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ABSTRACT

In this note we further develop the proposal made in preceding works of constructing the infinite family of Lie–isotopic liftings of Galilei's relativity for closed–isolated systems of particles possessing local, potential and selfadjoint, as well as nonlocal, nonhamiltonian and non selfadjoint internal forces. In particular, we show that the nonlinear and nonlocal generalization of the Galilei transformations introduced in a preceding note do indeed represent motion of extended particles within resistive media, but in such a way to coincide with the conventional transformations at the abstract, realization–free level. This allows the preservation of the basic, physical and mathematical axioms of Galilei's relativity under our liftings, and their realization in the most general possible nonlinear, nonlocal and nonhamiltonian way.

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As well known, the *Galilei relativity* (*see*, e.g., ref.s [1,2]) is a description of physical systems via their form-invariance under the Galilei's symmetry $G(3.1) = [O_\theta(3) \otimes T_{r^\circ}(3)] \times [T_{v^\circ}(3) \times T_{t^\circ}(1)]$, or, equivalently, under the celebrated *Galilei's transformations*

$$t' = t + t^\circ, \quad \text{translations in time,} \tag{1a}$$
$$r'_i = r_i + r^\circ_i, \quad \text{translations in space,} \tag{1b}$$
$$r'_i = r_i + t^\circ v^\circ_i, \quad \text{Galilei boosts,} \tag{1c}$$
$$r' = R(\theta)\, r, \quad \text{rotations.} \tag{1d}$$

The relativity is verified in our physical reality only for a rather small class of Newtonian systems, called *closed selfadjoint systems*. These are systems (such as our planetary system) which verify the conventional total Galilean conservation laws when isolated, and admit internal forces which are local (differential), potential and selfadjoint.

For all remaining Newtonian systems, Galilei's relativity is violated according to a number of mechanisms [3] (see the classification of the breakings of Galilei's relativity into *isotopic, selfadjoint, semicanonical, canonical and essentially selfadjoint* of ref. [4], Sect. A.12).

In the final analysis, the limitations of Galilei's relativity are inherent in its mathematical structure. In fact, as studied in the first (and most basic) note [5] of this series:

1) The *linear* character of Galilei's transformations is at variance with the generally *nonlinear* structure of Newtonian systems, as established by incontrovertible physical evidence;

2) The *local* (differential) character of Galilei's relativity is at variance with the generally *nonlocal* (integral) nature of the systems in our Earthly environment; and

3) The strictly *Hamiltonian* (canonical) structure of Galilei's relativity is at variance with the generally *nonhamiltonian* character of physical systems in our physical reality.

An infinite family of Lie-isotopic generalizations of the Galilei symmetry, under the name of *Galilei-isotopic symmetries* $\hat{G}(3.1)$ was submitted in Refs. [3,4] to represent a broader class of Newtonian systems, while the name of *Galilei-isotopic relativities* was submitted for the corresponding generalized relativities. In particular, it was shown that:

A) The Galilei-isotopic symmetries characterize *closed non-selfadjoint systems.* These are systems (such as Jupiter) which verify the conventional, total, Galilean conservation laws when isolated, while admitting the additional class of nonlocal, nonhamiltonian and nonselfadjoint internal forces. [See the second [6] and third [7] notes of this series).

B) The Galilei-isotopic symmetries possess the structure

$$\hat{G}(3.1) = [\hat{O}_\theta(3) \otimes \hat{T}_{r^\circ}(3)] \times [\hat{T}_{v^\circ}(3) \times T_{t^\circ}(1)] , \qquad (2)$$

and they result to be all locally isomorphic to the conventional symmetry G(3.1) under the positive-definiteness of the underlying isounit, by admitting the latter as a particular case. In this sense, $\hat{G}(3.1)$ provides an infinite family of *Lie-isotopic coverings* of G(3.1).

C) All symmetries $\hat{G}(3.1)$ can be explicitly constructed via the Lie-isotopic theory, that is, via the use of the same parameters and generators (conserved quantities) of the conventional symmetry, but via the most general possible, axiom-preserving realizations of Lie algebras and Lie groups. In this way, an infinite number of symmetries $\hat{G}(3.1)$ were identified for each given Hamiltonian H = T + V (i.e., for given potential-selfadjoint forces), as characterized by an infinite number of possible generalizations $\Omega^{\mu\nu}$ of the canonical Lie tensor $\omega^{\mu\nu}$, representing the additional infinite class of possible nonhamiltonian internal forces (infinite class of possible interior physical media and conditions).

In the remaining preceding notes [8,9,10] we have introduced the isotopic generalization $\hat{\mathfrak{R}}_t \times T^*\hat{E}_2(r,G,\hat{\mathfrak{R}})$ of the conventional space $\mathfrak{R}_t \times T^*E(r,\delta,\mathfrak{R})$ of ref. [3,4] which allows the incorporation of nonlocal forces, and identified the following most general possible nonlinear as well as nonlocal *Galilei-isotopic transformations* [10]

$$t' = t + t^\circ \hat{B}_4^{-2}(r,p,..), \quad \text{isotime translations,} \qquad (3a)$$

$$r'_i = r_i + r^\circ_i \hat{B}_i^{-2}(r,p,..), \quad \text{isospace translations,} \qquad (3b)$$

$$r'_i = r_i + t^\circ v^\circ_i \hat{B}_i^{-2}(r,p,...), \quad \text{isogalilei boosts,} \qquad (3c)$$

$$r' = \hat{R}(\theta) * r = \hat{R}(\theta)\, G(r,p,...)\, r, \quad \text{isorotations,} \qquad (3d)$$

where the $\hat{B}$'s are generally nonlinear as well as nonlocal functions of all variables, they vary from system to system, and they can be explicitly computed via the Lie-isotopic techniques for each system [10].

It is easy to see that transformations (3) leave invariant the following isoseparations in $\hat{\mathfrak{R}}_t \times T^*\hat{E}_2(r,G,\hat{\mathfrak{R}})$

$$t_a - t_b = \text{inv.}, \quad (r_{ia} - r_{ib})\, G_{ij}(r,p,...)\, (r_{ja} - r_{jb}) = \text{inv.}, \qquad (4)$$
$$i,j = 1,2,3\ (= x,y,z), \quad a,b, = 1,2,....,N,$$

the latter one applying for $t_a = t_b$.

The purpose of this note is to study the relativities characterized by the Galilei-isotopic symmetries. To begin, we can enlarge Definition 6.3.9, p. 243, of Ref. [4] via the results of the preceding notes of this series as follows.

DEFINITION: The general, nonlinear, nonlocal and nonhamiltonian, Galilei-isotopic relativities are the form-invariant description of physical systems characterized by the infinite family of Galilei-isotopic symmetries $\hat{G}(3.1)$ [3,4,10] when all locally isomorphic *to the conventional Galilei symmetry G(3.1), with corresponding, infinite family of general Galilei-isotopic transformations (3) on isospaces $\hat{\mathfrak{R}}_t \times T^*\hat{E}_2(r,G,\hat{\mathfrak{R}})$.*

The reader should be aware of the *uniqueness of transformations* (3) (up to the degrees of freedom of the Lie-isotopic theory which are broader than the conventional ones, and include, e.g., the Birkhoffian *gauge transformations* [3,4,6]).

The reader should also keep in mind the restriction $\hat{G}(3.1) \approx G(3.1)$ in the above definition. This is due to the fact that, if such restriction is lifted (i.e., if the isounits are not necessarily positive-definite), isosymmetries $\hat{G}(3.1)$ still formally exist, but they do not qualify for the characterization of covering relativities. See in this respect the classification of all possible compact and noncompact isotopes $\hat{O}(3)$ of O(3) of Ref. [11].

The form-invariant description of physical systems characterized by the Galilei-isotopic relativities has been studied in Refs. [3,4,10]. We therefore remain here with the problems of: a) identifying the physical laws characterized by the Galilei-isotopic relativities; b) prove their form-invariance under $\hat{G}(3.1)$; and c) prove their abstract-axiomatic equivalence to the conventional physical laws of Galilei's relativity.

The reader should recall that these properties are essentially assured by the ultimate, abstract identity of all conventional and generalized methodological tools and symmetries [6], such as the abstract identity among Hamiltonian and Birkhoffian mechanics, Lie and Lie-isotopic algebras, symplectic and symplectic-isotopic geometries, etc.. This abstract unity implies the abstract identity of the symmetries $\hat{G}(3.1)$ and $G(3.1)$ and, necessarily, that of the represented physical systems, the closed selfadjoint and nonselfadjoint systems. Properties a), b) andf c) above pertaining to physical laws must then be merely identified in the formalism.

For this purpose, consider the historical *Galilei's boosts*

$$r'_i = r_i + t^\circ v^\circ_i, \quad p'_i = p_i + mv^\circ_i, \qquad (5)$$

which, as well known, apply for the simple case of a particle with constant speed, under the (often tacit) assumption that motion occurs in vacuum.

Suppose now that the particle considered penetrates within a physical medium at a given instant of time t. Then, the Galilei transformations are evidently inapplicable, e.g., because of their linearity and locality, while the particle experiences a drag force, that is nonlinear and nonlocal.

Our generalized transformations

$$r'_i = r_i + t^\circ v^\circ_i \hat{B}_i^{-2}(r,p,...), \quad p'_i = p_i + mv^\circ_i \hat{B}_i^{-2}(r,p,...), \qquad (6)$$

are then applicable to represent the *deviations* from the original uniform motion. In particular, Eqs. (6) can represent a (monotonic) increase or decrease of speed depending on the sign of the v°-parameter (sice the $\hat{B}^{-2}$-terms are always positive definite). In the former case we have the usual drag force caused by motion within the physical medium. In the latter case we have instead a particle penetrating within a highly turbolent medium which causes an increase in the speed.

The important point is that, in the transition from the linear and local transformations (5) to their nonlinear and nonlocal

generalizations (6) we preserve the relativity axioms underlying the Galilean uniform motion.

In fact, recall that our isotopic liftings leave unchanged (by central assumption) the generators, that is, they leave unchanged the existing potential forces. Then, the isotopic liftings $G(3.1) \Rightarrow \hat{G}(3.\hat{1})$ characterize the transition from the original, $G(3.1)$-invariant system of particles moving in vacuum, to the same system moving within physical media which therefore acquires a $\hat{G}(3.1)$-invariance. The infinite number of isotopes $\hat{G}(3.1)$ is then needed for physical consistency in order to represent the infinite variety of possible, different physical media in which the original system can be immersed.

This establishes property a) above, namely, the generalized physical laws for the rectilinear motion.

We now remain with the identification of properties b) and c) above, namely, the form-invariance of the generalized physical laws (6) under $\hat{G}(3.1)$ and their axiomatic equivalence to the Galilean laws (5).

For this purpose, we note that laws (5) of the uniform motion in vacuum are geometrically expressed by the structures

$$T(v^\circ)\, r_i = r_i - t^\circ v^\circ_i, \quad T(v^\circ)\, p_i = p_i - mv^\circ_i, \qquad (7a)$$

$$T(v^\circ) = \exp\,(v^\circ_j\, \omega^{\mu\nu}\, (\partial_\nu G_j)\, (\partial_\mu), \qquad (7b)$$

namely *the structure of the the Galilean law of uniform motion is provided by the right modular (associative) action of the finite Galilei boosts $T(v^\circ)$ on the coordinates and momenta.*

But isotopic laws (6) are geometrically expressed by [9,10]

$$\hat{T}(v^\circ) * r_i = r_i - t^\circ v^\circ_i \hat{B}_i^{-2}, \quad \hat{T}(v^\circ) * p_i = p_i - mv^\circ_i B_i^{-2}, \qquad (8a)$$

$$\hat{T}(v^\circ) = \{\exp v^\circ_j\, \omega^{\mu\sigma}\, 1_2{}^{\sigma\nu}\, (\partial_\nu G_j)\, (\partial_\mu)\}\, 1_2. \qquad (8b)$$

Thus, *the structure of the variable motion within a uniform medium is characterized by the modular-isotopic (associative-isotopic) action of the finite isogalilei* boosts on *coordinates and momenta.*

But the modular action $T(v^\circ)r$ coincides with the modular-isotopic action $\hat{T}(v^\circ)*r$ at the abstract, realization-free level by construction. This shows that *the abstract axioms underlying the Galilean uniform motion are preserved by our covering Galilei-isotopic relativities,* and proves property c) above.

The proof of property b) is trivial and merely follows from the Lie-isotopic group composition law

$$\hat{T}(r^\circ) * \hat{T}(r'^\circ) = \hat{T}(r^\circ + r'^\circ), \qquad (9)$$

Note that the unity of physical and mathematical thought is such that we can introduce only one abstract law of rectilinear motion, say, **T(υ⁰)r**, with infinitely many different, but locally isomorphic realizations $\hat{T}(v^\circ)*r$ representing the infinitely many nonuniform motions within different physical media, and only one canonical realization $T(v^\circ)r$, representing uniform motion in vacuum.

The extension of the above results to other physical laws is straightforward, and is here left to the interested reader for brevity.

Note that, despite their nonlinearity, *all Galilei-isotopic transformations (3) locally coincide with the conventional transformations (1),* i.e., at a given, fixed value $\bar{r}, \bar{p}, \ldots$ of the local variables, we have

$$t^\circ \hat{B}_i^{-2}(\bar{r}, \bar{p}, \ldots) \equiv t'^\circ = \text{cost.}, \qquad r^\circ \hat{B}_i^{-2}(\bar{r}, \bar{p}, \ldots) \equiv r'^\circ, \qquad (10a)$$

$$v^\circ \cdot \hat{B}_i^{-2}(\bar{r}, \bar{p}, \ldots) \equiv v'^\circ = \text{const.}, \qquad \hat{R}(\theta)\big|_{\bar{r}, \bar{p}, \ldots} \equiv R(\theta'). \qquad (10b)$$

A similar situation must then hold for the physical laws, namely, we can state that *the physical laws characterized by our Galilei-isotopic relativities are locally equivalent to the conventional Galilean laws.*

We have reached in this way the most important result of the analysis of these notes, which can be expressed as follows:

THEOREM: All infinitely possible Galilei-isotopic relativities on $\hat{R}_t \times T^*\hat{E}_2(r,G,\hat{R})$ *coincide with the conventional Galilei relativity on* $R_t \times T^*E(r,\delta,R)$ *at the abstract, realization-free level, that is, not only all infinitely possible Galilei-isotopic symmetries* $\hat{G}(3.1)$ *coincide with the conventional Galilei symmetry* $G(3.1)$, *but also the infinite class of Galilei-isotopic transformations (3) coincide with the conventional Galilei transformations (1), and the same holds for the related physical laws.*

The above properties illustrate the ultimate physical and mathematical unity of the Galilei-isotopic relativities with the conventional one, exactly as anticipated earlier from the ultimate abstract unit of the underlying methodological tools [3,5,7].

Despite that, the physical differences between the Galilei-isotopic and the conventional relativity are nontrivial, as we shall illustrate in detail in future works, classically and operationally.

At this moment, let us recall that the Galilei relativity establishes the *equivalence of all inertial frames,* as well known. On the contrary, the Galilei-isotopic relativities establish *equivalence subclasses of noninertial frames, those with respect to the center-of-mass frame of the system,* each class being characterized by each relativity (i.e., by each physical medium). The understanding is that different systems imply different subclasses of isotopically equivalent frames.

To put it different, physical events can occur in the Universe according to a multiply infinite number of noninertial conditions The Galilei-isotopic relativities essentially indicates that all these noninertial frames cannot be reduced to one single Lie-isotopic class of equivalence, but require their classification into infinite subclasses of frames, isotopically equivalent to the observer's frame at rest with the event considered.

But the Galilei-isotopic relativities are coverings of the conventional one. This means that the conventional inertial aspects are not lost, but fully included and actually generalized in the boader Galilei-isotopic setting. This concept can be made more clear via the use of the Corollary of Ref. [10] under which we have the following

COROLLARY: The Galilei-isotopic relativities admit an infinite subclass of linear relativities on $R_t \times T^*\hat{E}_2(r,g,\hat{R})$, *here called "restricted Galilei-isotopic relativities", which are nontrivially* different *than the conventional Galilei's relativity, and are evidently given by constant isometrics* $g > 0$.

In fact, under the assumption of the Corollary, the general Galilei-isotopic transformations (3) assume the particular, manifestly linear form

$$t' = t + t^{\circ}, \quad \text{isotime translations,} \tag{11a}$$

$$r'_i = r_i + r^{\circ}_i\, b_i^{-2}, \quad \text{isospace translations} \tag{11b}$$

$$r'_i = r_i + t^{\circ}\, v^{\circ}_i\, b_i^{-2}, \quad \text{isogalilei boosts,} \tag{11c}$$

$$r' = \hat{R}(\theta) * r = \hat{R}(\theta)\, g\, r, \quad \text{isorotations,} \tag{11d}$$

$$g = \mathrm{diag.}\,(b_1^2, b_2^2, b_3^2), \quad b_k = \mathrm{cost.} > 0. \tag{11e}$$

The lack of equivalence between the restricted Galilei-isotopic relativities and the conventional one is soon illustrated by the fact that the former characterizes *deformable bodies* [11], while the latter characterizes *rigid bodies* [1,2]. For further comments, see Figure 1.

$$\begin{array}{|l|}\hline \text{Closed SA} \\ \text{systems;} \\ \mathfrak{R}_t \times T^*E(r,\delta,\mathfrak{R}) \\ \delta = I, \\ \text{Inertial} \\ \text{frames;} \\ \text{Lie's} \\ \text{theory;} \\ \text{Convent.} \\ \text{Galilei's} \\ \text{relativity;} \\ \hline \end{array} \subset \begin{array}{|l|}\hline \text{Closed NSA} \\ \text{systems;} \\ \mathfrak{R}_t \times T^*\hat{E}(r,g,\hat{\mathfrak{R}}) \\ g = \text{pos. const} \neq I \\ \text{Inertial} \\ \text{frames;} \\ \text{Lie-isotopic} \\ \text{theory;} \\ \text{Restricted} \\ \text{Gali.-isotopic} \\ \text{relativity;} \\ \hline \end{array} \subset \begin{array}{|l|}\hline \text{Closed NSA} \\ \text{systems;} \\ \hat{\mathfrak{R}}_t \times T^*\hat{E}_2(r,G,\hat{\mathfrak{R}}) \\ G = G(r,p,..) > 0, \\ \text{Noninertial} \\ \text{frames;} \\ \text{Lie-isotopic} \\ \text{theory;} \\ \text{General} \\ \text{Gal.-isotopic} \\ \text{relativity.} \\ \hline \end{array} \tag{12}$$

FIGURE 1: A classification of physical systems, with their carrier spaces, observer's frame (assumed at rest with respect to the center-of-mass of the system), and related methodology. The first column depicts the conventional linear-inertial-Hamiltonian setting; the second column depicts the first nontrivial isotopic generalization, that of linear-inertial-Birkhoffian type; and the third column depicts the most general possible nonlinear, nonlocal, nonhamiltonian *and noninertial* setting. The first two columns have equivalent inertial characterizations, because they are both defined on inertial frames. However, the first column treats rigid bodies, while the second represents deformable bodies. The third column represents instead the most general possible

conditions of extended-deformable bodies in regard to both the existing forces and the observer frames.

The most important examples of Galilei-isotopic systems are the two-body and three-body, closed nonselfadjoint systems outlined in Ref. [7]. The two-body case implies the simplest conceivable isotopy, the *scalar isotopy* of the canonical tensor, Eq. (8), ref. [7], i.e.,

$$\omega^{\mu\nu} \Rightarrow \Omega^{\mu\nu} = b^2\, \omega^{\mu\nu}, \quad b = \mathrm{const.} > 0. \tag{13}$$

Despite this simplicity, the acceleration-dependent forces which originate the lifting, imply such profound modifications of the conventional two-body bound states to permit a quantitative representation of *all* the intrinsic characteristics of the π^0 as a *"compressed positronium"*, i.e., as our generalized state of one ordinary electron and one ordinary positron isotopically bounded one inside the other at mutual distances of the order of 1F (i.e., at mutual distances smaller than the size of their wavepackets, thus activating our nonselfadjoint forces) [see the calculations of Sect. 5 of Ref.[12]). In turn, this offers realistic possibilities of identifying the π^0 constituents with ordinary particles freely produced in the spontaneous decays, as previously established for the nuclear and atomic cases. By comparison, such a model is fundamentally inconsistent within the context of the conventional two-body Galilean system, as the interested reader is encouraged to verify (see the various reasons identified in ref. [12].

For the three-body closed nonselfadjoint system in stable configurations (i.e., the straight line and triangular configurations of Ref. [7]), we also have a scalar isotopy of type (12), and merely in a higher number of dimension.

But, again, the physical implications are, by far, nontrivial. In fact, the Galilei-isotopic relativities permit a central nucleus with mass much smaller than that of the peripheral constituents, the so-called *isonucleus* of Fig. 2, Ref. [7]. In turn, such a structure (once implemented with our covering isorotational symmetry $\hat{SO}(2)$ for the spin) offers realistic hopes of achieving a quantitatively consistent representation of Rutherford's [13] historical conception of the neutron as a *"compressed hydrogen atom"*, that is, as an electron totally compressed (say, in a supernova explosion) in the center of the proton, thus acquiring our configuration of isonucleus (see ref. [14] for preliminary operator studies). In turn, if such a model is proved consistent in due time, it could allow the identification of the $\bar{d}$-quark with Rutherford's electron which, as such, can be freely produced in the spontaneous decays.

A particularly intriguing example of Galilei-isotopic system is given by the three-body systems in *Jannussis' configuration* [15] (see also Eqs. (15) of Ref. [7]), that with genuine subsidiary constraints, which we regret to be unable to study at this time for brevity.

We also regret to be unable to study at this time closed nonselfadjoint systems with a large number of constituents N. In fact, for a sufficiently high value of N, the systems acquire intriguing statistical aspects (e.g., an internal irreversibility fully compatible with the center-of-mass revisibility) with a number of expected connection with *Prigogine's statistics* [16]. Moreover, we have intriguing connections with Mach's principle originating in the interior acceleration-dependent forces (prior any Riemannian structure) [17] In the final analysis, the systems under consideration constitute our *Galilei-isotopic structure model of Jupiter*, as pointed out in our introductory words [5]. This completes our comments on many-particles Galilei-isotopic systems.

Explicit examples of individual Galilei-isotopic particles are given in the subsequent note of this series. A more detailed and comprehensive presentation of the studies of these notes can be found in ref. [18].

We cannot close this note without a few comments regarding the virtual complete restriction of the physical literature of this century to inertial frames.

On epistemological grounds, it should recalled here that *inertial frames are a philosophical abstraction, because they do not exist in our Earthly environment, nor can they be attained in our Planetary or Galactic systems.* The Galilei-isotopic relativities are therefore intended to identify the equivalence class of each *actual frame*, that is, of each *noninertial frame.*

While inertial frames are now part of the history of physics and, as such, they must be certainly preserved in any future development, one should be truly aware of the rather serious implications for any restriction of physical treatments to inertial frames only. In fact, such a restriction implies the necessary

linearity (as well as locality and selfadjoint character) of the theory. A restriction of this type

♣ *classically*, implies the implicit acceptance of the perpetual motion in a physical environment, and

♥ *operationally*, it could be the ultimate origin of the fundamental open problems of contemporary physics, such as our insufficient knowledge in the highly noninertial processes at the instant of initiation of the fusion process, or the inability to understand the equally noninertial (acceleration dependent [17]) processes in the hadronic structure which could permit the identification of the constituents with ordinary massive particles freely produced in the spontaneous decays

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