PROCEEDINGS OF THE THIRD INTERNATIONAL CONFERENCE ON LIE-ADMISSIBLE TREATMENT OF IRREVERSIBLE PROCESSES (ICLATIP - 3)
Kathmandu University, Nepal, April (2011) pages xx-xx

NONUNITARY LIE-ISOTOPIC AND LIE-ADMISSIBLE SCATTERING THEORIES OF HADRONIC MECHANICS, IV:
Reversible Electron-Proton and Electron-Positron Scatterings

A. O. E. Animalu[a,b] and R. M. Santilli[a]
([a]) Institute for Basic Research
P. O. Box 1577, Palm Harbor, FL 34682, U.S.A., ibr@gte.net,
([b]) University of Nigeria, Department of Physics and Astronomy
Nsukka, Nigeria, nascience@aol.com

Abstract

In the preceding three papers of this series ,we have presented a nonunitary, invariant, axiom preserving, Lie-isotopic broadening of the scattering theory, called *isoscattering theory,* for the representation within the context of hadronic mechanics of reversible particle events. In this paper we show, apparently for the first time, that the nonunitary character of the new scattering theory allows the representation of the synthesis and subsequent spontaneous decay of: the synthesis of the neutron from a proton and an electron, e.g., as occurring in stars, $e^- + p \to n + \nu \to p + e^- + \nu + \bar{\nu}$; the π^o meson from an electron-positron pair, $e^+ + e^- \to \pi^0 \to e^+ + e^-$; and similar events known as *synthesis of hadrons.* These events are beyond the representational capability of the conventional scattering theory because, as shown in preceding studies, they require a *positive binding energy* (since the rest energy of the synthesized particle is bigger than the sum of the rest energies of the original constituents) under which the Schröedinger equation and other unitary formalisms of quantum mechanics provides no physically meaningful solutions. By contrast, hadronic mechanics has allowed an exact, numerical and invariant representation of *all* characteristics of said hadron syntheses precisely in view of its nonunitary character. Consequently, in this paper we show that the isoscattering theory does allow, for the first time, a representation of the indicated hadron syntheses as *nonconservative events* requiring missing energy provided by the environment. The proposed isoscattering theory then emerges as the *only* known *invarioant* representation of these nonconservative scattering events.

Key words scattering theories, nonunitary theories, isounitary theories
PACS 03.65.Nk, 11.30.Cp, 03.65.-w

TABLE OF CONTENT

1. Introduction

In the preceding paper [1] of this series, we have presented the nonunitary, axiom-preserving, Lie-isotopic generalization of the conventional scattering theory under the name of *isoscattering theory,* for the treatment within the context of relativistic hadronic mechanics (HM) of reversible particle events. In particular, the new scattering theory was constructed via the isotopies of the conventional Feynman graphs/rules of quantum electrodynamics (QED) [2] within the context of conventional relativistic quantum mechanics (QM) for the specific purpose of studying non-Lagrangian, non-Hamiltonian, thus nonunitary effects expected in particle events since quite some time.

In this paper we formulate, apparently for the first time, the isoscattering theory for reversible events given by the synthesis and subsequent spontaneous decay of the neutron from a proton and an electron,

$$e^- + p \rightarrow n + \nu \rightarrow p + e^- + \nu + \bar{\nu} \tag{1.1}$$

the synthesis and subsequent spontaneous decay of the π^o meson from an electron-positron pair,

$$e^+ + e^- \rightarrow \pi^0 \rightarrow e^+ + e^- \tag{1.2}$$

and similar events occurring in the core of stars or in particle accelerators, generically referred to as *synthesis of hadrons,* much along the synthesis of the deuterium and otehr nuclei.

The conventional scattering theory is *inapplicable* for the above events (and certainly not "violated" because not developed for the events considered) for the following reasons. Recall that all consistent bound states of quantum mechanics at the particle, nuclear and molecular levels are characterized by a *negative binding energy* resulting in the well known *mass defect,* according to which the rest energy of the bound state is *smaller* than the sum of the rest energies of all constituents.

By contrast, the rest energy of synthesized hadrons is *bigger* than the sum of the rest energies of all original particles. It was shown in 1978 by Santilli [3] that the Schrödinger equation, as well as the unitary formalism at large pf quantum mechanics, provide no physically meaningful solutions for the syntheses here considered, because they would require a "positive binding energy" which is sheer anathema for quantum mechanics. Therefore, Santilli [3] proved that a nonunitary image of the Schrödinger equation provides a numerically exact representation of *all* characteristics of the π^o particle in its synthesis from an electron-positron pair. Following further developments of hadronic mechanics, Santilli achieved in 1990 [4a] a nonrelatiistic, exact and invariant representation of *all* characteristics of the neutron in its synthesis from a proton and an electron, and subsequently provided the relativistic formulation in papers [4b,4c] 9see also Refs. [4d,4e] for related works).

Therefore, Santilli has established in Refs. [3.4] the need for a nonunitary generalization of the conventional scattering theory predicted since some time, but first formulated in an invariant form in Ref. [1], for a quantitative representation of syntheses (1.1) and (1.2), as well as for the synthesis of hadrons at large verifying the general rule that *the rest energy of the synthesized hadrons is bigger than the sum of the rest energies of the original particles.*

On conceptual grounds, these events are *nonconservative* due to missing energy for the syntheses themselves which is provided by the environment (for non-experts in the field, we recall that the missing energy cannot be provided by the relative kinetic energy of the original particles or by the hypothetical neutrino for various inconsistencies, see Ref. [4d,e]). The nonconservative character of the events implies their non-Lagrangian and non-Hamiltonian character, in the sense that said events cannot any longer be entirely described via the sole knowledge of a Lagrangian or a Hamiltonian as it is typically the case for the conventional scattering theory, because the nonconservation originates from interactions violating the integrability conditions for the existence of a potential (the *conditions of vbariational self-adjointness* [4e]). In turn, these features establish beyond scientific doubt the need for a nonunitary covering of the scattering theory since the insufficiency of the Hamiltonian implies the need for an additional operator that breaks the unitary character of the theory.

On more technical grounds, the nonconservative character of the events implies the inapplicability of Lie's theory with the familiar time evolution of a (Hermitean) operator A

$$i\frac{dA}{dt} = [A, H] = AH - HA, \tag{1.3}$$

in favor of Santilli's Lie-isotopic time evolution 9first presented in Ref. [2] of 1978)

$$i\frac{dA}{dt} = [A\hat{,}H] = ATH - HTA \tag{1.4}$$

where T is a second operator (generally independent from, and non-commuting with H) characterizing the nonunitarity of the theory. In fact, the Santilli time evolution can be solely achieved from the Lie form via a nonunitary transform

$$UU^\dagger = \hat{I} = 1/\hat{T} \neq I, \tag{1.5}$$

for which

$$U[A, H]U^\dagger = [\hat{A}\hat{,}\hat{H}] = \hat{A}\hat{T}\hat{H} - \hat{H}\hat{T}\hat{A}, \tag{1.6a}$$

$$\hat{A} = UAU^\dagger, \quad \hat{H} = UHU^\dagger, \tag{1.6b}$$

by clarifying that, in the generally adopted notation, the "hats" in the top of operators are often omitted for simplicity as in Eq. (1.4). The isoscattering theory then emerges as the only known theory which is:

1) *Universal,* in the sense of admitting all possible syntheses of hadrons (under obvious smoothness and regularity conditions);

2) *Invariant,* in the sense of predicting the same numerical values under the same conditions at different times, despite its nonunitary structure (thanks to the the novel isomathematics bypasing the inconsistency theorems as reviewed in paper I); and

3) *Covering,* in the sense of admitting the conventional scattering theory as a trivial particular case when the nonunitary effects are null, i.e., $T = I$;
all above features being necessary for a viable generalized scattering theory, e.g., because the conventional Coulomb scattering of extended particles without collision is definitely unitary in structure. Hence, any generalized scattering theory not admitting the conventional Coulomb scattering as a particular case is disproved by experimental evidence. The isoscattering theory admits indeed said conventional Coulomb scattering, trivially, because unitarity is a mere particular case of the broader nonunitarity.

In closing these introductory lines, we would like to recall Barut's [5] model of the synthesis of the neutron, Eq. (1.1),

$$n \equiv (p, e^-, \bar{\nu})_{QM} \tag{1.7}$$

within conventional Feynman Lagrangian path-integrals based on the transition from the conventional $O(3,1)$ to the $O(4,2)$ dynamical group. According to this model, the three quarks used in the standard model for the representation of the structure of a *family* of baryons including the neutron are identified with the proton, the electron and the anti-neutrino.

By contrast, Santilli [4] structure model of the neutron within the context of hadronic mechanics is characterized by the proton and the electron in a mutated form $\hat{p}, \hat{e}$ resulting under the necessary nonunitary lifting of the Lorentz symmetry $\hat{O}(3.1)$

$$n \equiv (\hat{p}, \hat{e}^-)_{HM} \tag{1.8}$$

under the acceptance of, and compatibility with the $SU(3)$-color Mendeleev-type classification of hadrons into families.

In connection, Barut [5] achieved the synthesis of the neutron within the context of a *unitary theory.* However, this required the abandonment of the Lorentz symmetry $O(3.1)$ in favor of the broader conformal symmetry $O(4.2)$. In turn, the latter transition implied the addition of physically unidentified coordinates outside our spacetime, as well as the assumption (generally rejected by the scientific community for various technical reasons) that the anti-neutrino is an actual physical constituent of the neutron.

By comparison, Santilli [4] structure model of the neutron remains within our spacetime in (3+1)-dimensions and solely assumes mutated forms of the proton $\hat{p}$ and the electron, $\hat{e}$ (due to their total mutual penetration), as the physical constituents of the neutron, but requires a necessary nonunitary lifting of the entire formulation of quantum mechanics, thus including the dynamical equation, the Lorentz symmetry and the scattering theory. However, the basic mechanism of the new theory, the isotopies, preserve the original axioms to such an extent that the new theory can be formulated with the same symbols of the old, only subjected to a broader realization. As an example, the Lorentz $O(3.1)$ and the Lorentz-Santilli isotopic $\hat{O}(3.1)$ symmetries are not only isomorphic but they are indistinguishable at the abstract, realization-free level.

We should also mention for further needs Santilli [4d] hypothesis of the *etherino* with energy given by the value missing for the neiutron synthesis (1.1),

$$m_{a^0} \geq m_n - (m_p + m_e) = 0.78MeV \equiv 1.53m_e, \tag{1.9}$$

which has bveen specifically and clearly suggested by Santilli not as an additional hypothetical particle within the current "zoo" of unknown particles, but merely as a *vehicle representing within conventional unitary theories the transfer from the environment to the neutron of the missing energy, spiny and other quantities.* Consequently, in lieu pof Eq. (1.1), Santilli considers the alternative formulation

$$e^- + a + p \to n \to e^- + a + p, \tag{1.10}$$

where the antineutrino $\bar{\nu}$ (neutrino ν) is replaced by the etherino a (anti-etherino $\bar{a}$).

In considering the etherino hypothesis, the following aspects should be kept in mind:

a) The assumption of Santilli's etherino as a physical constituent of the neutron in lieu of Barut's anti-neutrino leads to a number of catastrophic inconsistencies, and we shall write

$$n \neq (p, e^-, a)_{QM}, \tag{1.11}$$

including the impossibility of a perennial confinement of the etherino inside the neutron, with its consequential necessary emission as a free particle in the spontaneous decay jointly with the proton and the electron. The clear lack of existence of the etherino as a free particle in our spacetime completely disproves model (1.11).

b) The etherino has been introduced for a *quantum mechanical* representation of the synthesis of the neutron. As such, the etherino no longer appears in the covering representation of the neutron synthesis via hadronic mechanics. Alternatively we can say that the nonunitary broadening of quantum mechanics, beginning with Santilli iso-Hilbert spaces over isofields, is a direct representation of the transfer of energy and other quantities from the environment to the neutron that, as such, require no actual particle for their realization.

c) Santilli's etherino replaces the Pauli-Fermi historical hypothesis of the neutrino without necessarily dismissing the so-called "neutrino experiments." In fact, the synthesis of the neutron not only requires energy, but also spin 1/2 as first identified by

Pauli and Fermi. Since the environment of the synthesis provides the missing quantities, the hypothesis of the neutrino as a physical particle in our spacetime becomes no longer necessary. Also, to account for the actual synthesis of the neutron, the Pauli-Fermi hypothesis should have been formulated with a *neutrino,* rather than an anti-neutrino in the left,

$$p + \nu + e^- \to n, \tag{1.12}$$

since recent studies [4d,4e] have established that the formulation via an anti-neutrino in the left comp[atible with reaction (1.1),

$$p + \bar{\nu} + e^- \to n \tag{1.13}$$

increases, rather than eliminate the value of the missing energy.

Additionally, the etherino hypothesis does not dismiss experimental data on "neutrino experiments" because the transfer of energy, spin and other quantities from the environment to the neutron is predicted as being a *longitudinal* impulse in our spacetime, thus distinct from the notoriously *transverse* photon and, as such, can can account for the measured data. Note finally, the rejection by an increasing number of physics of the notion of several different massive neutrinos since they imply that massive particles in our spacetime propagate through immense hyperdense hadronic media, such as entire planets or stars, without appreciable collisions. By comparison, being a longitudinal "impulse" (rather than a physical particle) through spacetime, Santilli etherino can indeed travel through vast hyperdense media without excessively hyperbolic assumptions because it propagates in the spacetime underlying the medium, rather than through the medium itself.

In this paper, particles and operators described via QM are represented with conventional symboils, e.g., p, e, S, etc., while particles and operators described via HM are represented witrh the same symbols complemented with a hat, $\hat{p}, hate, \hat{S}$, etc. To make this paper minimally self-0sufficient and readily understandable to non-experts in HM, we shall review in Sec. 2.1 the main characteristics of the generalized scattering theory developed from Ref.[1] based on the lifting $O(3,1) \to \hat{O}(3,1)$ and summarize the Feynman graphs/rules in QED of spin-$\frac{1}{2}$particles for computation of the S-matrix. We shall then present in Sec. 2.2 an explicit characterization of the scattering region in terms of the lifting $O(4,2) \to \hat{O}(4,2)$ for describing the isoscattering process (1.1) and realization of the structure models: $n \equiv (p, e^-, \bar{\nu})_{QM} \sim (p, e^-, a^0)_{QM} \to (\hat{p}, \hat{e}^-)_{HM}$.In Sec. 3, we shall apply the Feynman graphs/rules to the computation of the scattering cross section for the processes (1.1) and (1.2). In Sec. 4 we shall discuss the experimental implications and in Sec. 5 draw our conclusions.

2. Review/Characterization of the Isoscattering Fornalism

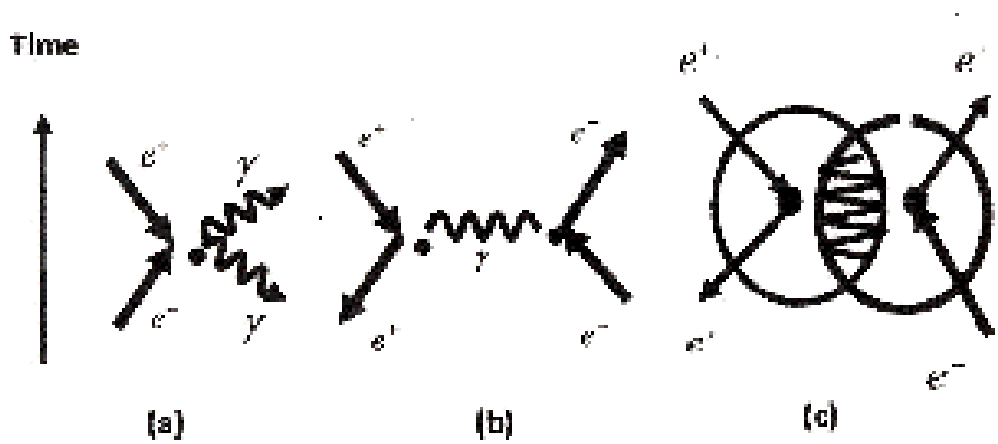

Figure 1: *A schematic view of the evolution of the scattering theory, from early formulations (a), to the more advanced Feynman's formulations in which the scattering is mediated by a particle (b), to the isoscattering formulation where, in addition to mediation by particles, we have internal nonunitary effects due to total m,mutual penetration of the hyperdense charge distributions of and/or wavepackets of the scattering particles (c).*

2.1. Review

As is well-known [2], the usual Feynman propagator in conventional QED of spin-$\frac{1}{2}$ particles can be characterized as follows in the O(3,1) carrier space of a relativistic quantum mechanics:

$$S_F(x) = (\gamma^\mu p_\mu + im)\Delta_F(x), \Delta_F(x) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ipx}}{p^2 - m^2 + i\epsilon} \qquad (2.1a)$$

with corresponding expression in momentum 4-vector space:

$$S_F(p) = (\gamma^\mu p_\mu + im)\Delta_F(p), \Delta_F(p) = \frac{\gamma^\mu p_\mu + im}{p^2 - m^2 + i\epsilon} \qquad (2.1b)$$

In terms of the "isounit" ($\hat{I}$)and isotopic element ($\hat{T} = \hat{I}^{-1}$)defined below and represented as $\hat{I}_{st}$ and T_{st},the generalized Feynman (which may be called iso-Feynman) propagator in the $\hat{O}(3,1)$ carrier space of hadronic mechanics is given by the corresponding expressions as follows

$$\hat{S}_F(\hat{x}) = (\hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}^\mu \times \hat{p}_\mu + i \times \hat{m}) \times \hat{T}_{st} \times \hat{\Delta}_F(\hat{x}),$$

$$\hat{\Delta}_F(\hat{x}) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip\times\hat{T}_{st}\times x}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i\times\hat{\epsilon}} \tag{2.2a}$$

with corresponding expression in iso-momentum 4-vector space

$$\hat{S}_F(\hat{p}) = (\hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}^{\mu} \times \hat{p}_{\mu} + i \times \hat{m}) \times \hat{T}_{st} \times \hat{\Delta}_F(\hat{p}),$$

$$\hat{\Delta}_F(\hat{p} = \frac{(\hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}^{\mu} \times \hat{p}_{\mu} + i \times \hat{m}) \times \hat{T}_{st}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i\times\hat{\epsilon}} \tag{2.2b}$$

In the presence of an external electromagnetic field, the solution of the (regular) Dirac-Santilli isoequation takes the form

$$\hat{\Psi} = \hat{\psi}(\hat{x}) + \hat{e}\hat{\times}\hat{\int}\hat{d}^4\hat{x'}\hat{\times}\hat{S}_f(\hat{x}-\hat{x'})\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x'})\hat{\times}\hat{\Psi}(\hat{x}$$

$$= \hat{\psi}(\hat{x}) + \hat{e}\hat{\times}\hat{\int}\hat{d}^4\hat{x'}\hat{\times}\hat{S}_f(\hat{x}-\hat{x'})\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x'})\hat{\times}\hat{\psi}(\hat{x})$$

$$+\hat{e}^{\hat{2}}\hat{\times}\hat{\int}\hat{d}^4\hat{x'}\hat{\int}\hat{d}^4\hat{x"}\hat{\times}\hat{S}_f(\hat{x}-\hat{x'})\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x'})\hat{\times}\hat{S}_f(\hat{x'}-\hat{x"})\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x"})\hat{\times}\hat{\psi}(\hat{x"}) + ... \tag{2.3}$$

This leads to a formal definition of the iso-Feynman propagator either as a series

$$\hat{S'}_f(\hat{x},\hat{x'}) = \hat{S}_f(\hat{x}-\hat{x'}) + \hat{e}\hat{\times}\hat{\int}\hat{d}^4\hat{x"}\hat{\times}\hat{S}_f(\hat{x}-\hat{x"})\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x"})\hat{\times}\hat{S}_f(\hat{x'}-\hat{x"}) + ... \tag{2.4a}$$

or as an integral equation

$$\hat{S'}_f(\hat{x},\hat{x'}) = \hat{S}_f(\hat{x}-\hat{x'}) + \hat{e}\hat{\times}\hat{\int}\hat{d}^4\hat{x"}\hat{\times}\hat{S}_f(\hat{x}-\hat{x"})\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x"})\hat{\times}\hat{S'}_f(\hat{x'}.\hat{x"}) \tag{2.4b}$$

where $\hat{\gamma}\hat{.}\hat{A}(\hat{x'}) = \hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}_{\mu} \times \hat{A}_{\nu}(\hat{x'})$ and $\hat{A}_{\nu}(\hat{x'})$ is the iso-electromagnetic four-vector potential given by the corresponding iso-gauge principle[7] to which we shall return in Sec. 4

Note that, in the limit of unitary transformation, we recover exactly the conventional expressions. For this reason, the primary interest of isoscattering theory lies in the formal relationship/differentiation of the two isoscattering profiles (1.1) and (1.2) for interpreting the existing and future scattering experimental data. To do this, we note that the isotopies of the Dirac matrices ($\hat{\gamma}_{\mu}$) have been explicitly defined in ref.[1] as follows:

$$\hat{\gamma}_k = b_k \times \begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \hat{\gamma}_4 = i \times b_k \times \begin{pmatrix} I_{2\times2} & 0 \\ 0 & -I_{2\times2} \end{pmatrix} \tag{2.5a}$$

$$\hat{\gamma}_\mu \hat{,} \hat{\gamma}_\nu = \hat{\gamma}_\mu \times T_{st} \times \hat{\gamma}_\nu + \hat{\gamma}_\nu \times T_{st} \times \hat{\gamma}_\mu = 2 \times \hat{\eta}_{\mu\nu},$$

where $b_k^2 = n_k^{-2} (k = 1, 2, 3), (b_1^2 \times b_2^2 \times b_3^2) = 1$ for an ellipsoidal scattering region. And whereas, without spin mutation, the generalized spin matrices are

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1 \times n_2 \\ n_1 \times n_2 & 0 \end{pmatrix}, \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times n_1 \times n_2 \\ i \times n_1 \times n_2 & 0 \end{pmatrix},$$

$$\hat{\sigma}_3 = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, \hat{T} = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix} \quad (2.5b)$$

they are, with spin mutation,

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1^2 \\ n_2^2 & 0 \end{pmatrix}, \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times n_1^2 \\ i \times n_2^2 & 0 \end{pmatrix}, \hat{\sigma}_3 = \begin{pmatrix} w \times n_1^2 & 0 \\ 0 & w \times n_2^2 \end{pmatrix} \quad (2.5c)$$

Consequently, the isotopies provide five additional quantities [the four ($b_k, k = 1, ...4$) for spacetime mutation and one (w) for the spin] for the representation of experimentally measureable features of the scattering region, such as shape, deformation, scaling, density, anisotropy, etc .

As our interest is to elaborate the basic physical concepts in terms of Feynman diagrams for electron scattering with an electromagnetic field, as well as remove divergences from the theory, we show in Table 1 the two characteristic differences in 1st and 2nd quantization schemes.

The generalized S- matrix is given in 1st quantization scheme by

$$\hat{S}_{f,i} = lim_{t \to \inf} \hat{\int} \hat{d}^3 \hat{x} \hat{\times} \hat{\psi}_{\hat{p}}^{+\hat{s'}} \hat{\times} \hat{\Psi}_{\hat{p}}^{\hat{s}} \quad (2.6)$$

where $\hat{\Psi}_{\hat{p}}^{\hat{s}}$ is the exact solution given as in Eq.(2.3) by

$$\hat{\Psi}_{\hat{p}}^{\hat{s}}(\hat{x}) = \hat{\psi}_{\hat{p}}^{\hat{s}}(\hat{x}) + \hat{e} \hat{\times} \hat{\int} \hat{d}^4 \hat{x'} \hat{\times} \hat{S}_f(\hat{x} - \hat{x'}) \hat{\times} \hat{\gamma} \hat{.} \hat{A}(\hat{x'}) \times \hat{\Psi}_{\hat{p}}^{\hat{s}}(\hat{x'}) \quad (2.7a)$$

with the normalization

$$\hat{\int} \hat{d}^3 \hat{x} \hat{\times} \hat{\psi}_{\hat{p}}^{\hat{+s}}(\hat{x}) \hat{\times} \hat{\psi}_{\hat{p'}}^{\hat{+s'}}(\hat{x}) = \hat{\delta}_{\hat{s}\hat{s'}} \hat{\times} \hat{\delta}^3(\hat{p} - \hat{p'}) \quad (2.7b)$$

Note that the correspondence principle in 1st quantization scheme involves a lifting of the Coulomb vertex in QED into the approximate Yukawa vertex in hadronic mechanics, and additionally involves the lifting from Bose-Einstein to Fermi-Dirac statistics in 2nd quantization scheme, i.e., mutation of spin under sufficiently high

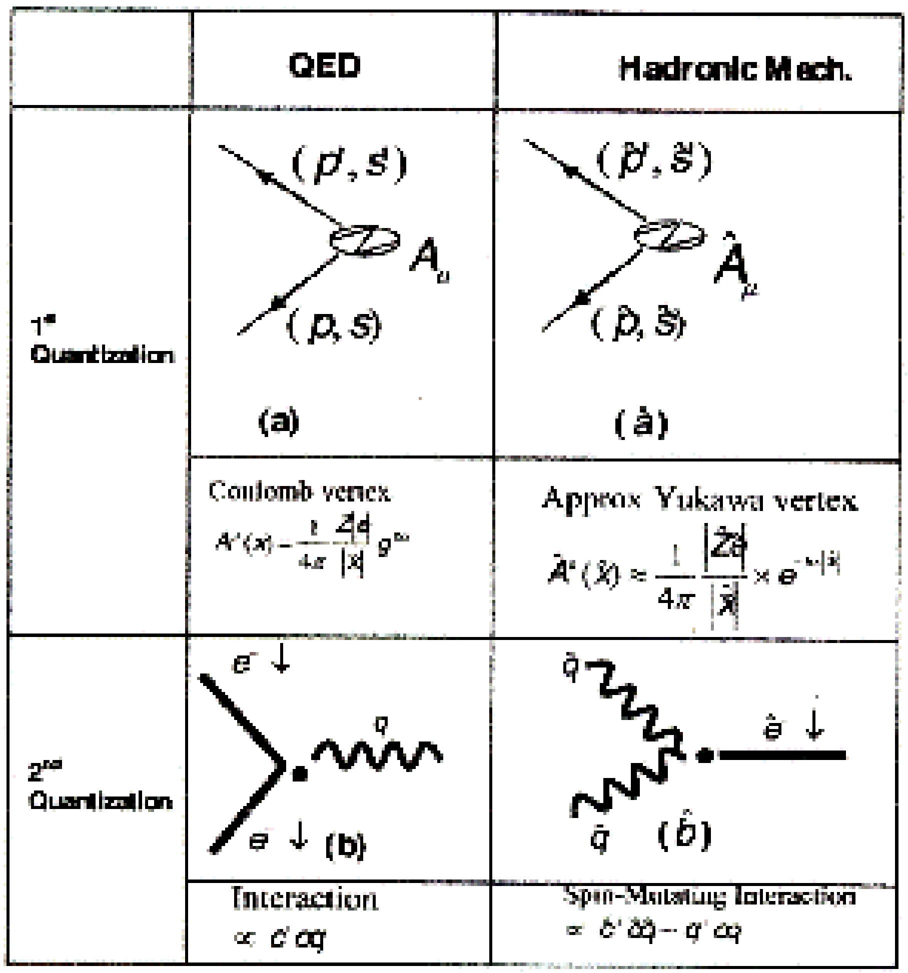

Figure 2: *(Table 1): Expected Modification of QED in HM*

energies. The correspondence between Feynman graphs/rules and their isotopic images for computation of contributions to the S-matrix in QED of spin-$\frac{1}{2}$ particles are summarized in table 2 which we intend to apply to the two scattering profiles (1.1) and (1.2) in Sec. 3.

2.2. $\hat{O}(4,2)$ Dynamical Symmetry of the Scattering Region.

We now turn to further specification of the structure of the scattering region introduced in Fig. 1. While the isotopies of Dirac matrices characterize the lifting of the Lorentz group $O(3,1) \to \hat{O}(3,1)$, in terms of five additional quantities, namely the four $b_k(k = 1, ..., 4)$ for spacetime mutation and one (w) for the spin, for analyzing experimentally measureable features of the scattering region, such as its shape, deformation, anisotropy, etc, the most important distinctive features of the scattering region for the three profiles of $e^- - p$ scattering shown in Fig. 3, are intriguingly realized by characterizing the scattering region in terms of isotopic lifting of the larger dynamical group,$O(4,2) \to \hat{O}(4,2)$ where, as is well-known, $O(4,2)$ contains $O93,1)$ as a subgroup. We shall discuss this feature before taking up the computation of the conventional and generalized S-matrices in Sec.3.

Since correlated pairs of spin-$\frac{1}{2}$ particles,e^-, ν and e^-, a^0 , can be subsumed and long-range 1/r-potential between pairs of particles (p, e^-) eliminated simultaneously in the representation of the conventional O(4,2) dynamical symmetry group, in terms of the most general parity-conserving current in the O(4,2) algebra of Dirac matrices[6] which includes certain "convective" currents proportional to the total momentum of the particle-antiparticle system, it is appropriate to characterize the scattering region of Fig. 1 by the lifting

$$J_\mu \equiv \bar{\psi}\gamma_\mu\psi \to \hat{\bar{\psi}} \times \hat{T} \times (\hat{\gamma}_\mu - i \times \kappa_0 \times \hat{\vec{\partial}}_\mu) \times \hat{T} \times \hat{\psi} = \hat{J}_\mu \tag{2.8}$$

The generalized wave equation that conserves $\hat{J}_\mu$ is given by the generalized Lagrangian density

$$\hat{L} = -\frac{1}{2}\bar{\psi}(\hat{x}) \times \hat{T} \times (-i \times \hat{\gamma}^\mu \times \hat{\vec{\partial}}_\mu + \kappa_1) \times \hat{T} \times \hat{\psi}(\hat{x}) -$$
$$\hat{\bar{\psi}}(\hat{x}) \times \hat{T} \times \kappa_0 \times \hat{\vec{\partial}}^\mu \hat{\vec{\partial}}^\mu \times \hat{T} \times \hat{\psi}(\hat{x}) \tag{2.9}$$

as (cf Eq.(3.2) of Barut, Cordero and Ghirardi[6])

$$(i \times \hat{\gamma}^\mu \times \hat{\vec{\partial}}_\mu + \kappa_0 \times \hat{\vec{\partial}}^\mu \hat{\vec{\partial}}_\mu - \kappa_1) \times T \times \hat{\psi}(\hat{x}) = 0 \tag{2.10}$$

	Feynman Graph&Factor in S-matrix		Iso-Feynman graph&Factor in $\hat{S}$-matrix	
External final Electron	p,s	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{mc_0}{p_0}}\bar{u}^{(s)}(p)$	$\hat{p},\hat{s}$	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{\hat{m}\times\hat{c}}{\hat{p}_0}}\times\hat{\bar{u}}^{(\hat{s})}(\hat{p})$
External final Positron	p,s	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{mc_0}{p_0}}v^{(s)}(p)$	$\hat{p},\hat{s}$	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{\hat{m}\times\hat{c}}{\hat{p}_0}}\times\hat{v}^{(\hat{s})}(\hat{p})$
External final photon	q,ϵ	$\frac{1}{(2\pi)^{3/2}}\frac{\epsilon^{\mu}(q)}{\sqrt{2q_0}}$	$\hat{q},\epsilon$	$\frac{1}{(2\pi)^{3/2}}\frac{\hat{\epsilon}^{\mu}(\hat{q})}{\sqrt{2\hat{q}_0}}$
External initial electron	p,s	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{mc_0}{p_0}}u^{(s)}(p)$	$\hat{p},\hat{s}$	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{\hat{m}\times\hat{c}}{\hat{p}_0}}\times\hat{u}^{(\hat{s})}(\hat{p})$
External initial positron	p,s	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{mc_0}{p_0}}\bar{v}^{(s)}(p)$	$p,\hat{s}$	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{\hat{m}\times\hat{c}}{\hat{p}_0}}\times\hat{\bar{v}}^{(\hat{s})}(\hat{p})$
External initial photon	q,ϵ	$\frac{1}{(2\pi)^{3/2}}\frac{\epsilon^{\mu}(q)}{\sqrt{2q_0}}$	$\hat{q},\epsilon$	$\frac{1}{(2\pi)^{3/2}}\frac{\hat{\epsilon}^{\mu}(\hat{q})}{\sqrt{2\hat{q}_0}}$
Internal electron	p	$S_F(p)$	$\hat{p}$	$\hat{S}_F(\hat{p})$
Internal photon	ν μ	$-i\eta^{\mu\nu}D_F(q^2)=\frac{-i\eta^{\mu\nu}}{q^2+i\epsilon}$	ν μ	$-i\times\hat{\eta}^{\mu\nu}\times D_F(q^2)=\frac{-i\times\hat{\eta}^{\mu\nu}}{\hat{q}^2+\hat{m}_\gamma+i\hat{\epsilon}}$
Electromagnetic int. Vertex	p' q p	$-i\sqrt{4\pi}e\gamma^{\mu}(2\pi)^4\times\delta^{(4)}(p'-p-q)$	$\hat{p}'$ $\hat{q}$ $\hat{p}$	$-i\sqrt{4\pi}e\hat{\gamma}^{\mu}\times(2\pi)^4\times\hat{\delta}^{(4)}(\hat{p}'-\hat{p}-\hat{q})$
Coulomb int. Vertex	p	$\frac{-i4\pi Ze^2}{\lvert\vec{p}'-\vec{p}\rvert^2}\gamma^0(2\pi)\times\delta^{(1)}(p'_0-p_0)$	$\hat{p}'$	$\frac{-i4\pi\hat{Z}\times e^2}{\lvert\vec{p}'-\vec{p}\rvert^2+\hat{m}_\gamma^2}\times\hat{\gamma}^0\times(2\pi)\times\hat{\delta}^{(1)}(\hat{p}'_0-\hat{p}_0)$

Figure 3: *(Table 2): Feynman and Generalized Feynman Graphs/Rules*

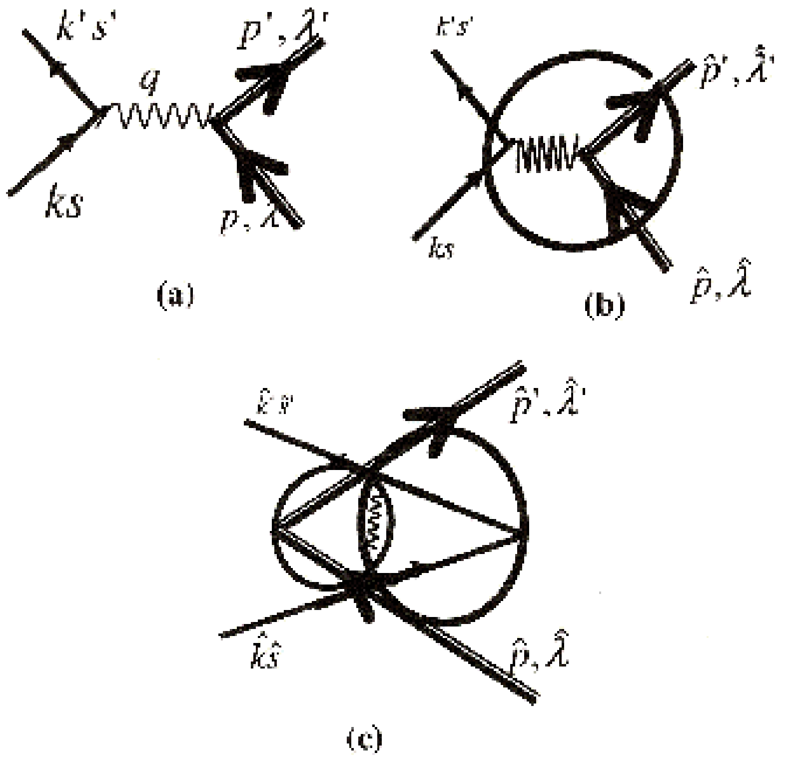

Figure 4: *Feynman diagrams for (a) conventional $e^- - p$ long-range Coulomb interaction via "virtual" photon exchange; (b) point-electron contact/penetration into an extended proton wave packet,$e^- + p \to n + \nu$ which implies $n \sim (pe^- \bar{\nu})_{QM} \to (pe^- a^0)_{QM}$; and (c) mutual overlap of extended electron and extended proton wave packets, $e^- + p \to n + \nu + \bar{\nu}$ which implies $n \equiv (\hat{e}^-, \hat{p})_{HM}$ involving mutation of spin and in the similar scattering process $e^+ + e^- \to \pi^0 \to e^+ + e^-$ which implies $\pi^0 \equiv (\hat{e}^-, \hat{e}^+)_{HM}$.*

where κ_0, κ_1 are constants. It is of interest to note that the last term in Eq.(2.9) (due to convective currents) gives rise to the Pauli magnetic transitions, inasmuch as for any Dirac spinor ψ, it is easy to establish from the relation

$$(\partial^\mu \bar{\psi})(\partial^\mu \psi) = (\partial^\mu \bar{\psi})\gamma^\mu \gamma^\nu (\partial^\nu \psi)$$

a connection with the intrinsic Pauli-moment coupling which is related to inclusion of a non-potential term,$-i\partial^\mu(\bar{\psi}\sigma_{\mu\nu}\partial^\nu)\psi$ in the free Dirac Lagrangian density[7]. Thus even the ($\hat{T} \to 1$) limit of $\hat{O}(4,2)$ corresponding to the conventional $O(4,2)$ provides a simple non-trivial profile of neutron production in $(e^-, p$) scattering as summarized in Table 3.

Table 3: O(4,2) profile of $e^- - p$ scattering & neutron production.			Wave Equations	Symmetry Group
$e^- + p \rightarrow n + \nu \Rightarrow$	$n \sim (pe^-\bar{\nu}_e)_{QM}$ (Barut's model[6])	p	$(i\gamma_\mu \partial_\mu - m_p)\psi_p = 0$	O(3,1)
		(e^-, ν_e)	$(i\gamma_\mu \partial_\mu - m_e^{-1}\partial_\mu \partial^\mu)\psi_e = 0$	O(4,2)
$e^- + p + a^0 \rightarrow n + \nu \rightarrow e^- + p + \bar{\nu} \Rightarrow$	$n = (p, e^- a^0)_{QM}$ (Santilli's "etherino" model [4])	(e^-, a^0)	$\{i\gamma_\mu \partial^\mu - 3m_e - (2m_e)^{-1}\partial_\mu \partial^\mu\}\psi = 0$	O(4,2)

Figure 5: *(Table 3) O(4,2) Profile of e-p scattering and neutron production*

According to this table, if in the scattering process, $e^- + p \to n + \nu$, the proton is treated as pointlike particle described by the conventional Dirac equation with O(3,1) symmetry, then the electron with an associated massless neutrino may be described by the simplest (scale-invariant[8]) equation with O(4,2) dynamical symmetry,

$$(i\gamma_\mu \partial_\mu - m_e^{-1}\partial_\mu \partial^\mu)\psi_e = 0 \tag{2.11}$$

whose mass equation has two roots,$m = 0, m_e$, and therefore leads to Barut's model[6] of neutron production, $n \sim (pe^-\bar{\nu})_{QM}$ (which is not compatible with negative binding energy). Alternatively, if one adopts Santilli's "etherino hypothesis"[4] (for compatibility with neutron decay and negative binding energy for $n = (pe^-a^0)_{QM}$), the electron with an associated massive "etherino" may be described by the more general equation

$$[i\gamma_\mu\partial^\mu - 3m_e - (2m_e)^{-1}\partial_\mu\partial^\mu]\psi = 0, \tag{2.12}$$

whose mass equation and its two non-zero roots are respectively given by:

$$m^2 + 2m_e m - 6m_e^2 = 0, \tag{2.13}$$

$$m_\pm = m_e(-1 \pm \sqrt{7}), i.e., \frac{m_+}{m_e} = 1,65; \frac{|m_-|}{m_e} = 3.6 \tag{2.14}$$

Consequently, since $0.78MeV = 1.53m_e$, it follows by setting $m_{a^0} \equiv m_+ = 1.65m_e$ that one may validly characterize a quantum mechanical bound state of $n = (p, e^-, a^0$ system with negative binding energy: $m_n - (m_p + m_e + m_{a^0}) \equiv -0.18m_e$. Intriguingly, the numerical coefficients in the wave equations (2.11) and (2.12) are uniquely related in terms of Gell-Mann SU(3) λ - generators,

$$\lambda_0 = \sqrt{\frac{2}{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \lambda_8 = \sqrt{\frac{1}{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \lambda_8^{-1} = \sqrt{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}, \tag{2.15}$$

and a triplet field

$$\Psi = \begin{pmatrix} \psi_\nu \\ \psi_e \\ \psi_{a^0} \end{pmatrix},$$

as the components of the wave equation

$$(i\gamma_\mu\partial^\mu - m_e\sqrt{\frac{3}{2}}(\lambda_(0) - \sqrt{2}\lambda_8) + (\frac{1}{m_e\sqrt{3}})\lambda_9^{-1}\partial_\mu\partial^\mu)\Psi = 0 \tag{2.16a}$$

where

$$m_e\sqrt{\frac{1}{2}}(\lambda_0 - \sqrt{2}\lambda_8) = (3m_e)\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

$$(\frac{1}{m_e\sqrt{3}})\lambda_8^{-1} \equiv (\frac{1}{3m_e})\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}. \tag{2.16b}$$

As the three equations in this system are uncoupled except insofar as there is only one characteristic mass,$m_(e)$, for the whole triplet, Eq.(2.16a) implies that in the absence of convective currents, the apparent chiral $SU(3)xSU(3)$ symmetry of the leptonic triplet (Ψ) is broken in the manner prescribed by Gell-Mann, Oakes and

Renner[9]. We shall return to the experimental verification of the predicted object described by the second mass ratio,$\frac{|m_-|}{m_e} = 3.6$ in Sec.4.

Of primary interest in the $\hat{O}(4,2)$ characterization of the hadronic mechanics scattering region is the case of contact/overlap of two extended wavepackets shown in Fig. 3c, which is also applicable to $e^- - e^+$ scattering. This leads to a visual (Feynman graph) representation of Rutherford-Santilli neutron production in $e^- - p$ scattering and neutral pion production in $e^- - e^+$ scattering as follows. The selection of the isotopic element $T = \hat{I}_{-1}$ appearing in the definition of the generalized $\hat{\delta}$ function[10] for this scattering profile is given in ref.[1] as

$$\hat{I} = \begin{pmatrix} n_{11}^2 & 0 & 0 & 0 \\ 0 & n_{12}^2 & 0 & 0 \\ 0 & 0 & n_{13}^2 & 0 \\ 0 & 0 & 0 & n_{14}^2 \end{pmatrix} \times \begin{pmatrix} n_{21}^2 & 0 & 0 & 0 \\ 0 & n_{22}^2 & 0 & 0 \\ 0 & 0 & n_{23}^2 & 0 \\ 0 & 0 & 0 & n_{24}^2 \end{pmatrix} \times$$

$$exp[N \times (\hat{\psi}/\psi) \times \int d^3r \times \psi_{\downarrow}^*(r) \times \psi_{\uparrow}(r)] \tag{2.17}$$

where $n_{ak}^2, a = 1, 2, k = 1, 2, 3$ are the semiaxes of the ellipsoids representing the two particles,$n_{a4}^2, a = 1, 2$represent their densities, ψ represents the isowavefunction, represents the conventional wavefunction (corresponding to $\hat{I} = 1$), and N is a positive constant. A two-dimensional (2D) elaboration of Feynman graph showing the ellipsoidal deformations of e^- and e^+ wave-packets in the neutral pion production process, $e^- + e^+ \to \pi^0 \to e^- + e^+$, is shown in Fig. 4a.

Such a *wave-and-particle* picture is obtained by representing each particle or antiparticle as a "point" on the (red) circum-ellipse of a triangle (ABC) defined by a pair of imaginary generating lines (AB and AC) $[br = \pm is\ ]$ of a "point-ellipse" $[b^2r^2 + s^2 = (ct)^2 = 0]$ in projective 2-dimensional (r, s, t)-space and the "line at infinity" (BC)$(t = 0)$ and locating the π^0 at the centre of the inscribed ellipse of the pair of triangles forming a hexagram so as to satisfy Brianchon's and Pascal's theorems (p. 64 of ref.[12]) as shown in Fig 4a. For $ct \neq 0$, we infer by rewriting the equation of the ellipse in the form,$(ct)^2 - (s - ibr)(s + ibr) = 0$, that the pair of imaginary lines defines by the linear homogeneous equations

$$\begin{pmatrix} ct & -(s - ibr) \\ -(s + ibr) & ct \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0, \tag{2.18}$$

are associated with the ellipse ABC (call it S) in Fig.4a. Moreover, by the geometric principle of duality, the lines AB, AC and BC envelope a conic (call it $\tilde{S}$) , the dual of S with respect to the triangle ABC) whose equation has the general form (p.62 of ref.[13])

$$as^2 + 2fsr + b^2r^2 = 0 = ct \tag{2.19a}$$

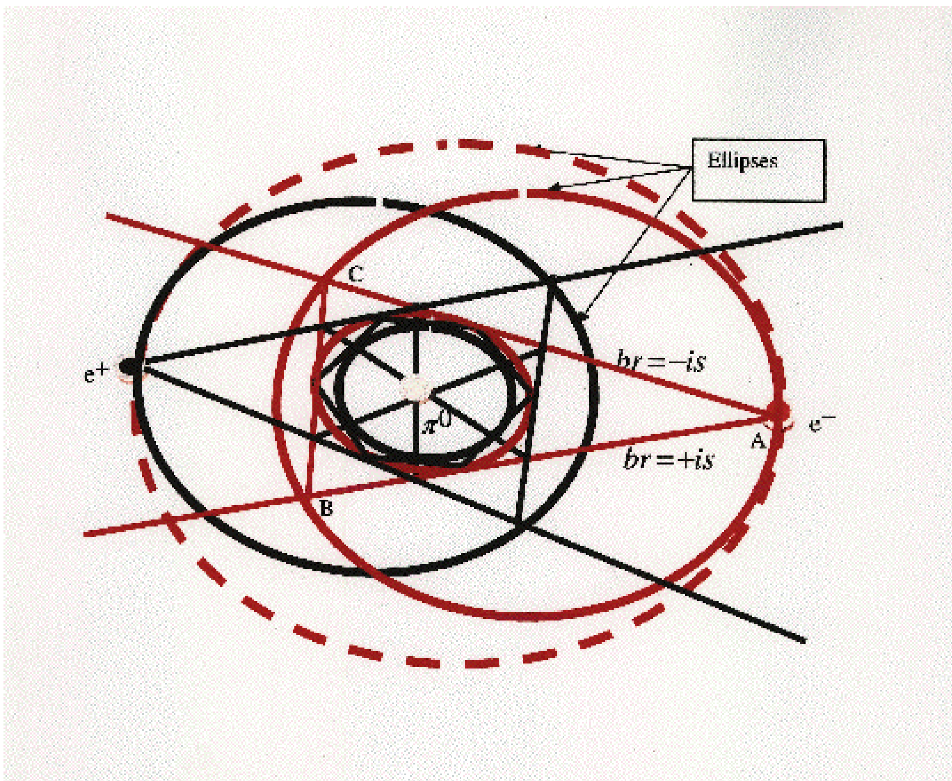

Figure 6: *Elaboration of Feynman graph for overlapping* $e^{(-)}$ *and* $e^{(+)}$ *ellipsoidal wave-packets in the scattering region for the process* $e^- + e^+ \to \pi^0 \to e^- + e^+$ *and the enveloping ellipsoid (i.e. ellipse in 2-dimensional space).*

and is a hyperbola, a parabola, or an ellipse according as $f^2 - ab^2$ is greater than, equal to, or less than zero. The concept of "isotopic" lifting arises naturally in this way, with the radii of the circular conics (S and $\tilde{S}$) providing two fundamental lengths, and the consequential "mutation" of Pauli spin comes about in the case where (for $ct \neq 0$) space-time dualism leads to the rectangular hyperbola,$(s+br)(-s+br)-(ct)^2=0$, whose pair of asymptotic lines are defined by the linear homogeneous equations:

$$\begin{pmatrix} s+br & -ct \\ -ct & -s+br \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = 0, or[(s+\tilde{\beta}_0 ct)(\sigma_3)_{\mu\nu} + br\delta_{\mu\nu}] = 0, \qquad (2.19b)$$

where

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \to \tilde{\beta}_0\sigma_3 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

implies that

$$\tilde{\beta}_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tilde{\beta}_0^2 = -I$$

is non-unitary! An example of the nesting of two ellipses in the scattering region may be constructed by generalizing Eq.(2.19b) to the 4x4 matrix form:

$$[(br + ct\tilde{\beta})\eta^0_{\mu\nu} + s\delta_{\mu\nu}]w_\nu = 0 \qquad (2.20)$$

which involves the metric tensors,

$$(\eta^0_{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, (\tilde{\beta}\eta^0_{\mu\nu}) = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad (2.21)$$

The sum of the two metrics defines an isometric tensor

$$\eta_{\mu\nu} \equiv \frac{1}{2}[(\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu) + (\tilde{\alpha}_\mu\tilde{\beta}\tilde{\alpha}_\nu + \tilde{\alpha}_\nu\tilde{\beta}\tilde{\alpha}_\mu)] \equiv (1+\tilde{\beta})\eta^0_{\mu\nu} \qquad (2.22)$$

which consists of the conventional Dirac's γ-matrices for spin-$\frac{1}{2}$ particles[14]

$$\gamma_0 = \beta, \gamma_r = \beta\alpha_r (r = 1,2,3); with \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\eta^0_{\mu\nu} \qquad (2.23a)$$

and the (dual) Dirac's$\tilde{\beta}, \tilde{\alpha}$ matrices for integral spin particles[14]

$$(\tilde{\alpha}_\mu\tilde{\beta}\tilde{\alpha}_\nu + \tilde{\alpha}_\nu\tilde{\beta}\tilde{\alpha}_\mu) = 2\tilde{\beta}\eta^0_{\mu\nu}; \tilde{\gamma}_\mu\tilde{\gamma}_\nu + \tilde{\gamma}_\nu\tilde{\gamma}_\mu = -2\eta^0_{\mu\nu}, \tilde{\gamma}_\mu = \tilde{\beta}\tilde{\alpha}_\mu \qquad (2.23b)$$

where,

$$\beta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \alpha_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

$$\alpha_2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix},$$

$$\tilde{\alpha}_0 = 1$$

$$\tilde{\beta} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \tilde{\alpha}_1 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

$$\tilde{\alpha}_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \tilde{\alpha}_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

As a result of the above relations, the visual (Feynman graph) image of $e^- - e^+$ scattering leading to production π^0 is provided by the two quadric surfaces, S and $\tilde{S}$:

$$x^\mu(1 \pm \tilde{\beta})\eta^0_{\mu\nu}x^\nu \equiv (ct)^2 - x^2 - y^2 - z^2 \pm 2cty = s^2 \tag{2.24}$$

We note that when $ct = s$, these are two spheres of radii ct in contact along the y-axis and that $\hat{\eta}_{\mu,\nu} = (1 + \tilde{\beta})\eta^0_{\mu\nu} \equiv \hat{I}\eta^0_{\mu\nu}$ defines a non-trivial "isounit" ($\hat{I}$) for such a generalized Feynman graph/rules for computing the S-matrix for extended particle scattering processes leading to fusion products like the π^0 . Note also that if $y/s = \pm isin\theta$ then (2.24) is a torus with parametric equations:

$$x = (is - ctsin\theta)cos\phi; y = (is - ctsin\theta)sin\phi; z = ctcos\theta \tag{2.25}$$

A corresponding two-dimensional wave-and-particle (extended wavepacket) picture of the proton and the toroidal orbit of the electron in McDonough's representation of the Rutherford-Santilli neutron which apparently relates it to Santilli's "etherino" model of the neutron is shown in Fig.4b.

3. Computation of the iso-S-matrix from Feynman Graphs/Rules.

In order to familiarize the reader with the use of the our generalized Feynman graphs/rules for computation of the $\hat{S}$-matrix we begin with the conventional electron-proton Coulomb scattering (Fig. 3a) that leads to the familiar Mott scattering cross-section. Fig. 3a is now elebaorated as shown in Fig. 5a,

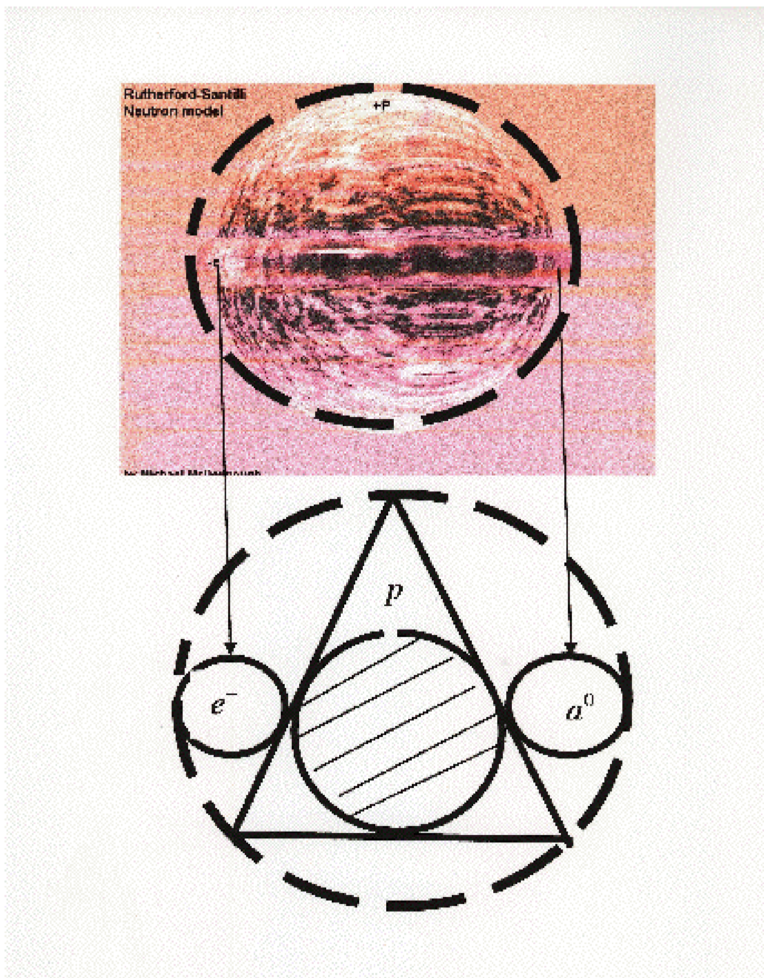

Figure 7: *Projection of the Macdonough representation of the Rutherford-Santilli neutron showing its relationship to Santilli's "etherino" model of the neutron*

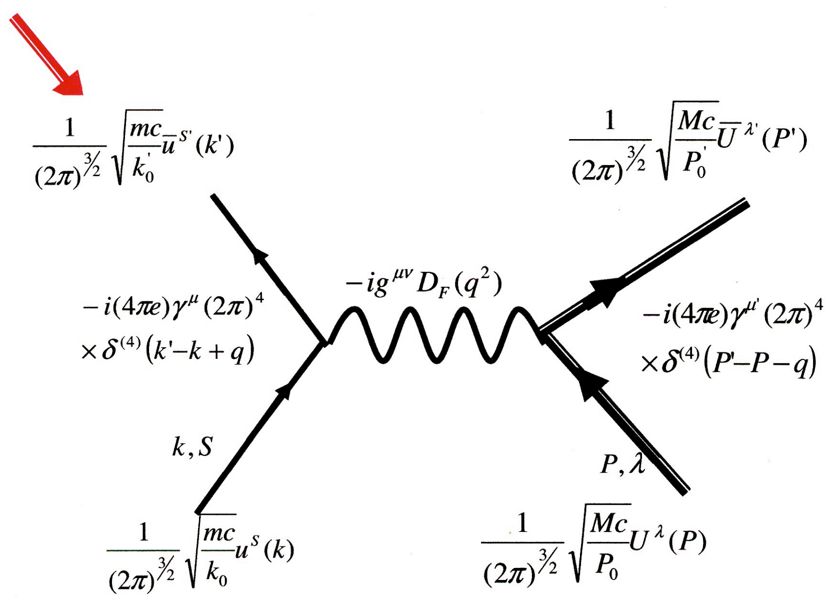

Figure 8: *Conventional Feynman Graph/Rule for e-p Coulomb Scattering*

To write down the S-matrix one starts in the direction of the (top left [red]) arrow from left to right as indicated in Fig. 5a and, at each vertex, inserts all other factors between the incoming and outgoing arrows. If loop closes, one takes trace to get

$$S_{fi} = -4\pi i \int \frac{d^4q}{(2\pi)^4} \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{mc}{k'_0}} \times \bar{u}^{S'}(k')] \times$$

$$[-i(4\pi e)\gamma^{\mu'}(2\pi)^4 \times \delta^{(4)}(k' - k + q)] \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{mc}{k_0}} \times u^S(k)] \times$$

$$[-ig^{\mu\nu}D_F(q^2)] \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{Mc}{P'_0}} \times \bar{U}^{\lambda'}(P')] \times$$

$$[-i(4\pi e)\gamma^{\mu'}(2\pi)^4 \times \delta^{(4)}(P' + P - q)] \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{Mc}{P_0}} \times U^\lambda(P)]$$

$$= -4\pi i \int \frac{d^4q}{(2\pi)^4}\sqrt{\frac{(mc)^2(Mc)^2}{k'_0 k_0 P'_0 P_0}} \times \bar{u}^{S'}(k')\gamma^\mu u^S(k) \times$$

$$\frac{-4\pi i\alpha}{(4\pi)^4}(2\pi)^4\delta^{(4)}(k' + P' - k - P) \times$$

$$[(k' - k)^2 + i\varepsilon]^{-2} \times \bar{U}^{\lambda'}(P')\gamma_\mu U^\lambda(P). \tag{3.1}$$

In terms of the electromagnetic current,

$$j^\mu(k'k, -q) = (2\pi)^4 \times \delta(k' - k + q)]\frac{e}{(2mc)^3} \times \sqrt{\frac{(mc)^2}{k'_0 k_0}} v^{S'}(k')\gamma^\mu u^S(k)$$

and the Moller potential

$$A_\mu(k' - k) = \frac{4\pi}{(k' - k)^2 + i\varepsilon} J_\mu(p', p; k' - k)$$

the S-matrix takes the form of current-current interaction

$$S_{fi} = -4\pi i \int \frac{d^4q}{(2\pi)^4} j^\mu(k'k, -q)\frac{1}{q^4} J_\mu(p'p; q) \tag{3.2}$$

The differential cross section with no polarization for initial particles in the laboratory frame,$p = (Mc_0; 0)$ is given by

$$d\sigma = \frac{1}{2}\sum_{ij} \frac{1}{|\frac{\mathbf{k}}{k_0}| \frac{1}{(2\pi)^3} \frac{V}{(2\pi)^3}} \frac{|S_{if}|^2}{T} d^3p' d^3k' \tag{3.3}$$

which, on extending the sum to all initial and final spin states, becomes

$$d\sigma = \frac{(4\pi\alpha)^2}{(2\pi)^4}(2\pi)^4\delta^{(4)}(k' + P' - k - P) \times \frac{m^2M^2c_0^4}{k_0'k_0P_0'P_0}\times$$

$$|\frac{\mathbf{k}}{k_0}|\frac{1}{[(k'^{\mu} - k^{\mu})(k'_{\mu} - k_{\mu})]^2} \times \frac{1}{4}Trace[\gamma^{\mu}\frac{mc_0 + \gamma.k}{2mc_0}\gamma_{\nu}\frac{mc_0 + \gamma.k'}{2mc_0}]\times$$

$$Trace[\gamma^{\mu}\frac{Mc_0 + \gamma.P}{2Mc_0}\gamma_{\nu}\frac{Mc_0 + \gamma.P'}{2Mc_0}]d^3k'd^3P'$$

$$= \frac{\alpha^2}{|\mathbf{k}|}\delta^{(4)}(k' + P' - k - P) \times \frac{1}{(k' - k}^4 \times$$

$$\frac{1}{4}Trace[\gamma^{\mu} \times (mc_0 + \gamma.k) \times \gamma_{\nu} \times (mc_0 + \gamma.k')]\times$$

$$\frac{1}{4}Trace[\gamma^{\mu} \times (Mc_0 + \gamma.P) \times \gamma_{\nu} \times (Mc_0 + \gamma.P')] \times \frac{d^3P'}{P_0'}\frac{d^3k'}{k_0'}$$

where $\alpha \equiv e^2/hc_0$.

We shall not proceed further with the explicit evaluation of this expression except to note that the product of the two traces is

$$\frac{1}{16}S_p().S_(p)() = [k^{\mu}k'^{\gamma} - g^{\mu\gamma}(kk') + (mc_0)^2g^{\mu\gamma}]\times$$

$$[P^{\mu}P'^{\gamma} - g^{\mu\gamma}(PP') + (Mc_0)^2g^{\mu\gamma}] \tag{3.5}$$

where $P' = P + k - k'$ and in the laboratory frame $[P = (Mc_0, 0)\ ]$ under the assumptions that the proton has no structure and that

$$mc_0/k_0 << 1, mc_0/k_0' << 1, q^2 = (k' - k)^2 \simeq -4k_0k_0'sin^2(\frac{\theta}{2}),$$

one obtains the standard expression[15] of "potential scattering theory" for electron-point-proton scattering (with unpolarized initial state and no observation of final spin):

$$\frac{d\sigma_{e-p}}{d\Omega} = \frac{\alpha^2E^2(1 - \beta sin^2(\theta/2))}{4P^4sin^4(\theta/2)} = \frac{\alpha^2cos^2(\theta/2)}{E^2sin^4(\theta/2)} \equiv \frac{d\sigma_{Mott}}{d\Omega} \tag{3.6}$$

where

$$\beta = \frac{|\mathbf{P}|}{E}, \frac{1}{2}(\mathbf{P'} - \mathbf{P})^{\mathbf{2}} = (\mathbf{q})^{\mathbf{2}} = \mathbf{2P^2(1 - cos}(\theta)) = \mathbf{4P^2sin^2}(\theta/\mathbf{2})$$

and σ_{Mott} is the Mott scattering cross section. A preliminary HM approach to deep-inelastic (irreversible) scattering has been reviewed by Animalu and Ekuma[16].

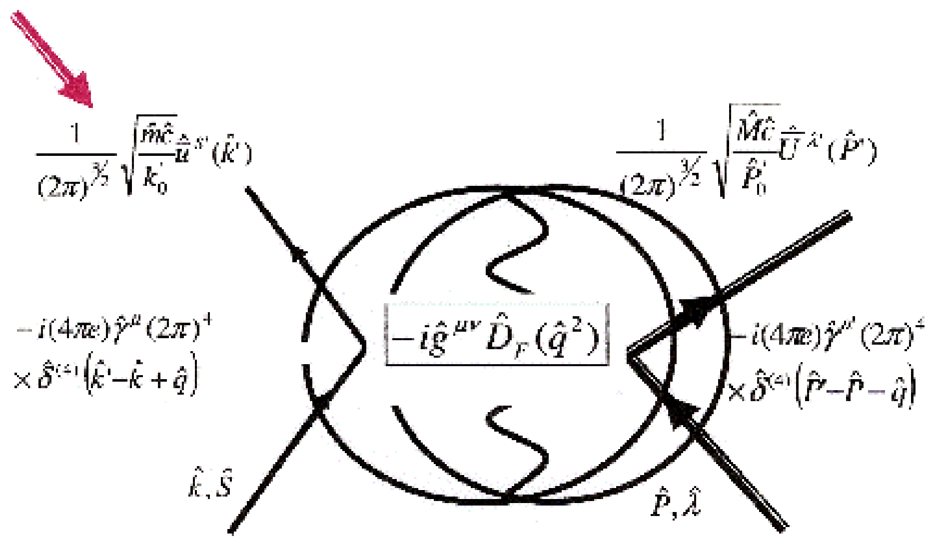

Figure 9: *Fig5b: Generalized Feynman Graph/Rule for e-p Scattering*

Turning next to the isoscattering profile represented by Fig.3(b and c) we consider the elaboration in Fig. 5(b).

Again, to write down the S-matrix one starts in the direction of the (red) arrow from left to right as indicated above and, at each vertex, inserts all other factors between the incoming and outgoing arrows. If loop closes, one takes trace to get

$$\hat{S}_{fi} = -4i\pi \int \frac{d^4\hat{q}}{(2\pi)^4} \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{\hat{m}\hat{c}}{\hat{k}'_0}} \times \hat{\bar{u}}^{\hat{S}'}(\hat{k}')]\times$$

$$[-i(4\pi e)\hat{\gamma}^{\mu'}(2\pi)^4 \times \hat{\delta}^4(\hat{k}' - \hat{k} + \hat{q})] \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{\hat{m}\hat{c}}{\hat{k}_0}} \times \hat{u}^{\hat{S}}(\hat{k})]\times$$

$$[-i\hat{g}^{\mu\nu}\hat{D}_F(\hat{q}^2)] \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{\hat{M}\hat{c}}{\hat{P}'_0}} \times \hat{\bar{U}}^{\hat{\lambda}'}(\hat{P}')]\times$$

$$[-i(4\pi e)\hat{\gamma}^{\mu'}(2\pi)^4 \times \hat{\delta}^{(4)}(\hat{P}' + \hat{P} - \hat{q})] \times [\frac{1}{\sqrt{(2\pi)^3}}\sqrt{\frac{\hat{M}\hat{c}}{\hat{P}_0}} \times \hat{U}^{\hat{\lambda}}(\hat{P})]$$

$$= -4\pi i \int \frac{d^4\hat{q}}{(2\pi)^4}\sqrt{\frac{(\hat{m}\hat{c})^2(\hat{M}\hat{c})^2}{\hat{k}'_0\hat{k}_0\hat{P}'_0P_0}} \times \hat{\bar{u}}^{\hat{S}'}(\hat{k}')\hat{\gamma}^{\mu}\hat{u}^{\hat{S}}(\hat{k})\times$$

$$\frac{-4\pi i\alpha}{(4\pi)^4}(2\pi)^4\hat{\delta}^{(4)}(\hat{k}' + \hat{P}' - \hat{k} - \hat{P})\times$$

$$[(\hat{k}' - \hat{k})^2 + (m_\phi\hat{c})^2 + i\hat{\varepsilon}]^{-2} \times \hat{\bar{U}}^{\hat{\lambda}'}(\hat{P}')\hat{\gamma}_\mu\hat{U}^{\hat{\lambda}}(\hat{P}). \tag{3.7}$$

This may similarly be rewritten in the form of generalized current-current interaction:

$$\hat{S}_{fi} = -4i\pi \int \frac{d^4\hat{q}}{(2\pi)^4} \times \hat{j}^\mu(\hat{k}'\hat{k}, -\hat{q}) \times \frac{1}{[\hat{q}^2 + (m_\phi\hat{c})^2]^2} \times \hat{J}_\mu(\hat{p}'\hat{p}; \hat{q}) \tag{3.8}$$

where

$$\hat{j}^\mu(\hat{k}'\hat{k}, -\hat{q}) = (2\pi)^4 \times \hat{\delta}(\hat{k}' - \hat{k} + \hat{q})] \times \frac{e}{(2\hat{m}\hat{c})^3} \times \sqrt{\frac{(\hat{m}\hat{c})^2}{\hat{k}'_0\hat{k}_0}}\hat{\bar{u}}^{S'}(\hat{k}')\hat{\gamma}^\mu\hat{u}^S(\hat{k}) \tag{3.9a}$$

$$\hat{A}_\mu(\hat{k}' - \hat{k}) = \frac{4\pi}{(\hat{k}' - \hat{k})^2 + (m_\phi\hat{c})^2 + i\hat{\varepsilon}}\hat{J}_\mu(\hat{p}', \hat{p}; \hat{k}' - \hat{k}) \tag{3.9b}$$

are, respectively, the generalized electromagnetic current$\hat{j}^\mu$ and generalized Moller current$\hat{J}_\mu$ associated with the generalized electromagnetic vector potential, $\hat{A}_\mu$.

We observe that three novel features arise: firstly, from the generalized internal photon line $\hat{D}_F(\hat{q}^2)$ which, as indicated in Table 1, is no longer divergent in the limit $\hat{q} \to 0$, secondly, from the generalized Dirac matrices $\hat{\gamma}^\mu$, and thirdly from the generalized currents,$\hat{j}^\mu$ and $\hat{J}_\mu$. The experimental verification of these novel features are readily streamlined by re-interpretation of the standard model current-current interaction model of the weak decay of the neutron to which we now turn.

4.Experimental Verification.

The experimental verification of isoscattering theory requires us to reconcile and re-interpret the various scattering profiles and models of neutron production considered in this paper with the standard (electroweak interaction) model[17] of neutron decay in terms of our sequence of representations of the carrier space-time symmetry of the scattering region in Fig. 1

$$QM \to O(3,1) \to O(4,2)$$

$$\downarrow$$

$$HM \to \hat{O}(3,1) \to \hat{O}(4,2)$$

as summarized in Table 4 and in Fig.6 as well as the iso-gauge principle for the lifting of the electromagnetic gauge field $A_\mu(x) \to \hat{A}_\mu(\hat{x})$, which we now proceed to discuss in turns,

Table 4: Models of neutron production in $e-p$ scattering & neutron weak decay processes			Wave Equations	Symmetry Group
$p+e^- \to n+\nu$ $\to p+e^- +\nu+\bar{\nu}$	$n=(p,e^-,\bar{a}^0)_{QM}$ $\to (p,\hat{e}^-)_{HM}$ $\Rightarrow \hat{e}^- \cong e^-\bar{a}^0$	p	$(i\gamma_\mu\partial_\mu - m_p)\psi_p = 0$	$O(3.1)$
		(e^-,a^0)	$\left(i\gamma_\mu\partial_\mu - 3m_e - (2m_e)^{-1}\partial_\mu\partial^\mu\right)\psi_e = 0$	$O(4,2)$
$n \xrightarrow[w]{} p+W^-$ $\to p+e^-+\bar{\nu}$	$n-(p,W^-)_{QM}$ $\Rightarrow W^- \approx \hat{e}^- \approx e^-a^0$	$(\hat{e}^-,\hat{a}^0)$	$\left(i\hat{\gamma}_\mu\hat{\partial}^\mu - 3\hat{m}_e - (2\hat{m}_e)^{-1}\hat{\partial}_\mu\hat{\partial}^\mu\right)\hat{I}\hat{\psi}_e = 0$	$\hat{O}(4,2)$

Figure 10: *Table 4: Models of neutron production*

Table 4 states that compatibility of scattering profiles and neutron structure (with negative binding energy) requires us to eliminate the neutrino and antineutrino ($\nu, \bar{\nu}$) and replace them with the etherino and antietherino $(a^0, \bar{a}^0)$ as constituent of the

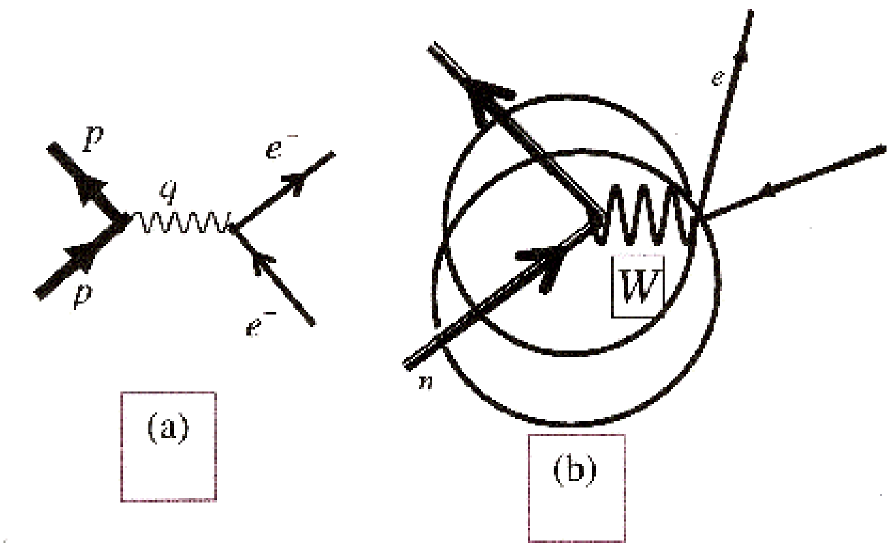

Figure 11: *Feynman graph for (a) p-e scattering and (b)neutron decay* $n \to p + e^- + \nu$

neutron in conventional QM, but build the neutron from the proton (p) and mutated electron ($\hat{e}^-$) in HM. Moreover, consistency of HM concept of electron mutation requires firstly that $\hat{e}^- \approx e^-\bar{a}^0$ to reach the Rutherford Santilli neutron, $n = (p, \hat{e}^-)_{HM}$ and secondly that$\hat{e}^- \sim W^- \approx e^-\bar{a}^0$, where $W^-$is the massive intermediate boson that mediates neutron weak decay as shown in Fig.6.

Consequently, a first experimental verification is expected to emerge from the lifting of the O(4,2) wave equation (2.12) for the pair e^-, a^0 to the following $\hat{O}(4,2)$ wave equation

$$[i\bar{\gamma}_\mu \hat{\partial}^\mu - 3\hat{m}_e - (2\hat{m}_e)^{-1}\hat{\partial}_\mu \hat{\partial}^\mu]\hat{T}\hat{\psi}_e = 0 \tag{4.1a}$$

for $\hat{e}^-, \hat{a}^0$. The iso-mass equation,$\hat{m}^2 + 2\hat{m}_e\hat{m} - 6\hat{m}_e^2 = 0$, has two roots

$$\hat{m}_\pm = \hat{m}_e(-1 \pm \sqrt{7}), i.e., \frac{\hat{m}_+}{\hat{m}_e} = 1.65, \frac{\hat{m}_-}{\hat{m}_e} = -3.65 \tag{4.1b}$$

Thus, $\hat{m}_+/\hat{m}_e = 1.65 \equiv \hat{m}_{a^0}/\hat{m}_e$ as before for compatibility with negative binding energy for neutron production. However, by observing that $|\hat{m}_-|/\hat{m}_e = 3.65 \approx m_\Lambda/m_d$, where$3m_d = m_n = 939MeV$ and $m_\Lambda \approx \frac{1}{2}(m_\Lambda + m^0_{Sigma}) = \frac{1}{2}(1116+1192) = 1142MeV$ we infer from Eq.(2.15) and (2.16) that$\hat{e}^-$ may be re-interpreted as d-isoquark. In addition, unlike the usual Dirac equation that has only positive mass, Eq.(4.1a) has both positive and negative masses, and the negative mass may be necessary for the binding of the correlated pairs of particles in O(4,2) theories.

With regards to the lifting of the electromagnetic gauge field,$A_\mu(x) \to \hat{A}_\mu(\hat{x})$, we observe that the divergence of the Feynman graph for the $e^- - p$ Coulomb interaction in Fig. 6(a) arises basically from the factor $1/|\mathbf{k}' - \mathbf{bfk}|^2 \equiv \mathbf{1/q^2}$ associated with the Fourier transform of the long-range Coulomb potential, $V_C(r) \equiv (e/c_0)A_0(r) = -e^2/r$ which is determined by the time-component (A_0) component of the electromagnetic 4-vector potential,$A_\mu(\mu = 0, 1, 2, 3)$. For this reason, the divergence is related to the structure of the electromagnetic gauge field. However, by expressing the interparticle Coulomb force $-dV_C/dr = -e^2/r^2$ as a functional of the potential energy V_C , and eliminating explicit r-dependence between dV_C/dr and V_C , a non-linear first-order differential equation results:

$$\frac{dV_C}{dr} = \frac{V_C^2}{e^2}, or \frac{\partial A_0}{\partial r} = (1/e^2)A_0^2 \tag{4.2}$$

As this is a special case of Riccati's equation, an obvious step to achieve a progressive generalization of the Coulomb potential is to lift Eq.(4.2) to the most general iso-Riccati's equation

$$\frac{\partial A_0}{\partial r} \to \frac{\partial \hat{A}_0}{\partial \hat{r}} = (1/\hat{e}^2)\hat{A}_0^2 + \zeta \hat{A}_0^2 + \kappa \tag{4.3}$$

ζ, κbeing constants (in general, functions of $\hat{r}$). We note that the derivative and nonlinear parts, $[\partial A_0/\partial r - (1/e)A_0^2]$, of Eq.(4.3) may be re-interpreted as appropriate component of the SU(2) Yang-Mills gauge field,

$$F_{\mu\nu}^a \equiv \frac{\partial A_\nu^a}{\partial x_\mu} - \frac{\partial A_\mu^a}{\partial x_\nu} + g_0 \varepsilon^{abc} A_\mu^b A_\nu^c \tag{4.4}$$

where g_0 is a coupling constant. Moreover, an exact solution of Eq.(4.3) given in the Appendix to ref.[11] has the form of a Hulthen potential which has an approximate Yukawa form:

$$V_H(r) = \frac{-Me^{-m_0 r}}{1 - e^{-m_0 r}} \approx \frac{-Me^{-m_0 r}}{m_0 r + O(r^2)} \approx \frac{-Me^{-m_0 r}}{r} \equiv \phi(r) \tag{4.5}$$

However, by using well-known standard transformation we may convert the nonlinear first-order Riccati Eq.(4.3) into a linear second-order differential equation for ϕ (see, p.201 of Piaggio[18]) :

$$\hat{A}_0 = -e\frac{dlog(\phi)}{dr} \equiv -e\frac{(d\phi/dr)}{\phi} \equiv -e\frac{\phi_1}{\phi}, \tag{4.6a}$$

where $\phi_1 \equiv d\phi/dr$. Note that this transformation may be rewritten as a (Weyl-like) gauge principle in the integral form:

$$\phi = exp(-(1/e)\int_0^r \hat{A}_0 dr). \tag{4.6b}$$

From (4.6a) we find

$$\frac{d\hat{A}_0}{dr} \equiv -e\frac{\phi_2}{\phi} + e\frac{\phi_1^2}{\phi^2}, \tag{4.7}$$

so that, on substitution in Eq.(4.3), the terms in ϕ_1^2 disappear, and hence, on multiplying the resulting equation through by ϕ/e , we obtain a linear second-order differential equation:

$$\phi_2 - \zeta\phi_1 - (\kappa/e)\phi = 0. \tag{4.8}$$

In addition, if we select $\zeta \equiv -2/r$ and put$m_\phi^2 = \kappa/e$, this equation takes the standard form

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{\kappa}{e}\phi \equiv (\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} - m_\phi^2)\phi(r) = 0 \tag{4.9}$$

which is the static limit of the Klein-Gordon equation for a spin-0 scalar field ϕ of mass m_ϕ in units such that $h/2\pi = c_0 = 1$. The fact that the Fourier transform of $\phi(r)$ given by $-g_0^2/[q^2 + (m_\phi\hat{c})^2/\hat{h})^2]$ eliminates the divergence of the Fourier transform of V_C in the limit $q \to 0$ was obtained originally from second-order Coulomb scattering S-matrix by Dalitz[19].

5. Concluding Remarks.

History of science has established that physical theories provide a mere approximation of nature due to its complexity generally beyond our understanding. Therefore., no matter how beautiful and correct a given theory may appear at a given time, its structural generalization is inevitable due to the advancement of scientific knowledge and the identification of conditions beyond those of the original conception. This is also the fate of the 20th century scattering theory, since the inevitability of its structural generalization, in due time, for higher and higher energies and more and more complex collisions of particles is beyond doubt.

In this and the preceding papers of this series, we have established that the 20th century scattering theory can indeed be lifted into an axiom-preserving isotopic formulation for reversible scattering processes which is universal for the class admitted, invariant over time. and admitting of the conventional theory as a simple particular case.

In particular, this and the preceding papers of this series, have established the:

1) **Conditions of exact validity of the 20th century scattering theory,** given by Coulomb and other scattering of particles under conditions admitting a valid point-like approximation (e.g., at sufficient mutual distance without collisions),

as necessary for the applicability of the local-differential topology and mathematics underlying relativistic quantum mechanics;

2) **Conditions of unknown validity of the 20th century scattering theory,** given by reversible scattering events entirely representable via negative binding energies under conditions of partial or total mutual p[penetration of the charge distributions and/or wave[packets of particles. In this case, vast preceding studies have established the non-Lagrangian and non-Hamiltonian character of the events with expected revisions of the "experimental results" claimed from the elaboration of measured quantities vis the 20th century theory. The unsettled character of this case, clearly stated since Paper I of this series, is that the 20th century and the isotopic scattering theory have exactly the same axioms, to such an extent of coinciding at the abstract realization-free level. Under this axiomatic identity, no scientific conclusion can be reached without a detailed scrutiny, whether for the validity or invalidity under isotopies of 20th century "exp[experimental results" for reversible scattering events entirely representable via a negative binding energy.

3) **Conditions of inapplicability (and not violation) of the 20th century scattering theory,** given by reversible scattering events requiring "positive binding energies," as it is the case for the synthesis of the neutron from a proton and an electron, the synthesis of the π^o meson from an electron-positron pair, and the syntheses of hadrons at large, as occurring in the core of stars, in particle accelerators or under other conditions. The inapplicability of the 20th century scattering theory for the latter class is beyond credible doubt due to its unitary character, while the events considered solely admit a quantitative representation via nonunitary theories as established by Santilli since 1978 [3].

In conclusion, this and the preceding papers of this series have indeed established the *necessity, consistency and validity* of the isoscattering theory of relativistic hadronic mechanics because the only known at this writing permitting quantitative representations of reversible syntheses of hadrons.

.

Acknowledgments

The authors has no words to express their deepest gratitude to Larry Horwitz of Tel-Aviv University for truly invaluable critical comments and insights without which this paper would not have possible. Additionally, the author would like to thank the participants of various meetings were the foundations of this paper were discussed in one form or another, including: the participants of the meeting of the International Association of Relativistic Dynamics (IARD) held on June 2007, in Thessaloniki, Greece; the participants to a seminar delivered at the European Laboratory in Ispra, Italy in January, 2009; and the participants of the inauguration ceremony of the Research Institute of Hypercomplex Systems in Geometry and Physics (RIHSGP) held in Moscow, Russia, on May, 2009. Finally, the second named author would like to thank the founder of the new Institute, D. G. Pavlov, its president V. Gladyshev, as well as Gh. Atanasiu, V. Balan, P. Rowlands and all other members of the new Institute for their kind invitation and hospitality in Moscow during which visit this paper was completed. The main aspects of these papers were also discussed during the recent *Third International Conference on the Lie-Admissible Treatment of Irreversible Processes* held at the University of Kathmandu, Nepal, from January 5 to 9, 2011. The author would like to thank all participants for invaluable comments. Additionally, special thanks are due to Dorte Zuckerman for editorial control and to Christian Corda, the Editor of the proceedings for editorial control.

References

[1] A.O.E. Animalu and R.M. Santilli, Nonunitary, Lie-isotopic and Lie-admissible scattering theopries of hadronic mechanics, I, II, III,

[2] R.P. Feynman, Quantum Electrodynamics (W.A. Benjamin, Inc. N.Y.) 1962

[3] Hadronic J. **1**, 574 (1978)

[4] R. M. Santilli: Hadronic J. 13, 513 (1990) [4a]; Communication of the JINR, Dubna, Russia E4-93-352, 1993 [4b]; Chinese J. Syst. Ing. and Electr. 6, 177 (1996) [4d]; Foundations of Physics 37, 670 (20070 [4d]; *Hadronic Mathematics, Mechanics and Chemistry.* International Academic Press 2008, Internationa;l Academic Press, http://www.i-b-r.org/Hadronic-Mechanics.htm

[5] A.O. Barut, in Group Theor. Methds in Physics (Edited by W. Beiglbock, A. Bohm and T. Takasugi), SLNP Vol. 94, 490 (1979); A.O. Barut, Ann. Physik Leipzig 43 83 (1986); see also, A.O.E. Animalu, Lepton and Hadron Currents in O(4,2) Current Algebra Letters Nuovo Cim. 3, 729 (1972).

[6] A. O. Barut, P. Cordero and G.C. Ghirardi, Phys. Rev. 182, 1844 (1969)and Nuov. Cim. 66A, 36 (1970); A.O.E. Animalu, Mass Ratio of Quarks, Phys. Rev. D3, 1106 (1971); Charge Spectrum of 4-Component Fields with O(4,2) Dynamical Symmetry, Phys. Rev. 4D, 1992-1925 (1971).

[7] J. Bernstein, in Elementary Particles and their Currents (Freeman, San Francisco, 1947) p. 41.

[8] A.O.E. Animalu,, Scale Symmetry, Physics Today p.76, June (1972)

[9] M. Gell-Mann, R.J. Oakes and B. Renner, Phys. Rev. 175,2195 (1968); M. Gell-Man, Phys. Letters 8, 214 (1964); G. Zweig, CERN Report 8419TH 412 (1964).

[10] H.C. Myung and R.M. Santilli, Foundations of the Hadronic Generalization of the Atomic Mechanics, II: Modular-Isotopic Hilbert Space Formulation of the Exterior Strong Problem, Hadronic J. 5, 1277-1366 (1982); M. Nishioka, Extension of the Dirac-Myung-Santilli Delta function to Field Theory, Lett. Nuov. Cim Ser. 2, Vol. 39, pp.369-372 (1984).

[11] A.O.E. Animalu, Gauge-Invariant Relativistic theory of the Rutherford-Santilli Neutron, Hadronic J 27,599-624 (2004).

[12] John Evans, Mind, Body and Electromagnetism (Element Books 1986), p.64-66.

[13] J. Todd, Projective and Analytical Geometry (Pitman, London 1958) p.62.

[14] P.A.M. Dirac P.A.M. Dirac, Quantum theory of the Electron, Proc. Roy. Soc. (Lond.) A117, 610 (1928), Part II, A118 351 (1928); A Positive-Energy relativistic Wave equation, Proc. Roy. Soc. (Lond.) A322, 435 (1971); A328, 438 (1972).

[15] J.D.Bjorken and S. Drell Advanced Quantum Mechanics Vols. 1 and 2 (1966).

[16] A.O.E. Animalu and C.E. Ekuma, Review of deep-Inelastic e-p Scattering: A Hadronic Mechanics Viewpoint, African J. Phys. Vol. 1, p.133-153 (2008).

[17] S.Weinberg, Phys. Rev. Lett 18, 1264 (1967).

[18] H.T.H Piaggio. An Elementary Treatise on Differential Equations and their Applications London) (1961).

[19] R.H Dalitz Proc Roy Soc 206A,509(1951)