

SANTILLI ISODIFFERENTIAL CALCULUS (1996)
Newton's differential calculus depends on the assumed base field

IsoDifferential

$$\hat{\mathbf{d}}\hat{\mathbf{r}} = \hat{\mathbf{T}} \times \mathbf{d}\hat{\mathbf{r}} = \hat{\mathbf{T}}\mathbf{d}(\mathbf{r} \times \hat{\mathbf{I}}) = \mathbf{d}\mathbf{r} + \hat{\mathbf{T}}\mathbf{d}\hat{\mathbf{I}}$$

IsoDerivative

$$\frac{\partial \hat{\mathbf{f}}}{\partial \hat{\mathbf{r}}} = \hat{\mathbf{I}} \times \frac{\partial[\mathbf{f}(\hat{\mathbf{r}}) \times \hat{\mathbf{T}}]}{\partial \hat{\mathbf{r}}} = \frac{\partial \mathbf{f}(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}} + \mathbf{f}(\hat{\mathbf{r}}) \times \hat{\mathbf{I}} \times \frac{\partial \hat{\mathbf{T}}}{\partial \hat{\mathbf{r}}}$$

$$\hat{\mathbf{d}}\hat{\mathbf{f}}(\hat{\mathbf{r}}) = \frac{\partial \hat{\mathbf{f}}(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}} \hat{\times} \hat{\mathbf{d}}\hat{\mathbf{r}}$$

Example

$$\hat{\mathbf{f}}(\hat{\mathbf{r}}) = \hat{\mathbf{r}}^2 = \hat{\mathbf{r}} \hat{\times} \hat{\mathbf{r}} = \hat{\mathbf{r}} \times \hat{\mathbf{T}} \times \hat{\mathbf{r}} = \hat{\mathbf{r}}^2 \times \hat{\mathbf{T}} = \mathbf{r}^2 \times \hat{\mathbf{I}}$$

$$\hat{\mathbf{d}}\hat{\mathbf{r}}^2 = \mathbf{2} \times \hat{\mathbf{r}} \times \mathbf{d}\mathbf{r} + \hat{\mathbf{r}}^2 \times \hat{\mathbf{I}} \times \mathbf{d}\hat{\mathbf{T}}, \quad \frac{\partial \hat{\mathbf{r}}^2}{\partial \hat{\mathbf{r}}} = \mathbf{2} \times \hat{\mathbf{r}} + \hat{\mathbf{r}}^2 \times \hat{\mathbf{I}} \times \frac{\partial \hat{\mathbf{T}}}{\partial \hat{\mathbf{r}}}$$

R. M. Santilli, "Nonlocal-Integral Isotopies of Differential Calculus, Mechanics and Geometries," in *Isotopies of Contemporary Mathematical Structures*, P. Vetro Editor, Rendiconti Circolo Matematico Palermo, Suppl. Vol. 42, 7-82 (1996),
<http://www.santilli-foundation.org/docs/Santilli-37.pdf>

S. Georgiev, *Foundations of the IsoDifferential calculus*, Volume I, Nova scientific publisher, in press (2014) <http://www.santilli-foundation.org/docs/isohandbook.pdf>