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Michele Barone

*Nuclear Research Centre*
*Demokritos, Greece*

and

Franco Selleri

*Università di Bari*
*Bari, Italy*

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# ISOMINKOWSKIAN REPRESENTATION OF COSMOLOGICAL REDSHIFTS AND THE INTERNAL RED-BLUE-SHIFTS OF QUASARS

**Ruggero Maria Santilli**
The Institute for Basic Research, Box 1577, Palm Harbor, FL 34682, USA
E-mail: ibrrms@pinet.aip.org

## 1. STATEMENT OF THE PROBLEM

**1.1: The main hypothesis.** In this paper we study the cosmological quasar redshift and their internal redshifts and blueshifts via a new geometry, called *isominkowskian geometrY*, which is constructed as a covering of the Minkowskian geometry for the representation of electromagnetic waves and extended particles propagating within inhomogeneous and anisotropic physical media. The complementary *isoeuclidean* and *isoriemannian geometries* are also indicated.

Recall that: 1) homogeneity and isotropy of empty space are the geometric pillars of the conventional Doppler law; 2) quasars chromospheres are *inhomogeneous* (because of the local variation of the density) and *anisotropic* (because of the intrinsic angular momentum which creates a preferred direction in the physical medium, the underlying vacuum remaining homogeneous and isotropic); and 3) light is emitted in the interior of the quasars and propagates in their large chromospheres (of the order of millions of radial km) before reaching empty space.

The isominkowskian geometry implies a generalization of the Doppler law, called *isodoppler law*, which predicts: 1) a *frequency-dependent redshift* for inhomogeneous and anisotropic media of low density such as atmospheres and chromospheres (in which case light *loses* energy to the medium); 2) a *frequency-dependent blueshift* for inhomogeneous and anisotropic media of very high densities, such as those in the core of the quasars (in which case light *acquires* energy from the medium); and 3) *lack* of any shift for light propagating in homogeneous and isotropic media such as water.

Our main hypothesis is that the difference between the cosmological redshift of quasars over that of the associated galaxies is entirely reducible to the redshift of light while traveling in the quasar chromospheres *before* reaching empty space, thus permitting the quasars to be at rest with respect to the associated galaxies (or being expelled at small, thus ignorable speeds), while the internal quasar red/blue/shifts is due to the particular frequency dependence of the redshift itself. According to this hypothesis, the quasar cosmological redshifts and their internal red/blue/shifts are due to interior physical characteristics of the quasars and, more specifically, to the inhomogeneity and anisotropy of their chromospheres, i.e., to the *departures from the geometry of empty space.*

**1.2: Experimental verifications.** In this paper we show that the isominkowskian geometry provides a numerical representation of: I) the data by Arp [1] on the cosmological redshift of quasars, thus reducing them at rest with respect to the associated galaxy, as confirmed by a number of gamma spectroscopic data establishing the physical connection of the quasars with the associated galaxy; II) the data by Sulentic and others (see [2] and quoted literature) on the quasar internal red/blue/shifts; and III) the redshift of Fraunhofer lines of light from the inhomogeneous and anisotropic chromosphere of our Sun (see Marmet's studies [3] and vast literature therein).

Moreover, the isominkowskian geometry identifies intriguing interconnections between the seemingly different data I, II, III, and permits the prediction of novel, experimentally verifiable effects, such as the prediction that the dominance of red of Sun light at sunset is partially (but not entirely) an isoredshift due to the inhomogeneity and anisotropy of our atmosphere. This prediction is supported by the fact that the sky at the zenith is not red, in which case the increase in redness at the horizon would be completely explainable with conventional means (scattering, absorption, etc.). Instead, the dominance of *blue* at the zenith and of *red* at the horizon supports the isominkowskian geometry.

In this paper we also present of a number of experimental verifications of the isominkowskian geometry in particle physics which are indirectly, yet significantly related to the quasar red/blue/shifts, such as the anomalous behaviour of the meanlife of unstable hadrons with speed whose structure is fully equivalent to the isodoppler law, the data on the Bose-Einstein correlation for the UA1 experiments at CERN, the anomalous total magnetic moment of few-body nuclei, and others.

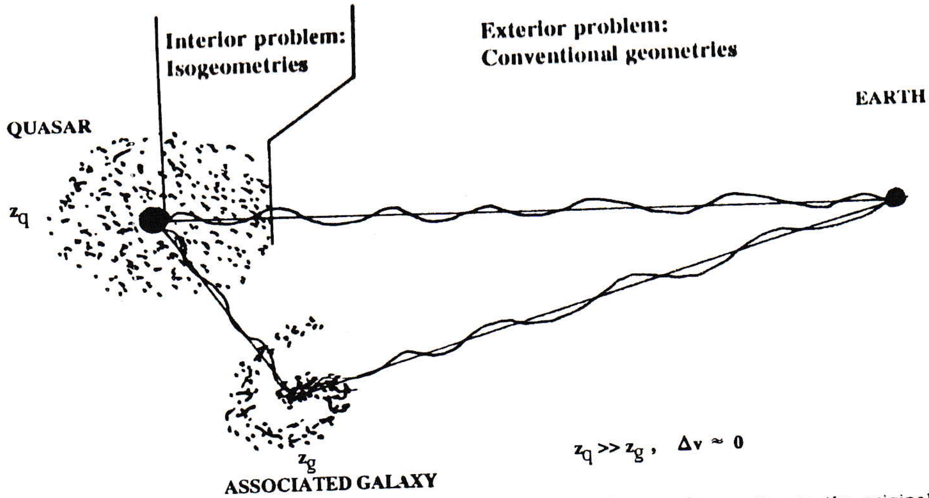


Fig. 1. A schematic view on the main hypothesis of this paper (Sect. 1.1) according to the original proposal [10].

Above all, this paper is intended to stimulate the experimental resolution of the now vexing problem of the quasar shifts via novel direct experiments, such as measure the isoredshift predicted for light from distant stars passing through the Sun's chromosphere, or a planetary atmosphere, or measure the predicted isoredshift component of the Sun's light at sunset by following a sufficient number of Fraunhofer lines from the zenith to the horizon.

All these measures, if confirmed, would provide final evidence that a portion (but not necessarily all) of the cosmological redshift of quasars is of interior geometric character due to the departures from the homogeneity and isotropy of space caused by the inhomogeneity and anisotropy in their environment. The separate problem of the cosmological redshift of galaxies is only briefly considered.

**1.3: Connection with alternative theories.** Numerous *alternative theories* (i.e., of non-Doppler character) have been submitted (see [1-3] and review [4]) such as Arp's theory of the *creation of matter* in the quasars, Marmet theory based on photon scattering, and others. These theories are capable of representing the cosmological quasar redshift, although their capability to represent the internal red/blue/shift and other recent evidence is under study.

The continuation of the study of these alternative interpretations is encouraged here because each one adds valuable information to the other and, in the final analysis, all quantitative interpretations may well result to be deeply interconnected.

For instance, Arp's theory emerges from our studies in a new light because the creation of matter may ultimately result to be an interplay between matter and antimatter which is prohibited in conventional geometries, but permitted in our isogeometries because of an inner conjugation indicated later on. Similarly, Marmet's representation of the data on the Sun's chromosphere [3] may essentially result to be an *operator* counterpart of our *classical* studies. We regret the inability to study these interconnections in detail at this time for lack of space.

**1.3: A historical distinction.** An aspect of fundamental relevance for the studies of this paper is the historical distinction between the *exterior dynamical problem* (i.e., electromagnetic waves and point-like test bodies moving in the homogeneous and isotropic vacuum), and the *interior dynamical problem* (i.e., electromagnetic waves and extended test bodies moving within inhomogeneous and anisotropic physical media). This distinction was introduced by the founders of analytic dynamics, and kept up to the first part of this century (see, e.g., Schwartzschild's *two* papers [4], the first famous one on the *exterior* problem and the second little known paper on the *interior* problem, or early treatises in gravitation, e.g., ref.s [6], the first with a preface by Einstein).

Regrettably, the above distinction was progressively relaxed, up to the current condition of virtual complete silence in the specialized literature. This is due to the belief that the interior problem can be reduced to to the exterior form, which is certainly admissible as an *approximation* (see Schwartzchild's [5] insistence on the approximate character of his solution for the *interior* problem). The point is that such a reduction cannot be exact, as established by the so-called *No-Reduction Theorems* [7] which prove that an interior system (such as a satellite during re-entry in Earth's atmosphere with a monotonically decaying angular momentum) simply cannot be consistently reduced to a finite collection of ideal elementary particles each in a stable orbit with conserved angular momentum.

With the clear understanding that the Minkowskian and Riemannian geometries are exactly valid in empty space, the above theorems establish their *inapplicability* (rather than "violation") for interior conditions on numerous, independent, topological, analytic, geometric and other grounds. For instance, interior systems are *nonlinear in the velocities* (a missile in atmosphere has a drag force nowadays proportional to the tenth power of the speed and more), *nonlocal-integral* (because the shape of the test body directly affects its trajectory, thus calling for integral terms), and *nonpotential* (because the notion of potential has no mathematical or physical meaning for contact interior forces and the systems are *variationally nonselfadjoint* [7]). The inapplicability of the Minkowskian and Riemannian geometries for these interior conditions is so evident to require no additional comment. The only scientifically meaningful issue is the construction of appropriate covering geometries specifically conceived for interior conditions.

**1.4: Insufficiencies of the conventional interpretation.** The conventional interpretation of quasars redshifts is based on the celebrated *Doppler law*

$$\omega = \omega_o (1 - v \cos \alpha / c_o) \gamma, \quad \gamma = (1 - v^2 / c_o^2)^{-\frac{1}{2}}, \tag{1.1}$$

where $\alpha$ is the angle between the direction of light and of motion of the source and $c_o$ is the speed of light in vacuum. The *redshift* $\Delta\omega = \omega - \omega_o < 0$, is therefore reduced to the computation of the speed v of the quasars with respect to Earth (see, e.g., ref.s [6]). Note that such interpretation is: 1) purely classical, 2) relativistic without gravitational corrections, and 3) based on the assumption that light is emitted by the quasars and propagates immediately in vacuum without any effect when passing through the chromospheres.

The theoretical insufficiencies of law (1.1) for interior conditions are beyond credible doubts. The homogeneity and isotropy of empty space are known to be the geometric pillars for the derivation of the law. Its inapplicability for light propagating within inhomogeneous and anisotropic atmospheres is then unquestionable.

Astrophysical insufficiencies of law (1.1) for the interpretation of the data on quasars redshift began to emerge with the discovery of the quasars themselves, and then progressively increased in time [1,2,3]. Among the most visible inconsistencies we recall [loc. cit.] galaxies younger than their stars, galaxies older than the life of the universe, discrete variations of redshift, quasars evolving into galaxies, speeds in excess of those permitted by Einsteinian theories, etc.

These and other inconsistencies have now reached such dimension and diversification to call for a revision of the fundamental *geometries* used in the description of the universe.

**1.5: Bibliographical notes.** The isogeometries were constructed by this author to satisfy the following conditions 1) have a structure which is nonlinear (in coordinates, velocities and any needed additional quantity), nonlocal-integral (in all needed variables), nonpotential, inhomogeneous and anisotropic; 2) preserve the axioms of the original geometry at the abstract level so as to permit a geometric unification of exterior and interior problems; and 3) be coverings of conventional geometries, thus admitting the latter as particular cases when motion returns to be in vacuum.

The methods for the construction of the isogeometries were proposed by the author back in 1978 [8] when at the Department of Mathematics of Harvard University under DOE support. They are called *isotopies* from Greek terms meaning "preserving the topology", and interpreted as axiom-preserving (the broader *genotopies* [8] are reviewed for brevity, see in this respect the contribution by Jannussis [24] in these proceedings). These methods essentially permit nonlinear-nonlocal-nonhamiltonian, but axiom-preserving generalizations (called *liftings*) of any given mathematical or physical structure, as outlined in Sect. 2.

Isotopies were first applied to the lifting of classical Hamiltonian mechanics and Lie's theory into covering theories [7,8]. The first isotopic lifting of the Minkowskian geometry was proposed is ref. [9] of 1982. The isotopic lifting of the Riemannian geometry was first proposed in ref. [10] of 1988, jointly with the proposal to elaborate Arp's data [1] (Fig. 1.1). Such elaboration was subsequently conducted by Mignani in ref. [11]. A detailed study of the isogeometries first appeared in ref.s [12]. Ref.s [13] provide a classical presentation of the isogeometries with ref.s [14] giving the operator counterpart. Mathematical reviews are available in ref.s [15-17], an independent physical review is available in ref. [18]. A review of the isogeometries is available in ref. [19]. Preliminary, yet significant verifications are provided in ref.s [25-36]. A comprehensive presentation of the content of this paper is provided in ref. [37] for flat and in ref. [38] for curved isogeometries.

## 2: BASIC NOTIONS ON ISOTOPIES

**2.1: Isotopies of the unit.** The fundamental isotopies are the liftings of the n-dimensional unit $I = \mathrm{diag.}\,(1, 1, \ldots, 1)$ of contemporary geometries into an n×n-dimensional matrix $\hat{1}$ whose elements have the most general possible, nonlinear and nonlocal dependence on time t, coordinates x, their derivatives of arbitrary order $\dot{x}$, $\ddot{x}$, ..., and any needed additional interior quantity, such as the frequency $\omega$ of the wave, the local density $\mu$, the local temperature $\tau$, the local index of refraction n, etc. [7,8]

$$I = \mathrm{diag.}\,(1, 1, \ldots, 1) \rightarrow \hat{1} = \hat{1}(t, x, \dot{x}, \ddot{x}, \omega, \mu, \tau, n, \ldots)\,. \qquad (2.1)$$

under the condition (necessary for an isotopy) of preserving the original axioms of I. The above liftings have been classified into five topologically significant classes called *Kadeisvili's Classes* I-V [19,20]. In this paper we shall only consider liftings of Kadeisvili's Class I (with generalized units $\hat{1}$ that are smooth, bounded, nowhere degenerate, Hermitean and positive-definite), which characterize isotopies properly speaking), and of Class II (the same as Class I but with negative-definite isounits). For brevity we shall limit ourselves to brief comments on the remaining Class III (the union of Class I and II), IV (holding for singular isounits representing gravitational collapse) and V (with arbitrary, e.g., discrete, isounits).

The isotopies of the unit demand, for consistency, a corresponding, compatible lifting of all associative products AB among generic quantities A, B, into the *isoproduct* [8]

$$A\,B \rightarrow A * B = A\,T\,B\,, \quad T = \text{fixed}\,, \; I\,A = A\,I \equiv A \rightarrow \hat{1} * A = A * \hat{1} \equiv A, \quad \hat{1} = T^{-1}, \qquad (2.2)$$

whose isotopic character is ensured by the preservation of associativity, $A(BC) = (AB)C \rightarrow A*(B*C) = (A*B)*C$. Under the above conditions, $\hat{1} = T^{-1}$ is called the *isounit* and T is called the *isotopic element*. Note the necessity, e.g., in number theory, of lifting the product whenever the (multiplicative) unit is lifted and viceversa.

**2.2: Isotopies of fields.** The isotopies of the unit $I \rightarrow \hat{1}$ and of the product $AB \rightarrow A*B$ demand the lifting of conventional fields $F(a,+,\times)$ of real numbers R, complex number C and quaternions Q with generic elements a, conventional sum + and product $a \times b := ab$, into the so-called *isofields* [12]

$$F(a,+,*) \rightarrow \hat{F}(\hat{a},+,*)\,, \quad \hat{a} = a\hat{1}\,, \quad \hat{a} * \hat{b} = \hat{a}\,T\,\hat{b} = (a\,b)\hat{1}, \quad \hat{1} = T^{-1}\,. \qquad (2.3)$$

with elements $\hat{a} = a\hat{1}$ called *isonumbers*, conventional sum + and isoproduct (2.2), under the condition (again necessary for an isotopy) of preserving the original axioms of F. All operations in F must be generalized for $\hat{F}$. We have *isosquares* $\hat{a}^{\hat{2}} = \hat{a}*\hat{a} = \hat{A}T\hat{a} = a^2\hat{1}$, *isoquotient* $\hat{a}\hat{/}\hat{b} = (a/b)\hat{1}$, *isosquare roots* $\hat{a}^{\hat{1/2}} = a^{1/2}\hat{1}$, etc. (see [12,14] for detailed studies).

The above liftings are nontrivial inasmuch as they imply the inapplicability under isotopies of the entire mathematical formulations of conventional geometries. As an illustration, statements such as "two multiplied by two equals four" are generally incorrect for isogeometries. In fact, for $\hat{1}$

= 3, "two multiplied by two equals twelve", with the understanding that the very notion of integer number is generally lost in favor of an integro-differential notion, e.g., $\hat{2} = 2\exp(N\int dx\psi^{\dagger}(x)\phi(x))$ as for the Cooper pair of electrons in superconductivity with wavefunctions $\psi$ and $\phi$ (see Sect. 5.5).

**2.3: Isotopy of metric spaces.** Liftings $I \to \hat{1}$, $AB \Rightarrow A*B$ and $F \to \hat{F}$ then require the isotopies of vector, metric and pseudo-metric spaces, evidently because they depend on the field in which they are defined. In fact, real metric or pseudo-metric spaces S(x,g,R) with Hermitean metric g over R must be subjected to the liftings into the so-called *isospaces* (first introduced in [9])

$$S(x,g,R) \Rightarrow \hat{S}(x,\hat{g},\hat{R}), \quad \hat{g} = Tg, \quad \hat{1} = T^{-1}, \quad x^{\hat{2}} = (x^t \hat{g} x)\hat{1} \in \hat{R}. \tag{2.4}$$

under the condition, again, of preserving the original axioms of S(x,g,R). In particular, *the basis of a metric (or, more generally, vector) space is preserved under isotopies* [12], thus including the preservation of the basis of a Lie algebra. This results in nonlinear and nonlocal (in x, $\dot{x}$, $\ddot{x}$, ....) generalization of the original space, yet such that $\hat{S}(x,\hat{g},\hat{R}) \approx S(x,g,R)$.

We have indicated earlier the loss of conventional numbers under isotopies. When passing to isospaces, one should keep in mind the loss of conventional functional analysis into a covering formulation called *functional isoanalysis* [20]. In fact, the very notion of angle is lost under isotopies (see next section), thus implying the consequential loss of trigonometry, Legendre polynomials, etc. in favor of suitable, unique (and intriguing) covering notions [14].

**2.3: Lie-isotopic theory.** The preceding liftings demand a corresponding compatible lifting of all branches of Lie's theory into the so-called *Lie-isotopic theory* first submitted in [8] and then studied in ref.s [12-20]). We can here mention only the lifting of the envelope $\xi(g)$ of a Lie algebra g and related exponentiation in terms of the original (ordered) basis $\{X_i\}$ of g

$$\hat{\xi}: \quad \hat{1}, \quad X_i * X_j \ (i \leq j), \quad X_i * X_j * X_k, \quad (i \leq j \leq k), \ \ldots\ldots, \quad i, j, k = 1, 2, \ldots, n, \tag{2.5a}$$

$$e_{\hat{\xi}}^{\,i\hat{w}*X} = \hat{1} + (i\,\hat{w} * X)/1! + (i\,\hat{w} * X) * (i\,\hat{w} * X)/2! + \ldots = \{e^{iXTw}\}\hat{1} = \hat{1}\{e^{iwTX}\}, \tag{2.5b}$$

the lifting of Lie algebra $g \approx [\xi(g)]^-$ with familiar Lie theorem, such as the 2-nd theorem $[X_i, X_j]_{\xi} = X_i X_j - X_j X = C_{ij}^{\ k} X_k$, into the *Lie-isotopic algebras* $\hat{g} \approx [\hat{\xi}(g)]^-$ with *Lie-isotopic theorems* [8], e.g.,

$$\hat{g}: [X_i, X_j]_{\hat{\xi}} = X_i * X_j - X_j * X_i = X_i T X_j - X_j T X_i = \hat{C}_{ij}^{\ k}(t, x, \dot{x}, \ddot{x}, \omega, \mu, \tau, n, \ldots) * X_k, \tag{2.6}$$

where the $\hat{C}$'s, called *structure isofunctions*, are restricted by the *Third Isotopic Theorem* [8,16]; the lifting of transformations and related (connected) Lie groups G into the *Lie-isotopic transformation groups* [8]

$$\hat{G}: x' = \hat{U}(\hat{w}) * x, \quad \hat{U}(w) = \prod_k e_{\hat{\xi}}^{\,i X_k * \hat{w}_k} = \hat{1}\{\prod_k e^{i w_k T X_k}\} = \{\prod_k e^{i X_k T w_k}\}\hat{1}, \tag{2.7b}$$

$$\hat{U}(0) = \hat{1}, \quad \hat{U}(\hat{w}) * \hat{U}(\hat{w}') = \hat{U}(\hat{w}') * \hat{U}(\hat{w}) = \hat{U}(\hat{w} + \hat{w}'), \quad \hat{U}(\hat{w}) * \hat{U}(-\hat{w}) = \hat{1}, \tag{2.7a}$$

$$(e_{\hat{\xi}}^{\,X_1}) * (e_{\hat{\xi}}^{\,X_2}) = e_{\hat{\xi}}^{\,X_3}, \ X_3 = X_1 + X_2 + [X_1, X_2]_{\hat{\xi}}/2 + [(X_1 - X_2), [X_1, X_2]_{\hat{\xi}}]_{\hat{\xi}}/12 + \ldots \tag{2.7c}$$

the lifting of the conventional representation theory into the *isorepresentation theory of Lie-isotopic algebras and groups* (which is structurally nonlinear, nonlocal and noncanonical); and other liftings [13,14].

Note the preservation of the Lie algebra axioms by the isotopic product $[A, B]_{\hat{\xi}} = ATB - BTA$. Note also the nontriviality of the isotopic theory from the appearance of the nonlinear-integral quantity T in its exponentiation (2.7a). We should also note that, even though structurally nonlinear, nonlocal and noncanonical, the Lie-isotopic theory verifies the axioms of linearity, locality and canonicity at the isotopic level and, for this reason, it is called *isolinear, isolocal and isocanonical*. Note finally that all nonlinear-nonlocal-noncanonical theories *always* admit an *identical* isolinear-isolocal-isocanonical reformulation with evident advantages.

**2.5: Isosymmetries.** The Lie-isotopic transformation groups are turned into symmetries of isospaces, called *isosymmetries*, via the following:

**Theorem 2.1** [21]: *Let G be an N-dimensional Lie group of isometries of an m-dimensional, metric or pseudo-metric, and real or complex space* S(x,g,F), F = R *or* C,

$$G: x' = A(w)\ x, \quad (x'-y')^{\dagger} A^{\dagger} g A (x'-y') \equiv (x-y)^{\dagger} g (x-y), \quad A^{\dagger} g A = A g A^{\dagger} = g, \tag{2.8}$$

and T is derived from the deformed metric $\hat{g} = Tg$ (see the example in the next section). Note also that there is no need to verify isoinvariance (2.9) because ensured by the original invariance (2.8).

It is also easy to prove that $\hat{G} \approx G$ for all Class I isotopies (but not so for other Classes for which in general $\hat{g} \approx [\xi(g)]^{-} \neq g$). This property identifies one of the primary applications of isosymmetries, the reconstruction of exact symmetries when believed to be conventionally broken. In fact, in ref.s [21] one can see the reconstruction of the exact rotational *symmetry* at the isotopic level $\hat{O}(3) \approx O(3)$ for all ellipsoidical deformations of the sphere. In ref. [9] one can see the reconstruction of the exact Lorentz *symmetry* at the isotopic level $\hat{O}(3.1) \approx O(3.1)$ for all signature preserving ($T > 0$) deformations of the Minkowski metric $\hat{\eta} = T\eta$. See ref.s [13,14] for the reconstruction of additional exact symmetries.

**2.6: Inequivalence of the Lie and Lie-isotopic theories.** Despite the isomorphism $\hat{G} \approx G$, Lie and Lie-isotopic symmetries are inequivalent on numerous counts, such as: 1) G is customarily linear-local-canonical, while $\hat{G}$ is nonlinear-nonlocal-noncanonical; 2) the mathematical structures underlying $\hat{G}$ and G (fields, spaces, etc.) are structurally different; 3) $\hat{G}$ can be derived from G via *nonunitary* transformations under which

$$UU^{\dagger} = \hat{1} \neq I, \; U(AB - BA)U^{\dagger} = A'TB' - B'TA', \; T = (UU^{\dagger})^{-1} = T^{\dagger}, \; A' = UAU^{\dagger}, \;\; B' = UBU^{\dagger}. \qquad (2.10)$$

The above inequivalence also emerges in the *isorepresentation theory* [14] , e.g., because the spectra of eigenvalues of the same operator are different in the two theories (due to the necessary isotopy of eigenvalue equations $H|b\rangle = E^{\circ}|b\rangle \rightarrow H{*}|\hat{b}\rangle = HT|\hat{b}\rangle = \hat{E}{*}|\hat{b}\rangle \equiv E|\hat{b}\rangle$, $E \neq E^{\circ}$). Also, weights, Cartan tensors, etc. acquire a nonlinear-nonlocal-noncanonical dependence on the base manifold, etc.

**2.7: Isodual conjugations and antimatter.** The generalization of the unit permits the identification of a new antiautomorphic conjugation $\hat{1} \rightarrow \hat{1}^{d} = -\hat{1}$ introduced in [21] under the name of *isoduality*. This map implies the existence of isodual images of all quantities of Class I (fields, spaces, algebras, groups, etc.) into corresponding forms of Class II.

In particular, any positive number m or isonumber $\hat{m} = m\hat{1}$ is mapped into the *isodual number* $m^{d} = m\,1^{d} = -\,m$ or *isodual isonumber* $\hat{m}^{d} = m\hat{1}^{d} = -\,\hat{m}$, while the *isodual isonorm* is given by $\lceil \hat{m} \rceil^{d} = (m\,T\,m)^{\frac{1}{2}}\hat{1}^{d} = -\lceil \hat{m} \rceil$ and it is *negative-definite*. The most intriguing properties of isodual spaces and isodual symmetries is that they describe particles with *negative-definite energy moving backward in time.*

Recall that antiparticles originated from the negative-energy solutions of conventional relativistic equations, although such solutions were abandoned because the behaviour of the systems was unphysical in our space-time. Isodual spaces and isodual symmetries provide a fundamentally novel approach because the interpretation of the same negative-energy solution in isodual spaces is now fully physical [13,14].

The isogeometries therefore permit a novel cosmological conception of the structure of the universe in which, for the limit case of an equal distribution of matter and antimatter, all total quantities, such as total energy, total time, etc., are identically null (see ref. [38] for brevity).

## 3: ISOMINKOWSKIAN GEOMETRY

**3.1: Isominkowskian spaces.** Consider an electromagnetic wave propagating first in empty space (exterior relativistic problem), then throughout our atmosphere (interior relativistic problem). As well known, *the Minkowski space*

*Then, the infinitely possible isotopes* $\hat{G}$ *of* G *characterized by the same generators and parameters of* G *and new isounits* $\hat{1}$ *(isotopic elements* T*), leave invariant the isocomposition on the isospaces* $\hat{S}(x,\hat{g},\hat{F})$, $\hat{g} = Tg$, $\hat{1} = T^{-1}$,

$$\hat{G}: \; x' = \hat{A}(w) * x, \; (x'-y')^{\dagger} * \hat{A}^{\dagger}\hat{g}\hat{A} * (x'-y') = (x-y)^{\dagger}\hat{g}(x-y), \; \hat{A}^{\dagger}\hat{g}\hat{A} = \hat{A}\hat{g}\hat{A}^{\dagger} = \hat{1}\hat{g}\hat{1}, \qquad (2.9)$$

The above results yield the "direct universality" of the Lie-isotopic symmetries, i.e., their capability of providing the invariance of all infinitely possible deformations $\hat{g} = Tg$ of the original metric g (universality), directly in the x-frame of the experimenter (direct universality). Note also the simplicity of the explicit construction of the desired isotransformations via rule (2.7) where w are the conventional parameters, X are the conventional generators in their adjoint representation

$$M(x,\eta,R): \quad x = (\mathbf{x}, x^4),\ x^4 = c_o t,\ x^2 = x^\mu \eta_{\mu\nu} x^\nu = x^1 x^1 + x^2 x^2 + x^3 x^3 - x^4 x^4,\ \eta = \text{diag.}(1, 1, 1, -1) \qquad (3.1)$$

*geometrizes the homogeneity and isotropy of empty space* and, as such, it is exactly valid for exterior conditions.

The *isominkowski space* (first submitted in [9]) is intended to geometrize the inhomogeneity and anisotropy of interior conditions. It is constructed via two simultaneous liftings, that of the Minkowski metric $\eta$ into the *isometric* $\hat{\eta} = T\eta$ of Class I and the joint lifting of the unit of $M(x,\eta,R)$, $I = \text{diag.}(1, 1, 1, 1)$, into the 4×4-dimensional isounit $\hat{1} = T^{-1}$, and we shall write

$$\hat{M}(x,\hat{\eta},\hat{R}): \qquad \hat{\eta} = T(x, \dot{x}, \ddot{x}, \mu, \tau, n, \dots)\, \eta, \quad \hat{1} = T^{-1} > 0, \quad x^2 = (x^\mu \hat{\eta}_{\mu\nu} x^\nu)\, \hat{1} \in \hat{R}. \qquad (3.2)$$

Note that isospaces $\hat{M}(x,\hat{\eta},\hat{R})$ have the most general possible nonlinear-nonlocal-noncanonical structure because the functional dependence of $\hat{\eta}$ remains unrestricted. The isometric can always be (although not necessarily) diagonalized for Class I, resulting in isoseparation of the type

$$x^2 = x^1 b_1^2(x, \dot{x}, \dots)\, x^1 + x^2 b_2^2(x, \dot{x}, \dots)\, x^2 + x^3 b_3^2(x, \dot{x}, \dots)\, x^3 - x^4 b_4^2(x, \dot{x}, \dots)\, x^4, \quad b_\mu > 0, \qquad (3.3)$$

Despite evident structural differences, the joint liftings $\eta \to \hat{\eta} = T\eta$ and $I \to \hat{1} = T^{-1}$ imply that the *isominkowskian space is locally isomorphic to Minkowskian space,* $\hat{M}(x,\hat{\eta},\hat{R}) \approx M(x,\eta,R)$ [9,12,14]. Owing to the positive-definiteness of the isotopic element T, it is easy to see that $\hat{M}(x,\hat{\eta},\hat{R})$ and $M(x,\eta,R)$ coincide at the abstract level. Exterior and interior descriptions are therefore *different realizations* of the same abstract geometric axioms. This is the central geometric property which is assumed for the description of both, exterior and interior relativistic problems, and which carries intriguing consequences, as we shall see.

**3.2: Characteristic quantities of physical media.** The b-quantities (at times also expressed in the form $b_\mu = 1/n_\mu$) are called the *characteristic quantities* of the medium considered. The inhomogeneity of the medium can be represented via an explicit dependence of the b's on the local density, and the anisotropy can be represented via different values among the b's, the factorization of a preferred direction of the medium, and other means.

When the local behaviour is needed at one given interior point, one needs the full nonlinear-nonlocal dependence of the b's. This is illustrated e.g., by the local speed of light at one given point when passing though our atmosphere which is given by $c = c_o b_4 = c_o/n_4$, where $n_4 = b_4^{-1}$ (the *local index of refraction*) has a rather complex functional dependence on local quantities.

When the global behaviour throughout a given physical medium is requested, the characteristic quantities can be averaged into constants, $b^\circ_\mu = \text{Aver.}(b_\mu)$, or $n^\circ_\mu = \text{Aver.}(n_\mu)$, $\mu = 1, 2, 3, 4$. This is evidently the case for the *average speed of light* throughout our atmosphere $c = c_o/n^\circ_4$, in which case $n^\circ_4$ is the *average index of refraction*. Note that $b_\mu \equiv b^\circ_\mu \equiv 1$ in vacuum.

A first intuitive understanding of the isominkowski spaces can be reached by noting that the characteristic functions $b_\mu = 1/n_\mu$ essentially extend the local index of refraction $1/n_4$ to all space-time components. Equivalently, by recalling that physical media are generally opaque to light, *the isotopies* $M(x,\eta,R) \to \hat{M}(x,\hat{\eta},\hat{R})$ *essentially extend to all physical media the geometric structure of light in vacuum.* In this sense, the characteristic constant $b^\circ_4$ geometrizes the density of a given medium, while the constants $b^\circ_k$ geometrize the internal nonlinear-nonlocal effects.

It is evident that different physical media necessarily require different isounits $\hat{1}$. This occurrence is similar to the need of infinitely possible Riemannian spaces in general relativity in order to represent the infinitely possible astrophysical masses. The point here is that each mass admits infinitely possible isounits, trivially, because each mass can be realized in infinitely possible different densities, sizes, chemical compositions, etc.

**3.3: Isominkowskian geometry.** It is the geometry of isospaces $\hat{M}(x,\hat{\eta},\hat{R})$ and possesses novel characteristics as compared to the conventional geometry. Their understanding requires the knowledge of the inapplicability mentioned in Sect. 2 of the notion of angles, trigonometry and functional analysis at large in favor of covering isotopic notions.

To study the main characteristics, let us consider first the *isoeuclidean geometry* which is evidently the space-component of the isominkowskian geometry. Consider the *isoeuclidean subspace* $E(x,\hat{\delta},\hat{R})$ in the 1-2 plane with diagonal isometric and separation

$$\hat{E}(x,\hat{\delta},\hat{R}): \quad x^2 = x^1 b_1^2(x, \dot{x}, \ddot{x}, ...)\, x^1 + x^2 b_2^2(x, \dot{x}, \ddot{x}, ...)\, x^2 = \text{inv}. \tag{3.4}$$

As one can see, this space is curved in the most general possible form, that is, with curvature dependent on local coordinates x, velocities $\dot{x}$, accelerations $\ddot{x}$, etc. (see next section). The loss of the conventional angles then follows from the evident loss of intersecting straight lines.

At this point, the isotopies play a central constructive role. Recall that the original space is flat. Its image under isotopy is then *isoflat*. Similarly, the images of straight lines are *isostraight* i.e., verify the axioms of straight lines in isospace. This implies the possibility of reconstructing angles under isotopies which is not possible for Riemann. The use of the isotopies of the group of rotation [21] permits the identification of the unique isotopic image $\hat{\alpha}$ of a conventional angle $\alpha$ in $\hat{E}(x,\hat{\delta},\hat{R})$ given by $\hat{\alpha} = \alpha b_1 b_2$. This permits the construction of the isotopies of conventional trigonometry, here called *isotrigonometry*, which is based on the following isofunctions and related properties

$$\text{isosin}\, \hat{\alpha} = b_2^{-1} \sin(\alpha\, b_1\, b_2), \quad \text{isocos}\, \hat{\alpha} = b_1^{-1} \cos(\alpha\, b_1\, b_2), \tag{3.5a}$$

$$b_1^2\, \text{isocos}^2 \hat{\alpha} + b_2^2\, \text{isosin}^2 \hat{\alpha} = \cos^2 \hat{\alpha} + \sin^2 \hat{\alpha} = 1. \tag{3.5b}$$

Note the deformation of the argument $\alpha \rightarrow \hat{\alpha}$ as well as of the magnitude of trigonometric functions $1 \rightarrow b_k^{-1}$ which are intriguing for certain (e.g., nuclear) deformations of potential wells and wavefunctions [14]. The rest of the isotrigonometry can then be constructed accordingly. The extension to the three-dimensional isoeuclidean case is consequential and it is omitted here for brevity [14].

We consider now the hyperbolic isoplane 3-4 with isoinvariant

$$\hat{M}(x,\hat{\eta},\hat{R}): \quad x^2 = x^3 b_3^2(x, \dot{x}, \ddot{x}, ..)\, x^3 - x^4 b_4^2(x, \dot{x}, \ddot{x}, ...)\, x^4 = \text{inv}. \tag{3.6}$$

The isotopic image $\hat{v}$ of a hyperbolic angle (speed) v is then given by $\hat{v} = v b_3 b_4$, as provable via the use of the isorepresentations of $\hat{O}(1.1)$ [13,14], with corresponding *isohyperbolic functions* and related properties

$$\text{isosinh}\, \hat{v} = b_4^{-1} \sinh(v\, b_3\, b_4), \quad \text{isocosh}\, \hat{v} = b_3^{-1} \cosh(v\, b_3\, b_4), \tag{3.7a}$$

$$b_3^2\, \text{isocosh}^2 \hat{v} - b_4^2\, \text{isosinh}^2 \hat{v} = \cosh^2 \hat{v} - \sinh^2 \hat{v} = 1. \tag{3.7b}$$

We are now equipped to indicate a most important feature of the isominkowskian geometry, the reconstruction at the isotopic level of exact straight lines, perfect circles and conventional light cones. The loss of the notion of straight line and its reconstruction under isotopy has been indicated earlier. The preservation of perfect circles can be seen as follows. Recall that, by conception, isotopies of Class I map the circle into the infinite families of ellipses (3.4) with semiaxes $b_k^2$. But the unit is jointly lifted from I = diag. (1, 1) to $\hat{1}$ = diag. $(b_1^{-2}, b_2^{-2})$. We then have the deformation of each semiaxis $1 \rightarrow b_k^2$ with the joint deformation of the unit $1 \rightarrow b_k^{-2}$. The original circle therefore remains a perfect circle in isospace, while the ellipses emerge only when the figure are projected in our space (see ref.s [13,14] for details).

We now outline the preservation of the light cone under isotopy. Let us first recall that, in the physical reality, the speed of light is not a "universal constant", but a locally varying quantity with a rather complex functional dependence on density, index of refraction, etc. As a result, the "light cone" in interior problems is not a "cone", but a rather complex hypersurfaces. The understanding of the isominkowskian geometry requires the knowledge that the "deformed cone" of the physical reality is mapped into a perfect cone in isospace, called *light isocone*, and the locally variable speed is mapped precisely into the original, constant speed of light in vacuum $c_0$. Consider the isolight cone $x^2 = 0$ in the 3-4 plane, Eq. (3.7). Then, the isotrigonometry yields $\Delta x = D\, b_4 \sin\hat{\alpha}$, $\Delta t = D\, b_3 \sin\hat{\alpha}$, and

$$\Delta x / \Delta t = D\, b_4 \sin\hat{\alpha} / D\, b_3 \sin\hat{\alpha} = (b_4 / b_3)\, c_0, \quad \hat{\alpha} = \alpha\, b_3\, b_4, \tag{3.8}$$

from which we recover the conventional expression in empty space tang $\hat{\alpha} = c_0$ = const. This occurrence is an expression of the overall unity of physical and mathematical thought achieved by isotopic technique because they allow the use of the *same* light cone for motion in vacuum with constant speed $c_o$ and motion in interior conditions with variable speed $c = c_o b_4$.

**3.4: Isolorentz and isopoincaré symmetries.** Necessary complements of the isominkowskian geometry are given by the isotopies $\hat{O}(3.1)$ and $\hat{P}(3.1)$ of the Lorentz O(3.1) and Poincaré P(3.1) symmetries, respectively. They were constructed for the first time in ref. [9] via the Lie-isotopic theory and then studied in details in monographs [13,14] to which we must refer for brevity. We can only recall the *isolorentz transformations* here presented for $\hat{\eta} = \mathrm{diag.}\,(g_{11}, g_{22}, g_{33}, -g_{444})$ with conventional functions for simplicity (rather than isofunctions)

$$x'^1 = x^1, \quad x'^2 = x^2, \tag{3.9a}$$

$$x'^3 = x^3 \cosh|v(\hat{g}_{33}\hat{g}_{44})^{\frac{1}{2}}| - x^4 g_{44}(g_{33}g_{44})^{-\frac{1}{2}} \sinh|v(g_{33}g_{44})^{\frac{1}{2}}| = \hat{\gamma}(x^3 - \hat{\beta} x^4), \tag{3.9b}$$

$$x'^4 = -x^3 g_{33}(g_{33}g_{44})^{-\frac{1}{2}} \sinh|v(g_{33}g_{44})^{\frac{1}{2}}| + x^4 \cosh|v(g_{33}g_{44})^{\frac{1}{2}}| = \hat{\gamma}(x^4 - \hat{\beta} x^3), \tag{3.9c}$$

$$\hat{\beta}^2 = v^k g_{kk} v^k / c_o g_{44} c_o, \qquad \hat{\gamma} = |1 - \hat{\beta}^2|^{-\frac{1}{2}}, \tag{3.9d}$$

which are easily constructed via rule (2.7) with w = v, X given by the conventional Lorentz generators in adjoint representation, and $T = \mathrm{diag.}\,(g_{11}, g_{22}, g_{33}, g_{444}) > 0$. Note the unity and mutual consistency of the algebraic and geometric isotopies. In fact, the latter predict the hyperbolic angle $\hat{v} = v\,(g_{33}g_{44})^{1/2}$ which turns out to be exactly that provided by the Lie-isotopic theory. The addition of the isorotations and isotranslations is done via similar rules and with similar algebraic-geometric consistencies (see [13,14,21] for brevity).

Note that the absolute value is necessary in the definition of $\hat{\gamma}$, Eq.s (3.9d) because $\hat{v}^2 = v_k b_k^2 v_k \gtreqless c_o^2$. This is the first contact we have in this paper with the joint representation of redshift *and* blue shift (see below).

As expected, isotransformations (3.9) have the most general possible nonlinear-nonlocal-noncanonical structure (in which case they are called *general isolorentz transforms*) because of the arbitrariness in the functional dependence of the $g_{\mu\mu}$ terms, as needed for the form-invariance of isoseparation (3.3). Yet the isolorentz symmetry is locally isomorphic to the original symmetry, as expressed by their formal similarities with conventional Lorentz transformations, and confirmed by the isotopic commutation rules [13,14].

Note finally that general isotransforms (3.9) are nonlinear and therefore *noninertial*, as expected for interior conditions. Nevertheless, when passing to the outside and studying the global behaviour via the average of the b's to constants $b°_\mu$, they reacquire the conventional linear and therefore inertial character (in which case they are called *restricted isolorentz transforms*).

**3.5: Isominkowskian classification of physical media.** Recall that there is an infinite variety of interior physical conditions for each given astrophysical mass. This variety is classified by the isominkowskian geometry into nine different types which play a fundamental role in practical applications (Sect. 5). Consider for simplicity the global interior cases with space isotropy $b°_1 = b°_2 = b°_3$. We then have the *isominkowskian classification* into: Type I for $b°_3 = b°_4$ ($\hat{\beta} = \beta$, $\hat{\gamma} = \gamma$), II for $b°_3 > b°_4$ ($\hat{\beta} > \beta$, $\hat{\gamma} < \gamma$) and III for $b°_3 < b°_4$ ($\hat{\beta} < \beta$, $\hat{\gamma} > \gamma$). Each of these types is then divided into three subcases depending on whether $b_4 = 1, < 1, > 1$.

The following identifications are known at this writing: Type I.1 ($b_3 = b_4 = 1$) is therefore empty space. Type I.2 ($b_3 = b_4 < 1$) represents the homogeneous and isotropic water with index of refraction $n° = b°_4^{-1}$ and speed of light $c = c_o/n° < c_o$. Type II.2 ($b_3 > b_4 < 1$) represents our inhomogeneous and anisotropic atmospheres with low density. Type II.3 ($b_3 < b_4 > 1$) represents the media of the highest possible density, such as those in the interior of a star (or, equivalently, in the interior of a hadron). Additional identifications are under study, e.g., for conductors (Type II.1), superconductors (Type I.3), intermediately heavy astrophysical atmospheres (Types III.1 and 2), etc. [13,14].

**3.6: Isospecial relativity.** The abstract identity between spaces and isospaces $M(x,\eta,R) \approx M(x,\hat{\eta},\hat{R})$ and between symmetries and isosymmetries $O(3.1) \approx \hat{O}(3.1)$, implies the isotopies of all basic postulated of the special relativity, called *isopostulates*, originally proposed in [9] and studied in detail at the classical level in [13] and at the operator level. in [14].

A new relativity for interior conditions therefore emerges from the isominkowskian geometry, the isopoincaré symmetry, and the isopostulates, called *isospecial relativity* [9,13,14]. It is a covering of the special relativity in the sense that: A) it describes structurally more general systems (nonlinear-nonlocal-noncanonical systems of the interior problem), B) via structurally more general methods (isotopic methods); and C) admits the conventional special relativity as a particular case whenever motion returns to be in vacuum for which $b_\mu = 1$. Moreover, the special and isospecial relativities coincides, by construction, at the abstract level. Readers not familiar with isotopic techniques should therefore be warned that possible criticisms on the isospecial relativity for interior conditions essentially are criticisms on the conventional relativity in vacuum.

A significant property of the isospecial relativity in its most general possible formulation of Kadeisvili Class V is its *direct universality* in the sense of applying for all possible deformations $\hat{\eta} = T\eta$ of the Minkowski metric (universality), directly in the frame of the observer (direct universality). This property has significant experimental relevance. As we shall see in Sect. 5, numerous noneinsteinian time evolutions exist in the literature which, being different, create evident problems in their experimental test. Such problems are eliminated by the geometric unification of all seemingly different laws into a unique isotopic law.

Another general property of the isospecial relativity of Class I is its abstract identity with the general relativity for the isotopic element dependent on the local coordinates only, $T = T(x)$, $\eta$ $T\eta = \eta(x)$. This property can be better seen from the fact that *the isopoincaré symmetry for the isotopic element* $T(x)$ *characterizes the symmetry of all possible Riemannian metrics* $\eta(x)$ As an illustration, the nonlinear symmetry of the Schwartzchild line element is given by merely plotting its $g_{\mu\mu}$ elements in isosymmetry (3.9). The same holds fro all possible Riemannian line elements. The geometric unification of the special and general relativities then follows. The point important for this paper is that such unification is a mere basis for broader interior treatments because isotopic methods naturally hold for arbitrary dependence $T(x, \dot{x}, \ddot{x}, \omega, \mu, \tau, n, \dots)$.

**3.7: Isodoppler red/blue/shifts.** The prediction of the isospecial relativity most important for this paper is that light propagating within inhomogeneous and anisotropic media experiences an alteration of its conventional Doppler's effect according to the *isodoppler law* (for $\alpha = 0$)

$$\hat{\omega} = \hat{\gamma}\,\omega_0 = \frac{\omega_0}{|1 - v_k\, b_k^2\, v_k / c_0\, b_4^2\, c_0|^{\frac{1}{2}}}. \tag{3.10}$$

As one can see, the isospecial relativity has the following predictions: **Types I.1, I.2, I.3** (empty space or water) have *no deviation from the Doppler shift*, as verified in the physical reality in which light does not lose energy to the medium; **Types II.1, II.2, II.3** (such as our atmosphere) have an *isoredshift*, that is, a shift toward the red *in addition* to the Doppler shift due to the *loss* of energy to the medium; and **Types III.1, III.2, III.3** (such as hyperdense quasars atmospheres or the interior of hadronic matter) have an *isoblueshift* due to the *acquisition* of energy from the medium.

In order to reach a form of the isodoppler law applicable to astrophysics, we assume for simplicity the space-isotropy $b_1 = b_2 = b_3 = b$, we recall the dependence of the index of refraction $b_4$ from the frequency and assume the factorizability of such a dependence in the $\hat{\beta}$ term. We can therefore write $\hat{\beta}^2 = \beta(b^2/b_4^2) = \beta[b^{\circ 2}/b^{\circ}{}_4^2]\, f(\omega_0)$, where $b^\circ$ and $b^\circ{}_4$ are constants and $f(\omega_0) \gtrless 1$ is the factorized frequency dependence. Law (3.10) for the global behaviour of light through quasar chromosphere can be written in one of the forms

$$\hat{\omega} = \hat{\gamma}\,\omega_0 = \frac{\omega_0}{|1 - \beta\,[b^{\circ 2}/b^{\circ}{}_4^2]\, f(\omega_0)|^{\frac{1}{2}}}. \tag{3.11a}$$

$$\hat{\omega}^2 - \omega_0^2 \equiv \beta\,[b^{\circ 2}/b^{\circ}{}_4^2]\, f(\omega_0)\,\hat{\omega}^2, \qquad \hat{\omega} - \omega_0 \approx -\tfrac{1}{2}\,\beta\,[b^{\circ 2}/b_4^2]\,\omega_0\, f(\omega_0). \tag{3.11b}$$

The *astronomical redshift of quasars* is then due to the property for a basic frequency, usually 4680 A° [2],

$$B^\circ = [b^{\circ 2}/b^{\circ}{}_4^2]\, f(\omega_0)|_{\omega_0 = 4680\ A^\circ} = \text{const.} > 1, \tag{3.12}$$

The *internal red/blue/shift of quasars* is then due to the full use of law (3.11a) which shows that frequencies smaller or bigger than the basic frequency 4680 A° have proportionately different shifts which are expected to have an approximate Gaussian behaviour owing to the condition $f(\omega_0) \gtrless 1$.

For the sun's chromosphere we recall the experimental information (Sect. 5) that the velocity dependence is restricted to the space components $b_k$. In this latter case, the global averaging must be done on the expression βB resulting in the form $K^\circ f(\omega_o) = \langle v^2 b(v)^2 / c_o b_4^2 \rangle f(\omega_o)$, $f(\omega_o) \geq 1$, with isodoppler law

$$\hat{\omega} = \hat{\gamma}\,\omega_o = \frac{\omega_o}{|1 - K^\circ f(\omega_o)|^{\frac{1}{2}}}, \quad \hat{\omega}^2 - \omega_o^2 \equiv K^\circ \hat{\omega}^2 f(\omega_o), \quad \hat{\omega} - \omega_o \approx -\tfrac{1}{2} K^\circ \omega_o f(\omega_o). \tag{3.13}$$

The comparison of the above laws with astrophysical data is done in Sect. 5.

**3.8: Other predictions.** The isospecial relativity has a number of other *novel* predictions for interior conditions (i.e., predictions not possible for the special relativity) which can be experimentally tested with contemporary technology, such as the *isodilation law* [9]

$$\tau = \hat{\gamma}\,\tau_o = \tau_o / |1 - v_k b_k^2 v_k / c_o b_4^2 c_o|^{\frac{1}{2}}, \tag{3.14}$$

which is confirmed by available experimental data on the behaviour of the meanlife of unstable hadrons with speed (Sect. 5), or the *isoequivalence law*

$$\hat{E} = m\,c^2 = m\,c_o^2\,b_4^2. \tag{3.15}$$

verified by preliminary experiments on the chemical synthesis of hadrons [36] and other data [36,14].

**3.9: Isodual relativities.** By recalling the antiautomorphic maps $1 \to 1^d = -1$, and $\hat{1} \to \hat{1}^d = -\hat{1}$ and their characterization of antiparticles (Sect. 2.7), isotopic methods identify *four* different relativities: the **conventional special relativity** on $M(x,\eta,R)$ with invariant P(3.1) for the description of particles in vacuum; the **isodual special relativity** on the isodual Minkowski space $M^d(x,\eta,R^d)$ with isodual Poincaré symmetry $P^d(3.1)$ for the description of antiparticles in vacuum; the **isospecial relativity** on isominkowski spaces $\hat{M}(x,\hat{\eta},\hat{R})$ with isopoincaré symmetry $\hat{P}(3.1)$ for the description of particles within physical media; and the **isodual isospecial relativity** on the dual isominkowski spaces $\hat{M}^d(x,\hat{\eta}^d,\hat{R}^d)$ with *isodual isopoincaré symmetry* $\hat{P}^d(3.1)$ for the description of antiparticles in interior conditions.

The working hypothesis in which the total matter is equal to the total antimatter then leads to a structurally novel view of the universe in which *the total energy, the total time and other total characteristics of the universe (as the sum of those for matter and antimatter) are identically null,* a view confirmed by the isotopies of Riemann [23].

**3.10. Connections with the studies by Arp, Sulentic, Marmet, and others.** We indicated in Sect. 1 that Marmet theory [3] can ultimately result to be an operator version of the isodoppler formulation. A similar interconnection exists with Sulentic studies [2], and with other approaches.

A most intriguing interconnection appears to exist between the isodoppler representation and Arp's theory [1] achieving a non-Doppler redshift via the creation of matter. This latter view is faced with known problematic aspects and understandable resiliency in the physics community when considered within the context of conventional relativities *alone*. This scenario is altered by the isodual relativities. In fact, conventional relativities represent both matter and antimatter in the same space-time, with ensuing difficulties for the creation of matter from nothing. In our covering isorelativities antimatter is represented in a *separate isodual universe*, which is known not to be isolated from our universe because of the finite transition probabilities between positive- and negative-energy solutions of conventional relativistic equations. Rather than the creation of something from nothing, Arp's theory on the creation of matter acquires a different light in a cosmology with null total energy, time and other quantities [35] because it may result to be an interchange of energies between the two universes. We regret the inability to study these interconnections in more details at this time.

**3.11: Applications.** Numerous applications of the isospecial relativity are now available at the classical, operator, statistical and levels [13,14]. In Sect. 5 we shall outline only those experimental applications which are directly or indirectly related to the quasars red/blue/shifts. It may be important for an overall view to outline below other applications.

The simplest possible application is a *static* one, the representation of a straight rod when penetrating in water [14]. As well known, the rod *appears* to bend when entering in water, but in the reality it remains straight. Thus, the angle α of rod bending in water measured from the outside does not coincide with the physical angle $\hat{\alpha}$ in the interior of water. This occurrence is directly represented by the simplest possible case of isoeuclidean geometry with line element (3.5) in which $b_1 = b_2 = b°$ owing to the homogeneity and isotropy of water. The value b° is then determined by the relation $\hat{\alpha} = \alpha b°^2$. In short, the isoeuclidean geometry corrects the error in our perception that the rod is bent by keeping it straight.

The simplest possible *dynamical* application is the classical relativistic particle in a resistive medium without potential interactions [13]. Consider a free, classical, extended, relativistic particle with Lagrangian L = mc and Minkowskian geodesic $d^2x^\mu/ds^2 = 0$. The penetration of the particle within a resistive medium is described by the *same* Lagrangian although now written in isominkowski space $L = mc = mc_o b_4$. The infinitely possible resistive forces due to shape, density, temperature, speed, etc. cannot be represented by central conception with the Lagrangian because they are nonpotential. They are then represented by the infinitely possible isotopies of the unit I → $\hat{I}$. The understanding of the isospecial relativity requires the additional knowledge that *the motion of the extended particle in interior conditions remains fully geodesic*, i.e., in isospace we still have $d^2x^\mu/ds^2 = 0$. In summary, the two structurally different trajectories (one free and the other with contact interactions, one linear-local-potential and the other nonlinear nonlocal nonpotential) are completely unified, and solely differentiated by the selection of the unit. The point is that all geometric, algebraic and analytic axioms are the same.

A deeper inspection soon reveals possibilities of physical applications for the isospecial relativity which are simply beyond any descriptive capacity of Einsteinian theories [13,14]. In fact, the isounit of the preceding example admits the factorization $\hat{1} = \hat{1}_o \mathrm{diag.}\,(b°_1{}^{-2}, b°_2{}^{-2}, b°_3{}^{-2}, b°_4{}^{-2})$. Thus, *the Lagrangian* L = mc *in isospace can directly represent the actual "nonspherical" shape of the test body considered,* such as a spheroidal ellipsoid with semiaxes $b°_1{}^2, b°_2{}^2, b°_3{}^2$ (or arbitrary shapes with a nondiagonal isounit). The term $b°_4$ geometrizes the density of the test body and the factor $\hat{1}_o$ represents the drag force. Such a representation is manifestly impossible with the conventional relativity even after quantization. But these are the beginning of the capabilities of the isospecial relativity. A still deeper inspection shows that *the same Lagrangian* L = mc *in isospace can represent all infinitely possible "deformations" of its original "nonspherical" shape,* e.,g., via a dependence of the $b°_k$-quantities on pressure, speed, etc., which is manifestly impossible for conventional relativities even after first, second or third quantization.

These and other features we cannot report here for brevity (see ref.s [13,14]) have permitted the isospecial relativity to resolve some of vexing problems in contemporary physics, such as the first achievement of an exact numerical representation of the total magnetic moments of few-body nuclei [14] which have still remained unexplained in their entirety despite studies over three quarter of a century. The isotopic treatment is simply given by representing protons and neutrons as extended and, therefore, deformable. This implies the deformability of their charge distributions depending on the physical conditions at hand and, thus, of their intrinsic magnetic moments. The anomalies in total magnetic moments then merely represent the (generally small) deformations of the constituents in a nuclear structure. The point is that these deformations are simply beyond any possibility of the special relativity.

We should also mention the resolution of another vexing problem of contemporary physics permitted by the isospecial relativity, that of quark confinement [14]. Current trends assume the same Minkowski and Hilbert spaces for the interior and exterior problems of hadrons. A finite probability of quarks tunneling free is then inescapable from the uncertainty principle irrespective of the infinite character of the potential barrier, which is contrary to experimental evidence. Now, the isotopic $\hat{SO}(3)$ symmetry is isomorphic to the conventional SU(3), and the quantum numbers of the two theories are identical, thus rendering the isotopic theory fully compatible with existing experimental data. Moreover, the use of the conventional relativity for the exterior and the isospecial one in the interior easily permits the two Hilbert spaces to be *incoherent*, in which case the transition probability for free quarks is rigorously proved to be identically null even for collisions with infinite energy and no potential barrier at all (as hinted by asymptotic freedom).

It is important also to understand that the isospecial relativity is applicable in fields *beyond* physics, e.g., in theoretical biology. An unexpected and suggestive application along the latter lines is in conchology [14]. Consider the growth of sea shells with minimal complexity, e.g., with one bifurcation [22]. Such a growth can indeed be inspected with our Euclidean perception of physical reality. Nevertheless, computer simulations show that sea shells should crack during their growth if strictly represented in our Euclidean or Minkowskian spaces [22]. On the contrary, their growth is normal if represented in isoeuclidean or isominkowskian space, that is, with a conventional Lagrangian over a generalized unit. The representation of the bifurcations themselves is controversial in Euclidean or Minkowskian spaces because requiring *discontinuous transformations into negative times* [22], while the same can be continuously represented via our isorelativities of Kadeisvili Class III. Note that the dimension of the the space is not altered. The generalization is in the structure of the geometry, as advocated in this paper.

The latter example clearly identified the limitation of our perception of Nature, and suggests caution before claiming final knowledge based on our manifestly limited three Eustachian tubes, not only in biophysics, but also in physics and astrophysics.

## 4: ISORIEMANNIAN GEOMETRY

**4.1: Isoriemannian geometry and its isodual.** The cosmological implications of this paper are studied in the separate paper [38]. We here merely mention that the isotopies and isodualities apply also to the Riemannian geometry resulting in covering structures admitting in the tangent space the isominkowskian geometry and its isodual.

**4.2: Gravitational isodoppler shifts.** The aspect important for this paper is that the isodoppler shift is also additive to the gravitational redshift as in the relativistic case. Our study of the quasar red/blue/shifts can therefore be restricted to the isodoppler law (3.11) because the gravitational treatment would yield conventional gravitational corrections (when appropriate).

**4.3: Isogeneral relativity and its isodual.** The above studies imply a step-by-step generalization of Einstein exterior gravitation for test particles in vacuum into a dual form, one called *isogeneral relativity* or *isogravitation* for short, for interior gravitational problems of matter, and the other called *isodual isogravitation* for the interior gravitational problem of antimatter. The interested reader may consult ref.s [13,35]. The aspect important for this paper is that conventional gravitational theories possess no universal symmetry, as well known. On the contrary, isogravitation is based on the same symmetry at the foundation of the isodoppler law, the isopoincaré symmetry. Experimental confirmations of the isodoppler law within physical media would therefore have direct gravitational and cosmological implications.

## 5: REPRESENTATION OF QUASARS COSMOLOGICAL AND INTERNAL SHIFTS

**5.1: Representation of Arp's data** [1]. Isodoppler law (4.10) was originally submitted by this author in memoir [10] of 1988 to avoid the violation of Einstein's relativities under Einsteinian exterior conditions in vacuum, e.g., to avoid speeds of matter in vacuum higher then the speed of light. The main hypothesis of Sect. 1.1 can now be more technically expressed via the characterization of quasars chromospheres with isominkowskian media of Type II.2 with $b_1 = b_2 = b_3 > b_4$, $b_4 < 1$, $\hat{\beta} > \beta$, $\hat{\gamma} < \gamma$ and average speed of light $c = c_o b_4 = c_o/n^\circ < c_o$, with consequential natural redshift $\hat{\omega}' = \hat{\gamma}\omega < \omega' = \gamma\omega$. The elaboration of Arp's data was then suggested in [10].

Numerical calculations along this proposal were done by Mignani in ref. [11] of 1992 by confirming that isodoppler's law (4.11) can indeed reduce the speed of the quasars all the way to that of the associated galaxies. This was submitted as a limiting case in which the difference between the quasars redshift and that of the associated galaxy is entirely of isotopic nature. It is understood that quasars can indeed be expelled from their associated galaxies but at Einsteinian speeds $v << c_o$. This latter case implies a small correction of the b-quantities and can therefore be ignored.

The isotopic elaboration of Arp's data was conducted in ref. [11] via the relation

$$B^\circ = \frac{b^\circ_3}{b^\circ_4} = \frac{(\Delta\omega' + 1)^2 - 1}{(\Delta\omega' + 1)^2 + 1} \times \frac{(\Delta\hat{\omega}' + 1)^2 - 1}{(\Delta\hat{\omega}' + 1)^2 + 1}. \tag{5.1}$$

where $\Delta\omega'$ represents the measured Einsteinian redshift for galaxies, and $\Delta\hat{\omega}'$ represents the isotopic redshift for quasars according to law (4.11a), with resulting numerical values [1]

| GAL. | $\Delta\omega'$ | QUASAR | B | $\Delta\hat{\omega}'$ |
|---|---|---|---|---|
| NGC | 0.018 | UB1 | 31.91 | 0.91 |
| | | BSO1 | 20.25 | 1.46 |
| NGC 470 | 0.009 | 68 | 87.98 | 1.88 |
| | | 68D | 67.21 | 1.53 |
| NGC 1073 | 0.004 | BSO1 | 198.94 | 1.94 |
| | | BSO2 | 109.98 | 0.60 |
| | | RSO | 176.73 | 1.40 |
| NGC 3842 | 0.020 | QSO1 | 14.51 | 0.34 |
| | | QSO2 | 29.75 | 0.95 |
| | | QSO3 | 41.85 | 2.20 |
| NGC 4319 | 0.0056 | MARK205 | 12.14 | 0.07 |
| NGC 3067 | 0.0049 | 3C232 | 82.17 | 0.53 |

(5.2)

The above results provide a clear confirmation of the isospecial relativity and underlying isominkowskian geometrization. In fact, the data show that all B values are positive and bigger than one, exactly as predicted by the geometrization of Type II.2.

The identification of the individual values $b^\circ_3 - \langle b_k \rangle$ and $b^\circ_4$ requires at least one additional experimental value, such as the average speed of light in the quasar chromospheres which would evidently fix $b^\circ_4$. Then $b^\circ_3$ could be computed from the B-ratios. As an indication, the assumption for quasar UB1 of the *average* speed of light in its chromosphere $c = 0.80\ c_o$ would yield the value $b_3 \approx 40$.

The problem of the apparent speed of the galaxies is not considered in the above analysis because it is a separate issue. The reader should be aware that isogeometries imply three independent corrections to the current estimates of the distance of galaxies from us: 1) A correction due to a possible isoredshift of light in the interior of the galaxies; 2) Another correction due to the fact that space can be considered as empty only at the local (say, planetary) level because at intergalactic distances space itself becomes an ordinary medium (since it is filled up with dust, electromagnetic waves, particles, etc.), thus requiring a second, relatively smaller isotopic correction in the redshift; and 3) The very notion of distance is altered by the isogeometries [37,38]. Intriguingly, *each* of the above corrections implies a *decrease* of the current estimates on the distance of galaxies from us.

Under *limiting* conditions, these corrections are indeed capable of interpreting the cosmological redshift itself as being of entirely isotopic origin, thus yielding a new cosmological conception of the Universe as being unlimited, composed of essentially stationary galaxies of matter and antimatter and with a number of novel features, such as without any need for "the 'missing mass" (from the isoequivalence law (3.15), see ref.s [37,38]).

It should be stressed that current data are insufficient to rule out the "big bang" theory, in which framework the isotopies merely yield corrections to the current estimates on the explosion of the Universe.

**5.2: Representation of Sulentic data** [2]. The *cosmological* redshift represented in ref. [11] is essentially that of isodoppler law (3.11a) under values (3.12) for a basic frequency such as 4680A°. The representation of Sulentic [2] *internal* red/blue/shift requires the full use of law (3.11a) with the explicit frequency dependence. The assumption of a Gaussian realization of $f(\omega)$ then leads to the isotopic behaviour

$$\hat{\omega}^2 - \omega^2 = k_1\, \omega^2 e^{-k_2 (\omega - \omega_o)^2}, \qquad (5.2)$$

where $k_1$ and $k_2$ are positive constants. Numerous fits of the experimental data are then possible. As an indication, the values $k_1 = 10$ and $k_2 = 1$ yield a preliminary, yet meaningful representation of Sulentic data of Table 4, p. 61, ref. [2]. Note the shift of the center of the Gaussian as indicated by current data. Needless to say, a more accurate representation can be derived when additional measures are available such to permit the identification of the function $f(\omega)$.

**5.3: Representation of Marmet's data** [3]. The data on the redshift of spectral lines from the sun's chromosphere as studied by Marmet [3] and others are some of the most direct experimental confirmations of the isotopic character of the quasars redshift.

The latter data can be interpreted via essentially the same isodoppler law, only referred to form (3.13) because of the need of the different average since the sun is moving at low speed with respect to our laboratory. In fact, in first approximation, law (3.13) reproduces Marmet's expression (6), p. 240, ref. [3] identically

$$\omega / \Delta\omega = \Delta\lambda / \lambda \approx -2 / K^\circ f(\omega)\big|_{\omega=\text{const.}} \approx -2.73 \times 10^{-21}\, T^2 N_c\,, \tag{5.3}$$

where T is the temperature of the sun's chromosphere, and $N_c$ is the average number of collisions of photons in a given column density. Note the emergence of a dependence on the frequency which is expected to be experimentally verifiable and which, if confirmed, would establish the possibility of resolving the problem of quasar red/blue/shifts via spectroscopic measures on the Sun.

**5.4: Representation of timelife behaviour.** The isospecial relativity has additional experimental verifications indirectly related to the quasar red/blue/shifts which, as such, are significant for this paper. The first one is the isotopic behaviour (3.14) of the meanlife of unstable hadrons with speed which, if confirmed, would provide a clear verification of the *structure* of the isodoppler law (3.10).

Blochintsev and his school [25] pioneered the hypothesis that the nonlocal internal effects expected in the hadronic structure from mutual penetrations of the wavepackets of the constituents can manifest themselves via departures from the Minkowskian behaviour of the meanlife of unstable particle with speed, and computed a generalized law. The problem was subsequently studied by several authors [26], resulting in additional different laws.

This author submitted in [9] the isominkowskian geometrization of the physical medium in the interior of hadrons with isotopic law (3.14) which was proved by Aringazin [27] to be "directly universal", i.e., including all possible generalizations [25,26] via different expansions in terms of different parameters and with different truncations.

The first phenomenological verification was provided in calculations [28] on deviations from the Minkowskian geometry inside pions and kaons conducted via standard gauge models in the Higgs sector. These phenomenological studies resulted in the deformed Minkowski metric inside hadrons $\hat{\eta} = \text{diag.}\{(1 - \alpha/3), (1 - \alpha/3), (1 - \alpha/3), -(1 - \alpha)\}$, which is precisely of the isominkowskian type with numerical values

$$\text{PIONS } \pi^{\pm}: \quad b^{\circ}{}_1{}^2 = b^{\circ}{}_2{}^2 = b^{\circ}{}_3{}^2 \cong 1 + 1.2 \times 10^{-3}\,, \quad b^{\circ}{}_4{}^2 \cong 1 - 3.79 \times 10^{-3}\,, \tag{5.4a}$$

$$\text{KAONS } K^{\pm}: \quad b^{\circ}{}_1{}^2 = b^{\circ}{}_2{}^2 = b^{\circ}{}_3{}^2 \cong 1 - 2\times 10^{-4}\,, \quad b^{\circ}{}_4{}^2 \cong 1 + 6.1\times 10^{-4}\,, \tag{5.4b}$$

Note the *change in numerical value* of the isotopic element in the transition from pions to kaons, which is necessary because of the change of the density (recall that all hadrons have approximately the same size, but different rest energies, thus having different densities and different isounits).

The first direct experimental verification was reached by Aroonson et al. [29] who measured a clear nonminkowskian behaviour of the meanlife of the $K^\circ$ in the energy range 30–100 GeV. Subsequent direct experiments conducted by Grossman et al. [30] confirmed the Minkowskian behaviour of the meanlife of the same particle in the *different* energy range 100–350 GeV (see review [18]).

These seemingly discordant experimental measures were proved to be unified by the isominkowskian geometrization of the K°–particle by Cardone et al. [31] via phenomenological plots of both measures [29,30] in the range 30–350 GeV resulting in the following characteristic b°–values

$$b^{\circ}{}_1{}^2 = b^{\circ}{}_2{}^2 = b^{\circ}{}_3{}^2 \simeq 0.909080 \pm 0.0004, \qquad b^{\circ}{}_4{}^2 \simeq 1.002 \pm 0.007\,, \tag{5.5a}$$

$$\Delta\, b^{\circ}{}_k{}^2 \cong 0.007, \qquad \Delta\, b^{\circ}{}_4{}^2 \cong 0.001\,. \tag{5.5b}$$

which are of the same order of magnitude of values (5.3b). Measures (5.4b) also confirm the prediction of the isominkowskian geometry in the range 30–400 GeV that the $b^\circ_4$ quantity, being a geometrization of the density, is constant for the particle considered (although varying from hadron to hadron with the density), while the dependence in the velocities rests with the $b_k$–quantities.

the latter analysis is important inasmuch as it establishes the possible existence of an isodoppler shift even for a medium at rest in which $< v^2/c_o^2> = 0$, but $<v^2 b(v)^2/c_o^2 b_4^2> \neq 0$.

**5.4: Representation of Bose-Einstein correlation.** Another important verification has been recently achieved via theoretical [32] and experimental [33] studies on the Bose–Einstein's correlation. These studies provide a direct verification of the basic isominkowskian geometrization of physical media and, as such, are significant for the quasars red/blue/shifts.

Evidence establishes that no correlation exists for particles interactions when admitting effective point-like approximations. The Bose-Einstein correlation therefore appears to be due precisely to the *extended* character of the wavepacket of particles, which results in an evident *nonlocal* structure of the interactions at very small distances. The use of the isominkowskian geometrization for the interior of the $p-\bar{p}$ fireball results in the two-point Boson isocorrelation function on $\hat{M}(x,\hat{\eta},\hat{R})$, ref. [32], Eq. (10.3), p. 122,

$$\hat{C}_{(2)} = 1 + \frac{K^2}{3} \sum_{\mu} \hat{\eta}_{\mu\mu} \left( e^{-q_t^2 / b^{\circ 2}_{\mu}} \right), \qquad \hat{\eta} = \mathrm{Diag.}\,(b^{\circ}{}_1^2, b^{\circ}{}_2^2, b^{\circ}{}_3^2, - b^{\circ}{}_4^2), \tag{5.6}$$

where $q_t$ is the momentum transfer and the term $K = b^{\circ}{}_1^2 + b^{\circ}{}_2^2 + b^{\circ}{}_3^2$ is normalized to 3, under the sole approximation, also assumed in conventional treatments, that the longitudinal and fourth components of the momentum transfer are very small. Phenomenological studies conducted in [33] via the UA1 data at CERN confirm model (5.5) in its entirety, and identify the numerical values

$$b^{\circ}{}_1 = 0.267 \pm 0.054, \quad b^{\circ}{}_2 = 0.437 \pm 0.035, \quad b^{\circ}{}_3 = 1.661, \quad b^{\circ}{}_4 = 1.653 \pm 0.015 \tag{5.7}$$

These measures have the following important implications: A) They confirm the nonlocal-nonhamiltonian origin of the correlation, which is at the foundation of these studies; B) They confirm the isominkowskian geometrization for the $p-\bar{p}$ fireball; C) they provide a numerical value of $b^{\circ}{}_4$ for particles of the density of the $p-\bar{p}$-fireball for use in isoequivalence principle (3.12) (see below); D) They confirm the capability of the isotopies of directly representing the nonspherical shape of the fireball and all its deformations; and E) They prove the reconstruction of the exact Poincare' symmetry under nonlocal-nonhamiltonian interactions.

**5.5: Cooper pair in superconductivity.** This is a clear physical systems beyond any realistic capability of Einsteinian theories because it consists of two electrons of the *same* charge experiencing an *attractive* interaction. Animalu [34] has shown that the use of the isominkowskian geometry representing the mutual wave-overlapping of the two electron (with isounit given in Sect. 2.2) permits a quantitative interpretation of the attractive interactions in the Cooper pair which is in excellent agreements with numerous experiments (see [34] for brevity).

**5.6: Chemical synthesis of hadrons.** The isominkowskian geometry also permits a speculative, yet intriguing prediction, the cold fusion/chemical synthesis of protons and electrons into neutrons (plus neutrinos). It is essentially allowed by the rest energy of the electron when inside the hyperdense medium in the interior of the proton and computed via isoequivalence principle (3.15) with numerical value $b'_4 = 1.653$ from data (5.6). This permits a representation of all characteristics of the neutron [35]. This prediction has received a preliminary, yet direct experimental verification by don Borghi et al [36]. If confirmed, the event would permit the chemical synthesis of all unstable hadrons from lither (massive) hadrons. Moreover, it would permit the artificial disintegration of unstable hadrons, such as the artificial disintegration of peripheral neutrons in a nuclear structure, with realistic possibilities of a new technology, called *hadronic technology*, because based on mechanisms in the interior of individual hadrons. See Vol. III of ref.s [14] for other experimental verifications.

**5.7: Proposed experiments.** A number of experiments have been proposed in classical mechanics, astrophysics and particle physics to test the isominkowskian geometry and related isospecial relativity such as:

**Experiment 1** [13]: measure the redshift of light from a quasar just before and then after passing through a planetary atmosphere or the sun's chromosphere. The isominkowskian geometry predicts in this case an additional redshift. The average data (5.2) yield $\langle B^{\circ} \rangle = 72.78$, $\langle \Delta\hat{\omega}' \rangle = 1.15$, $\langle \Delta\omega' \rangle = 0.01$, thus characterizing the average isoshift $\langle \Delta\hat{\omega}' \rangle - \langle \Delta\omega' \rangle = 1.14$. The assumptions that the quasar atmospheres are $10^5$ denser than the atmosphere of Jupiter (or of Earth), and that the isotopic effect is proportional to the density in first approximation, lead to the estimate of the isoredshift in Jupiter's atmosphere of the order of $\langle \Delta\hat{\omega}'_{Jupiter} \rangle \approx 1.14 \times 10^{-5}$ which is fully measurable. For smaller ratios of the densities of the quasars and planetary atmospheres, the effect evidently becomes bigger.

**Experiment 2** [13]: Follow a sufficient number of Fraunhofer lines of sun light from the zenith to the horizon to see whether or not the tendency toward the red is in part an isoredshift. The numerical estimates of the preceding experiment also apply to Earth's atmosphere, yielding a measurable effect.

**Experiment 3** [14]: Finalize the behaviour of the meanlife of unstable particles with speed [29,30]. As indicated earlier, any deviation from Minkowskian time dilation is a confirmation of the corresponding isodoppler behaviour for frequencies owing to its direct universality [27].

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