THE SEVENTH

MARCEL GROSSMANN MEETING

On recent developments in theoretical and experimental general relativity, gravitation, and relativistic field theories

Proceedings of the Meeting held at
Stanford University
24–30 July 1994

Editors

Robert T. Jantzen
Department of Mathematical Sciences
Villanova University
Villanova, PA 19085 USA

G. Mac Keiser
Gravity Probe B Relativity Mission
W. W. Hansen Experimental Physics Laboratory
Stanford University
Stanford, CA 94305 USA

Series Editor
Remo Ruffini

World Scientific
Singapore • New Jersey • London • Hong Kong

Dedicated to REMO RUFFINI in recognition of his efforts for the organization of the Marcel Grossmann Meetings on General Relativity

ISOTOPIC QUANTIZATION OF GRAVITY AND ITS UNIVERSAL ISOPOINCARE´ SYMMETRY

RUGGERO MARIA SANTILLI
The Institute for Basic Research, P. O. Box 1577, Palm Harbor, FL 34682, USA

ABSTRACT

We propose for the first time a novel isotopic quantization of gravity without Hamiltonian; we identify its universal symmetry as being isomorphic the Poincare´ symmetry; and we point out a number of intriguing implications.

We here outline a novel quantization of gravity without Hamiltonian first presented at the MG7 which is based on the so-called *isotopic methods* introduced by this author back in 1978[1], worked out in detail in monographs[2] and independently studied in ref.s[3].

The main idea is to lift the conventional associative product AB among generic quantities A, B, into the form $A*B = ATB$, where T is a fixed positive-definite quantity called *isotopic element*, while jointly lifting the original unit I of an amount equal but inverse of the deformation of the product, in which case $\hat{I} = T^{-1}$ is the correct left and right new unit, $\hat{I}*A = A*\hat{I} \equiv A$, called *isounit*. Such dual lifting is *isotopic* in the sense of preserving the original axioms[1a], e.g., the isotopic images of fields, vector spaces, algebras, geometries, etc., remain isomorphic to the original structures.

Isotopic liftings are physically non-trivial because the functional dependence of the isounit $\hat{I}$ remains unrestricted. As a result, the isotopic image of a linear-local-Lagrangian theory is given by a theory which is: a) arbitrarily nonlinear in the space-time coordinates x, wavefunctions $\psi(x)$, their derivatives of arbitrary order, $\dot{x}$, $\ddot{x}$, $\partial\psi$, $\partial\partial\psi$, ..., interior local density μ, temperature τ, etc.; b) arbitrarily nonlocal-integral; and c) non-(first-order)-Lagrangian (see ref.[2b] for the local-differential Birkhoffian/second-order-Lagrangian mechanic and ref.[2d] for the more general, nonlocal-integral isobirkhoffian mechanics). The lifting is also mathematically nontrivial because it requires the consequential isotopies of the *totality* of the mathematical structure of the original theory into a simple yet unique and nontrivial form admitting $\hat{I}$ as the new unit. This includes the lifting of: numbers; angles; fields; vector, metric and Hilbert spaces; trigonometry; functional analysis; Lie algebras, groups and symmetries; Euclidean, Minkowskian and Riemannian geometries; classical and quantum mechanics; etc.[2]

The above methods permit a new quantization of gravity hereon called *quantum isogravity*. Its carrier space is the *isominkowski space* introduced by this author back in 1983[4a]. Let $M(x,\eta,R)$ be a conventional Minkowski space in the chart $x = \{x^\mu\} = \{r, x^4\}$, $x^4 = c_o t$, where c_o is the speed of light in vacuum, $\eta = \mathrm{diag.}(1, 1, 1, -1)$, with invariant $x^2 = x^t\eta x$ on the field $R(n,+,\times)$ of real numbers n with conventional sum n+m and multiplication $n\times m = nm$. The lifting $\eta \rightarrow \hat{\eta} = T(x, \dot{x}, \ddot{x}, \psi, \partial\psi, \partial\partial\psi, \mu, \tau, ...)\eta$, where T is a 4×4 positive-definite matrix, while jointly lifting the unit $I \rightarrow \hat{I} = T^{-1}$, evidently preserves the original axioms of M, including flatness, resulting in the isospace $\hat{M}(x,\hat{\eta},\hat{R})$ over the *isofield* $\hat{R}(\hat{n},+,*)$ of *isonumbers* $\hat{n} = n\hat{1}$ with sum $\hat{n}+\hat{m} = (n+m)\hat{1}$ and isomultiplication $\hat{n}*\hat{m} = \hat{n}T\hat{m} = (nm)\hat{1}$. The lifting $M(x,\eta,R) \rightarrow \hat{M}(x,\hat{\eta},\hat{R})$ is geometrically nontrivial because the separation has the most general possible nonlinear integral form, e.g., of the diagonal type

$$x^2 = [\, x^1\, T_{11}(x, \dot{x}, ...)\, x^1 + x^2\, T_{22}(x, \dot{x}, ...)\, x^2 + x^3\, T_{33}(x, \dot{x}, ...)\, x^3 - x^4\, T_{44}(x, \dot{x}, ...)\, x^4\,]\,\hat{1} \in \hat{R}(\hat{n},+,*) \quad (1)$$

The primary application of the isominkowskian geometry is for the so-called *interior problem* (motion of extended relativistic particles or electromagnetic waves within inhomogeneous and anisotropic physical media such as planetary atmospheres or astrophysical chromospheres) studied in detail in ref.[2d] with a considerable number of exact-numerical representations of astrophysical data on quasars cosmological redshifts, internal redshifts and blueshifts, etc.

In this note we use for the first time the isominkowskian geometry for the *gravitational* characterization of the *exterior problem* (motion of point-like test bodies or electromagnetic waves within the homogeneous and isotropic vacuum). Its most fundamental implication is that *curvature is not necessary for the characterization of gravity because Riemannian metrics and equations are identically admitted by the isominkowskian geometry.* Let $\Re(x,g,R)$ be a conventional (3+1)-Riemannian space with symmetric and real-valued metric $g(x)$ and separation $x^2 = x^t g x$ over the reals R. It is easy to see that $g(x)$ is *identically* admitted *as a particular case* of the isominkowskian metric $\hat{\eta}(x, \dot{x}, \ddot{x}, ...)$ resulting in the local isomorphism $\Re(x,g,R) \approx \hat{M}(x,\hat{\eta},\hat{R})$, $g(x) \equiv \hat{\eta}(x)$.

The main idea of quantum isogravity is to embed gravitation in the unit of a conventional relativistic quantum field theory (RQFT). This is permitted by the isotopic methods via the factorization of any given Riemannian metric in the form $g(x) = T(x)\eta$, where $T(x)$ is always positive-definite from the locally Minkowskian character of $\Re$, and the lifting of the unit $I = \mathrm{diag.}\,(1, 1, 1, 1)$ of any given RQFT into the *gravitational isounit* $\hat{1} = [T(x)]^{-1}$ which evidently contains all the essential elements of the original curvature. Note that T can always be diagonalized from its positive-definiteness, the metric for raising and lowering the indices in $\hat{M}$ is $\hat{\eta}(x) \equiv g(x)$, and $\hat{1} = (\hat{1}^{\mu}{}_{\nu}) = (\hat{1}_{\mu}{}^{\nu}) = (\hat{1}_{\mu\nu}) = (\hat{1}^{\mu\nu})$.

A consistent isoquantization of gravity then requires the lifting of the totality of the mathematical structure of RQFT into that of the iso-RQFT, also known as *relativistic hadronic mechanics.*[2d] We here recall: the liftings $R(n,+,\times) \rightarrow \hat{R}(\hat{n},+,*)$ and $M(x,\eta,R) \rightarrow \hat{M}(x,\hat{\eta},\hat{R})$ outlined above; the lifting of the enveloping operator algebra ξ over the field of complex numbers $C(c,+,\times)$ with generic product AB into the isotope $\hat{\xi}$ with isoproduct $A*B = ATB$ over $\hat{C}(\hat{c},+,*)$; the lifting of the original Hilbert space $\mathcal{H}$ with inner product $<|> \in C$ into the *isohilbert space* with *isoinner product* $<\hat{|}> = <|T|>\hat{1} \in \hat{C}$ under which originally Hermitean-observable quantities remain Hermitean-observable; the lifting of eigenvalue equations $H|> = E_o|>$ into the isotopic form $H*|> = HT|> = \hat{E}*|> = E\hat{1}T|> \equiv E|>$, $E \neq E_o$ (necessary for isolinearity) indicating that the final numbers of the theory are the conventional ones; the lifting of the operator four-momentum $p_{\mu}|> = -i\,\partial_{\mu}|>$ into the isoform $p_{\mu}*|> = -i\,\hat{\partial}_{\mu}|>$, where $\hat{\partial}_{\mu} = \hat{1}\partial/\partial x^{\mu}$ is the *isoderivative*, and the compatible liftings of the remaining aspects of RQFT (see ref.[2d] for brevity).

Most important are the following properties: 1) the isotopic image of the original RQFT is invariant under its own time evolution; 2) the iso-RQFT admits the conventional theory as a particular case for $\hat{1} \equiv I$; and 3) iso-RQFT and RQFT coincide at the abstract level in which (from $T > 0$) $\hat{R}(\hat{n},\hat{+},\hat{*}) \equiv R(n,+,\times)$, $\hat{M}(x,\hat{\eta},\hat{R}) \equiv M(x,\eta,R)$, $\hat{\xi} \equiv \xi$, $\hat{\mathcal{H}} \equiv \mathcal{H}$, etc. In turn, these abstract identities assure the mathematical and physical consistency of iso-RQFT.

In conclusion, the main conjecture submitted in this note is that *a consistent operator form of gravity already exists. It did creep in un-noticed until now because it is embedded in the unit of conventional* RQFT. As an illustration, the embedding of gravity in Dirac's equation for a diagonal isounit (which is assumed hereon) can be written

$$(\hat{\gamma}^{\mu} * p_{\mu} + i\,\hat{m}) * |> = [\,\hat{\gamma}^{\mu}(x)\,T(x)\,\hat{\eta}_{\mu\nu}(x)\,p^{\nu} - i\,m\,\hat{1}\,]\,T(x)\,|> = 0, \qquad (2a)$$

$$\{\hat{\gamma}^{\mu} \hat{,} \hat{\gamma}^{\nu}\} = \hat{\gamma}^{\mu}\,T\,\hat{\gamma}^{\nu} + \hat{\gamma}^{\nu}\,T\,\hat{\gamma}^{\mu} = 2\,\hat{\eta}^{\mu\nu} \equiv 2\,g^{\mu\nu}, \quad \hat{\gamma}^{\mu} = T_{\mu\mu}{}^{1/2}\,\gamma^{\mu}\,\hat{1}, \qquad (2b)$$

where γ^{μ} are the conventional gammas and $\hat{\gamma}^{\mu}$ are called *isogamma matrices*. The important point is that at the abstract level the conventional and isogravitational Dirac's equations coincide, $(\gamma^{\mu}p_{\mu} + im)|> \equiv (\hat{\gamma}^{\mu}*p_{\mu} + i\hat{m})*|>$. Note that *the anticommutator of the isogamma matrices yields (twice) the Riemannian metric* $g(x)$, thus confirming the full embedding of gravitation. A similar isotopic realization of gravity can be formulated for any other RQFT. As an example, the *Dirac-Schwartzschild equation* (here presented for the first time) is given by Eq.s (2) with $\hat{\gamma}_k = (1 - 2M/r)^{-1/2}\gamma_k\hat{1}$ and $\hat{\gamma}_4 = (1 - 2M/r)^{1/2}\gamma_4\hat{1}$. Similarly one can construct the *Dirac-Krasner equation* and others.

By no means the above quantum gravity is a mere curiosity because it carries rather deep geometrical, theoretical and experimental implications, such as:

Consequence 1: Quantum gravity permits the introduction for the first time of a universal symmetry for gravitation called isopoincare' symmetry $\hat{P}(3.1)$, ***which results to be locally isomorphic to the conventional symmetry*** $P(3.1)$. The isosymmetry can be readily constructed via the Lie-isotopic theory[1-6] and consists in the reconstruction of $P(3.1)$ for the gravitational isounits $\hat{1} = [T(x)]^{-1}$, $g(x) = T(x)\eta$. Since $\hat{1} > 0$, one can see from the inception that $\hat{P}(3.1) \approx P(3.1)$. Under the the lifting $P(3.1) \to \hat{P}(3.1)$ the original generators $X = \{X_k\} = \{M^{\mu}{}_{\nu}, p_{\alpha}\}$, $M^{\mu}{}_{\nu} = x^{\mu}p_{\nu} - x^{\nu}p_{\mu}$ $k = 1, 2, \ldots, 10$, $\mu, \nu = 1, 2, 3, 4$, remain unchanged while the original parameters $w = \{w_k\} = \{(\theta, v^{\circ}), a^{\circ}\} \in R$ become isonumbers, $\hat{w} = w\hat{1} \in \hat{R}$. The connected component $\hat{P}_o(3.1)$ of the isopoincaré symmetry $\hat{P}(3.1)$ can then be written

$$\hat{P}_o(3.1): \; \hat{A}(\hat{w}) = \prod_k \hat{e}^{i\,X * \hat{w}} = (\prod_k e^{i\,X\,T\,w})\,\hat{1}, \qquad (3)$$

where $\hat{e}^{A} = (e^{A\,T})\hat{1} = \hat{1}(e^{TA})$ is the *isoexponentiation* as characterized by the *isotopic Poincaré-Birkhoff-Witt Theorem* originally derived in ref.[1a], while the preservation of the original dimension is ensured by the the *isotopic Baker-Campbell-Hausdorff Theorem* also originally derived in ref.[1a] (see the ref.s[3a,3b,3c] for algebraic, geometric and historical aspects, respectively). It is easy to see that structure (3) forms a connected Lie-isotopic transformation group with *isogroup laws* $\hat{A}(\hat{w})*\hat{A}(\hat{w}') = \hat{A}(\hat{w}')*\hat{A}(\hat{w}) = \hat{A}(\hat{w}+\hat{w}')$, $\hat{A}(\hat{w})*\hat{A}(-\hat{w}) = \hat{A}(0) = \hat{1} = [T(x)]^{-1}$. Note that $\hat{P}(3.1)$ acts *isotransitively* in $\hat{M}(x,\hat{\eta},\hat{R})$, i.e., $x' = \hat{A}(\hat{w})*x$, because the preservation of the original action Ax would now violate isolinearity.

To identify the isoalgebra $\hat{p}_o(3.1)$ of $\hat{P}_o(3.1)$, we note that the *canonical isocommutation rules* are $[x^{\mu} \hat{,} p_{\nu}] * |> = (x^{\mu} * p_{\nu} - p_{\mu} * x^{\nu}) * |> = i\,\hat{\delta}^{\mu}{}_{\nu} * |> = i\,\hat{1}^{\mu}{}_{\alpha}\,\delta^{\alpha}{}_{\nu} * |>$. The isotopic lifting of the conventional transition from a Lie group to a Lie algebra (see the recent study[5]) then yields the isocommutation rules of $\hat{p}_o(3.1)$

$$[M^{\mu}{}_{\nu} \,\hat{,}\, M^{\alpha}{}_{\beta}] = i(\hat{1}_{\nu}{}^{\alpha} M^{\mu}{}_{\beta} - \hat{1}^{\mu}{}_{\alpha} M^{\nu}{}_{\beta} - \hat{1}^{\nu}{}_{\beta} M^{\mu}{}_{\alpha} + \hat{1}^{\mu}{}_{\beta} M^{\alpha}{}_{\nu}), \tag{4a}$$

$$[M^{\mu}{}_{\nu} \,\hat{,}\, p_{\alpha}] = i(\hat{1}^{\mu}{}_{\alpha} p_{\nu} - \hat{1}^{\nu}{}_{\alpha} p_{\mu}), \qquad [p_{\alpha} \,\hat{,}\, p_{\beta}] = 0, \tag{4b}$$

where $[A \hat{,} B] = AT(x)B - BT(x)A$ is the *Lie-isotopic product* originally proposed in ref.[1a] which does indeed verify the Lie axioms as one can see. Since the elements $\hat{1}^{\mu}{}_{\mu}$ are positive-definite and $\hat{1}^{\mu}{}_{\nu} = 0$ for $\mu \neq \nu$, rules (4) confirm the local isomorphism $\hat{p}_o(3.1) \approx p_o(3.1)$. Note that *momentum operators become commutative in their isominkowskian representation* (while they are notoriously noncommutative in their Riemannian representation). This confirms the achievement of a *representation of gravitation in a flat space*. The *isocasimir invariants* are

$$C^{(0)} = \hat{1} = [T(x)]^{-1}, \quad C^{(1)} = p^{\hat{2}} = p_{\mu} * p^{\mu} = p_{\mu} * \hat{\eta}^{\mu\nu} p_{\nu}, \quad C^{(3)} = \hat{W}_{\mu} * \hat{W}^{\mu}, \; \hat{W}_{\mu} = \epsilon_{\mu\alpha\beta\rho} M^{\alpha\beta} * p^{\rho}. \tag{5}$$

Under sufficient boundedness and continuity properties of the $T_{\mu\mu}$ elements, the original convergence of $P_o(3.1)$ into finite transforms ensures the convergence of their isotopic images which can then be readily computed from Eq.s (3). The space components $S\hat{O}(3)$, called *isorotations*, were first computed in ref.[6a] and can be written for a rotation in the (x, y)-plane

$$x' = x \cos(T_{11}^{\frac{1}{2}} T_{22}^{\frac{1}{2}} \theta_3) - y T_{11}^{-\frac{1}{2}} T_{22}^{\frac{1}{2}} \sin(T_{11}^{\frac{1}{2}} T_{22}^{\frac{1}{2}} \theta_3), \tag{6a}$$

$$y' = x T_{11}^{\frac{1}{2}} T_{22}^{-\frac{1}{2}} \sin(T_{11}^{\frac{1}{2}} T_{22}^{\frac{1}{2}} \theta_3) + y \cos(T_{11}^{\frac{1}{2}} T_{22}^{\frac{1}{2}} \theta_3), \tag{6b}$$

(see ref.[2d] for general isorotations in the three Euler angles). Isotransforms (6) leave invariant all ellipsoidical deformations of the sphere in the Euclidean space $E(r,\delta,R)$, $r = \{x, y, z\}$, $\delta = \text{diag.}(1, 1, 1)$. Such ellipsoids become perfect spheres $r^{\hat{2}} = (r^t \hat{\delta} r)\hat{1}_s$ in *isoeuclidean spaces* $\hat{E}(r,\hat{\delta},\hat{R})$, $\hat{\delta} = T_s\delta$, $T_s = \text{diag.}(T_{11}, T_{22}, T_{33})$, $\hat{1}_s = T_s^{-1}$, called *isospheres*, because of the joint lifting of the semiaxes $1_k \to T_{kk}$ and of the related units $1_k \to T_{kk}^{-1}$. This perfect isosphericity is the geometric origin of the isomorphism $\hat{O}(3) \approx O(3)$

The space-time isosymmetry $S\hat{O}(3.1)$ is characterized by the above isorotations and the *isolorentz boosts* originally derived in ref.[4a] which can be written say, in the (z, t)-plane, in terms of the conventional parameter v

$$z' = z \sinh(T_{33}^{\frac{1}{2}} T_{44}^{\frac{1}{2}} v) - t T_{33}^{-\frac{1}{2}} c_o T_{44}^{\frac{1}{2}} \cosh(T_{33}^{\frac{1}{2}} T_{44}^{\frac{1}{2}} v) = \hat{\gamma}(x^3 - \hat{\beta} x^4), \tag{7a}$$

$$t' = z T_{33}^{\frac{1}{2}} c_o^{-1} T_{44}^{-\frac{1}{2}} \sinh(T_{33}^{\frac{1}{2}} T_{44}^{\frac{1}{2}} v) + t \cosh(T_{33}^{\frac{1}{2}} T_{44}^{\frac{1}{2}} v) = \hat{\gamma}(x^4 - \hat{\beta} x^3)/c_o, \tag{7b}$$

$$\beta = v/c_o, \; \hat{\beta} = v_k T_{kk}^{\frac{1}{2}} / c_o T_{44}^{\frac{1}{2}}, \; \hat{\gamma} = |1 - \hat{\beta}^2|^{-1/2}. \tag{7c}$$

Note that the above isotransforms are *nonlinear*, as expected for a correct symmetry of gravitation, and are formally similar to the Lorentz transforms, as expected from their isotopic character. For $T_{\mu\mu} = 1/n_{\mu}^2$ one can introduce the "isogravitational speed of light" $c = c_o/n_4$. Isotransforms (7) then characterize the *gravitational isolight cone*, i.e., the perfect cone in isospace $\hat{M}(x,\hat{\eta},\hat{R})$, including the conventional characteristic angle (the derivation of the latter property requires the isotrigonometry[2b] and it is omitted for brevity), which is the geometric origin of the isomorphisms $\hat{O}(3.1) \approx O(3.1)$.

The *isotranslations* can be written $x' = (\hat{e}^{ipa}) * x = x + a^{\circ}A(x)$, $p' = (\hat{e}^{ipa}) * p = p$, where $A_{\mu} = T_{\mu}{}^{1/2} + a^{\circ\alpha}[T_{\mu}{}^{1/2} \hat{,} p_{\alpha}]/1! + \ldots$ with "gravitational isoplanewave" $\psi = \hat{e}^{kx} = \{\exp[kT_s r -$

$k_4 T_t c_0 \hat{1} \} \hat{1}$. The full $\hat{P}(3.1)$ isosymmetry is then given by adding the *isoinversions*[6] $\hat{\pi} * x = (-r, t)$, $\hat{\tau} * x = (r, -t)$, $\hat{\pi} = \pi \hat{1}$ and $\hat{\tau} = \tau \hat{1}$ with π and τ conventional inversion operators (see ref.[4b] for a recent detailed study of the classical isopoincaré symmetry).

As one can see, the isopoincaré transforms provide the universal isosymmetry of all infinitely possible invariants (1).. In particular, there is *absolutely nothing to compute* in the sense that for any arbitrarily given (diagonal) Riemannian metric $g(x)$ (such as Schwartzschild, Krasner, etc.[7]) one merely *plots* the $T_{\mu\mu}$ terms in the decomposition $g_{\mu\mu} = T_{\mu\mu}\eta_{\mu\mu}$ (no sum) in the isotransforms. The invariance of the separation $x^t g x$ is then ensured by the above derivation.

One of the primary results of this note is the elimination of the historical difference between the special and general relativities whereby the former admits the universal P(3.1) symmetry, while the latter does not. In fact, in this note we have established the *universal* character of the isopoincaré symmetry $\hat{P}(3.1)$ for all possible (3+1)–dimensional gravitations (the (2+2)–de Sitter or other cases being given by a mere changes of signature or dimension). The gravitational field on $\hat{M}(x,\hat{\eta},\hat{R})$ must now be isocovariant under $\hat{P}(3.1)$ in essentially the same way as the electromagnetic field on $M(x,\eta,R)$ must be covariant under P(3.1). Note the *necessity* of the *flat* representation of gravity for the very formulation of its universal isopoincaré symmetry.

Consequence 2: The isotopic formulation of quantum gravity implies the geometric unification of the special and general relativity. This is evidently due to the fact that all related topological distinctions are now lost owing to the abstract identities $\hat{R}(\hat{n},\hat{+},\hat{*}) \equiv R(n,+,R)$, $\hat{M}(x,\hat{\eta},\hat{R}) \equiv M(x,\eta,R)$, $\hat{\mathcal{H}} \equiv \mathcal{H}$, $\hat{\xi} \equiv \xi$, $\hat{P}(3.1) \equiv P(3.1)$, etc.

Consequence 3: Quantum isogravity permits a novel approach to the unification of weak, electromagnetic and gravitational interactions via the embedding of gravity in the unit of conventional unified gauge theories. We can therefore submit the conjecture that gravitation is *already* contained in the existing unified gauge theories. It did escape identification until now because it is embedded in the *unit* of the theory (see also ref.[1b] for the isotopies of electromagnetic interactions). This *iso-grand-unification* will be studied elsewhere.

Consequence 4: Quantum isogravity permits a fundamentally novel approach to gravitational horizons as the zeros of (the space component of) the isounit and of gravitational singularities as the zeros of (the space component of) the isotopic element. In fact, at the Schwartzschild's horizon $r = 2M$ the space isounit $\hat{1}_s = (1 - 2M/r) \times \text{diag.}(1, 1, 1)$ of the isosphere $r^2 = (r^t \hat{\delta} r)\hat{1}_s$ is null, while at $r = 0$ the space isotopic element $T_s = (1 - 2M/r)^{-1} \times \text{diag.}(1, 1, 1)$ is null. Recall in this respect that the restriction of the isounits/isotopic elements to a sole x–dependence is grossly un–necessary for isotopic theories. The extension of the above *exterior quantum isogravity* to the corresponding *interior quantum isogravity* is merely given by assuming an unrestricted functional dependence of the isounit, $\hat{1} = \hat{1}(x, \dot{x}, \ddot{x}, \psi, \partial\psi, \partial\partial\psi, \mu, \tau, \ldots)$. A more adequate formulation of gravitational horizons and singularities is then given by the zeros of the (space component of) the latter isounits and isotopic elements. This extension appear to permit a second generation of studies on gravitational collapse, black holes and all that, which is essentially expected to produce contributions to the existing knowledge in the field[7] due to the local interior variation of the speed of light, the internal nonlinear–nonlocal–nonlagrangian effects due to deep particle overlappings, and other topics. These aspects too will be studied elsewhere for brevity.

Consequence 5: Space and time in quantum gravity acquire a local character in the sense that their isounits have an explicit dependence on the local gravitational field itself. In essence, the isotopic reformulation of gravity implies the redefinition $x^t g x = \bar{x}^t \eta \bar{x}$, $g = T\eta$, $\bar{x} = xT^{\frac{1}{2}}$, thus resulting in the *space isounits* $\hat{1}_k = T_{kk}^{-1/2}$ and *time isounit* $\hat{1}_t = T_{44}^{-1/2}$ (those of the isolight cone are instead T_{kk}^{-1} and T_{44}^{-1}). As an example, the space–time isounits for an observers in the exterior Schwartzschild field are given by $\hat{1}_k = (1 - 2M/r)^{1/2} 1_k$ and $\hat{1}_t = (1 - 2M/r)^{-1/2}$. Note that the isogravitational theory recovers the relativistic Einsteinian space–time for $M = 0$ or $r \to \infty$, for which $\hat{1}_k \equiv \hat{1}_t = 1$. However, for a non-null gravitational field the isounits are different then the conventional units thus resulting in novel notions of space and time which are different for observers throughout the Universe with the same speed relative to an inertial frame but different gravitational fields. This poses the intriguing *experimental* question whether time here on Earth's atmosphere and, say, time on Jupiter's atmosphere are different due to the difference of their gravitational fields predicted by the isounit $\hat{1}_t = (1-2M/r)^{-1/2}$, $r>2M$. Stated differently, the issue requiring experimental resolution is whether we live in conventional space–time, in which case the Riemannian description of gravitation is the physical one and the equivalent isominkowskian formulation has a mere mathematical character, or we live in isospace and isotime, in which case the isominkowskian description is the physical one and the Riemannian description has a mathematical value. An experimental proposal to resolve this issue by sending a probe to Jupiter will be studied in detail elsewhere.

As a final comment we note that the *Lie-isotopies* were proposed[1a] as closed-reversible particular cases of the more general *Lie-admissible genotopies* for open-irreversible conditions, the latter ones emerging when T is no longer Hermitean.[2c,2d]

1. R. M. Santilli, Hadronic J. 1, 223 (1978 (1a) and Phys. Rev. D 20, 555 (1979 (1b)
2. R. M. Santilli, *Foundations of Theoretical Mechanics*, Vol.s I (1978) (2a), II (1983) (2b), Springer-Verlag, Heidelberg-New York; and *Elements of Hadronic Mechanics*, Vol I (1993) (2c), II (1994) (2d) and III (to appear) (2e), Academy of Sciences of Ukraine, Kiev.
3. D. S. Sourlas and G. T. Tsagas, *Mathematical Foundations of the Lie-Santilli Theory*, Academy of Sciences of the Ukraine, Kiev (1993) (3a); J. V. Kadeisvili, *Santilli's Isotopies of Contemporary Algebras, Geometries and Relativities*, first edition (1991), second edition, Academy of Sciences of the Ukraine, Kiev (1994) (3b); J. Lôhmus, E. Paal and L. Sorgsepp, *Nonassociative Algebras in Physics*, Hadronic Press, Palm Harbor, FL, USA (1994) (3c)
4. R. M. Santilli, *Lett. Nuovo Cimento* 37, 545 (1983) (4a), and *J. Moscow Phys. Soc.*, in press (4b)
5. J. V. Kadeisvili in *Analysis, Geometry and Groups: A Riemann Legacy Volume*, H. Srivastava and Th. Rassias, Editors, Hadronic Press, Palm Harbor, FL, USA
6. R. M. Santilli, Hadronic J. 8, 25 and 36 (1985) (6a); JINR Rapid Comm. 6, 24 (1993)(6b)
7. C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation*, Freeman, San Francisco (1973) (7a); R. Giacconi and R. Ruffini, *Physics and Astrophysics of Neutron Stars and Black Holes*, North Holland, Amsterdam (1978)(7b); *Proceedings of the Seventh M. Grossmann Meeting on Gravitation*, World Scientific, Singapore, in press. (7c)