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ABSTRACT

In this note, we review and expand the current knowledge on two-body and three-body systems with action-at-a-distance, potential (selfadjoint) forces, as well as contact, nonlinear and nonhamiltonian (nonselfadjoint) internal forces. The stable configuration of the two-body system is reviewed, and two new stable configurations for the three-body system are introduced, one along a straight line (nonselfadjoint extension of the restricted three-body problem) and one along a triangle (nonselfadjoint extension of Lagrange's historical triangle) in preparation for their operator treatment.

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In the preceding note [1] we have reviewed the definition of closed-isolated systems with potential (selfadjoint) and nonhamiltonian (nonselfadjoint) internal forces, called *closed nonselfadjoint systems*, and we have outlined the generalized analytic, algebraic and geometrical methods for their treatment.

In this note we shall study the emerging generalization of Kepler's two-body and three-body systems.

The reader should be aware that the (classical) notion of closed nonselfadjoint systems is centered in the existence of an interior medium which is responsible for the contact nonhamiltonian interactions. In turn, such medium is classically created by a large number of constituents N, as in Jupiter's structure.

In this note we shall study closed nonselfadjoint systems in their smallest possible number of constituents N = 2 and 3, in which case the interior medium is evidently absent. The internal forces are in this case merely expressed by the condition of contact interaction among the constituents, that is, the (extended) constituents must be in physical contact among each other as a necessary condition to have two- and three-body closed nonselfadjoint systems.

Whenever such a physical contact is removed, and the constituents move freely in space, the systems considered reacquire their Keplerian, selfadjoint structure.

It is intriguing to anticipate since now that, in the transition to an operator treatment, the interior medium exists also for the case of two- and three-bodies closed nonselfadjoint systems. In fact, in these cases each constituent is an extended wavepacket moving *within* the wavepackets of the remaining constituents.

In different terms, at the classical level of this note, the constituents can indeed be in mutual physical contact, but evidently without mutual penetration. This results in the lack of underlying medium and, therefore, in special forms of the nonselfadjointy forces. In the particle case, instead, we do have indeed total mutual penetration of the wavepackets of the constituents, in which case each constituent is the medium of the others.

In conclusion, the analysis of this note should be essentially considered a rudimentary classical and nonrelativistic basis for future operator research.

TWO-BODY CLOSED NONSELFADJOINT SYSTEMS. Two-body closed nonselfadjoint systems were first studied in the original proposal [2], which also identified their stable configuration. The systems were then studied in ref. [3]. The first Birkhoffian representation of the systems was reached by Jannussis and collaborators [4].

For the case of two particles, motion is necessarily in a plane, say k = 1,2 (=x,y), and systems (3) of ref. [1] become

$$M\ddot{R} = 0, \qquad (1)$$
$$m\ddot{r} = f^{SA}(r) + F^{NSA}(r, \dot{r}, \ddot{r}),$$

where

$$M = m_1 + m_2,$$
$$m = \frac{m_1 m_2}{m_1 + m_2}, \qquad (2)$$
$$R = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} \text{ and } r = r_1 - r_2 ,$$

Conditions (4) of ref. [1] on the NSA forces then become

$$F_1^{NSA} = -F_2^{NSA} \stackrel{\text{def}}{=} F^{NSA}, \qquad (3)$$
$$\dot{r} \times F^{NSA} = 0,$$
$$r \wedge F^{NSA} = 0.$$

with general solution

$$F^{NSA} = g[r_1^{(2n)} - r_2^{(2n)}], \qquad (4)$$
$$n = 1, 2, \ldots,$$

where g = cost, and $r^{(2n)}$ represents the 2n-th derivative. For n = 1, the only admissible stable orbit is then the circle, in which case the nonselfadjoint force assumes form (3.4.11) of ref. [2], i.e.

$$F^{NSA} = g\ddot{r} \qquad (5)$$

with equations of motion (3.4.12) ref. (22), which we can write

$$\hat{m}_1 = -k(r_1 - r_2) / |r_1 - r_2|^3, \qquad (6a)$$

$$\hat{m}_2 = +k(r_1 - r_2) / |r_1 - r_2|^3, \qquad (6b)$$

$$\hat{m}_1 = b^2 m_1, \ \hat{m}_2 = b^2 m_2, \ b^2 = (m - g) / m > 0, \ g < m, \qquad (6c)$$

namely, *two-body closed nonselfadjoint systems essentially provide a renormalization of the masses of the conventional two-body Kepler's system.* What is remarkable is that this occurs already at the classical nonrelativistic level of this note. [The algebraic reasons for this alteration of the mass will however be clear only after studying, later on, the generalized notion of particle characterized by our Galilei-isotopic relativities].

The reader is warned against the appraisals of the above results via old notions that are inapplicable to the physical conditions considered. Specifically, if one keeps thinking at dimensionless points, the above results are evidently inconsistent because elliptic orbits are admissible too, as is well known.

However, the systems under consideration represent, by central assumption, *extended particles under mutual contact interactions,* such as two spheres in mutual contact (or, operationally, two extended wavepackets moving one inside the other). It is then evident that two spheres (or, for that matter, two extended objects of arbitrary shape) can rotate one with respect to the other under mutual contact only in a circle (and exactly the same situation is expected for wavepackets, as shown in ref. [2]).

Stated differently, one can indeed think at point-like particles for systems (1), with the understanding that such a conception directly implies the loss of the nonselfadjoint forces, with consequential recovering of elliptic trajectories (see Fig. 1 for more details).

Finally, the emergence of an acceleration-dependent force should not be dismissed lightly, because it appears to have rather intriguing (and basically unexplored) connection with MacgUs Principle, as well as with Ampère-Neumann electrodynamics (see in this respect ref.m [5] and quoted works).

We consider now *Jannussis' representation* [4] of system (1), which can be written in terms of the Pfaffian (5) of ref. [1]

$$\hat{A} = \int_{t_1}^{t_2} dt \, \{p_a b^2 \dot{r}_a - [p_a b^2 p_a / 2m_a + k / |r_1 - r_2|]\} , \; a = 1,2 \qquad (7)$$

In conclusion, the two-body nonselfadjoint generalization of the conventional Kepler's system is characterized by the simplest possible isotopic lifting of a Lie algebra, that characterized by the scalar isotopy of the canonical Lie tensor

$$\omega^{\mu\nu} \Rightarrow \Omega^{\mu\nu} = b^2 \omega^{\mu\nu}, \qquad (8)$$

Despite the simplicity of the isotopy, the physical implications in particle physics are rather intriguing on a number of counts (see Figure 1).

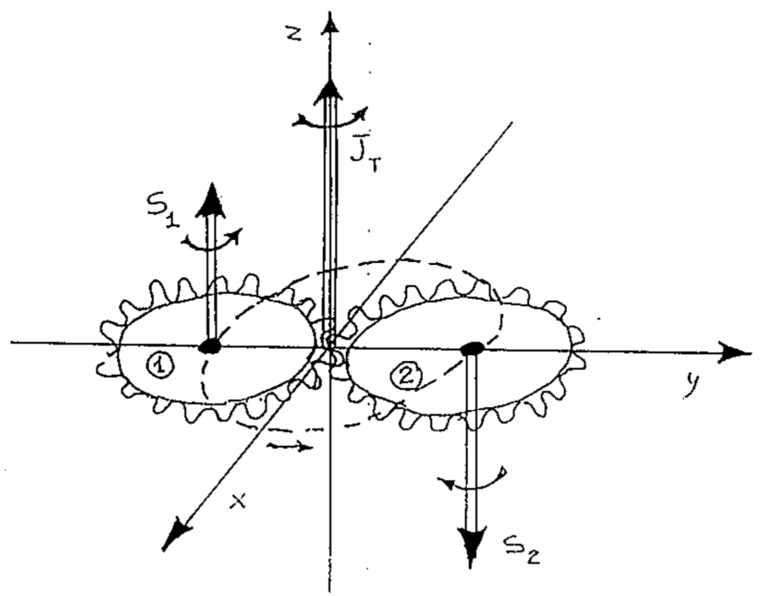

FIGURE 1: A symbolic view of the *"gear model"* for the representation of closed nonselfadjoint two-body systems introduced in ref. [2]. The model was suggested because effective in identifying the stable configurations under contact mutual interactions, classically and operationally. In fact, gears can rotate one around the other only in a circle, as represented by Eq.s (4) Moreover, when an intrinsic angular momentum is added, the gear model provides a visualization of the property that *only singlet states are stable.* In fact, gears rotates "in

phase" one inside the other in a singlet coupling, as shown in the figure. For triplet states, instead, we have an unstable system because the gears should rotate one against the other, thus creating mutually resistive forces which would push the gears one away from the other, by therefore rendering null thenonselfadjoint forces. In the transition to operator settings, the gear model is equally effective because in this case we have extended wavepackets rotating one around the other in conditions of total mutual penetration. The mutually resistive forces for triplet couplings then persist, resulting again in the singlet state with circular orbit as the only stable state. The model was used in ref. [2] to study the structure of the π^0 as a *"compressed positronium"*, i.e., as a closed nonselfadjoint system of one electron compressed inside the positron and rotating one with respect to the other at mutual distances of the order of 1F in a singlet state. It essentially emerged that such a model appears to be capable of providing a quantitative representation of *all* the total characteristics of the π^0 (total energy, meanlife, spin, charge radius, space and charge parity, etc.), thanks to the "renormalization" of the masses which is implicit in Eq.s (6). On the contrary, and this should be stressed here, the same model is inconsistent within the context of the conventional quantum mechanics for numerous reasons, such as: the inability to represent the total energy of the π^0 with very light constituents; the impossibility of achieving the relatively high meanlife of the particle; etc. (see ref. [2] for details). These studies are intended as a Newtonian basis for a reconsideration of the above operator models we hope to present at some future time. Their primary objective, as one can see, is to attempt the identification of the hadronic constituents (quarks) with suitably generalized forms of ordinary particles which can be freely produced in the spontaneous decays, as historically established for the preceding nuclear and atomic layers.

THREE-BODY CLOSED NONSELFADJOINT SYSTEMS. Their existence and consistency was also identified in the original proposal [2], and then reviewed in ref. [3]. Their first detailed study was provided by Jannussis and his collaborators [4] (see below).

For three-body closed nonselfadjoint systems in their general configuration, motion occurs in three dimension and the subsidiary constraints become essential.

The study of these systems requires a step-by-step generalization of the (rather vast) structure of the conventional three-body Kepler systems (for an excellent analytic treatment of the latter, see ref. [6].

Evidently, this task cannot be performed in this note. We shall therefore content ourselves with the identification of the most stable configurations without subsidiary constants.

The equations of motion in their second-order form in $T^*E(\hat{r},\delta,\mathfrak{R})$ can be written

$$m_1\ddot{r}_1 = -\frac{m_1 m_2}{r_{12}^3}(r_1 - r_2) - \frac{m_1 m_3}{r_{13}^3}(r_1 - r_3) + F_1^{NSA}, \quad (9a)$$

$$m_2\ddot{r}_2 = -\frac{m_2 m_1}{r_{12}^3}(r_1 - r_2) - \frac{m_2 m_3}{r_{23}^3}(r_2 - r_3) + F_2^{NSA}, \quad (9b)$$

$$m_3\ddot{r}_3 = -\frac{m_3 m_1}{r_{13}^3}(r_3 - r_1) - \frac{m_3 m_2}{r_{23}^3}(r_3 - r_2) + F_3^{NSA}, \quad (9c)$$

where

$$r_{12} = |r_1 - r_2|, r_{13} = |r_1 - r_3|, \text{ and } r_{23} = |r_2 - r_3|. \quad (10)$$

and the NSA forces are restricted by conditions (4) of ref. [1] to verify the identities

$$\textstyle\sum_a F_a^{NSA} = 0, \quad \sum_a p_a \times F_a^{NSA} = 0, \quad \sum_a r_a \wedge F_a^{NSA} = 0. \quad (11)$$

A straighforward generalization of the two-body solution of Eqs. (4) with n = 1 to the three-body case, leads to the following realization of the NSA forces (apparently introduced here for the first time)

$$F_1^{NSA} = c(\ddot{r}_1 - \ddot{r}_2) + d(\ddot{r}_1 - \ddot{r}_3), \quad (12a)$$

$$F_2^{NSA} = c(\ddot{r}_2 - \ddot{r}_3) + d(\ddot{r}_2 - \ddot{r}_1), \quad (12b)$$

$$F_3^{NSA} = c(\ddot{r}_3 - \ddot{r}_1) + d(\ddot{r}_3 - \ddot{r}_2). \quad (12c)$$

where, for simplicity, we have ignored higher, (even-order) derivatives; the quantities c and d are assumed to be independent constants; and we shall put hereon c = d.

The first stable configuration is that with one particle at rest at the center of the system with solution

$$r_1 = \text{cost.}, \quad r_2 = 0, \quad r_3 = \text{const.} \quad (13)$$

Moreover the centers-of-mass of the three bodies must be on a straight line. This is the *nonselfadjoint extension of the restricted three-body problem.* For further properties, see Fig. 2.

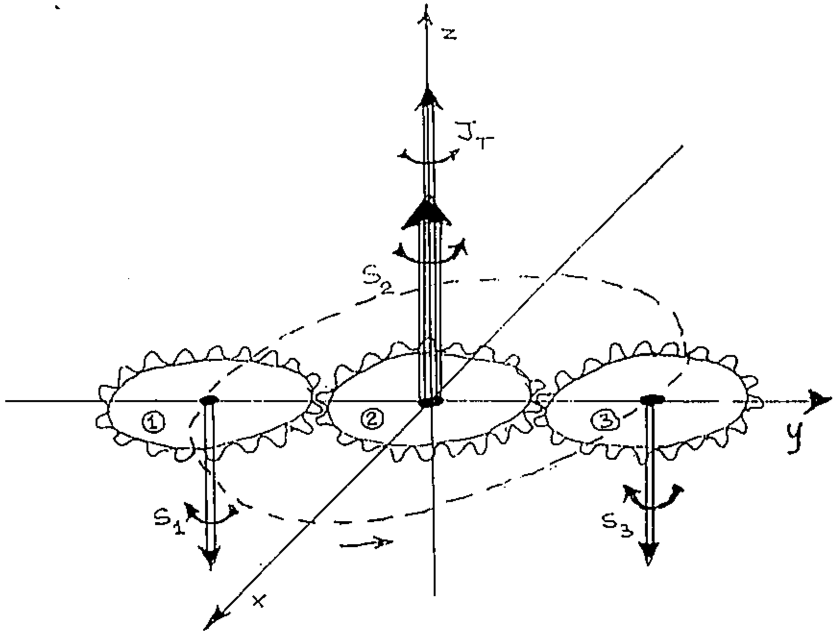

FIGURE 2: The *"gear-model"* for a stable configuration of the three-body, closed, nonselfadjoint systems [2]. The configuration here depicted is the simplest possible one consisting of the three bodies under mutual physical contact, disposed along a straight line, and rotating around the center-of-mass of the system, with the peripheral bodies having equal masses (for simplicity but without loss of generality). The configuration allows the introduction of a new notion of nucleus, here submitted under the name of *isonucleus,* which can essentially have an unrestricted (non-null) mass under contact nonselfadjoint interactions. By comparison, the conventional nuclus can occur in Kepler's systems only when its mass is much larger than the mass of the peripheral constituents. In fact, contact interactions merely

force the peripheral bodies against the central body which, as such, can have an arbitrary mass, including a mass much smaller than those of the peripheral bodies. For stability, the motion must necessarily occur along a straight line for much of the same reasons of the conventional Kepler case [6]. In fact, whenever the centers-of-mass of the three particles are no longer along a straight line, the configuration changes into that of *Lagrange's historical triangle* (see Figure 3). The conceptual value of the gear model becomes visible when adding the intrinsic angular momenta. In fact, the pairing of particles can no longer be a singlet or a triplet, as in the conventional Kepler case, but can only be of singlet type in order to prevent mutual resistive forces and their consequential instabilities. Moreover, the gear model becomes particularly valuable in spercial cases, e.g., when the central body has a null intrinsic angular momentum. In this case, the peripheral bodies must necessarily have non-null intrinsic angular momenta in order tobe able to have a stable rotation around the stationary central body. Note that the notion of isonucleus persists for closed nonselfadjoint systems of more than three particles. The above results are essentially unchanged when passing to three extended wavepackets in conditions of total mutual immersion. In this case, the configuration of this figure appears to be relevant for the study of Rutherford's historical conception of the neutron as a *"compressed hydrogen atom"* [7], according to which the peripheral electron is compressed (say, in the core of a star) inside the proton, all the way down to the center of the hyperdense medium in the proton structure (for preliminary operator studies of the model see ref. [8]). Note that *Rutherford's configuration of the electron at the center of the proton is permitted by our notion of isonucleus for a closed nonselfadjoint three-body system, but strictly prohibited by the conventional three-body system.* A central objective of these studies is to conduct a quantitative treatment of Rutherford's historical hypothesis, evidently following the operator formulation of the theory. The hope is the possibility of identifying the $\bar{d}$-quark with Rutherford's electron which, as such, can be freely produced in the spontaneous decays $\bar{d} \Rightarrow e + \bar{\nu}_e$. It is hoped that, in this way, the reader begins to see the reasons for the necessary priorstudy of the Newtonian setting.

The next stable configuration is that of the celebrated *Lagrange triangle* [6], which is also described by forces (12), with solution

$$r_1 = \text{const.} \equiv 0, \qquad r_2 = \text{const.} \equiv 0, \qquad r_3 = \text{const.} \equiv 0. \tag{14}$$

Again, we essentially have contact interactions forcing the three bodies one against the other. The system then rotates rigidiy around its center-of-mass. On historical grounds, we should recall the analytic difficulties Lagrange faced in achieving the stable triangular configuration (14). It is then significative to point out that these difficulties are readily resolved by contact interactions. For further comments see Fig. 3.

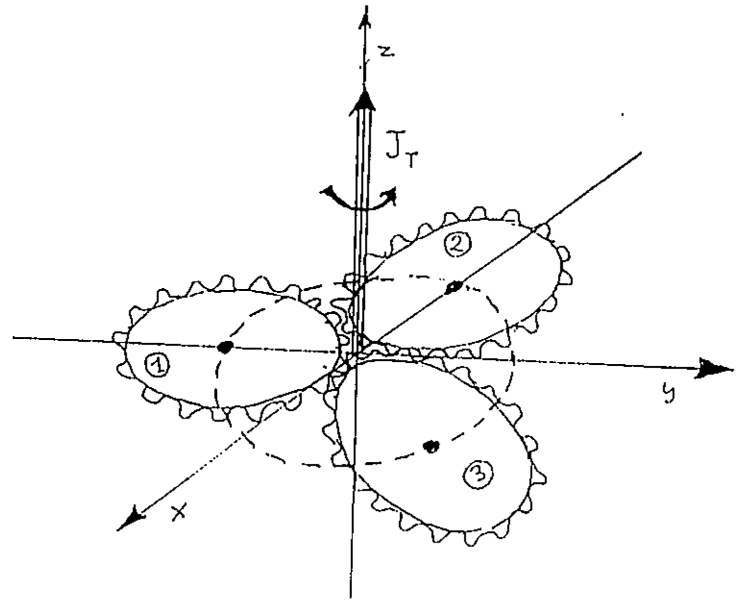

FIGURE 3: The *"gear model"* for the three-body, closed, nonselfadjoint system in *Lagrange's historical triangle configuration* [6]. In this case, the three bodies are forced into mutual contact and rotate rigidly around the center-of-mass of the system, each orbit being a circle. The conceptual value of the gear model becomes transparent in this case, because it clearly illustrates the fact that *a necessary condition for stability is that all three constituents have a null intrinsic angular momentum.* Intriguingly, the individual angular momenta are null, but the total angular momentum of the system is evidently not null. The configuration of this figure is significant for the study of heavier hadrons as closed three-bodynonselfadjoint bound states, as we hope to illustrate in subsequent papers.

The above two cases exhaust all possible configurations of closed nonselfadjoint systems with stable individual orbits. The remaining cases are predictably more involved than the corresponding conventional cases [6], and cannot possibly be treated in this note. They can be best illustrated via Jannussis' Birkhoffian representation [4]

$$B = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3} - \frac{m_1 m_2}{r_{12}} - \frac{m_1 m_3}{r_{13}} - \frac{m_2 m_3}{r_{23}} = E_{tot} , \tag{15a}$$

$$\{R_\mu\} = \{p_1 + \gamma r_3, p_2 + \gamma r_1, p_3 + \gamma r_2, 0, 0, 0\} , \tag{15b}$$

which represents the nonselfadjoint forces of clear resistive character

$$\begin{aligned} F_1^{NSA} &= \gamma(\dot{r}_2 - \dot{r}_3) , \\ F_2^{NSA} &= \gamma(\dot{r}_3 - \dot{r}_1) , \\ F_3^{NSA} &= \gamma(\dot{r}_1 - \dot{r}_2) . \end{aligned} \tag{16}$$

with Lie-isotopic tensor

$$(\Omega^{\mu\nu}) = \begin{pmatrix} (0)_{9\times 9} & & (1)_{9\times 9} & \\ & 0_{3\times 3} & \gamma(1)_{3\times 3} & -\gamma(1)_{3\times 3} \\ (-1)_{9\times 9} & -\gamma(1)_{3\times 3} & 0_{3\times 3} & \gamma(1)_{3\times 3} \\ & \gamma(1)_{3\times 3} & -\gamma(1)_{3\times 3} & 0_{3\times 3} \end{pmatrix} . \tag{17}$$

In this case, however, closedness requires the essential, irreducible constraint

$$r_1 \times r_2 + r_2 \times r_3 + r_3 \times r_1 = c. \tag{18}$$

For additional details in this intriguing case, we refer the interested reader to ref. [4].

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