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A NEW CONCEPTION OF LIVING ORGANISMS AND ITS REPRESENTATION VIA LIE-ADMISSIBLE H_v-HYPERSTRUCTURES

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Abstract

Recent studies have confirmed Einstein's 1935 legacy implying that quantum mechanics and chemistry are "incomplete" theories in the sense of being excellent for the description of systems composed by point-like constituents under potential interactions (such as the atomic structure), but said theories are "incomplete" for the description of complex time-irreversible systems of extended constituents with internal non-potential interactions (as expected in a cell). Sadi verifications were achieved thanks to the prior "completion" over the past half a century of quantum theories into the covering hadronic mechanics and chemistry with a time irreversible Lie-admissible structure. In this paper we present, apparently for the first time, a new conception of living organisms, solely permitted by the verifications of Einstein's legacy, composed by a very large number of extended wavepackets in conditions of continuous mutual penetration/entanglement and, therefore, of continuous communications via contact non-potential interactions. Due to the extremely large number of constituents and the extreme complexity of the multi-valued internal communications, in this paper we introduce, also apparently for the first time, the representation of the indicated new conception of living organisms via two hyperbimodular, Lie-admissible H_v-hyperstructures, the first with all hyperoperations ('hope') ordered to the right and the second with all hopes ordered to the left. The irreversibility of living organisms is represented by the inequivalence of the left and right hopes. The extremely large number of internal communications is represented by the extremely large number of solutions of the indicated hopes. We close the paper with the indication that new medical diagnostics and treatments are expected in the transition from the current quantum chemical conception of living organisms as collections of isolated point-like constituents to the indicated new conception.

1. INTRODUCTION ON HYPERSTRUCTURES

The largest class of hyperstructures are called H_v*-structures* and introduced in 1990 [33], [34]. These satisfy the *weak axioms* where the non-empty intersection replaces the equality. Some basic definitions are the following:
In a set H equipped with a hyperoperation (abbreviation *hyperoperation=**hope***)

$$\cdot: H\times H \to P(H)-\{\varnothing\},$$

we abbreviate by *WASS* the *weak associativity*: $(xy)z\cap x(yz) \neq \varnothing,\ \forall x,y,z\in H$ and by *COW* the *weak commutativity*: $xy\cap yx \neq \varnothing,\ \forall x,y\in H$.

The hyperstructure (H,·) is called ***H_v-semigroup*** if it is *WASS*, it is called ***H_v-group*** if it is reproductive H_v-semigroup, i.e. $xH = Hx = H,\ \forall x\in H$.

Motivation. I the classical theory the quotient of a group with respect to an invariant subgroup is a group. F. Marty from 1934, states that, the quotient of a group with respect to any subgroup is a hypergroup. Finally, the quotient of a group with respect to any partition is an H_v-group [34].

The *powers* of an element $h\in H$ are: $h^1=\{h\},\ h^2=h\cdot h,\ \ldots,\ h^n=h\circ\ldots\circ h$, where (°) is the *n-ary circle hope*: the union of hyperproducts, n times, with all patterns of parentheses put on them. An H_v-semigroup (H,·) is a *cyclic of period s*, if there is a *generator* g, and a natural n, such that $H=h^1\cup\ldots\cup h^s$. If there is an h and s, such that $H=h^s$, then (H,·) is called *single-power cyclic of period s*.

In a similar way more complicated hyperstructures can be defined:
$(R,+,\cdot)$ is ***H_v-ring*** if (+) and (·) are *WASS*, the reproduction axiom is valid for (+) and (·) is *weak distributive* with respect to (+):

$$x(y+z)\cap(xy+xz)\neq\varnothing,\quad (x+y)z\cap(xz+yz)\neq\varnothing,\quad \forall x,y,z\in R.$$

Let $(R,+,\cdot)$ H_v-ring, $(M,+)$ *COW* H_v-group and there exists an external hope

$$\cdot:\ R\times M \to P(M):\ (a,x) \to ax$$

such that, $\forall a,b\in R$ and $\forall x,y\in M$, we have

$$a(x+y)\cap(ax+ay) \neq \varnothing,\quad (a+b)x\cap(ax+bx) \neq \varnothing,\quad (ab)x\cap a(bx) \neq \varnothing,$$

then *M* is an H_v*-module* over *F*. In the case of an H_v-field *F* instead of an H_v-ring *R*, then the H_v*-vector space* is defined.

For more definitions and applications on H_v-structures one can see the books [2], [4], [5], [8], [31], [32], [34], [35], [38], [40], [45], [52].

Definition 1.1 The ***fundamental relations*** β^*, γ^* and ε^*, are defined, in H_v-groups, H_v-rings and H_v-vector spaces, respectively, as the smallest equivalences so that the quotient would be group, ring and vector spaces, respectively [33], [34], [35], [50], [51].

The way to find the fundamental classes is given by the following:

Theorems 1.2 Let $(H,\cdot)$ be H_v-group and denote by U the set of finite products of elements of H. We define the relation β in H by setting $x\beta y$ iff $\{x,y\}\subset u$ where $u\in U$. Then β^* is the transitive closure of β.

Let $(R,+,\cdot)$ be H_v-ring. Denote by U the set of finite polynomials of elements of R. We define the relation γ in R as follows: $x\gamma y$ *iff* $\{x,y\}\subset u$ where $u\in U$. Then the relation γ^* is the transitive closure of the relation γ.

An element is called *single* if its fundamental class is singleton.

Fundamental relations are used for general definitions. Thus, an H_v-ring $(R,+,\cdot)$ is called ***H_v-field*** if R/γ^* is a field.

Let $(H,\cdot)$, $(H,*)$ be H_v-semigroups defined on the same set H. $(\cdot)$ is called ***smaller*** than $(*)$, and $(*)$ ***greater*** than $(\cdot)$, iff there exists an

$$f\in Aut(H,*) \text{ such that } xy\subset f(x*y),\ \forall x,y\in H.$$

Then we say that $(H,*)$ *contains* $(H,\cdot)$. If $(H,\cdot)$ is a structure then it is called *basic structure* and $(H,*)$ is called H_b-*structure*.

Theorem 1.3 (The Little Theorem). Greater hopes than the ones which are *WASS* or *COW*, are also *WASS* or *COW*, respectively.

This Theorem leads to a partial order on H_v-structures.

A very interesting class of H_v-structures, is the following [9], [32]:

An H_v-structure is called ***very thin*** iff all hopes are operations except one, which has all hyperproducts singletons except only one, which is a subset of cardinality more than one.

A large class of H_v-structures is the following [9], [41]:

Let $(G,\cdot)$ be groupoid (resp., hypergroupoid) and $f:G\to G$ be a map. We define a hope (∂), called *theta-hope,* we write ***∂-hope***, on G as follows

$$x\partial y=\{f(x)\cdot y,\ x\cdot f(y)\},\quad \forall x,y\in G.\quad (resp.\ x\partial y=(f(x)\cdot y)\cup(x\cdot f(y)),\ \forall x,y\in G)$$

If $(\cdot)$ is commutative then ∂ is commutative. If $(\cdot)$ is *COW*, then ∂ is *COW*.

Let $(G,\cdot)$ be groupoid (or hypergroupoid) and $f:G\to P(G)-\{\varnothing\}$ be multivalued map. We define the (∂), on G as follows $x\partial y=(f(x)\cdot y)\cup(x\cdot f(y)),\ \forall x,y\in G.$

Motivation for the ∂-hope is the map *derivative* where only the multiplication of functions can be used. Basic property: if $(G,\cdot)$ is a semigroup then $\forall f$, the (∂) is *WASS*.

Another well known and large class of hopes is given as follows [9], [31], [47]:

Let $(G,\cdot)$ be groupoid, then $\forall P\subset G,\ P\neq\varnothing$, we define the following hopes called *P-hopes*: $\forall x,y\in G$

$$\underline{P}:\ x\underline{P}y=(xP)y\cup x(Py),\quad \underline{P}_r:\ x\underline{P}_ry=(xy)P\cup x(yP),\quad \underline{P}_l:\ x\underline{P}_ly=(Px)y\cup P(xy).$$

The $(G,\underline{P})$, $(G,\underline{P}_r)$ and $(G,\underline{P}_l)$ are called *P-hyperstructures*. If $(G,\cdot)$ is semigroup, then $x\underline{P}y=(xP)y\cup x(Py)=xPy$ and $(G,\underline{P})$ is a semihypergroup but we do not know about $(G,\underline{P}_r)$ and $(G,\underline{P}_l)$. In some cases, depending on the choice of P, the $(G,\underline{P}_r)$ and $(G,\underline{P}_l)$ can be associative or *WASS*.

A generalization of P-hopes is the following [6], [9]:

Construction 1.4 Let $(G,\cdot)$ be an abelian group and P any subset of G. We define the hope $\times_P$ as follows:

$$\begin{cases} x\times_P y = x\cdot P\cdot y=\{x\cdot h\cdot y\ /\ h\in P\} & \text{if } x\neq e \text{ and } c\neq e \\ x\cdot y & \text{if } x=e \text{ or } y=e \end{cases}$$

we call this hope P_e*-hope*. The hyperstructure $(G,\times_P)$ is an abelian H_v-group.

H_v-structures are used in Representation Theory of H_v-groups which can be achieved by generalized permutations or by H_v-matrices [34], [38], [49]. *H_v-matrix* is called a matrix if has entries from an H_v-ring. The hyperproduct of H_v-matrices is defined in a usual manner. The problem of the H_v-matrix representations is the following:

Definition 1.5 Let $(H,\cdot)$ be an H_v-group, find an H_v-ring $(R,+,\cdot)$, a set $M_R=\{(a_{ij})\ /\ a_{ij}\in R\}$ and a map

$$T:\ H\to M_R:\ h\mapsto T(h)\ \text{ such that }\ T(h_1h_2)\cap T(h_1)T(h_2)\neq\varnothing,\ \forall h_1,h_2\in H.$$

Then T is *H_v-matrix representation*. If $T(h_1h_2)\subset T(h_1)(h_2),\ \forall h_1,h_2\in H$ is valid, then T is an *inclusion representation*. If $T(h_1h_2)=T(h_1)(h_2)=\{T(h)\ /\ h\in h_1h_2\}$, $\forall h_1,h_2\in H$, then T is a *good representation*.

Hopes on any type of ordinary matrices can be defined [8], [49], [53] they are called *helix hopes*.

Definition 1.6 Let $A=(a_{ij})\in M_{m\times n}$ be matrix and $s,t\in N$, *with* $1\leq s\leq m,\ 1\leq t\leq n$. The helix-projection is a map $\underline{st}: M_{m\times n}\to M_{s\times t}: A\to A\underline{st}=(\underline{a_{ij}})$, where $A\underline{st}$ has entries

$$\underline{a}_{ij} = \{ a_{i+\kappa s, j+\lambda t} \,/\, 1 \le i \le s,\ 1 \le j \le t \text{ and } \kappa, \lambda \in N,\ i+\kappa s \le m,\ j+\lambda t \le n \}$$

Let $A=(a_{ij}) \in M_{m\times n}$, $B=(b_{ij}) \in M_{u\times v}$ be matrices and $s=min(m,u)$, $t=min(n,v)$. We define a hyper-addition, called ***helix-sum***, by

$$\oplus : M_{m\times n} \times M_{u\times v} \to P(M_{s\times t}) : (A,B) \to A \oplus B = A\underline{st} + B\underline{st} = (\underline{a}_{ij}) + (\underline{b}_{ij}) \subset M_{s\times t}$$

where $(\underline{a}_{ij})+(\underline{b}_{ij}) = \{(c_{ij})=(a_{ij}+b_{ij}) \,/\, a_{ij} \in \underline{a}_{ij} \text{ and } b_{ij} \in \underline{b}_{ij})\}$.
Let $A=(a_{ij}) \in M_{m\times n}$, $B=(b_{ij}) \in M_{u\times v}$ and $s=min(n,u)$. Define the ***helix-product***, by

$$\otimes : M_{m\times n} \times M_{u\times v} \to P(M_{m\times v}) : (A,B) \to A \otimes B = A\underline{ms} \cdot B\underline{sv} = (\underline{a}_{ij}) \cdot (\underline{b}_{ij}) \subset M_{m\times v}$$

where $(\underline{a}_{ij}) \cdot (\underline{b}_{ij}) = \{(c_{ij})=(\sum a_{it} b_{tj}) \,/\, a_{ij} \in \underline{a}_{ij} \text{ and } b_{ij} \in \underline{b}_{ij})\}$.

The helix-sum is commutative, *WASS*, not associative. The helix-product is *WASS*, not associative and not distributive to the helix-addition.

Using several classes of H_v-structures one can face several representations [48].

Definition 1.7 Let $M=M_{m\times n}$ be module of $m\times n$ matrices over a ring R and $P=\{P_i : i \in I\} \subseteq M$. We define, a kind of, a P-hope $\underline{P}$ on M as follows

$$\underline{P} : M\times M \to P(M) : (A,B) \to A\underline{P}B = \{ AP^t_i B : i \in I \} \subseteq M$$

where P^t denotes the transpose of the matrix P.

We present a proof for the fundamental relation analogous to Theorem 1.2 in the case of an H_v-module:

Theorem 1.8 Let $(M,+)$ be H_v-module over R. Denote U the set of expressions of finite hopes either on R and M or the external hope applied on finite sets of elements of R and M. We define the relation ε in M by: $x\varepsilon y$ *iff* $\{x,y\} \subset u$, $u \in U$. Then the relation ε^* is the transitive closure of the relation ε.

Proof. Let $\underline{\varepsilon}$ be the transitive closure of ε, and denote by $\underline{\varepsilon}(x)$ the class of the element x. First, we prove that the quotient set $M/\underline{\varepsilon}$ is a module over R/γ^*.
In $M/\underline{\varepsilon}$ the sum ($\oplus$) and the external product ($\otimes$), using the γ^* classes in R, are defined in the usual manner:

$$\underline{\varepsilon}(x) \oplus \underline{\varepsilon}(y) = \{\underline{\varepsilon}(z) : z \in \underline{\varepsilon}(x)+\underline{\varepsilon}(y)\},$$

$$\gamma^*(a) \otimes \underline{\varepsilon}(x) = \{\underline{\varepsilon}(z) : z \in \gamma^*(a)\cdot\underline{\varepsilon}(x)\}, \quad \forall a \in R,\ x,y \in M.$$

Take $x' \in \underline{\varepsilon}(x)$, $y' \in \underline{\varepsilon}(y)$. Then $x'\underline{\varepsilon}x$ *iff* $\exists x_1,\dots,x_{m+1}$ with $x_1=x'$, $x_{m+1}=x$, $u_1,\dots,u_m \in U$ such that $\{x_i, x_{i+1}\} \subset u_i$, $i=1,\dots,m$, and $y'\underline{\varepsilon}y$ *iff* $\exists y_1,\dots,y_{n+1}$ with $y_1=y'$, $y_{n+1}=y$ and $v_1,\dots,v_n \in U$ such that $\{y_j, y_{j+1}\} \subset v_j$, $j=1,\dots,n$. From the above we obtain

$$\{x_i, x_{i+1}\} + y_1 \subset u_1+v_1,\ i=1,\dots,m-1,\quad x_{m+1} + \{y_j, y_{j+1}\} \subset u_m+v_j,\ j=1,\dots,n.$$

The $u_i+v_1=t_i$, $i=1,..,m-1$, $u_m+v_j=t_{m+j-1}$, $j=1,..,n\in U$, so $t_k\in U$, $\forall k\in\{1,..,m+n-1\}$. Take $z_1,...,z_{m+n}$ with $z_i\in x_i+y_1$, $i=1,...,n$ and $z_{m+j}\in x_{m+1}+y_{j+1}$, $j=1,...,n$, thus, $\{z_k,z_{k+1}\}\subset t_k$, $k=1,...,m+n-1$. Therefore, $\forall z_1\in x_1+y_1=x'+y'$ is $\underline{\varepsilon}$ equivalent to $z_{m+n}\in x_{m+1}+y_{n+1}=x+y$. Thus, $\underline{\varepsilon}(x)\oplus\underline{\varepsilon}(y)$ is a singleton so we can write $\underline{\varepsilon}(x)\oplus\underline{\varepsilon}(y)=\underline{\varepsilon}(z)$, $\forall z\in\underline{\varepsilon}(x)+\underline{\varepsilon}(y)$. Similarly, using the properties of γ^* in R, we prove that $\gamma^*(a)\otimes\underline{\varepsilon}(x)=\underline{\varepsilon}(z)$, $\forall z\in\gamma^*(a)\cdot\underline{\varepsilon}(x)$.

The *WASS* and the weak distributivity on R and M guarantee that the associativities and the distributivity are valid for $M/\underline{\varepsilon}$ over R/γ^*. Therefore, $M/\underline{\varepsilon}$ is a module over R/γ^*.

Now let σ equivalence relation in M such that M/σ is module on R/γ^*. Denote $\sigma(x)$ the class of x. Then $\sigma(x)\oplus\sigma(y)$ and $\gamma^*(a)\otimes\sigma(x)$ are singletons $\forall a\in R$ and $x,y\in M$, i.e.

$$\sigma(x)\oplus\sigma(y)=\sigma(z),\ \ \forall z\in\sigma(x)+\sigma(y),\quad \gamma^*(a)\otimes\sigma(x)=\sigma(z),\ \ \forall z\in\gamma^*(a)\cdot\sigma(x).$$

Thus we write, $\forall a\in R$, $x,y\in M$ and $A\subset\gamma^*(a)$, $X\subset\sigma(x)$, $Y\subset\sigma(x)$

$$\sigma(x)\oplus\sigma(y)=\sigma(x+y)=\sigma(X+Y),\quad \gamma^*(a)\otimes\sigma(x)=\sigma(ax)=\sigma(A\cdot X).$$

By induction, extend these relations on finite sums and external products. Thus, $\forall u\in U$, we have $\sigma(x)=\sigma(u)$, $\forall x\in u$. Consequently $x'\in\varepsilon(x)$ implies $x'\in\sigma(x)$, $\forall x\in M$.

But σ is transitively closed, so we obtain: $x'\in\underline{\varepsilon}(x)$ implies $x'\in\sigma(x)$.

Thus, $\underline{\varepsilon}$ is the smallest equivalence on M such that $M/\underline{\varepsilon}$ is a module on R/γ^*, i.e. $\underline{\varepsilon}=\varepsilon^*$. ■

The general definition of an H_v-Lie algebra was given as follows [30], [44]:

Definition 1.9 Let $(L,+)$ H_v-vector space on $(F,+,\cdot)$, $\varphi:F\to F/\gamma^*$ canonical, $\omega_F=\{x\in F:\varphi(x)=0\}$, where 0 is zero of F/γ^*. Let ω_L the core of $\varphi':L\to L/\varepsilon^*$ and denote 0 the zero of L/ε^*. Consider the *bracket (commutator) hope*:

$$[\ ,\ ]: L\times L\to P(L): (x,y)\to[x,y]$$

then L is an *H_v-Lie algebra* over F if the following axioms are satisfied:

(L1) The bracket hope is bilinear, i.e.

$$[\lambda_1x_1+\lambda_2x_2,y]\cap(\lambda_1[x_1,y]+\lambda_2[x_2,y])\neq\varnothing$$

$$[x,\lambda_1y_1+\lambda_2y_2]\cap(\lambda_1[x,y_1]+\lambda_2[x,y_2])\neq\varnothing,\ \ \forall x,x_1,x_2,y,y_1,y_2\in L,\ \ \forall\lambda_1,\lambda_2\in F$$

(L2) $[x,x]\cap\omega_L\neq\varnothing$, $\forall x\in L$

(L3) $([x,[y,z]]+[y,[z,x]]+[z,[x,y]])\cap\omega_L\neq\varnothing$, $\forall x,y\in L$

The enlargement or reduction of hyperstructures are examined in the sense that an extra element appears in one result or we take out an element. In both directions most useful in representation theory, are those H_v-structures with the same fundamental structure [36], [37]:

Let $(H,\cdot)$ be H_V-semigroup and $v\notin H$. Extend $(\cdot)$ into the $\underline{H}=H\cup\{v\}$ as follows: $x\cdot v=v\cdot x=v$, $\forall x\in H$, and $v\cdot v=H$. The $(\underline{H},\cdot)$ is an h/v-group where $(\underline{H},\cdot)/\beta^*\cong Z_2$ and v is a single element. We call $(\underline{H},\cdot)$ the *attach h/v-group* of $(H,\cdot)$.

Let $(G,\cdot)$ be semigroup and $v\notin G$ be an element appearing in a product ab, where $a,b\in G$, thus the result becomes a hyperproduct $a\otimes b=\{ab,v\}$. Then the minimal hope $(\otimes)$ extended in $G'=G\cup\{v\}$ such that $(\otimes)$ contains $(\cdot)$ in the restriction on G, and such that $(G',\otimes)$ is a minimal H_v-semigroup which has fundamental structure isomorphic to $(G,\cdot)$, is defined as follows:

$$a\otimes b=\{ab,v\},\quad x\otimes y=xy,\quad \forall(x,y)\in G^2-\{(a,b)\}$$

$$v\otimes v=abab,\quad x\otimes v=xab \ \text{ and } \ v\otimes x=abx,\quad \forall x\in G.$$

$(G',\otimes)$ is very thin H_v-semigroup. If $(G,\cdot)$ is commutative then the $(G',\otimes)$ is strongly commutative.

Let $(H,\cdot)$ be hypergroupoid. We say that *remove* $h\in H$, if we consider the restriction of $(\cdot)$ on $H-\{h\}$. We say $\underline{h}\in H$ *absorbs* $h\in H$ if we replace h by $\underline{h}$. We say $\underline{h}\in H$ *merges* with $h\in H$, if we take as product of $x\in H$ by $\underline{h}$, the union of the results of x with both h, $\underline{h}$ and consider h and $\underline{h}$ as one class.

The ***uniting elements*** method, introduced by Corsini & Vougiouklis [3], is the following: Let G be algebraic structure and let d be a property, which is not valid and it is described by a set of equations; then, consider the partition in G for which it is put together, in the same class, every pair that causes the non-validity of d. The quotient G/d is an H_v-structure. Then, quotient out the G/d by β^*, a stricter structure $(G/d)/\beta^*$ for which the property d is valid, is obtained.

An application of the uniting elements is when more than one property is desired. The following Theorem is valid [3], [34].

Theorem 1.10 Let $(G,\cdot)$ be a groupoid, $F=\{f_1,\dots,f_m,f_{m+1},\dots,f_{m+n}\}$ be system of equations on G consisting of two subsystems $F_m=\{f_1,\dots,f_m\}$, $F_n=\{f_{m+1},\dots,f_{m+n}\}$. Let σ, σ_m the equivalence relations defined by the uniting elements procedure using the systems F and F_m, and let σ_n be the equivalence relation defined using the induced equations of F_n on the grupoid $G_m=(G/\sigma_m)/\beta^*$. Then

$$(G/\sigma)/\beta^*\cong(G_m/\sigma_n)/\beta^*.$$

In the paper [42], there is a first description on how Santilli's theories effect in hyperstructures and how new theories in Mathematics can be appeared by Santilli's pioneer research.

Hyperstructures have applications in mathematics and in other sciences. These applications range from biomathematics -conchology, inheritance- and hadronic physics or on leptons, in the Santilli's iso-theory, to mention but a few. The hyperstructure theory is closely related to fuzzy theory; consequently, can be widely applicable in linguistic, in sociology, in industry and production, too. For these applications the largest class of the hyperstructures, the class *H_v-structures*, is used, they satisfy the *weak axioms* where the non-empty intersection replaces the equality. The main tools of this theory are the fundamental relations which connect, by quotients, the H_v-structures with the corresponding classical ones. These relations are used to define hyperstructures as H_v-fields, H_v-vector spaces and so on, as well. The definition of the general hyperfield was not possible without the H_v-structures and their fundamental relations. *Hypernumbers or H_v-numbers* are called the elements of H_v-fields and they are important for the representation theory [6], [7], [29], [30], [39], [46].

The problem of enumeration and classification of hyperstructures, was started from the beginning, it is complicate in H_v-structures because we have very great numbers. The number of H_v-groups with three elements, up to isomorphism, is 1.026.462. There are 7.926 abelian; the 1.013.598 are cyclic. The partial order in H_v-structures and the Little Theorem, transfers and restrict the problem in finding the minimal, *up to isomorphisms*, H_v-structures.

2. LIE-SANTILLI ADMISSIBILITY IN HYPERSTRUCTURES

The *isofields* needed in the theory of *isotopies* correspond into the hyperstructures were introduced by Santilli & Vougiouklis in 1999 [6], [7], [29] and they are called *e-hyperfields*. The H_v-fields can give e-hyperfields which can be used in the isotopy theory in applications as in physics or biology. We present in the following the main definitions and results restricted in the H_v-structures.

Definitions 2.1 A hyperstructure $(H,\cdot)$ which contain a unique scalar unit e, is called e-hyperstructure. In an e-hyperstructure, we assume that for every element x, there exists an inverse x^{-1}, *i.e.* $e \in x\cdot x^{-1} \cap x^{-1}\cdot x$. Remark that the inverses are not necessarily unique.

A hyperstructure $(F,+,\cdot)$, where $(+)$ is an operation and $(\cdot)$ is a hope, is called *e-hyperfield* if the following axioms are valid:

1. $(F,+)$ is an abelian group with the additive unit 0,

2. $(\cdot)$ is *WASS*,
3. $(\cdot)$ is weak distributive with respect to $(+)$,
4. 0 is absorbing element: $0\cdot x = x\cdot 0 = 0,\ \forall x\in F$,
5. exist a multiplicative scalar unit 1, i.e. $1\cdot x = x\cdot 1 = x,\ \forall x\in F$,
6. for every $x\in F$ there exists a unique inverse x^{-1}, such that $1\in x\cdot x^{-1}\cap x^{-1}\cdot x$.

The elements of an e-hyperfield are called *e-hypernumbers*. If the the relation: $1=x\cdot x^{-1}=x^{-1}\cdot x$, is valid, then we say that we have a *strong e-hyperfield.*

Definition 2.2 [6], [7], [43]. *The Main e-Construction.* Given a group $(G,\cdot)$, where e is the unit, then we define in G, a large number of hopes $(\otimes)$ as follows:

$$x\otimes y = \{xy,\ g_1,\ g_2,\dots\},\ \ \forall x,y\in G\text{-}\{e\},\ \ g_1,\ g_2,\dots\in G\text{-}\{e\}$$

$g_1,\ g_2,\dots$ are not the same for each pair (x,y). Then $(G,\otimes)$ becomes an H_v-group, because it contains the $(G,\cdot)$. The H_v-group $(G,\otimes)$ is an e-hypergroup. Moreover, if for each x,y such that $xy=e$, so we have $x\otimes y=xy$, then $(G,\otimes)$ becomes a strong e-hypergroup.

Another important new field in hypermathematics comes straightforward from Santilli's Admissibility. We can transfer Santilli's theory in admissibility for representations in two ways: using either, the ordinary matrices and a hope on them, or using hypermatrices and ordinary operations on them [13], [15], [42], [43], [44], [47], [48].

The general definition is the following:

Definition 2.3 Let L be H_v-vector space over the H_v-field $(F,+,\cdot)$, $\varphi:F\rightarrow F/\gamma^*$, the canonical map and $\omega_F=\{x\in F:\varphi(x)=0\}$, where 0 is the zero of the fundamental field F/γ^*. Let ω_L be the core of the canonical map $\varphi':L\rightarrow L/\varepsilon^*$ and denote by the same symbol 0 the zero of L/ε^*. Take two subsets $R,\ S\subseteq L$ then a ***Lie-Santilli admissible hyperalgebra*** is obtained by taking the Lie bracket, which is a hope:

$$[\ ,\ ]_{RS}: L\times L\rightarrow P(L):\ [x,y]_{RS}= xRy-ySx= \{xry-ysx\ /r\in R,\ s\in S\}$$

Special cases, but not degenerate, are the 'small' and 'strict' ones:

(a) When only S is considered, then $[x,y]_S= xy-ySx= \{xy-ysx\ /s\in S\}$

(b) When only R is considered, then $[x,y]_R= xRy-yx= \{xry-yx\ /r\in R\}$

(c) When $R=\{r_1,r_2\}$ and $S=\{s_1,s_2\}$ then

$$[x,y]_{RS}= xRy-ySx= \{xr_1y-ys_1x,\ xr_1y-ys_2x,\ xr_2y-ys_1x,\ xr_2y-ys_2x\}.$$

(d) When $S=\{s_1,s_2\}$ then $[x,y]_S= xy-ySx= \{xy-ys_1x,\ xy-ys_2x\}$.

(e) When $R=\{r_1,r_2\}$ then $[x,y]_R= xRy-yx= \{xr_1y-yx, xr_2y-yx\}$.

(f) We have one case which is 'like' P-hope for any subset $S\subseteq L$:

$$[x,y]_S = \{xsy-ysx \,/\, s\in S\}.$$

On non square matrices we can define admissibility, as well:

Construction 2.4 Let $(L=M_{m\times n},+)$ be H_v-vector space of m×n hyper-matrices on the H_v-field $(F,+,\cdot)$, $\varphi:F\to F/\gamma^*$, canonical map and $\omega_F=\{x\in F:\varphi(x)=0\}$, where 0 is the zero of the field F/γ^*. Similarly, let ω_L be the core of $\varphi':L\to L/\varepsilon^*$ and denote by the same symbol 0 the zero of L/ε^*. Take any two subsets $R,S\subseteq L$ then a *Santilli's Lie-admissible hyperalgebra* is obtained by taking the Lie bracket, which is a hope:

$$[\ ,\ ]_{RS}: L\times L\to P(L): [x,y]_{RS}=xR^ty-yS^tx.$$

Notice that $[x,y]_{RS}=xR^ty-yS^tx=\{xr^ty-ys^tx \,/\, r\in R$ and $s\in S\}$.

Special cases, but not degenerate, are the 'small' and 'strict' ones:

(a) $R=\{e\}$ then $[x,y]_{RS} = xy-yS^tx = \{xy-ys^tx \,/\, s\in S\}$

(b) $S=\{e\}$ then $[x,y]_{RS} = xR^ty-yx = \{xr^ty-yx \,/\, r\in R\}$

(c) $R=\{r_1,r_2\}$ and $S=\{s_1,s_2\}$ then

$$[x,y]_{RS}=xR^ty-yS^tx=\{xr_1^ty-ys_1^tx,xr_1^ty-ys_2^tx,xr_2^ty-ys_1^tx,xr_2^ty-ys_2^tx\}$$

According to Santilli's iso-theory [9], [11], [13], [15], [22], [24], [25], [26], [27], [28], [39], [42], [46], [50], on a field $F=(F,+,\cdot)$, *a general isofield* $\hat{F}=\hat{F}(\hat{a},\hat{+},\hat{\times})$ is defined to be a field with elements $\hat{a}=a\times\hat{1}$, called *isonumbers*, where $a\in F$, and $\hat{1}$ is a positive-defined element generally outside F, equipped with two operations $\hat{+}$ and $\hat{\times}$ where $\hat{+}$ is the sum with the conventional additive unit 0, and $\hat{\times}$ is a new multiplication

$$\hat{a}\,\hat{\times}\,\hat{b}: = \hat{a}\times\hat{T}\times\hat{b}, \quad \text{with } \hat{1} = \hat{T}^{-1}, \quad \forall\hat{a}, \hat{b}\in\hat{F} \qquad \text{(i)}$$

called *iso-multiplication*, for which $\hat{1}$ is the left and right unit of F,

$$\hat{1}\,\hat{\times}\,\hat{a} = \hat{a}\times\hat{1} = \hat{a}\,, \quad \forall\hat{a}\in\hat{F} \qquad \text{(ii)}$$

called *iso-unit*. The rest properties of a field are reformulated analogously.

In order to transfer this theory into the hyperstructure case we generalize only the new multiplication $\hat{\times}$ from (i), by replacing with a hope including the old one. We introduce two general constructions on this direction as follows:

Construction 2.5 *General enlargement.* On a field $F=(F,+,\cdot)$ and on the isofield $\hat{F}=\hat{F}(\hat{a},\hat{+},\hat{\times})$ we replace in the results of the iso-product

$$\hat{a}\hat{\times}\hat{b}=\hat{a}\times\hat{T}\times\hat{b},\quad \text{with } \hat{1}=\hat{T}^{-1}$$

of the element $\hat{T}$ by a set of elements $\hat{H}_{ab}=\{\hat{T},\hat{x}_1,\hat{x}_2,\ldots\}$ where $\hat{x}_1,\hat{x}_2,\ldots\in\hat{F}$, containing $\hat{T}$, for all $\hat{a}\hat{\times}\hat{b}$ for which $\hat{a},\hat{b}\notin\{\hat{0},\hat{1}\}$ and $\hat{x}_1,\hat{x}_2,\ldots\in\hat{F}-\{\hat{0},\hat{1}\}$. If one of $\hat{a}$, $\hat{b}$, or both, is equal to $\hat{0}$ or $\hat{1}$, then $\hat{H}_{ab}=\{\hat{T}\}$. Thus, the new iso-hope is

$$\hat{a}\hat{\times}\hat{b}=\hat{a}\times\hat{H}_{ab}\times\hat{b}=\hat{a}\times\{\hat{T},\hat{x}_1,\hat{x}_2,\ldots\}\times\hat{b},\ \forall\hat{a},\hat{b}\in\hat{F}\qquad \text{(iii)}$$

$\hat{F}=\hat{F}(\hat{a},\hat{+},\hat{\times})$ becomes *isoH_v-field.* The elements of $\hat{F}$ are called *isoH_v-numbers* or *isonumbers*.

Remarks 2.6 More important hopes, of the above construction, are the ones where only for few ordered pairs $(\hat{a},\hat{b})$ the result is enlarged, even more, the extra elements $\hat{x}_i$, are only few, preferable exactly one. Thus, this special case is if there exists only one pair $(\hat{a},\hat{b})$ for which

$$\hat{a}\hat{\times}\hat{b}=\hat{a}\times\{\hat{T},\hat{x}\}\times\hat{b},\quad \forall\hat{a},\hat{b}\in\hat{F}$$

and the rest are ordinary results, then we have a hyperstructure called *very thin isoH_v-field.*

The assumption that $\hat{H}_{ab}=\{\hat{T},\hat{x}_1,\hat{x}_2,\ldots\}$, $\hat{a}$ or $\hat{b}$, is equal to $\hat{0}$ or $\hat{1}$, with that $\hat{x}_i$, are not $\hat{0}$ or $\hat{1}$, give that the isoH_v-field has one scalar absorbing $\hat{0}$, one scalar $\hat{1}$, and $\forall\hat{a}\in\hat{F}$, has one inverse.

Construction 2.7 *The P-hope.* Consider an isofield $\hat{F}=\hat{F}(\hat{a},\hat{+},\hat{\times})$ with $\hat{a}=a\times\hat{1}$, the isonumbers, where $a\in F$, and $\hat{1}$ is a positive-defined element generally outside F, with two operations $\hat{+}$ and $\hat{\times}$, where $\hat{+}$ is the sum with the conventional unit 0, and $\hat{\times}$ is the iso-multiplication

$$\hat{a}\hat{\times}\hat{b}:=\hat{a}\times\hat{T}\times\hat{b},\ \text{with } \hat{1}=\hat{T}^{-1},\ \forall\hat{a},\hat{b}\in\hat{F}.$$

Take a set $\hat{P}=\{\hat{T},\hat{p}_1,\ldots,\hat{p}_s\}$, with $\hat{p}_1,\ldots,\hat{p}_s\in\hat{F}-\{\hat{0},\hat{1}\}$, define the ***isoP-$H_v$-field***, $\hat{F}=\hat{F}(\hat{a},\hat{+},\hat{\times}_P)$, where the hope $\hat{\times}_P$ as follows:

$$\hat{a}\hat{\times}_P\hat{b}:=\begin{cases}\hat{a}\times\hat{P}^\wedge\times\hat{b}=\{\hat{a}\times\hat{h}^\wedge\times\hat{b}\,/\,\hat{h}^\wedge\in\hat{P}^\wedge\} & \text{if } \hat{a}\neq\hat{1} \text{ and } \hat{b}\neq\hat{1}\\ \hat{a}\times\hat{T}^\wedge\times\hat{b} & \text{if } \hat{a}=\hat{1} \text{ or } \hat{b}=\hat{1}\end{cases}\qquad \text{(iv)}$$

The elements of $\hat{F}$ are called *isoP-H_v-numbers*.

Remark. If $\hat{P} = \{\hat{T},\hat{p}\}$, that is that $\hat{P}$ contains only one $\hat{p}$ except $\hat{T}$. The inverses in isoP-H_v-fields, are not necessarily unique.

Example 2.8 In order to define a generalized P-hope on $\hat{Z}_7 = \hat{Z}_7(\hat{\underline{a}},\hat{+},\hat{\times})$, where we take $\hat{P}=\{\underline{\hat{1}},\underline{\hat{5}}\}$, the weak associative multiplicative hope is described by the table:

$\hat{\times}$	$\underline{\hat{0}}$	$\underline{\hat{1}}$	$\underline{\hat{2}}$	$\underline{\hat{3}}$	$\underline{\hat{4}}$	$\underline{\hat{5}}$	$\underline{\hat{6}}$
$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$
$\underline{\hat{1}}$	$\underline{\hat{0}}$	$\underline{\hat{1}}$	$\underline{\hat{2}}$	$\underline{\hat{3}}$	$\underline{\hat{4}}$	$\underline{\hat{5}}$	$\underline{\hat{6}}$
$\underline{\hat{2}}$	$\underline{\hat{0}}$	$\underline{\hat{2}}$	$\underline{\hat{4},\hat{6}}$	$\underline{\hat{6},\hat{2}}$	$\underline{\hat{1},\hat{5}}$	$\underline{\hat{3},\hat{1}}$	$\underline{\hat{5},\hat{3}}$
$\underline{\hat{3}}$	$\underline{\hat{0}}$	$\underline{\hat{3}}$	$\underline{\hat{6},\hat{2}}$	$\underline{\hat{2},\hat{3}}$	$\underline{\hat{5},\hat{4}}$	$\underline{\hat{1},\hat{5}}$	$\underline{\hat{4},\hat{6}}$
$\underline{\hat{4}}$	$\underline{\hat{0}}$	$\underline{\hat{4}}$	$\underline{\hat{1},\hat{5}}$	$\underline{\hat{5},\hat{4}}$	$\underline{\hat{2},\hat{3}}$	$\underline{\hat{6},\hat{2}}$	$\underline{\hat{3},\hat{1}}$
$\underline{\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{5}}$	$\underline{\hat{3},\hat{1}}$	$\underline{\hat{1},\hat{5}}$	$\underline{\hat{6},\hat{2}}$	$\underline{\hat{4},\hat{6}}$	$\underline{\hat{2},\hat{3}}$
$\underline{\hat{6}}$	$\underline{\hat{0}}$	$\underline{\hat{6}}$	$\underline{\hat{5},\hat{3}}$	$\underline{\hat{4},\hat{6}}$	$\underline{\hat{3},\hat{1}}$	$\underline{\hat{2},\hat{3}}$	$\underline{\hat{1},\hat{5}}$

The hyperstructure $\hat{Z}_7= Z_7(\hat{\underline{a}},\hat{+},\hat{\times})$, is commutative and associative on the multiplication hope.

Consruction 2.9 The generalized P-construction can be applied on rings to obtain H_v-fields. Thus for, $\hat{Z}_{10} = Z_{10}(\hat{\underline{a}},\hat{+},\hat{\times})$, and if we take $\hat{P}=\{\underline{\hat{2}},\underline{\hat{7}}\}$, then we have the table

$\hat{\times}$	$\underline{\hat{0}}$	$\underline{\hat{1}}$	$\underline{\hat{2}}$	$\underline{\hat{3}}$	$\underline{\hat{4}}$	$\underline{\hat{5}}$	$\underline{\hat{6}}$	$\underline{\hat{7}}$	$\underline{\hat{8}}$	$\underline{\hat{9}}$
$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$
$\underline{\hat{1}}$	$\underline{\hat{0}}$	$\underline{\hat{1}}$	$\underline{\hat{2}}$	$\underline{\hat{3}}$	$\underline{\hat{4}}$	$\underline{\hat{5}}$	$\underline{\hat{6}}$	$\underline{\hat{7}}$	$\underline{\hat{8}}$	$\underline{\hat{9}}$
$\underline{\hat{2}}$	$\underline{\hat{0}}$	$\underline{\hat{2}}$	$\underline{\hat{8}}$	$\underline{\hat{2}}$	$\underline{\hat{6}}$	$\underline{\hat{0}}$	$\underline{\hat{4}}$	$\underline{\hat{8}}$	$\underline{\hat{2}}$	$\underline{\hat{6}}$
$\underline{\hat{3}}$	$\underline{\hat{0}}$	$\underline{\hat{3}}$	$\underline{\hat{2}}$	$\underline{\hat{3},\hat{8}}$	$\underline{\hat{4}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{6}}$	$\underline{\hat{2},\hat{7}}$	$\underline{\hat{8}}$	$\underline{\hat{4},\hat{9}}$
$\underline{\hat{4}}$	$\underline{\hat{0}}$	$\underline{\hat{4}}$	$\underline{\hat{6}}$	$\underline{\hat{4}}$	$\underline{\hat{2}}$	$\underline{\hat{0}}$	$\underline{\hat{8}}$	$\underline{\hat{6}}$	$\underline{\hat{4}}$	$\underline{\hat{2}}$
$\underline{\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{0},\hat{5}}$
$\underline{\hat{6}}$	$\underline{\hat{0}}$	$\underline{\hat{6}}$	$\underline{\hat{4}}$	$\underline{\hat{6}}$	$\underline{\hat{8}}$	$\underline{\hat{0}}$	$\underline{\hat{2}}$	$\underline{\hat{4}}$	$\underline{\hat{6}}$	$\underline{\hat{8}}$
$\underline{\hat{7}}$	$\underline{\hat{0}}$	$\underline{\hat{7}}$	$\underline{\hat{8}}$	$\underline{\hat{2},\hat{7}}$	$\underline{\hat{6}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{4}}$	$\underline{\hat{3},\hat{8}}$	$\underline{\hat{2}}$	$\underline{\hat{1},\hat{6}}$
$\underline{\hat{8}}$	$\underline{\hat{0}}$	$\underline{\hat{8}}$	$\underline{\hat{2}}$	$\underline{\hat{8}}$	$\underline{\hat{4}}$	$\underline{\hat{0}}$	$\underline{\hat{6}}$	$\underline{\hat{2}}$	$\underline{\hat{8}}$	$\underline{\hat{4}}$
$\underline{\hat{9}}$	$\underline{\hat{0}}$	$\underline{\hat{9}}$	$\underline{\hat{6}}$	$\underline{\hat{4},\hat{9}}$	$\underline{\hat{2}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{8}}$	$\underline{\hat{1},\hat{6}}$	$\underline{\hat{4}}$	$\underline{\hat{2},\hat{7}}$

Then the fundamental classes are

$$(0)=\{\hat{\underline{0}},\hat{\underline{5}}\},\ (1)=\{\hat{\underline{1}},\hat{\underline{6}}\},\ (2)=\{\hat{\underline{2}},\hat{\underline{7}}\},\ (3)=\{\hat{\underline{3}},\hat{\underline{8}}\},\ (4)=\{\hat{\underline{4}},\hat{\underline{9}}\},$$

and the multiplicative table is the following

×	**(0)**	**(1)**	**(2)**	**(3)**	**(4)**
(0)	(0)	(0)	(0)	(0)	(0)
(1)	(0)	(1),(2)	(2),(4)	(3),(1)	(4),(3)
(2)	(0)	(2),(4)	(3)	(2)	(1)
(3)	(0)	(3),(1)	(2)	(3)	(4)
(4)	(0)	(4),(3)	(1)	(4)	(2)

Consequently, $\hat{Z}_{10} = Z_{10}(\hat{\underline{a}}, \hat{+}, \hat{\times})$, is an H_v-field.

3. APPLICATIONS TO A NEW CONCEPTION OF LIVING ORGANISMS

3.1 Einstein's argument that 'quantum mechanics is not a complete theory.'
As it is well know, Einstein accepted the validity of quantum mechanics for the representation of the atomic structure and other systems, but never accepted quantum mechanics as being a final theory capable of representing all possible elements of reality.

For this reason, Einstein expressed the view in 1935, jointly with his students Boris Podolsky and Nathan Rosen, that *'Quantum mechanics is not a complete theory'* (EPR argument) [10], in the sense that quantum mechanics (and we add nowadays quantum chemistry) could admit suitable enlargements for the representations of more complex systems.

Additionally, Einstein never accepted the uncertainties of quantum mechanics as being final in the sense that they are indeed valid for point-particles in vacuum but there could exist conditions in the universe recovering classical determinism. For this reason, Einstein's made his famous quote: *'God does not play dice with the universe.'*

3.2 Verification of Einstein's legacy by irreversible systems.
The most evident illustration if the validity of the lack of 'completeness' of quantum mechanics (and, therefore, of quantum chemistry) is given by the fact that *quantum mechanics and chemistry can only represent systems of point-like particles that are invariant under time-reversal*, such as the atomic structure. This is due to the invariance under anti-Hermiticity of the quantum mechanical Lie product between Hermitean operators

$$[A,B] = AB - BA = -[A,B]^\dagger,$$

where AB is the conventional classical associative product. In fact, the Lie product characterizes Heisenberg's time evolution of an observable A in terms of the Hamiltonian H,

$$idA/dt = [A,H] = AH - HA.$$

However, physical, chemical and biological processes such as nuclear fusion, combustion and living organisms, are irreversible over time.

The verification of Einstein's legacy via irreversible processes was first identified by R. M. Santilli during his Ph. D. Studies at the University of Torino, Italy, in the mid 1960s.

In fact, Santilli's Ph.D. thesis, published in the 1967 paper [13], provided the first known confirmation of the EPR argument (see also Ref. [13] of 1968) via the following Lie-admissible 'completion' of quantum mechanical Lie algebras for the representation of irreversible processes

$$(A,B) = ARB - BSA = (ATB - BTA) + (AJB + BJA), \;\; R = T - J,\; S = T + J \neq 0,$$

where the new product (A,B) is Lie-admissible according to A.A. Albert [1] when the attached antisymmetric product

$$[A,B]^* = (A,B) - (B,A) = ATB - BTA$$

verifies the Lie axioms whenever T is nowhere singular. Also, according to Albert [1] the product (A,B) is called *Jordan-admissible* when the attached symmetric product

$$\{A,B\}^* = (A,B) + (B,A) = AJB + BJA$$

verifies the axioms of Jordan algebras.

Santilli called *hadronic mechanics* [19] and *hadronic chemistry* [22] the 'completion' of quantum mechanics and chemistry, respectively, with a Lie-admissible structure for the representation of irreversible structures and processes.

3.3 Lie-admissible genomathematics

The mathematics underlying Lie-admissible formulations, collectively known as *genomathematics* [16], [17], [20], [23], can be summarized as follows. A generally tacit assumption of conventional, classical, numeric fields underlying Lie's theory is that *the multiplication of two numbers to the right n-->3 is equal to the multiplication of the same numbers to the left, 2<--3 = 2-->3.* Consequently, the indicated order of the multiplication is ignored in classical number theory, and we merely write 2×3 = 6.

In the transition from Lie theory to the covering Lie-admissible theory, the above ordering of the multiplication is no longer ignorable because the multiplication to the right 2>3 = 2S3 is no longer equal to the multiplication to the left $2<3 = 2R3 \neq 2>3$. This occurrence has permitted the identification of two, classical, numeric fields underlying Lie-admissible formulations [16]:

1) The *forward genofields* $F^{>}(n^{>},>,I^{>})$ with *forward genounit* $I^{>}=1/S$, *forward genonumbers* $n^{>}= nI^{>}$, and *forward genoproduct* $n^{>}>m^{>} = n^{>}Sm^{>}$, where *n, m* represent ordinary numbers; and

2) The *backward genofields* $F^{<}(n^{<},<,I^{<})$ with *backward genounit* $I^{<}=1/R$, *backward genonumbers* $n^{<} = I^{<}n$, and *backward genoproduct* $n^{<}< m^{<} = n^{<}Rm^{<}$.

Recall that Lie algebras can be constructed via the universal enveloping associative algebras ξ with classical, associative, modular product *AB*. The indicated inequivalence of the multiplications to the right and to the left implies the existence for Lie-admissible theories of *two universal, enveloping, genoassociative genoalgebars,* that to the right $\xi^{>}$ (left $\xi^{<}$) with genoassociative genoproduct to the right $A>B$ (left $A<B$), resulting in a non-trivial bimodular formulation.

The indicated bimodular formulations characterize the *time-irreversible, Lie-admissible, Heisenberg-Santilli genoequation* [13], [19]

$$idA/dt = (A, H) = ARH - HSA = A < H - H > A.$$

Recall that, in quantum mechanics, the modular associative multiplication to the right of an operator H to a Hilbert stat, $H_{\psi}(t,r) = E_{\psi}(t,r)$ yields the same eigenvalues E for the modular associative multiplication to the left $\psi(t,r)H = \psi(t,r)E$.

The Lie-admissible 'completion' of the above Schrödinger's equation yields the non-trivial bimodular structure:

1) The modular genoassociative action to the right representing the time evolution forward in time via the *Schrödinger-Santilli genoequation to the right* [19]

$$H(r,p) > \psi^{>}(t^{>},r^{>}) = H(r,p)S(\psi^{>},...)\psi(t,r) = E^{>}\psi^{>}(t^{>},r^{>})^{>},$$

and

2) The modular genoassociative action to the left representing motion backward in time via *Schrödinger-Santilli genoequation to the left*

$$\psi^{<}(t^{<},r^{<}) < H(r,p) = \psi^{<}R(\psi^{<},...)H(r,p) = \psi^{<}(t^{<},r^{<})E^{<}$$

where $E^{>} \neq E^{<}$.

The representation of irreversible processes from first axiomatic principles is then evident whenever $R \neq S$.

3.4 Verifications of Einstein's legacy.

Following the above mathematical studies, Santilli dedicated decades to experimental and industrial verifications of hadronic mechanics and chemistry. Following the achieved of such a mathematical and applied maturity, Santilli proved Einstein's legacy that 'quantum mechanics is not a complete theory' as well as the progressive recovering of Einstein's determinism in the interior of hadrons, nuclei and stars and its full recovering in the interior of gravitational collapse [21], [25], [26], [27], [28]. These results were achieved via the representation of the *extended and overlapping character of the constituents of irreversible systems in terms of the forward genotopic element* with realizations of the type

$$\hat{T} = \Pi_{k=1,\dots,N} Diag.(1/n_{1k}^2,\ 1/n_{2k}^2,\ 1/n_{3k}^2,\ 1/n_{4k}^2)e^{-\Gamma(\psi,\partial\psi,\dots)},$$

where n_{1k}^2, n_{2k}^2, n_{3k}^2, (called *characteristic quantities*) represent the deformable semi-axes of the k-particle normalized to the values $n_{\mu k}^2=1$, $\mu=1, 2, 3$ for the sphere; n_{4k}^2 represents the *density* of the k-particle considered normalized to the value $n_{4k} = 1$ for the vacuum; and $\Gamma(\psi,\partial\psi)$ represents non-linear, non-local and non-potential interactions caused by mutual overlapping/entanglement of the particles considered.

The aspects of studies [21], [25], [26], [27], [28] important for this paper are the following. Recall that particles originally in conditions of mutual overlapping/entanglement of their wave packets and then separated, have been experimentally proved to *instantly influence each other at a distance*, by therefore requiring superluminal communications that would violate special relativity. This is the very feature that prompted Einstein the argument that 'quantum mechanics is not a complete theory.' Santilli has achieved a quantitative representation of the indicated instantaneous communication at a distance via the representation of the extended character of the wavepacket of particles resulting in their continuous mutual penetration/entanglement at a distance of their center of mass, by therefore eliminating the need for superluminal communications. Above all, studies [21], [25], [26], [27], [28] have established that *the instantaneous communication of entangle particles at a distance occurs without any use of energy because the interaction are not derivable from a potential by basic assumptions.*

3.5 Application to a new conception of living organisms.

Note that all biological structures, including cells, viruses and large living organisms, are irreversible over time because they are born, grow and then die. Santilli introduced in monograph [18] of 1994 the representation of biological structures via *classical, multivalued, Lie-admissible formulations on a 3-dimensional Euclidean space,* namely, Lie-admissible formulations characterized by genounits, called *classical hyperunits*, with an ordered number of values all defined in the Euclidean space of our sensory perception

$$I^{>} = (I_1, I_2, ..., I_n) = 1/T = (1/T_1, 1/T_2, ...1/T_n) = 1/S.$$

Correspondently, the product of generic non-singular quantities a, b (such as numbers, functions, matrices, etc.), called *classical hyperproducts*, are equally multivalued, yet defined in our 3-dimensional Euclidean space

$$a > b = aSb = aT_1b + aT_2b + ... + aT_nb$$

in which all individual products are classical.

Correspondently, Ref. [18] introduced the notion of *classical hyperfields*, namely, sets of multi-valued elements, products and units which verify the axioms of numeric fields.

The transition from the classical, single-valued, Lie-admissible formulations outline in Section 3.3 to their multi-valued extension of was indicated in Ref. [18] as being necessary for the representation of the complexity of biological structures.

Note the fundamental character of the classical hyperunits and related hyperfields because the entire new formalism, including classical hyperalgebras, hyperspaces and hypertopology, are constructed via mere compatibility arguments with the base classical hyperfield.

In 1995, the Australian conchologist Chris Illert (see Part I of Ref. [12]) showed via computer simulations and direct calculations that the *growth of seashells over time cannot be consistently represented in a classical, 3-dimensional, single-valued Euclidean space $E(r,\delta,1)$* with classical coordinates $r=(x,y,z)$ metric $\delta=Diag.(1,1,1)$ and unit 1 over the field of real numbers $(R,n,\times,1)$, because, in said space, seashell grow irregularly and then crack. Illert then showed a consistent representation of seashell growth via the use of a *3-dimensional, two-valued Euclidean space* $(\hat{E}, \hat{r}, \delta, 1)$ where

$$\hat{r} = \{(x_1,x_2), (y_1,y_2), (z_1,z_2)\}$$

Santilli (see Part II of Ref. [18]) indicated that Illert's discovery confirms the need for hyperstructures in the representation of living organisms. In fact, the representation space used by Illert can be more accurately written as a *classical, 3-dimensional, two-valued, forward hyperspace* $(E^{>}, r^{>}, \delta^{>}, I^{>})$, over the forward, classical hyperfield $(R^{>}, n^{>}, >, I^{>})$ with classical forward hyperunit

$$I^{>} = \{(I_{1x}, I_{2x}), (I_{1y}, I_{2y}), (I_{1z}, I_{2z})\} = 1/T^{>} =$$
$$= \{(1/T_{1x}, 1/T_{2x}), (1/T_{1y}, 1/T_{2y}), (1/T_{1z}, 1/T_{2z})\}$$

and classical, 3-dimensional but two-valued products between arbitrary quantities a, b [18]

$$a > b = aT^{>}b = (a_xT_{1x}b_x + a_xT_{2x}b_x) + (a_yT_{1y}b_y + a_yT_{2y}b_y) + (a_zT_{1z}b_z + a_zT_{2z}b_z),$$

The Lie-admissible character of the representation and, therefore, its irreversibility, are assured when the backward hyperunit and, therefore, the hyperproducts, are different than the corresponding backward values.

A central notion of the above classical 2-valued, hyperstructural representation of seashells growth is the 3-dimensional character of the representation space, which is independent from the multi-valued character of each axis. Such a structure is necessary, on one side, to achieve compatibility of the mathematical representation with our sensory perceptions, while at the same time allowing an unlimited number if hidden degrees of freedom needed for a quantitative representation of the complexity of seashells. In fact, we inspect seashell growth with our three Eustachian tubes. Consequently, any *multi-dimensional representation, such as the use of a 6-dimensional space, would not be compatible with our sensory perception and, as such, not being experimentally verifiable.*

A major advance in the hyperstructural representation if biological structures was initiated by T. Vougiouklis in 1999 [39] with the lifting of the classical hyperstructures of 5 Ref. [18] to Vougiouklis H_v-structures (see also Ref. [42] and subsequent papers) which are formulated via hyperoperations (nicknamed 'hopes') including weak associativity (nicknamed '*WASS*'), weak commutativity (nicknamed '*COW*') and other hyperoperations.

The advantages of lifting the classical hyperstructures of Ref. [18] to Vougiouklis H_v-structures are several. The first advantage is a large increase of the representational capabilities which is necessary for a representation of biological structures such as the DNA, via a formulation that, at the abstract realization-free level, is compatible with the three-dimensional space of our sensory perception.

Other advantages are due to rather unique capabilities by Vougiouklis H_v-structures to characterize *bona fide* hyperfields on which the rest of the Lie-admissible formulation is expected to be built (see, e.g., Ref. [46]).

In this paper we introduce, apparently for the first time, a new conception of living organisms permitted by verifications [21], [25], [26], [27], [28] of the EPR argument according to which *a living organism, such as a cell, a virus or a human person, is composed by a very large number of extended constituents in conditions of continuous mutual entanglement of their wavepackets and, therefore, in continuous mutual communications.*

In view of the complexity and very large number of multi-valued internal communications, the best representation of the above conception of living organisms known to the authors, is given by two, hyperbimodular, Lie-admissible, H_v-structures, one for the representation of growth in time via hope, WASS and COW for ordered hypermodular hope to the right, and a second for the representation backward in time with hope, WASS and COW for ordered hypermodular hope to the left.

3.6 A specific hyperstructure formalism of living organisms.

As we present in section 3.3, in the transition from Lie theory to the covering Lie-admissible theory, we must specify an element S on the right and an element R to the left. In hyperstructure realization we can use as S and R, sets instead of elements. But in this case, we have hopes of constant length and the living organisms are not the case. Therefore, we suggest the use of a special case of the main e-construction to face the problem. Our construction equips the main product with an e-hope where the hyperproduct of two elements depend of those two elements. In fact, we keep the product and enlarge all the appropriate results.

Construction 3.1 *The Living Organism Construction.* In a set G equipped with several operations we take one product $(\cdot)$, where $(G,\cdot)$ is a group. Suppose that e is the unit, then we define in G, a large number of hopes $(\otimes)$ as follows:

$$e\otimes x = x\otimes e = x, \ \forall x\in G,$$

$$x\otimes y = \{xy, g_{xy1}, g_{xy2}, \dots\}, \ \forall x,y\in G\text{-}\{e\}, \text{ where } g_{xy1}, g_{xy1}, \dots \in G\text{-}\{e\}$$

$g_{xy1}, g_{xy2}, \dots$ depend on the pair (x,y). Then $(G,\otimes)$ becomes an H_v-group, because it contains the $(G,\cdot)$. The H_v-group $(G,\otimes)$ is an e-hypergroup. Moreover, if for each x,y such that $xy=e$, so we have $x\otimes y=xy$, then $(G,\otimes)$ becomes a strong e-hypergroup.

Remarks. **1.** In the H_v-group $(G,\otimes)$ the hope $(\otimes)$ is *WASS* and if the $(G,\cdot)$ is commutative, then the hope $(\otimes)$ is *COW*.
2. The Living Organism Construction can be used as S or R in forward genofields or backward genofields, respectively, according to section 3.3.

Recall that, according to the Schrödinger equation of quantum chemistry, living organisms are composed by a collection of isolated points. By contrast, according to the Schrödienger-Santilli genoequation of hadronic chemistry, living organisms are composed by the indicated large number of extended constituents in conditions of continuous entanglement and communication.

It is hoped that the proposed new conception of living organisms may allow new diagnostics, e.g., via the identification of possible miscommunications between different constituents, as well as new treatments, e.g., via the disruption of selected communications.

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