

Can Strong Interactions Accelerate Ordinary Particles Faster Than the Speed of Light? (*) (**)

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Summary. - In this note I formulate the hypothesis that the maximal possible speed of an ordinary massive particle or of a physical signal under strong and electromagnetic interactions is not necessarily equal to c , but can be smaller than c , bigger than c , or even infinite, depending on the local physical characteristics (density, temperature, etc.) of the hadronic matter in which the particle or signal propagates. I then present a number of arguments of plausibility. Those of experimental nature range from the recent indications in nuclear physics via neutron interferometers of a conceivable breaking of the SU_2 spin symmetry, to recent astrophysical data indicating the existence of ordinary matter traveling at speeds bigger than c . The theoretical arguments of plausibility are based on the so-called Lie-admissible formulations, and are given by a Minkowski space generalization of the recently proposed structure models under strong internal forces as closed non-self-adjoint systems. These are systems which, when seen from an outside observer, verify the conventional conservation laws (exterior problem). However, at the level of dynamical behaviour of the constituents, all conventional symmetries are broken to permit unrestricted forces and dynamical conditions (interior problem), as established, for instance, by interior motions in our Earth. The consistency of the model is proved. It is shown in this way that the strict compliance with the special relativity of a particle in exterior treatment, such as a proton in an accelerator, is fully compatible with the violation of the same relativity in the interior problem, including the achievement of speed higher than c by the constituents. A number of epistemological arguments of plausibility are then presented. As clearly expressed by Lorentz, Poincaré, and Einstein in their limpid writings, the special relativity was conceived for isolated, pointlike, particles moving in vacuum under long-range, action-at-a-distance interactions. The physical arena considered is fundamentally different because it refers to the motion of extended particles within the hadronic medium composed by other particles. The evident lack of homogeneity and isotropy of the medium then implies the inapplicability of the foundations of the special relativity, let alone the boosts responsible for the upper bound c of the speed.

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A further argument of plausibility is given by the nature of the forces considered, the nonpotential, nonlocal, integro-differential forces resulting from mutual penetration of the wave packets of particles as necessary for the strong interactions. In fact, these forces are of contact type and, as such, can accelerate objects without any need of potential energy. But the most direct epistemological arguments are of gravitational nature, and refer to the apparent need that the theory of gravitation for the interior problem is not locally Lorentz, in order to prevent excessive approximations of perpetual-motion-type, such as the motion of a satellite in our atmosphere with a conserved angular momentum. The lack of local Lorentz character of the theory then implies the breakdown of the conventional limit c of physical particles. The note concludes with a number of implications, such as the capability of the theory of turning the currently unphysical tachyons into physical, fully causal and ordinary particles.

As is well known, the pioneering papers by LORENTZ⁽¹⁾, POINCARÉ⁽²⁾ and EINSTEIN⁽³⁾, implied a dependence of the mass of an ordinary particle on the speed, $m = m_0/\sqrt{1 - v^2/c^2}$. Under the hypothetical availability of infinite energy, the maximal admissible speed is then that of light,

$$(1) \quad v_{\max} = c.$$

The validity of law (1) for an isolated particle moving in vacuum under long-range electromagnetic interactions has been established by an impressive amount of clear and direct experimental evidence, as daily proved, say, in particle accelerators.

Despite these achievements, we are not in a position to state that law (1) is « universal »; that is, it is valid under all possible physical conditions of particles. For instance, the law does not possess clear, direct, or otherwise final experimental verification for a particle under strong interactions. To put it explicitly, we can firmly state that the maximal possible speed of a proton in a particle accelerator (long-range electromagnetic interactions) is c . However, on grounds of scientific caution, we cannot state that the same upper bound on the speed must necessarily hold when the same particle is a member of a nuclear structure (strong nuclear interactions), or enters into deep inelastic collisions with other hadrons (strong hadronic interactions), or moves within the core of a star (strong astrophysical interactions).

In this paper I would like to submit a generalization of law (1) according to which the maximum possible speed of an ordinary massive particle depends on local quantities which can be defined in a Minkowski space $M(3,1)$ (such as proper time τ and co-ordinates x , as well as the density, temperature, and other characteristics of the medium in which the particle moves) and, depending on these local conditions, it can be smaller, equal, or bigger than c ; *i.e.*

$$(2) \quad v_{\max} = v_{\max}(\tau, x, \dots) \cong c.$$

More specifically, I submit the hypothesis that the maximal possible speed of an ordinary massive particle is different, depending on whether the particle is under nuclear, hadronic, or astrophysical interactions, and I shall write in self-evident notation,

$$(3) \quad v_{\max}^{\text{nucl}} \neq v_{\max}^{\text{hadr}} \neq v_{\max}^{\text{astr}}.$$

(¹) H. A. LORENTZ: *Amst. Proc.*, **6**, 809 (1904); *Verl.*, **12**, 986 (1904).

(²) H. POINCARÉ: *C. R. Acad. Sci.*, **140**, 1504 (1905); *Rend. Pal.*, **21**, 129 (1906).

(³) A. EINSTEIN: *Ann. Phys. (N. Y.)*, **17**, 891 (1905).

By keeping in mind that strong, hadronic and astrophysical structures also have electromagnetic interactions, hypotheses (2) and (3) are conceived as a generalization of law (1) for a particle under joint strong and electromagnetic interactions. Needless to say, the possible generalized law (2) must recover the established law (1) identically, when the strong interactions are absent. By recalling that the size (electromagnetic-charge radius) of all strongly interacting particles is of the order of magnitude of the range of the strong interactions ($\sim 10^{-13}$ cm = 1 fm), the hypothesis submitted in this paper is specifically conceived for an ordinary massive particle (wave packet) in conditions of mutual penetration and overlapping with other particles (other wave packets); that is for motion within a physical medium called «hadronic medium». As a result, the physical conditions underlying possible generalized law (2) or (3) are fundamentally different from those referred to by LORENTZ, POINCARÉ and EINSTEIN and, actually, they were unknown in the period 1904-1906.

In this note I would like to present a few experimental, theoretical, and epistemological arguments of plausibility for the generalized law (2). A more detailed study of the problem, as well as of the possible law (3) will be presented elsewhere.

A number of old and recent experimental indications in nuclear physics indicates the possibility that ordinary massive particles can experience an alteration of their structure as well as of their intrinsic physical characteristics in the transition from the electromagnetic to the strong interactions. I am referring here in particular to:

1) the apparent quite large deviation from the conventional values of the magnetic moments of nucleons while they are members of a nuclear structure, as suggested by the Schmidt limits (⁴);

2) an apparently consequential breaking of the SU_2 spin symmetry, as can be inferred via the fundamental experiment by RAUCH *et al.* (⁵) on the 4π spinorial symmetry via neutron interferometers, as well as the apparently considerable departure from the prediction of conventional theories in the test of optical activity of neutrons within matter (⁶); and

3) the apparent, also considerable, breaking of the T -symmetry under strong nuclear interactions, as indicated in the experiment by CONZETT *et al.* (⁷) (for a study of ref. (⁴⁻⁷), see ref. (⁸)).

The experimental argument for the plausibility of law (2) is then consequential. The experimental information under consideration implies that Einstein's special relativity, while exact for electromagnetic interactions according to current knowledge,

(⁴) See, for instance, J. M. BLATT and V. F. WEISSKOPF: *Theoretical Nuclear Physics* (New York, N. Y., 1963); E. SEGRÉ: *Nuclei and Particles* (New York, N. Y., 1964); G. EDER: *Nuclear Forces* (Cambridge, Mass., 1968), and other well-written treatises in nuclear physics.

(⁵) H. RAUCH, A. WILFING, W. BAUSPIESS and U. BONSE: *Z. Phys. B*, **29**, 281 [1978]. It should be indicated that the measure of the spin precession achieved in this experiment ($\alpha = (716.8 \pm 3.8)$ degrees) has been recently revised via up-dated physical constants, yielding the new value $\alpha = (715.87 \pm 3.8)$ degrees (private communication by Prof. H. RAUCH, Director, Atom-institut, Schöttelstrasse 115, A1020 Wien, Austria). Notice that the new measure does not include the 720 degrees needed to establish the exact character of the SU_2 -spin symmetry under strong interactions.

(⁶) M. FORTE, B. R. HECKEL, N. F. RAMSEY, K. GREEN, G. L. GREENE, J. BYRNE and J. M. PENDLEBURY: *Phys. Rev. Lett.*, **45**, 2088 (1980). The deviation of the measures conducted in this experiment from the conventional theoretical predictions are treated by L. STODOLSKY, SLAC preprint 2536 (May 1980).

(⁷) For a recent treatment of the experiment, see R. J. SLOBODRIAN: *Hadronic J.*, **4**, 1258 (1981).

is only approximate for the joint strong and electromagnetic interactions⁽⁸⁾. Therefore, if future experiments confirm these circumstances, the departure from law (1) would be established, and only the amount of departure (*i.e.*, the explicit functional structure of law (2) or the explicit upper bounds of law (3)) would be open to specific studies.

In order to attempt a dynamical differentiation between the electromagnetic and the strong interactions, in monographs⁽¹⁰⁾ I have studied discrete nonrelativistic systems with unrestricted forces via the integrability conditions for the existence of an action functional, the so-called conditions of variational self-adjointness. In ref.⁽⁹⁾ I introduced the notions of *closed self-adjoint* (CSA) system and *closed non-self-adjoint* (CNSA) system. These are systems which, when seen from an outside observer are seemingly the same on theoretical grounds, inasmuch as they both verify the familiar ten conservation laws of total quantities (closure). However, the systems are different in their structural dynamics, in the sense that the former are composed of pointlike constituents moving in vacuum under action-at-a-distance, potential, (SA) forces, while the latter are composed of extended constituents under the most general forces known at this time, the variational non-self-adjoint integro-differential forces (a combination of local-differential and nonlocal-integral forces which are derivable and nonderivable from a potential). A technical difference is that the total conservation laws are first integrals of the structural equations of motion for CSA systems, while they are *bona fide* subsidiary constraints for the CNSA systems. Examples of CSA systems are given by our planetary system in Newtonian approximation, or by isolated systems under electromagnetic internal forces (such as an atom). An example of a CNSA system is given by our Earth which, when considered as isolated from the rest of the universe, is closed. Nevertheless, a necessary condition to avoid excessive approximations of perpetual-motion type, is that the equations of motion for the constituents are not (directly⁽¹⁰⁾) derivable from an action principle as is the case, say, for trajectory problems in atmosphere, spinning tops with drag torques, etc. A fundamental hypothesis of paper⁽⁹⁾ is that hadrons, and, more generally, all closed systems under strong internal forces, are of the more general non-self-adjoint type owing to the mutual penetration of the wave packets of particles with expected, consequential, nonlocal-nonpotential forces. To put it in intuitive terms, our Earth was proposed in ref.⁽⁹⁾ as a rudimentary Newtonian image of a hadron in the same measure as that according to which our planetary system can be seen as a Newtonian image of an atom.

A theoretical argument of plausibility for hypothesis (2) can be given by a discrete, Minkowski-space formulation of the notion of CNSA system. In particular, this can be done via: a) the distinct separation of the theoretical formulation into the *exterior*

⁽⁸⁾ R. M. SANTILLI: *Hadronic J.*, **4**, 1166 (1981). New experiments on the fundamental test of SU_2 spin are also proposed by H. RAUCH and A. ZEILINGER: *Hadronic J.*, **4**, 1280 (1981). Rather crucial contributions have also been made by G. EDER (*Hadronic J.*, **4**, 634 (1981); and *ibidem*, **4**, in press (1981)) who proved that, in addition to the strong neutron-nuclei interactions, the electromagnetic field in the vicinity of nuclei is so intense that it could conceivably produce a breaking of the SU_2 spin symmetry of the measurable amount of 1%.

⁽⁹⁾ R. M. SANTILLI: *Hadronic J.*, **1**, 574 (1978). This paper provides the (apparently) first theoretical formulation and treatment of the hypothesis that the SU_2 spin symmetry is broken in the transition from electromagnetic to strong interactions due to the conditions of mutual penetrations of particles and expected nonlocal nonpotential forces. The treatment was done via the generalization of the associative envelope of SU_2 to a nonassociative Lie-admissible form (*loc. cit.*, p. 786-797). The hypothesis presented in this note is a consequence of this treatment (see below) as well as of the related experimental information^(8,9).

⁽¹⁰⁾ R. M. SANTILLI: *Foundations of Theoretical Physics*, Vol. I (New York, N. Y., and Heidelberg, 1978); and Vol. II (in press).

treatment (description of the system as a whole as seen by an outside observer), and the *interior treatment* (description of each individual constituent while considering the rest of the system as external); *b*) realization of the exterior treatment in strict compliance with the special relativity; and *c*) violation of the special relativity for the interior treatment in a progressively increasing way corresponding to the degree of mutual penetration of the constituents in the transition from nuclear, to hadronic, to astrophysical conditions.

The exterior treatment is essentially that available in the literature. Assume the metric tensor on $M(3.1)$ to be $(g^{\mu\nu}) = (- + + +)$, $\mu, \nu = 0, 1, 2, 3$. Let X^μ be the centre-of-mass four-vector of the system, and impose its verification of the familiar Lorentz invariant separation

$$(4) \quad dX^\mu dX_\mu = -c^2 d\tau^2,$$

where τ is the proper time of the system; let

$$(5) \quad P^\mu = M_0 c V^\mu, \quad V^\mu = dX^\mu/d\tau$$

be the total four-momentum, where M_0 is the total rest mass; and let

$$(6) \quad J^{\mu\nu} = X^\mu P^\nu - X^\nu P^\mu + N^{\mu\nu}$$

be the total angular-momentum tensor, with the conventional separation into the orbital and the intrinsic part. The closure is then expressed by the familiar ten conservation laws of total quantities⁽¹¹⁾

$$(7) \quad \frac{dP^\mu}{d\tau} = 0, \quad \frac{dJ^{\mu\nu}}{d\tau} = 0, \quad \mu, \nu = 0, 1, 2, 3.$$

A number of additional conditions can be imposed to ensure the compliance of the system with special relativity⁽¹²⁾, but they are inessential for this paper, and left as possible future refinements.

In the transition to the interior treatment the research attitude is drastically changed. A necessary condition for a particle to be in interaction is that at least one of its physical characteristics is *nonconserved*⁽⁹⁾. The condition is verified by the electromagnetic interactions via the nonconservation, in general, of the energy and of the linear momentum of the particle, of course, in such a way to be compatible with total conservation laws. This implies a form of breaking of the symmetry under space-time translations⁽¹³⁾ at the level of each constituent, without affecting its exact validity for the

⁽¹¹⁾ Equations (7) then imply the Poincaré invariants $d(P^\mu P_\mu)/d\tau = 0$ and $d(W^\mu W_\mu)/d\tau = 0$, $W^\mu = \frac{1}{2} \varepsilon^{\mu\alpha\beta\gamma} J_{\alpha\beta} P_\gamma$.

⁽¹²⁾ See, e.g., H. VAN DAM and TH. W. RUIJGROK: *Physica A (The Hague)*, **104**, 281 (1980).

⁽¹³⁾ To exclude abstract mathematical treatments which are physically vacuous, we assume that a symmetry is exact when, not only conventional mathematical conditions are verified, but also the first integrals represent directly physical laws. This definition of symmetry has a hierarchy of breakings classified into isotopic, self-adjoint, semi-canonical, canonical, and essentially non-self-adjoint. See in this respect Vol. II of ref. (10) or R. M. SANTILLI: *Phys. Rev. D*, **20**, 555 (1979). The breaking of the Poincaré symmetry presented in this note is one of the simplest possible (of semi-canonical type). The breaking can be progressively increased, up to the most general possible which is of essentially non-self-adjoint type.

system as a whole. However, the Lorentz symmetry is exact under electromagnetic interactions not only for the state as a whole, but also for each constituent. This feature is at the foundation of Dirac's equation for one electron of an atomic structure, in which, as is well known, the rest of the system is considered as external. The feature is expected to persist for all CSA systems⁽¹⁴⁾. In the transition to CNSA systems the situation is fundamentally different because these systems are conceived to admit the structurally most general interactions known at this time. In turn, this is reflected in the realization of the interactions via the most general possible nonconservations of the physical characteristics of each constituent, of course, in such a way as to be compatible with total conservation laws. Still, in turn, this produces a sufficient form of breaking of the Lorentz symmetry at the level of each constituent, as desired. The breaking of the symmetry then produces the intended theoretical argument of plausibility. In fact, the very theoretical tools for the derivation of law (1) are inapplicable under the circumstances for each constituent of a CNSA system, while the system as a whole verifies law (1) by construction.

The consistency of the model has been proved at the Newtonian level in ref. (9), and at the statistical level in ref. (15). The proof of the consistency at the level of Minkowsky space formulations is trivial. Let x_k^μ , p_k^μ and $j_k^{\mu\nu}$, $k = 1, 2, \dots, n$, be the four-co-ordinates, four-momentum, and angular-momentum tensor of the constituents and suppose that their relationship with the total quantities is of the familiar linear, type

$$(8) \quad P^\mu = \sum_{k=1}^n p_k^\mu, \quad J^{\mu\nu} = \sum_{k=1}^n j_k^{\mu\nu}.$$

Then the model is realized via the nonconservation laws

$$(9) \quad \frac{dp_k^\mu}{d\tau} \neq 0, \quad \frac{dj_k^{\mu\nu}}{d\tau} \neq 0, \quad k = 1, 2, \dots, n, \quad \mu, \nu = 0, 1, 2, 3,$$

which ensure the maximal possible conditions of interactions⁽¹⁶⁾. Under smoothness and other conditions inessential here, the system of functions characterized by eqs. (7) and (9) is consistent for $n > 1$. In fact, there always exist $10n$ functions (p_k^μ and $j_k^{\mu\nu}$) verifying the 10 equality conditions (7) and any number of inequality conditions such as (9)⁽¹⁷⁾.

A number of generalizations of model (7)-(9) via constrained systems of ordinary or partial, and differential or integro-differential equations will be presented in a forthcoming paper⁽¹⁸⁾. All these models are essentially based on the breakdown of Lorentz

(14) Even though no use of the conditions of variational self-adjointness was made, excellent studies on conventionally relativistic (CSA, in our language) systems can be found in F. ROHRlich: *Hadronic J.*, **4**, 831 (1981) and quoted papers.

(15) A. TELLEZ-ARENAS, J. FRONTEAU and R. M. SANTILLI: *Hadronic J.*, **5**, 177 (1979).

(16) Nonconservative relativistic systems have been studied by a number of authors. We refer here in particular to J. FRONTEAU: *Hadronic J.*, **2**, 727 (1979); H. BETTE: *Int. J. Theor. Phys.*, **19**, 877 (1980), and papers quoted therein.

(17) The reader should keep in mind the physical reality for which the model is intended. Closed systems are stable (e.g., in the sense of ref. (15)). Yet, the *instability* of the orbits of the constituents is the rule in nature, and their stability is the exception. The dichotomy stability/exterior problem vs. instability/interior problem of our model is intended as a rudimentary first representation of this physical reality.

(18) R. M. SANTILLI: *On a possible Lie-admissible generalization of Einstein's special relativity for strong interactions*, I, in preparation.

separation (4) at the level of each individual constituent. In turn, this breakdown is one way to express theoretically the transition from special relativity to more general settings, including generalizations of law (1).

A rather direct way of identifying the occurrence is via the use of the algebra of the time evolution. Let us recall some of the basic physical aspects which lead to the Lorentz separation for the constituents of a CSA system. For simplicity, we consider only the case of one particle under electromagnetic interactions represented via the familiar Hamiltonian

$$(10) \quad H = \frac{1}{2m_0} (p^\mu - eA^\mu)(p_\mu - eA_\mu).$$

The (proper) time evolution of x^μ is given by

$$(11) \quad \left\{ \begin{array}{l} \dot{x}^\mu = \frac{dx^\mu}{d\tau} = [x^\mu, H] = \frac{\partial x^\mu}{\partial a^i} \omega^{ij} \frac{\partial H}{\partial a^j} = \frac{\partial H}{\partial p_\mu} = \frac{1}{m_0} (p^\mu - eA^\mu), \\ (\omega^{ij}) = \begin{pmatrix} 0_{4 \times 4} & 1_{4 \times 4} \\ -1_{4 \times 4} & 0_{4 \times 4} \end{pmatrix}, \quad a^i = \begin{cases} x^\mu, & i = \mu, \\ p_\mu, & i = 4 + \mu. \end{cases} \end{array} \right.$$

The roots of the validity of the Lorentz separation

$$(12) \quad \frac{dx^\mu}{d\tau} \frac{dx_\mu}{d\tau} = 2m_0 H = -c^2$$

can then be seen in the Lie algebra character of the time evolution. In fact, this character implies the antisymmetry of the product which, in turn, implies the conservation of H ($\dot{H} = [H, H] = 0$). The constancy of H ($= -mc^2$) then implies law (12) trivially.

A number of technical refinements exist in the literature, the most important one for our analysis being that via Dirac's constrained treatment of relativistic systems. In our unified notation this essentially implies the transition from the fundamental, canonical, Lie (cosymplectic) tensor ω^{ij} to a more general tensor⁽¹⁰⁾

$$(13) \quad \left\{ \begin{array}{l} \omega^{ij} = \left(\left\| \frac{\partial R_i^0}{\partial a^j} - \frac{\partial R_j^0}{\partial a^i} \right\|^{-1} \right)^{ij} \rightarrow \Omega^{ij} = \left(\left\| \frac{\partial R_i}{\partial a^j} - \frac{\partial R_j}{\partial a^i} \right\|^{-1} \right)^{ij}, \\ \det(\omega^{ij}) = 1, \quad R^0 = (p_\mu, 0) \rightarrow R = R(\tau, a) \neq R^0, \quad \det(\Omega^{ij}) \neq 0. \end{array} \right.$$

The theory, however, remains strictly Lie in algebraic character.

In the transition to the CNSA generalization the situation is profoundly altered, first on conceptual grounds and, second, on theoretical-mathematical grounds. CNSA systems include the conventional potential forces of contemporary theoretical physics. However, they also admit additional forces of contact type for which the notion of potential energy has no physical basis. When a given CSA system, say, of type (10), is implemented into a CNSA form, a direct way of representing the generalized time evolution in the same local variables x^μ and p^μ , is given by the generalization of the

Lie algebra into the covering Lie-admissible algebras⁽¹⁹⁾ according to structures of the type

$$(14) \quad \left\{ \begin{array}{l} \dot{x}^\mu = \frac{dx^\mu}{d\tau} = (x^\mu, H) = \frac{\partial x^\mu}{\partial \alpha^i} S^{ij}(\tau, \alpha) \frac{\partial H}{\partial \alpha^j} \neq \frac{1}{m_0} (p^\mu - eA^\mu), \\ S^{ij} = \Omega^{ij} + T^{ij}, \quad T^{ij} = T^{ji}, \quad \det(S^{ij}) \neq 0, \end{array} \right.$$

where H is the energy of the particle which is now strictly nonconserved. The lack of Hamiltonian character of the forces, the lack of antisymmetric character of the Lie-admissible algebra, and the consequential nonconservation of the energy ($\dot{H} = (H, H) \neq 0$) then imply a generalization of the Lorentz separation for each constituent of the type

$$(15) \quad \frac{dx^\mu}{d\tau} \frac{dx_\mu}{d\tau} = f(\tau, x, \dots),$$

where the dots refer to additional conceivable dependences on the local conditions, such as the density and temperature of the hadronic medium in which the particle propagates. The generalization of the maximal possible speed of the particle of type (2) then follows. Clearly, its explicit computation demands the identification of the generalized relativity which is applicable for given local conditions, and, as such, it goes beyond the objectives of this first study of the problem.

Hypothesis (2) has been essentially presented in this paper as a consequence of the current experimental and theoretical studies on the nature of the strong interactions⁽⁴⁻⁹⁾. However, there are a number of epistemological arguments of plausibility which should be taken into account. One is given by the inspection of the foundations of special relativity which, as well known, can be seen in the homogeneity and isotropy of space and related aspects. These foundations are clearly applicable under the conditions limpidly expressed by LORENTZ, POINCARÉ and EINSTEIN (pointlike particles moving in vacuum under action-at-a-distance, long-range interactions). The same foundations are no longer applicable to the physical conditions considered in this paper. In fact, we have the *motion of an extended wave packet within a nonhomogeneous and nonisotropic medium*, the hadronic medium constituted by all other particles of the systems. The need to generalize special relativity under the conditions indicated then appears rather forceful. The dependence of the generalized law (2) on the local co-ordinates is also self-evident.

But, perhaps, the most direct way of arriving at law (2) is of gravitational inspiration. Although not sufficiently emphasized in the existing literature, gravitational theories must be divided into the treatment of the *exterior* and of the *interior problem*. The local Lorentz character of any gravitational model of the exterior problem is self-evident, as experimentally established in any way. In the transition to the interior problem, the situation is, again fundamentally different. In fact, in order to represent simple interior motions, such as the decay of the orbit of a satellite in Earth atmosphere, the theory is not expected to be locally Lorentz and, at any rate, must permit the local nonconservation of the angular momentum. Equivalently we can say that

⁽¹⁹⁾ The direct universality of the Lie-admissible algebras for the brackets of the evolution in Newtonian mechanics under the most general forces known at this time was established by R. M. SANTILLI: *Hadronic J.*, **1**, 233, 1279 (1978). Since that time, this universality has been extended to quantum mechanics, classical and quantum statistical mechanics, as well as classical and quantum field theory. Thanks to the invaluable participation by a number of mathematicians, the approach is currently studied at the yearly *Workshops on Lie-Admissible Formulations*; see the *Proceedings (Hadronic J.*, Vol. 4, No. 6, Vol. 5, No. 1, and Vol. 4, No. 2, 3, 4); see also H. C. MYUNG, S. OKUBO and R. M. SANTILLI (Editors): *Applications of Lie-Admissible Algebras in Physics*, Vol. I and II (1978), and others in preparation (Nonantum, Mass., U.S.A.).

the theory for the interior problem *must not* be (directly) derivable from an action principle. When «large constituents» such as a satellite are considered, this is an experimentally established reality of the classical treatment of the interior problem. When these constituents are reduced to their particle components, and a quantum-mechanical treatment is attempted, the situation persists. In fact, as indicated earlier, we must prevent the unphysical conditions of a proton orbiting in the core of a star with a locally conserved angular momentum, and this can be most directly done via a gravitational model for the interior problem which is not locally Lorentz. This is *per se* sufficient to ensure the lack of universality of law (1), and the plausibility of generalizations of type (2).

In closing this note, it may be of some interest to point out a few implications of the study. Hypothesis (2) implies that *physical signals can propagate with unbounded speeds and, at the extreme, even with infinite speed*. I am referring here to the *propagation of a signal through hadronic matter* such as a nucleus, a hadron, or a star. Of course, the notion of «signal» should not be restricted to photons, because even the mechanisms of emission of a photon within hadronic matter are unknown, let alone its propagation inside particles. On the contrary, a «signal» should be interpreted as any physically measurable process of propagation which is related by cause and effect. For instance, a signal is given by the process according to which a particle collides with a nucleus, and one or more particles are emitted in another point of the nuclear surface at a later time. The speed of the signal is then given by the special nuclear distance of the effect divided by the separation time. When physical signals are seen from this profile, the possibility of signals with infinite speed (*e.g.*, inside stars undergoing gravitational collapse) becomes rather natural.

Another implication of hypothesis (2) or (3) is that *the physical constituents of systems with strong internal forces can propagate with speeds not necessarily bounded by c* . For instance, there may be reasons whereby nuclear constituents cannot achieve the speed c even under the theoretical availability of infinite energies. However, there may be different reasons whereby the hadronic constituents travel faster than the speed of light, even under relatively low energies. In the final analysis, this is the most natural consequence of the new forces considered. In fact, non-self-adjoint forces accelerate objects via contact, instantaneous effects, without any need of potential energy⁽¹⁰⁾.

Another implication of hypothesis (2) or (3) is that *tachyons*⁽²⁰⁾, *under strong interactions, are physical, ordinary particles*. As a matter of fact, the hypothesis can apparently put in a different light the rather frequent, and often inevitable, appearance in physics of faster-than-light particles (*e.g.*, in field theory).

Intriguingly, the well-known case of the Čerenkov light can be considered as a particular case of law (2), rather than (1). In fact, the effect is fundamentally due to the presence of a medium (in this case, ordinary matter such as water) which permits the conventional physical electrons to travel faster than light.

But the most intriguing arguments are of experimental nature. In addition to those reported earlier⁽⁴⁻⁸⁾, a number of astrophysical data (particularly those related to the transfer of matter within binary systems) can apparently be interpreted only via the assumption of ordinary matter traveling faster than c .

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⁽²⁰⁾ A rather comprehensive list of references on tachyons has been kindly provided to me by Prof. R. MIGNANI (Università degli Studi, Istituto di Fisica G. Marconi, Roma, Italy), it has been omitted here for brevity, but it is available to interest colleagues.