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ISOTOPIC LIFTING OF SU(2)-SYMMETRY WITH APPLICATIONS TO NUCLEAR PHYSICS[1]

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We introduce the axiom-preserving, nonlinear, nonlocal and noncanonical isotopies/Q-operator deformations $\hat{SU}_Q(2)$ of the $SU(2)$-symmetry; construct their isorepresentations; and prove their lack of unitary equivalence to conventional representations under the local isomorphism $\hat{SU}_Q(2) \approx SU(2)$. We then apply the theory to the reconstruction of the exact isospin symmetry under electromagnetic and weak interactions and to the exact representation of total magnetic moments for the deuteron and few-body nuclei under the exact isospin symmetry.

Изотопическая $SU(2)$-симметрия в применении к ядерной физике

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Вводятся аксиомосохраняющиеся, нелинейные, нелокальные и неканонические изотопически/Q-операторные деформации $SU_Q(2)$ $SU(2)$-симметрии; создаются их изопредставления; и доказывается недостаток в них единой эквивалентности общепринятым представлениям в локальном изоморфизме $\hat{SU}_Q(2) \approx SU(2)$. Теория затем применяется к воссозданию точной изоспиновой симметрии в электромагнитных и слабых взаимодействиях и к точному представлению всех магнитных моментов дейтрона и малонуклонной системы в точной изоспиновой симметрии.

1. Statement of the Problem

It is generally assumed that the $SU(2)$-spin symmetry (see, e.g., [1]) can solely characterize the familiar eigenvalues $j(j+1)$ and $m, j = 0, \frac{1}{2}, ..., m = j, j-1, ..., -j$.

In this note we shall show that the isotopic/Q-operator deformation of $SU(2)$, herein denoted $\hat{SU}_Q(2)$, while being locally isomorphic to $SU(2)$, can characterize the more general eigenvalues

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$$J^{\hat{2}} \Rightarrow f(\Delta) j\,[f(\Delta)\,j + 1], \quad J_3 \Rightarrow f(\Delta) m, \tag{1.1}$$

and others, where j and m have conventional values and $f(\Delta)$ is a real valued, positive-definite function of $\Delta = \det Q$ such that $f(1) = 1$.

For the two-dimensional case, the condition $\det Q = 1$ for $Q = \mathrm{diag}(g_{11}, g_{22})$ is realized by $g_{11} = g_{22}^{-1} = \lambda$. This implies the preservation of the conventional value $\frac{1}{2}$ of the spin, but the appearance of a nontrivial generalization of Pauli's matrices, herein called *isopauli matrices*, with an explicit realization of the «hidden variable» λ in the structure of the spin $\frac{1}{2}$ itself.

As a first application, we construct the isotopies of the conventional isospin (see, e.g., [2,3]) and show that they permit the reconstruction of an exact $\hat{SU}(2)$-isospin symmetry under electromagnetic and weak interactions because protons and neutrons acquire equal masses in the underlying isospace.

It should be noted that the isotopic lifting $SU(2) \Rightarrow \hat{SU}_Q(2)$ can be interpreted as an application of the so-called q-deformations [4], although in their isotopic axiomatic formulation for the most general possible, integro-differential operator Q [5].

In the recent note [6] we have presented the isotopies of Dirac's equation and shown their capability to provide a numerical representation of the magnetic moment of few-body nuclei. As a second application, in this note we re-inspect this result under an exact isospin symmetry realized with the same magnitude of the magnetic moments of protons and neutrons in isospace. Additional applications in nuclear physics, such as for the introduction of a small nonlocal-nonhamiltonian term in the nuclear force, will be presented elsewhere.

2. Isotopies of $SU(2)$-Symmetry

The understanding of this note requires a knowledge of: the nonlinear-nonlocal-noncanonical, axiom-preserving isotopies of Lie's theory, originally introduced in [7] (see the recent review [8] and general presentation [9]); the isotopies $\hat{O}(3)$ of the rotational symmetry $O(3)$ submitted in [10]; the isotopies $\hat{O}(3.1)$ of the Lorentz symmetry $O(3.1)$ submitted in [11]; and the isotopies of quantum mechanics (QM), called *hadronic mechanics* [HM], originally submitted in [12] and then elaborated by various authors (see recent studies [5,8,13] and monographs [14]).

The fundamental notion is the *isotopy of the unit* of the theory considered [5—14], in this case, the generalization of the conventional trivial unit I = diag (1, 1) of $SU(2)$ into the most general possible, two-dimensional matrix $\hat{I}$ preserving the original axioms of I (smoothness, boundedness, nonsingularity, Hermicity and positive-definiteness) as a necessary condition for isotopy,

$$I = \mathrm{diag}(1, 1) > 0 \Rightarrow \hat{I} = \hat{I}(t, z, \bar{z}, \dot{z}, \dot{\bar{z}}\, \psi, \psi^{\dagger}, \partial\psi, \partial\psi^{\dagger}, \ldots) > 0. \quad (2.1)$$

The isotopy of the unit then demands, for consistency, a corresponding, compatible lifting of all associative products AB among generic QM quantities A, B, into the isoproduct

$$AB \Rightarrow A * B := AQB, \quad Q \text{ fixed}, \quad (2.2)$$

where the isotopic character of the lifting is established by the preservation of associativity, i.e., $A * (B * C) = (A * B) * C$.

The assumption $\hat{I} = Q^{-1}$ then implies that $\hat{I}$ is the correct left and right unit of the theory, $\hat{I} * A = A * \hat{I} \equiv A$, in which case Q is called the *isotopic element*, and $\hat{I}$ is called the *isounit*. Note the appearance of q-deformations in their Q-operator form at the very foundation of the theory [5].

The isotopies of the unit $I \Rightarrow \hat{I}$ and of the product $AB \Rightarrow A * B$ then imply the necessary lifting of *all* mathematical structures of QM into those of HM [5—18]. Here we mention the lifting of the field of complex numbers $C(x, +, \times)$, with elements c, ordinary sum + and multiplication $c \times c' = cc'$, into the infinitely possible isotopies $\hat{C}_Q(\hat{c}, +, *)$, with *isocomplex numbers* $\hat{c} = c\hat{I}$, conventional sum + and isomultiplication $\hat{c}_1 * \hat{c}_2 = \hat{c}_1 Q \hat{c}_2 = (c_1 c_2)\hat{I}$ (see [16,17] for details).

The isotopies of the unit, multiplication and fields then demand, for mathematical consistency, corresponding compatible isotopies of the basic carrier space, the two-dimensional complex Euclidean space $E(z, \bar{z}, \delta, C)$ with familiar metric δ = diag(1, 1) into the complex two-dimensional *isoeuclidean spaces* introduced in [11]

$$\hat{E}_Q(z, \bar{z}, \hat{\delta}, \hat{C}) : z = (z_1, z_2),$$

$$\hat{\delta} = Q\delta \equiv g = \mathrm{diag}\,(g_{11}, g_{22}) = g^{\dagger} > 0, \quad (2.3a)$$

$$z_i^{\dagger} g_{ij}(t, z, \bar{z}, \ldots) z_j = \bar{z}_1 g_{11} z_1 + \bar{z}_2 g_{22} z_2 = \mathrm{Inv.}, \quad (2.3b)$$

where the assumed diagonalization of Q is always possible (although not necessary) from its positive-definiteness.

The isotopic character (as well as novelty) of the generalization is established by the fact that, under the *joint* lifting of the metric $\delta \Rightarrow \hat{\delta} = Q\delta = g$ and of the field $C \Rightarrow \hat{C}_Q$, $\hat{I} = Q^{-1}$, all infinitely possible isospaces $\hat{E}_Q(z, \bar{z}, \hat{\delta}, \hat{C})$ are locally isomorphic to the original space $E(z, \bar{z}, \delta, C)$ under the condition of positive-definiteness of the isounit $\hat{I}$ [11]. In turn, this evidently sets the foundation for the local isomorphism of the corresponding symmetries.

Note that separation (2.3) is the most general possible nonlinear, nonlocal and noncanonical generalization of the original separation $z^{\dagger}z$ under the sole condition of remaining positive-definite, i.e., of preserving the topology sig δ = sig $\hat{\delta} = (+, +)$. The symmetries of invariant (2.3) are then expected to be nonlinear, nonlocal and noncanonical, as desired.

The preceding isotopies imply, for consistency, the isotopics of Hilbert spaces $\;:\langle \psi | \varphi \rangle \in C$ into the so-called isohilbert space $\hat{\ }_Q$ with *isoproduct* and *isonormalization*

$$\hat{\ }_Q : \langle \hat{\psi} \uparrow \hat{\varphi} \rangle = \langle \hat{\psi} | Q | \hat{\varphi} \rangle \hat{I} \in \hat{C}_Q; \ \langle \hat{\psi} \uparrow \hat{\psi} \rangle = \hat{I}. \quad (2.4)$$

Then, operators which are Hermitean (observable) for QM remain Hermitean (observable) for HM, as was first proved in [15].

The liftings of the Hilbert space then require corresponding isotopies of all conventional operations [13,14]. We here mention isounitarity $\hat{U} * \hat{U}^{\dagger} = \hat{U}^{\dagger} * \hat{U} = \hat{I}$; the isoeigenvalue equations $H * |\hat{\psi}\rangle = HQ|\hat{\psi}\rangle = \hat{E} * |\hat{\psi}\rangle \equiv E|\hat{\psi}\rangle$; the isoexpectation values $\hat{\langle A \rangle} = \langle \hat{\psi} | QAQ | \hat{\psi} \rangle / \langle \hat{\psi} | Q | \hat{\psi} \rangle$; etc.

The lifting of the unit, base field and carrier space then require, for mathematical consistency, the lifting of the entire structure of Lie's theory, that is, the isotopies of enveloping associative algebras ξ, Lie algebras L, Lie groups G, representation theory, etc. [7—9]. Here we mention the *isoassociative enveloping operator algebras* $\hat{\xi}_Q$ with isoproduct (2.2); $A * B \equiv AQB$ the *Lie-isotopic algebras* $\hat{L}_Q$ with isoproduct

$$[A, B]_{\hat{\xi}_Q} = [A \hat{,} B] = A * B - B * A \equiv AQB - BQA; \quad (2.5)$$

the (connected) *Lie-isotopic groups* $\hat{G}_Q$ of *isolinear isounitary transforms* on $\hat{E}_Q(z, \bar{z}, \hat{\delta}, \hat{C})$

$$z' = \hat{U}(w) * z = \hat{U}(w)Qz = \hat{U}(w)Q(z, \bar{z}, \dot{z}, \dot{\bar{z}}, \hat{\psi}, \hat{\psi}^{\dagger}, \ldots)z \quad (2.6a)$$

$$\hat{U}(w) = e^{iX * \hat{w}}\big|_{\hat{\xi}_Q} = \{e^{iXQw}\}\hat{I}, \quad (2.6b)$$

$$\hat{U}(w) * \hat{U}(w') = \hat{U}(w') * \hat{U}(w) = \hat{U}(W + w'),$$
$$\hat{U}(w) * \hat{U}(-w) = \hat{U}(0) = \hat{I}, \tag{2.6c}$$

where the reformulation in terms of the conventional exponentiation has been done for simplicity of calculations.

The *isounitary* $\hat{U}_Q(2)$ *symmetry* is the most general possible, nonlinear, nonlocal and noncanonical, simple, Lie-isotopic invariance group of separation (2.3b) with realization in terms of isounitary operators on Q

$$\hat{U} * \hat{U}^\dagger = \hat{U}^\dagger * \hat{U} = \hat{I} = Q^{-1}, \tag{2.7}$$

verifying isotopic laws (2.6).

$\hat{U}(2)$ can be decomposed into the *connected, special isounitary symmetry* $\hat{SU}_Q(2)$ for

$$\det(\hat{U}Q) = +1, \tag{2.8}$$

plus a discrete part which is similar to that for $\hat{O}(3)$ [10] and is here ignored for brevity.

The connected $\hat{SU}_Q(2)$ components admit the realization in terms of the generators $\hat{J}_k$ and parameters θ_k of $SU(2)$

$$\hat{U} = \prod_k e_{\hat{\xi}}^{i\hat{J}_k * \hat{\theta}_k} = \left\{ \prod_k e^{i\hat{J}_k Q \theta_k} \right\} \hat{I}, \tag{2.9}$$

under the conditions

$$\mathrm{tr}(\hat{J}_k Q) \equiv 0, \quad k = 1, 2, 3. \tag{2.10}$$

The *isorepresentations of the isotopic algebras* $\hat{SU}_Q(2)$ can be studied by imposing that the isocommutation rules have the same structure constants of $SU(2)$, i.e., for the rules

$$[\hat{J}_i \hat{,} \hat{J}_j] = \hat{J}_i Q \hat{J}_j - \hat{J}_j Q \hat{J}_i = i\varepsilon_{ijk} \hat{J}_k, \tag{2.11}$$

with isocasimir

$$\hat{J}^{\hat{2}} = \sum_k \hat{J}_k * \hat{J}_k, \tag{2.12}$$

and maximal isocommuting set $\hat{J}^{\hat{2}}$ and $\hat{J}_3$ as in the conventional case. These assumptions ensure the local isomorphisms $\hat{SU}(2) \approx SU(2)$ by construction.

Let $| \hat{b}_k^d \rangle$ be the d-dimensional isobasis of $\hat{SU}_Q(2)$ with iso-orthogonality conditions

$$\langle \hat{b}_i^d | * | \hat{b}_j^d \rangle := \langle \hat{b}_i^d | Q | \hat{b}_j^d \rangle = \delta_{ij}, \quad i,j = 1,2,\dots, n. \tag{2.13}$$

By putting as in the conventional case $\hat{J}_\pm = \hat{J}_1 \pm \hat{J}_2$, and by repeating the same procedure as the familiar one [1], we have

$$\hat{J}_3 * | \hat{b}_k^d \rangle = b_k^d | \hat{b}_k^d \rangle, \quad \hat{J}^{\hat{2}} * | \hat{b}_k^d \rangle = b_1^d (b_1^d - 1) | \hat{b}_1^d \rangle,$$
$$d = 1,2,\dots, \quad k = 1,2,\dots,d, \quad b_1^d \equiv -b_d^d, \quad b_1^d(b_1^d - 1) \equiv b_d^d(b_d^d + 1). \tag{2.14}$$

A consequence is that the *dimensions of the isorepresentations of* $\hat{SU}_Q(2)$ *remain the conventional ones*, i.e., they can be characterized by the familiar expression $n = 2j + 1$, $j = 0, \frac{1}{2}, 1, \dots$ as expected from the isomorphism $\hat{SU}_Q(2) \approx SU(2)$.

However, *the explicit forms of the matrix representations are different than the conventional ones*, as expressed by the rules

$$(\hat{J}_1)_{ij} = \frac{1}{2} i \; | \hat{b}_i^d \rangle * (\hat{J}_- - \hat{J}_+) * | \hat{b}_j^d \rangle, \tag{2.15a}$$
$$(\hat{J}_2)_{ij} = \frac{1}{2} i \; | \hat{b}_i^d \rangle * (\hat{J}_- - \hat{J}_+) * | \hat{b}_j^d \rangle, \tag{2.15b}$$
$$(\hat{J}_3)_{ij} = \langle \hat{b}_i^d | * \hat{J}_3 * | \hat{b}_j^d \rangle, \tag{2.15c}$$

under condition (2.10).

The isorepresentations of the desired dimension can then be constructed accordingly. In the next section we shall compute the two-dimensional isorepresentations, while those of higher dimensions are studied elsewhere [14].

A new image of the conventional $SU(2)$-symmetry is characterized by our isotopic methods via the antiautomorphic map $I = \mathrm{diag}(1,1) \Rightarrow I^a = -1$ called *isoduality*, first introduced in [10], which provides a novel and intriguing characterization of antiparticles [14]. The corresponding *isodual* $\hat{SU}_Q^d(2)$ *symmetry* will be studied in a separate work.

In summary, our isotopic methods permit the identification of four physically relevant isotopies of $SU(2)$ which, for the case of isospin, are given by: the broken conventional $SU(2)$ for the usual treatment of $p - n$; the exact isotopic $\hat{SU}_Q(2)$ for the characterization of $p - n$ (see next section); the broken isodual $SU^d(2)$ symmetry for the characterization of the antiparticles $\bar{p} - \bar{n}$ in isodual spaces; and the exact, isodual, isotopic $\hat{SU}_Q^d(2)$ for the characterization of antiparticles $\bar{p} - \bar{n}$ in isodual isospace.

The reader may be interested in knowing that, when the positive- (or negative-) definiteness of the isotopic elemnt Q is relaxed, the isotopes $\hat{SU}(2)$ unify all three-dimensional simple Lie groups of Cartan classification over a comples field (of characteristic zero). In fact, we have the compact isotopes $\hat{SU}(2) \approx SU(2)$ for $g_{11} > 0$, $G_{22} > 0$ and the noncompact isotopes $\hat{SU}(2) \approx SU(1,1)$ for $g_{11} > 0$ and $g_{22} < 0$ (see [10] for the corresponding unification of orthogonal groups over the reals). In this note we consider only positive-definite isotopic elements Q.

3. Isotopies of Pauli Matrices

Recall that the conventional Pauli matrices σ_k (see, e.g. [2]) verify the rules $\sigma_i \sigma_j = i\, \varepsilon_{ijk}\, \sigma_k$, $i, j, k = 1, 2, 3$. In this section we show that the isoalgebra $\hat{SU}_q(2)$ implies the existence of intriguing generalizations of these familiar matrices.

To have a guiding principle, we recall that [12], in general, *Lie-isotopic algebras are the image of Lie algebras under nonunitary transformations.* In fact, under a transformation $UU^\dagger \neq I$, a Lie commutator among generic matrices A, B, acquires the Lie-isotopic form

$$U(AB - BA)U^\dagger = A'Q\beta' - B'Q\,A', \tag{3.1a}$$

$$A' = UAU^\dagger,\ \ B' = UBU^\dagger,\ \ Q = (UU^\dagger)^{-1} = Q^\dagger. \tag{3.1b}$$

We therefore expect a first class of fundamental (adjoint) isorepresentations characterized by the maps $J_k = \frac{1}{2}\sigma_k \to \hat{J}_k = UJ_kU^\dagger$, $UU^\dagger \neq I$, $\pm\frac{1}{2} \to +\frac{1}{2}f(\Delta)$, $3/4 \to (3/4)\,f^2(\Delta)$, here called *regular adjoint isorepresentations of* $\hat{SU}_Q(2)$.

An example is readily constructed via Eqs. (2.15) resulting in the following generalization of Pauli's matrices here called *regular isopauli matrices*

$$\hat{\sigma}_1 = \Delta^{-1/2}\begin{pmatrix} 0 & g_{11} \\ g_{22} & 0 \end{pmatrix},\ \ \hat{\sigma}_2 = \Delta^{-1/2}\begin{pmatrix} 0 & -ig_{11} \\ +ig_{22} & 0 \end{pmatrix},$$

$$\hat{\sigma}_3 = \Delta^{-1/2}\begin{pmatrix} g_{22} & 0 \\ 0 & -g_{11} \end{pmatrix}, \tag{3.2a}$$

$$[\hat{\sigma}_i \hat{,} \hat{\sigma}_j] = \hat{\sigma}_i Q\, \hat{\sigma}_j - \hat{\sigma}_j Q\, \hat{\sigma}_i = 2i\, \varepsilon_{ijk}\, \hat{\sigma}_k,\ \ Q = \mathrm{diag}(g_{11}, g_{22}), \tag{3.2b}$$

where $\Delta = \det Q = g_{11}g_{22} > 0$, with generalized isoeigenvalues for $f(\Delta) = \Delta^{1/2}$ and $\hat{J}_k = \frac{1}{2}\hat{\sigma}_k$, $k = 1,2,3,$

$$\hat{J}_3 * |\hat{b}_i^2\rangle = \pm(1/2)\Delta^{1/2}\,|\hat{b}_i^2\rangle, \tag{3.3a}$$

$$\hat{J}^2 * |\hat{b}_i^2\rangle = (3/4)\Delta\,|\hat{b}_i^2\rangle,\ \ i = 1, 2 \tag{3.3b}$$

which confirm the «regular» character of the generalization here considered. The isonormalized isobasis is then given by a simple extension of the conventional basis, $|\hat{b}\rangle = g^{-1/2}\,|b\rangle$.

Recall that Pauli's matrices are essentially unique, in the sense that their transformations under unitary equivalence do not yield significant changes in their structure, as well known. The situation is different for the isopauli matrices, because isorepresentations are based on various degrees of freedom which are absent in the conventional $SU(2)$ theory, such as: 1) infinitely possible isotopic elements Q; 2) formulation of the isoalgebra in terms of structure functions [7]; 3) use of an isotopic element for the isohilbert space different than that of the isoalgebra [13,14]; and others.

We shall call *irregular adjoint isorepresentations of* $\hat{SU}_Q(2)$ those with generalized eigenvalues other than (3.3), e.g., those of type (1.1). A first example is given by the *irregular isopauli matrices*

$$\hat{\sigma}_{1'} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma_1,\ \ \hat{\sigma}_{2'} = \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix} = \sigma_2,$$

$$\hat{\sigma}_{3'} = \begin{pmatrix} g_{22} & 0 \\ 0 & -g_{11} \end{pmatrix} = \Delta\, \hat{I}\, \sigma_3 \tag{3.4}$$

which verify the isocommutation rules *with structure functions*

$$[\hat{\sigma}_{1'} \hat{,} \hat{\sigma}_{2'}] = 2i\,\hat{\sigma}_{3'},\ \ [\hat{\sigma}_{2'} \hat{,} \hat{\sigma}_{3'}] = 2i\,\Delta\,\hat{\sigma}_{1'},\ \ [\hat{\sigma}_{3'} \hat{,} \hat{\sigma}_{1'}] = 2i\,\Delta\,\hat{\sigma}_{2'}, \tag{3.5}$$

without evidently altering the local isomorphisms $\hat{SU}_Q(2) \approx SU(2)$.

The new isoeigenvalue equations are given by

$$\hat{J}_{3'} * |\hat{b}_i^2\rangle = \pm\frac{1}{2}\Delta\,|\hat{b}_i^2\rangle,\ \ \hat{J}'^2 * |\hat{b}_i^2\rangle = \frac{1}{2}\Delta(\frac{1}{2}\Delta + 1)\,|\hat{b}_i^2\rangle, \tag{3.6}$$

which comfirm the «irregular» character under consideration and provide an illustration of Eqs. (1.1).

Yet another realization of irregular isopauli matrices is given by

$$\hat{\sigma}'_1 = \begin{pmatrix} 0 & g_{22}^{-1/2} \\ g_{11}^{-1/2} & 0 \end{pmatrix}, \hat{\sigma}'_2 = \begin{pmatrix} 0 & -ig_{22}^{-1/2} \\ ig_{11}^{-1/2} & 0 \end{pmatrix}, \hat{\sigma}'_3 = \begin{pmatrix} g_{11}^{-1} & 0 \\ 0 & g_{22}^{-1} \end{pmatrix}, \qquad (3.7)$$

with isocommutation rules and isoeigenvalues

$$[\hat{J}'_1 \hat{,} \hat{J}'_2] = i\Delta \hat{J}'_3, \ [\hat{J}'_2, \hat{J}_3] = i\hat{J}'_1, \ [\hat{J}'_3 \hat{,} \hat{J}'_1] = i\hat{J}'_2, \qquad (3.8a)$$

$$\hat{J}'_3 * |\hat{b}^2_i\rangle = \pm \frac{1}{2} |\hat{b}^2_i, \hat{J}'^2 * |\hat{b}^2_i\rangle = \frac{1}{2}\left(\frac{1}{2} + \Delta^{\frac{1}{2}}\right) |\hat{b}^2_i\rangle. \qquad (3.8b)$$

Note that the regular isorepresentations (3.2) are characterized by *structure constants*; while irregular isorepresentations (3.4) and (3.7) are characterized by *structure functions*. Intriguingly, the former generally occur in the mathematical study of $\hat{SU}_q(2)$, to have the local isomorphism $\hat{SU}_q(2) \approx SU(2)$ by construction, as done in Sect.2. However, the latter generally occur in physical applications [13,14]. This is due to the fact that generators are not changed by isotopies [7—9] (recall that they represent physical quantities). Their embedding in an isotopic algebre then generally implies the appearance of the structure functions.

By no means the above two classes exhaust all possible, physically significant isorepresentations (in fact, we do not study here for brevity the isorepresentations with different isotopic elements for the isoenvelope and isohilbert space). We therefore introduce a third class under the name of *standard adjoint isorepresentations*, which occur when the eigenvalues are the conventional ones, but the algebra is isotopically nontrivial.

In fact, regular isopauli matrices (3.2) admit the conventional eigenvalue $\frac{1}{2}$ for $\Delta = 1$. This condition can be verified by putting $g_{11} = g_{22}^{-1} = \lambda$. We discover in this way the existence of the *standard isopauli matrices*

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & \lambda \\ \lambda^{-1} & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -i\lambda \\ i\lambda^{-1} & 0 \end{pmatrix}, \quad \hat{\sigma}_3 = \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & -\lambda \end{pmatrix}, \qquad (3.9)$$

which admit all *conventional* eigenvalues and structure constants,

$$[\hat{J}_i \hat{,} \hat{J}_j] = i\varepsilon_{ijk}\hat{\sigma}, \quad \hat{J}_3 * |\hat{b}\rangle = \pm \frac{1}{2} |\hat{b}, \quad \hat{J}^2 * |\hat{b}\rangle = (3/4)|\hat{b}\rangle \qquad (3.10)$$

yet exhibit a «hidden variable» λ in their very structure. Note however that the functional dependence of λ is left completely unrestricted by the isotopy.

Thus, λ can be an arbitrary, real-valued, nowhere null, nonlinear-integral function, $\lambda = \lambda(z, \varepsilon, \hat{\psi}, \hat{\psi}^\dagger, \ldots) = \bar{\lambda} \neq 0$.

Needless to say, irregular isorepresentations also become standard under the condition $\det g = 1$. We therefore have the following additional standard isopauli matrices

$$\tilde{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma_1, \ \tilde{\sigma}_2 = \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix} = \sigma_2, \ \tilde{\sigma}_3 = \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & -\lambda \end{pmatrix}, \qquad (3.11a)$$

$$\tilde{\sigma}'_1 = \begin{pmatrix} 0 & \lambda^{\frac{1}{2}} \\ \lambda^{-\frac{1}{2}} & 0 \end{pmatrix}, \quad \tilde{\sigma}'_2 = \begin{pmatrix} 0 & -i\lambda^{\frac{1}{2}} \\ i\lambda^{-\frac{1}{2}} & 0 \end{pmatrix}, \quad \tilde{\sigma}'_3 = \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & -\lambda \end{pmatrix}. \qquad (3.11b)$$

Isopauli matrices with generalized eigenvalues are useful for interior structural problems, i.e., the description of a neutron in the core of a neutron star or, along the same lines, for a hadron constituent. As such, the applications of the general case of the $\hat{SU}_Q(2)$ isosymmetry is studied elsewhere [18].

When studying conventional particles, e.g., those of nuclear physics, the subclass of $\hat{SU}_Q(2)$ which is physically relevant is the special one with conventional eigenvalues which is studied in the next sections. The image $\hat{\sigma}^d_k$ of (3.9) under isoduality, called *isodual Pauli matrices*, will be studied elsewhere.

4. Applications to Isospin in Nuclear Physics

As well known [2], the conventional $SU(2)$-isospin symmetry is broken by electromagnetic and weak interactions. One of the first applications of our isotopic/Q-operator deformation of $SU(2)$ is to show that the isospin symmetry can be reconstructed as exact at the isotopic level, namely, there exist a realization of the underlying isospace $\hat{E}_Q(z, \bar{z}, \hat{\delta}, \hat{C})$ in which protons and neutrons have the same mass, although the conventional values of mass are recovered under isoexpectation values.

The main idea is that the $SU(2)$-isospin symmetry is broken when realized in its simplest conceivable form, that via the Lie product $AB-BA$. However, when the same symmetry is realized via a lesser trivial product, such as our Lie-isotopic product $AQB-BQA$ [7], it can be proved to be exact even under electromagnetic and weak interactions. Actually, the constant Q-matrix acquires the meaning of a suitable average of these interactions.

The reader should be aware that, by no means, this is an isolated occurrence, because it represents rather general capabilities of the Lie-isotopic theory referred to as the *isotopic reconstruction of exact space-time and internal symmetries when conventionally broken*. For example, the rotational symmetry has been reconstructed as exact for all infinitely possible ellipsoidical deformations of the sphere [10]; the Lorentz symmetry has been reconstructed as exact at the isotopic level for all possible signature preserving deformations $\hat{\eta} = Q\eta$ of the Minkowski metric [11]; etc.

The reconstruction of the exact $\hat{SU}_Q(2)$-isospin symmetry is so simple to appear trivial. Consider a twelve-component isostate

$$\hat{\psi}(x) = \begin{pmatrix} \hat{\psi}_p(z) \\ \hat{\psi}_n(x) \end{pmatrix}, \qquad (4.1)$$

where $\hat{\psi}_p(x)$ and $\hat{\psi}_n(x)$ are solutions of the isodirac equation of note [6] which transfroms isocovariantly under $_Q(3.1) \times \hat{SU}_{Q'}(2)$ for the particular subclass with conventional eigenvalues. In this note we study only the $\hat{SU}_Q(2)$ part without any isominkowskian coordinates, thus restricting our attention to the isonormalized isostates

$$|\hat{\psi}_p\rangle = \begin{pmatrix} \lambda^{-\frac{1}{2}} \\ 0 \end{pmatrix}, \; |\hat{\psi}_n\rangle = \begin{pmatrix} 0 \\ \lambda^{\frac{1}{2}} \end{pmatrix}, \; \langle \hat{\psi}_k | Q | \hat{\psi}_k \rangle = 1, \; k = p, n, \qquad (4.2)$$

where $Q = \text{diag.}(\lambda, \lambda^{-1})$, $\hat{I} = Q^{-1} = \text{diag.}(\lambda^{-1}, \lambda)$.

We then introduce the $\hat{SU}_Q(2)$-isospin with realization (3.9) admitting conventional eigenvalues $\pm \frac{1}{2}$ and 3/4, defined over the isospace $\hat{E}_Q(z, \bar{z}, \hat{\delta}, \hat{C})$, $\hat{\delta} = Q\delta$.

We now select such isospace to admit the same masses for the proton and the neutron. This is readily permitted by the «hidden variable» λ when selected in such a way that

$$m_p \lambda^{-1} = m_n \lambda, \quad \text{i.e.,} \; \lambda^2 = m_p / m_n = 0.99862. \qquad (4.3)$$

The mass operator is then defined by

$$\hat{M} = \{\tfrac{1}{2}\lambda(m_p + m_n)\hat{I} + \tfrac{1}{2}\lambda^{-1}(m_p - m_n)\hat{\sigma}_3\}\hat{I} = \begin{pmatrix} m_p\lambda^{-1} & 0 \\ 0 & m_n\lambda \end{pmatrix} \qquad (4.4)$$

and manifestly represents equal masses $\hat{m} = m_p \lambda^{-1} = m_n \lambda$ in isospace.

The recovering of conventional masses in our physical space is readily achieved via the isoeigenvalue expression on an arbitrary isostate

$$\hat{M} * |\hat{\psi}\rangle = M\hat{I}Q|\hat{\psi}\rangle = M|\hat{\psi}\rangle = \begin{pmatrix} m_p & 0 \\ 0 & m_n \end{pmatrix} |\hat{\psi}\rangle, \qquad (4.5)$$

or, equivalently, via the isoexpectation values

$$\langle \hat{\psi}_p | Q\hat{M}Q | \hat{\psi}_p \rangle = m_p, \quad \langle \hat{\psi}_n | Q\hat{M}Q | \hat{\psi}_n \rangle = m_n. \qquad (4.6)$$

Similarly, the charge operator can be defined by

$$q = \frac{1}{2} e (\hat{I} + \hat{\sigma}_3) = \begin{pmatrix} e\lambda^{-1} & 0 \\ 0 & 0 \end{pmatrix}. \qquad (4.7)$$

Thus, the $\hat{SU}_Q(2)$ charges on isospace are $q_p = e\lambda^{-1}$ and $q_n = 0$. However, the charges in our physical space are the conventional ones,

$$\langle \hat{\psi}_p | QqQ | \hat{\psi}_p \rangle = e, \quad \langle \hat{\psi}_n | QqQ | \hat{\psi}_n \rangle = 0. \qquad (4.8)$$

The *isodual* $\hat{SU}^d_Q$*(2)-isospin* characterizes the antiparticle $\bar{p}$ and $\bar{n}$ will be studied elsewhere.

The entire theory of isospin and its applications [2] can then be lifted in an isotopic form which remains exact under all interactions [14]. This is not a mere mathematical curiosity, because it implies a necessary *isotopy of the nuclear force*, e.g., via $\hat{SU}_Q(2)$-isotopic exchange mechanism.

These dynamical implications are studied elsewhere. We only mention that their physical origin lies in the old hypothesis that nuclear forces have a (very small) nonlocal-nonhamiltonian component due to the overlapping of the charge distributions of nucleons. The «hidden variable» λ here introduced merely provides an average of these novel components of the nuclear force.

5. Application to Few-Body Nuclear Magnetic Moments

In the recent note [6] we have shown that the $\hat{SU}_Q(2)$ symmetry permits a direct representation of: 1) the expected *nonspherical* shapes of the charge distribution of nucleons; 2) all their infinitely possible *deformations* due to external forces; and 3) the consequential *alteration of the «intrinsic» magnetic moment of protons and neutrons* under sufficient conditions.

These results were then applied in note [6] to the apparently first, *exact* representation of the total magnetic moment of the deuteron and other few-body nuclei.

It is now recommendable to re-examine these results within the context of the exact isospin symmetry of this note.

Consider nuclei with A even and introduce $(A/2)$-dimensional isospaces $\hat{E}_k(z, \hat{\delta}, \hat{C})$, $\hat{\delta} = Q_k\delta$, isotopic elements $Q_k = \mathrm{diag.}(\lambda_k, \lambda_k^{-1})$, $\det Q_k = 1$, isounits $\hat{I}_k = Q_k^{-1}$, and related $\hat{SU}(2)$ isosymmetry for isospin $\frac{1}{2}$ (the extension to odd A is the same as in QM). As well known (see, e.g., [2,3]), total nuclear magnetic moments are computed via the familiar expressions

$$\mu^{(S)} = g^{(s)}(eh/2m_pc_0)\,S, \quad g_p^{(s)} = 5.585,$$

$$g_n^{(s)} = -3.816, \quad eh/2m_pc_0 = 1, \tag{5.1a}$$

$$\mu^{(L)} = g^{(L)}L, \quad g_p^{(L)} = 1, \quad g_n^{(L)} = 0. \tag{5.1b}$$

In the preceding section we selected the «hidden parameter» λ to identify the p and n masses. We here select the λ-parameter to render equal the (magnitude of the) $p-n$ magnetic moments via the model

$$\hat{\mu} = \{\tfrac{1}{2}\lambda(g_p + g_n)\hat{I} + \tfrac{1}{2}\lambda^{-1}(g_p - g_n)\,\hat{\sigma}_3\}\hat{I} = \mathrm{diag.}(g_p\lambda^{-1}, g_n\lambda), \tag{5.2a}$$

$$\tilde{g} = \lambda^{-1}g_p^{(S)} = -\lambda\, g_n^{(S)}, \quad \langle\hat{\psi}_k|Q\mu Q|\hat{\psi}_k\rangle = g_k, \; k = p, n. \tag{5.2b}$$

A simple isotopic lifting of the conventional QM isospin treatment (see [14] for details), then leads to the following alternative formulation of model (3.9) of note [6]

$$\hat{\mu}^{HM}_{\mathrm{Tot}} = \frac{1}{2}\sum_k \hat{I}_k \times L_{k3} + \sum_k (\tfrac{1}{2}L_{k3} + \tilde{g}_k\tau_{k3}\times S_{k3}). \tag{5.3}$$

Recall that, in the conventional treatment we have two terms, called scalar and vector components, with the latter being dominant over the former. The dominance of the latter becomes greater under isotopies, and constitutes the sole contribution for $L = 0$.

Consider now the case of the Deuteron (D) and the experimental value of its magnetic moment

$$\mu_0^{\mathrm{exp}} = 0.857, \quad (\mu_p = 1). \tag{5.4}$$

As well known, in QM we have the theoretical value

$$\mu_D^{QM} = g_p + g_n = 0.879, \tag{5.5}$$

which, as such, does not represent value (5.4) exactly, while significant differences persist under relativistic, $L = 2$ and other corrections ($L = 1$ is prohibited by parity [2,3], while many-body techniques are evidently inapplicable for the deuteron).

In note [6] we provided the *exact* representation of value (5.4) via a mutation $\hat{\mu}_p, \hat{\mu}_n$ of the $p-n$ magnetic moments due to a small deformation of their charge distributions. In this note we present the same result, but this time obtained in an isospace with exact isospin symmetry and equal magnitudes of the $p-n$ magnetic moments in the deuteron, which is achieved for value, in $\hat{E}(r, \bar{r}, \hat{\delta}, \hat{C})$

$$\lambda^2 = |g_n|/|g_p| = 3.816/5.5816 = 0.685, \quad \tilde{g} = 0.428. \tag{5.6}$$

The use of the HM formalism then yields the isoeigenvalues of (5.3)

$$\hat{\mu}_D^{HM} * \hat{\psi} = 2\hat{g}\,\tau_3 \times S_3 Q\hat{\psi} + 2\tilde{g}\hat{\psi} = 0.857\hat{\psi}. \tag{5.7}$$

This illustrates the possibility of exactly representing μ_D^{exp} as already done in Sect. 3B of [6], but under the additional condition of having the same magnitude of the $p-n$ magnetic moments in isospace.

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