

**NONLOCAL FORMULATION OF THE BOSE-EINSTEIN
CORRELATION WITHIN THE CONTEXT OF
HADRONIC MECHANICS,**

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Abstract

In this paper we submit the hypothesis that the Bose-Einstein correlation originates from a nonlocal component of the strong interactions in the interior of the fireball caused by the deep overlapping of the charge distributions-wavepackets of the $p\bar{p}$ collision at high energy, along the historical legacy of Bogoliubov, Fermi and others. Owing to their contact nature, the notion of potential energy has no meaning for the nonlocal internal interactions here considered which, as such, are structurally outside the representational capabilities of quantum mechanics on analytic, topological, operator and other grounds. We therefore study the Bose-Einstein correlation via the covering hadronic mechanics, which consists of axiom-preserving isotopies of quantum mechanics derived from the fundamental isotopy, the generalization of Planck's constant $\hbar = 1$ into an integrodifferential unit $\hat{1} = \hat{1}(t, r, p, \hat{p}, \psi, \psi^\dagger, \partial\psi, \partial\psi^\dagger, \dots)$, $\det. \hat{1} \neq 0, \hat{1} = \hat{1}^\dagger, \hat{1} > 0$. Because of to the disparate character of the existing literature, we first review for the reader's convenience the elements of: the classical foundations of the isotopies; the isotopic quantization, the elements of nonrelativistic and relativistic hadronic mechanics; the underlying generalized notions of isoparticle and isocomposite system; the isotopic

symmetries and relativities that are applicable to the fireball with internal nonlocal and nonhamiltonian interactions; and other aspects. We then study the interior problem of the Bose-Einstein correlation via isorelativistic hadronic mechanics, and shows that it is indeed directly derivable from first principles via the joint use of conventional Hamiltonians $H = H(r, p)$ and generalized units $\hat{I} = \hat{I}(t, r, p, \dot{p}, \psi, \psi^\dagger, \partial\psi, \partial\psi^\dagger, \dots)$. More specifically, we show that the Bose-Einstein correlation is outside the structural axioms of quantum mechanics, e.g., that of the conventional expectation values $\langle \psi | \psi \rangle$, while it is directly derivable from the corresponding generalized axioms, e.g., that of the isoexpectation values $\langle \psi | \hat{I} | \psi \rangle = \langle \psi | T | \psi \rangle$, $\hat{I} = T^{-1}$. We then study the exterior problem of the Bose-Einstein correlation via a direct representation of the prolate spheroidal ellipsoidal shape of the fireball, achieved by means of the factorization of the isounit $\hat{I} = \{ \text{diag. } (b_1^2, b_2^2, b_3^2, -b_3^2, b_4^2) \} \hat{I}(0)$ where the b 's are certain "characteristics quantities of the fireball" and $\hat{I}(0)$ is a certain nonlocal and nondiagonal operator. In the final section we identify our properly normalized, two-particle isocorrelation function in a form ready for plots with experimental data, as well as preliminary estimates of the value of its four characteristics parameters b_i .

In summary, our analysis shows that, on rigorous grounds, the Bose-Einstein correlation is outside the structure of quantum mechanics, while its representation via the covering hadronic mechanics can:

- 1) identify the origin of the Bose-Einstein correlation in the nonlocal internal effects due to deep wave-overlapping of the $p\bar{p}$ collision at high energy;
- 2) provide a quantitative representation of the correlation from basic principles under the sole conventional approximation that the q_0 and q_1 components of the momentum transfer are very small;
- 3) directly represent the actual shape of the fireball, a prolate spheroidal ellipsoid oriented along the original $p\bar{p}$ direction, as well as its rapid expansion in time, all this prior to any second quantization;
- 4) identify the maximal value 1.67 for the two-points correlation function on exact grounds from basic axioms without approximations; and
- 5) provide a satisfactory representation of available experimental data on the Bose-Einstein correlation.

But, in the opinion of the author, the most important result of this analysis is that the current experimental data on Bose-Einstein correlation can result to be a direct experimental evidence on the validity of the historical legacy of Bogoliubov, Fermi and others on the ultimate nonlocal structure of strong interactions, with consequential far reaching implications, such as the need for a new generation of covering relativities.

dedicated to the memory of

N. N. BOGOLIUBOV,

*because of his teaching on truly fundamental physical issues,
the only one which passes the test of time.*

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1. INTRODUCTION

The physical event generally known as the *Bose-Einstein correlation* (see, e.g., réf. [1] and quoted papers) constitutes an intriguing but vastly unexplained phenomenon, whereby two or more uncorrelated particles (such as the proton-antiproton annihilation in UA1 experiments at CERN) first interact, then coalesce into what is generally called a *fireball* [2], and finally decay in a variety of unstable particles whose final detectable product is a set of correlated bosons.

In this paper we submit the hypothesis that the correlation originates in nonlocal interactions expected in the $p\bar{p}$ annihilation, as well as in the interior of strong interactions at large, according to the historical legacy of Bogoliubov, Fermi and others. Stated differently, our hypothesis implies that the correlation is a phenomenon strictly due to the extended character of the charge distribution of hadrons and of their wavepackets, and that, whenever the interacting particles are at sufficiently large distances to permit their effective point-like approximation, no correlation is possible.

The above hypothesis also explains the inability to reach a final understanding of the phenomenon with ordinary quantum mechanics. In fact, quantum mechanics is strictly local-differential in topological structure. This implies the quantum mechanical inability to provide a quantitative treatment of nonlocal interactions via first principles, as well known.

Moreover, quantum mechanics is structurally of potential-Hamiltonian type, namely, it can only represent action-at-distance interactions described by a potential. On the contrary, nonlocal effects due to deep mutual penetration of wavepackets are well known at the classical level to be of *contact type*, namely, interactions for which the notion of potential energy has no physical meaning [3]. As such, contact, nonlocal interactions are conceptually, topologically and analytically outside the representational capabilities of quantum mechanics.

In an attempt to overcome these limitations, we submitted back in 1978¹ the proposal to construct the so-called axiom-preserving *isotopies* of the conventional Lie's theory, under the name of *Lie-isotopic theory* [3] and of quantum mechanics under the name of *hadronic mechanics* [4], while providing certain technical means to study nonhamiltonian forces, called *conditions of variational nonselfadjointness* [5].

¹ When at Harvard university under support from the U.S. Department of Energy, contract numbers ER-78-S-02-4742, AS02-78ER-4742, and DE-Ac02-8-ER10651.

These proposals were subsequently studied by numerous authors (see, e.g., the recent Proceedings [6] and quoted literature). In particular, the proposal to build hadronic mechanics reached sufficient maturity of formulation in papers [7,8] and in the more recent memoirs [9].

The classical foundations of the theory were first identified in the *Birkhoffian generalization of conventional Hamiltonian mechanics* [10], as a first step in the study of the so-called *interior dynamical problem*, i.e., extended and deformable particles moving within inhomogeneous and anisotropy physical media. In particular, Birkhoffian mechanics resulted to be "directly universal"² for all nonlinear and nonhamiltonian vector-fields in local-differential approximation verifying certain topological conditions.

The extension of these results to the most general possible (classical) nonlinear, nonlocal-integral and nonhamiltonian systems was first done in memoirs [11]. These latter studies reached mathematical maturity in memoirs [12,13] via the identification of axioms-preserving isotopies of the conventional symplectic, affine and Riemannian geometries, under the corresponding names of *isosymplectic*, *isoaffine* and *isoriemannian geometries*.

Sufficient physical maturity in the nonrelativistic, relativistic and gravitational treatment of the most general known, nonlinear, nonlocal and nonhamiltonian system of the interior problem was then reached in monographs [14,15] via the submission of certain isotopies of conventional relativities under the names of *isogalilean*, *isospecial* and *isogeneral relativities*. Independent reviews can be found in ref.s [16-18].

Additional studies have shown the existence of a unique and unambiguous *isotopic quantization* mapping the classical isotopic formulations [14,15] into the corresponding operator forms [7-9], today known under the name of *hadronization* [9,19,20]. This indicates the existence of a unity of thought between the classical and operator formulations, which is mathematically expressed in both cases by the notion of isotopy, and physically represented, also in both cases, by the treatment of nonlocal and nonhamiltonian interactions.

The operator formulation of the isotopic theories follows very closely the classical ones. A central conceptual idea is the admission that there exist physical conditions in the universe simply outside the representational capabilities of a Lagrangian or Hamiltonian operator. This is the case for: the deformation of shape of extended charge

² Throughout this analysis, by "direct universality" we shall mean the capability of representing all systems of the class admitted ("universality"), directly in the frame of the experimenter ("direct universality").

distributions; the inhomogeneity and anisotropy of physical media in the interior of stars, nuclei and hadrons; the nonlinear, nonlocal and nonpotential interactions originating from mutual overlappings of the wavepackets of particles, as expected in the structure of strong interactions.

For any given mathematical or physical structure, its *isotopic images* can be essentially conceived as the most general possible, nonlinear, nonlocal and noncanonical realizations of the original axioms.

The fundamental isotopy characterizing hadronic mechanics, from which all other aspects of the theory can be derived, is the generalization of Plack's unit $\hbar = 1$ of quantum mechanics into the most general possible unit $\hat{1}$, called *isounit*, which verifies the original axioms of \hbar (nonsingularity, Hermiticity and positive-definiteness), but possesses an unrestricted, integro-differential dependence on all local variables, wavefunctions and their derivatives, as well as any needed additional quantity,

$$\hbar = 1 \quad \Rightarrow \quad \hat{1} = \hat{1}(t, r, p, \dot{p}, \psi, \psi^\dagger, \partial\psi, \partial\psi^\dagger, \dots), \quad (1.1a)$$

$$\det. I \neq 0, \quad I = I^\dagger, \quad I > 0 \quad \Rightarrow \quad \det \hat{1} \neq 0, \quad \hat{1} = \hat{1}^\dagger, \quad \hat{1} > 0. \quad (1.1b)$$

In a way much similar to the classical case [14,15], all action-at-a-distance interactions are represented with the Hamiltonian operator $H = T + V$, while all nonlocal and nonpotential interactions are embedded in the isounit $\hat{1}$. Conventional operator formulations must then be generalized in such a way to admit $\hat{1}$ as the correct, right and left operator unit.

The latter aspect essentially requires a suitable generalization of the entire structure of quantum mechanics, such as: the enveloping operator algebra, the attached Lie algebra, the underlying Hilbert space, the Heisenberg's and Schrödinger's representations, the unitary transformation theory, etc.

However, since the new mechanics is an isotopy of quantum mechanics, the generalization is axiom-preserving. In fact, the emerging covering mechanics coincides, by construction, with the conventional quantum mechanics at the abstract, realization-free level, thus establishing its mathematical consistency.

As a result of these efforts we can say today that hadronic mechanics has reached sufficient mathematical maturity as a covering of quantum mechanics for the quantitative representation of extended-deformable particles with linear and nonlinear, local and nonlocal,

deformable particles with linear and nonlinear, local and nonlocal, Hamiltonian as well as nonhamiltonian interactions in a form suitable for experimental verification.

The Bose-Einstein correlation appears to be an ideal setting to test the physical effectiveness of the new mechanics, particularly when studied under our basic hypothesis of its nonlocal and nonhamiltonian origin.

This paper has been written to be as self-sufficient as possible, owing to the disparate nature of the existing literature, and to provide the interested phenomenologist in Bose-Einstein correlation with the necessary theoretical methods for experimental plots. In particular, a primary objective is to submit, apparently for the first time, an isotopic model of Bose-Einstein-correlation from basic principles in a form suitable for direct plotting with experimental data. The agreement of the model with available measures is rather impressive, with the understanding that it needs additional, independent experimental plots.

The paper is organized as follows. In Sect. 2 we identify the most important limitations of quantum mechanics, in general, and for the Bose-Einstein correlation, in particular. In Sect. 3 we outline the classical isotopic formulations, while in Sect. 4 we review the elements of their mapping into operators forms. In Sect. 5 we provide the elements of hadronic mechanics, with an outline of the apparent resolution of the limitations identified in Sect. 2. Sect. 6 is devoted to the identification of the generalized notion of composite hadronic systems characterizing the fireball. In Sect. 7 we outline the isotopic space-time symmetries and relativities applicable to a fireball with internal nonlocal and nonhamiltonian effects.

We then pass in Sect. 8 to the specific study of the Bose-Einstein correlation via *the isotopic description of the interior problem of the fireball with a nonlocal origin of correlation* ; then in Sect. 9 we introduce the *isotopic formulation of the exterior experimental detection of correlated bosons* ; and in Sect. 10 we present *the comparison of our isotopic model with experimental data* . The primary results are summarized in Sect. 11.

As we shall see, the results of this paper illustrate rather clearly the physical effectiveness, as well as the horizon of truly novel possibilities offered by a quantitative treatment of the historical legacy on the ultimate nonlocal structure of matter.

2. BASIC LIMITATIONS OF QUANTUM MECHANICS

As well known, *quantum mechanics* (see, e.g., ref. [21] and quoted literature) was conceived for the atomic structure, as well as, more generally, for the electromagnetic interactions at large, for which it subsequently resulted to be exact according to an overwhelming amount of experimental evidence.

The central notion of quantum mechanics is Planck's quantum of energy \hbar , which is the basic algebraic unit of the theory. The primary mathematical structures of the theory are given by:

A) The *universal, enveloping, associative, operator algebra* ξ with elements A, B, \dots (say, matrices or local-differential operators) and product given by the trivial associative product AB , under which Planck's constant assumes the meaning of the *left and right unit* of the theory

$$\xi: \left\{ \begin{array}{l} \hbar = 1, \\ AB = \text{assoc.}, \quad 1A \equiv A1 \equiv A \quad \forall A \in \xi; \end{array} \right. \quad (2.1a)$$

B) The *field* F of real numbers \mathbb{R} or of complex numbers \mathbb{C} .

C) The *Hilbert space* \mathcal{H} with states $|\psi\rangle, |\phi\rangle, \dots$, and inner product

$$\mathcal{H}: \quad \langle \psi | \phi \rangle = \int dr' \psi(t, r') \phi(t, r') \in \mathbb{C}; \quad (2.2)$$

All familiar formulations of quantum mechanics can be derived from the above primitive mathematical structures either in a direct or an indirect way. As an example, Schrödinger's equation for a given (Hermitian) Hamiltonian H

$$i \frac{\partial}{\partial t} |\psi\rangle = H |\psi\rangle = E |\psi\rangle, \quad (2.3)$$

is a consequence of the original associativity of the envelope ξ which results in the action of the operator H on the state $|\psi\rangle$ being right, modular and associative, i.e., such that

$$ABC |\psi\rangle = A(BC |\psi\rangle) = (AB)C |\psi\rangle = (ABC) |\psi\rangle. \quad (2.4)$$

Similarly, Heisenberg's celebrated equations for the time evolution of a physical quantity Q ,

$$i\dot{Q} = [Q, H]_{\xi} = QH - HQ, \quad (2.5)$$

are characterized by a Lie algebra L with brackets $[A, B]$ which is homomorphic to the antisymmetric algebra ξ^- attached to the envelope ξ , $L \approx \xi^-$.

Finally, we recall that the exponentiation of Eq.s (2.5) into a finite Lie group is a power series expansion in the envelope ξ , namely, it is technically permitted by the infinite-dimensional basis in ξ originating from the Poincaré-Birkhoff-Witt theorem, with familiar expansion

$$e_{|\xi}^{i\alpha X} = 1 + i\alpha X / 1! + (i\alpha X)(i\alpha X) / 2! + \dots, \quad \alpha \in F, \quad X = X^{\dagger} \in \xi, \quad (2.6)$$

under which the infinitesimal form (2.5) can be exponentiated to the finite group form

$$Q(t) = e_{|\xi}^{itH} Q(0) e_{|\xi}^{-Ht}. \quad (2.7)$$

All other aspects of quantum mechanics, such as linear operations on \mathcal{H} , Heisenberg's uncertainty principle, Pauli exclusion principle, representation theory, etc., can be derived via a judicious use of formulations derivable from or compatible with the above fundamental structures ξ , \mathfrak{A} (or \mathbb{C}) and \mathcal{H} .

It is important for our analysis to identify the primary limitations of quantum mechanics for the Bose-Einstein correlation.

LIMITATION 1: LACK OF REPRESENTATION OF EXTENDED CHARGE DISTRIBUTIONS As well known, the topological, geometrical and algebraic structures underlying quantum mechanics are strictly *local-differential*. As a result, *quantum mechanics can only represent extended charge distributions in a first point-like approximation*. Such a limitation is evidently not a problem for the electrons of an atomic structure, owing to their point-like charge structure, but it is a clear limitation for all hadrons in general (because they have a finite charge radius of about 1 Fermi), and, in particular, for the $p\bar{p}$ inelastic scatterings of the Bose-Einstein correlation.

Stated differently, a primary limitation of the quantum mechanics, which is particularly relevant for the problem of boson correlation, is that of its original conception, namely, for sufficiently large mutual distances of particles under which their actual size is ignorable. Despite

the large volume of experimental verifications recalled earlier, the limitation considered persists to this day because inherent in the very structure of the theory.

In order to reach a description of the *extended* character of the hadrons, one has to pass to the so-called second quantization. Even at that level, the theory cannot represent the *actual shape* of the charge distribution considered.

As an example, there are indications that the shape of the charge distribution of a nucleon is not perfectly spherical, but is instead an oblate spheroidal ellipsoid along, say, the z-axis with values for the semiaxes for the proton [22]

$$b_x^2 = b_y^2 = 1, \quad b_z^2 = 0.60, \quad (2.8)$$

which provide one (not necessarily unique) explanation of the anomalous magnetic moments of the nucleons.

It is evident that quantum mechanics *cannot* represent non-spherical shape (2.8), whether directly or indirectly. In fact, even passing to second quantization, the form factors only provide a remnant of the actual shape, and not a representation of the actual shape itself. At any rate, the rotational invariance of the theory would eliminate all nonspherical shapes of type (2.8).

In regard to the boson correlation, the above limitation implies that *quantum mechanics can only represent perfectly spherical fireballs*, with evident limitations in the quantitative description of the phenomenon considered. In fact, there is clear theoretical and experimental evidence indicating that the fireball of boson correlation is not perfectly spherical, but a prolate spheroidal ellipsoid oriented along the direction of the original p- \bar{p} collision (see Fig. 1).

LIMITATION 2: LACK OF REPRESENTATION OF THE DEFORMATIONS OF EXTENDED CHARGE DISTRIBUTIONS. Once the lack of representation of the actual shape of a charge distribution is understood, one can see that *quantum mechanics is intrinsically unable to represent the possible variations and/or deformations of given charge distributions*, e.g., because prohibited by the underlying rotational symmetry.

But the fireball is expanding immediately upon its formation. The above limitation therefore implies *the inability of quantum mechanics to represent the evolution of the fireball of Bose-Einstein correlation* (see Fig. 1).

In addition to the above *evolution*, we also have a possible *deformation* of the fireball caused by external, sufficiently intense external fields. It is important in this respect to recall that *rigid bodies do not exist in the physical reality*. Thus, shape (2.8) of a proton or of a neutron cannot be assumed to be perfectly rigid. Evidently the *amount* of the deformation for given external forces and/or collisions is unknown at this writing, and only tentatively indicated by neutron interferometric experiments (see ref. [23] and quoted papers for the experimental profile, and ref. [15], Chap. VII for a preliminary treatment). However, the basic concept which is relevant for this paper is that the *existence* of the deformations of shape (2.8) under sufficiently intense external forces or collisions is simply beyond scientific doubts.

When considered in the context of boson correlations, this second limitation implies that *quantum mechanics can only represent fireballs which, besides being perfectly spherical, are also perfectly rigid*. The ensuing limitations of the theory are also evident. In fact, the basic rotational symmetry is known to characterize a theory of *rigid bodies*.

LIMITATION 3: LACK OF REPRESENTATION OF NONLOCAL NONPOTENTIAL INTERACTIONS. Above all, the most important limitation for Bose-Einstein correlation is *the inherent inability of quantum mechanics to represent the nonlocal interactions expected in the hyperdense medium in the interior of the fireball*.

As well known, in the atomic structure we have constituents at large mutual distances with no appreciable overlapping of their wavepackets. Under these conditions, we have the lack of appreciable nonlocal effect (as well as the lack of effect nonlinear in the wavefunction [24]), with consequential *exact* validity of quantum mechanics.

In the interior of a fireball we have instead physical conditions fundamentally different than those of the original conception of quantum mechanics, inasmuch as we have *constituents in conditions of total mutual penetration, overlapping and compression of their wavepackets one inside the others*. It is evident that the latter conditions result in the most general physical conditions and interactions that are conceivable by contemporary mathematical knowledge.

These interactions are composed by conventional local-potential-Hamiltonian (e.g., external electromagnetic) interactions, plus the interactions caused by the mutual penetration, overlapping and compression of the wavepackets. The latter ones generally are:

a) *nonlinear* in all variables, i.e., nonlinear in the local coordinates r and wavefunctions ψ and ψ^\dagger as well as in their *derivatives* $\dot{r}(p)$, $\ddot{r}(\dot{p})$, $\partial\psi$, $\partial\psi^\dagger$, ...;

b) *nonlocal* in all variables, in the sense of having an integral dependence on the coordinates and wavefunctions, as well as in their *derivatives*; and, last but not least;

c) *nonpotential-nonhamiltonian*, namely, such to violate the integrability conditions for the existence of a Hamiltonian representation, the so-called *conditions of variational selfadjointness* [3,5] as typical for all contact interactions of our physical reality, whether in classical or particle physics³.

When seen within the context of boson correlation, the above features imply *the inherent impossibility of quantum mechanics to provide an effective description of the expected physical origin of the correlation*. In fact, the particles originating the fireball begin their interaction at large mutual distances and are generally uncorrelated. The correlation evidently originates in the expected nonlocal interactions inside the fireball. At any rate, what is significant here is the inability of conventional, action-at-a-distance interactions to provide a satisfactory interpretation of the correlation (see also Fig. 1).

LIMITATION 4: LACK OF REPRESENTATION OF THE BOSE-EINSTEIN CORRELATION FROM BASIC AXIOMS. Expectedly, *the very notion of correlation is outside the representational capabilities of the basic axioms of quantum mechanics*. Consider a system of n particles represented with the symbol $k = 1, 2, \dots, n$, each one possessing correlated and uncorrelated components represented with

³ The reader should be warned against not uncommon models based on the simplistic addition of an "integral potential" to the Hamiltonian for the evident attempt of preserving old knowledge, because they can be proved to be inconsistent on numerous counts. In fact, mathematically, these additions are not allowed by the underlying local-differential topology, and, physically, they imply granting a potential energy to contact interactions which cannot have any, thus resulting in unphysical trajectories (this latter point is clearly illustrated at the Newtonian level by the familiar drag forces which, if treated via a potential energy, imply major physical inconsistencies). As we shall see later, this is the ultimate physical motivation for our representation of contact nonlocal effects in the interior of the fireball via the generalized unit of the theory, that is, *with a quantity positively other than the Hamiltonian*.