

# SANTILLI'S LIE-ISOTOPIC GENERALIZATION OF GALILEI'S AND EINSTEIN'S RELATIVITIES

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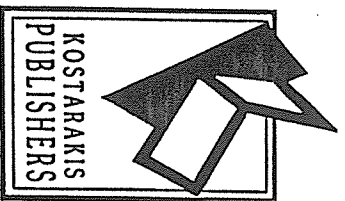
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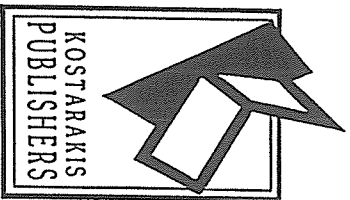


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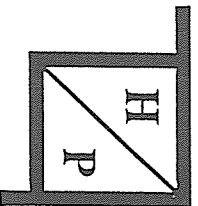
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**DEDICATION**

*This monograph is dedicated to*

**Mr. MICHAEL S. GORBACHEV**  
*President of the U.S.S.R.*

*because of his vision, courage and  
historical contributions to mankind*

*June 1, 1990*

This monograph presents an enlarged version of the lectures delivered by Prof. **Ruggero Maria Santilli** at the **INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS** of Trieste, Italy, in the first part of December 1990, following notes taken at the lectures by one of the Authors, Prof. **A. Jannussis**, and subsequently enlarged thanks to the assistance of all the other Authors, as well as to the editorial assistance of the staff of **THE INSTITUTE FOR BASIC RESEARCH** of Palm Harbor, Florida, U.S.A.

An invitation by Prof. **Abdus Salam**, Director of the Centre, to Prof.s Santilli and Jannussis must be here acknowledged with sincere gratitude, because it permitted the organization of the original material presented at the lectures and stimulated its subsequent enlargement. Penetrating comments by Prof. Salam at the lectures resulted to be invaluable for the achievement of sufficient maturity of presentation, and for stimulating subsequent research.

\* \* \*

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## PREFACE

Throughout this century, Lie's theory has been developed in both mathematical and physical literatures with respect to the simplest conceivable unit, say  $I = \text{Diag.}(1, 1, \dots, 1)$ , and the simplest conceivable product  $AB - BA$ , where  $AB$  is the trivial associative product. In a pioneering memoir written at Harvard University in 1978, Ruggero Maria Santilli identified, apparently for the first time, a generalized formulation of Lie's theory constructed with respect to the most general possible unit  $\hat{I}$ , in which case the Lie product assumes less trivial forms, such as  $A * B - B * A$  where  $A * B$  is still associative but of the more general type  $A * B = AgB$ , where  $g$  is fixed, sufficiently smooth and nonsingular, and  $\hat{I} = g^{-1}$ . The generalized theory was called the "Lie-isotopic theory" for certain historical reasons reviewed in the text. The original proposal of 1978 contains the development of the Lie-isotopic theory to a rather remarkable extent, including a generalization of: the theory of universal enveloping associative algebras (Poincaré-Birkhoff-Witt Theorem, etc.); Lie's celebrated First, Second and Third Theorems; Lie's transformation groups; and Lie's symmetries. The memoir concluded with the conjecture of a conceivable generalization of Galilei's Relativity in classical mechanics for extended particles moving within resistive media (which are not only Galilei-noninvariant, but also generally nonhamiltonian). This original proposal was subjected to a systematic study in subsequent years by Santilli as well as a number of independent authors, not only for the original classical profile, but also for a conceivable operator counterpart, as well as for relativistic, gravitational and gauge extensions.

This review is a guide through a considerable and disparate literature, devoted to: the identification of the state of the mathematical studies on the Lie-isotopic generalization of conventional formulations of Lie's theory; their primary applications, to classical mechanics, particle physics and astrophysics; and an outline of the proposed fundamental tests. Except for minor treatments, the studies on conceivable operator realizations are deferred to a possible separate review.

We begin with a review of the algebraic notion of isotopy and its application to associative and Lie algebras. We then pass to the notion of analytic isotopy in classical mechanics, that realized via the Birkhoffian generalization of Hamiltonian mechanics. We also indicate the notion of operator isotopy on Hilbert spaces, that realized via the hadronic generalization of quantum mechanics, as well as the methods of "hadronization," that is, the mapping of Birkhoffian into hadronic mechanics. The notion of isotopy in

symplectic geometry concludes our introductory chapter.

The second chapter is devoted to a detailed review of the mathematical studies on the Lie-isotopic formulations of: enveloping associative algebras; Lie's Theorems; Lie algebras; Lie groups; and the application of the generalized theory to space-time symmetries. The second chapter ends with a fundamental theorem by Santilli on the reconstruction of the exact nature of space-time symmetries at the Lie-isotopic level, when broken at the conventional level.

The third chapter is devoted to the applications of the Lie-isotopic theory. We begin with a review of Santilli's isotopic generalization of the group of rotations and some of its properties such as: the capability by the rotational symmetry to remain exact at the Lie-isotopic level when conventionally broken, say, for spheres undergoing deformations, or for any physical condition implying a topology-preserving alteration of the Euclidean metric. We then pass to the review of Santilli's Lie-isotopic generalization of Galilei's Relativity for systems of extended-deformable particles which are nonhamiltonian (but Birkhoffian) because of motion within a resistive medium. We review the property that, again, under certain topological restrictions, the Galilei symmetry remains exact at the Lie-isotopic level when conventionally broken by nonhamiltonian forces. A number of intriguing implications and open problems are also considered. We then pass to the review of Santilli's Lie-isotopic generalization of Einstein's Special Relativity and related properties, such as: the capability of incorporating all available studies on Lorentz "noninvariance" (universality), e.g., the several phenomenological calculations predicting deviations from Einstein's behavior on the mean life of unstable hadrons at different speeds; the capability of reconstructing the Lorentz symmetry as isotopically exact for all the above models (in which it is conventionally broken); the capability to represent a disparate variety of physical conditions outside the applicability of the conventional relativity, such as deformation of charged distributions, motion of electromagnetic waves in inhomogeneous and anisotropic media, motion of electrons in metals, propagation of causal signals within dense hadronic matter, etc.; the generalization of the various laws of the conventional relativity with intriguing implications although in need of experimental preliminary confirmations; and a number of other aspects. The third chapter then passes to a review of the construction by Gasperini and Santilli of a Lie-isotopic generalization of Einstein's gravitation which is, locally, Lorentz-isotopic and Galilei-isotopic, as well as capable of resolving at least some of the numerous problematic aspects of the conventional theory available in the literature. The need for

the conduction of certain basic tests on fundamental space-time symmetries (that have been regrettably ignored for decades) completes the third chapter.

In the Appendices we review a variety of topics that complement the main text, such as: Lie-isotopic generalization of gauge theories; computation of the maximal speed of causal signals within hadronic matter; Lie-isotopic field equations; and other aspects.

The situation emerging from this review is essentially as follows. From a mathematical viewpoint, there is little doubt that the Lie-isotopic theory is mathematically consistent and does provide a genuine covering of the conventional formulation of Lie's theory. The understanding is that the studies are at the beginning and so much remains to be done. From the viewpoint of theoretical physics, the classical formulations of the Lie-isotopic theory have clear applications in Newtonian mechanics, particularly for the physical systems of our everyday life, that is, with nonhamiltonian forces, for which the conventional formulations are simply inapplicable. In regard to relativistic settings, the isotopic theories are admittedly tentative, conjectural and in need of direct tests, although we are aware of no experimental or other information on the novel physical conditions considered capable of disproving the predictions of the new theory at this writing. As a matter of fact, all evidence currently available appears to favor the Lie-isotopic symmetries over the conventional ones, in a way, after all, predictable from the necessary compatibility with established Newtonian applications.

We are here referring to: phenomenological calculations on the behavior of the meanlife of unstable hadrons with energy conducted over the past several decades showing an apparent violation of the Einsteinian law, while they are clearly and directly interpreted by Santilli's covering law; the preliminary measures via neutron interferometry conducted by Rauch and his associates on the apparent deformation of the charge distribution of neutrons under external nuclear fields, with consequential alteration of the magnetic moments/rotational asymmetry, which are also directly and quantitatively interpreted by Santilli's exact,  $SU(2)$ -isotopic symmetry; and others. Not surprisingly, the astrophysical applications of Santilli's covering relativities appear to be in full agreement with their particle and classical counterparts. We are here referring, e.g., to the possibility of interpreting the quasar redshift as due to propagation of light within the hyperdense, inhomogeneous and anisotropic media surrounding the quasars, rather than to the currently unplausible quasars speeds of the order of ten time the speed of light in vacuum; and other very intriguing astrophysical applications.

As a result of all the above, a thrilling possibility of a new scientific edifice emerges from Santilli's pioneering studies, with predictable implications at every level of contemporary physics, most of which are still unexplored as of now. But, by far, the most important implications of Santilli's studies are from an experimental viewpoint. In fact, the studies focus the attention on considerably overdue, fundamental experiments which have been submitted in the technical literature for decades, but largely ignored until now. We are referring to experiments such as: final measures of the behavior of the mean life of unstable hadrons at different speeds; or to final measures of the expected deformation of the charge distributions of hadrons under sufficiently intense external fields; and others. All these experiments, in their currently available preliminary form, show clear deviations from the Einsteinian predictions, in favor of the prediction of Santilli's relativities and their exact, isotopic, Lorentz symmetry. This situation leaves the ultimate foundations of contemporary physics in a state of "suspended animation" which will evidently persist until the experiments are finally done, and the issue of conventional versus isotopic space-time symmetries resolved one way or the other.

This work will achieve one of its most important objectives if it succeeds in stimulating experimentalists to finally conduct these much overdue, fundamental tests.

June 1, 1990



# 1 INTRODUCTION

## 1.1 A Brief Survey of the Literature

Despite rather vast mathematical and physical studies, the formulation of Lie's theory has been essentially restricted until recently to that via the familiar Lie product  $[A, B] = AB - BA$ , where  $AB$  is the simplest possible associative product, e.g., that of matrices. The unit of the theory is then the trivial element, e.g.,  $I = \text{diag}(1, 1, \dots, 1)$ .

An inspection of the physical literature confirms this condition, which has its origin in the construction of quantum mechanics via the enveloping associative algebra of operators  $A, B, \dots$ , their simplest possible product  $AB$ , and Heisenberg's time evolution  $i\hbar\dot{A} = AH - HA$ . An inspection of the mathematical literature confirms the same condition which has its origin, this time, in the representation theory of enveloping associative algebras also realized via the product  $AB$ .

In a pioneering memoir of 1978 (written while at the Lyman Laboratory of Physics of Harvard University), Ruggero Maria Santilli [1] identified, apparently for the first time, a generalized formulation of Lie's theory which he called *Lie-isotopic theory* for certain historical reasons reviewed later on. The central idea is that of building the theory around the most general possible unit, say  $\hat{I} = (I_{ij})$ , where the elements  $I_{ij}$  have an arbitrary functional or operator dependence subject only to certain topological restrictions. This demanded, of course, a generalization of the enveloping algebra, from the form with trivial product  $AB$ , into a covering form with product of the type  $A*B = ATB$ , where  $\hat{I} = T^{-1}$ . The Lie product then takes the more general form  $A*B - B*A$ .

Santilli was the first to realize the mathematical and physical nontriviality of the theory and to develop it to a considerable extent already in the original proposal [1]. In fact, in this first memoir one can see several theorems generalizing enveloping associative algebras, the celebrated Lie's first, second and third theorems, and the conventional notion of Lie group, into forms compatible with the most general possible unit  $\hat{I}$ . Under the condition that the old unit  $I$  is contained as a particular case of the generalized unit  $\hat{I}$ , Santilli's theory becomes a *covering* of the conventional one, in the sense of being formulated on structurally more general foundations, while admitting the conventional formulation as a trivial particular case.

Remarkably, the Lie-isotopic theory was proposed by Santilli as a particular case of a structurally yet more general theory based on the so-called

*Lie-admissible algebras*, which will not be reviewed in this monograph. Nevertheless, the point is important for this review because some of the subsequent advances made by Santilli and others on the Lie-isotopic theory can be identified only as a particular case of the more general Lie-admissible formulations. Perhaps this is the reason why the Lie-isotopic theory has not received until now the attention it deserves in both physical and mathematical literatures.

The subsequent memoir also of 1978 [2] and paper [3] were primarily devoted to Lie-admissible algebras, although containing advances important also for the simpler Lie-isotopic theory such as the foundation of a conceivable operator realization of the algebras, including the generalization of Heisenberg's equations of the type  $i\hbar\dot{A} = A * B - B * A$ . Santilli completed the year 1978 with the release of the two monographs [4,5], the first setting up the methodological foundations of the classical realization of the Lie-isotopic theory (the so-called conditions of variational selfadjointness), and the second initiating the application of hadronic mechanics to particle physics.

In 1979 we see the appearance of the first review [6] [again for the Lie-admissible approach] followed by paper [7] on the initiation of the representation theory of the generalized algebras on suitable bimodular vector spaces. Paper [8] presents an intriguing application to gauge theories.

Paper [9] of 1980 studies the difficulties of conventional quantization, and suggests their reinspection under a broader algebraic structure. Paper [10] of 1981 studies the expected existence of a conceivable generalization of quantum mechanical laws for the interior of hadrons, with particular reference to Heisenberg's uncertainty principle. Paper [11] enters deeper into conceivable physical implications for particle physics, this time for the notion of particle under external strong interactions realized with nonlocal and nonhamiltonian terms due to mutual wave overlappings.

In 1982 we see the appearance of paper [12] which consists of a review of the physical implications of the generalized Lie structures for nonpotential nonhamiltonian interactions in Newtonian, statistical and particle mechanics. Paper [13] studies the conceivable generalization of Heisenberg's and Schrödinger's equations that are expected from the broader realizations of Lie's theory. Paper [14] presents another courageous analysis, the possibility that causal signals can propagate within dense hadronic matter at speed higher than  $c_0$ , the speed of light in vacuum. At the end of 1982 we also see the appearance of monographs [15,16] on the classical realizations of his algebraic theories, the so-called Birkhoffan [15] and Birkhoffan admissible

[16] mechanics. In these monographs one can see Santilli's extended presentations of the conceivable generalizations of Lie-isotopic and Lie-admissible type, respectively, of the classical Galilean relativity for extended particles with action-at-a-distance, potential forces, as well as contact, nonpotential and nonhamiltonian forces due to motion within a resistive medium.

In 1983 we see the appearance of three central contributions. Paper [17] presents a model on the reversibility of strong interactions for center-of-mass conditions, with irreversible dynamics for each individual constituent when considering the rest of the system as external. Paper [18] is, in our opinion, the most important paper under consideration here after refs. [1,2]. It presents the foundations of a conceivable Lie-isotopic covering of Einstein special relativity for generalizations of the Minkowski metric caused by motion of extended particles within generally inhomogeneous and anisotropic physical media. The paper also provides the explicit method for the construction of an infinite class of covering transformations from the original Lorentz ones and the given generalized metric. Paper [19] provides a generalization of Wigner's theorem on quantum mechanical symmetries within the broader Lie-isotopic setting representing nonpotential nonhamiltonian forces caused by mutual wave-overlappings of particles. This paper also signals the achievement of mathematical maturity of the generalized operator formulation, with the clear understanding that its physical validity is still basically open at this writing.

In 1984 we see the appearance of another important contribution [20]. In the preceding paper [18] Santilli shows that, under certain topological restrictions, the continuous part of the Lorentz symmetry can be proved to be exact at the abstract, Lie-isotopic level when generally considered as "broken" at the simplistic level of the product  $AB - BA$ . Paper [20] complements these results, this time, for the discrete part of the Lorentz symmetry. In fact, the paper indicates how parity may well be an exact symmetry under weak interactions, provided the theory is realized within the context of the covering Lie-isotopic approach, because all  $P$ -breaking terms can be incorporated in the generalized unit  $\hat{f}$  [as well as in other degrees of freedom]. The exact character of the  $P$ -[as well as other] symmetries then follows from the property that Lie algebras leave invariant their unit element.

In 1985 we see additional contributions by Santilli in the field. The year started with the inspiring "Journey in the Solar system" [21] (an invited contribution to the Calcutta conference). We then see the appearance of papers [22,23] specifically devoted to Lie-isotopic symmetries. These papers



(which had been written prior to paper [18] and presented at a meeting of 1983) essentially provide a rigorous mathematical formulation of the process according to which a given Lie symmetry, when broken at the simpler level  $AB - BA$ , can be “reconstructed” as exact at the higher Lie-isotopic level  $A * B - B * A$ . The papers also identify the means of constructing the (generally infinite family of) covering, exact, Lie-isotopic transformations via the sole knowledge of the old transformations and of the new metric. Papers [22,23] then apply the theory to a case of truly central physical relevance: the breaking of the rotational symmetry, say, for the deformation of a spherical charge distribution under external fields, and the recovering of the exact rotational symmetry for the deformed distribution at the covering Lie-isotopic level.

In 1988 we see the appearance of four memoirs [25] which, jointly with the original memoir [1], constitute Santilli’s most significant scientific contributions. In fact, these latter memoirs present a comprehensive isotopic generalization of contemporary algebras, geometries and mechanics for systems that are not only nonlinear and nonlocal (as those of the preceding contributions), but also nonlocal integral; the memoir then apply these broader mathematical tools for the construction of isotopic coverings of Galilei’s, Einstein’s Special and Einstein’s General Relativities for interior dynamical problems; the memoirs finally present a detailed study of the mutual compatibility of the emerging generalized formulations and propose a number of experimental verifications.

In 1989 we see the appearance of four additional memoirs [26] this time devoted to the operator formulation of the isotopic theories, including a study of the “hadronization” of classical into operator formulations; the construction of the spinorial  $SU(2)$ -isotopic symmetry and its representations; some isotopic generalizations of the various properties of the conventional theory of angular momentum and spin (such as the isotopic Clebsch-Gordon coefficients, etc); the construction of the operator formulation of the isotopic Galilean and special relativities; the foundations of the isotopic field theory, including the isotopic generalization of the Klein-Gordon and Dirac’s equations; the operator study of Rauch’s experiment on the spinorial symmetry of neutrons; and other important topics.

Paper [25] of 1990 tests the possibilities of hadronic mechanics via a quantitative study of the possible representation of the original Rutherford’s conception of the neutron as a generalized bound state of one ordinary proton and one electron, whose total angular momentum is represented via the isotopic  $SU(2)$ -symmetry to account for the expected nonlocal and non-

hamiltonian effects due to total mutual penetration of the wavepackets of the constituents.

In 1991 we use the appearance of a series of papers written at the ICTP [27, 28, 29] which develop in more details the operator formulation of the isotopic special relativity based on the isotopies of the Poincaré symmetry; the construction of the generalized field theory invariant under the isotopic Poincaré symmetry; and some applications (Rauch's experiment on the spinorial symmetry and Rutherford's conception of the neutron).

Monograph [30], currently under preparation, is expected to complete the series of the preceding volumes [4,5] and [15,16]. This completes the review of the contributions written by Santilli alone.

Papers [31–44] were written by Santilli in collaboration with several authors on numerous topics related to the precedings research (see below).

A number of physicists have studied Santilli's proposal of 1978.

R. Mignani [45] made seminal contributions in the operator realization of Lie-isotopic theories, such as: the independent identification of the Lie-isotopic generalization of Schrödinger's equation; the proposal to construct a nonpotential scattering theory; the construction of the Lie-isotopic  $ST(3)$  symmetry; and others.

M. Gasperini [46] made other equally seminal contributions, such as: the computation following hypothesis [14], that, within the context of contemporary gauge theories, the speed of causal signals within hadronic matter could indeed exceed  $c_0$ ; the foundations of a possible Lie-isotopic generalization of gauge theories; and the foundations of a possible Lie-isotopic generalization of Einstein gravitation for the interior problem.

A team headed by A. Jannussis made numerous contributions [47] in both classical and operator realizations of Santilli's algebras. M. Nishioka [48] also made several contributions in the field, such as the expected generalization of the delta function. A. J. Kalnay [49] succeeded in quantizing Nambu's mechanics for triplets. The algebra emerging at the operator level is exactly that of Santilli's type [2]. (This aspect, which we regrettably cannot review in this paper, opens the possibility of a true quark confinement with an identically null probability of tunnel effects into free states, besides an infinite potential barrier, as studied in papers [44].

Animalu [50] conducted several, additional, independent research, such as the study of possible contributions to conventional quark theories of the generalized setting offered by hadronic mechanics, and others.

A. Tellez Arenas, J. Fronteau and R. M. Santilli [31,32] studied the statistical profile of a generalized class of physical systems characterized by the