Life is a game of chess

against God.

God will be a winner,

of course.

The number of moves

is a question only.

GRAVITY PARTICLES AND SPACE-TIME

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ISOTOPIC UNIFICATION OF GRAVITATION AND RELATIVISTIC QUANTUM MECHANICS AND ITS UNIVERSAL ISOPOINCARÉ SYMMETRY

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Abstract

We propose a new quantization of gravity, called quantum-iso-gravity, via the unification of gravitation and relativistic quantum mechanics which is permitted by the isotopic (i.e., axiom-preserving) generalization of the unit. We show that its axiomatic consistency is ensured by the conventional axioms of relativistic quantum mechanics, merely realized in a more general way. We then introduce the universal symmetry for all possible interior and exterior gravitations, called isopoincaré symmetry, and prove its isomorphism to the conventional symmetry. We finally point out a number of intriguing implications.

1 Statement of the problem

The historical open problem of quantum gravity (QG) is the need, on one side, for relativistic quantum mechanics (RQM) to have a meaningful Hamiltonian while, on the other side, Einstein's gravitation in vacuum has a null Hamiltonian [1]. A second open problem is the achievement of a QG which is axiomatically consistent as the conventional RQM, i.e., invariant under its own time evolution with physical quantities which are Hermitean-observable at all times, etc. A third open problem has emerged from recent studies in interior gravitational problems of quasars [1c], that QG should be a nonunitary image of conventional quantum theories, as needed, e.g., for a representation of irreversibility.

In this note we propose a new QG based on the generalization of the unit of RQM which, as such, requires no Hamiltonian at all, thus resolving the first historical problem. The axiomatic consistency of the proposed QG is guaranteed by the preservation of the abstract axioms of the RQM only realized in a more general way, including form-invariance, Hermiticity of observables at all times, etc., thus resolving the second problem. Finally, the proposed QG is a rather natural nonunitary image of conventional RQM, thus verifying the third condition.

Our model is based on the isotopic methods introduced by this author back in 1978 [2], developed in the recent monographs [3] and independently studied by various authors [4] (see papers [5] for recent reviews). The main elements to render this note self-sufficient are reviewed below. The model submitted in this note was first presented at the Seventh M. Grossmann Meeting on General Relativity held at Stanford University in July 1994, see the contribution in the proceedings [1c] and then at other meetings. A comprehensive presentation will be available in the forthcoming monograph [6].

2 Elements of isotopies

The main idea of the isotopic methods is to lift the conventional 4-dimensional unit of relativistic quantum mechanics, $I = \text{diag.}(1,1,1,1)$ into a 4×4-matrix which is well behaved and positive-definite, but otherwise possesses the most general possible dependence on local space-time coordinates $x = \{x^\mu\} = \{r, x^4\}, x^4 = c_0 t$, where c_0 is the speed of light in vacuum, wavefunctions $\psi(x)$ and their derivatives of arbitrary order, as well as any needed additional quantity, such as the density μ. of the medium considered, its temperature, τ, etc.

$$I = \text{diag.}(1,1,1,1) \mapsto \hat{I}(x, \dot{x}, \ddot{x}, \psi, \partial\psi, \partial\partial\psi, \mu, \tau, \ldots). \tag{1}$$

Relativistic quantum mechanics is then reconstructed to admit $\hat{I}$, rather than I, as the left and right unit. This requires first the lifting of the conventional enveloping associative algebra ξ of RQM with unit I, generic elements $A, B, \ldots$, and trivial associative product AB into an envelope $\hat{\xi}$ with the same elements $A, B, \ldots$, but now equipped with a new product

$$AB \mapsto A * B = ATB, \quad T = \text{fixed}, \tag{2}$$

which is such that $\hat{I} = T^{-1}$ is the left and right unit of $\hat{\xi}$,

$$\hat{I} * A = A * \hat{I} = A, \quad \forall A \in \hat{\xi}, \tag{3}$$

in which case $\hat{I}$ is called the isounit and T is called the isotopic element.

The lifting then requires for consistence corresponding isotopies of the remaining mathematical methods of RQM. These lifting are isotopic in the sense that they preserve all original axioms by construction (for positive-definite isounits) [3e,3f], i.e., the isotopic images of fields, vector spaces, algebras, geometries, etc., remain isomorphic to the original structures by construction.

Despite their simplicity, isotopic liftings are nontrivial because they imply the mapping of linear-local-Lagrangian theories into some of the most general known nonlinear-nonlocal-nonlagrangian forms which are however such to reconstruct the linear, local and Lagrangian characters in isospace [3d,3f].

The most direct way to reach an isotopic structure is by submitting RQM to a nonunitary transforms. Let $\mathcal{H}$ be a conventional Hilbert space over a conventional field of complex numbers $C = C(c, +, \times)$. Then a nonunitary transform on $\mathcal{H}$ over C of the fundamental relativistic canonical commutation rules yields precisely the rules of the corresponding rules of the isotopic theory,

$$UU^\dagger = \hat{I} \neq I, \quad T = (UU^\dagger)^{-1} = \hat{I}^{-1}, \quad \overline{x} = Ux^\mu U^\dagger, \quad \overline{p}_\mu = Up_\mu U^\dagger, \tag{4}$$

$$U[x^\mu, p_\nu]U^\dagger = Ux^\mu p_\nu U^\dagger - Up_\nu x^\mu U^\dagger = \overline{x}^\mu T \overline{p}_\nu - \overline{p}_\nu T \overline{x}^\mu = i\delta^\mu_\nu UIU^\dagger = i\delta^\mu_\nu \hat{I}, \tag{5}$$

$$\mu, \nu = 1,2,3,4.$$

However, it is easy to see that such an isotopic theory is not form-invariant under an additional nonunitary transform. It is easy to se that the generalized unit $\hat{I}$ is not invariant under additional nonunitary transforms, $\hat{I}' = W\hat{I}W^\dagger \neq \hat{I}, WW^\dagger \neq I$, thus preventing the applicability of measurements. Also, the above theory does not preserve Hermiticity at all times. In fact, starting from the original condition of Hermiticity on $\mathcal{H}$,

$$\{<| H^\dagger\} |>=<| \{H |>\}, \qquad H^\dagger = H,$$

the condition of Hermiticity under nonunitary transforms still defined on a conventional Hilbert space $\mathcal{H}$ becomes

$$\{<| T\overline{H}^{\hat{\dagger}}\} |><| \{HT |>\}, \quad \overline{H}^{\hat{\dagger}} = T^{-1}\overline{H}T, \tag{6}$$

which, as such, is generally violated, thus preventing thre observability at all times. It then follows that the theory does not admit invariant special functions and physical laws. This general loss of for-invariance, Hermiticity-observability, etc. also holds for q- and k-deformations, quantum groups and all theories of quantum gravity possessing nonunitary time evolutions, yet defined on a conventional Hilbert space [5a].

The resolutions of the latter problems requires the necessary isotopies of the entire structure of RQM into a form called relativistic hadronic mechanics (RHM) [3f], admitting of $\hat{I}$s the correct left and right unit. This includes: A) the lifting of the field $F(a, +, \times)$ of real numbers R or complex numbers C with conventional sum $a+b$ and product $a \times b = ab$ into the isofields $\hat{F}(\hat{a}, +, \hat{*})$ with isonumbers $\hat{a} = a \times \hat{I}$ with sum $\hat{a} + \hat{b} = (a+b)\hat{I}$, isoproduct $\hat{a} * \hat{b} = \hat{a}T\hat{b} = (ab)\hat{I}$, and all generalized operations (see [7] for details); B) the lifting of the conventional Minkowski space $M(x, \eta, R)$ with metric $\eta = \text{diag.}(1,1,1,-1)$ and invariant $x^2 = x^t \eta x$ on $R(n, +, \times)$ into the isominkowski space [8a] $\widehat{M}(x, \hat{\eta}, \widehat{\mathrm{R}})$ with isometric $\hat{\eta} = T(x, \dot{x}, \ddot{x}, \psi, \partial\psi, \partial\partial\psi, \ldots)\eta$, and isoseparation on $\widehat{\mathrm{R}}(\hat{n}, +, *)$ among two points x, y (see ref. [5c] for topological aspects)

$$x^{\hat{2}} = [(x^1 - y^1)T_{11}(x, \ldots)(x^1 - y^1) + (x^2 - y^2)T_{22}(x, \ldots)(x^2 - y^2) + (x^3 - y^3)T_{33}(x, \ldots)(x^3 - y^3) - (x^4 - y^4)T_{44}(x, \ldots)(x^4 - y^4)]\hat{I}, \tag{7}$$

where the assumed diagonal form of the isotopic element T is always possible from its positive- definiteness; C) the lifting of the original Hilbert space $\mathcal{H}$ with states $|>$ and inner product $<|> \in \mathrm{C}(c, +\times)$ into the isohilbert space $\widehat{\mathcal{H}}$ with isoinner product and related isonormalization

$$<\hat{|}> = <| T |> \hat{I} \in \hat{\mathrm{C}}(\hat{c}, +, *), \qquad <| T |> = 1; \tag{8}$$

D) the lifting of eigenvalue equations $H\ |> = E_0\ |>$ into the isotopic form

$$H * |> = HT\ |> = \hat{E} * |> = E\hat{I}T\ |> \equiv E\ |>, \quad E \neq E_0, \tag{9}$$

indicating that the final numbers of the theory are the conventional ones; E) the lifting of the operator four-momentum $p_\mu\ |> = -i\partial_\mu\ |>$ into the for characterized by the isodifferential calculus

$$p_\mu * |> = -i\hat{\partial}_\mu\ |> = -iT_\mu^\nu \partial_\nu\ |>, \tag{10}$$

where $\hat{\partial}_\mu\ |> = -iT_\mu^\nu \partial_\nu$ the isoderivative and $dx = \hat{I}dx$ is the isodifferential and isotopic forms of the various axioms of conventional differential calculus, e.g.,

$$\hat{\partial}_\mu(f * g) = (\hat{\partial}_\mu f) * g + f * (\hat{\partial}_\mu \hat{g}), \quad \hat{\partial}_\mu^{\hat{2}} = \hat{\partial}_\mu * \hat{\partial}_\mu = \hat{I}_\mu^\nu \partial_\nu{}^2,$$

etc.; F) the lifting of expectation values $< A > = <| A |> / <|>$ into the form

$$\hat{<} A \hat{>} = <| TAT |> / <| T |>, \tag{11}$$

for which $\hat{<}\hat{I}\hat{>} = I$; and the compatible liftings of the remaining aspects of RQM. Since $\hat{I} = UU^\dagger$ is Hermitean we can hereon assume it to be positive definite and diagonal.

The most important properties emerging from the above liftings are the following. First, RHM is a fully axiomatic theory in exactly the same sense as RQM because nonunitary transforms can always be turned into the isounitary transform on $\mathcal{H}$,

$$WW^\dagger = \hat{I} \neq I, \quad W = \widehat{W}T^{1/2}, \quad WW^\dagger = \widehat{W} * \widehat{W}^\dagger = \widehat{W}T\widehat{W}^\dagger = \hat{w}^\dagger * \widehat{W} = \hat{I}, \tag{12}$$

under which RHM is form-invariant. In fact, the isounit of the theory is invariant

$$\widehat{W} * (x^\mu * p_\nu - p_\nu * x^\mu) * \widehat{W}^\dagger = \overline{x}^\mu * \overline{p}_\nu * \ x^\mu - \overline{p}_\nu * \overline{x}^\mu = i\delta^\mu_\nu \widehat{W} * \hat{I} * \widehat{W}^{\hat{\dagger}} = i\delta^\mu_\nu \hat{I};$$

all operators which are initially Hermitian remain so at all times. In fact, the condition of Hermiticity on $\widehat{\mathcal{H}}$ over $\hat{\mathrm{C}}(\hat{c}, +, *)$ now reads

$$\{<| TH^{\hat{\dagger}}\}\ |> = <|\ \{HT\ |>\}m \tag{13}$$

and, as such, it coincides with the Hermiticity on $\mathcal{H}$ over $\mathrm{C}(c, +, \times)$, $H^{\hat{\dagger}} \equiv H^\dagger = H$. All observables of RQM therefore remain observables for RHM. Moreover, RHM and RQM coincide at the abstract level (for $\hat{I} > 0$) where $\widehat{\mathrm{R}}(\hat{n}, +, \times) = \mathrm{R}(n, +, \times)$, $\hat{\xi} \equiv \xi$, $\widehat{M}(x, \hat{\eta}, \widehat{\mathrm{R}}) \equiv M(x, \eta, \mathrm{R})$, $\widehat{\mathcal{H}} \equiv \mathcal{H}$, etc. also, RHM can approximate RQM as close as desired for $\hat{I} \simeq I$ and admit the latter identically as a particular case for $\hat{I} \equiv I$. Finally, RHM admits all infinitely possible signature-preserving deformations $\hat{\eta} = T\eta$ of the Minkowski metric (universality) directly in the frame of the observer (direct universality).

3 Quantum-iso-gravity

We now apply RHM for the isotopic unification of gravitation and RQM. This is achieved via the isoquantization of gravity hereon called quantum isogravity (QIG). The first condition for its realization is the representation of gravity via the isominkowskian, rather than the Riemannian space. Let $R(x, g, \mathrm{R})$ be a conventional (3+1)-Riemannian space with symmetric, nonsingular and real-valued metric $g(x)$ and separation $x^2 = x^t g x \in R(n, +, \times)$. It is easy to see that $g(x)$ is identically admitted as a particular case of the isominkowskian metric $\hat{\eta}(x, \dot{x}, \ddot{x}, \ldots)$ resulting in the local isomorphism $R(x, g, \mathrm{R}) \approx \widehat{M}(x, \hat{\eta}, \widehat{\mathrm{R}})$, $g(x) \equiv \hat{\eta}(x)$.

The main idea of QIG is to embed gravitation in the unit of conventional RQM. This is permitted by the isotopic methods via the factorization of any given Riemannian metric in the Minkowskian metric and the assumption of the factor as the gravitational isounit of the theory

$$g(x) = T_{gr}(x)\eta, \qquad \hat{I}_{gr}(x) = [T_{gr}(x)]^{-1}, \tag{14}$$

where $T_{gr}(x)$ is always positive-definite from the locally Minkowskian character of R (evidently outside gravitational horizons). The desired unification of gravitation and RQM mechanics is then achieved via the reconstruction of the later for the isounit $\hat{I}_{gr}$. Since the isometric of RHM in this case is the Riemannian metric, this results in a novel quantization of gravity which resolves the three basic problems indicated earlier. In fact, the quantization is via the unit, rather then the Hamiltonian; it is invariant under its own time evolution; and it is indeed a nonunitary image of the RQM. Moreover, the preservation of the basic axioms of RQM at the abstract level ensures the mathematical consistency of the theory, the understanding being that its physical consistency requires specific studies.

In short, the main conjecture submitted in this note is that a consistent operator form of gravity already exists. It did creep in un-noticed until now because it is embedded in the unit of conventional RQM. In fact, the axioms of RHM imply that

$$\hat{<}\hat{I}_{gr}\hat{>} = <| T_{gr}T_{gr}^{-1}T_{gr} |> / <| T_{gr} |> = 1, \tag{15}$$

thus confirming the "hidden" character of gravitation in conventional RQM. As an illustration, the embedding of gravity in Dirac's equation can be written

$$(\hat{\gamma}^{\mu} * p_{\mu} + im) * |> = [\hat{\gamma}^{\mu}(x) T_{gr}(x) \hat{\eta}_{\mu\nu}(x) p^{\nu} - im\hat{I}] T_{gr}(x) |> = 0, \tag{16}$$

$$\{\hat{\gamma}^{\mu}, \hat{\gamma}^{\nu}\} = \hat{\gamma}^{\mu} T_{gr} \hat{\gamma}^{\nu} + \hat{\gamma}^{\nu} T_{gr} \hat{\gamma}^{\mu} = 2\hat{\eta}^{\nu\mu} \equiv 2g^{\mu\nu}, \qquad \hat{\gamma}^{\mu} = T_{\mu\mu}{}^{1/2} \gamma^{\mu} \hat{I}_{gr}, \tag{17}$$

where γ^{μ} are the conventional gammas and $\hat{\gamma}^{\mu}$ are called isogamma matrices. The important point is that at the abstract level the conventional and isogravitational Dirac equations coincide from the topological equivalence of I and $\hat{I}_{gr}$,

$$(\gamma^{\mu} p_{\mu} + im) |> \equiv (\hat{\gamma}^{\mu} * p_{\mu} + im) * |> .$$

Note that the anticommutator of the isogamma matrices yields (twice) the Riemannian metric $g(x)$, thus confirming the full embedding of gravitation. As an example, the Dirac-Schwartzschild equation (here presented for the first time) is given by Eq.s (2) with

$$\hat{\gamma}_k = (l - 2M/r)^{-1/2} \gamma_k \hat{I}_{gr}, \qquad \hat{\gamma}_4 = (l - 2M/r)^{1/2} \gamma_4 \hat{I}_{gr}. \tag{18}$$

Similarly one can construct the Dirac-Krasner equation and others or similar realizations of for the Klein-Gordon, Weyl and any other relativistic field equation.

In order to initiate the appraisal of the possible physical relevance of QIG, we here identify the following primary implications:

Consequence 1: QIG permits the introduction, apparently for the first time, of a universal symmetry for all possible exterior and interior gravitations called isopoincaré symmetry $\hat{P}(3.1) = \hat{L}(3.1) \times \hat{\tau}(3.1)$ [8]. It, where $\hat{L}(3.1)$ is the isolorentz symmetry and $\hat{\tau}(3.1)$ is the symmetry under isotranslations (i.e., Lorentz transformations and translations in isospace, respectively), which results to be locally isomorphic to the conventional symmetry $P(3.1) = L(3.1) \times \tau(3.1)$

The isosymmetry can be readily constructed via the Lie-isotopic theory [2a,3b,3f,4,5b] and consists in the reconstruction of $P(3.1) = L(3.1) \times \tau(3.1)$ for the generalized unit $\hat{I}_{gr} = [T_{gr}(x)]^{-1}$, $g(x) = T_{gr}(x)\eta$. Since $\hat{I}_{gr} > 0$, one can see from the inception that $\hat{P}(3.1) \approx P(3.1)$. This implies in particular that the $\hat{P}(3.1)$ and $P(3.1)$ have the same connectivity properties (for positive-definite isounits, but not so otehrwise [3f,6]).

Under the lifting $P(3.1) \rightarrow \hat{P}(3.1)$ the original generators $X = \{X_k\} = (M_{\mu\nu}, p_{\alpha}\}$, $M_{\mu\nu} = x_{\mu}p_{\nu} - x_{\nu}p_{\mu}$, $k = 1, 2, ..., 10$, $\mu, \nu = 1, 2, 3, 4$, remain unchanged while the original parameters $w = \{w_k\} = \{(\theta, v), a\} \in \mathbf{R}$ are lifted into the form $\hat{w} = w\hat{I} \in \hat{\mathbf{R}}(\hat{n}, +, *)$. The connected component $\hat{P}_0(3.1)$ can be written via the exponentiation in $\hat{\xi}$ characterized by the isotopic Poincaré – Birkhoff – Witt Theorem [2a]

$$\hat{P}_0(3.1) : \hat{A}(\hat{w}) = \prod_k \hat{e}^{iX*\hat{w}} = \prod_k e^{iX*Tw}\hat{I}, \tag{19}$$

while the preservation of the original dimension is ensured by the isotopic Baker – Campbell– Hausdorff Theorem [2a]. It is easy to see that structure (19) forms a connected Lie-isotopic transformation group with isogroup laws

$$\hat{A}(\hat{w}) * \hat{A}(\hat{w}') = \hat{A}(\hat{w}') * \hat{A}(\hat{w}) = \hat{A}(\hat{w} + \hat{w}'), \quad \hat{A}(\hat{w}) * \hat{A}(-\hat{w}) = \hat{A}(0) = \hat{I}_{gr} = [T_{gr}(x)]^{-1} \tag{20}$$

Note that $\hat{P}(3.1)$ acts isotransitively in $\hat{M}(x, \hat{\eta}, \hat{\mathbf{R}})$, i.e., $x' = \hat{A}(\hat{w}) * x$, because the preservation of the original transform $x' = Ax$ would now violate linearity in isospace. The isotopy of the discrete transforms is elementary, and reducible to the forms $\hat{\pi} * x = \pi x = (-r, x^4)$, $\hat{\tau} * x = \tau x = (r, -x^4)$, where $\hat{\pi} = \pi\hat{I}$, $\hat{\tau} = \tau\hat{I}$, where π, τ are the conventional inversion operators.

To identify the isoalgebra $\hat{p}_0(3.1)$ of $\hat{P}_0(3.1)$, we use the isodifferential calculus on $\hat{M}$ indicated earlier. By recalling that the coordinates in the covariant form in $\hat{M}$ are given by $x_{\mu} = \hat{\eta}_{\mu\nu}x^{\nu}$, we have the property $\hat{\partial}_{\mu}/\hat{\partial}x^{\nu} = \hat{\eta}_{\mu\nu}$. The fundamental isocommutation rules of RHM can therefore be written

$$[x_{\mu} \hat{,} p_{\nu}] * |> = i\hat{\eta}_{\mu\nu} |> . \tag{21}$$

The isocommutation rules of $\hat{p}_0(3.1)$ are then given by

$$[M_{\mu\nu} \hat{,} M_{\alpha\beta}] = i(\hat{\eta}_{\nu\alpha} M_{\mu\beta} - \hat{\eta}_{\mu\alpha} M_{\nu\beta} - \hat{\eta}_{\nu\beta} M_{\mu\alpha} + \hat{\eta}_{\mu\beta} M_{\alpha\nu}), \tag{22}$$

$$[M_{\mu\nu} \hat{,} p_{\alpha}] = i(\hat{\eta}_{\mu\alpha} p_{\nu} - \hat{\eta}_{\nu\alpha} p_{\mu}), \qquad [p_{\alpha} \hat{,} p_{\beta}] = 0, \tag{23}$$

where $[A \hat{,} B] = AT_{gr}(x)B - BT_{gr}(x)A$ is the Lie-isotopic product (originally proposed in [2a] which does indeed verify the Lie axioms as one can verify. The local isomorphism $\widehat{p}_0(3.1) \approx p_0(3.1)$ is then ensured by the positive-definiteness of $T_{gr}(x)$. The realization in which the generators are given by

$$M^\mu{}_\nu = x^\mu p_\nu - x^\nu p_\nu$$

implies the preservation by $\widehat{p}_0(3.1)$ of the same structure constants of $p_0(3.1)$.

The isocasimir invariants are then lifted into the forms

$$C^{(0)} = \widehat{I}_{gr} = [T_{gr}(x)]^{-1}, \qquad C^{(1)} = \widehat{p^2} = p_\mu * p^\mu = \widehat{\eta}^{\mu\nu} p_\mu * p\nu,$$
$$C^{(3)} = \widehat{W}_\mu * \widehat{W}^\mu, \qquad \widehat{W}_\mu = \epsilon_{\mu\alpha\beta\rho} M^{\alpha\beta} * p^\rho. \tag{24}$$

The local isomorphism $\widehat{p}_0(3.1) \approx p_0(3.1)$ is sufficient, per se, to guarantee the axiomatic consistency of QIG. Note also that the momentum operators become commutative in their isominkowskian representation (while they are notoriously noncommutative in their Riemannian representation). This confirms the achievement of a representation of gravitation in an isoflat space, i.e., a space possessing zero curvature in the isospace $\widehat{M}$, but not in its projection into $R(x, g, \mathbf{R})$.

Under sufficient boundedness, regularity and smoothness of the the isotopic element, the space components $S\widehat{O}(3)$, called isorotations [8a,8b] can be easily computed from isoexponentiations (19) yielding the explicit form in the (x, y)-plane

$$x' = x\cos(T_{11}{}^{1/2}T_{22}{}^{1/2}\theta_3) - yT_{11}{}^{-1/2}T_{22}{}^{1/2}\sin(T_{11}{}^{1/2}T_{22}{}^{1/2}\theta_3). \tag{25}$$
$$y' = xT_{11}{}^{1/2}T_{22}{}^{-1/2}\sin(T_{11}{}^{1/2}T_{22}{}^{1/2}\theta_3) + y\cos(T_{11}{}^{1/2}T_{22}{}^{1/2}\theta_3), \tag{26}$$

(see [3f] for general isorotations in all three Euler angles). Isotransforms (8) leave invariant all ellipsoidical deformations

$$xT_{11}x + yT_{22}y + zT_{33}z = r$$

of the sphere $xx + yy + zz = r$ in the Euclidean space $E(r, \delta, \mathbf{R})$, $r = \{x, y, z\}$, $\delta = \text{diag.}(1, 1, 1)$. Such ellipsoids become perfect spheres $\widehat{r^2} = (r^t, \widehat{\delta} r)\widehat{I}_s$ in isoeuclidean spaces $\widehat{E}(r, \widehat{\delta}, \widehat{\mathbf{R}})$, $\widehat{\delta} = T_s\delta$, $T_s = \text{diag.}(T_{11}, T_{22}, T_{33})$, $\widehat{I}_s = T_s^{-1}$, called isospheres [9a], because the deformation of the semiaxes $1_k \to T_{kk}$ is compensated by the deformation of the related units of the inverse amounts $1_k \to T_{kk}{}^{-1}$. This perfect isosphericity is the geometric origin of the isomorphism $\widehat{0}(3) \approx O(3)$, i.e., of the exact character of the rotational symmetry for deformed spheres when treated at the isotopic level.

The connected space-time isosymmetry $S\widehat{0}(3.1)$ is characterized by the isorotations and the isolorentz boosts [8a,8d] which can be written in the $(3, 4)$-plane

$$x^{3'} = x^3\sinh(T_{33}{}^{1/2}T_{44}{}^{1/2}v) - x^4 T_{33}{}^{-1/2}T_{44}{}^{1/2}\cosh(T_{33}{}^{1/2}T_{44}{}^{1/2}v)$$

$$= \widehat{\gamma}(x^3 - T_{33}{}^{-1/2}T_{44}{}^{1/2}\widehat{\beta}x^4), \tag{27}$$
$$x^{4'} = x^3 T_{33}{}^{1/2}c_0^{-1}T_{44}{}^{-1/2}\sinh(T_{33}{}^{1/2}T_{44}{}^{1/2}v) + x^4\cosh(T_{33}{}^{1/2}T_{44}{}^{1/2}v)$$
$$= \widehat{\gamma}(x^4 - T_{33}{}^{1/2}T_{44}{}^{-1/2}\widehat{\beta}x^3), \tag{28}$$
$$\widehat{\beta} = v_k T_{kk}{}^{1/2}/c_0 T_{44}{}^{1/2}, \qquad \widehat{\gamma} = (1 - \widehat{\beta}^2)^{-1/2}. \tag{29}$$

Note that the above isotransforms are nonlinear in x, as expected for a correct symmetry of gravitation, and are formally similar to the Lorentz transforms, as expected from their isotopic character. Isotransforms (27-28) characterize the gravitational isolight cone [9b], i.e., the perfect cone in isospace $\widehat{M}(x, \widehat{\eta}, \widehat{\mathbf{R}})$. In fact, in a way similar to the isosphere, we have the deformation of the original light cone $1_\mu \to T_{\mu\mu}$ while the corresponding units are deformed of the inverse amount $1_\mu \to T_{\mu\mu}^{-1}$ thus preserving the original cone as a necessary condition for an isotopy. The abstract identity of the light and isolight cones is the geometric origin of the isomorphisms $\widehat{O}(3.1) \approx O(3.1)$, that is, the exact character of the Lorentz symmetry for locally varying speeds of light $c(x, \mu, \tau, ...) = c_0 T_{44}(x, \mu, \tau, ...)$ when treated at the isotopic level.

In particular, the isolight cone possesses all properties of the conventional light cone, including the characteristic angle. The maximal causal speed in isospace therefore remains the speed of light in vacuum c_0. This is an evident important property for the physical consistency of quantum gravity.

The isotranslations can be written

$$x' = (\widehat{e}^{ipa}) * x = x + aA(x), \qquad p' = (\widehat{e}^{ipa}) * p = p, \tag{30}$$
$$A_\mu = T_{\mu\mu}{}^{1/2} + a^\alpha[T_{\mu\mu}{}^{1/2}\widehat{,}p_\alpha]/I! + ... \tag{31}$$

with "gravitational isoplanewave" $\psi = \widehat{e}^{kx} = \{\exp(\mathrm{k}T_s\mathrm{r} - k_4 T_t c_0 \widehat{t})\}\widehat{I}$. The extension of the above derivation to the isospinorial covering $\mathcal{P}(3.1) = S\widehat{L}(2, \widehat{\mathbf{C}}) \times \tau(3.1)$ has been studied in detail in [3f,6]. The above results imply the following:

Theorem: The isopoincaré symmetry $P(3.1)$ is the universal invariance of all infinitely passible separations (6) thus providing, as a particular case, the universal invariance for all possible exterior and interior gravitations.

Note that there is nothing to compute in the sense that for any arbitrarily given (diagonal) Riemannian metric $g(x)$ (such as Schwartzschild, Krasner, etc, [1a]) one merely plots the $T_{\mu\mu}$ terms in the decomposition $g_{\mu\mu} = T_{\mu\mu}\eta_{\mu\mu}$ (no sum) in the above given isotransforms. The invariance of the separation $x^t g x$ is then ensured by the construction of the isosymmetry, as one can easily verify. At any rate, Lie symmetries are known to leave invariance their own unit. Note also that the (2+2)-de Sitter or other cases can be derived from the theorem via mere changes of signature or dimension of the isounit. Note finally that the above theorem includes invariances for theories much broader than the Riemannian metric, such as the invariance for the isoriemannian metrics $\hat{g} = T(x, \dot{x}, \ddot{x}, \hat{\ }\partial\psi, \partial\partial\psi, ...)g(x)$ currently under study for interior gravitational problems [3f,3d].

Consequence 2: QIG implies the geometric unificarion of the special and general relativity.

This is evidently due to the fact that all distinctions between the special relativity in Minkowski space and the general relativity in isominkowski space are now lost owing to the abstract identities $\hat{\mathrm{R}}(\hat{n}, +, *) = R(n, +, \mathrm{R})$, $\hat{M}(x, \hat{\eta}, \hat{\mathrm{R}}) \equiv M(x, \eta, \mathrm{R})$, $\hat{\mathcal{H}} \equiv \mathcal{H}$, $\hat{P}(3.1) \equiv P(3.1)$, etc. An important implication is the elimination of the historical difference between the special and general relativities whereby the former admits the universal Poincaré symmetry, while the latter does not a Isogravitation emerges from QIG as possessing a universal symmetry which turns out to be locally isomorphic to the conventional Poincare symmetry. The gravitational field on $\hat{M}(x, \hat{\eta}, \hat{\mathrm{R}})$ must now be isocovariant under $\hat{P}(3.1)$ in essentially the same way as the electromagnetic field on $M(x, \eta, \mathrm{R})$ must be covariant under $P(3.1)$. Note the necessity of the isoflat representation of gravity for the very formulation of its universal isopoincaré symmetry. In fact, no isosymmetry can be constructed in the Riemannian space.

Consequence 3: QIG permits a novel approach to the unification of weak, electromagnetic and gravitational interactions via the embedding of gravity in the unit of conventional unified gauge theories here called "iso-grand-unification",which is planned for study elsewhere.

The conjecture here submitted is therefore that gravitation is already contained in the existing unified gauge theories. It did escape identification until now because it is embedded in the unit of the theory (for the isotopies of the electromagnetic interactions see ref. [2b] for the isotopies of gauge theories see ref. [10] and review [4d]).

Consequence 4: QIG permits a novel approach to gravitational horizons as the zeros of (the space component of) the isounit, and of gravitational singularities as the zeros of (the spaee component of) the isotopic element.

In fact, at the Schwartzschild's horizon $r = 2M$ the space isounit $\hat{I}_s = (1 - 2M/r) \times \mathrm{diag.}(1, 1, 1)$ of the isosphere $r^{\hat{2}} = (r^t \hat{\delta} r)\hat{I}_s$ is null, while at $r = 0$ the space isotopic element $T_s = (1 - 2M/r) \times \mathrm{diag.}(1, 1, 1)$ is null. Recall in this respect that the restriction of the isounits/isotopic elements to a sole x-dependence is grossly un-necessary for isotopic theories as illustrated by the above theorem. The extension of the above exterior quantum isogravity to the corresponding interior quantum isogravity is merely given by admitting a nonJinearity in the velocities and in the derivatives of the wavefunction, $\hat{I}_{gr}(x, \dot{x}, \ddot{x}, \hat{\ }\partial\psi, \partial\partial\psi, ...)$. A more adequate formulation of gravitational horizons and singularities is then given by the zeros of the (space component of) the latter isounits and isotopic elements. This extension evidently permits a second generation of studies on gravitational collapse, black holes and all that, because it permits a quantitative treatment of internal effects, such as interior nonlocal and non-(first)-order- Lagrangian effects expected in very high densities, etc., which are outside any realistic treatment via the Riemannian geometry [3f].

Also, recall that the "universal constancy of the speed of light" is a philosophical abstraction because in interior conditions (such as in our atmosphere) light has a locally varying speed. The isopoincaré symmetry can directly represent the actual speed of light in interior conditions via the more general isotopic elements $\hat{T}_{\mu\mu} = T_{\mu\mu}/n_\mu^2$ (no sum), where $T_{\mu\mu}$ the conventional gravitational term. Isoinvariant (7), when projected in our space-time, then yields the local speed of light $c = c_0/n_4$ where n_4 is the familiar local index of refraction, with the understanding that in isospace the maximal causal speed remains c_0 as indicated earlier. The space components n_k are evidently requested by isolorentz covariance which essentially provides a space-time symmetrization of the index of refraction. The latter symmetrization is important for a direct geometrization of the inhomogeneity and anisotropy of physical media (such as our atmosphere), e.g., via a dependence of the n's from the local density, the differentiation of the value of their space and time components, the factorization of a preferred direction in the medium (the underlying empty space remaining perfectly homogeneous and isotropic under isotopies), etc. (see ref.s [3b,3e] for specific applications and available experimental verifications).

These features are not merely formal because the immediate exterior of gravitational horizons is not empty, but composed of hyperdense chromospheres in which the speed of light is not c_0, thus implying the inapplicability of the conventional light cone. Our QIG resolves these problems too via the direct representation of the local variation of the light speed and the reconstruction in isospace of the perfect light cone, thus permitting quantitative studies.

Consequence 5: Space and time in QIG have a local character in the sense that their isounits have an explicit dependence on the local gravitational field itself.

In fact, the generalization of the unit in isoinvariant (7) directly implies the lifting

$$I = \mathrm{diag.}(I_s, I_t) \rightarrow \mathrm{diag.}(\hat{I}_s, \hat{I}_t), \tag{32}$$

$$I_s = \mathrm{diag.}(+1, +1, +1), \quad I_t = +1 \rightarrow \hat{I}_s = \mathrm{diag.}(T_{11}, T_{22}, T_{33}), \quad \hat{I}_t = T_{44}. \tag{33}$$

As an example, the space-time isounits for an observer in the exterior Schwartzschild field are given by $\widehat{I}_s = (1-2M/r)\text{diag.}(1,1,1)$ and $\widehat{I}_t = (1-2M/r)^{-1}(M > r)$. One may also assume the redefinition $x^t g x = x^t \widehat{\eta} x = \overline{x}^t \eta \overline{x}$, $g = T_{gr}\eta$, $\overline{x} = x T_{gr}^{1/2}$. Riemannian coordinates are therefore equivalent to space-time coordinates in our Minkowski space with space isounits $\widehat{I}_k = T_{kk}^{-1/2}$ and time isounit $\widehat{I}_t = T_{44}^{-1/2}$. Note that the isogravitational theory recovers the relativistic Einsteinian space-time for $M = 0$ or $r \to \infty$, for which $\widehat{I}_k \equiv \widehat{I}_t = I$. However, for a non-null gravitational field the isounits are different than the conventional units. QIG therefore predicts the capability of altering space and time via the alteration of their units, evidently in addition to the Einsteinian variation with speed.

The above results pose the intriguing experimental question whether time here on Earth and, say, time on Jupiter are different due to the difference of their gravitational fields as predicted by the isounit $\widehat{I}_t = (1-2M/r)^{-1}(r > 2M)$ (in addition to conventional gravitational corrections).

With the understanding that the mathematical consistency of QIG is established by its Poincare covariance, the resolution of the physical consistency of QIG requires experiments measures as to whether

- we live in space-time as conventionally understood, in which case the Riemannian description of gravitation is the physically correct one and the equivalent isominkowskian formulation has a mere mathematical character, or
- we live in isospace and isotime, in which case the isominkowskian description is the physically correct one and the Riemannian description has a mathematical value.

The above issue can be resolved with current technology by sending a probe to Jupiter's atmosphere capable of conducting comparative measures of time with respect to Earth.

We should also mention for completeness that quantum-iso-gravity provides a novel characterization of antiparticles via negative-definite isounits. In fact, the operator formulation of the antiautomorphic map $\widehat{I} > 0 \to \widehat{I}^d = -\widehat{I} < 0$, called isoduality [8b], has been proved to be equivalent to charge conjugation [3f,6]. Note that isoduality also applies for conventional units $I > 0 \to I^d = -I < 0$ The isodual representation of antiparticles then requires the reconstruction of all conventional and isotopic methods with respect, this time, to a negative-definite unit, including numbers, fields, spaces, symmetries, etc.

The universal symmetry of antiparticles results to be the isodual isopoincaré symmetry $\widehat{P}^d(3.1) = \widehat{L}^d(3.1) \times \widehat{\tau}^d(3.1)$ over the isodual isominkowskian spaces $\widehat{M}^d(x, \widehat{\eta}^d, \widehat{\mathrm{R}}^d)$, $\widehat{\eta}^d = -\widehat{\eta}$, $\widehat{\mathrm{R}}^d \approx -\widehat{\mathrm{R}}$, which is the isosymmetry $\widehat{P}^d(3.1)$ of this note

when referred to the isounit

$$\widehat{I}^d = \text{diag.}(-T_{11}{}^{-1}, -T_{22}{}^{-1}, -T_{33}{}^{-1}, -T_{44}{}^{-1}),$$

as well as its isodual isospinorial covering

$$\widehat{\mathcal{P}}^d(3.1) = S\widehat{L}^d(2.\widehat{\mathrm{C}}^d) \times \widehat{\tau}^d(3.1).$$

Note that conventional or isotopic space-time separations are iso-self-dual, i.e., invariant under isoduality,

$$x^{\widehat{2}d} = (x^t \widehat{\eta}^d x)\widehat{I}^d \equiv (x^t \widehat{\eta}^x)\widehat{I} = x^{\widehat{2}},$$

and this may explain the reason why isoduality has remain undetected until recently [8]. Note also the independence of isoduality from space-time reflections. In fact, the former map one space into another while the latter occur within the same space, the former reverse the sign of the units while the latter preserve it, etc.

In particular, isodual numbers (those with unit $I^d = -I$) and isodual isonumbers (those with isounit $\widehat{I}^d = -\widehat{I}$) have a negative-definite nonn. This implies that all physical characteristics of antiparticles in isodual representation are negative-definite, including energy, time, angular momentum, etc. These negative-definite quantities are however referred to negative-definite units, by resulting in this way to be equivalent to positive-definite quantities referred to positive-definite units. This resolves the historical problematic aspects of negative-energy solutions of Dirac's equation when referred to positive-definite units.

Also, the negative-definite units have emerged as originating from the very structure of the conventional Dirac equation and, more specifically, from $\gamma_4 = \text{diag.}(I, -I)$, where $I = \text{diag.}(1,1)$. The Dirac equation then results to be invariant under the total spinorial symmetry $\mathcal{P}(3.1) \times \mathcal{P}^d(3.1)$ [3f,6] which is iso-self-dual, because $(I \times I^d)^d = l^d \times l \approx I \times l^d$.

The above isodual representation of antiparticles has stimulated a "new physics of antimatter" because it permits quantitative studies suitable for experimental verifications with contemporary technology of antigravity, the space-time machine and other topics beyond the capabilities of conventional theories (see [3f,6] for brevity).

We should also mention that the iso-self-dual symmetry of Dirac equations $\mathcal{P}(3.1) \times \mathcal{P}^d(3.1)$ has stimulated studies on a new cosmology called isocosmology [3f,6] which, in the limit case of equal distributions of matter and antimatter, implies a Universe with null total physical characteristics, i.e., null total energy, null total time, null total angular momentum, etc. It should be noted that the isodual Riemannian geometry [3f,6] on spaces $R^d(x, g^d, \mathrm{R}^d)$, $g^d = -g$, $\mathrm{R}^d \approx -\mathrm{R}$, permits a classical treatment of stars, galaxies and quasars as made up of antimatter. Such a classical treatment then admits a unique and unambiguous operator formulation for antiparticles via the iso-quantum- gravity of this note.

We finally note that the l.ie-isotopies were proposed as closed-reversible particular cases of the more general Lie-admissible genotopies for open-irreversible conditions [2a,3]. The Lie- admissible quantum gravity, or quantum genogravity (QGG) can be constructed from the formalism of this note by merely relaxing the condition that the isotopic element T is symmetric. QGG, rather than QIG, is more appropriate for the geometrization of interior irreversible gravitational processes and, as such it is the geometrization more appropriate of the novel Lie- admissible black hole dynamics recently introduced in ref [11] (see also ref.s [3f,12] for additional Lie-admissible studies of gravitation).

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