Geno-hyper-fields and geno-hyper-numbers for a matter and antimatter model

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In this presentation we continue the results mainly of the papers:

R.M. Santilli, T. Vougiouklis, *Isotopies, Genotopies, Hyperstructures and their Applications*, New Frontiers Hyperstructures and Related Algebras, Hadronic (1996), 177-188.

B. Davvaz, R.M. Santilli, T. Vougiouklis, *Multi-valued hypermathematics for the characterization of matter-antimatter systems and their extension*, Algebras, Groups and Geometries 28(1), (2011), 105-116.

B. Davvaz, R.M. Santilli, T. Vougiouklis, *Studies of multi-valued hyper-structures for characterization of matter and antimatter systems*, J. Computational Methods in Sciences and Engineering 13, (2013), 37-50.

1. Introduction

All branches of Mathematics are used as models to express problems appeared in nature and life. Sometimes these models are used as an organized device. The research interests to specify the appropriate topic of mathematics to represent the problem. This fact leads to the collaboration of researchers from mathematics and applied sciences. A new branch of mathematics is the algebraic hyperstructures where in classical algebraic structures the multivalued replaces the single valued operations. Applications appeared already as in Hadronic mechanics, Biology, Conchology, Linguistics, Urban problems.

The hyperstructures are used in Hadronic Mechanics, more precisely on the characterization of matter-antimatter systems. We present some topics of hyperstructures related as: H_v-structures including the e-hyperstructures (e-hypernumbers, e-hypervector spaces), P-hyperoperations (including a P-like hyperoperation). The theory of Santilli's iso-mathematics and geno-mathematics are also presented and a new realization is used to represent a part of the Santilli's theory on matter and anti-matter.

As it is well known, antimatter was solely treated in the 20th century via charge conjugation on a Hilbert space where there exists the coordinate of the representation space, such as the Minkowski space-time.

The above approach caused a historical imbalance between matter and antimatter, because matter was treated at all known levels, from Newtonian mechanics to second quantization, while antimatter was solely treated at the level of second quantization. The resolution of this imbalance required the construction of a new mathematics, called Santilli isodual mathematics, which is constructed via an anti-Hermitean conjugation. The resulting isodual theory of antimatter has established a complete democracy in the treatment of matter and antimatter with intriguing implications, such as the prediction of gravitational repulsion (antigravity) for matter in the field of antimatter and vice-versa. Despite their simplicity, the physical and mathematical differences between charge and isodual conjugations are nontrivial. Under charge conjugation, antimatter is assumed to exist in the same spacetime of matter. The isodual conjugation maps, for consistency, each quantity used in the representation of matter into its isodual image, thus including a necessary conjugation of spacetime with coordinates into the novel isodual spacetime with isodual coordinates. This conjugation implies that, under isoduality, antimatter exists in a new spacetime which is physically distinct from yet coexisting with our spacetime.

From a mathematical viewpoint, the co-existence of the conventional and isodual spacetimes in the same region of space creates a number of intriguing problems. It is rather natural to see the matter and antimatter via multi-dimensional models. This mathematical formulation is easily seen as being unacceptable because our sensory perception deny the existence of spacetime bigger than those with four dimensions. The compatibility of the complexities of nature with our sensory perception has motivated the construction of multi-valued hyperstructures with hyperunits. In its most elementary possible formulation expressed via conventional operations, matter and antimatter can be represented via a two-valued hyperstructure characterized by the multiplicative hyperunit.

The resolution of the 20^{th} century imbalance between matter-antimatter via the isodual theory appears to produce physical evidence for the realization in nature of multi-valued hyperstructures with hyperunits, and therefore suggesting the mathematical study presented below.

The Lie-Santilli and Jordan-Santilli isoalgebras verify the attached products

$$[A,B]^*=ATB-BTA \quad and \quad \{A,B\}^*=AWB+BWA$$

respectively. This was introduced by Santilli in 1978 and it is the most general realization of products that are jointly Lie-admissible and Jordan-admissible

$$(A,B) = ARB–BSA = (ATB–BTA)+\{AWB+BWA\} =$$

$$[A,B]^*+\{A,B\}^* = (ATH–HTA)+\{AWH+HWA\}, R=T+W, S=W-T. \quad (1.1)$$

On the other side the Santilli's genomathematics theory try to express more complicate structures such as DNA turns into multivalued case therefore it lead to the equations

$$\hat{\imath}^{>} = \{1^{>}_{1}, 1^{>}_{2}, 1^{>}_{3},\ldots, 1^{>}_{n}\} = \{1/S_1, S_2, S_3 ,\ldots, S_n\}, \quad (2.1)$$

$$A>^{\wedge}B = AS_1B + AS_2B + AS_3B + \ldots + AS_nB, \quad (2.2)$$

$$^{<}\hat{\imath} = \{^{<}1_1, {}^{<}1_2, {}^{<}1_3,\ldots, {}^{<}1_n\} = \{1/S_1, S_2, S_3 ,\ldots, S_n\}, \quad (2.3)$$

$$A^{\wedge}< B = AR_1B + AR_2B + AR_3B + \ldots + AR_nB, \quad (2.4)$$

We can see all the above equations in the hyperstructure realization. So we have to present the related theory and the range that this theory offers for the realization or the representation of the iso-geno-theory.

2. The hyperstructures

We deal with *H_v-structures* introduced in 1990, which satisfy the *weak axioms* where the non-empty intersection replaces the equality. In a set H equipped with a hyperoperation (abbr. *hyperoperation*=***hope***)

$$\cdot : H\times H \to \mathcal{P}(H)-\{\emptyset\} : (x,y) \to x\cdot y \subset H$$

abbreviate by

WASS: *weak associativity*: $(xy)z\cap x(yz)\neq\emptyset,\ \forall x,y,z\in H$ and by

COW: *weak commutativity*: $xy\cap yx \neq \emptyset,\ \forall x,y\in H$.

The hyperstructure $(H,\cdot)$ is called ***H_v-semigroup*** if it is *WASS*, it is called ***H_v-group*** if it is reproductive H_v-semigroup, i.e.

$$xH=Hx=H,\ \forall x\in H.$$

The hyperstructure $(R,+,\cdot)$ is called ***H_v-ring*** if both $(+)$ and $(\cdot)$ are *WASS*, the reproduction axiom is valid for $(+)$ and $(\cdot)$ is ***weak distributive*** with respect to $(+)$:

$$x(y+z)\cap(xy+xz) \neq \varnothing, \quad (x+y)z\cap(xz+yz) \neq \varnothing, \quad \forall x,y,z\in R.$$

Let $(H,\cdot)$, $(H,*)$ be H_v-semigroups defined on the same set H. $(\cdot)$ is called ***smaller*** than $(*)$, and $(*)$ ***greater*** than $(\cdot)$, iff there exists an

$$f\in Aut(H,*) \text{ such that } xy\subset f(x*y), \forall x,y\in H.$$

Then we write $\cdot\leq *$ and we say that $(H,*)$ ***contains*** $(H,\cdot)$. If $(H,\cdot)$ is a structure then it is called ***basic structure*** and $(H,*)$ is called ***H_b-structure***.

Theorem *(The Little Theorem).* Greater hopes than the ones which are *WASS* or *COW*, are also *WASS* or *COW*, respectively.

So we have posets on H_v-structures.

In several books and papers one can find numerous applications. An new application is to replace in questionnaires the scale of Likert by the bar of Vougiouklis & Vougiouklis.

3. Two classes of hopes

Definition. Let $(G,\cdot)$ be a groupoid, then for every $\emptyset \neq P \subset G$, define the following hopes called ***P-hopes***: for all $x,y \in G$

$$\underline{P}: x\underline{P}y = (xP)y \cup x(Py),$$

$$\underline{P}_r: x\underline{P}_r y = (xy)P \cup x(yP),$$

$$\underline{P}_l: x\underline{P}_l y = (Px)y \cup P(xy).$$

The $(G,\underline{P})$, $(G,\underline{P}_r)$, $(G,\underline{P}_l)$ are called ***P-hyperstructures***. The most usual case is if $(G,\cdot)$ is semigroup, then

$$x\underline{P}y = (xP)y \cup x(Py) = xPy$$

and $(G,\underline{P})$ is a semihypergroup but we do not know about $(G,\underline{P}_r)$ and $(G,\underline{P}_l)$. In some cases, depending on the choice of P, the $(G,\underline{P}_r)$ and $(G,\underline{P}_l)$ can be associative or WASS. If in the set G, more operations are defined then obviously for each one operation several P-hopes can be defined.

Definitions. Let H a set equipped with n operations (or hopes) $\otimes_1,\ldots,\otimes_n$ and a map (or multivalued map)

$$f: H\to H \text{ (or } f:H\to \mathcal{P}(H)-\{\varnothing\}\text{, respectively)},$$

then n hopes $\partial_1,\partial_2,\ldots,\partial_n$ on H can be defined, called ***theta-hopes*** and we write ***∂-hope,*** by putting

$$x\partial_i y = \{f(x)\otimes_i y,\ x\otimes_i f(y)\},\ \forall x,y\in H,\ i\in\{1,2,\ldots,n\}$$

in case where $\otimes_i$ are hopes or f is multivalued, we have

$$x\partial_i y = (f(x)\otimes_i y)\cup(x\otimes_i f(y)),\ \forall x,y\in H,\ i\in\{1,2,\ldots,n\}.$$

If $\otimes_i$ is associative then ∂_i is WASS.

Let $(G,\cdot)$ groupoid and $f_i:G\to G$, $i\in I$, take $f_\cup:G\to \boldsymbol{P}(G)$: $f_\cup(x)=\{f_i(x) \mid i\in I\}$. We call *union ∂-hopes*, on G if we consider the $f_\cup(x)$. A special case is the union of f with the identity, i.e. $\underline{f} = f\cup(id)$, so $\underline{f}(x)=\{x,f(x)\}$, $\forall x\in G$, which is called ***b-∂-hope***. We denote the b-∂-hope by $(\underline{\partial})$, so

$$x\underline{\partial}y = \{ xy,\ f(x)\cdot y,\ x\cdot f(y) \},\ \forall x,y\in G.$$

Examples ► Polynomials $g_i(x)=a_ix+b_i$ have $g_1\partial g_2=\{a_1a_2x+a_1b_2, a_1a_2x+b_1a_2\}$, so it is a hope. All polynomials x+c, where c be a constant, are units.

► *The constant map*. $(G,\cdot)$ group and $f(x)=a$, then $x\partial y=\{ay,xa\}$, $\forall x,y\in G$. If $f(x)=e$, then $x\partial y=\{x,y\}$, the smallest incidence hope.

Properties. If $(G,\cdot)$ semigroup: $\forall f$, the ∂-hope is *WASS*. $\forall f$, the b-∂-hope $(\underline{\partial})$ is *WASS*. If f is projection and homomorphism, then (∂) is associative.

If $(\cdot)$ is reproductive then (∂) is also reproductive:

$$x\partial G=\cup_{g\in G}\{f(x)\cdot g,\ x\cdot f(g)\}=G, \qquad G\partial x=\cup_{g\in G}\{f(g)\cdot x,\ g\cdot f(x)\}=G.$$

If $(\cdot)$ is commutative then (∂) is commutative. If f is into the centre of G, then (∂) is commutative. If $(\cdot)$ is *COW* then, (∂) is *COW*. u is right unit if $f(u)=e$, e unit in $(G,\cdot)$. The elements of the kernel, are the units of (G,∂).

Proposition. Let $(G,\cdot)$ group then, for all $f: G\to G$, the (G,∂) is an H_v-group.

Hopes on any type of matrices can be defined.

4. The H_v-rings, the H_v-fields and representations

The main tool to study hyperstructures are the ***fundamental relations*** β^*, γ^* and ε^*, which are defined, in H_v-groups, H_v-rings and H_v-vector spaces, resp., as the smallest equivalences so that the quotient would be group, ring and vector space, resp. The relation β^* was introduced by M. Koskas in 1970. The relations γ^* and ε^*, were introduced by T. Vougiouklis and he named them Fundamental.

Theorem. Let $(H,\cdot)$ be an H_v-group and denote by $\boldsymbol{U}$ the set of all finite products of elements of H. We define the relation β in H by setting $x\beta y$ iff $\{x,y\}\subset \boldsymbol{u}$ where $\boldsymbol{u}\in \boldsymbol{U}$. Then β^* is the transitive closure of β.

An element is called *single* if its fundamental class is singleton.

Analogous theorems for γ^* in H_v-rings, ε^* in H_v-modules and H_v-vector spaces, are also proved.

Definition. Let $(R,+,\cdot)$ a ring and $f:R\to R$, $g:R\to R$ two maps. We define two hopes (∂_+) and $(\partial\cdot)$, called both *theta-hopes*, on R as follows

$$x\partial_+y=\{f(x)+y,x+f(y)\},\quad x\partial\cdot y=\{g(x)\cdot y,x\cdot g(y)\},\ \forall x,y\in G.$$

Propositions

1. Let $(R,+,\cdot)$ a ring and $f:R\to R$, $g:R\to R$ maps, then $(R,\underline{\partial_+},\partial\cdot)$, is H_v-ring.
2. In $(\mathbf{Z},+,\cdot)$ and $n\neq 0$ a natural. Take f with $f(n)=0$, $f(x)=x$, $\forall x\in\mathbf{Z}-\{n\}$. Then $(\mathbf{Z},\partial_+,\partial\cdot)$ is H_v-ring, moreover, $(\mathbf{Z},\partial_+,\partial\cdot)/\gamma^* \cong \mathbf{Z}_n$.

Fundamental relations are used for general definitions. Thus we have:

Definition. An H_v-ring $(R,+,\cdot)$ is called ***H_v-field*** if R/γ^* is a field. The elements of a hyperfield are called ***hypernumbers***.

In the above Proposition (2) we remark that in the special case for $n=p$, prime, then $(\mathbf{Z},\partial_+,\partial\cdot)$ is an H_v-field.

Using again the fundamental relations we may obtain more general hyperstructures. The H_v-semigroup $(H,\cdot)$ is called *h/v-group* if H/β^* is a group. The class of h/v-groups is more general than the class of H_v-groups.

Definitions. An H_v-field is called *additive* if its addition is a hope and the multiplication is an ordinary, single valued, operation. An H_v-field is called *multiplicative* if its multiplication is a hope and addition is an operation.

H_v-structures are used in Representation (abbreviated by ***rep***) Theory. Reps of H_v-groups can be considered by generalized permutations or by H_v-matrices. Reps by generalized permutations can be achieved using translations.

Definitions. ***H_v-matrix*** is called a matrix with entries elements of an H_v-ring or H_v-field. The hyperproduct of two H_v-matrices (a_{ij}) and (b_{ij}), of type $m\times n$ and $n\times r$ respectively, is defined, in the usual manner, and it is a set of $m\times r$ H_v-matrices. The sum of products of elements of the H_v-ring is the union of the sets obtained with all possible parentheses put on them, i.e. the *n-ary circle hope* on the hyperaddition. The hyperproduct of H_v-matrices does not necessarily satisfy *WASS*.

The problem of the H_v-matrix representations is the following:

Let $(H,\cdot)$ be H_v-group. Find an H_v-ring $(R,+,\cdot)$, a set $M_R=\{(a_{ij}) \mid a_{ij}\in R\}$ and a map $T: H\to M_R$: $h\mapsto T(h)$ with $T(h_1h_2)\cap T(h_1)T(h_2)\neq\emptyset$, $\forall h_1,h_2\in H$. T is an *H_v-matrix rep.* If $T(h_1h_2)\subset T(h_1)(h_2)$, $\forall h_1,h_2\in H$, then T is an *inclusion rep. If $T(h_1h_2)=T(h_1)(h_2)=\{T(h) / h\in h_1h_2\}$, $\forall h_1,h_2\in H$, then T is a* good rep. *If T is one to one and good then it is a* faithful rep.

The reps problem can be simplified in special cases:

(a) The H_v-matrices are over H_v-rings with 0 and 1 and if these are scalars.

(b) The H_v-matrices are over *very thin* H_v-rings.

(c) The H_v-rings contains singles, then these act as absorbings.

Theorem. A necessary condition to have an inclusion rep T of an H_v-group $(H,\cdot)$ by $n\times n$ H_v-matrices over the H_v-ring $(R,+,\cdot)$ is the following:

For all classes $\beta^*(x)$, $x\in H$ there must exist elements $a_{ij}\in H$, $i,j\in\{1,\dots,n\}$ such that

$$T(\beta^*(a)) \subset \{ A = (a'_{ij}) \mid a'_{ij} \in \gamma^*(a_{ij}),\ i,j\in\{1,\dots,n\} \}$$

5. The e-hyperstructures

The *iso-hyper-fields* needed in Lie-Santilli's theory on *isotopies* for the hyperstructures were introduced by Santilli & Vougiouklis in 1996 and they are called *e-hyperfields*.

Definition. A hyperstructure $(H,\cdot)$ which contain a unique scalar unit e, is called *e-hyperstructure*. In an e-hyperstructure, $\forall x$, there exists an inverse element x^{-1}, i.e. $e \in x \cdot x^{-1} \cap x^{-1} \cdot x$. The inverses are not necessarily unique.

Definition. A hyperstructure $(F,+,\cdot)$, where $(+)$ is an operation and $(\cdot)$ is a hope, is called *e-hyperfield* if the following axioms are valid:

1. $(F,+)$ is an abelian group with the additive zero 0,
2. $(\cdot)$ is WASS,
3. $(\cdot)$ is weak distributive with respect to $(+)$,
4. 0 is absorbing element: $0 \cdot x = x \cdot 0 = 0, \forall x \in F$,
5. there exists a scalar unit 1, i.e. $1 \cdot x = x \cdot 1 = x, \forall x \in F$,
6. $\forall x \in F$ there exists a unique inverse x^{-1}, such that $1 \in x \cdot x^{-1} \cap x^{-1} \cdot x$.

The elements of an e-hyperfield are called *e-hypernumbers*. In the case that the relation: $1=x\cdot x^{-1}=x^{-1}\cdot x$, is valid, then we have a *strong e-hyperfield*.

Definition. *The Main e-Construction*. Given a group $(G,\cdot)$, where e is the unit, then we define in G, a large number of hopes ($\otimes$) as follows:

$$x\otimes y = \{xy, g_1, g_2,\ldots\},\ \forall x,y\in G-\{e\},\text{ and } g_1, g_2,\ldots\in G-\{e\}$$

Then $(G,\otimes)$ becomes an H_v-group, in fact is H_b-group containing $(G,\cdot)$. The $(G,\otimes)$ is e-hypergroup. If xy=e, $\forall x,y$, then $(G,\otimes)$ is strong e-hypergroup.

A P-hope which is appropriate to obtain e-hyperstructures is:

Construction. Let $(G,\cdot)$ be an abelian group and P any subset of G with more than one elements. We define the hyperoperation $\times_P$ as follows:

$$x\times_P y = \begin{cases} x\cdot P\cdot y = \{x\cdot h\cdot y \mid h\in P\} & \text{if } x\neq e \text{ and } c\neq e \\ x\cdot y & \text{if } x=e \text{ or } y=e \end{cases}$$

we call this hope *P_e-hope*. $(G,\times_P)$ is an abelian H_v-group.

6. H_v-Lie algebras

Definitions. Let $(F,+,\cdot)$ be an H_v-field, $(\boldsymbol{V},+)$ be a COW H_v-group and there exists an external hope

$$\cdot : F\times \boldsymbol{V} \to \mathcal{P}(\boldsymbol{V})-\{\varnothing\} : (a,x) \to ax$$

such that, for all a,b in F and x,y in $\boldsymbol{V}$ we have

$$a(x+y)\cap(ax+ay) \neq \varnothing, \quad (a+b)x\cap(ax+bx) \neq \varnothing, \quad (ab)x\cap a(bx) \neq \varnothing,$$

then $\boldsymbol{V}$ is called an *H_v-vector space over F*. In the case of an H_v-ring instead of an H_v-field then the *H_v-modulo* is defined. The fundamental relation ε^* is the smallest equivalence relation such that the quotient $\boldsymbol{V}/\varepsilon^*$ is a vector space over the fundamental field F/γ^*.

The general definition of an H_v-Lie algebra over F is the following:

Definition. Let $(\boldsymbol{L},+)$ be an H_v-vector space over the H_v-field $(F,+,\cdot)$, take the canonical map $\varphi: F \rightarrow F/\gamma^*$ with $\omega_F=\{x\in F:\varphi(x)=0\}$, 0 is the zero of F/γ^*, ω_L the core of $\varphi':\boldsymbol{L}\rightarrow\boldsymbol{L}/\varepsilon^*$ and denote again 0 the zero of L/ε^*. Consider the *bracket (commutator)* hope:

$$[\ ,\ ] : \boldsymbol{L}\times\boldsymbol{L} \rightarrow \mathcal{P}(\boldsymbol{L})-\{\varnothing\}:\ (x,y) \rightarrow [x,y]$$

then $\boldsymbol{L}$ is an H_v-Lie algebra over F if the following axioms are satisfied:

(L1) The bracket hope is bilinear, i.e.

$[\lambda_1 x_1+ \lambda_2 x_2,y]\cap(\lambda_1[x_1,y] + \lambda_2[x_2,y]) \neq \varnothing$

$[x, \lambda_1 y_1+\lambda_2 y]\cap(\lambda_1[x,y_1]+\lambda_2[x,y_2])\neq\varnothing$, $\forall x,x_1,x_2,y,y_1,y_2\in\boldsymbol{L}$ and $\lambda_1,\lambda_2\in F$

(L2) $[x,x] \cap \omega_L \neq \varnothing$, $\forall x\in\boldsymbol{L}$

(L3) $([x,[y,z]]+[y,[z,x]]+[z,[x,y]]) \cap \omega_L \neq \varnothing$, $\forall x,y\in\boldsymbol{L}$

7. Santilli's hyper-admissibility

The Lie-Santilli admissibility on square matrices can be faced in two ways:

(a) using ordinary numbers and hopes for square matrices,

(b) using hypernumbers as entries and ordinary operations.

Definition. Let $\boldsymbol{L}=(\boldsymbol{M}_{n\times n},+)$ be an H_v-vector space of square hyper-matrices over $(\boldsymbol{F},+,\cdot)$, $\varphi:\boldsymbol{F}\to\boldsymbol{F}/\gamma^*$, the canonical map and $\omega_F=\{x\in\boldsymbol{F}:\varphi(x)=0\}$, where 0 is the zero of the fundamental field $\boldsymbol{F}/\gamma^*$. Similarly, let ω_L be the core of the canonical map $\varphi':\boldsymbol{L}\to\boldsymbol{L}/\varepsilon^*$ and denote by 0 the zero of $\boldsymbol{L}/\varepsilon^*$. $\forall \boldsymbol{R},\boldsymbol{S}\subseteq\boldsymbol{L}$ a *Santilli's Lie-admissible hyperalgebra* is obtained using the Lie bracket:

$$[\ ,\ ]_{\mathbf{RS}}:\ \ \boldsymbol{L}\times\boldsymbol{L}\to\boldsymbol{P}(\boldsymbol{L}):\ [x,y]_{\mathbf{RS}} = x\boldsymbol{R}y - y\boldsymbol{S}x = \{\, xry-ysx \mid r\in\boldsymbol{R} \text{ and } s\in\boldsymbol{S}\,\}$$

Special cases, but not degenerate, are the 'small' and 'strict' ones:

(a) $\boldsymbol{R}=\{e\}$ then $[x,y]_{\mathbf{RS}} = xy-ySx = \{xy-ysx \mid s\in\boldsymbol{S}\}$

(b) $\boldsymbol{S}=\{e\}$ then $[x,y]_{\mathbf{RS}} = xRy-yx = \{xry-yx \mid r\in\boldsymbol{R}\}$

(c) $\boldsymbol{R}=\boldsymbol{S}$

Now we transfer the Santilli's admissibility problem from the introduction: The Santilli's admissibility can be achieved in the following ways:

- ► The use of an H_v-field instead of an ordinary field.
- ► The replacement, or enlargement, of the single valued external or internal operations on vectors by multivalued ones.
- ► The replacement of the selected elements R and S by sets of elements.

(i) Therefore in Eq.(1.1),

- ► we can use elements λ from an H_v-field or the external operation λA be a hope or, of course, both can be used.
- ► we can use elements of an H_v-field or replace R and S (consequently T and W) by sets of elements. In the later case, we can use the P_e-hopes.

(ii) The Eqs.(1.2) are already a generalization into the multivalued case. In fact this generalization is an H_b-hope since it is an extension of single valued operation. However, if we replace $S_1,\ldots, S_n$ and $R_1,\ldots, R_n$ by sets of elements then we obtain enlarged hopes.

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