NONUNITARY LIE-ISOTOPIC AND LIE-ADMISSIBLE SCATTERING THEORIES OF HADRONIC MECHANICS, III:

Basic Lie-Isotopic Formulation without Divergencies

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Abstract

In the preceding Papers I and II, we have: introduced various arguments suggesting nonunitary coverings of the conventional unitary scattering; outlined the problematic aspects of nonunitary theories; presented the novel isomathematics allowing an invariant isounitary reformation; and specialized to scattering processes the isotopic branch of hadronic mechanics comprising the isotopies of Lies theory. special relativity and quantum mechanics. In this paper we present a solution of the Dirac legacy indicated in Paper II, namely, a nonunitary-isounitary scattering theory without divergencies ab initio indicated by one of the authors to Dirac prior to his death. Joint elaborations of measured quantities (cross section, scattering angles, etc.) via the conventional and isotopic scattering theories are presented in subsequent works to identify the expected implications in final experimental results of nonlinear, nonlocal and nonpotential effects in high energy scattering processes. As indicated in paper I, the presentation is restricted to reversible scattering processes, because the treatment of irreversible processes requires the yet broader Lie-admissible covering of the Lie-isotopic formulation.

Key words scattering theories, nonunitary theories, isounitary theories
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TBLE OF CONTENT

1. Identification of the Isounit.

1.1. Introduction. In the preceding Refs. [1,2], herein indicated as Papers I and II, respectively, in which:

1) We presented various mathematical, theoretical and experimental arguments suggesting a re-inspection of the elaboration of measured quantities (cross sections, scattering angles, etc.) via the conventional, unitary, Hamiltonian scattering theory due to expected nonlinear, nonlocal and contact-nonpotential effects beyond the representational capabilities of quantum mechanics;

2) We pointed out that the sole possibility for a quantitative representation of the above effects, collectively called in these papers *non-Hamiltonian effects,* is the existing from the class of unitary equivalence of quantum mechanics, thus bringing again into focus past attempts at a nonunitary generalization of quantum scattering theories;

3) We then recalled the *Theorems of Catastrophic Mathematical and Physical Inconsistencies of Noncanonical and Nonunitary Theories* (Refs. [6-12] of Paper I) that have established the lack of invariance over time of units of measurements, the inability of predicting the same numerical values under the same conditions at different times, violation of causality and other inconsistencies under nonunitary time evolution;

4) We then introduced the elements, specialized to the scattering problem, of the novel *isomathematics* that has been specifically constructed for the invariant treatment of non-Hamiltonian effects and the resolution of the Inconsistency Theorems via the *isounitary reformulation of nonunitary scattering theories;* and

5) we finally presented the elements, also specialized to the scattering problem, of the *isotopic branch of hadronic mechanics* including the invariant deformations-isotopies of Lie's theory, special relativity and relativistic quantum mechanics, with emphasis on Dirac's pioneering work in the field.

We are now sufficiently equipment to initiate the presentation of the proposed covering *nonunitary-isounitary scattering theory* beginning with the identification of the all fundamental isounit.

To prevent major misrepresentations of the content of this paper, that often remain undetected by renowned experts in quantum mechanics but non-experts in the covering hadronic mechanics, serious readers are suggested to acquire a technical knowledge of Papers I and II, hereon tacitly assumed, with particular reference to isofunctional analysis, isodifferential calculus, regular and irregular isorepresentations, regular and notions notions of spin, regular and irregular Dirac-Santilli isoequations, Dirac's generalization of Dirac's equation, and related aspects.

1.2. Main Conditions. As indicated in paper II, the isotopic branch of hadronic mechanics, also known as *isomechanics,* was built for the representation of the deep mutual penetration of the wavepackets and/or charge distributions of particles as occurring in the scattering region of Figure I.2. These conditions are of contact character, thus having zero range, and are expected as being nonlinear (in the wavefunctions), nonlocal (of integral type), and nonpotential (that is, not representable with a Hamiltonian). These non-Hamiltonian interactions are structurally beyond any possible representation by quantum mechanics due to its strictly linear, local-differential and potential-Hamiltonian character.

In view of these insufficiencies, the representation of the scattering region is done via *two* operators, the conventional Hamiltonian H as currently used in scattering theories, and Santilli's isounit $\hat{I}$ or isotopic element $\hat{T} = \hat{I}^{-1}$, for the representation of all non-Hamiltonian interactions and effects.

Following studies for decades, the isounit has been selected over any other alternative representations of non-Hamiltonian interactions because it is the only one assuring the crucial *invariance over time,* i.e., the characterization of the same numerical values under the same conditions at different times. After all, whether conventional or generalized, the unit is the fundamental invariant of all theories.

The main requirements for the isounit (or isotopic element) are the following (see Refs. [3] for detailed studies):

CONDITION I: Positive-definiteness.

$$\hat{I}(t, r, p, E, \xi, \omega, \psi, \partial\psi, ...) = \hat{I}^\dagger(t, r, p, E, \xi, \omega, \psi, \partial\psi, ...) = \\ 1/\hat{T}(t, r, p, E, \xi, \omega, \psi, \partial\psi, ...) > 0. \tag{1.1}$$

This condition is sufficient to assure the preservation of the original axioms under isotopic liftings at all levels, with consequential local isomorphism between the Hilbert space and the Hilbert-Santilli isospace, the Lorentz-Poincaré (LP) symmetry and the Lorentz-Poincaré-Santilli (LPS) isosymmetry, quantum mechanics and isomechanics, etc.

CONDITION II: Elimination of quantum levels,

$$Lim_{r\gg 1fm}\hat{I}(t,r,p,E,\xi,\omega,\psi,\partial\psi,...)=\hbar,$$

$$Lim_{r\gg 1fm}\hat{T}(t,r,p,E,\xi,\omega,\psi,\partial\psi,...)=1/\hbar. \quad (1,2)$$

This condition assures the existence of a unique and unambiguous limit under which hadronic mechanics recovers quantum mechanics for all mutual distances of particles bigger than 1 fm, e,g,. for mutual distances bigger than the size of particles wavepackets. Also, Hamiltonian interactions remain fully valid inside the scattering region. Hence, the above condition clarifies the fact that isomechanics essentially provides expected *corrections* to quantum treatments in the scattering region (only.

A primary function of condition (1.2) is to illustrate the main feature of hadronic mechanics, *the absence of conventionally quantized energy levels within a hyperdense hadronic medium.* Conceptually, this condition is illustrated by the evident impossibility that an electron in the core of a star (or, equivalently, in the interior of a high energy scattering region) cannot possibly have the same quantized levels as occurring when orbiting in vacuum around the nucleus in the hydrogen atom. The isounit then represents the integro-=differentia;l *deviations* from conventional quantum levels caused by the medium.

Physically, condition (1.3) is illustrated by the so-called "screened Coulomb potentials" used in quantum chemistry, namely, the multiplication of the Coulomb potential $V=e^2/r$ by an arbitrary function, $V^*=f(r)e^2/r$ that has resulted as being necessary for a numerically exact representation of the mutual penetration of valence electron pairs in molecular structures. However, in so doing, it is evident that the screened potential no longer admits the quantum energy levels of the conventional potential, as studied in detail in monograph [4].

CONDITION III: Elimination of quantum divergencies,

$$||\hat{I}|| \gg 1, \quad ||\hat{T}|| \ll 1. \quad (1.3)$$

As it is well known to experts in the field, the above condition assures that all perturbative and other series that are divergent (or weakly convergent) for quantum mechanics are turned into strongly convergent series in the covering hadronic mechanics.

In turn, this important feature, whose achievement escaped the best minds of 20th century physics, implies *numerical differences* between the sum of divergent quantum series (turned into convergent forms via cut-off, arbitrary parameter of unknown physical origin, and other manipulations) and the naturally convergent hadronic series. Moreover, these differences are expected to produce numerical differences between the elaboration of experimental data via scattering and isoscattering theories, a feature well known to experts in hadronic mechanics since the early 1980 (se Section 1 of paper I and references quoted therein), but often ignored by particle physics to their peril. Note that conditions (1.3) are fully compatible with conditions (1.2), as shown by all realizations assumed later on.

1.3. Explicit Realization. All isounits used in experimental verifications of hadronic mechanics to date (see Refs. [3d]) have emerged as verifying quite naturally Conditions I, II and III. that, therefore, *are not* subsidiary constraints, but seemingly natural occurrences.

Inspired by these experimental verifications, particularly for the representation of the Cooper pair and valence bonds [5], one can see the adoption of the following *realization of the isounit for the relativistic isoscattering theory*

$$\hat{I} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2) \times e^{N \times \frac{\psi^i(x)}{\psi^f(x)} \times \int d^4x \times \psi^{i\dagger}(x) \times \psi^f(x)}, \tag{1.4}$$

where

$$n_\mu = n_\mu(t, r, p, E, \xi, \omega, \psi, \partial\psi, ...) > 0, \ \mu = 1, 2, 3, 4, \tag{1.5}$$

where:

1) The characteristic quantities $n_k, \ k = 1, 2, 3$ provide the first known representation of the *generally nonspherical and deformable shape* of the scattering region normalized to the values $n_k = 1, k = 1, m2, 3$ for the sphere;

2) The characteristic quantity n_4 provides the first known representation of the *density of the scattering region* (i.e.m the ratio between its energy and

its volume) normalized to the value $n_4 = 1$ for the vacuum; and

3) the first known representation of the *inhomogeneity of the scattering region* is provided by a functional dependence of the characteristic quantities, e.g., on the local coordinates, while the first known representation of the *anisotropy of the scattering region* is provided, e.g., by different values of space and time characteristic quantities.

It should be indicated that the characteristic quantities have an explicit functional dependence when formulated in the *interior* of the scattering region, but they are generally averaged into constants when inspected from the outside, an assumption we shall tacitly make hereon.

It is easy to see that isounit (1.4) naturally verifies Conditions I, II, III. Additionally:

A) The integral $\int d^4x \times \psi^{i\dagger}(x) \times \psi^f(x)$ represent the intended nonlocal character of the scattering region as well as the verification of limits (1.2);

B) The ratio between initial and final wavefunctions, ψ^i/ψ^f, characterizes the intended nonlinear character of the scattering theory (with the possibility of additional embedding of nonlinear terms);

C) N is an isorenormalization constant to be identified later on;
and the *exponential* character of the isounit originates at the primitive Newtonian level (see Section II.3.2. In fact, the exponential character of the isounit emerged when turning nonconservative non-Hamiltonian Newtonian systems into an identical isotopic form, and this feature persists under operator formulations.

Alternatively, on mathematical grounds, one can see the emergence of the exponential structure of the isounit from the isodifferential of Section I.3.4. In this case, the use of the expressions of the type $\hat{I} = e^{A(r)}$ implies the cancellation between $\hat{T}$ and $\hat{I}$ in the additive term, namely, $r \times T \times d\hat{I} = r \times \partial_r A dr$.

it is hoped the reader can see that, besides all the arguments presented in Papers I and II, the LP symmetry cannot possibly be exact for the scattering region under the sole admission of its nonspherical, inhomogeneous and anisotropic character, in favor of the exact universal LPS isosymmetry. The isotopies of quantum mechanics of paper II and of the scattering theory presented in this paper, then follow rather inevitably.

2. Elements of Isoscattering Theory.

2.1. Isoscattering Amplitude. By assuming a knowledge of the content and terminology of Papers I and II, we can now expeditiously proceed with the formulation of the desired isoscattering theory. For clarity, we shall write most of the formalism in its projection on conventional spaces so as to show the differences with conventional formulations.

An important difference between the quantum and isotopic scattering theory is the assumption in the following. The conventional scattring theory assumes the speed of light *in vacuum* c and related LP symmetry as being unchanged in the *hyperdense medium* inside the scattering region. By contrast, the isoscattering theory assumes that the speed of light in the interior problem is the local avriable $C = c/n_4 = c \times b_4$ with universal LPS isosymmetry. However, when the scattering problem is studied on the Minkowski-Santilli isospacetime over the isoreals, the speed of light in the interior problems remains indeed that in vacuum, as recalled in Section II-2. These views imply that the characterization of the density of the scattering region indicated earlier is given by theindex of refraction $n_4 = 1/b_4$, a notion absent in quantum theories.

The isoscattering theory alsop assumes that the geometry in the interior of the scattering region is mutated by its medium. This mutation is represented with the transition from the Euclidean space $E(r, \delta, \mathcal{R}$ over the reals $\mathcal{R}$ characterizing the vacuum (intended as empty space), to the Euclid-Santilli isospace $\hat{E}(\hat{r}, \hat{\delta}, \hat{\mathcal{R}}$ over the isoreals $\hat{\mathcal{R}}$ characterizing the hyperdense medium inside the scattering region.

Additionally, we assume the conservation of the volume of the scattering region in the transition from the conventional to the isotopic treatment that can be expressed by the condition

$$Det\ \hat{\delta} = (b_1^2 \times b_2^2 \times b_3^2) = (b_1 \times b_2 \times b_3)^2 = 1 \tag{2.1}$$

However, it should be stressed that this condition can be easily relaxed later on, e.g., in the transition from low to high energy scattering in which the scattering volume definitely is not preserved. We finally assume for simplicity that the characteristic b-quantities have been averaged to constants because the scattering region is inspected from the outside.

The above assumptions require the replacement of the conventional spherical coordinates with the isospherical form (3.36), including the lifting of conventional angles into the isoangles

$$\hat{\theta} = \theta \times \hat{I}_\theta, \quad \hat{\phi} = \phi \times \hat{I}_\phi, \ \hat{I}_\theta = b_3, \ \hat{I}_\phi = b_1 \times b_2, \tag{2.2}$$

In the simple case here considered, we then have the following trivial identities between isodifferentials and differentials

$$\hat{d}\hat{\theta} = b_3^{-1} \times d(\theta \times b_3) = d\theta, \quad \hat{d}\hat{\psi} = (b_1 \times b_2)^{-1} \times d(\phi \times b_1 \times b_2) = d\phi \tag{2.3}$$

But we have the expression

$$\hat{d}c\hat{o}s\hat{\phi} = s\hat{i}n\hat{\phi} \times \hat{d}\hat{\phi} = (b_1 \times b_2)^{-1} \times sin(\phi \times b_1 \times b_2) \times d\phi, \tag{2.4}$$

that illustrates nontrivial departures between conventional and isotopic treatments despite the simplest possible assumptions made above.

The *solid isoangle* is evidently given by

$$\hat{d}\hat{\Omega} = \hat{d}\hat{\theta} \times \hat{d}c\hat{o}s(\hat{\phi}) =$$

$$= (b_1 \times b_2)^{-1} d\theta \times d\phi \times sin(\phi \times b_1 \times b_2), \tag{2.5}$$

with isointegral

$$\hat{\Omega} = \Omega \times \hat{I}_\Omega = \Omega \times (b_1 \times b_1 \times b_3)^{-1} = \hat{\int}\hat{\int} \hat{d}\hat{\theta} \times \hat{d}\hat{\phi} \times s\hat{i}n\hat{\phi} =$$

$$= b_3^{-1} \times \int\int d\theta \times d\phi \times sin(\phi \times b_1 \times b_2), \tag{52.6}$$

and final expression

$$\Omega = (b_1 \times b_2) \times \int\int d\theta \times d\phi \times sin(\phi \times b_1 \times b_2), \tag{52.7}$$

that also illustrates the differences between conventional and isotopic treatments despite the preservation of the scattering volume (but not necessarily of the surface).

The *isoscattering amplitude* $\hat{f}(\hat{\theta}, \hat{\psi})$ can be defined in its most elementary form via the expression

$$e^{i \times k \times z} + \frac{\hat{f}(\hat{\theta}, \hat{\phi})}{\hat{r}} \hat{\times} e^{i \times k \times r}, \tag{2.8}$$

where $e^{i\times k\times z}$ and $e^{i\times k\times r}$ are the *conventional* incoming wand scattered waves because we have assumed the exact validity of quantum scattering theories outside the scattering region.

The projection of Eq. (2.8) on a conventional Euclidean space can be written

$$e^{i\times k\times z} + \frac{\hat{f}(\theta \times b_3, \phi \times b_1 \times b_2)}{r} \times e^{i\times k\times r}, \tag{2.9}$$

where we assume the reader is aware that the isodivision in Eq. (2.8) allowing the replacement of $\hat{r}$ with r.

The *isodifferential cross section* is then given by

$$\hat{d}\hat{\sigma} - = |\hat{f}(\hat{\theta}, \hat{\phi})|^2 \hat{\times} \hat{d}\hat{\Omega}, \tag{2.10}$$

and the *total cross section* assumes the form

$$\sigma = \hat{\int} \hat{d}\hat{\sigma} = \hat{\int} |\hat{f}(\hat{\theta}, \hat{\phi})|^2 \hat{\times} \hat{d}\hat{\Omega} = \int |\hat{f}(\hat{\theta}, \hat{\phi})|^{\times} d\Omega. \tag{2.11}$$

As recalled in Section I.1, the cross section is a number that, as such, is independent from the selected elaboration. The novelty of the isoscattering theory is the mutation of the scattering amplitude, whose implications will be elaborated below.

2.2. Isoscattering Matrix. . According to our assumptions of Section I.2, the initial and final states, $|i>$, $|f>$, respectively, are defined on a conventional Hilbert space $\mathcal{H}$ over a conventional quantum field $\mathcal{C}$, as denoted by the lack of "hat" in these states. However, by central assumption, their interconnection is done via isomechanics on the Hilbert-Santilli isospace $\hat{\mathcal{H}}$ over the isofield of complex isonumbers $\hat{\mathcal{C}}$ with isoinner product (3.15) of Paper I and isounit (1.4). Therefore, the *isoscattering matrix* is defined by (see Ref. [6b], Chapter 12, for a review of earlier works and references)

$$< i|\hat{\times}\hat{S}\hat{\times}|f> \;\; = \;\; < i| \times T \times \hat{S} \times T \times |f>, \tag{2.12}$$

with basic isounitarity property

$$< i| \times T \times (\hat{S} \times T \times |f>) \; = \; (< i| \times T \times \hat{S}^{\dagger}) \times T \times |f> \tag{2.13}$$

namely

$$\hat{S}\hat{\times}\hat{S}^{\dagger} = \hat{S}\hat{\times}\hat{S} = \hat{I}, \tag{2.14}$$

or

$$\Sigma_f \hat{S}_{fi} \times T_{ik} \times \hat{S}_{fk} = \hat{\delta}_{ik} = \hat{I} \times \delta_{ik}, \tag{52.15}$$

expressing the conservation of probability on isospace over isofields.

It should be indicated that *the isoscattering matrix is an isomatrix,* namely, its elements are isonumbers $\hat{n} = n \times \hat{I}$. Consequently, the isotopic element in the isoinner product (5.11) can be eliminated with the reduction of type (4.35a)

$$\hat{S} = \tilde{S} \times \hat{I} \tag{2.16}$$

under which we can regain the conventional form

$$< i| \times T \times \hat{S} \times T \times |f > \;\; = \;\; < i| \times \tilde{S} \times |f > . \tag{2.17}$$

However, the nontrivial character of the isoscattering theory is that the reduced matrix $\tilde{S}$ is *nonunitary,*

$$\tilde{S} \times \tilde{S}^{\dagger} \neq I. \tag{2.18}$$

as it is the case also for Eq. (5.14). The latter property is crucial to guarantee the exiting put of the class of unitary equivalence of quantum scattering theories and, in its absence, novelty would only be illusory. Note that *without the isotopic formulation, nonunitary scattering theories would be inconsistent* [6=11].

The *isotransition probability* that states $|i >$ are turned into the states $|f >$ is then given by

$$\hat{P}_{fi} = \hat{S}^{\dagger}_{fi} \times T \times \hat{S}_{fi} = (\tilde{S}^{\dagger}_{fi} \times \tilde{S}_{fi}) \times \hat{I}, \tag{52.19}$$

with evident *total isoprobability*

$$\hat{P}_{tot} = \Sigma_f \hat{S}^{\dagger}_{fi} \times T \times \hat{S}_{fi}. \tag{2.20}$$

2.3. Isopropagators. We now construct the isotopic image of quantum propagators, here called *isopropagators,* and then compare the resulting isoseries with a conventional expansion. We maintain the conventional assumption of the Hamiltonian being composed by two parts,

$$H = H_0 + H_1, \tag{2.21}$$

and consider the Schrödinger-Santilli isoequation (4.14) under the simplified assumption that the isounit of time is 1, namely, that time is not lifted, $\hat{t} = t$,

$$i \times \partial_t \hat{\psi}(t,r) = [H_0(r,p) + H_1(r,p)] \times T(t,r,p,\psi,\partial\psi,...) \times \hat{\psi}(t.r). \tag{2.22}$$

To avoid the venturing of superficial technically unsubstantiated perceptions (to their peril), readers should be aware that the isoscattering theory allows the consistent inclusion, for the first time to our knowledge, of *nonlinear effects,* inclusion that would imply basic inconsistencies for conventional scattering theories indicated in reference to Eq. (4.15).

Additionally, the isoscattering theory allows the verification of causality via an irreversible treatment of irreversible scattering processes via the use of the broader genomathematics [15,16], a condition also impossible for quantum scattering theories, with the understanding that the proper treatment of irreversibility requires the covering Lie-admissible genoscattering theory.

The isopropagator $\hat{G}(t,r;t'r')$ is then defined by

$$i \times \partial_t \hat{G}(t,r;t',r') - (H_0 + H_1) \times T \times \hat{G}(t,r;t',r') = \hat{\delta}(t'-t) \times \hat{\delta}^3(r'-r), \tag{2.23}$$

where $\hat{\delta}^3$ and $\hat{\delta}$ denotes the Dirac-Myung-Santilli isodelta function of hadronic mechanics (Section 3.5).

Recall that quantities with an isoscalar structure show the factorized isounit when projected on conventional spaces, such as $\hat{g} = g \times \hat{I}$. In this case we have the simplification of the isoproduct of an isofunction by another quantity $\hat{g} \hat{\times} A = g \times \hat{I} \times T \times A = g \times A$.

However, the isodelta is an *isodistribution* and, as such, it does not admit the factorization of the isounit, as it is the case for the isodifferential. Since the isodelta has has no factorization of the isounit, the same holds for the isopropagator $\hat{G}(t,r;t',r')$.

Assuming that $\hat{G}_0(t,r;t',r')$ is the isopropagator for H_0, we then have

$$\hat{G}(t,r;t',r') = \hat{G}_0(t,r;t',r') +$$

$$+\int\int dt' \times d^3r' \times \hat{G}(t,r;t_1,r_1) \times T \times H_1(t,r) \times T \times \hat{G}_0(t,r;t',r'). \quad (2.24)$$

2.4. Convergent Isoexpansions. At this point, we subject Eq. (5.24) to a power isoexpansion in terms of H_1 as in the original case [1], however, without the conventional restriction that the interacting term H_1 is small. By recalling that the formulation is on $\hat{\mathcal{H}}$, we have the expression

$$\hat{G}(t,r;t',r') = \hat{G}_0(t,r;t',r')+$$

$$+\int\int dt' \times d^3r' \times \hat{G}_0(t,r;t',r') \times T \times H_1(t_1,r_1) \times T \times \hat{G}_0(t,r;t_1r_{,1})+$$

$$+\int\int dt' \times d^3r' \times \hat{G}_0(t,r;t_1,r_1) \times T \times H_1(t_1,r_1) \times T \times \hat{G}_0(t_1,r_1;t',r')+$$

$$+\int\int dt' \times d^3r' \times \hat{G}_0(t,r;t_1,r_1) \times T \times H_1(t_1,r_1)\times$$

$$\times T \times \hat{G}_0(t_1,r_1;t_2,r_2) \times T \times H_1(t_2,r_2) \times \hat{G}_0(t_2,r_2;t',r') + ... \quad (2.25)$$

Similarly, by assuming that $\hat{\psi}_0(t,r)$ is the isoeigenfunction of H_0,

$$i \times \partial_t \hat{\psi}_0(t,r = H_o \times T \times \hat{\psi}_0(t,r), \quad (2.26)$$

we have the isoexpansion for the wave isofunction

$$\hat{\psi}(t.r) = \hat{\psi}_0(t,r)+$$

$$+\int\int dt_1 \times d^3r_1 \times \hat{G}_0(t,r;t_1,r_1) \times T \times \hat{\psi}_0(t_1.r_1) + ... \quad (2.27)$$

We now assume for $t \to -\infty$

$$\psi_0(t,z) = \psi_i(t,r)e^{i\times k\times r} \quad (2.28)$$

Then we have the expression

$$\hat{\psi}(t.r) = \psi_0(t,r)+$$

$$+\int\int dt_1 \times d^3r_1 \times \hat{G}_0(t,r;t_1,r_1) \times T \times \psi_0(t_1.r_1) + ... \quad (2.29)$$

and we have the excplicit form of the isoscattering matrix

$$\hat{S}_{fi} = \hat{I} + i \times \int dt' \times d^3r \times \psi_0(tt', r')^\dagger \times T \times H_1(t', r') \times \psi_0(t', r + ... \quad (2.30)$$

It is evident that all the above expansions are strongly convergent, not only because the isotopic eleent verifies the condition

$$|T| \ll |H_1|. \quad (2.31)$$

but also because the isopropagator no longer admits the divergence of the conventional propagator for $t = t', r = r'$, thus confirming a main objective of this paper.

2.5. Prediction of Mini-Black-Holes. Let us consider the isogravitational content of the isoscattering theory outlined in Section 2.5 of Paper II. Recall the identification of the conventional Riemannian metric, such as the Schwartzschild metric in the coordinates (θ, ϕ, r, t) , with the isometric $\hat{\eta}$ in the very structure of the isogamma matrices, Eqs. (2.35) of Paper II,

$$ds^2 = r^2(d\theta^2 + sin^2 d\theta^2 + d\phi^2) + (1 - \frac{2 \times M}{r})^{-1} \times dr^2 -$$

$$-(1 - \frac{2 \times M}{r} \times dt^2 \equiv \hat{T}_{sch} \times \eta \equiv \hat{\eta}, \quad (2.32)$$

which is represented with *gravitational isounit and isotopic element*

$$\hat{T}_{sch} = Diag.[1, 1, (1 - \frac{2 \times M}{r})^{-1}, (1 - \frac{2 \times M}{r})], \quad (2.33a)$$

$$\hat{I}_{sch} = Diag.[1, 1, (1 - \frac{2 \times M}{r}), (1 - \frac{2 \times M}{r})^{-1}]. \quad (2.33b)$$

We also recall that the conventional Dirac equations represents an *electron in vacuum,* as well known, while the isotopically lifted equation represents *the same electron when immersed within a hyperdense hadronic medium.*

It is then easy to see that *the isoscattering theory predicts the possible creation of mini black holes under sufficiently high energies,* as evident from the singularities of isometric (2.32) that now constitutes the actual metric of the scattering region.

Needless to say, there is no possibility to predict at this stage of the knowledge the threshold of *energy density* needed to trigger a mini-black-hole, and this explains the importance of lifting the scattering theory into a form admitting of a quantitative representation of the energy density of the scattering region.

Recall that, for the conventional scattering theory, the metric for the scattering region is the conventional Minkowski metric, thus without any predictive capacity for mini-black-holes. Consequently, the above prediction of the isoscattering theory and related isoline element (2.32) is sufficient to illustrate the need for serious caution and scrutiny before embarking in extremely high energy scattering experiments based on insufficient quantum methods that can at best produce controversial results.

3. Isotopies of Feynman's Diagrams without Divergences

3.1 Conventional Feynman Diagrams We are now equipped to tackle a central aspect of our research, the isotopies of Feynman's diagrams, first studied by Animalu [7] and here referred to as *iso-Feynman diagrams.* These isotopies require a re-inspection in this paper because originally conducted without the use of isomathematics, thus having the shortcomings of Refs. [6-12] of Paper I.

We begin by recalling the main features of conventional Feynman diagrams. then construct their isotopic images, and outlining the rules for computing isoscattering cross-sections from iso-Feynman diagrams.

As is well-known (see, e.g., Ref. [8]), Feynman's diagrams (also called path-integral technique in QED) comprise a representation of fundamental elementary particles (e.g., electrons e^-), their antiparticles (e.g. positrons e^+), and their annihilation into two photons, e.g.,

$$e^- + e^+ \to \gamma + \gamma, \tag{3.1}$$

in which: the electron e^- is represented by a "point" moving forward in time and the positron e^+ by another "point" moving backwards in time; the pair annihilates at the intersection of the lines joining the two point-particle trajectories as shown in Figure 1(a)); the emitted photons are represented by wiggly lines radiating away from the point of annihilation, and Figure 1(b) represents Coulomb interactions via virtual photon exchange.

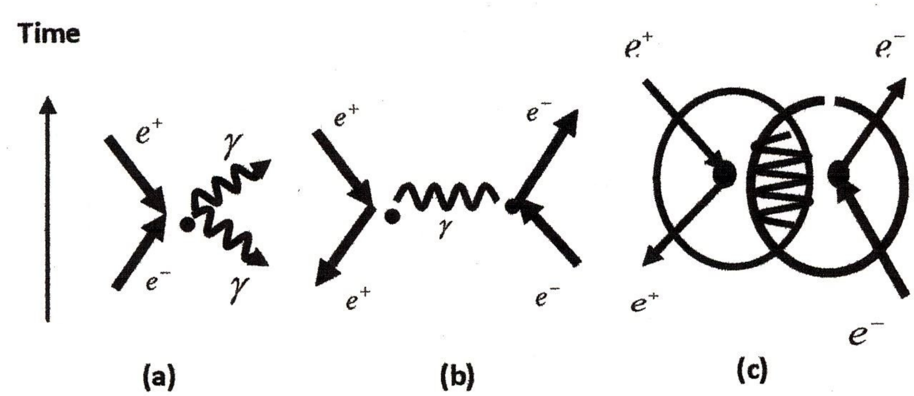


Figure 1: *Conventional Feynman's graphs for: (a)* $e^-e^+ \to \gamma + \gamma$ *annihilation into two photons; (b)* $e^- - e^+$ *Coulomb interaction via "virtual" photon exchange; and (c) nonunitary image of non-Hamiltonian interactions of extended wave packets due to overlapping in the annihilation process (3.1) or the' model of the fusion process* $e^+ + e^- \to \pi^0$

The scattering cross-sections is then calculated by following a number of *Feynman rules* designed to reproduce, at the lowest order, classical results found with conventional quantum mechanics. The full power of the method is realized, however, in calculating the radiative corrections and higher order terms and, as we shall see in this section, corrections from non-Hamiltonian forces activated in the hadronic scattering region.

3.2. The Problem of Antiparticles in Feynman's Diagrams. As indicated in paper I, Section 2, in these initial studies we shall continue to use the 20th century formalism for antiparticles. Nevertheless, the reader should be aware of the novel *isodual theory of antimatter* that has achieved the first known axiomatically consistent representation of antimatter at all levels, from Newtonian mechanics to second quantization, while being equivalent to charge conjugation in its operator form.

The new theory of antimatter has been motivated by clear insufficiencies of the classical theories of the 20th century, such as the absence of any differentiation between *neutral* matter and antimatter, problematic aspects in the classical treatment of *charged* antiparticles (because their operator image is a conventional "particle," rather than the needed charge conjugated "antiparticle"), and other problematic aspects requiring a resolution on serious scientific grounds.

The map from matter to antimatter at all levels is performed by an operation called *isoduality,* represented with the upper symbol d, essentially given by anti-Hermiticity, that must be applied, for consistency, to all possible quantities and their operations. Consequently, a central feature of the isodual theory is that *all physical quantities of antimatter are measured with negative units,* thus including negative units of time, energy, linear momentum, etc.

The above formulation of antimatter resolves the known uneasiness in Feynman's use of motions backward in time, because a motion backward in time referred to a negative unit of time is as causal as motion forward in time referred to a positive unit of time.

Additionally, this feature renders fully causal the negative energies of Dirac's equation when, again, referred to a negative unit, and provides a new interpretation of Dirac;s equation as directly representing an electron-positron pair at the level of first quantization without any need for the "hole theory" or second quantization, because the isodual theory of antimatter holds at the *classical* level, let alone for first quantization [9].

Additionally, these studies have identified a new fundamental symmetry of spacetime, called *isoselfduality* (essentially the invariance under anti-Hermiticity as verified by the imaginary unit $i \equiv i^d$), that essentially providing a deeper representation of the invariance of a particle-antiparticle pair under charge conjugation. This new symmetry is verified by the l.h.s. of Eq. (3.1), since $(e^- + e^+)^d = e^+ + e^-$, but it is violated in the r.h.s. because $\gamma^d \neq \gamma$.

This new spacetime symmetry (that could be verified by our entire universe in the event made up of matter and antimatter in equal amounts) suggests the verification of the same symmetry in the r.h.s. of Eq. (3.1). this has been achieved in Ref. [9] via the differentiation between the *conventional photon* γ emitted by matter and the *isodual photon* γ^d emitted by antimatter. The latter is predicted as being physically different than that emitted by matter, e.g., because repelled by the gravitational field of matter and having other experimentally verifiable features that may initiate, in due time, the new field of *experimental antimatter astrophysics.*

As a result,. the new isodual theory of antimatter replaces Eq. (3.1) with in the following isoselfdual invariant reaction [9]

$$e^- + e^{-d} \;\rightarrow\; \gamma + \gamma^d, \tag{3.2}$$

where $e^d \equiv e^+$ since isoduality is equivalent to charge conjugation.

It is evident that the above occurrence requires, alone, a re-inspection of the entire formulation of the Feynman's diagrams that we cannot possibly achieve in these first papers to prevent a prohibitive length and that has to be deferred to a subsequent study.

3.3 Isotopies of Feynman Diagrams. Figure 1(c) represents the proposed isotopic image of non-Hamiltonian interactions of extended wave packets due to overlapping, as predicted by hadronic mechanics since its inception, e.g., for model of theπ^o synthesis from electrons and positrons, the synthesis of neutrons from protons and electrons, or the synthesis of hadrons at large from lighter particles (see the excellent review by kadeisvili [10]).

In terms of the isounit $\hat{I}$ and isotopic element $\hat{T}$, here indicated as $\hat{I}_{st}$ and T_{st} for the carrier space of a relativistic hadronic mechanics, the correspondence between free-particle Feynman propagators in conventional relativistic theory and their isotopic image in hadronic mechanics, are can be characterized as follows:

$$[S_F(x) = (\gamma^\mu p_\mu + im)\Delta_F(x)] \longrightarrow [\hat{S}_F(\hat{x}) = (\hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}_\mu \times \hat{p}_\nu + i \times \hat{m}) \times T_{st} \times \hat{\Delta}_{\hat{F}}(\hat{x})], \tag{3.3a}$$

$$[\Delta_F(x) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-px}}{p^2 - m^2 + i\epsilon}] \longrightarrow [\hat{\Delta}_F(\hat{x}) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-p \times T_{st} \times x}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i \times \hat{\epsilon}}] \tag{3.3b}$$

with corresponding expression in momentum 4-vector space:

$$[S_F(p) = (\gamma^\mu p_\mu + m)\Delta_F(p^2) = \frac{\gamma^\mu p_\mu + m}{p^2 - m^2 + i\epsilon}] \longrightarrow$$

$$[\hat{S}_F(\hat{p}) = (\hat{\eta}_{st}^{\mu\nu} \times \hat{\gamma}_\mu \times \hat{p}_\nu + i \times \hat{m}) \times T_{st} \times \hat{\Delta}_F(\hat{p}^2) = \frac{(\hat{\eta}_{st}^{\mu\nu} \times \hat{\gamma}_\mu \times \hat{p}_\nu + i \times \hat{m}) \times T_{st}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i \times \hat{\epsilon}}] \tag{3.4}$$

3.4. Isotopicv Inclusion of Electromagnetic interactions. In the presence of an external electromagnetic field, the solution of the (regular) Dirac-Santilli isoequation takes the form

$$\hat{\Psi} = \hat{\psi}(\hat{x}) + \hat{e}\hat{\times}\hat{\int} \hat{d^4}\hat{x'}\hat{\times}\hat{S}_f(\hat{x} - \hat{x'})\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x'})\hat{\times}\hat{\Psi}(\hat{x}')$$

$$= \hat{\psi}(\hat{x}) + \hat{e}\hat{\times}\hat{\int} \hat{d}^4\hat{x'}\hat{\times}\hat{S}_f(\hat{x} - \hat{x'})\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x'})\hat{\times}\hat{\psi}(\hat{x}')$$

$$+\hat{e}^{\hat{2}}\hat{\times}\hat{\int} \hat{d}^4\hat{x'}\hat{\int} \hat{d^4}\hat{x''}\hat{\times}\hat{S}_f(\hat{x}-\hat{x'})\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x'})\hat{\times}\hat{\psi}(\hat{x}')\hat{\times}\hat{S}_f(\hat{x'}-\hat{x''})\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x''})\hat{\times}\hat{\psi}(\hat{x}'')+... \tag{3.5}$$

This leads to a formal definition of the Feynman-Animalu isopropagator either as a series:

$$\hat{S'}_f(\hat{x}, \hat{x'}) = \hat{S}_f(\hat{x} - \hat{x'}) + \hat{e}\hat{\times}\hat{\int} \hat{d}^4\hat{x''}\hat{\times}\hat{S}_f(\hat{x} - \hat{x''})\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x''})\hat{\times}\hat{S}_f(\hat{x'} - \hat{x''}) + ... \tag{3.6}$$

or as an integral equation:

$$\hat{S'}_f(\hat{x}, \hat{x'}) = \hat{S}_f(\hat{x} - \hat{x'}) + \hat{e}\hat{\times}\hat{\int} \hat{d}^4\hat{x''}\hat{\times}\hat{S}_f(\hat{x} - \hat{x''})\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x''})\hat{\times}\hat{S'}_f(\hat{x'}, \hat{x''}). \tag{3.7}$$

where $\hat{\gamma}\hat{.}\hat{A}(\hat{x'}) \equiv \hat{\eta}_{st}^{\mu\nu} \times \hat{\gamma}_\mu \times \hat{A}_\nu(\hat{x'})$ and $\hat{A}_\nu(\hat{x'})$ is the iso-electromagnetic field 4-vector potential given by the corresponding iso-gauge principle[xx].

To elaborate the basic physical concepts in terms of Feynman diagrams for electron scattering with an electromagnetic field in interior dynamical conditions, we show in Figure 4 the two characteristic differences in 1st

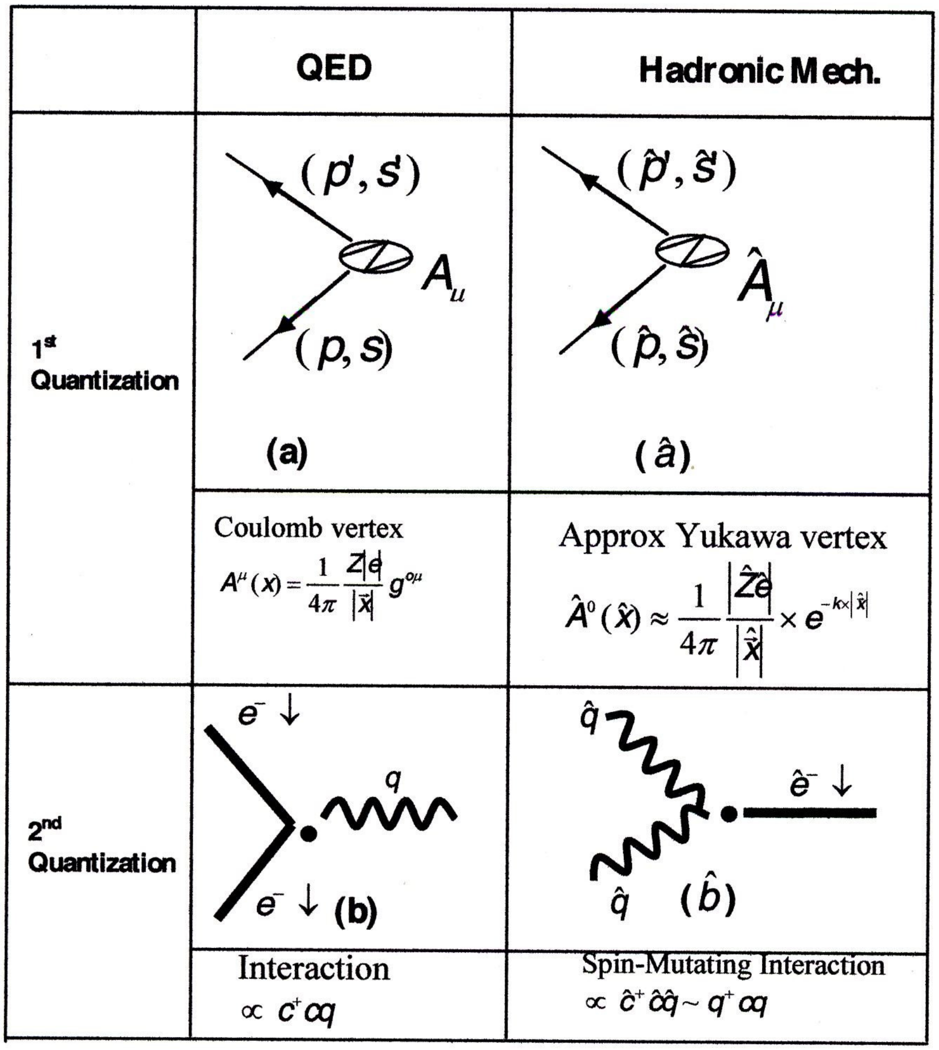


Figure 2: Correspondence between the Feynman graphs for fermion-fermion interaction vertex with boson emission in 1st and 2nd quantization schemes in QED and the isotopic images for the boson-boson interaction vertex with single fermion spin emission in HM.

	Feynman Graph&Factor in S-matrix		Iso-Feynman graph&Factor in $\hat{S}$-matrix	
External final Electron	p,s	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{mc_0}{p_0}}\bar{u}^{(S)}(p)$	$\hat{p},\hat{s}$	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{\hat{m}\times\hat{c}}{\hat{p}_0}}\times\hat{\bar{u}}^{(\hat{S})}(\hat{p})$
External final Positron	p,s	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{mc_0}{p_0}}v^{(S)}(p)$	$\hat{p},\hat{s}$	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{\hat{m}\times\hat{c}}{\hat{p}_0}}\times\hat{v}^{(\hat{S})}(\hat{p})$
External final photon	$q,\in$	$\frac{1}{(2\pi)^{3/2}}\frac{\in^{\mu}(q)}{\sqrt{2q_0}}$	$\hat{q},\hat{\in}$	$\frac{1}{(2\pi)^{3/2}}\frac{\hat{\in}^{\mu}(\hat{q})}{\sqrt{2\hat{q}_0}}$
External initial electron	p,s	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{mc_0}{p_0}}u^{(S)}(p)$	$\hat{p},\hat{s}$	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{\hat{m}\times\hat{c}}{\hat{p}_0}}\times\hat{u}^{(\hat{S})}(\hat{p})$
External initial positron	p,s	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{mc_0}{p_0}}\bar{v}^{(S)}(p)$	$\ddot{p},\bar{s}$	$\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{\hat{m}\times\hat{c}}{\hat{p}_0}}\times\hat{v}^{(\hat{S})}(\hat{p})$
External initial photon	$q,\in$	$\frac{1}{(2\pi)^{3/2}}\frac{\in^{\mu}(q)}{\sqrt{2q_0}}$	$\hat{q},\hat{\in}$	$\frac{1}{(2\pi)^{3/2}}\frac{\hat{\in}^{\mu}(\hat{q})}{\sqrt{2\hat{q}_0}}$

Figure 3: Feynman Graphs/Rules and their Isotopic Images

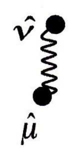
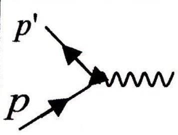
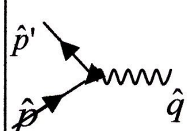
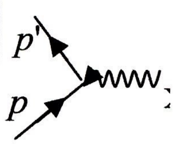
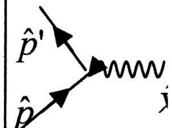
	Feynman Graph & factor in S-matrix		Iso-Feynman graph & factor in $\hat{S}$-matrix	
Internal Electron		$iS_F(p)$, given by the expression in Eq, (6.2)		$i\hat{S}_F(\hat{p})$ given by the expression in Eq.(6.2)
Internal photon		$-ig^{\mu\nu}D_F(q^2) = \frac{-ig^{\mu\nu}}{q^2+i\in}$		$-i\hat{g}^{\mu\nu}\hat{D}_F(\hat{q}^2) = \frac{-i\hat{g}^{\mu\nu}}{\hat{q}^2 + +(m_\phi\hat{c})^2 + i\in}$
Vertex		$-i\sqrt{4\pi}e\gamma^{\mu}(2\pi)^4\times$ $\gamma^{\mu}\times\delta^{(4)}(p'-p-q)$		$-i\sqrt{4\pi}e\hat{\gamma}^{\mu}(2\pi)^4\times$ $\hat{\gamma}\times\hat{\delta}^{(4)}(\hat{p}'-\hat{p}-\hat{q})$
Coulomb Vertex		$\frac{-i4\pi Z\lvert e\rvert e}{\lvert\vec{p}'-\vec{p}\rvert^2}\gamma^0\times$ $(2\pi)\delta(p_0'-p_0)$		$\frac{-i4\pi Z\lvert\hat{e}\rvert\hat{e}}{\lvert\hat{\vec{p}}'-\hat{\vec{p}}\rvert^2+(m_\phi\hat{c})^2}\times\hat{\gamma}^0\times$ $(2\pi)\hat{\delta}(\hat{p}_0'-\hat{p}_0)$

Figure 4: Continuation of Figure 5

and 2nd quantization schemes. The iso-scattering matrix is given in first quantization scheme by

$$\hat{S}_{f,i} = lim_{t\to\infty} \hat{\int} \hat{d}^3 \hat{x} \hat{\times} \psi_{\hat{p}'}^{+\hat{s}'} \hat{\times} \Psi_{\hat{p}}^{\hat{s}} \tag{3.8}$$

and$\Psi_{\hat{p}}^{\hat{s}}$ is the exact solution given as in Eq.(6.5) by

$$\Psi_{\hat{p}}^{\hat{s}}(\hat{x}) = \hat{\psi}_{\hat{p}}^{\hat{s}}(\hat{x}) + \hat{e} \hat{\times} \hat{\int} \hat{d}^4 \hat{x}' \hat{\times} \hat{S}_f(\hat{x} - \hat{x}') \hat{\times} \hat{\gamma}.\hat{A}(\hat{x}') \hat{\times} \hat{\Psi}_{\hat{p}}^{\hat{s}}(\hat{x}') \tag{3.9}$$

with the iso-normalization

$$\hat{\int} \hat{d}^3 \hat{x} \hat{\times} \hat{\psi}_{\hat{p}}^{+\hat{s}}(\hat{x}) \hat{\times} \hat{\psi}_{\hat{p}'}^{\hat{s}'}(\hat{x}) = \hat{\delta}_{\hat{s}\hat{s}'} \hat{\times} \hat{\delta}^3(\hat{\mathbf{p}} - \hat{\mathbf{p}}') \tag{3.10}$$

Note that the correspondence principle in 1st quantization scheme involves a lifting of the Coulomb vertex in QED into the approximate Yukawa vertex in hadronic mechanics, and additionally involves the lifting from Bose-Einstein to Fermi-Dirac statistics in 2nd quantization scheme, i.e., mutation of spin under sufficiently hiugh energies.

The correspondence between Feynman graphs/rules and their isotopic images for computation of contributions to the S-matrix in QED of spin-particles are summarized in the Figures 5, 6, as well as the rules:

(1) $\hat{\int} \frac{\hat{d}^4 p}{2\pi^4}$ for each internal line.

(2) Overall sign $(-)^{L+P}$ where L is the number of closed electron loops and P is the permutation of the external particles.

(3) Phase space of final particle involves $d^3p...$; and

(4) Flux of particles is $V \frac{\mathbf{V_1 - V_2}}{(2\pi)^6}$.

3.5. Concluding remarks. Lagrange, Hamilton, Jacobi and other founders of mechanics stated that nature cannot be entirely reduced to potential interactions solely representable with quantities we call today Lagrangians and Hamiltonians, for which reason they wrote their celebrated analytic equations with *external terms* representing interactions not admitting a potential energy (for historical references and comments, see Refs. [11]).

Due to the successes of purely Lagrangian and Hamiltonian theories, such as special relativity and quantum mechanics, the external terms were removed from the analytic equation in the mainstream physics of the 20th century, thus abstracting all events in the universe to potential interactions among point-like particles mving in vacuum, an abstraction that was also implemented for the scattering theories.

However, the *No Reduction Theorem* of paper I, Section 2, has confirmed the historical legacy of Lagrange, Hamilton and Jacobi, by establishing in particular that the nonlinear, nonlocal and nonpotential interactions represented with external terms necessarily originate at the ultimate level of nature, that of deep mutual penetration and overlapping of particles, thus requiring a broadening of the scattering theory for their inclusion.

At any rate, the rigorous implementation of the axioms of quantum mechanics without *ad hoc* manipulations, and their inherent point-like abstraction of particles, essentially reduce most scattering process among charged particles to Coulomb interactions. This causes uneasiness in the conventional explanation of multi-particle productions via second quantization, since the latter events are clearly visible in detectors, thus expected as being interpreted at the *semiclassical* level, let alone that of *first* quantization.

Mutatis mutandae, the admission of any particle dimension of the same order of magnitude of the scattering region, causes their deep mutual overlapping, with consequential need to include in scattering processes precisely the nonlinear, nonlocal and nonpotential interactions originated by Lagrange,. Hamilton and Jacobi, since these contacts effects are unavoidable for the dynamics of extended particles.

Predictably, the inclusion of external terms in the analytic equation has dramatic implications since it causes the loss of most conventional mathematical and physical knowledge. In fact, the brackets of the time evolution of Hamilton's equation with external terms violate the conditions to characterize an algebra, let alone causes the loss of all Lie algebras (see Refs. [6] for their analysis at the foundation of hadronic mechanics).

This occurrence mandated very laborious efforts lasted for decades to construct basically new mathematics and mechanics capable of incorporating Lagrange's and Hamilton's external terms. The sole representation of the historical external terms achieving invariance over time was their classical representation via Santilli's isounit and related isotopic element that can be presented in these concluding remarks in their original as well as ultimate meaning. This scenario explains the delay of decades prior to being in a position of addressing the scattering problem.

The reasons for a nonunitary-isounitary broadening of conventional unitary scattering theory have been indicated throughout these papers and need not be repeated here. The possible significance of the former theory over the latter can solely be established in subsequent papers over a significant period of time.

Hence, we can conclude these remarks by bringing to the attention of the curious reader that the ultimate origin of the new isoscattering theory rests indeed the historical legacy of Lagrange and Hamilton on the external terms of their celebrated analytic equations, which terms, following their popular suppression in the 20th century, have re-emerged in all their historical, mathematical and physical relevance.

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