ABSTRACTS for HYPERSTRUCTURES

Workshop 15

1. The hypernumbers (e-hypernumbers) in Santilli's sense

by Thomas Vougiouklis

Abstract

The largest class of hyperstructures is the one which satisfy the weak properties. These are called H_v-structures introduced in 1990 and they proved to have a lot of applications on several applied sciences. The basic question on Santilli's isotheory is 'what are the numbers?' Transferring this question into hyperstructures we ask 'what are the hyper-numbers?' We present an introduction on the special classes of hyperstructures used in the Lie-Santilli theory called e-hyperstructures focusing to the definition of the hypernumbers.

Definitions: In a set H equipped with a hyperoperation (abbr. *hope*) $\cdot: H\times H \rightarrow \mathcal{P}(H)-\{\emptyset\}$, we abbreviate by *WASS* the *weak associativity*: $(xy)z\cap x(yz)\neq\emptyset$, $\forall x,y,z\in H$ and by *COW* the *weak commutativity*: $xy\cap yx\neq\emptyset$, $\forall x,y\in H$. The hyperstructure $(H,\cdot)$ is called *H_v-semigroup* if it is *WASS*, it is called *H_v-group* if it is reproductive H_v-semigroup: $xH=Hx=H$, $\forall x\in H$. $(R,+,\cdot)$ is called *H_v-ring* if both (+) and $(\cdot)$ are *WASS*, the reproduction axiom is valid for (+) and $(\cdot)$ is *weak distributive* with respect to (+): $x(y+z)\cap(xy+xz)\neq\emptyset$, $(x+y)z\cap(xz+yz)\neq\emptyset$, $\forall x,y,z\in R$.

The main tool to study hyperstructures are the *fundamental relations* β^*, γ^* and ε^*, which are defined, in H_v-groups, H_v-rings and H_v-vector spaces, resp., as the smallest equivalences so that the quotient would be group, ring and vector space, resp. Fundamental relations are used for general definitions. Thus, an H_v-ring $(R,+,\cdot)$ is called *H_v-field* if R/γ^* is a field. Therefore, the elements of an H_v-field, are called *hyper-numbers*.

2. Notes on Hv-groups

by N. Lygeros

Abstract

Definition: (F. Marty) $\langle H,.\rangle$ is a hypergroup if (.) : $H \times H \to p(H)$ is an associative hyperoperation for which the reproduction axiom $hH = Hh = H$ is valid for any h of H.
Definition: An hyperoperation is weakly associative if for any
$x, y, z \in H, \quad x(yz) \cap (xy)z \neq \varnothing$

Definition: (Th. Vougliouklis). $\langle H,.\rangle$ is an H_v-group if (.): $H \times H \to p(H)$ is a weakly associative hyperoperation for which the reproduction axiom $hH = Hh = H$ is valid for any h of H.
Theorem: (R. Bayon, N. Lygeros) There exists, up to isomorphism, 20 H_v-groups of order 2.
Theorem: (S.C Chung, B.M. Choi): There exists up to isomorphism, 13 minimal H_v-groups of order 3 with scalar unit (see table 2)
Theorem: (R. Bayon, N. Lygeros): There exists, up to isomorphism, 292 H_v-groups of order 3 with scalar unit.
Theorem: (R. Bayon, N. Lygeros): There exists, up to isomorphism, 6494 minimal H_v-groups of order 3.
Theorem: (R. Bayon, N. Lygeros): There exists, up to isomorphism, 1026462 H_v-groups of order 3.
Theorem: (R. Bayon, N. Lygeros): There exists, up to isomorphism, 631 609 H_v-groups of order 4 with scalar unit.

Number of abelian H_v-groups of order 4 with scalar unit in respect with their automorphism group

|Aut(Hv)|	1	2	3	4	6	8	12	24
	_	_	_	32	_	46	5510	626021

Idea (N. Lygeros) 4| |Aut(e-H_v4)|
Proposition (R. Bayon, N. Lygeros) Let (R,+,.) be an hyperring then Aut(R)=Aut(+)UAut(.)
Corollary (R. Bayon, N. Lygeros) Let (R,+,.) be an hyperring then |Aut(R)|>=max(|Aut(+)|,Aut(.)|)
Theorem (R. Bayon, N. Lygeros) There are 63 isomorphism classes of hyperrings of order 2.
Theorem (R. Bayon, N. Lygeros). There are 875 isomorphism classes of H_v-rings of order 2.
Theorem (R. Bayon, N. Lygeros). There are 33277642 isomorphism classes of hyperring of order 3

Workshop 144

1. Geno-hyper-fields and geno-hyper-numbers for a matter and antimatter model

by B. Davvaz, P. Nikolaidou, R.M. Santilli, T. Vougiouklis

Abstract

The Lie-admissible mathematics, known as Santilli genomathematics, are used for the description of matter and antimatter situation. The hyperstructure theory is offered for more flexible and complicate models. Using the largest class of hyperstructures which are called H_v-structures we present hyperfields, more precisely H_v-fields, which also can be used in the hypermatric representation theory.

Basic definitions, on the H_v-structure theory, are the following: In the hyperstructure $(H,\cdot)$ the hyperoperation $(\cdot)$ is called *weak associative* if $(xy)z \cap x(yz) \neq \emptyset$, $\forall x,y,z \in H$ and *weak commutative* if $xy \cap yx \neq \emptyset$, $\forall x,y \in H$. The hyperstructure $(H,\cdot)$ is called *H_v-semigroup* if it is *WASS*, it is called *H_v-group* if it is reproductive H_v-semigroup: $xH=Hx=H$, $\forall x \in H$. $(R,+,\cdot)$ is called *H_v-ring* if both $(+)$ and $(\cdot)$ are *WASS*, the reproduction axiom is valid for $(+)$ and $(\cdot)$ is *weak distributive* with respect to $(+)$: $x(y+z) \cap (xy+xz) \neq \emptyset$, $(x+y)z \cap (xz+yz) \neq \emptyset$, $\forall x,y,z \in R$. The *fundamental relations* β^*, γ^* and ε^*, which are defined, in H_v-groups, H_v-rings and H_v-vector spaces, resp., as the smallest equivalences so that the quotient would be group, ring and vector space, resp. An H_v-ring $(R,+,\cdot)$ is called *H_v-field* if R/γ^* is a field. Therefore, the elements of an H_v-field, are called *hyper-numbers*. The above hypernumbers are used to describe the geno-theory.

References (basic)

[1] B.Davvaz, R.M.Santilli, T.Vougiouklis, *Studies of multi-valued hyper-structures for characterization of matter and antimatter systems,* J.Comp.Methods Sci.Eng.13, (2013),37-50

[2] R.M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry,* Volumes I, II, III, IV and V, Int. Academic Press, USA, 2008.

[3] R.M. Santilli, T. Vougiouklis, *Isotopies, Genotopies, Hyperstructures and their Applications,* Proc. New Frontiers in Hyperstructures and Related Algebras, Hadronic Press (1996), 177-188.

[4] T.Vougiouklis, *Hyperstructures and their Representations,* Monographs Math., Hadronic, 1994.

2. From Hv-groups to e-hyperstructures and e-hyperfields

by N. Lygeros, Th. Vougiouklis

Abstract

(a). Fundamental Definitions

Definition Santilli-Vougiouklis: A hyperstructure (H,·) which contain a unique scalar unit e, is called e-hyperstructure.

Definition Santilli-Vougiouklis: A hyperstructure (F,+,·), where (+) is an operation and (·) is a hyperoperation, is called *e-hyperfield* if the following axioms are valid:

1. (F,+) is an abelian group with the additive unit 0,
2. (·) is WASS,
3. (·) is weak distributive with respect to (+),
4. 0 is absorbing element: $0 \cdot x = x \cdot 0 = 0, \forall x \in F$,
5. there exists a multiplicative scalar unit 1, i.e. $1 \cdot x = x \cdot 1 = x, \forall x \in F$,
6. for every $x \in F$ there exists a unique inverse x^{-1}, such that $1 \in x \cdot x^{-1} \cap x^{-1} \cdot x$.

Definition Santilli-Vougiouklis: The Main e-Construction. Given a group (G,·), where e is the unit, then we define in G, a large number of hyperoperations (⊗) as follows:

$$x \otimes y = \{xy, g_1, g_2, \ldots\}, \forall x,y \in G-\{e\}, \text{ and } g_1, g_2, \ldots \in G-\{e\}$$

$g_1, g_2, \ldots$ are not necessarily the same for each pair (x,y). Then (G,⊗) becomes an H_v-group, in fact is H_b-group which contains the (G,·). The H_v-group (G,⊗) is an e-hypergroup. Moreover, if for each x,y such that xy=e, so we have $x \otimes y = xy$, then (G,⊗) becomes a strong e-hypergroup.

(b) Enumeration Theorems

Theorem Chung-Choi: There exists up to isomorphism, 13 minimal Hv-groups of order 3 with scalar unit.

Theorem Bayon-Lygeros: There exists, up to isomorphism, 292 H_v-groups of order 3 with scalar unit.

Theorem Bayon-Lygeros: There exists, up to isomorphism, 6494 minimal H_v-groups of order 3.

Theorem Bayon-Lygeros: There exists up to isomorphism 1026462 H_v-groups of order 3

Theorem Bayon-Lygeros: There exists, up to isomorphism, 631 609 H-$g_v$roups of order 4 with scalar unit.

Theorem Bayon-Lygeros: There are 63 isomorphism classes of hyperrings of order 2.

Theorem Bayon-Lygeros: There are 875 isomorphism classes of H_v-rings of order 2.

Theorem Bayon-Lygeros: There are 33277642 isomorphism classes of hyperring of order 3.

References

[1] R. Bayon, N. Lygeros, *Advanced results in enumeration of hyperstructures,* J. Algebra, 320 (2008), 821-835.

[2] P. Corsini, V.Leoreanu, *Applications of Hypergroup Theory,* Kluwer Academic Publ., 2003.

[3] B. Davvaz, *A brief survey of the theory of H_v-structures,* 8^{th} AHA Congress, Spanidis Press (2003), 39-70.

[4] F. Marty, *Sur une généralisation de la notion de groupe,* 8éme Congrés Math. Scandinaves, Stockholm, (1934), 45-49.

[5] R.M. Santilli, T. Vougiouklis, *Isotopies, Genotopies, Hyperstructures and their Applications,* Proc. New Frontiers in Hyperstructures and Related Algebras, Hadronic Press (1996), 177-188.

Workshop 145

1. The Lie-Santilli admissible hyperalgebras of type A_n

by Pipina Nikolaidou, Thomas Vougiouklis

Abstract

The representation theory by matrices gives to researchers a flexible tool to see and handle algebraic structures. This is the reason to see Lie-Santilli's admissibility using matrices or hypermatrices to study the multivalued (hyper) case. Using the well known P-hyperoperations we extend the Lie-Santilli's admissibility into the hyperstructure case. We present the problem and we give the basic definitions on the topic which cover the four following cases:

Suppose R, S be sets of square matrices (or hypermatrices). We can define the hyper-Lie bracket in one of the following ways:

1. $[x,y]_R = xRy - yx$ 2. $[x,y]_S = xy - ySx$
3. $[x,y]_{RR} = xRy - yRx$ 4. $[x,y]_{RS} = xRy - ySx$ (general case)

When the conditions, for all square matrices (or hypermatrices) x, y, x,

$$[x,x]_{RS} \ni 0, \quad [x,[y,z]_{RS}]_{RS} + [y,[z,x]_{RS}]_{RS} + [z,[x,y]_{RS}]_{RS} \ni 0$$

of a hyper-Lie algebra are satisfied?

We apply this generalization on the Lie algebras of the type A_n.

2. An extension of Santilli's admissibility of hyper-Lie algebras on non-square matrices

by Thomas Vougiouklis

Abstract

The largest class of hyperstructures is the one which satisfy the weak properties. These are called H_v-structures introduced in 1990 and they proved to have a lot of applications on several applied sciences such as linguistics, biology, chemistry, physics, and so on. The H_v-structures can be used as models mainly as an organized devise. In this paper we introduce and present a hyperproduct on non square matrices by using a generalization of the well known P-hyperoperations. We apply this on the special classes of hyperstructures used in the Lie-Santilli theory called e-hyperstructures. We connect this theory with the corresponding classical algebra, mainly for the Representation Theory by (hyper) matrices, through the fundamental relations defined on the hyperstructures.

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Key words: hyperstructures, H_v-structures, hopes, ∂-hopes, e-hyperstructures.

References (basic)

[1] B. Davvaz, R.M. Santilli, T. Vougiouklis, *Multi-valued hypermathematics for the characterization of matter-antimatter systems and their extension,* Algebras, Groups and Geometries 28(1), (2011), 105-116.

[2] B. Davvaz, R.M. Santilli, T. Vougiouklis, *Studies of multi-valued hyper-structures for characterization of matter and antimatter systems,* J. Computational Methods in Sciences and Engineering 13, (2013), 37-50.

[3] V. Kac, *Infinite Dimensional Lie Algebras*, Birkhauser, 1983.

[4] R.M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry,* Volumes I, II, III, IV and V, Int. Academic Press, USA, 2008.

[5] R.M. Santilli, T. Vougiouklis, *Isotopies, Genotopies, Hyperstructures and their Applications,* Proc. New Frontiers in Hyperstructures and Related Algebras, Hadronic Press (1996), 177-188.

[6] T. Vougiouklis, *Hyperstructures and their Representations,* Monographs in Mathematics, Hadronic Press, 1994.

[7] T. Vougiouklis, *∂-operations and H_v-fields,* Acta Math. Sinica, (Engl. Ser.), V.24, N.7 (2008), 1067-1078.