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INSTITUTE FOR THEORETICAL PHYSICS
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Ruggero Maria Santilli

THEORETICAL FOUNDATIONS
Volume II:
**ELEMENTS OF
HADRONIC MECHANICS**

ACADEMY OF SCIENCES OF UKRAINE
INSTITUTE FOR THEORETICAL PHYSICS

ELEMENTS of HADRONIC MECHANICS

Volume II:

THEORETICAL FOUNDATIONS

Ruggero Maria Santilli

The Institute for Basic Research

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EXCERPTS FROM THE REVIEWS

A. Jannussis (Univ. of Patras, Greece): "*Hadronic Mechanics supersedes all theories to date.*"

(opening address of the International Conference on the Frontiers of Physics. Olympia, Greece, 1993)

H. P. Leipholz (Univ. of Waterloo, Canada): "*Santilli's studies are truly epoch making.*"

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T. L. Gill (Howard Univ., Washington, D. C.): "*The three volumes on Hadronic Mechanics represent the most important contribution to physics in the last fifty years.*"

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Dedicated to my spiritual guide and friend

Father DOMINIC CALARCO, S. X., Dr. Miss.

*because of his teaching that
the problems of contemporary societies,
including problems in the current condition of physical research,
are of a primary ethical nature.*

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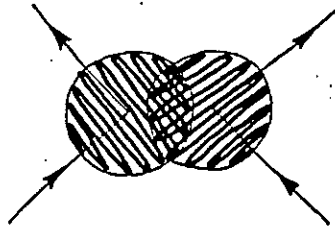
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PREFACE

These three volumes are the first books written on a generalization of quantum mechanics under the name of *hadronic mechanics* which I proposed back in 1978 when at Harvard University under support from the U. S. Department of Energy and then studied by a number of mathematicians, theoreticians and experimentalists.

The primary objective of the new mechanics is to reach a quantitative treatment in a form suitable for experimental verifications of the old legacy that the strong interactions in general, and the hadronic structure in particular, have a nonlocal component due to the experimentally established mutual penetration and overlapping of the wavepackets-wavelengths-charge distributions of hadrons, as symbolically represented below



under the condition that the representation preserves all the essential physical characteristics of quantum mechanics, such as observability, causality, etc.

I have presented the mathematical foundations of hadronic mechanics in Volume I. The scope of this Volume II is the identification of the theoretical foundations of the new mechanics, with particular reference to the basic physical laws for an axiomatically correct treatment of the above physical conditions. Applications and experimental verifications will be studied in Volume III.

The fundamental hypothesis of hadronic mechanics is the generalization of the basic unit of the enveloping operator algebra of quantum mechanics into a form with an arbitrary, integro-differential dependence

$$I = \text{diag. } (1, 1, \dots) \rightarrow \hat{I}(t, r, p, \psi, \partial\psi, \partial\partial\psi, \dots). \quad (1)$$

The description of physical systems in hadronic mechanics then requires *two* operators, the Hamiltonian $H = K + V$ for the representation of all interactions derivable from a potential V , and the generalized unit \hat{I} for the representation of all interactions and internal effects which are not derivable from a potential or a Hamiltonian by conception.

A first simple example of generalized unit is given by the diagonal and positive-definite form in three dimension $\hat{I} = \text{Diag. } (b_1^{-2}, b_2^{-2}, b_3^{-2})$ which permits a direct representation of *nonspherical* charge distributions of hadrons, such as ellipsoids with semiaxes b_k^2 , as well as all their infinitely possible *deformations*. In turn, these capabilities of hadronic mechanics (which are manifestly absent in quantum mechanics) permit the first exact-numerical representation on record of the total magnetic moment of the deuteron, tritium and of few-body nuclei.

An example illustrating the nonlocal-integral character of the theory is *Animalu's generalized unit* $\hat{I} = \exp\{N \int dV \psi^\dagger(r)\phi(r)\}$ for the electrons of the Cooper pair in superconductivity with wavefunctions ψ and ϕ , which permits the first representation on record of their *attractive interactions* in a way remarkably in agreement with experimental data. Numerous additional examples will be identified during the course of our analysis, such as the generalized unit permitting the first numerical representation on record of the experimental data on the Bose-Einstein correlation as originating from *nonlocal interactions* in the interior of the $p\bar{p}$ fireballs.

As studied in Vol. I, the generalization of the unit requires, for evident reason of compatibility, a consequential generalization of the entire mathematical formalism of quantum mechanics into a new formalism admitting of \hat{I} as the correct left and right unit. This includes a generalization of: fields of real and complex numbers; vector, metric and pseudo-metric spaces; Euclidean, Minkowskian and Riemannian geometries; ordinary functions (e.g., trigonometric functions), special functions (e.g., spherical functions), transform (e.g., Fourier transform), distributions (e.g., Dirac delta distribution); Banach and Hilbert spaces; Lie algebras, Lie groups and Lie symmetries; transformations and representation theories; classical Hamiltonian mechanics; etc.

Generalized units (1) are classified into *Hermitean generalizations* $\hat{I} = \hat{I}^\dagger$, characterizing the *Lie-isotopic branch of hadronic mechanics*, and *nonhermitean generalizations* $\hat{I} \neq \hat{I}^\dagger$, characterizing the more general *Lie-admissible branch of hadronic mechanics*.

The former methods are used for the treatment of closed-isolated systems of particles with Hamiltonian and nonhamiltonian internal interactions verifying conventional total conservation laws, including the reversibility of the center-of-mass trajectory. The latter formulations are used for the

characterization of open-nonconservative systems in irreversible conditions under the most general known, external, nonlinear-nonlocal-nonhamiltonian interactions.

Each of the above two branches is then classified into *Kadeisvili's five different classes* depending on the primary topological characteristics of the generalized unit. This illustrates the rather diversified structure of hadronic mechanics for the characterization of a hierarchy of physical systems with increasing complexity and methodological needs. By comparison, quantum mechanics has one single structure, as well known.

By using the above diversified structure, in this volume I establish the *direct universality* of hadronic mechanics, i.e., the capability of the theory of representing all possible operator systems (universality) directly in the frame of the experimenter (direct universality). As we shall see, this includes a representation of gravitational singularities represented via the zeros of the generalized units.

I should stress that the above direct universality refers to *systems* and does not include various other methods for the treatment of the same systems. As an example, hadronic mechanics represents all possible systems described by the so-called *q-deformations* (and actually much more as we shall see), but *hadronic mechanics and q-deformations are structurally inequivalent*, e.g., because defined on different fields, different Hilbert spaces, etc.

As we shall see, hadronic mechanics permits a variety of novel interpretation of existing data, applications and predictions which are simply beyond any hope of quantitative treatment with quantum mechanics. In particular, the new mechanics permits novel structure models of nuclei, hadrons and stars with a variety of new predictions all verifiable with current technology, such as: a quantitative formulation of *antigravity*; the prediction of a *space-time machine* based on the alteration of the *units* of space and time; an apparent new form of subnuclear energy I called *hadronic energy*; a new technology at distances smaller than 10^{-13} cm I called *hadronic technology*; and others. In this volume I present the *theoretical foundations* permitting these predictions. Their treatment and experimental verification will be considered in Vol. III.

The *algebraic origin* of hadronic mechanics can be traced back to A. A. Albert (Trans. Amer. Math. Soc. **64**, 552 (1948)) who introduced the notion of *Lie-admissible algebras* as generally nonassociative algebras U with elements $a, b, ..$ and abstract product ab which are such that the attached algebras U^- with product $[a, b]_U = ab - ba$ are Lie. Albert introduced the above notion for the primary purpose of studying the so-called *noncommutative Jordan algebras* with realization of the product

$$(a, b) = \lambda a b - (1 - \lambda) b a, \quad (2)$$

where ab is associative and λ is a scalar, which do not possess a well defined content of Lie algebra (i.e., there is no finite value of λ under which product (b) is Lie). As such, their possible physical relevance is unknown at this time.

The notion of Lie-admissible algebras U used in these volume was introduced by R. M. Santilli in his Ph. D. studies (Lett. Nuovo Cimento 51, 570 (1967)) according to the *dual condition* that U^- is Lie and that Lie algebras are contained in the classification of U . This turns Lie-admissible algebras into genuine coverings of Lie algebras. Santilli [loc. cit.] then introduced the realization

$$(a, b) = p a b - q b a, \quad (3)$$

where ab is associative and p, q are scalars.

Subsequently, Santilli (Hadronic J. 1, 223, 574 and 1279 (1978)) introduced the so-called *general realization of Lie-admissible algebras*

$$(A, B) = A R B - B S A, \quad (4)$$

where $R, S, R \pm S$ are nonsingular operators, whose attached antisymmetry product

$$[A, B]_U = (A, B) - (B, A) = A T B - B T A, \quad T = R + S, \quad (5)$$

does satisfy the Lie algebra axioms but it is not conventionally Lie and characterizes instead Lie-isotopic algebras. In this way I reached in 1978 the central dynamical equations of for closed-conservative and open-nonconservative systems, respectively [loc. cit.]

$$i dA / dt = [A, H]_U = A T H - H T A, \quad (6)$$

$$i dA / dt = (A, H)_U = A R H - H S A, \quad (7)$$

which are at the foundations of hadronic mechanics.

The *classical analytic origin* of hadronic mechanics can be traced back to G. D. Birkhoff (*Dynamical systems*, A.M.S., Providence, RI (1927)) who identified the following generalization of Hamilton's equations

$$\Omega_{\mu\nu}(a) \frac{da^\nu}{dt} = \frac{\partial H}{\partial a^\mu}, \quad (8)$$

$$\Omega_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu, \quad \det \Omega \neq 0, \quad a = \{ r^k, p_k \}, \quad k = 1, 2, 3, \quad \mu, \nu = 1, 2, \dots, 6,$$

which preserves the abstract axioms of Hamiltonian mechanics, thus being an *isotopy* of the latter, because the underlying two form $\Theta = \Omega_{\mu\nu} da^\mu \wedge da^\nu$ is an exact, nowhere degenerate and therefore symplectic as the canonical two-form, although of the most general possible exact type. Similarly, the brackets characterized by the contravariant tensor $\Omega^{\mu\nu} = |(\Omega_{\alpha\beta})^{-1}|^{\mu\nu}$ do verify the Lie

axioms, although they characterize the more general Lie-isotopic algebras.

Some fifty six years following their appearance, I conducted systematic studies of the largely forgotten Eq.s (8), which I called *Birkhoff's equations* (R. M. Santilli, *Foundations of Theoretical Mechanics*, Vol. II: *Birkhoffian Generalization of Hamiltonian mechanics*, Springer-Verlag, Heidelberg-New York (1983)) and prove them to be directly universal for all possible local-differential Newtonian systems verifying certain topological conditions.

However, the emerging step-by-step generalization of Hamiltonian mechanics, which I called *Birkhoffian mechanics*, resisted all attempts for quantization into a form usable for practical applications. This forced me to conduct a second laborious study of the *classical foundations* of hadronic mechanics (R.M. Santilli, *Hadronic J. Suppl.* **4B**, issue no. 1 (1988); *Isotopic Generalizations of Galilei's and Einstein's Relativities*, Vol.s I and II, Second Edition, Ukraine Academy of Sciences, Kiev (1994)) which resulted in the construction of: A) the more general *Birkhoff-isotopic mechanics* with basic equations

$$\Omega_{\mu\alpha}(a) T^{\alpha}_{\nu}(t, a, \dot{a}, \dots) \frac{da^{\nu}}{dt} = \frac{\partial H}{\partial a^{\mu}}, \quad (9)$$

where T is a nowhere degenerate *symmetric* matrix; B) the *symplectic isotopic geometry* with nowhere-degenerate exact two-isoforms $\Theta = \Omega_{\mu\alpha} T^{\alpha}_{\nu} da^{\mu} \wedge da^{\nu}$; and C) the classical realization of the Lie-isotopic algebra characterized by the contravariant tensor $\hat{\Gamma}^{\mu}_{\alpha} \Omega^{\alpha\nu} = |(\Omega_{\rho\sigma}(a) T^{\sigma}_{\tau})^{-1}|^{\mu\nu}$.

The above studies permitted the identification of the *classical origin of the fundamental quantity of hadronic mechanics, the generalized unit* $\hat{1}$. In turn, this permitted the identification of unique and unambiguous generalized methods for the mapping of classical into operator theories, and established the direct universality of hadronic mechanics as originating at the classical level. In fact, the Newtonian systems mapped into quantum mechanics must be restricted to admit a meaningful Hamiltonian, while such restriction is unnecessary for hadronic mechanics which provides the operator image of the most general possible Newtonian systems which are arbitrarily nonlinear, nonlocal-integral and nonhamiltonian.

The *statistical origin* of hadronic mechanics can be traced back to the studies by I. Prigogine and his Bruxelles School (see, e.g., Cl. George et al., *Hadronic J.* **1**, 520 (1978)) which are based on a *nonunitary* formulation of conventional quantum statistics. Subsequently, J. Fronteau, Tellez-Arenas and R. M. Santilli (*Hadronic J.* **3**, 130 (1979)) formulated a generalization of statistical mechanics with a Lie-admissible structure. More recently, A. Jannussis and R. Mignani (*Physica* **A187**, 575 (1992)) proved that the nonunitary irreversible statistics of Prigogine's school has an essential Lie-admissible structure.

The above results permitted hadronic mechanics to identify the origin

of irreversibility in the ultimate level of the structure of matter, that of elementary particles in open-nonconservative conditions, such as a proton in the core of a star considered as external. This elementary origin of irreversibility is studied in this volume at the nonrelativistic, relativistic and gravitational levels. Macroscopic irreversibility is then a mere collection of *elementary* nonunitary-irreversible systems.

These advances resolves recently identified inconsistencies of the conventional conception of irreversibility, such as those treated by the so-called *No-Reduction Theorems* which establish the impossibility of reducing a macroscopic body in irreversible conditions with continuously decaying angular momentum to an ideal collection of quantum mechanical particles in reversible conditions and with conserved angular momentum (and viceversa). The only way I know of resolving these inconsistencies is by identifying the origin of irreversibility in the ultimate elementary level of matter with a consequential, *necessary* generalization of quantum mechanics.

It is important in these introductory notes to indicate the reasons why hadronic mechanics was constructed via the Lie-isotopic and Lie-admissible methods rather than other generalized approaches existing in the literature. Consider, for instance, the so-called *q-deformations* by L. C. Biedenharn (J. Phys. A22, L873 (1989)), A. J. MacFarlane (J. Phys. A22, L4581 (1989)) and many others, with generic product

$$(a, b) = a b - q b a, \quad (10)$$

which, on one side, generalize quantum mechanics while, on the other side, are treated via conventional quantum mechanical methods (e.g., conventional fields, metric spaces, Hilbert spaces, etc.).

Even though I originated the *q-deformations* some twenty two years before Biedenharn, MacFarlane and the others,¹ I was forced to abandon them by the late 1970's because of a considerable number of rather serious problematic aspects of physical character, such as: lack of form-invariance under the time evolution of the theory; loss of Hermiticity, and therefore of observability, under the time evolution (see below why); loss of the measurement theory because of the lack of invariance of the assumed unit under the time evolution; lack of uniqueness of generalized operations such as exponentiation, with consequential

¹ It should be noted that Biedenharn and MacFarlane did not quote in their 1989 papers on *q-deformations* the historical paper by Albert (Trans. Amer. Math. Soc. 64, 552 (1948)) on the essential Lie-admissible character of their deformations, or the preceding more general *(p, q)-number* and *(R, S)-operator-deformations* by R.M. Santilli (Lett. Nuovo Cimento 51, 570 (1967); Hadronic J. 1, 574 (1978)), or any of the related literature on hadronic mechanics which, by 1989, was rather considerable. The quotation of the above prior literature was then ignored in the subsequent vast literature in *q-deformations* as the reader can verify.

ambiguities in the related generalized physical laws (such as q-uncertainties); lack of validity of the q-special functions at all times; and others.

In fact, the time evolution of q-deformations (10) is evidently noncanonical and therefore *nonunitary*. This implies: the lack of conservation in time of the basic unit of the theory, the conventional form $I = \text{diag. } (1, 1, 1, \dots)$, because nonunitary transforms are such that, by definition, $I' = UIU^\dagger \neq I$; the lack of form-invariance of the theory,

$$U(a b - q b a)U^\dagger = a' R b' - b' S a', \quad (11)$$

$$R = (U U^\dagger)^{-1}, \quad S = q R, \quad a' = U A U^\dagger, \quad b' = U b U^\dagger. \quad (12)$$

The other problematic aspects then follow.

Even though transforms (11) have the (R, S)-operator-structure (7), they are unacceptable for hadronic mechanics because again not form-invariant. In fact, a second nonunitary transform $WW^\dagger = \tilde{T}^{-1} \neq I$ establishes their lack of form-invariance,

$$W a' R b' W^\dagger - W b' S a' W^\dagger = a'' \tilde{T} R' \tilde{T} b'' - b'' \tilde{T} S' \tilde{T} a'', \quad (13)$$

thus leaving all original problematic aspects essentially unchanged.

The only way I know to achieve an *axiomatic Lie-admissible theory*, that is, a theory possessing the same axiomatic properties of quantum mechanics (form-invariance, Hermiticity-observability at all times, invariance of the basic unit, uniqueness of the various operations, validity of functional analysis at all times, etc.) is by reformulating the theory according to the basic axioms of the genotopic branch of hadronic mechanics studied in this volume (the most general possible branch over genofields, genospaces, genohilbert spaces, etc.).

Another line of inquiry which I had to abandoned for the construction of hadronic mechanics is that of the so-called *nonlinear theories*. I am here referring to theories studied by R. W. Hasse (J. Math. Phys. **16**, 2005 (1975)), N. Gisin (J. Phys. **A 14**, 2259 (1981)), H.-D. Doebner and G. A. Goldin (Phys. Lett. **A 162**, 397 (1992)) and several others with a *nonlinearity in the wavefunction* represented with the conventional Schrödinger's equation

$$i \partial_t \psi(t, r) = H(t, r, p, \psi, \psi^\dagger, \dots) \psi(t, r), \quad (14)$$

which also generalize quantum mechanics, yet are treated via conventional quantum methods.

In fact, the above approach generally represents open-nonconservative systems and, as such, it is expected to have nonhermitean Hamiltonians and *nonunitary time evolutions*, in which case they have the same problematic aspects of the q-deformations. Irrespective of whether the Hamiltonian is

Hermitean or not, Eq. (14) have the additional problematic aspects caused by the evident loss of the superposition principle.

The only way known to this author of resolving the above problematic aspects is via the axioms of hadronic mechanics which imply the factorization of all nonlinear contributions $H(t, r, p, \psi, \psi^\dagger, \dots) = H_0(t, r, p)T(\psi, \psi^\dagger, \dots)$, and then the reconstruction of the entire formalism of quantum mechanics with respect to the generalized unit $I = T^{-1}$.

Yet another line of research which I had to be abandoned for the construction of hadronic mechanics is the *theory with nonassociative Lie-admissible envelope* submitted by S. Weinberg (Ann. Phys. **194**, 336 (1989)) according to the following basic equation

$$i \frac{dA}{dt} = A \hat{\times} H - H \hat{\times} A = \frac{\partial A}{\partial \psi_k} \frac{\partial H}{\partial \psi_k^\dagger} - \frac{\partial H}{\partial \psi_k} \frac{\partial A}{\partial \psi_k^\dagger}, \quad (15)$$

where the product $A \hat{\times} H$ characterizes a nonassociative Lie-admissible algebra.² Note that the equations are also nonlinear in the wavefunctions as Eq.s (14).

The reasons why this latter approach had to be abandoned are known since the early studies of general Lie-admissible theories. In fact, *when used as the enveloping algebra* $A \hat{\times} H$ of the time evolution $idA/dt = A \hat{\times} H - H \hat{\times} A$ ³ nonassociative Lie-admissible algebras admit no unit at all, thus preventing any applicability of the measurement theory. Also they admit no known exponentiation, thus preventing the achievement of a consistent generalized symmetry as well as consistent generalized physical laws dependent on the exponentiations (such as Gaussians, the uncertainties, etc.). Moreover, Weinberg's theory violates the *No-Quantization Theorem* by S. Okubo (Hadronic J. **5**, 1667 (1989)) according to which the Heisenberg's type and the Schrödinger-type formulations are inequivalent for all theories with nonassociative envelopes.

By looking now in retrospective, the only known generalizations of quantum mechanics which are axiomatic in the sense indicated earlier can be essentially derived as follows. First, let us recall the axiomatic structure of quantum mechanics as embedded, say, in its fundamental commutation rules

² S. Weinberg abstained from quoting in his 1989 paper the contributions by Albert of 1948, Santilli of 1967 and 1978 quoted earlier, or other contributions in hadronic mechanics (which were rather numerous by 1989), in order to identify the *nonassociative Lie-admissible character of the envelope* of his his equations. All subsequent papers on Eq.s (15) also did not quote the above essential literature, as the reader can verify.

³ Recall that in quantum mechanics *the envelope is associative* with conventional product AH while *the brackets of the time evolution are nonassociative*, i.e., $idA/dt = AH - HA =$ nonassociative-Lie. I therefore refer in the text to to the problematic aspects suffered by nonassociative Lie-admissible envelopes with product $A \hat{\times} H$ and *not* to theories with Lie-admissible brackets $idA/dt = (A, H) = ARH - HSA =$ nonassociative Lie-admissible.

$$rp - pr = i\hbar I, \quad (16)$$

formulated on a conventional Hilbert space \mathcal{H} over a complex field. As it is well known, the enveloping operator algebra ξ with elements r, p and their polynomial combinations is *associative* with conventional product rp and fundamental unit $I = \text{diag. } (1, 1, \dots)$, $IA = AI = A, \forall A \in \xi$. The time evolution of the theory is unitary, i.e., represented by the operator U such that $UU^\dagger = U^\dagger U = I$. This implies: the invariance of the basic unit at all times, $I' = UIU^\dagger \equiv I$, with consequential applicability of the measurement theory at all times; the preservation of the Hermiticity of the Hamiltonian $H = UHU^\dagger$ at all times with consequential observability at all times; the form-invariance of the theory,

$$U(rp - pr)U^\dagger = r'p' - p'r' = i\hbar UIU^\dagger = i\hbar I, \quad (17)$$

and the remaining conventional axiomatic properties.

The Lie-isotopic generalization of quantum mechanics preserving all the above axiomatic properties can be constructed as follows. First, recall from Vol. I that the Lie-isotopic algebras are a *nonunitary* image of conventional Lie algebras. One can therefore subject rule (16) to a nonunitary transformation $UU^\dagger = \hat{1} \neq I$, for which we have

$$U(rp - pr)U^\dagger = r'Tp' - p'Tr' = i\hbar UIU^\dagger = i\hbar \hat{1} = i\hbar T^{-1}, \quad (18)$$

where one should note that $\hat{1} = UU^\dagger$ and $T = (UU^\dagger)^{-1}$ are *Hermitean*.

This first step renders *necessary* the following isotopic generalizations: 1) the enveloping algebra ξ with product AB is lifted into the form $\hat{\xi}$ with isoassociative product $A*B = ATB, T$ fixed; 2) the Lie product $[A, B]_{\mathcal{V}\xi} = AB - BA$ is lifted into the Lie-isotopic product $[A, B]_{\hat{\xi}} = ATB - BTA$; and 3) the fundamental unit I is lifted into the generalized quantity $\hat{1}$ of Eq. (1) which is indeed the correct left and right unit of the theory, $\hat{1}*A = T^{-1}TA \equiv A*\hat{1} \equiv A, \forall A \in \hat{\xi}$.

However, the above formulation is still insufficient because under an additional nonunitary time evolution $WW^\dagger = D \neq I$, the generalized unit $\hat{1}$ is not preserved, $\hat{1}' = W\hat{1}W^\dagger \neq \hat{1}$, and the Lie-isotopic product is not form-invariant. Also, the envelope is now *isoassociative* with Hermiticity condition on \mathcal{H} $H^\dagger = TH^\dagger T^{-1} \neq H^\dagger, T = (UU^\dagger)^{-1}$, and general loss of Hermiticity-observability according to *Lopez's Lemma* II.3.C.1 of p. 122 (D. F. Lopez, *Hadronic J.* **16**, 429 (1993)), etc.

The *only* solution I know resolving *all* these problematic aspects is that along the axioms of the isotopic branch of hadronic mechanics. In this case, nonunitary transforms $WW^\dagger \neq I$, can always be reformulated in the isounitary form

$$W = \hat{W} T^{1/2}, \quad W W^\dagger = \hat{W} T \hat{W}^\dagger = W * W^\dagger = \hat{W}^\dagger T \hat{W} = \hat{W}^\dagger * W = \hat{1} = \hat{1}^\dagger = T^{-1}, \quad (19)$$

yielding the *fundamental isocommutation rule* of hadronic mechanics

$$r T p - p T r = i \hbar T^{-1}, \quad (20)$$

first identified by Santilli (Hadronic J. Suppl. **4B**, 1 (1989)).

It is then easy to prove the form-invariance of: the generalized unit $\hat{1}' = \hat{W} * \hat{1} * \hat{W}^\dagger = \hat{W} T T^{-1} T \hat{W}^\dagger = \hat{1}$; the isoassociative product $\hat{W} * (A * B) * \hat{W}^\dagger = A * B'$; the Lie product; and, consequently the fundamental isocommutation rules

$$\hat{W} * (r * p - p * r) * \hat{W}^\dagger = r' * p' - p' * r' = i \hbar \hat{W} * \hat{1} * \hat{W}^\dagger = i \hbar \hat{1}. \quad (21)$$

The preservation of all other axiomatic features of quantum mechanics, including the Hermiticity-observability at all times, is then consequential, as we shall see. As a matter of fact, hadronic and quantum mechanics emerge as coinciding at the abstract, realization free level, as one can see from the abstract identity of Eq.s (16) and (20).

It should be noted that all structures which deviate from Eq. (20) violate one of the other axiom of hadronic mechanics, therefore resulting in problematic aspects for physical applications. For instance, structures with commutation rules of the generalized Lie type

$$r p - p r = i \hbar F(q, \dots), \quad r T(q, \dots) p - p T(q, \dots) r = i \hbar F(q, \dots), \quad F \neq T^{-1}, \quad (22)$$

do not possess an axiomatic structure and, as such, are afflicted by the problematic aspects indicated earlier. In fact, they are not form-invariant even when expressed in isofields, isospaces and isohilbert spaces, thus suffering all the shortcomings of the conventional q-deformations.

Note that this is the fate also for the so-called *quantum groups* because they preserve the conventional quantum-Lie brackets but generalize their eigenvalues, thus implying nonunitary transforms with all the problematic aspects of q-deformations.

The achievement of an axiomatic structure for the more general Lie-admissible formulations is evidently more complex. In this case the dynamical equations describe irreversible systems and therefore require the selection of a given direction in time. In fact, Eq. (7) can be written $idA/dt = ARH - HSA = A \langle H - H \rangle A$, where \langle represents motion forward to future time, and \rangle represents motion forward from past times. The generalized unit is necessarily nonhermitean, thus requiring two different units one for the isoproduct \langle and the other for the conjugate product \rangle .

The Lie-admissible theory which is axiomatic in the above sense to my

best knowledge at this writing is characterized by the equations

$$i \hbar dA / dt = A R H - H S A = A \langle H - H \rangle A = \begin{cases} i \hbar \hat{\Gamma} = i \hbar S^{-1} \\ \text{or} \\ i \hbar \hat{\Gamma} = i \hbar R^{-1} \end{cases} \quad (23)$$

$$R = S^\dagger, \quad (24)$$

formulated over a dual generalization of the entire quantum mechanical formalism, one per each direction of time (the genofields, genospaces and genohilbert spaces mentioned earlier), which were first identified by Santilli (Hadronic J. 1, 574 (1978) and Hadronic J. Suppl. 4B, 1 (1989)).

I should stress that the above structures are the *only* axiomatically consistent formulations which I know at this writing. In fact, structures of the type

$$r R p - p S r = i \hbar R^{-1} \text{ or } = i \hbar S^{-1} \text{ but } R \neq S^\dagger, \quad (25)$$

$$r R(q, \dots) p - p S(q, \dots) r = i \hbar T(q, \dots), \quad T \neq R^{-1} \text{ and } T \neq S^{-1}, \quad (26)$$

violate one of another axiom of hadronic mechanics therefore resulting in one or another problematic aspect for physical applications.

The following additional comments are recommendable in these introductory words. Hadronic mechanics studies physical conditions fundamentally different than those of quantum mechanics. In fact, the latter studies the motion of point-like particles in the homogeneous and isotropic vacuum, such as an electron of an atomic cloud (exterior problem), while the former studies the more general class of extended-nonspherical particles moving within inhomogeneous and anisotropic physical media, such as a proton in the core of a star (interior problem). In particular, hadronic mechanics has no impact for the atomic structure because, by construction, it recovers quantum mechanics identically for all mutual distances greater than 1 fm (10^{-13} cm). The differences in the mathematical structures between quantum and hadronic mechanics should therefore be interpreted as a representation of said physical differences.

A most insidious aspect in the study of hadronic mechanics is the rather widespread tendency of appraising it via the mathematical methods of quantum mechanics. This attitude leads to a host of misrepresentations and inconsistencies which often remain undetected, such as assuming that the the magnitude of the angular momentum is $J^2 = JJ$ with respect to the unit I (rather than the correct form $J^2 = JT(x, \dot{x}, \psi, \partial\psi, \dots)J$ with respect to the unit T^{-1}) which, for hadronic mechanics, violates linearity and all basic axioms of the theory. The appraisal of hadronic mechanics via the formalism of quantum mechanics is equivalent to the appraisal of quantum mechanics via the formalism of Newtonian mechanics.

In particular, the transition from Newtonian to quantum mechanics did

imply certain necessary mathematical generalizations, most notably the use of infinite-dimensional Hilbert spaces, although fundamental mathematical notions such as numbers, angles, vector spaces, trigonometry, special functions, integral transforms, etc., remained common to both classical and quantum disciplines.

In the transition from quantum to hadronic mechanics the *totality* of its mathematical structure must be generalized in a simple yet effective way, as indicated earlier. In view of this occurrence, I have attempted to render this volume self-sufficient for a first study of hadronic mechanics. However, its technical knowledge can only be acquired following a study of the mathematical foundations of Vol. I.

The theoretical foundations of hadronic mechanics are the result of the efforts of numerous scholars identified in the various chapters. Among the mathematicians I mention here the contributions to the Lie-isotopic theory by A. U. Klimyk, D. S. Surlas and G. F. Tsagas, and those to the Lie-admissible theory by H. C. Myung. Among the physicists who participated in the earlier study of the theory besides myself, I mention the contributions by A. O. E. Animalu, A. K. Aringazin, G. D. Brodimas, G. Eder, J. Fronteau, M. Gasperini, A. Kalnay, A. Jannussis (and other associates), R. Mignani, M. Mijatovic, M. Nishioka, S. Okubo, A. Tellez-Arenas, B. Veljanosky and others. The experimentalists who contributed to hadronic mechanics will be identified in Volume III.

In closing allow me to indicate that a primary objective of hadronic mechanics is the introduction of two, sequential, Lie-isotopic and Lie-admissible generalizations of the Galilean, special and general relativities for nonlinear, nonlocal-integral and nonhamiltonian systems in closed-reversible and open-irreversible conditions, respectively. Readers with the personal conviction that current relativities have a final character for all possible physical conditions existing in the Universe are discouraged from inspecting these volumes. On the contrary, readers with the "young minds of all ages" (mentioned in the preface of Volume I) may find the content of these volume stimulating.

All true scientists (in Einstein's definition) are encouraged to participate in the laborious scientific process of trial and error toward truly fundamental advances, not in marginal talks in academic corridors, but in the only way physical knowledge really advances, via publications.

Summer 1994

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PREFACE TO THE SECOND EDITION

In this second edition I have corrected a number of misprints and errors of the first edition which were kindly brought to my attention by a number of readers.

I have also updated a number of references, particularly those on key issues of hadronic mechanics, according to a number of papers recently appeared in various Journals.

I have finally updated a number of important aspects, such as: the prediction of the isodual theory that gravity is reversed for *elementary* antiparticles, such as the positrons, but bound states of particles and antiparticles, such as the positronium, are attracted in the field of Earth; the isominkowskian geometry permits a symbiotic representation of both the Minkowskian and the Riemannian geometries, which is at the foundation of the isotopic unification of gravity and relativistic quantum mechanics presented in the preceding edition of this volume; the indication (without treatment at this time) of different operator expressions of hadronic mechanics with the use of the isodifferential calculus with a nontrivial formulation of the isodifferentials $\hat{d}x = \hat{1}dx$, where $\hat{1}$ is the isounit; more adequate transformations of the Lie-admissible equations under genounitary transforms; and other aspects.

Any additional comment by interested colleagues would be sincerely appreciated.

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However, I am solely responsible for these volumes owing to the numerous
changes and expansions of the final version.

1: CLASSICAL FOUNDATIONS

1.1: STATEMENT OF THE PROBLEM

We begin the physical studies of this second volume by pointing out that:

1) Hadronic mechanics possesses a well defined classical image with unique interconnecting maps in a way fully parallel to the known classical foundations of quantum mechanics;

2) The primary difference between the classical foundations of quantum and hadronic mechanics is that the former are of potential-Hamiltonian type while the latter admit potential-Hamiltonian as well as nonpotential-nonhamiltonian forces;

3) The classical foundations of quantum mechanics admit a limited class of Newtonian systems, while those of hadronic mechanics are *directly universal*, that is, admitting of all possible classical systems (universality) in the frame of the experimenter (direct universality).

The above classical universality then sets the foundations of the corresponding direct universality of hadronic mechanics for all possible operator systems.

The classical foundations of quantum mechanics, the familiar *Hamiltonian mechanics*¹ are well known (see ref.s [1] and quoted literature). The classical foundations of hadronic mechanics have been studied in:

* Monographs [2] on the integrability conditions for the existence of a potential V , a Hamiltonian H or a Lagrangian L , the so-called *conditions of variational selfadjointness* (SA) and ensuing *Birkhoffian generalization of Hamiltonian mechanics*, or *Birkhoffian mechanics* for short;

* Monographs [3] on the *Hamilton-isotopic mechanics* and related covering called *Birkhoff-isotopic mechanics*, which possesses a Lie-isotopic structure (Ch.

¹ The term "Hamiltonian mechanics" is misleading because referred to the so-called *truncated analytic equations* which are not those originally conceived by Hamilton with *external terms* (see Sect. 1.7.1). Nevertheless, the term is now of general use and will be kept in this volume to denote the conventional canonical mechanics without external terms.

I.4); and

* Monographs [4] on the still more general *Hamilton-admissible mechanics* and related covering called *Birkhoff-admissible mechanics* which possesses a Lie-admissible structure (Ch. I.7).

In this chapter we shall outline the main structural lines of these classical studies with particular reference to those profiles which are at the classical foundations of hadronic mechanics. Particular attention is due to the *Hamilton-isotopic mechanics because it is the classical image of the Lie-isotopic branch of hadronic mechanics*, and to the *Hamilton-admissible mechanics because it is the classical image of the Lie-admissible branch* (Fig. 1.1.1).

It should be noted that we know today unique and unambiguous operator maps of Hamilton-isotopic mechanics into the Lie-isotopic hadronic mechanics, called *isoquantization*, and of Hamilton-admissible mechanics into the Lie-admissible hadronic mechanics called *genoquantization*.² We also know the operator image of the more general Birkhoff, Birkhoff-isotopic and Birkhoff-admissible mechanics but this latter knowledge is merely formal at this writing.

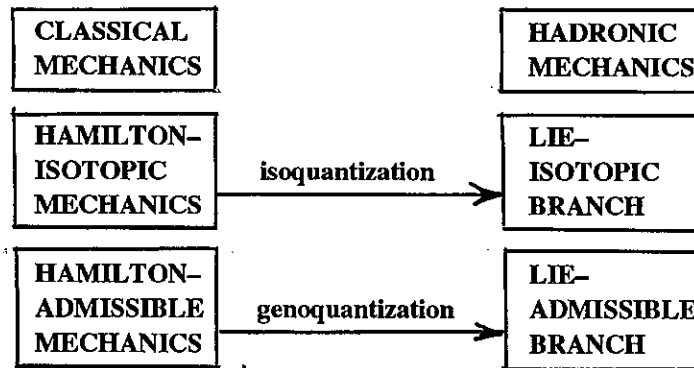


FIGURE 1.1.1: A schematic view of the classical foundations of hadronic mechanics.

It is important to know from the outset the physical differences between the classical foundations of quantum and hadronic mechanics. As well known, quantum mechanics was conceived for systems (such as the atomic structure)

² It appears recommendable for the reader to get accustomed from the beginning with the fact that the term "quantum" is conceptually misleading and technically inappropriate within the context of hadronic mechanics. Names such as "quantization", "quantum of energy", "quantized particles", etc., do have a correct meaning, conceptually and technically, but only for quantum mechanics, that is, for point-particles in vacuum. In the transition to the covering hadronic mechanics, as indicated from the Preface and studied throughout this volume, the very notion of quantum of energy must be generalized into the integral isounit, $\hbar \rightarrow \hbar = \hbar I$, representing exchanges of energy for extended wavepackets totally immersed within *hadronic media*, i.e., media composed of wavepackets of other particles with a density of the order of that of hadrons.

which are isolated from the rest of the Universe, admit only conservative internal forces and are characterized by only one operator, the Hamiltonian H or Lagrangian L. These systems are called *closed Hamiltonian* (or *variationally selfadjoint*) systems [2]. Classical Hamiltonian mechanics then provides the ideal classical foundations of quantum mechanics because it characterizes precisely closed Hamiltonian systems of N particles in Euclidean space and related total conserved quantities which we can write as

$$m_a \ddot{r}_{ka} = F_{ka}^{SA}(t, r, \dot{r}), \quad k = 1, 2, 3 (=x, y, z), \quad a = 1, 2, \dots, N, \quad (1.1.1a)$$

$$F_{ka}^{SA}(t, r, \dot{r}) = \frac{d}{dt} \frac{\partial V(t, r, \dot{r})}{\partial \dot{r}_a^k} - \frac{\partial V(t, r, \dot{r})}{\partial r_a^k} \quad (1.1.1b)$$

$$E = H = K + V, \quad K = \sum_a p_a^2 / 2 m_a, \quad P_k = \sum_a p_{ak}, \quad (1.1.1c)$$

$$M_k = \sum_a \epsilon_{kij} r_{ai} p_{aj}, \quad G_k = \sum_a (m_a r_{ak} - t p_{ak}) \quad (1.1.1d)$$

On the contrary, hadronic mechanics was conceived for systems with local-differential-potential as well as nonlocal-integral-nonpotential internal forces; that is, with forces which, by central assumption, *are not* representable with the single quantity H. As studied in detail in Vol. II of ref. [2], the latter systems can also be closed-isolated, thus verifying the same total conservation laws (1.1.1c) and (1.1.1d), in which case they are called *closed nonhamiltonian* (or *variationally nonselfadjoint*) systems [2] and can be written jointly with the closure conditions and ensuing total conserved quantities (see Appendix II.1.B for more details) as

$$m \ddot{r}_k = F_k^{SA}(t, r, \dot{r}) + F_k^{NSA}(t, r, \dot{r}, \ddot{r}, \dots). \quad (1.1.2a)$$

$$\sum_{a=1, \dots, N} F_a^{NSA} = 0, \quad \sum_{a=1, \dots, N} r_a \cdot F_a^{NSA} = 0, \quad \sum_{a=1, \dots, N} r_a \times F_a^{NSA} = 0, \quad (1.1.2b)$$

$$E = H = K + V, \quad K = \sum_a p_a^2 / 2 m_a, \quad P_k = \sum_a p_{ak}, \quad (1.1.2c)$$

$$M_k = \sum_a \epsilon_{kij} r_{ai} p_{aj}, \quad G_k = \sum_a (m_a r_{ak} - t p_{ak}) \quad (1.1.2d)$$

Then conventional Hamiltonian mechanics loses any validity as the classical counterpart of hadronic mechanics, in favor of the suitable generalized mechanics.

In summary, a primary physical difference of the classical foundations of quantum and hadronic mechanics is that the former are patterned along contemporary analytic trends, those representing systems with only one quantity H or L (*variationally selfadjoint interactions*), while the latter are patterned along the original analytic conception by Lagrange and Hamilton (Sect. 1.7.1), according to which the systems of our physical reality *cannot* be solely represented with one quantity H or L, but require 3N additional external terms F_k^{NSA} (*variationally*

nonseladjoint interactions).

In the language of these volumes we can say that:

1) the classical foundations of quantum mechanics are given by *exterior dynamical systems* (Ch. I.1); i.e., systems of particles which can be effectively approximated as being point-like when moving within the homogeneous and isotropic vacuum; while

2) the classical foundations of hadronic mechanics are given by *interior dynamical systems* (Ch. I.1); i.e., systems of particles which are extended and therefore deformable, while moving within inhomogeneous and anisotropic physical media.

An objective of this chapter is to illustrate that the hadronic representation of systems with the *two* quantities, the Hamiltonian $H = K + V$ (or Lagrangian $L = K - V$) and the isounit $\hat{1}$ (Ch.s I.2, I.4, I.7) is patterned precisely along the original conception by Lagrange and Hamilton to such an extent as to preserve even the number $(1 + 3N)$ of independent quantities. In fact, the independent elements of the isounit $\hat{1}$ (e.g., its diagonal terms) are precisely $3N$.

The original analytic equations with external terms are rewritten in the Hamilton-isotopic form for closed nonhamiltonian systems, or in the Hamilton-admissible form for open nonhamiltonian systems, because the analytic brackets with external terms violate the conditions for the existence of any algebra, let alone Lie algebras (Sect. I.7.1).

The additional knowledge recommendable from the outset pertains to the reasons why only one mechanics is sufficient for the classical image of quantum mechanics, while hadronic mechanics requires two different, yet complementary mechanics.

Closed variationally selfadjoint systems are composed by collections of particles each one in a stable orbit, as majestically illustrated by the Solar systems. Under these conditions, one mechanics only with totally antisymmetric brackets is evidently sufficient to represent the stability of both the system as a whole and each of its constituents [1].

The situation for the more general variationally nonselfadjoint systems (1.1.2) is fundamentally different. In fact, when studied from the outside as a whole the systems are closed as the Hamiltonian ones, thus requiring a mechanics with *totally antisymmetric brackets* as an evident necessary condition for the conservation of the total energy. The nonhamiltonian internal forces then requires that such brackets are of the generalized *Lie-isotopic* type, thus yielding in a unique way (up to isoequivalence) the Hamilton-isotopic mechanics [3].

However, global stability is achieved in systems (1.1.2) via a collection of particles each of which is in unstable conditions. We merely have internal exchanges of energy and other physical quantities but always such to satisfy total conservation laws. While the emphasis in the exterior global treatment is in the total conservation laws, the emphasis for the study of each individual constituent is instead in the characterization of the most general possible *time-rate-of-variation* of the energy, angular momentum and other physical quantities when