern Physics*. Special Issue: Issue II: Foundations of Hadronic Mechanics. Vol. 5, No. 2-1, 2016, pp. 143-160.

doi: 10.11648/j.ajmp.2016050201.19

Received: September 2, 2015; **Accepted:** September 2, 2015; **Published:** June 1, 2016

Abstract: In this paper, we outline the rapidly growing literature on arbitrary speeds within physical media and show that, contrary to a widespread belief for one century, arbitrary speeds for interior dynamical problems are compatible with the abstract axioms of special relativity, provided that they are realized with the covering isomathematics specifically developed for the conditions considered. We finally point out a number of intriguing implications in cosmology, particle physics, nuclear physics, chemistry, gravitation, and mathematical models of interstellar travel.

Keywords: Special Relativity, Superluminal Speeds, Isorelativity

1. Maximal Speeds for Exterior Problems in Vacuum

As it is well known, the advent of the Lorentz transformations [1]

$$x'^1 = x^1, x'^2 = x^2, \tag{1a}$$

$$x'^3 = \gamma(x^3 - \beta x^4), x'^4 = \gamma(x^4 - \beta x^3), \tag{1b}$$

$$\beta = \frac{v}{c}, \gamma = \frac{1}{\sqrt{1-\beta^2}}. \tag{1c}$$

and their extension by Poincaré [2] (hereon referred to as the *Lorentz-Poincaré (LP) symmetry*) leave invariant the line element in Minkowski space-time $M(x, \eta, I)$

$$x^2 = x_1^2 + x_2^2 + x_3^2 - t^2 c^2 = x^\mu \eta_{\mu\nu} x^\nu, \tag{2a}$$

$$x = (x^\mu) = (x_1^\mu - x_2^\mu), x^4 = t, \mu, \nu = 1, 2, 3, 4, \tag{2b}$$

$$\eta = \text{Diag.}(1, 1, 1, -c^2), I = \text{Diag.}(1, 1, 1, 1), \tag{2c}$$

and are at the foundation of axioms of Special Relativity (SR) [3].

As it is also well known, symmetry (1) identifies the *maximal causal speed* for the conditions clearly expressed by Lorentz, Poincaré and Einstein [1-3] and experimentally

confirmed, namely, for *exterior dynamical problems*, consisting of *point particles and electromagnetic waves propagating in vacuum* (conceived as empty space) when represented in an inertial reference frame.

In fact, the light cone, e.g., for infinitesimal displacements in (3, 4)-dimensions

$$(\delta x^3)^2 - (\delta t)^2 c^2 = 0, \tag{3}$$

establishes the maximal causal speed in vacuum

$$\frac{\delta x^3}{\delta t} = V_{max}^{vacuum} = c. \tag{4}$$

For decades, *faster than light speeds* (also called "superluminal speeds") were essentially ignored because they would violate causality and other physical laws. Nevertheless, with the passing of time the study of superluminal speeds became inevitable.

Nowadays, a search on superluminal speeds at the various archives in the internet shows the existence of a large number of papers published in refereed journals, thus suggesting a study on the problem of the causal and time invariant formulation of superluminal speeds.

Under a literature on superluminal speeds of such a dimension, we regret being unable to provide a comprehensive review, and are forced to quote a few representative illustrations of the studies considered in this

paper, *superluminal speeds of ordinary masses or electromagnetic waves*, by deferring the study of tachyons (see, e.g., contributions by E. Recami and his group [47, 48]) to a separate paper.

2. Superluminal Speeds in the Expansion of the Universe

To our knowledge, studies of superluminal speeds were first motivated by the Doppler interpretation of the Hubble law [4] on the cosmological redshift of light

$$z = \frac{\lambda_o}{\lambda_e} - 1 \approx Hd \equiv \frac{v}{c} \tag{5}$$

where λ_o (λ_e) is the wavelength of light at the origin (that observed on Earth), d is the distance of a galaxy from Earth, and H is the Hubble constant.

In fact, with the passing of the decades and the advances in telescopes, it became evident that the galaxies at the edge of the universe have values $z > 1$ with consequential superluminal speeds. This occurrence can be assumed as signaling the initiation of studies in faster than light speeds. As an example, in 1966, Rees [6] attempted the reconciliation of superluminal galactic speeds with special relativity limit (4) by studying the possibility that superluminal speeds are illusory.

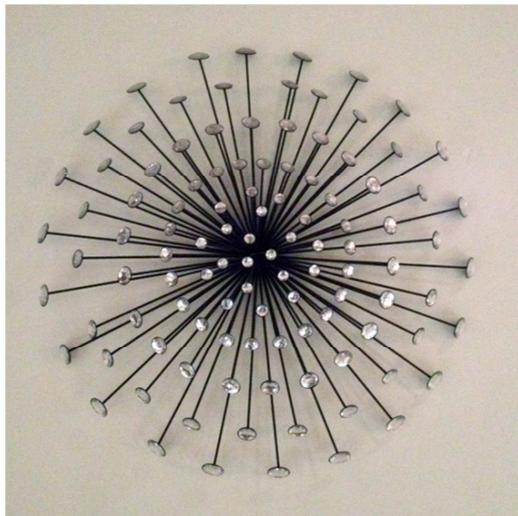


Figure 1. An artist rendering of the conjecture of the expansion of the universe (5) showing Earth necessarily at the center of the universe due to the dependence of the expansion speed $v = Hcd$ on the distance d in all "radial" directions from Earth. This return to the Middle Ages, which is inherent in the conjecture of the expansion of the universe, is the historical reason for which Einstein, Hubble, Hoyle, Zwicky, de Broglie, Fermi and other famous scientists died without accepting the Doppler interpretation of the cosmological redshift [43-47]. Said interpretation became fashionable despite such authoritative oppositions because of the tacit intent of imposing the validity of the conventional interpretation of special relativity for the large scale structure of the universe.

It should be recalled that Einstein, Hubble, Hoyle, Zwicky, de Broglie, Fermi and other famous scientists died without accepting conjecture (5) on the Doppler interpretation of the

cosmological redshift because the interpretation $Hd = v/c$ holds in all possible *radial* directions from Earth, thus implying a necessary return to the Middle Ages with Earth at the center of the universe (Figure 1).

The same conclusion is inevitable for the conjecture of the big bang because, as a necessary condition to represent experimental data on the radial character of the cosmological redshift, the big bang must have occurred in our galactic vicinity, thus implying an "explosion" of the type depicted in Figure 1.

For the intent of avoiding Earth at the center of the universe, supporters of special relativity ventured the additional conjecture that space itself is expanding. However, it is known that this conjecture would have achieved its intent in the event the expansion of the universe were uniform. In reality, conjecture (5) intrinsically implies the *acceleration of the expansion*, that is, the increase of the speed of galaxies with the increase of their distance, $v = Hcd$. This acceleration also occurs in all radial directions from Earth, thus implying again Earth at the center of the universe (see Figure 2 and Refs. [43-45]).



Figure 2. An illustration of the lack of the evidence that the conjecture of the expansion of space itself would provide a consistent representation of conjecture (5) in the event the expansion were uniform. However, conjecture (5) implies the increase of the speed $v = Hcd$ in all radial directions from Earth that cannot possibly be consistent represented via the inflation of balloon [43-47].

In any case, it is easy to see that the sole geometry representing conjecture (5) is that with the shape of a funnel (Figure 3). However, a necessary condition to represent experimental data is that Earth is at the tip of the funnel evidently because all speeds v are measured from Earth, thus implying again Earth at the center of the universe. Also, the funnel-type geometry causes a rather drastic departure from general relativity due to its irreconcilable incompatibility with the Riemannian geometry (Figure 3).

To confirm the sound vision by Einstein, Hubble, Hoyle, Zwicky, de Broglie, Fermi and other famous scientists, the implausibility of the expansion of the universe, is confirmed by the fact that the energy needed to accelerate billions and billions of galaxies is so disproportionately large to prevent any realistic model for its physical origin.

The yet additional conjecture that the universe is filled up with the mysterious and invisible dark energy was ventured to represent the expansion and inherent acceleration of the universe. However, no quantitative model has appeared to date

in the refereed scientific literature achieving such a goal due to several technical insufficiencies, including the fact that, according to Albert Einstein, energy is the source of gravitational *attraction*, and certainly not of *repulsion*.

The above orthodox models are based on the *conception of the large scale structure of the universe as an exterior dynamical problem consisting of particles and electromagnetic waves traveling in vacuum*. This conception is evidently mandatory for the tacit intent of imposing the cosmological validity of the conventional interpretation of special relativity.

Following decades of cosmological studies, the author's conclusion is that the inconsistencies or sheers insufficiencies of the conjecture on the expansion of the universe are due to the fact that *the cosmological redshift of galactic light characterizes a strictly interior dynamical problem consisting of particles and electromagnetic waves propagating within the intergalactic medium* [9-16], mostly composed of hydrogen and other gases at absolute zero degree temperature, dust, cosmic rays, besides including light emitted by all stars in the universe.

As established in Refs. [43-47], the latter conception of the universe implies necessary, experimentally established deviations from the conventional interpretation of special relativity, with particular reference to deviations from the Doppler shift. However, as we shall see in this paper, the abstract axioms of special relativity remain valid provided that they are elaborated with a mathematics more appropriate for interior conditions.

The conception of the universe as an interior dynamical problem was pioneered in 1929 by Zwicky [5] who suggested the interpretation of the cosmological redshift $z = Hd$ via the hypothesis that light loses energy during its long travel to reach Earth due to scattering with the intergalactic medium.

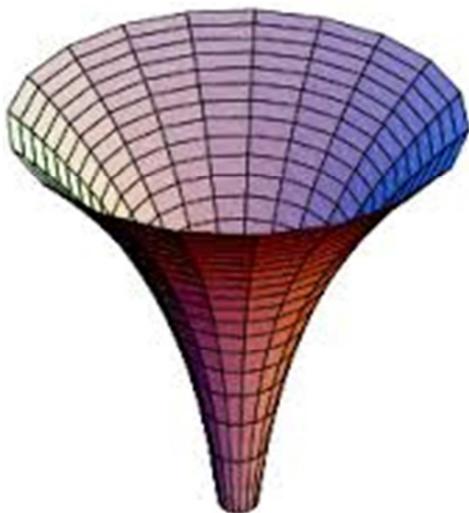


Figure 3. An illustration of the only known consistent representation of the expansion of the universe according to assumption (5) that represents the linear increase of the speed with the distance. However, a necessary condition for consistency is that Earth must be at the tip of the funnel, thus implying again Earth at the center of the universe. Additionally, the funnel geometry is irreconcilably incompatible with the Riemannian geometry of general relativity [43-47].

Unfortunately for scientific knowledge, Zwicky's hypothesis was "disqualified" by the orthodox physics community, and remains "disqualified" to this day, because it clearly violates Einstein's special relativity, evidently due to the fact that, for Zwicky's hypothesis, light is no longer immutable as required by special relativity axioms.

However, Zwicky's hypothesis is experimentally verifiable on Earth, while all conjectures on the expansion of the universe are individually conceived not to be testable on Earth so that they can be imposed via abuses of academic credibility. The author has spent decades of research in the field and confirmed experimentally the validity of Zwicky's hypothesis with the consequential lack of the universe (see for brevity Refs. [43-45]).

The conception of cosmology as an interior dynamical problem is best illustrated by the redshift of galactic stars, which is anomalous in the sense that it is generally smaller (bigger) than the redshift of the galaxy as a whole for stars near (far away from) the galactic center.

Always for the intent of reconciling physical evidence with special relativity, the scientific community coordinated the conjecture that galaxies (as well as their clusters) are filled up with the mysterious, invisible and undetectable dark matter.

However, no quantitative model has been published in the refereed literature showing that dark matter achieves a *quantitative* representation of the anomalous galactic redshift (Figure 5). Besides, according to Newton, galaxies should contract in the event they are filled up with any type of matter, contrary to astrophysical evidence.

In papers [46,47], the author has shown that the problem of the anomalous redshift of galactic stars is indeed a fully interior dynamical problem because the origin of the anomalies is entirely due to the loss (acquisition of energy of star light to (from) the actual material gas filling up all galaxies which is cold at the galactic periphery (very hot near its center).

The resulting frequency shift without any appreciable Doppler's contribution are today known as *Santilli isoredshift (isoblueshift)*, where the prefix "iso" stands to denote their derivation via the coveting of 20th century mathematics known as *asisomathematics*

Ref. [46,47] show in particular that, unlike the case for the conjecture of dark matter, the loss or acquisition of energy of star light from the innergalactic medium does indeed achieve a numerically exact and time invariant representation of the anomalous galactic star redshift without any appreciable Doppler contribution (Figure 6).

The above studies appear to provide sufficient experimental evidence acquired on Earth on the interior character of the large scale structure of the universe. In the next section, we shall show corresponding experimental evidence on the interior character of the structure of hadrons, nuclei and stars, thus suggesting the need to study interior problems for both the large and small scale structures of the universe.

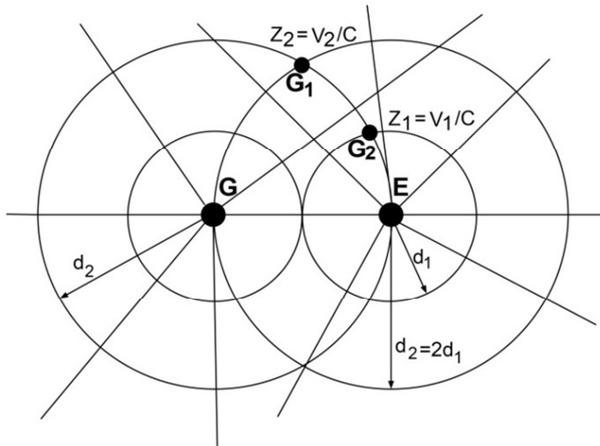


Figure 4. An illustration of the geometric inconsistency of all possible models on the expansion of the universe because galaxies G_1 and G_2 have a relative acceleration when seen from Earth E due to the double distance, while there exists an infinite number of observers in the universe for which the same galaxies G_1 and G_2 have the same distance, as it is the case for galaxy G , in which case there is no relative acceleration, with ensuing clear inconsistency of the very conjecture of the expansion of the universe [43-47].

3. Maximal Speeds for Interior Problems Within Physical Media

The author has essentially devoted fifty years of research to the mathematical, theoretical and experimental studies of interior dynamical problems beginning with his Ph. D. theses in the 1960s [9]. The general *irreversibility over time of interior dynamical problems* has requested the introduction since the mid 1960s of the *Lie-admissible covering of Lie's theory* which is at the foundation of the 20th century interpretation of special relativity [10-16].

The covering Lie-admissible formulations admit a particular case known as *Santilli Lie0isotopic formulations*, that apply for interior dynamical problems when considered as isolated from the rest of the universe, thus being reversible over time [23-34].

The conceptual foundations of these studies can be summarized as follows: when elementary particles move in vacuum as empty space, their only possible acceleration is that via action-at-a-distance potential interactions (more technically known as *variationally selfadjoint (SA) interactions* [15a]). In this case, it is easy to see that the achievement in vacuum of the speed of light c requires infinite energy and, therefore, the surpassing of the speed c in vacuum by ordinary masses or electromagnetic waves is excluded.

However, Santilli [10] showed in 1981 that the situation is substantially different when elementary particles move in interior conditions because, in this case, accelerations are the result, not only of conventional SA interactions, but also of *contact non-potential interactions* (technically known as *variationally nonselfadjoint (NSA) interactions*, [15a]) for which the notion of "potential energy" has no physical value or meaning.

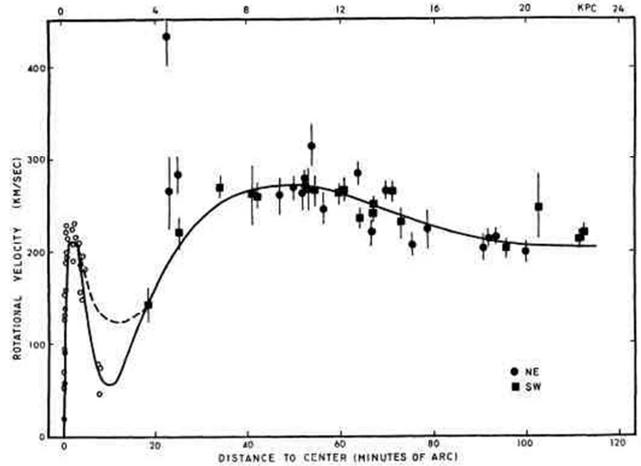


Figure 5. A plot of the anomalous redshift of galactic stars showing the decrease of the redshift for stars near the galactic center and its increase for peripheral stars. The conjecture of dark matter was ventured to represent the anomalous galactic stars, but no such a quantitative representation has appeared in the refereed literature to date, besides intrinsic inconsistencies of the conjecture of dark matter identified in Refs. [43-47].

It was then easy to see already in the 1980s that *under NSA interactions the local speeds of ordinary masses within physical media are unrestricted*, thus being arbitrarily bigger (or smaller) than c depending on local conditions of density, temperature, frequencies and other physical data.

The analytic background of the studies on interior conditions is given by the "true Hamilton's equations," those with external terms not derivable from a Hamiltonian

$$\frac{dr}{dt} = \frac{\partial H(r,p)}{\partial p}, \quad \frac{dp}{dt} = -\frac{\partial H(r,p)}{\partial r} + F^{NSA}(t, r, v, \dots), \quad (6)$$

and their operator counterpart, which are specifically set for the representation of open irreversible processes we cannot possibly review here [16].

The mathematical backgrounds of the studies is given by the *Lie-admissible covering of Lie's theory* since the true Hamilton's equations emerged since the 1960s [9] as admitting a Lie-admissible algebra in the brackets of their time evolution when properly written (see the more recent memoir [13] for details).

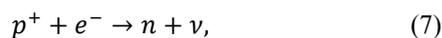
To understand the complexity of the problem, let us recall that physical theories can be claimed to have physical value if and only if they verify the *invariance over time*, namely, they predict the same numerical values under the same conditions at different times. It is easy to see that the true Hamilton's equations and their operator counterpart violate this crucial condition because they are non-canonical and non-unitary by conception.

The achievement of invariance over time for non-canonical and non-unitary theories required the construction of a new mathematics, today known as *Lie-admissible genomathematics* we cannot possibly review here (see mathematics studies [11-13] and monographs [16]).



Figure 6. An illustration of the evidence visible via telescopes that all galaxies are filled up indeed of matter, but of a conventional, real, gaseous medium which is very hot near the galactic center and very cold for peripheral stars. The deviations from the conventional interpretation of special relativity measured by Santilli's isoblueshift for hot gases and isoredshift for cold gases [43-45] has permitted a numerically exact and invariant representation of the anomalous redshift of galactic stars depicted in Figure 5 [46,47]. This and other representations, such as that of Arp's pair of connected quasars and galaxies with largely different cosmological redshifts, establishes that the large scale structure of the universe is an interior dynamical problem in which all cosmological redshifts are reduced to the experimentally established loss (absorption) of energy to (from) cold (hot) intergalactic or innergalactic media without any appreciable contributions from the Doppler's shift [43-47].

A main application of these studies has been the first achievement at both nonrelativistic and relativistic levels of an exact representation of all characteristics of the neutron in its synthesis inside stars according to Rutherford's "compression of the hydrogen atom," namely, from a proton and an electron according to the known reaction (for brevity, see review [14])



The main technical difficulty was due to the fact that the rest energy of the neutron is 0.872 MeV bigger than the sum of the rest energies of the proton and of the electron, under which condition we would need "positive binding energies" which are anathema for quantum mechanics, since they cause the physical inconsistency of the Schrödinger equation.

Santilli's main point is that, even though there exist indeed particles with "point-like charges" (such as the electron), there exist no "point-like wavepackets" in nature. Therefore, Rutherford's compression of the extended wavepacket of the electron within the hyperdense medium inside the proton generates NSA interactions under which a solution of synthesis (7) has been indeed found [14].

The main mechanism is that contact interactions are NOSA and, therefore, they are non-unitary. The non-unitary image of Schrödinger equation achieves consistency under "positive binding energies" thanks to a new renormalization of the rest energies of the constituents (called *isorenormalization*) [14,

16, 34]).

The aspect important for this paper is that an apparent necessary condition for the representation of all characteristics of the neutron in synthesis (7) is that the constituents of the neutron travel at (tangential) superluminal speeds.

Intriguingly, systematic plots of experimental data in hadron physics conducted in monograph [16d] without the aprioristic assumption of the Lorentz symmetry have confirmed superluminal speeds within the interior of hadrons in numerous cases, such as: phenomenological fits via gauge theories; anomalous meanlives of unstable hadrons with speed; the Bose-Einstein correlation; and other cases directly relevant for the study of superluminal speeds.

We regret not being able to review these phenomenological fits and related literature to avoid a prohibitive length of this paper. Nevertheless, their knowledge is important for a technical understanding of this paper.

We should add that, as clarified by Wall [17], the superluminal speeds of hadronic constituents here considered are not referred to tachyons as conventionally defined (see, e.g, Ref. [42]) because, as we shall see in the next sections, interior physical media cause a deformation (called "mutation") of the light cone with superluminal speeds of real valued masses. Therefore, the existence of tachyons (called in the field of this paper *isotachyons*) is shifted for speed beyond the maximal causal speed within physical media which are generally bigger than c , as shown in the next section.

More specifically, we are not excluding possible tachyonic contributions in the structure of hadrons or in other physical conditions [42]. The only point we would like to clarify is that, under the validity of isotopic theories for the hadronic structure, the speed characterizing tachyons has to be shifted beyond c (see, later on, Eq. (21) and related arguments).

Independently from Santilli's research, additional relevant studies on superluminal speeds are the experimental works initiated in 1992 by Enders and Nimtz [18] (see also the more recent paper [19] for additional references and paper [20]) suggesting apparent superluminal propagation of electromagnetic waves within certain physical guides.

The reconciliation of superluminal speeds with special relativity limit (4) is generally attempted by assuming that we are dealing with a "tunnel effect." However, in our view, tunnel effects generally refer to passages through a barrier, thus for distances of 1 fm covered by the uncertainty principle, and not for travel over lengths of several meters, as occurring for Refs. [18-20].

Hence, it is well possible that, in reality, Refs. [18-20] deal with an interior dynamical problem in which case superluminal speeds are due to NSA interactions of electromagnetic waves with the guides (including the so-called "stray fields" that are known to be NSA) under which interactions superluminal speeds are quite natural. Additional cases of superluminal speeds of ordinary masses can also be treated as interior dynamical problems, but we regret not being able to treat them here for brevity.

Yet an additional case relevant for this paper is the recent

controversy at CERN on superluminal or sub-luminal neutrinos, since the original 2011 announcements [21] indicated the detection of superluminal neutrino speeds, while the subsequent paper [22] indicated subluminal speeds.

We would like to point out that the truly fundamental issue for Refs. [21, 22] is the *identification of a causal and time invariant formulation of particle motion in interior conditions*, since for both views [21, 22] neutrinos travel *underground* from CERN to the Gran Sasso Laboratory.

The orthodox position is that neutrinos are point-like and travel underground without collisions, for which view interior conditions do not exist, and special relativity applies exactly.

However, *there exist no point-like wavepackets in nature*; to be physical, neutrinos must have an extended wavepacket that does interact with the wavepackets of peripheral atomic electrons in the dense underground conditions; special relativity cannot be even marginally formulated undergrounds, e.g., because light does not propagate there; and, therefore, the interior conditions of experiments [21, 22] are unavoidable on serious scientific grounds.

In the absence of the appropriate basic theory, the underground speed of neutrinos remains unsettled because the slightest modification of any form factor, parameter or data elaboration can produce subluminal results in Ref. [21] and superluminal results in Ref. [22], as experts in the field can verify.

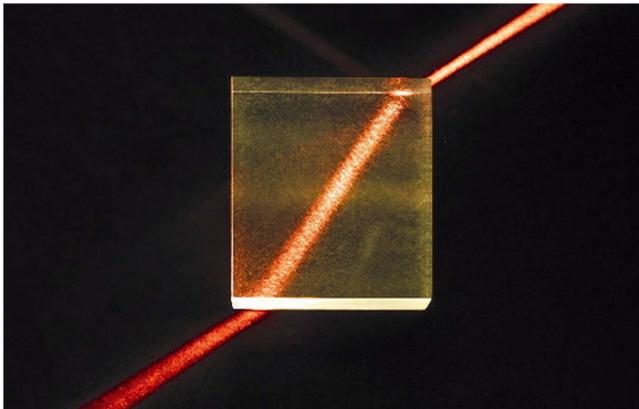


Figure 7. An illustration of the historical Lorentz problem at the foundation of this paper; the invariance of locally varying speeds of light within transparent physical media, here illustrated with the variation of speed from air to water and then back to air. The orthodox view is that light can be reduced to photons scattering among the water molecules, thus propagating in vacuum without any interior conditions. However, such an abstraction is known not to represent experimental data such as: the angle of refraction (because photons will scatter in all directions at the impact with water); the large reduction of speed for about 1/3 (because photons could at best account for a few percentages of speed reduction); the propagation of light in water as a straight beam (because photons would scatter light in all directions and the view of this picture would be impossible; and other experimental data. Hence, on serious scientific grounds, the representation of the propagation of light within transparent physical media requires a covering of special relativity specifically conceived for interior conditions.

Besides superluminal speeds, the study of interior dynamical problems inevitably requires the inclusion of *subluminal speeds*, namely, speeds of electromagnetic waves within transparent physical media *smaller* than c . This is

typically the case for the propagation in water of infrared or radio waves for which the reduction to photon is not possible (Figure 1) [16a].

In this paper, we shall show that, when formulated with the appropriate mathematics, *Special relativity axioms do indeed admit fully causal and time invariant arbitrary, that is whether superluminal or subluminal speeds depending on the physical characteristics of the medium at hand*.

Since the Lie-admissible genomathematics is excessively advanced for the limited scope of this paper, we shall restrict our study to the particular case of interior events that are reversible over time, namely, events in which we ignore in first approximation the dispersion of light (Figure 1). This class of reversible interior events can be well treated via *isomathematics*, with particular reference to *isorelativity*, referred to the *Lie-isotopic genomathematics of special relativity*.

Nevertheless, the reader should keep in mind that *the propagation of light within transparent physical media is an intrinsically irreversible event*, as established by the simple evidence that media become warmer when traversed by light. Consequently, we should stress that the isotopic formulations presented in this paper only permit a first approximation of interior conditions.

That the proper causal and time invariant representation of irreversible interior events requires the covering *genomathematics*, and related *genorelativity*, referred to the *Lie-admissible covering of isorelativity*. To avoid a prohibitive length, this broader treatment will be merely indicated for intended presentations in subsequent papers.

We should finally recall historical contributions [67-71] dealing with superluminal speeds (although not within the context of interior dynamical problems),

4. Solution of the Historical Lorentz Problem

As it is well known to physics historians, Lorentz first attempted the achievement of the invariance of the speed of electromagnetic waves of his time, namely, the locally *varying* speed within physical media here referred to infrared, radio and other large wavelengths not admitting a consistent reduction to photons (see Section 4 for the general case)

$$C = \frac{c}{n(t,r,v,e,\rho,\omega,\tau,\dots)}, \quad (8)$$

where n is the familiar index of refraction with a rather complex functional dependence on local variables, such as time t , coordinates r , speeds v , energy e , density ρ , frequency ω , temperature τ and other variables.

Due to insurmountable technical difficulties, Lorentz was solely able to achieve invariance for the *constant* speed c of electromagnetic waves in vacuum, resulting in the celebrated transformations (1) leaving invariant line element (2a).

Santilli has studied for decades the solution of the *historical Lorentz problem*, namely, the achievement of the universal

invariance of all possible locally varying speeds of electromagnetic waves within physical media, Eq. (8), which case evidently admits the constant speed c in vacuum as a particular case.

As a first step, when a member of MIT from 1974 to 1978, Santilli realized that Lorentz's inability to achieve the invariance of speeds (8) was due to insufficiencies of the basic theory, Lie's theory, because such a theory is strictly linear, local-differential and potential-Hamiltonian, while the invariance of speeds (8) is a strictly non-linear, non-local/integral and non-potential, thus non-Hamiltonian problem.

The results of these initial studies were released in monographs [15] (that originally appeared as MIT preprints to be subsequently published under affiliation to Harvard University). A main aspect of these studies is their conception as *isotopic* (intended in the Greek meaning of being "axiom preserving") lifting of the various branches of Lie's theory into such a form to admit the treatment of non-linear, non-local and non-Hamiltonian systems.

The proposal was centered in the isotopic lifting of the unit of the Lorentz symmetry, $I = \text{Diag.}(1,1,1,1)$, into a quantity \hat{I} (such as a function, a matrix, an operator, etc.) with an arbitrary functional dependence on all needed local variables, under the sole condition of being positive-definite, thus invertible,

$$I = \text{Diag.}(1,1,1,1) \rightarrow \hat{I} = \hat{I}(t, r, v, e, \rho, \omega, \tau, \dots) = 1/\hat{T}(t, r, v, \rho, \omega, \tau, \dots) > 0, \quad (9)$$

which lifting remains fixed for the interior problem considered.

For consistency, the lifting of the unit required the compatible lifting of the conventional associative product between arbitrary quantities A and B of the type

$$AB \rightarrow A \hat{\times} B = A\hat{T}B, \quad (10)$$

under which \hat{I} is indeed to the right and left unit of the theory,

$$\hat{I} \hat{\times} A = A \hat{\times} \hat{I} \equiv A, \quad (11)$$

for all elements A of the set considered.

Following basic assumptions (9)-(11), Santilli constructed a step by step isotopic generalization of the various branches of Lie's theory, including [15b]:

1) The isotopic lifting $\hat{\xi}(L)$ of the universal enveloping associative algebra $\xi(L)$ of a n -dimensional Lie algebra L with (Hermitean) generators $X_i, i = 1, 2, \dots, n$, and infinite-dimensional isotopic basis (today known as the *Poincaré-Birkhoff-Witt-Santilli isothorem* [35-42]):

$$\hat{I}, X_k, \hat{X}_i \hat{\times} \hat{X}_j, i \leq j, \hat{X}_i \hat{\times} \hat{X}_j \hat{\times} \hat{X}_k, i \leq j \leq k, \dots; \quad (12)$$

2) The isotopic liftings of Lie algebras with closure rules (today called *Lie-Santilli isoalgebras* [loc. cit.]

$$[X_i, \hat{X}_j] = X_i \hat{\times} X_j - X_j \hat{\times} X_i = C_{ij}^k X_k' \quad (13)$$

3) The corresponding isotopic lifting of Lie's

transformation groups (today called *Lie-Santilli isogroups* [loc. cit.]), e.g., here expressed for the time evolution

$$A(t) = U(t)A(0)U(t)^\dagger = [e^{H\hat{T}t}]A(0)[e^{-it\hat{T}H}]; \quad (14)$$

and the isotopies of the representation theory.

The above isotopies clearly show the non-linear, non-local (integral) and non-Hamiltonian character of the isotopic theory due to the appearance of a positive-definite but otherwise arbitrary quantity \hat{T} in the *exponent* of the group structure.

The representation of interior systems is then achieved via the representation of all SA interactions by means of the conventional Hamiltonian $H(r, p)$, and the representation of all NSA interactions by means of the generalized unit \hat{I} (see Refs. [16, 34] for concrete examples in classical and operator mechanics).

Following the construction of the isotopies of Lie's theory, Santilli introduced in letter [23] of 1983 the following isotopies of Minkowski space (2) (today known as the *Minkowski-Santilli isospace* [loc. cit.]) with the most general possible nonsingular and symmetric line element (thus including all possible Minkowskian, Riemannian, Fynslerian and other line elements in (3+1)-dimensions)

$$\hat{x}^2 = x^\mu (\hat{T}_\mu^\rho \eta_{\rho\nu} x^\nu = x^\mu \hat{\eta}_{\mu\nu} x^\nu = \frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2}, \quad (15a)$$

$$n_\mu = n_\mu(t, r, v, e, \rho, \omega, \tau, \dots) > 0, \mu = 1, 2, 3, 4, \quad (15b)$$

$$\hat{T} = \text{Diag.}(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2) > 0, \quad (15c)$$

$$\hat{I} = 1/\hat{T} = \text{Diag.}(n_1^2, n_2^2, n_3^2, n_4^2) > 0, \quad (15d)$$

where: the n 's are called the *characteristic quantities* of the medium considered; n_4 is the conventional index of refraction providing a geometrization of the density of the medium normalized to the value $n_4 = 1$ for the vacuum; n_1, n_2, n_3 provide a geometrization of the shape of the medium considered normalized to the values $n_1 = n_2, n_3 = 1$ for the sphere; the general *inhomogeneity* of the medium is represented by the dependence of the characteristic quantity on the local variables (e.g., the elevation for the case of our atmosphere); and the general *anisotropy* of the medium (e.g., the anisotropy of our atmosphere caused by Earth's rotation) is represented by different values of the type $n_4 \neq n_s$.¹

Note that, for exterior dynamical problems, homogeneity and isotropy equally occur in all directions. By contrast, within a physical medium inhomogeneity and anisotropy requires the selected of a given direction in space n_s due to variations for different directions.

Following the construction of the isotopies of Lie's theory and of Minkowski space-time, Santilli solved the historical Lorentz problem in page 551 of letter [23] via the lifting of the

¹Scientific caution is suggested before dubbing the n -characteristic quantities as "free parameters" because that would imply that, e.g., the index of refraction n_4 is a free parameter when in reality it is measured for a given medium, or that the $g_{\mu\nu}$ elements of the Schwartzchild metric (see Eqs. (17) below) are free parameters, etc.

Lorentz symmetry characterized by the isotopic element (15c). This resulted in the generalized transformations (Eqs. (15) of Ref. [23]), today known as the *Lorentz-Santilli (LS) isotransforms* [35-42] which we write in the currently used symmetrized form

$$x^{\wedge 1} = x^1, x^{\wedge 2} = x^2, \quad (16a)$$

$$x^{\wedge 3} = \hat{\gamma}(x^3 - \hat{\beta} \frac{n_3}{n_4} x^4), x^{\wedge 4} = \hat{\gamma}(x^4 - \hat{\beta} \frac{n_4}{n_3} x^3), \quad (16b)$$

$$\hat{\beta} = \frac{v_3/n_3}{c_0/n_4}, \hat{\gamma} = \frac{1}{\sqrt{1-\hat{\beta}^2}} \quad (16c)$$

leaving invariant the isoline element (15a), thus providing indeed the invariance of the varying speeds of light (8) (see Ref. [34b] for the general treatment).

Jointly with the above classical formulation, Santilli constructed the corresponding operator image [24] of the above isotransformations, and then constructed the isotopies of every main aspect of the LP symmetry, including the isotopies of: the rotational symmetry [25]; the SU(2)-spin symmetry [26]; the Poincaré symmetry [27]; the spinorial covering of the Poincaré symmetry [28]; the SU(2)-isospin symmetry and local realism [29]; and the isotopies of the Minkowskian geometry [30].

The resulting isosymmetry, today known as the *Lorentz-Poincaré-Santilli (LPS) isosymmetry*, was proved in Refs. [31,32] to be "directly universal" for all infinitely possible non-singular and symmetric space-times in (3 + 1)-dimensions, thus providing the universal symmetry of all possible Riemannian, Fynslerian, and other possible line elements, with a trivial extension to arbitrary space-time dimensions, such as those for the De Sitter symmetry.

Systematic studies can be found in monographs [33, 34], while independent studies can be found in monographs [35-42] and references quoted therein. A few comments are now in order to prevent possible misrepresentations that generally remain undetected by non-experts in the field.

To illustrate the universality of isoinvariant (15) and related isosymmetry (16) for all possible, symmetric and non-singular space-times in (3+1)-dimensions, let us note that they include as particular case all possible Riemannian line elements, such as the Schwartzchild's line element [27]

$$ds^2 = r^2(d\theta^2 + \sin^2 d\phi^2) + (1 - \frac{2 \times M}{r})^{-1} \times dr^2 - (1 - \frac{2 \times M}{r}) \times dt^2 \quad (17a)$$

$$\hat{T}_{sch} = \text{Diag.} [1, 1, (1 - \frac{2 \times M}{r})_r^{-1}, (1 - \frac{2 \times M}{r})_t], \quad (17b)$$

$$\hat{I}_{sch} = \text{Diag.} [1, 1, (1 - \frac{2 \times M}{r})_r, (1 - \frac{2 \times M}{r})_t^{-1}], \quad (17c)$$

where one can see that the gravitational singularity is that of the isotopy, namely, the infinite value of the isotopic element and the null value of the isounit. In fact, Ref. [27] was primarily intended to indicate the achievement of the universal symmetry for all possible (non-singular) Riemannian line elements.

A rather popular belief during the 20th century physics was that interior dynamical systems are not essential because they can be reduced to elementary particles moving in vacuum, thus recovering at the elementary level exterior conditions without non-linear, non-local and non-Hamiltonian interactions. This belief was disproved by the following:

NO REDUCTION THEOREM [13, 34]: A macroscopic, non-conservative system cannot be consistently reduced to a finite number of elementary particles all in conservative conditions and, vice versa, a finite number of elementary particles all in conservative conditions cannot consistently yield a macroscopic non-conservative system under the correspondence or other principles.

Stated in different terms, the reduction of interior to exterior systems implies the belief that entropy and thermodynamical laws are "illusory" because, when interior systems are reduced to elementary particle constituents, entropy and thermodynamical laws "disappear." The above No Reduction Theorem establishes that non-conservative (thus, NSA) interactions, rather than "disappearing," originate at the most elementary level of nature.

As a concrete example, the above No Reduction Theorem is verified by a spaceship during reentry in atmosphere because its non-linear, non-local and non-Hamiltonian interactions originate at the most elementary level, that of the deep mutual penetration of the wavepackets of peripheral atomic electrons of the spaceship with the wavepackets of the electrons of atmospheric atoms.

The next aspect needed for a serious understanding of the content of this paper is the necessity to verify the time invariance indicated in the preceding section, namely, the prediction of the same numbers under the same conditions at different times.

Santilli selected a generalization of the unit for the representation of non-linear, non-local and non-Hamiltonian interactions for the intent of achieving the much needed time invariance, since the unit is the basic invariant of any theory.

However, it is easy to see that the representation of non-Hamiltonian interactions via the isounit is not sufficient *per se* to achieve the needed time invariance because isotopic theories are non-canonical at the classical level and non-unitary at the operator level by conception and construction [16,33,34]. It is then easy to see that the isounit is not preserved by the time evolution of the theory, e.g., Eq. (14)

$$\hat{I} \rightarrow \hat{I}' = U \hat{I} U^\dagger \neq \hat{I}, U U^\dagger \neq I \quad (18)$$

But the isounit represent interior conditions. Therefore, its lack of conservation in time implies the transition over time from one interior system to another (e.g., from the synthesis of the neutron (7), for instance, to a nuclear fusion).

This occurrence is known under the name of *Theorem of Catastrophic Mathematical and Physical Inconsistencies of Non-Canonical and Non-Unitary Theories* when formulated with the mathematics of canonical and unitary theories, respectively. Regrettably, we cannot review this theorem for brevity and must refer the reader to works [13, 33, 34].

The resolution of the inconsistencies caused by the lack of

time invariance required decades of additional research by Santilli and a number of colleagues. The solution was finally achieved following the construction of a new mathematics, today known as *Santilli isomathematics*, characterized by the isotopic lifting of the *totality* of the quantities and their operation of the mathematics used for exterior problems.

When classical non-canonical and operator non-unitary theories are elaborated with the appropriate classical and operator isomathematics, respectively, the invariance over time of numerical predictions is regained, thus offering the mathematical and physical consistency needed for applications.

5. Compatibility of Arbitrary Interior Speeds with Special Relativity

The locally varying speeds of electromagnetic waves propagating within physical media left invariant by the LPS isosymmetry (16) are completely unrestricted and can, therefore, be smaller, equal or bigger than the speed of light in vacuum,

$$C = \frac{c}{n_4} \Leftrightarrow c. \tag{19}$$

This is due to the unrestricted functional dependence of the isotopic element (15c), except for the condition of being non-singular.

It is then easy to see that the *maximal causal speed* in Minkowski-Santilli isospaces is arbitrarily bigger, equal or smaller than c . In fact, the mutated light cone, called *light isocone*, in the $(s, 4)$ -dimensions is given by

$$\hat{x}^2 = \frac{x_s^2}{n_s^2} - t^2 \frac{c^2}{n_4^2} = 0, \tag{20}$$

and evidently characterizes the maximal causal speed within physical media

$$V_{max,s}^{media} = c \frac{n_s}{n_4} \Leftrightarrow c. \tag{21}$$

where, as indicated in Section 1, the selection of a space direction s is necessary since physical media are generally inhomogeneous and anisotropic.

Note the need to use a covering of the speed of light for maximal causal speed under the LPS isosymmetry because interior dynamical problems are generally opaque to light. Note also that the causal character of speeds (21) is guaranteed by the LPS isosymmetry in exactly the same way as the LP symmetry guarantees the causal character of c .

It has been popularly believed throughout the 20th century that any deviation from the speed of electromagnetic waves in vacuum implies a "violation of Einstein's special relativity." In this section, we show that this belief is not technically correct, because *Einstein's special relativity axioms do admit arbitrary causal speeds*, provided that they are realized via the appropriate mathematics.

To begin, Lorentz transformations (1) provide the invariance of the "constant" speed c without any

identification of its numerical value, which value is set by measurements. Therefore, *Lorentz transformations equally apply for an arbitrary constant speed $C = c/n_4$ within physical media*, with known value in water

$$C_{water} = \frac{c}{n_4} = \frac{2}{3}c, n_4 = \frac{3}{2}. \tag{22}$$

in which case no violation of Einstein special relativity axioms can be claimed.

Additionally, the replacement in the *conventional* transforms (1) of the speed of light c with maximal causal speed (21) yields, identically, the LPS isotransformations (16), as one can readily verify.

$$x'^1 = x^1, x'^2 = x^2, \tag{23a}$$

$$x'^3 = \gamma(x^3 - \beta x^4), \tag{23b}$$

$$x'^4 = \gamma(x^4 - \beta x^3), \tag{23c}$$

$$\beta = \frac{v}{v_{max}^{media}}, \gamma = \frac{1}{\sqrt{1-\beta^2}}. \tag{23d}$$

Also, *the Lorentz-Santilli isosymmetry (16) is locally isomorphic to the conventional Lorentz symmetry by conception and construction* to the point of preserving the original structure constants [23, 33, 34]. Therefore, no claim that isotransforms (16) violate Einstein's special relativity axioms can be consistently voiced due to the very conception and technical realization of the isotopies of special relativity.

More technically, when represented on Minkowski-Santilli isospace over the isofield of real numbers [11], light isocone (20) becomes the perfect cone with the same maximal causal speed c as that valid in empty space [33, 34].²

This is due to the fact that the cone axes are indeed mutated under isotopies from their original unit value to new values

$$(1_s, 1_4) \rightarrow (1/n_s^2, 1/n_4^2), \tag{24}$$

but, jointly, the related units are mutated by the *inverse* amounts,

$$(1_s, 1_4) \rightarrow (n_s^2, n_4^2), \tag{25}$$

thus preserving the original Minkowskian light cone identically.

We finally illustrate the compatibility of arbitrary speeds with special relativity axioms via the following transformation of the Minkowskian coordinates

$$x^\mu \rightarrow \frac{x'^\mu}{n_\mu}, \tag{26}$$

under which the Minkowski line element (2a) is transformed into the isotopic image (15a), and Lorentz transformations (1) are turned into the LS transform by keeping in mind transformation of the speed, e.g., along the third space

²The mathematically correct formulation of the Minkowski-Santilli isospace is given by the isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{t})$ defined over the field of isoreal numbers with unit \hat{t} given by Eq. (15c), with $\hat{x} = x\hat{t}$ as a condition to be an isonumber and $\hat{\eta} = \hat{T}\eta$ (see Ref. [30] for details).

direction

$$v = \frac{\delta x^3}{\delta t} \rightarrow v \frac{n_4}{n_3} \tag{27}$$

By recalling that the metric of isotopic theories $\hat{\eta} = \hat{T}\eta$ includes as particular case all possible Riemannian metrics g , in this section we have attempted to indicate that *Einstein's special relativity axioms have a representational capability dramatically broader than that believed in the 20th century because, in addition to admitting arbitrary maximal causal speeds, they also admit interior and exterior gravitational models.*

In fact, in Ref. [30] Santilli has shown the treatment of exterior gravitation via special relativity axioms on the metric $\hat{\eta}(x) = \hat{T}(x)\eta = g(x)$, while maintaining the machinery of the Riemannian geometry (covariant derivative, Christoffel's symbols, etc.) and Einstein-Hilbert field equations under the universal LPS symmetry as a condition to achieve the above indicated invariance over time of numerical predictions. In this exterior case, the maximal causal speed is evidently that in vacuum c .

In the same Ref. [3], Santilli has shown that special relativity axioms equally admit interior gravitational models, this time, with an unrestricted functional dependence of the metric $\hat{\eta}(x, \rho, -\tau, \omega, \dots) = \hat{T}(x, \rho, \tau, \omega, \dots)\eta = g(x, \rho, \tau, \omega, \dots)$ equally under the universal LPS symmetry, in which case the maximal local causal speed is arbitrarily bigger or smaller than c depending on local conditions.

Note that, under the full use of isomathematics, all the preceding enlargements of the conditions of applicability of Einstein's special relativity axioms can be formulated via conventional symbols as used in Eqs. (1)-(4), and merely subject them to different interpretations.

In fact, the variables x^μ can be interpreted as representing physical space-time coordinates with respect to the Lorentz unit $I = \text{Diag.}(1,1,1,1)$, in which case we have the 20th century formulation of special relativity for exterior problems invacuum.

Alternatively, we can consider the coordinates x^μ as being purely mathematical and assume the realization $x^\mu = x'^\mu/n_\mu$. In this case, when x'^μ are assumed as the physical coordinates and are referred to the isounit (15d), we realize special relativity axioms in such a way to have interior dynamical conditions with maximal causal speeds 921), exterior gravitation with the characteristic quantities representing conventional Riemannian metrics, interior gravitation, and other possibilities.

It should be stressed that the above studies apply to interior dynamical problems that are reversible over time. Their extension to irreversible problems can be first studied via isounit with an explicit but not invariant time dependence of the type

$$\hat{I}(t, \dots) - \hat{I}^\dagger(t, \dots) \neq \hat{I}(-t, \dots) = \hat{I}^\dagger(-t, \dots), \tag{28}$$

The reader should however be aware that under the above realization of the isounit the Lie-isotopic formulation is turned into a bimodular formulation with a Lie-admissible structure

[9-16].

We should finally indicate that the entire body of mathematical, theoretical and experimental formulations studied in this paper are nowadays known under the name of *isorelativity for matter* [50-54] and the *isodual isorelativity for antimatter*[55].

6. Expected Implications

6.1. Cosmology

As reviewed in Section 2, the conjecture of the expansion of the universe and its endless chain of subsequent conjectures were aimed at the unspoken intent (or *de facto* primary implication) of maintaining the validity of special relativity for the large scale structure of the universe because the alternative offered by Zwicky [5], that of galactic light losing energy to the intergalactic medium, would violate the special relativity.

Despite one century of efforts, the conjecture of the expansion of the universe is nowadays discredited in serious scientific circles due to excessive insufficiencies or sheer inconsistencies published in refereed journals, but none of them disproved also in refereed journals.

In fact, all current dominating cosmological models imply a return to the Middle Ages with Earth at the center of the universe due to the *radial* character in all directions from Earth of the conjectured expansion of the universe and its conjectured acceleration.

Isomathematics [38], the Lorentz-Santilli isosymmetry [23-30], its related isorelativity [51-54], and the vast experimental verifications on Earth of Zwicky hypothesis [43-47] are expected to restore the validity of special relativity for the large scale structure of the universe without inconsistent assumptions on the expansion of the universe.

This important occurrence is due to the fact that the Lorentz-Santilli isosymmetry uniquely and unambiguously characterize the *Doppler-Santilli isoshift law* [51, 43]

$$z \approx \pm \frac{v n_4}{c n_v} \tag{29}$$

where the characteristic quantities n_4, n_v depend on all local variables, including the speed v , the distance d traveled by light, etc., thus allowing the expansion

$$z \approx \pm \frac{v n_4}{c n_v} \approx \pm \frac{v}{c} (1 \pm \frac{Hc}{v} d) = \pm \frac{v}{c} \pm Hd, \tag{30}$$

where the first term is the conventional Doppler term, the second term is the Santilli isoredshift, and H is a constant (in first approximation) that can be assumed to be Hubble's constant for cosmological applications.

But experiments [43-47] have established that the conventional Doppler contribution is ignorable with respect to Santilli isoredshift for the propagation of light within physical media either of large density and short distances as it is the case for our atmosphere, or of low density but extremely large distances, as it is the case for intergalactic media.

Consequently, the original, experimentally verified

Hubble's law [4]

$$z = Hd \quad (31)$$

is uniquely, unambiguously and invariantly represented by the Lorentz-Santilli isosymmetry. But the Lorentz-Santilli isosymmetry is locally isomorphic to the conventional symmetry, and the axioms of isorelativity are identical to those of special relativity [51-54], thus implying the restoration of special relativity for the large scale structure of the universe, although in its broader isotopic realization, without any expansion of the universe.

However, the cosmological implications of the above possibilities are far reaching because all current numerical values in cosmology, beginning with currently assumed distances of stars and galaxies, should be re-inspected for possible revisions whenever admitting the absorption of light by innergalactic or intergalactic physical media.

In fact, current distances are estimated based on the luminosity of supernovae or large stars under the current assumption, necessary to maintain special relativity in cosmology, that light propagates in vacuum.

It is evident that the admission of innergalactic and intergalactic media as well as the admission of the experimental evidence [43-47] on the loss of energy by light to said media, are expected to imply a revision of the value of the intensity at the origin, with consequential revision of current estimates on cosmological distances.

The same implications are expected for other numerical values in cosmology since they are all derived via *theoretical conjectures*, since the same experimentally known values are those of the cosmological redshift.

6.2. Particle Physics

The unspoken intent of maintaining special relativity in cosmology was also applied to the structure of strongly interacting particles (hadrons) via the same approach, that based on conjecture that could not be directly verified in experiments, so that the conjectures would achieve collegial acceptance via coordinated supports.

In fact, the structure of hadrons was reduced to the hypothetical quarks that, by conception have to be permanently confined inside hadrons, thus reducing the structure of hadrons to imaginary spheres containing point-like hypothetical quarks moving in vacuum, as necessary for the validity of special relativity.

In particular, for the first time in history, the same model was used for both the *classification* of hadrons into family and the *structure* of individual hadrons belonging to a given multiplet. By contrast, atoms required *two* models, the Mendeleev classification of atoms into families, and the structure of individual atoms belonging to a given family.

Also, the classification of atoms was achieved via classical methods, while the structure of individual atoms required the construction of a generalization of classical theories, quantum mechanics.

The insufficiencies of such an *ad hoc* conduction of

physical research are numerous (see, e.g., paper [56] of 1981). Unfortunately for scientific knowledge, said insufficiencies are not disproved in refereed journals exactly as it has been the case for the insufficiencies of cosmological conjectures.

The most striking insufficiency with large societal implication is that belief, implicit in quark conjectures, that the permanently stable proton and electron "disappear" (sic) at the time of the synthesis of the neutron (n) top be replaced by the hypothetical quarks and, then, at the time of spontaneous decays of the neutron, the hypothetical quarks "disappear" and the permanently stable proton and electron "reappear."

Additionally, quarks cannot be defined in our spacetime due to incompatibilities with the Poincaré symmetry, as well known to quark experts. Therefore, any claim that quarks are "physical particles" has no serious physical ground. Additionally, the theoretical impossibility for quarks to be seriously confined inside hadrons (that is, to have an identical null probability of tunnel effects) is readily established by Heisenberg's uncertainty principle. In the final analysis, quarks are mere mathematical representations of a purely mathematical symmetry solely defined in a purely mathematical, unitary, complex-valued space.

In view of all the above insufficiencies (and numerous others), the view suggested by the author when at Harvard University in the 1980s [56] is that the $SU(3)$ -color classification of hadrons (now extended to the "Standard Model") is excellent; quarks are necessary for the mathematical elaboration of said classification; and the structure of individual hadrons requires a generalization of quantum mechanics into hadronic mechanics.

The latter view was suggested in the early 1980s because *there exist no point-like wavepackets in nature*. Consequently, hadrons are the densest medium identified by mankind to date caused by the total mutual penetration of the wavepackets of the constituents, with inevitable non-linear, non-local and non-Hamiltonian internal effects. Under these conditions, the intents at preserving the 20th century formulation of special relativity and quantum mechanics are non-scientific, besides being disproved by experimental evidence on the behavior of the main-life of unstable Kaons from $1q0$ to $100 GeV$ showing clear deviations from special relativity [57].

Isomathematics [38], the Lorentz-Santilli isosymmetry [23-30], and the additional experimental evidence on deviations from special relativity within physical media [43-47], offer realistic possibilities of restoring the validity within hadrons of the abstract axioms of special relativity and quantum mechanics, although in their generalized isotopic realizations, without the assumption of hypothetical constituents that have not been produced free even at the extremely high energies available at CERN.

According to this view, the constituents of all unstable particles are generally the massive particles produced in the spontaneous decays, although in a generalized form characterized by the Poincaré-Santilli isosymmetry under the law of relativistic hadronic mechanics due to their total mutual immersion (see review [42] and original papers quoted therein).

As it was the case for cosmology, the implications of the above new vistas in particle physics are rather deep because they require the re-inspection of the entire 20th century particle physics for possible revision.

As an example, the Pauli-Fermi conjecture of the neutrino, voiced to conserve the angular momentum in synthesis (7), is fundamentally dependent on the abstraction of the proton as a point-particle because necessary for the very applicability of quantum mechanics, as it is well known.

The representation of the proton as an extended charge distribution has eliminated the need for the neutrino to conserve the angular momentum because, during the "Rutherford's compression" of the hydrogen atom inside a star, the electron is constrained to orbit within the hyperdense medium inside the proton in such a fashion to have a null total angular momentum, in which case the spin of the neutron in synthesis (7) is the spin of the proton [14].

In any case, the Pauli-Fermi conjecture of the neutrino did not salvage quantum mechanics because the rest energy of the neutron is 0.782 MeV bigger than the sum of the rest energies of the proton and the electrons, in which case the Schrödinger equation is completely ineffective (for a bound state). The covering hadronic mechanics was proposed precisely for the representation of synthesis (7) and it proved to be correct at both the non-relativistic and relativistic level [14].

Despite the latter advances, our knowledge of the synthesis of the neutron remains Lilliputian at best and so much remains to be discovered by young minds of any age. For instance, we remain with the mystery of the origin of the missing 0.782 MeV needed to create the neutron which energy cannot be due to relative kinetic energy (because in this case the cross section of the scattering of protons and electrons is virtually null).

The author has submitted the hypothesis that the missing energy originates from space conceived as a universal substratum for all electromagnetic waves and all particles with a verity high energy density (due to the high value of the speed of light). In this case, the energy is expected to be transmitted from space to the neutron via a *longitudinal impulse* the author has called the *etherino*, by turning the neutron synthesis into a Lie-admissible interior dynamical problem [58].

The aspect intriguing for this paper is that, being a *longitudinal impulse* in a universal medium with physical characteristics similar to that of "rigidity" (due to the transversal character of electromagnetic waves), *the speed of transmission of energy from space to the neutron is expected to be a large multiple of the speed of light*. This illustrates again the possible existence of arbitrary speeds for interior dynamical problems in a way fully compatible with the abstract axioms of special relativity and relativistic quantum mechanics, although realized via the covering Lie-isotopic and then Lie-admissible mathematics.

As a final comment, it appears that the etherino hypothesis can represent experimental data on the so-called "neutrino experiments" in a more credible way than that provided by contemporary neutrinos that are believed to have mass. In fact,

the idea that massive particles can travel long distances within dense matter without collision is not plausible since a large number of neutrinos have to travel through a large number of nuclei without collision. By contrast, the same long travel through matter is more plausible for the etherino because the travel occurs *within the underlying universe substratum*, rather than through matter.

6.3. Nuclear Physics

The scientific scene depicted above for cosmology and particle physics occurred again in nuclear physics due to the restriction of nuclear physics research over one century to the unspoken (intent to be compatible with the conventional interpretation of special relativity and relativistic quantum mechanics despite truly vast insufficiencies.

In serious scientific circles, a theory can be assumed as being *exactly valid* for given, well defined conditions, if and only if said theory represents all experimental data from un-adulterated first principles. This has been indeed the case for the validity of quantum mechanics for the structure of the hydrogen atom.

Whenever a theory does not represent at least the main experimental data, said theory can be at best claimed to be *approximately valid*. In this conditions considered. Scientific ethics then requires the search for covering theories, as it was historically done for the atomic structure.

The conventional interpretation of special relativity and relativistic quantum mechanics can at best be considered as being approximately valid in nuclear physics because, following attempts over one century, they failed to achieve a consistent representation of the experimental data of the simplest nucleus, the deuteron, with embarrassing deviations for heavy nuclei such as the zirconium.

It is today known that the insufficiencies of standard theories in nuclear physics are due to the point-like abstraction of particles which is very effective for structures at large mutual distances of particles, such as the atomic structure, while being manifestly insufficient for the nuclear structure where the constituents are extended charge distributions in conditions of partial mutual penetration.

Isomathematics [38], the Lorentz-Santilli isosymmetry [23-30], and the experimental evidence on deviations from special relativity within physical media [43-47], offer realistic possibilities for new vistas in nuclear physics while preserving the abstract axioms of special relativity and quantum mechanics.

The central notion deals with the representation of nucleons as *extended, therefore deformable charge distribution* according to a conception dating back to Enrico Fermi, who indicated that the representation of nuclear magnetic moments may require the deformation of nucleons with consequential alteration of their magnetic moments. In view of their compatibility with the deformation theory [24-26], Santilli's Lie-isotopic formulations have provided a technical realization of Fermi's view, by achieving indeed the first known exact and invariant representations of nuclear magnetic moments [59,60] and spins [61].

Independently from the above, a structural insufficiency of standard theories is their lack of a "time arrow," with the consequential sole capability of representing isolated stable bound states that, as such, are reversible over time. Consequently, quantum mechanics is structurally insufficient for the representation of nuclear energies, because they are all based on *time irreversible processes*. Santilli Lie-Isotopic and, more appropriately, Lie-Admissible reformulations of special relativity and quantum mechanics offer a realistic possibility for quantitative studies of new nuclear energies that are already under way at U. S. corporations [52].

A novel aspect important for this presentation is that the above new vista in nuclear physics is based, not only in the representation of nucleons as extended and deformable under partial mutual penetration, but also in the consequential emergence of a new component in the nuclear force of contact, non-Hamiltonian type invariantly represented by Santilli isounit. In turn, the sole emergence of this new component in the nuclear forces implies possible advances in nuclear physics simply beyond our appraisal capabilities at this writing.

6.4. Chemistry

The author has always accepted the historical advances permitted by quantum chemistry at molecular distances, but could not accept quantum chemistry for the valence bond because valence electron pairs in singlet couplings should *repel*, rather than attract each other, due to the extremely large values of their Coulomb repulsion which is of the order of $10^{28}N$ at bond distances of 1 fm .

It is today known that this insufficiency is due to the impossibility of a consistent reduction of the universe to exterior conditions, intended to be everywhere composed of point-particles moving in vacuum (empty space). Chemistry establishes alone the impossibility in view of the non-Hamiltonian interaction originating from the deep mutual penetration of the wavepackets of valence electrons establishing the valence bond is an interior dynamical problem.

In fact, said contact non-Hamiltonian interactions have been able to overcome the repulsive Coulomb force and produce an *attraction* between the identical valence electrons in singlet coupling achieving an exact representation of the binding energies of the hydrogen and water molecules, as well as other chemical data [63].

Rather than being a mere academic curiosity, the achievement of an attractive force between valence electron pairs has permitted the identification of new bonds, called *Santilli magnecular bonds*, that, being *weaker* than the valence by conception and technical realization, allow full combustion, i.e., the absence of combustible contaminants in the exhaust such as CO, HT, etc., with evident environmental advances [64].

6.5. Biology

The insistence in the use of special relativity and quantum

mechanics in biology implies that biological structures are perfectly rigid, evidently due to the known incompatibility of said theories with the deformation theory, and eternal, evidently due to the known lack of a time arrow in the basic axioms.

By contrast, the Lie-isotopic, Lie-admissible and hyper-structural coverings of special relativity and quantum mechanics [50] are compatible with the deformation theory by conception and technical realization [25-27], and contain a time arrow in their ultimate foundations, the basic units [65].

It is hoped that these new vistas may permit more realistic representations of biological structures with expected advances perhaps not entirely conceivable at this writing.

6.6. Gravitation

Recent studies [54] have reviewed the historical objection against Einstein's gravitation, with particular reference to:

1. The lack of experimental evidence on the actual curvature of space since the bending of star light passing near the Sun is entirely due to the refraction of light in the Sun chromosphere and Newton's gravitation.

2. The impossibility to achieve a universal symmetry for Einstein's gravitation caused by the curvature of space, as studied in Section 3, with consequential adoption of "covariance" and ensuing lack of prediction of the same numerical values under the same conditions at different times.

3. The incompatibility of Einstein's gravitation with 20th century doctrines, including special relativity, electrodynamics, and quantum mechanics, which is also caused by the curvature of space.

The return of gravitation to a flat space via its formulation in the Minkowski-Santilli isospace and isogeometry presented in paper [54] under the name of *isogravitation*, has permitted the achievement of the universal invariance for all gravitational line elements [27], with a direct and unequivocal compatibility of isogravitation with 20th century theories.

The aspect of this new vista in gravitation important for this paper is the transition from the failed "unification" of the gravitational and electromagnetic fields, to their "identification" with ensuing possibility of the laboratory creation of the gravitational field according to proposals dating back to 1974 [66].

In turn, the capability to create an attractive gravitational field, combined with the predicted gravitational repulsion between matter and antimatter, is expected to initiate the study of fundamentally new propulsions beyond the rather primitive Newtonian propulsion of current use nowadays that are known as *isogeometric propulsion* [55].

6.7. Interstellar Travel

6.7.1. Foreword

As it is well known, the initiation of scientific studies on interstellar travel has been opposed by the orthodox physics community via discreditation and other means because interstellar travel requires arbitrary speeds that are popularly believed not to be permitted by Einstein's theories.

This posture is essentially based on the belief of the universal validity of Einstein's theories for all possible conditions existing in the large and small scale structure of the universe, expectedly, until the end of time.

In reality, it is the fate of all theories to have precise conditions of exact applicability beyond which the theories are only approximately valid or totally inapplicable. In the author's view, this is precisely the case of interstellar travel.

In this subsection we shall outline our current mathematical knowledge of interstellar travel, with a warning that its serious understanding requires a technical knowledge of our most advanced known mathematics for matter and antimatter, known as *hypermathematics* [50].

6.7.2. *Inapplicability of Einstein's Theories for Interstellar Travel*

Interstellar travel cannot be achieved in the event the necessary fuel has to be carried along in tanks. Consequently, in the author's view, interstellar travel can only be achieved by continuously extracting the necessary energy from space conceived as a *universal substratum* with a very large energy density, also known as *zero point energy* [72].

The process of energy extraction from space renders interstellar travel a strictly *interior* dynamical problem irreversible over time for which Einstein's theories cannot be minimally formulated in a consistent form due to the need for its Lie-admissible formulation for quantitative treatments.

In addition, we expect the need of the antimatter-type energy, known as *isodual energy*, and other aspects require the joint treatment of matter and antimatter which can be best done via of the most advanced mathematics known by mankind, the hypermathematics [50].

It should be indicated that, rather than being far fetched as popularly believed, the extraction of energy from space has resulted to be necessary for a consistent representation of the synthesis of the neutron from the hydrogen via a *longitudinal* impulse, called the *etherino* which is itself superluminal [58].

6.7.3. *The Universal Substratum for Particles and Electromagnetic Waves*

The first scientific work written by the author [73] (in 1957 when attending high school) dealt with the resolution of the historical problem of the "etherial wind," namely, the expectation that, in the event space has physical characteristics similar to medium, the orbiting of Earth around the Sun would be slowed down due to the resistance caused by said medium, which is contrary to evidence.

The argument of paper [73] is that electromagnetic waves as well as the elementary particles composing matter are "pure oscillations" of the universal substratum in the sense that the oscillations occur at specific points of space without any oscillating "little mass."

This is evidently the case for electromagnetic "waves" that, in the author opinion, cannot exist without a medium, but we have a similar occurrence also for the structure of the electron which is known to be a "pure oscillation" of a point of space with a frequency of 0.829×10^{10} Hz without any little mass" oscillating in its interior.

Consequently, the main point of Ref., [73] is that, when we move an object, we have no solid substance at all because we merely have the propagation through space of the oscillations characterizing the object without any possible "etherial wind."

Paper 73 then concluded that space should have characteristics similar to "rigidity" due to the *transverse* character of electromagnetic waves that prevent other structures of space, such as that as a "fluid" for which transverse waves cannot notoriously exist. The very high value of the speed of electromagnetic waves then suggested for space to have an extremely high energy density.

A central conception underlying the view here presented is therefore that *space is totally full and matter is totally empty*. This conception, which is clearly against our sensory perception, can be visualized by assuming the capability of "seeing" an electron. Under our ordinary time the electron appears as solid sphere due to the high value of the frequency of the oscillations. However, in the event said frequency goes to zero the electron would disappear. The same holds for the other elementary particles and, therefore, for all matter.

It should be noted that the conception of matter as being "pure oscillations" of space seems to be needed for other features of interstellar travel indicated below. It should be also noted that, according to a widely adopted view (see, e.g., Ref. [72]), matter is conceived as being "immersed" in the universal substratum.

6.7.4. *The Universal Substratum for Matter and Antimatter*

The above conception implies that the energy contained in space can be essentially conceived to be similar to energies in our environment, such as the potential energy, thus being a *positive* energy E measured with the conventional *positive* units, such as *erg*. This view is confirmed in particle laboratories since we can extract from space photons and/or particles via the use of conventional energies.

In the author's view, the above conception of space is basically insufficient to achieve interstellar travel due to the need for space to be the universal substratum also for antimatter photons and antimatter particles [74].

In the latter case, a number of consistency conditions require that the antimatter substratum should have *negative* energy, called *isodual energy* indicated as E^d although referred to *negative* units, known as *isodual units* indicated erg^d , where the upper symbol denotes the conjugation from matter to antimatter called *isoduality* and given by an anti-Hermitian map.

Additional requirements of compatibility with our current knowledge of matter and antimatter (such as the capability of extracting from space both, particles and antiparticles) imply that *space is a superposition of equal amounts conventional and isodual energy, each with extremely high density*, which can be best represented as a hyperstructure [75].

6.7.5. *Hyperspeeds*

A main assumption of the propulsion for interstellar travel, here [presented is *the capability by a "matter-spaceship" to extract from space antimatter/isodual energy* [74] because

matter and antimatter are predicted to experience gravitational repulsion at all levels of study from Newtonian mechanics to second quantization [55].

In fact, the author has not been able to achieve a mathematical propulsion model when a matter-spaceship extract conventional energy from space, since conventional (positive) energy is the origin of *gravitational attraction* according to Einstein.

By contrast, in the event a matter-spaceship is capable of extracting antimatter-energy from space propulsion in all directions *opposite* said extraction become possible at arbitrary speeds evidently proportional to the extracted antimatter energy.

It should be noted that this view is not generally adopted in current studies on interstellar travel, since they assume the achievement of propulsion via the extraction from space of positive energy from a matter-spaceship [72].

At this point, the mathematical, treatment of the propulsion here suggested under the name of *hypergeometric propulsion* becomes rather complex (see Chapter 4 of monograph [55] for technical details of the sublease of isogeometric propulsion, and the various references on antimatter quoted in this paper).

The main assumption of the proposed *hypergeometric propulsion* is that the spaceship 'achieves propulsion by changing the geometry in its environment without any motion being perceived by its operators, thus allowing instantaneous acceleration, singular trajectories, lack of sonic boom when traveling in atmosphere at supersonic speeds and other anomalies. Hypermathematics is needed for the joint use of matter and antimatter energies.

The simpler case of the *isogeometric propulsion* studied in details in Ref.[55] can be mathematically represented via the invariant lifting of the spacetime line element of special relativity

$$\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2 \equiv \Delta x'^2 \hat{I}_x + \Delta y'^2 \hat{I}_y + \Delta z'^2 \hat{I}_z - c^2 \Delta t'^2 \hat{I}_t \equiv \quad (32)$$

where

$$\Delta x = x_1 - x_2, \Delta y = y_1 - y_2, \Delta z = z_1 - z_2, \Delta t = -t_1 - t_2 \quad (33)$$

represents the motion in space and time as seen by an outside observer.

Isogeometric propulsion (for a spaceship composed of matter) occurs via the extraction from space of antimatter energy in a given direction, say Δx , resulting in a propulsion in the direction $-\Delta x$ opposite that of said extraction due to the repulsion of matter and antimatter.

The extraction of antimatter energy, say, in the x -direction, is mathematically represented via the transition from the conventional unit 1 of our spacetime to the isounit in the indicated direction, \hat{I}_x with consequential decrease of the distance $\Delta x'$ in the indicated direction due to the invariance $\Delta x = \Delta x' \hat{I}_x$.

Mathematically, there is no limit in the acquired speeds, which can be millions of times the speed of light c , since the

isounit in the x -direction can be extremely large in which case, the actual distance $\Delta x'$ covered by the spaceship is infinitesimally small.

Some of the consequences of the above isogeometric locomotion can be expressed as follow:

A. The extraction of any energy from space causes a deformation of the structure of space, called *mutation*.

B. The size of the spaceship perceived by external and internal observers can be dramatically different.

C. There cannot be mutation of space without a corresponding mutation of time, thus illustrating the name of "spacetime machine" in Ref. [55].

D. The visual detection of a spaceship by an external observer does not mean that the spaceship exists at his/her time.

E. The lapse of time measured by the spaceship operator can be much smaller than the lapse of time perceived by an external observer.

The generalization of the above isogeometric propulsion to the hypergeometric form is excessively complex for the limited scope of this presentation and will be presented elsewhere.

6.7.6. Hypercommunications

At interstellar distances, electromagnetic communications are like the smoke signals of the American Indians because they are excessively slow compared to the distances at hand. Since interstellar travel cannot even be conceived without effective communications, we find again the need of surpassing the speed of light which has been opposed for one century by the orthodox physics community.

Yet, there already exist in the refereed literature *experiments* showing the propagation of signals faster than light (see, e.g., Ref. [19]); signals propagating at a large multiple the speed of light have already been computed for the *longitudinal* propagation of energy in the synthesis of the neutron [58]; and, according to (unpublished) initial calculations, the Lie-Isotopic and Lie-admissible coverings of Maxwell's equations predict virtually instantaneous longitudinal signals particularly if space is assumed to have a characteristic similar to external high "rigidity."

6.7.7. Hyperfading

Trajectory corrections to avoid collisions can indeed be done for spaceships traveling at interplanetary speeds, but said corrections cannot be done when traveling at interstellar speeds that are expected to be millions of times the speed of light.

A conceivable solution, here suggested by the author under the name of *hyperfading*, is the capability of interstellar spaceships to alter their physical structure in such a way to cross through astrophysical bodies without collisions or damage. This also implies the capability to enter oceans without creating any wave.

Rather than being far fetched, this possibility is within current *mathematical* possibilities. In fact, the mutation of space which is inherent in the hypergeometric locomotion implies the alteration of the spacetime of the spaceship and of its inhabitants. Hyperfading is then mathematically possible

via the control of time permitted by antimatter, a conceivable dramatic reduction of the characteristic frequencies of the spaceship and its inhabitants under which the spaceship literally "fades away" from the universe, and other means.

Thanks to advances in antimatter with the ensuing *isogeometric propulsion* [55] in which hyperfading could be achieved due to the structural alteration of the local spacetime, the control of time predicted from a suitable use of *isodual energies* (negative energies referred to negative units), and other means.

It is hoped that young minds of any age will ignore biased obstructions from the orthodox physics community and initiate indeed scientific studies on the fascinating open problems connected to interstellar travel because, in the author's view, the achievement by mankind of interstellar travel is only a question of time.

7. Concluding Remarks

During the studies on axiom-preserving isotopies and genotopies of 20th century theories, the author has stated various times that:

Rather than abusing the names of Lorentz, Poincaré, Einstein and other founders of 20th century physics by applying their theories under interior dynamical conditions they were not intended for, and cannot be properly formulated and tested, the best way to honor their names is to maintain their axioms, and enlarge their conditions of exact applicability via the use of broader mathematics specifically built for interior conditions.

The implications in surpassing 20th century theories for interior dynamical conditions are so deep for all quantitative sciences to imply a "new scientific era," as indicated in the title of Ref. [42].

References

- [1] L. V. Lorentz, *Philos. Mag.* 34, 287 (1867).
- [2] H. Poincaré, *Archives Néerland. des Sciences Exactes et Naturelles*, 2, vol. 5, 252-278 (1900).
- [3] A. Einstein, *Ann. Phys. (Leipzig)* 17, 891 (1905).
- [4] E. Hubble, *Proceedings of the National Academy of Sciences of the United States of America*, 168 (1929).
- [5] F. Zwicky, *Proceedings of the National Academy of Sciences of the United States of America*, 773 (1929).
- [6] M. J. Rees, *Nature*, 468 (1966).
- [7] A. B. Whiting, "The expansion of space: free-particle motion and the cosmological redshift," *The Observatory*, 174 (2004) <http://adsabs.harvard.edu/abs/2004Obs...124174W>
- [8] P. Riess et al. *Astronomical Journal*, 1009 [1998]; S. Perlmutter et al. *Astrophysical Journal*, 565 (1999).
- [9] R. M. Santilli, "Embedding of Lie algebras in Lie-admissible structures," *Nuovo Cimento*, 570 (1967), available in free download from the link <http://www.santilli-foundation.org/docs/Santilli-54.pdf>
- [10] R. M. Santilli, "Can strong interactions accelerate particles faster than the speed of light?" *Lettere Nuovo Cimento*, 145 (1982) www.santilli-foundation.org/docs/Santilli-102.pdf
- [11] R. M. Santilli, "Isotopies and genotopies of dimension 1, 2, 4, 8, their isoduals and pseudoduals, and hidden numbers of dimension 3, 5, 6, 7," *Algebras, Groups and Geometries* Vol. 10, 273-321 (1993) <http://www.santilli-foundation.org/docs/Santilli-34.pdf>
- [12] R. M. Santilli, "Nonlocal-Integral Isotopies of Differential Calculus, Mechanics and Geometries," in *Isotopies of Contemporary Mathematical Structures*, P. Vetro Editor, *Rendiconti Circolo Matematico Palermo, Suppl. Vol. 42*, 7-82 (1996), <http://www.santilli-foundation.org/docs/Santilli-37.pdf>
- [13] R. M. Santilli, "Lie-admissible invariant representation of irreversibility for matter and antimatter at the classical and operator levels," *Nuovo Cimento B* 121, 443 (2006), available in free pdf download from the link <http://www.santilli-foundation.org/docs/Lie-admiss-NCB-I.pdf>
- [14] J. V. Kadeisvili, "The Rutherford-Santilli Neutron," *Hadronic J.* 2008; 31, 1, <http://www.i-b-r.org/Rutherford-Santilli-II.pdf> available in html version in the website <http://www.i-b-r.org/Rutherford-Santilli-neutron.htm>
- [15] R. M. Santilli, *Foundation of Theoretical Mechanics*, Volume I (1978) [15a], and Volume II (1982) [15b], Springer-Verlag, Heidelberg, Germany, <http://www.santilli-foundation.org/docs/Santilli-209.pdf> <http://www.santilli-foundation.org/docs/santilli-69.pdf>
- [16] R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry*, Volumes I [16a], II [16b], III [16c], IV [16d] and V [16e] International Academic Press 2008, <http://www.i-b-r.org/Hadronic-Mechanics.htm>
- [17] E. Wall, *The Physics of Tachions*, Hadronic Press (1995).
- [18] A. Enders and G. Nimtz, *J. Phys. I (France)*, 1693-1698 (1992)
- [19] G. Nimtz, *Foundations in Physics*, 1193-1199 (2011)
- [20] D. Sokolovski and E. Akhmatkaya, *Phys. Rev. A*, 052110 (2013).
- [21] A. Ereditato et al., "Measurement of the neutrino velocity with the OPERA detector in the CNGS beam," CERN preprint, <http://www.santilli-foundation.org/docs/OPERA-experiment.pdf>
- [22] N. Yu et al., "Measurement of the Velocity of Neutrinos from the CNGS Beam with the Large Volume Detector," *Phys. Rev. Lett.* v.109, 070801(2012) [70d]
- [23] R. M. Santilli, "Lie-isotopic Lifting of Special Relativity for Extended Deformable Particles," *Lettere Nuovo Cimento*, 545 (1983), <http://www.santilli-foundation.org/docs/Santilli-50.pdf>
- [24] R. M. Santilli, "Lie-isotopic Lifting of Unitary Symmetries and of Wigner's Theorem for Extended and Deformable Particles," *Lettere Nuovo Cimento* Vol. 38, 509 (1983), <http://www.santilli-foundation.org/docs/Santilli-51.pdf>
- [25] R. M. Santilli, "Isotopies of Lie Symmetries," I (basic theory) and II (isotopies of the rotational symmetry), *Hadronic J.* Vol. 8, 36 and 85 (1985), <http://www.santilli-foundation.org/docs/santilli-65.pdf>
- [26] R. M. Santilli, "Isotopic Lifting of the SU(2) Symmetry with Applications to Nuclear Physics," *JINR rapid Comm.* Vol. 6, 24-38 (1993), <http://www.santilli-foundation.org/docs/Santilli-19.pdf>

- [27] R. M. Santilli, "Nonlinear, Nonlocal and Noncanonical Isotopies of the Poincare' Symmetry," *Moscow Phys. Soc.* Vol. 3, 255 (1993), <http://www.santilli-foundation.org/docs/Santilli-40.pdf>
- [28] R. M. Santilli, "Recent theoretical and experimental evidence on the synthesis of the neutron," *Communication of the JINR, Dubna, Russia, No. E4-93-252* (1993), published in the *Chinese J. System Eng. and Electr.* Vol. 6, 177 (1995), <http://www.santilli-foundation.org/docs/Santilli-18.pdf>
- [29] R. M. Santilli, "Isorepresentation of the Lie-isotopic SU(2) Algebra with Application to Nuclear Physics and Local Realism," *Acta Applicandae Mathematicae* Vol. 50, 177 (1998), <http://www.santilli-foundation.org/docs/Santilli-27.pdf>
- [30] R. M. Santilli, "Isominkowskian Geometry for the Gravitational Treatment of Matter and its Isodual for Antimatter," *Intern. J. Modern Phys. D*, 351 (1998), <http://www.santilli-foundation.org/docs/Santilli-35.pdf>
- [31] J. V. Kadeisvili, "Direct universality of the Lorentz-Poincare'-Santilli isosymmetry for extended-deformable particles, arbitrary speeds of light and all possible space-times" in *Photons: Old problems in Light of New Ideas*, V. V. Dvoeglazov Editor Nova Science (2000), available as free download from <http://www.santilli-foundation.org/docs/Santilli-25.pdf>
- [32] A. K. Aringazin and K. M. Aringazin, "Universality of Santilli's iso-Minkowskian geometry" in *Frontiers of Fundamental Physics*, M. Barone and F. Selleri, Editors Plenum 91995), available as free download from <http://www.santilli-foundation.org/docs/Santilli-29.pdf>
- [33] R. M. Santilli, *Isotopic Generalizations of Galilei and Einstein Relativities*, Vols. I [14a] and II [14b] (1991), International Academic Press <http://www.santilli-foundation.org/docs/Santilli-01.pdf> <http://www.santilli-foundation.org/docs/Santilli-61.pdf>
- [34] R. M. Santilli, *Elements of Hadronic Mechanics*, Vol. I (1995) [15a], Vol. II (1995) [15b] Academy of Sciences, Kiev, <http://www.santilli-foundation.org/docs/Santilli-300.pdf> <http://www.santilli-foundation.org/docs/Santilli-301.pdf>
- [35] A. K. Aringazin, A. Jannussis, F. Lopez, M. Nishioka and B. Veljanosky, *Santilli's Lie-Isotopic Generalization of Galilei and Einstein Relativities*, Kostakaris Publishers, Athens, Greece (1991), <http://www.santilli-foundation.org/docs/Santilli-108.pdf>
- [36] J. Lohmus, E. Paal, and L. Sorgsepp, *Nonassociative Algebras in Physics* Hadronic Press, Palm Harbor, Florida (1994), <http://www.santilli-foundation.org/docs/Lohmus.pdf>
- [37] D. S. Sourlas and G. T. Tsagas, *Mathematical Foundation of the Lie-Santilli Theory*, Ukraine Academy of Sciences (1993), <http://www.santilli-foundation.org/docs/santilli-70.pdf>
- [38] R. M. Santilli, "Nonlocal-Integral Isotopies of Differential Calculus, Mechanics and Geometries," in *Isotopies of Contemporary Mathematical Structures*, P. Vetro Editor, *Rendiconti Circolo Matematico Palermo, Suppl.* Vol. 42, 7-82 (1996), <http://www.santilli-foundation.org/docs/Santilli-37.pdf>
- [39] J. V. Kadeisvili, *Santilli's Isotopies of Contemporary Algebras, Geometries and Relativities*, Ukraine Academy of Sciences, Second edition (1997), <http://www.santilli-foundation.org/docs/Santilli-60.pdf>
- [40] Raul M. Falcon Ganformina and Juan Nunez Valdes, *Fundamentos de la Isoteoria de Lie-Santilli*, International Academic Press (2001), <http://www.i-b-r.org/docs/spanish.pdf>
- [41] A. K. Aringazin, "Studies on the lie-Santilli IsoTheory with Unit of general Form," *Algebras, Groups and Geometries*, 299 (2011), <http://www.santilli-foundation.org/docs/Aringazin-2012.pdf>
- [42] I. Gandzha and J. Kadeisvili, *New Sciences for a New Era: The Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli*, Sankata Printing Press, Nepal (2011), <http://www.santilli-foundation.org/santilli-scientific-discoveries.html>
- [43] R. M. Santilli, "Experimental Verifications of IsoRedShift with Possible Absence of Universe Expansion, Big Bang, Dark Matter, and Dark Energy," *The Open Astronomy Journal*, 124 (2010), <http://www.santilli-foundation.org/docs/Santilli-isoredshift.pdf>
- [44] R. M. Santilli, G. Amato and G. West, "Experimental Confirmation of the IsoRedShift at Sun at Sunset and Sunrise with Consequential Absence of Universe Expansion and Related Conjectures," *Journal of Computational Methods in Sciences and Engineering*, Vol. 12, pages 165-188 (2012). <http://www.santilli-foundation.org/docs/Confirmation-sun-IRS.pdf>
- [45] H. Ahmar, G. Amato, J. V. Kadeisvili, J. Manuel, G. West, and O. Zogorodnia, "Additional experimental confirmations of Santilli IsoRedShift and the consequential absence of the universe expansion," *Journal of Computational Methods in Science and Engineering*, 321 (2013), <http://www.santilli-foundation.org/docs/IRS-confirmations-212.pdf>
- [46] R. M. Santilli, "Isominkowskian representation of cosmological redshifts and the internal redshift/blueshift of quasars," contributed paper in *Frontiers of Fundamental Physics*, M. Barone and F. Selleri, Editors, Plenum Press (1994) <http://www.santilli-foundation.org/docs/ire-obd-1994.pdf>
- [47] R. M. Santilli, "Representation of galactic dynamics via isoshifts without universe expansion, dark matter and dark energy," *American Journal of Modern Physics* Vol. 4, pages 26-43 (2015) <http://www.thunder-energies.com/docs/dark-matter-2015.pdf>
- [48] R. Folman and E. Recami, "On the phenomenology of tachyon radiation", *Found. Phys. Lett.*, 2 (1995).
- [49] E. Recami, M. Zamboni-Rached, C. A. Dartora, "Localized X-shaped field generated by a superluminal electric charge," *Phys. Rev. E*, 027602 (2004).
- [50] R. Anderson, "Outline of Hadronic Mathematics, Mechanics and Chemistry as Conceived by R. M. Santilli," *American Journal of Modern Physics* Vol. 4(5), pages 1-16 (2015) <http://www.santilli-foundation.org/docs/hadronic-math-mec-c-hem.pdf>
- [51] R. M. Santilli. *Isotopic Generalization of Galileo and Einstein Relativities*. Hadronic Press 1991, Volumes I [3a] and II [3b], <http://www.santilli-foundation.org/docs/Santilli-01.pdf> <http://www.santilli-foundation.org/docs/Santilli-35.pdf>
- [52] R. M. Santilli, "Isorelativity," invited plenary lectures (2014) <http://www.world-lecture-series.org/isorelativity-2014-i> <http://www.world-lecture-series.org/isorelativity-2014-ii> <http://www.world-lecture-series.org/isorelativity-2014-iii>

- [53] R. M. Santilli, "Isorelativity for matter and its isodual for antimatter," invited paper, American Journal of Modern Physics, in press (2015)
- [54] R. M. Santilli, "Rudiments of isogravitation for matter and its isodual for antimatter," American Journal of Modern Physics Vol. 4(5), pages 59-75 (2015) <http://www.santilli-foundation.org/docs/isogravitation.pdf>
- [55] R. M. Santilli, *Isodual Theory of Antimatter with Applications to Antigravity, Grand Unifications and Cosmology*, Springer (2006).
- [56] R. M. Santilli, "An intriguing legacy of Einstein, Fermi, Jordan and others: The possible invalidation of quark conjectures," Found. Phys. Vol. 11, 384-472 (1981) <http://www.santilli-foundation.org/docs/Santilli-36.pdf>
- [57] Yu. Arestov, R. M. Santilli and V. Solovianov, "Experimental evidence on the isominkowskian character of the hadronic structure," Foundation of Physics Letters Vol. 11, pages 483-492 (1998) <http://www.santilli-foundation.org/docs/Santilli-52.pdf>
- [58] R. M. Santilli, "The etherino and/or the neutrino hypothesis," Foundation of Physics, 670 (2007). <http://www.santilli-foundation.org/docs/EtherinoFoundPhys.pdf>
- [59] R. M. Santilli, "A quantitative isotopic representation of the deuteron magnetic moment," in *Proceedings of the International Symposium 'Dubna Deuteron-93*, Joint Institute for Nuclear Research, Dubna, Russia (1994), <http://www.santilli-foundation.org/docs/Santilli-134.pdf>
- [60] R. M. Santilli, "Nuclear realization of hadronic mechanics and the exact representation of nuclear magnetic moments," Intern. J. of Phys. Vol. 4, 1-70 (1998) <http://www.santilli-foundation.org/docs/Santilli-07.pdf>
- [61] A. A. Bhalekar and R. M. Santilli, "Exact and invariant representation of nuclear magnetic moments and spins according to hadronic mechanics," invited paper, to appear (2015).
- [62] Thunder Energies Corporation, website, <http://www.thunder-energies.com>
- [63] R. M. Santilli, *Foundations of Hadronic Chemistry, with Applications to New Clean Energies and Fuels*, Kluwer Academic Publishers (2001), <http://www.santilli-foundation.org/docs/Santilli-113.pdf> Russian translation <http://i-b-r.org/docs/Santilli-Hadronic-Chemistry.pdf>
- [64] C. P. Pandhurnekar, "Advances on Alternative Fuels with Santilli Magnetic Structure," International Journal of Alternative Fuels, ISSN: 2051-5987, Vol.17, 2014, <http://www.santilli-foundation.org/docs/Magnegas-2015.pdf>
- [65] R. M. Santilli, *Isotopic, Genotopic and Hyperstructural Methods in Theoretical Biology*, Ukraine Academy of Sciences, Kiev (1994) <http://www.santilli-foundation.org/docs/santilli-67.pdf>
- [66] R. M. Santilli, "Partons and Gravitation: some Puzzling Questions," (MIT) Annals of Physics, Vol. 83, 108-157 (1974), <http://www.santilli-foundation.org/docs/Santilli-14.pdf>
- [67] O. M. P. Bilanik, V. K. Deshpande, and E. V. C. G. Sudareshan, "Meta" Relativity, American Journal of Physics, Vol. 30, pages 718-723 (1962), <http://www.santilli-foundation.org/docs/1962-006.pdf>
- [68] A. O. Barut, *Electrodynamics and Classical Theory of Fields and Particles* (Dover, Nova Iorque, 1964), Classics of Science and Mathematics.
- [69] T. G. Pavlopoulos, "Are we observing Lorentz violation in gamma rays bursts?", Physics letters B, 625, 13-18 (2005), <http://www.santilli-foundation.org/docs/Extended-SR.pdf>
- [70] George P. Shpenkov, "On the Superluminal Speed in View of the Dialectical Model of the Universe," Infinite Energy, Vol. 77, pages 1-6, (2008); <http://www.santilli-foundation.org/docs/Shpenkov.pdf>
- [71] D. M. Bilaniuk and E. C. O. Sudarshan, "Particles beyond the light barrier," Physics Today, May 1969 <http://www.santilli-foundation.org/docs/Sudarshan.pdf>
- [72] Mark McCandlish, *Zero Point*, Documentary <https://vimeo.com/133170463>
- [73] R. M. Santilli, "Why space is rigid," (in Italian) Il Pungolo Verde, 1957 <http://www.santilli-foundation.org/docs/Santilli-49.pdf>
- [74] R. M. Santilli, "Antigravity, Hadronic Journal, 257-284 (1994) <http://www.santilli-foundation.org/docs/Santilli-13.pdf>
- [75] B. Davvaz, R. M. Santilli and T. Vougiouklis, Studies Of Multi-Valued Hyperstructures For The characterization Of Matter-Antimatter Systems And Their Extension Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes, C. Corda, Editor, Kathmandu University (2011), pages 45-57 <http://www.santilli-foundation.org/docs/Davvaz-Sant-Vou.pdf>

Santilli's Isodual Mathematics and Physics for Antimatter

P. M. Bhujbal

Department of Physics, Nutan Adarsh Arts, Commerce and Smt. Maniben Harilal Wegad Science College, Umrer, India

Email address:

prashantmbhujbal@yahoo.com

To cite this article:

P. M. Bhujbal. Santilli's Isodual Mathematics and Physics for Antimatter. *American Journal of Modern Physics*. Special Issue: Issue II: Foundations of Hadronic Mechanics. Vol. 5, No. 2-1, 2016, pp. 161-184. doi: 10.11648/j.ajmp.2016050201.20

Received: July 3, 2015; **Accepted:** July 4, 2015; **Published:** June 1, 2016

Abstract: In this paper, we study the isodual branch of hadronic mechanics achieved by the Italian-American scientist R. M. Santilli following decades of research. We show that, thanks to the background isodual mathematics (whose knowledge is assumed), Santilli's isodual theory of antimatter appears to be the only axiomatically consistent and time invariant theory permitting a classical representation of neutral (as well as charged) antimatter whose operator image is equivalent to charge conjugation, thus verifying all known experimental data in the field. We then show the important prediction of gravitational repulsion between matter and antimatter, with particular reference to the prediction that light emitted by antimatter, known as isodual light, experiences gravitational repulsion when in the field of matter. We finally point out that recent detections of antimatter galaxies via Santilli telescope with concave lenses may eventually result to be the first experimental evidence of antigravity.

Keywords: Antimatter, Isodual Light, Antigravity

1. Introduction

After being conjectured by A. Schuster in 1898, antimatter was predicted by P. A. M. Dirac [1] in the late 1920's in the negative-energy solutions of his celebrated equation. Dirac himself soon discovered that particles with negative-energy do not behave in a physical way and, for this reason, he submitted his celebrated "hole theory," which subsequently restricted the study of antimatter to the sole level of second quantization [2]. This occurrence created an imbalance in the physics of this century because matter is described at all levels of study, from Newtonian mechanics to quantum field theory, while antimatter is solely treated at the level of second quantization.

To initiate the study for the future removal of this imbalance in due time, Santilli presented a theory of antimatter which has been conceived to begin at the purely classical Newtonian level, and then to admit corresponding images at all subsequent levels of study [3] in which the guiding principle is to identify a map which possesses the main mathematical structure of charge conjugation, yet it is applicable at all levels, and not solely at the operator level.

The main characteristic of charge conjugation is that of being antiautomorphic (where the term "automorphic" is referred to the map of a given space onto itself). After studying

a number of possibilities, Santilli has selected a map which is anti-isomorphic (where the term "isomorphic" is referred to a map from one space onto another of equivalent topological characteristics to be identified later on) applicable at all levels of study, and given by the following isodual map here generically expressed to an arbitrary quantity Q (i.e. a function, or a matrix or an operator)

$$Q(x, \varphi, \dots) \rightarrow Q^d(x^d, \varphi^d, \dots) = -Q^\dagger(-x^\dagger, -\varphi^\dagger, \dots) \quad (1)$$

which, for consistency, must be applied to the totality of the mathematical structure of the conventional theory of matter, including numbers, fields, spaces, geometries, algebras, etc. This results in a new mathematics, called isodual mathematics, which is at the foundation of the classical isodual theory of antimatter of this paper.

The isodual map was first proposed by Santilli [4] in 1985 and then remained ignored for several years. According to it the isoduality can be represented as $+1 \rightarrow 1^d = -1$. The first hypothesis on the isodual theory of antimatter appeared for the operator version in 1993 [5] which also contains an initial study of the equivalence between isoduality and charge conjugation. The prediction of the isodual theory that antimatter in the field of matter experiences antigravity was first submitted in 1994 [6] which also proposed an experiment

for the measure of the gravity of elementary antiparticles in the gravitational field of earth. The experiment essentially consists of comparative measurements under the gravity of collimated, low energy beams of positrons and electrons in horizontal flight on a tube with sufficiently high vacuum as well as protection from stray and patch fields and of sufficient length to permit a definite result, e.g. the view by the naked eye of the displacements due to gravity of the positron and electron beams on a scintillator at the end of the flight.

Theoretical and experimental studies on the isodual theory of antimatter were conducted at the International Workshop on Antimatter Gravity and Anti-Hydrogen Atom Spectroscopy, held in Sepino, Italy, in May 1996 [7].

The motivations for the classical isodual theory of antimatter are rather numerous. First, there is the need indicated earlier to achieve a full equivalence in the treatment of matter and antimatter beginning at the classical level. In fact, far away galaxies and quasars may well be made up of antimatter. The absence of a classical theory of antimatter therefore implies the evident impossibility of quantitative studies of this important astrophysical issue.

Second, the current gravitational treatment of antimatter is afflicted by a number of problematic aspects. Current theories are based on only one map from classical to operator settings, the naive or symplectic quantization. Therefore, conventional classical representations of antimatter via positive energies do not yield antiparticles under quantization, but conventional particles with the mere reversal of the sign of the charge.

Third, there is a fundamental incompatibility between current theories of gravitation and unified gauge theories of electroweak interactions which is due precisely to antimatter. In fact, current gravitational theories characterize antimatter via a positive-definite energy-momentum tensor, while electroweak theories characterize antiparticles via negative energy states. Additional motivations have been identified in [8-10]. The need for a systematic study aiming at a resolution of these issues is then beyond scientific doubts.

The isodual theory emerged from the identification of negative units in the antiparticle component of the conventional Dirac equation and the reconstruction of the theory with respect to that unit. Isoduality therefore provides a mere reinterpretation of Dirac's original notion of antiparticle, while leaving all numerical predictions under electroweak interactions essentially unchanged.

Santilli was able to construct the isodual theory of antimatter which characterizes antimatter at all possible levels, from Newtonian mechanics to second quantization on the basis of his isodual mathematics [11-13].

In particular, isodual mathematics permitted the first known geometrically consistent representation of the gravitational field of a neutral antimatter body via the Riemann-Santilli isodual geometry defined over isodual fields [11, 13].

It should be stressed that the isodual theory of antimatter requires, for consistency requirement, the conjugation of all physical quantities of matter as well as, most importantly, all their units of measurements. Consequently, antimatter evolves along a time moving backward, the $t^d = -t$, and has

negative-definite energy $E^d = -E$ along Dirac's original conception [14]. The historical inconsistencies are resolved via the joint conjunction of the related units; in fact, negative time and negative energy referred to negative units of time and energy are as causal as our positive time and energies referred to positive units of time and energy.

Eventually, in this paper the author tried to focus on some important features of Santilli's Isodual Physics for antimatter based on Santilli's Isodual Mathematics.

2. Salient Features of Santilli's Isodual Mathematics

The Newton's equations, Galileo's relativity and Einstein's special and general relativities were conceived before the discovery of antimatter and therefore no classical representation of "neutral" antimatter could be generated since at that time only conjugation from matter to antimatter was the change of sign of the charge [15], which created one of the biggest scientific imbalance in the history because throughout the 20th century matter was studied at all possible levels, from Newtonian mechanics to second quantization, while antimatter was solely studied at the particle level. In essence, the prevalent stand still adopted is that, since Einstein's special and general relativities do not provide a proper description, antimatter does not exist in the universe in any appreciable amount. The sole generally admitted exception is that of man-made antiparticles created in laboratory, since their existence cannot be denied.

The above scientific imbalance was identified, apparently for the first time, by the Italian-American scientist Ruggero Maria Santilli. Santilli has been interested since his graduate studies to ascertain whether a far away galaxy is made up of matter or of antimatter. He soon learned that Newtonian, Galilean and Einsteinian theories had no value for the indicated problem since far away galaxies must be assumed to be neutral, in which case said theories had no distinction whatsoever between matter and antimatter. For this reason, Santilli initiated a long journey that first required the identification of mathematical means for the consistent classical distinction between neutral matter and antimatter prior to any possible physical application. Santilli discovered that a mathematics for the consistent classical treatment of neutral (or charged) antimatter did not exist and had to be built.

The 20th century position on antimatter implied the rather general belief that antimatter galaxies do not exist. This stringent stand eliminated altogether the problem of detecting antimatter asteroids on grounds that they do not exist due to the absence of the antimatter galaxies and related antimatter supernovas needed for their origination. This position was evidently based in the unspoken intent of maintaining the validity of Einstein's theories for all of the universe via the denial of the existence of antimatter galaxies, despite it being disproved by evidence since our Earth has indeed been hit in the past by devastating antimatter asteroids, and similar

asteroids have been detected by various observatories.

In fact, the catastrophic 1908 Tunguska explosion in Siberia with the power of one thousand Hiroshima nuclear bombs can be solely interpreted in a scientific way as being due to an antimatter asteroid annihilating in our atmosphere [15, 16]. This is due to various reasons, such as the complete absence of debris, let alone of a crater, in the ground. The Tunguska explosion excited the entire Earth's atmosphere for days, to such an extent those two days following the explosion; people could read newspapers in Sydney, Australia, at midnight without artificial light; and other reasons. Such a large excitation of the atomic and molecular constituents of our atmosphere can only be scientifically (i.e. quantitatively) represented as being due to huge radiations that, in turn, can only originate from the annihilation of an antimatter asteroid with our matter atmosphere. The widely accepted "interpretation" of the Tunguska explosion as being due to a (matter) comet has no scientific credibility due to the impossibility of such an origination to excite the entire Earth's atmosphere for days, and occur with the absence of debris in the grounds, let alone with the absence of a crater.

NASA has also reported explosions in our upper atmosphere that can only be due to small antimatter asteroids because annihilating at the time of contact with the upper portion of our matter atmosphere. Similarly, astronauts and cosmonauts have observed ashes in our upper atmosphere when on the dark side with respect to our Sun; these ashes can be best interpreted as being due to antimatter cosmic rays that annihilate in our atmosphere, because the only cosmic rays that can reach us at sea level being those due to matter cosmic rays.

As indicated above as well as earlier by Santilli and others, the existence of antimatter stars and galaxies is imperative and should not be ignored. As a representative example out of many, that recall antimatter is thought to exist in the Oort cloud in view of a possible explanation for gamma ray bursts. In fact, these phenomena can be explained by the annihilation of matter and antimatter asteroids or small comets. The explosion would create powerful gamma ray bursts and accelerate matter [17].

Besides antimatter asteroids, it is possible that Earth has been hit in the past by antimatter comets as indicated by the old observations, since the biblical times, not only of excessive brilliance but also of trajectories in our atmosphere that cannot be interpreted as being due to matter comets, e.g., because of slow penetration of the said objects in our atmosphere. In conclusion, the evidence on the existence of antimatter asteroids as well as of antimatter comets and their possibility of hitting Earth again is sufficiently serious [18].

Scientific studies in the detection of antimatter asteroids requires mathematical and physical theories suitable for the classical treatment of neutral antimatter evidently because antimatter asteroids are too large to be treated via operator theories and they must be assumed as being neutral since they are isolated in space. Santilli has repeatedly stated in his writings that: A protracted lack of solution of physical problems is generally due to the use of insufficient or

inadequate mathematics [15]. Additionally, Santilli stated that: There cannot exist a really new physical theory without a really new mathematics, and there cannot exist a really new mathematics without new numbers [loc. cit.]. For this reason, Santilli had spent decades in purely mathematical research, firstly, to identify new numbers and, secondly, to develop new mathematics that would allow a classical treatment of neutral or charged antimatter, because the entire body of applied mathematics is built on numbers.

Along these lines, the most fundamental and very first paper published by Santilli on antimatter is Ref. [19] of 1993 that introduced for the first time new numbers called "isodual" where the prefix "iso" was introduced in the Greek sense of indicating the preservation of conventional axioms used for matter and the term "dual" stands to indicate the map from matter to antimatter. The role of Santilli isodual numbers is such that his entire theory of antimatter that is called "isodual" precisely because of the main character of the new basic numbers.

It should be noted that Santilli discovered the new isodual number in Ref. [19] as a particular case of much more general numbers he called "isonumbers" and "genonumbers" and their isoduals.

Following the discovery of new numbers, Santilli constructed in Ref. [20] also of 1993 the isodualities of the Euclidean and Minkowski spaces which were evidently needed for any possible physical applications. He then proceeded in Ref. [21] to the construction of the isodual image of Lie's theory because it is evidently necessary for the construction of basic symmetries for antimatter, viz. the isodual images of the rotation, Galileo and Lorentz symmetries.

Finally, in the mathematical memoir [22] Santilli made a second fundamental mathematical discovery, a new formulation of the ordinary differential calculus that resulted in being crucial for the achievement of the first known formulation of Newton's equations for neutral or charged antiparticles. The complete formulation of the novel isodual mathematics was first presented by Santilli in monograph [8] of 1994 and then updated in monographs [15] of 2001.

Following the achievement of structural consistency of the new isodual mathematics, and only thereafter, Santilli initiated his physical studies with paper [23] of 1993 written on his original aim of the 1960s as a graduate student, how to detect possible antimatter stars and galaxies.

It is evident that the problem of detecting possible antimatter asteroids is of such a magnitude that it cannot be left unaddressed just to maintain the validity of Einstein's theories for antimatter. In an event North America is hit by an antimatter asteroid even with the size of a football, all North American communications will be disrupted, while the Military will be inoperative for days, due to extreme radiations absorbed and re-emitted by Earth's atmosphere. The same holds in the unfortunate event an antimatter asteroids hits India, Russia, China or other regions. Consequently, the problem of possible antimatter asteroids requires attention not only by the people at large, but also by the scientific and

military communities.

The multitude of open problems created by the detection of antimatter have been studied for decades by Santilli who has provided scientific arguments establishing that the threat to Earth caused by antimatter asteroids is more serious than what popularly believed in contemporary academia.

Therefore, Santilli had to confront the problem of identifying a mathematical conjugation (also called map or duality) capable of performing the transition from matter to antimatter at the purely classical Newtonian level, irrespectively of whether matter and antimatter are neutral or charged, under the condition that such a map recovers charge conjugation at the quantum level for the sake of consistency, which is evidently needed.

Following a decade of unpublished trials and errors, Santilli selected the following main assumption for the construction of the needed new mathematics. Recall that the conventional charge conjugation is defined on a Hilbert space H with states $\psi(r)$ over the field of complex numbers C and can be characterized by a conjugation of the type applied to the quantum representation of matter

$$C \psi(r) = -\psi^\dagger(r); \quad (2)$$

where r is the coordinate of the Euclidean representation space.

Consequently, Santilli introduced an anti-Hermitian conjugation called isoduality and denoted with the upper index d that, by central condition, has to be applied to all physical quantities, to all their units and to all their operations, and can be written as,

$$Q(t, r, v, \dots) \rightarrow Q^d(t^d, r^d, v^d, \dots) = -Q^\dagger(-t^\dagger, -r^\dagger, -v^\dagger, \dots) \quad (3)$$

where Q denotes a generic quantity depending on time t , coordinates r , velocities v , and any other needed variables [15].

For the trivial case of real numbers, isoduality reduces to the mere change of the sign of all quantities, all their units and the related operations. In the event a given body is charged, the isoduality evidently also applies by changing the sign of the charge. Hence, Santilli's isoduality applies irrespectively of whether the body is charged or not.

The main difference between conventional charge conjugation and Santilli's isoduality is that the former solely applies at the quantum level, while the latter applies, by central conception, at the classical Newtonian level, as well as at all subsequent levels of study, including the quantum level in which charge conjugation and isoduality are equivalent [15].

In terms as simple as permitted by the advanced nature of the topic, the classical isodual map from neutral matter to the corresponding neutral antimatter requires that all physical quantities change their sign. Consequentially, under the isodual map, time, energy, linear momentum, entropy, and other positive-definite physical quantities, become negative. There have been a number of proposals that antimatter must move backward in time as a condition to admit annihilation

into light. This rather natural assumption has been dismissed due to the violation of causality when motion backward in time is treated with the mathematics used for matter [15].

However, for Santilli's isodual mathematics, antimatter's motion backward in time $t^d = -t < 0$, when referred to a negative unit of time (e.g., $s^d = -1$ s), is as causal as matter moving forward in our time $t > 0$ when referred to the usual positive units of time (e.g., $+1$ s) [19].

Similarly, it is known since Dirac's time that negative energies also violate causality laws. However, Santilli has shown that negative-definite isodual energies $E^d = -E < 0$ referred to negative units (e.g., $\text{erg}^d = -1$ erg) are as physical as conventional positive energies $E > 0$ referred to positive units (e.g., $+1$ erg). Inconsistencies emerge only under crossovers of the two worlds, such as when positive energies are measured with negative units and vice versa [19].

In going deeper into the problem, Santilli discovered that the correct formulation of antimatter requires an entire new mathematics, today known as Santilli's isodual mathematics, which can be defined as the anti-Hermitian image of the entire mathematics used for matter, thus including isodual numbers, isodual functional analysis, isodual differential calculus, isodual Lie's theory, etc. In fact, any mix-up, even minute, of the mathematics for matter and that for antimatter leads to catastrophic inconsistencies that are generally not realized by non-experts in the field [19].

Santilli's research in antimatter was delayed for years by the classification of all "numbers" (namely, sets verifying the axioms of a numeric field) into real, complex and quaternion numbers that had been achieved by historic masters such as Gauss, Cayley, Hamilton and others. Finally, in 1993 Santilli re-inspected this historical classification and discovered that the axioms of a numeric field do not require the basic multiplicative unit to be necessarily positive definite, since said unit can be negative definite as well, provided that the conventional associative product of numbers $n \times m$ is redefined in such a way to admit the newly assumed negative unit [19].

We have in this way Santilli's isodual fields $F^d(n^d, x^d, 1^d)$ consisting of isodual real, isodual complex and isodual quaternion numbers with negative-definite isodual multiplicative unit and related isodual (associative) multiplication [19]

$$F^d(n^d, x^d, 1^d) : n^d = -n^\dagger, n^d x^d m^d = n^d x (1^d)^{-1} x m^d, 1^d = -1^\dagger, \quad (4)$$

under which 1^d is indeed the basic multiplicative unit at all levels (the additive unit 0 of a field remains evidently unchanged under isoduality because $0^d \equiv 0$ [19]),

$$1^d x^d n^d \equiv n^d x^d 1^d \forall n^d \in F^d, \quad (5)$$

Note, to prevent insidious misinterpretations, that is for the evident mathematical consistency, all real numerical values in isodual mathematics must be elements of Santilli's isodual fields and, as such, must be given by ordinary numbers multiplied by the isodual unit.

Consequently, Santilli undertook to the construction of

isodual functional analysis and isodual metric spaces, such as the Euclid-Santilli isodual space with line element [20]

$$E^d(r^d, \delta^d, I^d): r^d = r \times I^d, \delta^d = \text{Diag.}(-1, -1, -1) = \delta \times I^d, I^d = \text{Diag.}(-1, -1, -1), \quad (6)$$

$$r^{d2d} = (r^{di} \times^d \delta_{ij}^d \times^d r^{dj}) \times I^d \equiv r^2 = (r^i \times \delta_{ij} \times r^j) \times I^d, \quad (7)$$

where one should note the necessity of multiplying the line element by the isodual unit as a condition to be an isodual scalar, that is, an element of the isodual field. It then follows that the line element of the Euclid-Santilli isodual space coincides with that of the conventional Euclidean space, Eq. (7), and this explains the reason for the lack of detection of the isodual spaces for centuries [20].

Note that the study of the isodualities of the Euclidean space was a necessary pre-requisite to reach the yet unknown (in 1993) formulation of Newton's equation for neutral or charged antimatter [20].

Additionally, Santilli constructed the isoduality of Lie's theories, Minkowskian and Riemannian geometries and of virtually all mathematics used for the study of matter [21].

Thanks to his keen self-criticism, the emerging new mathematics continued to have hidden inconsistencies whose solution required additional years of study. Finally, in 1995, Santilli had the courage to re-inspect another pillar of 20th century applied mathematics, the ordinary differential calculus, by discovering that, contrary to popular belief since Newton's time, the differential calculus does indeed depend on the assumed basic unit and related field. We reach in this way the discovery of Santilli's isodual differential calculus with isodual differentials and isodual derivatives that, for the case of a real-valued new unit acquires the simple form [22]

$$d^d r^d = 1^d dr^d = 1^d d(r 1^d) \equiv dr, \quad (8)$$

$$\delta^d F^d(r^d) /^d \delta^d r^d = -\delta F / \delta r, \quad (9)$$

where one can see that (again for the case of a real-valued new unit) Santilli's isodual differential coincides with the conventional differential, and this explains the reason the isodual differential calculus remained unidentified for centuries.

Following the discovery of the isodual differential calculus, Santilli completed the construction of the isodual mathematics with a rigorous structural consistency, and passed only thereafter to physical applications.

A fundamental prediction of the isodual theory of antimatter is that light emitted by antimatter (antimatter light) is physically different than light emitted by matter (matter light) in a number of experimentally verifiable ways [9]. This important prediction was presented by Santilli at the International Conference on Antimatter held in Sepino, Italy, on June 1996.

Recall that light has no charge. But the isodual theory has been constructed to provide a differentiation between neutral matter and antimatter. Therefore, the physical distinction between matter and antimatter light is a direct and unavoidable consequence of the physical distinction between neutral matter and antimatter, as reflected in the distinction

between their representations.

Besides, the physical differentiations at advanced level we cannot review here, the most visible difference is the prediction by the isodual theory of antimatter that antimatter light is repelled by a matter gravitational field (see Figure 9).

The simplest way to illustrate this prediction is that at the primitive Newtonian level since all subsequent levels of study are merely consequential. Let us recall that Santilli has formulated Newtonian gravitation in a truly "universal" way via the "identical" representation of the historical equation in terms of "energy," rather than mass [8, 24].

Following the study of a number of alternatives, Santilli gave priority to the search for new numbers since all mathematics used for physics must be based on a numeric field as a condition for experimental verifications and, in any case, all aspects of applied mathematics can be built on a given numeric field via simple compatibility arguments. In 1993, Santilli [19, 25] finally identified the desired new number under the name of isodual real, complex and quaternionic numbers [17], which verify the condition of being anti-isomorphic to the conventional real, complex and quaternionic numbers, respectively. The word "isodual" was suggested to indicate a duality under the preservation of the conventional abstract axioms of numeric fields. The crucial condition of anti-isomorphism was achieved via the anti-Hermitean conjugation of all elements of a numeric field and all its operations. This implies that, given a field $F(n, \times, 1)$ with elements n, m, \dots , conventional associative product $n \times m = nm$ and trivial unit 1, Santilli isodual fields are indicated with the upper symbols $d, F^d(n^d, \times^d, 1^d)$, and are characterized by a negative basic unit $1^d = -1^\dagger = -1$, isodual numbers $n^d = n 1^d$ and isodual product $n^d \times^d m^d = n^d (1/1^d) m^d = nm 1^d$.

Following the identification of the desired numbers, Santilli passed to the systematic construction of the isodual image of all main mathematics used for the study of matter, including functional analysis, differential calculus, metric spaces, Lie algebras, symmetries, Euclidean, Minkowskian and Riemannian geometries, etc. These isodual formulations were first presented in the mathematical memoir [22] and first treated systematically in monographs [8]. The resulting mathematics is today known as Santilli isodual mathematics. It may be of some value to indicate that isoduality is a new transformation not reducible to parity and/or other conventional transformations. We should also recall the new symmetry identified by the isodual mathematics, called isoselfduality [8, 22], namely, the invariance under the isodual transformation, which is verified by the imaginary number $i \equiv i^d$ as well as by Dirac's equation.

Contrary to a possible perception of mathematical complexities, the isodual mathematics needed for applications can be constructed via the application of the simple anti-Hermitean map $Q \rightarrow Q^d = -Q^\dagger$, provided it is applied to the totality of quantities and to the totality of their operations

used for the treatment of matter. Readers should be alerted that, in the absence of even one isodual map, there are inconsistencies that generally remain undetected to non-experts in the field [22].

3. Santilli's Isodual Physics

3.1. Apparent Lack of Visibility of Antimatter Asteroids with Sun Light

Santilli has achieved a representation of antimatter at all possible levels, from Newtonian mechanics to second quantization and for conditions of increasing complexity, from fully conservative conditions to the most general possible irreversible non-Hamiltonian conditions, as well as hyperstructural conditions expected in possible antimatter living structures. These studies are far from trivial and have direct implications for the very safety of our planet, since they predict that antimatter asteroids are not visible with the light of our matter Sun. In fact, the studies predict that light emitted by a matter star annihilates when hitting an antimatter body without any refraction. Alternatively, the studies predict that light emitted by an antimatter star, called by Santilli isodual light, annihilates when hitting matter, thus not reaching us on Earth due to annihilation in the upper atmosphere, as it is the case for antimatter cosmic rays. In short, Santilli has initiated an entire new field called antimatter astrophysics whose primary aim is the identification of methods for the detection of antimatter stars, by noting that their isodual light is expected to annihilate even in lenses of telescopes orbiting in space, thus requiring a basically new conception of antimatter telescopes.

It should be noted that, Einstein special and general relativity have no means for differentiating between neutral matter and antimatter as expected for asteroids and stars. As a consequence, antimatter has been assumed as being nonexistent in the universe in any appreciable amount. Santilli's discoveries indicates that antimatter has not been detected because of the above indicated occurrences, namely, the annihilation of our Sun light in an antimatter asteroid, or the annihilation of light from an antimatter star in our atmosphere or in orbiting telescopes [26].

3.2. Newton-Santilli Isodual Equation for Antimatter

No consistent classical theory of antimatter existed prior to Santilli's research, to our best knowledge as yet. For instance, by resuming the use of the conventional associative multiplication $a \times b = ab$, the celebrated Newton's equation,

$$m \times dv/dt = F(t, r, v, \dots) \tag{10}$$

or the celebrated Newton's gravitation law

$$F = g \times m_1 \times m_2/r^2 \tag{11}$$

solely apply for matter, and have no means whatsoever to distinguish between matter and antimatter for the very simple reason that antimatter was inconceivable at Newton's times.

Prior discovery of his isodual mathematics, Santilli developed the isodual theory of antimatter that holds at all levels of study, thus restoring full democracy between matter and antimatter. In essence, in the 20th century antimatter was empirically treated by merely changing the sign of the charge, under the tacit assumption that antimatter exists in the same space as that for matter. Thus, both matter and antimatter were studied with respect to the same numbers, fields, spaces, etc. However, a correct classical representation of antimatter required a mathematics that is antiisomorphic to that used for matter as a necessary condition to admit a charge conjugated operator image.

Santilli represents antimatter via his anti-Hermitian isodual map that must be applied to the totality of quantities used for matter and all their operations. Hence, under isoduality, we have not only the change of the sign of the charge, but also the isodual conjugation of all remaining physical quantities (such as coordinates, momenta, energy, spin, etc.) and all their operations. This is the crucial feature that allows Santilli to achieve a consistent representation of antimatter also for neutral bodies.

We have in this way the Newton-Santilli isodual equation for antiparticles that we write in the simplified form

$$m^d \times^d d^d v^d / d^d t^d = F^d(t^d, r^d, v^d, \dots) \tag{12}$$

where "d" denotes isodual map, and the same conjugation holds for gravitation too.(see below).

Note that, after working out all isodual maps, antiparticle equation (12) merely yields minus the value of the conventional equation for particles in both the l.h.s. and the r.h.s, thus appearing to be trivial. However, a most important feature of the above equation is that it defines antiparticles in a new space, the Euclid-Santilli isodual space, which is coexistent but different than our own space. The Euclidean space and its isodual then form a two-valued hyperspace. In this section we describe how Santilli showed that, starting from the fundamental equation (12), the isodual theory of antimatter is consistent at all subsequent levels, including quantization and at that level it is equivalent to charge conjugation.

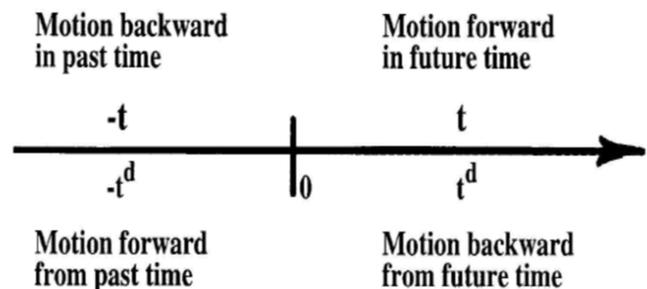


Figure 1. Contrary to popular beliefs, time has four directions as depicted by Santilli in this figure to illustrate the need for isoduality. In fact, time reversal can only allow the representation of two time directions. The remaining two time directions can solely be represented via the isodual map [26].

According to Santilli the isodual antiparticles have a negative energy. This feature is dismissed by superficial

inspections as being nonphysical, thus venturing judgments prior to the acquisition of technical knowledge. In fact, negative energies are indeed nonphysical, only when referred to our space time, that is, with respect to positive unit of time. By contrast, when referred to negative unit of time, all known objections on negative energies become inapplicable, let alone resolved.

Note also that isodual antiparticles move backward in time. This view was originally suggested by Stueckelberger in the early 1900s, and then adopted by various physicists, such as Feynman, but dismissed because of causality problems when treated with our own positive unit of time. Santilli has shown that the motion backward in time referred to a negative unit of time $t^d = -t$ is as causal as motion forward in time referred to a positive unit of time t , and this illustrates the nontriviality of the isodual map.

Moreover, the assumption that particles and antiparticles have opposing directions of time is the only one known aspect giving hopes for the understanding of the process of annihilation of particles and their antiparticles, a mechanism utterly incomprehensible for the 20th century physics [26].

3.3. Isodual Representation of the Coulomb Force

Santilli's assertion is that the isodual theory of antimatter verifies all classical experimental evidence on antimatter because it recovers the Coulomb law in a quite elementary way. Consider the case of two particles with the same negative charge and Coulomb Law

$$F = (-q_1) \times (-q_2) / (r \times r) \tag{13}$$

where the positive value of the r.h.s is assumed as representing repulsion, and the constant is assumed to have the value 1 for simplicity.

Under isoduality, the above expression becomes

$$F^d = (-q_1)^d \times^d (-q_2)^d /^d (r^d \times^d r^d) \tag{14}$$

thus reversing the sign of the equation for matter, $F^d = -F$. However, antimatter is referred to a negative unit of the force, charge, coordinates, etc. Hence, a positive value of the Coulomb force referred to a positive unit representing repulsion is equivalent to a negative value of the Coulomb force referred to a negative unit, and the latter also represents repulsion.

For the case of the electrostatic force between one particle and an antiparticle, the Coulomb law must be projected either in the space of matter

$$F = (-q_1) \times (-q_2)^d / (r \times r) \tag{15}$$

representing attraction, or in that of antimatter

$$F = (-q_1)^d \times^d (-q_2) /^d (r^d \times^d r^d) \tag{16}$$

in which case, again, we have attraction, thus representing classical experimental data on antimatter [26].

3.4. Hamilton-Santilli Isodual Mechanics

To proceed in his reconstruction of full democracy in the treatment of matter and antimatter, Santilli had constructed the isodual image of Hamiltonian mechanics because it is essential for all subsequent steps. In this way he reached what is today called the Hamilton-Santilli isodual mechanics based on the isodual equations

$$\begin{aligned} d^d r^d /^d d^d t^d &= \partial^d H^d(r^d, p^d) /^d \partial^d p^d, \\ d^d p^d /^d d^d t^d &= - \partial^d H^d(r^d, p^d) / \partial r \end{aligned} \tag{17}$$

and their derivation from the isodual action A^d (a feature crucial for quantization), from which the rest of the Hamilton-Santilli isodual mechanics follows [26].

3.5. Isodual Special and General Relativities

The special and general relativities are basically unable to provide a consistent classical treatment of antimatter. Santilli has resolved this insufficiency by providing a detailed, step by step isodual lifting of both relativities with a mathematically consistent representation of antimatter in agreement with classical experimental data. The reader should be aware that the above liftings required the prior isodual images of the Minkowskian geometry, the Poincare symmetry and the Riemannian geometry, as well as the confirmation of the results with experimental evidence [26].

3.6. Prediction of Antigravity

Studies on antigravity were dismissed and disqualified in the 20th century on grounds that "antigravity is not admitted by Einstein's general relativity." According to Santilli this posture resulted in a serious obscurantism because general relativity cannot represent antimatter, thus being disqualified for any serious statement pertaining to the gravity between matter and antimatter.

With his isodual images of special and general relativity, Santilli has restored a serious scientific process in the field, by admitting quantitative studies for all possibilities, and has shown that once antimatter is properly represented, matter and antimatter must experience antigravity (defined as gravitational repulsion) because of supporting compatible arguments at all levels of study, with no known exclusion. Thereby, all known "objections" against gravitational repulsion between matter and antimatter become inapplicable under Santilli isoduality. As a trivial illustration, in Santilli's isodual theory there have the repulsive Newton-Santilli force between a particle and an isodual particle (antiparticle) both treated in our space

$$F = g \times m_1 \times m_2 / r^2 = -g \times m_1 \times m_2 . r^2 \tag{18}$$

which is indeed repulsive. The same conclusion is reached at all levels of study.

Santilli further asserts that a very compelling aspect supporting antigravity between matter and antimatter is his identification of gravity and electromagnetism. In fact, the

electromagnetic origin of exterior gravitation mandates that gravity and electromagnetism must have similar phenomenologies, thus including both attraction and repulsion [26].

3.7. Test of Antigravity

Santilli has proposed an experiment for the final resolution as to whether antiparticles in the gravitational field of Earth experience attraction or repulsion. The experiment consists in the measure of the gravitational force of a beam of positrons in flight on a horizontal vacuum tube 10 m long at the end of which there is a scintillator. Then, the displacement due to gravity is visible to the naked eye under a sufficiently low energy (in the range of the 10^{-3} eV). The experiment was studied by the experimentalist Mills and shown to be feasible with current technologies and resolutory [12, 26].

3.8. Isodual Quantum Mechanics

Next, Santilli constructed a step-by-step image of quantum mechanics under his isodual map based on the Heisenberg-Santilli isodual time evolution for an observable Q

$$i^d x^d d^d Q^d / d^d t^d = [Q, H]^d = H^d x^d Q^d - Q^d x^d H^d \quad (19)$$

and related isodual canonical commutation rules, Schrodinger-Santilli isodual equations, etc.

He then proved that, at the operator level, isoduality is equivalent to charge conjugation. Consequently, the isodual theory of antimatter verifies all experimental data at the operator level too. Nevertheless, there are substantial differences in treatment, such as:

- 1) Quantum mechanics represents antiparticles in the same space of particles, while under isoduality particles and antiparticles exist in different yet coexisting spaces;
- 2) Quantum mechanics represents antiparticles with positive energy referred to a positive unit, while isodual antiparticles have negative energies referred to a negative unit;
- 3) Quantum mechanics represents antiparticles as moving forward in time with respect to our positive time unit, while isodual antiparticles move backward in time referred to a negative unit of time [26].

3.9. Santilli's Comparative Test of the Gravity of Electrons and Positrons in a Horizontal Supercooled and Supervacuum Tube: Proposed Experiments on the Gravity of Antimatter

The gravitational repulsion (antigravity) between matter and antimatter was suspected immediately following the discovery of antimatter, although without any possible theoretical treatment due to the absence of a theory capable of representing the gravitational field of neutral antimatter [15]. This insufficiency has been resolved by Santilli's works on antimatter. In fact, the isodual theory of antimatter predicts in a consistent and systematic way at all levels of study, from Newtonian mechanics to the Riemannian geometry, that matter and antimatter must experience gravitational repulsion

[6, 15].

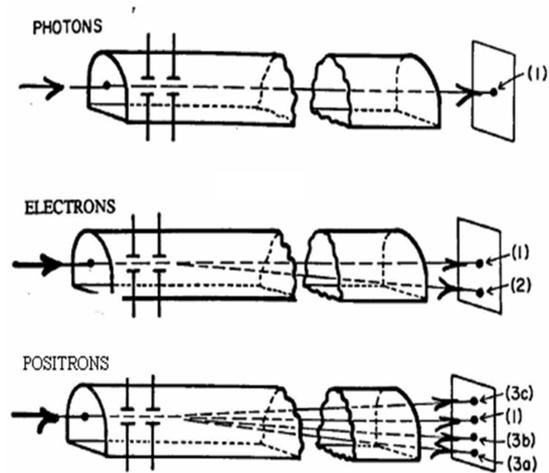


Figure 2. The original illustration used by Santilli for the 1994 proposal to test the gravity of positrons in horizontal light in a vacuum tube. The proposal has been qualified by experimentalists as being technically feasible nowadays and resolutory because the displacement due to gravity on a scintillator at the end of a 10 m light for positrons with milli-eV energy is visible to the naked eye. The usual criticisms based on disturbances caused by stray fields have been disqualified as political for a tube with at least 50 cm diameter. Virtually all major physics laboratories around the world have rejected even the consideration of the test, despite its dramatically lower cost and superior scientific relevance compared to preferred tests, on grounds that "Einstein theories do not admit antigravity," although with documented knowledge that said theories cannot consistently represent antimatter as reviewed in the test [26].

It can conceptually said that antigravity between matter and antimatter is a necessary consequence of the very existence of a "classical" gravitational representation of "neutral" antimatter because, since the charge is null, such a representation requires the sign conjugation of all physical quantities, thus including the sign of the gravitational force and, therefore, of the curvature tensor. On quantitative grounds, we refer to monograph [15] for the gravitational representation of antigravity via the Riemannian geometry for matter and its isodual for antimatter. For this writing it may sufficient to recall the most primitive prediction of antigravity, that in Newtonian mechanics, since all subsequent levels of study are evidently compatible to such a primitive one.

In fact, the Newton-Santilli isodual equation clearly predict gravitational repulsion between matter and antimatter both in our space as well as in the isodual space, according to the respective the laws,

$$F = g \times m_1 m_2 / r^2 < 0, \quad (20)$$

$$F^d = g^d x^d m_1^d x^d m_2 / r^{d2d} > 0, \quad (21)$$

where in our world we have a repulsion because the gravitational force is negative, $F < 0$, and referred to a positive unit of force, while in the isodual world we equally have a repulsion because the gravitational force is positive, $F^d > 0$, but it is referred to a negative unit of force.

Safety of our planet and, consequently, this class of experiments will not be considered herein.

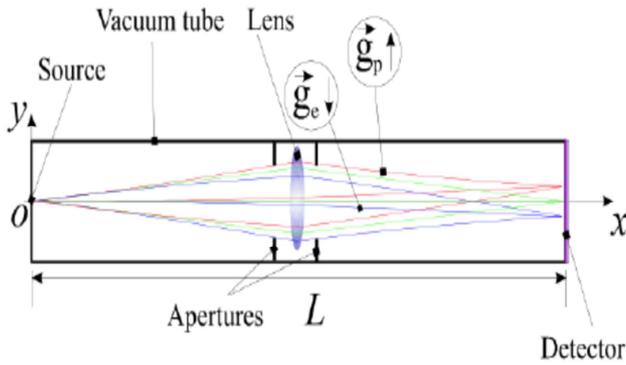


Figure 3. Principle set-up of Mills's adaptation of Santilli's comparative test of the gravity of electrons and Positrons. \rightarrow shows the gravitational attraction on a collimated beam of electrons that, when having a very low energy of the order of meV , is of the order of 1 cm following a flight of 10 m, thus being visible to the naked eye [12].

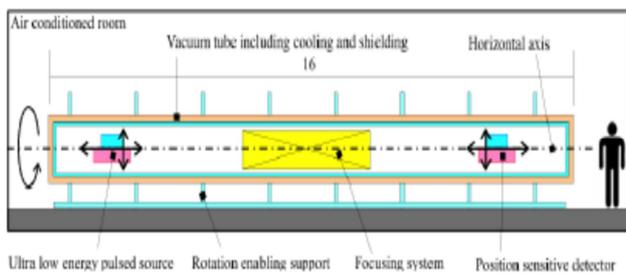


Figure 4. The possible alternatives for a collimated beam of positrons. Santilli's isodual theory of antimatter predicts gravitational repulsion (antigravity) at all its levels for positrons in a horizontal flight on Earth that, for very low energy of the order of meV , is of the order of 1 cm following a 10 m flight, thus being visible to the naked eye on the scintillator at the end of the tube. For that reason, Santilli's proposed experiment has been stated to be "resolatory" by experimentalists in the field [12, 27]. The lower two renderings are from the technical realization of the test [28] by the R. M. Santilli Foundation on the technical realization of proposal [6] (forth view from the top) and illustration of its size compared to a person [29].

The first experimental test of the gravity of positrons was formulated by W. E. Fairbanks and E. C. Witteborn at SLAC in 1967 [30] via the use of low energy positrons in vertical upward flight in a vacuum and cooled tube. Regrettably, the experiment could not be completed due to the unavailability at that time of detectors with the extreme sensitivity needed for meaningful measurements. Numerous additional experiments have been proposed to test the gravity of positrons in vertical flights, either upwards or downwards, such as the tests of Refs. [31, 32] and others. However, the gravitational force on particles is notoriously very weak, as a consequence of that the measurements with the most sophisticated neutron interferometric or other techniques are expected to remain ambiguous.

Thus, the class of proposed experiments to measure the gravity of positrons in vertical flight cannot possibly be as resolatory as necessary for the

In view of the indicated limitations of testing the gravity of positrons in a vertical flight, Santilli proposed in paper [6] of 1994 the experimental verification or dismissal of the predicted gravitational repulsion between matter and antimatter via measurements of the comparative behavior of

very low energy electrons and positrons moving in a 10 m long horizontal supercooled and super-vacuum tube (Figures 2 to 4).

It is evident that Santilli's gravity experiment via positrons in horizontal flight is strikingly better than preceding proposed tests [30-32] via positrons in a vertical flight. While the measurements in the latter tests are expected to remain ambiguous due to the smallness of the effect, in Santilli's experiment [6], for very low energy electrons and positrons of the order of meV in horizontal flight in a 10 m long supercooled and super-vacuum tube, the displacement due to gravity detected on a scintillator at the end of the tube is of the order of 1 cm, thus being visible to the naked eye. The preference of Santilli's test [6] over the tests of Refs. [30-32] is confirmed by a number of experimentalists in the field. For instance, during the International Conference on Antimatter held in Sepino, Italy, in June 1996, the experimentalist A. P. Mills declared Santilli's gravity experiment as being "resolatory" [12] and, therefore, is preferable over the others not equally resolatory experiments. Similarly, during the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes, held at the University of Kathmandu, Nepal, in January 2011, the experimentalist V. de Haan [27] confirmed Mills analysis and also declared Santilli's gravity experiment as being "resolatory".

Besides the above proposed experiments via the use of positrons, the only remaining proposed experiments are those based on anti-hydrogen atoms produced at CERN. Among the latter tests, we have pointed out the test proposed in Ref. [33] by the AEGIS Collaboration outlined in Figures 5 and 6, and the test proposed in Ref. [34] by the ALPHA Collaboration outlines in Figures 5 and 6. By assuming a technical knowledge of these proposed experiments, we here limit ourselves to the following comments.

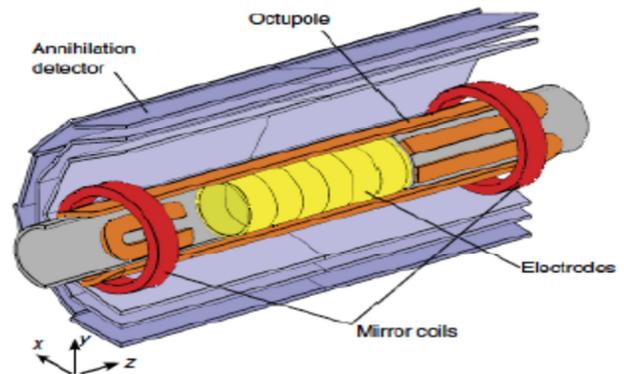


Figure 5. An illustration from Ref. [34] providing a cut-away diagram of the antihydrogen production and trapping of the ALPHA Collaboration, showing the relative positions of the cryogenically cooled Penning- Malmberg trap electrodes and other features.

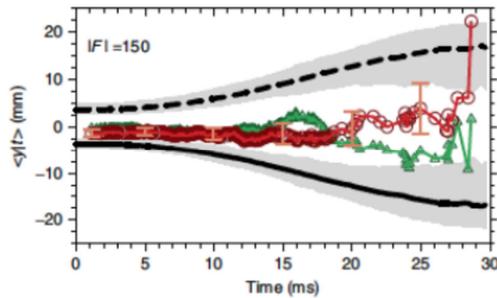


Figure 6. A second illustration from Ref. [34] on the proposed test of the gravity of antihydrogen atoms via their fall downward when released from the ALPHA antihydrogen trap of the preceding figures. The illustration depicts one of several simulated reverse cumulative average analysis. Compared to the even data to the reverse cumulative average. The green-triangle line is the reverse cumulative average of the x annihilation positions of the event data, and is included as a comparison. The black solid line is the represents 900,000 simulated antihydrogen atoms. The black dashed line mirrors the black-solid line, and is equivalent to a simulation study of antigravity. The grey bands separate the 90% confidence region. Again, the complexity of the apparatus and the high sensitivity of the detectors should be compared to corresponding data of Santilli's gravity test [6, 28].

To begin, the tests of Refs. [33, 34] have the same ambiguities in measurements as those of the tests with vertically moving positrons [30-32], since the former too deal with extremely small effects requiring extremely sensitive detectors under these conditions. The “experimental results” are inevitably prone to the approximations and/or manipulations that occurred in similar tests.

Besides that, the main problematic aspect of tests of Refs. [33, 34] is the one identified by Santilli [15] according to which, despite a popular beliefs at CERN and elsewhere, the “antiprotons” produced at CERN are not necessarily antiparticles, unless verified as such via annihilation processes, because at least in part, they can be anomalous protonic states created by the embedding of a singlet electron pairs inside ordinary protons. These states are called by Santilli the pseudoproton and denoted with the symbol \hat{p}^- .

Consequently, no gravity experiment based on “antihydrogen atoms” produced at CERN can be considered as being resolutory under such a serious ambiguity. Besides the study of antimatter, Santilli has dedicated decades of his research life also to the synthesis of neutrons inside a star according to Rutherford's historical conception that neutrons are synthesized by the “compression” of hydrogen atoms in the core of a star, nowadays represented with reaction



It is well known that the energies needed to achieve the synthesis of the neutron are fully available at CERN. In particular, Santilli has shown that neutrons can also be synthesized in laboratory from a hydrogen gas traverses by a DC arc, thus taking place at energies much smaller than those available at CERN. The experimental information important for the test of the gravity of antimatter obtained by Santilli is that Rutherford's compression is also achievable for an electron pair in singlet coupling (that occurs for valence electron pairs) resulting in the creation of pseudoproton

according to the reaction



where \hat{p}^- is predicted to have a mean life essentially similar (if not longer) than that of the neutron due to the similarities of the two syntheses.

As a matter of fact, Santilli has shown that synthesis [28] is more probable than synthesis [26] for various reasons, such as: synthesis [28] does not require the emission of a neutrino for the conservation of the total angular momentum as necessary for synthesis [30], Rutherford's compression of a single electron pair inside the proton is statistically more probable than the compression of the electron due to spin zero of the electron pairs (thus requiring no special proton-pair coupling), compared to the need for a singlet proton-electron coupling for synthesis [26] and other reasons.

It should be stressed that quantum mechanics does not allow a quantitative representation of synthesis [26] because the rest energy of the neutron is bigger than the sum of the rest energies of the proton and the electron, thus requiring a “positive binding energy” which is anathema for quantum mechanics, since in this case the Schrodinger equation no longer admits physically meaningful solutions [8]. Thanks to its non-unitary invariant character, hadronic mechanics has resolved these insufficiencies by achieving, for the first time to our knowledge, a numerically exact representation of “all” characteristics of the neutrons in synthesis [26] at both non-relativistic and relativistic levels [8, 26].

In particular, the use of Santilli's non-unitary invariant methods that have permitted a representation of synthesis [26] when applied to synthesis [28], show that the rest energy of the pseudoproton can be close to that of the antiproton, although expecting of exact numerical values are premature at this time since the sole experimentations to date have been conducted is by Santilli.

Therefore, Santilli stresses that the distinction between the antiproton and the pseudoproton cannot be solely based on their charge and rest energy, their only resolutory distinction being that based on annihilation processes. Needless to say, the antimatter nature of the “antiprotons” claimed at CERN cannot be denied. The point is that the antimatter character has to be proved beyond doubt prior to any true scientific claim. Now, as it is well known, the production of “antiprotons” at CERN is based on hitting a target with the 26 GeV proton beam produced by the old Proton Synchrotron (PS). It is then evident to all that, during the collision of protons with matter target, Santilli synthesis [28] is indeed possible, resulting in the synthesis of the pseudoproton. In fact, at the time of the impact, protons collide first with electrons clouds in general, including precisely the valence electron pairs of synthesis [28]. Once the pseudoproton has been synthesized, its capability to capture a positron in the anti-hydrogen trap is established by quantum mechanical laws, resulting in a neutral state ($\hat{p}^-; e^+$) which is similar to, but not necessarily, the anti-hydrogen atom (\hat{p}^-, e^+).

In short, the mathematical, theoretical and experimental

studies illustrate Santilli's main objection against the test of the gravity of antimatter via "antihydrogen atoms" currently produced at CERN because of the lack of clear proof that they are indeed antimatter and the absence of experiments for the resolution of the ambiguities because, being necessarily beyond quantum mechanics, the said experiments are notoriously not even plausible at CERN under current control. In conclusion, both classes of tests of the gravity of antimatter, those based on vertical motion of positrons and those based on the "anti-hydrogen atoms" produced at CERN, are not resolatory on grounds of our current knowledge. Consequently, Santilli's gravity test is and remains the best measurement of the gravity of antimatter since it is the only experiment whose results would be visible to the naked eye [29].

3.10. Experimental Detection of Antimatter Galaxies

The isodual theory of antimatter was born out of Santilli's frustration as a physicist for not being able to ascertain whether a far away star, galaxy or quasar is made up of matter or of antimatter. Santilli has resolved this uneasiness via his isodual photon γ^d namely, photons emitted by antimatter that have a number of distinct, experimentally verifiable differences with respect to photons γ emitted by matter, thus allowing, in due time, experimental studies on the nature of far away astrophysical objects.

$$\gamma^d \neq \gamma \tag{24}$$

A most important difference between photons and their isoduals is that the latter have negative energy, as a result of which, isodual photons emitted by antimatter are predicted to be repelled in the gravitational field of matter. A possibility for the future ascertaining of the character of a far away star or quasar is, therefore, the test via neutron interferometry or other sensitive equipment, whether light from a far away galaxy is attracted or repelled by the gravitational field of Earth [26].

3.11. The New Isoselfdual Invariance of Dirac's Equation

Santilli has released the following statement on the Dirac equation: I never accepted the interpretation of the celebrated Dirac equation as presented in the 20th century literature, namely, as representing an electron, because the (four-dimensional) Dirac's gamma matrices are generally believed to characterize the spin 1/2 of the electron. But Lie's theory does not allow the SU (2)-spin symmetry to admit an irreducible 4-dimensional representation for spin 1/2, and equally prohibits a reducible representation close to the Dirac's gamma matrices. Consequently, Dirac equation cannot represent an electron intended as an elementary particle since elementarily requires the irreducible character of the representation. In the event Dirac's gamma matrices characterize a reducible representation of the SU (2)-spin, Dirac's equation must represent a composite system.

I discovered the isodual theory of antimatter by examining with care Dirac's equation. In this way, I noted that its gamma

matrices contain a conventional two-dimensional unit $I_{2 \times 2} = \text{Diag.} (1, 1)$, as well as a conjugate negative-definite unit $-I_{2 \times 2}$. That suggested me to construct a mathematics based on a negative definite unit. The isodual map come from the connection between the conventional Pauli matrices $\sigma_k, k = 1, 2, 3$, referred to $I_{2 \times 2}$ and those referred to $-I_{2 \times 2}$. In this way I reached the following interpretation of Dirac's gamma matrices as being the tensorial product of $I_{2 \times 2}, \sigma_k$ times their isoduals,

$$\{I_{2 \times 2}, \sigma_k, k = 1, 2, 3\} \times \{I_{2 \times 2}^d, \sigma_k^d, k = 1, 2, 3\} \tag{25}$$

Therefore, I reached the conclusion that the conventional Dirac equation represents the tensorial product of an electron and its isodual, the positron. In particular, there was no need to use the "hole theory" or second quantization to represent antiparticles since the above re-interpretation allows full democracy between particles and antiparticles, thus including the treatment of antiparticles at the classical level, let alone in first quantization.

By continuing to study Dirac's equation without any preconceived notion learned from books, I discovered yet another symmetry I called isoselfduality, occurring when a quantity coincides with its isodual, as it is the case for the imaginary unit $i^d = i$. In fact, Dirac's gamma matrices are isoselfdual,

$$\gamma_\mu^d = \gamma_\mu, \mu = 0, 1, 2, 3. \tag{26}$$

This new invariance can have vast implications, all the way to cosmology, because the universe itself could be isoselfdual as Dirac's equation, in the event composed of an equal amount of matter and antimatter. In conclusion, Dirac's equation is indeed one of the most important discoveries of the 20th century with such a depth that it could eventually represent features at the particle level that actually hold for the universe as a whole [26].

3.12. Dunning-Davies Thermodynamics for Antimatter

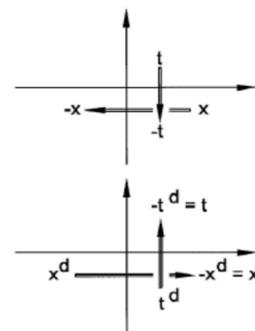


Figure 7. A schematic view of the additional peculiar property that the projection in our spacetime of the isodual space inversion appears as a time inversion and vice versa. In fact, a point in the isodual spacetime is given by $(x^d, t^d) = (-x, -t)$. The projection in our spacetime of the isodual space inversion $(x^d, t^d) \rightarrow (-x^d, t^d)$ is then given by $(x, -t)$, thus appearing as a time (rather than a space) inversion. Similarly, the projection in our spacetime of the isodual time inversion $(x^d, t^d) \rightarrow (x^d, -t^d)$ appears as $(-x, t)$, that is, as a space (rather than time) inversion. Despite its simplicity, the above occurrence has rather deep implications for all discrete symmetries in particle physics [35].

As well known, the sole formulation of thermodynamics of the 20th century was for matter. The first consistent formulation of thermodynamics for antimatter has been reached by J. Dunning-Davies with intriguing implications for astrophysics and cosmology yet to be explored, (see the original contribution by Dunning Davies quoted below) [26].

An important contribution to the isodual theory has been made by J. Dunning-Davies [35] who introduced in 1999 the first, and only known consistent thermodynamics for antimatter, here called Dunning-Davies antimatter thermodynamics with intriguing results and implications.

As conventionally done in the field, let us represent heat with Q, internal energy with U, work with W, entropy with S, and absolute temperature with T. Dunning-Davies isodual thermodynamics of antimatter is evidently defined via the isodual quantities

$$Q^d = -Q, U^d = -U, W^d = -W, S^d = -S, T^d = -T \quad (27)$$

on isodual spaces over the isodual field of real numbers $R^d = R^d(n^d, +^d, \times^d)$ with isodual unit $I^d = -1$.

It is also seen that isodual differentials are isoselfdual (that is, invariant under isoduality). Dunning-Davies then has the following theorem:

THEOREM [36]: Thermodynamical laws are isoselfdual.

Proof: For the First Law of thermodynamics we have

$$dQ = dU - dW \equiv d^d Q^d = d^d U^d - d^d W^d. \quad (28)$$

Similarly, for the Second Law of thermodynamics we have

$$dQ = T \times dS \equiv d^d Q^d = T^d \times^d S^d, \quad (29)$$

and the same occurs for the remaining laws.

Despite their simplicity, Dunning-Davies results [36] have rather deep implications. First, the identity of thermodynamical laws, by no means, implies the identity of the thermodynamics of matter and antimatter. In fact, in Dunning-Davies isodual thermodynamics the entropy must always decrease in time, since the isodual entropy is always negative and is defined in a space with evolution backward in time with respect to us. However, these features are fully equivalent to the conventional increase of the entropy tacitly referred to positive units.

Also, Dunning-Davies results indicate that antimatter galaxies and quasars cannot be distinguished from matter galaxies and quasars via the use of thermodynamics, evidently because their laws coincide, in a way much similar to the identity of the trajectories of particles and antiparticles of *Lemma: The trajectories under the same magnetic field of a charged particle in Euclidean space and of the corresponding antiparticle in isodual Euclidean space coincide* [8].

This result indicates that the only possibility known at this writing to determine whether far away galaxies and quasars are made up of matter or of antimatter is that via the predicted gravitational repulsion of the light emitted by antimatter called isodual light.

3.13. Isoselfdual Spacetime Machine

A "spacetime machine" is generally referred to a *mathematical* process dealing with a closed loop in the forward spacetime cone, thus requiring motions forward as well as backward in time. As such, the "machine" is not permitted by causality under conventional mathematical treatment, as well known.

Santilli discovered that *isoselfdual matter*, namely, matter composed by particles and their antiparticles such as the positronium, have a null intrinsic time, thus acquiring the time of their environment, namely, evolution forward in time when in a matter field, and motion backward in time when in an antimatter field.

Consequently, Santilli showed that *isoselfdual systems can indeed perform a closed loop in the forward light cone without any violation of causality laws*, because they can move forward when exposed to a matter and then move backward to the original starting point when exposed to antimatter [26].



Figure 8. An illustration of the serious implications of Santilli's isodual theory of antimatter: the need for a revision of the scattering theory of the 20th century due to its violation of the isoselfdual symmetry of Dirac's equation. The diagram in the left illustrates the isoselfduality of the initial particles (an electron and a positron) but its violation in the final particles (two identical photons). The diagram in the right illustrates one of the several needed revisions, the use for final particles of a photon and its isodual as a necessary condition to verify the new isoselfdual symmetry. Additional dramatic revisions are due to the purely action-at-a-distance, potential interactions of the conventional scattering theory (represented with a waving central line in the left diagram), compared to the non-Hamiltonian character of the scattering region caused by deep penetrations of the wave packets of particles (represented with a circle in the right diagram) [26].

3.14. Original Literature

Santilli's first paper on the isodual theory of antimatter is the one dating to 1994 [37] (following the 1993 paper on isodual numbers).

The first presentations of the classical isodual theory, antigravity, the isodual photon and the isoselfdual spacetime machine appeared in papers [3, 9, 38, and 39]. An independent study by an experimentalist on the feasibility and resolatory character of the proposed measurements of the gravity of positron in horizontal flight on Earth can be found in paper [12].

Comprehensive presentations of the isodual theory of antimatter are available in the monographs [8, 11]. The first formulation of thermodynamics for antimatter was reached by J. Dunning Davies in paper [35, 40].

3.15. Main Features of Santilli's Isodual Theory of Antimatter

Santilli initiated systematic applications of isodual mathematics to the study of antimatter resulting in the new theory today called *isodual theory of antimatter (or Santilli's Isodual Physics)* as one of the branches of the broader *hadronic mechanics* [11, 26]. A main feature is that all quantities that are positive (negative) for the study of matter become negative (positive) for the study of antimatter, with the clarification that all positive and negative matter quantities are referred to positive units of measurements for matter, while all negative and positive antimatter quantities are referred to negative units. In particular, antimatter is predicted to have negative energy $E^d = -E$ exactly as conceived by Dirac [41] and evolve along a negative time $t^d = -t$ according to an old attempt to understand annihilation of matter and antimatter. Causality and other physical problems are resolved by the isodual mathematics, since negative quantities are measured in terms of negative units. Hence, antimatter evolving backward in time with respect to negative units of time is as causal as matter evolving forward in time with respect to positive units of time. The same holds for negative energy referred to negative units, and of other negative quantities.

The first known formulation of Newton equation for antiparticles is based on the Newton-Santilli isodual equations, and confirmed their verification of all known experimental data on the classical behavior of antiparticles [22].

A systematic presentation of the isodualities of Euclidean, Minkowskian and Riemannian geometries, Lie theory, rotational, Galilean, Lorentz and Poincaré symmetries, Galilean and special relativities, and other basic formulations is provided which in particular, presented the first known consistent representation of the gravitational field of an antimatter body via the Riemann-Santilli isodual geometry [8].

New isoselfdual cosmology at the limit of equal amounts of matter and antimatter, in which case all total quantities of the universe, such as total time, total mass, total energy, etc., are identically null to avoid a discontinuity at creation and set up the basis for continuous creation [23].

The light emitted by antimatter, also called isodual light, resulting in a prediction of main character for the detection of antimatter galaxies according to which antimatter light is physically different than matter light in an experimentally verifiable way. Since the photon has no charge, the only possible conjugation is that for all other physical quantities. As a result, antimatter light is predicted to possess negative energy while all other characteristics are opposite to those of matter light. In particular, antimatter light is predicted to be repelled by matter gravity (Fig. 9), thus permitting the conception of experiments, e.g. via neutron interferometry, to verify whether one of the two photons emitted in electron-positron annihilation experiences repulsion in our gravitational field [9, 42-47].

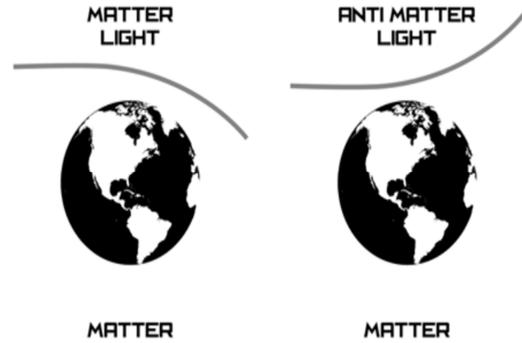


Figure 9. A view of the repulsion of antimatter light by a matter gravitational field predicted by the isodual theory of antimatter: The repulsion of antimatter light by a matter gravitational field which is a consequence of the classical conjugation of neutral matter into antimatter.

The first known hypothesis presented that the antimatter light possesses a negative index of refraction $n^d = -n$ when propagating within a transparent matter medium. Again, the consistent characterization of neutral antimatter requires the conjugation of all quantities with no exclusion to avoid catastrophic inconsistencies. This implies the necessary conjugation of the index of refraction into a negative value referred to our positive units of measurements since it is observed in our matter world (Fig. 10) [42-47].

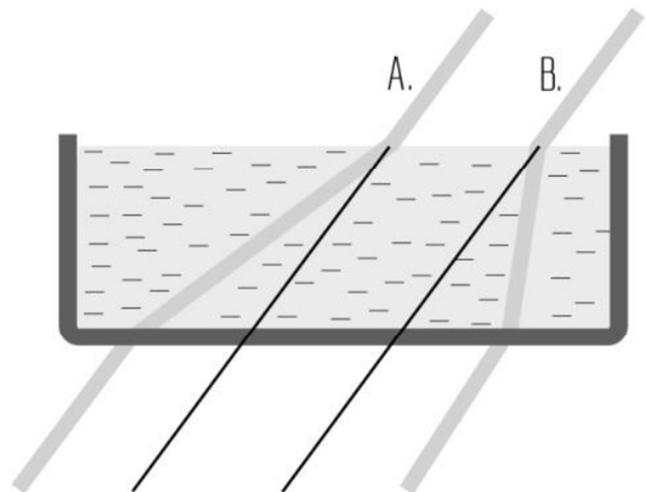


Figure 10. The prediction of negative index of refraction of antimatter light within matter water: The negative index of refraction of antimatter light which is a consequence of the repulsion of antimatter light from a matter gravitational field.

An important implication of the isodual theory of antimatter is the clarification that the conventional Dirac equation characterizes the tensorial product of one point-like particle with spin $\frac{1}{2}$ and its antiparticle without any need for second quantization [11]. Santilli could not accept the conventional 20th century view that Dirac's equations represents only one particle with spin $\frac{1}{2}$ because there exists no irreducible or reducible representation of the SU(2)-spin symmetry with the structure of Dirac's gamma matrices. Therefore, the author re-inspected Dirac's equation and showed that $\gamma^k = \sigma^k \times \sigma^{dk}$. And $\gamma^4 = \text{Diag.} (I_{2 \times 2}, -I_{2 \times 2})$ thus yielding the indicated

characterization of a spin $\frac{1}{2}$ particle and its antiparticle.

Dirac himself provided the true foundation of the isodual theory of antimatter by characterizing antiparticles with the negative unit $-I_{2 \times 2}$. Dirac merely missed the mathematics for the consistent physical treatment of negative energies. Note that there is no contradiction for a representation of antiparticle at the quantum mechanical level because the isodual theory of antiparticles applies at the classical level, let alone that of first quantization.

It should be aware that a negative index of refraction implies that antimatter light propagates within a transparent matter medium at superluminal speeds. A conceptual interpretation of this prediction is that the ordinary (positive) index of refraction for matter light propagating within a transparent matter medium is due to various, ultimately attractive interactions that slow down the speed of matter light. By contrast, when antimatter light propagates within a transparent matter medium, for consistency, all features of matter have to be conjugated, resulting in new repulsive interactions between antimatter light and the matter medium that, as such, accelerate antimatter light to superluminal speeds.

4. Application of Santilli's Isodual Theory for Detection of Antimatter Galaxies

During his Ph. D. in physics in the mid 1960s, the Italian American scientist Ruggero Maria Santilli decided to ascertain whether a far away galaxy was made up of matter or of antimatter and, in this way, initiated a fifty year long scientific journey. As a first step, Santilli proved that none of the 20th century mathematics, physics and optics were applicable for a classical study of antimatter, because the annihilation of matter and antimatter into light (when in contact with each other) requires a conjugation of all physical characteristics in the transition from matter to antimatter. Such a conjugation was absent in all 20th century sciences, since they were specifically built to treat matter. As an example, Einstein special and general relativities were conceived decades before the discovery of antimatter and, therefore, they were unable to represent matter-antimatter annihilation. Also, far away antimatter stars and galaxies have to be assumed as being neutral, thus implying the complete "inapplicability" (and not the "violation") of Einstein theories for the study of antimatter, since said theories only had the sign of the charge for conjugation.

In the early 1980s, Santilli constructed a new mathematics via a conjugation of conventional mathematics that was suitable for the "classical" description of "neutral" (or charged) antimatter bodies, technically known as anti-Hermiticity and called Santilli isoduality. Physical applications of conventional mathematics are based on positive units (such as +1 sec, +1 meter, etc.). In order to conjugate from neutral matter to neutral antimatter, Santilli constructed his new mathematics based on negative units (such as -1 sec, -1 meter, etc.). Since the charge cannot be used for conjugation of

neutral bodies, Santilli achieved a consistent representation of antimatter by conjugating all physical characteristics of matter, such as mass, energy, angular momentum, etc. and by conjugating for consistency also their units.

Santilli then spent decades of studies for the construction of the isodual image of the main aspects of 20th century mathematics, including the conjugation of number theory, functional analysis, differential calculus, symmetries, etc. The new mathematics has such a form as to admit negative left and right units at all levels [22]. The resulting new mathematics is today known as Santilli isodual mathematics [11].

Following the achievement of the appropriate new mathematics, Santilli conducted decades of studies on the construction of the corresponding physical theory, today known as Santilli isodual theory of antimatter, which includes the isodual image of all main parts of 20th century physics, including the isodual image of special and general relativities, by achieving in particular the first known consistent classical representation of the gravitational field of neutral (or charged) antimatter bodies. Additionally, Santilli constructed the isodual image of quantum mechanics, namely, an image of quantum mechanics compatible with isodual relativities. As a central part of the above studies, Santilli proved that the isodual theory of antimatter verifies "all" known experimental data on antimatter at both the classical and quantum levels [13].

Following, decades of research for the achievement of the appropriate mathematical and physical treats, Santilli initiated experimental test of his 50 year old dream: ascertain whether a far away star or galaxy is made up of matter or of antimatter. As an invited keynote speaker at the International Conference on Antimatter held in Sepino, province of Isernia, Italy in May 1996, Santilli presented the historical discovery that light emitted by antimatter (called antimatter light) is physically different than light emitted by matter (called matter light) in an experimentally verifiable way [9]. In particular, matter light is attracted by a matter gravitational field, while antimatter light is repelled by a matter field, namely, it experiences gravitational repulsion (Figure 9).

In 2012, at International Conference on Numerical Analysis and Applied Mathematics ICNAAM in Kos, Greece, Santilli presented a second historical discovery according to which, when propagating within a matter transparent medium such as glass, antimatter light has an index of refraction opposite that of matter light (see Ref. [42] and Figure 10). This property was derived as a consequence of the gravitational repulsion of Figure 9.

This second historical discovery established that a conventional Galileo refractive telescope cannot focus images from antimatter stars because its convex lenses, such as a Steinheil achromatic convex doublet, will disperse antimatter light in all directions as shown in Figure 11.

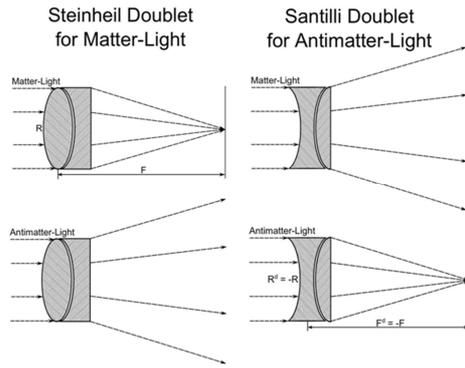


Figure 11. A view of the lenses used in the Galileo telescope to detect matter galaxies and their conjugate version used in the Santilli telescope to detect antimatter galaxies.

Consequently, Santilli conceived a conjugated doublet, called Santilli Achromatic Double Concave Doublet (international patent is pending), to focus images caused by antimatter light. Since antimatter light has an index of refraction in glass opposite that of matter light, the curvature of the lenses has to be conjugated from matter light, that is, has to be concave (see also Figures 13 and 14 of next section for details). In this way, Santilli established that none of the available telescopes can focus images of antimatter stars or galaxies because they are all based on the conventional law of refraction and related convex lenses. Consequently, images from far away antimatter stars or galaxies are dispersed in all directions by convex lenses without any focusing. Similarly, concave lenses will disperse in all directions images from matter light but they will converge images from antimatter light. Santilli also proved that we will never see antimatter images with our eyes because our iris is convex, thus dispersing antimatter light all over our retina without any focused view.

In 2012, Santilli constructed the first telescope with concave lenses, today known as Santilli Refractive Telescope or antimatter telescope (see Figure 14 of next section), and conducted systematic views of the night sky in the region of the Vega star, by achieving the first detection in scientific history of antimatter galaxies, antimatter cosmic rays and antimatter asteroids [43]. The above historical discovery has been confirmed twice by independent scientists [44] and [47].

This fifty years of mathematical, theoretical and experimental research of Santilli can provide an answer to his question of the mid 1960s, with the conclusions that: 1) All galaxies we see in the universe with the various available telescopes are solely made up of matter; 2) There exist indeed antimatter galaxies in the universe, but they are solely visible via special telescopes with concave lenses; and 3) We will never be able to focus images of antimatter with our eyes because our iris is convex.

The details of actual detections of far away antimatter galaxies and antimatter cosmic radiations are as given in the following sections.

4.1. Santilli’s Refractive Telescope with Concave Lenses or Antimatter Telescope and Experimental Method

Santilli has been constructed a new refracting telescope

with “concave” lenses; for detection of antimatter light from distant sources, because a conventional telescope with convex lenses (Galileo telescope) will disperse light with a negative index of refraction. For that Santilli secured the design and fabrication of two identical Galileo refracting telescopes; without the star diagonal viewer to avoid any unnecessary reflection of antimatter light.



Figure 12. The two identical Galileo telescopes and the camera [43].

One of the two telescopes converted to a concave version with identical but conjugated foci. The transformation of the telescope from the Galileo form with 100 mm effective convex primary lenses, to the Santilli’s antimatter telescope with features identical to those the Galileo one but conjugated based on Santilli’s isodual mathematics as described above. Since the camera is directly attached to the telescope without the eyepiece, this conversion essentially consisted in the fabrication and assembly of concave lenses as per the data of Figure 13 and Figure 14 provides a comparative view of the Galileo and the Santilli’s antimatter telescope.

He secured one single suitably selected camera (Canon: model EOS 600D with image sensor of type CMOS, and Bayer Filter) to obtain pictures from both the Galileo and the Santilli telescopes. He also secured a tripod with mount suitable for the parallel housing of the two telescopes. He optically aligned the two telescopes on the tripod by keeping in mind the evident impossibility of doing visual alignments with the antimatter telescope and conducted a number of day views with the so mounted and aligned pair of Galileo and Santilli telescopes to verify that astronomical objects visible in the former are not visible in the latter [43].

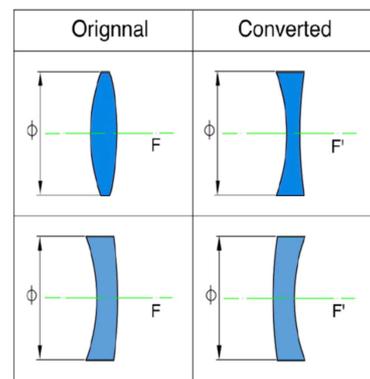


Figure 13. Main characteristics of the Galileo and antimatter primary lenses [43].

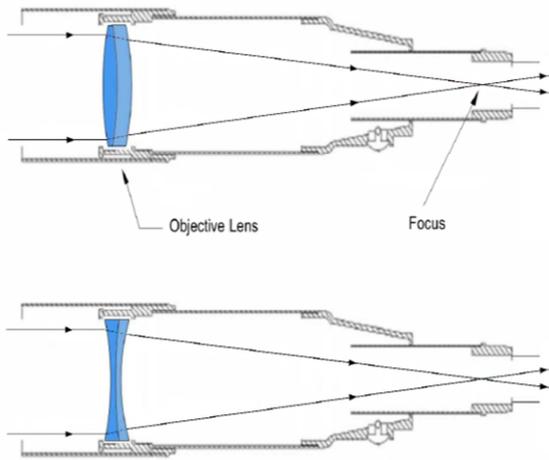


Figure 14. Schematic view of the telescopes with convex and concave lens [43].



Figure 15. The parallel mount of the Galileo and antimatter telescopes with related finder scopes [43].



Figure 16. A close up view of the mounting of the camera directly in the telescope in place of the eyepieces [43].

A number of night views of the same region of the sky via the so mounted and aligned Galileo and Santilli telescopes was conducted and obtained a number of pictures from both telescopes via the selected camera; and finally conducted a comparative inspection of the pictures from both telescopes under a variety of enlargements and contrasts to see whether

the pictures from the antimatter telescope contained focused images absent in the pictures from the Galileo telescope under the same enlargement and contrast.

Following the availability of the so mounted and aligned pair of telescopes, Santilli initiated night views by first confirming that, as expected, any celestial object visibly focused by the Galileo telescope was not focused at all with the antimatter telescope. In particular, the view of details of our Moon, which were very nicely focused by the Galileo telescope, resulted in a diffuse light when seen from the antimatter telescope without any possible identification. The same occurred for planets and nearby matter stars. Then Santilli finally initiated preliminary views of the sky at night with said pair of telescopes. He reported the tests conducted at the Gulf Anclote Park, Holiday, Florida, and GPS Coordinates: Latitude = 28.193, Longitude = - 82.786. The camera was set at the exposure of 15 seconds for the specific intent of having streaks of light from far away matter stars caused by Earth rotation, since streaks can be better identified with the limited capabilities of the available telescopes compared to individual dots of light in the pictures [43]. Additionally, streaks from matter stars have a clear orientation as well as length that are important for the identification of possible streaks from antimatter light. Following various tests, he selected the 10 setting of the camera at ISO 1600 because various tests with smaller and bigger ISO resulted inconclusive and ambiguous for various reasons. All pictures were analyzed (for details refer Ref. No. 43) with particular reference to the identification of the background as well as impurities in the camera sensors that are evidently present in both pictures from the Galileo and the Santilli telescope. The magnification has been obtained by Santilli via the Gimp 2.8 software [43]. In the succeeding sections pictures obtained by this pair of telescopes (Galileo telescope and Santilli telescope); in original (i.e. of Santilli) and two confirmative tests of independent researchers; are shown for the convenience of the readers. The focusing of images of antimatter galaxies, antimatter cosmic radiations etc. via a telescope with concave lenses is the first known experimental indication on the existence of antigenicity because a negative index of refraction is solely possible for the repulsion of antimatter light from matter.

4.2. Original Pictures of Santilli's Apparent Detection of Antimatter Galaxies

Following these preliminaries, Santilli oriented both telescopes at the indicated location and time toward the star Vega, and then specialized the orientation for the pair of matter stars Epsilon Alpha and Epsilon Beta near Vega. Some of the pictures taken by Galileo and Santilli telescopes of matter and antimatter galaxies respectively and of annihilation of antimatter cosmic rays [43] are as shown below.

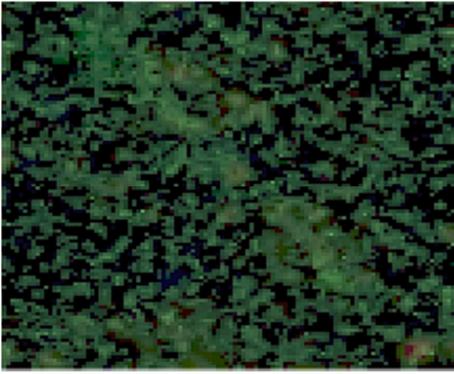


Figure 17. View of one of the streaks of matter light representing a far away matter star or galaxy identified in the Epsilon Alpha and Beta region of the night sky near Vega via the Galileo telescope [43].

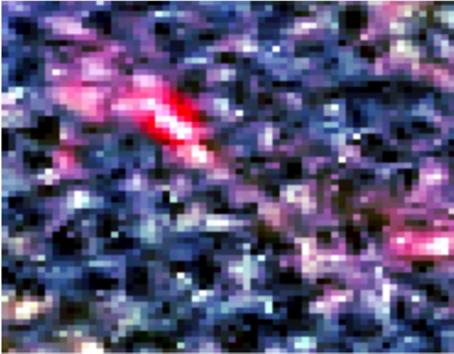


Figure 18. View of (a) First Streak of light detected in the Epsilon Alpha and Beta region with the Santilli telescope of antimatter galaxies [43].

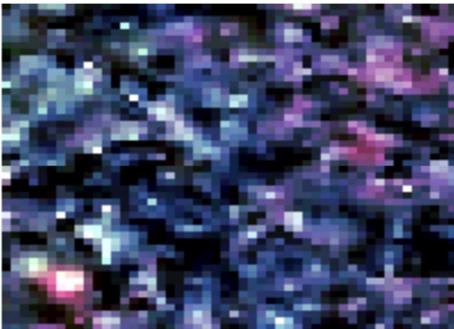


Figure 19. View of Second streak of light detected in the Epsilon Alpha and Beta region with the Santilli telescope of antimatter galaxies [43].

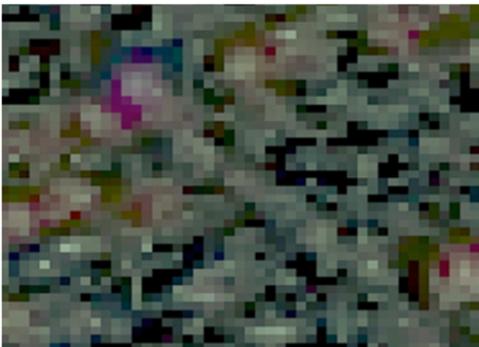


Figure 20. View of Third streak of light detected in the Epsilon Alpha and Beta region with the Santilli telescope of antimatter galaxies [43].

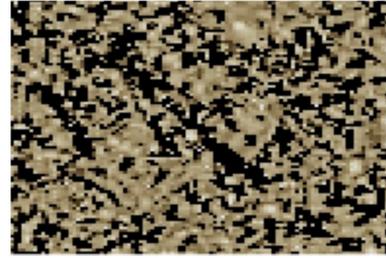


Figure 21. The first streak of darkness identified in the picture of the Epsilon Alpha and Beta region of the night sky taken with the antimatter telescope providing possible evidence of a far away antimatter star or galaxy as an alternative for the streaks of light [43].

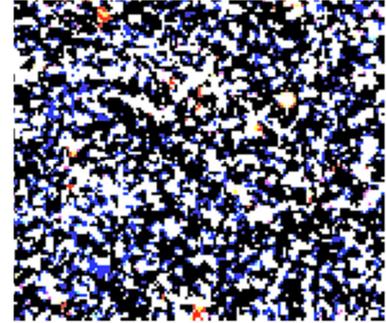


Figure 22. Another representative streak of darkness present in the antimatter telescope but absent in the Galileo telescope that may constitute an alternative to the streak of light [43].

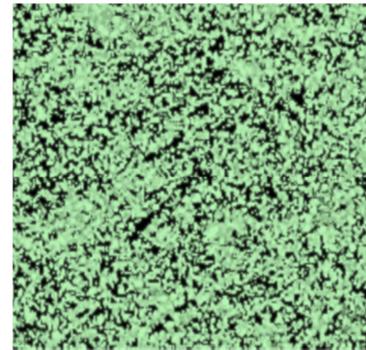


Figure 23. Seemingly connected streaks of darkness identified in a picture of the Vega region of the night sky taken with the antimatter telescope that could be due to the annihilation of a shower of small antimatter asteroids in our atmosphere, in a way much similar but the conjugate of the frequent view in the night sky of the streaks of light caused by the annihilation of a shower of small matter asteroids in our atmosphere [43].

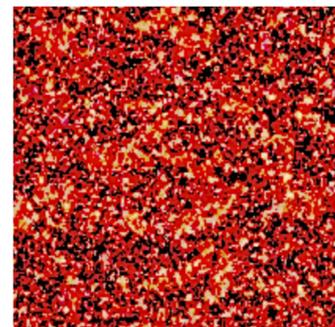


Figure 24. The first of numerous circular traces identified in a picture of Vega regions of the night sky on November 7, 2013, with the Santilli's antimatter telescope that could be due to the annihilation of an antimatter cosmic ray [43].

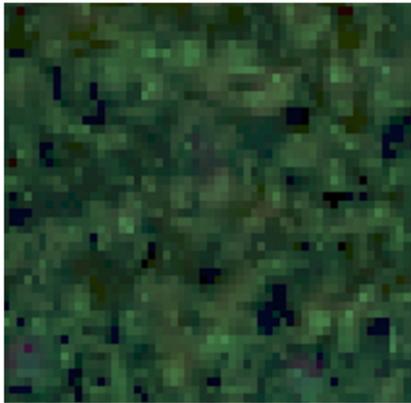


Figure 25. View of a circular trace identified in a picture of Deneb regions of the night sky with the Santilli's antimatter telescope [43].

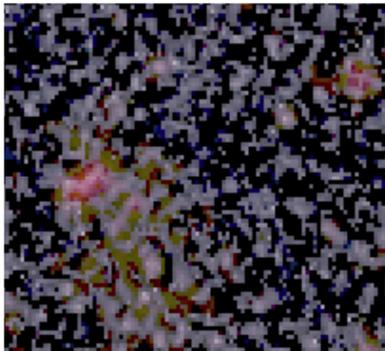


Figure 26. View of a circular trace identified in a picture of Altair regions of the night sky with the Santilli's antimatter telescope [43].

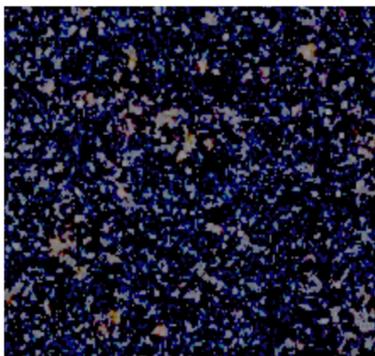


Figure 27. View of a circular trace identified in a picture of Sadr regions of the night sky With the Santilli's antimatter telescope [43].

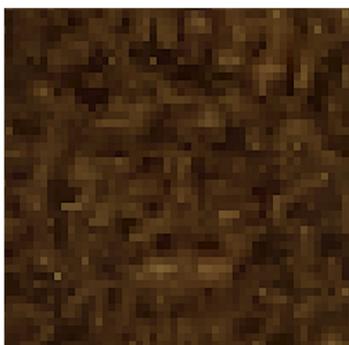


Figure 28. View of a circular trace identified in a picture of Gienah Cyngi regions of the Night sky with the Santilli's antimatter telescope [43].

4.3. Images of Antimatter for Preliminary Confirmations of Santilli's Apparent Detection of Antimatter

By using the same pair of Galileo and Santilli telescopes, the same camera, the same exposure of 15 seconds for ISO 1600, a team of scientist [44] went to Sebring, Florida and Enclote Gulf Park in Holiday, Florida and obtained pictures from both telescopes of the same region of the night sky studied by Santilli (that of Epsilon Alpha and Beta stars). Some of the original pictures are available from Ref. [48] in raw and tiff formats under the markings "Galileo-Epsilon-Sebring" and "Santilli-Epsilon-Sebring for readers". Figures 29 to 43 reports selected joint views from the Galileo and the Santilli telescopes showing clearly anomalous streaks that are present in the Santilli telescope but absent in the Galileo telescope, which streaks have essentially the same orientation and length of the streaks caused by matter stars, thus confirming the corresponding anomalous streaks first obtained by Santilli in a telescope with concave lenses i.e. Santilli's antimatter telescope.

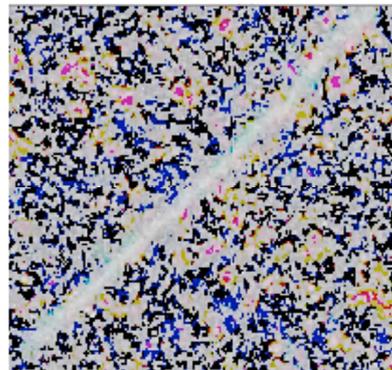


Figure 29. Picture from the Galileo Telescope of a star in the Epsilon region of the sky from Sebring, Florida [44].

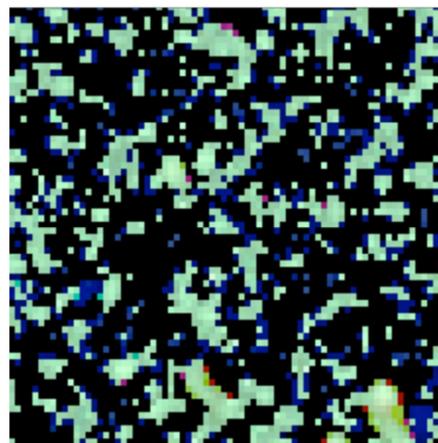


Figure 30. Picture from the Santilli telescope of a black streak in the Epsilon region of the sky from Sebring, Florida [44].

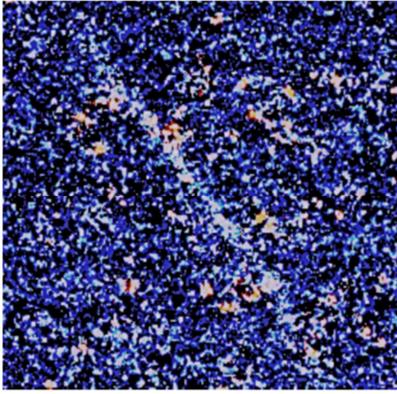


Figure 31. Picture from the Galileo telescope of a star from the Epsilon region of the sky from Sebring, Florida [44].

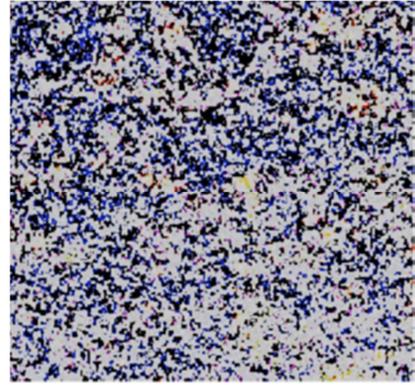


Figure 35. Picture from the Santilli telescope of another streak in the Vega region of the sky from Holiday, Florida [44].



Figure 32. Picture from the Santilli telescope of a black streak in the Epsilon region of the sky from Sebring, Florida [44].

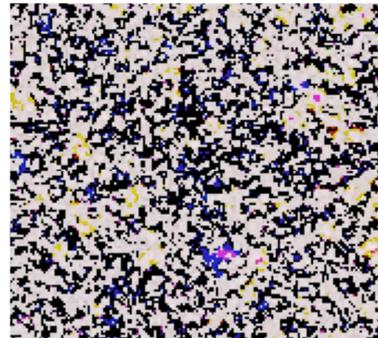


Figure 36. Picture from the Santilli telescope of an unknown event in the Epsilon sky region from Sebring, Florida [44].



Figure 33. Picture from the Galileo Telescope of a streak of a matter star in the Vega region of the sky from Holiday, Florida [44].

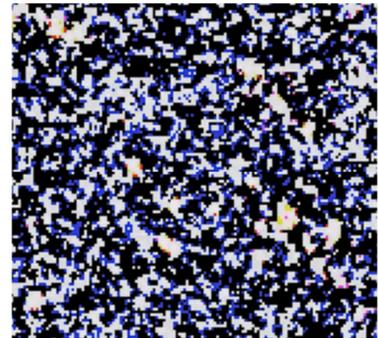


Figure 37. Picture from the Santilli telescope of a circular trace [44].

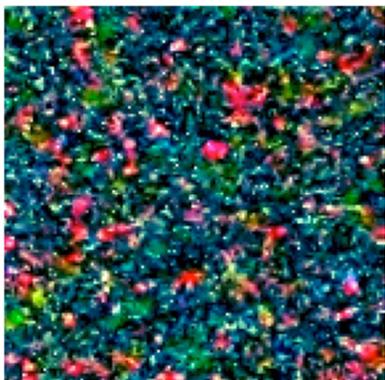


Figure 34. Picture from the Santilli telescope of a black streak in the Vega region of the sky from Holiday, Florida [44].

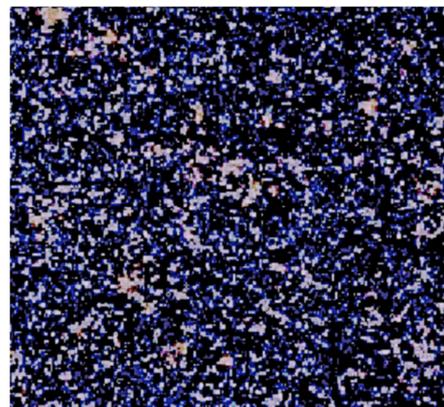


Figure 38. Picture from the Santilli telescope of another circular trace [44].

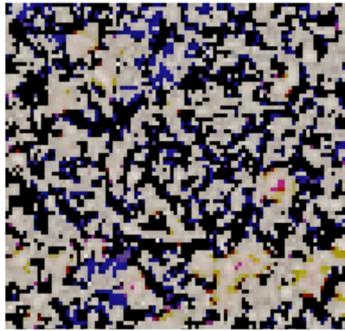


Figure 39. Picture from the Santilli telescope of yet another circular trace [44].



Figure 40. Picture from the Santilli telescope of yet another circular trace [44].

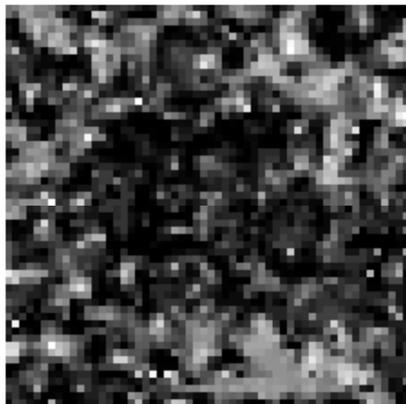


Figure 41. Picture from the Santilli telescope of yet another circular trace [44].



Figure 42. Picture from the Santilli telescope of yet another circular trace [44].

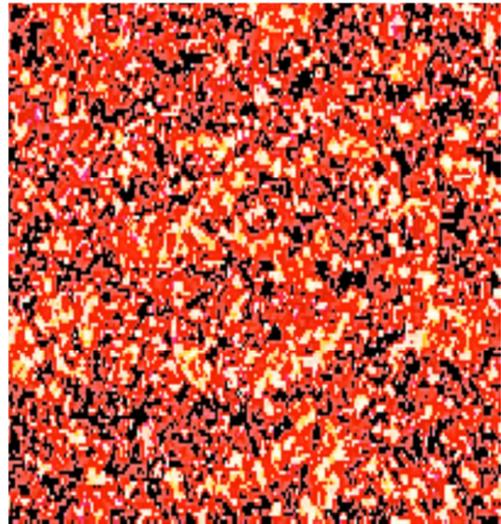
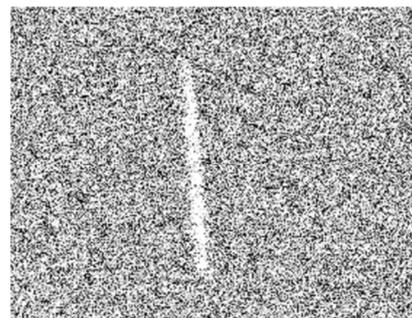


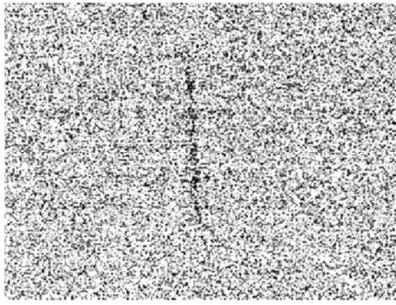
Figure 43. Picture from the Santilli telescope of yet another circular trace [44].

4.4. Pictures of Antimatter for Confirmation of Santilli's Detection of Antimatter Galaxies

Again some researchers [47] went to the same location of the preceding detections [43, 44], Gulf Anclote Park, Holiday, Florida, GPS: 28.193461, -82.786184, with the same pair of parallel Galileo and Santilli telescopes, and inspected the same region of the night sky, but this time via the use of a 35 mm Canon F-1N camera with SLR film, shutter speed B, Fujifilm roll ASA 400, and exposure Compensation 1. These telescopes were oriented toward the Draco and Vega regions of the night sky under a camera exposure of both telescopes for 15 seconds and captured numerous images on 35 mm Fujifilm Provia 400X ISO 400 out of which, for brevity we report in the link of Ref. [49] the following images: Vega-Gal-ISO400-019.tif, Vega-Sant-ISO400-020.tif, Draco-Gal-iso200-004.tif, Draco-Sant-Iso-ISO200-005.tif. The rolls containing all original images were developed by Zebra Color Company, 1763 1st Ave. North, St. Petersburg, FL 33713 (<http://zebracolor.com/index.html>) and the developed images were scanned at 5760 dpi. The scanned images were enlarged and inspected via the use of paint.net software for PC. These representative images of are listed as below, in Figures 44 to 50.

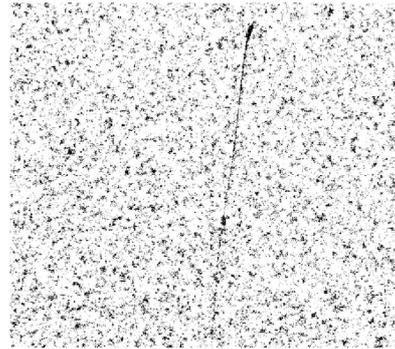


(a)



(b)

Figure 44. Images from the Vega region of the night sky showing a streak due to matter light from the Galileo telescope (a) and a streak of darkness from the Santilli telescope caused by antimatter light (b) [47].



(b)

Figure 46. Images from the Vega region of the night sky showing a streak of darkness expectedly from a far away antimatter light (a), and a streak of darkness expectedly from a small antimatter asteroid annihilating in our atmosphere (b) [47].

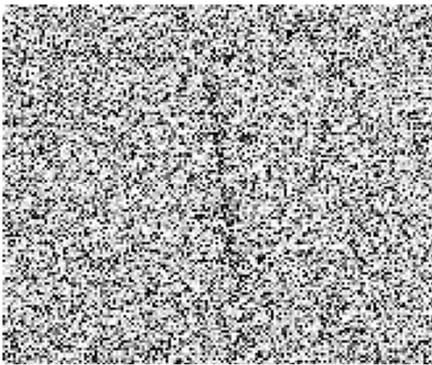
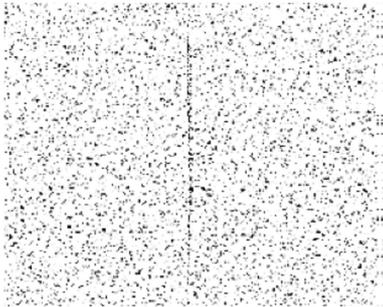
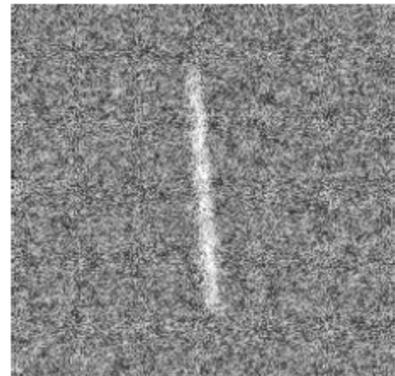
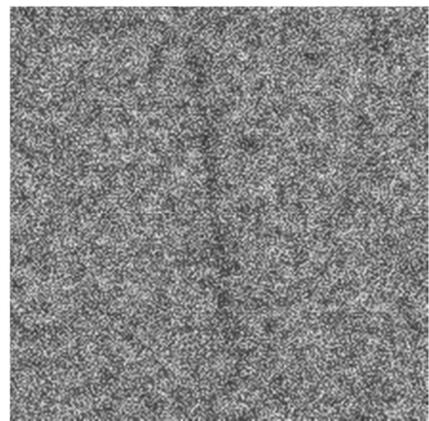


Figure 45. Images from the Vega region of the night sky from the Santilli telescope showing streaks of darkness caused by antimatter light [47].

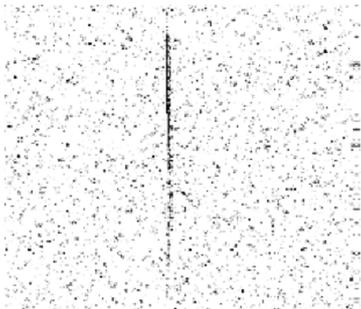


(a)



(b)

Figure 47. Images from the Draco region of the night sky showing a streak caused by matter light from the Galileo telescope (a) and a streak of darkness from the Santilli telescope caused by antimatter light (b) [47].



(a)

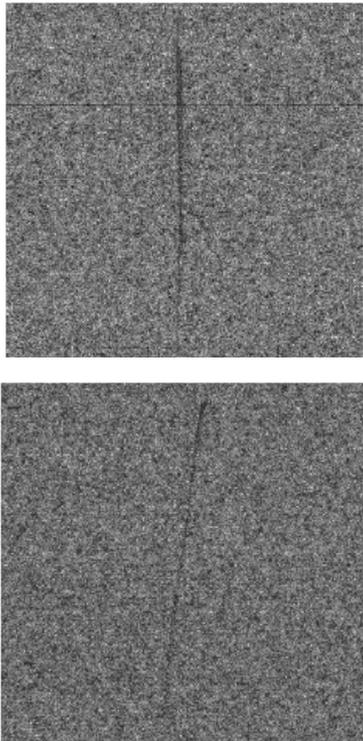
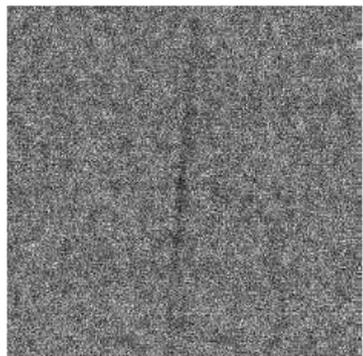
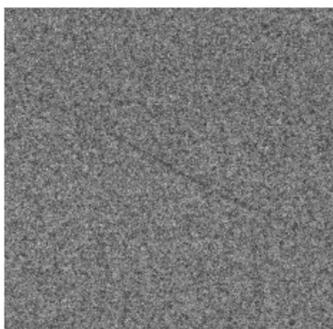


Figure 48. Images from the Draco region of the night sky via the Santilli telescope showing streaks of darkness caused by antimatter light [47].



(a)



(b)

Figure 49. Images from the Draco region of the night sky showing a streak of darkness expectedly from a far away antimatter light (a), and a streak of darkness expectedly from a small antimatter asteroid annihilating in our atmosphere (b) [47].

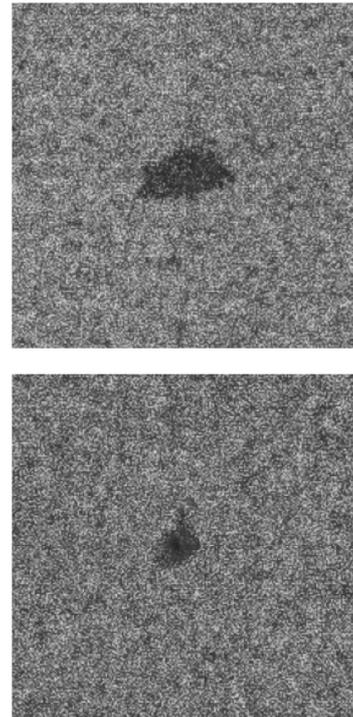


Figure 50. Images from the Draco region of the night sky via the Santilli telescope showing dots expected from the annihilation of antimatter cosmic rays in the upper regions of the atmosphere [47].

Figure 50 indicates anomalous dots (antimatter light produced by antimatter cosmic rays annihilating in the upper region of our atmosphere) present in the Santilli telescope but not in the Galileo telescope [47].

5. Conclusion

Based on the above analysis it is concluded that Santilli's isodual theory does indeed provide a consistent and time invariant, classical and operator description antimatter in a way compatible with available experimental data at both, the classical and quantum levels.

In particular, our analysis confirms that; 1) a consistent classical representation of neural antiparticles in a way compatible with the known quantum description can be achieved via negative-definite physical quantities such as energy, momentum, time, etc., under the consistency condition that they are measured with negative-definite units and 2) the focusing of images by a telescope with concave lenses appears to be the first experimental evidence of antigravity between matter and antimatter because the negative index of refraction for isodual light propagating within a matter medium necessary for said focusing can only be explained via repulsive interactions between the isodual light and the matter field.

Acknowledgements

The author expresses his gratitude to Prof. R. M. Santilli for encouragement and a critical reading of the paper. Thanks are also due to Prof. A. A. Bhalekar for his guidance.

References

- [1] P. A. M. Dirac, "The Principles of Quantum Mechanics", 4th ed. (Clarendon, Oxford, 1958).
- [2] R. L. Forward, in Antiprotons Science and Technology, eds. B. W. Augenstein, B. E. Bonner, F. E. Mills and M. M. Nieto (World Scientific, Singapore, 1988).
- [3] R. M. Santilli, *A Classical Isodual Theory of Antimatter and Its Prediction of Antigravity*, International Journal of Modern Physics A, 14 (1999) 2205-2238. <http://www.santilli-foundation.org/docs/Santilli-09.pdf>.
- [4] R. M. Santilli, Hadronic J. 8, (1985) 25-36.
- [5] R. M. Santilli, Commun. Theor. Phys. 3 (1993) 153.
- [6] R. M. Santilli, (a) Hadronic J. 17 (1994) 257; (b) Hadronic J. 17 (1994) 285
- [7] R. M. Santilli, *Antimatter Gravity and Antihydrogen Spectroscopy* (M. Holzschneider (editor), Baltzer Science Publ., 1997).
- [8] R. M. Santilli, Elements of Hadronic Mechanics, Volumes I and II, Ukraine Academy of Sciences, Naukova Dumka Publishers, Kiev, 1st edition (1994), 2nd edition (1995). <http://www.santilli-foundation.org/docs/Santilli-300.pdf> <http://www.santilli-foundation.org/docs/Santilli-301.pdf>
- [9] R. M. Santilli, *Does antimatter emit a new light?* Hyperfine Interactions Vol. 109 (1997) 63-81. <http://www.santilli-foundation.org/docs/Santilli-28.pdf>.
- [10] R. M. Santilli, Found. Phys. 27 (1996) 305.
- [11] R. M. Santilli, *Isodual Theory of Antimatter with Applications to Antigravity*, Grand Unification and Cosmology, Springer (2006). <http://www.santilli-foundation.org/docs/santilli-79.pdf>.
- [12] A. P. Mills, "Possibilities of measuring the gravitational mass of electrons and positrons in free horizontal flight", contributed paper for the Proceedings of the International Conference on Antimatter, held in Sepino, Italy, May 1996, published in the Hadronic J. Vol. 19 (1996) pp. 77-96 <http://www.santilli-foundation.org/docs/Santilli-11.pdf>
- [13] R. M. Santilli, *Isominkowskian Geometry for the Gravitational Treatment of Matter and its Isodual for Antimatter*, Intern. J. Modern Phys., D 7 (1998) 351-365. <http://www.santilli-foundation.org/docs/Santilli-35.pdf>
- [14] C. D. Anderson, Phys.Rev. 43 (1933) 491.
- [15] R. M. Santilli, "Isodual Theory of Antimatter with Application to Antigravity, Grand Unification and the Spacetime Machine", Springer, New York, (2001)
- [16] P. J. Wyatt, "Possible Existence of Anti-Matter in Bulk", Nature, Vol. 181, (1958) p 1194. doi:10.1038/1811194b0.
- [17] C. D. Dermer, "Gamma-Ray Bursts from Comet-Antimatter Comet Collisions in the Oort Cloud." In C. Kouveliotou, M. S. Briggs, & G. J. Fishman. 384. The Third Huntsville Symposium on Gamma-Ray Bursts, Huntsville AL, USA, October 1995, 2527, AIP Conf. Proc., Vol. 384, (1995) pp. 744-748. doi:<http://dx.doi.org/10.1063/1.51650>, Woodbury: American Institute of Physics.
- [18] V. Rojansky, "Cosmic Rays and Comets," Phys. Rev. Vol. 58, (1940) p 1010.
- [19] R. M. Santilli, "Isoduals and pseudoduals, and hidden numbers of dimension 3, 5, 6," 7, Algebras, Groups and Geometries 10, (1993) pp. 273-321. <http://www.santillifoundation.org/docs/Santilli-34.pdf>
- [20] R. M. Santilli, "Nonlocal Integral axiom-preserving isotopies and isodualities of the Minkowskian Geometry", in The Mathematical Legacy of Hanno Rund, J.V. Kadeisvili, Editor, Hadronic Press, Palm Harbor, (1993) pp. 383-430.
- [21] R. M. Santilli, "Isotopies, genotopies and isodualities of Lie's Theory", Talk delivered at the International Congress of Mathematicians, Zurich, August 3-11, unpublished (1994)
- [22] R. M. Santilli, "Nonlocal-Integral Isotopies of Differential Calculus, Mechanics and Geometries, in Isotopies of Contemporary Mathematical Structures" Rendiconti Circolo Matematico Palermo, Suppl. 42 (1996) pp. 7-82. <http://www.santillifoundation.org/docs/Santilli-37.pdf>
- [23] R. M. Santilli, "A new cosmological conception of the universe based on the isominkowskian geometry and its isodual", Part I pages 539-612 and Part II pages Contributed paper in Analysis, Geometry and Groups, A Riemann Legacy Volume, Volume II, H.M. Srivastava, Editor, (1993) pp. 539- 612
- [24] R. M. Santilli, "Can antimatter asteroids, stars and galaxies be detected with current means?" Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes, C. Corda, Editor, Kathmandu University, Nepal, (2011) pp 25-36. <http://www.santilli-foundation.org/docs/Antimatter-Asteroid.pdf>
- [25] Vougiouklis, "An Introduction to Santilli's Isodual Representation of Antimatter and the Ensuing Problem of Detecting Antimatter Asteroids", Numta Bulletin Issue 6-7, (2013). pp. 1-33. <http://www.santilli-foundation.org/docs/Antimatter-2013.pdf>
- [26] Ivan Gandzha and Jerdsey Kadeisvili, "New Sciences For A New Era, Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli", San-Marino draft dated June 28, 2011, Sankata Printing Press, Nepal, 2011. <http://www.santillifoundation.org/docs/RMS.pdf>
- [27] V.de Haan, "Proposal for the realization of Santilli comparative test on the gravity of electrons and positrons via a horizontal supercooled vacuum tube", Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes, C. Corda, Editor, Kathmandu University, (2011), pp. 57-67. <http://www.santilli-foundation.org/docs/deHaan-Arxiv.pdf>
- [28] R. Anderson, Santilli's comparative test of the gravity of electrons and positrons in a horizontal supercooled vacuum tube, Scientific document of the R. M. Santilli Foundation, February (2012). <http://www.santilli-foundation.org/docs/anitgravity-experiment-2012.pdf>
- [29] V. de Haan, "Santilli's comparative test of the gravity of electrons and positrons in a horizontal supercooled and supervacuum tube," <http://www.santilli-foundation.org/docs/Santilli-gravity-experiment.pdf>

- [30] W. M. Fairbanks and E. C. Witteborn, Phys. Rev. Letters, Vol. 19 (1967) pp. 1049.
- [31] G. testera, Hyperfine Interactions, Vol. 109 (1997) pp.333,
- [32] R. Poggiani, Hyperfine Interactions, Vol. 109 (1997) pp. 367,
- [33] M. G. Giammarchi et al. AEGIS Collaboration, "AEGIS at CERN: Measuring Antihydrogen Fall" in Proceedings of the Fifth Meeting on CPT and Lorentz Symmetry", held at Indiana University in Bloomington in June, 2010, <http://arxiv.org/abs/1106.0209>.
- [34] A. E. Charman et al, ALPHA Collaboration, Description and first application of a new technique to measure the gravitational mass of antihydrogen," Nature Communications, April 2013. DOI: 10.1038/ncomms2787
- [35] J. Dunning-Davies, "Thermodynamics of Antimatter via Santilli's isodualities", Foundations of Physics Letters, 12, (1999) pp. 593-599.
- [36] [R. M. Santilli, contributed paper to Proceedings of the International Workshop on Modern Modified Theories of Gravitation and Cosmology, E. I. Guendelman, Editor, Hadronic Press, pages 113-169 (1998).
- [37] R. M. Santilli, "Representation of antiparticles via isodual numbers, spaces and geometries", Comm. Theor. Phys., Vol. 3, (1994) 153-181. <http://www.santilli-foundation.org/docs/Santilli-12.pdf>.
- [38] R. M. Santilli, "Antigravity", Hadronic J., Vol. 17, (1994) 257-284 . <http://www.santilli-foundation.org/docs/Santilli-113.pdf>.
- [39] R. M. Santilli, "Space-time machine", Hadronic J., Vol. 17, (1994) 285-310. <http://www.santilli-foundation.org/docs/Santilli-10.pdf>.
- [40] J. Dunning-Davies, "Isodual thermodynamics for antimatter", to be completed and uploaded <http://www.santilli-foundation.org/docs/Dunning-Davies.pdf>.
- [41] P. A. M. Dirac, Proceedings of the Royal Society, 117(1928) pp. 610-624.
- [42] R. M. Santilli. "The Mystery of Detecting Antimatter Asteroids, Stars and Galaxies," American Institute of Physics Proceed. 2012, 1479, (2012) 1028-1032. <http://www.santillifoundation.org/docs/antimatter-asteroids.pdf>
- [43] Ruggero Maria Santilli, "Apparent detection of antimatter galaxies via a refractive telescope with concave lenses", Clifford Analysis, Clifford Algebras and their Applications (Cambridge, UK), Vol. 3, No. 1 (2014) pp 1-26 <http://www.santilli-foundation.org/docs/Antimatter-telescope-2013-final.pdf>.
- [44] P. M. Bhujbal, J. V. Kadeisvili, A. Nas, S. Randall, and T. R. Shelke, "Preliminary Confirmations of Antimatter Detection via Santilli's Telescope with Concave Lenses", Clifford Analysis, Clifford Algebras and their Applications (Cambridge, UK), Vol. 3, No. 1 (2014) pp. 27-39. <http://www.santilli-foundation.org/docs/Con-Ant-Tel-2013.pdf>
- [45] P. M. Bhujbal, "Santilli's Detection of Antimatter Galaxies: An Introduction and Experimental Confirmation", AIP Conference Proceedings Vol. 1648 (2015) pp. 510005-1-510005-5. <http://dx.doi.org/10.1063/1.4912710> [http://www.santilli-foundation.org/docs/1.4912710\(PM Bhujbal\).pdf](http://www.santilli-foundation.org/docs/1.4912710(PM%20Bhujbal).pdf)
- [46] P. M. Bhujbal, "Santilli's Apparent Detection of Antimatter Galaxies: An Introduction and Experimental Confirmation", Hadronic Journal [Submitted for Publication] ISSN: 0162-5519 <http://www.santilli-foundation.org/docs/bhujpal-antim-2014.pdf>
- [47] Simone Beghella-Bartoli, Prashant M. Bhujbal, and Alex Nas, "Confirmation of Santilli's detection of antimatter galaxies via a telescope with concave lenses", American Journal of Modern Physics, Special Issue: New Science Light Path on Cosmological Dark Matters, 4(1-1) (2015) 34-41 <http://www.santilli-foundation.org/docs/Antimatter-Detection.pdf>
- [48] P. M. Bhujbal, J. V. Kadeisvili, A. Nas, S. Randall, and T. R. Shelke, "Pictures of the Epsilon and Vega sky" (785 MB) <http://www.santilli-foundation.org/docs/Sebring-Holiday-Epsilon-Vega.zip>
- [49] S. Beghella-Bartoli, P. M. Bhujbal, and A. Nas, "Original images of the detections", 1 Giga Bytes <http://www.santilli-foundation.org/docs/original-images.zip>.

Possible Role of Antimatter Galaxies for the Stability of the Universe

S. Beghella-Bartoli, R. M. Santilli

Thunder Energies Corporation, 1444 Rainville Rd, Tarpon Springs, FL

Email address:

research@thunder-energies.com

To cite this article:

S. Beghella-Bartoli, R. M. Santilli. Possible Role of Antimatter Galaxies for the Stability of the Universe. *American Journal of Modern Physics*. Special Issue: Issue II: Foundations of Hadronic Mechanics. Vol. 5, No. 2-1, 2016, pp. 185-190.

doi: 10.11648/j.ajmp.2016050201.21

Received: January 19, 2016; **Accepted:** January 19, 2016; **Published:** June 1, 2016

Abstract: Recent mathematical, theoretical and experimental studies have confirmed via measurements on Earth Zwicky's hypothesis according to which the cosmological redshift is due to galactic light losing energy to intergalactic media without the expansion of the universe. The main problem of the ensuing return to a static universe is the inevitable prediction that the universe should collapse due to gravitational attractions among galaxies. In this paper, we review the historical inability by general relativity to achieve a stable universe solely composed of matter, and present apparently for the first time a cosmological model in which the universe achieves stability under the condition of admitting an equal number of matter and antimatter galaxies at such a large mutual distance for which gravitational interactions are ignorable.

Keywords: Antimatter, Static Universe, Universe Stability

1. Historical Notes

The study of the origin, evolution, and eventual fate of the universe has always been debated by philosophers and cosmologists, beginning with Aristarchus, Aristotle and Ptolemy. In particular, the geocentric theory of Ptolemy has been the dominant paradigm until it was put into question by Copernicus, Kepler, Galilei and Newton. But it was Albert Einstein, who essentially set up the foundations of what we nowadays call modern cosmology [1].

Given the relative very small velocities of stars in the Milky Way, the general belief in the early part of the 20th century was that we were living in a static universe. The main assumptions by Einstein were that there is a system of reference relatively to which matter may be looked upon as being permanently at rest, and that the large scale structure and evolution of such a universe should be determined by its finite matter density.

These assumptions implied a difficulty in the determination of border conditions at infinity, a difficulty that was solved by Einstein with the hypothesis of a closed universe without borders. But to achieve a finite matter density in a static universe, Einstein was forced to introduce the cosmological constant in his field equations for the intent

of achieve stability via a repulsive force counter-balancing the gravitational attraction.

Einstein himself didn't like the addition of the cosmological constant in his equations because it looked to him just like an ad-hoc hypothesis that achieved the desired stability but without explaining how the universe works. Numerous additional doubts on the validity of the model were expressed, such as those by Willem de Sitter who suggested Einstein to get rid of the lambda-term in his field equations [2, 3]. In essence, de Sitter argued that the cosmological constant is arbitrary and detracts from the elegance of Einstein's original theory of gravitation.

De Sitter also proposed a model that bypassed the problem of border conditions at infinity in which space is basically empty, with zero matter density, zero curvature and no division between time and space (unlike Einstein's model where an absolute time existed), thus achieving isotropy and homogeneity throughout space and time.

This solution was considered quite seriously by Einstein. De Sitter had used lambda to derive an empty model (so no finite density of matter) that actually satisfies boundary conditions at infinity. So the lambda-term went against Einstein's own beliefs (the field should be due to the matter, without which it cannot exist), and he persuaded himself he

needed to get rid of it one way or the other.

With the passing of time, experimental measurements started to become available, with particular reference to the historical measurements by Hubble [4] on the redshift of light coming from distant "nebulae," later on called galaxies, according to the law $z = Hd$, where z is the redshift, H is the Hubble constant, and d is the distance of the galaxy from Earth. This view was embraced by Eddington [5], Weyl [6, 7], Slipher [8], Friedmann [9, 10], Lemaitre [11] and others.

It appears that, quite likely, Einstein's main desire was the removal of the cosmological constant from his field equations because his main concern was to account for a finite density of matter in the universe, and since it seemed he could do that via other models, he was happy to cast away what he thought was a troublesome addition to his theory [12]. This doesn't mean he approved the expansion and related Big Bang, but he didn't like the use of an arbitrary *ad-hoc* term to obtain equilibrium in a matter-only universe.

Einstein and de Sitter eventually came up with a new model, known as Einstein-de Sitter model [13], that influenced cosmology for the next 50 years. The solution implied a homogeneous and isotropic universe, with zero space curvature, zero cosmological constant, and zero pressure, with asymptotically null expansion (see for more details, e.g., Refs. [13-15]).

The aspect most important for this paper is that, to our best understanding of historical profiles, Einstein never explicitly acknowledged the representation of the cosmological redshift via the conjecture of the expansion of the universe. Although he didn't participate actively to the cosmological debate after his 1917 paper, he continued to prefer a stationary universe, as it is clearly demonstrated by an unpublished paper of his [16], written in 1931 and recently rediscovered [17, 18], where, after Hubble's paper, he developed another steady-state model of the universe keeping into account Hubble's results, but, considering a pure matter universe, he apparently faced mathematical inconsistencies. The same position was adopted by Hubble and other cosmologists, such as Fritz Zwicky [19] who proposed the hypothesis that the redshift of galactic light is due to loss of energy by light to inter-galactic gases.

In summary, during the first part of the 20th century, jointly with the initiation of studies on the interpretation of Hubble's law via the expansion of the universe, there were also authoritative views that essentially implied a static conception of the universe, mostly according to Hoyle's cosmology [20]. It is important to note that Einstein himself basically anticipated Hoyle's view in his 1931 unpublished paper.

2. Recent Debates in Cosmology

Even today, after so many years the debate is still open, with the so-called Standard model suffering of multiple flaws, and many scientists looking back at models of the universe that don't need a Big Bang, if not completely static.

On May 22, 2004, an open letter was signed by many

scientists and published in New Scientist [21]. In this letter they complained about the many inconsistencies of the Big Bang theory and its corollaries, stating that "*the successes claimed by the Big Bang theory's supporters consist of its ability to retrospectively fit observations with a steadily increasing array of adjustable parameters, just as the old Earth-centered cosmology of Ptolemy needed layer upon layer of epicycles. Yet the big bang is not the only framework available for understanding the history of the universe. Plasma cosmology and the steady-state model both hypothesize an evolving universe without beginning or end. These and other alternative approaches can also explain the basic phenomena of the cosmos.*"

Since publication, the letter has been signed by more than 500 researchers worldwide.

A group of those researchers created an association called "Alternative Cosmology Group" [22], that, as stated in their website, "*is an open society of scientists from all over the world, dedicated to the advance in cosmology and basic research*". In this site it's possible to find a wide variety of papers and articles presenting both observational results and theoretical research suggesting an alternative point of view on the evolution and fate of our universe.

As an example, in one of the most recent articles [23], the authors compared the size and brightness of many galaxies at different distances from us, considering the most luminous spiral galaxies for comparisons, finding that, contrary to the prediction of the Big Bang theory, near and far galaxies have similar surface brightness. The authors "*conclude that available observations of galactic SB (surface brightness) are consistent with a static Euclidean model of the universe*", therefore "*the redshift is due to some physical process other than expansion*".

It may be also interesting for the purposes of this paper to mention some other researchers that, even in the "mainstream" interpretation of measured redshifts, thus considering an expanding universe, completely disagree with the idea of a Big Bang and consequent dark energy and matter, and find in the repulsive gravity of antimatter a possible solution of the inconsistencies met by Einstein and others. This is the case of Dr. Massimo Villata of the Italian National Institute for Astrophysics (INAF) and Dr. Dragan Hajdukovic, physicist at CERN.

The first one has developed a theory according to which repulsive gravity between matter and antimatter located in cosmic voids (and, important to stress, not detectable by our standard instruments) can account for universe expansion without need of dark energy or matter and Big Bang.

It is interesting to notice also that according to his calculations a reasonable antimatter mass, located in a particular void, could account for a recorded local velocity anomaly of the "Local Sheet," the part of the universe that includes the Milky Way and other nearby galaxies, by the mechanism of repulsive gravity. This local anomaly apparently couldn't be explained by a "dark energy" that acts uniformly throughout the space [24, 25]. As the author states "*Through simple dynamical considerations, we find that the*

Local Void could host an amount of antimatter roughly equivalent to the mass of a typical supercluster, thus restoring the matter-antimatter symmetry. Like matter, antimatter is self-attractive, so we can expect that it forms anti-galaxies and anti-stars, which would emit electromagnetic radiation, we could then detect. However antimatter, if emitting, should emit advanced radiation, which can be undetectable".

The second researcher also considers a repulsive gravitational interaction between matter and antimatter as an alternative to dark energy, dark matter and Big Bang, but he focuses on the microscopic interactions at the particle level. In his view repulsive gravity of virtual particles and antiparticles in quantum vacuum can explain several observations, including effects usually attributed to dark matter [26].

3. The Zwicky-Santilli Effect

The historical accounts outlined in the preceding section indicate that Einstein, Hubble, Zwicky, Hoyle, and other cosmologists died without accepting the conjecture of the expansion of the universe apparently because Hubble's law $z = Hd$ clearly establishes the same redshift for all galaxies at the same distance d , in all possible *radial* directions from Earth, thus essentially implying a return to a Ptolemaic view of the universe (Fig. 1).

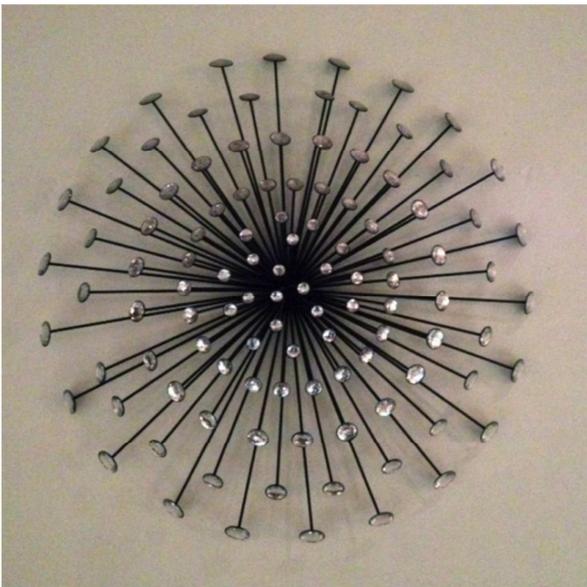


Figure 1. A sculpture illustrating the radial character of the interpretation of the Hubble Law $z=Hd$, via the expansion of the universe with an evident return to the Ptolemaic conception of Earth at the center of the universe.

In order to honor the above view, one of us (R. M. Santilli) conducted decades of mathematical, theoretical and experimental studies that have confirmed Zwicky's hypothesis via measurements on Earth resulting in an effect known as the Zwicky-Santilli effect (for brevity, see the collection of references [27]).

It may be interesting to mention that studies [27] are

complemented by the suggestive possibility of a *continuous creation of matter* in the universe occurring in the core of stars during their synthesis of the neutron from the hydrogen atom [28].

4. The Proposed Isoselfdual Universe

The main problem for a return to a static universe is its inherent prediction that the universe should collapse due to gravitational attractions between galaxies.

By remembering from Section 1 that could not be resolved via theories for matter, in this paper, we present apparently for the first time a cosmological model achieving stability of the universe via an appropriate distribution of matter and antimatter galaxies.

To avoid a prohibitive length, the understanding of the proposed model requires a knowledge of: the *isodual theory* for the *classical* representation of *neutral or charged antimatter* [29]; the prediction of matter-antimatter gravitational repulsion occurring at all levels of study, the recent apparent detection of antimatter galaxies via telescopes with concave lenses and the recent apparent detections of antimatter galaxies, antimatter asteroids, and antimatter cosmic rays (for brevity, see list [30] referred publications in the field).

Let us consider a distribution of point-like matter and antimatter masses in vacuum only subject to gravitational interactions. We can then study the stability of this simplified system in order to appraise the correctness of our hypothesis. As further simplification, we first consider masses in a one-dimensional space, and then extend the model to three dimensions.

In order to look for an equilibrium of the system, we have to study the first derivative of the state vector, which is characterized by position and velocity of the point masses. The derivative of position at the initial time is assumed to be zero, since masses are assumed to be initially at rest, so the only significant component to look at is the derivative of velocity.

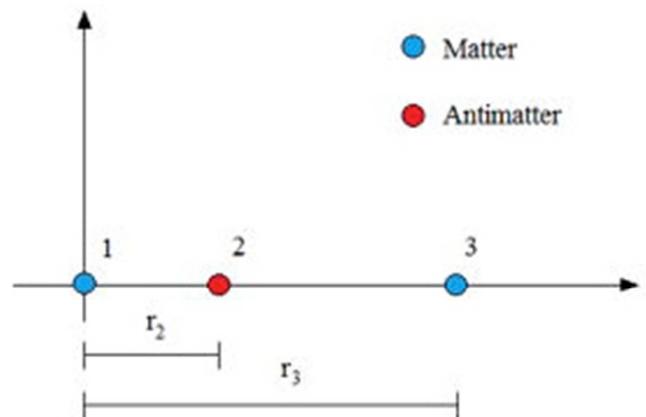


Figure 2. We illustrate the case of three unit masses in a one-dimensional, according to the distribution of one matter mass, one antimatter mass and one matter mass under the assumption of matter-antimatter gravitational repulsion.

Considering all possible permutations of up to 12 masses (half of them matter and half antimatter) at uniform finite distances, we easily find out that they are never in equilibrium, because the resultant acceleration on each mass is always different than zero. The use of variable distances for the same masses shows that equilibrium can be achieved only when distances themselves tend to infinity.

To better investigate the latter aspect, we study the equilibrium of 3 masses only (Fig 2), so that we can plot the results in a single graph. Being in one-dimension, there are only 2 free coordinates, which are the positions of the second and third mass with respect to the first one, and the accelerations can be plotted as a 3-D surface (Fig 3).

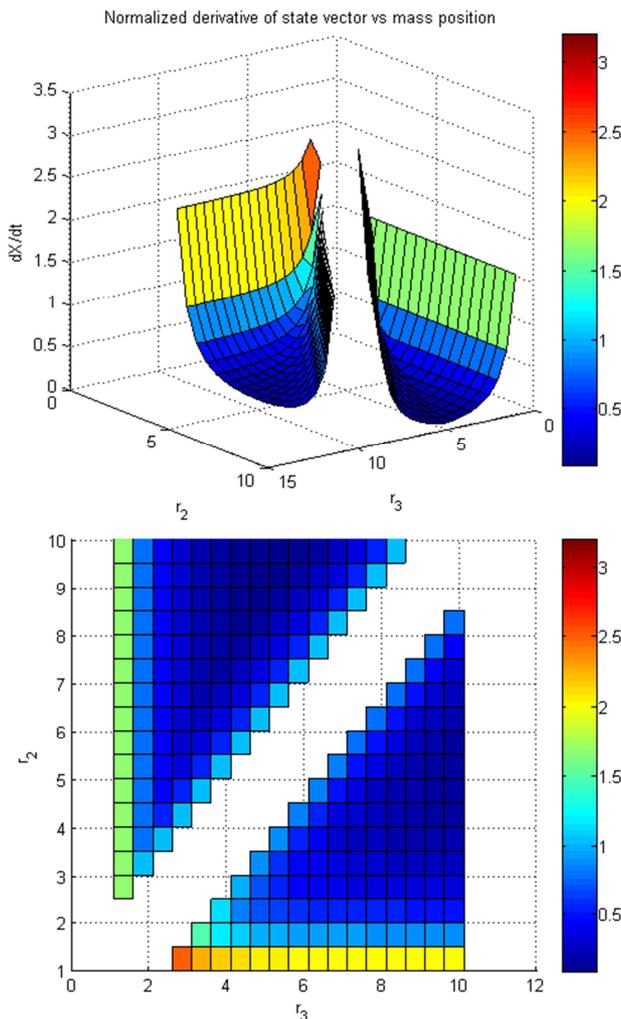


Figure 3. The graph shows the normalized gravity acceleration acting on the three unit masses in the one-dimensional case of Figure 2 versus the positions of the masses. The values shown are the sum of absolute values of the resultant accelerations acting on each mass, due only to gravitational interactions among the masses themselves. From the graph it is possible to see that the global acceleration never crosses the zero plane, but it tends to decrease asymptotically with the increasing of the distances. In the diagonal and at the borders the acceleration tends to infinity because in these cases the positions of two of the masses tend to coincide. We have the lowest accelerations in the blue parts of the graph, where the masses tend to have large equal distances.

As we can see from Figures 3 to 5, the resultant

accelerations between matter and antimatter decreases with the increase of their distance, and reach equilibrium at a mutual distance for which the gravitational repulsion is ignorable.

Therefore, for better readability, we did plot also the accelerations in the case of equidistant masses in the sequence matter-antimatter-matter (Fig 4). Having only one variable, the resulting graph (Fig 5) is two-dimensional, and it clearly shows that the accelerations decrease with the increasing of distance, tending asymptotically to zero at infinity.

Needless to say, the proposed cosmological model needs considerable additional studies. However, it is rather easy to show that the extension of the model to two and three dimensions readily the same results.

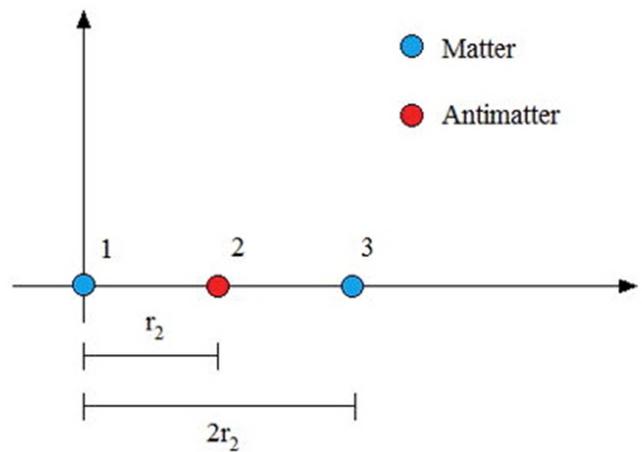


Figure 4. In this case the three u unit masses are considered again in one-dimension, but they are equidistant.

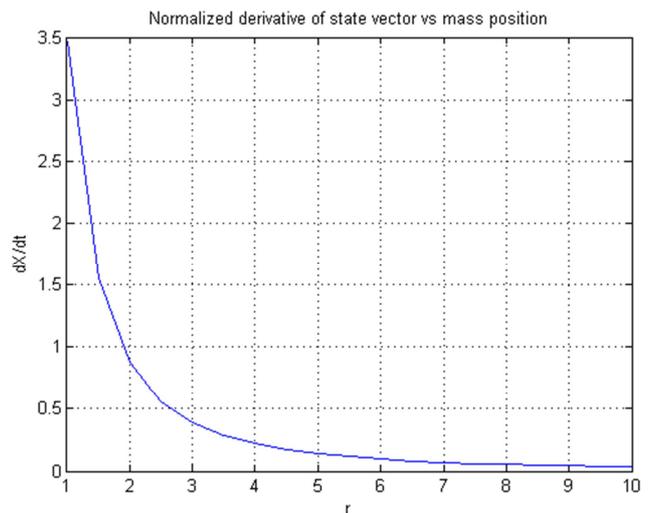


Figure 5. The graph shows the normalized gravity acceleration acting on the three equidistant unit masses with the sequence matter-antimatter-matter in one-dimension versus the positions of the masses themselves. We know from the previous graph that in this case the acceleration tends to be a minimum. From this figure, it is possible to see more clearly that the sum of absolute values of accelerations tends asymptotically to zero with the increasing of the distances among the masses themselves.

5. Conclusions

By remembering from Section 1 that the stability of the universe when solely composed by matter could not be consistently achieved via general relativity or other theories, we can conclude by saying that the universe can indeed achieve stability under the conditions of being composed by an equal number of matter and antimatter galaxies having a null total mass, matter and antimatter galaxies being at such large mutual distances caused by their gravitational repulsion at which all gravitational interactions are ignorable.

Needless to say, the above conditions merely represent limit values with large possibilities of local variations, such as: individual pairs of matter and antimatter galaxies need not necessarily have a null total mass since the gravitational repulsion exists also for different masses in absolute value; the conditions that the gravitational repulsion between matter and antimatter galaxies is ignorable implies very large variations of relative distances as well as (moderate) speeds, etc.

On more advanced grounds, we should note that the proposed cosmological model is isoselfdual, in the sense that it coincides with its isodual image [29], thus verifying the strict invariance under Santilli isoselfdual symmetry given by the Poincare' symmetry times its isodual [29]. The most rigorous proof of the consistency of our cosmological model (we can now call isoselfdual cosmology) can then be proved via the invariance under Santilli isoselfdual symmetry.

Recall that, in order to achieve a classical representation of neutral antimatter, it resulted necessary to assume that all characteristics of antimatter are opposite those of matter. Consequently, it is suggestive to note that all physical characteristics of the universe are identically null in our isoselfdual cosmology, thus including null total time, null total energy, etc.

The latter feature appears intriguing for quantitative studies on the origin of the universe without all calculations being lost at the act of creation due to singularities, since in the isoselfdual cosmology there is no change of numerical value of total physical quantities before and after creation. In turn, such a feature appears to support the suggestive hypothesis of a continuous creation in the core of matter and antimatter stars [28].

References

- [1] Einstein, A. 1917. "Kosmologische betrachtungen zur allgemeinen relativitätstheorie." Sitzungsber., K. Preuss. Akad. Wiss. 142-152. English translation in H. A. Lorentz, et al., eds. 1952. "The principle of relativity". Dover Publications, Mineola, New York, 175-188.
- [2] De Sitter, W. 1917a. "On the relativity of inertia. Remarks concerning Einsteins latest hypothesis.", Proc. R. Acad. Amsterdam 1217-1225.
- [3] De Sitter, W. 1917b. "Einsteins theory of gravitation and its astronomical consequences." Third paper. Mon. Not. R. Astron. Soc. 78: 3-28.
- [4] Hubble, E. 1929. "A relation between distance and radial velocity among extra-galactic nebulae.", Proc. Nat. Acad. Sci. 15: 168-173.
- [5] Eddington, A. S. 1923. "The mathematical theory of relativity", 2nd ed. Cambridge University Press, Cambridge.
- [6] Weyl, H. 1923. "Raum, zeit, materie: vorlesungen über allgemeine relativitätstheorie, fünft mgearbeitete auflage." Julius Springer, Berlin.
- [7] Weyl, H. 1923. "Zur allgemeinen relativitätstheorie." Phys. Z. 24: 230-232 (1923b). English translation: Weyl, H. 2009. "Republication of: On the general relativity theory". Gen. Relativ. Gravitat. 35: 1661-1666.
- [8] Slipher, V. M., 1913, "The radial velocity of the Andromeda Nebula." Lowell Observatory Bulletin 2: 56-57.
- [9] Friedmann, A. 1922: "Über die Krümmung des Raumes." Z. Phys. 10: 377-386. English translation: Friedman, A. "On the curvature of space." Gen. Rel. Gravitat. 31: 1991-2000.
- [10] Friedmann, A. 1924. "Über die Möglichkeit einer Welt mit konstanter negativer Krümmung des Raumes." Z. Phys. 21: 326-332. English translation: Friedmann, A. "On the possibility of a world with constant negative curvature of space." Gen. Rel. Gravitat. 31 2001-2008.
- [11] Lemaître, G. 1927. "Un Univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nbuleuses extra-galactiques." Annales de la Société Scientifique de Bruxelles A, 47: 49-59. English translation: Lemaître, G. 1931. "A Homogeneous universe of Constant Mass and Increasing Radius accounting for the Radial Velocity of Extra-Galactic Nebulae." Mon. Not. R. Astron. Soc. 91: 483-490.
- [12] Einstein, A. 1931. Letter to R. Tolman, Einstein Archives call no. 23-30. Post-dated June 27, 1931.
- [13] Einstein, A. and W. de Sitter. 1932 "On the relation between the expansion and the mean density of the universe" Proc. Nat. Acad. Sci. 18: 213-214.
- [14] P. Kerszberg, 1989, "The Einstein - de Sitter controversy of 1916-1917 and the rise of relativistic cosmology", Einstein and the history of general relativity, p. 325-366.
- [15] D. Janzen, 2016 "Einstein's cosmological considerations", 53 pages. Submitted to Eur Phy J (H), arXiv: 1402.3212.
- [16] Einstein, A. 1931. Zum kosmologischen Problem, Einstein Archives call no. 23-30.
- [17] C. O'Raiheartaigh and B. McCann, "Einstein's cosmic model of 1931 revisited: an analysis and translation of a forgotten model of the universe", European Physical Journal H 39 (2014) 63-85. DOI: 10.1140/epjh/e2013-40038-x and arXiv: 1312.2192.
- [18] Nature 506, 418-419 (27 February 2014).
- [19] Zwicky, F., 1929, Proceedings of the National Academy of Sciences of the United States of America, 773.
- [20] Hoyle, F., 1948 Mon. Not. R. Astron. Soc. 108, 372-382.
- [21] VV. AA., "Open Letter on Cosmology", New Scientist, issue 2448 May 22, 2004 <http://homepages.xnet.co.nz/~hardy/cosmologystatement.html>.
- [22] <http://www.cosmology.info/>.

- [23] Eric J. Lerner, Renato Falomo, and Riccardo Scarpa, "UV surface brightness of galaxies from the local universe to $z \sim 5$ ", *International Journal of Modern Physics D* Vol. 23, No. 6 (2014) 1450058 DOI: 10.1142/S0218271814500588.
- [24] Massimo Villata. "'Dark Energy' in the Local Void." *Astrophysics and Space Science* (2012) 339: 7-12. DOI: 10.1007/s10509-012-0994-9 and arXiv: 1201.3810v1 [astro-ph. CO].
- [25] Massimo Villata. "CPT symmetry and antimatter gravity in general relativity", *EPL (Europhysics Letters)*, Volume 94, Number 2, 2011.
- [26] Dragan Hajdukovic. "Quantum vacuum and virtual gravitational dipoles: the solution to the dark energy problem?" *Astrophysics and Space Science*. DOI: 10.1007/s10509-012-0992-y.
- [27] Fleming, 2014, P. Collected papers, interviews, seminars and international press releases on the lack of expansion of the universe, <http://www.santilli-foundation.org/docs/No-universe-expans.pdf>.
- [28] Santilli, R. M., 2007, "The etherino and/or the neutrino hypothesis," *Foundation of Physics* **37**, 670, <http://www.santilli-foundation.org/docs/EtherinoFoundPhys.pdf>.
- [29] R. M. Santilli, 2005, *Isodual Theory of Antimatter with Applications to Antigravity, Grand Unifications and Cosmology*, Springer, <http://www.santilli-foundation.org/docs/santilli-79.pdf>.
- [30] Fleming, 2015, P. Scientific references and PR Web News Releases on Santilli isodual Theory of Antimatter <http://www.santilli-foundation.org/docs/Santilli-Telescope-Refs-1-15.p>.

American Journal of Modern Physics (AJMP)

<http://www.sciencepublishinggroup.com/j/ajmp>

American Journal of Modern Physics (AJMP) aims to promote rapid communication and dialogue among the researchers, scientists, engineers and policy makers working in the areas of modern physics in the world. It brings the broad fundamental physics literature in established topical areas together and places it within the context of current trends in research and applications. *AJMP* welcomes the following tier 1 article types: Book Review, Editorial, General Commentary, Hypothesis & Theory, Methods, Mini Review, Opinion, Original Research, Perspective, Review, Specialty Grand Challenge and Technology Report.

Subject Coverage

American Journal of Modern Physics is a peer-reviewed, international, open access journal that publishes original research articles as well as review articles dealing with all aspects of research on physics. Subject areas may include, but are not limited to the following fields:

- General physics
- Applied biophysics
- Dielectrics and ferroelectricity
- Device physics
- Gravitation and astrophysics
- Elementary particles and fields
- Nuclear and hadronic physics
- Optical physics
- Fluid and plasma physics
- Statistical and nonlinear physics
- Condensed matter physics

Notes for Intending Authors

Publishing your research in a Science Publishing Group journal is simple and efficient. Please use online submission system:

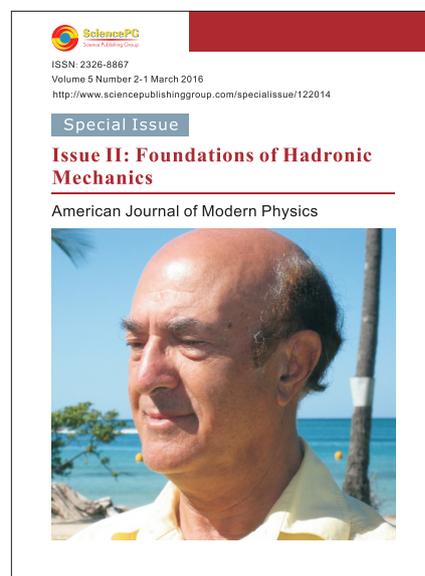
<http://www.sciencepublishinggroup.com/login>

Only the following word processor file formats are acceptable for the manuscript document for all of Science Publishing Group journals: •Microsoft word 2000/2003(doc) •Microsoft word 2007/2010(docx)

Website and Email

<http://www.sciencepublishinggroup.com/j/ajmp>

Email: service@sciencepublishinggroup.com



Editorial Board

- | | |
|--|--|
| Dr. M. Zemzi
Abdelaali Boudjemaa | Physics Department, University of Gabes, Gabes, Tunisia
Department of Physics, Faculty of Sciences, University of Chlef, Abou El Hassan, Chlef, Algeria |
| Prof. Adel Aschi | Laboratory of Physics of Soft Condensed Matter and Electromagnetic Modeling, Faculty of Sciences of Tunis, Department of Physics, Campus University, Belvedere, Tunis, Tunisia |
| Prof. Ahmed Maghraby | Ionizing Radiation Metrology Laboratory, National Institute of Standards, Ministry of Scientific Research, Alkharj, Saudi Arabia |
| Prof. Aleksandr Kamzin | Laboratory of Magnetism, Department of Ferroelectrics and Magnets, Ioffe Physical, Technical Institute of the Russian Academy of Sciences, Saint-Petersburg, Russia |
| Muataz Majeed | Department of Physics, University of Tikrit, Tikrit, Salahalddin, Iraq |
| Dr. Arezu Jahanshir | Department of Physics and Engineering Physics, Buein Zahra Technical University, Qazvin, Iran |
| Mustafa Erkovan | Nanoscience and Nanoengineering Department, Science Faculty, Sakarya University, Sakarya, Turkey |
| Deepak Kumar Sondhiya | Department of Physics, LNCT Group of Colleges, Bhopal, Madhya Pradesh, India |
| Dr. Dmitry Tsipenyuk | Department of Coherent and Nonlinear Optics, General Physics Institute, Russian Academy of Sciences, Moscow, Russia |
| Prof. Dumitru Baleanu | Department of Mathematics and Computer Sciences, Faculty of Art and Sciences, Cankaya University, Ankara, Balgat, Turkey |
| Dr. Elchin Huseynov | Department of Nanotechnology and Radiation Material Science, National Nuclear Research Center, Baku, Azerbaijan |
| Heba Saudi | Faculty of Science, Department of Physics, Al-Azhar University, Cairo, Egypt |
| Hossam Gomaa | Department of Physics, Sert University in Libya, Cairo, Egypt |
| Prof. Jing Ling Chen | Theoretical Physics Division, Chern Institute of Mathematics, Nankai University, Tianjin, China |
| Leila Marek-Crnjac | Department of Mathematics, Technical School Center Maribor, Maribor, Slovenia |
| Lioua Kolsi | Mechanical Engineering Department, College of Engineering, University of Hail, Hail, Saudi Arabia |

ISSN: 2326-8867(Print), 2326-8891(Online)



Science Publishing Group (SciencePG), an open access publisher with experienced and eminent reviewers and editorial board members, is mainly attaching importance to developing journals, books and conferences which have owned unique characters respectively. Here is the website of SciencePG: <http://www.sciencepublishinggroup.com>