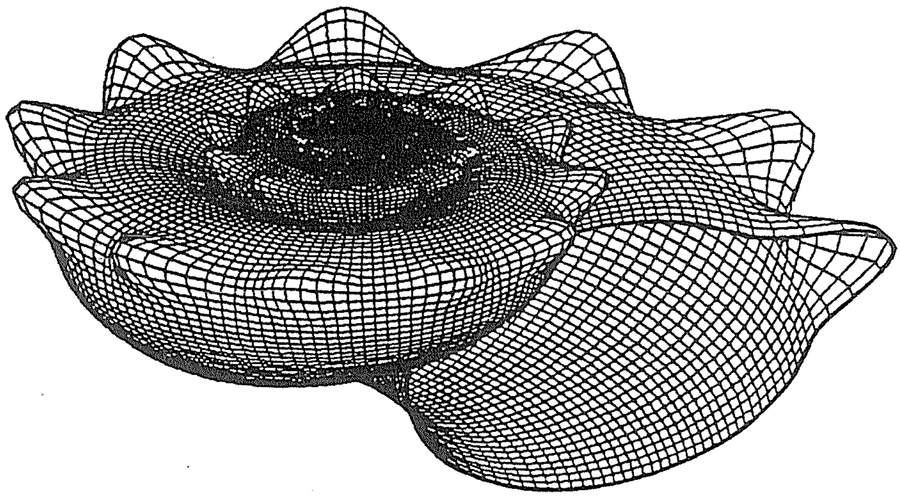
A PIONEERING MONOGRAPH IN CONCHOLOGY

foundations of THEORETICAL CONCHOLOGY

Second Edition

C. R. Illert and R. M. Santilli



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ABOUT THE FIRST AUTHOR

Christopher Roy Illert was born and educated in Australia. Currently, he is Associate Professor of Theoretical Biology at the Institute for Basic Research, Florida, U.S.A. He has served on the mathematics staff at Wollongong University in New South Wales and at the Computer Graphics Department of the Uki campus of the Science-Art Research Center. After graduating in 1973 from the Flinders University of South Australia, he scuba-dived extensively round the Australian coastline and became a member of the Marine Aquarium Research Institute of Australia, the Malacological Societies of South Australia and Australia, and the International Oceanographic Foundation. Over the years his studies of shells have been published in popular glossy magazines such as SEA & SHORE, SHELLS & SEA LIFE, UNDERWATER GEOGRAPHIC and AUSTRALIAN NATURAL HISTORY. His mathematical contributions have appeared in scholarly journals ranging from MATHEMATICAL BIOSCIENCES, IEEE COMPUTER GRAPHICS AND APPLICATIONS, IL NUOVO CIMENTO, HADRONIC JOURNAL and HADRONIC JOURNAL SUPPLEMENT and have been well received. Two of his publications have been selected as amongst the most outstanding contributions to a field of science this century and reprinted in Vol. 15 of the SPIE MILESTONE SERIES, Washington, D.C., USA, 1990. Prof. Illert is the organizer of various scientific meetings, including the First International Conchology Conference held in Australia in 1995.

For the second author see the inside back cover.

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and

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TABLE OF CONTENTS

PART I :

MATHEMATICAL REPRESENTATIONS OF SEA SHELLS FROM SELF-SIMILARITY IN NON-CONSERVATIVE MECHANICS

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CONTENTS

INTRODUCTION

Seashells are remarkable geometrical objects, being approximately self-similar throughout growth, and governed by laws and physical principles which are scale-invariant. Collectively shells pose a non-trivial dynamical problem which can be rigorously formulated (in complex-space) and completely understood. And, once we understand this general problem, it becomes evident that shell-geometries are of interest beyond conchology, malacology and paläontology, with broad implications for theoretical mechanics generally.

Shells are non-trivial geometrical shapes: the end-product of 500 million years of evolution in the oceans of our planet ... perhaps one of the greatest simulations of all time, dwarfing anything done by humans on super-computers. Shells are static real-world "snapshots" of dynamic forces and torques in action, providing solid examples that can be held in the hand: examples of, amongst other things, the onset of "chaos" (including "temporal chaos", see Section 3.2) in "dissipative" scale-invariant systems.

The fossil shell NIPPONITES MIRABILIS is a wonder of nature and a magnificent example: its principal growth-trajectory starts off in a predictable planar coil but becomes increasingly loopy, like the suture-line on a tennis ball, ultimately executing wild serpentine meanders resembling turbulent fluid flow (i.e. a vortex-street wrapped in a spiral). And the growth-trajectory that we see (hereafter called a CLOCKSPRING) is only the real part of a more general (perhaps even geodesic) curve through a multi-dimensional complex-space. Even the underlying physical principles (such as HOOKE'S LAW) only emerge coherently, and seem to make sense, within our full complex-space formalism (as in Section 1). Real-space E^3 *just doesn't seem adequate.*

So are seashell geometries profound enough to tell us that we live in a world that doesn't quite make sense unless we assume that it has <u>at least</u> five space-like and one time-like dimensions? How <u>seriously should</u> we take them? Certainly, if we do take shell geometries seriously, our insights are all the more powerfull because they emerge from totally classical, non-quantum, reasoning.

To some extent, much of this could be shrugged-off were it not for the fact that even in the purely real formalism (as given in Section 3.1) the critical physical constants associated with trajectory "curvature" and "torsion" often have to be complex numbers. Furthermore, a whole class of branching shell-geometries exist, exhibiting a feature called "temporal chaos" that implies hidden dimensions, acausal influences, or both. In this class are shells such as JANOSPIRA NODUS, and others beside, all required by theory and observed in nature despite the fact that their geometries could only be produced by forces that "act-at-a-distance" (both backward and forward) through time itself, thereby violating causality as we "know" it!

Such oddities, suspected by theoreticians such as Kanatani[7] *but ignored for lack of real-world examples, can now be convincingly demonstrated simply by reaching into the nearest shell cabinet. The proof is in the "seeing", so to speak, and as this is the way that Nature itself "does" theoretical mechanics, we might do well to learn from it ... no matter how surprizing.*

Not only do shells teach the significance of complex curvature and torsion in differential geometry (results not mentioned in any standard text on the subject), and the causal implications of self-similar non-conservative systems, but they also show that sensible, physically-meaningfull Lagrangians exist in nonconservative mechanics. Just because things sometimes happen "dissipatively" in the real-world, does not mean that there isn't an optimal fashion for them to proceed. This has been known for most of our present century, but denied by a small portion of the modern mathematical community concerned with Liapunov Function theory which, of course, advocates the use of conservative Lagrangians to model non-conservative systems ... a total contradiction in terms, rather like hammering a square peg into a round hole!

Santilli's book[29] *in 1979 gave a history of this misconception as it relates to the inverse problem in variational calculus yet, still, by 1982, research papers (such as the one by Chen & Russel*[30]*) were still appearing in the mathematical literature advocating the use of conservative Lagrangians for nonconservative systems. In the cited case*[30] *it was admitted that the approach was unsatisfactory, in their words "primitive and ad hoc", and yet the* Quarterly of Applied Mathematics *refused to print the correct Lagrangian for the problem (when I supplied it) and, surprizingly enough, even produced a spirited referee response which echoed almost word-perfect the notions of Bauer and Synge from the 1930's. In fact, there's absolutely nothing in several centuries of theoretical mechanics to even suggest that anything "more general than the concept of energy" either exists or makes sense. Liapunov Functions are a nonsense.*

Mistakes happen all the time, we've all made them, and that's how knowledge progresses. But when at least a portion of the mathematical community don't seem to want to correct a serious oversight, one which has been perpetuated on and off since the 1920's, then there is an interesting sociological phenomenon in progress. Why would Liapunov Function theorists simply choose to ignore *nonconservative Lagrangians, or arbitrarily discount them as "purely mathematical" objects devoid of physical meaning? Perhaps it is because mathematical equations, and logic alone, simply aren't enough to convince some people. What may be needed is a physically convincing, real-world example of a non-conservative system, one whose Lagrangian can be derived from first principles. If this can be achieved then we may overcome the psychological barriers, and progress most of the way toward convincing all mathematicians that there is a better way to deal with "dissipative" systems. If the seashell problem is able to provide such an example, then it will indeed have served an important role in theoretical (as well as applied) mechanics.*

In any case, from a solid empirical base encompassing 100,000 or so (living and extinct) molluscan shell varieties, the horizons of seashell mathematics reach outwards cutting into the fabric of other, better-known branches of modern physics ranging from elasticity theory and fluid dynamics, perhaps to subatomic particles. Indeed, the incremental nature of shell growth, emphasised in Sections 1.2 to 1.4, conjures notions of a succession of "force impulses" discretely deflecting straight-line trajectory increments into continuous-looking spirals. The whole metaphysical atmosphere resembles Feynman Diagram interpretations, and quantum-electrodynamic accounts of charged particle motion in external fields. Just how far this analogy extends, and how relevant it is, still remains largely to be seen but we make a start in Section 3.2 by offering the "example" of charged Lepton decay and Neutrino production in terms of discrete vector sums (1.3) which, in the limit, become circle-integrals.

Although constants associated with various modes of complicated shell-coiling are already being determined to several significant figure accuracy[16, 15], THEORETICAL CONCHOLOGY itself is a relatively new science which has emerged in the last two decades. Its literature is sparse and scattered, and no textbook on the subject has yet been made available by large publishers. For this reason we discuss literature from a diversity of sources in more detail than might otherwise be the case, as well as presenting new and previously unknown results. Such coverage should provide the best basis from which to grasp the logical development and significance of the subject, its relevance to other areas of science, and the theoretical problems which still lurk unresolved at the frontiers.

Chris ILLERT (1991).

1 CLOCKSPRING MECHANICS

1.1 LIVING CLOCKSPRINGS?

In 1908 *Harold Sellers Colton*[1] communicated, to the Philadelphian Academy of Sciences, the results of his extensive field and aquarium studies of the feeding-habits of marine snails. He found that some carnivores used their shell-aperture to force open the oysters and cockles on which they fed. It was a complicated process with the snail's soft-body muscles, coiled as they are about the shell's central axis (the columella), being able to create a torque that rotated the shell as a whole thereby enabling the current shell-aperture to be used as a tool to "bulldoze" bivalves open.

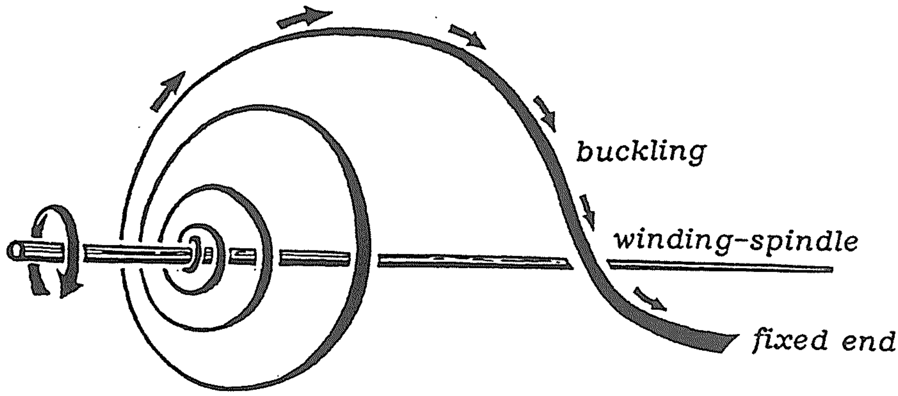
The dynamics of this process is rather like twirling the winding spindle of a tensile clockspring whose outermost end is anchored (satisfying a "fixed-end" boundary condition; see Case 3, in Section 3.1). Indeed, this is the kind of image suggested in 1914 by *Theodore Andrea Cook*[2], in his book "the Curves of Life", though he was apparently unaware of *Colton's* earlier fieldwork and apologised to the reader for making the analogy!

A succession of conchologists and malacologists have since confirmed *Colton's* finding that many marine snails (including volutes, bonnets, helmets, olives, harps and whelks) all to some extent physically twirl their shells, using them like clocksprings during the normal course of feeding. Some of these researchers include *Warren* (1916), *Copeland* (1918), *Clench* (1939), *Megelhaes* (1948), *Carricker* (1951), *Nielsen*[3] (1975) and *Illert*[4] (1979/80/81). The history of this strand of thought is summarized in a four-part series of articles by *Illert*[6] (1985) which also supplies a number of less well known literature references.

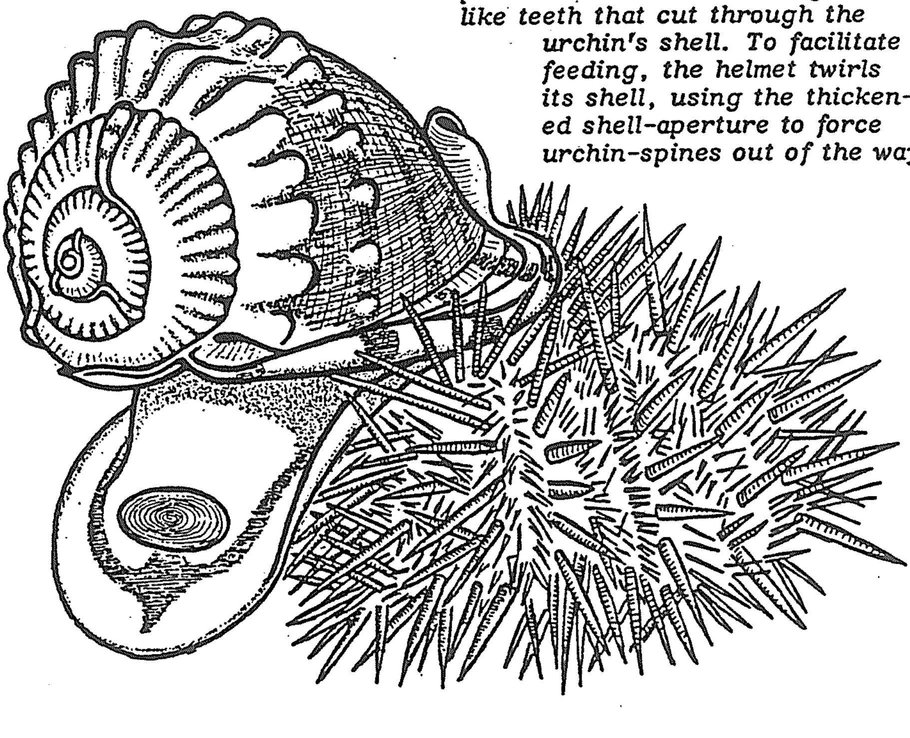
There can be little doubt that the forces acting on the shell are considerable, and the spring analogy appropriate, as some snails use their shells to bore through sand, wood or solid rock. The common abalone has a muscular foot capable of supporting 4000 times its body-weight ... have you tried to pry one off a rock? *Wainwright*[5] (1969), realizing that the stresses were so great that small cockles or scallops sometimes just shattered, implanted strain-gauges to physically measure the forces involved. *Nielsen*[3] (1975) found that even the predatory snails sometimes fractured their own shell-apertures, and *Illert*[4] (1979/80) realized that this was why some snails such as the South Australian Helmet, *Cassis bicarinata*, have a specially thickened apertural band (called a "varix") to reinforce their shell and prevent it from breaking when it is used to "bulldoze" seaurchin spines away during the course of feeding.

Another kind of "clockspring" geometry is represented by the cowrie shell. The juvenile starts coiling in a regular spiral way but, by maturity, the aperture curves inward and growth stops. The cowrie wanders about with its shell exterior covered by a fleshy skin, called the "mantle", emanating through the shell's apertural slit. If a fish or other predator grabbed at the cowrie's exposed mantle-skin, and would not let go, then the cowrie would try to withdraw back through the apertural slit by levering on the rigid shell. In this grim tug-of-war the fulcram of

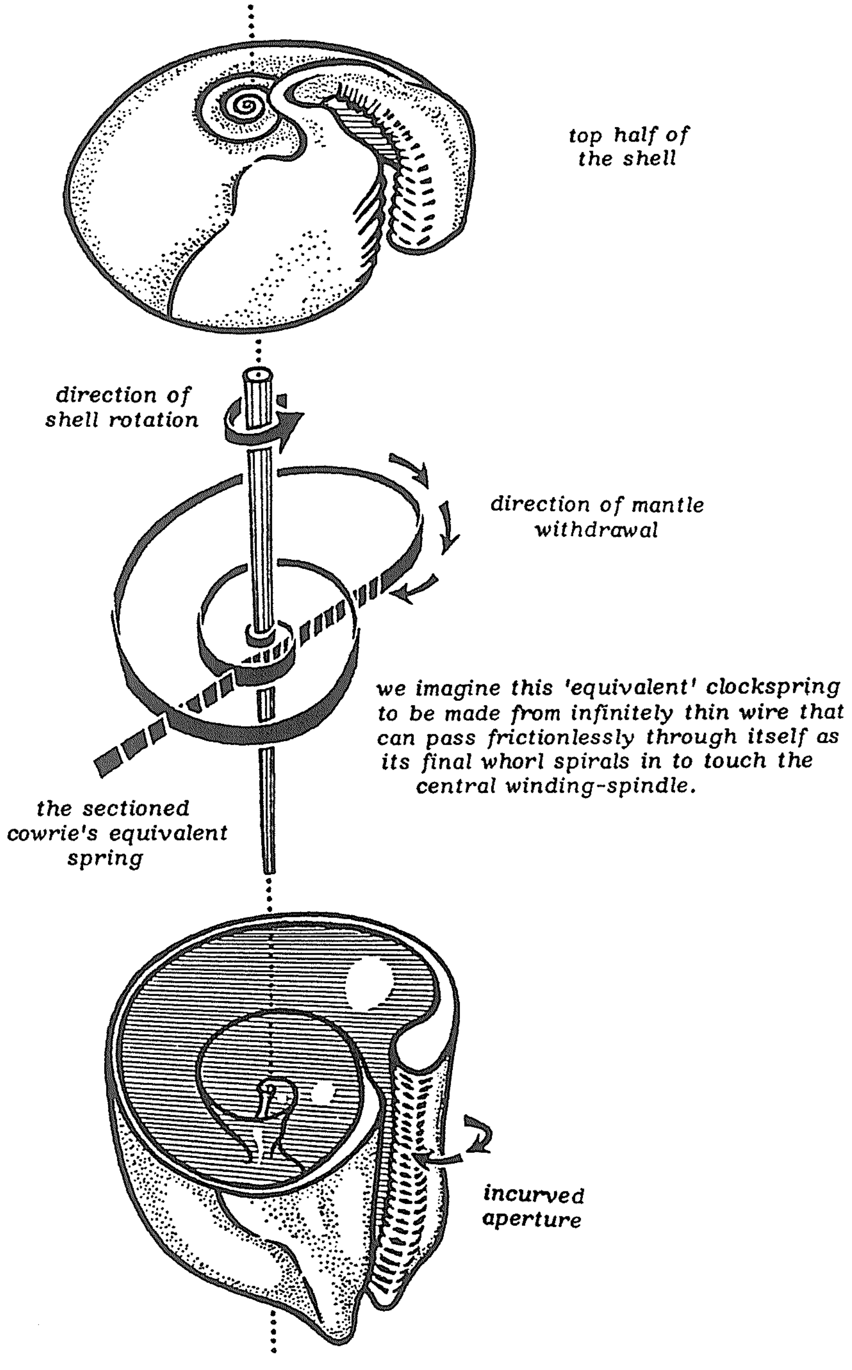
CLOCKSPRING OF THE FIRST KIND



the South Australian helmet, CASSIS BICARINATA, is carnivorous; eating sea-urchins with an extendable elephant-like trunk containing saw-like teeth that cut through the urchin's shell. To facilitate feeding, the helmet twirls its shell, using the thickened shell-aperture to force urchin-spines out of the way



CLOCKSPRING OF THE SECOND KIND



trans-axially sectioned cowrie shell

forces is the shell aperture; this is probably why most cowries have apertural crenelations ("teeth") enhancing their "grip" during such attacks.

All this biological evidence suggests that, although different species have different reasons for doing so, they produce shells that don't just coincidentally resemble optimal tensile clocksprings, as originally suggested by *Cook*, but that they actually function as such in order to best dissipate stresses incurred during the normal course of life. We would expect some version of *Hooke's Law*, for elastic springs, to underlie all naturally occurring shell geometries: it seems self-evident from the life-styles of the creatures themselves.

It is, of course, one thing to claim that this mathematics exists, and quite another thing to find it! The rest of Section 1 is devoted to the task of creating an internally consistent, logical and powerful "clockspring mechanics". By the end of Section 1 we have a mechanics which is too powerful and too general; the Hookean (matrix) constant of proportionality k, and one other (matrix) constant Ω, will remain undetermined. This is why Section 2 is devoted to deriving, and explaining, two symmetry constraints (respectively equations (2.41) and (2.42)) arising in a natural way from self-similarity, from which the structure of the two matrices can be deduced (as in (2.45)) or inferred (as in (2.46) & (2.47)). But, although we know the form of these two matrices, the constant terms within them are still arbitrary. Section 3 therefore shows various classes of clockspring trajectories which satisfy the Euler equation ((1.38) = (3.1)) for different values of the arbitrary constants λ and μ.

1.2 FIRST ORDER DISCRETE MECHANICS

Consider a vector-valued function $\underset{\sim}{\xi}(\phi) = (\xi_1(\phi), \xi_2(\phi), \xi_3(\phi))$ describing a continuous, twice-differentiable trajectory through some generalised multi-dimensional space: say $\underset{\sim}{\xi} : [0,\infty) \rightarrow \mathbb{C}^3$. A physically meaningfull formulation of the seashell problem does not seem attainable solely in E^3. It was attempted by *Illert*[6] in 1983 but even the real-space equations themselves suggested a complex-space formalism (the Euler equations becoming diagonalised only in complex coordinates $\underset{\sim}{\xi}$). Of course, all our complex space equations have their real-space counterparts ("projections") but something of the logical rationale is lost in them. The actual problem to which real-world seashells are the solution can only be fully appreciated in more than three real-space dimensions. We will consider ourselves to have done well if we can, by imposing a geometrically simplifying symmetry, reduce the space in which we formulate the problem down from $\mathbb{C}^3$ to $\mathbb{C}^2 \times E^1$.

Continuing then, with the above-discussed trajectory $\underset{\sim}{\xi}$, a suitable quantity of arc-length is defined as follows

$$\frac{d\underset{\sim}{\xi}}{d\phi}\, d\phi \;\cong\; \dot{\underset{\sim}{\xi}}\,\Delta \;. \qquad \ldots (1.1)$$

The diagram opposite is a geometrical "cartoon" whose purpose is to convey a visual image of what our equations mean.