Axiomatic inconsistencies of grand unifications and their possible isotopic resolution

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ABSTRACT. In this note we study the possibility that the historical problematic aspects in the achievement of a consistent grand unification are due to axiomatic incompatibilities between gravitation, as currently formulated on curved spaces, and electroweak interactions. Since the latter theories have proved to have a majestic axiomatic and physical consistency while the former theories have been afflicted by a large number of unresolved problematic aspects since their inception, in this note we identify the modifications of gravitational theories that are necessary to achieve an axiomatic compatibility with electroweak theories, thus permitting a consistent grand unification. The result of the study is a tribute to Henri Poincaré because of the emergence of the Poincaré symmetry as the universal symmetry of nature, including the reformulation of gravity in a form that is Poincaré invariant (rather than covariant).

1 INTRODUCTION

In this note we study the possibility that the historical problematic aspects in the achievement of a consistent grand unification are due to axiomatic incompatibilities between gravitation, as currently formulated on curved spaces [1], and electroweak interactions [2].

Since the latter theories have proved to have a majestic axiomatic and physical consistency while the former theories have been afflicted by a large number of unresolved problematic aspects since their inception (see, for instance, Ref. [3d] and papers quoted therein), in this note we identify the modifications of gravitational theories that are necessary to achieve axiomatic compatibility with electroweak theories with consequential straightforward axiomatically consistent grand unification.

The content of this note was first presented in the *Proceedings of the VIII Marcel Grossmann Meeting on General Relativity* held in Jerusalem in 1998, Ref. [6h], whose appearance is here indicated with appreciation.

The primary axiomatic incompatibilities between gravitation, as currently formulated on a curved space, and electroweak theories are the following:

(1) **Incompatibility due to curvature.** Electroweak theories are structured on *Minkowskian* axioms, while gravitational theories are formulated via *Riemannian* axioms, a disparity that is magnified at the operator level because of known technical difficulties of quantum gravity [3], e.g., to provide a PCT theorem comparable to that of electroweak interactions.

(2) **Incompatibility due to antimatter.** Electroweak theories are *bona fide* relativistic theories, thus characterizing antimatter via *negative-energy solutions,* while gravitation characterizes antimatter via *positive-definite energy-momentum tensors.* Fundamental inconsistencies then occur, such as the impossibility to explain why, within the context of gravitation on a curved space, how one photon can produce an electron-positron pair.

(3) **Incompatibility due to fundamental space-time symmetries.** Electroweak interactions are based on the axioms of special relativity, thus verifying the fundamental *Poincaré symmetry* $P(3.1)$, while such a basic symmetry is absent in contemporary gravitation. It is then evident that the inclusion of gravitation on a curved space requires a necessary breaking of the Poincaré symmetry with consequential catastrophic implications for the otherwise majestic beauty of electroweak theories.

Without any claim of uniqueness (see, e.g., the recent studies on unified theories of monograph [2m] and references quoted therein), the modifications of gravitational theories that are necessary to resolve the above incompatibilities can be outlined as follows:

(A) Isotopies. The view here submitted is that the above structural incompatibilities *are not* necessarily due to insufficiencies of Einstein-Hilbert field equations, but rather to *insufficiencies in their mathematical treatment.* Stated in plain language, we believe that the achievement of axiomatic compatibility between gravitation and electroweak interactions requires a basically new mathematics, that is, basically new numbers, new spaces, new geometries, new symmetries, etc.

In the hope of resolving in due time this first structural incompatibility, Santilli [4a] proposed back in 1978, when at the Department of Mathematics of Harvard University under DOE support, a new mathematics based on the so-called *isotopies*, and today known as *Santilli isomathematics* as studied in numerous works by the author and several other researchers [4-11] (for original mathematical aspects see in particular memoir [4e]).

The isotopies are nowadays referred to *liftings* of any given linear, local and canonical or unitary theory into its most general possible nonlinear, nonlocal and noncanonical or nonunitary extensions, that are nevertheless capable of reconstructing linearity, locality and canonicity or unitarity on certain generalized spaces and fields, called *isospaces and isofields.* From their Greek meaning, isotopies are therefore "axiom-preserving."

The fundamental isotopy of this note is that of the 4-dimensional unit I = diag (1, 1, 1, 1) of the Minkowskian and Riemannian spacetimes into a 4x4-dimensional, everywhere invertible, Hermitean and positive-definite matrix $\hat{I}$ whose elements have an arbitrary functional dependence on the local space-time coordinates x, as well as any other needed variable,

$$I = dia.(1,1,1,1) \rightarrow \hat{I}(x,...) = (\hat{I}^{\mu}_{\nu}(x,...)) = \hat{I}^{\dagger} = [\hat{T}(x,...)]^{-1} > 0, \quad (1)$$

with corresponding lifting of the conventional associative product

$$A \times B \rightarrow A \hat{\times} B = A \times \hat{T} \times B, \quad (2)$$

under which $\hat{I}(x,...) = [\hat{T}(x,...)]^{-1}$ is the correct left and right unit of the new theory called *isounit*, in which case $\hat{T}(x,...)$ is called the *isotopic element.*

When applicable, liftings (1) and (2) require, for consistency, the reconstruction of *all* mathematical methods of contemporary physics, with no exception known to this author.

In a communication at the *VII Marcel Grossmann Meeting on General Relativity* held in 1994 at Stanford University, Santilli [5a] showed that isomathematics permits a novel classical and operator treatment of gravitation that, on one side, preserves *all Riemannian* metrics, Einstein-Hilbert field equations and related experimental verifications while, on the other side, verifies the abstract *Minkowskian* axioms.

The main mechanism [5a] is that based on the factorization of any given Riemannian metric (e.g., Schwarzschild metric [1c]) g(x) into the Minkowski metric $\eta = Diag.(+1, +1, +1, -1)$

$$g(x) = T(x) \times \eta(formoredetails, seeRef.[5e, 6g]) \tag{3}$$

where the *gravitational isotopic element* $T(x)$ is evidently a 4-dimensional matrix which is always positive-definite from the locally Minkowskian character of Riemann. The entire theory must then be reconstructed with respect to the *gravitational isounit*

$$\hat{I} = [\hat{T}(x)]^{-1} = \eta \times [g(x)]^{-1} > 0, \tag{4}$$

It should be stressed for clarity that we are here referring to a mere *mathematical* reformulation of Einstein-Hilbert field equations on the *mathematical* isominkowskian spaces (i.e., refer said equations to a new unit $\hat{I}$) because the projection of the treatment into the conventional spacetime (i.e., when referred to the conventional spacetime unit I) recovers the said equations in their totality.

The reader should be aware that the above classical and operator isotopies are supported by two, hitherto unknown symmetries, first presented in memoir [4f] under the tentative name of *isoselfscalar symmetries*, which are characterized by the transforms

$$\eta \rightarrow \hat{\eta} = n^{-2} \times \eta, I \rightarrow \hat{I} = n^2 \times I, \tag{5}$$

where n is a parameter, and yield the symmetry of the conventional Minkowskian interval

$$x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I = (x^\mu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I} = \hat{x^2}, \tag{6}$$

with a corresponding invariance for the Hilbert space

$$< \phi| \times |\psi > \times I =< \phi| \times n^{-2} \times |\psi > \times (n^2 \times I) =< \phi|\hat{\times}|\psi > \times \hat{I}. \tag{7}$$

The isominkowskian representation of gravity then emerges from the above classical and quantum isosymmetries via the axiom-preserving addition of an x-dependence in the n-parameter, much along the transition from Abelian to non-Abelian gauge theories.

(B) Isodualities. Structural incompatibility (2) is only the symptom of deeper problems in the contemporary treatment of antimatter.

To begin, matter is treated nowadays at *all* levels, from Newtonian to electroweak interactions, while antimatter is treated only at the level of *second quantization.* Since there are serious indications that half of the universe could well be made up of antimatter, a more effective theory of antimatter must apply at *all* levels.

The dramatic disparity in the treatment of matter and antimatter also has its predictable problematic aspects. Since we currently use only one type of quantization (whether naive of symplectic), it is easy to see that *the operator image of the contemporary treatment of antimatter is not the correct charge conjugate state, but merely a conventional state of particles with a reversed sign of the charge.*

At any rate, stars can be safely assumed to be neutral. Therefore, the current Riemannian treatment of antimatter via the mere change of the sign of the charge prohibits any serious differentiation between matter and antimatter, resulting in predictable inconsistencies at deeper levels of research.

In an attempt to resolve these additional inconsistencies, Santilli entered into the search for a second novel mathematics under the condition of being an anti-isomorphic image of the preceding isomathematics in order to be equivalent to charge conjugation. After inspecting a number of alternatives, this author submitted in Ref.s [6] what is today known as *Santilli isodual theory of antimatter,* characterized by the lifting called the *isodual map:*

$$Q(x, \psi, ...) \rightarrow Q^d = -Q^\dagger(-x^\dagger, -\psi^\dagger ...). \tag{8}$$

When applied to the *totality* of quantities and their operations of a given theory of matter, map (8) yields an anti-isomorphic image, as axiomatically needed for antimatter. Moreover, while charge conjugation is solely applicable within operator settings, isoduality (8) is applicable at *all* levels of study, beginning at the *Newtonian* level. For brevity, we refer the reader to Refs. [7] for details.

It is evident that isodualities offer a realistic possibility of resolving the second structural problem between electroweak and gravitational interactions because antimatter can be treated in both cases with *negative-energy.*

The reader should also be aware that the isodual theory of antimatter was born from properties of the conventional Dirac equation

$$[\gamma^\mu \times (p_\mu - e \times A_\mu/c) + i \times m] \times \Psi(x) = 0, \tag{9a}$$

$$\gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix} \qquad (9b),$$

$$\gamma^4 = i \times \begin{pmatrix} I_s & 0. \\ 0 & -I_s \end{pmatrix}. \qquad (9c)$$

In fact, as one can see, *the negative unit* $I_s^d = Diag.(-1,-1)$ *appears in the very structure of* γ_4. The isodual theory was then constructed precisely around Dirac's unit I_s^d

In essence, Dirac assumed that the negative-energy solutions of his historical equation behaved in an unphysical way because tacitly referred to the conventional mathematics of his time, that with *positive units* $I_s > 0$. Santilli [7] showed that, when the same negative-energy solutions are referred to the *negative units* $I_s^d < 0$, they behaved in a fully physical way. This eliminates the need of second quantization for the treatment of antiparticles (as expected in a theory of antimatter beginning at the Newtonian level), and permits the reformulation of Dirac's equation in the form

$$[\tilde{\gamma}^\mu \times (p_\mu - e \times A/c) + i \times m] \times \tilde{\Psi}(x) = 0, \qquad (10a)$$

$$\tilde{\gamma}_k = \begin{pmatrix} 0 & \sigma_k^d \\ \sigma_k & 0 \end{pmatrix}, \quad \tilde{\gamma}^4 = i \begin{pmatrix} I_s & 0, \\ 0 & I_s^d \end{pmatrix}, \qquad (10b)$$

$$\{\tilde{\gamma}_\mu, \tilde{\gamma}_\nu\} = 2\eta_{\mu\nu}, \quad \tilde{\Psi} = -\tilde{\gamma}_4 \times \Psi = i \times \begin{pmatrix} \Phi \\ \Phi^d \end{pmatrix}, \qquad (10c)$$

where $\Phi(x)$ is now two-dimensional. Note that the above reformulation of Dirac's equation is fully symmetrized between particles and antiparticles.

It should be indicated that the isodual theory of antimatter was constructed to resolve the inconsistency according to which *Lie's theory prohibits the existence of a four dimensional representation of spin 1/2.* In fact, in the representation now becomes a two-dimensional regular representation of spin 1/2 time its isodual.

In so doing, we also reach a *yet new basic symmetry,* called *isoselfduality,* namely, the invariance under isoduality (8), a feature clearly possessed by Dirac's gamma matrices, $\gamma_\mu \equiv \gamma_\mu^d$. Despite its simplicity, the implications are rather deep. In fact, if the universe is isoselfdual, it is composed of equal amounts of matter and antimatter, the expansion of the universe becomes a natural consequence of the necessary gravitational repulsion between matter and isodual antimatter, all its total

characteristics (time, mass, energy, angular momentum, etc.) are identically null without any discontinuity at creation.

As was the case for the preceding isotopies, the isodual theory of antimatter also sees its solid roots in two additional novel symmetries, also unknown until recently, and first presented in memoir [4f], the first holding for the conventional Minkowski interval

$$x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I = [x^\mu \times (-n^{-2} \times \eta_{\mu\nu}) \times x^\nu] \times (-n^2 \times I)$$

$$= (x^\mu \times \hat{\eta}^d_{\mu\nu} \times x^\nu) \times \hat{I}^d = x^{d2d} \tag{11}$$

and the second holding for the Hilbert space

$$< \phi|\times|\psi > \times I =< \phi|\times(-n^{-2})\times|\psi > \times(-n^2\times I) =< \phi|\times\hat{T}^d\times|\psi > \times\hat{I}^d. \tag{12}$$

The above isodual symmetries ensure that all physical laws of matter also hold for antiparticles under our isodual representation, with corresponding symmetries for the isodual expressions.

(C) Isotopies and Isodualities of the Poincaré symmetry. A resolution of fundamental incompatibility (3) of grand unifications called for a third series of studies presented in Refs. [6,7] on the *isotopies and isodualities of the Poincaré symmetry* $\hat{P}(3.1)$, today called the *Poincaré-Santilli isosymmetry and its isodual* [8-12].

We are here referring to the reconstruction of the conventional symmetries with respect to an arbitrary positive-definite unit (1), for the isotopies, and with respect to an arbitrary negative-definite unit, for the isodualities. This reconstruction yields the most general known nonlinear, nonlocal and noncanonical liftings of conventional symmetries, while being locally isomorphic (for isotopies) or anti-isomorphic (for isodualities) to the original symmetries.

It is evident that the Poincaré-Santilli isosymmetry and its isodual have fundamental character for this note. In fact, one of their primary applications has been the achievement of the universal *symmetry* (rather than covariance) of all possible Riemannian line elements in their isominkowskian representation [6]. Once the unit of gauge theories is lifted to represent gravitation, electroweak interactions will also obey the isopoincare' symmetry for matter and its isodual for antimatter, thus offering realistic hopes for the resolution of the most difficult problem of compatibility for grand unifications, that for space-time symmetries.

The fundamental space-time symmetry of the grand unified theory inclusive of gravitation submitted in this note is the total symmetry of the *conventional* Dirac equation, here written with their underlying spaces and units

$$S_{Tot} = \{SL(2.C) \times T(3.1)\} \times \{SL^d(2.C^d) \times^d T^d(3.1)\}, \qquad (13a)$$

$$M_{Tot} = \{M(x, \eta, R) \times S_{spin}\} \times \{M^d(x^d, \eta^d, R^d) \times^d S^d_{spin}\}, \qquad (13b)$$

$$I_{Tot} = \{I_{orb} \times I_{spin}\} \times \{I^d_{orb} \times^d I^d_{spin}\}, \qquad (13c)$$

Note that the Poincaré symmetry emerges from these studies as being *eleven dimensional* in view of symmetries (6) and not ten dimensional as popularly believed throughout the 20-th century, and the same eleven dimensionality holds for the isodual symmetry in view of isodual symmetries (11).

The reader should not be surprised that the four new invariances (6)-(7) and (11)-(12) remained undetected throughout the 20-th century because their identification required the prior discovery of *new numbers*, first the numbers with arbitrary positive units for invariances (6)-(7), and then the additional new numbers with arbitrary negative units for invariances (11)-(12).

2 ISOTOPIC GAUGE THEORY

The isotopies of gauge theories were first studied in 1980's by Gasperini [11a], followed by Nishioka [10b], Karajannis and Jannussis [11c] and others, and ignored thereafter. However, these studies were defined on conventional spaces over conventional fields and used the conventional differential calculus. As such, these studies are not invariant, as we learned only in memoirs [4f].

The correct isotopies of (Abelian or non-Abelian) gauge theories requires their formulation via the entire use of Santilli isomathematics, thus including: *isofields* [4d] $\hat{C}(\hat{c}, \hat{+}, \hat{\times})$ with: additive isounit $\hat{0} = 0$; generalized multiplicative isounit $\hat{I}$ given by Eq. (1); elements, isosum, isoproduct and related generalized operations,

$$\hat{a} = a \times \hat{I}, a = n, c, \hat{a} \hat{+} \hat{b} = (a+b) \times \hat{I}, \hat{a} \hat{\times} \hat{b} = \hat{a} \times \hat{T} \times \hat{b} = (a \times b) \times \hat{I}, \quad (14a)$$

$$\hat{a}^{\hat{n}} = \hat{a} \hat{\times} \hat{a} \hat{\times} ... \hat{\times} \hat{a}, \hat{a}^{\hat{1/2}} = a^{1/2} \times \hat{I}^{1/2}, \hat{a} \hat{/} \hat{b} = (\hat{a}/\hat{b}) \times \hat{I}; \qquad (14b)$$

isominkowski spaces [6a] $\hat{M} = \hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ with isocoordinates $\hat{x} = x \times \hat{I} = \{x^\mu\} \times \hat{I}$, isometric $\hat{N} = \hat{\eta} \times \hat{I} = [\hat{T}(x, \ldots) \times \eta] \times \hat{I}$, and *isointerval* over the isoreals $\hat{R}$

$$(\hat{x} - \hat{y})^{\hat{2}} = [(\hat{x} - \hat{y}^\mu \hat{\times} \hat{N}_{\mu\nu} \hat{\times} (\hat{x} - \hat{y})^\nu = [(x - y)^\mu \times \hat{\eta}_{\mu\nu} \times (x - y)^\nu] \times \hat{I}, \quad (15)$$

equipped with isocontinuity [10a] and isotopology [10b,4f,10e] (see also Aslander and Keles [10d]); *isodifferential calculus* [4e] characterized by the following isodifferentials and isoderivatives

$$\hat{d}\hat{x}^\mu = \hat{I}^\mu_\nu \times d\hat{x}^\nu, \hat{d}\hat{x}_\mu = \hat{T}^\nu_\mu \times \hat{x}_\nu, \hat{\partial}_\mu \hat{f} = \hat{\partial}\hat{f} \hat{/} \hat{\partial}\hat{x}^\mu = (\hat{T}^\nu_\mu \times \partial_\nu f) \times \hat{I}, \quad (16a)$$

$$\hat{\partial}^\mu \hat{f} = (\hat{I}^\mu_\nu \times \partial_\nu f) \times \hat{I}, \hat{\partial}\hat{x}^\mu \hat{/} \hat{\partial}\hat{x}^\nu = \hat{\delta}^\mu_\nu = \delta^\mu_\nu \times \hat{I}, etc.; \quad (16b)$$

and all remaining aspects of isomathematics, such as the *isofunctional isoanalysis* [6g]; *isominkowskian geometry* [5e]; *relativistic hadronic mechanics* [4f]; and the*Lie-Santilli isotheory* [4,6,8d,10c].

This new mathematics permits the definition of the *isogauge transform*

$$\hat{\psi}' = \hat{U} \hat{\times} \hat{\psi} = \left(e^{-i \times X_k \times \hat{T}(x, \ldots) \times \theta(x)_k}\right) \times \hat{\psi}; \quad (17)$$

$$\hat{U} = \hat{e}^{-i \times X_k \times \theta(x)_k} = \left(e^{-i \times X_k \times \hat{T} \times \theta(x)_k}\right) \times \hat{I}, \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}, \quad (18)$$

whose nontriviality is expressed by the fact that *the gravitational isotopic element appears in the exponent,* as well as by the reconstruction of unitarity on isospaces over isofields; *isocovariant derivatives* [5e]

$$\hat{D}_\mu \hat{\psi} = (\hat{\partial}_\mu - i \hat{\times} \hat{g} \hat{\times} \hat{A}(\hat{x})^k_\mu \hat{\times} \hat{X}_k) \hat{\times} \hat{\psi}; \quad (19)$$

iso-Jacobi identity

$$[\hat{D}_\alpha \hat{,} [\hat{D}_\beta \hat{,} \hat{D}_\gamma]] \hat{+} [\hat{D}_\beta \hat{,} [\hat{D}_\gamma \hat{,} \hat{D}_\alpha]] \hat{+} [\hat{D}_\gamma \hat{,} [\hat{D}_\alpha \hat{,} \hat{D}_\beta]] = 0, \quad (20)$$

where one should note the reconstruction of unitarity on isospaces over isofields, g and $\hat{g} = g \times \hat{I}$ are the conventional and isotopic coupling constants, $A(x)^k_\mu \times X_k$ and $\hat{A}(\hat{x})^k_\mu \hat{\times} \hat{X}_k = [A(x)^k_\mu \times X_k] \times \hat{I}$ are the gauge and isogauge potentials; the *isocovariance*

$$(\hat{D}_\mu \hat{\psi})' = (\hat{\partial}_\mu \hat{U}) \hat{\times} \hat{\psi} \hat{+} \hat{U} \hat{\times} (\hat{\partial}_\mu \hat{\psi}) \hat{-} \hat{i} \hat{\times} \hat{g} \hat{\times} \hat{A}'(\hat{x})_\mu \hat{\times} \hat{\psi} = \hat{U} \hat{\times} \hat{D}_\mu \hat{\psi}, \quad (21a)$$

$$\hat{A}(\hat{x})'_\mu = -\hat{g}^{-\hat{1}} \hat{\times} [\hat{\partial}_\mu \hat{U}(\hat{x})] \hat{\times} \hat{U}(\hat{x})^{-\hat{1}}, \quad (21b)$$

$$\hat{\delta} \hat{A}(\hat{x})^k_\mu = -\hat{g}^{-\hat{1}} \hat{\times} \hat{\partial}_\mu \hat{\theta}(\hat{x})^k \hat{+} \hat{C}^k_{ij} \hat{\times} \hat{\theta}(\hat{x})^i \hat{\times} \hat{A}(\hat{x})^j_\mu, \quad (21c)$$

$$\hat{\delta}\hat{\psi} = -\hat{i}\hat{\times}\hat{g}\hat{\times}\hat{\theta}(\hat{x})^k\hat{\times}\hat{X}_k\hat{\times}\hat{\psi}; \tag{21d}$$

the *non-Abelian Yang-Mills isofields*

$$\hat{F}_{\mu\nu} = \hat{i}\hat{\times}\hat{g}^{-\hat{1}}\hat{\times}[\hat{D}_{\hat{\mu}}, \hat{D}_\nu]\hat{\psi}, \tag{22a}$$

$$\hat{F}^k_{\mu\nu} = \hat{\partial}_\mu\hat{A}^k_\nu\hat{-}\hat{\partial}_\nu\hat{A}^k_\mu\hat{+}\hat{g}\hat{\times}\hat{C}^k_{ij}\hat{\times}\hat{A}^i_\mu\hat{\times}\hat{A}^j_\nu; \tag{22b}$$

with related isocovariance properties

$$\hat{F}_{\mu\nu} \to \hat{F}'_{\mu\nu} = \hat{U}\hat{\times}\hat{F}_{\mu\nu}\hat{\times}\hat{U}^{-1}, \tag{23a}$$

$$Isotr(\hat{F}_{\mu\nu'}\hat{\times}\hat{F}^{\mu\nu'}) = Isotr(\hat{F}_{\mu\nu}\hat{\times}\hat{F}^{\mu\nu}), \tag{23b}$$

$$[\hat{D}_\alpha\hat{,}\hat{F}_{\beta\gamma}]\hat{+}[\hat{D}_\beta\hat{,}\hat{F}_{\gamma\alpha}]\hat{+}[\hat{D}_\gamma\hat{,}\hat{F}_{\alpha`\beta}] \equiv 0; \tag{29c}$$

derivability from the isoaction

$$\hat{S} = \hat{\int}\hat{d}^{\hat{4}}\hat{x}(-\hat{F}_{\mu\nu}\hat{\times}\hat{F}^{\mu\nu}\hat{/}\hat{4}) = \hat{\int}\hat{d}^{\hat{4}}\hat{x}(-\hat{F}^k_{\mu\nu}\hat{\times}\hat{F}^{\mu\nu}_k\hat{/}\hat{4}), \tag{24}$$

where $\hat{\int} = \int \times \hat{I}$, plus all other familiar properties in isotopic formulation.

The *isodual isogauge theory* is the image of the preceding theory following the application of the isodual map (8) to the totality of quantities and their operations. The latter theory is characterized by the isodual isogroup $\hat{G}^d$ with isodual isounit $\hat{I}^d = -\hat{I}^\dagger = -\hat{I}$. The base fields are the field $\hat{R}^d(\hat{n}^d, \hat{+}^d, \hat{\times}^d)$ of isodual isoreal numbers $\hat{n}^d = -\hat{n} = -n \times \hat{I}$ and the field $\hat{C}^d(\hat{c}^d, \hat{+}^d, \hat{\times}^d)$ of isodual isocomplex numbers $\hat{c}^d = -(c \times \hat{I})^\dagger = (n_1 - i \times n_2) \times \hat{I}^d = (-n_1 + i \times n_2) \times \hat{I}$.

For the reader not familiar with the new isomathematics, it should be noted that the above isotopy of Yang-Mills fields implies no variation of their numerical value, that is, *Yang-Mills isofields on isospaces over isonumbers have the same numerical values of conventional Yang-Mills fields computed on ordinary spaces over ordinary numbers* and, similarly, *isodual Yang-Mills fields or isofields are equivalent to their charge conjugate forms for the representation of antiparticles* (for brevity, see [*loc. cit.*].

3 ISO-GRAND-UNIFICATION

Iso-Grand-Unification (IGU), first indicated in Ref. [6h], is the direct product of Yang-Mills isofields and their isoduals as characterized by the the the total isoselfdual symmetry

$$\hat{S}_{Tot} = (\hat{\mathcal{P}}(3.1)\hat{\times}\hat{G}) \times (\hat{\mathcal{P}}(3.1)^d\hat{\times}^d\hat{G}^d) =$$

$$= [\hat{SL}(2,\hat{C})\hat{\times}\hat{T}(3.1)] \times [\hat{SL}^d(2,\hat{C}^d)\hat{\times}^d\hat{T}^d(3.1)], \tag{25}$$

where $\hat{\mathcal{P}}$ is the Poincaré-Santilli isosymmetry [10c] in its isospinorial realization [6f].

It should be indicated that we are referring here to the *axiomatic* consistency. The *physical* consistency is a separate problem which cannot possibly be investigate in this introductory note and will be investigated in future works. At this point we merely mention the general rule according to which isotopic liftings preserve not only the original axioms, but also the original numerical values [6g] (as an example, the image in isominkowskian space over the isoreals of the light cone, not only is a perfect cone, but a cone with the original characteristic angle, thus preserving the speed of light in vacuum as the maximal causal speed in isominkowskian space). This peculiar property of the isotopies implies the expectation that the proposed Iso-Grand-Unification preserves the numerical results of both the conventional unified gauge theories and the conventional treatment of gravitation.

The reader should be aware that the methods of the recent memoir [4f] permit a truly elementary, explicit construction of the proposed IGU. As well known, the transition from the Minkowskian metric η to Riemannian metrics $g(x)$ is a *noncanonical transform* at the classical level, and, therefore, a *nonunitary transform* at the operator level. The method herein considered for turning a gauge theory into an IGU consists in the following representation of the selected gravitational model, e.g., Schwarzschild's model:

$$g(x) = T(x) \times \eta, T(x) = (U \times U^\dagger)^{-1}, \tag{26a}$$

$$U \times U^\dagger) = Diag.[(1 - 2 \times M/r) \times diag(1,1,1), (1 - 2 \times M/r)^{-1}), \tag{26b}$$

and then subjecting the *totality* of the gauge theory to the nonunitary transform $U \times U^\dagger$. The method then yields: the isounit $I \to \hat{I} = U \times I \times U^\dagger$; the isonumbers $a \to \hat{a} = U \times a \times U^\dagger = a \times (U \times U^\dagger) = a \times \hat{I}, a = n, c$;

the isoproduct with the correct expression and Hermiticity of the isotopic element, $A \times B \rightarrow U \times (A \times B) \times U^\dagger = (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times B \times U^\dagger) = \hat{A} \times \hat{T} \times \hat{B} = \hat{A} \hat{\times} \hat{B}$; the correct form of the isohilbert product on $\hat{C}$, $< \phi| \times |\psi > \rightarrow U \times < \phi| \times |\psi > \times U^\dagger = (< \psi| \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi >) \times (U \times U^\dagger) =< \hat{\phi}| \times \hat{T} \times |\hat{\psi} > \times \hat{I}$; the correct Lie-Santilli isoalgebra $A \times B - B \times A \rightarrow \hat{A} \hat{\times} \hat{B} - \hat{B} \hat{\times} \hat{A}$; the correct isogroup $U \times (e^X) \times U^\dagger = (e^{X \times \hat{T}}) \times \hat{I}$, the Poincaré-Santilli isosymmetry $\mathcal{P} \rightarrow \hat{\mathcal{P}}$, and the isogauge group $G \rightarrow \hat{G}$.

It is then easy to verify that it the proposed Iso-Grand-Unification is indeed invariant under all possible additional nonunitary transforms $W \times W^\dagger = \hat{I}$, provided that, for evident reasons of consistency, they are written in their identical isounitary form, $W = \hat{W} \times \hat{T}^{1/2}, W \times W^\dagger = \hat{W} \hat{\times} \hat{W}^\dagger = \hat{W}^\dagger \hat{\times} \hat{W} = \hat{I}$. In fact, we have the invariance of the isounit $\hat{I} \rightarrow \hat{I}' = \hat{W} \hat{\times} \hat{I} \hat{\times} \hat{W}^\dagger = \hat{I}$, the invariance of the isoproduct $\hat{A} \hat{\times} \hat{B} \rightarrow \hat{W} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{W}^\dagger = \hat{A}' \hat{\times} \hat{B}'$, etc. Note that the isounit is *numerically* preserved, as it is the case for the conventional unit I under unitary transform, and that the selection of a nonunitary transform $W \times W^\dagger = \hat{I}'$ with value different from $\hat{I}$ evidently implies the transition to a different gravitational model.

It should be noted that the isounit representing gravitation verifies all the properties of the conventional unit I of gauge theories, $\hat{I}^{\hat{n}} = \hat{I}, \hat{I}^{\hat{1/2}} = \hat{I}, d\hat{I}/dt = \hat{I} \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{I} = \hat{H} - \hat{H} = 0$, etc. The "hidden" character of gravitation in conventional gauge theories is then confirmed by the isoexpectation value [4f] of the isounit which recovers the conventional unit I of gauge theories, $\hat{<} \hat{I} \hat{>} =< \hat{\psi}| \times \hat{T} \times \hat{I} \times \hat{T} \times |\hat{\psi} > / < \hat{\psi}| \times \hat{T} \times |\hat{\psi} >= I$.

It then follows that the proposed IGU constitutes an explicit and concrete realization of the theory of "hidden variables" [13a] $\lambda = T(x) = g(x)/\eta, \hat{H} \hat{\times} |\hat{\psi} >= \hat{H} \times \lambda \times |\hat{\psi} >= E_\lambda \times |\hat{\psi} >$, and the theory is correctly reconstructed with respect to the new unit $\hat{I} = \lambda^{-1}$, in which von Neumann's Theorem [13b] and Bell s inequalities [13c] do not apply, evidently because of the nonunitary character of the theory (see [13d or Vol. II of Refs. [6g] for details).

In summary, the main aspect conveyed in this note is the possibility that *gravitation has always been present in unified gauge theories and it did creep in un-noticed until Ref. [6h] because embedded where nobody looked for, in the "unit" of gauge theories.*

References

[1] B.Riemann, it Gött. Nachr. **13,** 133 (1868) and *Collected Works*, H.Weber, ed. (Dover, New York, 1953) [1a]. D.Hilbert,*Nachr. Kgl. Ges. Wissench. Gottingen,* 1915, p. 395 [2b]. A.Einstein, *Sitz. Ber. Preuss. Akad. Wissssench Berlin*, 1915, p. 844 [1c]. K.Schwartzschild, *Sitzber. Deut. Akad. Wiss. Berlin, K1. Math.–Phys. Tech.*, 189 and 424 (1916) [1c]. H.Weyl, *Raum–Zeit–Materie* (Springer, Berlin, 1916) [1d]. A.Einstein, H.Minkowski and H.Weyl, *The Principle of Relativity: A collection of original memoirs* (Methuen, London, 1923) [1e].

[2] C.N. Yang and R. Mills, *Phys. Rev.* **96**, 191 (1954) [2a]. S.L. Glashow, *Nuc. Phys.* **22**, 579 (1961) [2b]. S. Weinberg,*Phys. Rev.Lett.* **19**,1264 (1967) [2c]. A. Salam, in *Elementary Particle Physics (Nobel Symp. No. 8))*, N. Svartholm, Almquist and Wiksell, ed. (Stockholm, 1968) [2d]. J.C. Pati and A. Salam, *Phys. Rev.* **D10**, 275 (1974) [2e]. M. Günaydin and F. Gürsey, *J. Math. Phys.* **14**,1651(1973) [2f]. L.P. Horwitz and L.C. Biedenharn, *J. Math. Phys.* **20**, 269 (1979) [2g]. R.N. Mohapatra and J.C. Pati, *Phys. Rev.* **D11**, 2558 (1975) [2h]. G. Senjanovic and R.N. Mahopatra, *Phys. Rev.* **D12**, 1502 (1975) [2i]. H. Georgi and S.L. Glashow, *Phys. Rev Lett.* **32**, 438 (1974) [2j]; H. Georgi, in *Particles and Fields*, C.E. Carlson, ed.(A.I.P., 1975) [2k]; H. Fritzsch and P. Minkowski,*Ann. Phys.* **93**, 193 (1975 [2l]; V. de Sabbata and C. Sivaran, *Spun and Torsion in Gravitation* (World Scientific, Singapore, 1994) [2m]. S. Adler, A. Bazin and L. Schieffer, *Introduction to General Relativity* (Mc.Graw-Hill, New York, 1984) [2n].

[3] M.J.G.Veltman, in *Methods in Field Theory*, R. Ballan and J. Zinn–Justin, eds. (North–Holland, Amsterdam, 1976) [3a]. C.J.Isham, R.Penrose and D.W.Sciama, Editors, *Quantum Gravity 2* (Oxford University Press, Oxford, 1981) [3b]. *Proceedings of the VII M. Grossmann Meeting on General Relativity*, R. T. Jantzen, G. Mac Kaiser and R. Ruffinni, eds. (World Scientific, Singapore, 1996) [3c]; R. M. Santilli, "Unresolved axiomatic inconsistencies of gravitation on curved spaces." preprint IBR-TH-04-39, submitted for publication [3d].

[4] R.M.Santilli,*Hadronic J.* **1,** 224, 574 and 1267 [4a]; *Phys. Rev. D* **20,** 555 (1979) [4b]; *Foundation of Theoretical Mechanics*, Vol. **I** (1978) and **II** (1983), Springer, New York (1983) [4c]; *Algebras, Groups and Geometries* **10,** 273 (1993) [4d]; *Rendiconti Circolo Matematico Palermo, Suppl.* **42,** 7 (1996) [4e]; *Found. Phys.* **27**, 691 (1997) [4f].

[5] R.M.Santilli, in *Proceedings of the Seventh M. Grossmann Meeting on General Relativity*, R. T. Jantzen, G. Mac Kaiser and R. Ruffinni, Editors, World Scientific, Singapore (1996), p. 500 [5a]; in *Gravity, Particles*

and Space–Time, P. Pronin and G. Sardanashvily, eds. (World Scientific, Singapore, 1995), p. 369 [5b]; *Comm. Theor. Phys.* **3,** 47 (1994) [5c]; *Comm. Theor. Phys.* **4,** 1 (1995) [5d]. Int. J. Mod. Phys. D **7**, 351 - 407 (1998) [5e].

[6] R.M.Santilli, *Nuovo Cimento Lett.* **37,** 545 (1983) [6a]; *Lett. Nuovo Cimento* **38,** 509, (1983) [6b]; *Hadronic J.* **8,** 25 and 36 (1985) [6c]; *JINR Rapid Comm.* **6,** 24 (1993) [6d]; *J. Moscow Phys. Soc.* **3,** 255 (1993) [6e]; JINR Comm. No. E4-93-352 [1993] and *Chinese J. Syst. Eng.and Electr.* **&** Electr. *6*, 177 (1996) [6f]; *Elements of Hadronic Mechanics*, Vol. I, II 2nd edn, (Ukraine Academy of Sciences, Kiev, 1995) and Vol. III, in preparation, [6g]; "Unification of gravitation and electroweak interactions," Contributed paper in the *Proceedings of the Eight Marcel Grossmann Meeting in Gravitation,* T. Piran, and R. Ruffini, Editors, World Scientific, pages 473-475 (1999 [6h]); Foundations of Physics Letters **10**, 307 (1997) [6i].

[7] R.M.Santilli, *Comm. Theor. Phys.* **3,** 153 (1994) [7a]; *Hadronic J.* **17,** 257 (1994) [7b]; JINR Comm. No. E2-96-259 [1996] and *Hyperfine Interaction,* **109,** 63 (1997) [7c]; in *New Frontiers of Hadronic Mechanics*, T.L.Gill, ed. (Hadronic Press, Palm Harbor, Florida, 1996) [7d]; Intern. J. Modern Phys. A **14**, 2205 (1999) [7e].

[8] *Tomber's Bibliography and Index in Nonassociative Algebras* (Hadronic Press, Palm Harbor, Florida, 1984) [8a]. A.K. Aringazin, A.Jannussis, D.F.Lopez, M.Nishioka and B.Veljanosky, *Santilli's Lie–Isotopic Generalization of Galilei's Relativities* (Kostarakis Publisher, Athens, Greece, 1980) [8b]. J.V.Kadeisvili, *Santilli's Isotopies of contemporary Algebras, Geometries and Relativities*, 2nd ed. (Ukraine Academy of Sciences, Kiev, 1997) [8c]. D.S.Sourlas and G.T.Tsagas, *Mathematical Foundations of the Lie–Santilli Theory* (Ukraine Academy of Sciences, Kiev, 1993) [8d]. J.Lohmus, E.Paal and L.Sorgsepp, *Nonassociative Algebras in Physics* (Hadronic Press, Palm Harbor, Florida, 1994) [8e].

[9] S. L. Adler,*Phys. Rev.* **17**, 3212 (1978) [9a]; Cl. George, F. Henin, F. Mayne and I. Prigogine, *Hadronic J.* **1**, 520 (1978) [9b]; S. Okubo, *Hadronic J.* **3**, 1 (1979) [9c]; J. Fronteau, A. Tellez Arenas and R. M. Santilli, *Hadronic J.* **3**, 130 (1978) [9d] H. C. Myung and R. M. Santilli, *Hadronic J.* **5**, 1277 (1982) [9e]; C. N. Ktorides, H. C. Myung, and R. M. Santilli, *Phys. Rev. D* **22**, 892 (1982) [9f]. A. J. Kalnay, *Hadronic J.* **6**, 1 (1983) [9g]. R. Mignani, *Nuovo Cimento Lett.* **39**, 413 (1984) [9h]. J. D. Constantoupoulos and C. N. Ktorides, *J. Phys. A* **17**, L29 (1984) [9i]. E. B. Lin, *Hadronic J.* **11**, 81 (1988) [9l]. M. Nishioka, *Nuovo Cimento A* **82**, 351 (1984) [9m]. A. K. Aringazin, *Hadronic J.* **12**, 71 (1989) [9n]. D. Rapoport-Campodonico, *Algebras, Groups and Geometries* **8**, 1 (1991) [9o]. A. Jannussis, G. Brodimas, and R. Mignani, *J. Phys. A* **24**, L775 (1991) [9p]. A. Jannussis, M. Miatovic and B. Veljanowski, *Physics*

Essays **4**, 202 (1991) [9q]. R. Mignani,*Physics Essays* **5**, 531 (1992) [9r]; F. Cardone, R. Mignani and R. M. Santilli, *J. Phys. G* **18**, L61 and L141 (1992) [9s]. T. Gill, J. Lindesay, and W. W. Zachary, *Hadronic J.* **17**, 449 (1994) [9t]; A. O. E. Animalu, *Hadronic J.* **17**, 349 (1995) [9u]. A. O. E. Aniamalu and R. M. Santilli, *Int. J. Quantum Chemistry* **29**, 175 (1995) [9v]. D. Schuch, *Phys. Rev. A* **55**, 955 (1997) [9x]. M. Battler, M. McBee, and S. Smith, http://home1.gte.net/ibr/ [9y].

[10] J. V. Kadeisvili, *Algebras, Groups and Geometries* **9**, 283 and 319 (1992) [10a]. Gr. T. Tsagas and D. S. Sourlas, *Algebras, Groups and Geometries* **12**, 1 and 67 (1995) [10b]. J. V. Kadeisvili, *Math. Methods in Applied Sciences* **19**, 1349 [1996] [10c]. R. Aslaner and S. Keles, Algebras, Groups and Geometries **14,** 211 (1997) [10d]. S. Vacaru, Algebras, Groups and Geometries **14**, 225 (1997) [10e]. R. M. Falcón Ganfornina and J. Núñez Valdés, Algebras, Groups and Geometries **20**, 1 (2003) [10e].

[11] M. Gasperini, Hadronic J. **6**, 935 and 1462 (1983) [11a]. M. Nishioka, Hadronic J. **6**, 1480 (1983) and Lett. Nuovo Cimento **40**, 309 (1984) [11b]. G. Karayannis and A. Jannussis, Hadronic J. **9**, 203 (1986) [11c].

[12] A.P.Mills, Hadronic J. 19 (1996), 79 [12a]. M.Holzscheither,ed., *Proceedings of the International Worksop on Antimatter Gravity and Antihydrogen Atom Spectroscopy*, Sepino, Italy, May 1996,it Hyp.Int. **109** no.s 1-4 (1997) [12b]. J. P. Costella, B. H. J. McKellar and A. A. Rowlison, *Amer. J. Phys.* (1997), in press.

[13] D.Bohm, *Quantum Theory* (Dover Publications, New York, 1979) [13a]. J.Von Neumann, *The Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, 1955) [13b]. J.S. Bell,*Physics* **1,** 195 (1965) [13c]; R. M. Santilli Acta Applicandae Mathematicae **50**, pp. 177 (1998).

[14] A.Einstein, B.Podolsky, and N.Rosen, *Phys. Rev.* **47,** 777 (1935).

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