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WITH NONHAMILTONIAN INTERNAL FORCES**

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CLOSED SYSTEMS WITH NONHAMILTONIAN INTERNAL FORCES

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ABSTRACT

In a preceding paper, we have treated the inequivalence between the exterior (local, potential and selfadjoint) problem, and the interior (nonlocal, nonhamiltonian and nonselfadjoint) problem. In this note, we treat their compatibility via the notion of closed nonselfadjoint systems, i.e. systems which verify all conventional total conservation laws when isolated from the rest of the Universe; yet their interior structure is nonlinear, nonlocal and nonhamiltonian. The generalized analytic and geometrical formulations needed for their treatment are identified, jointly with their direct universality for the case of nonlinear and nonhamiltonian internal forces in local approximation. This allows the technical identification of the open problems for subsequent consideration.

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It is generally assumed that the global stability of a composite system is due to the stability of the individual orbits of its constituents, as is the case for the planetary and atomic structures. These systems are *closed* in the sense that they verify the ten total, Galilean, conservation laws (when isolated from the rest of the Universe), and are *variationally selfadjoint* (SA), in the sense that the internal forces verify the integrability conditions for the existence of a potential [1] (for a comprehensive treatment see also refs.[2,3]).

By no means, these systems exhaust all possible composite systems in our Universe. Another class is provided by *closed nonselfadjoint systems* [4]. These are systems which verify all total, conventional conservation laws (closedness) as it is the case for any isolated system; nevertheless, their internal forces are *variationally nonselfadjoint* (NSA), i.e., they violate the integrability conditions for the existence of a potential [1,2,3]

The latter systems are considerably more complex than the former. In fact, global stability is achieved, this time, via a collection of orbits each of which is generally unstable. We merely have internal local exchanges of energy, linear momentum and other physical quantities, but in such a way to verify total conservation laws.

An illustration of the latter systems is provided by an individual member of the Solar system, such as Jupiter. As one can see, its global stability is evident. Equally evident is the instability of the orbits of its individual constituents. In fact, direct visual observation via telescopes establishes that Jupiter's total angular momentum is evidently conserved, but its internal structure is characterized by particles with monotonically nonconserved (i.e., continuously decreasing or increasing) angular momenta, and a similar situation occurs for all other quantities.

At a deeper analysis, one can see that Jupiter's interior dynamical equations violate the integrability conditions not only for the existence of a potential, but also for the existence of a (conventional) Lagrangian or Hamiltonian representation in the frame of the observer. The inapplicability of the conventional canonical realizations of Lie's theory and of the symplectic geometry then follows.

This establishes the starting grounds for this series of studies: the inapplicability of conventional space-time symmetries and relativities for the interior dynamical problem of closed systems with nonlinear, nonlocal and nonhamiltonian internal forces.

Following proposal [4], the generalized analytic, algebraic and geometrical formulations for closed nonselfadjoint systems were studied in ref. [3], evidently as a basis for possible, relativistic, gravitational and operator extensions. The systems were then studied from a statistical viewpoint in refs. [5,6] with rather intriguing implications, e.g., the possibility of introducing a new notion of internal irreversibility which is compatible with a reversible exterior dynamics, exactly as occurring for Jupiter. Specific classical examples of generalized two- and three-body systems were studied in ref. [7] jointly with their generalized analytic formulations and symmetries.

The predictable (and rather intriguing) connections with Prigogine's statistics [8], which have remained unexplored as of today, are scheduled for subsequent works.

The implications of closed nonselfadjoint systems are non-trivial, mathematically and physically.

From a mathematical viewpoint, the systems considered require the construction of covering analytic, algebraic and geometric formulations [2], besides implying a host of intriguing and fundamental, open mathematical problems (such as the achievement of global topological stability via local instabilities).

From a physical viewpoint, the implications of closed nonselfadjoint systems are equally deep, inasmuch as they imply a necessary generalization of conventional relativities at all levels of study, Newtonian, relativistic and gravitational, as well as classical and operational. In fact, the insistence in the validity of conventional relativities for the interior of closed nonselfadjoint systems would imply truly excessive approximations, such as the acceptance of the perpetual motion in a physical environment, without any possibility of its consistent reduction to conservative settings [9].

All studies [1-4] were arbitrarily nonlinear, and therefore nonlagrangian-nonhamiltonian by conception. Nevertheless, they were based on a local approximation of the interior nonselfadjoint forces because of the local-differential character of the underlying symplectic geometry.

An objective of these studies is to extend the above results to a full nonlocal treatment, as needed to identify the classical foundations of the historical open legacy of the ultimate nonlocality of the strong interactions. For this purpose, we shall first review and expand in this note the main lines of the local-

differential treatment of closed nonselfadjoint systems. Their extension to nonlocal-integral formulations will be studied in the subsequent notes of this series.

Let us begin with a representation of closed selfadjoint systems as vector-fields on a manifold. Let $E(r, \delta, \mathfrak{R})$ be the conventional Euclidean space in three-dimension where $r = (r_k)$, $k = 1, 2, 3$ ($= x, y, z$) are the physical coordinates of the experimenter and the metric is given by the familiar form $\delta = \text{diag.}(1, 1, 1)$ over the reals \mathfrak{R} . Introduce in $E(r, \delta, \mathfrak{R})$ a system of N particles denoted with the symbol $a = 1, 2, \dots, N$. Let $T^*E(r, \delta, \mathfrak{R})$ be the cotangent bundle (the conventional phase space) with local chart (coordinates) $a = (a^\mu) = (r, p) = (r_{ka}, p_{ka})$, $\mu = 1, 2, \dots, 6N$, where the p 's are the physical linear momenta, i.e., $p_{ka} = m_a v_{ka}$, $v_{ka} = \dot{r}_{ka} = dr_{ka} / dt$. For simplicity of notation, all indices of the coordinates and momenta will be treated as subindices while the distinction between covariant and contravariant indices will be kept in $T^*E(r, \delta, \mathfrak{R})$. Then, *closed selfadjoint systems* can be defined as the *Hamiltonian vector-field*

$$(\dot{a}^\mu) = \begin{pmatrix} \dot{r}_{ka} \\ \dot{p}_{ka} \end{pmatrix} = (\Xi^\mu(a)) = \begin{pmatrix} p_{ka} / m_k \\ \int_{k_a}^{S_A} (r) \end{pmatrix}, \quad (1a)$$

$$\dot{X}_i(t, a) = \frac{\partial X_i}{\partial a^\mu} \dot{a}^\mu - \frac{\partial X_i}{\partial t} = 0, \quad (1b)$$

$$k = 1, 2, 3, \quad a = 1, 2, 3, \dots, N, \quad \mu = 1, 2, \dots, 6N$$

where the X 's represent the familiar, total, Galilean, conserved quantities

$$X_1 = H = T(p) + V(r), \quad (2a)$$

$$(X_2, X_3, X_4) = (P_k) = \sum_a p_{ka}, \quad (2b)$$

$$(X_5, X_6, X_7) = (M_k) = \sum_a r_{ka} \wedge p_{ka}, \quad (2c)$$

$$(X_g, X_g, X_{10}) = (G_k) = \sum_a (m_a r_{ka} - t p_{ka}), \quad (2d)$$

The more general *closed nonselfadjoint systems* in their regular, local and analytic formulation are the nonhamiltonian *vector-fields* on $T^*E(t, \delta, \mathfrak{R})$

$$\begin{aligned} (\dot{a}^\mu) &= \begin{pmatrix} \dot{r}^{ka} \\ \dot{p}_{ka} \end{pmatrix} = (\Gamma^\mu(t, a)) = \\ & \begin{pmatrix} f_{ka}^{SA}(r) + F_{ka}^{SA}(t, r, p) + F_{ka}^{NSA}(t, r, p, \dot{p}, \dots) \\ \partial X_i / \partial a^\mu - \partial X_i / \partial t \end{pmatrix}, \quad (3a) \\ \dot{X}_i(t, a) &= \frac{\partial X_i}{\partial a^\mu} \dot{a}^\mu - \frac{\partial X_i}{\partial t} = 0, \quad (3b) \end{aligned}$$

$\mu = 2, 1, \dots, 6N$, $\alpha = 1, 2, \dots, N$, $K = x, y, z$, $i = 1, 2, \dots, 10$, where the X 's are exactly the same as in Eqs. (2), and F^{SA} represents internal forces that are still of potential type, but Galilei-noninvariant.

Systems (3) constitute the physical foundations of these studies. They provide a primitive, classical and nonrelativistic representation of the structure of Jupiter, as well of our proposed structure of hadrons [4].

Note that, while systems (1) are unconstrained, Eqs. (3) characterize a system with subsidiary constraints, in the sense that conservation laws (3b) are now, in general, subsidiary constraints to vector-fields (3a).

Finally, while systems (1a) are Galilei-invariant, a necessary condition for the existence of closed nonselfadjoint systems (3) is that they are *not invariant under the conventional Galilei symmetry* [3]. This is evident on numerous counts, e.g., the necessary nonhamiltonian character of the vector-fields, their arbitrary nonlinearity in the coordinates and velocities, the inapplicability of canonical formulations by assumption, etc.

It is easy to see that, under the assumed topological conditions (regularity, locality and analyticity), systems (3) are consistent because they are under-determined.

Moreover, it is possible to show that *systems (3) admit unconstrained solutions in the nonselfadjoint forces for given*

Galilean selfadjoint forces. In fact, by ignoring hereon the selfadjoint Galilei-noninvariant forces (or by absorbing them in the nonselfadjoint ones), after simple calculations (see ref. [3] p. 236), subsidiary constraints (3b) can be reduced to the following seven conditions in the nonselfadjoint forces

$$\sum_a F_a^{NSA} = 0, \quad \sum_a p_a \times F_a^{NSA} = 0, \quad \sum_a r_a \wedge F_a^{NSA} = 0, \quad (4)$$

Unconstrained solution in the nonselfadjoint forces therefore always exist for $N > 1$, including the case $N = 2$, as we shall see in the subsequent note of this series. The case $N = 1$ is impossible because one isolated particle is free and cannot experience nonselfadjoint forces.

We now outline the methodological tools used in refs. [1-4] for the treatment of closed nonselfadjoint systems.

ANALYTIC FORMULATIONS. A step-by-step generalization of Hamiltonian mechanics under the name of *Birkhoffian mechanics* was conceived for systems (3) [1] and subsequently worked out in detail in ref. [3]. In particular, the new mechanics was proven to be *directly universal* for (regular, local and analytic) Newtonian systems [3], namely, a representation of all systems considered always exists (universality) directly in the a -coordinates of the experimenter (direct universality).

The analytic representation occurs via the following first-order *Pfaffian variational principle*

$$\delta \dot{A} = \delta \int_{t_1}^{t_2} dt [R_\mu(a) \dot{a}^\mu - B(t,a)] = 0, \quad \mu = 1, 2, \dots, 6N, \quad (5)$$

where the R_μ and B functions are computed from the given equations (3a) via one of the several techniques of ref. [3], and $B(t,a) = B(t,r,p)$ is called the *Birkhoffian*, because it is generally different than the total energy. Principle (5) characterizes the *covariant Birkhoff equations*

$$\Omega_{\mu\nu}(a) \dot{a}^\nu = \partial_\mu B(t,a), \quad (6)$$

where

$$\Omega_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu \quad (7)$$

called the *covariant Birkhoff's tensor*, is nowhere degenerate in the considered (star-shaped) region of the local variables.

The *contravariant Birkhoff's equations* are evidently given by

$$\dot{a}^\mu = \Omega^{\mu\nu}(a) \partial_\nu B(t,a), \quad (8)$$

where

$$\Omega^{\mu\nu} = (|\Omega_{\alpha\beta}|^{-1})^{\mu\nu}, \quad (9)$$

is the *contravariant Birkhoff's tensor*.

Pfaffian principle (5) also implies the following *Birkhoffian generalization of the Hamilton-Jacobi equations*

$$\partial_t \dot{A} + B(t,a) = 0, \quad \partial_\mu \dot{A} = R_\mu(a), \quad (10)$$

which have a predictably crucial role for the operator formulation of systems (3), as we shall see.

The rest of the Birkhoffian generalization of Hamiltonian mechanics follows. The reader interested in these studies is urged to acquire a technical knowledge of Birkhoffian mechanics [3], because numerous aspects will be tacitly assumed as known, some of which are rather insidious.

For instance, the computation of the R_μ and B functions from the equations of motion generally yields *nonautonomous representations*, i.e., one with $R_\mu = R_\mu(t,a)$ and $B = B(t,a)$ (see the examples of ref. [3]). This implies the still more general *nonautonomous Birkhoff's equations*, which are not considered in these studies because they violate the Lie algebra axioms in favor of the more general Lie-admissible axioms [3].

Nevertheless, all nonautonomous representations can be reduced to the *semiautonomous form*, i.e., that with $R_\mu = R_\mu(a)$, $B = B(t,a)$ of Eq.s (6) and (8) via the degrees of freedom of the theory, e.g., the so-called *Birkhoffian gauge transformations*

$$R'_\mu(a) = R_\mu(t,a) + \partial_\mu G(t,a), \quad B'(t,a) = B(t,a) + \partial_t G(t,a). \quad (11)$$

For numerous additional aspects we are regrettably forced to refer the interested reader to ref. [3].

Birkhoffian mechanics is a covering of Hamiltonian mechanics in the sense that: 1) the former mechanics is based on formulations structurally more general than those of the latter; 2) the former mechanics represents physical conditions structurally more general than those of the latter; and 3) the former mechanics admits the latter as a particular case for $R = R^0 = (p,0)$.

In fact, under values $R = R^0 = (p,0)$, the covariant Birkhoff's tensor (7) assumes the familiar canonical form

$$(\omega_{\mu\nu}) = \begin{pmatrix} \partial_\mu R^0_\nu & -\partial_\nu R^0_\mu \\ 0_{3n \times 3n} & -I_{3n \times 3n} \\ I_{3n \times 3n} & 0_{3n \times 3n} \end{pmatrix} \quad (12a)$$

$$R^0 = (p, 0) = (p_{ka}, 0_{ka}), \quad (12b)$$

with contravariant form

$$(\omega^{\mu\nu}) = (|\omega_{\alpha\beta}|^{-1}) = \begin{pmatrix} 0_{3n \times 3n} & I_{3n \times 3n} \\ -I_{3n \times 3n} & 0_{3n \times 3n} \end{pmatrix} \quad (13)$$

Birkhoff's equations then recover Hamilton's equations identically

$$\omega_{\mu\nu} \dot{a}^\mu = \partial_\mu H(t,a), \quad \dot{a}^\mu = \omega^{\mu\nu} \partial_\nu H(t,a), \quad H \equiv B. \quad (14)$$

We finally recall that the various aspects of Birkhoffian mechanics can be constructed via a judicious use of *noncanonical* transformations of the corresponding aspects of Hamiltonian mechanics. As an example, Birkhoff's equations can be constructed via noncanonical transformations of Hamilton's equations

$$l \rightarrow l' \equiv l, \quad a'' \rightarrow a'''(a),$$

$$\left[\omega_{\mu\nu} \dot{q}^\nu - \frac{\partial H(t, q)}{\partial \dot{q}^\mu} \right]_{SA} \equiv \left\{ \left[\frac{\partial R_\nu^0(a)}{\partial \dot{q}^\mu} - \frac{\partial R_\mu^0(a)}{\partial \dot{q}^\nu} \right] \dot{q}^\nu - \frac{\partial H(t, q)}{\partial \dot{q}^\mu} \right\}_{SA} \quad (15)$$

$$= \left\{ \frac{\partial a''}{\partial \dot{q}^\mu} \left\{ \left[\frac{\partial R'_\nu(a')}{\partial \dot{q}^\mu} - \frac{\partial R'_\mu(a')}{\partial \dot{q}^\nu} \right] \dot{q}^\nu - \frac{\partial B'(t, a')}{\partial \dot{q}^\mu} \right\}_{SA} \right\}_{NSA} = 0,$$

$$(R_\mu^0) = (p, 0), \quad R'_\mu(a') = \left(\frac{\partial a''}{\partial \dot{q}^\mu} R_a^0 \right)(a'), \quad B'(t, a') = H(t, a(a'))$$

and the same occurs for variational principles, Hamilton-Jacobi theory, etc. As a result, *Birkhoff's equations preserve their form under the most general possible transformations*, trivially, because they already have the most general possible form.

These features are important for the subsequent studies of this series. As an example, our generalization of Galilei's symmetry G(3.1) for systems [3], submitted in ref. [3] under the name of *Galilei-isotopic symmetry* G(3.1), can be constructed via noncanonical transformation of G(3.1).

Similarly, the operator image of Birkhoffian mechanics, tentatively submitted in ref. [4] under the name of *hadronic mechanics*, can be expectedly built via nonunitary transformations of conventional quantum mechanics, and then preserve its structure under the most general possible transformations.

ALGEBRAIC FORMULATIONS. The construction of a step-by-step generalization of the conventional formulation of Lie's theory [10,11,12] under the name of *Lie-isotopic theory* was suggested in ref. [1] as the algebras underlying Birkhoffian mechanics. The studies were then reviewed and expanded in ref. [3], with the understanding that they are still at the beginning and so much remains to be done.

The central idea is the generalization of the trivial unit 1 of current use in both mathematical and physical formulations into the most general possible unit $\hat{1}$, called *isotopic unit* which is nonsingular, Hermitian and positive-definite, but possesses an otherwise arbitrary dependence on all local variables and quantities [1,3]

$$\hat{1} = \hat{1}(t, a, \dots). \quad (16)$$

The generalization of the unit then implies a corresponding generalization of all major structural aspects of Lie's theory. In fact, consider the conventional *universal enveloping associative algebra* A [12] with elements a, b, c, \dots and trivial associative product ab over a field F (hereinafter assumed of characteristic zero),

$$A : \quad ab = \text{assoc.}, \quad |a = a| = a \quad \forall a \in A. \quad (17)$$

Then, for $\hat{1}$ to remain the (left and right) unit of the theory, the algebra A has to be generalized into the algebra \hat{A} , called *associative-isotopic algebra* (or *isoassociative algebra*), which is the same vector space as A , but equipped with the new product [1]

$$\hat{A} : \quad a*b = aTb = \text{assoc.}, \quad T = \text{fixed}, \quad (18a)$$

$$\hat{1}*a = a*a' = a \quad \forall a \in \hat{A}, \quad \hat{1} = T^{-1}. \quad (18b)$$

In turn, the lifting $A \Rightarrow \hat{A}$ implies a corresponding generalization of all structural theorems on envelopes (the Poincaré-Birkhoff-Witt Theorem on the infinite basis, etc. [1,3]).

Similarly, Lie algebras L are currently conceived as the antisymmetric algebra attached to A , i.e., $L \approx A^-$, with the simplest conceivable realization of the Lie product

$$L : \quad [a, b]_A = ab - ba. \quad (19)$$

Under the lifting $l \Rightarrow \hat{l}$ the Lie algebras must be defined as the antisymmetric algebras \hat{L} attached to \hat{A} , i.e., $\hat{L} \approx \hat{A}^-$, called *Lie-isotopic algebras* [1,3], in which case the product assumes the less trivial form

$$\hat{L} : \quad [a, b]_{\hat{A}} = a*b - b*a = aTb - bTa. \quad (20)$$

The lifting $L \Rightarrow \hat{L}$ then implies a generalization of all structural theorems of Lie's algebras, such as the celebrated Lie's First, Second and Third Theorems, etc. [1,3].

Finally, conventional connected Lie groups G are defined via the exponentiation in A (i.e., with respect to l), as permitted by the Poincaré-Birkhoff's-Witt Theorem

$$G : g(w) = \exp_A(iwX) \\ = 1 + iwX / 1! + (iwX)^2 / 2! + \dots \quad (21)$$

where $w \in F$ represents the parameters and the X 's are the (Hermitian) generators, and verify the familiar group laws

$$g(0) = 1, \quad g(w)g(w') = g(w+w'), \quad g(w)g(-w) = 1, \quad (22)$$

where the composition is evidently associative.

The lifting $1 \Rightarrow \hat{1}$ implies a necessary generalization of structure (21) into a form \hat{G} , called *Lie-isotopic group* [1,3], with exponentiation now in \hat{A} (i.e., with respect to $\hat{1}$) which is permitted by the isotopic generalization of the Poincaré-Birkhoff-Witt Theorem [1,3]

$$\hat{G} : \hat{g}(w) = \exp_{\hat{A}}(iwX) \\ = \hat{1} + (iwX) / 1! + (iwX)^2 / 2! + \dots \\ = \hat{1} (\exp_A(iwTX)) = (\exp_A(XTW)) \hat{1} \quad (23)$$

with *Lie-isotopic group laws*

$$\hat{g}(0) = \hat{1}, \quad \hat{g}(w)\hat{g}(w') = \hat{g}(w+w'), \quad \hat{g}(w)\hat{g}(-w) = \hat{1}, \quad (24)$$

The need for a compatible generalization of the structural theorems on topological groups then follows.

The name *'isotopy'* was suggested in ref. [1] (on the basis of certain historical accounts we are regrettably forced to ignore here for brevity) to emphasize the fact that, by construction, \hat{A} remains associative, \hat{L} remains a Lie algebra, and \hat{G} remains a Lie group, as one can verify.

As a matter of fact, by construction, all distinctions between the conventional Lie structures $A, L,$ and G and their isotopic generalizations $\hat{A}, \hat{L},$ and \hat{G} cease to exist at the abstract, realization-free level.

Stated differently, the Lie-isotopic theory preserves all the basic axioms of the conventional Lie's theory, and simply realizes them in their most general possible form. This is the algebraic counterpart of the abstract, analytic unification of Birkhoffian

and Hamiltonian mechanics we shall discuss shortly.

The proof that Birkhoffian mechanics possesses a Lie-isotopic structure was conducted in ref. [1] by showing that the isotopic generalization of Lie's First, Second and Third Theorems provide a direct characterization of Birkhoff's equations in exactly the same way as the conventional Lie's Theorems provided the historical, direct characterization of the (conventional) Hamilton's equations [10].

For the limited scope of this note, it is sufficient to recall that the classical realization of the conventional Lie's theory is characterized by the conventional Poisson brackets among functions A and B in $T^*E(r,\delta,\mathfrak{F})$

$$[A, B] = (\partial_\mu A) \omega^{\mu\nu} (\partial_\nu B) \quad (25)$$

and canonical group structure

$$G: g(w) = \exp_A(w \omega^{\mu\nu} (\partial_\nu X) (\partial_\mu)) \quad (26)$$

In the transition to Birkhoffian mechanics, the brackets underlying Eqs. (8) are given instead by the most general possible (regular, unconstrained) brackets verifying the Lie algebra axioms, the *Birkhoff's brackets* (also called *generalized Poisson brackets* [11])

$$[A, B] = (\partial_\mu A) \Omega^{\mu\nu}(a) (\partial_\nu B), \quad (27)$$

(see the analytic, algebraic and geometric proofs of ref. [3]). The group structure is then given by

$$\hat{G} : \hat{g}(w) = \exp_A(w \Omega^{\mu\nu} (\partial_\nu X) (\partial_\mu)), \quad (28)$$

It is evident that the liftings $(A,B) \Rightarrow (\hat{A}, \hat{B})$ and $g(w) \Rightarrow \hat{g}(w)$ are isotopic.

GEOMETRICAL FORMULATIONS They are given by the *conventional symplectic geometry* [13] in its exact but most general possible form.

In fact, the Birkhoffian functions R_μ characterize the most

general possible one-form in a local chart,

$$\hat{\theta}_1 = R_{\mu}(a) da^{\mu}, \quad (29)$$

Then, Birkhoff's tensor (7) characterizes the most general possible exact symplectic two-form in a local chart

$$\hat{\Omega}_2 = d\hat{\theta}_1 = \Omega^{\mu\nu}(a) da^{\mu} \wedge da^{\nu}, \quad (30)$$

As a result, Eq.s (29) can be interpreted as being *Birkhoffian vector-fields* [3], in the sense that

$$\Gamma \lrcorner \hat{\Omega}_2 = 0. \quad (31)$$

The most general possible exact formulation the symplectic geometry is then applicable for the characterization of closed nonselfadjoint systems. In particular, the symplectic geometry was proved to be directly universal for nonlinear but local and analytic Newtonian systems. For further details, see ref. [3].

Under the restriction $R = R^0 = (p,0)$, one obtains the canonical formulation of the symplectic geometry, i.e., Eq. (29) becomes the canonical one-form, Eq. (30) becomes the canonical two-form, and Eq. (31) becomes a conventional Hamiltonian vector-field.

In conclusion, *the conventional symplectic geometry, in its most general possible exact formulation, is the geometry underlying the Lie-isotopic algebras and the Birkhoffian mechanics.*

A major difference however emerges between the Lie-isotopic algebras and the symplectic geometry. In essence, the Lie-isotopic algebras and groups are insensitive to the topology of their (positive-definite) units 1. As a result, they can represent nonlocal (integrals) interactions in a topologically consistent way, provided that they are all incorporated in unit 1.

On the contrary, the symplectic geometry is strictly local-differential, as well known [13]. As such, it cannot incorporate nonlocal-integral interactions without serious topological inconsistencies. All studies of refs. [1-4] were based on the conventional symplectic geometry and, as such, they treated closed nonselfadjoint systems in their nonlinear, but local

approximation.

However, a central objective of these studies is to attempt a quantitative treatment of the historical legacy on the ultimate interior nonlocality of the strong interactions (in a way compatible with the local-differential character of their center-of-mass).

This creates the problem of identifying a suitable reformulation of the symplectic geometry which does indeed admit a full nonlinear, nonlocal and nonhamiltonian treatment of systems (3) in a way compatible with that permitted by the complementary Lie-isotopic theory. This problem will be studied in the subsequent notes of this series.

The primary results of the above studies are therefore the following [3]:

- 1) In the transition from closed selfadjoint to closed nonselfadjoint systems there is no need to abandon conventional analytic, algebraic and geometric formulations, because
 - 1a) both systems are derivable from a first-order variational principle;
 - 1b) the contravariant algebraic tensor of both systems is Lie; and
 - 1c) the covariant geometric tensor of both systems is symplectic.
- 2) closed nonselfadjoint systems emerge in their Birkhoffian representation when one assumes the most general possible realization of the above structures, while closed selfadjoint systems in their Hamiltonian representation emerge when one assumes the simplest possible (canonical) realization of the same structure. And

- 3) All distinctions between Birkhoffian and Hamiltonian formulations (and, thus, between closed nonselfadjoint and selfadjoint systems) cease to exist at the abstract, realization-free level. This property is evident within the context of the symplectic geometry, where there is no geometric distinction between Hamiltonian and Birkhoffian vector-fields, but it equally holds at the algebraic and analytic levels (see ref. [3] for technical details).

The above properties are sufficient to anticipate our primary results or, equivalently, to provide advance guidelines for their achievement.

In fact, Properties 1, 2 and 3 above require that the space-time symmetries of closed nonselfadjoint systems must be constructed in such a way to be locally isomorphic to the conventional space-

time symmetries, as a necessary condition for their identity at the abstract, realization-free level, in a way compatible with the above abstract identity between closed selfadjoint and nonselfadjoint systems and their methodologies.

Similarly, for the case of closed selfadjoint systems, the total conservation laws are not subsidiary constraints, but first integrals originating from their Galilean symmetry $G(3,1)$. From property 3) above, we must therefore expect that the same Galilean total conservation laws follow from the invariance of closed nonselfadjoint systems, this time, under our covering Galilei-isotopic symmetries $\hat{G}(3,1)$ [1,3].

In fact, by central condition, the Lie-isotopic symmetries preserve the parameters and generators of the conventional symmetries [1,3]. As a result, the ten generators of our Galilei-isotopic symmetry $\hat{G}(3,1)$ are expected to be precisely the ten conventional conserved quantities of $G(3,1)$ -invariant closed nonselfadjoint systems.

Another central objective of this series of papers is therefore that of identifying the unconstrained subclass of systems (3), that is characterized by our Galilei-isotopic symmetry.

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