A Conceivable Lattice Structure of the Coulomb Law.

E. PAPP (*) and R. M. SANTILLI (**)

The Institute for Basic Research - 96 Prescott Street, Cambridge, Massachusetts 02138

(ricevuto il 19 Maggio 1983)

PACS. 03.65. – Quantum theory; quantum mechanics.

Summary. – We present a few heuristic remarks on recent extensions of the Coulomb law via effective potentials and other means, which appear to admit a lattice structure in time and space whose spacings are given by the characteristic period of the electron and its Compton wave-length, respectively.

As is well known, since the original contributions of the period 1971-1975 [1], the formulation of quantum field theory on a lattice has developed into one of the most promising contemporary treatments of strong interactions [2]. Significantly, the foundations of the theory rest on the consistency (as well as aesthetic beauty) of the lattice formulation of the electromagnetic interactions. In this note we shall indicate the possibility of further extensions of the foundations of the theory, via a coinceivable lattice formulation of the Coulomb law. To prevent excessive expectations, it should be indicated from outset that we shall limit ourselves to a few heuristic comments specifically intended for the electromagnetic interactions, whose possible extension to strong interactions remains to be worked out.

To begin with a brief historical perspective, let us recall that the celebrated Coulomb law

$$F_{\text{Coul}}(r) = \pm e^2/r^2 \tag{1}$$

was proposed by C. AUGUSTIN DE COULOMB in 1785 for the treatment of macroscopic charges $\pm e$. In 1897, J. J. THOMPSON extended the application of the law to the static interactions of elementary charges $\pm e$. As a result of authoritative endorsements, such as that by E. RUTHERFORD, the Coulomb law became subsequently established for the elementary charges according to exactly the same structure (1) conceived for macroscopic charges some two centuries ago.

(*) Permanent address: D-3570 Stadt Allendorf, Von-Brentano-Str. 1, West Germany.

(**) Supported by the U.S. Department of Energy under contract no. DE-AC02-80ER10651.A002.

(1) F. WEGNER: *J. Math. Phys. (N.Y.)*, **12**, 2259 (1971); K. G. WILSON: *Phys. Rev. D*, **14**, 2455 (1974) and A. M. POLYAKOV: *Phys. Lett. B*, **59**, 82 (1975).

(2) See, for instance, the recent review by J. B. KOGUT: University of Illinois at Urbana-Champaign report no. ILL-(TH)-82-42 (December 1982).

Nevertheless, the development of quantum mechanics and of quantum field theory pointed out quite naturally the need for refinements of the Coulomb law for particle interactions. In fact, we knew since 1935 [3] that quantum effects may produce a modification of the Coulomb potential. Fluctuations and other aspects produce additional modifications, resulting in what are called today effective potentials [4].

A first contact with lattice theories is provided by the frequent appearance in effective potentials of a length-scale which, for the case of electrons, is predictably given by the Compton wave-length $l_C = \hbar/m_0$ $(c = 1)$. As an example, second-order polarization effects can be expressed by the generalized potential

$$V_{\text{pol}}(r) \simeq \pm \frac{2e^2\alpha}{3\pi r} \ln \frac{r\sqrt{2}}{l_C}\,, \tag{2}$$

acting within the region $r \leqslant l_C$. As another example, fourth-order quantum effects of the electrostatic interactions of two points charges of masses m_1 and m_2 yield the generalized potential [5]

$$V_{\text{Gen}}(r) = \pm e^2/(r + l_{\text{e.m.}}) \tag{3}$$

acting in the region $r \geqslant l_{\text{e.m.}}$, where $l_{\text{e.m.}} = e^2/2(m_1 + m_2)$ is the electromagnetic radius of the interacting system.

A number of other generalizations/modifications of the Coulomb law along lines (2) and (3) can be found in the literature. As far as our objective is concerned, generalizations of this type identify a length-scale, but they do not appear to exhibit a lattice structure.

A generalization of the Coulomb law which identifies a length-scale and exhibits a form of *lattice structure in time* has been proposed by one of us [6] according to the following main ideas. All dynamic-geometric models on the structure of the electron (such as its interpretation as an oscillation of the geometry of space [7]) imply that the particle force-field at a given space distance is periodic in time, while law (1) implies a force-field at a given distance which is constant in time.

To attempt a resolution of such an apparent incompatibility, the following hypothesis was submitted in ref. [6].

Hypothesis 1. *The electrostatic interactions between two electrons, or two positrons, or one electron and one positron, have an explicit time dependence of pulsating type, e.g., according to the generalized Coulomb law* [8]

$$F_{\text{Gen}}(r, t) = \pm \frac{e^2}{r^2}\, 2 \sin^2 \omega_0 t\,, \tag{4}$$

---

[3] E. A. UEHLING: *Phys. Rev.*, **48**, 55 (1935) and R. SERVER: *Phys. Rev.*, **48**, 49 (1935).

[4] See, for instance, M. R. BROWN and M. J. DUFF: *Phys. Rev. D*, **11**, 2124 (1975).

[5] S. N. GUPTA and S. F. RADFORD: *Phys. Rev. D*, **21**, 2213 (1980).

[6] R. M. SANTILLI: *Hadronic J.*, **4**, 770 (1981).

[7] J. A. WHEELER: *Geometrodynamics* (New York, N.Y., 1962).

[8] For completeness, we should recall that realization (4) is not unique. In fact, a number of alternative forms were proposed in ref. [6], *e.g.*, $F_{\text{gen}}(r, t) = \pm (e^2/r^2)(1 \pm w \sin \omega_0 t)$, where w is a suitable (dimensionless) parameter to be determined from experimental data. These additional forms preserve the periodicity of (4) and are therefore inessential for the limited objectives of this note. Note that law (4), besides admitting no free parameter, permits a « structure model of the elementary charge », as elaborated in ref. [6].

whose period of pulsation is given by the characteristic frequency of the electrons (at rest)

$$T_0 = \frac{2\pi}{\omega_0} = 2\pi\frac{\hbar}{m_0} = 0.829\cdot 10^{-20}\,\text{s}\,. \tag{5}$$

According to the hypothesis, the field of an isolated electron has precisely the periodic time dependence expected from geometric models, *e.g.*, $E = \pm (e/r^2)\sqrt{2}\sin\omega_0 t$. As such, the field contains both attractive and repulsive forces. The electrostatic interactions are then responsible for their separation into the attractive or repulsive form (4).

A preliminary study of the plausibility of the hypothesis was presented in ref. (6) for the nonrelativistic case only (see below for comments on the relativistic extension). In essence, there are realistic hopes that the hypothesis is compatible with available data on electrons' phenomenology and permits the prediction of new effects (such as the manifestation—and detection—of the pulsating structure via resonating effects induced by gammas with characteristic periods multiple of T_0^6). In fact, the generalized law (4) recovers law (1) identically for time averages over periods $T \gg T_0$ (the pulsating behaviour itself is not manifest for a sufficient multiple of T_0). Also, the conventional law is recovered for a sufficient superposition of pairs of elementary interactions (even without time average), owing to their different phases. The recovering of law (1) for a collection of elementary charges is, therefore, quite natural for hypothesis I. Finally, the transition probability for scattering data of law (4) computed via the time-dependent theory, say, W_{if}, coincides with that of the conventional, time-dependent case, say W_{if}^0, for sufficient values of time at all orders. For instance, the zero-order contribution of the difference $W_{if} - W_{if}^0$ is proportional to the quantity (6)

$$d = \frac{2}{\pi T}\int\limits_{-\infty}^{+\infty} \mathrm{d}\omega_{km}\,\frac{\sin^2\frac{1}{2}\omega_{km}T}{\omega_{km}^2[1-(\omega_{km}/\omega_0)^2]^2} - 1\,, \tag{6}$$

which is sufficiently well approximated by zero already for periods of time T of the order of 10^{-20} s. Similar cases occur for higher-order contributions.

In this note, we would like to point out that generalized Coulomb law (4) has a particular form of lattice structure in time. In fact at the values

$$t_n = \frac{\pi}{4\omega_0} + nT_0\,, \qquad n = 0, \pm 1, \pm 2, \ldots, \tag{7}$$

the generalized law and the conventional one coincide. Thus, *the conventional Coulomb law can be interpreted as emerging from the generalized law* (4) *on a time lattice with spacing* T_0.

Intriguingly, such interpretation appears to admit a space counterpart. As an example, consider the quasi-potential generalization of ref. (9).

$$V_{\text{Gen}}(r) = \pm\frac{e^2}{r}\,\text{ctgh}\,\pi\frac{r}{l_C}\,. \tag{8}$$

Then, at the values $r = is_n$,

$$S_n = \pm\frac{l_C}{4} + nl_C\,, \qquad n = 0, \mp 1, \mp 2, \ldots, \tag{9}$$

generalized law (8) coincides with the conventional one. Thus, *the Coulomb law can*

(9) N. B. SKACHKOV and I. L. SOLOVTSOV: *Sov. J. Nucl. Phys.*, **26** (4), 367 (1977).

also be interpreted as emerging from the formulation of generalized law (8) *on a space lattice with spacing* l_C *(although absolute values are needed in this case).*

Intriguingly, the space and time spacings can be unified by rewriting law (4) in the form ($\hbar = c = 1$)

$$F_{\mathrm{Gen}}(r, t) = \pm \frac{e^2}{r^2} 2 \sin^2 \pi \frac{t}{l_C}, \tag{10}$$

thus permitting a formal, space-time, « Coulomb plaquette » with spacing l_C. The understanding is that the actual spacing in time remains T_0 and that in space remains the Compton wave-length.

The differences between the type of lattice structure considered here and that currently used in strong interactions [1,2] are self-evident. For instance, the lattice spacing of the former can only decrease via the increase of the speed of the particles, while that of the latter can be made to decrease according to computational needs.

Note that the spacing l_C becomes particularly intriguing if the measurements themselves are expressed in terms of intervals [10]. In fact, the Compton wave-length can be interpreted as the quantum-mechanical length-scale for the effective validity of the point-like characterization of the electron, which is subjected to finer subdivisions for higher-order effects, as expressed by eq. (2). Also, the spacing l_C agrees with the quantum-mechanical fluctuations of the proper-time operator [11] $t = p^\mu x_\mu / m_0$. This opens up the possibility that, under relativistic extensions, lattice structure (7) belongs, strictly speaking, to the proper time.

To conclude this note, we would like to recall that the pulsating generalization of the Coulomb law has been submitted for the actual behaviour of the electrostatic interactions of individual electron pairs, and not as higher-order corrections. As a result, the validity of the generalization cannot be claimed prior to a detailed study of its compatibility with available experimental data at the level of a full QED treatment. In this respect, we should recall the well-established validity of QED down to distances of the order of 10^{-16} cm. Relativity arguments might therefore be used to imply a corresponding validity for periods of time of the order of 10^{-26} s. Nevertheless, the reader is discouraged from accepting these arguments as a final character prior to a detailed study of the problem. In fact, a number of aspects may well imply the compatibility of the QED treatment of hypothesis 1 with available experimental data, such as the high value of the electromagnetic coupling constant, or the space-time fluctuations due to the quantum indeterminacy. In the final analysis, the validity of QED at small distances has been established via suitable modifications of the Feynman propagators, that is, via suitable effective potentials [12]. Then, a reformulation of these studies in a way compatible with hypothesis 1 cannot be excluded *a priori*. After all, we should not forget that the compatibility seems to be quite possible for quantum-mechanical nonrelativistic settings [6], by therefore rendering plausible the existence of a consistent, relativistic, and field-theoretical extension.

Despite these unsettled aspects, the implications for strong interactions remain intriguing and deserving further studies. In fact, hypothesis 1 might imply that the notion of lattice, which is currently restricted to computational functions [1,2] is in actuality deeply rooted in the basic physical laws. Even in case the validity of hypothesis 1 cannot be established for electron pairs, the hypothesis might still be valid for the hadronic constituents, with self-evident promising implications for a variety of unsettled issues of our current knowledge of the hadronic structure.

---

(10) A. J. KALNAY and B. P. TOLEDO: *Nuovo Cimento*, **48**, 997 (1967).
(11) E. PAPP: *Nuovo Cimento B*, **10**, 471 (1972).
(12) See, *e.g.*, B. L. BERON *et al.*: *Phys. Rev. D*, **17**, 2187 (1978).