



## Foundations of isomathematics

A. S. Muktibodh

Citation: [AIP Conference Proceedings](#) **1558**, 707 (2013); doi: 10.1063/1.4825589

View online: <http://dx.doi.org/10.1063/1.4825589>

View Table of Contents: <http://scitation.aip.org/content/aip/proceeding/aipcp/1558?ver=pdfcov>

Published by the [AIP Publishing](#)

---

# Foundations of Isomathematics

A.S.Muktibodh

Mohota College of Science, Nagpur  
India  
E-mail: amukti2000@yahoo.com

**Abstract.** Santilli's isomathematics has a strong foundation in the early literature of mathematics surveyed by R.H.Bruck in his landmark book 'A Survey of Binary Systems' [1] dating back to 1958. This work aims at exploring the very basics of Isomathematics as suggested by Santilli [7] and [8]. The concept of 'Isotopy' plays a vital role in the development of this new age mathematics. Starting with Isotopy of groupoids we develop the study of Isotopy of quasi groups and loops via Partial Planes, Projective planes, 3-nets and multiplicative 3-nets.

**Keywords:** Partial plane, k-net, loops and isotopy,  
**PACS:** 11.10.Nx

## INTRODUCTION

Starting with a field  $(F, +, \cdot)$  of characteristic zero, Santilli constructs a new algebraic structure called 'Isofield'  $\hat{F} = (\hat{F}, \hat{+}, \hat{\times})$  which is isotopic to the original field  $F$ , with a prescribed element  $\hat{I} = \frac{1}{T} > 0$ , called isounit, as the multiplicative unit (unity) which may not be from the original field, with multiplication appropriately defined in the form  $n \hat{\times} m = n \times T \times m$  with  $n, m \in \hat{F}$ , under which  $\hat{I}$  remains the correct left and right unit with  $\hat{I} \hat{\times} n = n \hat{\times} \hat{I} = n$ ,  $n \in \hat{F}$  for all elements of the set. Lifting of the elements to isoelements is done via  $n \times \hat{I} = \hat{n}$ .

It is important to note that isoaddition is defined to be same as the addition in the original field  $F$ . Hence we may write  $+$  instead of  $\hat{+}$ . The lifting of the trivial multiplicative unit  $I$  to  $\hat{I}$  is the first basic step of Isomathematics. In this paper we investigate the nature of isofield [7] as the generalization of field. The remarkable transition from field to isofield is the key to the whole 'isomathematics' developed by Santilli. We also propose three propositions which follow from the definition of Isofield.

## PARTIAL PLANES

**Definition 1.** A *partial plane* is a system consisting of a non-empty set  $G$  partitioned into two disjoint subsets (one of which may be empty), namely the point-set and the line-set together with a binary operation, called incidence, such that (i) (Disjuncture) If  $x$  is incident with  $y$  in  $G$  then one of  $x, y$  is a line of  $G$  and the other is a point, (ii) (Symmetry) If  $x$  is incident with  $y$  in  $G$  then  $y$  is incident with  $x$  in  $G$ , and (iii) If  $x, y$  are distinct elements of  $G$  there is at most one  $z$  in  $G$  such that  $x$  and  $y$  are both incident with  $z$  in  $G$ .

**Definition 2.** A *Projective plane* is a special kind of a partial plane  $G$  such that; (iv) if  $x$  and  $y$  are distinct points or distinct lines of  $G$ , there exists a  $z$  in  $G$  such that  $x$  and  $y$  are both incident with  $z$  in  $G$ ; (v) there exists at least one set of four distinct points of  $G$  no three of which are incident in  $G$  with the same element.

It is easy to show that in the presence of (i)-(iv), postulate (v) is equivalent to; (vi) there exists at least one set of four distinct lines of  $G$  no three of which are incident in  $G$  with the same element.

**Definition 3.** A *k-net* is a partial plane whose line-set has been partitioned into into  $k$  disjoint classes such that (a)  $N$  has at least one point, (b) Each point of  $N$  is incident in  $N$  with exactly one line of each class, and (c) Every two lines of distinct classes in  $N$  are both incident in  $N$  with exactly one point.

Since a net is a partial plane, every net may be embedded in at least one projective plane. Every projective plane contains nets and, of these, two types additive 3-net and multiplicative 3-net have special significance.

Second type is a 3-net  $N$ , a multiplicative net of a projective plane  $P$ . We select three distinct points  $A, B, C$  of  $P$  which form a triangle; that is, the lines  $AB, BC, CA$  are distinct. The points of  $N$  are the points of  $P$  not incident in  $P$  with  $AB, BC$  or  $CA$ ; the lines of  $N$  are the lines incident in  $P$  with exactly one of  $A, B, C$ ; and elements  $a, b$ , of  $N$  are incident in  $N$  if and only if they are incident in  $P$ . Again not every 3-net is a multiplicative net of some projective plane but ever 3-net is a sub-3-net of such a multiplicative net.

**Definition 4.** A *Quasigroup* is a groupoid  $G$  such that, for each ordered pair  $a, b \in G$ , there is one and only one  $x$  such that  $ax = b$  in  $G$  and one and only one  $y$  such that  $ya = b$  in  $G$ .

In other words a quasigroup is groupoid whose composition table is a Latin square.

**Definition 5.** A *loop* is a quasigroup with an identity.

### 3-NETS AND ISOTOPY

#### *Isotopy of Groupoids*

Let  $(G, \cdot)$  and  $(H, \circ)$  be two groupoids. An ordered triple  $(\alpha, \beta, \gamma)$  of one-to-one mappings  $\alpha, \beta, \gamma$  of  $G$  upon  $H$  is called an *isotopism* of  $(G, \cdot)$  and  $(H, \circ)$ , provided  $(x\alpha) \circ (y\beta) = (x.y)\gamma$ .  $(G, \cdot)$  is said to be isotopic with  $(H, \circ)$  or  $(G, \cdot)$  is said to be an *isotope* of  $(H, \circ)$ .

We can have an **additive 3-net** and **multiplicative 3-net** of a projective plane. Every 3-net  $N$  of order  $n$  gives rise to a class of quasigroups  $(Q, \circ)$  of order  $n$  by defining one-to-one mappings  $\theta(i)$  with  $i = 1, 2, 3$  of  $Q$  upon the class of  $i$ -lines of  $N$ . Two quasigroups obtainable from the same 3-net by different choices of the set  $Q$  or of the mappings  $\theta(i)$  are said to be **isotopic**. For any  $Q$ , the  $\theta(i)$  can be so chosen that  $(Q, \circ)$  is a loop with a prescribed element  $e$  of  $Q$  as identity element. This lifting of the multiplicative quasigroup to a loop with the prescribed identity gives rise to the 'isofield' [4] for which associativity and commutativity for multiplication may not hold.

From here onwards we start developing all the properties of such isofields and give detailed account of Isomathematics and its special status.

Thus the new realization of a field, called as isofield is due to Santilli. This new algebraic structure has revolutionized contemporary mathematics and found its applications in so far unexplored ( unexplained) and unknown territories of quantum mechanics and quantum chemistry.

The study of isomathematics was motivated by specific physical needs. This new theory of numbers has become the basis of recent studies of nonlinear-nonlocal, nonhamiltonian systems in nuclear particle and statistical physics.

Isofields are of two types, **isofield of first kind**; wherein the isounit does not belongs to the original field, and **isofield of second kind**; wherein the isounit belongs to the original field. The elements of the isofield are called as **isonumbers**. This leads to a plethora of new terms and parallel development of conventional mathematics.

Isonumbers is the generalization of conventional numbers formed by lifting conventional unit 1 to  $\hat{1}$ . In fact this lifting leads to a variety of algebraic structures which are often used in physics. The following flowchart is self explanatory.

Isonumbers  $\longrightarrow$  Isofields  $\longrightarrow$  Isospaces  $\longrightarrow$  Isotransformations  $\longrightarrow$  Isoalgebras  $\longrightarrow$  Isogroups  $\longrightarrow$  Isosymmetries  $\longrightarrow$  Isorepresentations  $\longrightarrow$  Isogeometries etc.

In view of the definition of an isofield [8], we can say that an isofield is a an additive abelian group equipped with a new unit (called isounit ) and isomultiplication defined appropriately so that the resulting structure becomes a field which is non-commutative and non-associative, i.e. this structure is not isoassociative and isocommutative. In fact this structure satisfies weaker **isoalternative laws** as follows.  $\hat{a} \hat{\times} (\hat{b} \hat{\times} \hat{b}) = (\hat{a} \hat{\times} \hat{b}) \hat{\times} \hat{b}$  and  $\hat{a} \hat{\times} (\hat{a} \hat{\times} \hat{b}) = (\hat{a} \hat{\times} \hat{a}) \hat{\times} \hat{b}$ .

We mention an important proposition by Santilli.

**Proposition 1.** The necessary and sufficient condition for the lifting (where the multiplication is lifted but elements are not)  $F(a, +, \times) \longrightarrow (\hat{F}, +, \hat{\times}), \hat{\times} = \times T \times, \hat{1} = T^{-1}$  to be an isotopy (that is for  $\hat{F}$  to verify all axioms of the original field  $F$ ) is that  $T$  is a non-null element of the original field  $F$ .

We propose three propositions which directly follow from the definition of isofield.

**Proposition 2.** *If  $(F, +, \times)$  is a field and  $(\hat{F}, \hat{+}, \hat{\times})$  is the corresponding isofield such that the isounit  $\hat{I} \in F$  then  $(F, +, \times) \cong (\hat{F}, \hat{+}, \hat{\times})$ .*

**Proposition 3.** *If  $(F, +, \times)$  is a field and  $(\hat{F}, \hat{+}, \hat{\times})$  is the corresponding isofield such that the isounit  $\hat{I} \notin F$  then  $(F, +, \times)$  is isotopic to  $(\hat{F}, \hat{+}, \hat{\times})$ .*

In this case the resulting isofield is not necessarily commutative and associative.

**Proposition 4.** *Isofield corresponding to anon-associative and non-commutative field is isomorphic to the original field.*

## REFERENCES

1. R.H. Bruck, *A Survey of Binary Systems*, Springer Verlag, (1958).
2. Bates, Grace E., and Fred Kiokemeister, *A note on homomorphic mappings of quasigroups in to multiplicative systems*. Bull. Amer. Math. Soc. 54, 1180-1185 (1948). MR10 353.
3. Bates, Grace E. *Free loops and nets and their generalizations*. Amer. j. Math. 69,499-550 (1947).MR 10,12.
4. Chun-Xuan Jiang, *Foundations of Santilli's Isonumber Theory, with applications to New Cryptograms, Fermat's Theorem and Goldbach's Conjecture*, International Academic Press, America-Europe-Asia, 2002. bibitemkadJ.K.Kadeisvili and N.Kamiya, *A characterization of Isofields and their isoduals*, Hadronic J. 16, 155-172 (1993).
5. Hall Marshall, *Projective planes*. Trans. Amer. Math. Soc. 54, 229-277 (1943). MR5,72.
6. Pickert, Gunter, *Projective Ebenen*. Berlin-Gottingen-heidelberg, Springer-Verlag, 1955, MR 17,399.
7. Ruggero Maria Santilli, *isonumbers and genonumbers of dimension 1,2,4,8, their isoduals and pseudoduals, and "hidden numbers" of dimension 3,5,6,7*, algebras, groups and geometries 10, 273-322 (1993).
8. R.M.Santilli, *Foundations of Hadronic Chemistry*, Kluwer Academic Publisher, Dordrecht, 2001.
9. R.M.Santilli, *Isotopies of contemporary mathematical structures, II; Isotopies of symplectic geometry, affine geometry, Riemannian geometry and Einstein gravitation*, Algebras, Groups and Geometries, 8, 275-390 (1991).