COMMUNICATIONS IN
THEORETICAL PHYSICS
Vol. 4, No. 1, 1995, 123-153

ISOTOPIC LIFTING OF QUARK THEORIES WITH EXACT CONFINEMENT AND CONVERGENT PERTURBATIVE EXPANSIONS

Ruggero Maria Santilli*
Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980, Dubna, Russia

(Received 28 February, 1994)

In this paper we study the axioms-preserving isotopies of quark theories, there called Isoquark theories, which essentially are realizations of the Poincaré and unitary symmetries with respect to the most general possible integro-differential unit, while being locally isomorphic to the original symmetries. We show that: 1) the Lie-isotopic theory permits the preservation of all conventional quantum numbers, thus rendering the isotopic quark theory indistinguishable with the conventional theory; 2) the isotopies imply a differentiation between the conventional exterior and generalized interior Hilbert spaces to such an extent that they are incoherent, thus permitting the achievement of an exact confinement of quarks, that is, a theory with a probability of free quarks which is identically null even in the absence of a potential barrier (as hinted at by asymptotic freedom); and 3) the mechanism which permits exact confinement, an operator form of Nambu's mechanics for triplets, also permits the regaining of convergence of perturbative expansions which are conventionally divergent. In short, this paper indicates that the isotopic generalization of the Poincaré symmetry for nonlocal internal conditions of hadrons, rather than being a drawback, permits apparently fundamental advances in strong interactions which are not possible with the conventional symmetry.

1. Introduction

Despite outstanding achievements (see, e.g., the collected original works[1]), quark theories still remain with unsettled aspects, such as the

* This paper is the extended version of the presentation by the author at the International-conference on High Energy Physics and Quantum Field Theory held in Protvino, Russia, in September 1993.

lack of a rigorous confinement[2].

In this paper we introduce the isotopies of conventional quark theories, or *isoquark theory* for short, which preserves the (flavor and color) $SU(3)$-symmetry including all conventional quantum numbers; verifies an exact confinement, that is, it admits transition probabilities for free quarks which are explicitly computed and proved to be identically null prior to any QCD extension and in the absence of any potential barrier; and admit convergent perturbative expansions.

This formulation is essentially based on the generalization of the internal Hilbert space of hadrons into a form which is incoherent with the external Hilbert space of our physical reality. The generalization is motivated by expected nonlinear, nonlocal and nonhamiltonian effects due to deep waveoverlappings, and is realized via the isotopies of the unit of conventional quark theories into intergo-differential forms which preserve the original positive-definite character. The $SU(3)$-symmetries constructed with respect to such generalized units result to be locally isomorphic to the conventional symmetry, thus permitting the preservation of all conventional physical results[1], while avoiding problematic aspects of type[2].

Intriguingly, the isoquark model submitted in this paper is, in the final analysis, an explicit realization of the theory of <<hidden variables>> (see the recent review[3]) and, as such, it is essentially based on a <<*completion*>> of quantum mechanics much along the historical argument of Einstein-Podolsky-Rosen[4] and others.

An understanding of this paper requires a knowledge of: the isotopies of Lie's theory, also called *Lie-Santilli theory*, originally submitted in ref. **[5]** and then worked out in ref. **[6]** (see the recent mathematical works[7,8]); the isotopies of conventional space-time symmetries, originally submitted in refs. **[8, 9, 10]** and then worked out in refs. **[12, 13]**; and the isotopies of quantum mechanics, originally submitted in ref. **[14]**, and then worked out by a number of authors (see, e.g. refs. **[15-17]**, the review in the recent paper[18] and monograph[19]).

For notational convenience, we here briefly recall the main lines.

The fundamental notion is the *isotopy of the unit*, that is, the generalization of the conventional unit I = diag. (1, 1, 1) of the $SU(3)$-symmetry into the most general possible, three-dimensional matrix with integro-differential elements depending on time t, all possible local variables z, wave-functions ψ and their derivatives of arbitrary order, as well as any other needed quantity, under the conditions (necessary to qualify as an isotopy) of preserving the original axioms of I, i.e. no where degeneracy, Hermiticity and positive-definiteness,

$$I = \mathrm{diag.}(1,1,1) > 0 \Rightarrow \hat{I} = \hat{i}(t, z, \bar{z}, \dot{z}, \bar{z}, \psi, \psi \partial^{\dagger}\partial\Psi, \partial\Psi^{\dagger}. \ldots) > 0 \quad (1.1)$$

All conventional mathematical and physical structures are then modified in such a way to admit $\hat{I}$ as the correct right and left unit, in which case $\hat{I}$ is called the *isounit*.

The isotopy of the unit $I \Rightarrow \hat{I}$ implies the necessary lifting of the field of complex numbers C into the infinitely possible isotopes $\hat{C} = \{\hat{c} \mid \hat{c} = c\hat{I}, c \in C\}$, which are characterized by the same additive unit 0 of C, but the generalization of the trivial multiplicative unit 1 into the isounit $\hat{I}$. This implies that sums in $\hat{C}$ are conventional, but products must be generalized into the form

$$C: c_1 c_2 \Rightarrow \hat{C}: \hat{c}_1 * \hat{c}_2 = \hat{c}_1 T \hat{c}_2, \quad T = \text{fixed}, \ \hat{I} = T^{-1}$$

$$c_1, c_2 \in C, \hat{c}_1 \hat{c}_2 \in \hat{C} \quad (1.2)$$

where T is called the *isotopic element*. Then, $\hat{I}$ is the correct left and right unit of $\hat{C}, \hat{I}; * \hat{c} = \hat{c}\hat{I} \equiv \hat{c}, \forall \hat{c} \in \hat{C}$ in which case $\hat{C}$, is called an *isofield*. The isotopic (that is, axiom-preserving) character of the lifting $C \Rightarrow \hat{C}$ is readily established by the fact that $\hat{C}$ is locally isomorphic to C for all positive-definite isounits $\hat{I} > 0$. The isofield $\hat{\Re}$ of real numbers is defined accordingly.

The isotopies of the unit and of the field then demand corresponding compatible isotopies of the basic carrier space, that is, the isotopic lifting of the conventional complex, three-dimensional Euclidean space $E(z,\bar{z},\delta,C)$ of $SU(3)$ with familiar metric δ = diag. (1, 1, 1) into the complex, three-dimensional *isoeuclidean spaces*

$$E(z,\bar{z},\delta,C) \Rightarrow \hat{E}(z,\bar{z},\hat{\delta},\hat{C}): z = (z_1, z_2, z_2, z_3),$$

$$\hat{\delta} = T\delta \equiv g = \text{diag.}(g_{11}, g_{22}, g_{33}) = g^{\dagger} > 0, \quad (1.3a)$$

$$x^{\dagger} z = \bar{z}_1 z_1 + \bar{z}_2 z_2 + \bar{z}_3 z_3 \Rightarrow z_i^{\dagger} g(t, z, \bar{z}, \ldots) z_j =$$
$$= \bar{z}_1 g_{11} z_1 + \bar{z}_2 g_{22} z_2 + \bar{z}_3 g_{33} z_3, \quad (1.3b)$$

$$C \Rightarrow \hat{C} \approx C\hat{I}, \hat{I} = T^{-1} = g^{-1} = \text{diag.}(g_{11}^{-1}, g_{22}^{-1}, g_{33}^{-1}) > 0,\ g_{kk} > 0, \quad (1.3c)$$

where the assumed diagonalization of $\hat{I}$ is always possible (although not necessary) from its positive definiteness. The isotopic character (as well as novelty) of the generalization is established by the fact that, under the *joint* lifting of the metric $\delta \Rightarrow \hat{\delta} = T\delta = g$ and of the field $C \Rightarrow \hat{C} \approx C\hat{I}, \hat{I} = g^{-1}$, all infinitely possible isospaces $\hat{E}(z,\hat{z},\hat{\delta},\hat{C})$ are locally isomorphic to the original space $E(z,\bar{z},\delta,C)$ under the sole condition of positive-definiteness of the isounit $\hat{I}$[7]. In turn, this evidently sets the foundation for the local isomorphism of the corresponding symmetries.

Note that separation (1.3b) is the most general possible nonlinear, nonlocal and noncanonical generalization of the original separation $z^{\dagger}_i \delta_{ij} z_j$ under the sole condition of remaining positive-definite.

The lifting of the unit and of the base field also requires, for mathematical consistency, the lifting of the conventional associative envelope ξ of $SU(3)$ with generic elements A, B, C and trivial associative product AB, into the *isoassociative enveloping algebra* $\hat{\xi}$

$$\xi: AB = \text{assoc.} \Rightarrow \hat{\xi}: A * B \equiv AgB, \quad g = \text{fixed}, \quad (1.4)$$

where the isotopic character is readily established by the preservation of the original associativity of the product $A * B$. The Poincaré-Birkhoff-Witt Theorem then admits a consistent isotopic generalization[5,6] which ensures the existence of a consistence isotopy $\hat{\xi}(SU(3))$ of the universal enveloping associative algebra $\xi(SU(3)$ of $SU(3)$ and related infinite-dimensional basis.

Isotopies (1.1)-(1.4) then necessarily require compatible liftings of Lie algebras L into *Lie-Santilli algebras* $\hat{L}$[5, 6]

$$L: [A,B]_{\xi} = AB - BA \Rightarrow \hat{L}: [A,B]_{\hat{\xi}} = [A\hat{,}B] = A * B - B * A$$
$$= AgB - BgA. \quad (1.5)$$

In order to identify the corresponding isotopies of Lie groups one should first note the necessity under isotopies to lift conventional linear transformations $x' = Ux$ into the so-called *isolinear transformations*[5,6]

$$x' = U(w)x \Rightarrow x' = \hat{U}(w) * x = \hat{U}gx =$$
$$= \hat{U}g(s, x, \dot{x}, \ddot{x}, \psi, \psi^{\dagger}, \partial\psi, \partial\psi^{\dagger}, \mu, \tau, n, \ldots)x, \hat{w} \in \hat{\Re}$$

In fact, the conventional transformations $x' = \hat{U}x$ are no longer linear under isotopies, while the transformations $x' = \hat{U} * x$ do verify the axioms of linearity, of course, in their isotopic form. Nevertheless, as shown by equations (1.6), when projected in the original space, isotransformations are nonlinear (as well as nonlocal and noncanonical). The isotopies of quark theories under study in this paper therefore constitute an attempt at representing strong interactions in their most general possible nonlinear form (including nonlinearity in the derivatives of the wavefunctions).

Note also that any nonlinear theory can always be written in an infinite number of identical isolinear forms. Isotopies can therefore be conceived as methods for the reduction of nonlinear theories to identical, although more manageable isolinear forms.

Moreover, conventional exponentiation has no mathematical or physical meaning under isotopies, evidently because of the loss of the conventional associative envelope ξ in which the exponentiation is defined via the usual power series expansion. Nevertheless, the isotopic Poincaré-Birkhoff-Witt theorem permits a consistent generalization of the exponential mapping, called *isoexponentiation.*

These occurrences then permit the lifting of (connected) Lie groups G into the so-called *Lie-Santilli groups* $\hat{G}$[5,6] characterized by the isoexponentiatons in ξ

$$G: U(w) = e^{iXw} \Rightarrow \hat{G}: \hat{U}(w) = e_{|\xi}^{iXw} = \{e_{|\xi}^{iXTw}\}\hat{I} = \{e^{iXgw}\}\hat{I} \tag{1.7}$$

where the reformulation in terms of the conventional exponentiation has been done for simplicity of calculations. Isogroups $\hat{G}$ then verify the *isotopic group laws* **[5,6]**

$$\hat{U}(\hat{w}) * \hat{U}(\hat{w}') = \hat{U}(\hat{w}') * \hat{U}(\hat{w}) = U(\hat{w} + \hat{w}'),$$

$$\hat{U}(0) = \hat{U}(w) * \hat{U}(-w) = \hat{I} = g^{-1}. \tag{1.8}$$

The nontriviality of the isotopy is then illustrated by the fact that, even though the isotopic character can be eliminated in practical calculations via the rule

$$z' = \{e_{|\xi}^{iXw}\} * z = \{e^{iXgw}\}\hat{I}gz = \{e^{iXgw}\}z, \tag{1.9}$$

we have the appearance of the integro-differential quantity g in the exponent of the group structure. The reader should be aware of the existence of consistent isotopic generalizations of the main structural theorems of Lie algebras and Lie groups, such as: Lie's First, Second and Third Theorems, the Baker-Campbell-Hausdorff Theorem, etc.[5-8].

The preceding isotopies require the lifting of the conventional Hilbert space $\mathcal{H}$ of nonrelativistic or relativistic quantum mechanics with generic states $|\mu\rangle$ and conventional inner product $\langle\mu|\nu\rangle \in C$, into the *isohilbert space* $\hat{H}$ with generic isostates $|\hat{\mu}\rangle$, isoinner product and related iso normalization[16-19]

$$\mathcal{H}: \langle u|v\rangle \in C \Rightarrow \hat{\mathcal{H}}: \langle\hat{u}|\hat{v}\rangle = \langle\hat{u}|G|\hat{v}\rangle\hat{I} \in \hat{C}, \quad \langle\hat{u}|G|\hat{u}\rangle = 1, G>0 \tag{1.10}$$

where the isotopic element G is generally independent of g. The reader should therefore keep in mind that the general formulation of the isoquark theories implies the use of two *<<degrees of freedom>>*, the isotopic element $T = g$ of the isofield $\hat{C}$ and of Lie-isotopic theory (enveloping algebra, Lie-isotopic algebra and Lie-isotopic group), plus the independent isotopic element G of the isohilbert space.

The axiom-preserving character of isotopies (1.10) is established by the fact that the composition $\langle\hat{u}|G|\hat{v}\rangle$ is still <<inner>> under the assumed positive-definiteness of G, i.e. $\hat{\mathcal{H}}$ is Hilbert. One central problem of this paper is the identification of a realization of the isotopic elements g and G such to render identically null the transition probabilities between the isohilbert space $\hat{\mathcal{H}}$ and the conventional one $\mathcal{H}$

The above assumptions imply the generalization of the operation of Hermitean conjugated into the form

$$H^{\dagger} = g^{-1}GH^{\dagger}gG^{-1} \tag{1.11}$$

However, the general case with $g \neq G$ is excessively broad for the analysis of this paper, and can be kept in mind for possible future studies, such as the reconstruction at the isotopic level of exact symmetries when broken at the conventional level (see attempt[20] at reconstructing the *exact parity for weak interactions* via the embedding of all symmetry breaking terms in the isotopic elements, in which the maximal possible isotopic degrees of freedom are needed).

In fact, all the results of this paper can be derived for the simpler case when $g \equiv G$ which is hereon assumed, for which the conventional notion of Hermiticity is preserved, $\hat{H}^\dagger \equiv H^\dagger$ (see, e.g. ref. **[18]** for a review). Thus, *all quantities that are observable for conventional quark theories remain observable for the isoquark theories studied in this paper.*

The conventional eigenvalue equations for Hermitean operators $H{=}H^\dagger$ are however lifted into the *isoeigenvalue equations*

$$H * |u\rangle = E_0 |u> \Rightarrow Hg|\hat{u}\rangle = \hat{E} * |\hat{u}\rangle \equiv E|\hat{u}\rangle, \quad E \in \Re, \quad E \neq E^0 . (1.12)$$

where the reality of the eigenvalues is proved from the Hermiticity of the operator as in the conventional case. Note the *same* Hermitean operator H admits *different* eigenvalues under isotopies, which illustrates again, this time from an eigenvalue viewpoint, the nontriviality of the isotopies here considered.

The casuality of the isotopic treatment of nonlocal-integral interactions is then evident because of their embedding in the isounit $\hat{I}$ of the theory, its isocommutation with all possible generators, $[X \hat{,} \hat{I}] = X - X \equiv 0$, and its conventional expectation value $\langle \hat{u} | * \hat{I} * | \hat{v} \rangle = \langle \hat{v} | * | \hat{u} \rangle = 1$ (see ref. **[19]** for a detailed treatment).

Note that structure (1.12) implies the emergence of an operator realization of the <<hidden variables>>[3] characterized precisely by the isotopic element T[22]. The isotopies of quark theories studied in this paper can then be interpreted as providing an explicit realization of the theory of <<hidden variables>>, this time of unitary character. The emerging completion of quantum mechanics much along the historical argument by Einstein, Podolsky and Rosen[4] indicated earlier, is therefore given precisely by the isotopes (see ref. **[22]** for details).

As a final comment, note that, by construction, all isotopic structures coincide with the conventional ones at the abstract, realization-free level, for which all distinctions disappear between I and $\hat{I}, C$ and $\hat{C}, E(z,\bar{z},\delta,C)$ and $\hat{E}(z,\bar{z},\hat{\delta},\hat{C}), \xi$ and $\hat{\xi}$, L and $\hat{L}$, G and $\hat{G}$,

Uz and $U * z$, etc. In turn, this abstract unity guarantees the mathematical consistency of the isotopic formulations. In this paper we shall use the above isotopic methods for the construction of isoquark theories verifying the following primary conditions.

Condition I. Identically preserve all quantum number of conventional quark theories, such as charge, hyperchange, strangeness, baryonic number, color, etc.

Condition II. Verify an exact quark confinement, defined as an identically null transition probability of isoquarks into free states (or identically null probability of tunnel effect for free isoquarks), even in the absence of a potential barrier.

Condition III. Permit convergent isoperturbative expansions for strong interactions when conventionally divergent.

2. Isotopies of Quark Theories

A first study of the isotopic $S\hat{U}(3)$ symmetry was conducted in ref. **[23]** with the proof of the isomorphism $S\hat{U}(3) \approx SU(3)$, while general lines on the representation theory of $SU(n)$ where studied in ref. **[24]**, with particular emphasis on the $SU(2)$ case.

In this section we shall first identify the flavor $S\hat{U}(3)$ symmetry, here called *isoflavor symmetry*, construct its fundamental representations, impose Condition I of Section 1, and then perform the extension to the case of the isotopic color, here called *isocolor*. The analysis will be conducted for isohilbert spaces (1.10) under the condition $T = g \equiv G$, and consequential preservation of the conventional Hermiticity.

The *isotopic* $S\hat{U}(3)$ *symmetries* can be defined as the largest possible, nonlinear, nonlocal and noncanonical Lie-Santilli invariance group of the three-dimensional complex isoeuclidean spaces (1.3).

Since isotopies preserve the simplicity (or semisimplicity) as well as the dimensionality of the original Lie symmetry[7,8], we can construct the *isotopic* $S\hat{U}(3)$ *algebra* by searching for eight generators $\hat{\lambda}_i$

verifying the *isocommutation rules*

$$S\hat{U}(3):[\hat{\lambda}_i \hat{,} \hat{\lambda}_j] = \lambda_j g \lambda_j - \lambda_j g \hat{\lambda}_i = 2if_{ijk}\hat{\lambda}_k, i,j, \quad k=1,2,\ldots,8$$

namely, under the condition of preserving the conventional structure constants f_{ijk} of $SU(3)$[1]. This ensures the local isomorphisms $S\hat{U}(3) \approx SU(3)$ *ab initio*.

The *isotopic* $S\hat{U}(3)$ can be defined in terms of the following isoexponentiations on a isohilbert space $\hat{\mathcal{H}}$

$$\hat{U}(w) = e_{\xi}^{iw_k\hat{\lambda}_k} = \hat{I}\{e^{iw_k g \hat{\lambda}_k}\} = \{e^{i\hat{\lambda}_k g w_k}\}\hat{I}, \quad w \in \Re$$

under the condition that they are isounitary, i.e.

$$\hat{U} * \hat{U}^{\dagger} = \hat{U}^{\dagger} * \hat{U} = \hat{I} = g^{-1} \qquad (2.3),$$

which is reducible to the conditions

$$tr(\hat{\lambda}_k g) \equiv 0, \quad k=1,2,\ldots,8. \qquad (2.4)$$

The isorepresentations of $S\hat{u}(3)$ can be constructed via the use of the *isocreation and isoannihilation operators* $\hat{a}_i, \hat{a}_j^{\dagger}, i=u,d,s$ first introduced in ref. [**25, 26**] with isocommutation rules

$$[\hat{a}_i \hat{,} \hat{a}_j^{\dagger}] = \delta_{ij}, [\hat{a}_i \hat{,} \hat{a}_j] = [\hat{a}_i^{\dagger} \hat{,} \hat{a}_j^{\dagger}] = 0, \ i,j, = u, d, s,$$

where the product is defined by equations (1.5), and isocasimir invariant

$$\hat{N} = \hat{a}_u^{\dagger} * \hat{a}_u + \hat{a}_d^{\dagger} * \hat{a}_d + \hat{a}_s^{\dagger} * \hat{a}_s = \hat{N}_u + \hat{N}_d + \hat{N}_s.$$

Introduce now the isobasis and related isonormalization

$$|n_u, n_d, n_s\rangle, \qquad n_u, n_d, n_s = 0, 1 \qquad (2.7a)$$

$$\langle n_u, n_d, n_s | n_u \hat{\uparrow} n_{d'}, n_{s'}\rangle = \langle n_u, n_d, n_s | g | n_{u'}, n_{d'}, n_{s'}\rangle \hat{I}$$

$$= \delta_{uu'}\delta_{dd'}\delta_{ss'}\hat{I}, \hat{I} = \hat{G}^{-1} \qquad (2.7b)$$

The following isoeigenvalue equations then hold

$$\hat{N}_u * |n_u, n_d, n_s\rangle = g_u |n_u, n_d, n_s\rangle, \qquad (2.8a)$$

$$\hat{N}_d * |n_u, n_d, n_s\rangle = g_d |n_u, n_d n_s\rangle, \qquad (2.8b)$$

$$\hat{N}_s * |n_u, n_d, n_s\rangle = g_s |n_u, n_d, n_s\rangle, \qquad (2.bc)$$

where g_u, g_d, g_s are certain functions of the isometric elements g_{kk} to be determined shortly.

The action of the isocreation and isoannihilation operators can be easily computed yielding

$$\hat{a}_u * |n_u, n_d, n_s\rangle = (g_u n_u)^{1/2} |n_u - 1, n_d, n_s\rangle, \qquad (2.9a)$$

$$\hat{a}_u^{\dagger} * |n_u, n_d, n_s\rangle = \left[(g_u n_u)^{1/2} + 1\right] || n_u + 1, n_d, n_s\rangle, \qquad (2.9b)$$

with similar expressions for the other operators.

This allows the computation of the matrix elements

$$M_{ij} = \langle n_u, n_d, n_s | * \hat{a}_i^{\dagger} * \hat{a}_j * n_{u'}, n_{d'} n_{s'}\rangle. \qquad (2.10)$$

Simple calculations then yield the following *fundamental isorepresentations of* $S\hat{U}(3)$

$$\hat{\lambda}_1 = (\hat{a}_u^{\dagger} * \hat{a}_d + \hat{a}_d^{\dagger} * \hat{a}_u) = \begin{pmatrix} 0 & g_d^{1/2} & 0 \\ g_u^{1/2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\hat{\lambda}_2 = (\hat{a}_u^{\dagger} * \hat{a}_d + \hat{a}_d^{\dagger} * \hat{a}_u) = \begin{pmatrix} 0 & -ig_d^{1/2} & 0 \\ ig_u^{1/2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\hat{\lambda}_3 = (k_1\hat{a}_u^{\dagger} * \hat{a}_u - k_2\hat{a}_d^{\dagger} * \hat{a}_d) = \begin{pmatrix} k_1 g_u & 0 & 0 \\ 0 & -k_2 g_d & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\hat{\lambda}_4 = (\hat{a}_u^{\dagger} * \hat{a}_s + \hat{a}_s^{\dagger} * \hat{a}_u) = \begin{pmatrix} 0 & 0 & g_s^{1/2} \\ 0 & 0 & 0 \\ g_u^{1/2} & 0 & 0 \end{pmatrix},$$

$$\hat{\lambda}_5 (i\, \hat{a}_u^{\dagger} * \hat{a}_s - \hat{a}_s^{\dagger} * \hat{a}_u) = \begin{pmatrix} 0 & 0 & -ig_s^{1/2} \\ 0 & 0 & 0 \\ ig_u^{1/2} & 0 & 0 \end{pmatrix},$$

$$\hat{\lambda}_6 = (\hat{a}_d^{\dagger} * \hat{a}_s + \hat{a}_s^{\dagger} * \hat{a}_d) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & g_s^{1/2} \\ 0 & g_d^{1/2} & 0 \end{pmatrix},$$

$$\hat{\lambda}_7 = -i(\hat{a}_d^{\dagger} * \hat{a}_s - \hat{a}_s^{\dagger} * \hat{a}_d) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -ig_s^{1/2} \\ 0 & ig_d^{1/2} & 0 \end{pmatrix},$$

$$\hat{\lambda}_8 = \frac{1}{\sqrt{3}}(k_3\hat{a}_u^{\dagger} * \hat{a}_u + k_4\hat{a}_d^{\dagger} * \hat{a}_d - 2k_5\hat{a}_s * \hat{a}_s) =$$

$$= \frac{1}{\sqrt{3}} \begin{pmatrix} k_3 g_u & 0 & 0 \\ 0 & k_4 g_d & 0 \\ 0 & 0 & -2k_5 g_s \end{pmatrix}, \tag{2.11}$$

where the additional k-quantities are unknown functions of the isometric elements g_{kk}.

To compute these quantities, we first consider the eight conditions $tr(\hat{\lambda}_i g) \equiv 0$ which yield

$$k_1 g_u g_{11} = k_2 g_d g_{22},\ k_3 g_u g_{11} + k_4 g_d g_{22} = 2k_5 g_s g_{33}.$$

The remaining conditions are given by the isocommutation rules

$$|\hat{\lambda}_1 \hat{;} \hat{\lambda}_{21} = 2 | \lambda_3 : (g_u g_d)^{1/2} g_{22} = k_1 g_u, (g_u g_d)^{1/2} g_{11} = k_2 g_d,$$

$$|\hat{\lambda}_4 \hat{;} \hat{\lambda}_5| = i\lambda_3 + i\sqrt{3}\,\lambda_8 : (g_u g_s)^{1/2} g_{33}$$

$$= \frac{1}{2}(k_1 + k_3) g_u, (g_u g_s)^{1/2} g_{11} = k_5 g_s, \tag{2.13b}$$

$$|\hat{\lambda}_6 \hat{;} \hat{\lambda}_7| = -i\lambda_3 + 1\sqrt{3}\,\lambda_8 : (g_d g_s)^{1/2} g_{33} =$$

$$= \frac{1}{2}(k_2 + k_4) g_d, (g_d g_s)^{1/2} g_{22} = k_5 g_s, \tag{2.13c}$$

with solutions

$$k_1 = k_3 = g_{11},\ k_2 = k_4 = g_{22},\ k_5 = g_{33}, \tag{2.14a}$$

$$g_u = g_{22}^2,\ g_d = g_{11}^2, g_s = g_{11}^2 g_{22}^2 / g_{33}^2, \tag{2.14b}$$

The *fundamental isorepresentation of* $S\hat{U}(3)$ is then given by expressions (2.11) with values (2.14) for the unknown quantities. As an example, the diagonal matrices are given by

$$\hat{\lambda}_3 = \begin{pmatrix} g_{11} g_{22}^2 & 0 & 0 \\ 0 & -g_{22} g_{11}^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{2,15a}$$

$$\hat{\lambda}_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} g_{11} g_{22}^2 & 0 & 0 \\ 0 & g_{22} g_{11}^2 & 0 \\ 0 & 0 & -2\dfrac{g_{11}^2 g_{22}^2}{g_{33}} \end{pmatrix}, \tag{2.15b}$$

The verification of the isocommutation rules (2.1) by the above isorepresentation is instructive for the reader not familiar with isotopic techniques. Equally instructive is the proof that *the isorepresentations of* $S\hat{U}(3)$ *are not unitarily equivalent to those of the conventional* $SU(3)$. We are here referring to the lack of existence of a unitary transformation $UU^\dagger = U^\dagger U = I = \mathrm{diag.}\ (1, 1, 1)$ such that $\lambda_k = U \hat{\lambda}_k U^\dagger$, $k = 1, 2, ..., 8$. Note also that isorepresentations (2.11) are not reducible to the form $\hat{\lambda}_k = \lambda_k \hat{I}$ called the trivial isotopy[10].

A few comments are now in order. First, let us recall the isotopies $\hat{G}(3.1)$ and $\hat{P}(3.1)$ of Galilei's and Poincaré's symmetries, respectively[6,12,13]. We shall then define as a *nonrelativistic or relativistic isoquark* a representation of the tensorial products

$$\hat{G}(3.1) \times S\hat{U}(3), \hat{P}(3.1) \times S\hat{U}(3),$$

respectively. Equivalently, we can say that an isoquark is an ordinary quark, although defined with respect to the most general possible, positive-definite isounits $\hat{I}$

Note that quarks and isoquarks coincide by construction at the abstract realization-free level because, in addition to the local isomorphisms $S\hat{U}(3) \approx SU(3)$[23], we also have $\hat{G}(3.1) \approx G(3.1)$ and $\hat{P}(3.1) \approx P(3.1)$[6,12]. This points should be kept in mind because criticisms on the isoquark theory, unless treated with care, could result to be criticisms on the basic axioms of conventional quark theories.

We next point-out the degrees of freedom of isoquarks. Note in this respect that, except for their positive-definiteness, the isometrics g are left completely unrestricted by the isotopic theory. This means that they can have indeed the most general possible nonlinear and nonlocal dependence on all variables and quantities. But their elements enter into the very definition of the isorepresentations, i.e.,

$$S\hat{U}(3): \hat{U} * z = \hat{U}(t, z, \bar{z}, \dot{z}, \bar{z}, \psi, \psi^\dagger, \partial\psi, \partial\psi^\dagger, ...) * z$$
$$= \left\{ e^{i\hat{\lambda}_k g(t,z,\bar{z},\psi,\psi',\partial\psi,\partial\psi',...) g(t,z,\bar{z},\psi,\psi',\partial\psi,\partial\psi',...) w}_{\xi}\ {}_k \right\} z. \tag{2.17}$$

As a result, *the isotopic* $S\hat{U}(3)$ *theory provides an infinite class of nonlinear, nonlocal and noncanonical generalizations of the conventional* $SU(3)$ *symmetry while preserving the local isomorphisms* $S\hat{U}(3) \approx SU(3)$.

Note also that the use of the plural in $S\hat{U}(3)$ <<symmetries>> is evidently due to the infinitely possible isometrics g. This means that, at this stage of the analysis, we have infinitely possible, geometrically equivalent, but physically different isoquarks.

We now impose Condition I of Section I on the identity of the quantum numbers of the isoquark theory with those of the conventional one. For this purpose note that the Cartan isosubalgebra remains characterized by the maximal set of isocommuting, diagonal elements $\hat{T}=\hat{\lambda}_3$ and $\hat{Y}=\hat{\lambda}_8/\sqrt{3}$ with equations

$$\hat{T}*|n_u,n_d,n_s\rangle=t|n_u,n_d,n_s\rangle, \hat{Y}*|n_u,n_d,n_s\rangle=y|n_u,n_d,n_s\rangle \quad (2.18)$$

and isoeigenvalues

$$t=g_{11}^2 g_{22}^2\{1,-1,0\}, y=g_{11}^2 g_{22}^2\left\{\frac{1}{3},\frac{1}{3},-\frac{2}{3}\right\}. \quad (2.19)$$

Condition I is therefore verified by the particular expression

$$g_{11}\equiv g_{22}^{-1}=\chi_1=\chi_2^{-1}$$

under which the eigenvalues of the isotopic $\hat{T}$ and $\hat{Y}$ quantities coincide with those of the conventional operators T and Y,

$$\hat{T}*|n_u,n_d,n_s\rangle\equiv T|n_u,n_d,n_s\rangle=\{1,-1,0\}|n_u,n_d,n_s\rangle, \quad (2.21a)$$

$$\hat{Y}*|n_u,n_d,n_s\rangle\equiv Y|n_u,n_d,n_s\rangle=\left\{\frac{1}{3},\frac{1}{3},-\frac{2}{3}\right\}|n_u,n_d,n_s\rangle. \quad (2.21b)$$

Equivalent results could be reached bv imposing the direct identities

$$T\equiv\hat{T}g=\hat{\lambda}_3\hat{g}, \quad Y\equiv\hat{Y}g=\lambda_8\hat{g}/\sqrt{3}. \quad (2.22)$$

and use of condition (2.20). The nontriviality of the realization is illustrated by the fact that, from equations (2.16) $\hat{T}\neq T\hat{I}$ and $\hat{Y}\neq Y\hat{I}$.

The above results ensure that the isoflavor SU (3) symmetry possesses exactly the same quantum numbers of the conventional theory. A primary result of this paper is that current experimental evidence cannot distinguish between conventional and isotopic quark theories.

Along these lines we here recall that the isotopies do not change the dimensionally of a given irreducible isorepresentation[6,7,8] or the

dimensionality in the decomposition of the reducible ones. More explicitly, let us denote isorepresentation (2.11) with the symbol $\hat{3}$ and its conjugate with the symbol $\hat{3}^c$. Then we have the tensorial product of isoflavor isorepresentations

$$\hat{3}\otimes\hat{3}^c=\hat{I}\otimes\hat{8}, \ \hat{3}\otimes\hat{3}\otimes\hat{3}=\hat{I}\otimes\hat{8}\otimes\hat{8}\otimes\hat{1}\hat{0}, \quad (2.23)$$

etc. which is fully equivalent to the conventional ones, yet exhibiting novel internal degrees of freedom.

Next, we derive the notion of color from first isotopic principles, here called *isocolor*. Recall[1] that we were forced to generalize the original flavor SU (3) via the introduction of color to avoid the violation of Pauli's exclusion principle in states such as $\Omega^-=sss$, the quarks being Fermions. This problem was resolved via the introduction of a <<hidden label>> which would differentiate quarks with the same flavor and spin, thus permitting the verification of Pauli's principle.

In this paper we submit the hypothesis that *the isocolor is characterized by the isometric element* g_{33} hereon denoted with χ. Consider the isoflavour $SU(3)$-symmetry as introduced above, under condition (2.20), where the isometric element $g_{11}=g_{22}^{-1}$ is assumed as fixed (see next section). Such a theory still exhibits one additional degree of freedom given by $g_{33}=\chi_3$. In fact, the eigenvalues of $\hat{T}\equiv T$ and $\hat{Y}\equiv Y$ as well as of all other isoflavor quantum numbers are explicitly dependent on g_{11} and g_{22}, but completely independent from g_{33}. As a result, it appears that the <<hidden label>>[1] can indeed be derived from first principles, that is, from the very structure of the isorepresentations of $S\hat{U}(3)$.

The isotopies of $S\hat{U}(3)$ considered so far allow us to introduce a family of isoflavor isoquarks $\hat{q}_k, k=u,d,s$ depending on the assumed value of $g_{33} = \chi_{33}$ for each fixed $g_{11} = g_{22}^{-1}$, i.e.,

$$\hat{q}_u(\chi_3), \hat{q}_d(\chi_3), \hat{q}_s(\chi_3), \chi_3 = g_{33}. \qquad (2.24)$$

Note that the above identification is prohibited for conventional flavor quarks q_k, trivially, because in this case the element g_{33} is assumed as being one. Thus, our isocolor has no counterpart in conventional color theories. Note also that, while in conventional settings the $SU(3)$ symmetry is essentially duplicated to introduce color, this is no longer the case under isotopy because color is characterized by the third component of the isometric of the *isoflavor* $S\hat{U}(3)$. In this sense, the *isotopies permit a unification of flavor and color theories into one single mathematical structure, the isotopies* $S\hat{U}(3)$.

The restriction of the permissible values of the isocolor can be done as in the conventional case, via the condition that only isocolor singlets exist, thus resulting in only three different values, here symbolically indicated with

$$\chi = \hat{1}, \hat{2}, \hat{3} \text{ (or isored, isowhite and isoblue)}.$$

The above theory can then be completed with the *isocharmed and isobottom isoquarks,* although this extension is here left to the interested reader jointly with numerous additional aspects we cannot possibly study in this first paper.

The reader can now see the realization of the <<hidden variables>> provided by the isoquark theory. In fact we have essentially shown in this section via isorepresentation (2.11) that the abstract $SU(3)$ symmetry possesses three <<hidden variables>>, the diagonal elements $g_{kk} = \chi_k, k = 1, 2, 3$ of the isometric g, which are reduced to only two free parameters from condition (2.20). The underlying <<completion>> of quantum mechanics is then characterized by the isotopies $I \Rightarrow \hat{I}, C \Rightarrow \hat{C}$, $\xi \Rightarrow \hat{\xi}, L \Rightarrow \hat{L}, G \Rightarrow \hat{G}, \mathcal{H} \Rightarrow \hat{\mathcal{H}}$, etc.

3. Exact Confinement of Isoquarks

In this Section we shall identify a suitable realization of the isometrics of the isoquark theory of the preceding section which verifies condition II of Section 1, the achievement of an *exact confinement of isoquarks,* i.e., the identically null transition probability of isoquarks to be free even in the absence of a conventional potential barrier.

Recall that a nonrelativistic isoquark is an isorepresentation of $\hat{G}(3.1) \times S\hat{U}(3)$ in isospaces $\Re_t \times \hat{E}_{\text{space}}(r, \delta, \Re) \times \hat{E}_{\text{int.}}(z, g, \hat{C})$, with corresponding expressions for the relativistic case. Thus, the total isounit of the theory is given by the tensorial product

$$\hat{I}_{\text{tot}} = \hat{I}_{\text{space-time}} \times \hat{I}_{\text{int}}, \hat{I}_{\text{space-time}} \neq \hat{I}_{\text{int}} = g^{-1}$$

with corresponding tensorial products for total isohilbert spaces, isofields, isoassociative algebras, etc. (see refs,[**18, 19**] for details).

In this section we shall study the exact confinement of isoquarks via the sole use of the internal part $\hat{I}_{\text{int}}$. This is due to the fact that the isotopies are essentially recommended for the internal structural problem of hadrons (i.e. at mutual distances of particles smaller than 1 fm), evidently because quantum mechanics is exact for the arena of its inception and experimental verification. Thus, all exterior units (i.e., units for distances bigger than 1 fm) must be the conventional ones.

More particularly, the model submitted in this section is based on the incoherence of the internal unitary isohilbert space with the corresponding external space, while the Hilbert spaces related to the spacetime symmetries can remain coherent. Moreover, we shall study the exact confinement of isoquarks for the simpler case in which the isotopic element $T = g$ of the enveloping algebra coincides with the isotopic element G of the isohilbert space (Section 1).

The reader should therefore be aware that isotopic techniques can provide even stronger means for the achievement of exact confinement based on: 1) the incoherence of the total internal and external Hilbert spaces; 2) the use of two independent isotopic elements g and G; 3) the addition of an infinite potential barrier; and others.

By keeping these additional possibilities in mind, the main result of this section can be formulated as follows.

LEMMA 1 *Let $\mathcal{H}$ be a conventional Hilbert space of the exterior $SU(3)$ symmetry with states $|\phi\rangle, |\psi\rangle, \ldots$ and inner product*

$$\langle\phi|\psi\rangle = \int dr' \phi^\dagger(r')\psi(r') \in C, \tag{3.2}$$

and let $\hat{\mathcal{H}}$ be an isohilbert space of the interior $S\hat{U}(3)$ symmetry with isostates $|\hat{\phi}\rangle, |\hat{\psi}\rangle, \ldots$ and isoinner product

$$\langle\hat{\psi}\uparrow\hat{\psi}\rangle = \langle\hat{\phi}|g|\hat{\psi}>\hat{I}_{\text{int}} = \{\int dr' \hat{\phi}^\dagger(r') g \hat{\psi}(r')\}\hat{I}_{\text{int}} \in \hat{C},$$

$$\hat{I}_{\text{int}} = g^{-1} > 0. \tag{3.3}$$

Then, there always exists at least one realization of the isometric g under which the probability amplitude of an interior isoquark $|\hat{q}\rangle$ to propagate into the exterior as a free quark $|q\rangle$ is identically null even in the absence of a potential barrier, i.e.,

$$P(q;\hat{q}) = |\langle q(t)|g|\hat{q}(t_0)\rangle|^{\hat{2}} \equiv 0. \tag{3.4}$$

PROOF. Consider the isostate $|\hat{q}(t_0)> \in \hat{\mathcal{H}}$. Its time evolution is characterized by the isounitary operator

$$|\hat{q}(t)\rangle = \hat{U}(t) * |\hat{q}(t_0)\rangle = \hat{U}(t) g |\hat{q}(t_0)\rangle. \tag{3.5}$$

The proof of the above lemma is then reduced to the orthogonality $\langle q(t)|g|\hat{q}(t)\rangle \equiv 0$ which evidently always admit at least one solution in g.

For this, introduce the new ordinary state in $\mathcal{H}$

$$|\tilde{q}(t)\rangle = g|\hat{q}(t)\rangle. \tag{3.6}$$

It is then easy to see that, for a nontrivial expression of g, the eigenvalues of a Hermitean operator H are different (Section. 1), i.e.,

$$H\,|q(t)\rangle = E|q(t)\rangle,\ H|\tilde{q}(t)\rangle = \tilde{E}|\tilde{q}(t)\rangle, E \neq \tilde{E},$$

But then

$$0 \equiv \langle q(t)\rangle| \leftarrow H|\tilde{q}(t)\rangle - \langle q(t)|H \rightarrow |\tilde{q}(t)\rangle = (E-\tilde{E})\langle q(t)|\tilde{q}(t)\rangle, \tag{3.8}$$

which can hold identically iff

$$\langle q(t)|\tilde{q}(t)\rangle \equiv 0. \quad \text{QED} \tag{3.9}$$

In different terms, the null transition probability is inherent in the very structure of the isotopies, that is, in their capacity of being a realization of the <<hidden variables>> for the inferior structural problem only, with the consequential alteration of the interior eigenvalues recalled in Section 1. Still equivalently, we can say that the lack of an exact confinement until now may eventually result as being due to the lack of an explicit and quantitative realization of the theory of <<hidden variables>>.

Our next objective is to identify a realization of the isometry g of $S\hat{U}(3)$ which ensures the achievement of the above exact confinement. The model was submitted in talk[28], and has not appeared in print until now. It is based on *the quantization of Nambu's mechanics for triplets*[29]*which emerged as possessing an intriguing and essential isotopic structure*[30,31]. In fact, the admission of an isoquantized Nambu triplet space for the interior of hadrons is so structurally incoherent with the conventional Hilbert space for our exterior space-time continuum, to result indeed in an evident exact confinement under the most extreme high energy collisions, and in the absence of any potential barrier.

Let x_{ai} be the coordinates of a triplet in Nambu's generalized phase space with $i = 1,2,3$, and α representing a number of space-time components which remain unrestricted by the isotopic theory, and are thereforre ignored. As well known,. *Nambu's classical dynamical* *equations* for an arbitrary quantity F can be writen for the case of triplets [29]

$$\frac{dF}{dt} = \{F, H_1, H_2\}, \qquad (3.10)$$

where H_1 and H_2 are <<Nambu's Hamiltonians>>, and the brackets are given by the Jacobian

$$\{F, H_1, H_2\} = \sum_a \frac{\partial(F, H_1, H_2)}{\partial(x_{\alpha 1}, x_{\alpha 2}, x_{\alpha 3})} \qquad (3.11)$$

with a well-known extension to more than three particles (e.g. nine colour isoquarks) here ignored for brevily, while the rest of the properties of the mechanics are assumed as known.

The isoquantization of Nambu's mechanics [30,31] can be summarized as follows. Assume an isohilbert space $\hat{\mathcal{H}}$, and let us preserve the same classical symbols for quanitites now defined in $\hat{\mathcal{H}}$. Then *the operator image of Nambu's equation of motion* is given by

$$ic\frac{dF}{dt} = [F, \hat{}H_1, \hat{}H_2], c \in C, c \neq 0, \qquad (3.12)$$

under the requirement that the operator brackets verify the following conditions evidently equivalently to the corresponding classical ones:

a) If $F_k, k = 1, 2, 3$, are members of the algebra U characterized by the generalized brackets, then

$$[F_1, \hat{}F_2, \hat{}F_3] \in U; \qquad (3.13)$$

b) The generalized brackets verify the alternative laws

$$[F_1, \hat{}F_2, \hat{}F_3] = -[F_2, \hat{}F_1, \hat{}F_3], \text{ etc.} \qquad (3.14)$$

c) The generalized brackets characterize a derivation, i.e. verify the differential rule under the product $A \odot B$ of the underlying enveloping algebra

$$[A \odot B, F_1, \hat{}F_2] = A \odot [B, \hat{}F_1, \hat{}F_2] + [A, \hat{}F_1, \hat{}F_2] \odot B, \text{ etc.} \qquad (3.15)$$

The realization of the generalized brackets verifying the above properties is given by

$$[A, \hat{}B, \hat{}C] = (A \odot B) \odot C + (B \odot C) \odot A + (C \odot A) \odot B - \\ - (B \odot A) \odot C - (A \odot C) \odot B - (C \odot B) \odot A, \qquad (3.16)$$

where the product $A \odot B$ is given by

$$A \odot B = ARB - BSA = AH_1^{-1}B - BH_2^{-1}A, \qquad (3.17)$$

and results to characterize a (nonassociative) *Lie-admissible algebra* precisely of the type assumed at the foundation of the isotopies of this paper [5,14].

The point important for this paper is that *Nambu's classical mechanics for triplets is an intringuing particular case of the isotopies of clasiccal Hamiltonian mechanics called Birkhoffian mechanics*[6] *and, consequently, its operator image* (3.12) *with realization* (3.16) *is an intriguing particular case of the isotopic <<completion>> of quantum mechanics.*

In fact, we have the following

LEMMA 2 **[31]**. *The operator time evolution* (3.12) *formulated with respect to the triplet bracket* $[F, \hat{}H_1, \hat{}H_2]$ *of equation* (3.16) *are characterized by the following isotopy of Heisenberg equations*

$$ic\frac{dF}{dt} = [F, \hat{}H_1, \hat{}H_2] \equiv [F, \hat{}H] = FTH - HTF, \qquad (3.18a)$$

$$H = H_1, + H_2, \quad T = H_1^{-1} + H_2^{-1}, \qquad (3.18b)$$

Ref. **[31]** then introduced the classical isosymmetries of the Birkhoffian realization of Nambu's mechanics, and the corresponding isosymmetries of the operator formulation.

Evidently, *a most salient application of equation* (3.18) *for operator* *triplets is the identification of the isometric g of the isoquark theory of Section 2 with the isotopic element* (3.18b) [28]

$$g = T = H_1^{-1} + H_2^{-1} \tag{3.20}$$

with the diagonalization

$$g = H_1^{-1} + H_2^{-1} = \begin{pmatrix} \chi_1 & 0 & 0 \\ 0 & \chi_1^{-1} & 0 \\ 0 & 0 & \chi_3 \end{pmatrix}. \tag{3.21}$$

Note, that since each Hamiltonian can be assumed to be positive definite, so is the sum of their inverses. Thus, g is positive definite, by therefore guaranteeing the local isomorphisms $S\hat{U}(3) \approx SU(3)$. The expression $g = H_1^{-1} + H_2^{-1}$ is therefore a fully acceptable realization for isoquark theories (additional important properties of such identification will be indicated in the next section).

The achievement of an exact isoquark confinement without potential barrier then follows from the structural difference between $\mathcal{H}$ and $\hat{H}$ indicated earlier, the former being based on an Eucledian sapce, while the latter being based on Nambu's triplet space.

We reach in this way a model in which the isoquarks are exactly confined, yet are free in the sense that have no binding energy. This view apepars to support current studies on <<asymptotic freedom>>, although at all possible low and high energies, which we leave for investigation by the interested reader.

4. Convergtent Perturbative Isoexpansions

An additional important application of our isotopies is that of permitting the regaining of isotopic convergence in perturbative expansions for strong interactions, called *isoperturbation theory*. This possibility was pointed out in the first proposal to build the isotopies of quantum mechanics [14], and was later investigated in refs. **[24, 36]**,

while a detailed presentation is appearing in ref. **[19]**. In this section we shall reinspect this aspect, specifically, for the isoquark theory of the preceding sections. The main property can be formulated as follows:

LEMMA 3. *Given a conventionally divergent, quantum mechanical, perturbative series, there always exsits an infinite number of isotopies of complex numbers, operator algebras and Hilbert spaces characterized by*

$$|\hat{I}| = |g^{-1}| >> -1, \text{ or } |g| << 1 \tag{4.1}$$

under which the series becomes convergent.

The above property is so evident to need no formal proof. We shall merely illustrate it from the viewpoints of Lie algebras, conventional expansions, and Lie groups. Consider a conventional, divergent, canonical series in terms of a positive-definite parameter $k >> 1$.

$$A(k) = A(0) + k[A,H]_\xi/1! + k^2[[A,H]_\xi, H]_\xi/2! + \ldots \Rightarrow \infty, \tag{4.2a}$$

$$[A,H]_\xi = AH - HA, \tag{4.2b}$$

where the operators A and H are assumed to be Hermitean and sufficiently bounded.

Lemma 3 is then readily proved by the isotopy

$$\hat{A}(k) = \hat{A}(0) + k[A,H]_\xi/1! + k^2[[A,H]_\xi, H]_\xi/2! + \ldots \le |N| < \infty, \tag{4.3a}$$

$$[A,H]_\xi = ATH - HTA, \ |T| \le 1. \tag{4.3b}$$

As an example, the simple case in which T is a sufficiently small positive definite constant $T = \varepsilon k^{-1}$ turns expansion (4.2) into the convergent form

$$\hat{A}(k) = \hat{A}(0) + k[A,H]_\xi/1! + k^2[[A,H]_\xi, H]_\xi/2! + \ldots, \tag{4.4a}$$

$$\equiv \hat{A}(0) + \varepsilon[A,H]_\xi/1! + \varepsilon^2[[A,H[_\xi, H]_\xi/2! + \ldots,$$

$$k>1,\ T<<1,\ \varepsilon = kT<<1. \tag{4.4b}$$

Even though more involved, the same result can also be proved for a positive-definite operator T verifying condition (4.1), e.g., with sufficiently small eigenvalues.

The proof via conventional perturbative expansions is also so simple to appear trivial. Consider a Hermitean operator of the type appearing in strong interactions,

$$H(k) = H_0 + kV, \quad H_0|\psi\rangle = E_0|\psi\rangle,$$

$$H(k)|\psi(k)\rangle = E(k)|\psi\rangle(k)\rangle, \quad k>>1. \tag{4.5}$$

Assume, for simplicity at this first stage, that H_0 has a nondegenerate discrete spectrum, and consider the time-independent perturbation. Then, conventional perturbative series are divergent, as is well known. In fact, the eigenvalue $E(k)$ of $H(k)$ up to second order is given by (see, e.g., ref. **[37]**).

$$E(k) = E_0 + kE_1 + k^2E_2 = E_0 + k\langle\psi|V|\psi\rangle + k^2\sum_{p\neq n}\frac{|\langle\psi_p V\psi_n\rangle|^2}{E_{0n}-E_{0p}}. \tag{4.6}$$

But under isotopies we have

$$H(k) = H_0 + kV, \quad H_0T|\tilde{\psi}\rangle = \tilde{E}_0|\tilde{\psi}\rangle,$$

$$H(k)T|\psi(k)\rangle = \tilde{E}(k)|\psi(k)\rangle, k>1, \tag{4.7}$$

By keeping in mind that under isotopies expectation values $\langle y|H|\psi\rangle$ are lifted into the form $\langle\psi|THT|\psi\rangle$ (see, e.g., ref. **[16]**), then tedious but simple liftings of the conventional perturbation expansion yield[24,36]

$$\tilde{E}(k) = \tilde{E}_0 + k\tilde{E}_1 + k^2\tilde{E}_2 + \tilde{O}(k^2)$$

$$= \tilde{E}_0 \cdot k\langle\tilde{\psi}|TVT|\tilde{\psi}\rangle + k^2\sum_{p\neq n}\frac{|\langle\tilde{\psi}_p|TVT|\tilde{\psi}_n\rangle|^2}{\tilde{E}_{0n}-\tilde{E}_{0p}}.$$

whose convergence can be evidently reached via a suitable selection of the isotopic element verifying conditions (4.1).

As an example for a positive-definite constant $T<<1$, expression (4.8) becomes

$$\tilde{E}(k) = \tilde{E}_0 + kT^2\langle\psi|VT|\psi\rangle + k^2T^5\sum_{p\neq n}\frac{|\langle\psi_p|V|\psi_n\rangle|^2}{\tilde{E}_{0n}-\tilde{E}_{0p}} \tag{4.9}$$

This shows that the original divergent coefficients $1, k, k^2, \ldots$ are now turned into the manifestly convergent coefficients $1, kT^2, k^2T^5, \ldots$, with $k>1$ and $T<<1$, thus ensuring isoconvergence for a suitable selection of T for each given k and V.

Note that the above results hold under the particular case of the isotopies of quantum mechanics (Section 1) for which $T = G$. Even broader possibilities then exist for the general case $T\neq G$. For such general treatment, we refer the interested reader to the ref. **[19]**.

A third and perhaps more elegant way of illustrating the isoconvergence is via the isosymmetries. Suppose that the original Hamiltonian has the familiar Galilei-invariant form for N particles represented with the index k = 1, 2, ..., N on Euclidean three-dimensional space $E(r,\delta,\Re)$

$$H = \sum_k p_k\delta p_k/2m_k + kV(r), r = |(r_i-r_j)\delta(r_i-r_j)|, V_2. \tag{4.10}$$

Its extension in the isoeuclidean space $\hat{E}(r,\hat{\delta},\hat{\Re}), \hat{\delta} = T\delta, \hat{\Re} \approx \Re\hat{I}$, $\hat{I} = T^{-1} > 0$[12] is given by

$$\hat{H} = \sum_k p_k\hat{\delta}(t,p,\ldots)p_k/2m_k + kV(r),$$

$$r = |(r_i - r_j)\hat{\delta}(t, p, \ldots)(r_i - r_j)|, \ k > 1.$$

and results to be automatically invariant under the isogalilean symmetry $\hat{G}(3.1)$ constructed for the isounit $\hat{I} = T^{-1} \equiv \hat{\delta}^{-1}$.

It is then easy to see that the conventional group theoretical approach to perturbation (see, e.g., ref. [38], Section 6.3) admits a consistent (and intriguing) lifting precisely into the isoperturbation theory. In fact, the eigenvectors of the expression

$$\tilde{H}_0 * |\tilde{\psi}_a\rangle = \tilde{E}_{0a} |\tilde{\psi}\rangle, \tilde{H}(k) * |\ \tilde{\psi}(a)\rangle = \tilde{E}_a(k) |\tilde{\psi}_a\rangle, \ k > 1 \tag{4.12}$$

now constitute a basis of the isorepresentation of $\hat{G}(3.1)$ which is either irreducible (non-degenerate case) or reducible (degenerate case). The deviation of the new eigenvalues from the old is then a measure of the isotopy $G(3.1) \Rightarrow \hat{G}(3.1)$ and the <<isorenormalization>> of the divergent series under a suitable selection of the isotopic elements T then follows.

In fact, the lifting of the group method via the use of the isogalilean symmetry yields to first order the expression

$$E(k) = E_{0a} + k\langle\psi_a|V|\psi_a\rangle \Rightarrow \tilde{E}(k) = \tilde{E}_{0a} + k\langle\tilde{\psi}_a| * V * |\tilde{\psi}_a\rangle, \tag{4.13}$$

which is fully equivalent to result (4.8).

After the above preliminary analysis, we now consider the isoquark theories of the preceding sections with fundamental realization (3.20) of the isotopic element,

$$T = g = H_1^{-1} + H_2^{-1}. \tag{4.14}$$

It is easy to see that the above realization verifies the conditions of Lemma 3, thus offering, besides exact confinement, the additional convergence of perturbative series under strong interactions.

In fact, the absolute value (e.g., the numerical value of the matrix elements) of the total Hamiltonian $H \doteq H_1 + H_2$ can be readily assumed to be much bigger than one for strong interactions. It is therefore always possible to assume that each to the two operators H_1 and H_2 is much bigger than one. An important property of realization (4.14) is therefore that the absolute value of the isotopic element T can indeed be much smaller than one,

$$|T| = |g| = |H_1^{-1} + H_2^{-1}| << 1 \tag{4.15}$$

exactly as requested by Lemma 3.

Thus, rather remarkably, the isoquantization of Nambu's mechanics for triplets permits via one single algorithm the combined achievement of: 1) the isotopies of quark theories with $\hat{SU}(3) \approx SU(3)$; 2) the exact confinement of isoquarks; and 3) a convergent isoperturbative theory.

We can therefore say that a *convergent* perturbation theory for *strong* interactions appears to be within technical reach.

5. Concluding Remarks

The most important aspect we attempted to convey in this paper is the possibility that the current problematic aspects of strong interactions are due to the lack of serious consideration of the teaching of the founders of contemporary physics.

The first teaching with a fundamental relevance for the isoquark theory is the celebrated Einstein-Podolsky-Rosen argument on the lack of completion of quantum mechanics. In fact, the isoquark theory is a completion of the quark theory of isotopic type, while the achievement of exact confinements is centrally dependent on the explicit isotopic realization of <<hidden variables>>.

The second teaching that has been instrumental for the studies of this paper is the old expectation that strong interactions admit a nonlocal component due to deep mutual penetrations and overlappings of the wavepackets/wavelengths/charge distributions of hadrons.

Intriguingly, current experimental evidence appears to support such expectation independently from any quark theory. In fact, the isorelativistic representation of the Bose-Einstein correlation as originating from the nonlocality of the interior of the fireball of the $p-\bar{p}$ annihilation submitted in ref. **[18]**, appears to have preliminary, yet impressive experimental verifications in ref. **[36]** via the data of the UAI experiments at CERN. A number of other experimental verifications are emerging in a variety of other field (see ref. **[19]** for a review).

The studies of this paper are completed in ref. **[39]**, where we construct the isotopic $S\hat{U}(2)$ theory, classify its isorepresentations, and *reconstruct the exact isospin symmetry under electromagnetic and weak interactions* with equal *p-n* masses in isospaces, and physical masses in our conventional space.

REFERENCES

1. D.B. Lichtenberg and S.P. Rosen, *Developments in the Quark Theory of Hadrons*, (Hadronic Press; 1980).
2. L. Chattarjee and S. Bhattacharryya, *Hadronic J.* **9**, 95 (1986).
3. D. Home and F. Selleri, *Rivista Nuovo Cimento* **9**, 9 (1991).
4. A. Einstein, B. Podolsky and N. Rosen, *Phys. Rev.* **47**, 777 (1935).
5. R.M. Santilli, *Hadronic J.* **1**, 228 and 1279 (1978).
6. R.M. Santilli, *Foundations of Theoretical Mechanics*, Vol. II, (Springer Verlag; 1983).
7. R.M. Santilli, Algebras, groups and geometries, **8**, 169-287 (1991).
8. D.S. Sourlas and G.T. Tsagas, *Mathematical Foundations of the Lie-Santilli Theory*, Ukraine Academy of Sciences, in press.
9. R.M. Santilli, *Phys. Rev.* **D20**, 555 (1979).
10. R.M. Santilli, *Hadronic J.* **8**, 25 and 36 (1985).
11. R.M. Santilli, *Lettere Nuovo Cimento* **37**, 545 (1983); *ibidem*, **38**, 509 (1983).
12. R.M. Santilli, *Isotopic Generalizations of Galilei's and Einstein's Relativistics*, Vol. II, 2nd Edition, Ukraine Academy of Scienced, in press.
13. J.V. Kadeisvill, *Santilli's Isotopies of Contemporary Algebras, Geometries and Relativistics*, (Hadronic Press, Tarpon Springs, FL; 1992).
14. R.M. Santilli, *Hadronic J.* **1**, 574 (1978).
15. C.N. Ktorides, H.C. Myung and R.M. Santilli, *Phys. Rev.* **D22**, 892 (1980).

16. H. Myung and R.M. Santilli, *Hadronic J.* **5**, 1277 and 1367 (1982).
17. R. Mignani, H.C. Myung and R.M. Santilli, *Hadronic J.* **6**, 1878 (1983).
18. R.M. Santilli, *Hadrcnic J.* **15**, 1 (1992).
19. R.M. Santilli, *Elements of Hadronic Mechanics*, Vols. I (1973) and II (1994), Ukraine Academics of Sciences, K. ev.
20. R.M. Santilli, *Hadronic J.* **7**, 1680 (1984).
21. R.M. Santilli, *Hadronic J. Suppl.* **4B**, 1 (1989).
22. R.M. Santilli, In: *Proceedings of the International Conference of the Frontiers of Physics*, M. Banore et al, Editors, Plenum, 1993.
23. R. Mignani, *Lettere Nuovò Cimento* **39**, 413 (1984); R. Mignani and R.M. Santilli, ICTP preprint, 1991, unpublished.
24. R.M. Santilli. *Hadronic J. Suppl.* **4B**, 102 (1989).
25. A. Jannussis and D. Vaougios, *Hadronic J.* **10**, 75 (1987).
26. A.K. Aringazin, *Hadronic J.* **13**, 183 (1990).
27. R.M. Santilli, *JINR Comm.* E4-93-352 and *Chinese J. Syst. Eng. and Electr.* **6**, No. 2 (1995).
28. R.M. Santilli, Talk at the Second Workshop on Hadronic Mechanics, Como, Italy, 1984, unpublished.
29. Y. Nambu, *Phys. Rev.* **D7**, 2405 (1973).
30. A.J. Kalnay, *Hadronic J.* **6**, 1 (1983).
31. A.J. Kalnay and R.M. Santilli, *Hadronic J.* **6**, 1798 (1983).
32. A. Jannussis et. al., *Lettere Nuovo Cimento* **38**, 181 (1983).
33. F. Cardone, R. Mignani and R.M. Santilli, *J. Phys.* **G18**, L61 (1992).
34. F. Cardone, R. Mignani and R.M. Santilli, *J. Phys.* **G18** (1992) L141.
35. F. Cardone and R. Mignani, *Nonlocal Approach to the Bose-Einstein Correlation*, Univ. of Rome preprint No. 894, 1992, unpublished.
36. A. Jannussis, R. Mignani and D. Skatsas, *Physica* **A187** (1992) 575.
37. W.S.C. Williams, *An Introduction to Elementary Particles*, (Academic Press, New York; 1971).
38. P. Roman, *Advanced Quantum Mechanics*, (Addison Wesley Publ. Co., Reading, MA; 1965).
39. R.M. Santilli, *JINR Rapid Comm.* **6**, 24 (1993).