

**NONUNITARY LIE-ISOTOPIC AND LIE-ADMISSIBLE
SCATTERING THEORIES OF HADRONIC MECHANICS, I:
Conceptual and Mathematical foundations**

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Abstract

A nonunitary extension of the conventional, unitary scattering theory has been advocated by various authors as an effective way to incorporate nonpotential effects expected in dissipative nuclear events, deep mutual penetrations of the wavepackets of scattering particles, and other events. Nevertheless, these efforts had to be abandoned because of the violation of causality, lack of conservation of probabilities, and other problems emerging under nonunitary time evolutions. We show that the reformulation of a nonunitary scattering theory permitted by the isotopic branch of hadronic mechanics and its underlying Lie-isotopic theory, here presented under the name of *isoscattering theory*, reconstructs unitarity on iso-Hilbert spaces over isofields, a property known as *isounitariness*, thus resolving said problematic aspects, while having no divergencies *an initio*, and providing a significant broadening of the quantum scattering theory, although the Lie-isotopic theory is expected as being solely applicable to *reversible scattering events*. This first paper is devoted to the conceptual and mathematical foundations of the Lie-isotopic scattering theory, including the resolution of the inconsistencies of nonunitary theories. The physical foundations, the absence of divergencies from primitive axioms, and initial comparisons of the elaboration of measured quantities (cross sections, scattering angles, etc.) via the Lie and the Lie-isotopic scattering theories for reversible scattering events are studied in subsequent papers. Deep inelastic events are *irreversible over time*, thus requiring the further Lie-admissible broadening of the formalism studied in subsequent papers. ..

Key words scattering theories, nonunitary theories, isounitary theories

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1. Introduction

As it is well known, the conventional scattering theory of quantum mechanics (see, e.g., Ref. [1] and literature quoted therein), has permitted historical advances in the 20th century particle physics. Nevertheless, physics is a discipline that will never admit final theories because all theories are a mere approximation of the complexities of nature. No matter how accurate a given theory may be perceived, its broadening for a more accurate representation of nature is only a question of time.

In fact, numerous authoritative doubts on the final character of the conventional quantum scattering theory have been expressed, such as:

1) P. A. M. Dirac (see, e.g., Ref. [2]) expressed in 1981 serious concerns for the infinities in scattering theories and indicated the need for a revised theory avoiding divergences *ab initio*, rather than via *ad hoc* procedures of unknown physical origin;

2) B. Davies (see Ref. [3] and papers quoted therein) voiced in 1981 the need to extend the scattering theory into a nonunitary form so as to incorporate imaginary potentials as used in dissipative nuclear effects and other events;

3) W. Heisenberg (see the review in Ref. [5])voiced the need for for a nonlinear extension of quantum mechanics, due to the known nonlinear character of nature;

4) Einstein, Podolsky and Rosen expressed their celebrated doubt on the "lack of completion" of quantum mechanics (see later on for comments);

5) R. M. Santilli [4,5] suggested in 1978 the construction of a nonunitary covering of quantum mechanics under the name of *hadronic mechanics* in order to lift the quantum assumption of point-like particles into a form admitting a representation of the actual extended, thus generally nonspherical and deformable character of the wavepackets and/or charge distributions of particles, a representation of contact non-Hamiltonian interactions expected in deep overlapping of scattering particles, and other effects beyond the representational capabilities of quantum mechanics.

The above initial efforts subsequently resulted as being afflicted, in their original formulation, by fundamental inconsistencies. In essence, a theory along the above lines generally requires non-Hamiltonian effects (i.e., effects not representable with a Hermitean Hamiltonian), a feature causing

the time evolution of the theory as being no longer unitary. In turn, the loss of unitarity implies: the loss of Hermiticity, thus observability, over time (an occurrence known as *Lopez lemma* [6]); the violation of causality; the lack of conservation of probabilities; the inability to predict the same numerical values under the same conditions at different times; and other basic problematic aspects known under the name of *Theorems of Catastrophic Inconsistencies of Nonunitary Theories* [6-11] (see also the review in Ref. [16a]).

A resolution of the above inconsistencies required the construction of a new mathematics, today known as *isomathematics*, based on the isotopic (i.e., axiom-preserving) lifting of the basic unit $\hbar = 1$ of quantum mechanics into the most general possible, positive-definite, integro-differential operator with an explicit dependence on any desired local quantity $\hat{I}(t, r, p, \psi, \dots) = 1/\hat{T}(t, r, p, \psi, \dots) > 0$ known as *isounit*; its inverse \hat{T} being known as the *isotopic element*. The isotopic lifting of the basic (left and right) unity $\hbar \rightarrow \hat{I}(t, r, p, \psi, \dots)$ then required corresponding compatible isotopies of the entire mathematics of quantum mechanics, including the isotopic lifting of fields, spaces, functional analysis, differential calculus, topology, geometries, algebras, groups, symmetries, representation theory, etc. [12-25].

While quantum systems are entirely represented by the sole knowledge of the Hamiltonian $H(r, p) = p^2/2m + V(r)$, the representation with hadronic mechanics of extended particles at short mutual distances requires the knowledge of *two* quantities, the usual Hamiltonian $H(r, p) = p^2/2m + V(r)$ for the representation of action-at-a-distance, potential interactions, plus the isounit $\hat{I}(t, r, p, \psi, \dots)$ for the representation of the actual size, shape and density of particles, their contact nonpotential interactions and other features beyond any hope of representation via a Hamiltonian. Note that, being an operator by assumption, the isounit does not commute with the Hamiltonian and, therefore, it is not generally a constant (although it is at times averaged into a constant).

By remembering that the unit is the basic invariant of any theory, the representation of non-Hamiltonian features and interactions via the isounit is the only form known to the authors permitting nonunitary theories to achieve the crucial invariance over time as possessed by the majestic axiomatic consistency of unitary quantum theories. The resolution of the

remaining inconsistencies of early nonunitary theories was achieved via the reconstruction of unitarity over iso-Hilbert spaces over isofields, a property known as *isounitariness* (see the review below).

Mathematical maturity was achieved with: the discovery in 1993 that the conventional axioms of numerical fields admit basically new realizations of real, complex and quaternionic numbers with arbitrary (left and right, positive-definite) units, thus resulting in basically new numbers [12]; the discovery in 1995 of the dependence by the conventional differential calculus on the assumed basic unit with the consequential emergence of new calculi [13]; the isotopies in 1998 of the fundamental $SU(2)$ spin and isospin symmetries with consequential revision of Bell's inequality and all that [14]; and other advances identified later on. The achievement of physical maturity was then consequential, and so were numerous applications and experimental verifications (see monographs [15] of 1995, updates [16] of 2008, books [17-24] and vast literature quoted therein).

In these papers, we present the reformulation of nonunitary scattering theories permitted by the isotopic branch of hadronic mechanics that is based on the *Lie-Santilli isothery* and related *isomathematics* [4,14-23]. Since all isounits assumed in these papers are Hermitean from their positive-definiteness, $\hat{I} = \hat{I}^\dagger > 0$, such a reformulation is primarily intended for *scattering processes that are reversible over time*, hereon called *isoscattering theory*, whose prefix "iso" is intended to indicate the preservation of the abstract axioms of the quantum scattering theory and merely present a *broader realization*.

Hence, the reader should be aware from these introductory lines that a main effort of these initial papers on isotopies is that of *preserving the abstract axioms of special relativity, quantum mechanics and the conventional scattering theory*, and studying their broader realization permitted by the novel mathematics. The non triviality of these isotopic liftings will then be illustrated by showing that the scattering theory: 1) Resolves the inconsistencies of nonunitary theories; 2) Avoids divergences ab initial; and 3) Broadens the representational capability of the conventional scattering theory with the representation of conventional potential interactions represented by the conventional Hamiltonian H , plus nonpotential interactions represented by the isotonic \hat{I} caused by the deep mutual penetration of

particles as customary in high energy scattering events, and the direct geometric representation of the size, shape and density of the scattering region. The issue as to whether the numerical values characterized by the scattering theory are different than those characterized by the conventional theory for the same measured quantities, can only be addressed subsequently.

It should be stressed that *the extension of the formalism to irreversible processes requires a yet broader irreversible mathematics*, known as *Lie-admissible mathematics*, which is characterized by *two non-symmetric units*, $\hat{I}^>, \hat{I}^<$ for motions forward and backward in time, respectively. In turn, such basic assumptions require a step-by-step Lie-admissible lifting of the entire isotopic formalism [4,15]. Due to its complexity, this broader formulation cannot possibly be presented in these first papers, and will be presented at some later time (see monographs [15] and the latest memoir [25]).

Hence, *the reversible scattering theory presented in these papers is a mandatory intermediate step prior to the construction of the irreversible Lie-admissible scattering theory and related new mathematics* known under the names of *scattering theory* and *mathematics* where the prefix “geno” was suggested since the original proposal of 1978 [4.5] to indicate from its Greek meaning that, this time, the axioms of special relativity, quantum mechanics and the scattering theory are abandoned in view of their notorious reversible character (see next section) in favor of new, structurally broader, irreversible axioms.

It should be indicated that in these papers we present, apparently for the first time, the axiomatic foundations of the isoscattering theory, although the main elements of the new theory have been known for some time, but often ignored by physicists dealing with scattering processes to their peril. In fact, the following basic results have been available in the scientific literature for some time:

1) *Convergent perturbation theory*. Recall that quantum mechanics is based on the well known Lie product $[A, B] = AB - BA$ between generic matrices or operators A, B , while the isotopic branch of hadronic mechanics is based on the Lie-Santilli isoproduct $[A, B] = \hat{A}\hat{T}B - B\hat{T}\hat{A}$, first presented in Ref. [4] of 1978 and then studied by the authors in various works (see the review in Refs. [15,16]), where T is the inverse of the isounit. It was

then easy to see since the original proposal to build hadronic mechanics [5] that any divergent (or weakly convergent) canonical series $A(w) = A(0) + w(AH - HA)/1! + \dots \rightarrow \infty$ can be turned into a strongly convergent form under the lifting $A(w) = A(0) + w(\hat{A}\hat{T}H - H\hat{T}\hat{A})/1! + \dots$ for all isotopic element \hat{T} sufficiently smaller (in absolute value) than w , a feature naturally verified by actual models, as we shall see. This feature was then studied by A. Jannussis and other authors (for brevity, see Chapter 11 of monograph [15b] for review and references).

2) *Conservation of probability.* As it is well known, the quantum S -matrix is unitary as a condition to preserve probabilities [1]. Hence, it was popularly believed that nonunitary theories violate the conservation of probabilities. The recovering of the conservation of probability under an isounitary reformulation of nonunitary theory was well established by 1995 [15b].

3) *Absence of divergencies.* Recall that divergencies in quantum scattering theories mainly originate from Dirac's delta function $\delta(x - x_0)$ since the latter is divergent at $x = x_0$ [1]. The absence of divergencies in the scattering theory of hadronic mechanics was identified in 1982 by Myung and Santilli [26] with the introduction of the isotopic covering of the Dirac delta function called by Nishioka [27] the *Dirac-Myung-Santilli isodelta function* and denoted $\hat{\delta}(x - x_0) = \delta[\hat{T}(x - x_0)]$ which, as one can see, removes the divergency of the delta function at $x = x_0$ under a judicious choice of the isotopic element T , as reviewed later on in Section 3.8.

4) *Nonpotential scattering theory.* The extension of the quantum scattering theory to incorporate interactions not entirely represented with a Hamiltonian, as expected in deed inelastic scattering, was sufficiently voiced in the original proposal [5], and subsequently studied by R. Mignani [28] and others. Additional more recent studies on nonpotential scattering theory have been conducted by A. K. Aringazin et al [29] (again for brevity, see Chapter 12 of monograph [15b] for reviews and additional references).

5) *Inequivalence of Hamiltonian and non-Hamiltonian data elaborations.* It is popularly believed that, since cross sections, scattering angles and other quantities are measured, the numerical values produced by data elaborations via unitary scattering theories have a final experimental character. In reality, nature is not as simple as all of us tend to believe. Santilli showed in

1989 (see the review in Chapter 12 of monograph [15b]) that the *elaboration of measured quantities via quantum and hadronic scattering theories are generally inequivalent*, thus warranting serious comparative studies. This is due to the fact known since 1978 [5] that, if the Hamiltonian H of a given scattering theory has the eigenvalue E , $H|\psi\rangle = E|\psi\rangle$, *the same Hamiltonian H has a generally different eigenvalue E' for the isoscattering theory*, $HT|\psi'\rangle = E'|\psi'\rangle$, $E' \neq E$, trivially, in view of the general lack of commutativity between H and T (see Section 3.6 for details). Irrespective of all preceding aspects, the latter occurrence, alone, warrants a reinspection of the conventional, reversible, Hamiltonian, unitary scattering theory.

The reader should be aware from these introductory lines of the existence of preliminary, yet rather vast experimental support of deviations from conventional Lie theories in virtually all quantitative sciences when dealing with the main assumption of the scattering theory, that is, extended particles and electromagnetic waves moving within physical media. Among these experimental data, we mention:

1) The need for contact non-Hamiltonian interactions to achieve an actual *attractive* force between the *identical* electrons in molecular valence couplings since, as expected to be known although rarely voiced, identical electrons repel each other according to quantum mechanics and chemistry [32];

2) Deviations from the geometry of spacetime have been, again preliminarily, yet directly measured in the experimental verification of the *isoredshift*, [31]. We are here referring to a shift toward the red of the frequency of light propagating within a transparent physical medium without any relative motion between the source, the medium and the detector, the shift being merely due to the loss of energy $E = h\nu$ by light to the medium due to inevitable interactions, with consequential evident reduction of frequency.

3) The elaboration of numerous particle physics experiments dealing with the hyperdense interior of hadrons, when elaborated without *ad hoc* parameters or arbitrary functions of unknown physical origin, show the clear presence of non-Hamiltonian effects [16d]. This is typically the case of the two-point amplitude of the Bose-Einstein correlation whose quantum fit of experimental data requires *four* arbitrary parameters (the so-called “chaoticity parameters”), while vacuum expectation values admit at

best *two* parameters. These effects can be fully representable via a four-dimensional isounit of which the three space components represent the actual, very elongated shape of the proton-anti proton fireball, and the fourth component represents its density, in remarkable agreement with experimental data [*loc. cit.*].

In any case, as part of the ongoing efforts to appraise the experimental claims based on the conventional scattering theory, a rather significant experimental effort is under way at this writing (Spring 2010) to repeat within physical media the historical experiments that have established the validity of special relativity, all done in vacuum, as well known. This significant experimental effort on the disciplines actually holding within physical media at large, and within the scattering region in particular, combined with the theoretical efforts herein considered, will eventually provide the necessary elements for the resolution of fundamental open issues in scattering experiments, of course, in due time.

Above all, the reader should keep in mind that special relativity and quantum mechanics are reversible theories, thus having manifest limitation for all energy releasing processes, due to their strict irreversibility. Therefore, the conception, quantitative treatment and experimental verification of much needed new clean energies, such as the novel *Intermediate Controlled Nuclear Fusions* (ICNF) [37] are crucially dependent on the covering formulations treated in these papers. Their possible confirmation in particle accelerators via the covering isoscattering theory would then acquire a primary significance for the resolution of alarming environmental problems.

For additional historical data and a comprehensive literature in the field, interested colleagues may inspect Refs. [16], particularly the *General Bibliography of Hadronic Mechanics* in Volume [16a].

In closing these introductory lines, we should recall that the conventional scattering theory achieved maturity only following decades of collegial studies presented in a large number of refereed publications. Consequently, it is hoped the reader is not expecting a final resolution of the scattering problem in these initial papers, but merely the *initiation* of the studies leading to a future collegial resolution following a predictable large number of additional papers.

2. Basic Physical Assumptions

2.1. Exterior and Interior Dynamical Problems. Until the earlier part of the 20th century, there was a clear distinction between (see Refs. [30] for technical characterizations):

- 1) *exterior dynamical problems*, referred to systems of point-particles and electromagnetic waves propagating in empty space; and
- 2) *interior dynamical problems*, referred to extended particles and electromagnetic waves propagating within physical media.

As a historical note, we recall that Schwarzschild wrote *two* papers, the first on the exterior gravitational problem containing his celebrated solution, and a second, virtually ignored paper on the interior gravitational problem (for review and references, see Ref. [15a]).

The primary difference between exterior and interior problems is that the former verify the integrability conditions for the existence of a Lagrangian or a Hamiltonian (the so-called *conditions of variational selfadjointness*), while the latter systems (called *variationally nonselfadjoint*) violate these conditions due to the presence of contact, nonconservative and nonpotential interactions, thus not being representable with Lagrangian or Hamiltonian mechanics [30].

With the advent of special and general relativities, the distinction between exterior and interior dynamical problems was eliminated via the reduction of interior problems to a finite number of point-particles that, as such, move in vacuum, thus recovering the conditions of exterior problems.

2.2. No Reduction Theorems. In the second half of the 20th century, it became known that interior dynamical problems cannot be consistently reduced to exterior problems, an occurrence known under the name of *No reduction Theorems*, such as:

NO REDUCTION THEOREM 2.1 [5,25]: A macroscopic system in nonconservative and irreversible conditions cannot be consistently reduced to a finite collection of point-like particles all in conservative and reversible conditions and, vice versa, the latter system cannot consistently reconstruct the former under the correspondence or other principles.

A number of additional No Reduction Theorems were also proved based

on the violation of thermodynamical laws due to the evident loss of entropy when passing from a real physical system to an ideal collection of point-particles moving in empty space all in conservative conditions, as necessary to verify special relativity, quantum mechanics and the conventional scattering theory.

An additional popular belief disproved by the No Reduction Theorems is that total conservation laws for an isolated system are solely verified by a system of particles in conservative conditions. In fact, it was proved in Ref. [30b] that, since they have no potential energy, nonconservative forces are in essence exchange forces, as a result of which they cancel each other when the system is isolated, resulting in the full verification of the conventional total conservation laws.



Figure 1: *A suggestive view from NASA of a spaceship during reentry in our atmosphere. Recent No reduction Theorems have established that the nonlinear, nonlocal-integral and nonpotential non-Hamiltonian forces experienced by the spaceship originate at the ultimate elementary level of nature, thus being also present in the interior of the scattering region at high energies.*

Yet another popular belief dispelled by the above No reduction Theorems is that the nonlinear, nonlocal-integral and nonpotential forces of our macroscopic environment "disappear" in the reduction of an interior system to its elementary constituents. As a matter of fact, No Reduction

Theorem 2.1 establishes that *the nonlinear, nonlocal and nonpotential forces experienced, for instance, by a spaceship during reentry in our atmosphere originate at the most primitive possible level, that of elementary particles,* and are evidently due to the interactions of the electron orbitals of the peripheral atoms constituting the spaceship with the electron orbitals of the resistive medium (see Figure 1).

The particular type of non-Hamiltonian interactions here referred to deals with the deep overlapping of the wave packets of and/or the charge distribution of particles and are referred to as *nonlocal-integral interactions* (or merely nonlocal for brevity) in the sense that they occur over a surface or volume integral. As such, the nonlocal interactions at the basis of the scattering theory cannot be reduced, by conception, to a finite set of isolated points.

Note that we are including nonlocal interactions experienced by electrons, namely, by particles with a notorious *point-like charge*. Nevertheless, electrons do not have a “point-like wavepacket,” thus experiencing indeed nonlinear, nonlocal and nonpotential interactions when in conditions of deep mutual penetration, as it is the case for valence electron coupling in molecular structures [32].

The studies reported in Refs. [30] have also established that the time evolution of systems with nonlinear, nonlocal and nonpotential interaction are necessarily noncanonical at the classical level and nonunitary at the operator level. We are now minimally equipped to formulate the following:

ASSUMPTION 2.1: The scattering region is an interior dynamical system, thus characterized by a nonlinear (in the wavefunction), nonlocal (integral) and nonpotential (nonunitary) time evolution.

Note that the No Reduction Theorems prohibit the exact reduction of the scattering region to a finite set of isolated points, which is considered a mere first approximation of a rather complex reality. The same theorems identify the evident need for covering formulations. Note finally that the No Reduction Theorems are not bypassed by the reduction of the scattering particles to point-like quarks, since elementary constituents with a point-like wavepacket do not exist.

2.3. Insufficiencies of the Lorentz-Poincaré Symmetry. The breaking of the *Lorentz-Poincaré (LP) symmetry* for interior dynamical problems at large, and particularly for the interior of the scattering region, is rather plausible and should be studied seriously because no scattering theory can claim final results until the basic spacetime symmetry is established beyond scientific doubt. Among a number of symmetry breaking aspects, we quote [15,16]:

1) The axiomatic foundations of the Lorentz-Poincaré symmetry requires the equivalence of all inertial reference frames. This feature is certainly valid in empty space, but it is unresolved for the interior of the scattering region because of the impossibility of even defining inertial reference frames in *interior* conditions. Inertial reference frames are indeed used in quantum scattering theories, but they constitute an *exterior* treatment, thus reducing an interior to an exterior problem. Additionally, in vacuum there is no known experimental way to detect a privileged reference frame, as well known (Michelson-Morley experiment). By contrast, the sole reference frame that can be consistently defined for the scattering region is the privileged reference frame at rest with the interior region itself, since other frames would require motion of a hypothetical observer within a hyperdense medium.

2) The Lorentz-Poincaré symmetry is exactly valid for *Keplerian systems*, that is, systems of point-particles moving in vacuum around a heavier particle known as the *Keplerian nucleus*. By contrast, *the scattering region has no Keplerian nucleus*. This aspect alone may cause a breaking of the Lorentz-Poincaré symmetry.

3) There are serious reasons to expect that the historical experiments that have established the validity of special relativity in vacuum are invalid in interior conditions [31]. For instance, it is easy to see that., in the event the known Fizeau experiment is repeated entirely underwater, there are contributions for the travel of light in water outside the traditional pipes with opposite water velocities that violate Lorentz-transformations. By contrast, the repetition of the Michelson-Morley experiment under complete underwater conditions is expected to retain the original result, this time confirming the constancy of the speed of light with respect to the privileged reference frame at rest with the water, by therefore no longer confirming the Lorentz

symmetry. Needless to say, a problem of such a fundamental character cannot be resolved in a few sentences one way or another, and requires the systematic repetition in interior conditions of all historical experiments that have established the validity of the Lorentz-Poincaré symmetry in vacuum [31].

The reader should be aware that a rather vast effort has been conducted over decades for the construction of a covering spacetime symmetry applicable to interior problems at large, and the scattering region in particular. These efforts required first the construction of the covering *Lie-Santilli isothory* [4,15,16,18-34] capable of reducing nonlinear, nonlocal and non-canonical (or nonunitary) interior problems to equivalent *isolinear, isolocal and isocanonical (or isounitary) forms* (see next section for details). Only following the achievement of the Lie-Santilli isothory, the efforts could be concentrated in the construction of a covering of the Lorentz-Poincaré symmetry applicable to interior conditions [33-43] which is known today as the *Lorentz-Poincaré-Santilli (LPS) isosymmetry*. We reach in this way the following:

ASSUMPTION 2.2: The scattering region is characterized by the Lorentz-Poincaré-Santilli isosymmetry.

The noninitiated reader should know that, by conception and construction, *the LPS isosymmetry is locally isomorphic to the conventional LP symmetry*. This feature may have deep implications for the scattering problem because, in the final analysis, it may imply that the data elaboration of existing high energy experiments with the conventional and the isotopic scattering theory yields the same numerical value. Rather than being a drawback, if established by future collegial works, this possible outcome alone warrants this study, e.g., because it would established the validity of special relativity for interior conditions nowadays considered inapplicable.

In any case, as shown in paper III of this series, even assuming that the elaboration of past experiments via the conventional and isotopic scattering theory yields the same numerical results, the broader representational capabilities of the isotopic theory are beyond doubt, thus offering the possible prediction and representation of scattering events beyond the capability of the conventional theory; It is only hoped the reader does not expect the

final resolutions of these complex issues in these initial papers.

2.4. Insufficiencies of Quantum Mechanics. Following the historical successes of quantum mechanics for the structure of the hydrogen atoms and numerous other systems, quantum mechanics has been applied to all possible particle conditions existing in the universe, thus including interior conditions, as typically occurring in the scattering region as well as in the structure of hadrons, nuclei and stars.

Despite the achievement of historical results, serious doubts have emerged in regard to the *exact* character of quantum mechanics for *interior* problems, such as [15,16]:

1) Quantum mechanics has permitted the achievement of a numerically *exact* representation from first principles of *all* experimental data of exterior dynamical problems, By contrast, when passing to interior problems, quantum mechanics has only permitted an *approximate* representation of experimental data, an occurrence that, per se, is a direct indication of the merely approximate character of quantum mechanics for interior conditions. For instance, quantum mechanics has provided an exact representation of the structure of the hydrogen atoms, while it misses 2% of the binding energy of the hydrogen molecule from unadulterated quantum principles [32]. In nuclear physics, quantum mechanics misses an exact representation of the simplest possible nucleus, the deutneriom, since there are insufficiencies in the representation of its spin, magnetic moment, stability and other features, with dramatic insufficiencies for heavy nuclei such as the zirconium [16].

2) The No Reduction Theorems establish that the nonlinear, nonlocal and nonpotential character of our macroscopic systems originate at the ultimate level of elementary particles, thus requiring a covering of quantum mechanics. As an example, the approximate character of quantum mechanics for the hydrogen molecule originates from the conditions of deep mutual penetration of the wavepackets of the two valence electrons by characterizing interactions dramatically beyond a possible representation by quantum mechanics. Similar occurrences hold for other interior problems.

3) The representation of interior conditions via quantum mechanics is generally done with the use of completely arbitrary parameters or functions

of unknown physical origin that are fitted from the data, and quantum mechanics is then claimed as being exactly valid. As an example, an exact representation of the binding energy of the hydrogen molecule is achieved via the so-called "screened Coulomb potentials," that is, the multiplication of the Coulomb potential by an arbitrary function such as $V(r) = f(r)e^2/r$, and then the fitting of the arbitrary function ($f(r)$) from the experimental data. However, it is known that "screened Coulomb potentials" do not admit quantized levels and, therefore, the very name "quantum chemistry" becomes questionable [32]. In particle physics, the use of *ad hoc* parameters and functions for interior conditions has reached at time paradoxical characters. For instance, the experimental data of the two-points function of the Bose-Einstein correlation are fitted via the use of *four* arbitrary parameters (called the "chaoticity parameters") and then the claim that relativistic quantum mechanics is exact. However, the quantum axioms for the expectation value of a two-dimensional Hermitean operator may admit, under debatable assumptions, a maximum of *two* arbitrary parameters, the use of four parameters being excluded by the very axioms of quantum mechanics [16a]. A deeper inspection has shown that the missing two parameters must originate from *off-diagonal elements* in the vacuum expectation values thus casting shadow on the consistent representation of observables.

In view of the above and numerous other insufficiencies [16a,24], a vast effort has been conducted by numerous scientists over decades for the construction of a nonlinear, nonlocal and nonpotential covering of classical and quantum mechanics known under the name of *hadronic mechanics* with the following main results [15,16,32,33]:

A) The construction of the so-called *iso-, geno-, and hyper mathematics* for the representation of variationally nonselfadjoint interior systems of *matter* that are single-valued reversible, single valued irreversible, and multi-valued irreversible, respectively, and their isoduals for *antimatter*, these new mathematics being characterized by different generalized units as outlined in Section 3;

B) The construction of corresponding new classical mechanics, known as *iso-, geno- and hyper-Lagrangian or Hamiltonian mechanics for matter*, and their isoduals for *antimatter*, achieving the representation of interior dynamical systems via an action principle, as outlined in paper II; and

C) The *isotopic, genotopic and hyperstructural branches of hadronic mechanics* for the operator representation of the above identified interior systems of *matter*, and their isoduals for *antimatter*, possessing progressively increasing complexity and methodological needs, as also outlined in paper II.

The above formulations have indeed allowed exact representations of interior problems from unadulterated first axioms, such as an exact representation of the binding energy and other features of the hydrogen molecule from first principles without arbitrary functions [32], an exact representation of the experimental data of the Bose-Einstein correlation from first principles without arbitrary parameters, and other interior problems in classical physics, particle physics, nuclear physics, superconductivity, chemistry, astrophysics and cosmology (see Vol. [16d] and Chapter 5 of Ref. [24] for a review).

We are now equipped to formulate the following:

ASSUMPTION 2.3: Quantum mechanics is assumed as being exactly valid everywhere in the exterior of the scattering region, while the covering hadronic mechanics is assumed as being exactly valid in the interior region.

The smooth transition from the interior (hadronic mechanics) to the exterior (quantum mechanics) is simply achieved via realizations of the generalized unit of the type

$$\text{Lim}_{r>1fm} \hat{I}(t, t, r, p, \psi, \dots) = \hbar. \quad (2.1)$$

As we shall see in paper II, the above condition is quite naturally verified by all meaningful realizations of the generalized unit.

In view of the general inequivalence of $\hat{I}(t, r, p, \psi, \dots)$ and I , the evident lack of general commutativity of $\hat{I}(t, r, p, \psi, \dots)$ and $H(r, p)$, and other aspects, the isoscattering theory requires a reinspection of the data elaboration of experimental data achieved with the conventional scattering theory to ascertain whether said data elaborations persist under nonlinear, nonlocal and nonpotential internal effects, or the final numerical values themselves need a revision.

2.5. Restrictions for Irreversibility and Antimatter. Recall that

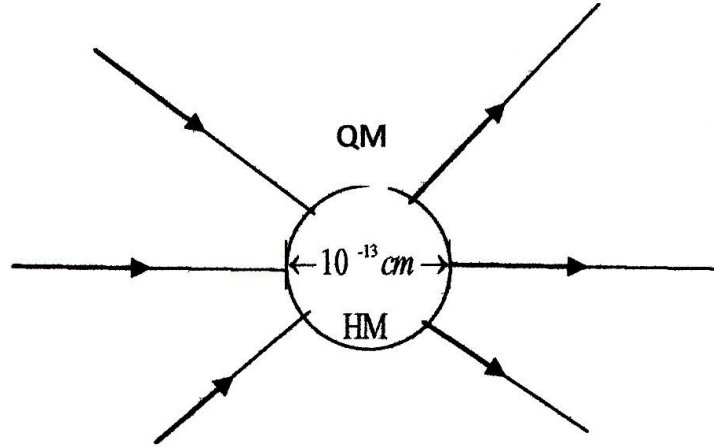


Figure 2: A schematic view of the main assumptions of these papers, the validity of conventional quantum mechanics everywhere in exterior conditions, and the validity of the covering hadronic mechanics for interior conditions.

the formalism of the covering scattering theory includes that of quantum mechanics, plus three covering formalisms of hadronic mechanics with progressively increasing complexity, and all their isoduals for antimatter. To avoid the initiation of the study with excessive complexities, in these three papers we shall restrict our formulations to *isomathematics and isomechanics*, resulting in the suggested name of *isoscattering theory*, where the reader should keep in mind that the prefix "iso" indicates the preservation of the axioms of the conventional theory, and merely the use of *broader realizations*.

This restriction implies that, by conception and construction, *the isoscattering theory does not generally represent irreversible processes*, unless under certain conditions, as we shall see, such as isounits that are Hermitean but time noninvariant

$$\hat{I}(t, r, p, \psi, \dots) \neq \hat{I}(-t, r, p, \psi, \dots). \quad (2.2)$$

In other words, we shall essentially study scattering processes in the way they are treated by quantum mechanics, without a quantitative representa-

tion of their irreversibility, and shall address the latter issue in a subsequent paper based on *Lie-admissible genomathematics and genomechanics* [25]. In any case, the construction of the Lie-isotopic isoscattering theory is a recommendable pre-requisite for the much broader *Lie-admissible irreversible genoscattering theory*.

Additionally, *the isoscattering theory of these first papers does not include antiparticles* also to avoid excessive complexities at start up. This additional restriction is due to recent advances in antimatter that have achieved full scientific democracy between matter and antimatter at all levels of study, from Newtonian mechanics to second quantization, thus ending the scientific imbalance of the 20th century of treating antimatter at the sole quantum or quantum field theoretical levels [33].

These advances have been stimulated by E. C. G. Stueckelberg conception of *antimatter with a negative time*, but the achievement of consistency required the use of a conjugation of all physical and mathematical quantities, thus leading to *negative time, energy, and other physical quantities referred to corresponding negative units*, that are as causal as conventional positive time, energy and other physical quantities referred to corresponding positive units.

The treatment of this new setting required the construction of the new *isodual mathematics* that is anti-isomorphic to conventional mathematics in all its parts and operations. In turn, these advances have identified a new symmetry, called *isoselfduality*, essentially given by invariance under anti-Hermiticity trivially verified by the imaginary unit $i = -i^\dagger$, but less trivially verified by the Dirac equation and related gamma matrices (see Ref. [33] for details)

$$\gamma_\mu \equiv -\gamma_\mu^\dagger, \quad (2.3)$$

and other cases.

Physically, isoselfduality has emerged as representing systems of particles and their anti[particles], thus permitting a new interpretation of the Dirac equation as providing a direct *quantum* representation of an electron and its antiparticle (the positron) without any need for the "hole theory," since the the isodual theory applies at the *classical*, let alone purely quantum level, where it reaches equivalence with the conventional charge conjugation [33].

Therefore, the inclusion of antiparticles in our study of scattering processes requires a reinspection of the very structure of the *conventional* Feynman's diagrams so as to achieve a full democracy of treatment between particles and antiparticles, thus suggesting a separate treatment to avoid excessive complexities at start up.

It should be noted, as we shall see in paper II., that *the invariance under isoselfduality is generally violated by quantum scattering treatments inclusive of particles and antiparticles*. This occurrence alone mandates a reinspection *ab initio* of scattering theories in general, let alone when including particles and antiparticles.

In these papers, we shall use the terms "quantum mathematics," "quantum scattering theory," etc. to denote aspects pertaining to quantum mechanics and use the terms "hadronic mathematics," "hadronic scattering theory," etc. to denote their corresponding coverings as characterized by hadronic mechanics.

A number of divergent terminologies exist in the literature of this paper as compared to that of the quantum scattering theory. For instance, the term "potential" is used in the literature of hadronic mechanics as a synonym of "Hamiltonian" or, more technically, referring to the verification of all integrability conditions for the existence of a Hamiltonian [30], while systems of that class are not necessarily called "potential" in the quantum literature.

This is the case for the interaction term $H_1 = J * A$ that is generally considered as being of nonpotential character in the quantum literature, while it verifies the conditions of variational self-adjointness (see monographs [30]), thus being of a true potential for the hadronic literature, as confirmed in any case by the fact that said interaction term is fully "Hamiltonian" and *additive* to the kinetic term and other potentials, e. g., $H = H_0 + H_1$.

By comparison, the terms "nonpotential" is used in the hadronic literature to stress the impossibility of representing the novel "nonpotential" interactions with a Hamiltonian, technically referring to the *violation of the conditions of variational self-adjointness in the frame of the experimenter*, thus requiring new vistas.

3. Elements of Isomathematics.

3.1. Introduction. As indicated in Sections 1 and 2, numerous aspects warrant the broadening of the scattering theory to incorporate non-Hamiltonian effects, that is, effects that cannot be represented via the conventional Hamiltonian. Any meaningful broadening of the conventional scattering theory requires the existing from the class of unitary equivalence of quantum mechanics. However, the ensuring nonunitary theories are afflicted by a litany of problems known under the name of *Theorems of Catastrophic Inconsistencies of Nonunitary Theories* [6-12]. Consequently, the central objective of this section is to *identify an equivalent formulation of nonunitary theories resolving the inconsistency problems.*

Following decades of research, the solution of the above problem required the construction by various authors of a *new mathematics*, known as *isomathematics*, originally proposed by Santilli [4] in 1978, subsequently studied by the same author in disparate works, as well as by numerous pure and applied mathematicians, including (in chronological order of contributions) R. M. Santilli, S. Okubo, H. C. Myung, M. L. Tomber, Gr. T. Tsagas, D. S. Sourlas, J. V. Kadeisvili, A. K. Aringazin, A. Kirhukin, R. H. Ohemke, G. F. Wene, G. M. Benkart, J. M. Osborn, D. J. Britten, J. Lohmus, E. Paal, L. Sorgsepp, D. B. Lin, J. V. Voujouklis, P. Broadbridge, P. R. Chernoeff, J. Sniatycku, S. Guiasu, E. Prugovecki, A. A. Sagle, C.-X. Jiang, R. M. Falcon Ganfornina, J. Nunez Valdes, A. Davvaz, and others (see the comprehensive bibliography at the end of Ref. [16a]). To illustrate the complexity of the problems to be addressed, following the original proposal of 1978, initial mathematical maturity was solely achieved in memoir [13] of 1996, thus warranting this review and specialization to the scattering region so as to avoid possible insidious misinterpretations.

For the benefit of experimentalists we indicated that, as a result of the above efforts, the new mathematics can be constructed via the systematic application of *axiom-preserving liftings*, called *isotopies*, of the *totality* of the mathematics of quantum mechanics, including all its operators and all its operations, thus including the isotopic lifting of numbers, functional analysis, differential calculus, geometries, topologies, Lie theory, symmetries, etc. [13,15,16]. As we shall see in paper II, said isotopies can be very easily constructed via the application of nonunitary transforms to the totality

of the formalism of the conventional scattering theory, thus being indeed accessible to experimentalists.

The physical needs for isomathematics have been indicated in Sections 1 and 2, and consists in the necessity for a representation of non-Hamiltonian scattering effects in a form that is *invariant over time* so as to admit the same numerical predictions under the same conditions at different times. Following the study of all possible alternatives, the latter condition required the representation of non-Hamiltonian scattering effects with an axiom-preserving generalization of the trivial (positive-definite) unit of quantum mechanics $\hbar = 1$ into the most general possible (positive-definite as a condition to characterize an isotopy), integro-differential operator \hat{I} . Since the unit is the fundamental (left and right) invariant of any theory, whether conventional or generalized, the representation of non-Hamiltonian effects via the isounit has indeed achieved the desired time invariant representation.

However, the assumption of a generalized unit has requested the compatible reconstruction of the entire mathematics used in quantum mechanics with no exception known to the authors. In fact, the sole elaboration of the isoscattering theory, e.g., with conventional trigonometric functions, activates the Theorems of Catastrophic Inconsistencies because it would be the same as elaborating the conventional scattering theory, e.g., with isotrigonometric functions.

Since no formulation of isomathematics specialized intended for scattering problems has been presented to date, it is important to outline it in this first paper for minimal self-sufficiency of the presentation, as well as to minimize possible insidious misinterpretations that may be caused by insufficient technical knowledge of the field. In this section we shall outline the rudiments of isomathematics for a positive-definite but otherwise arbitrary isounit \hat{I} and show the resolution of the inconsistency problem under isotopies.

We should also indicate the distinction between *deformations* and *isotopies*. The former are alterations of conventional quantum formulations defined over conventional fields, thus being catastrophically inconsistent on mathematical and physical grounds (see Refs. [6-11] for brevity), while the latter can be characterized as deformations defined over isofields, thus avoiding the inconsistency theorems.

Note that isofields were introduced in 1993 [12]. Consequently, the contemporary formulation of deformations coincide with previously proposed isotopies, as it is the case for the *isotopies of the Lorentz symmetry* first proposed by Santilli in 1983 [34], at that time, over conventional fields, and subsequently reintroduced identically, even in the symbols and terms, as deformations, unfortunately, without the quotation of the original derivation [34]. Similar occurrence hold for other deformations (see Ref. [15a] for brevity).

In these papers, conventional terms, such as numbers, spaces, etc. are referred to conventional notions of quantum mathematics. The corresponding notions of hadronic mathematics are indicated isonumbers, isospaces, etc. We regret a perhaps excessive use of the prefix "iso," but it appears recommendable in a first presentation of *applied mathematics* to prevent insidious inconsistencies.

Within the context of *pure mathematics*, we shall show that *both the conventional and the isotopic mathematics can be presented with the same symbols and operations*, since they coincide at the abstract level by conception and construction. However, the latter formulation requires, in any case, an in depth knowledge of the isotopic realization of conventional abstract axioms, thus warranting again the use of the prefix "iso" in this first presentation, with the understanding that pure mathematicians may subsequently achieve the necessary mathematical rigor.

It is at times indicated that, due to the above abstract identity, isomathematics is trivial, a view perhaps correct. but only following its discovery. However, the implications solely permitted by isomathematics. such as the extension of Lie's theory, the Lorentz-Poincaré symmetry and Einstein's axioms for the treatment of nonlinear, nonlocal and non-Hamiltonian systems, are far from being trivial.

3.2. Isounits, Isoproducts and Isofields. As indicated earlier, isomathematics is based on the following isotopic, thus axiom-preserving lifting of the trivial unit into the most general possible positive-definite integro-differential operator

$$\hbar = 1 > 0 \rightarrow \hat{I}(t, r, p, E, \xi, \omega, \psi, \partial\psi, \dots) = 1/\hat{T}(t, r, p, E, \xi, \omega, \psi, \partial\psi, \dots) > 0. \quad (3.1)$$

first introduced in 1978 [4,5] and known as *Santilli isounit*, while \hat{T} is known as the *isotopic element*. We shall use the notation T when the isotopic element is projected on quantum spaces, but keep the notation \hat{I} to avoid confusion with I .

The isotopic lifting of the (multiplicative) unit evidently requires a corresponding compatible lifting of *all* multiplications between arbitrary quantities A, B , from the simple associative form used in quantum mechanics, herein denoted $AB = A \times B$, to the new form first introduced by Santilli in Ref. [4] of 1978

$$AB = A \times B \rightarrow A \hat{\times} B = A \times T \times B, \quad (3.2)$$

which is also isotopic, because verifying the associativity law of the original product. It is easy to see that, under lifting (3.2), \hat{I} is indeed the correct left and right unit of the theory, $\hat{I} \hat{\times} \hat{A} = A \hat{\times} \hat{I} = A$ for all elements A of the set considered.

Fundamental assumptions (3.1) and (3.2) have permitted the isotopic lifting of numerical fields $F(a, \times, I)$, such as the field of real numbers $R(n, \times, I)$, complex numbers $C(c, \times, I)$ and quaternions $Q(q, \times, I)$ into the *Santilli isofields* $\hat{F}(\hat{a}, \hat{\times}, \hat{I})$ [12], consisting of the original numbers $a = n, c, q$ lifted into the form of *Santilli isonumbers* $\hat{n} = n \times \hat{I}$ equipped with isounit (3.1) and isoproduct (3.2), $\hat{n}_1 \hat{\times} \hat{n}_2 = (n_1 \times n_2) \times \hat{I}$, as well as with the *conventional* sum $\hat{n}_1 \hat{+} \hat{n}_2 = \hat{n}_1 + \hat{n}_2$ and related conventional additive unit 0, $\hat{n} + \hat{0} = \hat{0} + \hat{n} = \hat{n}$, i.e., $\hat{0} = 0 \times \hat{I} \equiv 0$.

To avoid inconsistencies, it should be stressed that *all* operations with numbers have to be lifted in an isotopic form we cannot possibly review here (see [15]). We merely mention for use in the isoscattering theory the *isodivision* given by $\hat{\int} = / \times \hat{I}$ so that we have simplifications in isomultiplications of the type $(\hat{a}/\hat{b}) \hat{\times} (\hat{c}/\hat{d}) = [(a/b) \times (c/d) \times \hat{I}]$.

Also, and very importantly, conventional numbers expressing numerical values of physical quantities such as coordinates $r.$, momenta p , energy E , etc. have no meaning for isomathematics and must be lifted into the isotopic form $\hat{r} = r \times \hat{I}$, $\hat{p} = p \times \hat{I}$, $\hat{E} = E \times \hat{I}$, etc. as a necessary condition to be elements of a Santilli isofield, that is, to be *isoscalars*.

Readers should, however, be reassured that conventional numbers, as needed for experiments, are indeed recovered by the isoscattering theo-

ries. As an example, the (right, modular, associative) eigenvalue expression $E \times |\psi\rangle$ becomes for isomathematics $\hat{E} \hat{\times} |\hat{\psi}\rangle$ that can be simplified in the form $E \times \hat{I} \times \hat{T} \times |\hat{\psi}\rangle = E \times |\hat{\psi}\rangle$, thus recovering the conventional real number E needed for measurements.

It should be indicated that *isofields are isomorphic to ordinary fields, by conception and construction*, a property necessary for the consistent application of the isoscattering theory to experimental measurements. In fact, Santilli merely provided in Ref. [12] a *broader realization* of the conventional field axioms. The nontriviality of the realization is indicated by the fact that *the isounit of a Santilli isofield $\hat{F}(\hat{a}, \hat{\times}, \hat{I})$ is generally outside the original field $F(a, \times, I)$* . In this case, $\hat{F}(\hat{a}, \hat{\times}, \hat{I})$ are called *isofields of the first type*. When $\hat{I} \in F$, we have *isofields of the second type*.

Despite the simplicity of the isonumber theory, readers should be warned against predictable perceptions of triviality because, for instance, under the assumption of the isounit $\hat{I} = 3$, thus dealing with isofields of the second type, we have " 2×3 " = 18 and the *number 4 becomes a prime number*.

For in depth knowledge of Santilli isofield theory and its intriguing implications, interested readers are suggested to study the original paper [12], Ref. [15a] and Jiang's monograph [22].

3.3. Isofunctional Analysis. Any elaboration of the isoscattering theory with conventional functions, such as sine, cosine, exponential, etc. leads to inconsistencies [6-11,15]. Even though not clearly indicated in the mathematical literature, all functions crucially depend on the assumed basic unit and multiplication. Therefore, liftings (3.1) and (3.2) have required the laborious reconstruction of functional analysis into a form compatible with the basic axioms of isomathematics.

Studies on the *isofunctional analysis* were initiated by Santilli [4] and continued by Myung and Santilli [26], Kadeisvili [21], Nishioka [27] Aringazin [29] and others (see the general bibliography of Ref. [16a] for a comprehensive listing). A presentation of isofunctional analysis sufficient for the isoscattering theory is available in monograph [15a]. For completeness we recall the following notions:

3.3.1) *Isopowers*,

$$\hat{a}^{\hat{n}} = \hat{a} \hat{\times} \hat{a} \hat{\times} \dots \hat{a} = (a^n) \times \hat{I}, \quad (3.3)$$

for which $\hat{I}^{\hat{n}} = \hat{I}$;

3.3.2) *Isoexponentiation*,

$$\hat{e}^a = \hat{I} + a/\hat{1}! + a \hat{\times} a/\hat{2}! + \dots = (e^{a \times T}) \times \hat{I} = \hat{I} \times (e^{T \times a}), \quad (3.4)$$

where one should note the emergence of the integro-differential quantity T in the *exponent*;

3.3.3) *Isologarithm*,

$$\hat{\log}_{\hat{e}} \hat{a} = \hat{I} \times \log_{\hat{e}} a, \quad (3.5)$$

which expression is indeed the inverse of the isoexponentiation, as one can verify, as well as yields a correct isonumber for result;

3.3.4) *Isotrigonometric functions* (for isosphericval coordinates see later on Section 3.8),

$$\hat{\sin} \hat{\theta} = T_{\theta} \times \sin(\theta \times \hat{I}_{\theta}), \quad (3.6a)$$

$$\hat{\cos} \hat{\phi} = T_{\phi} \times \cos(\phi \times \hat{I}_{\phi}), \quad (3.6b)$$

where evidently the isounits for angles are generally different than those for space.

Note that the use of conventional angles would have no sense for the isoscattering theory because all numbers must be isonumbers for consistency. We shall identify later on specific realizations of the various isounits.

A rather intriguing and unexpected feature of isotopies is that of preserving on isospaces over isofields the *numerical values* of the quantities prior to lifting. This feature has been crucial for the reconstruction of the exact light cone and special relativity on isospace over isofield when light becomes a local variables, thus requiring in conventional spaces deformed light cones.

According to this feature, the isoscattering theory is expected to preserve the numerical value of the angles θ and ϕ as measured in experiments. However, the preservation is for the new isoangles $\hat{\theta}$ and $\hat{\phi}$. Consequently, the correct identification is

$$\theta = \hat{\theta} = \theta' \times \hat{I}_{\theta}, \quad \phi = \hat{\phi} = \phi' \times \hat{I}_{\phi}. \quad (3.7)$$

The above rules indicate the expected differences in the elaboration of experiments via the scattering and isoscattering theories.

3.3.5) *Isomatrices*, given by conventional matrices whose elements are isoscalars, such as for the diagonal case

$$\hat{M} = \text{Diag.}(\hat{a}_1, \hat{a}_2, \dots, \hat{a}_n), \quad (3.8)$$

where $\hat{a}_k = a_k \times \hat{I}$;

3.3.6) *Isodeterminant*,

$$\hat{D}et\hat{M} = [\text{Det}(\hat{M} \times \hat{T}) \times \hat{I}, \quad (3.9)$$

where one should note that $\hat{M} \times \hat{T}$ is an ordinary matrix. Hence, the value of the isodeterminant is indeed an isonumber.

3.3.7) *Isotrace*,

$$\hat{T}r\hat{M} = \text{Tr}(\hat{M} \times \hat{T}) \times \hat{I}, \quad (3.10)$$

etc. It should be stressed that the above elements of isofunctional analysis are merely introductory and a study of at least Chapter 6 of monograph [15a] is necessary for a serious knowledge of the isoscattering theory.

3.4. Isodifferential Calculus. It was believed for centuries that the differential calculus is independent of the assumed basic unit, since the latter was traditionally given by the trivial number 1.

Santilli [13] has disproved this belief by showing that the differential calculus can be dependent on the assumed unit, by formulating the *isodifferential calculus* with basic *isodifferential*, for instance, of an isocoordinate \hat{r}

$$\hat{d}\hat{r} = \hat{d}[r \times \hat{I}(r, \dots)] = \hat{T} \times d[r \times \hat{I}(r, \dots)], \quad (3.11)$$

that does indeed coincide with the conventional differential for all isounits independent from r , $\hat{d}\hat{r} \equiv dr$, while yielding structural differences for all cases relevant for the isoscattering theory, namely, when the isounit depends on the local coordinates. In the latter case we have

$$\hat{d}\hat{r} = T \times d[r \times \hat{I}(r, \dots)] = dr + r \times T \times d\hat{I}(r, \dots). \quad (3.12)$$

The compatible formulation of the *isoderivative* is then given by

$$\frac{\hat{\partial}}{\hat{\partial}\hat{r}} = \hat{I} \times \frac{\partial}{\partial r}. \quad (3.13)$$

The *isointegral* is defined as the inverse of the isodifferential and can be written for simplicity

$$\int \hat{d}\hat{r} = \int d\hat{r}, \quad \int \hat{d}\hat{r} \hat{\times} \hat{f}(\hat{r}) = \int d\hat{r} \times f(\hat{r}), \quad (3.14)$$

where we have used the isofunction $\hat{f}(\hat{r}) = \hat{I} \times f(\hat{r})$.

Note that, as formulated above for simplicity, isodifferentiation and isointegration yield ordinary scalars and *not* isoscalars, a feature assumed later on in Section 2.3 of paper II to reach a formulation accessible to experimentalists.

It should be indicated that the use of the conventional differential calculus leads to catastrophic mathematical and [physical inconsistencies particularly in the dynamical equations [6-11], thus mandating the use of the covering isodifferential calculus. Consequently, the sole functional differences between the conventional and isodifferential calculus are sufficient to warrant a reinspection of the quantum scattering theory.

As an illustration, the realizations of the isounit of primary physical relevance are based on exponentials, e.g., $\hat{I} = \hat{M} \hat{\times} \exp[f(r, \dots)]$, where \hat{M} is a matrix or operator not dependent on r . In this case, the isounit and the isotopic element disappear from the projection of the isodifferential in our space. This results in significant deviations between conventional and isotopic differentials, e.g., $dr \neq \hat{d}\hat{r} = dr \times (1 + r \times \partial f \partial r)$ thus providing additional expectations of possible numerical differences in the final elaboration of the same experiment with the conventional and the isotopic scattering theory.

3.5 Iso-Hilbert Spaces. The fundamental representation space of hadronic mechanics is a *new realization* of the abstract axioms of the conventional Hilbert space \mathcal{H} over the field of complex numbers \mathcal{C} , first proposed by Santilli [5] in 1978, then studied by Myung and Santilli [26] and other authors (see the review in Ref. [15a] and quoted references), today known as *iso-Hilbert space* or *Hilbert-Myung-Santilli isospaces*, and denoted $\hat{\mathcal{H}}$ over the isofield $\hat{\mathcal{C}}$. The new space is characterized by *isostates* $|\hat{\psi}\rangle$ with *isoinner product*, and related *isonormalization*,

$$\langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle \times \hat{I} = \langle \hat{\psi} | \times \hat{T} \times | \hat{\psi} \rangle \times \hat{I} \in \hat{\mathcal{C}}, \quad (3.15a)$$

$$\langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle \times \hat{I} = \hat{I}, \quad (3.15b)$$

isoexpectation values of an iso-Hermitean operator $\hat{Q} = \hat{Q}^\dagger$

$$\langle \hat{\psi} | \hat{\times} \hat{Q} \hat{\times} | \hat{\psi} \rangle \times \hat{I} = \langle \hat{\psi} | \times T \times Q \times T \times | \hat{\psi} \rangle \times \hat{I}, \quad (3.16)$$

isounit under isonormalization (3.16)

$$\langle \hat{\psi} | \hat{\times} \hat{I} \hat{\times} | \hat{\psi} \rangle \times \hat{I} = \langle \hat{\psi} | \times \hat{T} \times \hat{T}^{-1} \times | \hat{\psi} \rangle \times \hat{I} = \hat{I}, \quad (3.17a)$$

$$\hat{I} \hat{\times} | \hat{\psi} \rangle \equiv | \hat{\psi} \rangle; \quad (3.17b)$$

isoeigenvalue equation for iso-Hermitean operators

$$\hat{H} \hat{\times} | \hat{\psi} \rangle = H \times T \times | \hat{\psi} \rangle = \hat{E} \hat{\times} | \hat{\psi} \rangle = E \times | \hat{\psi} \rangle, \quad \hat{E} \in \hat{\mathcal{R}}, \quad E \in \mathcal{R}; \quad (3.18)$$

and additional properties we cannot possibly review here. We limit ourselves to quote the following main properties (see Ref. [15a] for details):

3.5.1) Hilbert-Santilli isospaces are isomorphic to conventional Hilbert spaces by conception and construction, as illustrated by the fact that the isoinner product (3.15) is still inner from the positive-definite character of the isounit. This property is crucial to ensure the covering character of hadronic over quantum mechanics, as well as the existence of a unique and unambiguous interconnecting maps indicated below.

3.5.2) Operators that are Hermitean on \mathcal{H} over \mathcal{C} are also iso-Hermitean, namely, they remain hermitean under lifting to the Hilbert-Santilli isospace over the isofield of isocomplex numbers, and we shall often write $\hat{Q} = \hat{Q}^\dagger = \hat{Q}^\dagger$. Therefore, all quantities that are observable for quantum mechanics remain observable for hadronic mechanics, although the opposite is not generally true because of the existence of Hermitean operators representing irreversible process that are well defined for hadronic mechanics but cannot be even formulated for quantum mechanics due to its simpler structure.

3.5.3) The conventional Hilbert space admits a new symmetry discovered by Santilli [13,14] called *isoscalar symmetry*, given by a rescaling of the unit under which the conventional inner product is invariant,

$$\begin{aligned} & \langle \psi | \times | \psi \rangle \times I \equiv \\ \equiv & \langle \psi | \times w^{-1} \times | \psi \rangle \times (w \times I) = \langle \psi | \hat{\times} | \psi \rangle \times \hat{I}, \quad w \in \mathcal{C}. \end{aligned} \quad (3.19)$$

Evidently, the property persists for the Hilbert-Santilli isospace and we have

$$\begin{aligned} & \langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle \times \hat{I} = \langle \hat{\psi} | \times T \times | \hat{\psi} \rangle \times \hat{I} \equiv \\ \equiv & \langle \hat{\psi} | \times (w^{-1} \times T) \times | \hat{\psi} \rangle \times (w \times \hat{I}) = \langle \hat{\psi} | \times T' \times | \hat{\psi} \rangle \times \hat{I}'. \end{aligned} \quad (3.20)$$

The lack of discovery of symmetry (3.19) for over one century should not be surprising, because the new symmetry required the prior discovery of new numbers, those with arbitrary units [12]. In fact, isosymmetry (3.19) requires the reformulation of numbers as isonumbers $\hat{n} = n \times 1$.

Despite its apparent triviality, the discovery of isosymmetry (3.19) has permitted the achievement of a new grand unification of gravitational and electroweak interactions essentially based on the embedding of gravitation where nobody looked for, in the *unit* of electroweak theories. The new grand unification includes the first known axiomatically correct inclusion of antimatter in grand unified theories also nobody cared for since gravitation on a Riemannian space cannot represent neutral antimatter. gthis suggests the use of the isodual theory of antimatter to achieve a grand unifications with a degree of democracy between matter and antimatter (see papers [44-46] for original words and monograph [33] for comprehensive treatment).

3.6. Isolinearity, Isolocality and Isounitariness. We are now equipped to introduce the following important:

DEFINITION 5.6.1: ISOLINEARITY.

Operators that are nonlinear on \mathcal{H} over \mathcal{C} (that is, *nonlinear in the wavefunction*) can be *identically* rewritten in a form that is linear on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$, a property called *isolinearity*. The reformulation is simply done by embedding all nonlinear terms in the isounit, In fact. hadronic mechanics was proposed [5] to reformulate complex nonlinear models, e.g., $H(r, p, \psi) \times |\psi \rangle = E \times |\psi \rangle$, into an identical isolinear form $H_o(r, p) \times T(r, p, \psi) \times |\psi \rangle = E \times |\psi \rangle$, $H = H_o \times T$. Despite its simplicity, the reformulation is not trivial because the conventional nonlinear formulation generally violates the superposition principle, thus being generally inapplicable to composite systems, while the isotopic formulation verifies the superposition principle on isospace over isofield, thus allowing consistent studies of nonlinear composite systems.

DEFINITION 5.6.2: ISOLOCALITY.

Operators that are nonlocal on \mathcal{H} over \mathcal{C} , e.g., of nonlocal-integral type, can be *identically* reformulated in a form on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$ that is local-differential everywhere except at the isounit, a property known as *isolocality*. Again, the reformulation is done via the embedding of all nonlocal terms in the isounit. It should be noted that the technical understanding of isolocality requires a technical knowledge of the *isotopology* of hadronic mechanics initiated by the mathematicians Gr. Tsagas and D. S. Surlas [34] (see also monograph [19]) and completed by the mathematicians M. Falcon Ganfornina and J. Nunez Valdes [35] (see also monograph [23]).

DEFINITION 5.6.3: ISOUNITARITY.

All operators U that are nonunitary on \mathcal{H} over \mathcal{C} can be *identically* reformulated in a form verifying unitarity on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$, a property called *isounitariness*. The reformulation is done via the simple identity

$$U \times U^\dagger \neq I, \quad U = \hat{U} \times \hat{I}^{1/2}, \quad (3.21)$$

under which we have the *isounitariness law*

$$\hat{U} \hat{\times} \hat{U}^\dagger = \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}. \quad (3.22)$$

This is the property indicated in Section 1 that assures nonunitary S -matrices to preserve probabilities under the condition that the matrices are not treated via the mathematics of quantum mechanics.

3.7. Resolution of the Inconsistency Theorems. We are now sufficiently equipped to show the resolution of the Theorems of catastrophic Inconsistencies of Nonunitary Theories [6-11]:

INVARIANCE OF THE BASIC UNIT.

The units of the conventional scattering theory characterize a geometrization of basic units of measurements. For instance, the unit of the three-dimensional Euclidean space is a geometrization of the units of length along each axis, e.g., $I = \text{Diag}(1\text{cm}, 1\text{cm}, 1\text{cm})$. When expressed in dimensionless form, the unit acquires the familiar version $I = \text{Diag.}(1, 1, 1)$. All quantum units are invariant under unitary time evolution, $I \rightarrow U \times I \times U^\dagger \equiv I$, thus confirming the majestic axiomatic consistency of quantum mechanics.

However, these units are no longer invariant under nonunitary time evolutions $U \times U^\dagger \neq I$ because, in this case, we can have maps of the type $I \rightarrow U \times I \times U^\dagger = \text{Diag.}(231\text{cm}, 1.36\text{cm}, 0.3\text{cm}) \neq I$. This illustrates a first inconsistency of nonunitary scattering theories, the lack of preservation over time of the basic units of measurements, with consequential lack of consistent applicability of nonunitary theories to experiments.

A central features of the isoscattering theory is the invariance of the isounits \hat{I} under the isounitary time evolution of the theory. In fact, under isounitariness law (22) we have, for instance, the invariance $\hat{I} = \text{Diag.}(231\text{cm}, -1.36\text{cm}, 0.3\text{cm}) \rightarrow \hat{U} \hat{\times} \hat{I} \hat{\times} \hat{U}^\dagger \equiv \hat{I}$, thus resolving the first inconsistency of nonunitary theories

INVARIANCE OF OBSERVABLES.

Another central property of quantum mechanics is that, when a quantity is observable at a given time, it remains observable at all subsequent times. This feature is verified by the preservation of Hermiticity under unitary time evolutions and provides another illustration of the majestic consistency of quantum mechanics.

When the time evolution is no longer unitary, Hermiticity is not necessarily preserved over time (this is the *Lopez lemma* [6] indicated in Section 1). In fact, the transformed eigenvalue equation for an operator H that is Hermitean at the initial time t_o under nonunitary transforms $U = U(t)$ is given by $H \times |\psi \rangle \rightarrow (U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi \rangle \times U^\dagger)$. Consequently, the initial Hermiticity of H is not necessarily preserved over time due to the lack of general commutativity of $U \times H \times U^\dagger$ and $(U \times U^\dagger)^{-1}$.

It is an instructive exercise for the reader interested in acquiring a knowledge of the isoscattering theory to prove that iso-Hermiticity is indeed preserved under isounitary transformations [6,12].

INVARIANCE OF NUMERICAL PREDICTIONS.

Yet another important feature of the axiomatic consistency of quantum mechanics is that, if a Hermitean operator H has the eigenvalue E (e.g., $E = 5\text{MeV}$) at the initial time, $H \times |\psi \rangle = E \times |\psi \rangle$, said eigenvalue is preserved at all times, as shown by the transformation $(U \times H \times U^\dagger) \times (U \times |\psi \rangle \times U^\dagger) = H' \times |\psi' \rangle = U \times (E \times |\psi \rangle \times U^\dagger) = E \times |\psi' \rangle$.

Under nonunitary time evolutions, the eigenvalue at the initial time of

a Hermitean operator is not necessarily preserved at subsequent times, as shown by the transformation $(U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi\rangle \times U^\dagger) = H' \times T \times |\psi'\rangle = U \times (E' \times |\psi\rangle \times U^\dagger) = E' \times |\psi'\rangle, T = (U \times U^\dagger)^{-1}$, where the lack of preservation of the eigenvalue, $E' \neq E$, follows from the fact that $|\psi'\rangle$ is now the eigenstate of the new operator $H' \times T$. It is an instructive exercise for interested readers to verify that isoeigenvalues are indeed preserved under isounitary time evolutions. The resolution of the remaining inconsistencies then follows [16a,16c].

The property important for the isoscattering theory is that *eigenvalues of Hermitean operators are numerically altered under nonunitary-isounitary lifting*. This occurrence suggests, alone, a reinspection of the conventional scattering theory because the possible presence of nonunitary effects in deep inelastic scattering could imply numerical results different than those currently assumed.

3.8. Delta Isofunction. As well known, Dirac's delta function, here expressed for the case of a one-dimensional coordinate r ,

$$\delta(r - r_0) = \frac{1}{2\pi} \times \int_{-\infty}^{+\infty} e^{i \times k \times (r - r_0)} \times dk, \quad (3.23)$$

is divergent at $r = r_0$, by therefore constituting the origin of divergences in quantum scattering theories [1].

In view of the above, Myung and Santilli [26] introduced in 1982 the isotopic lifting of Dirac's delta function, today known as the *Dirac-Myung-Santilli delta isofunction*, or *DMS isodelta* for brevity (see, e.g., Nishioka [27]) that, by using the notions of isointegral (3.14), and isoexponentiation (3.4), can be written

$$\hat{\delta}(r - r_0) = \frac{\hat{I}}{2\pi} \hat{\int}_{-\infty}^{+\infty} \hat{e}^{i \times k \times (r - r_0)} \hat{\times} \hat{dk}, = \frac{1}{2\pi} \times \int_{-\infty}^{+\infty} e^{i \times k \times T \times (r - r_0)} \times dk, \quad (3.24a)$$

$$T = \sum_{k=1}^n c_k \times (r - r_0)^k, \quad c_k \in \mathcal{C}. \quad (3.24b)$$

where we write the isotopic element T without a "hat" to denote its formulation on conventional spaces, and example (3.24b) an illustration of the possible removal of the singularity at r_0 . We then have the evident property

$$\hat{\delta}(r - r_0) = \delta[T \times (r - r_0)]. \quad (3.25)$$

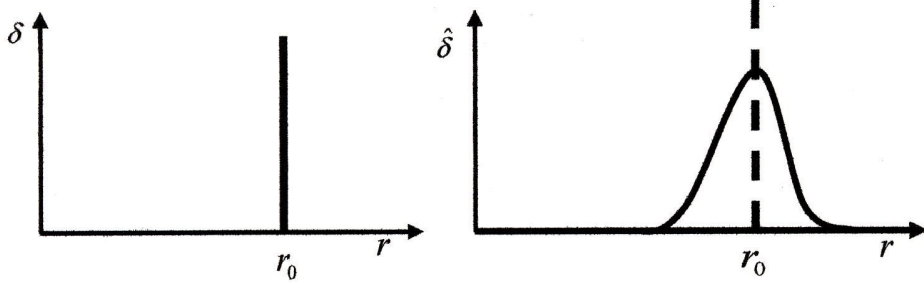


Figure 3: A schematic view in the left of the conventional delta function $\delta(r-r_0)$ illustrating its divergent character at r_0 , and a schematic view in the right of the Dirac-Myung-Santilli isodelta function of hadronic mechanics $\hat{\delta}(r-r_0) = \delta[T(r-r_0)]$, illustrating the absence of the above divergency, a feature allowing the removal of divergencies in the isoscattering theory from primitive axioms.

As illustrated in Figure 2, under the appropriate realization of the isotopic element T , the DMS isodelta eliminates the divergent character of the delta function, thus setting up the foundations for a new scattering theory without divergencies *ab initio*, which is a main objective of this paper.

Note that for (3+1)-dimensional spaces each coordinate is multiplied by its isotopic element (see next section). For numerous additional properties, e.g., the derivation of the isodelta via isotransforms, the reader is encouraged to study monograph [xx]. Section 6.4.

3.9. Isospherical Isocoordinates. An additional mathematical notion needed for the elaboration of the isoscattering theory is given by the *isospherical coordinates* [15] here considered for in the Euclid-Santilli isospace with isounit

$$\hat{I} = \text{Diag.}(1/b_1^2, 1/b_2^2, 1/b_3^2) = 1/T > 0, \quad (3.26)$$

isometric

$$\hat{\delta} = T \times \delta = \text{Diag.}(b_1^2, b_2^2, b_3^2), \quad (3.27)$$

and isoinvariant

$$\hat{r}^2 = x^2 \times b_1^2 + y^2 \times b_2^2 + z^2 \times b_3^2. \quad (3.28)$$

Under the assumption of the conventional orientation of the angles θ, ϕ with respect to the z -axis, we have the isounits

$$\hat{I}_\theta = b_3, \quad \hat{I}_\phi = b_1 \times b_2, \quad (3.29)$$

and the projection of the isocoordinates on the conventional Euclidean space

$$x = r \times b_1^{-1} \times \sin(\theta \times b_3) \times \cos(\phi \times b_1 \times b_2), \quad (3.30)$$

$$y = r \times b_2^{-1} \times \sin(\theta \times b_3) \times \sin(\psi \times b_1 \times b_2), \quad (3.31)$$

$$z = r \times b_3^{-1} \times \cos(\theta \times b_3). \quad (3.32)$$

Understanding of the isoscattering theory requires the knowledge that Eq. (3.28) represents an ellipsoid only when considered on the Euclidean space with respect to the trivial unit 1, because the same invariant represents the perfect sphere in Euclid-Santilli isospace over isofield called *isosphere*. This is due to the fact that the k -axis is mutated by the quantity $1 \rightarrow b_k^2$, but the corresponding unit is mutated by the *inverse* amount $1 \rightarrow b_k^{-2}$, thus preserving the perfect sphericity.

Similarly, the rotational symmetry has been popularly believed in the 20th century as being broken for ellipsoid (3.41), while in reality such a breaking is due to insufficient treatment since the rotational symmetry is reconstructed as exact on Euclid-Santilli isospaces, as shown by the perfect sphericity of the isosphere.

4. Concluding Remarks

In this paper, we have suggested the re-inspection of the conventional, potential, unitary scattering theory of relativistic quantum mechanics on grounds of the following aspects:

1) The apparent inapplicability (rather than violation) of the Lorentz-Poincaré symmetry and special relativity within physical media at large, and within the scattering region in particular, due to: difficulties for a consistent formulation of their axioms (impossibility of introducing inertial systems

within a medium, the sole existence of the privileged reference frame at rest with the medium, difficulties in the verification of all axioms within a transparent medium, and others); deviations predicted in the repetition within physical media of the historical experimental verifications of special relativity in vacuum (repetition of Fizeau experiment entirely within water, and others); difficulties in reaching a *numerical* (rather than solely conceptual) representation of *all* data for *all* frequencies in the *entire* reduction to photons of electromagnetic waves propagating within physical media (inability to reach a numerical representation of the angle of refraction and the index of refraction; impossibility for a large number of photons to pass through a large number of nuclei as needed to maintain the main nonscattered part of a light beam along a straight line; difficulties in reducing to photons electromagnetic waves with one meter wavelength propagating within physical media; impossibility of representing with photons traveling in vacuum seemingly unavoidable superluminal causal speeds within physical media; etc.); and other insufficiencies;

2) Impossibility of reducing to photons traveling in vacuum the electromagnetic phenomena within the scattering region due to its hyperdense character, thus implying the locally varying speed $C = c/n$, suggesting a return to the Maxwellian interpretation of light and photon wavepackets as transversal electromagnetic waves propagating in the ether as a universal substratum without conflict with special relativity in vacuum (due to our impossibility of identifying a privileged system at rest with the ether), and consequential relevance of the *Lorentz problem*, namely. the achievement of the universal symmetry for all locally varying speeds of light $C = c/n$;

3) The strict *reversibility* over time of the Lorentz-Poincaré symmetry and special relativity compared to the strict *irreversibility* over time of high energy inelastic scattering processes, with ensuing difficulties for rigorous verifications of causality and other laws, and the need for covering theories as irreversible as the scattering process being represented;

4) The need advocated by Heisenberg for a covering of quantum mechanics which is nonlinear in the wavefunction and other quantities due to the expected nonlinearity of high energy scattering processes, compared to the linear character of quantum mechanics, the breaking of the superposition principle for Hamiltonians dependent on wavefunctions and consequential

inapplicability of nonlinear quantum models to composite scattering processes;

5) Einstein-Podolsky-Rosen historical doubts on the final character of quantum mechanics; Dirac's call for a reformulation of the scattering theory that is convergent *ab initio* so as to avoid the achievement of numerical results in high energy scattering experiments via *ad hoc* procedures to achieve mathematical convergence of unknown physical origin or content; and other authoritative doubts;

6) The *No Reduction Theorems* preventing a consistent reduction of macroscopic irreversible systems to a finite set of particles all in nice conservative conditions, with consequential impossibility of reducing highly irreversible scattering processes to point-like quantum particles verifying the rotational and Lorentz symmetries, thus identifying the origin of irreversibility in the total mutual penetrations of the wave[packets and/or charge distributions of particles in the scattering region, essentially as occurring for macroscopic irreversible systems (such as a spaceship during reentry in atmosphere);

7) The unavoidable non-Hamiltonian and, therefore, nonunitary character of the contact effects due to total mutual penetration of extended wavepackets and/or charge distributions of particles in the scattering region, with consequential exiting from the class of unitary equivalence of quantum mechanics;

8) The numerical alteration of the eigenvalues of scattering operators under non-Hamiltonian, thus nonunitary internal effects, with consequential possible lack of final character of the data elaboration of measured quantities (cross sections, scattering angles, etc.) via unitary scattering theory;

9) The recent discovery of the invariance of particle-antiparticle systems under the new symmetry called *isoselfduality* (invariance under anti-Hermiticity) that is verified by the Dirac equation, resulting in its direct representation of an electron and a positron without need for the "hole theory," said new invariance not being generally verified by the scattering amplitude for particle-antiparticle processes;

and other aspects all concurring in a return to the old need for a nonunitary covering of the conventional unitary scattering theory.

In this paper, we have then recalled the *Theorems of Catastrophic Math-*

ematical and Physical Inconsistencies of Noncanonical and Nonunitary Theories, implying the lack of invariance over time of the units of measurements, the lack of conservation over time of observable, the general inability to predict the same numerical results under the same conditions at different times, and others serious insufficiencies.

In order to avoid excessive complexities at start up, in this and the following papers we have restricted our analysis to *reversible* scattering processes *without antiparticles*. We have then, apparently for the first time, specialized to the scattering region the new mathematics known as *isomathematics*, that has been specifically built over decades of efforts by various authors to bypass said inconsistency theorems; we have outlined their resolution; and restricted the study to a time reversal invariant formulation of the nonunitary scattering theory without antiparticles under the name of *isoscattering theory*.

In this paper, we have also indicated the possibility that, in the final analysis, the *elaboration* via the scattering and isoscattering theories of the same *measured* data may lead to the same numerical results. This possibility should not be excluded due to the indicated preservation under isotopies of both Einsteinian and quantum axioms and, in case confirmed, would be quite valuable because it would confirm the broadening of their applicability under nonlinear, nonlocal and nonunitary internal effects.

However, even under the assumption that the data elaboration of *past* experiments are the same for the conventional; and the isotopic scattering theories, the latter is expected to admit the representation of events precluded to the former, such as the synthesis of neutrons from protons and electrons as occurring in stars, or the synthesis of hadrons at large from lighter particles that, as we shall see in Paper IV of this series, can best be treated via a nonunitary-isounitary theory due to the need for a *negative binding energy* under which the Schrödinger equation no longer admits physically meaningful solutions [36].

Above all, the reader is suggested to keep in mind that the ultimate aim of all studies herein considered is the conception, quantitative treatment and experimental verification of much needed new clean energies, such as the novel *Intermediate Controlled Nuclear Fusions* (ICNF) [37], due to their strictly irreversible, as well as nonlinear, nonlocal and non-Hamiltonian

character.

The proof of the convergence from primitive axioms without *ad hoc* manipulations, the comparison of the data elaboration of measured quantities via the scattering and isoscattering theory is done in subsequent papers. Similarly, the inclusion of antiparticles and the extension to irreversible scattering processes requires additional new mathematics (known as *isodual mathematics* and *Lie-admissible genomathematics*, respectively), thus requiring separate studies.

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