Published in
Clifford Analysis, Clifford Algebras and their Applications
Vol. 6, No. 1, pp. 1-119, 2017

Work on an invited keynote talk at the ICNPAA Conference
University of La Rochelle, France, July 4-8, 2016 AND
and on an Invited Plenary Talk at SIPS 2-16. Hainan island, China,
November 5 to 11, 2016

ELEMENTS OF HADRONIC MECHANICS, III: Experimental verifications

Ruggero Maria Santilli
Thunder Energies Corporation, 1444 Rainville Rd, Tarpon Springs, FL 34689
Email: research@thunder-energies.com

Abstract.
tp be converted into a Preface

In this paper, we outline the following branches of hadronic mechanics: *isomathematics*, with ensuing *isomechanics* and *isochemistry*, which have been constructed for the representation of time reversal invariant systems of extended and deformable particles in interior dynamical conditions as occurring, e.g., in stable nuclides; the covering *genomathematics*, with ensuing *genomechanics* and *genochemistry* which has been constructed for the representation of time irreversible systems as occurring, e.g., in nuclear syntheses and all energy releasing processes; and *isodual conventional, isotopic and genotopic mathematics* with ensuing it isodual theory of antimatter, which has been constructed for the *classical* (as well as operator) representation of *neutral* (as well as charged) antimatter systems in conditions of increasing complexity, under the condition of compatibility with matter-antimatter annihilation, that requires the surpassing of charge conjugation in favor of new vistas. We review the main experimental verifications of the various branches of hadronic mechanics, and point out expected, novel, mathematical, physical, chemical and industrial applications for matter and antimatter, with particular reference to new, clean, sustainable forms of energy.

PACS 03.75.Be, 29.30.H, 67.65.+z
Key words: isomathematics, genomathematics, isodual mathematics

TABLE OF CONTENTS

1. Introduction

As it is well known, quantitative studies require an appropriate mathematical treatment. Consequently, all scientific advances achieved by mankind during the past centuries were due to the use of appropriate mathematics.

Despite these historical achievements, mathematics will never admit final formulations capable of describing all possible natural phenomena, since the possibilities of conceiving new mathematics with ensuing new applications are virtually endless.

In this paper, we outline three new mathematics and their applications in physics, chemistry and angineering for the representation of systems with increasing complexity that were presented in an invitedengineering keynote talk delivered at the ICNPAA congress 2016 held at the University of La Rochelle, France, on July 4 to 8, 2016.

It should be indicated that the literature in the field is rather vast since it has long passed the mark of $30,000$ pages of published research. In this paper, we can only outline and quote the original contributions and some of the reviews. A comprehensive list of references up to 2007 is available in Vol. I of Refs. [8], while primary contributions from 2008 on are available from the scientific archives [65]. All publications are available in free pdf download.

2. IsoMathematics and its Applications

2.1. Basic Assumptions

We can introduce *isomathematics* as a new mathematics with a Lie-isotopic algebraic structure which is a covering of 20th century mathematics which intended to perform the transition:

▷ from the Newtonian, Galilean and Einsteinian notion of *point particles* under potential, therefore Hamiltonian interactions,

▷ to the covering notion of *extended particles* under the most general known linear and non-linear, local and non-local and potential-Hamiltonian as well as contact non-Hamiltonian interactions

,

under the condition of being *time reversal invariant* (the extension to time irreversible systems is treated in the next section with *genomathematics.*

To illustrate the need for such a new mathematics, let us recall that a central feature of 20th century mathematics is given by the local character of the Newton-Leibnitz differential calculus that, as such, can only be defined at a finite number of isolated points.

The most important physical implication of such a local-differential character is the sole capability of 20th century mathematics to characterize *point particles* (also called *massive points*), as originally expressed by Newton via his celebrated equation:

$$m\frac{dv}{dt} = F(t, r, v). \tag{2.1}$$

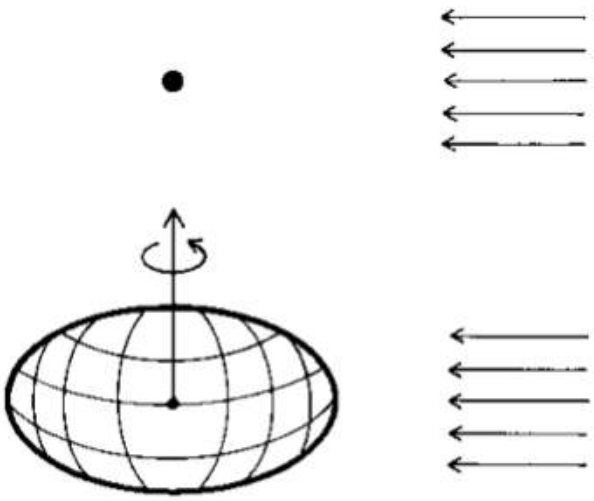

Figure 1: *An illustration of the central objective of the novel isomathematics, consisting in the transition from the representation of the Newtonian notion of massive points moving in vacuum under solely Hamiltonian interactions (top view), to the representation of the broader notion of extended particles moving within physical media under Hamiltonian and non-hamiltonian interactions.*

The implications of the above central feature are illustrated by the fact that Galileo and Einstein were forced to adopt in full the Newtonian notion of point particle since they had no other choice than that of formulating their relativities for via the Newton-Leibnitz differential calculus.

Despite the appearance of the notion of *wave packet,* the Newtonian notion of pointy particle remained at the foundation of quantum mechanics because, for instance, the celebrated Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\psi(t,r) = [-\frac{\hbar^2}{2m}\nabla + V(t,r)]]osi(t,r), \tag{2.2}$$

remains solely defined at a finite number of isolated points in space and time.

We can safely state that the indicated 20th century mathematics and its physical applications are fully effective for the quantitative representation of the so-called (Figure 1):

EXTERIOR DYNAMICAL SYSTEMS: consisting of point particles and electromagnetic waves propagating in vacuum (conceived as empty space).

As an illustration, the trajectory of Jupiter in our solar system can be fully represented by considering Jupiter a massive point since Jupiter's extended character does not affect its dynamics due to motion in empty space. Similarly, the relativistic motion of a proton in a particle accelerator can indeed be fully represented by approximating the proton as a massive point since, again, its motion occurs through the vacuum of particle accelerators.

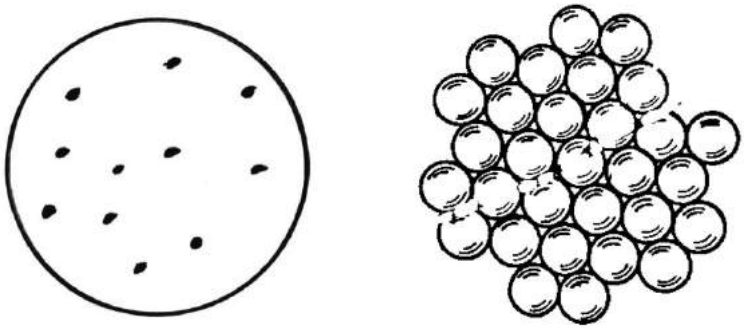

Figure 2: *A view in the left of the abstraction of the nuclear structure essentially permitted by the mathematics underlying quantum mechanics as an aggregate of point-particles due to its local-differential character, and a view in the right of the extended character of nuclear constituents permitted by isomathematics.*

Nevertheless (as also illustrated in Section 3), the reduction of the entire universe to massive points is outside the boundaries of serious science due to the existence of the broader systems limpidly advocated by Lagrange, Hamilton and other founders of analytic mechanics (Figure 1):

INTERIOR DYNAMICAL SYSTEMS: consisting of extended particles and electromagnetic waves propagating within physical media.

The lack of exact character of 20th century mathematics for the representation of interior systems can be easily seen from the necessity of abandoning the Newtonian notion of point particles in favor of a covering mathematics suitable for the representation of their actual extended character, since the latter cannot be consistently reduced to a finite number of isolated points.

Additionally, whether moving in vacuum or within physical media, point particles can only experience action-at-a-distance interactions derivable from a potential, thus being Hamiltonian. By contrast, extended particles moving within physical media experience *contact interactions* that have, by conception, *zero range,* thus being *non-Hamiltonian.*

Finally, it should be mentioned that the local-differential character of quantum mechanics essentially implies the restriction of composite systems solely to internal interactions that are linear in the wavefunction and other quantities, while it is widely expected that composite system of extended particles at short mutual distances are expected to have non-linear internal interactions.

In summary, the clear existence in nature of interior dynamical systems establishes the need for a mathematics for the description of *extended particles under linear and non-linear, local and non-local and Hamiltonian as well as non-Hamiltonian forces (or interactions).*

It should be clarified that, due to the lack of a time direction, *isomathematics is solely intended for interior dynamical problems that are invariant under time reversal,* such as the structure of stable nuclei. The broader mathematics needed for time irreversible pro-

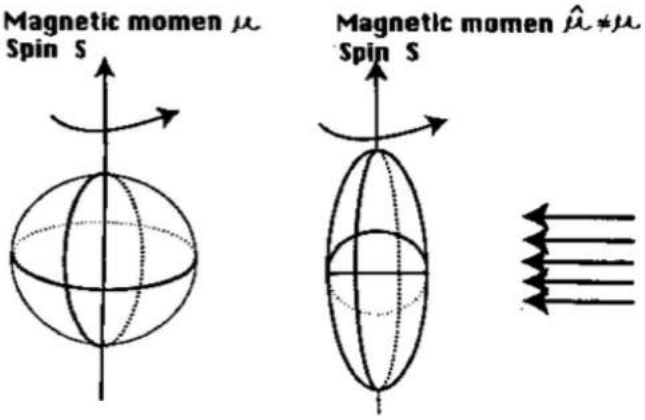

Figure 3: *Enrico Fermi predicted that a reason for the inability of quantum mechanics to represent nuclear magnetic moments is the expected deformation of the charge distribution of protons and neutrons under strong nuclear forces, with consequential deformation of their magnetic moments. Isomathematics has permitted the first known numerically exact and time invariant representation of nuclear magnetic moments because the regular Lie-isotopic rotational $\hat{SO}(3)$ and spin $\hat{SU}(2)$ symmetries preserve conventional eigenvalues, while admitting an internal degree of freedom representing Fermi's hypothesis [57,48]. It should be noted that, despite its simplicity, the representation of Fermi's hypothesis required, firstly, a generalization of 20th century mathematics, and secondly, the compatible generalization of quantum mechanics into hadronic mechanics much along the celebrated argument by Einstein, Podolsky and Rosen on the 'lack of completion of quantum mechanics' [14].*

cesses, such as particles and nuclear reactions, will be studied in the next section.

An extensive library search conducted by the author in the 1970's established that the mathematics needed for quantitative treatment of interior dynamical systems simply did not exist, and therefore, it had to be constructed as a condition for serious advances in problems of direct societal relevance, such as the interior of hadrons, nuclei and stars.

In the late 1970's, the author joined the faculty of Harvard University under financial support from the U. S. Department of Energy (DOE) with contracts numbers ER-78-S-02-47420.A000, AS02-78ER04742, DE-AC02-80ER1065, DE-AC02-80ER-1065.A001, and DE-AC02-80ER.1065.

The author's main objective is the study of new clean energies expected to be of nuclear character. As part of these studies, the author accepted the full validity of quantum mechanics for the atomic structure, but assumed quantum mechanics to be merely approximately valid for the nuclear structure due to the lack of exact representation of nuclear data, including nuclear magnetic moments and spin.

The study of this occurrence indicated that the origin of the insufficiencies of quantum mechanics were not due to its axions, but actually to the local-differential character of its underlying mathematics since it ultimately implies the abstraction of nuclei to a sphere with isolated point particles in it, while in reality nuclei are composed by extended charge distributions in conditions of partial mutual penetration (Figure 2).

2.2. Rudiments of isomathematics

2.2.1. Isoproduct

In order to attempt a representation of protons and neutrons as extended charge distributions, the author proposed in memoirs [1,2] (see monographs [3] for a comprehensive treatment) the author suggested in Refs. [1.2] of 1978 (see monographs [3] for a comprehensive treatment) the following generalization of the conventional associative product $AB = A \times B$ between generic quantities matrices, operators, etc.

$$A \times B > A\hat{\times}B = A \times \hat{T} \times B, \quad \hat{T} > 0, \tag{2.3}$$

where $\hat{T}$ is restricted to be positive-definite, but possess otherwise an unrestricted functional dependence on local quantities, such as coordinates x, velocities v, energy E, density μ, temperature τ, pressure ξ, wavefunction ψ, their derivatives $\partial_x \psi$, etc.

$$\hat{T} = \hat{T}(x, v, E, \mu, \tau, \xi, \psi, \partial_x \psi, ...) > 0. \tag{2.4}$$

The above assumption immediately allowed an explicit representation of protons and neutrons as extended, non-spherical and deformable charge distributions via realizations of the new quantity $\hat{T}$ of the type for one particle

$$\hat{T} = Diag.(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, \frac{1}{n_4^2}) e^{-\Gamma}, \tag{2.5a}$$

$$n_\mu = n_\mu((x, v, E, \mu, \tau, \xi, \psi, \partial_x \psi, ...) > 0, \quad \mu = 1, 2, 3, 4, \tag{2.5b}$$

$$\Gamma = \Gamma((x, v, E, \mu, \tau, \xi, \psi, \partial_x \psi, ...) > 0, \tag{2.5c}$$

with elementary extension to several particles, where: $n_k^2,\ k = 1, 2, 3$ represents the semiaxes of the particle considered assumed to be a spheroid ellipsoid for simplicity; n_4^2 represents the density of the particle considered; and Γ represent non-linear, non-local and non-Hamiltonian interactions due to mutual penetrations, interactions known as *variationally non-selfadjoint* [3a].

Since the new product $A\hat{\times}B$ is as associative as the original one, Carla Santilli suggested for the emerging new mathematics the name of *isomathematics* where the prefix "iso" is intended in the Greek sense of preserving the original axioms. The new product $A\hat{\times}B$ was then called *isoproduct* and the quantity $\hat{T}$ was called *isotopic element* [1-3].

Monographs [3b] presented the initial formulation of isomathematics essentially consisting in the reformulation of various branches of 20th century mathematics via isoproduct (2.3), with particular reference to the isotopies of functional analysis, metric spaces and Lie's theory, although the mathematics was formulated over conventional numeric fields and was elaborated via the conventional differential calculus.

2.2.2. Isonumbers

Studies subsequent to Refs. [1-3] indicated that the physical theories characterized by the above initial formulation of isomathematcs did not predict the same numerical values

under the same conditions at different times due to the lack of invariance under time evolution and other reasons, thus preventing meaningful physical applications.

In 1993, while visiting the *Joint Institute for Nuclear Research* in Dubna, Russia, the author had no other choice than that of reinspecting the historical classification of numbers into real, complex and quaternions. In this way, the author discovered that the axioms of a numeric field do not necessarily require that the basic multiplicative unit of the field should be the trivial number 1, because said axioms admit an arbitrary multiplicative unit under the conditions of being positive-definite and of compatible reformulation of numbers and their product [4].

The above discovery lead to a new class of numbers and numeric field, today known as *Santilli isonumbers and isofields,* respectively (see, e.g., monograph [5] and literature quoted therein) that can be essentially outlined as follows:

Let $F(n, \times, 1)$ be a conventional field of real, complex or quaternionic numbers n with product $nm = n \times m$, multiplicative unit 1 and related axiom $1 \times n = n \times 1 \equiv n \; \forall n \in F$, and additive unit 0 and related sum $n + m \in F, \; \forall n, m \in F$.

The infinite family of isotopies $\hat{F}(\hat{n}, \hat{\times}, \hat{1})$ of $F(m, \times, 1)$ is characterized by an invertible and positive-definite *multiplicative isounit*

$$\hat{1} = 1/\hat{T} > 0, \tag{2.6}$$

and *isoreal, isocomplex and isoquaternionic isonumbers*

$$\hat{n} = n \times \hat{1}, \tag{2.7}$$

equipped with the isoproduct

$$\hat{n}\hat{\times}\hat{m} = \hat{n} \times \hat{T} \times \hat{m} = (n \times m) \times \hat{1} \in \hat{F}, \tag{2.8}$$

and conventional *isoadditive unit*

$$\hat{0} = 0 \times \hat{1} = -0, \tag{2.9}$$

and *isosum*

$$\hat{n}\hat{+}\hat{m} \equiv \hat{n} + \hat{m}. \tag{2.10}$$

Isofields were then distinguished into those of the first (second) kind depending on whether the isounit is (is not) an element of the original field.

Needless to say, all operations on numbers were suitably generalized for isonumbers. For instance, the *isosquare* is defined by $\hat{n}^{\hat{2}} = \hat{n}\hat{\times}\hat{n} = n^2 \times \hat{I}$ and similar definitions holds for the remaining operations on numbers [4].

It is evident that isofield are isomorphic to conventional fields by conception. Despite that, the lifting from fields to isofields is not trivial, as illustrated by the fact that, for the case of isofield of the first kind, the use of the isounit $\hat{1} = 3$ implies that the number 4 is prime, thus indicating that the notion of prime number depends on the assumed multiplicative unit (see, later on Monograph).

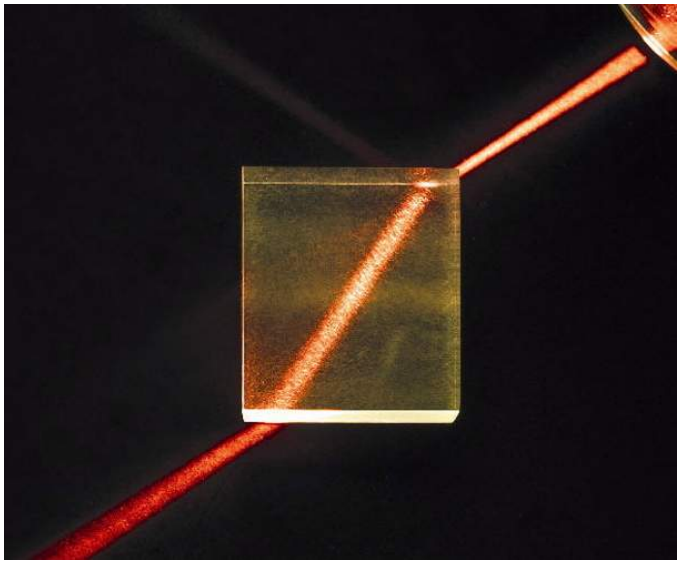

Figure 4: *As it is well known, Albert Einstein obtained a fully justified Nobel Prize for his hypothesis of the photon for the quantized 'absorption' of light by matter. The reduction to photons for light 'propagating' in water for the intent of recovering special relativity at the level of photons scattering among water molecules is afflicted by a host of inconsistencies, such as: A) The impossibility of representing the angle of refraction of light (since photons will scatter in all directions at the point of impact with the water surface); B) The impossibility to represent the reduction of the speed of light by about 1/3 (the scattering of photons among the water molecules can at best represent the reduction of the speed of light by a few percentages); C) The impossibility to represent the propagation of light along a straight beam illustrated in this figure (because photons will scatter in all directions and certainly not propagate along a straight line); and other inconsistencies. Additionally, inconsistencies in water are given by the fact that electrons traveling in water at speeds bigger than the local speed of light (Cherenkov light); have the violation of the relativistic sum of speeds; the basic axioms of special relativity cannot be properly formulated, let alone experimentally verified due to the absence in water of inertial reference frames. Finally, the reduction to photons of light propagating in water dismissed by electromagnetic waves of large wavelength, such as infrared and radio waves, which have the same phenomenology as that of visible light, yet their reduction to photon has no physicals sense [31].*

Figure 5: *Special relativity is unable to provide a quantitative representation of the fact that objects submerged in water appear to have dimensions bigger than those occurring on the outside. Such a difference is readily represented by isorelativity and its Minkowski-Santilli isogeometry [7,21,26]. In fact, isotopies preserve not only the original axioms, but also the original numeric values according to the rule* $L_{ext} \times 1 = L_{int} \times \hat{1}$. *But for water* $\hat{1} < 1$. *Consequently,* $L_{int} > L_{ext}$. *This geometric effect has resulted to be rather general for interior conditions as expressed by the transition from the conventional axioms for exterior point conditions, to the isoaxioms for interior conditions, Eqs. (2.33) to (2.38).*

The first commercial application of isomathematics has been its use in cryptology via the new class of *isocryptograms* consisting of cryptograms with an infinite number of possible multiplicative units that can be automatically changed every second or less, thus requiring an infinite time for their resolution [7a].

With the understanding that commercial uses of cryptograms cannot be publicly disclosed, it appears that U. K. credit cards and bank accounts may be protected by isocryptograms following the purchase of the rights from the author by a British company in the field in the late 1990's.

2.2.3. Isospaces

Following the 1993 discovery of the isonumbers [4], the original formulation of isomathematics on conventional fields [3b] was extended to a form admitting the multiplicative isounit $\hat{1}$ at all levels, thus including the reformulation of functional analysis, metric spaces, algebras, geometries, etc.

In particular, the isotopies of metric and pseudo-metric spaces as well as manifolds were reformulated over isofields. As an illustration, consider a conventional Minkowski space $M(x, \eta, I)$ on the field of real numbers R with spacetime coordinates $x = (x^1, x^2, x^3, x^4 = ct)$, metric $\eta = Diag.(1, 1, 1, -1)$ and unit $I = Diag(1, 1, 1, 1)$.

Then the infinite family of isotopic Minkowski spaces $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$ on the isofield of isoreal isonumbers $\hat{R}$, today known as *Minkowski-Santilli isospaces,* are characterized by the *spacetime isocoordinates* $\hat{x} = x \times \hat{I}$ (where the multiplication of the isounit is necessary

to be elements of the isofield), isounit (2.6) with realization (2.5) of the isotopic element *isometric* $\hat{\eta} = \hat{T} \times \eta$, and *spacetime isoinvariant*

$$\hat{x}^{\hat{2}} = \hat{x}^{\mu} \hat{\times} \hat{\eta}_{\mu\nu} \hat{\times} \hat{x}^{\nu} =$$

$$= (\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2}) \times \hat{I}, \tag{2.11}$$

where the final multiplication by the isounit is again necessary for the invariant to be an isoscalar.

It should be noted that invariant (2.11) is the most general possible symmetric (non-singular) invariant in $(3+1)$-dimensions, thus including as particular cases all possible Minkowskian, Riemannian, Fynslerian and to all other possible symmetric spacetime invariants.

2.2.4. Santilli-Georgiev isodifferential calculus

Despite these advances, physical applications still lacked the crucial property of predicting the same numerical values under the same conditions at different times.

In order to resolve this impasse, during the *Second International Conference on Lie-Admissible Treatment of Irreversible Systems* held in June 1995 at the Castle Prince Pignatelli, Italy, the author had no other choice than that of reinspecting the ultimate essence of 20th century mathematics, namely, the *Newton-Leibnitz differential calculus.*

In so doing, the author discovered that, contrary to a rather popular belief in mathematics and physics for four centuries, *the differential calculus depend on the multiplicative unit of the base field* because, when said unit depends on the variable of differentiations, said calculus has to be lifted into the infinite family of *isodifferential* first formulated in memoir [6] of 1996

$$\hat{d}\hat{r} = \hat{T} d[r\hat{I}(r, ...)] = dr + r\hat{T} d\hat{I}, \tag{2.12}$$

with corresponding *isoderivative* [6]

$$\frac{\hat{\partial} \hat{f}(\hat{r})}{\hat{\partial} \hat{r}} = \hat{I} \frac{\partial \hat{f}(\hat{r})}{\partial \hat{r}}. \tag{2.13}$$

The covering character of the isodifferential calculus over the conventional calculus is illustrated by the fact that, whenever the isounit is independent from the differentiation variable or it is a constant, the conventional calculus is recovered entirely and unambiguously.

Twenty nine years following the author's discovery of the isodifferential calculus, the mathematician Svetlin Georgiev has published a series of six volumes [9] (to date, with others forthcoming) essentially reformulating all main aspects of 20th century mathematics in terms of the isodifferential calculus and related isomathematics.

In particular, monographs [9] identify all possible mathematically consistent formulations of the isoproduct (2.3) that are particularly important for physical and industrial applications when dealing with multiple variables since they generally require different isotopic element (e.g., because of different dimensionality).

Georgiev's monographs [9] also apply for the covering Lie-admissible genomathematics for the representation of irreversible processes via the specialization of the isotopic element according to rule (3.27). Consequently, monographs [9) provide indeed the pure mathematical structure of all studies presented in this paper.

In view of the historically unprecedented character of monographs [9], *the author recommends for the new differential calculus the name of "Santilli-Georgiev isodifferential calculus."*

The 1996 discovery of the isodifferential calculus signaled the achievement of mathematical, and physical maturity of *isomathematics intended as the infinite family of axiom-preserving isotopies of 20th century mathematics defined on isospaces (or isomanifolds) over isofields and elaborated via the isofunctional analysis and the isodifferential calculus.*

On conceptual grounds, *isomathematics has been conceived and constructed to replace point particles with extended particles, as by the isodifferential calculus in which the isotopic element $\hat{T}$ representing the shape of particles enters in its very definition (Figure 1).*

Monographs [7[provide the first comprehensive presentation of isomathematics following the advent of the isodifferential calculus, with 2006 upgrade available in Refs. [8]. A general list of references up to 2007 is available in the first volume of Refs. [8].

2.2.5. Lie-Santilli isotheory

In the author's view, the physically most important part of isomathematics is given by the isotopies of the various branches of Lie's theory, first formulated in Refs. [1-3], upgraded in Refs. [7] following the advent of the isodifferential calculus, and today known as the *Lie-Santilli isotheory* (see e.g., Ref.[10]).

Let L be an n-dimensional Lie algebra with Hermitean generators $X_k,\ k = 1, 2, ...n$ defined on a conventional numeric fields. Then, the infinite family of isotopies of L are characterized by the following main features:

THEOREM 1: (Poincaré-Birkhoff-Witt-Santilli theorem): The isocosets of the isounit and of the standard isomonomials

$$I,\ \ X_k,\ \ \hat{X}_i\hat{\times}\hat{X}_j,\ i \leq j,\ \ \hat{X}_i\hat{\times}\hat{X}_j\hat{\times}\hat{X}_k,\ i \leq j \leq k,\ \ldots, \tag{2.14}$$

form an (infinite dimensional) basis of the universal enveloping isoassociative algebra $\hat{E}(\hat{L})$ (also called isoenvelope for short) of a Lie-Santilli isoalgebra $\hat{L}$.

The above theorem was used in 1978, for the characterization of the *isoexponentiation*

$$\hat{e}^{\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}} =$$

$$= \hat{I} + \hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}\hat{/}\hat{1}! + (\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X})\hat{\times}(\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X})\hat{/}\hat{2}! + \ldots =$$
$$= \hat{I} \times (e^{i\times w\times T\times X}) = (e^{i\times w\times X\times T}) \times \hat{I}. \tag{2.15}$$

By recalling that Lie's theory can only characterize linear, local and Hamiltonian systems, the presence in the exponent of the isotopic element $\hat{T} > 0$ with an arbitrary functional

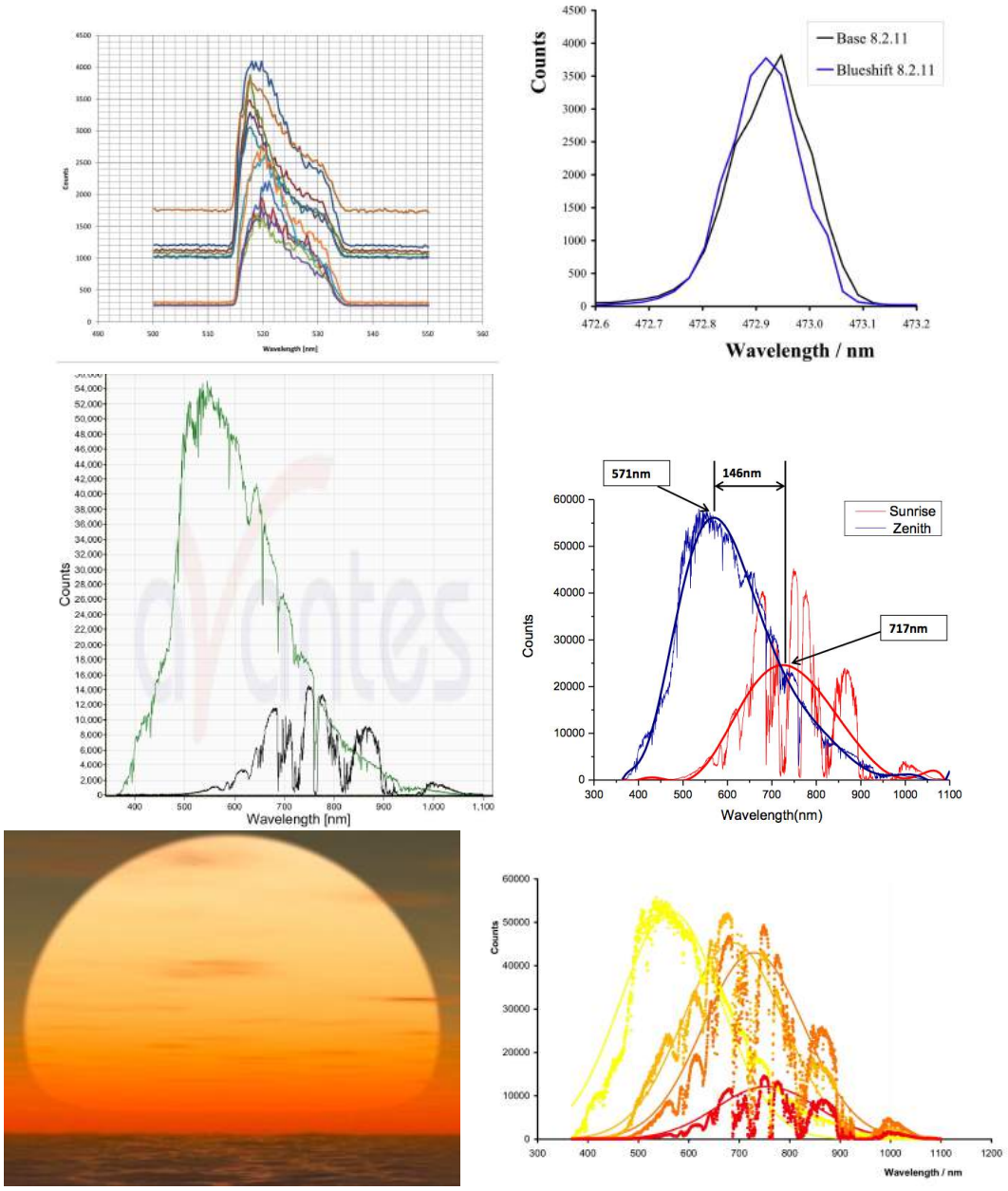

Figure 6: *The first two figures represent the first measurements of the redshift of laser light propagating within compressed air without any relative motion, an effect known as Santilli isoshift, while the remaining two figures represent the redshift of 'direct' Sunlight in the transition from the Zenith to the horizon which is an isoredshift since it occurs without any relative motion between the Sun, the atmosphere and the observer [31-36].*

dependence of type (2.5c) then confirmed the capability of the covering Lie-Santilli isotheory to characterize non-linear, non-local and non-Hamiltonian system as occurring in the physical reality.

THEOREM 2: The antisymmetric isoalgebras $\hat{L}$ attached to the isoenveloping algebras $\hat{E}(\hat{L})$ verify the isocommutation rules

$$[\hat{X}_i,\hat{X}_j] = \hat{X}_i\hat{\times}\hat{X}_j - \hat{X}_j\hat{\times}\hat{X}_i = \tag{2.16a}$$

$$= \hat{C}_{ij}^k(x,v,\xi,\omega,\psi,\partial\psi,...)\hat{\times}\hat{X}_k = C_{ij}^k(x,v,\xi,\omega,\psi,\partial\psi,...)\times X_k. \tag{2.16b}$$

The above isocommutation rules clearly show the axiom-preserving character of the isotopies due to their evident verification of the axioms of a Lie algebra, although via a broader realization.

We should indicate that, nowadays the Lie-Santilli isotheory is classified into *regular* and *irregular* isotheory depending on whether the structure quantities $C_{i,j}^k$ are constants or functions, respectively. This classification is rather important because it indicates the impossibility of a transform capable of mapping the Lie into the entire Lie-Santilli isotheory.

THEOREM 3: The isogroup characterized by finite (integrated) form $\hat{G}$ of isocommutation rules (1.12) on an isospace $\hat{S}(\hat{x},\hat{F})$ over an isofield $\hat{F}$ with common isounit $\hat{I} = 1/\hat{T} > 0$ is a group mapping each element $\hat{x} \in \hat{S}$ into a new element $\hat{x}' \in \hat{S}$ via the isotransformations

$$\hat{x}' = \hat{g}(\hat{w})\hat{\times}\hat{x}, \;\; \hat{x},\hat{x}' \in \hat{S}, \;\; \hat{w} \in \hat{F}, \tag{2.17}$$

with the following isomodular action to the right:

1) The map $\hat{g}\hat{\times}\hat{S}$ into $\hat{S}$ is isodifferentiable $\forall \hat{g} \in \hat{G}$;

2) $\hat{I}$ is the left and right unit $\hat{I}\hat{\times}\hat{g} = \hat{g}\hat{\times}\hat{I} \equiv \hat{g}$, $\forall \hat{g} \in \hat{G}$;

3) the isomodular action is isoassociative, $\hat{g}_1\hat{\times}(\hat{g}_2\hat{\times}\hat{x}) = (\hat{g}_1\hat{\times}\hat{g}_2)\hat{\times}\hat{x}$, $\forall \hat{g}_1,\hat{g}_2 \in \hat{G}$;

4) in correspondence with every element $\hat{g}(\hat{w}) \in \hat{G}$ there is the inverse element $\hat{g}^{-\hat{I}} = \hat{g}(-\hat{w})$ such that $\hat{g}(\hat{0}) = \hat{g}(\hat{w})\hat{\times}\hat{g}(-\hat{w}) = \hat{I}$;

5) the following composition laws are verified $\hat{g}(\hat{w})\hat{\times}\hat{g}(\hat{w}') = \hat{g}(\hat{w}')\hat{\times}\hat{g}(\hat{w}) = \hat{g}(\hat{w}+\hat{w}'), \forall \hat{g} \in \hat{G}$, $\hat{w} \in \hat{F}$; with corresponding isomodular action to the left, and general expression $\hat{g}(\hat{w}) = \prod_k \hat{e}^{\hat{i}\hat{\times}\hat{w}_k\hat{X}_k}\hat{\times}\hat{g}(0)\hat{\times}\prod_k \hat{e}^{=\hat{i}\hat{\times}\hat{w}_k\hat{X}_k}$.

Nowadays, *isomathematics* is referred to the infinite family of isotopies of 20th century mathematics, with particular reference to the isotopies of Lie's theory, when strictly formulated on isospaces over isofields and elaborated via the Santilli-Georgiev isodifferential calculus.

2.3. Isosymmetries and Isoparticles

It is evident that the above isotransformations verify the axioms of a Lie group, although in a broader realization. This feature has stimulated a rather vast literature for the isotopies of conventional spacetime and internal symmetries, including the isotopies:

2.1) $\hat{SO}(3)$ of the rotational symmetry $SO(3)$ [11,12];

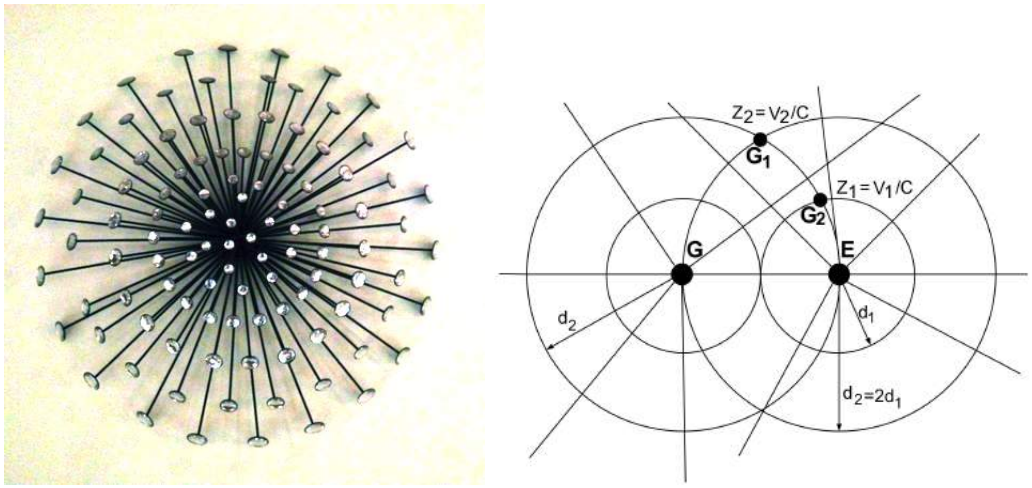

Figure 7: *A. Albert Einstein, Edwin Hubble, Fritz Zwicky, Fred Hoyle, Enrico Fermi and other illustrious scientists died without accepting the expansion of the universe because the representation of the cosmological redshift via the Doppler effect, $z = Hd, = v/c$ (where H is Hubble's constant and d is the distance of galaxies) is 'radial' in all possible directions from Earth, thus implying a return to the Middle Ages with Earth at the center of the universe, as illustrated by the sculpture on the left. Additionally, the conjecture of the expansion of the universe and its inherent acceleration are geometrically inconsistent because, as illustrated by the diagram on the right, the two galaxies G_1 and G_2 are at double distance from Earth E, thus have a relative acceleration, while there exist an infinite number of observers in the universe for which said two galaxies have the same distance, as it is the case for observer G, thus preventing the conjecture of the expansion of the universe from occurring in the physical reality. In any case the author has established via numerous experiments in the U.S.A. and in Europe that the redshift of the Sun at Sunset is due to loss of energy by light to the atmosphere (Santilli isoredshift), thus establishing that the redness of the Sun at Sunset is visual evidence of the lack of expansion of the universe. In particular, the pro-rating of the isoredshift of Sunlight in our atmosphere (for about* 90 *nm) to intergalactic media provides an exact and invariant representation of Hubble's law $z = Hd$ without any conjectural motion of galaxies moving away from us at incredible speeds [37,38].*

2.2) $\hat{SU}(2)$ of the spin SU(2) symmetry [13,14];

2.3) $\hat{SO}(3.1)$ of the Lorentz symmetry $SO(3.1)$ at the classical [15] and operator [16] levels;

2.4) $\hat{P}(3.1)$ of the Poincare' symmetry $P(3.1)$ [17,18];

2.5) $\hat{P}(3.1)$ of the spinorial covering $P(3.1)$ of the Poincare' symmetry [19,20],
as well as the Minkowskian geometry [21].

A most important implication of the above spacetime isosymmetries in their characterization of the notion of *isoparticle* as the regular irreducible isorepresentation of the Lorentz-Poincaré-Santilli isosymmetry $\hat{P}(3.1)$ which characterizes the transition from motion of point particles in vacuum to motion of extended particles within physical media.

Some of the main results of the above isotopies can be summarized as follows:

2.I) The proof of the universality of the isosymmetry $\hat{P}(3.1)$ for all possible, symmetric (non-singular) isoinvariant (2.11) (see, e.g., Ref. [22]), including the universal invariance of all possible Riemannian line elements [18] and ensuing invariant reformulation of gravitation [23,24].

2.II) The solution of the historical *Lorentz problem,* namely, the achievement of the universal invariance of locally varying speeds of light

$$C = \frac{c}{n_4}, \tag{2.18}$$

achieved by the *Lorentz-Santilli isosymmetry* $\hat{SO}(3.1)$ in paper [16] of 1983, here expressed in its projection on a conventional Minkowski space

$$x^{1'} = x^1, \;\; x^{2'} = x^2, \tag{2.19a}$$

$$x^{3'} = \hat{\gamma}(x^3 - \beta x^4), \quad x^{4'} = \hat{\gamma}(x^4 - \hat{\beta} x^3), \tag{2.19b}$$

$$\beta = \frac{v}{c}, \;\; \hat{\beta} = \frac{v_3/n_3}{c/n_4}, \quad \hat{\gamma} = \frac{1}{\sqrt{1 - \hat{\beta}^2}}. \tag{2.19c}$$

It should be noted that Lorentz solely achieved the invariance for a *constant* speed of light c due to insurmountable difficulties in achieving the invariance of locally varying speeds of light due to its inherent non-linear character. The author constructed the isotopies of Lie's theory [1-3] precisely for the treatment of non-linear systems as a prerequisite for the solution of the Lorentz problem in Ref. [16]. It should be finally noted that, by cancellation and construction, the Lorentz-Santilli isosymmetry is locally isomorphic to the conventional symmetry in full line with the very basic requirements of isomathematics.

2.III) The proof of the compatibility of arbitrary speeds with the abstract axioms of special relativity, since the Lorentz symmetry does not identify the explicit value of a constant speed c [25]. Moreover, the replacement of the Minkowski coordinates x_μ with their

isoinvariant version $\hat{x}_\mu = x_\mu/n_\mu$ transforms the conventional Lorentz transformations into their isotopic covering with local speed of light (2.18).

The use of isomathematics and related isosymmetries has permitted the isotopies of main 20th century physical theories, that are today known as the *isotopic branch* of the covering of quantum mechanics proposed by the authors in 1978 [1-3] under the name of *hadronic mechanics,* namely, a mechanics intended for the structure of hadrons and therefore of nuclei and stars [7,8].

To avoid a prohibitive length, the most salient aspects of these studies, also known under the name of *isomechanics,* is outlined below.

2.4. Isotopies of Newtonian mechanics

With basic equations defined on the iso-Euclidean space over isofields, known as the *Newton-Santilli isoequations,* which provide the first known classical representation of extended particles since Newton's times (Figure 1), equations here presented for the simplest possible subclass with the sole isodifferential for the velocities

$$\hat{m}\hat{\times}\frac{\hat{d}\hat{v}}{\hat{d}\hat{t}} - \hat{F}^{SA}(\hat{t},\hat{r},\hat{v}) = m \times \frac{dv}{dt} + v\hat{T}\frac{d\hat{I}}{dt} - F^{SA}(t,r,v) =$$

$$= m \times \frac{dv}{dt} - F^{NSA}(t,r,v,a,....) - F^{SA}(t,r,v) = 0, \tag{2.20a}$$

$$F^{NSA} = v\hat{T}\frac{d\hat{I}}{dt}, \hat{I}_v = \hat{I}_v(t,...).\ \hat{I}_t = \hat{I}_r = \hat{I}_F = 1. \tag{2.20b}$$

In the above equations, conventional forces are represented with the variationally selfadjoint force F^{SA} [3a] and all non-linear, non-local and non-Hamiltonian forces F^{NSA} are embedded in the isodifferential, thus illustrating its fundamental character for the mathematical and physical study of interior dynamical problems. It should be noted that the above equations are characterized by the *Galileo-Santilli isosymmetry* [26].

2.5. Isotopies of variational principle

With fundamental isoaction principle applicable for the first time to non-Hamiltonian systems

$$\hat{\delta}\hat{A} = \hat{\delta}\hat{\int}(\hat{p}\hat{\times}\hat{d}\hat{r} - \hat{H}\hat{\times}\hat{t}) = 0, \tag{2.21}$$

where, again, all non-Hamiltonian forces are embedded in the isodifferential. Among various novel applications of the above new isovariational principle, we mention the first known application of the optimal control theory to actual extended shapes under resistive forces.

2.6. Isotopies of Hamiltonian mechanics

With basic equations, today known as the *Hamilton-Santilli isoequations*

$$\frac{\hat{d}\hat{r}}{\hat{d}\hat{t}} = \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{p}}, \quad \frac{\hat{d}\hat{p}}{\hat{d}\hat{t}} = -\frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{r}}, \tag{2.22}$$

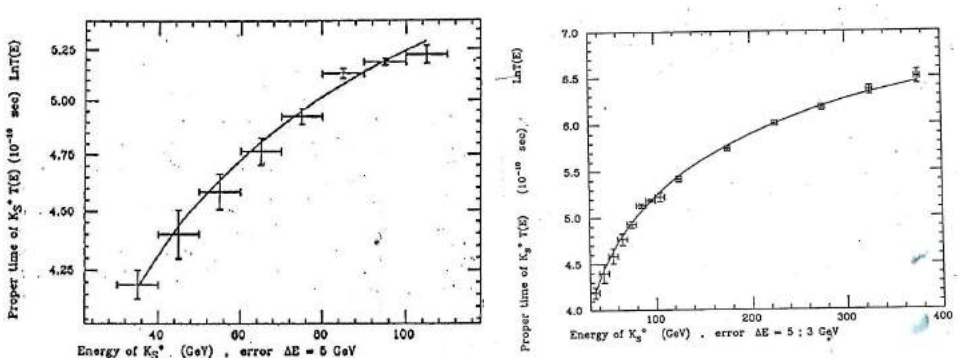

Figure 8: *The isoredshift of Sunlight propagating in our atmosphere establishes the inapplicability of special relativity within a medium of light density, such as our atmosphere (Figure 7), with the expected inapplicability of special relativity for denser media. In fact, Refs. [42-46] have shown that the relativistic time dilation law is violated by the behavior with speed of the mean life of unstable hadrons, thus confirming the inapplicability of special relativity within the hyperdense medium inside hadrons. In any case, the basic axioms of special relativity cannot even be defined, let alone be directly tested (that is tested without their usual assumption in the data elaboration), within a hyperdense medium such as that in the interior of hadrons. By contrast, as also shown by Refs. [42-47], the covering isorelativity results to be exactly valid.*

providing the only known generalization of Hamilton's equation to non-conservative systems which is derivable from a variational principle under the validity of the Galileo-Santilli isosymmetry.

2.7. Isotopies of Hamilton-Jacobi equations

With covering *Hamilton-Jacobi-Santilli isoequations* also defined on an iso-Euclidean space over the isofield of isoreal isonumbers

$$\frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{t}} = -\hat{H}, \quad \frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{r}} = \hat{p}, \quad \frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{p}} = 0. \tag{2.23}$$

The above equations are uniquely characterized by isovariational principle (2.21) and have allowed the first known consistent operator map of non-conservative systems due to the independence of the isoaction $\hat{A}$ from the isolinear momentum $\hat{p}$ established by the last equation (2.23). In fact, this is the feature that allows, after the operator map, the resulting wavefunction to be solely dependent on isotime and isocoordinates $\hat{\psi} = \hat{\psi}(\hat{t}, \hat{r})$. It should be noted that an operator theory with wavefunction additionally depending on isomomenta, $\hat{\psi} = \hat{\psi}(\hat{t}, \hat{r}, \hat{p})$ is beyond our current capability for quantitative applications.

2.8. Isotopies of Schrödinger's representation

with the following isoequations defined on a *Hilbert-Myung-Santilli isospace* $\hat{H}$ [27,28] over the isofield of isocomplex isonumbers $\hat{C}$ today known as the *Schrödinger-Santilli isoequations*

$$\hat{i}\hat{\times}\hat{\partial}_{\hat{t}}\hat{\psi} = \hat{H}\hat{\times}\hat{\psi} = \hat{H}(r,p) \times \hat{T}(\psi, \partial\psi, ...) \times \psi, \tag{2.24a}$$

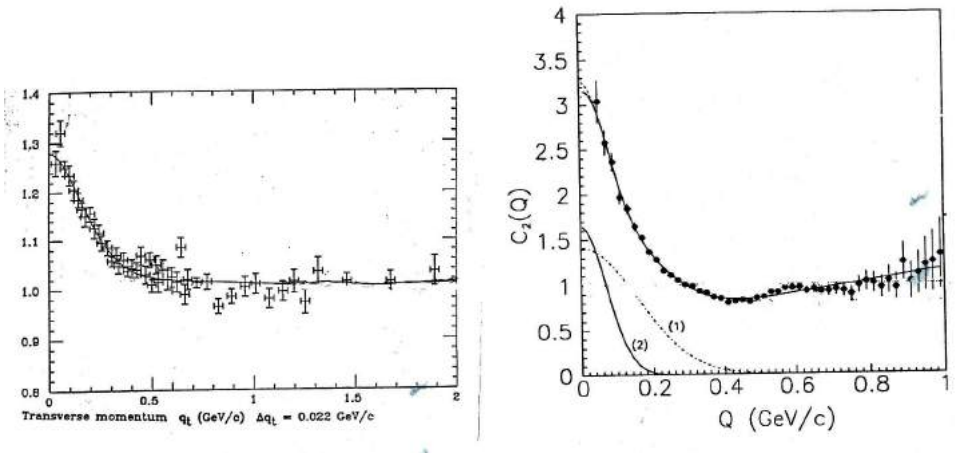

Figure 9: *The excellent fits at high energy (left) and at low energy (right) achieved by isorelativity for the experimental data of the Bose-Einstein correlation in which the space characteristic quantities acquire the direct physical meaning of representing the semiaxes of the proton-antiproton fireball, while the fourth characteristic quantity represent its density [45-46].*

$$\hat{p}\hat{\times}\hat{\psi} = -\hat{i}\hat{\times}\hat{\partial}_{\hat{r}}\hat{\psi} = -i \times \hat{I} \times \partial_{\hat{r}}\hat{\psi}, \tag{2.24b}$$

where one should note the truly fundamental role of the isodifferential calculus for the consistent definition of the *isolinear isomomentum,* Eqs. (2.24b) first achieved in memoir [6] of 1996. In fact, the entire structure of hadronic mechanics remained incomplete until Ref. [6] due to the lack of a consistent definition of the isolinear isomomentum.

2.9. Isotopies of Heisenberg's representation

With dynamical isoequations in the following infinitesimal and finite forms, today known as the *Hamilton-Santilli isoequations*

$$\hat{i}\hat{\times}\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = [\hat{A},\hat{H}] = \hat{A}\hat{\times}\hat{H} - \hat{H}\hat{\times}\hat{A} = \hat{A} \times \hat{T} \times \hat{H} - \hat{H} \times \hat{T} \times \hat{A}, \tag{2.25a}$$

$$\hat{A}(\hat{t}) = \hat{e}^{\hat{H}\hat{\times}\hat{t}\hat{\times}\hat{i}}\hat{\times}\hat{A}(0)\hat{\times}\hat{e}^{-\hat{i}\hat{\times}\hat{t}\hat{H}} = e^{\hat{H}\times]hatT\times t\times i} \times e^{-i\times t\times\hat{T}\times\hat{H}}, \tag{2.25b}$$

$$[\hat{r}_i,\hat{p}_j] = \hat{i}\hat{\times}\hat{\delta}_{i.j}, \;\; [\hat{r}_i,\hat{r}_j] = [\hat{p}_i,\hat{p}_j] = 0. \tag{2.25c}$$

Note that the above equations have been proved to be "universal" for the representation of closed-isolated systems with linear and non-linear, local and non-local and Hamiltonian as well as non-Hamiltonian internal forces [7]. Note also the characterization of the central dynamical equations of hadronic mechanics via the Lie-Santilli isoalgebras for the infinitesimal form (2.26a) and of the isogroups for the finite form (2.26b).

2.10. Isotopies of the Dirac's equation

With the following isoequations defined on the Minkowski-Santilli isospace $\hat{M}(\hat{x},\hat{\eta},\hat{I})$ on the isofield $\hat{R}$, first achieved in Ref. [20] of 1995

$$(\hat{\eta}^{\mu\nu}\hat{\times}\hat{\gamma}_\mu\hat{\times}\hat{p}_\nu - \hat{i}\hat{\times}\hat{m}\hat{\times}\hat{c})\hat{\times}\hat{\psi} = 0, \tag{2.26a}$$

$$\{\hat{\gamma}_\mu, \hat{\gamma}_\nu\} = \hat{\gamma}_\mu \times \hat{T} \times \hat{\gamma}_\nu + \hat{\gamma}_\nu \times \hat{T} \times \hat{\gamma}_\mu = 2\hat{\eta}_{\mu\nu} \tag{2.26b}$$

The above isoequations allow the first known study of an electron in the interior of a star under realizations of the isometric $\hat{\eta}$ for interior dynamical problem [7]. The same isoequations also allow the first known study of an electron under the gravitational field of the Sun (e.g., an electron in the chromosphere of the Sun) under gravitational realizations of the isometric [23].

Some of the main implications of isomathematcs and isomechanics, whose knowledge is essential to appraise their applications and experimental verifications, can be summarized as follows:

2.11. Rapid convergence of isoseries

Consider a divergent quantum mechanical series, such as the canonical series

$$A(w) = A(0) + (AH - HA)/1! + ... > \infty, \;\; w > 1. \tag{2.27}$$

Note from definition (2.6) that (by ignoring the terms representing the shape of particles) the isotopic element is much smaller than 1, and therefore, it is much smaller than the value of the parameter w,

$$|\hat{T}| = |e^{-\Gamma}| \ll w, \;\; \Gamma \gg 1. \tag{2.28}$$

It is then easy to see that the isocanonical image of series (2.27)

$$A(w) = A(0) + (A\hat{\times}T \times H - H \times \hat{T} \times A)/1! + ... > N < \infty, \tag{2.29}$$

is strongly convergent.

2.12. Consistent treatment of nonlinear interactions

As recalled earlier, nuclei are expected to admit internal interactions non-linear in the wavefunction and on other quantities due to the partial mutual penetration of the charge distribution of protons and neutron in a nuclear stricture.

One of the known insufficiencies of quantum mechanics is the inability to provide a quantitative representation of composite structures with said non-linear interactions because the ensuing Schrödinger equation

$$H(r, p, \psi, ...) \times \psi = E \times \psi \tag{2.30}$$

violates the superposition principle, thus prohibiting a quantitative representation of the individual constituents of said bound state.

By contrast, isomathematics and isomechanics readily solve the above insufficiency because they reconstruct linearity at the covering level of the Hilbert-Myung-Santilli isospace over isofields. Alternatively, this property can be seen from the fact that the following identical redefinition of Eq. (2.30)

$$\hat{H}(r, p) \times \hat{T}(\psi, ...) \times \psi = E \times \psi, \quad \hat{H}(r, p) \times \hat{T}([si, ...) = H(r, p, \psi, ...) \tag{2.31}$$

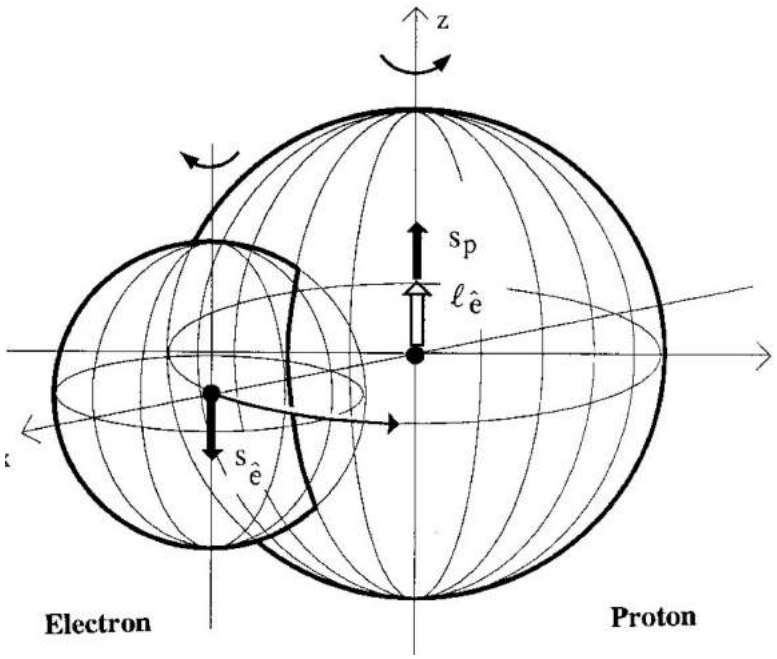

Figure 10: *The construction of hadronic mechanics was proposed by the author [1-3] to achieve a quantitative representation of the synthesis of the neutron in the core of a star from the hydrogen,* $p^+ + e^- > n+?$ *due to the complete inapplicability of quantum mechanics and special relativity because the rest energy of the neutron is* $0.783\ MeV$ *'bigger' than the sum of the rest energies of the proton and the electron. In this picture we indicate the most salient advancement permitted by isomathematics consisting in the representation of the proton as an extended particle, in which case there is the emergence of the constrained angular motion of the electron within the hyperdense proton that simply cannot exist for the point-like description of the proton by quantum mechanics. As a result of this new angular motion, the total angular momentum of the electron compressed inside the proton is null, and the spin of the neutron is that of the proton [50-58]. Intriguingly, there is no possibility for the introduction of a neutrino because of the lack of the needed angular momentum, the needed energy and other reasons in favor of Santilli etherino [59].*

is *isolinear,* that is linear on isospaces over isofields, since all non-linear interactions are embedded in the isotopic element (or, equivalently, in the isounit), with ensuing evident verification of the isosuperposition principle and consistently the consistent definition of the constituents as, of course, as *isoparticles.*

2.13. Elimination of the divergencies of quantum mechanics

As it is well known, the divergencies of quantum mechanics originate from the singularity existing at the origin of the *Dirac delta distribution.* When the latter is compared to the *Dirac-Myung-Santilli isodelta isofunction*

$$\delta(r) = \int e^{kr} dk, \quad \hat{\delta} = \int \hat{e}^{k \times \hat{T} \times r} dk \tag{2.32}$$

one can see the emergence in the latter of the isotopic element in the exponent of the integrand, with consequential possibility to eliminate said singularity.

The above feature has permitted the formulation of the *isoscattering theory* [29] which is strongly convergent as indicated earlier, thus avoiding the infinities of 20th century particle physics and their removal by subtracting infinities to infinities.

2.14. Isorelativities

The ultimate embodiment of all preceding mathematical and physical advances is provided by the *isotopies of the Galileo and special relativities,* nowadays known as *isorelativities,* first presented by the author in paper [16] of 1983, then studied in details in various works, such as monographs [26] and [7,8] (see also the forthcoming monograph [30]).

To avoid a prohibitive length, we here present the basic *isoaxioms* for relativistic motion in the k-space direction of a physical medium with local density n_4 which isoaxioms are applicable to time reversal invariant systems of isoparticles in interior conditions, and are uniquely and unambiguously characterized by the Lorentz-Santilli isosymmetry (2.19) [16] (see Refs. [23,24] for their gravitational version).

ISOAXIOM I: The speed of light within (transparent) physical media is given by:

$$C = \frac{c}{n_4}. \tag{2.33}$$

ISOAXIOM II: The maximal causal speed within physical media is given by:

$$V_{max,K} = c \frac{n_k}{n_4}. \tag{2.34}$$

ISOAXIOM III: The addition of speeds within physical media follows the isotopic law:

$$V_{tot} = \frac{\frac{V1.k}{n_k} + \frac{c_{2.k}}{n n_k}}{1 + \frac{v_1 v_2}{c^2} \frac{n_4^2}{n_k^2}}. \tag{2.35}$$

ISOAXIOM IV: The dilation of time, the contraction of lengths, the variation of mass and the energy equivalence within physical media follow the isotopic laws:

$$t'_k = \hat{\gamma}_k \, t, \tag{2.36a}$$

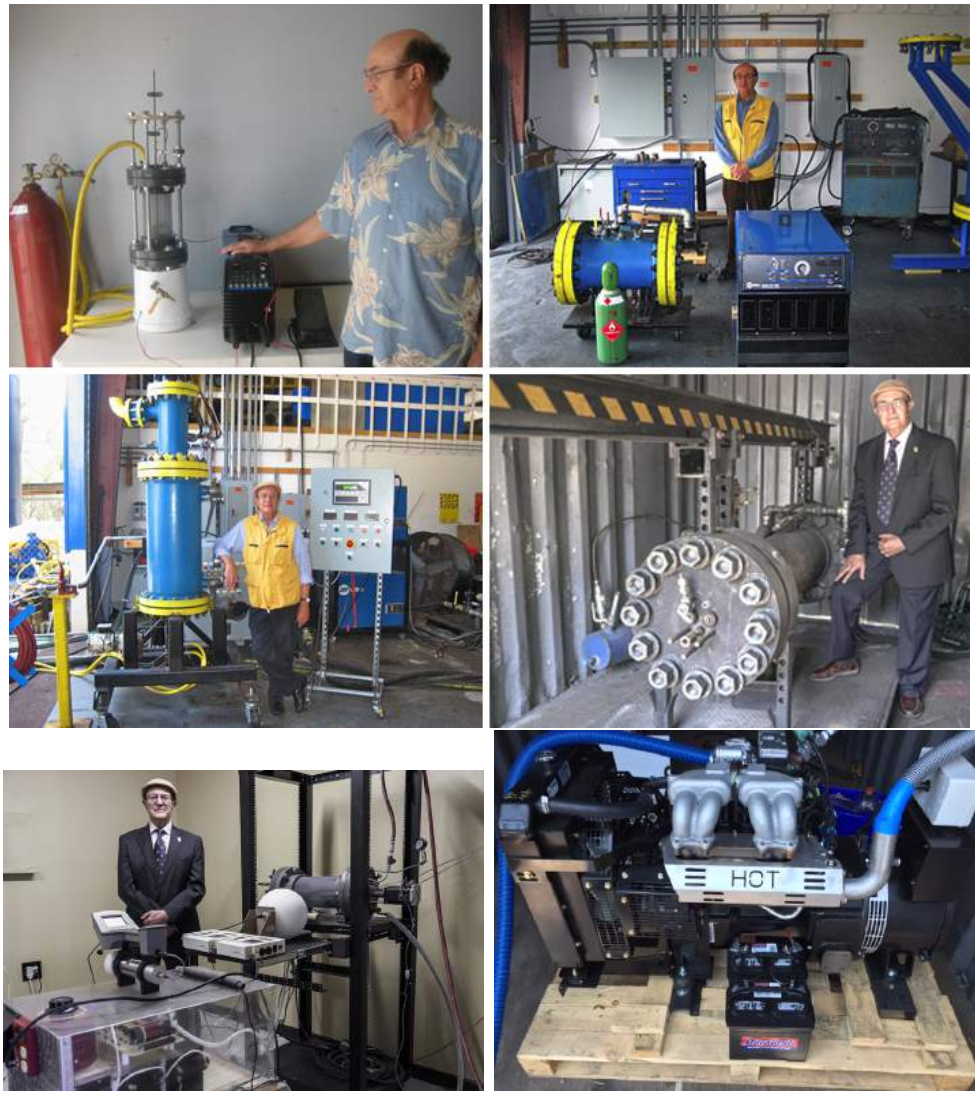

Figure 11: *A view of some of the equipment for the industrial synthesis of the neutron from a hydrogen gas according to isomathematics and isomechanics for the possible detection of nuclear material smuggled into container or suitcases under development by Thunder Energies Corporation, a publicly traded U.S. company with symbol TNRG with website http://thunder-energies.com/*

$$\ell'_k = \hat{\gamma}_k^{-1}\, \ell, \tag{2.36b}$$

$$m'_k = \hat{\gamma}_k\, m, \tag{2.36c}$$

$$\hat{E}_k = m\, V^2_{max,k} = mc^2 \frac{n_k^2}{n_4^2}, \tag{2.36d}$$

where

$$\hat{\beta}_k = \frac{v_3/n_k}{c_o/n_4},\ \ \hat{\gamma}_k = \frac{1}{\sqrt{1-\hat{\beta}_k^2}}. \tag{2.37}$$

ISOAXIOM V: The frequency shift within physical media follows the isotopic law (for null aberration)

$$\omega' = \hat{\gamma}\, [1 - \hat{\beta}\, cos(\hat{\alpha})]\, \omega. \tag{2.38}$$

Recall that the axioms of special relativity are valid in all space directions, trivially, because of the inhomogeneity and isotropy of the Minkowski spacetime. By contrast, the isoaxioms of isorelativity are solely valid in a given k-direction because physical media are generally inhomogeneous and anisotropic.

Note that the above isoaxioms are "universal" for all possible symmetric spacetimes (2.11) [22] and that they verify the crucial condition of predicting the same numerical values under the same conditions at different times due to the basic isoinvariance under the Lorentz-Santilli isosymmetry [11-21].

It should be finally noted that the isoaxioms are presented above in their *projection* on conventional Minkowski space because, when they are written on the Minkowski-Santilli isospace over isofields, they coincide with the conventional axioms to such a extent of admitting c as the maximal causal speed [7,25]. Hence, criticisms of the isoaxioms may eventually result to be criticisms on the conventional axioms of special relativity.

2.15. Experimental verification in water [7,8,26,30]

An instructive experimental verification of isorelativity is the dynamics in the simplest possible physical medium, ordinary water, where special relativity is afflicted by a number of insufficiencies of sheer inconsistencies (see Figures 4 and 5), including the inability to confirm experimentally the constancy of the speed of light, the existence of electrons traveling in water faster than the local speed of light (Cherenkov light), the violation of the relativistic sum (because the sum of two speeds of light in water does not yield the speed of light in water), and others.

It is easy to see that isorelativity resolves these insufficiencies. In fact, water is homogeneous and isotropic, as a result of which $n_\mu = 1,\ \mu = 1, 2, 3, 4$. In this case, the maximal causal speed in water, Eq. (2.34), is the speed of light in vacuum by therefore avoiding the violation of the principle of causality, the isorelativistic sum of two local light speeds c/n_4, Eq. (2.35) yields the high speed c/n_4, etc.

A suggestive verification is the representation by isorelativity of the apparent increase of the dimension of objects in water when inspected from the outside (Figure 4). This is an important application of isorelativity because the geometric dichotomy between interior and exterior observations is at the foundation of the deepest implications of isorelativity.

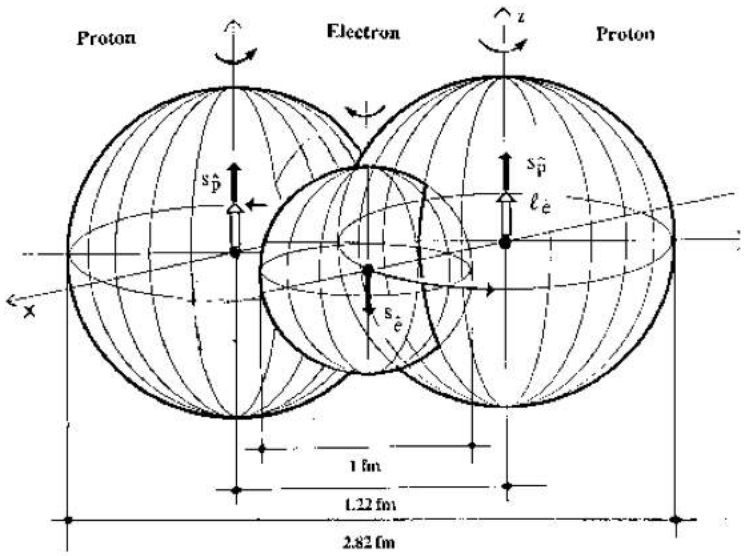

Figure 12: *Yet another reason for the 1979 proposal to construct hadronic mechanics is the inability by quantum mechanics to achieve a consistent structure model of the simplest nucleus, the deuteron, due to the lack of exact representation of its magnetic moment and other features. Most significant is the inability by quantum mechanics to represent the 'ground state' of the deuteron as a bound state of a proton and a neutron in the expected singlet coupling (because in this case the spin of the deuteron would be* 0*). The achievement of a representation of the synthesis of the neutron, and its reduction to a generalized bound state of one isoproton and one isoelectron, has resolved the above insufficiencies. In particular, the spin* 1 *of the deuteron in its true ground state is clearly represented via the three-body bound state of this figure, consisting of two isoprotons and one isoelectron [60]. It should be noted that these studies provide a realistic possibility of reducing all matter in the universe to isoprotons and isoelectrons.*

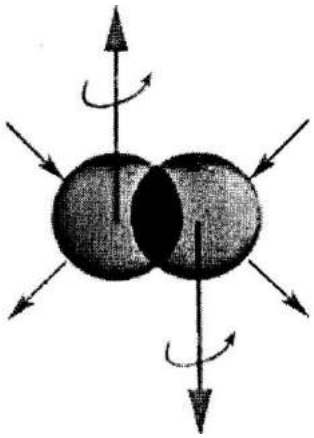

Figure 13: *A further reason for the 1978 proposal to build hadronic mechanics [1-3] is the impossibility for quantum mechanics and chemistry to represent molecular structure because identical valence electrons experience a 'repulsive' Coulomb force which becomes very large at* $1\ Fermi$ *mutual distance. The covering hadronic mechanics has resolved this additional impasse by showing that the non-linear, non-local, and non-Hamiltonian interactions due to total mutual penetration of the wavepackets in singlet coupling (depicted in this figure) produce such a strongly attractive force to overcome said Coulomb repulsion. The emerging new valence model, known as 'Santilli strong valence bond', has been shown to allow the first known exact representation from first principle of the binding energy and other characteristics of the hydrogen and water molecules [61-64].*

In fact, the geometric effect of Figure 5 is a visible occurrence of the more general the notion of *isorenormalization* introduced by the author in paper [2] of 1978 which indicates the mutation of the *intrinsic* characteristics of particles when in interior conditions, such as the mutation of time (2.36a), dimension (2.36b), mass (2.36c) and energy (2.36d) in the transition from motion in vacuum to motion within physical media. In turn, these isorenormalizations are crucial to achieve a representation of basic problems, such as the synthesis of the neutron from the hydrogen in the core of stars indicated below.

2.16. Experimental verification in Earth's atmosphere [7,8,26,31-37]

The insistence in the exact applicability of special relativity in our atmosphere is beyond the boundary of science because the axioms of special relativity require our homogeneous and isotropic spacetime, while our atmosphere is inhomogeneous and anisotropic, thus implying the inapplicability of the ultimate geometric foundations of special relativity.

Due to the dependence of the characteristic n-quantities in local variables, Eqs. (2.5), the Doppler-Santilli Isoaxiom V admits the expansion for the frequency shift of light propagating in our atmosphere

$$\Delta\omega = \pm\frac{v_k}{c} \pm H_k d, \tag{2.39}$$

where: H_l is (approximately) a constant; d is the distance covered by light; $\pm v/c$ is the standard Doppler effect, and $\pm Hd$ is the new *Santilli isoshift* describing the decrease (increase) of the frequency for light propagating within a gaseous medium at low (high)

temperature without any relative motion between the source, the medium and the observers.

Let us recall that, the latter effects are known as *isoredshift* (*isoblueshifts*), they merely consist of the release of energy by light to (acquisition of energy by light from) the medium, thus requiring no relative motion, and these new effects have received a rather wide experimental verification, firstly, for individual laser lights, and secondly, for the solar spectrum.

2.17. Experimental verification in astrophysics [8,38]

Einstein, Hubble, Hoyle, Zwicky, Fermi, de Broglie and other famous scientists died without accepting the expansion of the universe because the representation of Hubble's law on the cosmological redshift of galactic light with the Doppler axiom of special relativity, $z = Hd = v/c$ (where now H is Hubble's constant and d is the travel of galactic light to reach us) implies a return to the Middle Ages with Earth at the center of the universe due to the dependence of the redshift from all possible *radial* directions from Earth.

Isorelativity has provided an exact and time invariant representation of the cosmological redshift as being due to the loss of energy by galactic light to the very cold intergalactic medium (Zwicky's hypothesis of the *Tired Light*), without any need conjecturing trillions of galaxies moving away from Earth at even increasing speeds, and without the need for the very hyperbolic and unverifiable conjecture of the expansion of space itself proffered to avoid Earth at the center of the universe. In any case, the geometric inconsistency in the r.h.s. of Figure 7 equally applies for the expansion of space due to the inherent acceleration.

It should be noted that the isoshift provides also a quantitative representation of the anomalous redshift of galactic stars without the conjecture of dark matter that, in any case, never represented said anomalous redshift for which it was proposed. The return to the stationary universe advocated by Einstein, Hubble, Hoyle, Zwicky, Fermi, de Broglie and other scientists eliminates the need for the additional unverifiable conjecture of the dark energy.

It should be also noted that *the sole, actually measured numerical value in astrophysics is the cosmological redshift of galactic light according to Hubble's law* $z = Hd$. All other numerical values in astrophysics are derived via a plethora of assumptions, including the validity of special relativity in the intergalactic medium, the conjecture of the expansion of the universe, the conjecture of the acceleration of he expansion, etc.

It should be stressed that *the experimental verification provided in measurements [31-37] of Zwicky's historical hypothesis of light losing energy to the intergalactic medium during its long travel to reach Earth, will inevitably require a revision of the virtual entirety of the numerical values of 20th century astrophysics, beginning with revisions of the distance of galaxies, due to the need for adjustments of the luminosity of supernova and other means currently used to compute astronomical distances.*

2.18. Experimental verification in meanlives of unstable hadrons [8,39-44]

Hadrons are the densest media measured by mankind in laboratory to date. Within such

media, the widespread assumption of the exact validity of special relativity is also beyond the boundary of science due to the impossibility for even defining the basic axioms, let alone subjecting them "directly," that is, without their usual assumptions in the data elaboration.

Yet protons in particle accelerators are known to verify special relativity because they can be well approximated as point particles moving in vacuum. Under these conditions, the internal deviations from special relativity that can be measured in the outside are given to deviations from the Einsteinian time dilation law that are measurable for the behavior of the meanlives of unstable hadrons with speed.

Experimental works [39-44] have indeed verified the expected deviations from special relativity in the interior of hadrons and confirmed the validity of the isotime isodilation law (2.36a).

2.19. Experimental verification in the Bose-Einstein correlation [8, 44-46]

The Bose-Einstein correlation requires *four* arbitrary parameters for the fit of experimental data (called the "chaoticity parameters"), but the two-point correlation function can only provide *two* arbitrary parameters (under certain manipulations).

Refs. [44-46] have shown that the chaoticity parameters are , in reality, a measure of the *deviations* of special relativity from the experimental data of the Bose-Einstein correlation, and have shown that isorelativity is exact.

In particular, the fit of the experimental data establishes *the space characteristic quantities represent the semi-axes of the very elongated proton-antiproton fireball, while the fourth characteristic quantity represents its density.*

2.20. Experimental verification in nuclear magnetic moments [47-48]

The degrees of freedom achieved by the Lie-Santilli isotopies of the rotational and spin symmetries have permitted the first known, numerically exact and time invariant representation of the magnetic moments of stable nuclei in a way fully aligned with Fermi's historical hypothesis of Figure 3.

2.21. Experimental verification in nuclear spins [49]

The isotopic degrees of freedom that have permitted a representation of nuclear magnetic moments, has also permitted the first known exact and time invariant representation of the spin of stable nuclei.

2.22. Experimental verification in the synthesis of the neutron from the hydrogen [19,20,50-59]

One of the topics, whose study is carefully avoided at Ph. D. courses in physics, is the representation of the synthesis of the neutron from the hydrogen in the core of stars because of the complete inapplicability of special relativity and quantum mechanics because the rest energy of the neutron is $0.782\ MeV\ bigger$ than the sum of the rest energies of the proton and the electron, under which conditions the Schrödinger equations for a bound state and the Dirac equations provide no physical results at all.

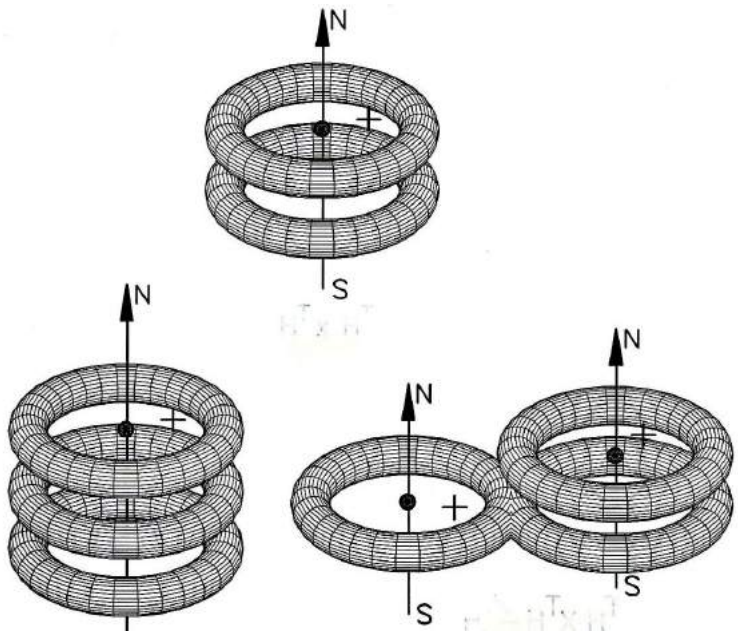

Figure 14: *Another application of isomathematics and isomechanics that should be mentioned in this paper, is the discovery in 1998 of the new chemical species of 'magnecules' consisting of individual atoms, dimers and ordinary molecules bonded together by opposing polarities of the magnetic field created by the toroidal polarization of the electron orbits. This picture illustrates: in the top an 'elementary magnecule' such as $C \times O$, where $\times$ denotes the new magnecular bonds; on the bottom left a magnecule such as $H \times H \times H$; and in the bottom right the magnecular bond of a conventional molecule, such as $C - O$ (where $-$ denotes the conventional valence bond) with an atom, such as H. The environmental importance of magnecules is that their bond is stable at ambient temperature, but 'weaker' than the valence bond, thus allowing the first known complete combustion, that is, a combustion without combustible contaminants in the exhaust.*

As clearly expressed in Refs. [1-3], the author proposed in 1978 the construction of a non-unitary covering of quantum mechanics under the name of hadronic mechanics precisely for the intent of achieving a quantitative representation of the synthesis of the neutron inside stars.

The efforts provided by numerous mathematicians, theoreticians and experimentalists around the world during the past thirty eight years have indeed permitted isomathematics at large and isomechanics in particular to achieve an exact and invariant representation of all characteristics of the neutron in its synthesis from the hydrogen, firstly, at the non-relativistic [50-59] and then at the relativistic level [19,20].

These scientific results have permitted the birth of a new technology, that of the laboratory synthesis of the neutron from the hydrogen which is under development by the U. S. publicly traded company $Thunder\ Energies\ Corporation$ (with stock symbol TNRG) to achieve means suitable to detect fissionable materials that could be smuggled into containers or suitcases.

2.23. Experimental verification in nuclear structures [60]

In the author's view, one of the strongest evidence on the lack of exact character of quantum mechanics in nuclear physics is its inability to provide a consistent representation of the experimental data of the simplest nucleus, the deuteron, including the inability to achieve an exact representation of the deuteron magnetic moment, spin, rest energy, and other data.

Monograph [60] has shown that, the reduction of the neutron to a bound state of one isoproton and one isoelectron, allows a consistent representation of all characteristic of the deuteron. In particular, the spin 1 of the deuteron emerges as being strong evidence that $the\ deuteron\ is\ a\ three\ body\ bound\ state$ of two isoprotons and one isoelectron, rather than being a two-body bound state of one proton and one neutron assumed for about one century.

It should be kept in mind that the reduction of the deuteron to two isoprotons and one isoelectron implies the reduction of all matter in the universe to isoprotons and isoelectrons in various conditions of complexity.

2.24. Experimental verification with molecular structures [61-64]

One of the best kept secrets during Ph. D. courses in chemistry, is the impossibility for quantum chemistry to represent molecular structures because two identical valence electron experience the known Coulomb repulsions that reach very big values at $1Fermi$ mutual distances, thus preventing any bond.

Refs. [61-64] have shown that the non-unitary covering of quantum chemistry proposed by the author under the name of $hadronic\ chemistry$ does indeed achieve a strongly attractive force between two identical electrons in singlet couplings, thanks to the nonlinear, non-local and non-Hamiltonain forces emerging in deep mutual penetration of the wavepackets that are so strong to overcome said Coulomb repulsion (Figure 13).

Additionally, Refs. [61-64] have shown that the emerging isovalence bond permits the only known numerically exact and time invariant representation of the binding energies of the hydrogen and water molecules from unadulterated first principles. These studies

Figure 15: *A view of some of the equipment for the industrial production of a gaseous fuel with complete combustion, including its use as automotive fuel meeting federal standards without catalytic converters that are being use by Magnegas Corporation traded at NASDAQ under the symbol MNGA with website http://magnegas.com*

have also confirmed the lack of divergencies in isomechanics and isochemistry, as well as the rapid convergence of isoseries indicated earlier.

2.25. Experimental verification with magnecular structures [61,64]

One of the most significant scientific and industrial applications of isomathematics and isochemistry has been the prediction and quantitative study of a new chemical species, known as *Santilli magnecules,* allowing the industrial production of the first known fuel with complete combustion, that is, without combustible components in its exhaust.

In turn, the new chemical species has permitted significant environmental applications at the U. S. publicly traded company *Magnegas Corporation* (traded at NASDAQ under the symbol MNGA), such as the use as fuel of stock cars operating without catalytic converters, while meeting all federal requirements in their exhaust (see the last picture of Figure 15).

2.26. Experimental verifications with new clean energies

A primary reason for the construction of isomathematics, isomechanics and isochemistry by scholars from various countries, including advance in molecular and magnecular structures, has been that of developing much needed, new, environmentally acceptable energies.

However, as indicated earlier, all energy releasing processes are irreversible over time, while isomathematics, and therefore, isomechanics and isochemistry, are strictly reversible over time since they have been conceived to treat *isolated* composite systems of extended particles at short mutual distances, with ensuing non-Hamiltonian internal forces.

Consequently, the quantitative treatment of new clean energies requires isotopic elements (2.50 with an explicit dependence on time in a time-reversal non-invariant way (see later on, Eqs. (3.27)) which characterize in reality the covering Lei-admissible genomathematics, genomechanics and geochemistry. As such, new clean fuels will be discussed in the next section.

2.27. Formulation of isomathematics via Vougiouklis hyperstructural axioms

All preceding sections and related references [1-67] deal with *single valued* mathematics, physics and chemistry in the sense that one given operation (for instance, the multiplication of two numbers or isonumbers) yields one single result.

Despite its covering character with respect to 20th century mathematics, said single-valuedness resulted to be too restrictive in face of the complexity of nature, e.g., for the study of biological structure.

The above limitation was resolved by the mathematicians by extending isomathematics to the most general possible formulation with an anti-symmetric algebraic bracket, that of hyperstructures based on *Vougiouklis hyperstructural axioms,* see representative Ref. [68] and numerous papers by T. Vougiouklis and his associates listed in free pdf download in the archives [65].

In the author's view, the hyperstructural isomathematics constitutes the most general possible, multi-valued mathematics for the representation of complex systems reversible

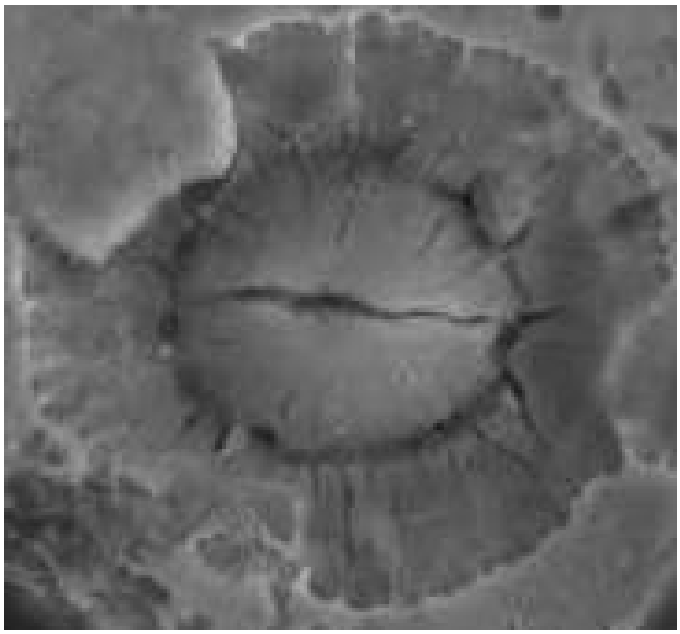

Figure 16: *One of the expected new applications of isomathematics discussed, following the keynote talk by the author at the 2016 ICNPAA congress, is the study of surface stress in metal. As it is well known, such a study is today based on the Newton-Leibnitz differential calculus, thus being restricted to individual points of the metal surface. By comparison, isomathematics allows the study of entire areas or volumes surrounding individual points, with evident increase of the representational capability, plus increased rapidity of calculations due to the fast convergence of isoseries of type (2.27).*

over time, while the hyperstructural formulation of the covering lie-admissible mathematics indicated at the end of the next section constitutes the most general mathematics that can be conceived nowadays by the human mind.

In this section, we have been regrettably forced solely to quote, to our best knowledge, the originating literature per each subsections so as to avoid a prohibitive length as well as inevitable discriminatory listings. However, comprehensive listing of references can be found in Vol. I of Refs. [8] and in the scientific archives [65]. Comprehensive independent review of isomathematics, isomechanics and isochemistry can be found in Refs. [66,67] and in other independent reviews quoted later on.

3. GenoMathematics and its applications

3.1. Basic assumptions

We can introduce *genomathematics* as a Lie-admissible covering of the Lie-isotopic isomathematics for the representation of *time irreversible* systems of extended and deformable particles under the most general known linear and non-liner, local and non-local and Hamiltonian as well as non-Hamiltonian interactions.

The research here reported was initiated by the author during his Ph. D. studies in theoretical physics at the University of Torino, Italy, in the mid 1960's. As part of these studies, the author had to identify methods for the representation of systems that are irreversible over time, as they occur in the physical reality.

According to a widespread view that persists to these days, the problem of irreversibility does not exist because macroscopic irreversible systems can be reduced to their particle constituents at which level the time reversibility of quantum mechanics and special reality is recovered in full.

However, as part of his Ph. D. thesis, the author proved the following property [72-74]:

THEOREM 3.1. An irreversible macroscopic system cannot be consistently reduced to a finite number of elementary particles all in reversible conditions, and vice versa, a finite number of elementary particles verifying the laws of quantum mechanical and special relativity, cannot under the correspondence or other principles, characterize a time irreversible macroscopic system, or achieve compatibility with entropy and thermodynamical laws.

The proof of the above theorem showed that the insufficiencies did not occur in the physical laws of quantum mechanics and special relativity, but instead they occurred in the mathematics underlying said. In fact, the ultimate origin of the time reversibility of 20th century theories emerged as being due to the *invariance of the Lie commutators under anti-Hermiticity*

$$[A, B] = -[A.B]^\dagger, \quad A = A^\dagger, \;\; B = B^\dagger \tag{3.1}$$

Since quantum mechanics and special relativity are crucially dependent on Lie's theory for the formulation of their laws, their time invariance then followed.

3.2. Irreversibility of Lagrange's and Hamilton's mechanics

In view of the above theorem, the author initiated his search for a mathematics that which is time irreversible in its basic structure, as needed for an effective representation of irreversibility. Following a search in various libraries, no mathematics verifying such a requirement was identified. Therefore, the needed new mathematics had to be built.

The first step in the search here considered was the study of the *original Lagrange [69] and Hamilton [70] equations, those with the external terms*

$$\frac{d}{dt}\frac{\partial L(t,r,v)}{\partial v^k_a} - \frac{\partial L(t,r,v)}{\partial r^k_a} = F_{ak}(t,r,v), \tag{3.2a}$$

$$\frac{dr^k_a}{dt} = \frac{\partial H(t,r,p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t,r,p)}{\partial r^k_a} + F_{ak}(t,r,p), \tag{3.2b}$$

$$L = \Sigma_a \frac{1}{2} \times m_a \times v_a^2 - V(t,r,v), \quad H = \Sigma_a \frac{\mathbf{p}_a^2}{2 \times m_a} + V(t,r,p), \tag{3.2c}$$

$$V = U(t,r)_{ak} \times v_a^k + U_o(t,r), \quad F(t,r,v) = F(t,r,p/m). \tag{3.2d}$$

Since the above equations did indeed represent irreversibility thanks to the external terms that had been "truncated" at the advent of quantum mechanics and special relativity.

However, the brackets of the time evolution of an observable represented via Hamilton's equations with external terms

$$\frac{dA}{dt} = (A,H,F) = \frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ka}} - \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ka}} + \frac{\partial A}{\partial r_a^k} \times F_{ka}, \tag{3.3}$$

characterizes a *triple system* $(A, H.F)$ THAT, AS SUCH, violates the axioms to characterize an algebra as currently understood in mathematics, precisely in view of the external terms.

In the absence of a consistent algebra in the brackets of the time evolution, it was not evidently possible to achieve a representation of irreversible systems as a covering of 20th century conventional methods for reversible systems.

3.3. Albert's Lie-admissible and Jordan-admissible algebras

Following extensive research in European mathematical libraries, the author identified paper [71] of 1948 by the American mathematician A. A. Albert presenting the notions of *Lie- admissible and Jordan admissible algebras* that can be characterized as follows:

DEFINITION 3.1: Let U be a generally non-associative algebra over a field F of characteristic zero with abstract elements a, b and product ab. The algebra U is called a Lie-admissible algebra when the attached anti-symmetric algebra U^-, with is the same vector space as U equipped with the product $[a,b] = ab - ba$, is a Lie algebra. The algebra U is called a Jordan-admissible algebra when the attached symmetric algebra U^+, which is the same vector space as U equipped with the product $\{a,b\} = ab + ba$, is a Jordan algebra.

It should be noted that Lie-admissible algebras are the most general possible algebras, since they include as particular cases all known algebras, such as associative, Lie, Jordan, flexible, nilpotent and other algebras. Therefore, Lie-admissible algebras are ideal candidates for the construction of a covering of quantum mechanics and special relativity as well as of their isotopic generalizations.

3.4. Lie-admissible mutations of Lie algebras and mechanics

In paper [71], Albert merely introduced the notion of Lie-admissible algebras without specific realizations as needed in physics. Therefore, as part of his Ph. D. thesis, the author published paper [72] of 1967 with the following specific realization of the following product as a covering of the Lie product

$$(A,B) = \mu AB - \nu BA \neq \pm (A,B)^\dagger, \tag{3.4a}$$

$$[A,B]* = (A,B) - (B,A) = (\mu + \nu)(AB - BA), \tag{3.4b}$$

Figure 17: *Another expected engineering application of isomathematics is the application of the optical control theory to actual shapes under resistive forces as depicted in this figure, which application is permitted, for the first time by isovariational principle (2.21). In this case, non-linear differential equations are turned into identical isolinear forms with evident advantages in their solution.*

$$\{A, B\}* = \{A.B\} + \{B, A\} = (\mu - \nu)(AB - BA), \tag{3.4c}$$

which is jointly Lie-admissible and Jordan-admissible under the conditions that $\mu,\ \nu$, and $\mu \pm \nu$ are non-null scalars, where the product AB is generally non-associative, with the understanding that its associative realization is a trivial particular case.

In papers [73,74] (that are also parts of his Ph. D. thesis), the author introduced the following generalization of Hamilton's equations

$$\frac{dr_a^k}{dt} = \mu \times \frac{\partial H(t, r, p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\nu \times \frac{\partial H(t, r, p)}{\partial r_a^k}, \tag{3.5}$$

under which the brackets of the time evolution characterize indeed a consistent algebra and that algebra results to be Lie-admissible

$$\frac{dA}{dt} = (A, H) = \mu \times \frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ka}} - \nu \times \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ka}}. \tag{3.6}$$

In the same Ph. D. thesis [73,74], the author also introduced the corresponding operator image of the above equations given by the generalized Heisenberg's equation in their infinitesimal and finite forms

$$i \times \frac{dA}{dt} = (A, H) = \mu \times A \times H - \nu \times H \times A, \tag{3.7a}$$

$$A(t) = e^{H \times t \times \nu \times i} \times A(0) \times e^{-i \times \mu \times t \times H}, \tag{3.7b}$$

which can be considered as the birth of hadronic mechanics [1-3,7].

Subsequently, the author reformulated the time evolution of the density matrix in statistical mechanics via a Lie-admissible bracket

$$\frac{d\rho}{dt} = (\rho, H). \tag{3.8}$$

Subsequently, due to a widespread lack of interest in the representation of irreversible systems, the author passed to the publication of "Phys. Rev" type of paper.

As recalled in Section 2, in 1978 the author joined the faculty of Harvard University under support from the Department of Energy that required the study of irreversibility since all energy releasing processes are irreversible over time.

In this way the author resumed full time research in Lie-admissible formulation, beginning with the identification of the most general known, jointly Lie-admissible and Jordan-admissible covering of Eqs. (3.8), first published in Refs. [1-3]

$$i \times \frac{dA}{dt} = (A, H) = A \times R \times H - H \times S \times A, \tag{3.9a}$$

$$A(t) = e^{H \times S \times t \times i} \times A(0) \times e^{-i \times t \times R \times \times H}, \tag{3.9b}$$

where, this time, $R,\ S$ and $R \pm S$ are invertible matrices or operators.

Jointly with works [1-3], the author published monographs [76] on the Lie-admissible structure of hadrons which illustrate the reason for the suggested name "hadronic mechanics."

The initiation of studies at Harvard University on the Lie-admissible representation of irreversible systems attracted world wide interest (as well as organized opposition indicated in Section 5), with a flurry of activities, including; five *Workshops on Lie-admissible Formulations* held at Harvard University; the *First International Conference on the Lie-admissible Treatment of Irreversible Systems* held in 1981, at the University of Orleans, France; the first five of the series of *Workshops on Hadronic Mechanics* held at the Institute for Basic Research and other meetings.

To avoid excessive length, detailed references on the proceedings of these meetings can be found in pages 159 to 161, Vol. I of Refs. [8]. Some of the most important initial contributions on Lie-admissible formulations are given by Refs. [77-85]. A comprehensive list of mathematical papers pertinent to Lie-admissible formulations is given by *Tomber's bibliography* [86]. A mathematical study on Lie-admissible algebras as provided by the mathematician H. C. Myung in monograph [87] who was also a co-organizer of the indicated mathematical meetings. Collection [88] provides reprints of the most important papers in the Lie-Admissible treatment of irreversible systems up to 1989.

3.5. Catastrophic inconsistencies of non-unitary theories

Following these initial studies, various authors [89-95] became aware of the fact that, as formulated in the preceding references, Lie-admissible formulations were afflicted by a number of inconsistencies that can be expressed via the following:

THEOREM 3.2 [89-95]: All operator theories possessing a non-unitary time evolution formulated on conventional Hilbert spaces over conventional fields, thus including Lie-admissible

and Lie-isotopic theories, are afflicted by mathematical and physical structural inconsistencies. In particular, said theories:

1) Do not leave invariant the multiplicative unit of the base field, and consequently, do not possess time invariant units of time, space, energy, etc., thus lacking physically meaningful application to measurements;

2) Do not conserve Hermiticity over time, thus lacking physically meaningful observables;

3) Do not possess unique and invariant numerical predictions;

4) Generally violate probability and causality laws;

5) Violate the basic axioms of Galileo's and Einstein's relativities.

Among a rather large class of non-unitary theories verifying the insufficiencies of Theorem 3.2 (see the classification and treatment in Ref. [99]), we mention the so-called *q-deformations of Lie algebras* with the product

$$(A, B) = A \times B - q \times B \times A, \tag{3.10}$$

that were introduced in 1989, by L. C. Biedenharn [96] and R. M. Macfairlane [97] in complete oblivion of Theorems 3.2 and of the origination of the deformation by Santilli in 1967 [72], both information being documentedly known to them (among various documents, Biedenharn and Santilli also applied for a joint DOE grant in the early 1980's on Lie-admissible deformations of Lie algebras).

Following the appearance of Refs. [96,97] there was a flurry of over ten thousand papers on q-deformations, all published without the quotation of their origination [72], and those quoting Ref. [72] (such as Ref. [84]) were rejected by mainstream journals. As a result of this occurrence, the author was dubbed "the most plagiarized physicist of the 20th century."

Ironically, the author never objected against this plagiarism because he did not want his name be associated to theories with catastrophic inconsistencies fully known in the early 1990's, as a result of which said theories will not resist the test of time.

3.6. The need for a new mathematics

The above scenario established the need to complete the construction of Lie-admissible formulations into such a form achieving the crucial invariance over time despite its tire irreversibility, which was a manifestly non-trivial task.

Following numerous trials and errors, the needed completion of Lie-admissible formulations was achieved, thanks to the discovery of *genonumbers* in Ref. [4] of 1993 and of the *genodifferential genocalculus* in memoir [6] of 1996. These advances permitted the achievement in Ref. [98] of 1997 of the first known time invariant formulation of q-deformations and Lie-admissible formulations at large, despite their structural irreversibility. The latest comprehensive mathematical and physical study of Lie-admissible formulation is available in memoir [99] of 2006.

The above studies have produced *two* complementary generalizations of isomatheatics for irreversible systems known under the names of *forward and backward genomathematics* whose main aspects are outlined in the following sections.

Figure 18: *An additional expected application of isomathematics is the possible development of a new generation of drone controls based on the transition from the current drone modeling based on their center of gravity, to the representation of their volumes represented by the isotopic element T in the isodifferential calculus (2.12) as perceived by birds when flying in flock formation with evident advantages in safety. Note that the same model is applicable to planes when flying in proximity of each other, e.g. when near busy airports.*

3.7. Forward and backward genoproducts

The central assumption proposed by the author in 1978 [1,2,3,76] for the representation of irreversible systems is the use of *two ordered multiplications* between arbitrary nonsingular quantity A, B (numbers, functions, matrices, etc.), including the *multiplication for motion forward in time,* called motion forward in time called *forward genoproduct*

$$A\hat{>}B = A \times \hat{T}^{>} \times B, \ , \tag{3.11a}$$

$$\hat{T}^{>} = Diag.(\frac{1}{n_1^{>2}}, \frac{1}{n_2^{>2}}, \frac{1}{n_3^{>2}}, \frac{1}{n_4^{>2}}) \times e^{-\Gamma^{>}}, \tag{3.11b}$$

$$n_\mu^{>} = n_\mu((t, r, v, E, \mu, \tau, \xi, \psi, \partial\psi, ...) > 0, \ \ \mu = 1, 2, 3, 4, \tag{3.11c}$$

$$\Gamma^{>} = \Gamma^{>}(t, r, v, E, \mu, \tau, \xi, \psi, \partial\psi, ...) > 0. \tag{3.11d}$$

where one recognizes the representation of extended shapes and non-conservative forces as in Eqs. (2.5) and the *multiplication for motion backward in time,* called *backward genoproduct*

$$A\hat{<}B = A \times^{<} \hat{T} \times B, = A, \tag{3.12a}$$

$${}^{<}\hat{T} = Diag.(\frac{1}{{}^{2<}n_1}, \frac{1}{{}^{<2}n_2}, \frac{1}{{}^{<2}n_3}, \frac{1}{{}^{<2}n_4}) \times e^{-{}^{<}\Gamma^{>}} \tag{3.12b}$$

$${}^{<}n_\mu = n_\mu((-t, r, v, E, \mu, \tau, \xi, \psi, \partial\psi, ...) > 0, \ \ \mu = 1, 2, 3, 4, \tag{3.12c}$$

$$^{<}\Gamma =^{<} \Gamma(-t, r, v, E, \mu, \tau, \xi, \psi, \partial\psi, ...) > 0, \tag{3.12d}$$

where the quantities $\hat{T}^{>}$ and $^{<}\hat{T}$, called *forward and backward genotopic elements,* respectively, are solely restricted to be invertible, but otherwise possesses an unrestricted functional dependence on all needed local quantities, as it is the case for the isotopic element (2.5).

Assumptions (3.11) and (3.12) *assure* a representation of irreversibility under the conditions that the forward and backward genotopic elements have very different values

$$\hat{T}^{>} \neq^{<} \hat{T}, \tag{3.13}$$

for instance, via simple realization of the *characteristic genoquantities* of the typo $n_\mu(t, ...) \neq n_\mu(-t, ...)$.

As an illustration, we indicate that, under assumptions (3.11)-(3.13), the *forward and backward geno-Hilbert spaces* [6] are incoherent to each other, thus preventing any meaningful conversion of events from motion forward to motion backward in time.

As indicated in Section 2, the prefix "iso" was suggested in 1978, for isomathematics and isomechanics by Carla Santilli to indicate the *preservation* of 20th century axioms. By contrast, the prefix "geno" was also suggested by Carla Santilli in 1978, but this time, to indicate the *generation* of new axioms as a condition to represent irreversibility, due to the above indicated reversibility of 20th century axioms.

3.8. Forward and backward genonumbers

The re-inspection of the axioms of numeric fields done in Ref. [4] established that:

A) The multiplicative unit can be an arbitrary invertible quantity with compatible lifting of the multiplication, resulting in the discovery of the isonumbers reported in Section 2.

B) The multiplication of numbers $n,\ m$ is not required to be both, to the right here denoted

$$n > m, \tag{3.14}$$

(meaning n multiplies m to the right) and to the left, here denoted

$$n < m, \tag{3.15}$$

(namely, m multiplies n to the left).

In fact, Ref. [4] proved that, given a numeric field $F(n, \times, 1)$, the ordering of all products to the right $F(n, \hat{>}, 1)$ and separately, to the left $F(n, \hat{<}, 1)$ continued to verify all axions of a field.

The combination of properties A and B then permitted the discovery in paper [4] of the *forward and backward genofields* $\hat{F}^{>}(\hat{n}^{>}, >, \hat{I}^{>})$ and $^{<}\hat{F}(^{<}\hat{n}, <, ^{<}\hat{I})$, respectively, with *forward genoreal, genocomplex and genoquarternionic genonumbers*

$$\hat{n}^{>} = n > \hat{I}^{>}, \tag{3.16}$$

corresponding *backward genonumbers*

$$^{<}\hat{n} =^{<} \hat{I} < n, \tag{3.17}$$

Figure 19: *It is widely believed that systems irreversible over time, such as the spaceship during reentry of this figure, are illusory in the sense that when the systems are reduced to their elementary particle constituents, the time reversibility of 20th century theories is recovered in full. The author proved a theorem as part of his Ph. D. thesis in the mid 1960's according to which such a reduction is inconsistent as well as incompatible with entropy and thermodynamical laws, thus identifying the need for new structurally irreversible methods that eventually lead to the construction of the new genomathematics.*

forward genounits

$$\hat{I}^{>} = 1/\hat{T}^{>}, \tag{3.18}$$

with related *forward genoidentity*

$$\hat{n}^{>}\hat{>}\hat{I}^{>} = \hat{I}^{>}\hat{>}n^{>} \equiv \hat{n}^{>} \ \forall \, \hat{n}^{>} \in \hat{F}^{>}, \tag{3.19}$$

backward genounits

$$^{<}\hat{I} = 1/^{<}\hat{T} \neq \hat{I}^{>}, \tag{3.20}$$

with related *backward genoidentity*

$$^{<}\hat{n}\hat{<}^{<}\hat{I} =^{<} \hat{I}\hat{<}^{<}\hat{n} \equiv^{<} \hat{n} \, \forall \, ^{<}\hat{n} \in^{<} \hat{F}, \tag{3.21}$$

forward genoproducts

$$\hat{n}^{>}\hat{>}\hat{m}^{>} = (n > m) > \hat{I}^{>} \in \hat{F}^{>}, \tag{3.22}$$

and *backward genoproducts*

$$^{<}\hat{n}\hat{<}^{<}\hat{m} =^{<} \hat{I}\hat{<}(n < m) \in^{<} \hat{F}. \tag{3.23}$$

As it is the case for isofields, genofields are classified into genofields of the first (second) kind depending on whether the genounits are (are not) elements of a conventional field. Note that for genofields of the first kind, not only $2 > 3$ does not necessarily yield 3, but also $2 > 3 \neq 2 < 3$.

Evidently, all operations for isonumbers, such as the isosquare, isoroot, etc., are lifted in the above dual form.

Note finally that, by conception and construction, forward (backward) genofields are isomorphic (anti-isomorphic) to conventional field. The representation of irreversibility is also assured by the lack of equivalence of forward and backward genofields.

As a final comment, we recall that the conventional sum is not generalized under genotopies as a condition to verify the axioms of a numeric field, which verification is needed, in turn, to verify the predictions of genomathematics with experimental measurements.

3.9. Forward and backward genofunctional genoanalysis

The *genofunctional genoanalysis* is readily achieved via a dual lifting of the isofunctional isoanalysis via the use of isoproduct and must be defined, for consistency on a genofield. As an illustration, the *forward genoexponentiation* can be written

$$\hat{e}_{>}^{\hat{X}^{>}} = \hat{I}^{>} + \hat{X}^{>} \hat{>} \hat{X}^{>}/1! + ... =$$

$$= (e^{\hat{X}^{>} > \hat{T}^{>}}) > \hat{I}^{>}. \tag{3.24}$$

The backward genoexponentiation can be easily identified and the same holds for other genofunctions.

The non-initiated reader should be made aware that *different variables generally require different genounits.* Physically, this occurrence is evident from the fact that different variables have different dimensions. Mathematical formulations with multiple genounits are rigorously treated by S. Georgiev [9].

3.10. Forward and backward genodifferential genocalculus

Despite the genotopies of virtually all aspects of 20th century mathematics and of isomathematics, the achievement of the crucial time invariance of irreversible processes remained elusive.

This basic impasse was resolved only in 1996 thanks to the discovery of the *forward genodifferential* of a genovariable

$$\hat{d}^{>} \hat{r}^{>} = \hat{T}^{>} > \hat{d}[r > \hat{I}^{>}(r, ...)] = dr + r > \hat{T}^{>} > d\hat{I}^{>}, \tag{3.25}$$

and corresponding genotopies of the isoderivative, resulting in the *forward genoderivative*

$$\frac{\hat{\partial}^{>} \hat{f}^{>}(\hat{r}^{>})}{\hat{\partial}^{>} \hat{r}^{>}} = \hat{I}^{>} > \frac{\partial \hat{f}^{>}(\hat{r}^{>})}{\partial \hat{r}^{>}}. \tag{3.26}$$

The entire conventional and isotopic differential calculus has to be subjected to genotopies, thus explaining the need for *six* volumes of Refs. [9].

It should be noted that, even though presented as part of isomathematics, the six volumes of the historical work by Georgiev [9] are indeed applicable to genomathematics with the simple assumption that *isotopic elements are explicit dependent on time in a way not invariant under time reversal*,— i.e.

$$T = T(t, ...) = T)t, ...)^{\dagger} \neq T(-t, ...) = T(= t, ...)^{\dagger}. \tag{3.27}$$

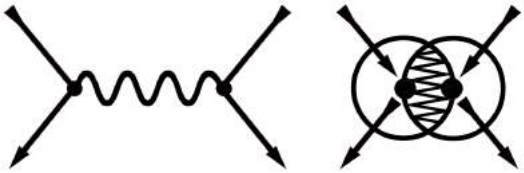

Figure 20: *One of the most significant implications of a quantitative representation of irreversibility is the inapplicability of the 20th century scattering theory to the data elaboration of high energy scattering experiments as currently done at CERN, FERMILAB, SLAC, DESY, and other particle laboratories. In fact, a quantitative representation of the irreversibility of said scatterings requires the abandonment of Feynman's diagrams (left view) in favor of covering scattering theory including a hyperdense scattering region (right view) representing Lagrange's and Hamilton's external forces responsible for irreversibility (see papers [29] for details).*

Additionally, the physical application of volumes [9] to represent irreversibility forward in time require the ordering of all products to the right.

3.11. Forward and backward genospaces

As an illustration of genospaces, let $\hat{M}(\hat{x},\hat{\eta},\hat{I})$ be one of the infinite family of Minkowski-Santilli isospaces as in Eqs. (2.19). The covering infinite family of *forward Minkowski-Santilli genospace* $\hat{M}^{>}(\hat{x}^{>},\hat{\eta}^{>},\hat{R}^{>})$ over the genoreal genofield $\hat{R}^{>}$ is characterized by the *forward genospacetime genocoordinates*

$$\hat{x}^{>}=x>\hat{I}^{>}, \tag{3.28}$$

(where the multiplication by the forward genounit is mandatory for consistency), the *forward geometric*

$$\hat{\eta}^{>}=\hat{T}^{>}>\eta, \quad \eta=Diag.(1,1,1,-1), \tag{3.29}$$

and the *forward genoline genointerval*

$$\hat{x}^{\hat{2}>}_{>}=\hat{x}^{\mu}_{>}\hat{>}\hat{\eta}^{>}_{\mu\nu}\hat{>}\hat{x}^{\nu}_{>}=(x^{\mu}>\hat{\eta}^{>}_{\mu\nu}>x^{\nu})>\hat{I}^{>}, \tag{3.30}$$

where the last expression represents the projection on a conventional Minkowski space.

A most important feature is that *genospaces permit, apparently for the first time, the representation of irreversibility directly via the basic geometric due to its non-symmetric character*

$$\hat{\eta}^{>}_{\mu\nu}\neq\hat{\eta}^{>}_{\nu\mu}, \tag{3.31}$$

in which case irreversibility is embedded in the ultimate geometric structure, namely, in the geometry of spacetime itself.

It should be indicated that *genogeometries* [7,8] are particularly intriguing due to their covering with respect to the Riemannian and other geometries. Recall that Minkowski-Santilli isospaces have an arbitrary functional dependence on local variables, including spacetime coordinates, thus have all the machinery of the Riemannian spaces, although

reformulated via isomathematics [21]. The same functional dependence evidently occur also for genogeometries which, therefore, allow the lifting of conventional and isotopic geometries into a form representing irreversibility directly via the non-symmetric character of their genometrics.

3.12. Forward and backward genorelativities

the Lie-admssible lifting of the Lie and Lie-Santilli's isotheories, first proposed in Refs. [1-3] of 1978 (and then studied in various works, e.g. [7,8]) are particularly important for quantitative studies of irreversible processes, with particular reference to the construction of *forward and backward genorelativities,* here referred to the forward and backward coverings, respectively, of isorelativities of Section 2.14.

The main structural elements of Lie-admissible theories are given by the following genotopies:

(1) The *forward and backward universal enveloping genoassociative algebra* $\hat{\xi}^{>}$, $^{<}\hat{\xi}$, with infinite-dimensional basis characterizing the *Poincaré-Birkhoff-Witt-Santilli genotheorem*

$$\hat{\xi}^{>} : \hat{I}^{>},\ \hat{X}_i,\ \hat{X}_i\hat{>}\hat{X}_j,\ \hat{X}_i\hat{>}\hat{X}_j\hat{>}\hat{X}_k,\ \ldots,\ i \leq j \leq k, \tag{3.32a}$$

$$^{<}\hat{\xi} : \hat{I},\ ^{<}\hat{X}_i,\ \hat{X}_i\hat{<}\hat{X}_j,\ \hat{X}_i\hat{<}\hat{X}_j\hat{<}\hat{X}_k,\ \ldots,\ i \leq j \leq k, \tag{3.32b}$$

where the "hat" on the generators denotes their formulation on genospaces over genofields and their Hermiticity implies that $\hat{X}^{>} =^{<} \hat{X} = \hat{X}$.

(2) The *Lie-Santilli genoalgebras* characterized by the universal, jointly Lie- and Jordan-admissible brackets

$$^{<}\hat{L}^{>} : (\hat{X}_i,\hat{X}_j) = \hat{X}_i\hat{<}\hat{X}_j - \hat{X}_j\hat{>}\hat{X}_i = C_{ij}^{k} > \hat{X}_k, \tag{3.33}$$

here formulated formulated in an invariant form (see below);

(3) The *Lie-Santilli genotransformation groups*

$$^{<}\hat{G}^{>} : \hat{A}(\hat{w}) = (\hat{e}^{\hat{i}^{>}\hat{>}\hat{X}\hat{>}\hat{w}})^{>}\hat{>}\hat{A}(\hat{0})\hat{<}(\hat{e}^{-\hat{i}\hat{<}\hat{w}\hat{\times})^{<}\hat{X}}) =$$

$$= (e^{i>\hat{X}>\hat{T}^{>}>w}) > A(0) < (e^{-i<w<^{<}\hat{T}<\hat{X}}), \tag{3.34}$$

where $\hat{w}^{>} \in R^{>}$ are the *genoparameters;* the *genorepresentation theory,* etc.

The above notions then allow the construction of *genosymmetries* first introduced in ref.[1] for the characterization of *time rate of variation* of physical quantities. Forward and backward genorelativities are then characterized by the simple genotopies of the isoaxioms of Section 2.14. here ignored for simplicity.

It should be noted on mathematical grounds that the Lie-Santilli genoalgebras and genogroups are "directly universal" in the sense of admitting as particular cases without the use of the transformation theory, all possible algebras and transformation groups, such as semigroups.

Recall that the representation theory of Lie algebras has a trivial bimodular structure in the sense that the modular action to the right is equivalent to that to the left. By contrast,

the *genorepresentation genotheory has a non-trivial bimodular structure* in equivalent that the genomodular action to the right is different than that to the left.

Physically, the invariance under genogroup, called *genosymmetries,* permits the characterization for the first time of *non-conservation laws,* namely, the characterization of the time rate of variation (rather than conservation) of physical quantities, that was first identified in the very title of Refs. [1,2].

3.13. Forward Newton-Santilli genomechanics

It is appropriate to recall that Newton was forced to construct new mathematics, the differential calculus, as a necessary condition to formulate his celebrated equations for a system of point particles.

The extension of Newton's equations to a form representing a system of extended and deformable particles in irreversible conditions has equally requested the author the prior construction of the necessary new mathematics outlined in the preceding sections under the name of Lie-admissible genomathematics.

Additionally, it should be noted that, in their general form including non-conservative forces, Newton's equations are variationally non-selfadjoint [3a] and hereon, they are not derivable from a variational principle.

This feature has implied the lack of a consistent operator image of non-conservative forces for about one century because quantization is crucially dependent on the derivability of a classical system from a variational principle, the ensuing Hamilton-Jacobi equation, and then their map into an operator image.

In order to solve this historical problem, the new Lie-admissible mathematics has been constructed in such a way to be indeed derivable from a variational principle as the foundation for the construction of a consistent operator image, as reviewed hereon.

Additionally, the Lie-admissible genomathematics has been constructed in such a way to provide a *covering* of isotheories as well as of 20th century theories. In fact, genotheories uniquely and unambiguously reduce to isotheories whenever all terms representing irreversibility are null, while 20th century theories are identically recovered when all genounits have the trivial value 1.

In this section we consider the *forward genomechanics,* since the constriction of its backward image is trivial. The fundamental assumption is their formulation on the *forward genoreal genonumbers* $R^>$ for the various variables. Consequently, the conventional time t, coordinates into the corresponding *forward genotime* $\hat{t}^>$, *genocoordinates* $\hat{r}^>$ *and genovelocities* $\hat{v}^>$ with explicit expressions

$$\hat{t}^> = t > \hat{I}_t^>, \quad \hat{r}^> = r > \hat{I}_r, \quad \hat{v}^> = v > \hat{I}_v^>, \tag{3.35}$$

where one should note that genounits have different values for different variables (e.g., because they have different dimensionalities), and the symbol $>$ represents the conventional multiplication under the condition of being ordered to the right.

The extended and deformable shape of particles is represented by the genounits as in Eq. (3.11). Irreversibility is then assured not only by all multiplications to be ordered to the right but also by the different numerical values of forward and backward genounits.

Isotheories are clearly recovered identically whenever forward and backward genounits are equal between themselves but different than 1, while 20th century theories are then recovered whenever all (forward and backward) genounits have the trivial value 1.

The central representation space of the forward genomechanics are then given by the Kronecker product of the *forward genospaces for genotimes, genocoordinates and genovelocities* (see Ref. [6] for more details)

$$\hat{E}^{>}_{tot} = \hat{E}^{>}_{t}(\hat{t}^{>}, >_t, \hat{I}^{>}_{t}) > \hat{E}^{>}_{r}(\hat{r}^{>}, >_r, \hat{I}^{>}_{v}) \times \hat{E}^{>}_{v}(\hat{v}^{>}, >_v, \hat{v}^{>}_{t}). \tag{3.36}$$

The generalization of Newtonian mechanics emerging from the above assumptions is dramatically beyond the limited objectives of this presentation. Therefore, we here assume that time and coordinates are not lifted, and only the velocities are generalized, thus implying the assumptions

$$\hat{I}^{>}_{t} = \hat{I}^{>}_{r} = 1, \quad \hat{I}^{>}_{v} = \hat{I}^{>}_{v}(t, r, a, ...) = 1/\hat{T}^{>} \neq 1. \tag{3.37}$$

Next, we here assume at leave a minimal knowledge of the forward and backward genodifferential calculus and genofunctional analysis.

Under the above assumptions, the *forward Newton-Santilli genoequations* [6] can be written

$$\hat{m}^{>} \hat{>} \frac{\hat{d}^{>}\hat{v}^{>}}{\hat{d}^{>}\hat{t}^{>}} - \hat{F}^{>SA}(\hat{t}^{>}, \hat{r}^{>}, \hat{v}^{>}) = m > \frac{dv}{dt} + v > \hat{T}^{>} > \frac{d\hat{I}^{>}}{dt} - F^{>SA}(t, r, v) =$$

$$= m > \frac{dv}{dt} - F^{>NSA}(t, r, v, a, ...) - F^{>SA}(t, r, v) = 0, \tag{3.38a}$$

$$F^{>NSA} = -v > \hat{T}^{>} > \frac{d\hat{I}^{>}}{dt}. \tag{3.38b}$$

The "direct universality" of the above equations for all possible, non-singular, NSA systems has been proved in memoir [6] and its knowledge is assumed hereon.

3.14. Forward genovariational principle

A general form of genounits is that via *non-symmetric matrices.* Their most effective representation is that in $6N$-dimensional *forward genospace* (mathematically expressible in terms of *forward genocotangent bundle* with local forward expressions

$$\hat{a}^{>\mu} = a^{\rho} > \hat{I}^{>\mu}_{1\,\rho}, \ \ (\hat{a}^{>\mu}) = \begin{pmatrix} \hat{x}^{>k}_{\alpha} \\ \hat{p}^{>}_{k\alpha} \end{pmatrix} \tag{3.39a}$$

$$\hat{R}^{>}_{\mu} = R_{\rho} > \hat{I}^{>\rho}_{2\,\mu}, \ (\hat{R}^{>}_{\mu}) = (\hat{p}_{k\alpha}, \hat{0}), \tag{3.39b}$$

$$\hat{I}^{>}_{1} = 1/\hat{T}^{>}_{1} = (\hat{I}^{>}_{2})^{T} = (1/\hat{T}^{>}_{2})^{T}, \tag{3.39c}$$

$$k = 1, 2, 3; \ \alpha = 1, 2, \ldots, N; \ \ \mu, \ \rho = 1, 2, \ldots 6N. \tag{3.39d}$$

In this case we have the following *genoaction principle* [6]

$$\hat{\delta}^{>}\hat{\mathcal{A}}^{>} = \hat{\delta}^{>}\hat{\int}^{>}[\hat{R}^{>}_{\mu}\hat{>}_{a}\hat{d}^{>}\hat{a}^{>\mu} - \hat{H}^{>}\hat{>}_{t}\hat{d}^{>}\hat{t}^{>}] =$$

$$= \delta\int[R_{\mu} > \hat{T}^{>\mu}_{1\,\nu}(t,x,p,\ldots) > d(a^{\nu} > \hat{I}^{>\nu}_{1\,\beta}) - H > dt] = 0, \tag{3.40}$$

where the second expression is the projection on conventional spaces over conventional fields and we have assumed for simplicity that the time genounit is 1.

3.15. Classical forward Lie-admissible genomechanics

It is easy to prove that the above genoaction principle characterizes the following *forward Hamilton-Santilli genoequations,* originally proposed in Ref. [1] of 1978 (see Refs. [7] for comprehensive treatment)

$$\hat{\omega}^{>}_{\mu\nu}\hat{>}\frac{\hat{d}^{>}\hat{a}^{\nu>}}{\hat{d}^{>}\hat{t}^{>}} - \frac{\hat{\partial}^{>}\hat{H}^{>}(\hat{a}^{>})}{\hat{\partial}^{>}\hat{a}^{\mu>}} =$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} > \begin{pmatrix} dr/dt \\ dp/dt \end{pmatrix} - \begin{pmatrix} 1 & K \\ 0 & 1 \end{pmatrix} > \begin{pmatrix} \partial H/\partial r \\ \partial H/\partial p \end{pmatrix} = 0, \tag{3.41a}$$

$$\hat{\omega}^{>} = \Big(\frac{\hat{\partial}^{>}R^{>}_{\nu}}{\hat{\partial}^{>}\hat{a}^{\mu>}} - \frac{\hat{\partial}^{>}\hat{R}^{>}_{\mu}}{\hat{\partial}^{>}\hat{a}^{\nu>}}\Big) > \hat{I}^{>} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} > \hat{I}^{>}, \tag{3.41b}$$

$$K = F^{NSA}/(\partial H/\partial p), \tag{3.41c}$$

where one should note the "direct universality" of the simple algebraic solution (3.22c).

The time evolution of a quantity $\hat{A}^{>}(\hat{a}^{>})$ on the forward geno-phase-space can be written in terms of the following brackets

$$\frac{\hat{d}^{>}\hat{A}^{>}}{\hat{d}^{>}t^{>}} = (\hat{A}^{>}, \hat{H}^{>}) = \frac{\partial\hat{A}^{>}}{\partial\hat{a}^{>\mu}} > S^{\,\mu\nu} > \frac{\partial\hat{H}^{>}}{\partial\hat{a}^{>\nu}} =$$

$$= \Big(\frac{\partial\hat{A}^{>}}{\partial\hat{r}^{>k}_{\alpha}} > \frac{\partial\hat{H}^{>}}{\partial\hat{p}^{>}_{ka}} - \frac{\partial\hat{A}^{>}}{\partial\hat{p}^{>}_{ka}} > \frac{\partial\hat{H}^{>}}{\partial\hat{r}^{>k}_{a}}\Big) + \frac{\partial\hat{A}^{>}}{\partial\hat{p}^{>}_{ka}} > F^{NSA}_{ka}. \tag{3.42}$$

Where the non-symmetric tensor

$$S^{>\mu\nu} = \omega^{\mu\rho} > \hat{I}^{2\,\mu}_{\rho}, \omega^{\mu\nu} = (||\omega_{\alpha\beta}||^{-1})^{\mu\nu}, \tag{3.43}$$

is clearly Lie-admissible (although not jointly Jordan-admissible at this classical level for reasons still unknown).

As one can see, the important consequence of genomathematics and its genodifferential calculus is that of turning the triple system (A, H, F) of Eqs. (3.3) into the bilinear form $(A\hat{,}H)$, thus verifies all axioms to be characterized an algebra, and that algebra results to be Lie-admissible.

The most important result of this section, is the proof that *the original Hamilton equations, those with external NSA terms, Eqs. (3.2), have indeed a Lie-admissible structure when properly written.* This is the central aim for which genomathematics has been built.

Note that *irreversible systems for extended particles is characterized by two quantities, the Hamiltonian H and the external forces F^{NSA} exactly along the teaching by Lagrange and Hamilton, Eqs. (3.2).* The invariance of such a formulation will be proved shortly.

It is easy to verify that the above identical reformulation of Hamilton's historical time evolution correctly recovers the *time rate of variations of physical quantities* in general, and that of the energy in particular

$$\frac{dA^{>}}{dt} = (A^{>}, H^{>}) = [\hat{A}^{>}, \hat{H}^{>}] + \frac{\partial \hat{A}^{>}}{\partial \hat{p}^{>}_{k\alpha}} > F^{NSA}_{k\alpha}, \tag{3.44a}$$

$$\frac{dH}{dt} = [\hat{H}^{>}, \hat{H}^{>}] + \frac{\partial \hat{H}^{>}}{\partial \hat{p}^{>}_{k\alpha}} > F^{NSA}_{ka} = v^{k}_{\alpha} > F^{NSA}_{ka}. \tag{3.44b}$$

It is easy to show that genoaction principle (3.40) characterizes the following *Hamilton-Jacobi-Santilli genoequations* [6,7]

$$\frac{\hat{\partial}^{>} \mathcal{A}^{>}}{\hat{\partial}^{>} \hat{t}^{>}} + \hat{H}^{>} = 0, \tag{3.45a}$$

$$\Big(\frac{\hat{\partial}^{>} \mathcal{A}^{>}}{\hat{\partial}^{>} \hat{r}^{>}}\Big) = \hat{p}^{>}_{i}, \tag{3.45b}$$

$$\frac{\hat{\partial}^{>} \mathcal{A}^{>}}{\hat{\partial}^{>} p^{>}_{k}}\Big) = 0. \tag{3.45c}$$

The above equations confirm the property (which is crucial for genoquantization as shown below) that the genoaction is indeed independent of the linear momentum.

Note the *direct universality* of the Lie-admissible equations (3.41) for the representation of all infinitely possible Newton equations (1.3) (universality) directly in the fixed frame of the experimenter (direct universality).

Note also that, *at the abstract, realization-free level, Hamilton-Santilli genoequations do not coincide* with Hamilton's equations without external terms due to the anti-symmetry of the Lie-admissible tensor $\hat{S}^{>}$. This feature illustrates the reason for selecting the prefix "geno- " rather than "iso-".

Therefore, *Hamilton-Santilli genoequations (3.41) are indeed irreversible over time for all possible reversible Hamiltonians,* as desired. The origin of irreversibility rests in the contact non-potential forces F^{NSA} according to Lagrange's and Hamilton's teaching that is merely reformulated in an invariant way.

The complementary *backward Hamilton-Santilli genomechanics* can be easily constructed via the use of backward genomathematics, and it is ignored for simplicity.

3.16. Operator forward Lie-admissible genomechanics

A simple genotopy of the naive or symplectic quantization applied to Eqs. (3.45) yields the *forward Lie-admissible branch of hadronic mechanics,* [7] also known as *forward genomechanics,* which is characterized by the following main assumptions:

1) The fundamental unit of quantum mechanics, Planck constant $\hbar$, is replaced with a nowhere singular (thus everywhere invertible) non-Hermitean *forward genounit* for the representation of all effects causing irreversibility, such as contact non-potential interactions among extended particles, etc. (see the subsequent chapters for various realizations)

$$\hbar \to \hat{I}^{>} = 1/\hat{T}^{>} \neq^{<} \hat{I} = 1/^{<}\hat{T}, \tag{3.46}$$

with all products being generalized and ordered to the right

$$\hat{A} \hat{>} \hat{B} = A > \hat{T}^{>} > B, \tag{3.47}$$

generally defined on genocomplex genonumbers $^{>}$ with genounit $\hat{I}^{>}$.

2) The *forward geno-Hilbert space* $\hat{\mathcal{H}}^{>}$ on $^{>}$ with *forward genostates* $|\hat{\psi}^{>})$ and *forward genoinner product*

$$(^{<}\hat{\psi}|\hat{>}|\hat{\psi}^{>}) > \hat{I}^{>} = (^{<}\hat{\psi}| > \hat{T}^{>} > |\hat{\psi}^{>}) > \hat{I}^{>} \in \hat{\mathcal{C}}^{>}, \tag{3.48}$$

fundamental property

$$\hat{I}^{>} \hat{>} |\hat{\psi}^{>} >= |\hat{\psi}^{>} >, \tag{3.49}$$

by therefore confirming that $\hat{I}^{>}$ is indeed the correct unit for motion forward in time, and *forward genounitary transforms*

$$\hat{U}^{>} \hat{>} (^{<}\hat{U})^{\dagger >} = (^{<}\hat{U})^{\dagger >} \hat{>} \hat{U}^{>} = \hat{I}^{>}. \tag{3.50}$$

3) The fundamental *Heisenberg-Santilli Lie-admissible equations* [2,3,7] for irreversible time evolutions of observables in the finite form

$$\hat{A}(\hat{t}) = \hat{U}^{>} \hat{>} \hat{A}(0) \hat{<}^{<} \hat{U} = (\hat{e}^{\hat{i}\hat{>}\hat{H}\hat{>}\hat{t}}_{>}) > \hat{A}(\hat{0}) < (_{<}\hat{e}^{-\hat{i}\hat{>}\hat{t}\hat{>}\hat{H}}) =$$

$$= (e^{i>\hat{H}>\hat{T}^{>}>t}) > A(0) > (e^{-i>t>^{<}\hat{T}\times\hat{H}}), \tag{3.51}$$

with corresponding infinitesimal version

$$\hat{i}\hat{>}\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = (\hat{A}\hat{,}\hat{H}) = \hat{A}\hat{<}\hat{H} - \hat{H}\hat{>}\hat{A} =$$

$$= \hat{A} >^{<} \hat{T}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \ldots.) > \hat{H} - \hat{H} > \hat{T}^{>}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \ldots) > \hat{A}. \tag{3.52}$$

Note that the conventional Heisenberg representation provides values of the observables at a fixed time, and the same situation occurs for the Heisenberg-Santilli genoequations. Irreversibility is established by the property that the time reversal image of the Heisenberg-Santilli genoequations is inequivalent to Eqs. (3.41).

4) The equally fundamental *forward Schrödinger-Santilli genoequations*, first suggested in the original proposal [2.3] (which are genounitarily equivalent to the Heisenberg-Santilli genoequations)

$$\hat{i}^{>}\hat{>}\frac{\hat{\partial}^{>}}{\hat{\partial}^{>}\hat{t}^{>}}|\hat{\psi}^{>}) = \hat{H}^{>} > |\hat{\psi}^{>}) =$$
$$= \hat{H}^{>}(\hat{r},\hat{v}) > \hat{T}^{>}(\hat{t},\hat{r},\hat{p},\hat{\psi},\hat{\partial}\hat{\psi}\ldots) \times |\hat{\psi}^{>}) = E^{>}\hat{>}|\psi^{>}). \tag{3.53}$$

5) The *forward genolinear genomomentum* that escaped identification for two decades and was finally identified thanks to the genodifferential calculus in Ref. [6] of 1996

$$\hat{p}_k^{>}\hat{>}|\hat{\psi}^{>}) = -\hat{i}^{>}\hat{>}\hat{\partial}_k^{>}|\hat{\psi}^{>}) = -i > \hat{I}_k^{>\,i} > \partial_i|\hat{\psi}^{>}). \tag{3.54}$$

6) The *fundamental genocommutation rules* also first identified in Ref. [14]

$$(\hat{r}_i^{>}\hat{,}\hat{p}_j^{>}) = \hat{i}^{>}\hat{>}\hat{\delta}_{i,j}^{>}, \quad (\hat{r}_i^{>}\hat{,}\hat{r}_j^{>}) = (\hat{p}_i^{>}\hat{,}\hat{p}_j^{>}) = 0, \tag{3.55}$$

7) The *genoexpectation values* of an observable for the forward motion $\hat{A}^{>}$ [14,19]

$$\frac{(<\hat{\psi}|\hat{>}\hat{A}^{>}\hat{>}|\hat{\psi}^{>})}{(<\hat{\psi}|\hat{>}|\hat{\psi}^{>})} > \hat{I}^{>} \in \hat{\mathcal{C}}^{>}, \tag{3.56}$$

under which the genoexpectation values of the genounit recovers the conventional Planck's unit as in the isotopic case

$$\frac{(<\hat{\psi}|\hat{>}\hat{I}^{>}\hat{>}|\hat{\psi}^{>})}{(<\hat{\psi}|\hat{>}|\hat{\psi}^{>})} = \hat{I}^{>}. \tag{3.57}$$

The following comments are now in order. Note first in the genoaction principle the crucial independence of genoaction $\hat{\mathcal{A}}^{>}$ from the genolinear genomomentum, as expressed by the Hamilton-Jacobi-Santilli genoequations (3.48c). Such independence assures that genoquantization yields a genowavefunction solely dependent on time and coordinates, $\hat{\psi}^{>} = \hat{\psi}^{>}(t,r)$.

Other geno-Hamiltonian mechanics do not verify such a condition, thus implying genowavefunctions with an explicit dependence also on linear momenta, $\hat{\psi}^{>} = \hat{\psi}^{>}(t,r,p)$ that are beyond our current knowledge.

This latter feature occurs for the operator image of the Birkhoffian mechanics of Ref. [3b] and explain the reason for abandoning such a mechanics, despite its direct universality for non-conservative systems, for the genomechanics adopted in this paper. It should be noted that the resolution of this issue requested years of research.

Note that *forward geno-Hermiticity coincides with conventional Hermiticity.* As a result, *all quantities that are observables for quantum mechanics remain observables for the above genomechanics.*

However, unlike quantum mechanics, physical quantities are generally *non-conserved,* as it must be the case for the energy

$$\hat{i}^{>} > \frac{\hat{d}^{>}\hat{H}^{>}}{\hat{d}^{>}\hat{t}^{>}} = \hat{H} > (^{<}\hat{T} - \hat{T}^{>}) > \hat{H} \neq 0. \tag{3.58}$$

Therefore, *the genotopic branch of hadronic mechanics is the only known operator formulation permitting non-conserved quantities to be Hermitean as a necessary condition to be observability.*

Other formulations attempt to represent non-conservation, e.g., by adding an "imaginary potential" to the Hamiltonian, as it is often done in nuclear physics. In this case the Hamiltonian is non-Hermitean with consequential loss of the Lie algebra in nuclear calculations because the bracket of Heisenberg time evolution becomes a triple system

$$i\frac{dA}{dt} - (A, H, H^{\dagger}) = A \times H^{\dagger} - H \times A, \quad H \neq H^{\dagger}. \tag{3.59}$$

Besides the above inconsistency, "non-conservative models" with non-Hermitean Hamiltonians are non-unitary and theories formulated on conventional Hilbert spaces over conventional fields, thus suffering all the catastrophic inconsistencies of Theorem 3.2.

It should be indicated that *genomechanics provide another explicit and concrete realization of "hidden variables" [26], thus constituting another "completion" of quantum mechanics in the E-P-R sense* (see Ref. [14] for details).

We finally note that, isomechanics of Section 2.8 and 2.9 is readily turned into the genomechanics of this section under the assumption that the isotopic element is dependent on time in an irreversible way, Eq. (3.26). However, to avoid insidious inconsistencies that generally remain unknown to non-expert, the entire formulation has to be restricted to an ordering to the right for forward motion in time.

3.17. Simple Construction of Lie-admissible genomodels

The reader seriously interested in the representation of the irreversibility of the physical reality should know the existence of a simple method for the mapping of a conventional quantum mechanics reversible model into its irreversible covering. The method, first identified in Ref. [98] of 1998 consists in the identification of *two* non-unitary transforms for the following characterization of the forward and backward genounit here assumed for simplicity to be real-valued but non-symmetric

$$\hat{I}^{>} = (^{<}\hat{I})^{\dagger} = U \times W^{\dagger}, \quad ^{<}\hat{I} = W \times U^{\dagger}, \tag{3.60a}$$

$$U \times U^{\dagger} \neq 1, \quad W \times W^{\dagger} \neq 1, \quad U \times W^{\dagger} = \hat{I}^{>}, \tag{3.60b}$$

and subjecting the totality of quantities and their operations of conventional models (expressed in terms of the conventional product $\times$) to said dual transform,

$$I > \hat{I}^{>} = U \times I \times W^{\dagger}, \quad I >^{<} \hat{I} = W \times I \times U^{\dagger}, \tag{3.61a}$$

$$\hat{a}^{>} = U \times a \times W^{\dagger} = a \times \hat{I}^{>}, \tag{3.61b}$$

$$a >^{<} \hat{a} = W \times a \times U^{\dagger} =^{<} \hat{I} \times a, \tag{3.61c}$$

$$\hat{a}^{>}\hat{>}\hat{b}^{>} = U \times (a \times b) \times W^{>} =$$

$$= (U \times a \times W^{\dagger}) \times (U \times W^{\dagger})^{-1} \times (U \times b \times W^{\dagger}), \tag{3.61d}$$

$$\partial/\partial x > \hat{\partial}^{>}/\hat{\partial}^{>}\hat{x}^{>} = U \times (\partial/\partial x) \times W^{\dagger} = \hat{I}^{>} \times (\partial/\partial x), \tag{3.61e}$$

$$< \psi| \times |\psi >><^{<} \psi| > |\psi^{>} >= U \times (< \psi| \times |\psi >) \times W^{\dagger}, \tag{3.61f}$$

$$\hat{H}^{>}\hat{>}|\psi^{>} >= (U \times H \times W^{\dagger}) \times (U \times W^{\dagger})^{-1} \times (U \times \psi > W^{\dagger}), \text{ etc.} \tag{3.61g}$$

Note that the above construction implies that *all physical quantities, including energy, coordinates, moments, etc., acquire a well defined direction in time.* In fact, under irreversibility, the value of the non-conserved energy at a given time t for motion forward in time is generally different than the corresponding value of the energy for $-t$ for motion backward in past times.

This explains the reason for having represented in this section energy, momentum and other quantities with their upper arrow of time $^{>}$. Such an arrow can indeed be omitted for notational simplicity, but only after the understanding of its existence.

Note finally that a conventional, one-dimensional, unitary Lie transformation group with Hermitean generator X and parameter w can be transformed into a covering Lie-admissible group via the following non-unitary transform

$$Q(w) \times Q^{\dagger}(w) = Q^{\dagger}(w) \times Q(w) = I, \; w \in R, \tag{3.62a}$$

$$U \times U^{\dagger} \neq I, \quad W \times W^{\dagger} \neq 1, \tag{3.62b}$$

$$A(w) = Q(w) \times A(0) \times Q^{\dagger}(w) = e^{X \times w \times i} \times A(0) \times e^{-i \times w \times X} >$$

$$> U \times (e^{X \times w \times i} \times A(0) \times e^{-i \times w \times X}) \times U^{\dagger} =$$

$$\equiv [U \times (e^{X \times w \times i}) \times W^{\dagger} \times (U \times W^{\dagger})^{-1} \times A \times A(0) \times$$

$$\times U^{\dagger} \times (W \times U^{\dagger})^{-1} \times [W \times (e^{-i \times w \times X}) \times U^{\dagger}] =$$

$$= (e^{i \times X \times X})^{>} > A(0) <^{<} (e^{-1 \times w \times X}) = \hat{U}^{>} > A(0) <^{<} \hat{U}. \tag{3.62c}$$

The above transform indicates the existence of a full Lie-admissible covering of Lie's theory that has yet to be developed to a considerable extent.

3.18. Invariance of Lie-Admissible genotheories

Recall that a fundamental axiomatic feature of quantum mechanics is the invariance under time evolution of all numerical predictions and physical laws, which invariance is due to the *unitary structure* of the theory.

However, quantum mechanics is reversible and can only represent in a scientific way beyond academic beliefs reversible systems verifying total conservation laws due to the antisymmetric character of the brackets of the time evolution.

As indicated earlier, the representation of irreversibility and non-conservation requires theories with a *non-unitary structure*. However, the latter are afflicted by the catastrophic inconsistencies of Theorem 3.2.

The only resolution of such a basic impasse known to the author has been the achievement of invariance via the use of genomathematics, provided that said genomathematics is applied to the *totality* of the formalism to avoid evident inconsistencies caused by mixing different mathematics for the selected physical problem.

Such an invariance was first achieved by Santilli in Ref. [99] of 1997 and can be illustrated by reformulating any given non-unitary transform in the *genounitary form*

$$U = \hat{U} \times \hat{T}^{>1/2}, W = \hat{W} \times \hat{T}^{>1/2}, \tag{3.63a}$$

$$U \times W^\dagger = \hat{U} > \hat{W}^\dagger = \hat{W}^\dagger > \hat{U} = \hat{I}^> = 1/\hat{T}^>, \tag{3.63b}$$

and then showing that genounits, genoproducts, genoexponentiation, etc., are indeed invariant under the above genounitary transform in exactly the same way as conventional units, products, exponentiations, etc., are invariant under unitary transforms

$$\hat{I}^> \to \hat{I}^{>'} = \hat{U} \hat{>} \hat{I}^> \hat{>} \hat{W}^\dagger = \hat{I}^>, \tag{3.64a}$$

$$\begin{aligned} \hat{A} \hat{>} \hat{B} \to \hat{U} \hat{>} (A > B) \hat{>} \hat{W}^\dagger = \\ = (\hat{U} \times \hat{T}^> \times A \times T^> \times \hat{W}^\dagger) \times (\hat{T}^> \times W^\dagger)^{-1} \times \hat{T}^> \times \\ \times (\hat{U} \times \hat{T}^>)^{-1} \times (\hat{U} \times T^> \times \hat{A} \times T^> \times \hat{W}^>) = \\ = \hat{A}' \times (\hat{U} \times \hat{W}^\dagger)^{-1} \times \hat{B} = \hat{A}' \times \hat{T}^> \times B' = \hat{A}' \hat{>} \hat{B}', \text{ etc.}, \end{aligned} \tag{3.64b}$$

from which properties, all remaining invariances follow, thus resolving the catastrophic inconsistencies of Theorem 3.2.

Recall that the representation of irreversible systems via Lie-admissible formulations requires *two* quantities, the conventional Hamiltonian $\hat{H}^>$ for conventional, reversible, potential interactions, and the genotopic element $\hat{T}^>$ for the representation of irreversible external terms.

Therefore, the numerical invariance of the genotopic element $\hat{T}^>$ or equivalently by the genounit $\hat{I}^> = 1/\hat{T}^>$, established by Eqs. (3.64) assures the invariant representation over time of irreversible systems, as desired.

3.19. Lie-admissible treatment of particles with linear dissipative force

In this section, we present classical and operator representations of non-conservative systems by omitting hereon for simplicity of notation all "hats" on quantities (denoting isotopies not considered in this section), by omitting the symbol $>$ denoting the ordered associative product to the right, but preserving the forward (backward) upper symbols $^>$ ($^<$) denoting forward (backward) motion in time.

Recall that massive points cannot experience any resistive force. Therefore, resistive forces, and irreversibility at large, are crucially dependent on the *extended* character of particles which is hereon tacitly ignored, but assumed to be known.

In this section, we assume for simplicity that all particles are perfectly spherical for simplicity with constant unit radius. In this way, the shape of the particle factors out and can be ignored for simplicity.

Let us begin with the representation of the simplest possible, classical dissipative system consisting of (an extended) particle moving within a physical medium under the sole action of a linear, velocity-dependent resistive force without any potential force at all

$$m\frac{dv}{dt} = F^{NSA} = -kv, \tag{3.65}$$

for which we have the familiar *variation (dissipation) of the energy*

$$\frac{d}{dt}(\frac{1}{2}mv^2) = -kv^2. \tag{3.66}$$

Progressively more complex examples will be considered below.

The representations of system (3.65) via the *Newton-Santilli genoequations* (3.38) for the simple case of conventional time $\hat{t}^> = t$ and coordinates $\hat{r}^> = r$ is given by

$$m^> \hat{>} \frac{d^>v^>}{d^>t^>} = m\frac{d(v\hat{I}_v^>)}{dt} = 0. \tag{3.67}$$

Due to the simplicity of the case and the velocity dependence of the force, the simplest possible genounit in the velocity is given by

$$I_v^>(t) = e^{\frac{k\times t}{m}} = 1/T_v^>(t) > 0, \tag{3.68a}$$

$$m^> > \frac{d^>v^>}{d^>t^>} = m\frac{d(vI_v^>)}{dt} = m\frac{dv}{dt}I^> + kv\frac{dI_v^>}{dt} = 0. \tag{3.68b}$$

The representation with *Hamilton-Santilli genoequations* (3.41) is also straightforward and can be written in disjoint $r^>$ and $p^>$ notations

$$H^> = \frac{p^{2>}}{2^> > m^>} = \frac{p^2}{2m}I_p^>, \tag{3.69a}$$

$$v^> = \frac{\partial^>H^>}{\partial^>p^>} = \frac{p^>}{m}, \quad \frac{d^>p^>}{d^>t^>} = -\frac{\partial^>H^>}{\partial^>r^>} = 0. \tag{3.69b}$$

The last equation then reproduces equation of motion (3.65).

The representation important for operator map is that provided by the genovariational principle (3.41) and the Hamilton-Santilli genoequations (3.41) in the unified notation $a^> = (r^{>k}, p_k^>)$ that become for the case at hand

$$\frac{da^{>\mu}}{dt} = \begin{pmatrix} dr^>/dt \\ dp^>/dt \end{pmatrix} = S^{>\mu\nu}\frac{\partial^>H^>}{\partial^>a^{>\nu}} = \begin{pmatrix} 0 & -1 \\ 1 & \frac{-kv}{(\partial H/\partial p)} \end{pmatrix} \begin{pmatrix} \partial^>H^>/\partial^>r^> \\ \partial^>H^>/\partial^>p^> \end{pmatrix}, \tag{3.70}$$

under which we have the genoequations in the separate notion of genocoordinates and genomoments

$$\frac{dr^{>}}{dt} = \frac{\partial^{>} H^{>}}{\partial^{>} p^{>}} = \frac{p^{>}}{m}, \quad \frac{dp^{>}}{dt} = -kv, \tag{3.71}$$

where one should note that the derivative can be assumed to be conventional.

We also have the following *classical, finite, Lie-admissible transformation genogroup*

$$A(t) = (e^{-t\frac{\partial H}{\partial a^{\mu}} S^{>\mu\nu} \frac{\partial}{\partial a^{\nu}}}) A(0) (e^{\frac{\partial}{\partial a^{\nu}} {}^{<} S^{\nu\mu} \frac{\partial H}{\partial a^{\mu}} t}), \tag{3.72}$$

defined in the $12-$dimensional *bimodular genophasespace* $^{<}T^*M \times T^*M^{>}$, with *infinitesimal Lie-admissible time evolution*

$$\frac{dA}{dt} = \frac{\partial A}{\partial a^{\mu}} (^{<} S^{\mu\nu} - S^{>\mu\nu}) \frac{\partial H}{\partial a^{\nu}} =$$

$$= (\frac{\partial A}{\partial r^k} \frac{\partial H}{\partial p_k} - \frac{\partial H}{\partial r^k} \frac{\partial A}{\partial p_k}) - (\frac{kv}{(\partial H/\partial p)}) \frac{\partial H}{\partial p} \frac{\partial A}{\partial p}) =$$

$$= [A, H] - kv \frac{\partial A}{\partial p}, \tag{3.73}$$

where we have dropped the forward arrow for notational convenience, and $\omega^{\mu\nu}$ is the canonical Lie tensor, thus proving the Lie-admissibility of the S-tensors. In fact, the attached antisymmetric brackets $[A, H]$ are the conventional Poisson brackets, while $\{A, H\}$ are indeed symmetric brackets (as requested by Lie-admissibility), but they do not characterize a Jordan algebra (Section 1.3).

It is easy to see that *time rate of variation of the (kinetic) energy* is given by

$$\frac{dH}{dt} = -kv \frac{\partial H}{\partial p} = -kv^2, \tag{3.74}$$

thus correctly reproducing behavior (3.66).

The operator image of the above dissipative system is straightforward. Physically, we are referring to a first approximation of a massive and stable particle (such as an electron) penetrating within hadronic matter (such as a nucleus).

The problem for the desired operator image is to identify forward and backward genounits and related genotopic elements $I^{>} = 1/T^{>}, ^{<} I = 1/^{<} T$ for which the following *operator Lie-admissible genogroup* now defined on a genomodule $^{<}\mathcal{H} \times \mathcal{H}^{>}$

$$A(t) = (e^{iHT^{>}t}) A(0) (e^{-it^{<}TH}), \tag{3.75}$$

and related infinitesimal form, the *Heisenberg-Santilli genoequations*

$$i\frac{dA}{dt} = A < H - H > A = A^{<} TH - HT^{>} A, \tag{3.76}$$

correctly represent the considered dissipative system.

We assume the realization of the genotopic element

$$T^{>} = 1 - \Gamma, \quad {}^{<}T = 1 + \Gamma, \tag{3.77}$$

for which we have

$$i\frac{dA}{dt} = (AH - HA) - (A\Gamma H + H\Gamma A) = \\ = [A, H] - \{A\hat{,}H\}, \tag{3.78}$$

where $[A, H]$ are the conventional Lie brackets, and $\{A\hat{,}H\}$ are Jordan-isotopic brackets. The desired representation then occurs for

$$I^{>} = e^{(k/m)H^{-1}} = 1/T^{>}, \quad {}^{<}I = e^{-H^{-1}(k/m)} = 1/{}^{<}T, \tag{3.79a}$$

$$i\frac{dH}{dt} = -\frac{kp^2}{m^2} = -kv^2. \tag{3.79b}$$

Note that the achievement of the above operator form of system (3.65) without the Lie-admissible formulation would have been impossible, to our knowledge.

The latter occurrence may illustrate the reason for the lack of achievement of a consistent operator formulation of non-conservative systems throughout the 20th century until the advent of the Lie-admissible formulations.

3.20. Direct universality of the Lie-admissible representation for non-conservative systems

We now show that Lie-admissible formulations are "directly universal," namely they provide a classical and operator representation of all infinitely possible (well behaved) non-conservative systems of N extended particles in irreversible conditions ("universality")

$$m_n\frac{dv_{nk}}{dt} + \frac{\partial V}{\partial r_n^k} = F_{nk}^{NSA}(t, r, p, \dot{p}, ...), \quad n = 1, 2, 3, ..., N, \quad k = 1, 2, 3, \tag{3.80}$$

directly in the frame of the observer, thus without transformations from the coordinates of the experimenter to mathematical frames ("direct universality").

A classical illustration is given by an extended massive object moving at high speed within a resistive medium, such as a missile moving in our atmosphere. In this case the resistive force is approximated by a power series expansion in the velocity up to the 10th power for the high speeds of contemporary missiles

$$m\frac{dv}{dt} = \Sigma_{\alpha=1,2,...,10}k_\alpha v^\alpha, \tag{3.81}$$

for which any dream of conventional Hamiltonian representation is beyond the boundary of science.

The direct universality of the genoaction principle and of Hamilton-Santilli genomechanics was proved in Section 3.15. The representation in geno-phase-space is characterized by the conventional Hamiltonian representing the physical total energy, and the

genounit for forward motion in time representing the NSA forces, according to the equations

$$H = \Sigma_{n,k} \frac{p_{nk}^2}{2m_n} + V(r), \quad I^> = \begin{pmatrix} 1 & (\frac{F^{NSA}}{(\partial H/\partial p)}) \\ 1 & 0 \end{pmatrix}, \tag{3.82}$$

under which we have the equations of motion (for $\mu, \nu = 1, 2, 3, ...6N$)

$$\frac{da^{>\mu}}{dt} = \begin{pmatrix} dr_n^{>k}/dt \\ dp_{nk}^>/dt \end{pmatrix} = S^{>\mu\nu} \frac{\partial^> H^>}{\partial^> a^{>\nu}} = \begin{pmatrix} 0 & -1 \\ 1 & (\frac{F^{NSA}}{(\partial H/\partial p)}) \end{pmatrix} \begin{pmatrix} \partial^> H^>/\partial^> r_n^{>k} \\ \partial^> H^>/\partial^> p_{nk}^> \end{pmatrix}, \tag{3.83}$$

the classical, finite, Lie-admissible genogroups

$$A(t) = e^{-t\frac{\partial H}{\partial a^\mu} S^{>\mu\nu} \frac{\partial}{\partial a^\nu}} A(0) e^{\frac{\partial}{\partial a^\nu}{}^< S^{\nu\mu} \frac{\partial H}{\partial a^\mu} t}, \tag{3.84}$$

with infinitesimal time evolution

$$\frac{dA}{dt} = \frac{\partial A}{\partial a^\mu} (^< S^{\mu\nu} - S^{>\mu\nu}) \frac{\partial H}{\partial a^\nu} =$$

$$= (\frac{\partial A}{\partial r_n^k} \frac{\partial H}{\partial p_{nk}} - \frac{\partial H}{\partial r_n^k} \frac{\partial A}{\partial p_{nk}}) - (\frac{km}{(\partial H/\partial p)})^{nk} \frac{\partial A}{\partial p_{nk}} \frac{\partial H}{\partial p_{nk}}) =$$

$$= [A, H] + \{A, H\}, \tag{3.85}$$

yielding the correct *non-conservation of the energy*

$$\frac{dH}{dt} = v^k F_k^{NSA}. \tag{3.86}$$

The operator image can be characterized by the genounits and related genotopic elements

$$I^> = e^\Gamma = 1/T^>, \quad {}^< I = e^{-\Gamma} = 1/{}^< T, \quad \Gamma = H^{-1}(v_n^k F_{nk}^{NSA}) H^{-1}, \tag{3.87}$$

with finite Lie-admissible time evolution

$$A(t) = exp(iHe^{-\Gamma} t) A(0) exp(-ite^{+\Gamma} H), \tag{3.88}$$

and related Heisenberg-Santilli genoequations

$$i\frac{dA}{dt} = A\hat{<}H - H\hat{>}A = [A, H] + \{A\hat{,}HA\} =$$

$$= (AH - HA) + (A\Gamma H + H\Gamma A), \tag{3.89}$$

that correctly represent the time rate of variation of the non-conserved energy

$$i\frac{dH}{dt} = v_n^k F_{nk}^{NSA}. \tag{3.90}$$

The non-initiated reader should be incidentally aware that generally different genounits may be requested for different generators, as identified since Ref. [2].

In the latter operator case we are referring to an extended, massive and stable particle, such as a proton, penetrating at high energy within a nucleus, in which case the rapid decay of the kinetic energy is caused by contact, resistive, integro-differential forces of nonlocal type, e.g., because occurring over the volume of the particle.

The advantages of the Lie-admissible formulations over pre-existing representation of nonconservative systems should be pointed out. Again, a primary advantage of the Lie-admissible treatment is the characterization of the *non-conserved* Hamiltonian with a *Hermitean,* thus *observable* quantity, a feature generally absent in other treatments.

3.21. Genotopies of Pauli Matrices

Following the study of the non-conservation of the energy, the next important topic is to study the behavior of the conventional quantum spin under contact nonconservative forces. For this objective, it is most convenient to use the methods of Section 3.18, namely to subject the conventional Pauli's matrices to two different non-unitary transformations. To avoid un-necessary complexity, we select the following two matrices:

$$A = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}, \quad AA^\dagger \neq I, \ ] \ BB^\dagger \neq I, \tag{3.91}$$

where a and b are non-null real numbers, under which we have the following forward and backward genounits and related genotopic elements

$$I^> = AB^\dagger = \begin{pmatrix} 1 & b \\ a & 1 \end{pmatrix}, \quad T^> = \frac{1}{(1-ab)} \begin{pmatrix} 1 & -b \\ -a & 1 \end{pmatrix}, \tag{3.92a}$$

$$^<I = BA^\dagger = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}, \quad ^<T = \frac{1}{(1-ab} \begin{pmatrix} 1 & -a \\ -b & 1 \end{pmatrix}. \tag{3.92b}$$

The *forward and backward Pauli-Santilli genomatrices* are then given respectively by

$$\sigma_1^> = A\sigma_1 B^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & (a+b) \end{pmatrix}, \quad \sigma_2^> = A\sigma_2 B^\dagger = \begin{pmatrix} 0 & -i \\ i & (a+b) \end{pmatrix}, \tag{3.93a}$$

$$\sigma_3^> = A\sigma_3 B^\dagger = \begin{pmatrix} 1 & b \\ a & -1 \end{pmatrix}, \quad ^<\sigma_1 = B\sigma_1 A^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & (a+b) \end{pmatrix}, \tag{3.93b}$$

$$^<\sigma_2 = B\sigma_2 A^\dagger = \begin{pmatrix} 0 & -i \\ i & (a+b) \end{pmatrix}, \quad ^<\sigma_3 = A\sigma_3 B^\dagger = \begin{pmatrix} 1 & a \\ b & -1 \end{pmatrix}, \tag{3.93c}$$

in which the direction of time is embedded in the structure of the matrices.

It is an instructive exercise for the interested reader to verify that *conventional commutation rules and eigenvalues of Pauli's matrices are preserved under forward and backward genotopies,*

$$\sigma_i^> \hat{>} \sigma_j^> - \sigma_j^> \hat{>} \sigma_i^> = 2i\epsilon_{ijk}\sigma_k^>, \tag{3.94a}$$

$$\sigma_3^> \hat{>} | > = \pm 1 | >, \quad \sigma^{2>} > | > = 2(2+1)| >, \tag{3.94b}$$

$$^{<}\sigma_i \hat{>}^{<}\sigma_j -^{<} \sigma_j \hat{>}^{<}\sigma_i = 2i\epsilon^{<}_{ijk}\sigma_k, \tag{3.94c}$$

$$^{<}|\hat{<}^{<}\sigma_3 =^{<} |\pm 1, \ ;< |<^{<} \sigma^{2>} =< |(2(2+1). \tag{3.94d}$$

We can, therefore, conclude by stating that *Pauli's matrices can indeed be lifted in such an irreversible form to represent the direction of time in their very structure.*

3.22. Genotopies of the Minkowski Spacetime

One of the fundamental axiomatic principles of hadronic mechanics is that irreversibility can be directly represented with the background geometry, and more specifically, with the metric of the selected geometry. This requires the necessary transition from the conventional *symmetric metrics* used in the 20th century to covering *non-symmetric geometries* first presented by the author in memoir [99] of 2006.

To show this structure, we study in this section the genotopy of the conventional Minkowskian spacetime and related geometry with the conventional metric $\eta = Diag.(1,-1,1,-1)$ and related spacetime elements $x^2 = x^\mu \eta_{\mu\nu} x^\nu, x = (x^1, x^2, x^3, x^4), x^4 = ct, c = 1$. For this purpose, we introduce the following four-dimensional non-Hermitean, nonsingular and real-valued forward and backward genounits

$$I^{>} = CD^\dagger = 1/T^{>}, \quad {}^{<}I = DC^\dagger = 1/{}^{<}T, \quad CC^\dagger \neq I, \quad DD^\dagger \neq I, \tag{3.95}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ p & 0 & 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ q & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{3.96}$$

where $p \neq q$ are non-null real numbers, under which we have the following forward and backward genotopy of the Minkowskian line element

$$x^2 \to x^{2>} = Cx^2 D^\dagger = C(x^t \eta x) D^\dagger =$$

$$= (C^t x^t D^{t\dagger})(CD^\dagger)^{-1}(C\eta D^\dagger)(CD^\dagger)^{-1}(CxD^\dagger) =$$

$$= (x^t I^{>}) T^{>} \eta^{>} T^{>} (I^{>} x) = x^\mu \eta^{>}_{\mu\nu} x^\nu =$$

$$= (x^1 x^1 + x^1 q x^3 + x^2 x^2 + x^3 x^3 + x^1 p x^4 - x^4 x^4), \tag{3.97a}$$

$$Dx^2 C^\dagger = D(x^t \eta x) C^\dagger =$$

$$= (x^{<} I)^{<} T^{<} \eta^{<} T (^{<} I x) = x^\mu \eta_{\mu\nu} x^\nu =$$

$$= (x^1 x^1 + x^1 p x^3 + x^2 x^2 + x^3 x^3 + x^1 q x^4 - x^4 x^4), \tag{3.97b}$$

resulting in the forward and backward non-symmetric genometrics

$$\eta^{>} = \begin{pmatrix} 1 & 0 & q & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ q & 0 & 0 & 1 \end{pmatrix}, \quad {}^{<}\eta = \begin{pmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ q & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{3.98}$$

exactly as desired.

Note that irreversibility determines a mutation of the line elements along a pre-selected direction of space and time.

Note also that the quantities p and q can be functions on the local spacetime variables, in which case the resulting *Minkowskian genogeometry* can be equipped by a suitable lifting of the machinery of the Riemannian geometry (see Ref. [7] for the isotopic case).

It should be indicated that the above irreversible formulation of spacetime has intriguing implications for the mathematical model known as *geometric locomotion* studied in detail in monograph [8] via the isotopies of the Minkowskian geometry.

A main unresolved problem is the directional mutation of the geometry as needed to permit the geometric locomotion in one preferred direction of space. An inspection of the mutated line elements (3.97) clearly shows that the genotopies are preferable over the isotopies for the geometric locomotion, as well as, more generally, for a more realistic geometric characterization of irreversible processes.

In a subsequent paper the author hopes to present the explicit form of the Lorentz-Santilli genotransformation leaving invariant genoelements (3.94) and related genotopy of special relativity, again for the purpose of showing that irreversibility can indeed be embedded in the basic axioms (see Ref. [7] for isospecial relativity).

3.23. Genotopies of Dirac's Equation

To complete the illustrations of Lie-admissible formulations in particle physics, we outline the simplest possible genotopy of Dirac's equation via the genotopies of the preceding two sections, one for the spin content of Dirac's equation and the other for its spacetime structure.

Also, we shall use Dirac's equation in its isodual re-interpretation representing a direct product of one electron and one positron, the latter without any need of second quantization (see monograph [7] for detail).

In turn, the latter re-interpretation requires the use of the *isodual transform* discussed in the next section $A \to A^d = -A^\dagger$) as being distinct from Hermitean conjugation.

Under the above assumptions, by eliminating all unnecessary factors, the *forward Dirac-Santilli genoequations* first presented in Ref. [99] of 2006, can be written

$$\eta^{>\mu\nu}\gamma_\mu^> T^> p_\nu^> - im)T^>|\psi^>) = 0, \tag{3.99a}$$

$$p_\nu^> T^>|\psi^>) = -i\frac{\partial^>}{\partial^> x^{>\nu}}|\psi^>) = -iI^>\frac{\partial}{\partial x^>}|\psi^>), \tag{3.99b}$$

with *forward genogamma matrices*

$$\gamma_4^> = \begin{pmatrix} A & 0 \\ 0 & B^d \end{pmatrix}\begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}\begin{pmatrix} A^d & 0 \\ 0 & B \end{pmatrix} = \begin{pmatrix} AA^d & 0 \\ 0 & -B^d B \end{pmatrix}, \tag{3.100a}$$

$$\gamma_k^> = \begin{pmatrix} A & 0 \\ 0 & B^d \end{pmatrix}\begin{pmatrix} 0 & \sigma_k \\ \sigma_k^d & 0 \end{pmatrix}\begin{pmatrix} A^d & 0 \\ 0 & B \end{pmatrix} = \tag{3.100b}$$

$$= \begin{pmatrix} 0 & A\sigma_k B^\dagger \\ B\sigma_k^d A^d & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma_k \\ \sigma_k^d & 0 \end{pmatrix} = \begin{pmatrix} 0 & \sigma_k^> \\ {}^<\sigma_k^d & 0 \end{pmatrix}. \quad (3.100c)$$

$$\{\gamma_\mu^>, \gamma_\nu^>\} = \gamma_\mu^> T^> \gamma_\nu^> + \gamma_\nu^> T^> \gamma_\mu^> = 2\eta_{\mu\nu}^>, \quad (3.100d)$$

where $\eta_{\mu\nu}^>$ is given by the same genotopy of Eqs. (5.36a).

The full understanding of the Dirac-Santilli genoequations requires the knowledge of the isodual theory of the next section because it establishes a new fundamental symmetry of Dirac's equation that remained undiscovered throughout the 20th century, namely its *isoselfduality* (invariance under isoduality) that is now playing an increasing role for realistic cosmologies, those inclusive of antimatter, or serious unified theory that must also include antimatter to avoid catastrophic inconsistencies [73].

In particular, the isodual theory establishes that, while the electron is moving forward, the positron is moving backward in time although referred to a negative unit of time, as a necessary condition to avoid the inconsistencies for negative energies that requested the conjecture of the "hole theory" (see Section 4).

3.24. P. Roman, J. Dunning-Davies and A. A. Bhalekar studies on Lie-admissible statistics and thermodynamics

A scientific imbalance of the 20th century has been the lack of interconnections between thermodynamics, on one side, and classical and quantum Hamiltonian mechanics, on the other side.

This historical occurrence is due to the fact that the very notion of entropy in nonequilibrium, let alone the versions of all thermodynamical laws tenable in nonequilibrium, are centrally dependent on irreversibility, while classical and quantum Hamiltonian mechanics are structurally reversible (since all known potentials are reversible in time) irrespective of the system (a collection of very large number of molecules) being static, in dynamical equilibrium or nonequilibrium.

As recalled in Section 3.1, the lack of interconnection was justified in the 20th century via the belief that the non-conservative forces responsible for irreversibility according to Lagrange and Hamilton, are "fictitious" in the sense that they only exist at the classical level and they "disappear" when passing to elementary particles, since the latter were believed to be completely reversible. In this way, thermodynamics itself was turned into a sort of "fictitious" discipline.

This manifestly political belief was disproved by Theorem 3.1 by the author during his Ph. D. thesis in the mid 1960's, propagated to various scalars but generally ignored at best, and often dismissed with discreditation of the author, since Theorem 3.1 sets incontrovertible limitations for Einstein theories, that by contrast, are currently assumed as being universally valid for the totality of the large scale and small scale structure of the universe.

The imbalance has been resolved, for the first time to our knowledge by Lie-admissible formulations beginning from its historical foundation. In fact, Theorems 1.1 has established that far from being "fictitious," nonconservative forces originate at the ultimate level of nature, that of elementary particles in conditions of mutual penetration causing

contact non-potential (NSA) interactions.

The insufficiency rested in the inability by quantum mechanics to represent nonconservative forces, rather than in nature. In fact, hadronic mechanics was proposed and developed precisely to reach an operator representation of the nonconservative forces originating irreversibility along the legacy of Lagrange and Hamilton.

As a result of the efforts presented in this paper, we now possess not only classical and operator time irreversible theories, but more particularly we have a $new\ mathematics,$ genomathematics, whose basic axioms are not invariant under time reversal beginning which is mathematically represented as a lack of invariance of the reversal of the multiplication from a modular action to the right to that to the left.

The evident line of research to interconnect irreversible mechanics with the notoriously irreversible thermodynamics is via the passing through the intermediate step of a formulation of statistical mechanics that does not exist in a form compatible with Lie-admissible mechanics, and therefore, the new Lie-admissible statistical mechanics had to be built.

The latter studies were initiated in 1969, by P. Roman and R. M. Santilli [75]. The main idea is that, in a way similar to the case of the time evolution of Hamilton's equations with external terms, Eq. (3.), the time evolution of the density matrix ρ of 20th century statistical mechanics violates the axioms to characterize an algebra due to a variety of NSA external terms

$$\frac{\partial \rho}{\partial t} - [\rho, H] + \text{external terms}. \tag{3.101}$$

The lack of a consistent algebra in the brackets of the time evolution then prevents any formulation of an irreversible statistical mechanics compatible to a mechanics, that in turn, to be a covering of the reversible Hamiltonian mechanics, must necessarily have a consistent algebra in the time evolution.

Still in turn, the latter algebra must be a covering of the Lie algebra, thus uniquely characterizing Lie-admissible algebras, not only for their covering character of Lie algebra's, but also in view of their direct universality.

The main idea of Ref. [75] is that of $identically$ reformulating existing models on the time evolution of the density matrix of statistical mechanics into a Lie-admissible form essentially along the rules of Section (3.15)

$$\frac{\partial \rho}{\partial t} = (\rho, H) = 0, \tag{3.102}$$

where (ρ, H) is Lie-admissible, in the sense that, the attached anti-symmetric brackets are Lie (we would say Lie-isotopic for contemporary knowledge)

$$[\rho, H]* = (\rho, H) - (H, \rho) = \ Lie. \tag{3.103}$$

Unfortunately, the above 1969 studies by P. Roman and R. M. Santilli had to be terminated because of organized academic obstructions outlined in more detail in Section 5.

Studies on a consistent connection between Lie-admissible mechanics and thermodynamics were pioneering in 2006, by J. Dunning-Davies with paper [101] that can be outlined as follows. By following Dunning-Davies, let us use conventional thermodynamical symbols, a classical form of thermodynamics, and the simple construction of irreversible formulations via two different complex valued quantities A and B.

Then, the first law of thermodynamics can be lifted from its conventional formulation, that via reversible mathematics, into the form permitted by genomathematics

$$Q \to Q^{>} = AQB^{\dagger} = QI^{>}, \quad U \to U^{>} = AUB^{\dagger} = UI^{>}, \quad etc., \tag{3.104a}$$

$$dQ = dU + pdV \quad \to \quad d^{>}Q^{>} = d^{>}U^{>} + p^{>} > d^{>}V^{>}, \tag{3.104b}$$

where, in the absence of operator forms, Hermitean conjugation is complex conjugation. For the second law we have

$$dQ = TdS \quad \to \quad d^{>}Q^{>} = T^{>} > d^{>}S^{>}, \tag{3.105}$$

thus implying that

$$TdS = dU + pdV \quad \to \quad T^{>} > d^{>}S^{>} = d^{>}U^{>} + p^{>} > d^{>}V^{>}. \tag{3.106}$$

As one can see, genomathematics permits the *first known formulation of entropy with a time arrow,* the only causal form being that forward in time. When the genounit does not depend on the local variables, the above genoformulation reduces to the conventional one identically, e.g.,

$$T^{>} > d^{>}S^{>} = (TI^{>})I^{>-1}[I^{>-1}d(SI^{>}) = TdS =$$

$$= I^{>-1}d(VI^{>}) + (pI^{>})I^{>-1}d(VI^{>}) = dU + pdV. \tag{3.107}$$

This confirms that genomathematics is indeed compatible with thermodynamical laws.

However, new vistas in thermodynamics are permitted when the genounit is dependent on local variables, in which case reduction (3.107) is no longer possible. An important case occurs when the genounit is explicitly dependent on the entropy. In this case the l.h.s. of Eq. (3.107) becomes

$$TdS + TS(I^{>-1}dI^{>}) = dU + pdV + (U + pV)(I^{>-1}dI^{>}). \tag{3.108}$$

We then have new thermodynamical models of the type

$$I^{>} = e^{f(S)}, \;\; T^{>} > d^{>}S^{>} = T(1 + S\frac{\partial f(S)}{\partial S})dS = dU + pdV, \tag{3.109}$$

permitting thermodynamical formulations of the behavior of anomalous gases (such as magnegases [21]) via a suitable selection of the $f(S)$ function and its fit to experimental data. Needless to say, equivalent models can be constructed for an explicit dependence of the genounit from the other variables. For these and other aspects we have to refer the interested reader to ref. [74].

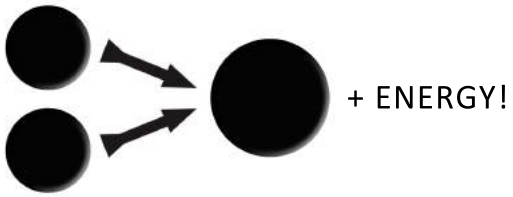

Figure 21: *Another important implication of quantitative studies on irreversibility is the abandonment of quantum mechanics and chemistry for nuclear synthesis (fusion) in favor of a covering theories due to the notorious reversibility over time of said theories compared to the structural irreversibility of any energy releasing processes. The technical understanding of this occurrence requires the knowledge that irreversibility mandates the abandonment of the vert notion of quantized transitional states in favor of integro-differential transitional states Eq. (3.46).*

The above pioneering studies have been continued by A. A. Bhalekar [102] who indicates that in applying time reversal symmetry test to various fluid dynamical equations including the Boltzmann integro-differential equation we are led to the ambiguous results.

Indeed, there is a distinction between thermodynamic and dynamic reversibility, that is in the former case, the identically same states are encountered on reversing the direction because the state of the system doesnt depend on the sign of the exchange differentials.

Whereas in the latter case the direction of a vector associated with the system determines its dynamic state. In view of this we recall the no reduction theorem of Santilli, namely the reversible molecular dynamics cannot produce macroscopic irreversible description and vice-versa the irreversible macroscopic motion cannot be reduced to reversible molecular motion of its constituents [13]. This then demands a mathematical apparatus inherited with unidirectionality of the description.

This demand is met by Santillis genomathematics and the corresponding mechanics. In Santillis genomathematics for forward and backward motions one uses different multiplying genounits namely, for forward motion it is $\hat{I}^{>}$ and for the backward motion it is $i^{<}\hat{I}$ and $\hat{I}^{>} \neq^{<} \hat{I}$.

3.25. Verification with Intermediate Controlled Nuclear Syntheses without harmful radiations

3.25.1. Introduction

Readers of this section are discouraged to appraise its content via 20th century sciences due to insidious inconsistencies that remain generally undetected. The content of this section can be solely appraised via the use of isomathematics, isomechanics and isochemistry. This illustrates the need for the experimental verification of the new disciplines presented in preceding section, prior to their application to new clean nuclear energies.

The scientific journey outlined so far was done for one primary objective of achieving a consistent quantitative prediction and treatment of controlled nuclear syntheses that, to be environmentally acceptable, should not release neutrons or other harmful radiations.

The main preparatory steps toward the achievement of the desired nuclear syntheses (see Section 3.25.6 for additional ones) are the following:

Mathematical and theoretical studies were initiated by the author during his Ph. D. Thesis in the 1960's with the Lie-admissible treatment of time-irreversible processes, Refs. [72-76]. The studies were continued when the author was at Harvard University from 1977 to 1962 under DOE contracts numbers ER-78-S-02-47420.A000, AS02-78ER04742, DE-AC02-80ER1065, DE-AC02-80ER-1065.A001, and DE-AC02-80ER.1065), that resulted in Refs. [1-3,76] plus additional works quoted later on.

Mathematical and theoretical studies were then continued at the Institute for Basic Research from 1983 to 2012 as well as at the Joint Institute for Nuclear Research, Dubna, Russia, the International Center for Theoretical Physics, Trieste, Italy and other institutions [19-20,50-66], (see Ref. [67] for a general review).

The construction of the reactors for the desired nuclear syntheses was initiated in the late 1900 at the laboratory of the Institute for Basic Research, located at 150 Rainville Rd. Tarpon Springs, Florida, where some of the original reactors are still available at this writing.

Industrial research for the control of the orbits of peripheral atomic electrons were conducted from 2003 to 2012 at the U. S. company *Magnegas Corporation* (www.magnegas.com), a company public traded at NASDAQ with stock symbol MNGA, via the construction of a series of reactors partially depicted in Figure 15 for the synthesis of *gaseous fuels* (known as *magnegases*) from liquid feedstock. Magnegas corporation owns all intellectual rights for magnecular gases that are needed for nuclear syntheses (see Section 2.5.6). Said magnegases and their reactors are in regular production and sale.

From 2013 on, all mathematical, theoretical and experimental research on the desired *nuclear syntheses* have been conducted at *Thunder Energies Corporation* (www.thunderenergies.com) also a publicly traded company with stock symbol TNRG), which company owns all intellectual rights on the new *nuclear syntheses* (and *not* on new magnegases), including patents, patent applications, copyright, domain names, know how, etc.

3.25.2. The crucial role of the neutron synthesis from the Hydrogen

By keeping in mind that the investment of trillions of dollars has produced no industrially relevant controlled nuclear syntheses, the author initiated his studies on nuclear synthesis with an in depth appraisal of the insufficiencies of the strictly reversible, 20th century nuclear physics for a consistent representation of the strictly irreversible energy releasing processes.

Besides the Lie-admissible representation of irreversibility reported in the preceding sections, the studies resulted in the following advances in nuclear physics that are at the true foundation of the nuclear syntheses prior to the additional studies presented in Section 2.25.6:

1. Representation of the synthesis of the neutron from the Hydrogen atom. The studies in this problem were initially conducted according to the historical reaction (see Refs.

[19-20,50-59], independent review [67,103-105] and lectures [106,107])

$$p^+ + e^- \ \rightarrow \ n + \nu. \tag{3.110}$$

These studies established that (Section 2.2.22) *the synthesis of the neutron is incompatible with quantum mechanics, quantum chromodynamics, special relativity and other 20th century theories* because the rest energy of the neutron is bigger than the sum of the rest energies of the proton and of the electron, as established by the known data

$$E_p = 938.272 \ MeV, \ \ E_e = 0.511 \ MeV, \ \ E_n = 939.565 \ MeV, \tag{3.111a}$$

$$E_n - (E_p + E_e) = 0.782 \ MeV > 0, \tag{3.111b}$$

Under the above conditions, the Schrödinger equation does not yield physically consistent results due to the need for a "positive binding energy" resulting in a "mass excess" that is beyond any descriptive capacity of quantum mechanics for bound states.

The inapplicability of quantum chromodynamics for the neutron synthesis is easily seen from the impossibility of transforming electrons into quarks and for numerous other technical reasons.

The inapplicability (and certainly not the violation) of special relativity for the neutron synthesis is clearly established by the impossibility of achieving a quantitative representation of synthesis (3.110) via Dirac's equations, by insurmountable technical difficulties of defining the axioms of special relativity within the hyperdense medium inside the proton and by other reasons.

The construction of hadronic mechanics, including its Lie-admissible genomathematics and genomechanics as well as their Lie-isotopic particular cases, were proposed in Refs. [1-3] as necessary pre-requisite for a quantitative representation of the synthesis of the neutron from the Hydrogen.

These studies established that the physical constituents of the neutron are the original particles, one proton and one electron, although in a mutated state due to their total mutual compression and overlapping, technically known as $isoproton$ and $isoelectron$ (i.e. isounitary isoirreducible isorepresentations of the Lorentz-Poincaré-Santilli isosymmetry outlined in Section 2).

It should be stressed that quarks are excluded as hadronic constituents because such a conjecture would imply that the permanently stable proton and electron "disappear" at the time of the neutron synthesis to be replaced by the hypothetical quarks, and at the time of the neutron decay, the hypothetical quarks would "disappear" while the proton and the electron would "reappear."

It should be recalled for completeness that the exact and time invariant representation of the neutron synthesis implied difficulties for the existence of the neutrino for numerous reasons, including the lack of energy needed for its creation, the lack of spin $1/2$ needed to accommodate the neutrino whenever the proton is represented as an extended and hyperdense particle, and other reasons (see Figure 10 and Refs. [19-20,50-59,103,105]).

Therefore, the author suggested the hypothesis of the $etherino$ as an $impulse$ (rather than a particle), represented with the symbol "a" which propagates the missing energy

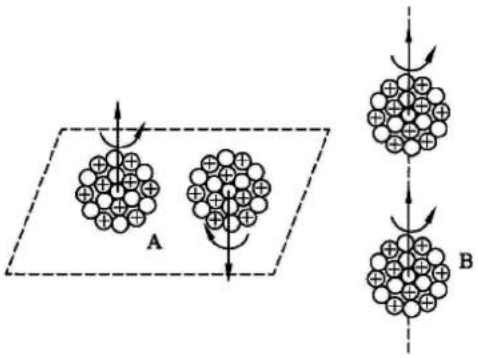

Figure 22: *A generally ignored condition for controlled nuclear syntheses is the proper coupling of nuclear spins. According to hadronic mechanics, said couplings must be either the "planar singlet coupling" (in the left of this picture) or the "axial triplet axial coupling" (in the right) because all other spin couplings cause strongly "repulsive" forces between nuclei [110,111].*

from the ether (conceived as a medium with extremely high energy density) to the neutron, according to the reaction [108]

$$p^+ + a + e^- \ \rightarrow \ n, \tag{3.112}$$

Th where one should note the presence of the etherino in the *left* (rather than the right as suggested by W. Pauli) of the reaction as a necessary condition to deliver the missing energy, since the latter cannot be carried by the proton and the electron because their cross section for relative energies of the order of $1 \ MeV$ is virtually null [19-20,50-59, 67,103-108].

Independently from the above studies, the physics community is suggested to face the reality that, after about one century of attempts and the expenditure of billions of taxpayer's money, no serious experimental evidence has been achieved on the actual existence of the neutrino because:

1) We first had the historical failure to detect "solar neutrinos" (neutrinos created inside the Sun by the synthesis of the neutron according to reaction (3.110) predicted at the rate of $10^3030 \ per \ second$) which solar neutrinos are predicted by 20th century nuclear physics to hit us on Earth in extremely big numbers per square centimeter.

2) In view of the preceding failure, there has been an increasing number of hypothetical neutrinos without any actual experimentally verifiable differentiation besides nomenclatures, all conjectures intended to maintain the validity of special relativity for electroweak interactions.

3) There was the need, again to maintain special relativity, that neutrinos have masses, thus creating evident uneasiness in the scientific community for accepting that massive particles can traverse entire planets without any appreciable collision.

4) There was the need, also to maintain special relativity, that neutrino mysteriously "oscillate" (presumably referred to neutrino changing one into another) with additional

uneasiness by the serious scientific community.

5) There were systematic failure to actually detect neutrinos in experiments at CERN except for excessively few presumed events out of an extremely large number of events;

6) Then there was the termination of neutrino experiments at the Gran Sasso Laboratory in Italy which termination was mandated by the Italian Government due to the lack of any serious detection of neutrinos in an extremely large detector over a number of years.

7) Despite the plethora of pre-existing neutrinos, there has been the conjecture of an additional one, this time called "sterile neutrino, and the recent public release in the U.S.A. of its lack of detection.

It is hoped that theoretical and experimental physicists have the accountability toward taxpayers of initiating the study of structural revisions of their theories via the inevitable abandonment of special relativity in favor of covering theories, because *whenever the proton is represented as an extended and hyperdense particle during the synthesis of the neutron from the Hydrogen the neutrino "cannot" exist since the electron acquires a constrained angular momentum* $1/2$ *after its compression inside the proton, thus leaving no spin for the hypothetical neutrino, in addition the lack of energy for its formation* [19-20,50-59, 67,103-108]

2. Representation of the structure of nuclei as bound states of isoprotons and isoelectrons. Despite historical achievements, the author considered 20th century representations of the nuclear structure (see, e.g., Refs. [100]) to be basically insufficient for consistent quantitative predictions and elaborations of controlled nuclear syntheses.

One of the reasons for said insufficiencies is that 20th century nuclear models failed to achieve over one century a consistent representation of the simplest nucleus, the deuterium, due to: the inability to represent the deuterium spin 1 for the ground state of two particles (the proton and the neutron) both having spin $1/2$; the lack of an exact relativistic representation of the deuterium magnetic moment and other insufficiencies.

For the evident intent of preserving quantum mechanics, these insufficiencies could be somehow bypassed for the deuterium via a number of *ad hoc* assumptions, such as the manifestly implausible assumption that the *ground state* of the deuterium is a combination of *excited states* since they are the sole to have non-null angular momenta, plus additional *ad hoc* assumptions [100].

However, the same adaptations became impossible for more complex nuclei where the deviations of quantum mechanics from nuclear experimental data are simply embarrassing, thus mandating a structural revision of the entirety of 20th century nuclear physics [8].

The reduction of the neutron to one isoproton and one isoelectron under the laws of isomathematics and isomechanics allowed the achievement of a three-body structure model of the deuterium in monograph [60] of 1998 (see independent review [109]) with the first known, numerically exact and time invariant representation of *all* experimental data for the deuterium from unadulterated first principles.

In particular, the spin 1 of the deuterium emerged as a clear experiential evidence on

the *three-body* structure of the deuterium (Figure 12).

The reduction of the deuterium to isoprotons and isoelectrons then allowed corresponding structure models for all stable nuclei, thus allowing the reduction of all matter in the universe to protons and electrons in conditions of progressively increasing complexities and methodological needs [60] (see Ref. [67] for a review).

3. Approximation of nuclear structures as bound states of extended and deformable isoprotons and isoneutrons under conventional Hamiltonian, as well as non-linear, non-local and non-Hamiltonian internal forces. In this case, nuclei are generally referred to as "isonuclei" to stress the necessary use of isomathematics and isomechanics.

To avoid a prohibitive length, the nuclear syntheses outlined herein are based on the *approximation* of nuclei as being composed of extended and deformable isoprotons and isoneutrons. In this case, the primary symmetries are: the isotopies of the $SO(3)$ for the angular momentum [11,12]; the isotopies of $SU(2)$ for the spin [13-14], and the isotopies of the Lorentz-Poincaré symmetry [7,15,16].

However, the reader should be aware that, whenever nuclear syntheses require an exact representation of magnetic moments [47,48] and spins [49], the use of isoprotons and isoelectrons as nuclear constituents is necessary for quantitative, numerically exact representation of nuclear data (Sections 2.20 and 2.21).

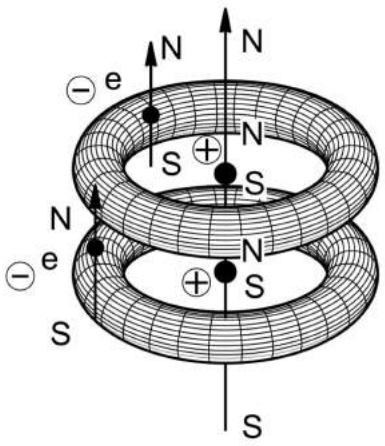

Figure 23: *A view of an elementary magnecule, namely, a two-atom magnecular bond as possible between Carbon and Oxygen, denoted $C \times O$, which was specifically conceived by the author as a necessary premise for Intermediate Controlled Nuclear Syntheses via the systematic and controlled exposure of nuclei out of their peripheral electron shell, as well as to achieve the necessary spin alignment, in this case of axial triplet type (Figure 22) in whose absence no nuclear synthesis is expected to acquire industrial relevance [61,64].*

3.25.3. Intermediate Controlled Nuclear Syntheses (ICNS) without harmful radiations

Following, and only following the acquisition of mathematical, theoretical and experimental knowledge on the synthesis of the neutron from the Hydrogen, the author initiated in the late 1990's systematic studies on nuclear syntheses.

By noting that, following the expenditure of large public funds, Low Energy Nuclear Syntheses (LENS) and High Energy Nuclear Syntheses (HENS), have not achieved industrially valuable results, the author showed that the energy available for the LENS is insufficient to service all operating and control systems, while the energy available for HENS is excessive, thus triggering known uncontrollable instabilities.

Therefore, the author proposed the novel *intermediate Controlled Nuclear Synthesis* (ICNS) of two light, nature, stable nuclei into a third light, natural and stable nucleus without the emission of harmful radiations and without the release of radioactive waste, also known under the nickname of *warm fusion* (Figure 21).

ICNS were first introduced by the author at the 1998 *Salt Lake City International Meeting on New Energies* [60], and then studies in Refs. [110-119] and with some of the large number of chemical analyses by independent laboratories available from Refs. [120-130] (see later on independent verifications).

The basic isonuclear reactions of ICNS are given by [60]

$$N_1(A_1,\, Z_1,\, J_1^{p_1},\, u_1) + N_2(A_2,\, Z_2,\, J_2^{p_2},\, u_2) + \mathrm{TR} \to$$
$$\to N_3(A_3,\, Z_3,\, J_3^{p_3},\, u_3) + \mathrm{Heat}, \tag{3.113}$$

under the verification of verifying all conventional nuclear conservation laws, such as

$$A_1 + A_2 = A_3,\ Z_1 + Z_2 = Z_3,\ J_1 + J_2 = J_3,\ p_1 + p_2 = p_3, \tag{3.114a}$$

$$U_1 + u_2 = u_3 + \Delta E,\ \ \Delta E = E_3 - (E_1 + E_2) > 0. \tag{3.114b}$$

Two light, natural and stable elements verifying all the above conditions are called *hadronic fuels* and the reactors providing their engineering realization are called *hadronic reactors,* to stress the essential use of hadronic mechanics at large, and its Lie-admissible branch in particular for their very connection, let alone technical realization.

Note the selection of hadronic fuels in such a way to produce no radiations at all, except heat, with the understanding that the operation of the hadronic reactors may indeed produce beta and electromagnetic radiations that are absorbed by the reactor walls and shielding.

Note also that, *the use of Hydrogen is strictly prohibited as hadronic fuel,* for the evident reason that, in the event Hydrogen is used, the synthesis and release of neutrons appear to be inevitable, thus violating the very central conditions for the development of ICNS.

3.25.4. Physical laws of ICNS according to hadronic mechanics

An inspection of available experiments on low and high energy nuclear syntheses reveals the lack of verification of basic laws necessary for the achievement, and then the optimization of truly controlled, industrially valuable nuclear syntheses.

Additionally, available experiments on nuclear syntheses were based on the use of quantum mechanics and special relativity, resulting in insidious inconsistencies as well as

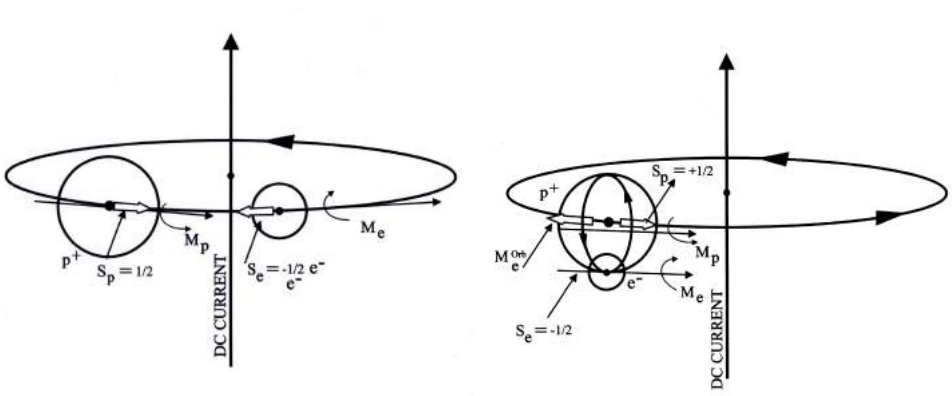

Figure 24: *A view of the main engineering principle for Intermediate Controlled Nuclear Syntheses which is based on the use of a suitable DC electric arc that: 1) Separates the original molecules into their individual atoms. 2) Polarizes the atomic electron orbits into toroids. 3) Couples them into the magnecular configuration of Figure 23. After the systematic achievement of such a configuration, engineering realizations of the "trigger" are activated to push nuclei at* 1 $Fermi$ *mutual distances against their repulsive Coulomb force (see Section 2.26 for its possible resolution), at which mutual distance the synthesis is unavoidable to the activation of attractive nuclear forces [61,64].*

misrepresentation of the data that generally remain unknown to non-experts in hadronic mechanics.

For instance, existing attempts to achieve controlled nuclear syntheses miss any means for the systematic exposure of nuclei out of their protective electron shells, in which case any systematic nuclear synthesis is known to be impossible.

Similarly, available experiments miss theoretical and engineering means for the proper coupling of nuclear spins, thus implying the occurrence of nuclear syntheses at random, because it is known that nuclei with opposing spin experience a very large *repulsion* when at 1 $Fermi$ mutual distance.

In view of the above insufficiencies, the author conducted systematic studies of nuclear syntheses via the use of genomathematics, genomechanics, and genochemistry resulting in the following basic laws first introduced in 1998 [60] (see also ref. [110]):

HADRONIC LAW 1: The electron clouds protecting hadronic fuels must be deformed in such a way to expose their nuclei in a controlled and systematic way, in which absence no ICNS is possible.

HADRONIC LAW 2: The spins of exposed nuclei according to Hadronic Law 1 must be controlled in a systematic way to have either the "planar singlet coupling" or the "axial triplet coupling" as defined in Figure 22, as an additional necessary condition to achieve ICNS.

HADRONIC LAW 3: Following the controlled exposure of nuclei and the controlled coupling of nuclear spins, ICNS requires a "trigger" intended as an engineering mean pushing the

exposed nuclei to be in contact with each other, in the absence of said conditions no systematic ICNS is possible due to the known Coulomb nuclear repulsion.

HADRONIC LAW 4: The energy used for the systematic and controlled implementations of Hadronic laws 1, 2 and 3 must have a "threshold value" per each selected pair of hadronic fuels, in the sense of having the "minimal" energy necessary to service all engineering components and controls. In the event of insufficient or excessive energy, no systematic ICNS is possible.

HADRONIC LAW 5: The most probable reactions for ICNS are those between a pair of hadronic fuels for which there is no emission of harmful radiations and no release of radioactive waste.

It should be indicated that, while Hadronic Laws 1, 2, 3 and 4 were derived via the use of hadronic mechanics, Hadronic Law 5 was based on the observation of nature.

Bubble of air contained in amber indicates that about 1 hundred million years ago, our atmosphere contained about 30% of Nitrogen, thus indicating the existence on Earth of a continuous process synthesizing Nitrogen as well as other elements.

In the author's view, the most plausible process is that of lightening since only nuclear syntheses can provide a numerical representation of thunder. The lack of emission of neutrons and the lack of release of radioactive waste is then established by our very existence.

All engineering efforts currently being conducted by Thunder Energies Corporation aim at the implementation and optimization of the above five hadronic laws.

Note that the trigger is a macroscopic engineering means forcing the two nuclei together that, as such, can cause no contribution at all for reaction (3.113).

3.25.5. Examples of ICNS

Inspection of the Table of Nuclides [131], establishes the existence of numerous pairs of light, natural and stable nuclei qualifying as hadronic fuels, that is, suitable for nuclear syntheses (3.113) under the verification of all conservation laws (3.114).

We review here a few illustrative cases from Refs. [60, 61,110-120] via the use of standard conversions

$$1\ u = 931\ MeV = 1.1415 \times 10^{-13}\ BTU,\ \ 1\ g = 114,000\ BTU, \tag{3.115}$$

where "g" stands for one standard, U. S. gasoline gallon.

The simplest ICNS subjected to extensive tests and independent experimental verification (see Sections 3.25.7) is the synthesis of the Nitrogen 7-N-14 from Deuterium 1-D-2 and Carbon 6-C-12 plus the "trigger" hereon ignored for simplicity

$$\begin{aligned} D(2,1,1^+,2.0141) + C(12,6,0^+,12.0000) + TR \quad \rightarrow \\ \rightarrow \quad N(14,7,1+,14.0030) + \Delta E_{heat}, \end{aligned} \tag{3.116}$$

Figure 25: *The experimental set up used by the author for the first known synthesis of the Nitrogen from carbon and Deuterium [110-111].*

with the production of energy per synthesis

$$\Delta E = (E_D + E_C) - E_N = 0.0111\ u = 10.2231\ MeV =$$

$$= 0,0157 \times 10^{-13} BTU/synthesis. \tag{3.117}$$

Consequently, under the assumption of 10^9 nuclear syntheses *per hour* the above synthesis would yield the energy production

$$E/h = 15,700\ BTU/h, \tag{3.118}$$

that in order to be industrially meaningful, has to be *bigger* than the total used energy by the related hadronic reactor. This is clearly an unlikely possibility, as a result of which the above synthesis is mainly use to establish the *existence* of ICNS, and certainly not for the industrial production of new clean nuclear energies.

Note that no neutron emission is conceivably possible for the above ICNS because, in case the Nitrogen synthesis occurs, no neutron can possibly be emitted and in the event the synthesis does not occur, no neutrons can possibly be created because the energy used by the hadronic reactors is about 10^{-6} times the energy needed to split the Carbon nucleus as a condition to emit neutrinos.

A second ICNS of particular significance for environmental issues (see Section 3.28), also subjected to extensive tests and independent verification, is the synthesis of Silicon 16-Si-28 from Oxygen 8-O-16 and Carbon 6-C-12 according to the reaction

$$O(16,8,0^+,17.9991) + C(12,6,0^+,12.0000) + TR \rightarrow$$

$$\rightarrow Si(28,14,0^+,27.9769) + \Delta E, \tag{3.119}$$

producing the energy per synthesis

$$\Delta E = 2.0222\ u = 1,882.6682\ MeV = 2.8604 \times 10^{-13} BTU/reaction \tag{3.120}$$

that, under the reasonable assumption of 10^9 nuclear syntheses $per\ hour$ would yield the energy production

$$E/h = 2.8604 \times 10^6\ BTU/h \tag{3.121}$$

corresponding to the significant energy content of about $25\ g$ per hour.

This author is conducting research and development for a publicly traded and funded company (Thunder Energies Corporation), thus requiring particular caution. Despite the predicted rather significant release of clean energy, the synthesis of the Silicon from Oxygen and Carbon is currently studied (under patent pending) for the cleaning and $enhancement$ of the energy output of the combustion of fossil fuels (Section 3.28), and it is not used (at this time) for the independent production of clean nuclear energy.

A third ICNS of environmental value is that of Sulfur 16-S-32 from two Oxygen atoms 8-O-16

$$O(16,8,0^+,17.9991) + O(16,8,0^+,17.9991) + TR \rightarrow$$
$$\rightarrow S(32,16,0^+,31.9720) + \Delta E, \tag{3.122}$$

producing per synthesis

$$\Delta E = 4.0262\ u = 3,7483,3922\ MeV = 5.6950 \times 10^{-13} BTU/synthesis \tag{3.123}$$

that, under the assumption of 10^9 nuclear syntheses $per\ hour$ would yield the energy production

$$F_h = 5.6950 \times 106\ BTU/h, \tag{3.124}$$

corresponding to about $72\ g$ per hour, namely, an mount of clean energy indeed of industrial relevance.

An additional ICNS of significant environmental value (since it can be a secondary synthesis following the synthesis of the Silicon, which is a rather frequent occurrence for ICNS) is the synthesis of Titanium 22-Ti-44 from the Oxygen 8-O-16 and Silicon 14-Si-28

$$O(16,8,0^+,15.9949) + Si(28,14,0^+,29.9737) + TR \rightarrow$$
$$\rightarrow\ Ti(44,22,0^+43.9596) + \Delta E, \tag{3.125}$$

also carrying a significant energy release per synthesis (although with a lesser frequency compared to the primary synthesis of the Silicon)

$$\Delta E = 2.0090\ u = 1870.3790\ MeV = 2.8417 \times 10^{-13}\ BTU. \tag{3.126}$$

Additional possible INCS worth mentioning are the synthesis of 9-F-18 from 1-D-2 and 8-O-16

$$D(2,1,1^+,2.0141) + O(16,8,0^+,15.9949) + TR \quad \rightarrow \tag{3.127}$$

$$\rightarrow \quad F(18, 9, 1+, 18.0009), \tag{3.128}$$

with energy release per synthesis

$$\Delta E = 0.0081\ u = 7.5411\ MeV = 0.0115 \times 10^{-13}\ BTU/.synthesis, \tag{3.129}$$

and the synthesis of 47-Ag-107 from 1-H-1 and 46-Pd-106

$$H(1, 1, 1/2^+, 1.0078) + Pd(106, 46, 0^+, 105.9034 + TR \rightarrow$$

$$\rightarrow Ag(107, 47, 1/2^+, 106.9050) + Heat \tag{3.130}$$

$$\Delta E = 0.0062\ u = 5.7722\ MeV = 0.0088 \times 10^{=13}\ BTU \tag{3.131}$$

and numerous others easily identifiable from the Table of Nuclides [131].

Figure 26: *A view in the left of the team from Princeton Gamma Spectroscopy that conducted extensive verifications of Santilli's synthesis of the Nitrogen from the Deuterium and Carbon, and a view in the right of the confirmation of the lack of emission of neutrons and the lack of release of radioactive waste [134].*

The above illustrative examples have been selected under the strict implementation of the central condition for ICNS, namely that two light, natural and stable nuclei are synthesized into a third light, natural and *stable* nuclei, without the emission of harmful radiation and without the release of radioactive waste.

Needless to say, there exist numerous pairs of light, natural and stable nuclei admitting their synthesis into a third light and natural but *unstable* nuclei that, as such, naturally decays with the possible release of neutron and other harmful radiation.

Among the latter group, there exist indeed ICNS that are environmentally acceptable, e.g., because of the lack of emission of neutron, alpha and other harmful radiation, although their study is left to the interested reader, since the studies reported in this paper solely deal with ICNS according to the rigorous implementation of their definitions and physical laws.

A third additional class of ICNS is possible with the admission in the l.h.d. of the reaction (3.113) of the etherino [110-120], although their study too is left to the interested reader to avoid a prohibitive length of this paper.

As indicated earlier, Hydrogen is *excluded* as hadronic fuel. The first reason is that the hadronic reactors for ICNS are equally capable of synthesizing neutrons when filled up with Hydrogen.

Additionally, the use of Hydrogen for nuclear syntheses is afflicted by serious mathematical, theoretical and experimental difficulties. As an example, the direct synthesis of the Helium from two Hydrogen is manifestly impossible

$$H(1,1,1/2^+,1.0078) + H(1,1,1/2^+,1.0078) + TR \neq He(4,2,0^+,4.0026), \qquad (3.132)$$

because of the violation, in general of the conservation of the total angular momentum and for other reasons.

Of course, there may be suitable engineering means allowing the spin of the two protons to have opposite orientation. However, the complexity of these engineering requirements should be compared with the comparatively easier realizations of ICNS (3.122)-(3.141) where no polarization of nuclear spin is needed at all, besides the problem of neutron emission.

3.25.6. Necessary premises for ICNS

In Section 2.15.1 we have identified the background requirements for the prediction and quantitative treatment of ICNS. In this section we outline the additional, rather considerable, requirements for their engineering realization.

It should be stressed upfront that in this section, we report experimental evidence on the *existence* of ICNS. Their optimization for a new type of combustion is reported Section 3.28 in support of (rather than the "replacement" of) fossil fuels. Research currently under way for the use of ICNS as a new source of clean energies cannot be reported at this writing.

An inspection of expected ICNS (3.116) to (3.131) reveals that the original hadronic fuels are generally given by ordinary *molecules,* such as Hydrogen $H_2 = H - H$, Oxygen $O_2 = O - O$, Carbon Monoxide $C - O$, etc., where the symbol "$-$" represents conventional valence bonds. It is then evident that, except for random events, no truly controlled nuclear synthesis is possible without:

1) Controllable and efficient engineering means for molecular separation;

2) Controllable and effective engineering means for the systematic control of the exposure of nuclei out of their electron shells;

3) Controllable and effective engineering means for the proper spin alignment;

4) Controllable and effective engineering means for the realization of the "trigger";

5) All necessary electronic means for remote control of all operations;

and numerous additional requirements.

It is evident that a resolution of the above engineering problems essentially required a re-inspection of all 20th century sciences, including a re-inspection and suitable reformulation of 20th century mathematics, physics and chemistry.

3.25.7. Engineering realization of ICNS

The necessary use of hadronic chemistry

Since most of the hadronic fuels are ordinary molecules, the author initiated his studies by reinspecting 20th century chemistry with particular reference to quantum chemistry and discovered that, despite advances and discoveries of historical proportions, *quantum chemistry is not recommendable for the studies of new energies and fuels,* for numerous reasons.

The first reason is that, indicated earlier, namely, that quantum chemistry is strictly reversible over time while all energy releasing processes are irreversible, thus mandating the construction of an irreversible covering of quantum chemistry, that was achieved achieved via the Lie-admissible genochemistry [61].

The second reason is that, *molecules cannot exist for quantum mechanics and chemistry because the identical valence electrons in singlet coupling cannot possibly bond together as occurring in nature, because their repulsive Coulomb force because extremely big at mutual distance of valence bonds*

$$F_{r=1\ F} = N\frac{e^2}{r^2} \approx 10^{26}\ N. \tag{3.133}$$

Consequently, the author had no other choice than that of identifying a *new notion of valence bond* today known as *santilli isovalence bond* (where the prefix "iso" denotes again its derivation via the necessary use of isomathematics). The novel isovalence has been able to provide a fully identified force between identical electrons in singlet coupling, which force resulted to be "attractive" as well as verifying all experimental data on molecular structures [61-63].

Since molecules exist in nature, since nuclei do not participate appreciably in molecular bonds, since molecular bonds are indeed created by peripheral valence electrons, and since all possible potential forces had field to achieve an attraction between valence electrons, it was evident that the "attractive" force sufficiently strong to overcome the Coulomb repercussion is due to a *basically new interaction in nature.*

The new strongly attractive force was first identified in paper [2] of 1978, then studied extensively in various papers and finally presented in a comprehensive way in monograph [61[of 2001, and resulted to be the operator version of Lagrange's and Hamilton's external terms caused by the zero-range total mutual penetration of the wavepackets.

Among numerous independent and important contributions, we quote here that by A. O. E. Animalu [132] who recognized that the force between the electrons and positrons in the π^0 synthesis of Ref. [2] is so strongly attractive, to remains attractive even for the electron pairs in superconductivity, and proved its validity via the new discipline known as *Animalu isosuperconductivity* [60,61].

The analytic solution presented in monograph [61] for the novel isovalence bond is identical to that originally presented in paper [2], namely, the deep overlapping of the wavepackets of identical valence electrons in singlet coupling generates non-linear, nonlocal and non-Hamiltonian forces that can be represented via strongly attractive a Hulten potential. The Hulten potential notoriously behaves like the Coulomb potential at very short distance, by therefore "absorbing" the latter and resulting in a strong attraction.

Unpublished advances since 2001 have shown that the mechanism for the creation of an attraction between identical valence electrons is much more complex then the above

view since it is structurally dependent on the *isorenormalization* (normalization due to non-Hamiltonian forces) of the charge.

According to the latter view, the very notion of charge as defined for an isolated electron is no longer valid to the same electron when in conditions of deep mutual compression with other particles, exactly as it is the case for the synthesis of the neutron.

In turn, quantitative studies on the isorenormalization of the elementary charge of the electron requires the use of *structure models of the electron itself*, such as the rudimentary model by the author according to which the electron is a "pure oscillation" (i.e., oscillation without oscillating mass) of one point of the ether conceive as the universal substratum for the existence and propagation of elementary particles as well as electromagnetic waves [133] (see [67] for a review).

Figure 27: *A view of the Hadronic Reactor II used by the author for the first synthesis of the Silicon from Carbon and oxygen, of Titanium from Silicon and Oxygen and other ICNS [112].*

The necessary use of Santilli magnecules

Following a more accurate *quantitative* representation of the structure of molecules, the author conceived, proposed and verified experimentally in 1998, the new chemical species of *Santilli magnecules* (see Section 2.25 and Ref. [61,64]) for the specific purpose of achieving a realization of Hadronic Laws 1 and 2.

In particular, the new chemical species was based precisely on the polarization of the "orbits" (and *not* the "orbitals") of valence electrons, from their natural spherical distribution protecting nuclei to toroidal distributions that exposed nuclei and properly align them along the axial triplet coupling, exactly as needed for ICNS (Figure 23).

Said toroidal distribution creates a *new magnetic force* along its symmetry axis that does not exist in the natural spherical distribution, thus allowing the coupling of polarized atoms with opposite magnetic polarities. The magnecular bond was conceived to be stable at ambient temperature, but be much *weaker* than the isovalence bond as a necessary condition to allow full combustion [61,64] (see Figure 30).

Engineering realization of magnecules

Following the experimental verification of the existence of magnecules in refereed journals [64], the author initiated in the early 2000 their industrial realization in preparation for subsequent studies in nuclear syntheses.

As correctly studied by A. K. Aringazin [134] (see also the review in monograph [61]), the industrial realization of magnecules was faced with the rather serious engineering problem according to which the control of the orbits of peripheral atomic electrics requires very strong magnetic fields of the order of $10^{10}\ G$ or more, which fields are not attainable even at the most advanced magnetic laboratories

An additional engineering problem (among others) was the *stability* of the new magnecular bond (indicated with notations of the type $C \times O$), in view of the fact that, by conception, their magnetic attraction was much weaker than the isovalence bond, and that in the absence of said magnecular field, the magnecular bond terminates instantly due to temperature, collisions and other factors.

In order to achieve an industrially viable solution the author founded in 2003 *Magnegas Corporation* (a U. S. company publicly traded at NASDAQ with symbol MNGA) which company owns all intellectual rights on the new chemical species of magnecules, including patents, patent applications, trademarks, domain names, copyrights, know how, etc.

The solution of the indicated engineering problem was provided by the use of suitably realized DC electric arcs submerged within a liquid since their magnetic field is inversely proportional to the square of the distance

$$M = N\frac{I}{r}. \tag{3.134}$$

Therefore, at distance of $10^{-8}\ cm$ and for currents I with $10^3\ A$, the local magnetic field is of the order of $10^{11}\ G$, thus being sufficient for the needed control of the orbits of peripheral electrons into toroids (see Figure 24).

The stability of the magnecular bond at ambient temperature was established by gas chromatographic measurements. It essentially resulted that, once a magnecular bond of type $C \times O$ has been created by the DC arc, the spherical distributions is indeed instantly recovered at the termination of said arc, however, for the magnecules as a whole.

It essentially emerged that magnecular bonds are indeed weak but sufficient to survive at ambient temperatures. However, at a sufficiently high temperature (called the Curie temperature), said bonds cease to exist by releasing the constituents in their individual atomic form, exactly as needed for ICNS.

3.25.8. Experimental confirmation ICNS

Following the above scientific journey that essentially lasted for half a century, the author finally initiated the construction in the early 2000's of hadronic reactors specifically conceived for ICNS, predictably, specialized for the considered ICNS due to the variety of hadronic fuels, although all reactors essentially comprised (see the figures of numerous configurations in Refs. [60,61] as well as patents and published patent applications):

1. A metal pressure vessel containing in its interior the selected pair of hadronic x fuels

Figure 28: *A view the Hadronic Reactor III built and used by the author for the synthesis of Silicon from Carbon and Oxygen, the synthesis of Titanium from Silicon and oxygen and other ICNS [110-120] (see Ref. [114] for a video on the reactor and ref. [115] for its sound during operation).*

and suitably shielded to contain electromagnetic radiations (beta radiation being trapped by the metallic walls of the vessel);

2. A specially built high voltage DC power unit to delivery in the interior of vessel a DC arc having special features, as well as engineering means for the realization of the "trigger" (that cannot be disclosed at this moment due to patent pending);

3. Numerous remotely operated electronic controls of the power, voltage, current, pressure, "trigger," and other features; plus additional features, too numerous and specialized for identification in this paper.

The totality of the intellectual properties on ICNS, including the variety of hadronic reactors for their realization, patents, patent applications, copyrights, domain names, know how, etc., are owned by *Thunder Energies Corporation* (www.thunder-energies.com), a publicly traded U. S. company with stock symbol TNRG.

It should be indicated that the reactors for the production of magnegases and those for ICNS are conceptually and structurally different on numerous groups as more precisely identified in the related patents and patent applications, such as: the former reactors are designed to produce a gaseous fuel, while the latter are desired to produce usable heat; the feedstock of the former are liquids while those of the latter are gaseous; DC arcs submerged in liquids are substantially different than those submerged in gases; etc.

Years of tests by the author have been reported in Refs. [60,61,110-119] while some of the chemical analyses by independent laboratories are available in Refs. [120-130]. Hadronic reactors for ICNS are friendly called "Dragons" because in the event one of their ports is open during operation, they literally spit fire.

Figure 26 depicts Dragon I that established for the first time to our knowledge the existence of the laboratory synthesis of the Nitrogen from Carbon and Deuterium, according to reactions (1.116). In this case, the reactor is filled up with Deuterium gas at pressure, while the submerged DC arc occurs between commercial grace carbon electrodes supplying the needed Carbon (see Refs. [110,11] for details).

Figures 28 and 29 depict Dragon II and III, respectively, for the laboratory synthesis of: Silicon from from Oxygen and Carbon according to reaction (3.119); Sulfur from two Oxygen atoms according to reaction (3.122); and Titanium from Silicon and Oxygen according to reaction (3. 125) (see Refs. [60,61,112] for details, video [114] on the operation of D=Dragon III, and VLC link for its sound when in operation).

Figure 30 illustrates one of the independent chemical analyses showing a peak on the syntheses of the Silicon which is about twenty times background values. Inspection of chemical analyses also establishes the syntheses of the Sulfur and Titanium.

Additional information on hadronic reactors for ICNS that can be released to the public is available from the corporate website.

3.25.9. Independent confirmations of ICNS

Refs. [134-141] provide independent verification of Santilli's ICNS with particular reference to the joint verification of the confirmation of the nuclear synthesis of a light, natural and stable element that did not exist prior to the operation of the reactor, jointly with the all important verification of the complete lack of detection of any emission of

Sample ID	%Na	%Al	%Si	%S	%Cl	%K	%Ca	%Ti
8.23.13.A x	0.55	0.015	0.024	0.010	0.015	0.005	0.006	0.001
8.23.13.A (-)	0.44	0.11	0.47	0.028	--	0.006	0.041	0.002
8.23.13.A (+)	0.30	0.020	0.15	0.014	--	--	0.013	--

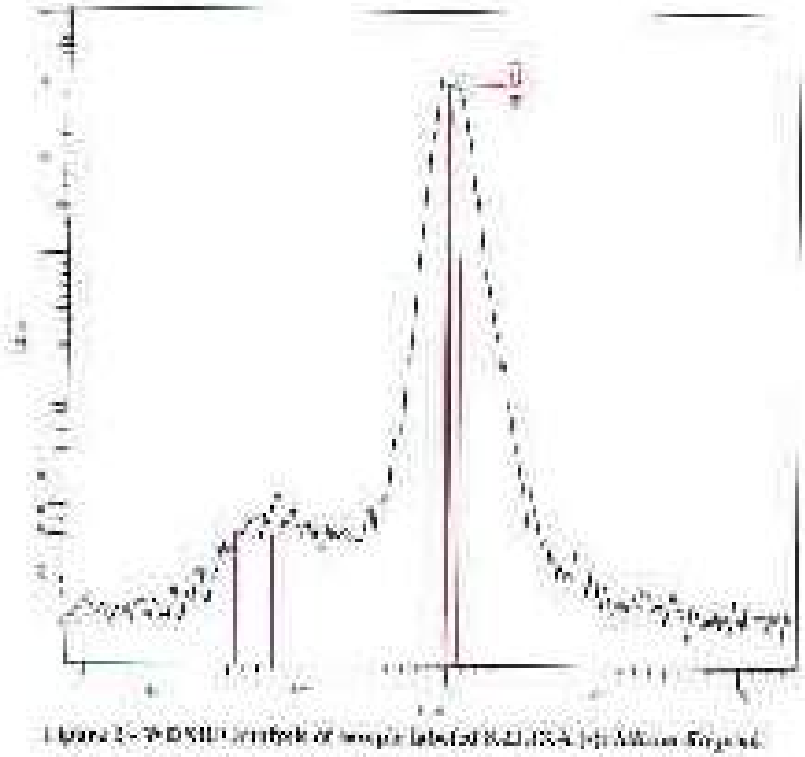

Figure 29: *Views of a report by Constellation Laboratory, Largo, Florida, signed by the director [130] showing a twenty-fold increased of Silicon over background values obtained via the use of Hadronic Reactor III of the proceeding figure (see Ref. [129] for details)*

neutrons.

The l. h. s. Figure 27 illustrates the team of independent verification [134] of the synthesis of the Nitrogen from Carbon and Oxygen without the emission of harmful radiations via the use of the same Dragon I used in the original tests (Figure 26). The r.h.s. of Figure 27 illustrates one of the detectors used to establish the lack of emission of any neutron.

Figure 30: *Pictures in the exterior of the NASDAQ building in Time Square, New York, of the "ringing of the bell" at the closing of trade in June 20, 2013, in recognition of the environmental and industrial achievement of Magnegas Corporation, showing the corporate team in the left and the author in the right.*

3.26. Pseudo-proton, pseudo-nuclides and their nuclear transmutations with Coulomb attraction

As it is well known, the biggest obstacle to overcome for the achievement of any nuclear synthesis of the two nuclei into a third is the Coulomb repulsion between the two original

nuclei due to their positive charge,

$$F = k\frac{Z_1\ Z_2`e^2}{r^2}, \tag{3.135}$$

which repulsion acquires very big value at the mutual distance needed to achieve the synthesis (about $10^{-13}`cm$).

In this section, we briefly outline ongoing research at Thunder Energies Corporation to achieve new clean nuclear energies *without* said Coulomb repulsion.

The main idea is that the achievement of the laboratory synthesis of the neutron from a proton and an electron (Sections 2.22 and 3.25) implies the evident possibility of a subsequent synthesis, that of the neutron and one additional electron, resulting in a new particle, first proposed by the author in Ref. [143], Appendix A in particular (see also ref. [144] and independent study [145]) called *pseudo-proton* and indicated with the symbol $\bar{p}^-$

$$n + e^- \to \bar{p}^-, \tag{3.136}$$

where we assume the null value of the total spin of the electron compressed inside the hyperdense proton (Figure 10) or via the alternative synthesis

$$p^+_\downarrow + e^-_\uparrow + e^-_\downarrow \to \bar{p}^-_\downarrow \tag{3.137}$$

where one cans see more clearly the complete absence of any need for the conjecture of the neutrino.

The important point is that *the pseudo-proton is negatively charged, thus avoiding the Coulomb repulsion and actually being being attracted by nuclei.* In this case we speak of *nuclear transmutations,* rather than syntheses, under the laws of hadronic mechanics.

As it is the case for the neutron, the pseudo-proton is unstable but its mean-life is predicted to be sufficiently long to trigger nuclear *transmutations* (rather than nuclear syntheses) into a new class of nuclides introduced by the author under the name *pseudo-nuclides* [144[according to the general law of hadronic mechanics

$$\bar{p}(1,\ 0,\ -1,\ u_{\bar{p}*-}) + N(A,\ Z,\ J,\ M) \to \bar{N}(A+1,\ Z-1,\ J,\ M + U_{\bar{p}^-} - BE). \tag{3.138}$$

that are generally untabulated, where BE represents the binding energy caused by the opposite charges of the pseudo-proton and of the nucleus.

In regard to the rest energy of the pseudo-proton, it should be indicated that traditional quantum mechanical thinking would tend to suggest that the rest energy of the pseudo-proton is considerably *smaller* than that of the proton ($1.008\ u$) due to they strongly attractive Coulomb interactions.

The author suggests caution for "any" use of quantum mechanical thinking whenever dealing with hyperdense media under the laws of hadronic mechanics. In fact, the negative charge of the pseudo-proton is the total value of a bound state under non-linear, non-local and non-Hamiltonian interactions that imply the isorenormalization of the internal charges.

Consequently, the serious statement at this writing in regard to the rest energy of the pseudo-proton is that it is unknown and, as it was historically the case for the proton, said rest energy has to be measured due to the complexity of the theoretical aspects and the paucity of knowledge at this writing of the new vistas.

Needless to say, the new pseudo-nuclides $\bar{N}$ of transmutation (3.138) are generally unstable and decay with the release of energy as well as of harmful radiations and/or radioactive waste that are unacceptable for the sustainable new nuclear energies under development by Thunder Energies Corporation.

However, particularly when dealing with light pseudo-nuclides said energy can be released without thee mission of radiations and without the release of radioactive waste, In fact, this is the case for the stimulated transmutation of 3-Li-7 [144]

$$\begin{gathered}\mathbf{\bar{p}^{-}(1,-1,1/2\uparrow,1.008)+Li(7,\,3,\,3/2_{\downarrow},\,7.016)\rightarrow}\\ \mathbf{\rightarrow\tilde{Li}(8,\,3,\,3/2,8.0225)+\beta^{-}\rightarrow}\\ \mathbf{\rightarrow Be(8,\,4,\,2,8.0225)+\beta^{-}\rightarrow}\\ \mathbf{2He(4,\,2,\,1,\,4.0026)+\Delta E}\end{gathered}\tag{3.139}$$

where there is the sole emission of electrons easily captured by the metal walls of hadronic reactors, thus without any harmful radiation, yet with the release of the following considerable energy per transmutation

$$\mathbf{\Delta E=2.887\times10^{-12}\,J},\tag{3.140}$$

Additionally, we have the following possible transmutation [144] suggested for experimental study: 3-Li-7 into two 2-He-4

$$\begin{gathered}\bar{p}^{-}(1,-1,1/2\uparrow,1.008)+Li(7,\,3,\,3/2_{\downarrow},\,7.016)\rightarrow\\ \rightarrow\tilde{Li}(8,\,3,\,3/2,8.0225)+\beta^{-}\rightarrow\\ \rightarrow Be(8,\,4,\,2,8.0225)+\beta^{-}\rightarrow\\ 2He(4,\,2,\,1,\,4.0026)+\Delta E,\end{gathered}\tag{3.141}$$

(where one should note the difference with Eq. (3.130) given by the replacement of p^{+} with $\bar{p}^{-}$ and ensuing different reaction); of 46-Pd-106 into 47-Au-107

$$\begin{gathered}\bar{p}^{-}(1,-1,1/2,1.008)+Pd(106,46,0^{+},105.903)\rightarrow\\ \rightarrow\tilde{Pd}(107,46,0^{+},105.911+\beta^{-}\rightarrow\\ \rightarrow Ag(107,47,1/2^{+},106.9050)+\beta^{-}+\Delta E\end{gathered}\tag{3.142}$$

or of 79-Au-197 into 79-Au-198

$$\bar{p}^{-}(1,-1,1/2,1.008)+Au(197,\,79,\,3/2,\,196.966)\rightarrow$$

$$\tilde{Au}(198,\ 79,\ 3/2,\ 196.974) + \beta^- \rightarrow$$
$$\rightarrow Au(198,\ 79,\ 2,\ 197.972) + \beta^- + \Delta E \rightarrow \quad (3.143)$$

where $79 - Au - 198$ is unstable and decay in 2.69 days into betas and $Hg - 198$ with the release of $1.372 MeV$.

Intriguingly, pseudo=protons are predicted to be produced jointly with the laboratory synthesis of the neutron (Sections 2.22 and 3.25). Possibilities for the experimental confirmation or denial of the existence of the pseudoproton are studied in Ref.[145].

It should be noted that the possible existence of the pseudo-proton casts shadows on the firm claim released by CERN, FERMILAB and other particle laboratories of having synthesized the "anti-proton."

These problematic aspects occur not only because of lack of exclusions in said claims of reactions (3.136) and/or (3.167), but also because of the violation of the PCT theorem in the production mode and other reasons (see Section 4.14 on antimatter for details).

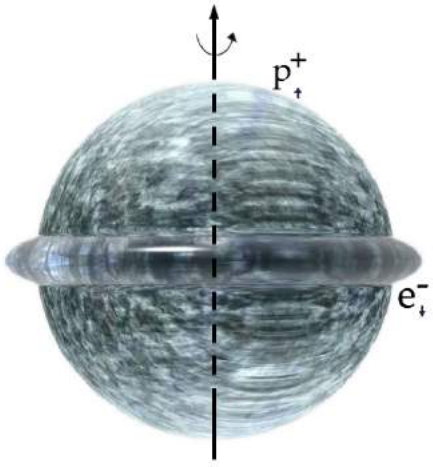

Figure 31: *It is generally believed that, according to quantum mechanics, the proton and the electron solely admit the various energy levels of the hydrogen atom. However, the proton and the electron have opposite charges producing an extremely big "attractive" force of the order of* 10^{26} *N at mutual distances of the order of* 10^{-13} *cm. Simple quantum mechanical calculations then predict the existence of an unstable bound state of a proton and an electron in singlet coupling under the contact contact depicted in this figure. the new bound state was submitted by the author as an intermediate state prior to the full synthesis of the neutron under the name of "nucleoid" [50-55,144] because significant for a new class of clean nuclear energies characterized by transmutations without Coulomb repulsion.*

3.27. Neutroids, nucleoids and their transmutations without Coulomb repulsion

During the systematic tests on the laboratory synthesis of the neutron from a hydrogen [50-58], the author noted that neutrons were indeed synthesized under sufficient power, e.g. of $50\ kW$.

However, for insufficient power, the author noted that there was the production of "entities" that were not neutrons because they did not actuate neutron detectors, although they stimulated delayed nuclear transmutations in substances away from the reactor, which delayed transmutation released neutrons fully identified as such as various detectors. The same anomalous, delayed, nuclear reactions had been detected by Son Borghi [42]

Following systematic tests, the author noted that the "entities" should be *neutral,* since they traverse the walls of the reactor, have approximately the same size of the neutron and have spin 0 , its rest energy being hereon approximate as being the same as that of the neutron.

. Following systematic tests, the author called these "entities" *neutroids* denoted with the symbol $\tilde{n}$,and suggested their synthesis via the reaction [50-55,144] (see Figure 30)

$$p^+ + e^- \rightarrow \tilde{n}(1,\, 0,\, 0,\, 1.008). \tag{3.144}$$

It should be stressed that *neutroids are mostly quantum mechanical states* due to the extremely big Coulomb attraction between the proton and the electron at their mutual distance of $10^{-13}\ cm$. In fact, such a quantum mechanical attraction is so big that the contribution due to wave-overlapping of the proton and the electron can be ignored in first approximation.

A most important feature is that *neutroids are neutral and, as such, they can stimulate nuclear transmutations without the Coulomb repulsion.* This is the very reason neutroids were proposed and are recommended for study.

It is evident that the absorption of a neutroid by a tabulated nuclide generally produces new, unstable, non-tabulated nuclides suggested for study under the name of *nucleoid* [50,111, 144] denoted with the symbol $\tilde{N}$ (see also reviews [67,103]) according to the reaction

$$\tilde{n}(1,\, 0,\, 0,\, 1.008)N(A,\, Z,\, J,\, M) \rightarrow \tilde{N}(A+1,\, Z,\, J,\, M+1.006). \tag{3.145}$$

Among a virtually endless possibility of nuclear transmutations triggered by nucleoids we restrict to the indication of the following one

$$\begin{gathered} \tilde{n}(1,\, 0,\, 0,\, 1.008)Li(7,\, 3,\, 3/2,\, 7.016)+ \rightarrow \\ \rightarrow \tilde{Li}(8,\, 3,\, 3/2,\, 8.022) + e^- \rightarrow Li(8,\, 3,\, 2,\, 8.022) \rightarrow \\ \rightarrow Be(8,\, 4,\, 0,\, 8.005) \rightarrow 2\alpha, \end{gathered} \tag{3.146}$$

An important independent study of nucleoids and their possible use for the production of new clean nuclear energies has been conducted by U. Abundo [143].

3.28. Apparent nuclear origin of planetary heat

As it is well known, heat is very difficult to maintain in our Earthly environment due to dissipation via infrared radiation and other processes. Consequently, due to their travel in empty space, planets should lose their internal energy via infrared radiation under sufficient time, and be cold.

One of the author's graduate students at the Department of Physics of Boston University in the mid 1970s conducted supervised calculations according to which our Earth should have dissipated all its internal, and be totally cold, heat about one billion years ago.

By keeping in mind that our Earth is geologically very active, the sole rational conclusion is that *the continued internal heat of Earth over billions of years, as well as of other planets and of planetary satellites, is due to nuclear syntheses in their core.* the rest of the planet then provides all needed shielding from possible radiations originating in their core.

Since nuclear syntheses are popularly believed to occur solely in the interior of stars, the above hypothesis on the origin of planetary heat was dismissed and never published.

To our knowledge, no quantitative representation of the continued heat of Earth has and other planetary bodies appeared in refereed journals since the 1970s, we decided to release it for the first time in this paper.

Earth's core is quite similar to the interior of the Sun thus having all the necessary conditions to realize the hadronic laws for nuclear syntheses 9Section 3.25.4), including high pressure to expose nuclei, strong magnetic field for their proper alignment, etc.

Among a virtually endless number of possible nuclear synthesis in Earth;s core, we mention a representative synthesis among the simplest possible ones, those with initial and final null spin

$$Mo(92, 42, 0, 91.9068) + Si(28, 14, 0, 27.9769) \rightarrow$$

$$\rightarrow Ba(120, 56, 0, 119.9260) \tag{3.147}$$

with moderate but positive energy output

$$\Delta E = 0.0423u, \tag{3.148}$$

without any emission of radiations or release of radioactive waste.

The study of endless other possible nuclear syntheses in Earth;s core is left to the interested reader.

Needless to say, the realization of all hadronic laws of Section 3.25.4 does not generally occur in all planets, e. g., because of the lack of a magnetic field or other reasons, thus explaining that only a number of planets or satellites continue to produce internal nuclear syntheses as a condition to remain hot over very extended period of time.

It should be stressed that the hypothesis of the nuclear origin of Earths's heat has no scientific value if studied via quantum mechanics and special relativity. It's existence crucially depends on the validity of the covering genomechanics and genorelativity in the hyperdense conditions existing in the core of planetary bodies.

3.29. Stimulated decay of radioactive nuclear waste

As it is well known, one of the biggest environmental problems of nuclear power plants is the accumulation and storage of their radioactive waste, currently done at the plant themselves, with evident environmental problems as well as huge costs.

This problem has stimulated numerous research, including the granting of a number of patents. In line with these efforts, the author conducted considerable research in the late 1990s on the possibility to recycler radioactive nuclear waste via their stimulated decay [146].

The main argument is that said waste is composed of large nuclei that are *naturally unstable,* thus rendering plausible the possibility if triggering their decay with the consequential reduction of their mean-life from current values of the order of thousands of years down to days or less.

It should be stressed that such a stimulated decay is impossible for relativistic quantum mechanics since it would imply a structural change of nuclear characteristics that are prohibited by the Lorentz-Poincaré symmetry, and for other reasons.

Nevertheless, a central feature of the the new vistas in nuclear physics permitted by relativistic hadronic mechanics, particularly by its Lie-admissible branch for irreversible processes, is that *the nuclear force has a component of non-Hamiltonian type* generated by the partial mutual penetration of the charge distribution of nucleons.

The possibilities for stimulated decay of large, naturally unstable nuclei are full permitted by the Lie-admissible covering of the Lorentz-poincaré symmetry and are offered by resonating means exciting precisely said non-Hamiltonian component of the nuclear force, with ensuing decay.

Regrettably, the author had to terminate research in the field because of life threats [146] (see also Section 5).

3.3-. A new combustion of fossil fuels

In the author's view, the combustion of fossil fuels everywhere on Earth nowadays is essentially the same as that at the down of civilization fifty thousand years ago, namely, we strike a spark and lit the fuel with consequential well known environmental problems.

It is reasonable to assume that, at the down of the third millennium, there should be a basically new conception of combustion of fossil fuels under development at Thunder Energies Corporation, essentially consisting of achieving combustion jointly with ICNS outlined in Section 3.25, thus having complete combustion as already achieved by magnegas [64], but by jointly achieving a considerable increase of energy output.

Colleagues interested in technical details are suggested do contact Thunder Energies Corporation via the email research@thunder-energies.com.

3.31. Formulation of genomathematics via Vougiouklis hyperstructural axioms

As it was the case for isomathematics (Section 2.27), the most general possible formulation of Lie-admissible genomathematics is that via *Vougiouklis hyperstructural axioms,* Ref. [157] and literature quoted therein, resulting in the most general mathematics that ca be formulated by the human mind at this writing, which appears to be particularly important for new vistas in biology due to its irreversible as well as multi-valued character be necessary for quantitative studies, e.g., of the DNA code.

4. Isodual mathematics, isodual isomathematics, isodual genomathematics and their applications

4.1. The fifty years old dream

Exactly fifty years ago, during Ph. D. studies at the university of Turin, Italy, in the mid 1960's, the author wanted to know whether a far away galaxy is made up of matter or of antimatter.

It became soon evident that the entirely of the mathematics and physics available at that time was inapplicable to the problem considered, because the study of antimatter galaxies required a formulation of *classical and neutral* antimatter, while the sole formulations available were for *charged particles in second quantization*.

Besides the lack of physical methods, the biggest insufficiencies of the time was the lack of a *mathematics* suitable for a quantitative classical representation of neutral (as well as charged) antimatter.

Therefore, the quantitative study of the problem considered required:

1) The construction *ab initio* of the needed mathematics.

2) The construction of the corresponding theory of antimatter.

3) The verification of their validity for all available experimental data in antimatter prior to the direct study of expected antimatter galaxies.

Rather than being discouraged by the above tasks, the author decided to initiate a long term scientific journey that, after half a century of research, can at best be considered to be at its initiation due to the complexity of nature.

In this section, we report the most salient aspects of the new vistas in antimatter with the understanding that a technical knowledge can be solely acquired with the study of the original papers all published in refereed journals and available in free pdf download.

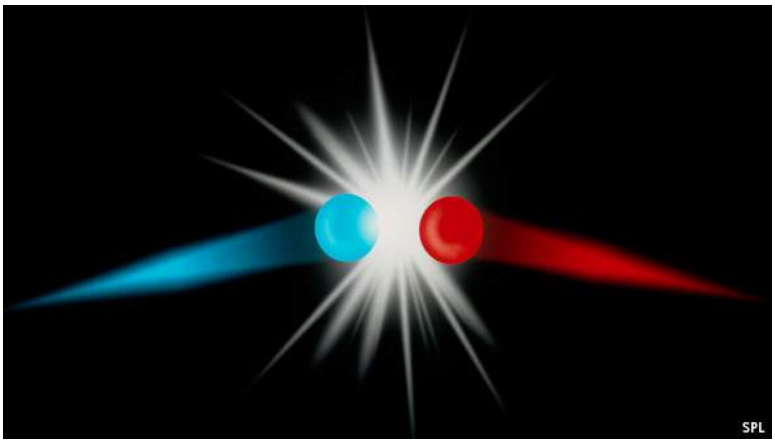

Figure 32: *A conceptual view of the majestic event in nature given by matter-antimatter annihilation, in which the entire matter and antimatter masses are converted into electromagnetic radiation generally in all possible frequencies. The achievement of a quantitative representation of matter-antimatter annihilation was the guiding principle of the studies in antimatter reported in Section 4.*

4.2. Matter-antimatter annihilation

The author's guiding principle for the studies hereon reported has been the incontrovertible experimental evidence that *matter and antimatter annihilate at contact.*

We are here referring to an event in which the entire *mass* of matter and antimatter is transformed into electromagnetic radiation generally in all possible frequencies (Figure 32).

The studies on antimatter hereon reported have been conducted under the condition of achieving a quantitative representation of the above transmutation.

It should be noted that the physical origin of matter-antimatter annihilation was widely ignored in 20th century physics and represented via the *ad hoc* selection of the scattering matrix representing the transition from the initial to the final state.

Even though mathematically valid, the scattering matrix provides no representation whatsoever of the fashioning mechanisms permitting the complete elimination in nature of masses and their transformations into light.

4.3. Dirac's historical conception of antimatter

The most important contribution toward the representation of matter-antimatter annihilation and the ensuing construction of the needed new theory of antimatter, came from the originator of the very notion of antimatter, P. A. M. Dirac [148] and more specifically, from his celebrated equation

$$(\eta^{\mu\nu} \times \gamma_\mu \times p_\nu - i \times m \times c) \times \psi = 0, \tag{4.1}$$

where the γs are the historical Dirac's matrices

$$\{\gamma_\mu, \gamma_\nu\} = \gamma_\mu \times \gamma_\nu + \gamma_\nu \times \gamma_\mu = 2\eta_{\mu\nu}. \tag{4.2}$$

In fact, Dirac's fourth gamma matrix

$$\gamma_4 = \begin{pmatrix} I_{2\times 2} & -0 \\ 0 & -I_{2\times 2} \end{pmatrix} \tag{4.3}$$

clearly implied Dirac's conception that *antimatter carries negative energy,* became mandated by the negative value $-I_{2\times 2}$ of the second component of the wavefunction.

The negative value of the unit in Dirac's fourth gamma matrix can be considered the very birth of the notion of antimatter with deep implications that were ignored during the 20th century.

It was evident that Dirac's original conception of matter having positive mass and antimatter having negative mass sets the foundation for a quantitative representation of matter-antimatter annihilation due to the null value of the total mass.

However, as it is well known, Dirac soon discovered that *negative energies violate causality.* Consequently, Dirac was forced to seek out models, such as that of the "hole theory," avoiding causality violations. These models were subsequently reformulated via second quantization and achieved wide acceptance in the 20th century physics community.

In this way, Dirac's original conception of antimatter became restricted to the level of charged particles in second quantization, without any possible use for the classical study of neutral antimatter.

Despite its unquestionable mathematical value, said restriction caused the biggest scientific imbalance in the history of science because matter was treated from Newton to second quantization, while antimatter was solely treated in second quantization.

Consequently, the author remained faithful to Dirac's original conception of antimatter as having negative masses by never accepted 20th century restrictions to be solely valid in second quantization, and spent decades of studies to regain full democracy between matter and antimatter in which both are treated at all levels of study.

The biggest imbalance in the history of science

MATTER	ANTIMATTER
Newtonian mechanics	no
Hamiltonian mechanics	no
Special relativity	no
Quantum mechanics	no
Quantum field theory	no
General relativity	no
Second quantization	YES

Figure 33: *A view of what is known as "The biggest scientific imbalance of the 20th century," because matter is studied with all possible methods, from Newtonian mechanics to second quantization, while antimatter is solely studied at the level of second quantization. The imbalance originated from causality violations of negative energy solutions of Dirac's equation that forced the restriction to elementary particles solely studied in second quantization. In Section 4, we report fifty years of research to re-establish scientific democracy between matter and antimatter at all levels of study.*

4.4. Insufficiencies of charge conjugation

The biggest difficulties encountered by the author against the conduction of the antimatter studies herein reported have been of political nature based on the claim that the conventional *charge conjugation* in its various forms, such as that for the wavefunction

$$\psi(t,r) \to \psi^C(t,r) = -\psi^\dagger(t,r), \tag{4.4}$$

needs no revision because representing antimatter in its entirety.

In reality, charge conjugation is solely applicable to charged particles represented in a Hilbert space $\mathcal{H}$ over a field of complex numbers $\mathcal{C}$. The validity of charge conjugation for neutral particles is merely assumed by *fiat* and never seriously inspected in the 20th century.

Hence, charge conjugation is dramatically insufficient for the classical study of neutral galaxies moving in a conventional Euclidean or Minkowskian space over the real numbers.

While respecting clear advances achieved in particle physics, the author never accepted charge conjugation as being the final conjugation from matter to antimatter because of a umber of insidious insufficiencies or sheer inconsistencies that are ignored in the best Ph. D. courses in physics around the world, such as:

1. *Charge conjugation is incompatible with matter-antimatter annihilation.* In fact, to avoid causality violation, charge conjugation preserves the positive character of the mass of antiparticles. It is then easy to prove that two particles both having positive masses simply cannot experience annihilation.

2. *Charge conjugation is not fully compatible with Dirac's original conception of antimatter.* In fact, charge conjugation correctly represents the transition from one component of the wavefunction of Dirac's equation (4.1) into the other, but in this process the mass remains positive, contrary to Dirac's original conception of antimatter.

3. *Charge conjugation maps antiparticles into the same Hilbert space of particles,* namely, charge conjugation is an inner authomorphisms of the Hilbert space. This seemingly innocuous mathematical occurrence causes serious physical inconsistencies at sufficiently deep levels of study, e.g., for grand unifications of electroweak and gravitational interactions, which unification is the ultimate test of the validity of the conjugation from matter to antimatter.

A clear identification of the political difficulties experience by the author is necessary for the understanding of this section. The reason for the rather unusual attachment to charge conjugation, despite its evident shortcomings and historical imbalances, is that *charge conjugation has conceived, implemented and maintained because it allows the extension of Einstein special relativity to antimatter.*

Problems of scientific ethics (addressed in Section 5) are then inevitable for numerous reasons such as:

A) Antimatter did not exist at the time of formulation of special relativity. Therefore, the extension of special relativity to antimatter is a potential abuse of Einstein's name.

B) Special relativity has no clue whatsoever for the transition from matter to antimatter. Therefore, the extension of special relativity to antimatter without a serious scrutiny raises issues of scientific ethics and accountability.

C) In the event adopted, special relativity prohibits a quantitative representation of matter-antimatter annihilation as an evident consequence of the lack of their proper differentiation besides the sign of the charge.

Needless to say, the author supports the continuation of studies in antimatter via charge conjugation, and as editor of various journals, the author has routinely accepted qualified papers on antimatter based on charge conjugation.

Problems of scientific ethics emerge when charge conjugation is used to suppress scientific democracy for qualified inquiries via the discreditation, rather than the technical disproofs in refereed journals, of new approaches in antimatter.

Figure 34: *The incontrovertible evidence establishing the insufficiency of charge conjugation is given by the 1908 Tunguska explosion in Siberia that flattened an area as big as France without any crater. Academia has propagated for one century that the Tunguska explosion was due to an ice comet exploding in our atmosphere This interpretation is dismissed as intended to deny the existence of antimatter in cosmology for the intent of maintaining Einstein's special relativity for antimatter restricted at the particle level. In reality, the Tunguska explosion ionized the entire Earth atmosphere by allowing people two days later to read newspapers at midnight without artificial light. Such a global ionization of our atmosphere cannot be explained, even minimally, with the explosion of an ice comet. The sole "scientific" interpretation of the Tunguska explosion (and many similar explosions in history) is that the Tunguska explosion is due to an antimatter asteroid annihilating in our atmosphere, with the consequential release of hard electromagnetic radiation that are known to ionize a gas. This evidence establishes the existence of antimatter at the astrophysical level, by therefore mandating its study for the very safety of our planet.*

4.5. The evidence suggesting the existence of antimatter galaxies

In the author's view, the strongest experimental evidence establishing the insufficiencies of charge conjugation is the 1908 Tunguska explosion in Siberia that had an energy equivalent to one thousand Hiroshima bomb and flattened an area as big as France without any crater at the epicenter (Figure 34).

The interpretation of the Tunguska explosion that has been voiced for one century by academia is that it was due to an "ice comet" that penetrated Earth's atmosphere and exploded at mid air, thus creating no crater.

Unfortunately, evidence establishes that such a view is of political character because aimed at, or implying in any case, the lack of existence if antimatter at the macroscopic level as a necessary condition the validity of Einstein's special relativity for antiparticles in second quantization.

In fact, a search on the internet establishes clear evidence that, two days following the Tunguska explosion, people in Sidney, Australia, as well as throughout Earth, could read newspapers at midnight without any artificial light.

This evidence establishes that *our entire atmosphere had been ionized,* an occurrence that can be soley caused by an enormous nuclear explosion or an antimatter asteroid annihilating in the upper atmosphere.

The nuclear origin of the Tunguska explosion has to be excluded for obvious reason, thus leaving the annihilation of an antimatter asteroid in our upper atmosphere as the sole quantitative interpretation.

The Tunguska explosion is only one of numerous big explosions in our upper atmosphere without craters that has been recorded throughout our history, that provides clear evidence of the existence of antimatter galaxies, evidently because an antimatter asteroid can only be caused by an antimatter supernova or similar astrophysical explosions.

Additional evidence on the existence of antimatter in the large scale structure of the universe is given by "flashes" of light seen by astronauts and cosmonauts in the dark side of Earth which can be solely interpreted in a quantitative way as being due to the annihilation of *antimatter cosmic rays* in our upper atmosphere, as subsequently confirmed experimentally.

In view of the clear possibility that our Earth can be hit again by antimatter asteroids with the consequential disruption of all our civilian, industrial and military communications, the author felt a "society duty" as a scientist to ignore political obstructions from academia, and intensify the study of antimatter at the purely classical and neutral level, thus seeking an alternative to charge conjugation.

4.6. Basic requirements of new mathematics for antimatter

The insufficiencies identified above established quite clearly the need for a basically new mathematics, specifically conceived for antimatter to achieve the following primary objectives:

1. Avoid causality violations in Dirac's conception of antimatter as having negative energy.

2) Extend Dirac's conception of negative energy to all other physical characteristics of antimatter, as a necessary condition to represent matter-antimatter annihilation.

3. Establish full democracy between matter and antimatter with the equal study of both at all levels, thus requiring a formulation for antimatter of all physical methods available for matter.

4. Verify all available experimental evidence for antimatter at both the classical and operator levels.

5. Permit the astrophysical study of antimatter at the classical (as well as operator) level for neutral (as well as charged) antimatter.

Following the above introductory aspects identified by the author in the mid 1970's, in the remaining parts of this section we shall outline the new vistas in antimatter in their chronological order of achievement.

4.7. Isodual mathematics, isodual isomathematics, and isodual genomathematics

4.7.1. Isodual numbers, isodual isonumbers, and isodual genonumbers

The birth of the new mathematics for antimatter can be traced to the negative unit $-I_{2\times 2}$ in Dirac's fourth gamma matrix, Eq. (4.3).

Unpublished initial attempts were aimed at a mere reformulation of 20th century physics in such a way to achieve compatibility with Dirac's conception of antimatter, although all these attempts failed for various inconsistencies.

These failures established that the new mathematics for antimatter had to be built starting from its most fundamental notion, that of numbers.

As recalled in Section 2.2.1, while visiting the Joint Institute for Nuclear Research, Dubna, Russia, in summer of 1993, the author re-inspected the axioms of numeric fields $F(n, \times, 1)$ and discovered that said axioms do not require the basic multiplicative unit to be the positive number 1, since that unit can be an arbitrary positive-definite quantity, provided that the product is suitably redefined [4]. This discovery set the foundation of isonumbers, and subsequently, to isomathematics and isomechanics.

During the same re-inspection [4], the author discovered that the axioms of a numeric field do not require that the unit is positive, because the following *negative multiplicative unit* denoted with the upper symbol ^{d}

$$I^d = -1^\dagger = -1, \tag{4.5}$$

is fully admitted, provided that the multiplication is redefined in the form

$$n \times^d m = n \times (-1)^{-1} \times m, \tag{4.6}$$

(were the term $(-1)^{-1}$ is requested by topological reasons) admitting I^d as the correct left and right unit of the theory,

$$I^d \times^d n = n \times^d I^d \equiv n \; \forall \; n \; \in \; F. \tag{4.7}$$

In this way, Dirac's negative unit $-I_{2\times 2}$ was assumed as the ultimate foundation of the entire mathematics and physics for antimatter, namely, as the basic multiplicative unit of all formulations.

The characterization of negative units was done in Ref. [4] via a map, today called *Santilli isodual map,* which is an anti-Hermitean map exactly like charge conjugation (4.4), with the fundamental difference that the latter map solely occurs at the quantum level, while the former map is applicable at all levels, beginning from Newtonian mechanics and then passing to quantum mechanics, as we shall see.

Quantities (4.5) and (3.6) are then called *Santilli isodual unit and isodual multiplication,* respectively. *Santilli isodual real, isodual complex and isodual quaternionic numbers* are then defined by

$$n^d = (n \times 1)^d = I^d \times^d n^d = -n^\dagger. \tag{4.8}$$

The entirety of number theory must then be subjected to the isodual map resulting in the *isodual fields* $F^d(n, \times^d, 1^d)$ [4]. The *isodual mathematics* is then constructed via isodual map of the totality of the mathematics used for matter with no exclusion (see monograph [149] for a comprehensive presentation).

It should be noted that the discovery of the isodual numbers established the existence of a third degree of freedom in the definition of numbers, besides the isonumbers of Section 2 and the genonumbers of Section 3.

The author also introduced in Ref. [4] the fourth degree of freedom of the axioms of numeric fields, that of *isodual isofields* $\hat{F}^d(\hat{n}^d, \hat{\times}^d, \hat{I}^d)$ (where the "hat" denotes isotopy) characterized by the isodual image of the isofields of Section 2.2.1 , with *isodual isomultiplicative isounit*

$$\hat{I}^d = -\hat{I}^\dagger, \tag{4.9}$$

isodual isoreal, isodual isocomplex and isodual isoquaternionic isonumbers

$$\hat{n}^d = (n \times \hat{I})^d = \hat{I}^d \times^d \hat{n}^d = -\hat{n}^\dagger \times \hat{I}^\dagger = \hat{n}^\dagger \times \hat{I}^d, \tag{4.10}$$

and *isodual isoproduct*

$$\hat{n}^d \hat{\times}^d \hat{m}^d = \hat{n}^d \times (\hat{I}^d)^{-1} \times \hat{m}^d. \tag{4,11}$$

The study of isodual isonumbers identified a new feature with very deep physical implications for antimatter indicated later on, consisting of the fact that *the isodual (i.e., anti-Hermitean) map must be applied for consistency the totality of the quantities Q used for matter, to the totality of their functional dependence and to the totality of their operations, with no exclusion to prevent insidious inconsistencies.*

Hence, when explicitly written, isodual isounit (4.9) reads

$$\hat{I}(t, r, v, \psi, ...) \to \hat{I}^d(t^d, r^d, v^d, \psi^d, ...) =$$
$$= -\hat{I}^\dagger(-t^\dagger, -r^\dagger, -v^\dagger, -\psi^\dagger, ...). \tag{4.12}$$

Note that isodual isonumbers constitute a fourth degree of freedom for the definition of numbers. The *isodual isomathematics* is constructed via the application of isodual map (4.11) to the totality of isomathematics [149].

Finally, Ref. [4] introduced the fifth new degree of freedom in the definition of fields given by *Santilli forward isodual genofields* $F^{>d}(\hat{n}^{>d}, \hat{>}^{>d}, \hat{I}^{>d})$ (where $>$ represents the ordering of all ordered to the right from Section 3) with *forward isodual genounits*

$$\hat{I}^{>d} = -\hat{I}^{>\dagger}, \tag{4.13}$$

forward isodual genoreal, genocomplex and genoquarternionic genonumbers

$$\hat{n}^{>d} = (n \times \hat{I})^{>d} = \hat{I}^{>d} \hat{>}^d \hat{n}^{>d} = \hat{n}^{>\dagger} > \hat{I}^{>d}, \tag{4.14}$$

and *forward isodual genoproduct*

$$\hat{n}^{>d}\hat{>}^{d}\hat{m}^{>d} = \hat{n}^{>d} > (\hat{I}^{>d})^{-1} > \hat{m}^{>d}, \tag{4.15}$$

and with corresponding *backward isodual genofields* here ignored for simplicity [4,149].

The *forward isodual genomathematics* is then constructed via the systematic application of the isodual map (4.11) to the totality of the forward genomathematics of Section 3 [149].

It should be recalled that only the multiplication and related unit were subjected to five generalizations in Ref. [4] because verifying the axioms of a numeric field, but no generalizations of the sum and related additive unit were proposed because they would violate said axioms.

Note also that, all five new fields could be introduced in one single abstract form. Their indicated differentiation has been done primarily for physical applications in order to achieve methods of progressively increasing complexity for the description of antimatter in conditions of correspondingly increasing complexity, namely:

1. Isodual numbers and isodual mathematics are used for time reversible systems of point-like antiparticles moving in vacuum.

2. Isodual isonumbers and isodual isomathematics are used for time reversal systems of extended antiparticles moving within an antimatter medium.

3. Forward isodual genonumbers and forward isodual genomathematics are used for time irreversible systems of extended antiparticles moving within an antimatter medium.

Note finally that Ref. [4] contains *no* treatment of antimatter due to the evident need of a considerable number of additional studies prior to meaningful application to antimatter.

4.7.2 Isodual, isodual isotopic and isodual genotopic differential calculus

Following the identification of the new isodual numbers in Ref. [4], the formulation of the isodual mathematics (as well as of the isodual isomathematics and isodual genomathematics) were presented in memoir [6] of 1996 with the formulation of *isodual functions* whose values must be, for consistency, isodual numbers

$$f^d(r^d) = -f^\dagger(-r^\dagger) \ \in \ F^d, \tag{4.16}$$

e.g.,

$$r^{d2d} = r^d \times^d r^d = -3^d, \tag{4.17}$$

the *isodual differential calculus,* here suggested to be called the *Santilli-Georgiev isodual differential calculus* from pioneering studies [9], with main *isodual differential*

$$d^d r^d \equiv dr \tag{4.18}$$

and *isodual derivative*

$$\frac{\partial^d f^d(r^d)}{\partial^d r^d} \equiv \frac{\partial f(r^d)}{\partial r^d} \tag{4.19}$$

(where one should conjugate also the fraction).

The identity of the main expressions of the isodual calculus with the conventional calculus may illustrate the reason for the lack of discovery of the isodual calculus prior to Ref. [6].

The *isodual isodifferential calculus* was introduced in ref. [6] with basic expression

$$\hat{d}^d\hat{r}^d = \hat{d}^d(\hat{r}^\dagger \times \hat{I}^d) = dr^\dagger + r^\dagger \times \hat{T}^d d\hat{I}^d, \tag{4.20}$$

$$\hat{I}^d = 1/\hat{T}^d < 0. \tag{4.21}$$

Finally, Ref.[6] introduced the *forward and backward Isodual genofunctions and forward isodual genodifferential calculi* with basic equation for the forward case

$$\hat{d}^{>d}\hat{r}^{>d} = \hat{d}^{>d}(\hat{r}^\dagger > \hat{I}^{>d}) = dr^\dagger + r^\dagger > \hat{T}^{>d} d\hat{I}^{>d}, \tag{4.22a}$$

$$\hat{I}^{>d} = 1/\hat{T}^{>d} < 0, \tag{4.22b}$$

and the condition assuring the representation of irreversibility (Section 3)

$$\hat{I}^{>d} \neq {}^{>d}\hat{I}. \tag{4.23}$$

Note that the identity of the differential with the conventional differential, Eq. (4.16), may be the reason for the lack of discovery of the isodual differential calculus until memoir [6] of 1996.

4.7.3. Isodual spaces, isodual isospaces and isodual genospaces

Next, Ref.[6] introduced the formulation of metric and pseudo-metric spaces which is compatible with the isodual numbers, functions and differential calculi.

Let $M(x, \eta, I)$ be the conventional Minkowski space (Section 2.2.3). The *Minkowski-Santilli isodual space* $M^d(x^d, \eta^d, I^d)$ is characterized by the isodual variables $x^d = -x$ and isodual metric $\eta^d = -\eta$ and isodual unit $I^d - -I$ with its isodual line element given by the simple conjugation

$$x^{d2} = [(x^\mu \times \eta_{\mu\nu} \times x^d) \times I]^d = -x^2, \tag{4.24}$$

with the understanding that lesser trivial isodual maps occurs for the *Minkowski-Santilli isodual isotopic and isodual genotopic spacetimes*.

4.7.4. Isodual Lie theory, isodual Lie-Santilli isotheory and isodual Santilli Lie-admissible theory

It is evident that isodualities of Lie's theory and its generalizations into the isoduality of the Lie-Santilli isotheory and of Santilli's Lie- admissible theory are crucial for any physical application. Their study was initiated in memoir [6] and conducted systematically in refs. [7]

Their progressive study is recommended as an instructive exercise for readers interested in acquiring a technical knowledge the new vista in gravitation.

4.7.5. Isodual geometries, isodual isogeometries and isodual genogeometries
One of the biggest insufficiencies of 20th century physics was the absence of a consistent formulation of the gravitational field of antimatter, evidently due to the restriction of antimatter to particle physics, with consequential inability to use the Riemannian geometry.

In view of these clear limitations, the ongoing classical formulation of the gravitational field of antimatter is, unquestionably, one of the most questionable models of the 20th century because applying Einstein general relativity to all possible gravitational conditions existing in nature, thus including antimatter, without any serious scrutiny or inspection of alternatives.

One of the most important aims of the isodual mathematics was a consistent formulation of the *gravitational field of antimatter* that was achieved for the first time in memoir [6] via the formulation of the *isodual Riemann-Santilli geometry.* The first isodual representation of the gravitational field of antimatter was reached in monographs [7], and then updated in monograph [149].

Comprehensive studies on the *isodual iso-Minkowskian geometry* for the alternative formulation of the gravity of antimatter were initiated in Ref. [21] and continued in Ref. [23] due to the visual, geometrical and experimental evidence on the apparent lack of actual curvature of space [24].

Regrettably we cannot outline these advances to prevent a prohibitive length of this paper, and are forced to refer the interested reader to the study of the original literature [21,23,24,149].

4.7.6. Hyperstructural formulation of isoduality via Vougiouklis axioms
As noted for isomathematics genomathematics, the broadest possible formulation of isodual mathematics is that via Vougiouklis hyperstructural axioms [177].

Recall that antimatter is predicted to exist in a space different than, yet coexisting with that of matter. This duality is a simple but very significant case of two-valued hyperstructures studied in Ref. [178] we regret not being able to review here for brevity.

4.8. Isodual, isodual isotopic and isodual genotopic theory of antimatter

4.8.1. Basic Isodual map
The *isodual, isodual isotopic, and isodual genotopic theories of antimatter* are characterized by the application of the corresponding mathematics for the description of antimatter in conditions of increasing complexity indicated in Section 4.7.1.

Alternatively, the isodual, isodual isotopic and isodual genotopic theories of antimatter can be derived by applying the *Santilli isodual map*

$$Q(t,r,v,]psi,...) \to Q^d(t^d,r^d,v^d,\psi,^d...) = -Q^\dagger(-t^\dagger,-r^\dagger,-v^\dagger,-\psi^\dagger,...), \tag{4.25}$$

to the totality of 20th century quantities Q, to the totality of their functional dependence and to the totality of their operations.

As shown below, the lack of application of the isodual map to only one quantity or one variable or only one operation, or only one unit leads to insidious inconsistencies that generally remains undetected by non-expert in the field.

To avoid a prohibitive length, we shall restrict our review hereon to the isodual theory for the description of point-like antimatter moving in vacuum and suggest Ref. [149] for the remaining formulations.

As one can see, the emerging formulation of antimatter is mathematically quite simple, although not trivial for the isodual isotopic and isodual genotopic cases for extended antimatter. Under the indicated broader theories, antimatter galaxies can indeed be represented with their actual dimensions.

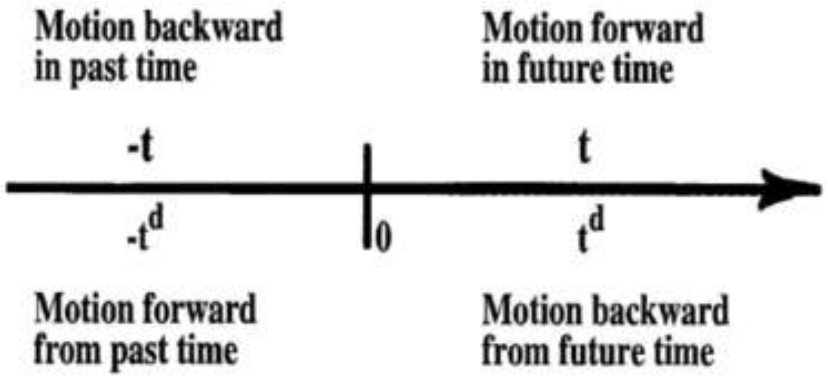

Figure 35: *It is popularly believed that there exist only two directions of time, for motion forward and backward in time. In reality, as illustrated in this figure, there exist "four" directions of time given by "motion forward and backward in future and past times" which cannot be achieved via time inversion, but can indeed be achieved via the use of the isodual map [149].*

4.8.2. Isodual representation space

Following, and only following the achievement of sufficient mathematical maturity in Ref. [6], the first classical formulation of the isodual theory of antimatter was presented in Ref.[150], and then studied in detail in monograph [17,49].

The classical non-relativistic theory of antimatter is formulated on the isodual space which is the direct product of the space for the isodual time, the isodual coordinates and the isodual velocities

$$E^d_{tot} = E^d_{t^d}(t^d, 1^d_t) \times^d E^d_{r^d}(r^d, \delta^d, I^d_r) \times^d E^d_{v^d}(v^d, \delta^d, I^d_v), \tag{4.26}$$

where one should note that the isodual units of time, coordinates and velocities, besides being negative-definite, are all dimensionally and numerically different among themselves.

The first nontrivial implication of the theory is that *the isodual map is basically new since there exist no map in 20th century physics (such as parity) transforming the Euclidean*

space into its isodual

$$E(r,\delta,I) \ \rightarrow \ E^d(r^d,\delta^d,I^d), \ \ \delta = Diag.(1,1,1), \ \ \delta^d = Diag.(-1,-1,-1). \tag{4.27}$$

The proof of this property is instructive for the interested reader [149].

The second non-trivial consequence is that *the isodual theory of antimatter implies that matter and antimatter exist in physically distinct, yet co-existing spaces, the Euclidean space for the former and its isodual for the latter.*

The relativistic representation space of the isodual theory of antimatter is the *isodual Minkowski space* which is easily constructed via the isoduality of conventional formulations [7,149].

4.8.3. Isodual classical mechanics

As indicated in Section 4.1, a central objective of the research reported in this section is the study of the expected existence of antimatter galaxies that requires a classical formulation of neutral antimatter. Under the above assumptions, the main dynamical equations are given in Refs. [21.150]

1. The *Newton-Santilli isodual equation*

$$m^d \times^d \frac{d^d v^d}{d^d t^d} = F^d(t^d, r^d, v^d), \tag{4.28}$$

which constitutes the first and only equations available for the classical description of point-like charged or neutral antiparticles moving in vacuum, thus solely having potential interactions (see Ref. [148] for the case of extended antiparticles in reversible conditions) (see the Newton-Santilli isodual genoequations for the irreversible covering [149]).

2. The *isodual variational principle*

$$\delta^d A^d = \delta^d \int^d (p^d \times^d d^d r^d - H^d \times^d d^d t^t) = 0. \tag{4.29}$$

2. The *isodual Lagrange equations*

$$\frac{d}{dt}\frac{\partial^d L^d(r^d, v^d)}{\partial^d v^d} - \frac{\partial^d L^d(r^d, v^d)}{\partial^d r^d} = 0, \tag{4.30}$$

and the *isodual Hamilton's equations*

$$\frac{d^d r^d}{d^d t^d} = \frac{\partial^d H^d(r^d, p^d)}{\partial^d p^d}, \quad \frac{d^d p^d}{d^d t^d} = -\frac{\partial^d H^d(t^d, r^d, p^d)}{\partial^d r}. \tag{4.31}$$

4.8.4. Isodual quantum mechanics

The first known study representation of antiparticles via the operator version of the isodual theory of antimatter was presented in Ref. [151].

It is easy to see that, despite its simplicity, isodual quantum mechanics for the description of point-like antiparticles in vacuum constitutes a basically new mechanics because it cannot be achieved with any known map other than isodual map.

It should be stressed that the *isodual isomechanics* for the description of extended antiparticles and the *isodual genomechanics* are not trivial.

The basic assumption of the isodual quantum mechanics is that its basic unit is the *negative value of Planck's constant*

$$\hbar^d = -\hbar = 1^d. \tag{4.32}$$

By assuming that

$$\psi^d = \psi^d(t^d, r^d) = -\psi^\dagger(-t, -r) \tag{4.33}$$

is an *isodual wavefunction* in the *isodual Hilbert space* $\mathcal{H}^\urcorner$, one can see that 1^d is indeed the correct multiplicative unit of the theory due to the identity

$$1^d \times^d \psi^d \equiv \psi^d. \tag{4.34}$$

Note the mathematical equivalence of charge conjugation (4.4) and the isodual conjugation (4.33), jointly with their physical differences indicated earlier.

Under the above basic assumptions, the basic equations of the isodual quantum mechanics are the following ones:

1. *Isodual Schrödinger equation* on $\mathcal{H}^\urcorner$ over $\mathcal{C}^\urcorner$

$$H^d(r^d, p^d) \times^d \psi^d(t^d, r^d) = E^d \times^d \psi^d(t^d, r^d) = -E \times \psi^\dagger(-t, -r). \tag{4.35}$$

2. *isodual linear momentum*

$$p^d \times^d \psi^d(t^d, r^d) = -i^d \times^d \hbar^d \times^d \partial^d_{r^d} \psi^d(t^d, r^d). \tag{4.36}$$

3. *Isodual Heisenberg equations*

$$i^d \times^d \frac{d^d A^d}{d^d t^d} = [A, H]^d = H^d \times^d A^d - A^d \times^d H^d. \tag{4.37}$$

4.8.5. The total energy of the positronium

It is generally believed that the positive value of the total energy of the positronium is a disproof of the isodual theory of antimatter. Unfortunately, the claim is due to the expression of judgment without an advance inquiry and without a lack of technical knowledge of isodual mathematics.

Since the positronium is studied in our Hilbert space, $\mathcal{H}$ over $\mathcal{C}$, we have to assume for calculating the ordinary wavefunction ψ. Let H be the Hamiltonian of the electron in the positronium. Then the total energy of the positronium is given by

$$(H \times + H^d \times^d)\psi = E \times \psi. \ E > 0l, \tag{4.38}$$

and it is manifestly positive [149].

Suppose now that the positronium is observed from the antimatter world. In this case, we need to use the isodual wavefunction ψ^d resulting in the new the expression

$$(H \times + H^d \times^d) \psi^d = E^d \times^d \psi^d \;\; E^d < 0, \tag{4.39}$$

for which the total energy of the positronium is negative, as necessary for the consistency of the isodual theory of antimatter.

4.8.6. Verification of experimental data on antimatter

It is easy to see that the use of the conventional Newton equation for particles and its isodual for antiparticles represents all experimental data on antiparticles at the classical level [149].

It is also easy to see that the isodual map is mathematically equivalent to charge conjugation, and therefore, isodual quantum mechanics verifies all known experimental data on antiparticles thanks to the operational equivalence of charge and isodual conjugation (see Ref. [149] for detailed studies).

4.9. Verification of causality by Dirac's negative energies

A central requirement of the correct formulation of the isodual theory of antimatter is that the isodual conjugation must be applied, not only of all physical quantities and all their operations, but also of their units.

Therefore, a physical system with negative energies referred to negative units of energy is exactly as causal as a physical system with positive energy referred to positive units of energy.

Similarly, a physical system with positive energy referred to negative units, and a system with negative energy referred to positive units violate causality.

Another necessary requirement for the consistent formulation of isodual theory is that *antimatter must evolve backward in time when referred to our evolution forward in time.*

Needless to say, this notion of "negative time" emerges only in the comparison between matter and antimatter, but it does not exist in the antimatter world for which events move forward in their own time.

The above comparative notion of opposite times do not violate causality because a system moving backward in time referred to negative units of time is exactly as causal as a system moving forward in time referred to positive units of time.

Similarly, a system moving forward in time referred to negative units of time and a system moving backward in time referred to positive units of time violate causality.

The same comparative opposite values occurs for all other physical quantities.

4.10. Representation of matter-antimatter annihilation

Following decades of studies of the problem, the sole quantitative representation of mater-antimatter annihilation known to the author, is that provided by the use of conventional 20th century theories for matter and the isodual theory for antimatter.

Recall that matter and antimatter are assumed to exist in physically different, yet coexisting spaces. Matter-antimatter annihilation is then represented at the classical level by the null value of the total mass, time and other physical quantities.

The operator representation is given by the expression for the masses

$$H \times \psi + H^d \times^d \psi^d = 0, \tag{4.40}$$

with similar expressions for other physical quantities, where one should note the difference between Eq. (4.40) and the representation of a "bound state" of matter and its antimatter, Eq. (4.38) and (4.39). The conservation of energy in each space then leaves no other alternative than the conversion of masses into electromagnetic radiations.

We should note that the above representation of matter-antimatter annihilation *prohibits the emission of massive particles in the annihilation process.*

In turn, this cast shadow on the actually anti-particle character of the antiprotons claims to be provided at CERN, FERMILAB and other laboratories, e.g., because of the very large number of particles produced in the claimed proton-antiproton annihilation of the Bose-Einstein annihilation (Section 2.19).

In turn, the above occurrence casts shadow on the test of antigravity currently conducted at CERN and other laboratories, as discussed below.

Particularly suggestive is the representation of matter-antimatter annihilation via the use of the behavior of time, which is left for study by the interested reader.

4.11. Isodual reformulation of Dirac's equation

An important implication of the isodual theory of antimatter is that Dirac's equation has not been properly written since its inception in the 1930's due to the lack of proffer mathematical differentiation between particles and their antiparticles.

The identical reformulation of Dirac's equation with full democracy in the description of particles and their antiparticles has been studied for the first time in Monograph [149] and can be written

$$(\eta^{\mu\nu} \times \gamma_\mu \times p_\nu - i \times m \times c) \times \psi = 0, \tag{4.41a}$$

$$\gamma_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k^d & 0 \end{pmatrix}, \ \gamma_4 = i \times \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & I_{2\times 2}^d \end{pmatrix}, \tag{4.41b}$$

$$\psi = \begin{pmatrix} \psi \\ \psi^d \end{pmatrix}, \tag{4.41c}$$

that eliminates any need for second quantization, and actually represents a spin $1/2$ particle, the electron, and its antiparticle, the positron.

Note the full compatibility of the isodual map with Dirac's original conception of antimatter compares to the insufficiencies of charge conjugation indicated in Section 4.4.

4.12. The novel isoselfdual symmetry

The isodual theory of antimatter implies the existence of a basically new symmetry in spacetime called $isoselfduality,$ consisting of the invariance under the isodual map (4.25).

It should be stressed that this is a basically new symmetry because, as indicated earlier, no existing spacetime symmetry or map, including all discrete symmetries, can represent isoduality due to the joint map of all quantities, all their operations and all their units.

The simplest example of isoselfduality is the positronium since isoduality changes the pair of electron and positron into itself.

The most important application of the new symmetry is that, if and only if reformulated into form (4.41), $Dirac's\ equation\ is\ isoselfdual.$

The new symmetry applies to all matter and antimatter systems, including cosmology. In the event the universe is composed by equal amounts of matter and antimatter, we have the $isoselfdual\ cosmology$ first presented in Ref. [152], in which the universe has null total characteristics of energy, time, momentum, etc., as a necessary condition to avoid a singularity at the act of its creation afflicting other models.

It should be indicated the new symmetry under isoselfduality is generally violated by the reactions leading to the production of the antiprotons claimed at CERN, FERMILAB and other laboratories, thus casting additional doubts on their existence in favor of the pseudo-protons of Section 3.26.

In any case, said process for the production of the claimed antiprotons also violate, in general, the PCT theorem. Additionally, as noted earlier, protons and claimed antiprotons do not entirely annihilate into electromagnetic radiation, and have additional problematic aspects identified in Ref.[149].

4.13. The prediction of antigravity

The most important prediction of the isodual theory of antimatter is that $matter\ and\ antimatter\ experience\ a\ gravitational\ repulsion,$ also known as $antigravity.$ It should be noted that the same prediction is shared by essentially all scholars in the field.

Antigravity was predicted via the use of the isodual theory for the first time in Ref. [153] and then studied in detail in Ref. [7,149].

Ref. [153] proposed the experimental verification or denial of the prediction of antigravity via the horizontal flight of very low energy electrons in a $20\ m$ long supervacuum and supercooled tube, and then the horizontal flight of equally low energy positrons. In the event of meV energies, the displacement due to gravity via a scientilloscope at the end of the tube is visible to the naked eye. Therefore, the proposed comparative test of the gravity of electrons and positrons is resolutory.

The experimentalists P. Mills [154] and V. de Haan [155] independently confirmed that the experiment is indeed feasible with current technology. Technical presentation of Santilli tests of antigravity is available in Ref. [146].

4.14. Problematic aspects of claimed antiprotons at CERN

CERN and other particle laboratories claim that antiprotons $\bar{p}^-$ can be produced by hitting a matter target MT with very high energy protons with various reactions essentially of

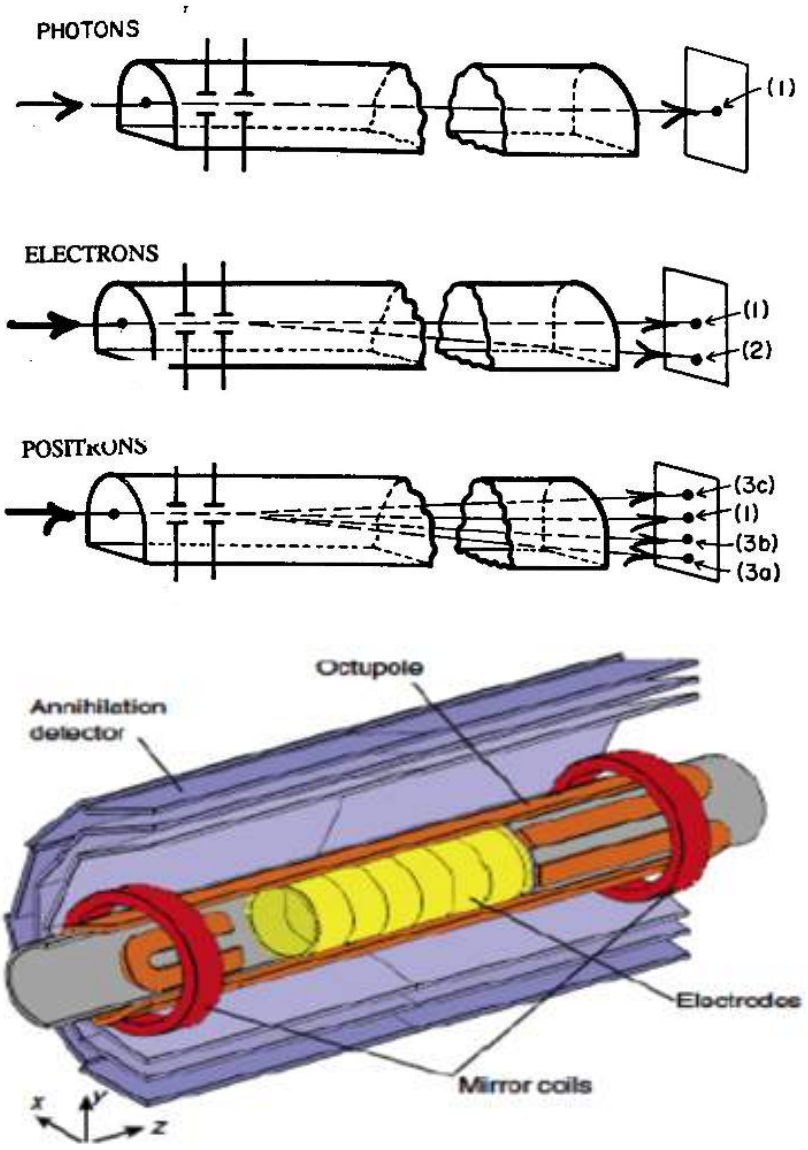

Figure 36: *A reproduction of the original 1994 conception of the comparative measurement of the gravity of very low energy electrons and positrons in horizontal flight in a supercooled and supervacuum tube [153,154] subsequently studied by experimentalists in papers [155-157] and declared to be resolutory.*

the type

$$p^+ + MT \ \rightarrow p^+ + \bar{p}^- + \ particles; es, \tag{4.42}$$

and then separating the claimed antiprotons via standard techniques.

Regrettably, the above claim is afflicted by a number of insufficiencies or sheer inconsistencies because reaction (4.42) violates the PCT theorem as well as the symmetry under isoselfduality.

Additionally matter and antimatter must necessarily annihilate into electromagnetic radiation, as established by the Tunguska explosion and other clear evidence (Section 4.5.)

By contrast, protons and the claimed antiprotons do not annihilate into light, as established by the plethora of massive particles produced in the Bose-Einstein correlation (Section 2.19). Additional insufficiencies or inconsistencies of the claimed antiprotons can be found in Ref. [149].

It then follows that the test of antigravity under study at CERN [157] via the so-called "anti-Hydrogen atom" is questionable because its nucleus, the claimed antiproton, is not necessarily an antiparticle until established by a considerable number of direct tests.

This scenario should be compared with Santilli's test of the comparative gravity of very low energy electrons and positrons in horizontal flight in a supervacuum and supercooled tube [153] that has been declared to be resolutory by independent experimentalists [154-156].

Regrettably for scientific knowledge and democracy, CERN Directorate has rejected to date (September 2016) various proposals at least to"consider" tests [153-156] jointly with others, thus creating the fear of an ongoing obscurantism indicated in Section 5.

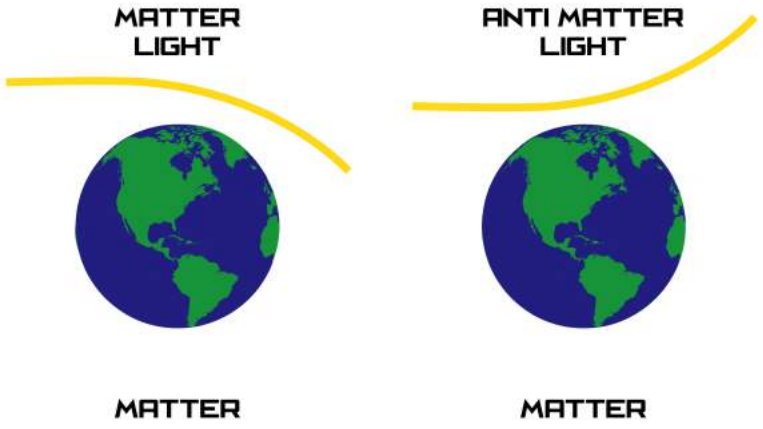

Figure 37: *Antimatter was conceived by Dirac as having negative energy, thus implying that light has negative energy. Once the violation of causality by negative energies is resolved thanks to the isodual mathematics, antimatter-light with negative energy is predicted to be repelled by a matter-gravitational field, as illustrated in this figure [157].*

4.15. The prediction of the isodual light

During the 1996 *International Workshop on Antimatter* held in Sepino, Italy, the author submitted the hypothesis that *light emitted by antimatter, called "isodual light" (or antimatter-light), is physically different than ordinary matter-light in an experimentally verifiable way* [158].

The proposal originated from the fact that, while Dirac's equation for the electro-positron pair verifies the symmetry under isoselfduality, the conventionally assumed electron-positron annihilation

$$e^- + e^+ \ \rightarrow \ 2\gamma, \tag{4.43}$$

violates isoselfduality.

Consequently, it was natural to assume under the iso that in the electron-positron annihilation there is the emission of *two* different ,photons, the conventional photon γ and its isodual γ^d,

$$e^- + e^+ \ \rightarrow \ \gamma + \gamma^d. \tag{4.44}$$

A detailed study[157] revealed that, as a necessary condition to avoid the violation of causality, *the antimatter-light must have "all" characteristics opposite those of matter-light,* thus have negative energy, negative time, negative polarization, etc.

The negative character of the energy then suggested that *antimatter-light is repelled by a matter gravitational field,* thus allowing its experimental verification (Figure 37).

Needless to say, the prediction of a new light emitted by antimatter has deep implications for all of physics. At this stage, we limit ourself to the indication that matter and antimatter *do not* annihilate into only one type of light, i.e., matter-light, but they annihilate into matter-light and antimatter-light with opposite characteristics, as necessary for general compatibility with matter-antimatter annihilation.

Hence, the illustration of matter-antimatter annihilation of Figure 32 is incomplete, due to the absence of a representation of antimatter-light.

4.16. Isodual special relativity, isodual isorelativity and isodual genorelativity

The most important application of the isodual Lie theory has been the construction of the isodual image of all conventional spacetime symmetries, including the *isodual Lorentz and Poincaré symmetries* on the *isodual Minkowski space* $M^d(x^d, I^d)$ over the isodual field $\mathcal{R}^\uparrow$. These new symmetries have been studied in details in monographs [7], that are crucial for the derivation of the isodual special relativity of the next section.

Throughout the 20th century, Einstein special relativity was applied to all conceivably possible conditions existing in the universe, thus including antimatter.

However, antimatter did not exist at the time of the inception of special relativity. Therefore, the adaptation of antimatter studies to comply with special relativity is a potential abuse of Einstein's name.

This section outlines mathematical, theoretical, experimental and industrial advances establishing that antimatter requires a basically new mathematics, with ensuing basically new physical theory, thus requiring the inevitable reformulation of special relativity known as *isodual special relativity* studied in details by the author in various works, see

Refs. [7,140,159,160] whose axioms are given by the isodual image of the conventional axioms.

More significant is the *isodual isorelativity* [7,149] and the *isodual genorelativity* [99] characterized by the isoduality of the isorelativity of Sections 2.14 and 3.12 respectively, which cannot possibly be outlined here to prevent a prohibitive length.

A technical knowledge of the indicated isodual relativities is essential for a true understanding of this section, with particular reference to an understanding of the antimatter detections outlined in Section 4.18.

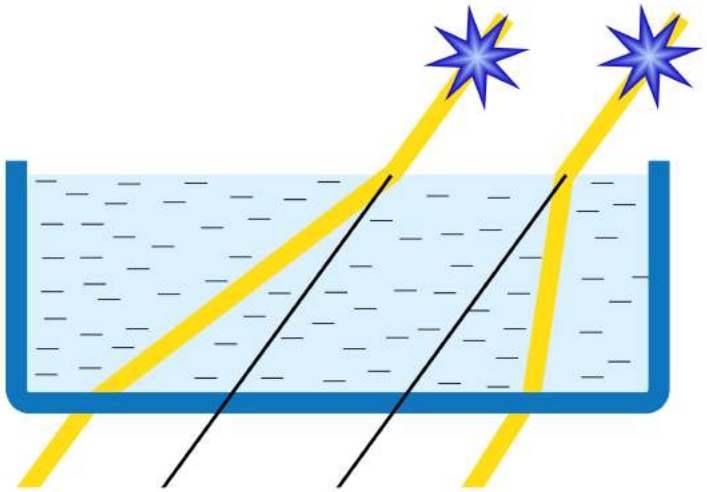

Figure 38: *An illustration of the central assumption of the isodual theory of antimatter, namely, the prediction that antimatter-light propagates within a matter-medium with an index of refraction opposite that of matter-light in the same medium [163,164].*

4.17. Dunning-Davies isodual thermodynamics for antimatter

It is important to indicate the pioneering formulation by J. Dunning-Davies [161] of 1999 of the first known *isodual thermodynamics for antimatter*, due to its intriguing feature that confirm the consistency of the isodual theory of antimatter, e.g., because seemingly inconsistent characteristics, such as a *negative entropy*, are in reality necessary for the general consistency.

4.18. The need of antimatter galaxies for the stability of the universe

In Section 2.17, we have reviewed the rejection of the expansion of the universe by Einstein, Hubble, Hoyle, Zwicky, Fermi, de Broglie and other famous scientists due to the fact that Hubble's law on the redshift of galactic light, $z = -Hd$ is "radial" in all directions from Earth. Consequently, its interpretation via the motion of galaxies away from Earth

$$x = Hd = \frac{v}{c}, \tag{4.45}$$

even though verifying special relativity, implies a return to the Middle Ages with earth at the center of the universe.

In Section 2.17 we have also reviewed the geometric and other inconsistencies of the extremely hypothetical assumption (venture to prevent Earth at the venture of the universe), that space itself is expanding despite the inconsistency originating, this time, from the acceleration of galaxies which is inherent in assumption (4.45).

Finally in Section 2.17 we have outlined the numerous experimental verifications on Earth of Zwicky hypothesis that light loses energy during its long travel through the intergalactic medium [31-36].

These combined mathematically, theoretical and experimental studies have established the lack of expansion of the universe, with ensuing return to the *steady state cosmology* advocated by Einstein, Hoyle, Hubble and others.

In turn, the return to a stable universe creates the problem of the stability of the universe addressed by Einstein, since a stable universe should collapse over sufficient time due to the gravitational attraction of galaxies.

The latter problems were studied during the earlier part of the 20th century, beginning with Einstein, without the achievement of the needed objective. It is sufficient to recall Einstein's attempt to introduce a cosmological constant in his field equations for gravitational "attraction" in the hope of achieving gravitational "repulsion" at large distance which was judged by Einstein himself "The biggest blunder of my life,"

The achievement of a stable non-expanding and non-contracting universe has been one of the major motivations for the author's search of antimatter galaxies, because, as shown in Ref. [162], a sufficient distribution of antimatter galaxies in the universe not only accounts for its stability due to matter-antimatter repulsion, but also accounts for the reason galaxies are at such large mutual distances.

4.19. Apparent detection of antimatter galaxies, antimatter asteroids and antimatter cosmic rays

4,19.1. Santilli telescope

In view of the fact that antimatter-light is predicted to be repelled by a matter (Figure 37), during the 2012 conference on Lie-admissible formulations [163] and the 2012 workshop on antimatter [164], the author presented the hypothesis that light propagating within a matter-medium experience a refraction opposite that of ordinary light [153], as illustrated in Figure 38.

In the same papers [163,164], the author indicated that the only possibility for detecting images with an index of refraction opposite that of light is that of using *concave lenses*, as illustrated in Figure 39. Telescopes with concave lenses are nowadays internationally known as *Santilli telescopes.*

Within the contest of the *isodual optics,* that is, the optics intended to study isodual light, conventional index of refraction n are assumed to have a positive value, $n > 0$, while the index of refraction of the isodual light, the isodual index of refraction n^d, is assumed to have a negate value, $n^d < 0$.

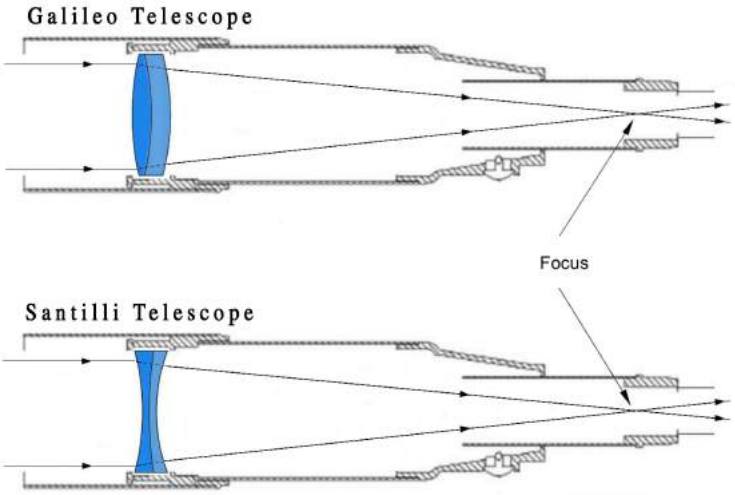

Figure 39: *A comparative view of the Galileo and Santilli telescopes. The former uses "convex" lenses for the focusing of images caused by matter-light, while the latter uses "concave" lenses for the focusing of images caused by antimatter-light. Note that the antimatter-light is dispersed by the convex lenses into the internal walls of the Galileo telescope, thus causing no interference in the image at the focal distance. Similarly, matter-light is dispersed by the concave lenses into the internal walls of the Santilli telescope, thus allowing fully defined detections.*

As it is well known, matter-light propagates in water at *subluminal* speeds $v = c/n$ for which

$$v = c/n < c, !\ \ n > 1. \tag{4.46}$$

Similarly, antimatter-light propagates in water with the speed $v^d = c^d/^d n^d = -v$ that erroneously appears to be *superluminal* while in reality

$$v^d = c^d/^d n^d, n^d < -1, \tag{4.47}$$

thus being subluminal.

The frequent "criticism" that the isodual theory of antimatter "violates" Einstein's special relativity emerges as being due to lack of technical knowledge of isodual mathematics, with particular reference to the fact that *a subluminal speed in the antimatter world may appear as superluminal when inspected from the matter world* due to the inversion of all signs of all operations.

Another criticism of the isodual theory of antimatter is that "negative index of refract do not exist in physics" without the knowledge that in reality antimatter-light treated with a positive index of refraction would violate causality.

Nowadays, Santilli telescopes are in production and sale by the U. S. publicly listed company *Thunder Energies Corporation* with $70\ mm,\ 100\ mm,\ 150\ mm$ and $200\ mm$ diameter, see Amazon.com and Figure 40.

Figure 40: *Views of the 70 mm, 100 mm, 150 mm and 200 mm pairs of Galileo and Santilli telescopes in production and for sale by the U. S.publicly traded company Thunder Energies Corporation (see also Amazon.com)*

4.19.2. Detection withs Santilli telescope

Recall that no image can be seen in the Santilli telescope with our naked eyes due to its concave lenses. Therefore, the proper way for using the Santilli telescope is its joint use with a Galileo telescope of the same diameter and focal distance.

In this way, focus is first reached by adjusting the focuser in the Galileo telescope, and then its setting are applied to the focused of the Santilli telescope.

Again in view of our inability to see images caused by antimatter-light, the sole possible detection is that by equipping the Galileo and the Santilli telescopes with suitably sensitive digital (pr film) cameras, such as Cannon 600D or equivalent. Images for both telescopes become visible in the camera screens.

Since antimatter galaxies are expected at extreme distances from Earth, still views in the digital camera are expected to yield small dots easily confused with lens impurities or background. Hence, the most reliable detections are done with a time exposure of at least $15\ s$ for the joint pair of Galileo and Santilli telescopes.

In this case, Earth's rotation causes a far away matter-galaxy to create a *bright streak* in the digital camera of the Galileo telescope. A far away antimatter-galaxy is then expected to create a *dark streak* over the camera background, that to be considered for analysis, has to have the same length and the same orientation as that of the bright streak for the matter-galaxy.

Intriguingly, *the darkness of the streaks created on a digital camera by antimatter-galaxies is a direct experimental verification of Dirac's conception of antimatter as having negative energy.*

In fact, the sole possibility for the creation of a dark streak is that antimatter-light carries negative energy, that as such, annuls the positive energy of the camera pixels, essentially, the energy of a photovoltaic processes.

Recall that astrophysical detections via the use of Galileo telescopes are generally done in remote areas at high elevation. By contrast, detections with the Santilli telescope are generally done in urban areas for the specific purpose of having a well defined backgrounds caused by ordinary light, in which absence, dark streaks are not visible.

Also, lenses of Galileo telescopes are generally kept as clean as possible. By contrast, impurities in the lenses of the Santilli telescope are kept (or created) and recorded at various enlargements as a condition to assure the proper focus of the Santilli telescope and to ascertain the novelty of dark streaks.

4.19.3. Santilli antimatter galaxy in the Vega region of the night sky

Following extensive tests in 2012-2013 with a pair of 100 mm Galileo and Santilli telescopes equipped with a Cannon camera model 600D, the author achieved the apparent first detection of an antimatter galaxy [165] today known as *santilli antimatter galaxy* [167-174] (see Figure 41 for representative pictures and Ref. [166] for comprehensive listing).

Ref. [165] also detected apparently for the first time, asteroids annihilating in our upper atmosphere and antimatter cosmic rays (see Figure 42 for representative pictures out of the comprehensive listing [166])

In the author's view, *the most significant result of the extensive tests reported in Ref.*

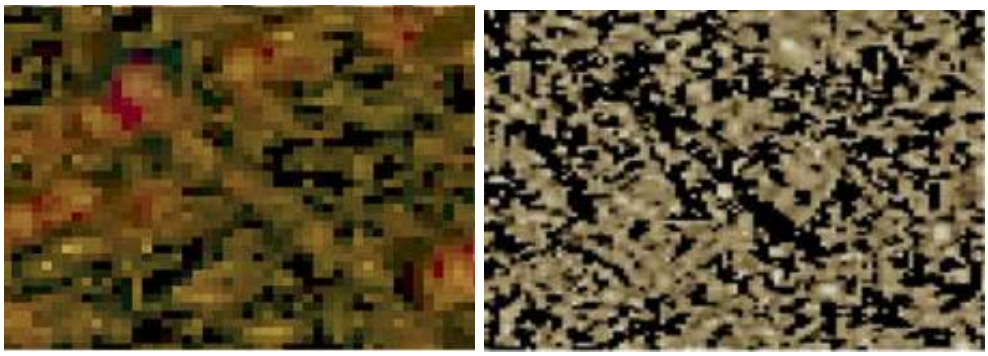

Figure 41: *Representative pictures obtained via a* 100 *mm pair of Galileo and Santilli telescopes equipped with a Cannon Camera Model 600D with* 15 *s exposure of Vega region (left) and of the first detection of Santilli antimatter galaxy (right) obtained on June 27, 2013 [165]. The first picture shows a bright streak, while the second picture shows a a dark streak over conventional background that confirms Dirac's conception of antimatter as having negative energy (see Ref. [166] for a collection of pictures from Ref. [165]).*

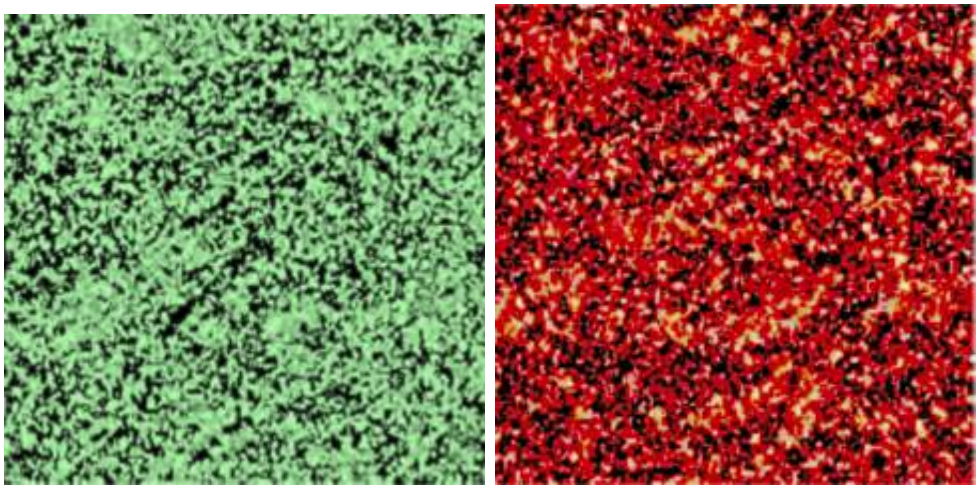

Figure 42: *Representative picture from the first independent verification [167] of detections [165] also obtained via a pair of* 100 *mm Galileo and Santilli telescopes equipped with a Cannon camera model 600D under* 15 *s exposure showing: the Vega star (upper left); Santilli antimatter galaxy (upper right); antimatter asteroids (lower left); and antimatter cosmic rays (lower right).*

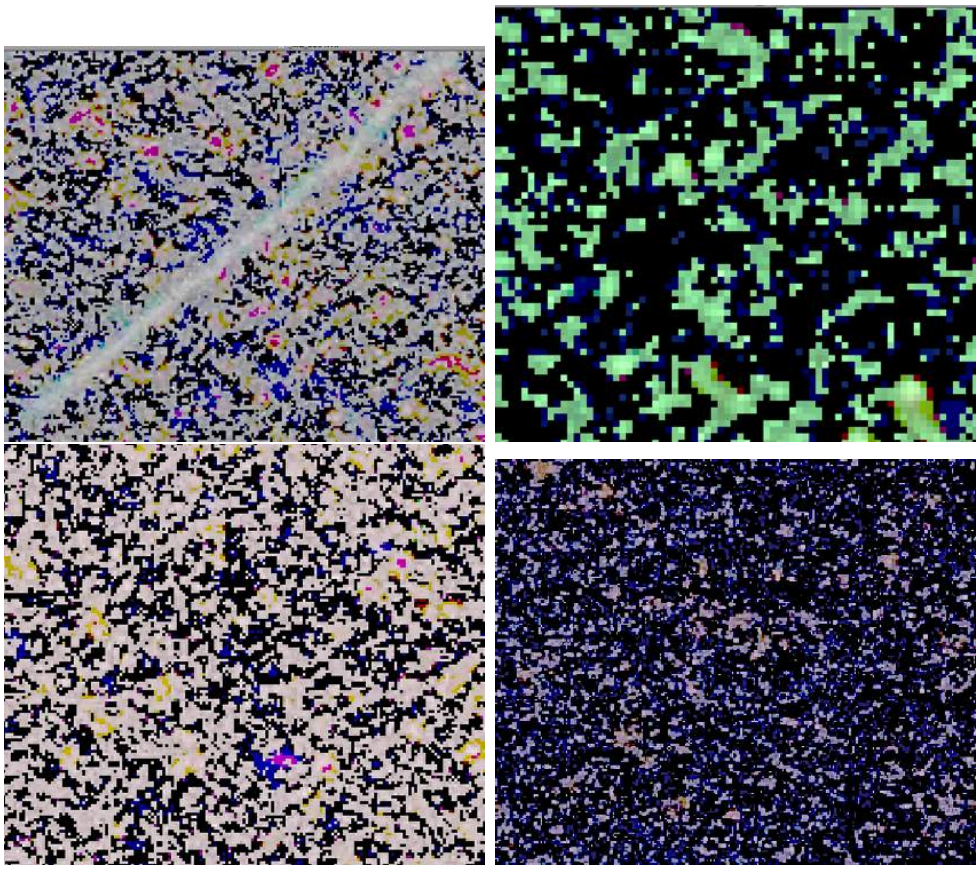

Figure 43: *Representative pictures from the first independent confirmatory paper [167] taken with the 100 mm pair of Galileo and Santilli telescopes equipped with a Cannon camera 600D in the Vega region of the night sky showing: Vega (top left); Santilli antimatter galaxy (top right); antimatter asteroid (bottom left); and antimatter cosmic ray (bottom right).*

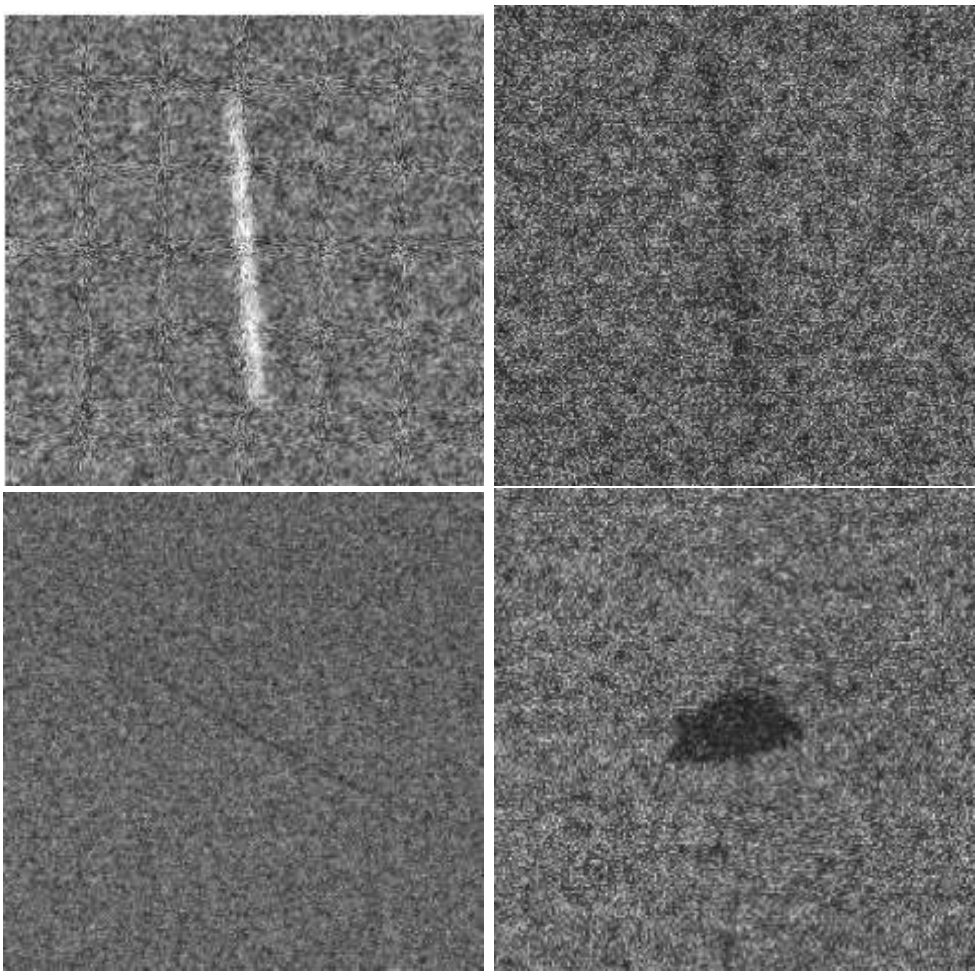

Figure 44: *Representative picture from the second independent verification [168] of detections [165] also obtained via a pair of* 100 *mm Galileo and Santilli telescopes equipped with a Cannon camera model 600D under* 15 *s exposure showing: the Vega star (upper left); Santilli antimatter galaxy (upper right); antimatter asteroids (lower left); and antimatter cosmic rays (lower right).*

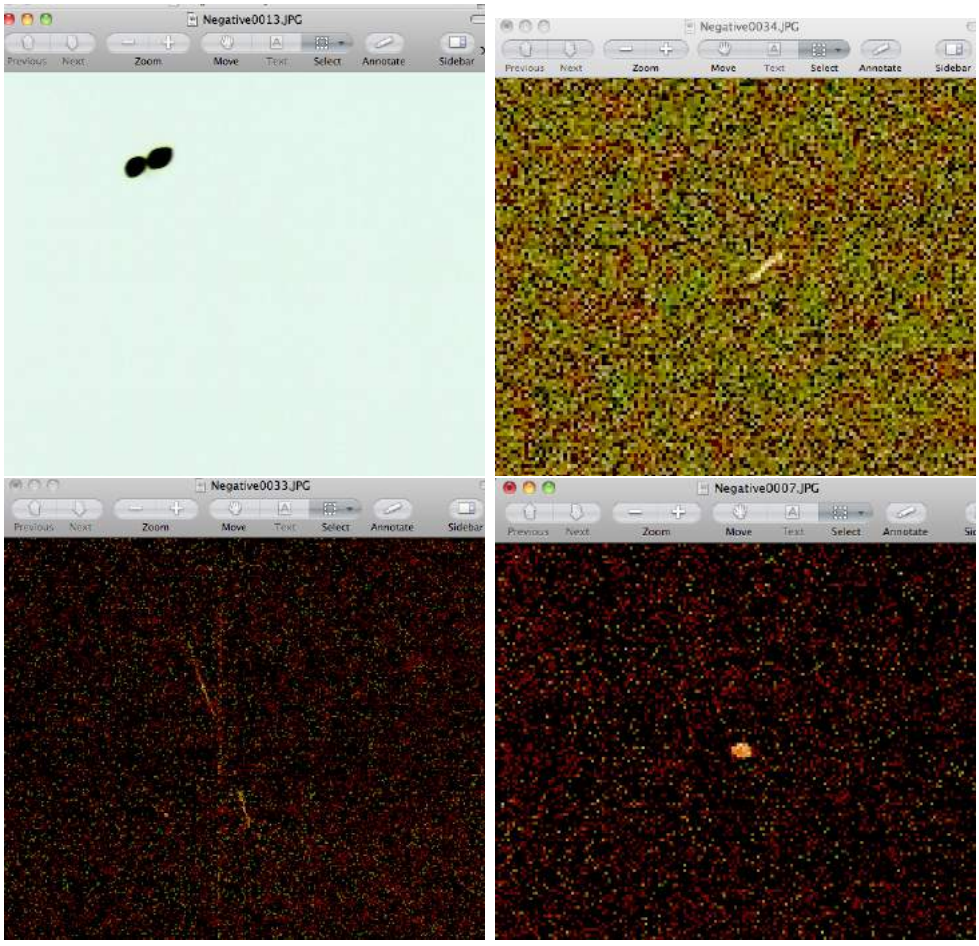

Figure 45: *Representative picture from the second detection of an antimatter galaxy in the Capella region of the night sky [169], called "Rak antimatter galaxy," obtained via a pair of* 150 *mm Galileo and Santilli telescopes equipped with a Cannon camera model 600D under* 15 *s expositor showing: Capella double star (upper left); Rak antimatter galaxy (upper right); antimatter asteroids (lower left); and antimatter cosmic rays (lower right).*

[165] is the clear repeated and systematic focusing via a concave lense of images of far away astrophysical objects. In fact, this is such a hard evidence to establish the entire isodual mathematics and isodual theory of antimatter.

The capability to detect astrophysical objects with concave lenses has been confirmed by systematic detections of clear images of terrestrial objects via Santilli telescopes [178] (see also Amazon.com and Thunder-energies.com) which are not reviewed here for brevity.

It is hoped for their own sake that interested colleagues will not behave like Cardinal Ballarmino (who refused to look into Galileo telescope), and use indeed readily available pairs of Galileo and Santilli telescope prior to venture "judgments."

4.19.4. Independent experimental confirmations of Santilli antimatter galaxy

Immediately following the appearance of Ref. [165], the independent astrophysicists, P. Bhujbal, J. V. Kadeisvili, A. Nas, S. Randall, and T. R. Shelke repeated Santilli detections and released paper [167] providing a "preliminary confirmation" of Santilli antimatter galaxy, antimatter asteroids and antimatter cosmic rays (see representative pictures in Figure 43).

In 2015, the independent astrophysicists S. Beghella-Bartoli, P. M. Bhujbal, A. Nas, repeated again all measurements in the Vega region of the night sky via the used of a pair of 100 mm Galileo and Santilli telescopes this tine equipped with a very sensitive film camera, and provided a second independent confirmation of Santilli antimatter galaxy, antimatter asteroids and antimatter cosmic rays [168] (see Figure 44 for representative pictures).

4.19.5. Rak antimatter galaxy in the Capella region of the night sky

During the years 2014-2015, the author conducted systematic scans of the Capella region of the night sky, and reported in Ref. [169] providing evidence on the existence of a second antimatter galaxy, as well as confirmatory evidence on the existence of antimatter asteroids and antimatter cosmic rays (see representative pictures in Figure 45).

The author suggests that the new antimatter galaxy detected in the Capella region of the night sky be named *Rak antimatter galaxy* in recognition of the dedication to the advancement of basic scientific knowledge by Professor Jan Rak, of the University of Jyväskylä, Finland (see the asbtract and concluding remarks of Ref. [169]).

4.19.6. Detection with Santilli telescope

Since matter and antimatter are expected to repel each other, antimatter asteroids are expected to be repelled by Earth's gravitational field, thus explaining the rarity of an antimatter asteroid colliding with Earth.

Despite such a repulsion, S. Beghella-Bartoli [175] has conducted detailed studies of the trajectories of antimatter asteroids in our Solar system and concluded that, under certain well defined conditions of relative speeds, directions, etc., antimatter asteroids can collide with Earth despite said gravitational repulsion, by therefore annihilating in our upper atmosphere (Section .4.5) .

In Ref. [165], the author has reported apparently for the first time, numerous black streaks under $15\ s$ exposure that can only be interpreted as being due to the annihilation

of small antimatter asteroids penetrating at high speed in our atmosphere since said black streaks do not have the orientation of matter and antimatter streaks, thus being virtually instantaneous (Figures 41, 42).

Refs. [176,167] have confirmed said findings with the detection of numerous instantaneous black streaks with arbitrary orientation and length under $15\ s$ exposure, thus confirming their creation by the annihilation of small antimatter asteroids in our upper atmosphere (Figures 43, 44). Additional confirmations have been provided in Ref. [169] (see Figure 45).

A team of scientist emphasized in Ref.[176] the risk caused to Earth by antimatter asteroids, and stressed the need for their study since we do not know at this writing whether Sun light is reflected or absorbed by a cold antimatter asteroid, thus establishing the current lack of knowledge for their detection.,

It should be indicated that the annihilation in our upper atmosphere of antimatter an asteroid of the size of a football is expected to disrupt all civilian, industrial and military communications for days.

Needless to say, the existence of antimatter galaxies implies the existence of antimatter asteroids due to events such as antimatter supernova. Vice versa, the detection of antimatter asteroids establishes the existence of antimatter galaxies.

4.19.2. Antimatter cosmic rays

The existence of antimatter galaxies and antimatter asteroids establishes the existence of *antimatter cosmic rays,* essentially given by high speed antiprotons, which annihilate at contact with the upper layers of our atmosphere.

In Ref. [165], the author reported, apparently for the first time, a number of *dark circles* or dark spots (not due to impurities in the lenses) under $15\ s$ exposure that, as such, can only be due to instantaneous events. The darkness of the circles or large spots establishes that they are due to light with negative energy. The fact that they are focused by concave lenses establishes that they have a negative index of refraction.

Santilli's detection of antimatter cosmic rays [165, have been confirmed by the systematic scans of Refs. [167,168] (see Figures 43, 44). Additional confirmations have been provided in Ref. [169] (see Figure 45).

As indicated in Section 4.5, the author predicted the existence of antimatter cosmic rays from the fact that both astronauts and cosmonauts have seen "flashes of light" in the dark side of our upper atmosphere.

Note that these detections are done by our eye with convex cornea. Therefore, they are caused by ordinary light. The circles and dots of darkness detected at sea level [165-169], are instead due to isodual light,

By contrast, no ordinary light can reach us at sea level. Consequently, the dark circles or dots detected at sea level are caused by isodual light.

Among a considerable number of independent studies on the isodual mathematics and isodual theory of antimatter, we quote Refs. [170-176] and literature quoted therein with regret to be unable to outline their content.

5. Concluding remarks

In this work, we updates and outlined the following branches of *hadronic mechanics* introduced in the preceding monographs [3,8,149]:

two volumes of this series [8] released in 1995:

1. *Isomathematics,* with ensuing *isomechanics* and *isochemistry*, which have been constructed for the representation of time reversal invariant systems of extended and deformable particles in interior dynamical conditions as occurring, e.g., in stable nuclides;

2. The covering *genomathematics,* with ensuing *genomechanics* and *genochemistry* which has been constructed for the representation of time irreversible systems as occurring, e.g., in nuclear syntheses and all energy releasing processes; and

3. *Isodual conventional, isotopic and genotopic mathematics* with ensuing it isodual heory of antimatter, which has been constructed for the *classical* (as well as operator) representation of *neutral* (as well as charged) antimatter systems in conditions of increasing complexity, under the condition of compatibility with matter-antimatter annihilation, that requires the surpassing of charge conjugation in favor of new vistas.

We have then reviewed the main experimental verifications and industrial applications of the indicated branches of hadronic mechanics, and pointed out expected, additional, mathematical, physical, chemical and industrial advances for matter and antimatter, with particular reference to new, clean, sustainable fuels and energies.

All in all, , it is hoped that the outline of fifty years of research by numerous scholars presented in this work may stimulate young minds of all ages to participate in the thrill of new discoveries and promote, in due time, a new scientific renaissance along the lines of ref. [67] with possible scientific advances beyond our imagination at this writing.

Acknowledgments

Thanks are due to Prof. Seenith Siva, organizer of the 2016 ICNPAA Conference held at the University of La Rochelle France, on July 4 to 8, for the possibility of delivering an invited keynote talk on the content of this paper. Particular thanks are due to all participants of said conference for penetrating comments and suggestions. Special thanks are also due to Prof. A. A. Bhalekar for an in-depth critical examination of the entire manuscript. Finally, thanks are due to Mrs. Sherri Stone for a detailed linguistic control. Needless to say, the author is solely responsible for the content of this work due to numerous additions and revisions implemented in the final version.

References

[1] R. M. Santilli, "On a possible Lie-admissible covering of Galilei's relativity in Newtonian mechanics for nonconservative and Galilei form-non-invariant systems," Hadronic J. Vol. 1, 223-423 (1978), available in free pdf download from http://www.santilli-foundation.org/docs/Santilli-58.pdf

[2] R. M. Santilli, "Need of subjecting to an experimental verification the validity within a hadron of Einstein special relativity and Pauli exclusion principle,"

Hadronic J. Vol. 1, 574-901 (1978), available in free pdf download from
http://www.santilli-foundation.org/docs/santilli-73.pdf

[3] R. M. Santilli, *Foundation of Theoretical Mechanics,* Volume I [3a] (1978) and II [3b] (1982) Springer-Verlag, Heidelberg, Germany,
http://www.santilli-foundation.org/docs/Santilli-209.pdf
http://www.santilli-foundation.org/docs/santilli-69.pdf

[4] R. M. Santilli, "Isonumbers and Genonumbers of Dimensions 1, 2, 4, 8, their Isoduals and Pseudoduals, and "Hidden Numbers," of Dimension 3, 5, 6, 7," Algebras, Groups and Geometries Vol. 10, 273 (1993)
http://www.santilli-foundation.org/docs/Santilli-34.pdf

[5] Chun-Xuan Jiang, *Foundations of Santilli Isonumber Theory,* International Academic Press (2001),
http://www.i-b-r.org/docs/jiang.pdf

[6] R. M. Santilli, "Nonlocal-Integral Isotopies of Differential Calculus, Mechanics and Geometries," in Isotopies of Contemporary Mathematical Structures," Rendiconti Circolo Matematico Palermo, Suppl. Vol. 42, 7-82 (1996),
http://www.santilli-foundation.org/docs/Santilli-37.pdf

[7] R. M. Santilli, *Elements of Hadronic Mechanics*, Vol. I and Vol. II (1995) [15b], Academy of Sciences, Kiev,
http://www.santilli-foundation.org/docs/Santilli-300.pdf
http://www.santilli-foundation.org/docs/Santilli-301.pdf

[8] R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry,*, Volumes I to V, International Academic Press, (2008),
http://www.i-b-r.org/Hadronic-Mechanics.htm

[9] S. Georgiev, *Foundations of the IsoDifferential Calculus,* Volumes, I, II, III, IV,V, and VI Nova Scientific Publisher (2015 on).

[10] D. S. Sourlas and G. T. Tsagas, *Mathematical Foundation of the Lie-Santilli Theory,* Ukraine Academy of Sciences (1993),
http://www.santilli-foundation.org/docs/santilli-70.pdf

[11] R. M. Santilli, "Rotational isotopic symmetries," ICTP communication No. IC/91/261 (1991),
http://www.santilli-foundation.org/docs/Santilli-148.pdf

[12] R. M. Santilli, "Isotopies of Lie Symmetries," I (basic theory) and II (isotopies of the rotational symmetry), Hadronic J. Vol. 8, 36 and 85 (1985),
http://www.santilli-foundation.org/docs/santilli-65.pdf

[13] R. M. Santilli, "Isotopic Lifting of the SU(2) Symmetry with Applications to Nuclear Physics," JINR rapid Comm. Vol. 6. 24-38 (1993),
http://www.santilli-foundation.org/docs/Santilli-19.pdf

[14] R. M. Santilli, "Isorepresentation of the Lie-isotopic SU(2) Algebra with Application to Nuclear Physics and Local Realism," Acta Applicandae Mathematicae Vol. 50, 177 (1998),
http://www.santilli-foundation.org/docs/Santilli-27.pdf

[15] R. M. Santilli, "Lie-isotopic Lifting of Special Relativity for Extended Deformable Particles," Lettere Nuovo Cimento **37**, 545 (1983),
http://www.santilli-foundation.org/docs/Santilli-50.pdf

[16] R. M. Santilli, "Lie-isotopic Lifting of Unitary Symmetries and of Wigner's Theorem for Extended and Deformable Particles," Lettere Nuovo Cimento Vol. 38, 509 (1983),
http://www.santilli-foundation.org/docs/Santilli-51.pdf

[17] R. M. Santilli, "Lie-isotopic generalization of the Poincare' symmetry, classical formulation," ICTP communication No. IC/91/45 (1991),
http://www.santilli-foundation.org/docs/Santilli-140.pdf

[18] R. M. Santilli, "Nonlinear, Nonlocal and Noncanonical Isotopies of the Poincare' Symmetry," Moscow Phys. Soc. Vol. 3, 255 (1993),
http://www.santilli-foundation.org/docs/Santilli-40.pdf

[19] R. M. Santilli, "Recent theoretical and experimental evidence on the synthesis of the neutron," Communication of the Joint Institute for Nuclear Research, Dubna, Russia, No. E4-93-252 (1993).

[20] R. M. Santilli, "Recent theoretical and experimental evidence on the synthesis of the neutron," Communication of the JINR, Dubna, Russia, No. E4-93-252 (1993), published in the Chinese J. System Eng. and Electr. Vol. 6, 177 (1995),
http://www.santilli-foundation.org/docs/Santilli-18.pdf

[21] R. M. Santilli, "Isominkowskian Geometry for the Gravitational Treatment of Matter and its Isodual for Antimatter," Intern. J. Modern Phys. D **7**, 351 (1998),
http://www.santilli-foundation.org/docs/Santilli-35.pdf

[22] A. K. Aringazin and K. M. Aringazin, "Universality of Santilli's iso-Minkowskian geometry" in *Frontiers of Fundamental Physics,* M. Barone and F. Selleri, Editors, Plenum (1995),
http://www.santilli-foundation.org/docs/Santilli-29.pdf

[23] R. M. Santilli, "Rudiments of IsoGravitation for Matter and its IsoDual for AntiMatter," American Journal of Modern Physics Vol. 4, No. 5, 2015, pp. 59, in the Special Issue I; *Foundations of Hadronic Mathematics,* Dedicated to the 80th Birthday of

Prof. R. M. Santilli
http://www.santilli-foundation.org/docs/10.11648.j.ajmp.s.2015040501.18.pdf

[24] J. Feldman, "Debate on General Relativity,"
http://galileoprincipia.org/physics

[25] R. M. Santilli, "Compatibility of Arbitrary Speeds with Special Relativity Axioms for Interior Dynamical Problems," American Journal of Modern Physics, in press (2016)
http://www.santilli-foundation.org/docs/ArbitrarySpeeds.pdf

[26] R. M. Santilli, *Isotopic Generalizations of Galilei and Einstein Relativities,* Vol. I (1991) [12a] and Vol. II (1991) [12b], Hadronic Press, Palm Harbor, Florida, available in free pdf download from
http://www.santilli-foundation.org/docs/Santilli-01.pdf
http://www.santilli-foundation.org/docs/Santilli-61.pdf

[27] H. C. Myung and R. M. Santilli, Hadronic Journal Vol. 5, pages 1277-1366 (1982). available as free download from
http://www.santilli-foundation.org/docs/Santilli-201.pdf

[28] M. Nishioka, "Expansion of the Dirac-Myung-Santilli Delta Function to Field Theory,"Lettere Nuovo Cimento Vol. 39, pages 369-372 (1984),
http://www.santilli-foundation.org/docs/Santilli-202.pdf

[29] R. M. Santilli and A. O. E. Animalu, "Nonunitary Lie-isotopic and Lie-admissible scattering theories of hadronic mechanics, in the *Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes,* C. Corda, Editor, Kathmandu University (2011) pages 163-177
I: http://www.santilli-foundation.org/docs/Isoscattering-I.pdf
II: http://www.santilli-foundation.org/docs/Isoscattering-II.pdf
III: http://www.santilli-foundation.org/docs/Isoscattering-III.pdf
IV: http://www.santilli-foundation.org/docs/Isoscattering-IV.pdf
V: http://www.santilli-foundation.org/docs/Isoscattering-V.pdf

[30] R. M. Santilli, *Isorelativity for matter and its isodual for antimatter*, to appear.

[31] R. M. Santilli, "Experimental Verifications of IsoRedShift with Possible Absence of Universe Expansion, Big Bang, Dark Matter, and Dark Energy," The Open Astronomy Journal **3**, 124 (2010),
http://www.santilli-foundation.org/docs/Isoredshift-Letter.pdf

[32] R. M. Santilli, "Experimental Verification of IsoRedShift and its Cosmological Implications," American Institute of Physics Conference Proceedings Vol. 1281, pp. 882-885 (2010)
http://www.santilli-foundation.org/docs/Santilli-isoredshift.pdf

[33] R. M. Santilli, "Experimental verifications of isoredshift with possible absence of universe expansion, big bang, dark matter and dark energy," The Open Astronomy Journal **3**, 124 (2010), available as free download from
http://www.santilli-foundation.org/docs/Santilli-isoredshift.pdf

[34] R. M. Santilli, G. West and G. Amato, "Experimental Confirmation of the IsoRed-Shift at Sunset and Sunrise with Consequential Absence of Universe Expansion and Related Conjectures, " Journal of Computational Methods in Sciences and Engineering, Vol. 12, pages 165-188 (2012)
http://www.santilli-foundation.org/docs/Confirmation-sun-IRS.pdf

[35] G. West and G. Amato, "Experimental Confirmation of Santilli's IsoRedShift and IsoBlueShift," contributed paper to the *Proceedings of the San Marino Workshop on Astrophysics and Cosmology for Matter and Antimatter,* Republic of San Marino, September 5 to 9, 2011, in press (2012),
http://www.santilli-foundation.org/docs/Confirmation-IRS-IBS.pdf

[36] H. Ahmar, G. Amato, J. V. Kadeisvili, J. Manuel, G. West, and O. Zogorodnia, "Additional experimental confirmations of Santilli's IsoRedShift and the consequential lack of expansion of the universe,"Journal of Computational Methods in Sciences and Engineering, Vol. 13, page 321 (2013),
http://www.santilli-foundation.org/docs/IRS-confirmations-212.pdf

[37] R. M. Santilli, "Representation of galactic dynamics via isoshifts without universe expansion, dark matter and dark energy," American Journal of Modern Physics Vol. 4, pages 26-43, 2015
http://www.thunder-energies.com/docs/dark-matter-2015.pdf

[38] P. Fleming, "Collected papers, interviews, seminars and international press releases on the lack of expansion of the universe."
http://www.santilli-foundation.org/docs/No-universe-expans.pdf

[39] A. K. Aringazin, Hadronic J. 12 71 (1989).

[40] S. H. Aronson et al., Phys. Rev. D 28, 495 (1983).

[41] N. Grossman et al., Phys. Rev. Lett. 59, 18 (1987).

[42] F. Cardone, R. Mignani and R. M. Santilli Lie-isotopic energy dependence of the KS lifetime, J. Phys. G: Nucl. Part. Phys. 18 [1992], L141-L152.

[43] F. Cardone and R. Mignani, Nonlocal approach to the Bose-Einstein correlation, Univ. of Rome Preprint No. 894, July 1992.

[44] Yu. Arestov, R. M. Santilli and V. Solovianov, "Experimental evidence on the isominkowskian character of the hadronic structure," Foundation of Physics Letters Vol. 11, pages 483-492 (1998)
http://www.santilli-foundation.org/docs/Santilli-52.pdf

[45] R. M. Santilli, "Nonlocal formulation of the Bose-Einstein correlation within the context of hadronic mechanics," Hadronic J. **15**, 1-50 and 81-133 (1992), http://www.santilli-foundation.org/docs/Santilli-116.pdf

[46] F. Cardone and R. Mignani, "Nonlocal approach to the Bose-Einstein correlation, Europ. Phys. J. C 4, 705 (1998). See also "Metric description of hadronic interactions from the Bose-Einstein correlation," JETP Vol. 83, p.435 (1996) http://www.santilli-foundation.org/docs/Santilli-130.pdf

[47] R. M. Santilli, "A quantitative isotopic representation of the deuteron magnetic moment," in *Proceedings of the International Symposium 'Dubna Deuteron-93,* Joint Institute for Nuclear Research, Dubna, Russia (1994), http://www.santilli-foundation.org/docs/Santilli-134.pdf

[48] R. M. Santilli, "Nuclear realization of hadronic mechanics and the exact representation of nuclear magnetic moments," R. M. Santilli, Intern. J. of Phys. Vol. 4, 1-70 (1998) http://www.santilli-foundation.org/docs/Santilli-07.pdf

[49] A. A. Bhalekar and R. M. Santilli, "Exact and Invariant Representation of Nuclear Magnetic Moments and Spins According to Hadronic Mechanics," American Journal of Modern Physics, 2016; 5(2-1): 56-118 http://www.santilli-foundation.org/docs/nuclear-MM-spins.pdf

[50] R. M. Santilli, "Apparent consistency of Rutherford's hypothesis on the neutron as a compressed hydrogen atom, Hadronic J.**13**, 513 (1990). http://www.santilli-foundation.org/docs/Santilli-21.pdf

[51] R. M. Santilli, "Apparent consistency of Rutherford's hypothesis on the neutron structure via the hadronic generalization of quantum mechanics - I: Nonrelativistic treatment", ICTP communication IC/91/47 (1992) http://www.santilli-foundation.org/docs/Santilli-150.pdf

[52] C. Borghi, C. Giori C. and A. Dall'Olio, Communications of CENUFPE, Number 8 (1969) and 25 (1971), reprinted via Santilli;'s proposal in the (Russian) Phys. Atomic Nuclei, ¡b¿ 56¡/b¿, 205 (1993).

[53] R. M. Santilli, Confirmation of Don Borghis experiment on the synthesis of neutrons, arXiv publication, August 15, 2006 http://arxiv.org/pdf/physics/0608229v1.pdf

[54] R. M. Santilli, The synthesis of the neutron according to hadronic mechanics and chemistry," Journal Applied Sciences **5**, 32 (2006)

[55] R. M. Santilli, "Apparent confirmation of Don Borghis experiment on the laboratory synthesis of neutrons from protons and electrons, Hadronic J. **30**, 29 (2007) http://www.i-b-r.org/NeutronSynthesis.pdf

[56] R. M. Santilli, "Documentation of scans from the Polimaster and SAM 935 detectors during tests [11-13]
http://www.neutronstructure.org/neutron-synthesis-3.htm
www.neutronstructure.org/neutron-synthesis-2.htm

[57] R. M. Santilli and A. Nas, "Confirmation of the Laboratory Synthesis of Neutrons from a Hydrogen Gas," Journal of Computational Methods in Sciences and Eng, in press (2014)
www.thunder-energies.com/docs/neutron-synthesis-2014.pdf

[58] R. M. Santilli and A. Nas, 12 minutes Film on the Laboratory Synthesis of Neutrons from a Hydrogen Gas"
www.world-lecture-series.org/newtron-synthesis-08-14

[59] R. M. Santilli, "The etherino and/or the Neutrino Hypothesis?" Found. Phys. **37**, p. 670 (2007)
www.santilli-foundation.org/docs/EtherinoFoundPhys.pdf

[60] R. M. Santilli, *The Physics of New Clean Energies and Fuels According to Hadronic Mechanics,* Special issue of the Journal of New Energy, 318 pages (1998)
http://www.santilli-foundation.org/docs/Santilli-114.pdf

[61] R. M. Santilli, *Foundations of Hadronic Chemistry, with Applications to New Clean Energies and Fuels,* Kluwer Academic Publishers (2001),
http: //www.santilli-foundation.org/docs/Santilli-113.pdf
Russian translation by A. K. Aringazin
http://i-b-r.org/docs/Santilli-Hadronic-Chemistry.pdf

[62] R. M. Santilli and D. D. Shillady,, "A new isochemical model of the hydrogen molecule," Intern. J. Hydrogen Energy Vol. 24, pages 943-956 (1999)
http://www.santilli-foundation.org/docs/Santilli-135.pdf

[63] R. M. Santilli and D. D. Shillady, "A new isochemical model of the water molecule," Intern. J. Hydrogen Energy Vol. 25, 173-183 (2000)
http://www.santilli-foundation.org/docs/Santilli-39.pdf

[64] J. Feldman, "First open nomination for chemistry," website
http://nobelprizeweb.com/index.php/nominations-1

[65] R. Anderson, "Scientific archives of the R. M. Santilli Foundation,," website,
http://www.santilli-foundation.org/news.html

[66] J. Feldman, "First open nomination for physics," website
http://nobelprizeweb.com/index.php/nominations-3

[67] I. Gandzha and J. Kadeisvili, *New Sciences for a New Era: Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli,* Sankata Printing Press, Nepal (2011), http://www.santilli-foundation.org/docs/RMS.pdf

[68] T. Vougiouklis, "The Santilli theory 'invasion' in hyperstructures," Algebras, Groups and Geometries Vol. 28, pages 83-104 (2011) http://www.santilli-foundation.org/docs/santilli-invasion.pdf

[69] J. L. Lagrange, *Mecanique Analytique* (1788), reprinted by Gauthier-Villars, Paris (1888).

[70] W. R. Hamilton, *On a General Method in Dynamics* (1834), reprinted in *Hamilton's Collected Works,* Cambridge Univ. Press (1940).

[71] A.A. Albert, Trans. Amer. Math. Soc. **64**, 552 (1948).

[72] 1] R. M. Santilli, "Embedding of Lie-algebras into Lie-admissible algebras," Nuovo Cimento **51**, 570 (1967), http://www.santilli-foundation.org/docs/Santilli-54.pdf

[73] R. M. Santilli, "Dissipativity and Lie-admissible algebras," Meccanica **1**, 3 (l969).

[74] [2] R. M. Santilli, "An introduction to Lie-admissible algebras," Suppl. Nuovo Cimento, **6**, 1225 (1968).

[75] P. Roman and R. M. Santilli, "A Lie-admissible model for dissipative plasma," Lettere Nuovo Cimento Vol. 2, 449-455 (l969)

[76] R. M. Santilli, *Lie-Admissible Approach to the Hadronic Structure,* Vol. II and II (1982) [11b], Hadronic Press, available as free downloads from the links http://www.santilli-foundation.org/docs/Santilli-71.pdf http://www.santilli-foundation.org/docs/Santilli-72.pdf

[77] J. Fronteau, A. Tellez-Arenas and R. M. Santilli, Lie-admissible structure of statistical mechanics, Hadronic J. 3 (1979), 130-176.

[78] J. A. Kobussen, Lie-admissible structure of classical field theory, Hadronic J. 3 (1979), 79-129.

bibitemr2 R. H. Ohemke, Some elementary structure theorems for a class of Lie-admissible algebras, Hadronic J. 3 (1979), 293-219.

[79] R. M. Santilli, An introduction to the Lie-admissible treatment of nonpotential interactions in Newtonian, statistical and particle mechanics, Hadronic J. 5 [1982], 264-359

[80] S. Okubo, Hadronic J. **5**, 1667 (1982).

[81] Y. Ilamed, On realizations of infinite-dimensional Lie-admissible algebras, Hadronic J. 3 (1979), 327-338. A. Jannussis, Noncanonical quantum statistical mechanics, Hadronic J. Suppl. 1 [1985], 576-609.

[82] A. Jannussis, Noncanonical quantum statistical mechanics, Hadronic J. Suppl. 1 [1985], 576-609.

[83] A. Jannussis and R. Mignani, Algebraic structure of linear and nonlinear models of open quantum systems, Physica A 152 [1988], 469-476.

[84] A. U. Klimyk and R. M. Santilli, Standard isorepresentations of isotopic Q- operator deformations of Lie algebras, Algebras, Groups and Geometries 10 [1993], 323-333

[85] D. Schuch, "Nonunitary connectioon between ex[plicitly time-dependent and nonlinear approaches for the description of dissipative quantum systems," Phys. Rev A **55** (1997), 955-966.

[86] M. L. Tomber, A short history of nonassociative algebras, Hadronic J. 2 (1979), 11252-1387.

[87] H. C. Myung, Editor, Mathematical Studies on Lie-admissible Algebras, Volumes I, II, III, IV, and V, Hadronic Press (1985-1987).

[88] A. Schoeber, Editor, *Irreversibility and Non-potentiality in Statistical Mechanics,* Hadronic Press (1984),
http://www.santilli-foundation.org/docs/Santilli-110.pdf

[89] D. F. Lopez, in *Symmetry Methods in Physics*, A. N. Sissakian, G. S. Pogosyan and X. I. Vinitsky, Editors (JINR, Dubna, Russia, 1994).

[90] D. F. Lopez, Hadronic J. **16**, 429 (1993).

[91] A. Jannussis and D. Skaltsas, Ann. Fond. L. de Broglie **18**, 137 (1993).

[92] A. Jannussis, R. Mignani and R. M. Santilli, Ann. Fond. L. de Broglie **18**, 371 (1993).

[93] D. Schuch, Phys. Rev. A **55**, 955 (1997).

[94] R. M. Santilli, Modern Phys. Letters **13**, 327 (1998).

[95] R. M. Santilli, Found. Phys. **32**, 1111 (2002).

[96] L. C. Biedernharn, J. Phys. A **22**, L873 (1989).

[97] A. J. Macfarlane, J. Phys. A **22**, L4581 (1989).

[98] R. M. Santilli, "Invariant Lie-admissible formulation of quantum deformations," Found. Phys. **27**, 1159- 1177 (1997)
http://www.santilli-foundation.org/docs/Santilli-06.pdf

[99] R. M. Santilli, "ˋLie-admissible invariant representation of irreversibility for matter and antimatter at the classical and operator levels," Nuovo Cimento B 121, 443 (2006),
http://www.santilli-foundation.org/docs//Lie-admiss-NCB-I.pdf

[100] A. de Shalit and H. Feshback, *Theoretical Nuclear Physics,.* Vols. I and II, Jphn Wioley & Sons (1974).

[101] J. Dunning-Davies¡br¿ ¡b¿The Thermodynamics Associated with Santillis Hadronic Mechanics¡/b¿¡br¿ Progress in Physics,. Vol. 4, pp 24-26 (2006),¡br¿ http://www.santilli-foundation.org/docs/Dunning-Davies-Thermod.PDF

[102] A. A. Balekhari, "Santilli's Lie-Admissible Mechanics. The Only OptionCommensurate with Irreversibility and Nonequilibrium Thermodynamics," AIP Conference Proceedings 1558, 702 (2013); doi: 10.1063/1.4825588 http://www.santilli-foundation.org/docs/bhalekar-lie-admissible.pdf

[103] . V. Kadeisvili, The Rutherford-Santilli Neutron," Hadronic J. Vol. 31, Number 1, pages 1-125, 2008
http://www.i-b-r.org/Rutherford-Santilli-II.pdf
also available in in html format at
http://www.i-b-r.org/Rutherford-Santilli-neutron.htm

[104] Chandrakant S. Burande, "On the Rutherford-Santilli Neutron Model," AIP Conference Proceedings 1648, 510006 (2015); doi: 10.1063/1.4912711
http://www.santilli-foundation.org/docs/1.4912711(CS-Burande(1)).pdf

[105] Chandrakant S. Burande, "Santilli Synthesis of the Neutron According to Hadronic Mechanicss," in the special issue dedicated to the 80th birthday of R. M. santilli entitled *Foundations of Hadronic Mechanics,* American Journal of Modern Physics 2016; 5(2-1): 17-36
http://www.santilli-foundation.org/docs/pdf3.pdf

[106] R. M. Santilli, "Laboratory synthesis of the neutron from a proton and an electron," lecture
http://www.world-lecture-series.org/level-v

[107] R. M. Santilli, Lectures, *IsoLinear IsoInvariant isoRelativity,*
Limitations of Special Relativity and Quantum Mechanics
http://www.world-lecture-series.org/isorelativity-2014-i
Experimental Verifications in Particle Physics
http://www.world-lecture-series.org/isorelativity-2014-ii
Experimental Verifications in Nuclear Physics
http://www.world-lecture-series.org/isorelativity-2014-iii

[108] R. M. Santilli, "Studies on the etherino hypothesis" Found. Phys. **37**, p. 670 (2007) www.santilli-foundation.org/docs/EtherinoFoundPhys.pdf

[109] Arun S. Muktibodh, "Studies on Santilli Three-Body Model of the Deuteron According to Hadronic Mechanics," in the special issue dedicated to the 80th birthday of R. M. santilli entitled *Foundations of Hadronic Mechanics,* American Journal of Modern Physics 2016; 5(2-1): 17-36,
http://www.santilli-foundation.org/docs/pdf4.pdf

[110] R. M. Santilli. "The novel "Controlled Intermediate Nuclear Fusion:" A report on its industrial realization as predicted by hadronic mechanics," Hadronic J. **31**, 1, 2008
http://www.i-b-r.org/CNF-printed.pdf

[111] R. M. Santilli, "Experimental Confirmation of Nitrogen Synthesis from deuterium and Carbon without harmful radiations," New Advances in Physics **4**, issue no.1, (2010
http://www.santilli-foundation.org/docs/Niitrogen-Synthgesis-NAP-2010.zip

[112] R. M. Santilli, "Additional Confirmation of the "Intermediate Controlled Nuclear Fusions" without harmful radiation or waste," ¡i¿Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes,¡/i¿ Kathmandu University (2011) pages 163-177 http://www.santilli-foundation.org/docs/ICNF-3.pdf

[113] R. M. santilli, "Video presentation of the third hadronic reactor for the Nitrogen and Silicon syntheses."]] http://www.world-lecture-series.org/dragon-iii

[114] R. M. Santilli, "Video on the Hadronic Reactor III for the synthesis of the Silicon,"
http://www.world-lecture-series.org/dragon-iii

[115] R. M. santilli, "Sound of Intermediate Controlled nuclear syntheses." www.santilli-foundation.org/Thunder-Fusions.amr

[116] R. M. Santilli, "Invariant Lie-Admissible Classical and Operator Mechanics for Matter, Lecture
http://www.world-lecture-series.org/level-iii

[117] R. M. Santilli, "Intermediate Controlled Nuclear Fusions without the emission of radiations and without the release of radioactive waste," Lecture
http://www.world-lecture-series.org/level-v

[118] R. M. Santilli, "Third Hadronic Reactor for Intermediate Controlled Nuclear Fusions without Radiations,"" lecture
http://www.world-lecture-series.org/level-v

[119] R. M. Santilli, "Introduction to Intermediate Fusions without Radiations," lecture delivered at the US-China Summit, October 30, 2011,
http://www.world-lecture-series.org/level-v

[120] D. Rossiter, Director, "IVA Report 184443 on comparative Nitrogen counts" (for Ref. [78]), www.santilli-foundation.org/docs/IVA-Report-184443.pdf

[121] D. Rossiter, Director, "IVA Report 184445 on comparative Nitrogen counts on samples of the Nitrogen synthesis, www.santilli-foundation.org/docs/Spectral-analysis-Ref-[79].png

[122] R. Brenna, T. Kuliczkowski and L. Ying, "Report on Test for Silicon" on the Nitrogen synthesis
http://www.santilli-foundation.org/docs/PGTI-Anal-test1.pdf

[123] D. Rossiter, Director, "IVA Report 189920 on comparative Silica counts,"
www.santilli-foundation.org/docs/IVAReport 189920.pdf

[124] D. Rossiter, Director, "IVA Report 189920 on comparative Silica counts," oxygen synthesis.www.santilli-foundation.org/docs/IVAReport 189920.pdf

[125] D. Rossiter, Director, "IVA Report 200010 on comparative Nitrogen counts.
http://www.santilli-foundation.org/docs/Oneida-analyses-2013.zip

[126] D. Swartz, "Constellation Technologies first report on comparative Silica counts,"
http://www.santilli-foundation.org/docs/Constellation-Si-10-13.zip

[127] D. Swartz, "Constellation technologies second report on comparative Silica counts,"
http://www.santilli-foundation.org/docs/Constellation-Rep-Si-2.zip

[128] D. Swartz, "Constellation technologies Third report on comparative Silica counts,"
http://www.santilli-foundation.org/docs/Constell-Si-3.pdf

[129] A. Nas, "Data on Constellation technologies tests 1 and 2 on comparative Silica counts,",
http://www.santilli-foundation.org/docs/Data-Constell-tests.docx

[130] D. Swartz, "Constellation technologies Third report on comparative Silica counts,"
http://www.santilli-foundation.org/docs/Constell-Silicon-10-14.pdf

[131] Nuclear Cata center, *Table of nuclides,* website
http://rarfaxp.riken.go.jp/ gibelin/Nuchart/
http://atom.kaeri.re.kr:8080/ton/index.html

[132] A. O. E. Animalu and R. M. Santilli, "Nonlocal isotopic representation of the Cooper pair in superconductivity," Intern. J. Quantum Chemistry Vol. 29, 185 (1995)
http://www.santilli-foundation.org/docs/Santilli-26.pdf

[133] R. M. Santilli, "A structure model of the elementary charge, Hadronic J. Vol. 4, pages 770-784 (19810
http://www.santilli-foundation.org/docs/Santilli-03.pdf

[134] A. K. Aringazin, "Toroidal configuration of the orbit of the electron of the hydrogen atom under strong external magnetic fields," Hadronic J. 24, 134 (2001),
http://www.santilli-foundation.org/docs/landau.pdf

[135] R. Brenna, T. Kuliczkowski, L. Ying, "Verification of Santilli intermediate Controlled Nuclear Fusions without harmful radiations a and the production of magnecular clusters," New Advances in Physics, **5**, 9 (2011)
http://www.santilli-foundation.org/docs/ICNF-2.pdf

[136] J. V. , C. Lynch and Y. Yang, "Confirmations of Santilli Intermediate Nuclear Fusions of Deuteron and Carbon into Nitrogen without Radiations." The Open Physical Chemistry HJournal **5**, 17 2013
http://www.santilli-foundation.org/docs/ICNF-Conf-2013.pdf

[137] L. Ying, W. Cai, J. , C. Lynch, S. Marton, S. Elliot and Y. Yang, "Experimental verification for Intermediate Controlled Nuclear Fusion," City College of New York Preprint, to appear,
http://www.santilli-foundation.org/docs/ICNF-Cai-paper-Ying.pdf

[138] Leong Ying, "Verification of Santillis Intermediate Nuclear Harmful Radiation and the Production of Magnecular Clusters," Lecture VD of the website
http://www.world-lecture-series.org/level-v

[139] Leong Ying, "Verification of Santillis Intermediate Nuclear Harmful Radiation and the Production of Magnecular Clusters Leong Ying," lecture
http://www.world-lecture-series.org/level-v

[140] R. B. Lanjewar, "A Brief Review of Intermediate Controlled Nuclear Syntheses (ICNS) without Harmful Radiations," AIP Conference Proceedings 1648, 510012 (2015); doi: 10.1063/1.4912717
http://www.santilli-foundation.org/docs/1.4912717(RB-Lanjewar).pdf

[141] I. B. Das Sarma, "Hadronic Nuclear Energy: An Approach Towards Green Energy," AIP Conference Proceedings 1648, 510008 (2015); doi: 10.1063/1.4912713
http://www.santilli-foundation.org/docs/1.4912713(IB-Das Sarma).pdf

[142] U. Abundo, "Interpretation n and enhancement of the excess energy of Rossi's reactor via Santilli neutroids and nucleoids," Hadronic Journal Vol. 37, pages 697-737 (2014, pdf link 106 MB
http://www.thunder-fusion.com/docs/abundo-paper-2014.pdf

[143] R. M. Santilli, "The hypothesis of the pseudoproton," Clifford Analysis, Clifford Algebras and their Applications Vol. 3, pages 1-26, 2014 (Cambridge, UK)
http://www.santilli-foundation.org/docs/Antimatter-telescope-2013-final.pdf

[144] R. M. Santilli, "Apparent nuclear transmutations without neutron emission triggered by pseudoprotons," American Journal of Modern Physics Vol. 4, p. 15-18 (2015)
http://www.thunder-energies.com/docs/pseudoproton-2014.pdf

[145] V.-Otto de Haan, "Possibilities for the detection of Santilli's neutroids and pseudoprotons,"American Journal of Modern Physics, in press (2015)
http://www.santilli-foundation.org/docs/Pseudo-proton-deHaan.pdf

[146] R. M. Santilli, "The problem of recycling nuclear waste," website
http://www.nuclearwasterecycling.com/

[147] T. Vougiouklis, "Hypermathematics, Hv-Structures, Hypernumbers, Hypermatrices and Lie-Santilli Addmissibility," American Journal of Modern Physics, Vol. 4, No. 5, 2015, pp. 38-46.11, in the Special Issue I; *Foundations of Hadronic Mathematics* Dedicated to the 80th Birthday of Prof. R. M. Santilli
http://www.santilli-foundation.org/docs/10.11648.j.ajmp.s.2015040501.15.pdf

[148] P. A. M. Dirac, Proceedings of the Royal Society 1928,,**117**, 610624.

[149] R.M. Santilli, . *Isodual Theory of Antimatter with Applications to Antigravity, Grand Unification and Cosmology,* Springer (2006).
http://www.santilli-foundation.org/docs/santilli-79.pdf

[150] R. M. Santilli, A Classical Isodual Theory Of Antimatter And Its Prediction Of Antigravity, International Journal of Modern Physics A, 14 (1999) 2205-2238.
http://www.santilli-foundation.org/docs/Santilli-09.pdf

[151] R. M. Santilli, "Representation of antiparticles via isodual numbers, spaces and geometries, " Comm. Theor. Phys. 1994 **3**, 153-181
http://www.santilli-foundation.org/docs/Santilli-112.pdfAntigravity

[152] R. M. Santilli, "A new cosmological conception of the universe based on the isominkowskian geometry and its isodual, Part I pages 539-612 and Part II pages Contributed paper in Analysis, Geometry and Groups, A Riemann Legacy Volume, Volume II, H.M. Srivastava, Editor, pp. 539-612 (1993)

[153] R. M. Santilli, "Antigravity," Hadronic J. 1994 **17**, 257-284
http://www.santilli-foundation.org/docs/Santilli-113.pdfAntigravity

[154] P. Mills, A. "Possibilities of measuring the gravitational mass of electrons and positrons in free horizontal flight," contributed paper for the proceedings of the

In- international Conference on Antimatter, held in Sepino, Italy, May 1996, published in the Hadronic J. 1996 **19**, 77-96
http://www.santilli-foundation.org/docs/Santilli-11.pdf

[155] V. de Haan, "Proposal for the realization of Santilli comparative test on the gravity of electrons and positrons via a horizontal supercooled vacuum tube, Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes, C. Corda, Editor, Kathmandu University, 2011, pages 57-67
http://www.santilli-foundation.org/docs/deHaan-Arxiv.pdf

[156] V.de Haan, "Proposal for the realization of Santilli comparative test on the gravity of electrons and positrons via a horizontal supercooled vacuum tube, Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes, C. Corda, Editor, Kathmandu University, (2011), pp. 57-67.
http://www.santilli-foundation.org/docs/Santilli-gravity-experiment.pdf

[157] A. E. Charman et al, ALPHA Collaboration, Description and first application of a new technique to measure the gravitational mass of antihydrogen," Nature Communications, April 2013. DOI: 10.1038/ncomms2787 [35] J. Dunning-Davies, Thermodynamics of Antimatter via Santillis isodualities, Foundations of Physics Letters, 12, (1999) pp. 593-599.

[158] R. M. Santilli, "Does antimatter emit a new light?" Invited paper for the proceedings of the International Conference on Antimatter, held in Sepino, Italy, on May 1996, published in Hyperfine Interactions 1997, **109**, 63-81
http://www.santilli-foundation.org/docs/Santilli-28.pdf

[159] R. M. Santilli, "Isotopic relativity for matter and its isodual for antimatter, Gravitation 1997, **3**, 2.

[160] P. S. Muktibodh, "Introduction to Isodual Mathematics and its Application to Special Relativity," American Institute of Physics Proceedings 2013
http://www.santilli-foundation.org/docs/P-Muktibodh.pdf

[161] J. Dunning-Davies, Thermodynamics of antimatter via Santilli's isodualities. Found. Phys./ Vol. 12, page 593 (1999)
http://www.santilli-foundation.org/docs/Isodual-therm.pdf

[162] S. Beghella-Bartoli, R. M. Santilli, "Possible Role of Antimatter Galaxies for the Stability of the Universe," American Journal of Modern Physics 2016; 5(2-1): 185-190
http://www.santilli-foundation.org/docs/pdf11.pdf

[163] R. M. Santilli, "Can antimatter asteroids, stars and galaxies be detected with current means?" in *Proceedings of the Third International Conference on the Lie-Admissible Treatment of Irreversible Processes,* C. Corda, Editor, Kathmandu University (2011) pages 25-36
http://www.santilli-foundation.org/docs/Antimatter-Asteroid.pdf

[164] R. M. Santilli,"The Mystery of Detecting Antimatter Asteroids, Stars and Galaxies," American Institute of Physics, Proceed. 2012, **1479**, 1028-1032 (2012) http://www.santilli-foundation.org/docs/antimatter-asteroids.pdf

[165] R. M. Santilli, "Apparent detection of antimatter galaxies via a telescope with convex lenses," Clifford Analysis, Clifford Algebras and their Applications vol. 3, 2014, pages 1-26 (Cambridge, U.K., http://www.santilli-foundation.org/docs/Antimatter-telescope-2013-final.pdf

[166] R. M. Santilli, "Representative pictures from the Antimatter telescope," http://www.santilli-foundation.org/docs/Antimatter-Vega-system-2013

[167] P. Bhujbal, J. V. Kadeisvili, A. Nas, S Randall, and T. R. Shelke. "Preliminary confirmation of antimatter detection via Santilli telescope with concave lenses," Clifford Analysis, Clifford Algebras and their Applications Vol. 3, pages 27-39, 2014 (Cambridge, UK) http://www.santilli-foundation.org/docs/Con-Ant-Tel-2013.pdf

[168] S. Beghella-Bartoli, P. M. Bhujbal, A. Nas, "Confirmation of Santilli's Detection of Antimatter Galaxies via a Telescope with Concave Lenses," American Journal of Modern Physics, Vol. 4, pp 34-41 (2015) http://www.santilli-foundation.org/docs/antimatter-detect-2014.pdf

[169] R. M. Santilli, "Apparent Detection of a New Antimatter Galaxy in the Capella Region of the Night Sky," Clifford Analysis, Clifford Algebras and their Applications, in press (2016)]] http://www.santilli-foundation.org/docs/capella-antimatter-galaxy.pdf

[170] P. M. Bhujbal, "Santillis Detection of Antimatter Galaxies: An Introduction and Experimental Confirmation, AIP Conference Proceedings Vol. 1648 (2015) pp. 510005-1-510005-5. http://dx.doi.org/10.1063/1.4912710 http://www.santilli-foundation.org/docs/1.4912710(PM Bhujbal).pdf

[171] P. M. Bhujbal, Santillis Apparent Detection of Antimatter Galaxies: An Introduction and Experimental Confirmation, Hadronic Journal [Submitted for Publication] ISSN: 0162-5519 http://www.santilli-foundation.org/docs/bhujpal- antim-2014.pdf

[172] A. A. Bhalekar,"Studies of Santilli's isotopic, genotopic and isodual four directions of time," American Institute of Physics proceedings, 2013 http://www.santilli-foundation.org/news.html.

[173] R. Anderson, A. A. Bhalekar, B. Davvaz, P. Muktibodh, R. M. Tangde, and T. Vougiouklis, "An introduction to Santilli isodual theory of antimatter and the ensuing problem of detecting antimatter asteroids," Numta Bulletin 2012-2013, **6**, 1-33 http://www.santilli-foundation.org/docs/Antimatter-2013.pdf

[174] P. M. Bhujbal,"Santilli's isodual mathematics and and physics for antimatter," American Journal of Modern Physics Vol.5, No.20-1, pp 1610194 (2016)
http://www.santilli-foundation.org/docs/bhujbal-2016.docx

[175] S. Beghella Bartoli, "Trajectories of antimatter bodies in our solar system according to the isodual theory of antimatter," Hadronic Journal Vol. 37, pages 1-27, 2014
http://www.santilli-foundation.org/docs/Simone-FINAL.pdf

[176] R. Anderson, A. A. Bhalekar, B. Davvaz, P. Muktibodh, V. M. Tangde, and T. Vougiouklis, "An introduction to Santilli's isodual theory of antimatter and the ensuing problem of detecting antimatter asteroids," Numta Bulletin Issue 67, pages 1-33, 2013
http://www.santilli-foundation.org/docs/Antimatter-2013.pdf

[177] Vougiouklis, An Introduction to Santilli's Isodual Representation of Antimatter and the Ensuing Problem of Detecting Antimatter Asteroids, Numta Bulletin Issue 6-7, (2013). pp. 1-33. http://www.santilli-foundation.org/docs/Antimatter-2013.pdf

[178] B. Davvaz, B. M. Santilli, and T. Vougiouklis, "Studies of Multi-Valued Hyperstructures for the Characterization of Matter-Antimatter Systems and their Extension, in Proceedings of the 2011 International Conference on Lie-admissible Formulations for Irreversible Processes, C. Corda, editor, Kathmandu University, Nepal, 2011, pages 45-57,
http://www.santilli-foundation.org/Hyperstructures.pdf

[179] R. M. Santilli, "Apparent Detection via New Telescopes with Concave Lenses of Otherwise Invisible Terrestrial Entities," American Journal of Modern Physics Vol. 5, issue 3, pages 45-53, 2016
http://www.thunder-energies.com/docs/ITE-paper-12-15-15.pdf

[180] R. M. Santilli Il Grande Grido - - Ethical Problem of Einstein followers in the U.S.A: An Insider's View, (1984) and Documentation of Il Grande Grido, Vols. I, II and III (1985)m Alpha Publishing
http://www.scientificethics.org/IlGrandeGrido.htm
1985-2008 update
http://www.scientificethics.org/Santilli-ethical-decay.htm
2008-2016 update
http://www.i-b-r.org
http://thunder-energies.com/docs/la-rochelle-keynote-paper.pdf

[181] Jeremy Dunning Davies, *Exploding a myth: "Conventional Wisdom" or Scientific Truth,* Horwood Publishing Limited, 2007

[182] Karl Popper, *Quantum Theory and the Shism in Physics,* Cambridge University Press (1982).

[183] F. Zwicky, "On the Red Shift of Spectral Lines through Interstellar Space," Proceedings of the National Academy of Sciences of the United States of America, **15**, 773 (1929)
http://www.pnas.org/cgi/reprintframed/15/10/773

[184] H. Arp. *Quasars Redshift and Controversies.* Interstellar Media, Berkeley (1987).

[185] John Ross, editor, *The Harvard Crimson,* editorial "The Politics of Science" (1985)
http://www.santilli-foundation.org/docs/HarvardCrimson.pdf