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An Intriguing Legacy of Einstein, Fermi, Jordan, and Others: The Possible Invalidation of Quark Conjectures[1]

Ruggero Maria Santilli[2]

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The objective of this paper is to present an outline of a number of criticisms of the quark models of hadron structure which have been present in the community of basic research for some time. The hope is that quark supporters will consider these criticisms and present possible counterarguments for a scientifically effective resolution of the issues. In particular, it is submitted that the problem of whether quarks exist as physical particles necessarily calls for the prior theoretical and experimental resolution of the question of the validity or invalidity, for hadronic structure, of the relativity and quantum mechanical laws established for atomic structure. The current theoretical studies leading to the conclusion that they are invalid are considered, together with the experimental situation. We also recall the doubts by Einstein, Fermi, Jordan, and others on the final character of contemporary physical knowledge. Most of all, this paper is an appeal to young minds of all ages. The possible invalidity for the strong interactions of the physical laws of the electromagnetic interactions, rather than constituting a scientific drawback, represents instead an invaluable impetus toward the search for covering laws specifically conceived for hadronic structure and strong interactions in general, a program which has already been initiated by a number of researchers. In turn, this situation appears to have all the ingredients for a new scientific renaissance, perhaps comparable to that of the early part of this century.

1. THE QUARK MODELS

Truly outstanding achievements have occurred in the study of the strongly interacting particles (hadrons) during the last decades. Beginning with the pioneering proposal by Gell–Mann[(1)] and Zweigh[(2)] of using the special

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[2] Department of Mathematics, Harvard University, Cambridge, Massachusetts.

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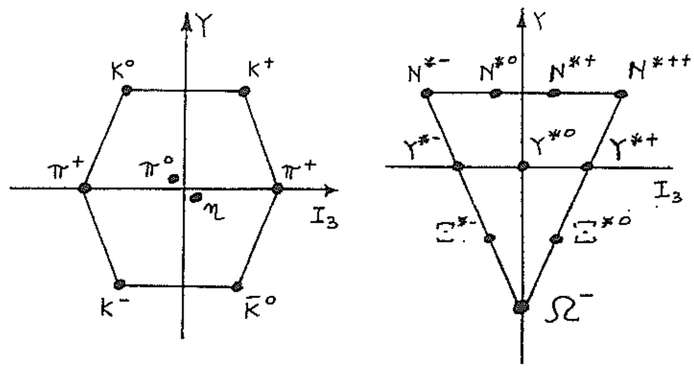

Fig. 1

unitary Lie group $SU(3)$ for the study of hadrons, the celebrated "eightfold way" provided the first comprehensive treatment of hadrons, with achievements such as the octet representation of the light mesons and the decuplet representation of baryons (Fig. 1). In particular, this approach to hadrons predicted the existence of the particle Ω^-, which was subsequently established by experiments.

The decomposition of the tensorial product of the fundamental three-dimensional representation of $SU(3)$ and its conjugate $\bar{3}$

$$3 \times \bar{3} = 8 + 1, \qquad 3 \times 3 \times 3 = 10 + 8 + 8 + 1 \tag{1}$$

suggested the hypothesis that mesons are a bound state of one quark and an antiquark, while baryons are a bound state of three quarks.

This original hypothesis, later called "the naive quark model," was subjected to a number of sequential, conceptual, and technical implementations. We mention only (i) the introduction of the so-called "sea of gluons" to attempt a field-theoretic description of quark models; (ii) the introduction of the notion of "color," in order to bypass an inconsistency created by the spin-statistics theorem for the Δ^{++} state (among other reasons); and (iii) the reformulation of the quark models in terms of so-called "bags" (e.g., the MIT and SLAC bag models).

More recently, a brilliant advance was achieved via the introduction of a new notion, called "charm," and new quarks. These advances led to the discovery of a new heavy hadron, the J/ψ particle, and, still more recently, to a new series of heavy hadrons sometimes referred to as "charmonium states."[3]

By incorporating a number of outstanding achievements (such as the non-Abelian gauge theory by Yang and Mills and the unified gauge theory of weak and electromagnetic interactions by Salam and Weinberg), a compre-

[3] See Ref. 3 for a review of the early stages of quark models, Ref. 4 for a more recent account, Ref. 5 for a review of charmonium and related topics, and the series of reprint volumes[6] for the annual progress in the field.

hensive theory of hadrons, called quantum chromodynamics (QCD), was proposed. According to this theory, hadron physics is described by the Lagrangian (density)[5,6][4]

$$\mathscr{L}_{\text{tot}} = \mathscr{L}_{\text{free}} + \mathscr{L}_{\text{int}} = -\tfrac{1}{4}G_a^{\mu\nu}G_{a\ \nu} + i(\bar{\psi}_q D\psi_q - m\bar{\psi}_q\psi_q) \tag{2}$$

whose fields satisfy the Lorentz covariance law

$$\psi'(x') = S(\Lambda)\psi(x), \qquad \Lambda \in SL(2.C) \tag{3}$$

as well as the unitary gauge symmetry. The results achieved by QCD are equally impressive and outstanding.

Despite these achievements, the problem of the structure of hadrons has remained elusive and controversial, to the point that no conclusive model of hadron structure can be claimed at this moment. Physics is a discipline with an absolute standard of values: the physical veritas. Until theoretical ideas (such as the hypothesis that quarks are physical particles) are experimentally established on unequivocal grounds, they constitute conjectutes or beliefs, but not the manifestation of physical truth. This is the reason for the use of the term "quark conjectures" in the title of this paper and in the following analysis.

The objective of this paper is to present an outline of the criticisms of quark models which have been present in the community of basic research for some time. As we shall see, these criticisms are numerous, and most of them are of a delicate nature because they involve an assessment of the validity of our current theoretical knowledge when applied to the description of the strong interactions. The treatment of these issues is not an easy task. Therefore, I appeal to the understanding of the receptive reader in case my presentation is sometimes deficient in completeness and rigor.

[4] It should be indicated here that Lagrange's equations in the form conventionally used in QCD and in the contemporary physics literature

$$\frac{\partial}{\partial x^\mu}\frac{\partial \mathscr{L}}{\partial \phi^{k;}{}_\mu}\frac{\partial \mathscr{L}}{\partial \phi^k} = 0; \qquad \phi^{k;}{}_\mu = \frac{\partial \phi^k}{\partial x^\mu}; \qquad k = 1, 2,\dots, N;\ \mu = 0, 1, 2, 3$$

are *erroneous* whenever the symbol $\partial/\partial x^\mu$ is interpreted in its conventional mathematical meaning, that of representing a *partial* derivative. The correct form of the equation is that with a *total* derivative,[7] i.e.,

$$\frac{d}{dx^\mu}\frac{\partial \mathscr{L}}{\partial \phi^{k;}{}_\mu} - \frac{\partial \mathscr{L}}{\partial \phi^k} = \frac{1}{2}\left(\frac{\partial^2 \mathscr{L}}{\partial \phi^{k;}{}_\mu\,\partial \phi^{i;}{}_\nu} + \frac{\partial^2 \mathscr{L}}{\partial \phi^{k;}{}_\nu\,\partial \phi^{i;}{}_\mu}\right)\phi^{i;\mu}{}_\nu$$

$$+ \frac{\partial^2 \mathscr{L}}{\partial \phi^{k;}{}_\mu\,\partial \phi^i}\phi^{i;}{}_\mu + \frac{\partial}{\partial x^\mu}\frac{\partial \mathscr{L}}{\partial \phi^{k;}{}_\mu} - \frac{\partial \mathscr{L}}{\partial \phi^k} = 0$$

In any case, my hope is that these criticisms will be considered by experts on quark models and that countercriticisms will be presented for a possible future resolution of this rather crucial aspect of basic research. Also, the current compartmentalized studies along quark and nonquark lines with little (if any) interplay is rather unappealing in terms of scientific effectiveness. It appears advisible that supporters and opponents of quark models join forces for an effective resolution of their differences, in the traditional pursuit of scientific knowledge.

As indicated earlier, the disagreements between supporters and opponents of quark models are varied and numerous ranging, from conceptual issues to differences in research attitudes and specific technical aspects. In this section we shall deal with preliminary issues only. Specific arguments will be considered in the subsequent sections.

The most visible problematic aspect of the quark models is that, despite rather large investments of financial and human resources through the years, the existence of quarks has not yet been experimentally established.

A first point of controversy is created by indications of the possible experimental detection of fractional charges.[8,9] This is generally (although not universally) seen by quark supporters as potential evidence of the possible existence of quarks in nature. Quark opponents disagree. The term "quarks" refers to a number of different particles carrying different quantum numbers. In order to establish unequivocally the quarks as the physical constituents of hadrons, *all* quarks must be *individually* established on clear experimental grounds. Besides epistemological reasons for the needed scientific rigor, there are technical reasons in support of this attitude. Indeed, as we shall indicate later, *fractional charges are allowed within a hadron by models fundamentally different from the quark models.*[10]

A second area of current disagreement is related to the so-called problem of confinement. Hadrons generally exhibit spontaneous decays. Since quarks are not produced free in these decays, and since their experimental detection in inelastic scatterings under all available (high and low) energies has remained inconclusive, a new trend has been initiated and lately intensified, consisting in the construction of a model whereby quarks, while being the physical constituents of hadrons, are confined within their charge volume and cannot be produced free. Despite brilliant contributions to a rather formidable theoretical problem, no conclusive model of confinement which can be accepted by the scientific community at large has been achieved.[11]

The controversies on the problem of confinement are numerous and range from epistemological to conceptual and technical issues. On epistemological grounds, the possible achievement of a final model of confinement is considered by some (but not all) quark supporters as virtually conclusive evidence of the validity of the quark models. Quark opponents disagree.

The idea is that the achievement of a conclusive model of confinement will simply shift the need of establishing on clear experimental grounds each and all quarks from outside to within a hadron. According to this view, the quark models will then remain fundamentally unestablished until a new technology capable of detecting particles under strong interactions is available. It is appropriate to point out here that the experimental capability available at this time for the direct detection of particles involves long-range electromagnetic interactions, e.g., as in bubble chambers. Predictably, a number of generations will pass until we are able to achieve the new technology indicated here of directly detecting particles under the short-range ($1\ F = 10^{-13}$ cm) strong interactions.

On conceptual grounds the disagreements are perhaps deeper. It is rather generally acknowledged that the constituents of hadrons are lightly bound. Since these bound states are unstable and exhibit spontaneous decays, the idea that they are confined meets with an understandable uneasiness. Indeed, one nonquark line of studies currently under investigation (which we shall indicate later) is based on the idea that, under the assumption that the constitutents are lightly bound and the states are (highly) unstable, the constituents can be produced free in spontaneous decays. In conclusion, there exists a considerable uneasiness in one segment of our community toward the very idea of confinement.

On technical grounds, the disagreements may be even deeper. They are due to the fundamental discipline used in quark models, quantum mechanics. Tunnel effects are a natural consequence of the very foundations of this discipline. The achievement of an unequivocal model of confinement therefore touches the foundations of this discipline, and calls for the construction of a model of an unstable bound state in which the probability of the tunnel effect is literally (and not approximately) zero. This does not appear to be an easy task within the context of the rather inflexible laws of quantum mechanics.

Another area of disagreement is due to the trend toward a progressive increase in the number of different quarks. There is no doubt that this trend has sound technical motivations within the context of quark models. Indeed, it is needed to properly accommodate new particles and their data, as made available by the experimenters or as predicted by theorists. Yet, an increase in the number of different, unidentified, quarks without a direct experimental backing is seen by quark opponents as an increase (rather than a decrease) of the problematic aspects of these models. Actually, the possibility that there exist infinitely different hadrons (and leptons) does not appear unrealistic at this moment. The concern is that such a situation might demand an infinite number of different quarks for these models to be consistent.[5] In conclusion,

[5] The efforts to limit this number to 30 should be indicated here. This point is controversial per se. As we shall elaborate in Sections 3 and 4, there appears to be a disagreement

whether quark models need a finite or an infinite number of quarks, the increase of the number of quarks has met with an understandable uneasiness and concern by one sigment of our community.

These introductory remarks are sufficient to indicate the delicate status of the studies of a rather fundamental physical problem. But these remarks are related to easily identifiable aspects only. As we shall see, the reasons for disagreement are of a much more substantial nature.

Before entering into these issues, it appears advisable to clarify one aspect of potential misunderstanding between quark supporters and opponents, which is apparently basic to the current lack of scientific cooperation between these groups. In the simplest possible terms, quark supporters often interpret criticisms of quark models as intended to invalidate the physical value of the unitary groups for hadrons. One of the objectives of this paper is to dispel this possible misunderstanding as clearly as possible.

According to a by now historical process, new insights in physics never "destroy" previous accomplishments of proved physical value. They simply identify their arena of applicability and implement them in a broader conceptual and technical context. For instance, Albert Einstein did not claim that Galileo Galilei was wrong in his relativity ideas. He simply identified the physical arena in which Galilei's relativity applies, and proposed a covering relativity for a broader physical arena.

We have stressed earlier in this section the brilliant achievements of unequivocal physical value of the unitary models in hadron physics. These results are now part of the history of pkysics and no possible future advances can invalidate them. Therefore, the issue of whether unitary models are valid or invalid has no scientific value.

A scientifically valuable problem, instead, is the identification of the part of the hadronic phenomenology to which the unitary models unequivocally apply, and the different or broader part in which they are unestablished and problematic, and for which the search for possible more general conceptual and technical approaches may be scientifically productive.

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on the very definition of what a model of the structure of hadrons should be, and what are the data to be quantitatively represented. The issue referred to here is whether 15 quarks and 15 antiquarks are capable of representing *all* hadrons currently known and *all* hadrons expected to be identified in the future, as well as *all* their total characteristics and, last but not least, *all* their decay modes and *all* the related fractions. In conclusion, we are referring here to a rather substantial volume of data already existing and which is expected to multiply in the near and far future, which should be *all* quantitatively represented by 15 quarks and 15 antiquarks, according to quark supporters. Other researchers expect instead the need to multiply the number of different quarks in a measure proportional to the increase of data, in order to avoid inconsistencies.

A tentative answer is the following.[12]

(A) The unitary models are unequivocally valid for the Mendeleev-type, exterior, "chemical" classification of hadrons only.

(B) The quark models are conjectural at this time only when taken as constituting a structure model for each individual element of a given unitary hadronic multiplet.

(C) In relation to the open problem of hadronic structure, studies using quark conjectures should be continued. Studies using fundamentally different conjectures should be continued also, under the condition that they achieve full compatibility with the established unitary models of Mendeleev-type classification.

Aspect (A) is self-evident. A simple inspection of the diagrams of Fig. 1 indicates that the validity of the unitary models for hadronic classification is simply incontrovertible. In Ref. 12 the unitary models were therefore assumed as the final classification of hadrons.

Aspect (B) is not equally self-evident. As a matter of fact, it may appear contradictory to aspect (A) under a first inspection. This second aspect emerged after a comparative anamnesis of the current status of the art in hadron physics with the historical solution of the problem of atomic phenomenology.

Atomic phenomenology demanded two different, yet compatible models, one of classification (Mendeleev) and one of structure (Bohr–Thomas–Fermi). These two models are profoundly different in conception, methods, and insights. Yet they are compatible, in the sense that the model of classification produced invaluable elements for the structure, while the structure model recovered the classification.

At the atomic level, the idea that one single model could serve for both the classification and structure of atoms was extraneous to the analytic mind of the founding fathers of contemporary physics.

At the hadronic level, the situation does not appear to be necessarily the same. Even though not explicitly stated in the available literature, quark models are intended to serve for both the classification *and* structure of the hadrons. A reinspection of the diagrams of Fig. 1 with tensorial products (1) and the quark hypothesis establishes this dual *intent* of the quark models of providing the classification of hadrons into unitary multiplets, and, jointly, the structure of each individual element of a unitary multiplet.

Points (A) and (B) are intended to stress the fact that, while the physical value of the unitary models of classification is unequivocal, the character of these models as also providing the structure of hadrons is conjectural at this time. The idea was to identify the arena in which unitary models are unequivocal and the different arena in which they are conjectural, in the hope of minimizing or otherwise avoiding unnecessary controversies. Apparently, this identification had not been considered in the (rather vast) literature prior to Ref. 12.

Point (C) was inspired by the traditional pursuit of open physical problems. There is no a priori reason available at this time to expect that the dichotomy classification/structure which was necessary at the atomic level will eventually be necessary also for the hadronic phenomenology. Thus, studies along the line of quark conjectures are encouraged. But there is equally no a priori reason that the hadronic phenomenology should not eventually demand (at least) two different, yet compatible models, one of classification and one of structure, in exactly the same way as occurred for atoms. Thus, the studies of models of structure fundamentally different than those of quark inspiration should be continued also. The need for these latter models to recover the unitary classification (which is unnecessary for the quark models) is then self-evident.

It is hoped that the reader can see the lack of contradictions in point (C) per se, as well as in relation to (A) and (B). These reason is quite simple. If experiments eventually rule out the existence of quarks as physical particles,[6] this will leave the physical validity and consistency of the unitary models of classification completely unaffected. In fact, at the level of classification, a quark is a mere mathematical quantity: *a representation of a Lie group*. It becomes a *particle* only when the same models are implemented to represent the totality of the hadronic phenomenology (classification, *and* structure, *and* scattering).

It is hoped that these remarks provide final reassurance to quark supporters that no physicist intends to (or actually can) invalidate the achievements of the unitary models. At the same time, it is hoped that this paper will provide arguments to stress that the restriction of all research on the fundamental problem of hadron structure to only quark conjectures may turn out to be a historical error.

We now present the main arguments of doubt about quarks, with the strict understanding that *the following parts of this paper solely refer to the interpretation of quark models as structure models of hadrons*. Under no circumstances are they intended to refer to the unitary classification of hadrons. For this reason we shall use the term "unitary models" when referring to the Mendeleev-type classification of hadrons, and the term "quark models" when they are assumed to constitute structure models

[6] As we shall see, this possibility is considerably greater than is conventionally acknowledged in the quark literature, provided that the experimenter considers on equal grounds feasible experimental proposals for either the proof or the disproof of the quark conjectures.

Also, we shall use the term "quark models" rather than "quark model," due to the current lack of selection of one single model among a rather considerable variety of different models.[13]

As a final comment, it should be clearly indicated that, to avoid prohibitive length, this paper is not intended to be a technical treatment of the new, Bohr-type structure model of hadrons[12] (which is not of quark inspiration). In fact, these studies are rather numerous and rapidly expanding,[7] to the point of rendering prohibitive a technical treatment in a paper devoted to a related, but ultimately different issue.

In summary, this paper is devoted to an outline of the numerous problematic aspects of the conjecture that quarks are physical particles, under the assumption that the unitary models provide a final Mendeleev-type classification of hadrons. The paper will present only incidental comments on the new, Bohr-type structure model of hadrons, which is incompatible with quark conjectures, yet is capable of achieving compatibility with the established unitary models of classification.[12,14–16]

2. THE FUNDAMENTAL ASSUMPTIONS OF THE QUARK MODELS

The issue of whether quarks exist or not is, in the final analysis, of rather limited physical depth. A more scientifically worthwhile issue is the classification of all the assumptions of quark models and the identification of their interrelations, where the term "assumptions" indicates hypotheses, laws, and disciplines lacking a clear and unequivocal experimental backing in the physical context to which they refer.

This broader approach to the problem is clearly essential, first, to avoid unnecessary controversies, and, second, to initiate a systematic theoretical study for the formulation of a series of experiments to determine whether the quark models do indeed constitute the true model of hadron structure or not.

We begin in this way to enter into the actual topics of disagreement. The idea is that, until the basic assumptions of the quark models have been experimentally established on clear grounds, the search for quarks may be sterile. Indeed, it is possible that the basic assumptions of the quark models

[7] The new model of hadron structure here referred to is based on the so-called Lie-admissible approach to strong interactions, and will be indicated in Sections 3 and 4. For more detailed treatment the interested reader is referred to the review memoir[14] and to the proceedings of the workshops on Lie-admissible formulations.[15] The series[16] reprints all mathematical, theoretical, and experimental papers directly or indirectly related to the model.

can be invalidated by experiments (see the subsequent sections), in which case the search for quarks would turn out to be meaningless.

This situation is created by the fact that, at a deeper analysis, the quark models are based on a rather complex "topology" of assumptions, each one of rather fundamental physical character, but without experimental backing at this time. As a matter of fact, the assumption that quarks are the constituents of hadrons is of secondary nature with respect to other emerging assumptions.

A classification of the basic assumptions of quark models and a study of their relationships has been conducted by in Ref. 12 (see Refs. 17 for a detailed treatment and Refs. 18 for basic tools). An outline of the results of these studies is presented below with the understanding that it is tentative and in need of assessment by independent researchers.

The assumptions of quark models can be classified into two groups, here called "primary assumptions" and "secondary assumptions." This classification is intended to identify the assumptions of primary physical character and those of derived character only, that is, consequential to the primary assumptions. In this way, the classification should be effective for the identification of the interrelationships among different assumptions.

The primary assumptions of quark models are the following.

Primary Assumption 1. Einstein's special relativity, as currently known, is valid for the hadronic constituents (or structure) as well as, more generally, for the strong interactions.

This is not the place to recall the unequivocal validity of Einstein's special relativity for electromagnetic interactions, as proved, say, in particle accelerators. Nevertheless, the validity of the same relativity under strong interactions is a mere conjecture at this time, lacking direct and incontrovertible experimental evidence. In fact, a detailed analysis of the structure of current experiments in high-energy physics[14] clearly indicates that special relativity is merely *assumed* in the data elaborations. Since these experiments are not intended to test the relativity considered but instead are devoted to other topics (identification of new particles, measurement of their physical properties, etc.), the results of currently available experiments cannot provide final evidence on the validity of Einstein's special relativity under strong interactions. At best, they can provide *plausibility arguments*. We reach in this way a first fundamental open problem of quark models as well as of contemporary physics at large. Einstein's special relativity is essential for the consistent definition of quarks. Jointly with the continuation of current experimental trends, it therefore appears advisable to establish the validity of this relativity in the arena of interest via direct and conclusive experiments. The formulation of these experiments is one of the primary objectives of the

Lie-admissible approach to strong interactions[12,14–16] As we shall review in Section 3, there exist a number of arguments indicating a conceivable inapplicability of Einstein's special relativity for the strong interactions, and the need of generalized relativity specifically conceived for the arena considered. If this possibility is eventually established by experiments, the problem of the structure of hadrons must be clearly reinspected ab initio (Section 4).

Primary Assumption 2. Quantum mechanics, in its currently available, nonrelativistic, relativistic, and field-theoretic versions, is valid for the hadronic constituents, as well as, more generally, for the strong interactions.

This is also not the place to recall the unequivocal validity of quantum mechanics for atomic structure, as well as for electromagnetic interactions. Nevertheless, this is not sufficient to establish the universal validity of the discipline for the entire microscopic world, irrespective of the physical conditions of the particles. An inspection of the currently available experiments indicates again[14] that the basic laws and principles of quantum mechanics (Pauli's exclusion principle, Heisenberg's indeterminacy principle, etc.) are merely *assumed* as valid in the data elaborations of current experiments in strong (nuclear and high-energy) interactions. On grounds of necessary scientific caution, we must therefore identify quantum mechanics as being conjectural for hadronic structure. As a matter of fact, a number of arguments exist leading to a conceivable inapplicability of quantum mechanics (in an exact form) for the strong interactions, and to the need of a covering discipline specifically conceived for the arena at hand (Section 4). Again, these remarks should not be interpreted as denying the *plausibility* of quantum mechanics for the strong interactions. They are merely intended to identify a second fundamental open problem of quark conjectures and of contemporary physics at large. It is evident that, in case the basic laws of quantum mechanics need suitable generalizations to represent particles under strong interactions, the problem of the structure of hadrons must also be reinspected ab initio, and the hypothesis that quarks are physical particles becomes a mere historical episode (Section 4).[8] The formulation of experiments showing the validity or invalidity (exact or approximate validity) of quantum mechanics under strong interactions is another basic objective of Lie-admissible studies.[12,14–18] The primary arguments will be reviewed in Sections 3–5. Some authoritative, historical voices of doubt will be recalled in Section 6.

[8] Due to the deep impact of any form of relativity for the (classical and) quantum mechanical description of nature, a conceivable inapplicability of Einstein's special (and Galilei's) relativity for the strong interactions will inevitably call for a revision of quantum mechanics. Conversely, a generalization of conventional quantum mechanical laws will call for a revision of the underlying relativity. As a result, the primary assumptions 1 and 2 are, in the final analysis, deeply interrelated.

Primary Assumption 3. The quarks (and the antiquarks) are the elemental constituents of hadrons.

It is hoped that the remarks of Section 1 are sufficient to identify the current conjectural character of quarks as the constituents of hadrons. By and large, this assumption is acknowledged in the current literature. The apparent novelty of the analysis of Ref. 12 is the identification of the fact that this assumption, even though independent from assumptions 1 and 2, carries a physical weight smaller than those of the preceding assumptions. In other words, the fact that the quark models are based on the conjecture of the validity of the basic relativity and quantum mechanical disciplines for the hadronic structure is of a much more fundamental character than the assumption that quarks are the constituents of hadrons. As a matter of fact, it was submitted that perhaps the current problematic aspects of quark models are only the symptoms of a conceivably much more fundamental problem of consistency at the level of the basic laws.[12,14,17]

Each of the primary assumptions indicated above possesses a variety of secondary assumptions of derived character. Without any claim of completeness, we quote the following ones.

Secondary Assumption 1a. The notion of constituent of atomic structure identically applies to hadronic structure, apart from unitary implementations.

A truly crucial prerequisite to properly formulate the problem of structure, let alone to properly treat it, is to achieve a quantitative characterization of the notion of constituent particles. A "particle" is nowadays technically identified via (representations of) the applicable relativity (Galilei's or Einstein's special relativity). This (nonrelativistic or relativistic) notion of particle is incontrovertible when it refers to atomic structure or to electromagnetic interactions. Nevertheless, the issue of whether the same notion identically applies to hadronic constituents and to strong interactions in general (apart from the additional unitary internal degrees of freedom) appears to be fundamentally open at this time on both theoretical grounds (Sections 3 and 4) as well as experimental grounds (Section 5). Clearly, this situation is a consequence of the primary assumption 1.

Secondary Assumption 1b. The familiar linear, local, Lorentz covariance law of fields, Eq. (3), applies to the fields representative of the hadronic constituents or of particles under strong interactions.

As we shall see, despite the unequivocal validity of this law for electromagnetic interactions, the validity of the same law for strong interactions is questionable on rather numerous grounds. In any case, law (3) does not possess a clear, direct, and unequivocal experimental backing. As such, it must be considered as of conjectural character at this time, on grounds of necessary scientific caution. Again, this situation is a consequence of the primary assumption 1. The relationship with secondary assumption 1a is self-evident.

Secondary Assumption 2a. Pauli's exclusion principle and the spin-statistics theorem are valid for the hadronic constituents as well as, more generally, for particles under strong interactions.

An inspection of the original (by now historic) papers on this subject[19] reveals that Pauli's principle was specifically conceived for atomic structure, in which it subsequently proved to be essential for the representation of a number of fundamental aspects of the Mendeleev classification (this is an aspect of the interplay between the model of classification and that of structure at the atomic level). In more recent times, the principle has been applied to nuclear structures, with results in excellent agreement with experimental data. Nevertheless, while the experimental backing of the principle at the atomic level is simply incontrovertible, the situation does not appear to be the same at the nuclear level. The experimental verification of Pauli's exclusion principle in nuclear physics was proposed[12] to establish whether the principle is valid in nuclear physics in the same quantitative measure as in atomic physics, or whether very small deviations are detectable. In different terms, the question which was submitted in Ref. 12 is whether our current knowledge of the validity of Pauli's principle in nuclear physics is quantitatively comparable to current knowledge of the validity of the PCT symmetry in particle physics, or is at a stage prior to the discovery of, say, parity violation in weak interactions. It is understood that, due to the good agreement with experimental data, possible departures from Pauli's principle in nuclear physics can at most be very small. The situation at the hadronic level appears to be different due to the rather profound physical differences of these two layers of microscopic reality. In any case, Pauli's principle for the hadronic constituents is a *conjecture* at this moment, without direct experimental backing; that is, aside from indirect experimental indications based on a variety of different assumptions. Again, these remarks are not intended to deny the plausibility of Pauli's principle in hadron physics. Instead, they are intended to stimulate the experimental resolution of the issue, as the only basis for the conduct of sound physical studies. The spin-statistics theorem is, to a considerable extent, the field-theoretic image of Pauli's principle in discrete quantum mechanics, and its lack of final experimental verification for strong interactions can also be seen by inspecting available experimental data. The derivative character of the assumption under consideration from the primary assumption 2 is self-evident.

Secondary Assumption 2b. Canonical quantization rules, in their nonrelativistic, relativistic, and field-theoretic versions, apply to hadronic structure and, more generally, to strong interactions.

Again, the validity of these rules for the electromagnetic interactions is established. Nevertheless, the validity of the same rules for the strong interactions is not equally clear and a number of arguments indicating their conceivable invalidity will be indicated in Section 4. The implications of this assumption are rather deep, and so are its relationships with the other assumptions under consideration. Indeed, if these rules are invalid for the strong interactions, the Lie algebra realization of the basic relativity laws in terms of quantum mechanical operators is invalid. The deep compatibility and inflexible relationship of the various aspects of contemporary theoretical physics begins to emerge. Pauli's exclusion principle, the spin-statistics theorem, the Lorentz covariance law, the notion of particle, and numerous other insights (e.g., Heisenberg's indeterminacy principle) are all links of one, single, rigid chain of mutually compatible insights. The possibile invalidity of only one link of this chain may imply the invalidity of the entire construction. A similar argument applies for the possible validity. This remark is made in the hope that it can assist the experimenter in the selection of that link of the chain whose validity or invalidity can be more readily established for the strong interactions in a way independent from its validity for electromagnetic interactions.

Secondary Assumption 3a. Quarks are composite constituents of hadrons.

As indicated by recent studies[6] the question of whether the hadronic constituents (for the case of heavy or superheavy hadrons) cluster into granules with fractional charges or not does not appear to be readily resolvable on the basis of our knowledge at this time. Therefore, it appears advisable to differentiate primary assumption 3 into a secondary assumption in which quarks are assumed as composite. In different terms, the problem of whether quarks exist or not is in actuality of multifold nature. First, there is the problem of whether the quarks are the final, elemental, indivisible constituents of hadrons. If experiments disprove this conjecture, there still remains the possibility that quarks are composite constituents of hadrons. As we shall see, a crucial function for the possible resolution of this issue is given by the problem of the structure of the light hadrons. This is why the problem of the structure of the octet of light mesons of Fig. 1 was considered to be of fundamental character in Ref. 12, and the same attitude is preserved in this paper.

Secondary Assumption 3b. Quarks are confined.

It is hoped that the remarks of Section 1 are sufficient to indicate the

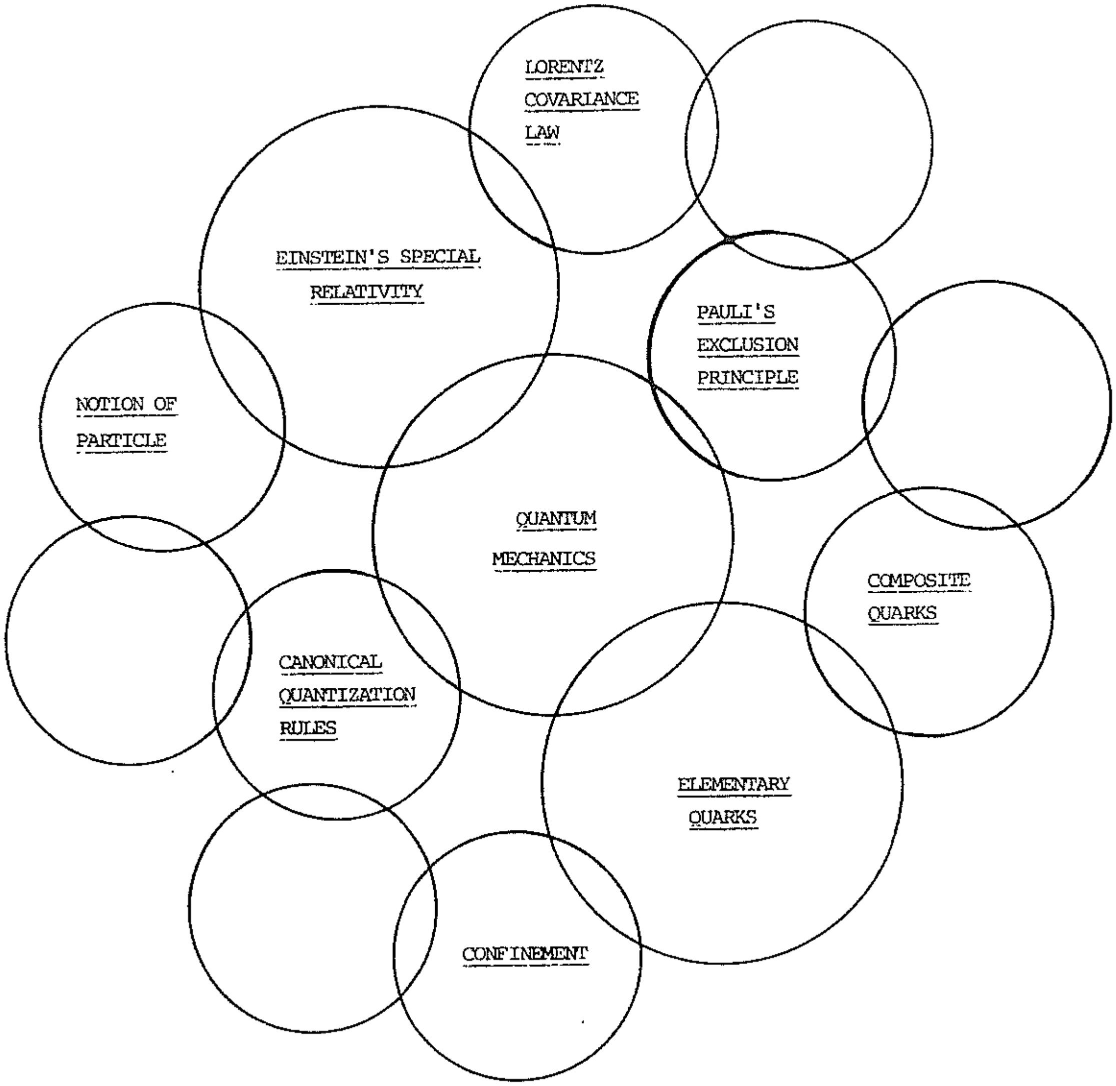

Fig. 2. The rather complex "topology" of assumptions of the quark models, schematically presented via "overlapping polycircles." According to the analysis of Ref. 12, quark models and QCD, when interpreted as dealing with the structure of hadrons, are based on rather numerous assumptions of fundamental physical character, all lacking a clear, direct, and unequivocal experimental backing. The experimental search for quarks then becomes of secondary relevance, when related to the need for a prior experimental resolution of the validity or invalidity of the basic physical laws of the models considered in the arena considered. The analysis is not intended to deny the plausibility of the laws considered in the arena considered, but merely intended to stimulate the experimental resolution, in due time, of these fundamental problems. Undoubtedly, the emergence of the conjectural character of the basic physical laws of quark models of hadron structure casts shadows on the hypothesis that the quarks are the constituents of hadrons and, in any case, calls for a comprehensive study of the formulation and conduct of a variety of experiments to eventually resolve the validity or invalidity of these models of hadron structure. This situation has been lingering in the community of basic research for some time. The reader may be amused to know that quarks have been referred to in the literature as the "Yeti particles," or that the case of quarks has been amiably related to the historical case of the ether, or that the role of the (rather powerful) quark supporters in contemporary theoretical physics has been compared to the control of science by priests during Galilei's time.

need of considering this an assumption at this time on grounds of scientific caution.

We have listed here only some of the most significant assumptions of the quark models which are implied by the primary assumptions 1–3. The reader can easily complete this classification with all additional aspects considered valuable.

The attitude implemented in this section should be indicated. It essentially consists in a critical examination of each and every methodological tool used in the actual realization of the quark models. The objective is that of identifying those tools that are unequivocally valid (e.g., a Fourier transform) and those that are conjectural (e.g., Feynman diagrams[9]) as a necessary prerequisite for the sound conduct and presentation of research.

A summary view of the content of this section is provided in Fig. 2

3. THE PROBLEM OF THE ARENAS OF VALIDITY AND INVALIDITY OF EINSTEIN'S SPECIAL RELATIVITY IN PARTICLE PHYSICS

The belief that Einstein's special relativity, as currently known, constitutes the final form of relativity applicable to particle physics, with unlimited applicability to all conceivable physical conditions of particles has no scientific value.

A more scientifically productive issue is the identification of the arena of particle physics in which Einstein's special relativity unequivocally applies, and the different or broader arena in which it is a conjecture at this time, and for which a covering relativity becomes conceivable.

This latter issue is scientifically productive on more than one ground. First, the identification of the arena of conjectural character of the relativity considered is a prerequisite for the subsequent problem of formulating and conducting experiments aiming at the verification of its validity or invalidity. Second, and equally important, the identification of the arena of conjectural validity of the relativity considered is a prerequisite for stimulating studies on a conceivable covering relativity. Predictably, these studies are expected to involve the identification of new mathematical tools. Whether such a conceivable covering relativity can be consistently constructed or not, and whether it will conform to physical reality or not, studies of this nature can only be valuable for the advancement of mathematical and physical knowledge.

[9] As we shall see, Feynman diagrams are conjectural for the strong interactions because (unlike the case of the electromagnetic interactions) the Lagrangian representations are conjectural for the interactions considered.

The issue under consideration has been studied in detail in Refs. 12, 14, 17, 18, and 20. The answer which has been submitted is the following.

1. The arena of unequivocal validity of Einstein's special relativity in particle physics is that of the electromagnetic interactions only.[10]

2. An arena of current conjectural character of the relativity considered in particle physics is that of the strong interactions in general and of the strong hadronic forces in particular.

3. Studies based on the validity of the relativity considered for strong interactions and for the structure of hadrons should be continued. Studies based on the invalidity of the relativity considered in this latter arena should also be continued, jointly with the search for a possible covering relativity specifically conceived for the strong interactions.

The argument in favor of the applicability of Einstein's special relativity for the strong interactions is known, and it is provided by the quark models, as well as QCD.[(6)] It may be of some value for the interested reader to briefly outline here the main arguments of Refs. 12 and 14, which suggest a possible invalidity of the relativity considered in the arena of interest.

The folloling are well-established experimental facts.

I. The "size" (charge radius) of hadrons does not increase appreciably with mass (contrary to the corresponding occurrence at the nuclear level), and it is of the order of 1 F for all hadrons.

II. The size of hadrons coincides with the range of the strong interactions.

III. Hadrons are constituted by wave packets.

A picture of the strong interactions profoundly different than that of the electromagnetic interactions results from these experimental data. In essence, the latter interactions are long range, and thus can occur at distances much greater than the charge radius of the particles, as is typically the case for atomic structure. Under these conditions, the particles can be effectively approximated as being pointlike (Fig. 3). This approximation has several important consequences. First, (long-range) *electromagnetic interactions admit an effective treatment via local models.* The latter terms refer, physically, to interactions occurring among a collection of isolated points and, mathematically, to interactions admitting a representation via ordinary or partial differential equations. Second, pointlike particles can only experience "action at a distance" forces, that is (local) forces satisfying the integrability conditions

[10] One may add also the weak interactions. We have excluded them from point 1 because, in our view, the problem demands a detailed study (rather than an a prioristic assumption).

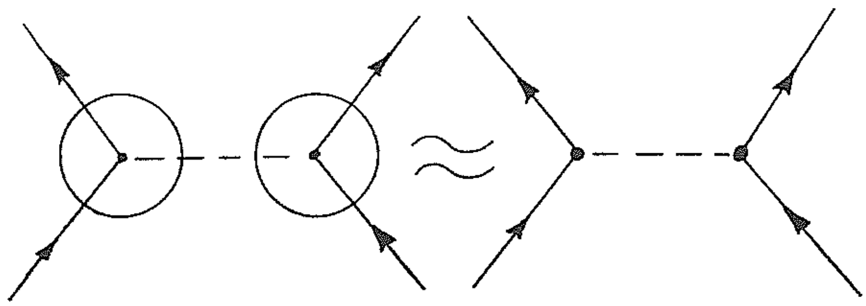

Fig. 3

for the existence of a potential function (the so-called conditions of variational self-adjointness; see Appendix A). As a result, *the electromagnetic interactions admit an effective treatment via (local) Lagrangian or Hamiltonian models.* Third, the Hamiltonian character of the equations of motion implies the applicability of conventional (Lie) algebraic and (symplectic) geometrical methods. As a result, *Lie algebra and symplectic geometry are applicable for the quantitative treatment of electromagnetic interactions.* The applicability of Einstein's special (or Galilei's) relativity then results. It should be stressed here that the entire physical and mathematical edifice rests on the capability, under the conditions considered, to approximate the particles as being massive points, according to the notion originally conceived by Galilei and Newton, and subsequently embraced by Einstein.

In the transition to the strong interactions the virtual entirety of the physical and mathematical treatment of the electromagnetic interaction appears questionable on a number of grounds. Experimental facts I and II establish that, as a necessary condition to activate the strong interactions, hadrons enter into a state of mutual penetration or overlapping of their charge volumes (Fig. 4a). As a consequence, *particles cannot be effectively approximated as being pointlike.* Furthermore, experimental fact III establishes that hadrons under strong interactions actually consist of wave packets under a condition of mutual overlapping (Fig. 4b).

The inappropriateness of the pointlike approximation of particles has a number of important consequences. First, it implies that the *strong interactions are expected to demand a suitable form of nonlocal model.* This form can be identified via experimental fact III. It is known that, in classical continuum mechanics, overlapping waves demand integrodifferential equations, that is, equations possessing a local (ordinary or partial) component, and an integral component representing the interactions at *all* points of overlapping. Apart from problems of quantization, there is no reason to expect that overlapping quantum waves can be treated via differential equations without the integral component.

The term "nonlocal models" suggested by experimental facts I–III can

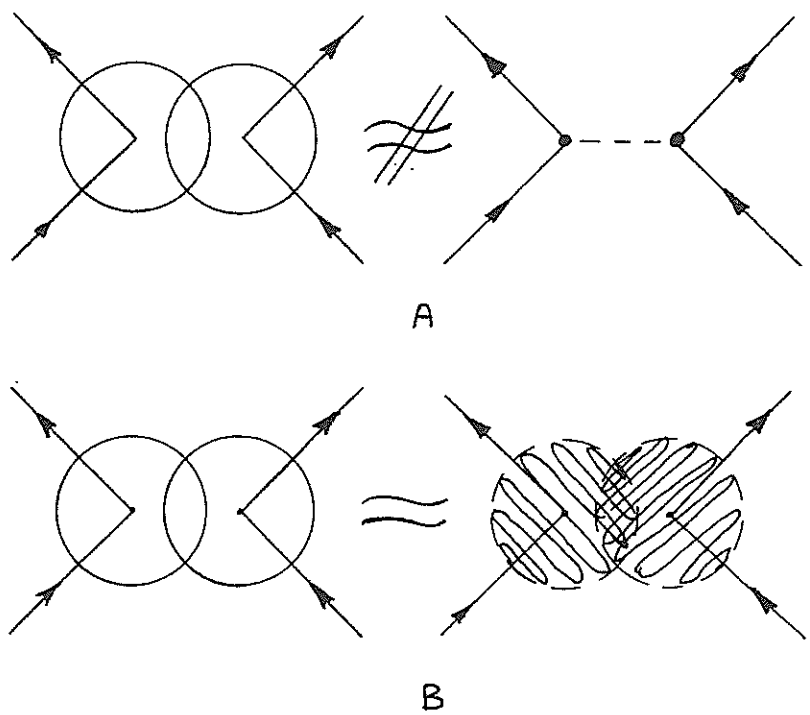

Fig. 4

now be made more precise. It refers, physically, to interactions occurring at all points of the volume of wave overlapping, and, mathematically, to interactions admitting a representation via ordinary or partial integrodifferential equations. The local-differential part is expected to represent the electromagnetic interactions as well as a component of the strong interactions (e.g., as in nuclear physics). The nonlocal-integral part is intended to represent the effect resulting from wave overlapping.

A further, rather crucial point that needs to be made precise concerns the nature of the nonlocal term. Contemporary high-energy physics is virtually dominated by Lagrangian (or Hamiltonian) models. This situation, however, is not sufficient to warrant that all interactions must be of Lagrangian or Hamiltonian type. In fact, it is well established in continuum mechanics that nonlocal forces resulting from wave overlapping are, in general, of nonpotential (i.e., non-Lagrangian or non-Hamiltonian) type. Again, there is no a priori reason to expect that, in the transition to quantum overlapping waves, this analytic character should be changed.

The alternative (potential or nonpotential character of the nonlocal forces) can be better illustrated in the arena where our intuition is surest, Newtonian mechanics. When a (local) Newtonian system admits only potential forces, the constituents are pointlike and admit only action at a distance. In order to represent the extended character of the objects responsible for contact (e.g., frictional) effects, the forces are selected to be of nonpotential type. We can equivalently say that the *nonpotential* (nonself adjoint; see Appendix A) character of the nonconservative forces is represen tative of the *extended structure* of the objects considered. In the transitioı to classical overlapping waves we have an equivalent situation. In fact, th nonpotential character of the nonlocal forces is representative of the extende character of the waves as well as of the continuum generalization of th contact forces of Newtonian mechanics. In the transition to quantum over lapping waves, gain, there is no a priori reason to expect that this physica characteristic, so deeply related to the actual representation of particles a extended objects, should be abandoned, and more simplistic equation should be assumed.

We reach in this way the representation of the strong interactions vi nonlocal (integrodifferential) field equations as suggested by experimenta facts I, II, and III and as proposed in Ref. 12:

$$\{[(\square + m_{(k)})\phi_k - f_k(\phi, \phi^{\cdot}{}_{\mu})]_{SA} - \int d^4x\, F_k(x, \phi, \phi^{\cdot}{}_{\mu})\}_{NSA} = 0 \qquad (4$$

$$K = 1, 2, \ldots, N; \quad \mu = 0, 1, 2, 3$$

with Newtonian image[11]

$$\left\{[m_{(k)}\ddot{\mathbf{r}}_k - f_k(t, \mathbf{r}, \dot{\mathbf{r}})]_{SA} - \int_V d\mathbf{r}'\, F_k(t, \mathbf{r}, \mathbf{r}', \dot{\mathbf{r}}, \dot{\mathbf{r}}')\right\}_{NSA} = 0 \qquad (5$$

where SA stands for self-adjointness (existence of a Lagrangian) and NSA stands for nonself-adjoiness (lack of general existence of a Lagrangian (Appendix A).

The argument of Ref. 12 is that *Einstein's special relativity become incompatible with the strong interactions when represented via nonlocal force nonderivable from a potential.*

A study of this intriguing situation has indicated that the breakdown o Einstein's special relativity for systems of type (4) in actuality occurs at th level of the mathematical foundations of relativity.

At the analytic level there is the general impossibility of even defining th canonical formalism, trivially, because of the general lack of existence of a Hamiltonian. Thus, the entire conventional analytic context (including it canonical part) is in question for the strong interactions already at the classica level, let alone the quantum mechanical level. It is hoped that the reader car

[11] The following formulation also can be considered:

$$\left\{[m_{(k)}\ddot{\mathbf{r}}_k - \mathbf{f}_k(t, \mathbf{r}, \dot{\mathbf{r}})]_{SA} - \int_V d\mathbf{r}\, \mathbf{K}_k(t, \mathbf{r}, \dot{\mathbf{r}})\right\}_{NSA} = 0$$

begin to see why the canonical quantization rules have been identified as a conjecture in Fig. 2, although the true technical situation will appear at the quantum mechanical level (Section 4).

At the algebraic level there is a general incompatibility of the interactions considered with Lie's theory as currently known, in the sense that the Lie algebras cannot be introduced via the brackets of the Hamiltonian time evolution law. Thus, for the systems considered, the applicability of *all* Lie algebras is in question, let alone the Lie algebra of the Poincaré group.

At the geometrical level there is an incompatibility of the systems considered with the symplectic or contact geometry as currently known, trivially, because the realizations of this geometry in the admissible charts demand local differential equations.

These breakdowns have been indicated in the hope of preventing a simplistic attitude toward making Einstein's special relativity compatibility with systems of type (4). Before such an attitude can acquire scientific value, the interested researcher must first solve a rather substantial array of problems of compatibility of the systems considered with the mathematical foundations of the relativity considered.

In actuality, the inapplicability of Einstein's special relativity for the strong interactions is seen to be rather natural when the *experimentally established* conditions of Fig. 4 are inspected without prejudice. As clearly expressed by Einstein in his limpid writings, his relativity was conceived for pointlike charged particles moving in vacuum (no contact forces!) under an external, long-range electromagnetic field. There is no reason to expect that the relativity that is so effective for such a physical arena must necessarily apply to a fundamentally different physical context. After all, the strong interactions were unknown in 1905! When the fundamental character of the pointlike nature of particles in special relativity is identified, the need for a possible covering relativity for extended particles in conditions of mutual penetration becomes rather natural. Here the term "covering relativity" is intended to express the compatibility condition that a possible generalized relativity must recover special relativity at the limit when all particles are approximated as being pointlike, with the resulting null value of all nonlocal or local nonself-adjoint forces.

A first direct consequence of experimental facts I, II, and III is that the *conventional Lorentz transformation law of fields, Eq. (4), is expected to be incompatible with the strong interactions, first, because it is a local transformation law, and, second, because it is linear. If locality is preserved, at least the linearity should be abandoned to avoid excessive approximations.* In short, the best research attitude is that of searching for a generalization of special relativity and of the transformation law of fields specifically conceived for particles in the experimentally established conditions of Fig. 4.

To initiate this predictably laborious and long-term research project, the attitude of Refs. 7, 12, 14, 17, 18, and 20 has been as pragmatic as possible. It is a fact that virtually all of the methods of contemporary theoretical physics are strictly local in character. The direct transition to nonlocal settings appears to face rather substantial technical difficulties, assuming that they can be treated with existing mathematical and physical knowledge. The attitude advocated was therefore that of approximating nonlocal strong interactions with local models, but in such a way as to account for the extended size of the particles. The answer was provided by knowledge well established in Newtonian mechanics. It is known that nonlocal forces are well approximated in mechanics via local (nonlinear) forces nonderivable from a potential (e.g., via polynomial expansions in the velocities) yielding the Newtonian systems of everyday experience, such as damped oscillators, spinning tops with drag torques, trajectory problems in the atmosphere, etc. All these systems belong to the class of local, variationally nonself-adjoint systems

$$\{[m_k\ddot{\mathbf{r}}_k - f_k(t, \mathbf{r}, \dot{\mathbf{r}})]_{\mathrm{SA}} - F_k(t, \mathbf{r}, \dot{\mathbf{r}})\}_{\mathrm{NSA}} = 0 \tag{6}$$

The nonself-adjoint forces then represent precisely the motion of extended objects in a resistive medium, while the self-adjoint forces represent the action-at-a-distance forces.

Of course, before reaching a position where one can even partially confront the problem of the relativity laws, one has to identify mathematical methods for the treatment of systems of type (6), *at the pure classical level first*. The problem of quantization is only second.

The study of these methods has identified the existence of two different, yet compatible generalizations of Lie's theory, tentatively called:

(A) *Lie-isotopic generalization*, in the (algebraic) sense that the theory is still Lie in character, although the product is the most general possible (regular) form of the Lie algebra product.

(B) *Lie-admissible generalization*, in the (algebraic) sense that the theory is no longer Lie in character. Instead, it is of the covering Lie-admissible[12] character.

Both generalizations of Lie's theory admit a classical (and quantum mechanical) realization, based on corresponding generalizations of Hamil-

[12] A *Lie-admissible algebra* U is an algebra with elements $a, b, c, \ldots$ and (abstract) product ab over a field F (assumed of characteristic zero throughout this paper) such that the attached algebra U^-, which is the vector space U equipped with the product $[a, b]_U = ab - ba$, is a Lie algebra. When ab is associative, U is a conventional Lie algebra. Thus, the notion of associative Lie-admissibility is at the foundation of Lie's theory in its conventional form. When ab is nonassociative, we have the Lie-admissible generalization of Lie's theory studied in Refs. 12, 14–17, 20, 21, 23–33, and 42–46.

ton's (Heisenberg's) equations. The classical analytic equations characterized by the Lie-isotopic generalization of Lie's theory were called *Birkhoff's equations*(18,20) for historical reasons. The classical analytic equations characterized by the Lie-admissible generalization of Lie's theory turn out to be the equations originally conceived by Hamilton, those *with* external terms, although written in an algebraically adequate form (see below). The corresponding quantum mechanical versions of these equations were apparently proposed for the first time in Ref. 12 (see next section).

Both generalizations of Lie's theory are "directly universal" for systems (6), that is, applicable to *all* systems of the class admitted (universality) without redefinition of the coordinates of the experimental detection system (direct universality). But only the Lie-admissible generalization of Lie's theory is directly universal for the most general systems known at this time, the variationally nonself-adjoint integrodifferential systems.

By using the methods available, we proposed (20) a *generalization of Galilei's relativity* for systems (6) which is of Lie-admissible algebraic character. A corresponding quantum mechanical generalization was proposed in Ref. 12. The studies for relativistic (classical and quantum mechanical) extensions are in progress.

Furthermore, by using the broader (bimodular) representation theory of the Lie-admissible algebras of operators, we proposed(12,21) a *generalized notion of particles under strong interactions*, that is, of extended particles under conditions of mutual penetration and overlapping of the wave packets. Finally, this generalized notion of particle was used to propose a *new Lie-admissible model of the structure of hadrons*(12,17) in which the constituents are physical particles that can be produced free under spontaneous decays.

A technical review of these studies would be impossible in this paper. Nevertheless, for the reader's convenience, as well as for subsequent notational needs, it is advisable to outline the truly essential ideas.

3.1. The Main Idea of the Birkhoffian Generalization of Hamiltonian Mechanics(18,20)

When studying systems of type (6) a first step is the identification of the integrability conditions for the existence of a Lagrangian, or, independently,[13] of a Hamiltonian representation. These are the conditions of variational self-adjointness indicated earlier. The second step is the identification of the methods for the construction of a Lagrangian, or, independently, of a Hamiltonian from the equations of motion under the integrability conditions. The third step is the identification of the property that the Lagrange and Hamilton equations of contemporary literature (sometimes called the

[13] That is, without a prior knowledge of a Lagrangian.

"truncated equations"[14]) *are not* directly universal in Newtonian mechanics. In fact, they can represent only a subclass of Newtonian systems in the coordinates of the experimental detection system (Appendix B).

A generalization of Hamiltonian mechanics is clearly necessary to avoid perpetual-motion-type approximations and to achieve the (much needed) direct universality.[15] The generalized mechanics identified by the conditions of variational self-adjointness has the following most salient properties.

Analytic Profile. Second-order systems can be turned into equivalent first-order systems (vector fields) via generally noncanonical prescriptions for the characterization of new independent variables. By using the linear momentum $p = m\dot{r}$, systems (6) can be written

$$(\Omega_{\mu\nu}(t, a)\dot{a}^{\nu} + \Gamma_{\mu}(t, a)) = \begin{pmatrix} h_1 & h_2 \\ h_3 & h_4 \end{pmatrix} \begin{pmatrix} \dot{r} - p/m \\ p - f_{\mathrm{SA}}(t, r, p) - F_{\mathrm{NSA}}(t, r, p) \end{pmatrix} = 0 \quad (7)$$

$$a = (r, p) \in T^*M, \quad r, p \in R^n; \qquad \mu = 1, 2, \ldots, 2n$$

[14] Lagrange and Hamilton appeared to be fully aware that Newtonian forces are generally *nonderivable* from a potential. In fact, they conceived their equations with external terms representing precisely all the forces that cannot be cast into a Lagrangian or a Hamiltonian form. Oddly, it has only been since the beginning of this century that the Lagrange and Hamilton equations have been "truncated" with the removal of the external terms, acquiring the more simplistic form of contemporary use. The researcher, however, should be aware that the truncated equations, unless properly treated and presented, may literally imply the belief in perpetual motion in our environment. Intriguingly, the equations originally conceived by Hamilton *do not* have a Lie algebra structure (see below). In fact, a Lie algebra structure emerges only after the removal of the external terms.

[15] The following theorem of indirect universality of Hamilton's equations for local Newtonian systems has been proved in Vol. II of Ref. 18:

Theorem. All local, analytic, and regular Newtonian systems admit an indirect Hamiltonian representation in a star-shaped region of their variables.

The "indirect Hamiltonian representation" refers here to the representation (in terms of the "truncated" Hamilton equations) of equivalent equations of motion obtained via the use of smoothness and regularity-preserving transformations of the local variables. In essence, when representations (B2) of Appendix B do not exist (in which case the systems are called essentially nonself-adjoint) a conventional Hamiltonian representation can still be obtained via the use of the transformation theory. In this case the coordinates of the representation are (necessarily) dependent in a generally nonlinear way on the old coordinates and the momenta. As such, the coordinates are not realizable with experiments, and the representation acquires a purely mathematical meaning. *This use of Hamilton's equations will be excluded throughout this paper.* Our attitude is that, to avoid insidious traps in the physical interpretation, all abstract mathematical algorithms (H, r, p, etc.) should possess a direct physical interpretation as well as admit the coordinates actually used in experiments (which is not necessarily the case for the theorem reviewed here).

where we have included the multiplication of a regular matrix of factor terms. The conditions of variational self-adjointness for systems (7) are given by

$$\Omega_{\mu\nu} + \Omega_{\nu\mu} = 0 \tag{8a}$$

$$\frac{\partial \Omega_{\mu\nu}}{\partial a^{\tau}} + \frac{\partial \Omega_{\nu\tau}}{\partial a^{\mu}} + \frac{\partial \Omega_{\tau\mu}}{\partial a^{\nu}} = 0 \tag{8b}$$

$$\frac{\partial \Omega_{\mu\nu}}{\partial t} = \frac{\partial \Gamma_{\mu}}{\partial a^{\nu}} - \frac{\partial \Gamma_{\nu}}{\partial a^{\mu}} \tag{8c}$$

Whenever they are satisfied in a (star-shaped) region of the local variables, system (7) is derivable from the most general possible (regular) variational principle for first-order systems, the Pfaffian principle

$$\delta^1 \int_{t_1}^{t_2} dt \, [R_{\mu}(t, a)\dot{a}^{\mu} - B(t, a)] = 0 \tag{9}$$

The underlying analytic equations are *Birkhoff's equations*

$$\begin{gathered} \left[\left(\frac{\partial R_{\nu}}{\partial a^{\mu}} - \frac{\partial R_{\mu}}{\partial a^{\nu}}\right) \dot{a}^{\nu} - \left(\frac{\partial B}{\partial a^{\mu}} + \frac{\partial R_{\mu}}{\partial t}\right)\right]_{\mathrm{SA}} = 0 \\ \det \left(\frac{\partial R_{\nu}}{\partial a^{\mu}} - \frac{\partial R_{\mu}}{\partial a^{\nu}}\right) \mapsto 0 \end{gathered} \tag{10}$$

which admit Hamilton's equations as a particular case, i.e.,

$$\begin{aligned} \left[\left(\frac{\partial R_{\nu}}{\partial a^{\mu}} - \frac{\partial R_{\mu}}{\partial a^{\nu}}\right) \dot{a}^{\nu} - \left(\frac{\partial B}{\partial a^{\mu}} + \frac{\partial R_{\mu}}{\partial t}\right)\right]_{\mathrm{SA}} \Big|_{R=R^0} &= \left(\omega_{\mu\nu}\dot{a}^{\nu} - \frac{\partial H}{\partial a^{\mu}}\right)_{\mathrm{SA}} \\ &= \begin{pmatrix} -\dot{p} - \partial H/\partial r \\ \dot{r} - \partial H/\partial p \end{pmatrix}_{\mathrm{SA}} = 0 \\ B = H; \qquad R^0 = (p, 0); \qquad (\omega_{\mu\nu}) &= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \end{aligned} \tag{11}$$

It is possible to prove that, under sufficient smoothness conditions, there always exists a regular matrix (h) of integrating factors capable of rendering system (7) self-adjoint. This establishes the following property,[18,20] the *direct universality of Birkhoff's equations for local Newtonian systems*:

Theorem 1. All local (real) analytic, regular, Newtonian systems always admit, in a star-shaped neighborhood of a regular point of their variables, a representation in terms of Birkhoff's equations.

The Birkhoffian functions can be computed from (self-adjoint) equations of motion according to

$$
\begin{aligned}
R_\mu(t, a) &= a^\nu \int_0^1 d\tau \, \Omega_{\mu\nu}(t, \tau a), \qquad \Omega_{\mu\nu} = \frac{\partial R_\nu}{\partial a^\mu} - \frac{\partial R_\mu}{\partial a^\nu} \\
B(t, a) &= -a^\mu \int_0^1 d\tau \left(\Gamma_\mu + \frac{\partial R_\mu}{\partial t}\right)(t, \tau a)
\end{aligned}
\tag{12}
$$

Algebraic Profile. A most important property of the *autonomous* Birkhoff equations is that the brackets of the time evolution law

$$
\dot{A}(a) = \frac{\partial A}{\partial a^\mu} \dot{a}^\mu = [A, B]^* = \frac{\partial A(a)}{\partial a^\mu} \Omega^{\mu\nu}(a) \frac{\partial B(a)}{\partial a^\nu}; \quad \Omega^{\mu\nu} = (\| \Omega_{\mu\nu} \|^{-1}) \tag{13}
$$

satisfy the Lie algebra laws. In particular, subsets (8a) and (8b) of the conditions of self-adjointness are equivalent to the necessary and sufficient conditions for brackets (13) to be Lie, i.e.,

$$
\begin{aligned}
\Omega^{\mu\nu} + \Omega^{\nu\mu} &= 0 \\
\Omega^{\mu\rho} \frac{\partial \Omega^{\nu\tau}}{\partial a^\rho} + \Omega^{\nu\rho} \frac{\partial \Omega^{\tau\mu}}{\partial a^\rho} + \Omega^{\tau\rho} \frac{\partial \Omega^{\mu\nu}}{\partial a^\rho} &= 0
\end{aligned}
\tag{14}
$$

The conventional Poisson brackets are clearly obtained as a particular case for $R = R^0$. Birkhoff's equations can therefore be interpreted as a Lie-algebra-preserving generalization of Hamilton's equations. The transition from the Hamilton to the Birkhoff equations is the analytic counterpart of the algebraic notion of *Lie isotopy* of Refs. 18 and 20.

The preservation of the Lie algebra character implies the existence of the integrated form of the (autonomous) Birkhoff equations, i.e.,

$$
G^*(t): \quad a' = \left[\exp\left(t\Omega^{\rho\sigma} \frac{\partial B}{\partial a^\sigma} \frac{\partial}{\partial a^\rho}\right)\right] a \tag{15}
$$

which is clearly a group-preserving generalization of the conventional canonical form

$$
G(t): \quad a' = \left[\exp\left(t\omega^{\rho\sigma} \frac{\partial H}{\partial a^\sigma} \frac{\partial}{\partial a^\rho}\right)\right] a \tag{16}
$$

The Lie-isotopic generalization of Lie's theory proposed in Ref. 20 (see Ref. 18 for a more detailed treatment and Ref. 14 for a recent review) consists of a Lie-isotopic generalization of Lie's first, second, and third theorems, complemented by a Lie-isotopic generalization of the Poincaré–

Birkhoff–Witt theorem.[16] According to this view, the Hamiltonian form (16) is a realization of the conventional formulation of Lie's theory, while the Birkhoffian form (15) is a realization of the Lie-isotopic generalization. Needless to say, these results merely establish the *existence* of a consistent, Lie-isotopic generalization of Lie's theory. Its actual detailed construction in the needed diversified form will call for contributions by a number of researchers.

Geometric Profile. Another important property of Birkhoff's equations is that subsets (8a) and (8b) of the conditions of self-adjointness coincide with the integrability conditions for an exterior two-form Ω_2 on the cotangent bundle T^*M in the local chart $a = (r, p)$ to be an exact symplectic form

$$\Omega_2 = \tfrac{1}{2}\Omega_{\mu\nu}\, da^\mu \wedge da^\nu = dR_1 = d(R_\mu\, da^\mu); \quad d\Omega_2 = 0; \quad \det(\Omega_{\mu\nu}) \neq 0 \quad (17)$$

More generally, the entire set of conditions of self-adjointness (8) coincide with the integrability conditions for a two-form $\hat{\Omega}_2$ on $\mathbb{R} \times T^*M$ to be an exact contact form, i.e.,

$$\begin{gathered}\hat{\Omega}_2 = \tfrac{1}{4}\hat{\Omega}_{ij} da^i \wedge da^j = d\hat{R}_1 = d(\hat{R}_i d\hat{a}^i); \\ d\hat{\Omega}_2 = 0; \quad d\Omega_2 = 0; \quad \hat{\Omega}_2 \,|_{T^*M} = \Omega_2 \\ \hat{a} = (t, a); \quad i = 0, 1, 2, \ldots, 2n; \quad \hat{\Omega}_{0\mu} = \frac{\partial B}{\partial a^\mu} + \frac{\partial R_\mu}{\partial t}; \quad \hat{\Omega}_{\mu\nu} = \Omega_{\mu\nu}\end{gathered} \quad (18)$$

As expected, Birkhoff's equations preserve the geometrical character of Hamilton's equations. We simply have the transition from the exact, fundamental (canonical) two-form

$$\omega_2 = \tfrac{1}{2}\omega_{\mu\nu}\, da^\mu \wedge da^\nu = dR_1{}^0 = d(R_\mu{}^0\, da^\mu) = dp_k \wedge dr^k \quad (19)$$

to the most general possible, but still exact,[17] symplectic structure.

The Birkhoffian generalization of Hamiltonian mechanics therefore can be equipped with a considerable methodological basis, consisting not only of a Lie-isotopic generalization of Lie's theory, but also of the symplectic and contact geometries.(22) Under these conditions, it is an easy prediction

[16] It should be noted here that the name "Birkhoff" in "Birkhoff's equations" and the "Poincaré–Birkhoff–Witt theorem" refer to father and son, respectively, the latter being my colleague at the Department of Mathematics, Harvard University, Prof. Garrett Birkhoff.

[17] The exactness of the forms is needed for the derivability from a variational principle as well as to preserve the geometric character of Hamilton's equations.

that the various parts of Hamiltonian mechanics will be, in due time, generalized into a Birkhoffian form. For initial studies on the Birkhoffian generalization of the canonical transformation theory, symmetries, first integrals, and related topics, we refer the reader to Ref. 18. For initial studies on a Birkhoffian generalization of the Hamilton–Jacobi theory, we refer the reader to Sarlet and Cantrijn.[23] The potential relevance of these studies for the quantum mechanical treatment of systems (6) will be indicated in the next section.

Despite these intriguing and promising properties, the Birkhoffian generalization of Hamiltonian mechanics does not appear to be the final form of mechanics, due to a number of insufficiencies. The most relevant ones are the following.

1. The brackets of the time evolution law for the *nonautonomous* Birkhoff equations

$$\dot{A}(a) = \frac{\partial A}{\partial a^{\mu}} \dot{a}^{\mu} = \frac{\partial A}{\partial a^{\mu}} \Omega^{\mu\nu} \frac{\partial B}{\partial a^{\nu}} + \frac{\partial A}{\partial a^{\mu}} \Omega^{\mu\nu} \frac{\partial R_{\nu}}{\partial_{t}} \stackrel{\text{def}}{=} A \times B \tag{20}$$

do not characterize an algebra as commonly understood (because they violate the joint, right, and left distributive laws and the scalar law). The loss of a consistent algebraic structure is clearly a drawback for the quantitative generalization of a number of conventional notions of Hamiltonian mechanics, particularly those based on Lie algebras (e.g., angular momentum). This deficiency is compounded by the fact that, even for autonomous systems (6), the computable Birkhoffian representations often depend explicitly on time.[18] In this case we cannot treat the systems considered via any algebraic structure (whether Lie or not).

2. Even though Theorem 1 ensures the existence of a Birkhoffian representation, its existence is in practice so complex (even for simple systems with low dimensionality) as to discourage even one most devoted to Lie's theory and symplectic geometry [the systems to be solved, Eq. (8) in the unknown elements $h_{\mu}{}^{\nu}$ for fixed forces, is a quasilinear, often hyperbolic system of partial differential equations].

3. The Newtonian interactions do not necessarily occur at a point, as implied by systems (6) and Eqs. (10). In actuality, physical reality demands the identification of methods which are readily and effectively applicable to arbitrary *nonlocal* systems.

Due to these (and other[18]) problematic aspects, we have sought and identified a covering of Birkhoffian (and, thus, of Hamiltonian) mechanics.

[18] For instance, the function B of Eqs. (10) cannot be, in general, the total energy. It is a mere mathematical quantity assisting in the treatment of the system. In order to differen-

3.2. The Main Idea of the Lie-Admissible Generalization of Hamiltonian Mechanics[17,20]

The most general known unconstrained Newtonian systems in Euclidean space can be written[19]

$$\{[m_k \ddot{\mathbf{r}}_k - \mathbf{f}_k(t, \mathbf{r}, \dot{\mathbf{r}})]_{SA} - \mathbf{F}_k(t, \mathbf{r}, \dot{\mathbf{r}}) - \iiint_V dv' \, \mathbf{G}(t, \mathbf{r}, \mathbf{r}', \dot{\mathbf{r}}, \dot{\mathbf{r}}', \ldots)\}_{NSA} = 0 \tag{21}$$

with a self-evident separation of the local and nonlocal forces derivable and nonderivable from a potential. Typical examples are satellites in the Earth's atmosphere, spinning tops with drag torques, etc.

The equations most effective for the representation of systems (21) are those originally conceived by Hamilton, i.e.,

$$\begin{gathered} \dot{a}^\mu - \omega^{\mu\nu} \, \partial H / \partial a^\nu - F^\mu = 0 \\ a = (r, p), \quad H = T + V, \quad F = (0, F_{NSA}), \quad (\omega^{\mu\nu}) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \\ \mu = 1, 2, \ldots, 2n \end{gathered} \tag{22}$$

rather than their truncated version (11) used in the contemporary physical and mathematical literature. In fact, Eqs. (22) possess the following properties:

(a) They are directly universal for all Newtonian systems, irrespective of their smoothness, regularity, locality, and other properties.

(b) The representation of systems (21) via Eqs. (22) is truly simple. The Hamiltonian represents the kinetic energy and the potential energy of all self-adjoint forces. All nonpotential forces, whether local or not, are represented via the external terms.

(c) Unlike the Birkhoffian case (see footnote 18), all mathematical algorithms of Eqs. (22) possess a clear and direct physical meaning. In particular, the Hamiltonian H of Eqs. (22) can always represent the total energy. Also, the variables t, r, and p can represent time, coordinates, and linear momenta in the actual experimental detection of the system.

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tiate the function B of Eqs. (10) from the function H of Eqs. (11), we called the former the *Birkhoffian.*[18,20] The lack of a direct physical meaning of the Birkhoffian is clearly insidious on physical grounds, particularly for quantum mechanical considerations, as we shall see better in Section 4.

[19] The notion of nonself-adjointness is considered here to be inclusive of that of self-adjointness. Thus, nonlocal, non-self-adjoint forces include nonlocal, self-adjoint forces.

In Ref. 20 it is noted that the brackets of the time evolution law of Eqs. (22)

$$\dot{A}(a) = \frac{\partial A}{\partial a^\mu} \dot{a}^\mu = \frac{\partial A}{\partial a^\mu} \omega^{\mu\nu} \frac{\partial H}{\partial a^\nu} + \frac{\partial A}{\partial a^\mu} F^\mu \stackrel{\text{def}}{=} A \times H \tag{23}$$

do not constitute an algebra as commonly understood.[20] We therefore suggested a simple modification of the way in which the external forces are written, in order to ensure the existence of a consistent algebraic structure. The reformulation of Eqs. (22) proposed in Ref. 20 reads

$$\left(\dot{a}^\mu - S^{\mu\nu}(t, a) \frac{\partial H(t, a)}{\partial a^\nu}\right) = \begin{pmatrix} \dot{r} - \partial H/\partial p \\ \dot{p} + \partial H/\partial r + s\, \partial H/\partial p \end{pmatrix} = 0,$$

$$r_k = s_{ki} \frac{\partial H}{\partial p_i}, \qquad s = s^T$$

$$S^{\mu\nu} = \omega^{\mu\nu} + T^{\mu\nu}; \qquad T^{\mu\nu} = T^{\nu\mu}; \qquad (T^{\mu\nu}) = \begin{pmatrix} 0 & 0 \\ 0 & -s \end{pmatrix} \tag{24}$$

with a solution for all systems (21)

$$s = \text{diag} \frac{F + \iiint dv\, G}{p/m} \tag{25}$$

and equivalent covariant form

$$S_{\mu\nu} \dot{a}^\nu - \frac{\partial H}{\partial a^\mu} = 0; \qquad S_{\mu\nu} = \omega_{\mu\nu} + T_{\mu\nu} = (\| S^{\mu\nu} \|^{-1})_{\mu\nu}; \qquad (T_{\mu\nu}) = \begin{pmatrix} -s & 0 \\ 0 & 0 \end{pmatrix} \tag{26}$$

It is easy to see that the brackets of the time evolution law of Eqs. (24)

$$\dot{A}(a) = \frac{\partial A}{\partial a^\mu} \dot{a}^\mu = \frac{\partial A}{\partial a^\mu} S^{\mu\nu} \frac{\partial H}{\partial a^\nu} \stackrel{\text{def}}{=} (A, H) \tag{27}$$

satisfy the right and left distributive laws as well as the scalar law. Thus, they characterize a consistent algebra. Notice that, unlike the Birkhoffian case, this consistent algebraic structure exists for all systems (21).

Next, we proceeded to the identification of the algebra characterized by brackets (A, H), which are Lie-admissible because the attached algebra with brackets

$$(A, H) - (H, A) = 2[A, H]_{\text{Poisson}} \tag{28}$$

[20] This is a situation similar to that of Eqs. (20).

is Lie. In this algebraic way, the truncated Hamilton equations are the attached equations of the true Hamilton equations. As a result, the truncated equations lose their fundamental role in mechanics in favor of the equations along the original unrestricted vision by (Lagrange and) Hamilton.

Since Lie-admissible algebras are known to be algebraic coverings of Lie algebras, the Lie-admissible character of Eqs. (24) permitted the identification of a new generalization of Hamiltonian mechanics. The most salient aspects are the following.

Analytic Profile. Equations (26) are derivable from the following variational principle (under sufficient smoothness and other conditions ignored here):

$$\begin{aligned} &\delta^* \int_{t_1}^{t_1} dt\, [R_\mu(t, a)\dot{a}^\mu - B(t, a)] \\ &= \int_{t_1}^{t_2} dt \left[\left(\frac{\partial R_\nu}{\partial a^\mu} - \frac{\partial R_\mu}{\partial a^\nu}\right) \dot{a}^\nu - \left(\frac{\partial B}{\partial a^\mu} + \frac{\partial R_\mu}{\partial t}\right)\right]_{\text{SA}} \delta^* a^\mu \\ &= \int_{t_1}^{t_2} dt \left(S_{\rho\sigma}(t, a)\dot{a}_\sigma - \frac{\partial H}{\partial a^\rho}\right)_{\text{NSA}} \delta a^\rho = 0; \qquad \delta^* a = g(t, a)\, \delta a \qquad (29) \end{aligned}$$

tentatively called a *genotopically mapped Hamilton principle* (in the sense that the transition $\delta \to \delta^*$ induces a nonself-adjoint structure). The matrix (g) of Eqs. (29) is the inverse of the matrix (h) of factor functions which are needed to turn nonself-adjoint equations (26) into an equivalent self-adjoint form. The universality of principle (29) follows from the universality of the existence of this equivalence transformation.[18]

Algebraic Profile. The integrability conditions for a tensor $S^{\mu\nu}$ to characterize Lie-admissible brackets (A, H) are given by

$$\begin{aligned} &(S^{\mu\rho} - S^{\rho\mu}) \frac{\partial}{\partial a^\rho} (S^{\nu\tau} - S^{\tau\nu}) + (S^{\nu\rho} - S^{\rho\nu}) \frac{\partial}{\partial a^\rho} (S^{\tau\mu} - S^{\mu\tau}) \\ &\quad + (S^{\tau\rho} - S^{\rho\tau}) \frac{\partial}{\partial a^\rho} (S^{\mu\nu} - S^{\nu\mu}) = 0 \qquad (30) \end{aligned}$$

with general solution

$$S^{\mu\nu} = \Omega^{\mu\nu} + T^{\mu\nu}; \qquad \Omega^{\mu\nu} = \left(\left\| \frac{\partial R_\nu}{\partial a^\mu} - \frac{\partial R_\mu}{\partial a^\nu} \right\|^{-1}\right)^{\mu\nu}; \qquad T^{\mu\nu} = T^{\nu\mu} \qquad (31)$$

Thus, the Lie brackets attached to the Lie-admissible brackets are given, ir general, by Birkhoff brackets, i.e.,

$$(A, H) - (H, A) = 2[A, H]^*_{\text{Birkhoff}} \tag{32}$$

In this sense, Eqs. (24) are a generalization not only of Hamilton's equations but also of Birkhoff's equations. In particular, the latter emerge as being th*attached* equations to the most general possible Lie-admissible equations

Despite the nonpower-associativity of the algebra,[14] Eqs. (5) admit ar integration to the connected group

$$G_1(t): \quad a = \left[\exp\left(tS^{\mu\nu}\frac{\partial H}{\partial a^\nu}\frac{\partial}{\partial a^\mu}\right)\right] a \tag{33}$$

which *is not of Lie type, because it admits a non-Lie, Lie-admissible algebr in the neighborhood of the origin*, as the reader can verify.

The existence of connected groups of transformations of this typ permitted the identification of a further generalization of Lie's theory Initial studies on the Lie-admissible generalization of Lie's first, second, anc third theorems were conducted in Ref. 20 (see Ref. 17 for a detailed treatmen and Ref. 14 for a recent review), following the generalization of the Poincaré-Birkhoff-Witt theorem to nonassociative, flexible, Lie-admissible algebra by Ktorides.[24] Subsequent studies of the generalization were conducted b Ktorides *et al.*[25] and Myung and Santilli.[26] The initiation of the representation theory of the Lie-admissible algebras was done in Ref. 21. The studies or the nonassociative, Lie-admissible generalization of Nelson's integrabilit conditions were conducted by Ghikas *et al.*[27]

Needless to say, these results establish the existence of a consistent Lie-admissible generalization of Lie's theory. Its actual construction in the neede diversified form will call for the participation of a number of researchers Nevertheless, this line of study is in rapid expansion, and several additiona advances are expected.

Geometric Profile. The symplectic geometry is incompatible witl nonassociative Lie-admissible algebras. In fact, the geometry can only b realized via totally antisymmetric forms, while the product of Lie-admissibl algebras is neither totally antisymmetric nor totally symmetric.

For this reason there was proposed[20] (see Ref. 14 for a recent review a generalization of the symplectic geometry tentatively called *symplectic admissible geometry*. It is the geometry of the tensorial two-forms whos attached exterior forms are (exact and) symplectic, and whose contravarian versions characterize Lie-admissible algebras. Explicitly, we have the two-forms

$$S_2 = S_{\mu\nu} da^\mu \otimes da^\nu = \tfrac{1}{2}(S_{\mu\nu} - S_{\nu\mu})\, da^\mu \wedge da^\nu + \tfrac{1}{2}(T_{\mu\nu} + T_{\nu\mu})\, da^\mu \times da^\nu$$
$$\det(S_{\mu\nu}) \neq 0; \qquad \det(S_{\mu\nu} - S_{\nu\mu}) \neq 0; \qquad dS_2 \neq 0 \tag{34}$$

with attached, exact, symplectic two-forms

$$S_2 - S_2^{\text{Alt}} = (S_{\mu\nu} - S_{\nu\mu})\, da^\mu \wedge da^\nu = 2\Omega_2 = 2dR_1\,; \qquad d\Omega_2 = 0 \tag{35}$$

The integrability conditions for symplectic-admissibility are given by

$$\frac{\partial}{\partial a^\tau}(S_{\mu\nu} - S_{\nu\mu}) + \frac{\partial}{\partial a^\mu}(S_{\nu\tau} - S_{\tau\nu}) + \frac{\partial}{\partial a^\nu}(S_{\tau\mu} - S_{\mu\tau}) = 0 \tag{36}$$

while Eqs. (30 are the integrability conditions for joint Lie-admissibility.[21] The extension to a *contact-admissible geometry* is self-evident and will be ignored here.

Some initial properties of these generalized geometries were identified in Ref. 20 (see also Refs. 14 and 17). The classification of all tensors satisfying the joint conditions of symplectic-admissibility and Lie-admissibility was achieved by Sarlet.[(28)] A study of the diffeomorphisms on the cotangent bundle which preserve symplectic-admissible two-forms was conducted by Cantrijn.[(29)] Again, these studies merely indicate the *existence* of a consistent generalization of the symplectic (and contact) geometry capable of geometrizing the Lie-admissible algebras, with the understanding that much work remains to be done.

What is important for subsequent needs (particularly for quantum mechanical considerations) is that the symplectic geometry turns out to be the *attached geometry* to a more general geometry. In different terms, the studies considered here confirmed that the symplectic geometry is not the final geometry of mechanics, as expected. Even though not necessarily unique, the geometry capable of treating directly the most general known systems is nonsymplectic, but symplectic-admissible. In particular, the terms of this broader geometry which break the symplectic character [the *symmetric* terms of two-forms (34)] are representative of local and nonlocal forces

[21] If a covariant tensor $S^{\mu\nu}$ is symplectic-admissible, its contravariant version $S^{\mu\nu} = (\| S_{\mu\nu} \|^{-1})^{\mu\nu}$ is not necessarily Lie-admissible and vice versa. Nevertheless, the joint symplectic-admissibility and Lie-admissibility exists and it is directly universal for systems (21). In fact, this is the case for Eqs. (24).

nonderivable from a potential, that is, of the forces outside conventional treatments of the symplectic geometry.[22]

Equations (24) with a joint symplectic-admissible and Lie-admissible character were tentatively called in Ref. 20 *Hamilton-admissible equations*. The following theorem of direct universality of Hamilton-admissible equations in mechanics was established in Ref. 20 for local Newtonian systems, and later extended in Ref. 14 to arbitrary nonlocal systems.

Theorem 2. The most general possible variationally nonself-adjoint integrodifferential Newtonian systems always admit a representation in terms of Hamilton-admissible equations.

The direct universality of the Lie-admissible algebra for the characterization of the time evolution law in Newtonian mechanics was subsequently extended to (1) classical statistical mechanics, by Fronteau *et al.*[(30)]; (2) classical field theory, by Kobussen[(31)]; and (3) quantum mechanics, by Santilli.[(14)] It is likely that the direct universality of the Lie-admissible algebras also will be established in other branches of physics.

The main ideas of the Lie-admissible covering of Galilei's relativity for local, nonconservative, and Galilei form-noninvariant systems are essentially the following.

Canonical group (16) *is* the "time component" of Galilei's group in canonical realization, i.e.,

$$G(3.1):\quad a' = \left[\exp\left(\theta_k \omega^{\mu\nu} \frac{\partial X_k}{\partial a^\nu} \frac{\partial}{\partial a^\mu}\right)\right] a, \qquad k = 1, 2, \ldots, 10 \tag{37}$$

where the X's are the familiar generators (total energy, linear momentum, etc.) and the θ's are the corresponding parameters (time, coordinates, etc.). For *local, autonomous, and conservative* systems, group (6) is a *relativity group*, that is, it provides a form-invariant description of physical laws. In particular, the time component $G_1(t)$ of $G(3.1)$ satisfies the following properties.

[22] The validity of the symplectic geometry for the electromagnetic interactions, as well as for the unified gauge theories of weak and electromagnetic interactions, is well known. We proposed that the symplectic-admissible geometry represent the joint weak, electromagnetic, and strong interactions in such a way that the former (latter) are realized as local (nonlocal) interactions. According to this proposal, the *antisymmetric* part of the symplectic-admissible forms represents the weak and electromagnetic interactions (as currently known) plus the self-adjoint component of the strong. The symmetric part of the two-forms represents the nonself-adjoint local and nonlocal components of the strong interactions, that is, the part representative of the *extended character* of hadrons. The proposal was motivated by the intent to *differentiate* (rather than unify) electromagnetic and strong interactions, according to the rather profound physical differences manifested in nature.

(A) The equations of motion in their first-order form are form-invariant under the group $G_1(t)$, i.e.,

$$\Xi^\mu(a) \to \Xi'^\mu(a') = \left(\frac{\partial a'^\mu}{\partial a^\rho} \Xi^\rho\right)(a(a')) \equiv \Xi^\mu(a'), \qquad (\Xi^\mu) = \begin{pmatrix} p/m \\ f_{\mathrm{SA}}(r) \end{pmatrix} \tag{38}$$

(B) The Hamiltonian H is form-invariant under $G_1(t)$, i.e.,

$$H'(a') = H(a(a')) \equiv H(a') \tag{39}$$

(C) When restricted in the neighborhood of the identities ($t \to dt \approx 0$), the symmetry $G_1(t)$ characterizes conservation laws, e.g.,

$$\dot{H} = [H, H] \approx \left\{H - \left[\exp\left(dt\, \omega^{\mu\nu} \frac{\partial H}{\partial a^\nu} \frac{\partial}{\partial a^\mu}\right)\right] H\right\} / dt \equiv 0 \tag{40}$$

When passing to local nonconservative systems, properties (A), (B), and (C) are generally inapplicable on both conceptual and technical grounds. On conceptual grounds, the emphasis is now on the representation of the *time rate of variation* of physical quantities (rather than their conservation).[23] On technical grounds, nonconservative Newtonian forces are generally noninvariant under Galilei's transformations. In short, when excessive approximations are abandoned, one can see that the violation of Galilei's relativity is the rule in Newtonian mechanics, and its preservation is the exception. This creates the need to identify the relativity which is applicable in *Newtonian* mechanics (no relativistic or gravitational extensions!) under unrestricted forces.

The proposed covering of Galilei's relativity, called *Galilei-admissible relativity*,[(20)] is based on the transition from connected, space–time, Lie symmetries to connected, space–time, Lie-admissible symmetries, that is, connected transformation groups constituting symmetries of vector fields which admit non-Lie, Lie-admissible algebras in the neighborhood of the identity. The transition from Lie to Lie-admissible symmetries has the following implications. First, it allows the representation of the time rate of variation of physical quantities (which is precluded in Lie symmetries). Second, it allows the realization of a covering relativity, that is, a generalized relativity capable of recovering conventional relativity identically at the limit for null Galilei-relativity-breaking forces (the external forces of the true Hamilton equations).

[23] Notice that the conservation is a particular case of the time rate of variation (but the converse does not hold). This is an important point of the covering character of generalized relativity.

The proposed Galilei-admissible group reads[14,17,20]

$$G(3.1):\quad a' = \left[\exp\left(\theta_k S_k^{\mu\nu} \frac{\partial X_k}{\partial a^\nu} \frac{\partial}{\partial a^\mu}\right)\right] a \tag{41}$$

where the generators X_k and the parameters θ_k coincide with those of Galilei's group,[24] and $S_k^{\mu\nu}$ are Lie-admissible tensors (generally different for differen generators).

When (41) is a symmetry of the system considered, it is assumed as a covering relativity group, that is, a symmetry of the vector field capable o characterizing nonconservative physical laws.

For the local, autonomous, nonconservative systems, Hamilton-admissible structure (33) is the time component of the group (41). It is possible tc prove[14,17,20] that for *all* systems of this type, the group (33) exists (universality) and satisfies the following properties (covering relativity).

(A′) The nonconservative equations of motion in their vector fielc form are invariant under $\hat{G}_1(t)$ (form-invariant description)

$$\hat{\Xi}^\mu(a) \to \hat{\Xi}'^\mu(a') = \left(\frac{\partial a^\mu}{\partial a^\rho} \hat{\Xi}^\rho\right)(a(a')) \wedge \hat{\Xi}(a'), \qquad \hat{\Xi} = \begin{pmatrix} p/m \\ f_{\text{SA}} + F_{\text{NSA}} \end{pmatrix} \tag{42}$$

(B′) The Hamiltonian H is noninvariant under $\hat{G}_1(t)$ (trivially, becaus it is now nonconserved), i.e.,

$$H'(a') = H(a(a')) \neq H(a') \tag{43}$$

(C′) When restricted to the neighborhood of the identity, the symmetry $\hat{G}_1(t)$ characterizes the time rate of variation of physical quantities as experimentally established in Newtonian mechanics; e.g., for the energy we have

$$\begin{aligned} \dot{H}(a) &\approx \left\{H - \left[\exp\left(dt\, S^{\mu\nu} \frac{\partial H}{\partial a^\nu} \frac{\partial}{\partial a^\mu}\right)\right] H\right\} \Big/ dt \\ &\approx (H, H) = \dot{\mathbf{r}} \cdot \mathbf{F}_{\text{NSA}} \end{aligned} \tag{44}$$

with similar expressions for other quantities.

Notice that when all nonself-adjoint forces are null, the Lie-admissible algebras, groups, and relativity recover identically the conventional Lie

[24] Physical quantities, such as the angular momentum, are *unique* in the sense that they do not depend on possible nonpotential forces. These forces merely affect the *behavior in time* but not the definition of, say, $\mathbf{M} = \mathbf{r} \times m\dot{\mathbf{r}}$. For this reason Galilei-admissible relativity was constructed in terms of generators with a direct physical meaning, those of conventional relativity.

algebras, groups, and relativity. For explicit examples see the quoted references. For the construction of the Lie-admissible symmetry from the equations of motion see Prince *et al.*[32]

We return now to the main topic of this section, the problem of the validity or invalidity of Galilei's and Einstein's special relativity for the strong interactions.

The studies via the Lie-isotopic and the Lie-admissible generalizations of Lie's theory have indicated that, at the classical level, *Galilei's and Einstein's special relativity are violated whenever the actual size, shape, and structure of the constituents of a system cannot be ignored.* In particular, the following different physical situations have emerged.

(A) When a system of classical "particles" is considered in vacuum under only action-at-a-distance forces, Galilei's (or Einstein's special) relativity apply unequivocally. This is the case, say, for our solar system in Newtonian approximation. The actual size, shape, and structure of the constituents do not affect the dynamics. Then, along the ideas of Newton and Galilei, the sun and the planets can be approximated as being *massive points.* In turn, points can only have local, action-at-a-distance forces. The applicability of Galilei's relativity is the result. A fully similar situation occurs for the motion of charged particles *in vacuum* under external (and/or mutual) electromagnetic fields. The validity of Einstein's special relativity is, in this case, incontrovertible.

(B) When a system according to specifications (A) enters a dissipative medium (say a gas), conventional relativities are violated conceptually and technically. In fact, Galilei noninvariance must be now admitted as a necessaty condition to allow forces more general than $f = -\partial V/\partial r$. The inapplicability of Einstein's special relativity can then be derived via simple compatibility arguments. Independently, one can prove that the solutions of the nonself-adjoint field equations (4) do not necessarily transform covariantly under the Lorentz group.[12] Conceptually, the inapplicability of Einstein's special relativity can be read off in available pictures of particle tracks. These pictures show the motion of charged particles in an external magnetic field along down-spiraling trajectories. This establishes the nonconservation of the angular momentum and, thus, a form of the breaking of the Lorentz symmetry, trivially, because the $SU(2)$ symmetry is contained in the $SL(2.C)$ symmetry.

In the transition to quantum mechanical or quantum field-theoretic settings, the situation is expected to remain the same because of the correspondence principle. In any case, if Eqs. (4) do not necessarily transform covariantly under the Lorentz group, this feature is not affected by the interpretation of the fields as operator-valued distributions.

In conclusion, the quantum mechanical version of Newtonian systems (6) and the quantum field-theoretic version of systems (4) are expected to violate Galilei's and Einstein's special relativities, respectively, and to demand the construction of covering quantum mechanical relativities specifically conceived under a genuine representation of the extended character of the particles (see next section).

All the remarks of this section are intended as classical, priliminary considerations for the strong interactions in general (e.g., a hadron going through nuclear matter, thereby experiencing forces more general than the trivial $f = -\partial V/\partial r$ due to wave overlappings). More specifically, nonconservative Newtonian systems are considered in the Lie-admissible literature as a classical limit of dissipative nuclear phenomena.

The problem of hadronic structure is different and more complex due to the need to achieve a closed system (i.e., a system satisfying the usual conservation laws of total quantities) despite the presence of nonself-adjoint internal forces. This problem also has been studied in the literature and will be considered in the next section.

In this section we have essentially considered only one argument indicating the possible invalidity of Einstein's special relativity for the strong interactions: the impossibility of ignoring the extended character of hadrons for the strong interactions. The reader should be informed that there exist a number of additional arguments, all leading to the same conclusions, some of which consist of a nonlocal reinterpretation of the experimentally established violation of discrete symmetries. For brevity, we refer the interested reader to the article by Kim.[34]

The situation is such that these arguments simply cannot be ignored in the literature of strong interactions. The most scientifically effective appraoch is that both quark supporters and quark opponents submit these arguments to a close theoretical scrutiny, with the understanding that the final resolution will be produced by experiments (Section 5) and not by theoretical considerations only. This approach appears necessary to avoid the ineffective, compartmentalized conduct of research.

We close this section with some comments on gravitational considerations. One of the criticisms of quark models is the following. The assumption that Einstein's special relativity is valid within a hadron can be, in the final analysis, unrealistic from simple gravitational considerations. Indeed, the assumption considered is equivalent to the assumption that the description of, say, a neutron star undergoing a phase transition into hadronic constituents must be described via Einstein's *special* relativity. After all, the transition from one hadron to such a neutron star is purely a matter of size. Einstein's special relativity should at least be abandoned in favor of the general form. But this demands the abandonment of the Lorentz group as a covariance group of physical meaning, in favor of an arbitrary covariance group. In turn, this recovers again, this time from gravirational considerations, the arguments for the invalidity of the Lorentz covariance law for hadronic constituents.

But the gravitational situation may well turn out to be a topic of controversy even greater than that of quarks, and would call for a rather long separate paper. Here we simply caution the open-minded and receptive researcher that the transition from Einstein special to general relativity does not appear to be sufficient to truly account for the nature of the strong interactions. A simple inspection of the equations of available gravitational models indicates that *nonlocal strong interactions nonderivable from a potential are directly incompatible with Einstein's general theory for the interior problem (only), as well as with all available models for the interior problem of which we are aware (e.g., of Weyl type, of supersymmetric type, etc.).* All these models are simply not set for the representation of integrodifferential systems, at least not in the form conventionally referred to. A study conducted in Ref. 17 indicates that this incompatibility persists also when the interactions considered are approximated with local nonself-adjoint forces, because of the breakdown of a number of technical ingredients of conventional models (e.g., curvature as deviation from geodesic motion; Riemannian character of the geometry; etc.).

A dichotomy fully parallel to that for a hadron then emerges for the problem of gravitation. The experimental evidence for the validity of the exterior problem of Einstein's general theory is considerable.[25] Yet, there is no direct and incontrovertible evidence on the validity of the corresponding model for the interior problem. The case of the interior problem must therefore be considered open at this time on both theoretical and experimental grounds. On the latter grounds, after all, we are not in a position to conduct an experiment, say, inside a star. On the former grounds the issue appears to be primarily a question of research attitude. Current models are by and large constructed via the use of mass tensors. But these tensors are a technical expedient to avoid the problem of structure. When a researcher attempts the construction of a model for the interior problem of gravitation under the *condition* that mass terms should not be used and the problem of the structure is instead confronted, a new perspective emerges.[26] The strong

[25] It should be recalled here that, even for the *exterior* problem, Einstein's general relativity is affected by numerous and rather substantial problematic aspects. For a resolution and treatment of these aspects we refer the interested reader to Yilmaz.(35)

[26] In Ref. 36 we studied the gravitational field of π^0 approximated as two oppositely charged particles in highly dynamical conditions. In the exterior of the π^0 the electromagnetic field is *nonnull* (for charges in motion), even though the total charge is null. This is in *conflict* with Einstein's gravitation (for which the source of the field is null

interactions become dominant, on methodological grounds, for the interior problem, in the sense that one does not first postulate a gravitational model and then search for a compatible realization of the strong interactions. Instead, one first identifies the most general possible nature of the strong interactions as suggested by physical reality, and then searches for a compatible gravitational model. When the gravitational problem for the interior of astrophysical objects is approached from this point of view, the abandonment of available models and the search for covering models appears to be unavoidable. In any case, *no model of the interior problem of gravitation should be considered physically consistent unless it provides an effective representation of "simple" interior systems, such as a satellite in the Earth's atmosphere.* This test is failed by all gravitational models known to this author (because of nonself-adjoint forces, lack of derivability from a conventional variational principle, etc.).

It is hoped that the reader is aware of the implications of these remarks For instance, most of the recent advances in astrophysics related to the interior problem (e.g., black holes) become questionable in their current formulation.

This state of affairs suggested completing the analysis of Vol. I of Ref. 1 (p. 487) with the following statement.

Contention 1. Galilei's relativity, Einstein's special relativity, and Einstein's general relativity for the interior problem (only) are incompatible with the strong interactions.

4. THE PROBLEM OF THE ARENAS OF VALIDITY AND INVALIDITY OF QUANTUM MECHANICS IN PARTICLE PHYSICS

We have outlined a number of criticisms of quark models. All these criticisms were of a qualitative, epistemological nature. Our presentation would be incomplete without an outline of criticisms of a quantiative nature This is the area of the greatest disagreement in the community of basic research. I hope that the receptive reader will appreciate my effort to present these criticisms in as clear a way as possible. The sole intent is that the clarity of presentation may be of assistance in identifying the disagreements, as a prerequisite for possible resolutions by quark supporters via counterarguments.

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in the exterior of neutral matter), but in *agreement* with Yilmaz' gravitation.[35] Therefor Ref. 36 established that Einstein's gravitation can hold only under the condition tha the charges of matter constituents are at rest, contrary to experimental evidence.

The objectives of this section are the following:

(I) To present the view that, under the validity of the unitary, Mendeleev-type classification of hadrons, quark models do not provide a quantitative, consistent structure model of each individual element of a unitary multiplet.

(II) To indicate that the primary difficulties of quark models are due to problematic aspects of the underlying discipline, quantum mechanics, when applied to the strong interactions.

(III) To recall recent studies according to which, under a covering of quantum mechanics specifically conceived for the strong interactions, a quantitative, consistent structure model has been apparently achieved for the lightest hadrons,[12,14] with a conceivable extension to the remaining hadrons.[10,17] Intriguingly, this structure model appears to be compatible with the unitary models of classification, but incompatible with the conjecture that quarks are physical particles.

Objective I can be expressed via the following:

Contention 2. Under the assumption of the validity of quantum mechanics for the strong interactions, the quark models do not provide a consistent, quantiative structure model of hadrons.

The first source of controversy may result from the requirements that a hadron model be of (a) structure type, (b) quantitative, and (c) consistent. We shall therefore attempt to elucidate this definition as clearly as possible, beginning with its area of applicability. The model we shall use as reference is one established on rather solid experimental and theoretical grounds, the model of atomic structure.

When the problem of the structure of atoms was confronted, the idea of first studying heavy or superheavy atoms, say, palladium, was foreign to the minds of the founding fathers of contemporary physics. Instead, utmost priority was given to the problem of the lightest atom known at that time, the hydrogen atom. The attitude implemented in Contention 2 is exactly the same. The idea is that the fundamental character of the problem of hadronic structure is possessed by the lightest hadron known at this time, the π^0. To be more in line with quark models, we can say that the fundamental information on hadronic structure is possessed by the octet of light mesons. Only after the problem of the structure of these particles has been resolved in final form can the problem of the *structure* (rather than classification) of heavy and superheavy hadrons be attacked in a truly effective way, that is, other than of conjectural character.

This is an area of great controversy. Indeed, a number of (although not all) quark supporters favor the idea that superheavy hadrons have a simpler structure than do light mesons. This attitude is understood and respected by quark opponents. Nevertheless, the opposite possibility, that the heavy or superheavy hadrons will eventually turn out to possess a structure substantially more complex than that of the light mesons, cannot be ruled out on the basis of our current knowledge. By now, it is a historical rule that the complexity of physical systems increases with mass, as already established at the atomic and nuclear levels. There is little evidence to suspect that a different situation will emerge for the hadrons. In any case, at the level of superheavy hadrons the question of whether quarks are elementary or composite does not appear to be readily solvable, while a different situation exist at the level of the light mesons, trivially, because of the lack of energies to cluster constituents.

In summary, the first step for the intended definition of a structure model is its restriction to the light mesons only, with the exclusion of baryons and all other handrons. The problem of the structure of the light mesons is considered to be of fundamental character on the basis of a comparison with the corresponding atomic and nuclear problems, as well as in the hope of gaining valuable insights for the problem of whether quarks can exist as elementary constituents of hadrons.

After having identified the arena of applicability of the definition considered, the next problem is that of identifying the desired meaning of the terms "structure model," "quantitative model," and "consistent model." Let us begin by pointing out a distinction between "classification model" and "structure model."

The predictive power of the Mendeleev classification of atoms is an historically established fact. The predictive power of the unitary classification of hadrons is equally established, in our view. In both the atomic and the hadronic classifications we had specific cases of predictions of new structures. In particular, the predictions included specific values of physical quantities of the states, such as mass, total angular momentum, etc. Also, and most importantly, in both cases the predictions were based on available information on *different* states, as typical of the classification. In summary, total physical quantities of a state (say, the helium or the $\pi^{\pm}$) can be represented within a pure classification setting.[27] On the contrary, a structure model is provided

[27] One of the questions raised in the recent literature[12,14] is whether the physical predictions, results, and insights of the unitary models truly need the assumption that quarks are physical particles (joint model of classification and structure) or not (model of classification only). In different terms, the prediction of the Ω^- particle can be apparently reconstructed without any assumption whatever that quarks are physical particles (Fig. 1). Is this implementable to the J/ψ particle and other insights? The view expressed in this paper is that *lack* of identification of quarks with physical particles is *essential* to avoid a number of inconsistencies (see later) which would otherwise cast an unnecessary shadow on the results of the unitary models of classification.

by the assumption of specific physical particles obeying specific dynamical equations and specific physical laws. For instance, we have a structure model of the hydrogen atom whenever we assume that it is a bound state of one physical electron and one physical proton obeying a given dynamics (say, nonrelativistic quantum mechanics or quantum electrodynamics and related physical laws). Quark models, as conventionally referred to, are intended to be structure models in this strict sense. For instance, a (naive) nonrelativistic structure model of the π^0 is a bound state of one quark and an antiquark assumed as physical particles.

Next, we pass to the definition of a "quantitative structure model." When the problem of the structure of the hydrogen atom was confronted, priority was given to a quantitative representation of the model of structure, that is, a representation of the model via equations which could be then confronted with physical reality.

The statement of the quantitative model of structure of the hydrogen atom can then be formulated as follows. It consisted of (A) the identification of the constituents (one electron and one proton); (B) the identification of the nature of their intereactions (the self-adjoint Coulomb force); and (C) the construction of a quantitative model of structure via Schrödinger's equation (as well as, later on, its relativistic extension) capable of representing *all* total physical quantities of the atom (e.g., mass, angular momentum, size), as well as *all* spectral lines (the Lyman, Balmer, Paschen, Brackett, and Pfund series).[28]

Finally, we consider the definition of a "consistent structure model." The achievement of a quantiative representation of *all* known data on the hydrogen atom *was not yet* sufficient for the scientific community of the early part of this century. In fact, the underlying discipline used in the model, quantum mechanics, was subjected to a critical scrutiny for theoretical self-consistency. This rather important critical process is vividly depicted in a number of accounts, such as the book dedicated to Alfred Landé.[37] Only *after* the discipline considered had passed a critical inspection for its applicability in the arena considered did the model of the hydrogen atom became finally established. The addition of the term "consistent" in Contention 2 is primarily intended to stress that the critical inspection of theoretical self-consistency must be resumed for the hadrons, because the self-consistency of quantum mechanics for the electromagnetic interactions can under no

[28] A rather crucial historical point should be recalled here for later need. Bohr's model became established because it represented *all* the data considered. For instance, a model of structure which represented, say, only the Lyman and Balmer series, but not the remaining series, would have been rejected by the scientific community of the time. The advent of Einstein's special relativity, Dirac's equations, and the Bethe–Salpeter equation provided all the well-known technical refinements.

circumstance be considered as evidence of its self-consistency for the *different* physical arena of the strong interactions. And in fact, rather substantial reasons for doubt exist, as we shall see.

A further reason for adding the term "consistent" in Contention 2 is related to quark confinement. This issue is multifold. First, the *spontaneous decays*

$$\text{mesons} \to q + \bar{q} + \cdots; \qquad \text{baryons} \to q + q + q + \cdots \tag{45}$$

do not exist according to all available experimental evidence. As a result, quark models of hadron structure admitting spontaneous decays (45) without a null (or at least sufficiently small) fraction are in disagreement with available physical knowledge and should be considered inconsistent. Second, the inelastic scatterings

$$\text{hadrons} + \text{hadrons} \to \text{quarks} + \text{antiquarks} + \cdots \tag{46}$$

do not occur according to all available low- and high-energy studies. As a result, quark models permitting scatterings (46) within current experimental feasibility should also be considered as invalid.

A still further reason for adding the term "consistent" in Contention 2 is the need of a detailed study of compatibility among different aspects, methods, and insights of the quark models. For instance, a possible radical modification of the structure of space–time (to attempt confinement) should be proved to be compatible with the notion of fermion (to preserve the currently desired notion of quark), or the methods for unitary symmetry-breaking used for the construction of the Gell-Mann–Okubo mass formula should be compatible with the dynamics.[(14)]

To summarize, according to the view expressed in this paper, a quantitative, consistent, structure model of the light mesons should satisfy the following (rather reasonable) conditions patterned along the solid grounds of what is held for atomic structure:

(A) The hadronic constituents are specific physical particles obeying specific dynamical equations and specific physical laws.

(B) The dynamical equations provide a quantitative representation of all the intrinsic data of the particles as listed by the Particle Data Group[(38)] plus the unlisted[29] charge radius, as well as all modes of the spontaneous decays, and all related fractions.

[29] The Particle Data Group does not currently list the charge radius (or size) of hadrons. This is regrettable on technical and historical grounds. On technical grounds, no structure model of hadrons can be considered as conclusive unless it represents quantitatively not only the size of hadrons, but also their lack of increase with mass. On historical

(C) The model is consistent both intrinsically and with regard to the assumed methodology. In particular, the fractions of decays (45) and (46) should be explicitly computed and proved to be either null or very small, to reach any level of credibility.

We present now a number of arguments supporting the view that, first, quark models do not constitute "quantitative structure models" in the sense indicated above, and, second, whether this is the case or not, quark models do not constitute "consistent structure models."

The masses of the light mesons can be quantitatively represented via the Gell-Mann–Okubo mass formula. Nevertheless, a Schrödinger-type equation capable of representing the same masses has not yet been achieved, despite a considerable search.[(39)] This problem is exactly along the dichotomy classification/structure focused upon in Ref. 12. In fact, the Gell-Mann–Okubo mass formula is a manifestation only of the classification approach (because it does not need physical constituents obeying given dynamical equations and physical laws).

Let us recall the historical point mentioned earlier according to which a structure model of the hydrogen atom that would have accounted for only some, but not all, spectral lines would have been rejected by the scientific community of the early part of this century.

Along these lines, the problem is whether or not (nonrelativistic) quark models represent quantitatively *all* experimental data currently available for *all* hadrons. This issue is (at least) two-fold. Indeed, we have the problem of representing all the intrinsic quantities and all data related to decay modes.

In relation to the first problem, there is no doubt that available quark models can represent several intrinsic characteristics both physical (e.g., spin) and chemical (isospin). However, there is equally no doubt that available quark models *do not* represent *all* these characteristics. One which is crucial for the resolution of the issue if the charge radius. This physical quantity is a genuine representative of the structure.[30] Its quantitative representation therefore demands structure-type (rather than classification-type) equations. In different terms, the size of hadrons is a manifestation of the internal dynamics of each individual hadron and not of the relationship of the particle

grounds, one of the first physical characteristics of the hydrogen atom that Bohr attempted to represent was the size. It is of course true that the charge radius is known sufficiently well only for a few hadrons. But, then, its listing is valuable to stimulate experiments on the missing values, as well as to emphasize the need that theoreticians include this physical quantity in their models. I have requested the Particle Data Group to include the charge radius of hadrons in their future listings.

[30] Recall that the Mendeleev classification of atoms is not expected to produce a quantitative representation of the size of, say, palladium.

with the others of the same multiplet of classification. We therefore conclude that quark models do not account for all known intrinsic characteristics of hadrons. It should be stressed that this is independent of the issue of whether we have a classification model only, or a joint classification and structure model. But then, the only possibility to save unitary models is that of restricting their physical meaning to classification only, and complementing them with *different*, yet compatible structure models (see later in this section).

But the resistance of quark supporters to even consider these arguments is well known. We therefore pass to the question of the quantitative representation of *all* decay models and *all* related fractions of (at least) *all* light mesons. An inspection of the available experimental data clearly indicates that this is not an easy task. Actually, a nonrelativistic quark model capable of representing all these data simply does not exist.[31]

But all these problematic aspects are still of preliminary character. To reach a deeper insight we have to inspect the current two-body nonrelativistic quark models.

The Hamiltonians are essentially constructed via (1) the use of potentials admitting Regge behavior (from stringlike considerations), (2) a gradual potential interpolating smoothly the short- and long-range regions as well as representing the nonrelativistic limit of gluon exchange; and (3) fine structure terms (which are essential for meaningful mass spectra). Apart from minor differences, the Hamiltonians have structures of the type[(40)]

$$\begin{aligned} G^{\text{quark}} &= T(\mathbf{p}) + V(\mathbf{r}) + H_{\text{FS}}(\mathbf{r},\ \mathbf{p},\ \mathbf{s},\ \mathbf{M},\ldots) \\ &= \frac{1}{2}\,\mathbf{p}^2 - \frac{8}{27}\,\frac{1}{\log(2/r+2)}\,\frac{1}{r} + (1-e^{-ar})\,\frac{r}{2\pi} \\ &\quad -\frac{1}{8}\left(\frac{1}{m_1{}^3}+\frac{1}{m_2{}^3}\right)(\mathbf{p}^2)^2 - \frac{4}{3}\,g_{\text{strong}}\left[-\frac{1}{2m_1m_2}\left(-\frac{\mathbf{p}^2}{r}\right) - \frac{(\mathbf{r}\cdot\mathbf{p})^2}{r^3}\right)\Big] \\ &\quad -\frac{\pi}{2}\,\delta^3(r)\left(\frac{1}{m_1{}^2}+\frac{1}{m_2{}^2}\right) + \frac{16\mathbf{S}_1\cdot\mathbf{S}_2}{3m_1m_2} \\ &\quad -\frac{1}{2r^3}\left[\frac{1}{m_1{}^2}\,\mathbf{S}_1 + \frac{1}{m_2{}^2}\,\mathbf{S}_2 + \frac{2}{m_1m_2}\,(\mathbf{S}_1+\mathbf{S}_2)\right] \\ &\quad \cdot\mathbf{M} + 2\mathbf{S}_1\cdot\mathbf{S}_2 + \frac{6(\mathbf{S}_1\cdot\mathbf{r})(\mathbf{S}_2\cdot\mathbf{r})}{r^2}\Big] + \cdots \end{aligned} \tag{47}$$

A number of inconsistencies of these models have recently come to light. The first is that the models imply the violation of the conservation of total

[31] The available data establish the existence of 45 different decays for the octet of light mesons, plus an additional 32 different decays which are expected, have been looked for, but have not yet been detected (see Table I).

Table I. The Technical Origin of Some of the Controversies in Hadron Physics[a]

$\{\pi^0 : 1^-, 0^-, +, 135, 0.8 \times 10^{-16}, \sim 1\ F\}, \{\pi^\pm : 1^-, 0^-, ..., 139, 2.6 \times 10^{-8}, \sim 1\ F\}$
$\{K^\pm : \frac{1}{2}, 0^-, ..., 439, 1.2 \times 10^{-8}, \sim 1\ F\}, \{K_S^0 : \frac{1}{2}, 0^-, ..., 498, 0.9 \times 10^{-10}, \sim 1\ F\}$
$\{K_L^0 : \frac{1}{2}, 0^-, ..., 498, 5.1 \times 10^{-8}, \sim 1\ F\}, \{\eta : 0^+, 0^-, +, 549, \Gamma = 0.8\ \text{keV}, \sim 1\ F\}$

$\pi^0 \to \gamma\gamma$, 98.8 %	$K^\pm \to e\pi^0\nu\gamma$, 3.7×10^{-4}
$\pi^0 \to \gamma e^+e^-$, 1.15 %	$K^\pm \to \pi e^+e^-$, 2.6×10^{-7}
$\pi^0 \to \gamma\gamma\gamma$, $<5 \times 10^{-6}$	$K^\pm \to \pi^\mp e^\pm e^\pm$, $<1.5 \times 10^{-5}$
$\pi^0 \to e^+e^-e^+e^-$, 3.3×10^{-5}	$K^\pm \to \pi\mu^+\mu^-$, $<2.4 \times 10^{-6}$
$\pi^0 \to \gamma\gamma\gamma\gamma$, $<6 \times 10^{-5}$	$K^\pm \to \pi\gamma\gamma$, $<3.5 \times 10^{-5}$
$\pi^0 \to e^+e^-$, $<2 \times 10^{-6}$	$K^\pm \to \pi\gamma\gamma\gamma$, $<3.0 \times 10^{-4}$
$\pi^\pm \to \mu\nu$, 100 %	$K^\pm \to \pi\nu\nu$, $<0.6 \times 10^{-6}$
$\pi^\pm \to e\nu$, 1.2×10^{-4}	$K^\pm \to \pi\gamma$, $<4 \times 10^{-6}$
$\pi^\pm \to \mu\nu\gamma$, 1.2×10^{-4}	$K^\pm \to e\pi^\mp\mu^\pm$, $<2.8 \times 10^{-8}$
$\pi^\pm \to \pi^0 e\nu$, 1.02×10^{-8}	$K^\pm \to e\pi^\pm\mu^\mp$, $<1.4 \times 10^{-8}$
$\pi^\pm \to e\nu\gamma$, 3×10^{-8}	$K^\pm \to \mu\nu\nu\nu$, $<6 \times 10^{-6}$
$\pi^\pm \to e^+e^-\nu$, $<3.4 \times 10^{-8}$	$K_S^0 \to \pi^+\pi^-$, 68.7 %
$K^\pm \to \mu\nu$, 63.6 %	$K_S^0 \to \pi^0\pi^0$, 31.3 %
$K^\pm \to \pi\pi^0$, 21.05 %	$K_S^0 \to \mu^+\mu^-$, $<3.2 \times 10^{-7}$
$K^\pm \to \pi\pi^-\pi^+$, 5.6 %	$K_S^0 \to e^+e^-$, $<3.4 \times 10^{-4}$
$K^\pm \to \pi\pi^0\pi^0$, 1.7 %	$K_S^0 \to \pi^+\pi^-\gamma$, 2.0×10^{-3}
$K^\pm \to \mu\pi^0\nu$, 3.2 %	$K_S^0 \to \gamma\gamma$, $<0.4 \times 10^{-3}$
$K^\pm \to e\pi^0\nu$, 4.8 %	$K_L^0 \to \pi^0\pi^0\pi^0$, 21.4 %
$K^\pm \to \mu\nu\gamma$, 5.8×10^{-3}	$K_L^0 \to \pi^+\pi^-\pi^0$, 12.2 %
$K^\pm \to e\pi^0\pi^0\nu$, 1.8×10^{-5}	$K_L^0 \to \pi\mu\nu$, 27.1 %
$K^\pm \to \pi\pi^\pm e^\pm\nu$, 3.7×10^{-5}	$K_L^0 \to \pi e\nu$, 39.0 %
$K^\pm \to \pi\pi^\pm e^\pm\nu$, $<5 \times 10^{-7}$	$K_L^0 \to \pi e\nu\gamma$, 1.3 %
$K^\pm \to \pi\pi^\pm\mu^\pm\nu$, 0.9×10^{-5}	$K_L^0 \to \pi^+\pi^-$, 0.2 %
$K^\pm \to \pi\pi^\pm\mu^\pm\nu$, $<3.0 \times 10^{-6}$	$K_L^0 \to \pi^0\pi^0$, 0.09 %
$K^\pm \to e\nu$, 1.5×10^{-5}	$K_L^0 \to \pi^+\pi^-\gamma$, 6.0×10^{-5}
$K^\pm \to e\nu\gamma$, 1.6×10^{-5}	$\eta \to \gamma\gamma$, 38.0 %
$K^\pm \to \pi\pi^0\gamma$, 2.7×10^{-4}	$\eta \to \pi^0\gamma\gamma$, 3.1 %
$K^\pm \to \pi\pi^+\pi^-\gamma$, 1×10^{-4}	$\eta \to 3\pi^0$, 29.9 %
$K^\pm \to \mu\pi^0\nu\gamma$, $<6 \times 10^{-5}$	$\eta \to \pi^+\pi^-\pi^0$, 23.6 %
$K_L^0 \to \pi^0\gamma\gamma$, $<2.4 \times 10^{-4}$	$\eta \to \pi^+\pi^-\gamma$, 4.9 %
$K_L^0 \to \gamma\gamma$, 4.9×10^{-4}	$\eta \to e^+e^-\gamma$, 0.5 %
$K_L^0 \to e\mu$, $<2.0 \times 10^{-9}$	$\eta \to \pi^0 e^+e^-$, <0.04 %
$K_L^0 \to \mu^+\mu^-$, 1.0×10^{-8}	$\eta \to \pi^+\pi^-$, <0.15 %
$K_L^0 \to \mu^+\mu^-\gamma$, $<7.8 \times 10^{-6}$	$\eta \to \pi^+\pi^-e^+e^-$, 0.1 %
$K_0^L \to \mu^+\mu^-\pi^0$, $<5.7 \times 10^{-5}$	$\eta \to \pi^+\pi^-\pi^0\gamma$, $<6 \times 10^{-4}$
$K_L^0 \to e^+e^-$, $<2.0 \times 10^{-9}$	$\eta \to \pi^+\pi^-\gamma\gamma$, <0.2 %
$K_L^0 \to e^+e^-$, $<2.8 \times 10^{-5}$	$\eta \to \mu^+\mu^-$, 2.2×10^{-5}
$K_L^0 \to \pi^0\pi^\pm e^\pm\nu$, $<2.2 \times 10^{-3}$	

[a] In this table we have listed most of the experimental data on the octet of light mesons from the particle data[38] with only one addition, the inclusion of the (approximate) charge radius of the particles. The data are divided into two groups: those for total

angular momentum. The proof of this is straightforward. Assume Hamiltonians (47) as classical and compute explicitly the equation of motion. It is then easy to prove that, along these equations,

$$\frac{d}{dt}\mathbf{M}_{\text{tot}} = \frac{d}{dt}(\mathbf{r} \times m\dot{\mathbf{r}}) \neq 0 \tag{48}$$

For consistency under the correspondence principle, the nonconservation of the angular momentum must persist at the quantum mechanical level.

The reason for (48) is that the mathematical algorithm "**p**" of the models by no means represent the *physical* linear momentum $\mathbf{p}_{\text{phys}} = m\dot{\mathbf{r}}$. Instead, it is the *canonical* momentum $\mathbf{p}_{\text{can}} = \partial L/\partial\dot{\mathbf{r}}$, which (for the case considered) is a rather complex operator functional on the physical linear momentum, i.e., $\mathbf{p}_{\text{can}} = \mathbf{f}(\mathbf{p}_{\text{phys}}, \mathbf{r})$, as the reader is encouraged to verify.

Also, the belief that invariance under the group of rotations necessarily implies the conservation of the physical angular momentum has been disproved in the literature.[41] In essence, the $SO(3)$ invariance of Hamiltonians (47) does imply the existence of first integrals (from Noether's theorem). The point is that these first integrals *do not* represent the physical, total angular momentum.

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quantities of each particle $\{I^G, I^P, C_n, m, \tau, R\}$, and the decay modes with related fractions. Most of the data have been approximated here because decimal values are inessential for the topic under consideration. The reason for the controversy under consideration is that, irrespective of whether quarks exist or not, quark models do not account, in their currently available form, for *all* the data listed in this table via a quantitative model, that is, via a solvable equation of structure. The solubility of the model is inferred from the fact that, for quark models at the nonrelativistic level, all the particles considered are a two-body system of one quark and an antiquark. Quark supporters infer that the light hadrons should be treated via relativistic and field-theoretic models. Quark opponents infer that a consistent model should be reached first at the nonrelativistic level, for the relativistic and field-theoretic extensions to acquire the needed credibility. In addition, to reach a final character, the model should be able to provide a quantitative representation of all total quantities, as well as all decay modes and the related fractions known. Last, but not least, in order to avoid a primary inconsistency, the model should provide confinement, that is, should provide a quantum mechanical representation of an unstable bound state with an identically null (and *not* approximately null) probability of tunnel effect of the constituents. The analysis of Ref. 12 has expressed the view that all these requirements are not only incompatible with the quark models, but are actually incompatible with the fundamental discipline used in the quark models, the conventional quantum mechanics, as indicated by the lack of consistent equations of structure, as well as the lack of confinement in the necessary strict sense defined above. In conclusion, the reasons for technical disagreements on quark models are not of minor character. Instead, they relate to the truly fundamental physical laws used in quark models, in this case, the conventional quantum mechanical laws.

An additional inconsistency of models (47) is that they *violate the conservation of total linear momentum*, i.e.,

$$\frac{d}{dt}\,\mathbf{p}^{\text{tot}}_{\text{phys}} = \frac{d}{dt}\,m\dot{\mathbf{r}} \neq 0 \tag{49}$$

as the reader is encouraged to verify.

A still further inconsistency is that the total physical energy $H^{\text{tot}}_{\text{phys}}$ (the sum of the kinetic energy and the potential energy of all self-adjoint forces) *is not* conserved for models (47). As a result, the nonrelativistic quark models currently under study represent open, nonconservative, bound states of conjectural) particles (under an equally conjectural confinement).

An understanding of the latter demands a knowledge of the integrability conditions for the existence of a potentiel.[18] Once these conditions are known, one can see that, to be consistent with Newtonian mechanics, a potential can at most be linear in the velocities (or canonical momenta), i.e. (Appendix *A*),

$$V = A^k(t, \mathbf{r})\,p_k + B(t, \mathbf{r}) \tag{50}$$

as typical for the electromagnetic interactions. As a result, the fine structure terms $H_{\text{fs}}(\mathbf{r}, \mathbf{p}, \mathbf{s}, \mathbf{M}, \ldots)$ of Hamiltonians (47) are by no means "potentials." On the contrary, they are bona fide generalized Hamiltonians (due to their polynomial structure, higher than two in the canonical operators). More generally, Hamiltonians (47) have a generalized structure of the (classical, nonsymmetrized) type (Appendix *B*),

$$\begin{aligned} H_{\text{can}} &= H_{\text{int,I}}(t, \dot{\mathbf{r}}, \mathbf{p})H(\mathbf{p}) + H_{\text{int,II}}(t, \dot{\mathbf{r}}, \mathbf{p}) \\ H_{\text{free}} &= (1/2m)\,\mathbf{p}^2, \qquad H_{\text{int,II}} = A^k(t, \dot{\mathbf{r}})p_k + B(t, \dot{\mathbf{r}}) \end{aligned} \tag{51}$$

with the *necessary* presence of multiplicative interactions terms (to attempt mass formulas). In turn, the multiplicative terms are representative precisely of nonconservative forces nonlinear in the velocities, as the reader can verify via *the explicit computation of the equations of motion* (*rather than the often tacit restriction to the Hamiltonian only*) or the study of the methodology of the inverse problem[18] (see Appendices A and B and footnote 15 for some basic elements). To summarize, the available nonrelativistic quark models do not provide a quantiative representation of all the intrinsic characteristics of hadrons and are unable to represent all decay modes and related fractions, and further, the fine structure terms which are necessary to attempt mass spectra imply the nonconservation of the total physical angular momentum, linear momentum, and energy.

Notice that these inconsistencies hold for all hadrons, whether light or heavy. Notice also that these are still preliminary inconsistencies of the models considered. Additional inconsistencies, which are considered more substantial by quark opponents, are those related to the expected lack of confinement in the strict form indicated earlier. In fact, the probability of tunnel effects of quark constituents for models (47) is not expected to be identically null, *once computed explicitly*. As a result, models (47) are expected to predict decays (45) and scatterings (46) with free quarks according to a fraction within current experimental capabilities, contrary to experimental evidence.[32]

But, as we shall see in a moment, all these inconsistencies and problematic aspects are still of preliminary character, because of the existence of still deeper problems at the level of the basic physical laws.

The problematic aspects of nonrelativistic quark models are generally dismissed by quark supporters on the grounds that a "true" structure model must be, first of all, relativistic, and second, quantum field theoretic. Quark opponents disagree. Nonrelativstic quantum mechanics provides an approximate, but fully consistent description of the structure of atoms, even though the peripheral electrons are definitely in relativistic conditions. Unless consistency is achieved at the nonrelavistic level, there is little hope of establishing it at only the quantum field-theoretic level.[33] This is due to the difficulties of solving (generally nonlinear) field equations and achieving a quantitative structure model in the sense recalled earlier. As a matter of fact, in the transition from nonrelativistic quark models to *QCD* the controversies on the problem of structure *increase*, rather than decrease.[34] To avoid a prohibitive length of this paper, we shall therefore abstain from entering into this topic, and we refer the interested reader to Section 2.4 of Ref. 14.

The situation emerging from these considerations is clearly such as to warrant a moment of reflection on the fundamental discipline used in quark models: Quantum mechanics. Recall that the application, in the early part of this century, of previously established (classical) knowledge to the new problem of atomic structure resulted in a chain of insurmountable controversies and inconsistencies. The view advocated in this paper is that the

[32] This expectation is due to the fact that tunnel effects are a natural consequence of the basic laws of quantum mechanics, particularly for gradual potentials.

[33] By contrast, the new structure model for hadrons proposed in Ref. 12 (to be reviewed later in this section) is based on the existence not only of a consistent nonrelativistic limit, but also of a meanigful *classical* limit, in exactly the same way as it occurs for the electromagnetic interactions.

[34] To begin with, the way in which the fundamental analytic equations, Lagrange's equations, are written in QCD is *erroneous*. See footnote 4.

current studies on hadron structure are much along the same historical pattern. In fact, the application to the *new* problem of hadronic structure of the quantum mechanical knowledge previously established for atomic structure is resulting in a proliferation of perhaps insurmountable controversies and inconsistencies.

As is well known, the resolution of the problem of atomic structure demanded the courageous construction of a new discipline, specifically conceived for the problem considered. It may well be that the resolution of the problem of hadronic structure will also call for the construction of a generalization of quantum mechanics (and related relativities), specifically conceived for the new problem considered. In different terms, the view advocated in Ref. 12 is that the now vexing problem of the existence or nonexistence of quarks is only the *symptom* of a much more fundamental problem of consistency at the level of the basic physical laws.

We reach in this way the primary objective of this section: the review of the arguments indicating a conceivable invalidity for the hadronic structure of the discipline so effective for the atomic structure. These arguments can be effectively presented by outlining the main ideas of the structure model of hadrons proposed in Ref. 12. The understanding is that the proposal was based on the acknowledgment of a substantial increase in complexity, and, thus, in *departures* from conventional lines, in the transition from atomic to hadronic structure. Another understanding is that the arguments are tentative and, as such, in need of critical inspection by independent researchers.

Some reliable information on a hadron is that the particle is composite (i.e., it is a bound state of constituents) and, when isolated from the rest of the universe, it constitutes a closed system (i.e., it satisfies the conservation laws of total physical quantities).

By returning for a moment to the Newtonian level, it is generally believed that closed composite systems necessarily have conservative (self-adjoint) internal forces, i.e., the equations describing the system are given by

$$(m_{(k)}\ddot{\mathbf{r}}_k + \partial V/\partial \mathbf{r}^k)_{\mathrm{SA}} = 0 \qquad k = 1, 2, \dots, N \tag{52a}$$

$$\phi_s(t, \mathbf{r}, \dot{\mathbf{r}}) = 0, \qquad s = 1, 2, \dots, 10 \tag{52b}$$

where the ϕ's represent the conventional total conservation laws (i.e., are first integrals of the equations of motion).

This belief was disproved in Ref. 12, Section 3.4. The argument is quite simple. There is no doubt that closed self-adjoint systems of type (52) exist in nature (e.g., our planetary system for an outside observer). Nevertheless, Newtonian physical reality establishes the existence of more general closed systems (called in Ref. 12 closed nonself-adjoint systems) with *nonpotential internal forces*. A clear example is provided by the Earth considered as isolated from the rest of the universe. The total energy and other total physical quantities are conserved. Nevertheless, the internal forces are generally of nonpotential type (different views are equivalent to a belief in perpentual motion in our environment).

The technical realization of this situation proposed in Ref. 12 [Eqs (3.4.2)] is given by

$$[(m_{(k)}\mathbf{r}_k + \partial V/\partial \mathbf{r}^k)_{\mathrm{SA}} - \mathbf{F}_k(t, \mathbf{r}, \dot{\mathbf{r}})]_{\mathrm{NSA}} = 0, \qquad k = 1, 2,..., N \tag{53a}$$

$$\phi_s(t, \mathbf{r}, \dot{\mathbf{r}}) = 0, \qquad s = 1, 2,..., 10 \tag{53b}$$

in which case the total conservation laws are bona fide subsidiary constraints to the equations of motion. The consistency of the model for the case of two bodies was studied in detail, and that for n bodies was indicated via the existence theory of overdeterlined systems of differential equations. Subsequently, the model was reinspected[30,33] and its consistency established (asymptotically) also on statistical grounds.

In short, the conservation of the total energy can be accounted for not only via conservative internal forces, but, more generally, via nonpotential internal forces. These latter forces merely express a form of exchange of energy (and other quantities) among the constituents of a composite system in a way more general than that allowed by the trivial forces $f = -\partial V/\partial r$ and in a way compatible with total conservation laws.

In the transition to "quantum mechanics" the situation is fully analogous. As a matter of fact, the generalized structure model of hadrons proposed in Ref. 12 was patterned in such a way as to admit the Earth as a crude classical image. In essence, when the bound state is such that the mutual distances are greater than the charge radius of the constituents (e.g., the view of positronium in Fig. 5) we have a closed quantum mechanical system with potential internal forces (in fact, it can be described via a Hamiltonian and the use of Heisenberg's equations). On the contrary, when the bound state occurs under conditions of mutual penetration (e.g., the schematic representation of the π^0 in Fig. 5), the internal forces are expected to be of nonlocal nonpotential type to account for wave overlapping (Section 3). In this case we have a closed "quantum mechanical" system with nonpotential internal forces, which is conceptually equivalent (apart from problems of quantization) to the structure of the Earth. The rather wide spread prejudice that only potential forces can occur in the microscopic world was ignored in Ref. 12 and it will be ignored here. In fact, the prejudice restricts all forces to action at a distance and prevents a quantitative treatment of the physical reality of wave overlapping, as elaborated in Section 3.

One way to indicate the expected existence of closed nonpotential systems in the microscopic world is the following. When a classical particle is under

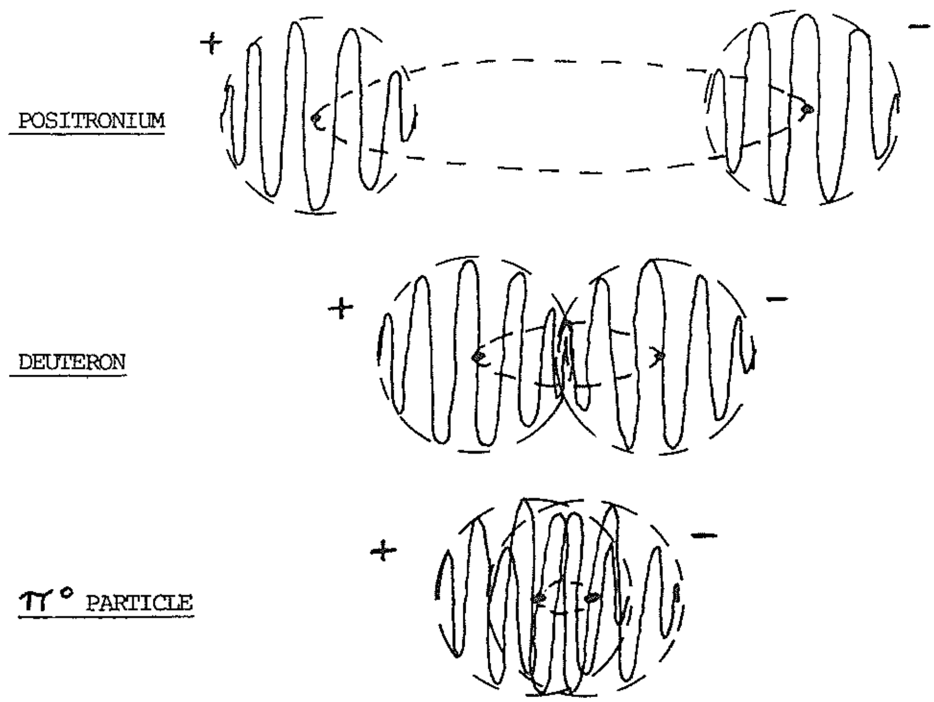

Fig. 5. The epistemological origin of some of the controversies in contemporary hadron physics. In essence, quantum field theory, which is strictly local in the available formulation, has proved to possess an unequivocal value as well as a clear physical effectiveness for the electromagnetic interactions, with particular reference to the part of this discipline called quantum electrodynamics (QED). After a number of implementations during the recent decades, the discipline has been applied to the strong interactions, resulting in more recent times in a theory called quantum chromodynamics (QCD). These implementations, however, have left completely unchanged the original local character of the theory, as well as its relativity character, that of satisfying Einstein's special relativity. The primary reason for controversy under the considerations here is that such a methodological context can be considered valid for the Mendeleev-type classification of hadrons, but it provides an excessive approximation when used to attempt a joint model of structure. This is due to the fact that, on the one hand, incontrovertible experimental data establish that strong interactions occur under conditions of penetration of the charge volumes and of overlapping of the wave packets. This necessarily calls for a nonlocal theory representing the interaction at all points of wave overlapping, and the local approximation representing the interactions at a few isolated points becomes excessive. On the other hand, Einstein's special relativity necessarily calls (for self-consistency) for pointlike approximations of particles. Equivalently, the forces permitted by relativity (the covariant potential forces) can only represent action at a distance, and cannot permit an effective representation of the new contact effects resulting from mutual penetration of particles. In summary, the true reasons for controversy are not of minor character. Instead, they relate to the fundamental physical laws used by QCD for the definition of quarks, in this case, Einstein's special relativity.

the action of potential forces, its vector field is Hamiltonian, and the time evolution law is canonical, i.e., of type (16). Thus, for a closed self-adjoint system, not only the system as a whole, but actually each one of its constituents evolves according to a canonical time evolution.

The quantum mechanical image of a canonical transformation is given, for consistency under the correspondence principle, by a unitary transformation. We reach in this way an important property of atomic structure: *Not only the atom as a whole evolves in time according to a unitary law, but actually each constituent evolves according to the same law.* For instance, if we consider one electron of an atomic cloud (and consider the rest of the system as external), its physical quantity $\tilde{A}$ (say, the spin) evolves according to the unitary law[35]

$$\begin{aligned} \tilde{A}'_{\text{const}} &= [\exp(it\tilde{H}_{\text{const}})]\tilde{A}_{\text{const}} \exp(-it\tilde{H}_{\text{const}}) \\ \tilde{H}_{\text{const}} &= \tilde{H}^{\dagger}_{\text{const}} \end{aligned} \tag{54}$$

In the transition to closed, nonself-adjoint systems we have an equivalent situation, although of generalized character. When a classical particle is under the action of nonpotential forces, its vector field in the physical variables $\mathbf{r}$ and $\mathbf{p} = m\dot{\mathbf{r}}$ is non-Hamiltonian, and the time evolution law is noncanonical, i.e., of the type (33). Thus, *each constituent of the closed, nonself-adjoint systems* (53) *evolves according to a noncanonical time evolution law.* Of course, the evolution of the system as a whole must be computed on the hypersurface of the constraints (53b) and turns out to be canonical (for consistency with the conservation of the total energy) (Appendix C). The point is that a total canonical time evolution by no means necessarily implies that each constituent of the system also evolves according to a canonical law.

The "quantum mechanical" image of systems (53) must therefore imply that, for consistency under the correspondence principle, *each constituent of a bound state of mutually overlapping particles evolves according to a nonunitary time evolution of the type*

$$\begin{aligned} \tilde{A}'_{\text{const}} &= \exp(it\tilde{Z}_{\text{const}})\, \tilde{A}_{\text{const}} \exp(-it\tilde{Z}_{\text{const}}) \\ \tilde{Z}_{\text{const}} &\neq \tilde{Z}^{\dagger}_{\text{const}} \end{aligned} \tag{55}$$

Notice that, since the forces admitted are of non-Hamiltonian type, the operator $\tilde{Z}$ of Eq. (55) *cannot* be interpreted as the Hamiltonian (energy) of the particle. The total time evolution of the closed system as a whole of course is unitary (Appendix C). The point is that a total unitary time evolution law

[35] Quantum mechanical operators will be denoted with a tilde.

by no means necessarily implies that each constituent also must evolve according to a unitary law. For instance, for the case of a closed, nonpotential, two-body system one may have

$$\begin{gathered}\tilde{Z}_i = \tilde{H}_i + \tilde{Y}_i\,, \quad i = 1, 2; \qquad \tilde{H}_{\text{tot}} = \tilde{H}_1 + \tilde{H}_2 = \tilde{H}^{\dagger}_{\text{tot}} \\ \tilde{H}_i = \tilde{H}_i^{\dagger}; \qquad \tilde{Y}_i \neq \tilde{Y}_i^{\dagger}; \qquad \tilde{Y}_1 = -\tilde{Y}_2\end{gathered} \tag{56}$$

In this simple case the total time evolution is unitary; yet, that of each constituent is not.

Next, it is important to understand the relationship between the character of the time evolution and that of the forces admitted.

When the time evolution in the physical coordinates $\mathbf{r}$ and $\mathbf{p} = m\dot{\mathbf{r}}$ is restricted to be canonical, all forces must be of conservative type. A *necessary condition* for the admission of nonpotential forces is that the time evolution law is noncanonical.[36] A fully equivalent situation exists at the quantum mechanical level. In fact, if the time evolution law is restricted to be unitary (and the algorithm "**p**" actually represents the physical linear momentum[(12)]), all forces are restricted to be of conservative type. In this case we have only action at a distance, and we fail to represent the interaction at all points of wave overlapping. A *necessary* condition to admit nonpotential (nonlocal) forces is therefore that the time evolution law is nonunitary, in full analogy with the classical situation.

Once these factors are understood, the fundamental question is: *What are the implications of nonunitary time evolutions for the basic physical laws of quantum mechanics?* The answer is intriguing and distressing at the same time: *All conventional physical laws of quantum mechanics (e.g., Pauli's exclusion principle, Heisenberg's indeterminacy principle, de Broglie's wavelength principle, Einstein's frequency law, etc.) are invalidated by nonunitary time evolutions.*

Consider the case of Pauli's principle. As is well known, a prerequisite fot the applicability of the principle is that the particles are fermions. In turn, the notion of fermion implies that the $SU(2)$ spin symmetry

$$\begin{gathered}[\tilde{J}_i\,, \tilde{J}_j] = \epsilon_{ijk}\tilde{J}_k \\ \tilde{\mathbf{J}}^2\,|\rangle = \tilde{J}_k\tilde{J}_k\,|\rangle = s(s+1)|\rangle; \qquad s = 0, 1/2, 1,...; \qquad \hbar = 1\end{gathered} \tag{57}$$

is exact for each particle, and the spin is characterized by half-odd-integer

[36] If the condition that the local coordinates $\mathbf{r}$ and $\mathbf{p} = m\dot{\mathbf{r}}$ have a direct physical meaning if lifted, the condition is no longer necessary. In fact, one can have in this case indirect Hamiltonian representations as originating from the theorem of Appendix B. But then "**p**" does not represent the physical linear momentum. To avoid insidious traps, this possibility is excluded throughout this paper, classically and quantum mechanically.

value of s (say, $s = 1/2$). However, under a nonunitary transformation, the $SU(2)$ spin symmetry is *broken.*[12,42] In fact, we have the rules of Lie algebra isotopy

$$\exp(it\tilde{Z})\,[\tilde{J}_i, \tilde{J}_j]\exp(-it\tilde{Z}_\dagger) = [\tilde{J}_i', \tilde{J}_j']^* = \tilde{J}_i'\tilde{C}\tilde{J}_j' - \tilde{J}_j'\tilde{C}\tilde{J}_i'$$
$$\tilde{J}_i' = \exp(it\tilde{Z})\,\tilde{J}\exp(-it\tilde{Z}^\dagger); \qquad \tilde{C} = \exp(it\tilde{Z}^\dagger)\exp(-it\tilde{Z}) \neq 1 \tag{58}$$

which are based on the isotopy of the enveloping algebra

$$\tilde{J}_i\tilde{J}_j \to \tilde{J}_i\tilde{C}\tilde{J}_i \tag{59}$$

In turn, this implies a change in the value of the magnitude of the spin according to a rule of the type [Ref. 12, Eqs. (4.19.11)]

$$\tilde{J}_k\tilde{J}_k\,|\rangle = s(s+1)|\rangle \to \tilde{J}_k\tilde{C}\tilde{J}_k\,|\rangle = f(s; t, \mathbf{r}, \mathbf{p},...)|\rangle \tag{60}$$

which is said to characterize a "spin mutuation." Vice versa, if the magnitude of the spin is preserved under the time evolution law, this necessarily implies the preservation of the conventional associative envelope of $SU(2)$. In turn this necessarily implies the preservation of the $SU(2)$ commutation rules which can only occur under unitary time evolutions.

The fate of the remaining laws of quantum mechanics can now be predicted. In fact, a nonunitary time evolution implies the following isotopy of the canonical commutation rules:

$$\tilde{a} = (\tilde{\mathbf{r}}, \tilde{\mathbf{p}}); \qquad [\tilde{a}^\mu, \tilde{a}^\nu] = i\omega^{\mu\nu} \to [\tilde{a}'^\mu, \tilde{a}'^\nu]^* = i\tilde{\Omega}^{\mu\nu}(\tilde{a}) \tag{61}$$

that is, the value of the rules is not preserved in time. The lack of preservation in time of Heisenberg's indeterminacy principle then follows from the role of the canonical commutation rules for its very derivation. A very similar situation occurs for the Broglie's principle and for Einstein's frequency law (for details, see Ref. 12, Section 4.10).

At this point the attitude of individual researchers may differ sharply On one side, the desire to preserve established knowledge may suggest to one group of researches the dismissal of nonunitary time evolutions and underlying nonpotential forces. On the other side, one may acknowledge the expectation that quantum mechanics is by no means the final discipline of the microscopic world. It is expected to be simply one stage of an ever-continuing scientific evolution.

Along the latter lines, the relevant issues become: (a) the initiation of the construction of a *generalization* of quantum mechanics specifically conceived under conditions of wave overlapping and unrestricted forces; (b) the construction of a *new* structure model of hadrons and its confrontation with available experimental data (in such a way as to achieve compatibility with the established unitary models of Mendeleev classification); and, last but not least, (c) the calculation of specific predictions of the generalized theory which can be subjected to *experiments* feasible with current technology.

Since the appearance of Ref. 12 in June 1978, considerable research has been conducted along each of problems (a), (b), and (c) by a number of theoreticians and experimentalists. A detailed review of these studies would render the length of this paper prohibitive. Nevertheless, in order to reach the desired objectives (implications for quark conjectures), it appears advisable to outline the truly essential steps.

4.1. The Main Ideas of the Lie-Isotopic and of the Lie-Admissible Generalizations of Quantum Mechanics

As is by now evident, one of the implications of the use of nonpotential (local or nonlocal) forces is the removal of the restriction that the transformation theory should be canonical at the classical level and unitary at the quantum level. In particular, the generalized dynamical equations should be form-invariant under unrestricted transformations, as a necessary condition to admit unrestricted forces. Since Hamilton's (Heisenberg's) equations are form-invariant *only* under canonical (unitary) transformations, a most direct way of constructing the Lie-isotopic generalization is that via the use of the transformation theory. In fact, at the classical level, the transition from the Hamilton to the Birkhoff equations is precisely given by local, smoothness-preserving and invertible, but otherwise unrestricted (i.e., noncanonical) transformations, according to the rule (see Vol. II of Ref. 18 for a detailed treatment)[37]

$$\hat{\omega}_{ij} d\hat{a}^j = \frac{\partial \hat{a}'^k}{\partial \hat{a}^i} \Omega_{ks}(\hat{a})\, d\hat{a}^s = 0, \qquad \hat{a} = (t, a), \qquad \hat{a}' = \hat{a}'(\hat{a}) \tag{62}$$

The existence of a Birkhoffian generalization of Hamiltonian mechanics is sufficient to indicate the existence of a corresponding generalization of Heisenberg's mechanics. A most direct way of reaching the desired "quantum" image of Birkhoff's equations is therefore that via the transformation theory, in full analogy with Eqs. (62). In fact, assuming that Heisenberg's equations

[37] We use here the notation of Eqs. (18).

hold at a fixed value of time, under a nonunitary transformation they are mapped into the Lie-isotopic form[38]

$$\dot{\tilde{a}}^{\mu} = (1/i)[\tilde{a}^{\mu}, \tilde{B}]^{*} = (1/i)(\tilde{a}^{\mu}\tilde{C}\tilde{B} - \tilde{B}\tilde{C}\tilde{a}^{\mu}) \tag{63a}$$

$$[\tilde{a}^{\mu}, \tilde{a}^{\nu}]^{*} = i\Omega^{\mu\nu}(\tilde{a}) \qquad \mu = 1, 2, \ldots, 2n; \qquad \tilde{a} = (\mathbf{r}, \mathbf{p}) \tag{63b}$$

which was apparently proposed for the first time in Ref. 12, p. 752, Eqs (4.15.49).

Notice that the envelope of Eqs. (63) can be interpreted in a dual way First, the equations can be derived via an isotopy of the envelope of type (59). Second, the equations can be derived via a Lie-admissible genotopic mapping of the envelope of the type

$$\tilde{Y}_i\tilde{Y}_j \to (\tilde{Y}_i, \tilde{Y}_j) = \tilde{Y}_i\tilde{F}\tilde{Y}_j - \tilde{Y}_j\tilde{G}\tilde{Y}_i; \quad (\tilde{Y}_i, \tilde{Y}_j) - \tilde{Y}_j, \tilde{Y}_i) = [\tilde{Y}_i, \tilde{Y}_j]^{*} \tag{64}$$
$$\tilde{F} \text{ and } \tilde{G} \text{ fixed}; \qquad \tilde{C} = \tilde{F} + \tilde{G}$$

In the first case, the envelope is still associative, while in the second it is a bona fide nonassociative Lie-admissible algebra. What is important for late needs (particularly for the historical remarks of Section 6) is that *to achieve a genuine generalization of quantum mechanics which is capable of admitting forces more general than $f = -\partial V/\partial r$, the generalization must occur at the level of the envelope*. This latter property has been independently identified and treated in detail also by Ktorides.[(43)]

Independently of these studies, Okubo[(44,45)] has worked out a generalization of Heisenberg's equations of Lie-isotopic character which is based on a flexible Lie-admissible generalization[39] of the envelope. For brevity we refe the interested reader to the quoted papers.

As occurred at the classical level, the studies soon revealed the *insufficiency* of a Lie-algebra-preserving generalization of Heisenberg's equations In fact, equations of type (63) do not achieve the needed universality for al possible nonlocal forces, in exactly the same way as occurs at the classica level for Birkhoff's equations.

To bypass this difficulty, Santilli[(12)] studied directly the algebrai structure of the nonunitary time evolution (55), that is, the algebraic structur was identified without any assumption that Heisenberg's equations hold at particular instant of time. Let $\tilde{H}$ be the *nonconserved* energy of *one* hadroni

[38] Notice that the $\tilde{B}$ operator of Eqs. (63) *does not* represent the energy (Appendix B

[39] A Lie-admissible algebra with (abstract) product ab is flexible when it satisfies the so called flexibility law $(ab)a = a(ba)$. Notice that Lie algebras are flexible.

constituent under conditions of wave overlapping with all the other constituents and nonlocal nonpotential forces. Then, under topological conditions inessential here, there always exist non-Hermitian, integrodifferential operators $\tilde{R}$ and $\tilde{S}$ such that $\tilde{Z} = \tilde{H}\tilde{S}$ and $\tilde{Z}^{\dagger} = \tilde{R}\tilde{H}$, under which law (55) becomes

$$\tilde{A}' = \exp(it\tilde{H}\tilde{S}\ \tilde{A}\exp(-it\tilde{R}\tilde{H}) \tag{65}$$

Now, in the neighborhood of the identity ($t \rightarrow dt$) we have[40]

$$\begin{aligned}\dot{\tilde{A}} = d\tilde{A}/dt &= (\tilde{A}' - \tilde{A})/dt \\ &= [\tilde{A}' - \exp(idt\tilde{H}\tilde{S})\,\tilde{A}\exp(-idt\tilde{R}\tilde{H})]/dt \\ &= (1/i)(\tilde{A}\tilde{R}\tilde{H} - \tilde{H}\tilde{S}\tilde{A}) \overset{\text{def}}{=} (1/i)(\tilde{A}, \tilde{H})\end{aligned} \tag{66}$$

The product $(\tilde{A}, \tilde{H})$ is strictly non-Lie, as desired, and is Lie-admissible because the attached algebra is Lie according to rule (64) (Ref. 12, p. 783).

In this way we reach the following property, tentatively called the *direct universality of the Lie-admissible algebras in quantum mechanics* (see Ref. 14, pp. 1820–1829):

Theorem 3. The time evolution law in quantum mechanics can always be expressed in the neighborhood of the identity, under sufficient topological conditions, via a nonassociative Lie-admissible algebra.

The reader should keep in mind that the Lie algebras are a particular case of the Lie-admissible algebras. As a result, the theorem contains trivially Heisenberg's time evolution law (for $\tilde{R} = \tilde{S} = \tilde{1}$).

The desired Lie-admissible generalization of Heisenberg's equations can then be written (Ref. 12, p. 746)

$$\begin{gathered}\dot{\tilde{a}}^{\mu} = (1/i)(\tilde{a}^{\mu}, \tilde{H}) = (1/i)(\tilde{a}^{\mu}\tilde{R}\tilde{H} - \tilde{H}\tilde{S}\tilde{a}^{\mu}) \\ (\tilde{a}^{\mu}, \tilde{a}^{\nu}) = i\tilde{S}^{\mu\nu}(\tilde{a})\end{gathered} \tag{67a}$$

$$\mu = 1, 2, \ldots, 2n, \qquad \tilde{R}, \tilde{S}, \tilde{H} = \text{fixed} \tag{67b}$$

The mathematical and physical similarities between the "quantum mechanical" equations (67) and the classical equations (24) are remarkable. Both sets of equations are based on the abandonment of Lie's theory in favor of the covering Lie-admissible algebras. Also, both sets of equations

[40] Lie-admissible algebras are used in this section for a dual application, one for the envelope, Eqs. (64), and one for the time evolution, Eqs. (66). In the former case we use the operators $\tilde{F}$ and $\tilde{G}$, while in the latter we use the operators $\tilde{R}$ and $\tilde{S}$. Clearly, the two sets of operators need not be the same.

are form-invariant under arbitrary transformations.[14] Finally, both sets of equations are capable of representing in a direct way (Section 3) non-Hamiltonian forces. The representation occurs in both cases via the algebraic product itself. The capability to effectively represent wave packets under conditions of mutual overlapping then results.

The reader should keep in mind that Eqs. (67) were developed for the dynamical behavior of *one* hadronic constituent while considering the rest of the system as external. Equivalently, Eqs. (67) were developed to achieve the *highest possible degree of nonconservation* of all physical quantities, i.e.,

$$\begin{aligned} \dot{\tilde{H}} &= (1/i)(\tilde{H}, \tilde{H}) \neq 0; \qquad \dot{\mathbf{p}} = (1/i)(\mathbf{p}, \tilde{H}) \neq 0 \\ \dot{\mathbf{M}} &= (1/i)(\mathbf{M}, \tilde{H}) \neq 0; \qquad \dot{\tilde{\mathbf{J}}} = (1/i)(\tilde{\mathbf{J}}, \tilde{H}) \neq 0; \qquad \text{etc.} \end{aligned} \tag{68}$$

In fact, this degree of nonconservation is one way to characterize interactions (trivially, when all physical characteristic of one particle are conserved, that particle is free). The conservation of specific quantities of a constituent should, however, not be excluded as a particular case.

Notice that generalized equations (67) [as well as (63)] permit the preservation of the Hilbert space, although interpreted as a bimodule.[21] In fact, the left and right actions are now no longer trivially related as in Heisenberg's representation. The representation theory of Lie-admissible algebras of operators on a bimodular Hilbert space is therefore a bona fide two-sided (left and right) generalization of the conventional one-sided representation theory of Lie algebras. As we shall see in a moment, these features have rather crucial implications. In fact, the capability to admit a Hilbert space is essential for the formulation of experiments, while the two-sided generalization of the representation theory implies that of particle constituent.

On epistemological grounds, the term "quantum mechanics" becames questionable for the mechanics characterized by Eqs. (67) [or (63)]. In fact, the departures from the conventional setting are such as to render questionable the notion of "quantum of energy." On physical grounds, this notion was originated to refer and still refers to the emission (or absorption), propagation, and detection of energy by the electrons of an atomic cloud in the transition from one orbit to another. Clearly, this notion implies that the underlying medium is the physical vacuum (rather than the vacuum of quantum field theory). When the electrons penetrate hadronic matter and reach a state of complete overlapping of their wave packets with those of the surrounding particles, each aspect (emission, propagation, and detection) of the atomic notion of quantum of energy is questionable, and calls for suitable generalizations. On mathematical grounds, the realization of the quantum of energy is crucially dependent on the property that Planck's constant $\hbar$ is the unit of the enveloping algebra (apart from multiplicative scalars). This is indeed the case for the envelope of Heisenberg's equations with associative product $\tilde{Y}_i\tilde{Y}_j$. But, whenever the envelope is generalized, Planck's constant ceases to be the unit. This is the case for *all* generalizations identified until now for the tratment of nonpotential forces, that is, for the isotopic generalization $\tilde{Y}_i\tilde{C}\tilde{Y}_j$ and for the genotopic Lie-admissible generalization $\tilde{Y}_i\tilde{F}\tilde{Y}_j - \tilde{Y}_j\tilde{G}\tilde{Y}_i$. To stress the departures from conventional approaches, it should be indicated that no new constant is expected to appear in the generalized mechanics. In fact, the unit of an isotopically or genotopically mapped envelope (when it exists) is a bona fide operator. As a result, the relatively simple atomic emission of a quantum of energy in vacuum is expected to be replaced by a rather complex event when the emission is within hadronic matter. Then, Planck's constant is expected to be replaced by an operator whose explicit form depends on the acting forces, that is, on the $\tilde{C}$ or $\tilde{F}$ and $\tilde{G}$ operators. The covering nature of these ideas over those of atomic mechanics is self-evident. In fact, when particles reach distances greater than their size, all nonself-adjoint forces are null, and the conventtional associative envelope of Heisenberg's equations is recovered in its entirety. Then, the operators $\tilde{C}$ or $\tilde{F}$ and $\tilde{G}$ characterizing the process of emission (or absorption) of energy acquire the values $\tilde{C} = 1$ or $\tilde{F} = 1$ and $\tilde{G} = 0$.

For these (and other) reasons, as well as to prevent possible confusions, a new terminology was introduced in Ref. 12. The conventional quantum mechanics was called *atomic mechanics* not only for historical reasons, but also because of its arena of incontrovertible applicability. The mechanics for closed, nonself-adjoint systems (hadronic structure) or, more generally, for wave overlappings (strong interactions) was called *hadronic mechanics*. An intermediate mechanics was also identified in the same reference and called *nuclear mechanics*. In essence, wave overlappings in the nuclear structure exist (as clearly established by experimental data on volumes), but they are of small value (of the order of $10^{-3}F$). Under these conditions, all the arguments reviewed in this paper for wave overlappings apply; yet, the departures from the atomic mechanics are expected to be minimal. This allows the $\tilde{R}$ and $\tilde{S}$ operators of Eqs. (67) to be averaged to constants λ and μ [or to be approximated via functions of time $\lambda(t)$ and $\mu(t)$].

Under these conditions, the Lie-admissible product acquires the simpler form

$$(\tilde{A}, \tilde{B}) = \lambda\tilde{A}\tilde{B} - \mu\tilde{B}\tilde{A} \qquad (69)$$

which characterizes a flexible Lie-admissible algebra.

For this reason, the type of algebra was assumed in Ref. 12 as characterizing the underlying physical arena and mechanics, according to the following classification (Ref. 12, p. 756):

(a) Lie algebras ↔ atomic mechanics.

(b) Flexible Lie-admissible algebras ↔ nuclear mechanics.

(c) General Lie-admissible algebras ↔ hadronic mechanics.

Needless to say, these possibilities have been presented here in the speculative spirit of this paper, and a long way remains to be covered, theoretically and experimentally, before reaching final conclusions.

4.2. The Main Ideas of the Proposed New Model of Hadronic Structure

Quantum mechanics was conceived to represent the atomic spectral lines within the context of the problem of atomic structure. By following this approach, the efforts of Ref. 12 (and of subsequent research) to construct a hadronic mechanics were motivated by the desire to achieve a quantitative interpretation of the decays of hadrons and related fractions. In fact, these decays can be interpreted as the hadronic constituents of the atomic spectral lines[41] (which are *absent* in hadron physics). The reasons are simple. The atomic spectral lines provided invaluable information on the nature of the atomic constituents, of course, when treated with the proper mechanics (quantum rather than classical). The argument is than that the "hadronic spectral lines," that is, the decays, are equally crucial for the fundamental open problem of hadron physics: the identification of the constituents with physical particles.

In particular, rather than reducing the problem of hadronic structure to a variety of more complex and unknown particles, the hypothesis of the new model proposed in Ref. 12 was as simple as possible. In fact, it was assumed that *the hadronic constituents are physical, already known, massive and charged particles which can be produced free in the spontaneous decays, or in inelastic scatterings.* The hope was to avoid ab initio the problematic aspects of unidentified multiple constituents, as well as the uneasiness produced by the idea that the constituents of unstable bound states under continuous forces and low binding energies, should not admit a finite, nonnull probability of free production.

To state this in different and more simplistic terms, the fundamental

[41] The reader should keep in mind that this idea is already contrary to the "quark philosophy," according to which the hadronic equivalent of the spectral lines is given by the "hadronic spectroscopy," that is, by the series of masses of different particles. This idea is rejected here because of our strict *classification* inspiration.

hypothesis of Ref. 12 is that the constituents of hadrons can be identified with massive and charged particles present in the already available decay data. A simple inspection of these data soon reveals that *the hypothesis is incompatible with the atomic mechanics*,[42] in a way similar to how the atomic spectral lines were incompatible with classical mechanics

At this point, the conduct of further studies depends on the attitude of the individual researcher If one believes that the atomic mechanics is the final discipline of the microscopic world, the hypothesis above must be rejected But then, the problematics of quarks and their confinement result. On the contrary, other researchers may acknowledge the fact that the validity of the atomic mechanics for the strong interactions in general and for the hadronic structure in particular, is a mere conjecture at this time, deprived of any direct and incontrovertible experimental support (Section 5). Under these conditions, scientific caution is essential to avoid personal prejudices, and scientific courage is needed to attempt possible nonincremental advances, as well as experimental resolutions.

Along the latter lines, the hypothesis above was subjected to a quantitative study of plausibility via the use of the hadronic mechanics. The results of Ref. 12 were positive, and were subsequently confirmed by other studies.[(15)] A conceptual review of the main arguments is the following.

The problem of the structure of hadrons is, in the final analysis, of *secondary* nature, and it is subordinate to a more fundamental problem: the quantitative identification of the hadronic constituents, or, more generally, of a particle under strong interactions. Only when this latter problem has been solved (even in a preliminary, but consistent form), can one study problems such as the existence or lack of existence of the free production of the hadronic constituents.

The main ideas are now familiar. When a particle is under long-range electromagnetic interactions, it can be approximated as being pointlike, and (Galilei's or) Einstein's special relativity applies. We reach in this way the familiar characterization of a particle moving in vacuum (*no wave overlappings*) under an external long-range electromagnetic field

$$\exp(i\theta_k \tilde{X}_k)\, \psi(x) \exp(-i\theta_k \tilde{X}_k) = D(\Lambda)\psi(\Lambda^{-1}x); \qquad \tilde{X}_k \in SL(2.C) \tag{70}$$

This notation of particle is that used for the atomic constituent. Notice, in particular, that *particles (or constituents) under electromagnetic interactions*

[42] For instance, the hypothesis implies that the bound state of three massive particles, which exhibit a spin 1/2 under electromagnetic interactions, has a null value of the total angular momentum.

have a perennial value of the spin, mass, and other physical characteristics, that is, they preserve their identity in time. In fact, to characterize a particle, one must fix a suitable representation of the Poincaré (or Galilei) group.

There is no a priori reason to expect that this notion of particle (or of constituent) should identically apply under strong interactions. As a matter of fact, the physical differences between the electromagnetic and the strong interactions are expected to result in a differentiation of the corresponding notions of particles.

When pointlike abstractions are ignored, and the physical reality of Fig. 4 is confronted, a generalization of the notion of particle (70) appears to be unavoidable. The tentative, generalized notion of particle under strong interactions proposed by Santilli[12,21] is given by the following nonunitary law:

$$\exp(i\theta_k \tilde{X}_k \tilde{S}_k)\, \psi(x) \exp(-i\theta_k \tilde{R}_k \tilde{X}_k) = D(\Lambda, \psi) \tag{71}$$
$$\tilde{X}_k \in SL(2.C); \qquad \tilde{R}_k\,,\, \tilde{S}_k \in A(SL(2.C))$$

where the $\tilde{X}$'s are the same as in law (70), the $\tilde{R}$ and $\tilde{S}$ operators are different for different generators, and $D(\Lambda, \psi)$ represents a suitable *nonlinear* (eventually *nonlocal*) transformation law. Notice that the $\tilde{R}$ and $\tilde{S}$ operators represent precisely those forces that are outside the applicability of conventional relativities. Notice also that the time component of law (71) is exactly along the lines of the integrated form of the Lie-admissible equations (67). The transition from law (70) to (71) has been called a *mutation* in Ref. 12. The reader should keep in mind that the *mutation operators* $\tilde{R}$ and $\tilde{S}$ can differ from one by a finite or infinitesimal amount, depending on the degree of wave overlapping. In the former case we have a *finite mutation*, otherwise we have an *infinitesimal mutation.*

The results of Ref. 12 and of the subsequent studies are that the hadronic constituents can indeed be produced free under spontaneous decays or inelastic scatterings, provided that the notion of "constituent" is defined according to the generalized law (71). In this case we simply have the transition of already known, physical particles under generalized dynamical conditions created by the wave overlapping, to conventional dynamical conditions. Correspondingly, the $\tilde{R}$ and $\tilde{S}$ operators perform the transition from a value different than one to the unit value.

To illustrate the "mechanics" of the model, it is sufficient here to indicate only the behavior of the spin. Mutation (71) implies a finite change in the value of the spin of a particle in the transition from the conditions experimentally measured (under electromagnetic interactions only) to strong interactions. The primitive idea is that the intrinsic angular momentum of an *extended* particle cannot preserve its conventional atomic value when the particle is within a hadronic medium. In fact, interferences in the intrinsic rotation are expected to arise from the dense wave overlappings, resulting in a mutation of the spin. The value of the mutated spin is expected to be dependent on the local dynamical conditions [see the t, $\mathbf{r}$, and $\mathbf{p}$ dependence in Eq. (70)]. The understanding is that, whenever the particle exits the hadronic medium, the conventional spin is reacquired, apart from secondary effects (e.g., emission of neutrinos).

The simplest possible spin mutation is the *constant mutation*, i.e., a mutation whereby the dynamics of wave overlappings is such as to produce a fixed value of the spin, although different than the conventional one under electromagnetic interactions. The constant spin mutation is produced by a flexible Lie-admissible generalization of the envelope of $SU(2)$ (approximation of the $\tilde{F}$ and $\tilde{G}$ operators to constants λ and μ) according to the rules

$$\begin{aligned} \tilde{J}_k\tilde{J}_k\,|\rangle = s(s+1)|\rangle \rightarrow (\tilde{J}_k\,, \tilde{J}_k)|\rangle &= (\lambda+\mu)\tilde{J}_k\tilde{J}_k\,|\rangle \\ &= (\lambda+\mu)s(s+1)|\rangle = s'(s'+1)|\rangle \end{aligned} \tag{72}$$

The computation of the s' value in terms of λ and μ then provides the mutated value of the spin. It is clear that, starting from a given value of the spin under electromagnetic interactions, one can reach via a mutation an *arbitrary value of the spin*, e.g., for $s = 1/2$, s' can be given by 0.351692... . This mechanism of spin mutation removes, at least in principle, one of the main obstacles for the hypothesis that the hadronic constituents are massive and physical particles which can be produced free under spontaneous decays (see footnote 42). Other objections can be removed via the mutation of other physical quantities,[43] as well as via other processes (such as pair creation within hadronic matter).

The case of the structure of the π^0 and of the $\pi^{\pm}$ particles was considered in detail in Ref. 12 via a *nonself-adjoint* force dependent on spin–spin couplings. It was indicated that the model provides a quantiative representation (via a generalized form of Schrödinger's equation) of *all* physical characteristics of the particles (such as mass, size, spin, mean life, charge, space and

[43] In Appendix C we indicate the duality of the relativity situations for closed, nonself-adjoint systems. The attentive reader has by now identified the existence of a corresponding duality for the gauge symmetry. The charge of an isolated hadron is conserved. Thus, the gauge symmetry is exact. But this does not necessarily imply that the gauge symmetry is exact also for each individual constituent. In fact the forces which break conventional relativities can be easily proved to break also the gauge symmetry.[17] For the structure model considered here, the breaking of the gauge symmetry at the structure level (only) appears to be *necessary* to identify the hadronic constituents with physical particles beginning with the kaons. For brevity we refer the interested reader to Section 5 of Ref. 12.

charge parity, dipole and quadrupole moments, etc.), as well as, most importantly, of the decays. The extension of the model to other (light) mesons was also discussed.

The problem of the ultimate nature of the constituents of hadrons was also studied. According to the model, the constituents of a given hadron are given by a suitable selection of hadrons with lower mass. For instance, the constituents of the kaons are the mesons, of course, in a mutated form. But these particles are per se unstable and, as such, they are not the "elementary constituents." The only elementary, massive, and charged particles produced in meson decays are the electrons. According to the contemporary view (based on the assumption of the validity of the atomic mechanics within a hadron), the electrons are "created" at the time of the decays. The hypothesis formulated in Ref. 12, and permitted by the Lie-admissible generalization of Heisenberg's equations, is that *the ultimate elementary constituents of hadrons are the ordinary electrons and positrons, although under conditions of mutual wave overlappings, and a nonunitary time evolution law.* The mutation of their spin is then a consequence of the nonunitarity of the time volution. The applicability of the Lie-admissible formulations is also a consequence (because of Theorem 3 of this section). The neutrinos emerged as the particles emitted when a mutated electron leaves the hadron and reacquires its conventional identity. Intriguingly, the model revived the very first structure model of the neutron, according to which it is a bound state of one proton and one electron, by (apparently) resolving the technical difficulties which led, historically, to the abandonment of the model (these difficulties were *all* due to the assumption that the atomic mechanics applies within the neutron, and are removed when a generalized mechanics is used—for details see Ref. 14, Section 2.4). The analysis of Ref. 12 concluded with the view that *the Lie-admissible generalization of atomic mechanics apparently allows the reduction of the physical universe to only three stable particles: the electron, the photon, and the neutrino.*

In the final analysis, the validity or invalidity of this (rather suggestive) view can be reduced to the issue of whether electrons are always subjected to a unitary time evolution under whatever dynamical conditions are realized in nature; or whether the unitary character of the time evolution is peculiar to the electromagnetic interactions, and more general, nonunitary (and, thus, Lie-admissible) evolutions are possible under the conditions of wave overlapping or other conceivable dynamical conditions. If the unitarity of the time evolution is universal in nature, the electrons *cannot* be the hadronic constituents. On the contrary, if the electron can have a nonunitary time evolution, the mutation of their intrinsic characteristics is an unavoidable consequence. In this case, the electrons *can* be the ultimate elementary constituents of hadrons (see also Appendix D).

In the speculative spirit of this paper we therefore conclude the section with the following statement:

Contention 3. Quantum mechanics, in its current nonrelativistic, relativistic, and field-theoretic versions, is incompatible with the strong interactions.

5. THE EXPERIMENTAL PROFILE

As stressed throughout this paper, the current controversies in the community of basic research do not concern matters of a minute technical character. Instead, they deal with fundamental issues. They arise from the fact that the validity or invalidity for the strong interactions of the basic relativity and quantum mechanical laws of the electromagnetic interactions is, at this moment, a mere personal view by individual researchers without clear and final experimental support.

It is appropriate to stress here that quark models are unable, in their present form, to resolve this rather fundamental problem of contemporary physics. Any different view is equivalent to the belief that the assumption that quarks are physical particles, complemented by the assumption that they confine, complemented by the further assumption that conventional quantum mechanical laws apply within a hadron, etc., together produce the resolution of the validity of Einstein's special relativity within a hadron.

Independently from this situation, currently available experiments on strong interactions are also unable to resolve the problem. In fact, the experiments are conducted by *assuming* conventional relativity and quantum mechanical laws in the data elaborations (Fig. 6). Under these conditions, the "experimental results" simply cannot test the assumptions.

This situation is considerably more severe than that conventionally acknowledged. In fact, unprejudicial inspection of, say, experiments in hadron–nuclei scatterings reveals the assumption of the validity of Pauli's

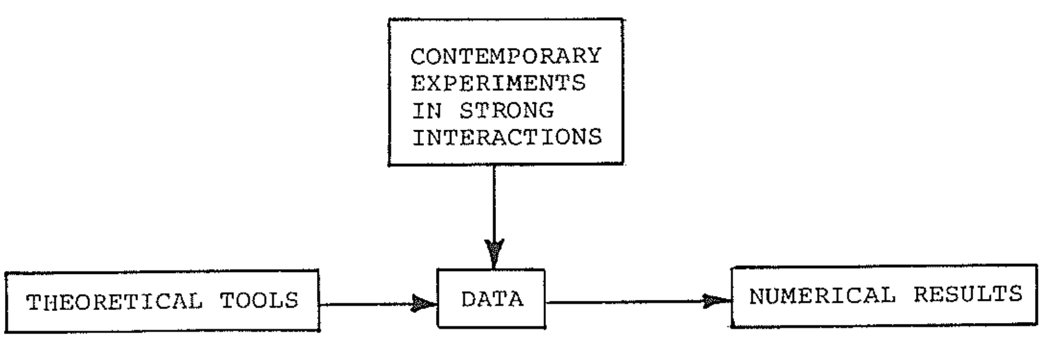

Fig. 6

exclusion principle in the data elaboration. Similarly, currently availabl data elaborations of deep inelastic scattering experiments of leptons o hadrons are crucially dependent on the assumption of the validity of Einstein' special relativity under conditions of wave overlapping. More distressingly the very fundamental notion of cross section currently used by experimenter is in question for the strong interactions. In fact, this notion, as is well known is a product of the "potential scattering theory." As a result, the curren notion of cross section is based on the (often tacit) assumption that the stron interactions are of potential type. This should be compared with the variou arguments (reviewed in this paper) according to which the strong interaction are not of potential type. The reader can now see the implications for dat elaborations. In fact, the possible construction of a "nonpotential scatterin theory" (currently under way by a number of researchers) is expected t imply cross sections different than those of current use. The use o different theoretical models in the data elaborations is then expected t produce *different numerical results for the same experiment* (a concret example will be given below).

Clearly, a situation of this type cannot be protracted indefinitely. Th only way to resolve the controversies is by the initiation of experimenta studies specifically conceived to resolve the problem of the validity o invalidity of conventional laws for the strong interactions.

In the hope of stimulating the initiation of these studies, the experimenta test of Pauli's principle under strong interactions was recommended i Ref. 12.

On theoretical grounds, the issue is now familiar. Recall that contempo-rary relativity and quantum mechanical laws are deeply interrelated, to th point that the possible invalidity of one of them implies that of the others A similar argument is conceivable for the case of possible validity. A elaborated in Section 4, the issue is crucially dependent on the problem whether the time evolution of hadrons under strong interactions is unitar of not. If it is unitary, there is little reason (if any) to doubt the validity o conventional settings. On the contrary, if the time evolution is nonunitary we have a mutation of the conventional intrinsic quantities of particles unde electromagnetic interactions. In this case, we have a breaking of the $SU(2)$-spin symmetry, in the sense that the conventional value of the spin cannot b preserved in time. This has a number of consequences. On dynamical grounds it would imply the violation of Pauli's exclusion principle for the stron interactions (actually, its inapplicability[12]), trivially, because of the loss o the notion of fermion. With regard to relativity, we would have the break-down of Einstein's special relativity, trivially, because $SU(2)$ is the littl group of the Poincaré group.

On experimental grounds, the problem is therefore that of achieving a *direct measure of the intrinsic characteristics of hadrons (the spin in particular) while under strong interactions.* If clear experimental data establish the preservation of the electromagnetic characteristics of particles, there is no reason to expect deviations from established laws. On the contrary, if experiments show deviations, the invalidation of conventional laws, and the search for covering laws is unavoidable, in our view.

To illustrate the current experimental situation in the field, consider the proton. Its spin has been measured countless times, but always while the particle is in vacuum under long-range electromagnetic interactions. A direct measure of the spin of the proton while it is a member of a nuclear structure simply does not exist at this time, to our knowledge.

What is important is that we have indeed achieved the necessary technology to resolve the problem. In fact, a pioneering experiment has already been conducted by Rauch and his associates[(47)] on the 4π spinor symmetry of neutrons via interferometer experiments. This experiment can be reinterpreted as providing a direct measure of the spin of the neutron while under low-energy nuclear (and thus strong) interactions. The reasons are quite simple. With reference to Fig. 7,[44] a neutron interfereometer essentially allows the wide-angle splitting of a monochromatic neutron beam. Both branches of the beam have been subjected to a magnetic field for spin precession. Fortunately (for the problem under consideration), the experimenters filled up the magnet gaps with mu–metal to prevent stray fields. It is this that renders the experiment fundamental. In fact, the experiment actually measures the spin of the neutron while under the joint interaction due to the magnet *and* the strong interactions with the mu-metal nuclei.

The experimental results are quite intriguing. Under the *assumption* of an exact $SU(2)$-spin symmetry for the interaction of the neutron with the mu-metal nuclei, theory predicts the intensity modulation of the exiting beam[(47)]

$$I'(P = 0) = \frac{I}{2}\left(1 + \cos\chi \cos\frac{\theta}{2}\right) \tag{73}$$

where χ is the nuclear phase shift and θ is the angle of precession. Since the intensity modulation can be directly tested, the angle θ can indeed be measured, thus yielding a test of the spin.

The best fit under an exact $SU(2)$-spin symmetry was found in Ref. 47 and is reproduced in Fig. 7. As one can see, the angle for the best fit is given by

$$\theta = 716.8 \pm 3.8 \text{ deg} \tag{74}$$

[44] I would like to thank Prof. Rauch and his associates for authorization to reproduce Fig. 7 from their paper,[(47)] as well as for invaluable comments.

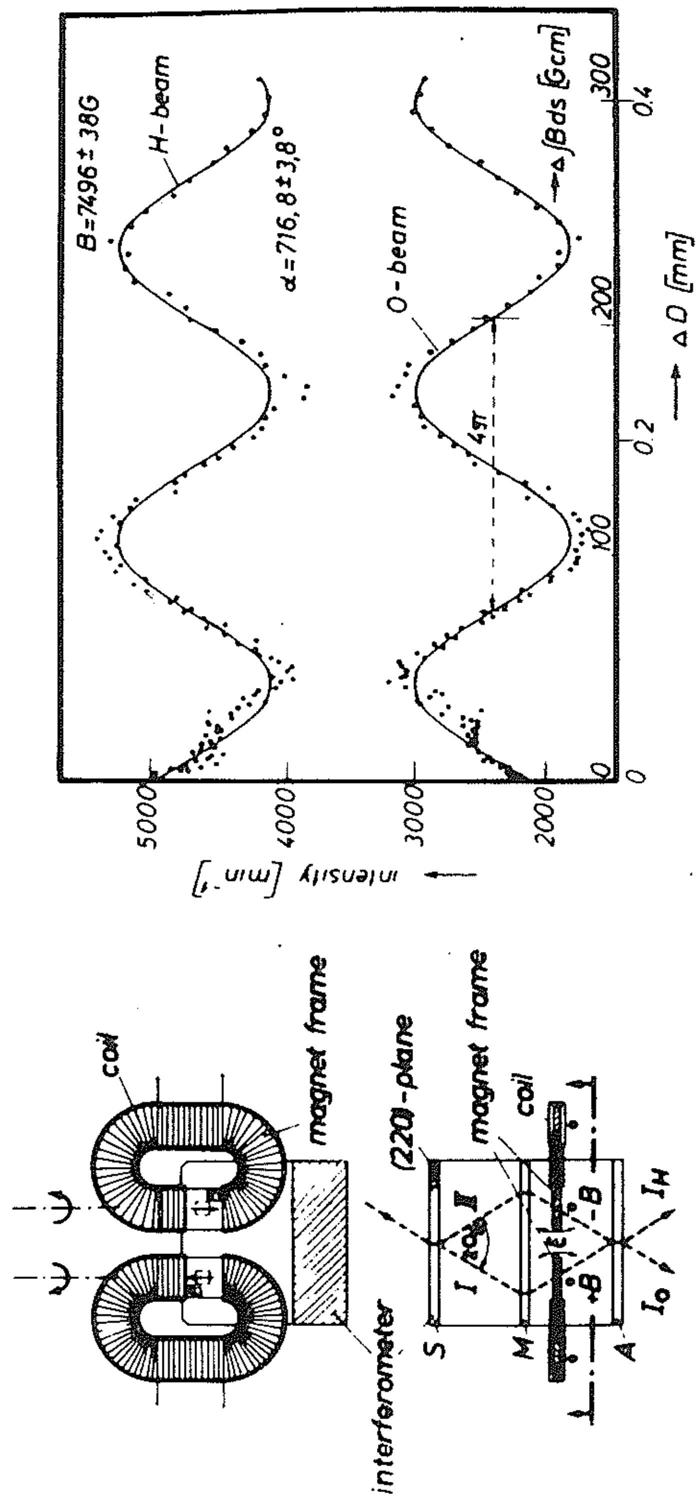
coil
magnet frame
(220)-plane
magnet frame
coil
interferometer
S
M
A
-B
+B
B=7496 ± 38G
H-beam
α=716,8±3,8°
O-beam
4π
intensity [min⁻¹]
ΔD [mm]
Δ∫Bds [Gcm]

Fig. 7

and, as such, it contains the angle 720 deg, which is *necessary* for the exact symmetry. Nevertheless, the 720 deg barely enters within the experimental error; the median value is below 720 deg; and, last but not least, there is a clustering of experimental points clearly out of the cosinusoidal behavior, that is, over and below the maxima and minima of the curve.

An alternative interpretation of the experimental data has been provided in Ref. 42. During the interaction of the neutrons with the nuclei of the mu-metal in the magnet gap, we have a wave overlapping, although of very small nature. Thus allows the implementation of the Lie-admissible mutation of the spin, but in its simplest possible (flexible) form, that characterized by the mutation of the associative envelope of $SU(2)$

$$Y_i Y_j \to (Y_i\,,\, Y_j) = Y_i Y_j + \epsilon Y_j Y_i\,; \qquad \epsilon \approx 0 \tag{75}$$

This essentially characterizes a quantitative treatment of the *broken* $SU(2)$-spin symmetry under strong interactions. The term "quantitative" is used here to indicate that the theory permits the computation of deviations which can be subjected to experiments. In fact, the mutated form of the intensity modulation can be computed via the methods of Ref. 25, and it is given by[42]

$$I'(\mathbf{P} = 0) = \frac{I}{2}\left\{1 + \frac{\epsilon^2}{2} + \epsilon \cos\chi + (1 + \epsilon)\cos\chi \cos\left((1 + \epsilon)\frac{\theta}{2}\right)\right\} \tag{76}$$

The third component of the spin (that considered in the experiment) is then mutated according to the rule[42]

$$s_3' \approx \tfrac{1}{2} + \tfrac{3}{8}\epsilon \tag{77}$$

This confirms the breaking of the $SU(2)$-spin symmetry, as desired, although in a very small measure.

Intriguingly, the mutated modulation (76) and spin (77) are compatible with the experimental data. As a matter of fact, they apparently permit the interpretation of the "slowdown effect" of the median angle (which is smaller than that for the exact symmetry) as well as a deformation of the fit capable of accommodating better the clusters of points outside the solid curve of Fig. 7.[42]

We therefore conclude by indicating that the available experimental information does not exclude the possibility of a spin mutation and, thus, of the nonunitarity of the time evolution under strong interactions. Needless to say, the information is still inconclusive because it does not exclude the exact symmetry either (although this would call for theoretical efforts to try to interpret the data via the exact symmetry, and the use of effects which are unknown or not understood at this time). Additional experimental measurements are therefore much needed and encouraged.

Notice the different numerical values which can be obtained with the same experiment. In fact, if one accepts the validity of the $SU(2)$-spin symmetry, the angle θ is given by 716.8 ± 3.8 deg. On the contrary, if one accepts a small breaking of the $SU(2)$-spin symmetry and its (nonunitary) Lie-admissible treatment, this numerical value must be corrected by a factor of $1/(1 + \epsilon)$. This illustrates the comments made earlier in this section in regard to the dependence of current experimental results on the assumed laws.

This situation can be resolved in a number of experimental ways:

I. By repeating the experiment with a better accuracy. Since the 720 deg barely fits within the data range, it is conceivable that a reduction (even minimal) of the current experimental error might establish that the setting does not permit the angle of the exact symmetry.

II. By repeating the experiment with a spin precession in only one branch, while the other moves in vacuum. Coincidence measures may then establish comparative deviations from the $SU(2)$-spin symmetry in the two branches.

III. By repeating the experiment for multiples of 4π and/or with increased strong interactions (e.g., via a gap metal with a higher nuclear absorption coefficient, or via magnet gaps of greater thickness). In this case, even under the current error, we might well have the desired experimental resolution.

But perhaps the most suggestive experimental information is that related to the deviation from the conventional value of the magnetic moment of the neutron while within a nuclear structure. This deviation has been established via the deviation of the total nuclear magnetic moment from the Schmidt limit, and can reach considerable values (up to 50 % and aven more). Once reinterpreted in our Lie-admissible treatment,[45] *this experimental information establishes the mutuation of the magnetic moment of the nucleons in the transition from the electromagnetic to the strong interactions*, which is precisely the thesis advocated by our studies.

Clearly, a mutation of the magnetic moment will likely imply a mutation of the intrinsic angular momentum, with consequential breaking of the $SU(2)$-spin [and, thus, $SL(2.c)$] symmetry for the strong interactions. In fact,

[45] It should be indicated here that the treatment of Eqs. (75)–(77) is the simplest conceivable. In fact, the mutation is approximated to a constant. In reality, the mutation quantity ϵ is an integrodifferential operator with a time, space, and velocity dependence such as to be null outside the region of wave overlappings.[42]

it is hardly conceivable (or, at any rate, quite difficult to explain quantitatively) that the magnetic moment mutates, while the spin remains unchanged.

The experiment by Rauch and his associates[47] now assumes a new light. In fact, the magnetic moment enters into the data elaboration via the angle θ. *The deviation (mutation) from conventional values of spin advocated in Ref.* 42 *is then nothing more than a dynamical counterpart of the deviation (mutuation) of the magnetic moment of the neutron under strong interaction as independently established in nuclear physics.*

6. THE LEGACY OF EINSTEIN, FERMI, JORDAN, AND OTHERS: THE POSSIBLE INVALIDATION OF QUARK CONJECTURES

It is appropriate to recall, for completeness, that the doubts on conventional laws, principles, and insights of quantum mechanics considered in this paper *are not new*. In fact, they were raised by the very founding fathers of contemporary physics, such as Einstein, Fermi, Jordan, and others. As a matter of fact, and as will be self-evident in a moment, the studies considered in this paper have been conducted by following this teaching by the founding fathers as closely as possible.

Einstein expressed mostly clearly his belief in the lack of a final character of the conventional indeterminacy of quantum mechanics. This historical point is touchingly and effectively reported in Heisenberg's book, *Physics and Beyond.*[48] A few excerpts from Heisenberg's presentation are appropriate here (Ref. 48, pp. 80–81), in relation to his recollection of the Solvay congress held in Brusselles in 1927, and attended by the leading scientists of the time:

> Einstein "refused point-blank to accept the uncertainty principle, and tried to think up cases in which the principle would not hold. The discussion usually started at breakfast, with Einstein serving us up with yet another imaginary experiment by which he thought he had definitely refuted the uncertainty principle. We would at once examine his fresh offering, and on the way to the conference hall, to which I generally accompanied Bohr and Einstein, we would clarify some of the points and discuss their relevance. Then, in the course of the day, we would have further discussions on the matter, and as a rule, by suppertime we would have reached the point where Niels Bohr could prove to Einstein that even his latest experiment failed to shake the uncertainty principle. Einstein would look a bit worried, but by the next morning he was ready with a new imaginary experiment more complicated than the last, and this time, so he avowed, bound to invalidate the uncertainty principle." ... "Later in life, also, when quantum theory had long since become an integral part of modern physics, Einstein was unable to change his attitude—at best, he was prepared to accept the existence of quantum theory as a temporary expedient."

A reconstruction of this historical episode has indicated that, in all the attempts to invalidate the uncertainly principle, Einstein used only variationally self-adjoint conservative forces. We know nowadays that, for these forces, the uncertainty principle does hold. Thus, Bohr and Heisenberg were indeed right in refuting Einstein's examples. But this does not close the issue nor exlude Einstein's vision of a genuine advancement of a fundamental character. It is regrettable that, at the time of the Solvay Congress of 1927, as well as later in life, Einstein was apparently unaware of the studies by Helmholtz (1887), Mayer (1896), Hirsch (1897, 1898), Bohem (1900), Könisberger (1901), Hamel (1903), Kurshak (1906), and others on the notions of variationally self-adjoint and nonself-adjoint systems (see the Foreword, Vol. I, Ref. 18 for details). It is rather tempting to speculate that perhaps if Einstein had known of these studies and the structurally more general forces they imply, he would have been in a position to present counterexamples to the conventional uncertainty principle for which Bohr's and Heisenberg's criticisms did not apply, trivially, because the physical arenas considered are different (Bohr's and Heisenberg's criticisms were based on the motion of atomic constituents, that is, particles moving in vacuum with action-at-a-distance forces and no appreciable overlapping of the wave functions, while counterexamples exist[12,14] for the motion of particles within dense hadronic matter, forces structurally more general than those of atomic type, and related nonunitary time evolutions).

Fermi also expressed quite clearly his doubts on the final character of conventional formulations. For instance, in his lecture notes in *Nuclear Physics*,[49] one can read (p. 111), in relation to the strong interactions and their short range,

> "...there are doubts as to whether the usual concepts of geometry hold for such small region of space."

We assume the reader is familiar (as Fermi was) with the fact that doubts on the underlying geometry necessarily imply doubts on the applicable relativity and quantum mechanical laws.

A reconstruction of this second historical episode has indicated that Fermi apparently expected the strong interactions (and more generally the forces responsible for the structure of the strongly interacting particles) to be local but nonderivable from a potential, of course, as an approximation of expected nonlocal forces. Apparently, Fermi was also unaware of the inverse problem. Thus, he argued that the lack of a potential would imply the lack of a Hamiltonian. This, in turn, would imply the inapplicability of conventional formulations and the need to search for broader formulations.

It is now clear that the studies considered in this paper have followed this teaching by Fermi ad litteram.

Jordan also expressed quite clearly his doubts on the final character of conventional quantum mechanics. For instance, in his celebrated paper with von Neumann and Wigner[50] one can read

> "One of us has shown that the statistical properties of the measurements of a quantum mechanical system assume their simplest form when expressed in terms of a certain hypercomplex algebra which is commutative but not associative. This algebra differs from the noncommutative but associative matrix algebra usually considered in that one is concerned with the commutative expression $\frac{1}{2}(A \times B + B \times A)$ instead of the associative product $A \times B$ of two matrices. It was conjectured that the laws of this commutative algebra would form a suitable starting point for a generalization of the present quantum mechanical theory. The need for such a generalization arises from the (probably) fundamental difficulties resulting when one attempts to apply quantum mechanics to questions in relativistic and nuclear phenomena."

The teaching of Jordan, von Neumann, and Wigner has also been invaluable for the studies considered here. In essence, their doubt concerned a fundamental algebraic structure of quantum mechanics, the universal enveloping associative algebra. It is remarkable that the recent studies outlined in Section 4 have confirmed this additional historical doubt in its entirety. It is appropriate to indicate here that the approach by Jordan, von Neumann, and Wigner has been implemented in full in the Lie-admissible generalization of Heisenberg's equations, Eqs. (67), because the product of the time evolution law, $(\tilde{f}, \tilde{H}) = \tilde{f}\tilde{R}\tilde{H} - \tilde{H}\tilde{S}\tilde{f}$, is jointly Lie-admissible *and Jordan-admissible*, that is, the attached products

$$[\tilde{f}, \tilde{H}]^* = (\tilde{f}, \tilde{H}) = (\tilde{H}, \tilde{f}) = \tilde{f}\tilde{C}\tilde{H} - \tilde{H}\tilde{C}\tilde{f}, \qquad \tilde{C} = \tilde{R} - \tilde{S}$$

$$\{\tilde{f}, \tilde{H}\}^* = \tfrac{1}{2}((\tilde{f}, \tilde{H}) + (\tilde{H}, \tilde{f})) = \tfrac{1}{2}(\tilde{f}\tilde{D}\tilde{H} + \tilde{H}\tilde{D}\tilde{f}), \qquad \tilde{D} = \tilde{R} + \tilde{S}$$

characterize a Lie algebra and a Jordan algebra, respectively, although in a generalized way. In conclusion, Eqs. (67) realize in full the idea by Jordan, von Neumann, and Wigner of alterning the *associative* character of the envelope. This is precisely the point advocated in this paper.

The reason for selecting *noncommutative* Jordan-admissible algebras, rather than commutative Jordan-admissible algebras (i.e., the Jordan algebras themselves), is that the latter prohibit the realization of a covering of conventional formulations, while the former do allow it. In turn, it is this ideas of generalizing the envelope that, when properly developed, leads to a generalization of the notion of intrinsic quantities (i.e., the spin) and thus of the notion of particle under joint electromagnetic and strong interactions.

By no means is this list of authoritative historical voices of doubt exhaustive. For instance, Pauli made it quite clear in his lectures and papers that his exclusion principle was conceived under the condition of *lack of overlapping* of the wave packets (the atomic structure). In fact, this condition is necessary to avoid forces that prohibit the separability of the wave function of identical particles and thus the study of the very basic property of whether the wave function is totally antisymmetric or not under certain permutations. This additional historical teaching has been taken into full account in the studies reported here. In fact, by the central condition, the area of inapplicability of Pauli's principle is that for which the wave packets overlap, thus producing precisely those forces that did not allow Pauli to separate the wave function (the variational nonself-adjoint forces).

Along similar lines, Wigner stated in Ref. 51, p. 18, and for additional, different arguments,

> "Let us not forget, the problem of interactions is still a mystery."

It is remarkable that once (1) the pointlike abstractions of particles are abandoned under the conditions of mutual penetration; (2) their extended character is accounted for, at least in a preliminary but effective way, via forces structurally more general than those of atomic type; and (3) the dynamical implications of these broader conditions are examined; all the different and seemingly unrelated doubts of Einstein, Fermi, Jordan, von Neumann, Wigner, and others become deeply interrelated, self-consistent, and mutually compatible.

Clearly, these authoritative doubts constitute open legacies. It is regrettable that the genuine speculative spirit underlying these legacies has been largely abandoned in the contemporary conduct of research, and the physical laws of atomic structure have assumed the current status of final character. However, according to Galilei,

> "In questions of science, the authority of a thousand is sometimes not worth the humble reasoning of one single individual."

Irrespective of the number of researchers who favor the final character of the atomic laws in nature, the historical legacies remain more open than ever, as clearly indicated by the fundamental experiment by Rauch and his associates[47] (Section 5). In fact, if the expected future refinements of these experiments establish the breaking of the $SU(2)$-spin symmetry under strong interactions, *all* the historical legacies would be experimentally verified.

But the legacy of Einstein, Fermi, Jordan, von Neumann, and others which is crucial for this paper is that a possible experimental verification of their vision would imply the irreconcilable invalidation of the conjecture that quarks are the elementary constituents of hadrons.

This point should be clearly stated in the literature for the following reason. As recalled in Section 1, quark models have proven capable of accommodating new particles and their data via suitable implementations (essentially given by an increase in the number of different quarks and/or of the dimension of the unitary group). This may create the expectation that the quark models of hadronic structure can be preserved under whatever new circumstances.

One of the primary objectives of this paper is to stress that a possible experimental verification of the invalidity of the $SU(2)$-spin symmetry under strong interactions, with consequential invalidation of Einstein's special relativity and quantum mechanics, would imply the breakdown of all the fundamental technical ingredients to even vaguely allow the definition of a quark as a physical particle, let alone to treat it consistently.

7. A SCIENTIFIC RENAISSANCE STIMULATED BY STRONG INTERACTIONS?

The receptive and open-minded reader has been exposed to a number of mathematical and physical incompatibilities such as:

(a) The incompatibility of Einstein's special relativity with the strong interactions.

(b) The incompatibility of quantum mechanics with the dynamical behavior of the hadronic constituents.

(c) The incompatibility of the Lie algebras with the time evolution law, under nonlocal nonself-adjoint forces.

(d) The incompatibility of the symplectic geometry with nonlocal strong interactions.

(e) The incompatibility of the Riemannian geometry with the interior problem of gravitation under nonlocal strong interactions.

Under no circumstances should the presentation of these contentions be interpreted as motivated by a lack of respect for existing outstanding achievements in mathematics and physics. On the contrary, the presentation is intended only in the hope of stimulating new achievements.

The interplay between mathematics and physics is well known. Mathematicians develop mathematical tools at the pure, abstract, level. Physicists apply them for the description of physical reality. Conversely, specific physical problems have often promoted the development of new mathematical studies.

The application of new mathematical tools in physics is, however, of pragmatic character; that is, new tools are used only when the insufficiency of previously used tools is either indicated as conceivable, or established. The idea of the efforts of this paper in indicating possible insufficiencies of currently used mathematical tools for the strong interactions is precisely along these lines. Indeed, the indication of these potential insufficiencies appears to be prerequisite for the promotion of the use of broader mathematical tools.

For instance, the Riemannian geometry was considered by physicists of the first part of this century as a mere mathematical curiosity. It became a fundamental tool for the study of gravitation only after Einstein indicated the insufficiency of geometries previously used in physics. Even before the experimental verification of the validity of this Riemannian approach to the exterior problem of gravitation was available, the (at that time) speculative approach served two purposes. First, it broadened the capability of treating physical systems via a more general mathematical tool. Second, it constituted an invaluable thrust for the development of geometry at the mathematical level, which is still felt today. A similar situation and interplay between mathematics and physics occurred for numerous other mathematical tools, such as the Lie algebras, the symplectic geometry, etc.

An objective of this paper is to appeal to young minds of all ages to continue this nonincremental scientific process, jointly with the conduct of more conventional research. After all, physics is and will remain an approximation of physical reality and, as such, it will never admit final descriptions. All our models simply constitute one stage of a continuing scientific process of sequential implementations.

When the current status of the art in strong interactions is inspected in this spirit, and without prejudicial views, it appears to possess all the ingredients needed to stimulate a new scientific renaissance of both mathematical and physical character.

A necessary prerequisite for this possibility to actually materialize is the acknowledgment that the virtual totality of the physical models currently available and rigorously established from classical mechanics to gravitation, to quantum mechanics, and to quantum field theory, are based on the existence of an action functional and, as such, are based on local forces derivable from a potential.

Once this situation has been identified, two layers of sequential implementations of these models become visible, one for local forces nonderivable from a potential, and one for nonlocal forces nonderivable from a potential.

Each of these two layers has great potentials for advancements in both mathematics and physics, as tentatively indicated in Table II. Irrespective of whether the strong interactions are local and derivable from a potential or not, the study of the methods for the treatment of forces more general

Table II. A Schematic View of the Hopes for a Scientific Renaissance in Mathematics and Physics[a]

Objective: search for possible coverings of the relativity and quantum mechanical disciplines of the electromagnetic interactions specifically conceived for the strong interactions

↓

Case of local nonself-adjoint strong interactions

Some mathematical topics	Some physical topics
Reinspection of Lie's theory for isotopically mapped products	Study of the Lie isotopic generalization of Heisenberg's equations
Possible generalization of the theory of Lie algebras via Lie-admissible algebras	Study of the Lie-admissible generalization of Heisenberg's equations
Lie-admissible generalization of Lie's deformation theory	Formulation of experiments to test Pauli's principle in nuclear physics
Abstract formulation of a possible nonpotential scattering theory	Formulation of experiments to test Einstein's special relativity in hadron scatterings
Possible generalization of the symplectic geometry of symplectic-admissible type	Study of the possible existence of a covering of Galilei and Einstein relativities

↓

Case of nonlocal strong interactions

Integrodifferential geometry	Interior problem of gravitation

[a] The topics listed in the table are purely indicative. Since the early part of this century, the study of the electromagnetic interactions of particles in general and the problem of the atomic structure in particular has promoted the well-known scientific renaissance of this century which has resulted in contemporary theoretical physics. The hopes under consideration here are that the study of the strong interactions in general and the problem of hadronic structure in particular appears to have all the ingredients for a new renaissance, this time aiming at a covering of the electromagnetic formulations specifically conceived for the new layer of physical reality under consideration. Initial studies along these lines are already under way by mathematicians and physicists.[15,46]

than $\mathbf{f} = -\partial V/\partial \mathbf{r}$, or of systems not necessarily derivable from a potential, can only be beneficial to mathematics, physics, and engineering. After all, whether in higher energy physics or in applied mathematics or in engineering, local systems derivable from an action functional constitute the exception and not the rule.

But, again, a necessary condition for possible advancements of genuine nonincremental character is the acknowledgment of the limitations of our knowledge as compared to the complexity of the physical universe, that is, an act of human and scientific humility.

APPENDIX A

The conditions of variational self-adjointness are studied in detail in Ref. 18. For the reader's convenience (as well as for subsequent use) we quote here only the following property (Ref. 18, Vol. I, pp. 67–68, 194–195):

Theorem. A necessary and sufficient condition for a local, class C^1 Newtonian force $F_i(t, r, \dot{r})$, $i = 1, 2, 3, \ldots, n$, to be derivable from a potential according to the familiar rule

$$F_i = -\frac{\partial U}{\partial r^i} + \frac{d}{dt}\frac{\partial U}{\partial \dot{r}^i}, \qquad r \in \mathbb{R}^n \tag{A1}$$

is that the force is at most linear in the velocity, i.e., it is of the type

$$F_i = \rho_{ij}(t, r)\,\dot{r}^j + \sigma_i(t, r) \tag{A2}$$

and all the following conditions of variational self-adjointness

$$\begin{gathered} \rho_{ij} + \rho_{ji} = 0 \\ \frac{\partial \rho_{ij}}{\partial r^k} + \frac{\partial \rho_{jk}}{\partial r^i} + \frac{\partial \rho_{ki}}{\partial r^j} = 0 \\ \frac{\partial \rho_{ij}}{\partial t} = \frac{\partial \sigma_i}{\partial r^j} - \frac{\partial \sigma_j}{\partial r^i}, \qquad i, j, k = 1, 2, \ldots, n \end{gathered} \tag{A3}$$

are identically satisfied in a star-shaped region of the variables (t, r). In this case the potential can be computed from the force according to the rule

$$\begin{aligned} U(t, r, \dot{r}) &= -r^k \int_0^1 d\tau\, F_k(t, \tau r, \tau \dot{r}) \\ &= B_k(t, r)\dot{r}^k + C(t, r) \end{aligned} \tag{A4}$$

When at least one of conditions (A2) and/or (A3) are violated, the force is called (variationally) nonself-adjoint, i.e., it is of nonpotential type; otherwise it is called self-adjoint (of potential type). The Lorentz force satisfies all the conditions of the theorem and therefore it is self-adjoint (Ref. 18, Vol. I, pp. 105–107). However, Newtonian forces generally *violate* the conditions of the theorem. Notice that the notion of nonself-adjoint force is a generalization of that of the Lorentz force. In fact, the latter is linear in the velocity while the former is generally nonlinear in the velocity. This generalization of the structure of the Lorentz force is assumed in this paper, following Ref. 12, at the foundation of the theory of the strong interactions, and it is interpreted as an analytic language for the differentiation between electromagnetic and strong interactions. The relativistic generalization of the variational approach to self-adjointness has been worked out in Ref. 17, Vol. I, and the field-theoretic generalization is treated in Refs. 7. As an incidental remark, the reader should keep in mind that *a necessary condition for a function $U(t, r, \dot{r})$ to qualify as a potential in Newtonian mechanics is that it is linear in the velocity, i.e., it is of type (A4)*. This property is a consequence of the theorem above as well as of other arguments presented in (well written) treateses in mechanics. Oddly, the condition of the linearly of the potential in the velocity (or in the momenta) is often ignored in the contemporary high-energy physics literature. For instance, in the nonrelativistic limit of QCD, functions of arbitrary powers of the momenta are often interpreted (erroneously) as "potentials" (see Section 4 for more details).

APPENDIX B

The problem of the representation of nonconservation systems via the "truncated" Lagrange and Hamilton equations has been studied in detail in Vol. II of Ref. 18. For the reader's convenience, we reproduce here only the following property.

Theorem. A necessary and sufficient condition for a local, class C^2, N-dimensional, unconstrained, nonself-adjoint Newtonian system in a three-dimensional Euclidean space

$$\{[m_{ij}\ddot{q}^j - f_i(t, q, \dot{q})]_{\mathrm{SA}} - F_i(t, q, \dot{q})\}_{\mathrm{NSA}} = 0, \qquad i = 1, 2, \ldots, 3N \quad \text{(B1)}$$

$$\det(m) \neq 0$$

(where m_{ij} is the mass tensor and the q's represent the Cartesian coordinates in a given ordering) to admit the ordered indirect analytic representation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}^k} - \frac{\partial L}{\partial q^k} \equiv g_k{}^i(t, q, \dot{q})[m_{ij}\ddot{q}^j - f_i)_{\mathrm{SA}} - F_i]_{\mathrm{NSA}} \qquad \text{(B2)}$$

$$\det(g) \neq 0$$

is that all the following conditions of variational self-adjointness in th integrating factors $g_k{}^i$

$$A_{ij} = A_{ji}\,; \qquad A_{ij} = g_i{}^k m_{kj}$$

$$\frac{\partial A_{ik}}{\partial \dot{q}^j} = \frac{\partial A_{jk}}{\partial \dot{q}^i}\,; \qquad \frac{\partial B_i}{\partial \dot{q}^j} + \frac{\partial B_j}{\partial \dot{q}^i} = 2\left(\frac{\partial}{\partial t} + \dot{q}^k \frac{\partial}{\partial q^k}\right) A_{ij} \tag{B3}$$

$$\frac{\partial B_i}{\partial q^j} - \frac{\partial B_j}{\partial q^i} = \frac{1}{2}\left(\frac{\partial}{\partial t} + \dot{q}^k \frac{\partial}{\partial q^k}\right)\left(\frac{\partial B_i}{\partial \dot{q}^j} - \frac{\partial B_j}{\partial \dot{q}^i}\right)$$

$$B_k = -g_k{}^i(f_i + F_i)$$

are identically satisfied in a star-shaped region of points $(t, q, \dot{q})$. Whe these conditions are met, a Lagrangian exists, has a generalized structure, an can be computed from the equations of motion according to the equations

$$L = L_{\text{int,I}}(t, q, \dot{q})L_{\text{free}}(\dot{q}) + L_{\text{int,II}}(t, q, \dot{q})$$

$$= -q^k \int_0^1 d\tau\, \{g_k{}^i(t, \tau q, \tau\dot{q})[m_{ij}\tau\ddot{q} - f_i(t, \tau q, \tau\dot{q}) - F_i(t, \tau q, \tau\dot{q})]\}$$

$$+ \frac{d}{dt}\int_0^1\int_0^1 d\tau\, d\tau'\, \tau q^k g_k{}^i(t, \tau q, \tau\tau'\dot{q}) m_{ij}\dot{q}^j \tag{B4}$$

$$L_{\text{free}} = \tfrac{1}{2}\dot{q}^i m_{ij}\dot{q}^j;\ L_{\text{int,II}} = B_k(t, q)\dot{q} + C(t, q)$$

Notice the necessary emergence of a generalized Lagrangian structure,[18 that is, of a Lagrangian with interaction terms which are both additive an multiplicative to the free term (when the multiplicative interaction tern reduces to one, the Lagrangian reacquires the conventional structure $L = L_{\text{free}} + L_{\text{int}}$, and all nonself-adjoint forces are null). Notice also tha Eqs. (B3) are quasilinear partial differential equations in the unknow integrating factors $g_k{}^i$ (for given equations of motion). As such, they do no necessarily admit a solution. This property establishes the fact that Lagrange' equations are not directly universal for local Newtonian systems. The lack o direct universality of Hamilton's equations then results. The theoren essentially identifies the subclass of Newtonian systems (called nonessentiall nonself-adjoint) with nonpotential forces F_i which admit an equivalent self adjoint form via a regular matrix of integrating factors. Intriguingly, it i precisely the class of systems with generalized Lagrangian structures consi dered here that have emerged as the nonrelativistic limit of QCD (se Section 4).

APPENDIX C

The situation referred to following Eq. (54) creates a duality of applicable relativities, classically and quantum mechanically. We are referring here to the applicability of conventional relativities when the system is considered as a whole by an outside observer, and the applicability of covering relativities for each constituent. This point is important for the consistency of the proposed new model of structure with available experimental data and therefore deserves a few comments. A hadron under at most electromagnetic interactions strictly obeys (Galilei's or) Einstein's special relativity. This property is satisfied by the new model of structure *by construction*. In fact, the compliance with conventional relativities is literally imposed via subsidiary constraints. More specifically, when a closed, nonself-adjoint system is seen from an outside observer, it exhibits all the conventional total conservation laws, and no deviation from conventional relativities can be detected.

In the transition to the interior problem the situation is fundamentally different. At this level, the *violation* of conventional relativity is imposed as a *necessary condition* to incorporate unrestricted forces. In fact, a central idea of the model is that of generalizing the internal forces to the point of rendering inapplicable the fundamental mathematical tools of conventional relativities, the Lie algebra and the symplectic geometry, let alone to lift all conventional covariance restrictions on the form of the structure forces. Intriguingly, the model is the only one conceivable at this time which, on the one hand, exhibits compatibility with available experimental information on the electromagnetic interactions of hadrons, and on the other, allows the representation of the hadronic constituents as extended wave packets under conditions of mutual overlapping. In fact, if conventional relativities are imposed, not only to a hadron as a whole, but also to each of its constituents (as customary in current studies on the problem), this implies *a rather substantial restriction on the structure forces*. A detailed study of the problem[17] has revealed that, under these conditions, the only admissible structure forces are of self-adjoint type. In this case, we have only internal action at a distance, and no representation of the extended size of the hadronic constituents nor of the (internal) dynamical effects resulting from wave overlapping.

In summary, the methods of the inverse problem[18] have permitted the following results of direct relevance for the problem of the hadronic structure.[17] (1) The identification and technical treatment of the forces resulting from wave overlapping (variationally non-self-adjoint forces), as well as of the action-at-a-distance forces among pointlike constituents (variationally self-adjoint forces). (2) The identification and technical treatment of the structure of conventional closed models (closed, self-adjoint models). (3) The discovery and technical treatment of a broader class of closed models with unrestricted internal forces (closed, non-self-adjoint models). (4) The treatment of the property that the imposition of conventional relativities to a structure model as a whole as well as to each of its constituents implies a closed, self-adjoint model. (5) The treatment of the property that closed, nonself-adjoint models are compatible with conventional relativities when the states are considered as a whole, while they demand covering relativities for the constituents, as a necessary condition to permit unrestricted internal forces. To avoid excessive approximation and pointlike abstractions, the only structure model of hadrons which is conceivable on grounds of available knowledge is apparently that along the idea of a closed, nonself-adjoint system. But, as we review in Section 4, this implies a profound change in the notion of particle constituent in the transition from atomic to hadronic structure. The experimental studies under way are centered on the experimental verification of whether these changes exist or not (Section 5).

APPENDIX D

A number of controversies on quark models have been indicated in Section 4 with regard to the quantitative interpretation of the hadronic phenomenology. There exist a number of additional controversies which are more directly related to the structure problem. As is well known, quarks must be pointlike particles for consistency of the theory (e.g., as demanded by the special relativity, or by the local character of QCD; see Fig. 5). But pointlike particles can only experience action-at-a-distance (self-adjoint) forces. Thus, the analytic character of the structure models provided by quarks is that of closed, self-adjoint systems (Section 4). A first controversy originates from the fact that the pointlike approximation of the constituents is certainly admissible for atomic structure, but it is questionable for hadronic structure. In fact, according to clear experimental evidence, *all* strongly interacting particles have a size of the order of 1 *F*. It is then rather natural to expect that the hadronic constituents, to be strongly interacting, also have a size of the order of 1 *F*. But this appears to be in irreconcilable disagreement with the conjecture that quarks are physical pointlike particles. Independently from that, clear experimental data establish that hadrons have a wave structure. Any structure model of hadrons, to be acceptable by the scientific community at large, must recover this physical characteristic in an effective way. The conjecture that quarks are pointlike particles does not appear to achieve this objective without considerable controversies. In fact, the conjecture naturally leads to the abstraction of a hadron as an ideal empty sphere with points in it, as typical, say, of the bag models. The attempts to achieve a "quark size" via renormalization and form factors rather than resolving the controversies create additional ones. In fact, for consistency, these attempts must refer to a size much smaller than that of a hadron (e.g., of the order of $10^{-10}F$). This creates a host of as yet unresolved technical problems (e.g., the localization of high-energy wave packets in extremely small regions of space). If the wave packet of the quark is assumed to be of the order of 1 F, one necessarily obtains the wave overlappings of Section 3. In this case the very notion of quark becomes intrinsically inconsistent because of the breakdown of the mathematical and physical foundations needed to consistently define a quark (unless one believes that a local theory can effectively treat waves in deep overlapping conditions, or that the wave aspect of particles can be ignored).

Quite significatively, the controversies on quarks have *increased* (rather than decreased) in time. All currently available information indicate that *the controversies will continue to increase in the future* (e.g., because of the tendency to increase the plurality of unidentified quarks). The view advocated in this paper is that this situation cannot be continued indefinitely. It is time to review the problem of the structure of hadrons from its physical and mathematical foundations, and search for possible alternatives, as a complement to the continuation of research along quark lines. An alternative model of hadronic structure has been proposed by Santilli[12] based on the following ideas. First, the hadronic phenomenology is divided into three distinct aspects: Mendeleev-type classification, Bohr-type structure, and scattering. It is extremely unlikely that one single model (whether of quark or nonquark inspiration) will succeed in solving the entirety of this phenomenology. In fact, the atomic phenomenology demanded different (yet campatible) models. In the transition to the more complex problem of the nuclear phenomenology, the need for different models emerged more strongly. When we pass to the still more complex hadronic phenomenology, the insufficiency of the human mind to unify the complexity of nature is expected to emerge in a more forceful way. We thus assumed that the hadronic phenomenology calls for (at least) three different models, one for the classification, one for the structure, and one for the scattering. The unitary models were then assumed as providing the final, Mendeleev-type classification of hadrons. However, to eliminate ab initio the current proliferating controversies, quarks were assumed as mere mathematical quantities (representations of unitary Lie groups of classification). In particular, the parameters currently associated to particle quarks in the unitary classification (e.g., mass, spin etc.) were reinterpreted in a purely mathematical way, without *any* intrinsic physical meaning (e.g., as parameters mixing different representations). After abandoning the possibility that quarks might be physical particles, the problem of structure was confronted by following Bohr's teaching, that is, by constructing the structur of one entity (say the π^0) without any use of the features of other entities o the same family (say, the $\pi^{\pm}$). This led to the *prohibition* of using in th structure problem the chemical numbers of the classification, such as isospin hypercharge, charm, etc. (for instance, one does not construct a structur model of palladium by using the valence of the palladium family). In thi way, the structure problem was removed of any constraint originating fron the tools of the classification. The understanding is that the *independen* structure model must achieve *compatibility* with the classification (e.g. independent structure models of the π^0 and $\pi^{\pm}$ must recover compatibilit with the isotopic triplet via similarities in nuclear scatterings).

These premises (see Section 2 of Ref. 12 or Vol. I of Ref. 17) permitte the confrontation of the problem of hadronic structure in a new way. First the hadronic constituents were assumed as *extended particles* with a size o the order of 1 *F*. This led to a structure model with a deep overlapping of th wave packets of the constituents and resulting nonlocal, nonpotentia internal forces, resulting in a closed, nonself-adjoint generalization of atomi models (Section 3, Ref. 12). Next, the problem of the mechanics capable o representing the structure was confronted. It soon emerged that, unde nonpotential forces, the constituents evolve according to a nonunitary law The invalidation of conventional laws, principles, and insights of quantun mechanics (the atomic mechanics) was then a mere consequence (Section 4) Next, the rudiments of a generalization of quantum mechanics (calle hadronic mechanics) for nonunitary time evolutions was attempted (Section of Ref. 12). Also, the problem of a hadronic constituent (or of a particl under strong interactions) was confronted as yet another prerequisite for th problem of structure. The studies revealed that a massive and charge particle exhibits variations of the intrinsic characteristics (called mutations in the transition from long-range electromagnetic interactions to stron interactions realized via wave overlappings and nonpotential forces. Thi generalization of the notion of particle (technically realized via a two-side generalization of the representation theory[21]) was then assumed as th notion of hadronic constituents.

After achieving these background tools, the problem of the structure o the light mesons was finally confronted (Section 5 of Ref. 12). It essentiall emerged that the generalization of quantum mechanics for closed, nonself adjoint systems permits the construction of unstable bound states with masse of the constituents generally lower than that of the total state, and with ver low binding energies. These features permitted the identification of th hadronic constituents with physical, already known, particles which can b produced free in the spontaneous decays. More particularly, the ultimat elementary constituents of hadrons, called *eletons* and *antieletons*, wer assumed to be given by the ordinary electrons and positrons under conditions of total wave overlapping and nonunitary time evolutions. When the energies are sufficient, eletons and antieletons cluster into more complex constituent states. In the spontaneous decays, we see the production of either clusters of eletons and antieletons, or of these particles, or of the product of their annihilation (gammas). When an eleton or a cluster of electons exists the hadron, the conventional intrinsic quantum mechanical features are re-acquired because the time evolution resumes the unitary form. Intriguingly, the neutrinos emerged in a new light, that of the particles emitted in the reduction of eletons to electrons. In this way, one elementary physical constituent and its antiparticle, jointly with their clusters, can account for a virtually infinite number of different hadrons. However, the total number of elementary constituents increases with the mass, by recovering a rule already established at the atomic and the nuclear levels. For instance, the total number of eletonic constituents of the π^0 is two, while that of the $\pi^{\pm}$ is three, and this illustrates the differences of the model from quark models (for which all light mesons have the *same* number of constituents). Another relevant aspect is that clusters of eletons can have, in principle, an arbitrary charge. This illustrates the point mentioned in Section 1, that nonelementary charges (e.g., fractional) are admitted by models fundamentally different than the

Table III

	Quark model of light mesons	Eletonic model of light mesons
Atomic mechanics	Assumed valid	Generalized
Constituent	Unidentified	Physical particles
Constituent mass	Very light	Very light
Binding energy	Very low	Very low
Confinement	Assumed valid	Absent
Nonrelativistic structure equations	Absent	Consistent
Quantitative representation of total characteristics via structure equations	Absent	Consistent
Decays	Assumed via intermediary processes	Realized via tunnel effects or annihilation processes
Newtonian limit	Unknown	Consistent

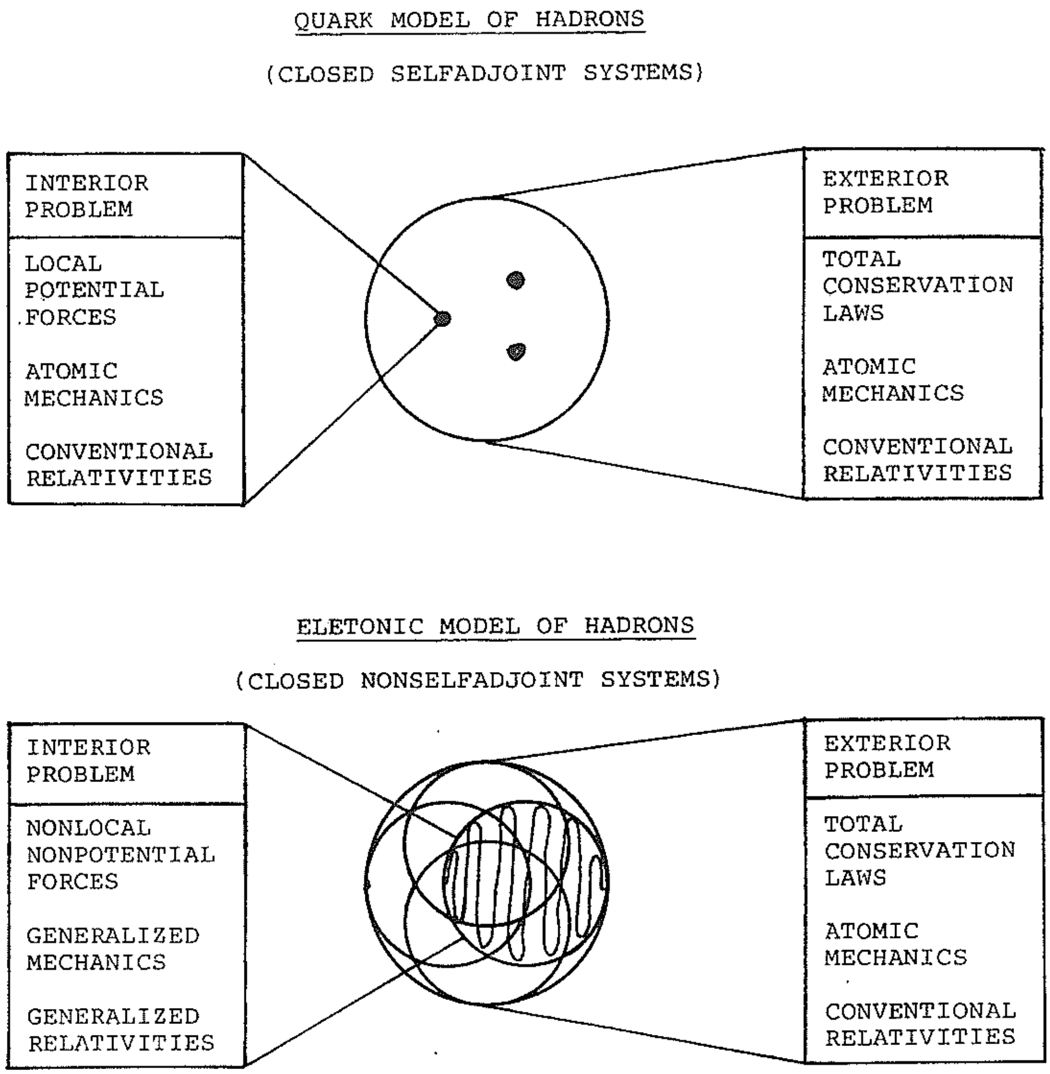

Fig. 8

quark models. A comparative view of the quark and eletonic models is giver in Table III and Fig. 8. For technical details, we refer the interested reade to Section 5 of Ref. 12 or to the forthcoming Vol. III of Ref. 17.

NOTE ADDED IN PROOF

Since the time of writing this paper, a number of additional experiment: have emerged in nuclear physics with clear indications of deviations fron conventional quantum mechanical views. A first experiment concerns th measure of the angle of precession of neutron in matter due to optica activity, in which the measured angle is three thousand times the predictec angle. A second experiment is related to the *T*-symmetry (also under stron nuclear interactions), in which, according to the words of the experimenters the deviation from the exact symmetry is "astonishing large." These and other experimental aspects are treated in Ref. 52.

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