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# ISOTOPIC LIFTINGS OF $SU(3)$ WITH INTEGER QUARK CHARGES *

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## ABSTRACT

In this note we show that the fractional charges of quarks may well be due to the current assumption of the simplest conceivable realization of the $SU(3)$ symmetry. In fact, under an isotopic lifting of the symmetry, quarks can apparently possess integer charges, although the theory will require predictable revisions that are considered elsewhere.

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As is well known, current $SU(3)$ theories lead to the fractional charges of quarks $(2/3, -1/3, -1/3)$ [1] which, despite numerous investigations for over two decades, have remained essentially unexplained as of today, theoretically and experimentally. In this note we submit the possibility that these fractional charges may be due to the selection of the simplest conceivable realization of the $SU(3)$ symmetry, that in terms of the enveloping associative algebra $\mathcal{A}(SU(3))$ with elements I = Diag. (1,1,1), the familiar basis $\lambda_k, k = 1, 2, \cdots 8$, [2] and all their possible polynomials, with the simplest possible associative product, that of matrices $\lambda_i \lambda_j$. The algebra $SU(3)$ is then realized in the familiar form

$$SU(3) : [\lambda_i, \lambda_j]_{\mathcal{A}} = \lambda_i \lambda_j - \lambda_j \lambda_i = 2i f_{ijk} \lambda_k, \tag{1}$$

where the $f's$ are the $SU(3)$ structure constants [2].

In a memoir of 1978 [3], one of us (RMS) introduced the so-called *Lie-isotopic lifting* of the (conventional formulation of) Lie's theory. It is essentially centered in the generalization of the trivial unit I of current use into the form $\hat{I}$ which, besides being non-singular and Hermitean, has otherwise an arbitrary functional dependence on all local variables and quantities (see below).

The lifting $I \rightarrow \hat{I}$ renders *necessary* the construction of a corresponding, compatible, generalization of the conventional Lie's theory in its central structures: enveloping associative algebras, Lie algebras and Lie groups [3].

In fact, for $\hat{I}$ to remain the unit of the theory, the algebra $\mathcal{A}$ must be lifted to the form

$$\hat{\mathcal{A}} : \begin{array}{ll} a * b \stackrel{\text{def}}{=} agb, \ g = \text{ fixed, non-singular } \alpha \text{ Hermitean}, & \text{(2a)} \\ \hat{I} * a = a * \hat{I} = a, \ \hat{I} = g^{-1}, \ \forall a, b \in \hat{\mathcal{A}}, & \text{(2b)} \end{array}$$

called ***isotopic-associative*** (or ***isoassociative***) because the new product $a * b$ remains associative [3]. Similarly, the antisymmetric algebra $\hat{A}^{-}$ attached to $\hat{A}$ is now characterized by the product

$$\hat{L} : [a,b]_{\hat{A}} = a * b - b * a = agb - bga, \tag{3}$$

called ***Lie-isotopic*** product because still Lie [3], as the reader is encouraged to verify. To see the corresponding group structure, introduce the ***isotransformations*** on a manifold $M$ with local chart $z$

$$z' = \hat{U}(w) * z = \hat{U}(w)gz, \tag{4}$$

where $w$ is a parameter. Then, the set of all possible $\hat{U}(w)$ forms a ***Lie-isotopic group*** when it verifies the rules [3]

$$\hat{U}(w) * \hat{U}(w') = \hat{U}(w') * \hat{U}(w) = \hat{U}(w + w'), \tag{5a}$$

$$\hat{U}(0) = \hat{U}(w) * \hat{U}(-w) = \hat{I} = g^{-1}. \tag{5b}$$

Moreover, the existence of consistent isotopies of the various structure theorems, including that of the Poincaré-Birkhoff-Witt theorem [3], allow the ***isoexponentiation*** of the algebra $\hat{L}$ into the corresponding (connected) group $\hat{G}$ according to

$$\hat{G} :\ \hat{U}(w) \stackrel{\text{def}}{=} \hat{I} + \frac{i\lambda}{1!} + \frac{(i\lambda w) * (i\lambda w)}{2!} + \cdots \stackrel{\text{def}}{=} e_{\hat{A}}^{i\lambda w} = e^{i\lambda g w}\hat{I} = \hat{I}e^{iwg\lambda}. \tag{6}$$

The isotopic theory was subsequently applied for a preliminary study of the lifting of several, conventional, space-time symmetries, such as: general theory of space-time isosymmetries on manifolds [4]; isorotational symmetries $\hat{0}(3)$ [5]; isospinorial symmetries $\widehat{SU}(2)$ [6]; Lorentz-isotopic symmetries $\hat{0}(3.1)$ [7]; Poincaré-isotopic symmetries $\hat{P}(3.1)$ [8]; isounitary symmetries $\widehat{SU}(3)$ [9]; the isogauge symmetries [10]; the discrete space-time symmetries [11]; the isocreation and isoannihilation algebra [12]; and others. In these studies, it essentially emerged that all possible isotopes $\hat{G}$ of a Lie symmetry $G$ are (locally) isomorphic to $G$ under the sole condition that the isounit $\hat{I}$ preserves the topological characteristics of the original unit I (positive-definiteness). The Lie-isotopic theory then permitted the reconstruction of exact space-time symmetries at the covering isotopic level, when believed to be broken at the conventional level (see the reconstruction of the exact rotational [5], Lorentz [7] and Poincaré [8] symmetries when believed to be broken, e.g., by a modification of the underlying metric). The reconstruction of the exact $\widehat{SU}(3)$ and isogauge symmetries when conventionally broken is under study at this writing. A review written for mathematicians can be found in the recent reference [13], which also presents a list of rather intriguing, fundamental, open problems.

In this note we shall construct the fundamental isorepresentation of the isotopic $\widehat{SU}(3)$ symmetries as a simple generalization of the corresponding isorepresentation of $\widehat{SU}(2)$ [6]; confirm the (local) isomorphism $\widehat{SU}(3) \approx SU(3)$ first identified in reference [9]; and identify a few preliminary implications, particularly for the charges of the quarks. A detailed presentation of the results of this note is presented elsewhere [14], jointly with additional developments and comments. For operators realizations of the formalism see references [15-20].

We shall define the (infinite) family of isotopes $\widehat{SU}(3)$ of $SU(3)$ the Lie symmetries of the (infinite) family of all possible invariants in complex three-dimensional space

$$z_i^{\dagger} g_{ij} z_j = z_1^* g_{11} z_1 + z_2^* g_{22} z_2 + z_3 g_{33} z_3 = \text{ inv}, \tag{7a}$$

$$g = g^{\dagger} = g(z, z^{\dagger}, \cdots) = \text{Diag.}(g_{11}, g_{22}, g_{33}),\ g_{kk} > 0,\ k = 1,2,3. \tag{7b}$$

By following the original proposal [3], the $\widehat{SU}(3)$ symmetries are constructed by assuming as unit the generalized quantity $\hat{I} = g^{-1}$. Isoenvelope (2) is then given by the isounit $\hat{I}$, the expected new basis, say, $\hat{\lambda}_k, k = 1, 2, \cdots 8$, and all their possible polynomials, where all original associative products (including powers) are now replaced with the isotopic product $a * b = agb$, where $g$ is given by Eq.(7b).

The algebra $\hat{\mathcal{A}}$ is defined over the ***iso-Hilbert*** space with inner product [19]

$$\hat{\mathcal{H}} :< n\hat{|}n' > \stackrel{\text{def}}{=} < n| * |n' > \hat{I} = < n|g|n' > \hat{I} \in \hat{C}, \tag{8}$$

over the ***isofield*** $\hat{C} = \{\hat{c}|\hat{c} = c\hat{I}, c \in C, \hat{I} \in \hat{\mathcal{A}}\}$ [4] with ordinary sum and product $\hat{c}_1 * \hat{c}_2 = \widehat{c_1 c_2} = c_1 c_2 \hat{I}$. By following the prescriptions of references [3,4], the original manifold underlying the $SU(3)$ symmetry, the Euclidean space $E(z, \delta, C), \delta =$ Diag (1,1,1), is then lifted into the (infinite) family of isotopes $\hat{E}(z, g, \hat{C})$. By recalling that the conventional operator algebra $\mathcal{A}$ with product $ab$, the conventional Hilbert space $\mathcal{H}$ with inner product $< n|n' >$, and conventional field $C$ constitute the structures at the basis of quantum mechanics, the isotopes $\hat{\mathcal{A}}, \hat{\mathcal{H}}$ and $\hat{C}$ characterize a generalization of quantum mechanics under the name of "hadronic mechanics" [15]. Note that, from condition $\hat{I} > 0, \hat{\mathcal{A}}$ is an associative algebra, $\hat{H}$ is a Hilbert space, and $\hat{C}$ is a field. Thus, as it occurs for the underlying algebraic structure [3], "hadronic mechanics" coincides with the conventional quantum mechanics at the abstract, coordinate free-level [15]. One has the latter when the simplest conceivable realizations of the structures, $\hat{\mathcal{A}}, \hat{H}$ and $\hat{C}$ are assumed; when more general realizations of the same structures are instead preferred, the former mechanics holds [16,17]. The generalization of conventional operations in Hilbert spaces was studied in reference [18] (see also reference [19]) to which we refer for brevity.

We now look for a realization of $\widehat{SU}(3)$ via isotransformations (4) forming a Lie-isotopic group (5), under the condition that they are ***isounitary***, i.e., [6]

$$\hat{U} * \hat{U}^{\hat{\dagger}} = \hat{U}^{\hat{\dagger}} * \hat{U} = \hat{I} = g^{-1},\ \hat{U} = e_{\hat{\mathcal{A}}}^{i\lambda w}, \tag{9}$$

which can hold iff [6]

$$Tr(\hat{\lambda}_k g) = 0,\ k = 1, 2, \cdots, 8. \tag{10}$$

To stress "ab initio" the local isomorphism between $\widehat{SU}(3)$ and $SU(3)$ [9], we now *impose* that the isocommutation rules of $\widehat{SU}(3)$ preserve the original structure constants $f_{ijk}$ of Equation (1) (the reader should be aware that this is not necessarily the case, because the structure constants under lifting are generally replaced by structure ***functions*** [3,4]). This implies that we shall search for the realization

$$\widehat{SU}(3) : [\hat{\lambda}_i, \hat{\lambda}_j]_{\hat{\mathcal{A}}} = \lambda_i * \lambda_j - \lambda_j * \lambda_i = 2i\hat{f}_{ijk} * \hat{\lambda}_k \equiv 2if_{ijk}\hat{\lambda}_k. \tag{11}$$

To construct the $\hat{\lambda}$'s, we introduce the isocreation and isoannihilations operators $\hat{a}_i, \hat{a}_i^{\dagger}, i = u, d, s$, with isocommutation rules [12]

$$[\hat{a}_i, \hat{a}_i^\dagger]_{\hat{A}} = 1, \ [\hat{a}_i, \hat{a}_j]_{\hat{A}} = [\hat{a}_i^\dagger, \hat{a}_j^\dagger] = 0, \ i, j = u, d, s, \tag{12}$$

and iso-Casimir invariant $\hat{N} = \hat{a}_u^\dagger * \hat{a}_u + \hat{a}_d^\dagger * \hat{a}_d + \hat{a}_s^\dagger * \hat{a}_s \stackrel{\text{def}}{=} \hat{N}_u + \hat{N}_d + \hat{N}_s$.

Introduce the isobasis in $\hat{\mathcal{H}}$

$$|n_u, n_d, n_s>, \ n_u, n_d, n_s = 1, 0, \ < n_u, n_d, n_s \hat{|} n_{u'}, n_{d'}, n_{s'} >= \delta_{uu'} \delta_{dd'} \delta_{ss'} \ \hat{I}. \tag{13}$$

We can then assume the existence of the following isoeigenvalue [18] equations

$$\begin{gathered} \hat{N}_u * |n_u, n_d, n_s > \ = g_u |n_u, n_d, n_s >, \hat{N}_d * |n_u, n_d, n_s >= g_d |n_u, n_d, n_s >, \\ \hat{N}_s * |n_u, n_d, n_s >= g_s |n_u, n_d, n_s >, \end{gathered} \tag{14}$$

where $g_u, g_d$ and $g_s$ are certain functions of the matrix elements $g_{11}, g_{22}$ and $g_{33}$ to be determined later on. It is then easy to prove the following actions of the isocreation and isoannihilation operators on the isobasis

$$\hat{a}_u * |n_u, n_d, n_s >= (g_u n_u)^{1/2} |n_u - 1, n_d, n_s >, \tag{15a}$$

$$\hat{a}_u^\dagger * |n_u, n_d, n_s >= [(g_u n_u)^{1/2} + 1] |n_u + 1, n_d, n_s >, \tag{15b}$$

with similar expressions for the remaining operators $\hat{a}_d, \hat{a}_d^\dagger, \hat{a}_s$ and $\hat{a}_s^\dagger$.

The matrix elements in the bilinear operators are of the type

$$(M) =< n_u, n_d, n_s| * \hat{a}_i^\dagger * \hat{a}_j * |n_{u'}, n_{d'}, n_{s'} >, \tag{16}$$

and all the non-null ones are given by

$$< 100| * \hat{a}_u^\dagger * \hat{a}_u * |100 >= g_u, \ < 100| * \hat{a}_u^\dagger * \hat{a}_d * |010 >= g_d^{1/2}, \ < 100| * \hat{a}_u^\dagger * \hat{a}_s * |001 >= g_s^{1/2}, \tag{17a}$$

$$< 010| * \hat{a}_d^\dagger * \hat{a}_u * |100 >= g_u^{1/2}, \ < 010| * \hat{a}_d^\dagger * \hat{a}_d * |010 >= g_d, \ < 010| * \hat{a}_d^\dagger * \hat{a}_s * |001 >= g_s^{1/2}, \tag{17b}$$

$$< 001| * \hat{a}_s^\dagger * \hat{a}_u * |100 >= g_u^{1/2}, \ < 001| * \hat{a}_s^\dagger * \hat{a}_d * |100 >= g_d^{1/2}, \ < 001| * \hat{a}_s^\dagger * \hat{a}_s * |001 >= g_s. \tag{17c}$$

Simple calculations then yield the isorepresentation

$$\hat{\lambda}_1 = (\hat{a}_u^\dagger * \hat{a}_d + \hat{a}_d^\dagger * \hat{a}_u) = \begin{pmatrix} 0 & g_d^{1/2} & 0 \\ g_u^{1/2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ \hat{\lambda}_2 = -i(a_u^\dagger * \hat{a}_d - \hat{a}_d^\dagger * \hat{a}_u) = \begin{pmatrix} 0 & -ig_d^{1/2} & 0 \\ ig_u^{1/2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{18a}$$

$$\hat{\lambda}_3 = (k_1 \hat{a}_u^\dagger * \hat{a}_u - k_2 \hat{a}_d^\dagger * \hat{a}_d) = \begin{pmatrix} k_1 g_u & 0 & 0 \\ 0 & -k_2 g_d & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{\lambda}_4 = (\hat{a}_u^\dagger * \hat{a}_s + \hat{a}_s^\dagger * \hat{a}_u) = \begin{pmatrix} 0 & 0 & g_s^{1/2} \\ 0 & 0 & 0 \\ g_u^{1/2} & 0 & 0 \end{pmatrix}, \tag{18b}$$

$$\hat{\lambda}_5 = -i(\hat{a}_u^\dagger * \hat{a}_s - \hat{a}_s^\dagger * \hat{a}_u) = \begin{pmatrix} 0 & 0 & -ig_s^{1/2} \\ 0 & 0 & 0 \\ ig_u^{1/2} & 0 & 0 \end{pmatrix}, \quad \hat{\lambda}_6 = (a_d^\dagger * \hat{a}_s + \hat{a}_s^\dagger * \hat{a}_d) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & g_s^{1/2} \\ 0 & g_d^{1/2} & 0 \end{pmatrix}, \tag{18c}$$

$$\hat{\lambda}_7 = -i(\hat{a}_d^\dagger * \hat{a}_s - \hat{a}_s^\dagger * \hat{a}_d) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -ig_s^{1/2} \\ 0 & ig_d^{1/2} & 0 \end{pmatrix}, \tag{18d}$$

$$\hat{\lambda}_8 = \frac{1}{\sqrt{3}}(k_3 \ \hat{a}_u^\dagger * \hat{a}_u + k_4 \hat{a}_d^\dagger * \hat{a}_d - 2k_5 \hat{a}_s^\dagger * \hat{a}_s) = \frac{1}{\sqrt{3}} \begin{pmatrix} k_3 g_u & 0 & 0 \\ 0 & k_4 g_d & 0 \\ 0 & 0 & -2k_5 g_s \end{pmatrix}, \tag{18e}$$

where the quantities $k_1, k_2, \cdots k_5$ are additional unknown functions of the metric elements $g_{11}, g_{22}$, and $g_{33}$.

To compute the unknown functions of the metric elements, we first consider the eight isotrace conditions (10) which result in the following two conditions in the unknown quantities

$$k_1 g_u g_{11} = k_2 g_d g_{22}, \ k_3 g_u g_{11} + k_4 g_d g_{22} = 2k_5 g_s g_{33}. \tag{19}$$

The remaining conditions are provided by the following isocommutators involving diagonal elements

$$[\hat{\lambda}_1, \hat{\lambda}_2]_{\hat{A}} = 2i\hat{\lambda}_3 : \tag{20a}$$

$$(g_u g_d)^{1/2} g_{22} = k_1 g_u, \ (g_u g_d)^{1/2} g_{11} = k_2 g_d, \tag{20b}$$

$$[\hat{\lambda}_4, \hat{\lambda}_5]_{\hat{A}} = i\hat{\lambda}_3 + i\sqrt{3}\hat{\lambda}_8 : \tag{20c}$$

$$(g_u g_s)^{1/2} g_{33} = (k_1 + k_3)\frac{g_u}{2}, \ (g_u g_s)^{1/2} g_{11} = k_5 g_s, \tag{20d}$$

$$[\hat{\lambda}_6, \hat{\lambda}_7]_{\hat{A}} = -i\hat{\lambda}_3 + i\sqrt{3}\hat{\lambda}_8 : \tag{20e}$$

$$(g_d g_s)^{1/2} g_{33} = (k_s + k_4)\frac{d_d}{2}, \ (g_d g_s)^{1/2} g_{22} = k_5 g_s, \tag{20f}$$

The desired solution is then given by

$$k_1 = k_3 = g_{11}, \ k_2 = k_4 = g_{22}, \ k_5 = g_{33}, \tag{21a}$$

$$g_u = g_{22}^2, \ g_d = g_{11}^2, \ g_s = \frac{g_{11}^2 g_{22}^2}{g_{33}^2}. \tag{21b}$$

The fundamental isorepresentation of $\widehat{SU}(3)$ is then given by Equations (18) under values (21) for the unknown quantities. As an example, the diagonal matrices are

$$\hat{\lambda}_3 = \begin{pmatrix} g_{11}g_{22}^2 & 0 & 0 \\ 0 & -g_{22}g_{11}^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \hat{\lambda}_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} g_{11}g_{22}^2 & 0 & 0 \\ 0 & g_{22}g_{11}^2 & 0 \\ 0 & 0 & -2\frac{g_{11}^2 g_{22}^2}{g_{33}} \end{pmatrix}. \tag{22}$$

The verification that the above fundamental isorepresentation verifies all isocommutation rules (11) is an instructive exercise for the reader not familiar with Lie-isotopic theories. Note that the metric elements $g_{11}, g_{22}, g_{33} > 0$ remain completely unrestricted.

We would like now to close this note with a few comments. As it had been the case for $\hat{O}(3)$ [5], $\widehat{SU}(2)$ [6], $\hat{O}(3.1)$ [7] and other symmetries, our analysis indicates that there exists *one* abstract $\mathcal{SU}(3)$ symmetry, with infinitely many realizations all isomorphic to $SU(3)$. The conventional realization $SU(3)$ emerges as the simplest conceivable one according to rules (1). Our infinitely many isotopes $\widehat{SU}(3)$ emerge when the same abstract symmetry is realized with the less trivial rules (11), one isotope per each possible metric $g$. This result immediately raises the question of: which of these infinitely many different realizations of $\mathcal{SU}(3)$ is the one occurring in the physical reality? Our tentative answer is that *all* of them may have physical relevance depending on the degree of approximation desired, as elaborated below (and better in reference [14] where we show that the approach can be extended consistently to $\widehat{SU}(4)$ and related charmed quarks.)

Second, the reader should be aware that the isotopes $\widehat{SU}(3)$ are not trivial on numerous counts. First, the fundamental isorepresentation cannot be reduced to the conventional one, within the context of the conventional realization (1), as the reader is encouraged to verify. Second, while the conventional $SU(3)$ theory is *linear*, our covering $\widehat{SU}(3)$ theory is, in general, not only *nonlinear*, but also *nonlocal* in all variables and quantities desired [20]. This can be seen by writing transformations (4) in their explicit form $z' = \hat{U}(w) * z = \hat{U}(w)gz = \hat{U}(w)g(z, z^\dagger, \cdots)z$. Third, the underlying isotopy $\delta \rightarrow g$ results to be a geometrization of the apparent deviations from the Euclidean (as well as Minkowski) metric indicated by several phenomenological predictions on the behaviour of the meanlife of unstable hadrons with speed (see ref. [7] and quoted references for brevity).

One of the authors (RMS) has argued since some time [15] that this nonlinear and nonlocal internal structure of hadrons is expected to be due to the fact that the hadronic constituents, even though possessing a point-like charge, are expected to have wavepackets of the order of the dimension of all hadrons ($\sim 1F$). According to this view, hadrons are not ideal empty spheres with points in them, but hyperdense media (called ***hadronic media***) composed by the wavepackets of the constituents in condition of total mutual immersion.

This leads to the conjecture that the hadronic medium may be "granulated" inside hadrons, i.e., that the isotopic metric, in first approximation, has predominant values along certain regions of space (time), each "granule" representing a quark.

This conjecture apparently permits the achievement of integer quark charges in first approximation for a constant $g$. In fact, the *isocharge* is given by

$$\hat{Q} = \frac{1}{2}\hat{\lambda}_3 + \frac{1}{2}\hat{Y} = \frac{1}{2}\hat{\lambda}_3 + \frac{1}{2\sqrt{3}}\hat{\lambda}_8 = \begin{pmatrix} \frac{2}{3} \; g_{11}g_{22}^2 & 0 & 0 \\ 0 \quad -\frac{1}{3} & g_{22}g_{11}^2 & 0 \\ 0 \quad\; 0 & -\frac{1}{3} & \frac{g_{11}^2 g_{22}^2}{g_{33}} \end{pmatrix}, \tag{23}$$

and, for the values

$$g_{11} = \sqrt[3]{6},\ g_{22} = \frac{3}{\sqrt[3]{12}},\ g_{33} = \frac{3}{\sqrt[3]{4}}, \tag{24}$$

one indeed obtains integer values $(1,-1,-1)$ of the quark charges. Needless to say, the transition from charges $(2/3,-1/3,-1/3)$ to $(1,-1,-1)$ requires a revision of the current interpretation of the quark structure of hadrons. The study whether this is indeed possible in a consistent way, is presented in refs.[23,24] .The purpose of this note is restricted to the submission of the *existence* of the infinite isotopes $\widehat{SU}(3)$ and of the particular one (24) for integral charges.

It should be indicated that Eq. (24) is nothing but an approximation of a physical reality expectedly much more complex. In fact, we already know that the full use of the Lie-isotopic theory in general and, of the Poincaré-isotopic symmetry in particular [8], leads to the *necessary* alteration (called *mutation* [3]) of conventional charges [21]. In this case, *the integer charges would be anomalous* because generally not occurring within the hadronic medium represented by geometrization $g$.

These generalized notions, rather than being a drawback, open up rather intriguing possibilities, such as the apparent possibility of representing quarks with mutated forms of ordinary massive particles produced freely in the spontaneous decays. For preliminary studies along these lines, see papers [14,22].

In the final analysis, the identification of the infinite family of coverings $\widehat{SU}(3)$ of $SU(3)$ will evidently call, sooner or later, for a reinspection of the entire theory.

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