

# Santilli's Isoprime Theory

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**Abstract:** We study Santilli's isomathematics for the generalization of modern mathematics via the isomultiplication  $a \hat{\times} a = ab\hat{T}$  and isodivision  $a \hat{\div} b = \frac{a}{b}\hat{I}$ , where the new multiplicative unit  $\hat{I} \neq 1$  is called Santilli isounit,  $\hat{T}\hat{I} = 1$ , and  $\hat{T}$  is the inverse of the isounit, while keeping unchanged addition and subtraction, , In this paper, we introduce the isoaddition  $a \hat{+} b = a + b + \hat{0}$  and the isosubtraction  $a \hat{-} b = a - b - \hat{0}$  where the additive unit  $\hat{0} \neq 0$  is called isozero, and we study Santilli isomathematics formulated with the four isooperations ( $\hat{+}, \hat{-}, \hat{\times}, \hat{\div}$ ). We introduce, apparently for the first time, Santilli's isoprime theory of the first kind and Santilli's isoprime theory of the second kind. We also provide an example to illustrate the novel isoprime isonumbers.

**Keywords:** Isoprimes, Isomultiplication, Isodivision, Isoaddition, Isosubtraction

## 1. Introduction

Santilli [1] suggests the isomathematics based on the generalization of the multiplication  $\times$  division  $\div$  and multiplicative unit 1 in modern mathematics. It is epoch-making discovery. From modern mathematics we establish the foundations of Santilli's isomathematics and Santilli's new isomathematics. We establish Santilli's isoprime theory of both first and second kind and isoprime theory in Santilli's new isomathematics.

### 1.1. Division and Multiplication in Modern Mathematics

Suppose that

$$a \div a = a^0 = 1, \tag{1}$$

where 1 is called multiplicative unit, 0 exponential zero.

From (1) we define division  $\div$  and multiplication  $\times$

$$a \div b = \frac{a}{b}, b \neq 0, a \times b = ab, \tag{2}$$

$$a = a \times (a \div a) = a \times a^0 = a \tag{3}$$

We study multiplicative unit 1

$$a \times 1 = a, a \div 1 = a, 1 \div a = 1/a \tag{4}$$

$$(+1)^n = 1, (+1)^{a/b} = 1, (-1)^n = (-1)^n, (-1)^{a/b} = (-1)^{a/b} \tag{5}$$

The addition  $+$ , subtraction  $-$ , multiplication  $\times$  and division  $\div$  are four arithmetic operations in modern mathematics which are foundations of modern mathematics. We generalize modern mathematics to establish the foundations of Santilli's isomathematics.

### 1.2. Isodivision and Isomultiplication in Santilli's Isomathematics

We define the isodivision  $\hat{\div}$  and isomultiplication  $\hat{\times}$  [1-5] which are generalization of division  $\div$  and multiplication  $\times$  in modern mathematics.

$$a \hat{\div} a = a^{\bar{0}} = \hat{I} \neq 1, \bar{0} \neq 0, \tag{6}$$

where  $\hat{I}$  is called isounit which is generalization of multiplicative unit 1,  $\bar{0}$  exponential isozero which is generalization of exponential zero.

We have

$$a \hat{\div} b = \hat{I} \frac{a}{b}, b \neq 0, a \hat{\times} b = a\hat{T}b, \tag{7}$$

Suppose that

$$a = a \hat{\times} (a \hat{\div} a) = a \hat{\times} a^{\hat{0}} = a \hat{T} \hat{I} = a. \quad (8)$$

From (8) we have

$$\hat{T} \hat{I} = 1 \quad (9)$$

where  $\hat{T}$  is called inverse of isounit  $\hat{I}$ .

We conjectured [1-5] that (9) is true. Now we prove (9). We study isounit  $\hat{I}$

$$a \hat{\times} \hat{I} = a, a \hat{\div} \hat{I} = a, \hat{I} \hat{\div} a = a^{-\hat{I}} = \hat{I}^2 / a, \quad (10)$$

$$(+\hat{I})^{\hat{n}} = \hat{I}, (+\hat{I})^{\frac{\hat{a}}{b}} = \hat{I}, (-\hat{I})^{\hat{n}} = (-1)^{\hat{n}} \hat{I}, (-\hat{I})^{\frac{\hat{a}}{b}} = (-1)^{\frac{\hat{a}}{b}} \hat{I} \quad (11)$$

Keeping unchanged addition and subtraction,  $(+, -, \hat{\times}, \hat{\div})$  are four arithmetic operations in Santilli's isomathematics, which are foundations of isomathematics. When  $\hat{I} = 1$ , it is the operations of modern mathematics.

### 1.3. Addition and Subtraction in Modern Mathematics

We define addition and subtraction

$$x = a + b, y = a - b \quad (12)$$

$$a + a - a = a \quad (13)$$

$$a - a = 0 \quad (14)$$

$$\hat{\times} = \times \hat{T} \times, \hat{+} = + \hat{0} +, \hat{\div} = \div \hat{I} \div, \hat{-} = - \hat{0} -; a \hat{\times} b = ab \hat{T}, a \hat{+} b = a + b + \hat{0};$$

$$a \hat{\div} b = \frac{a}{b} \hat{I}, a \hat{-} b = a - b - \hat{0}; a = a \hat{\times} a \hat{\div} a = a, a = a \hat{+} a \hat{-} a = a;$$

$$a \hat{\times} a = a^2 \hat{T}, a \hat{+} a = 2a + \hat{0}; a \hat{\div} a = \hat{I} \neq 1, a \hat{-} a = -\hat{0} \neq 0; \hat{T} \hat{I} = 1. \quad (19)$$

$(\hat{+}, \hat{-}, \hat{\times}, \hat{\div})$  are four arithmetic operations in Santilli's new isomathematics.

Remark,  $a \hat{\times} (b \hat{+} c) = a \hat{\times} (b + c + \hat{0})$ , From left side we have

$$a \hat{\times} (b \hat{+} c) = a \hat{\times} b + a \hat{\times} \hat{+} + a \hat{\times} c = a \hat{\times} (b + \hat{+} + c)$$

$$= a \hat{\times} (b + \hat{0} + c), \text{ where } \hat{+} = \hat{0} \text{ also is a number.}$$

$$a \hat{\times} (b \hat{-} c) = a \hat{\times} (b - c - \hat{0}). \text{ From left side we have}$$

$$a \hat{\times} (b \hat{-} c) = a \hat{\times} b - a \hat{\times} \hat{-} - a \hat{\times} c$$

$$= a \hat{\times} (b - \hat{-} - c) = a \hat{\times} (b - \hat{0} - c), \text{ where } \hat{-} = \hat{0} \text{ also is a number.}$$

It is satisfies the distributive laws. Therefore  $\hat{+}, \hat{-}, \hat{\times}$  and  $\hat{\div}$  also are numbers.

It is the mathematical problems in the 21st century and a new mathematical tool for studying and understanding the law of world.

Using above results we establish isoaddition and isosubtraction

### 1.4. Isoaddition and Isosubtraction in Santilli's New Isomathematics

We define isoaddition  $\hat{+}$  and isosubtraction  $\hat{-}$ .

$$a \hat{+} b = a + b + c_1, a \hat{-} b = a - b - c_2 \quad (15)$$

$$a = a \hat{+} a \hat{-} a = a + c_1 - c_2 = a \quad (16)$$

From (16) we have

$$c_1 = c_2 \quad (17)$$

Suppose that  $c_1 = c_2 = \hat{0}$ ,

where  $\hat{0}$  is called isozero which is generalization of addition and subtraction zero

We have

$$a \hat{+} b = a + b + \hat{0}, a \hat{-} b = a - b - \hat{0} \quad (18)$$

When  $\hat{0} = 0$ , it is addition and subtraction in modern mathematics.

From above results we obtain foundations of santilli's new isomathematics

## 2. Santilli's Isoprime Theory of the First Kind

Let  $F(a, +, \times)$  be a conventional field with numbers  $a$  equipped with the conventional sum  $a + b \in F$ , multiplication  $ab \in F$  and their multiplicative unit  $1 \in F$ . Santilli's isofields of the first kind  $\hat{F} = \hat{F}(\hat{a}, \hat{+}, \hat{\times})$  are the rings with elements

$$\hat{a} = a \hat{I} \quad (20)$$

called isonumbers, where  $a \in F$ , the isosum

$$\hat{a} + \hat{b} = (a + b) \hat{I} \quad (21)$$

with conventional additive unit  $0 = 0 \hat{I} = 0, \hat{a} + 0 = 0 + \hat{a} = \hat{a}$ ,

$\forall \hat{a} \in \hat{F}$  and the isomultiplications is

$$\hat{a} \hat{\times} \hat{b} = \hat{a} \hat{T} \hat{b} = a \hat{I} \hat{T} b \hat{I} = (ab) \hat{I}. \quad (22)$$

Isodivision is

$$\hat{a} \hat{\div} \hat{b} = \hat{I} \frac{a}{b} \tag{23}$$

We can partition the positive isointegers in three classes:

- (1) The isounits:  $\hat{I}$ ;
- (2) The isonumbers:  $\hat{1} = \hat{I}, \hat{2}, \hat{3}, \hat{4}, \hat{5}, \dots$ ;
- (3) The isoprime numbers:  $\hat{2}, \hat{3}, \hat{5}, \hat{7}, \dots$ .

Theorem 1. Twin isoprime theorem

$$\hat{P}_1 = \hat{P} + \hat{2}. \tag{24}$$

Jiang function is

$$J_2(\omega) = \prod_{3 \leq P} (P-2) \neq 0, \tag{25}$$

where  $\omega = \prod_{2 \leq P} P$  is called primorial.

Since  $J_2(\omega) \neq 0$ , there exist infinitely many isoprimes  $\hat{P}$  such that  $\hat{P}_1$  is an isoprime.

We have the best asymptotic formula of the number of isoprimes less than  $\hat{N}$

$$\pi_2(\hat{N}, 2) \sim \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{N}{\log^2 N}, \tag{26}$$

where

$$\hat{P}_1, \hat{P}_2 = \hat{P}_1 + \hat{d}, \hat{P}_3 = \hat{P}_1 + \hat{2} \times \hat{d}, \dots, \hat{P}_k = \hat{P}_1 + (\hat{k} - 1) \times \hat{d}, (\hat{P}_1, \hat{d}) = \hat{I}. \tag{32}$$

Let  $\hat{I} = 1$ . From (32) we have arithmetic progressions of primes:

$$P_1, P_2 = P_1 + d, P_3 = P_1 + 2d, \dots, P_k = P_1 + (k-1)d, (P_1, d) = 1. \tag{33}$$

We rewrite (33)

$$P_3 = 2P_2 - P_1, P_j = (j-1)P_2 - (j-2)P_1, 3 \leq j \leq k. \tag{34}$$

Jiang function is

$$J_3(\omega) = \prod_{3 \leq P} [(P-1)^2 - \chi(P)], \tag{35}$$

$\chi(P)$  denotes the number of solutions for the following congruence

$$\prod_{j=3}^k [(j-1)q_2 - (j-2)q_1] \equiv 0 \pmod{P}, \tag{36}$$

$$\pi_{k-1}(N, 3) = |\{(j-1)P_2 - (j-2)P_1 = \text{prime}, 3 \leq j \leq k, P_1, P_2 \leq N\}|$$

$$\sim \frac{J_3(\omega)\omega^{k-2}}{2\phi^k(\omega)} \frac{N^2}{\log^k N} = \frac{1}{2} \prod_{2 \leq P < k} \frac{P^{k-2}}{(P-1)^{k-1}} \prod_{k \leq P} \frac{P^{k-2}(P-k+1)}{(P-1)^{k-1}} \frac{N^2}{\log^k N}. \tag{38}$$

Theorem 4. From (33) we obtain

$$P_4 = P_3 + P_2 - P_1, P_j = P_3 + (j-3)P_2 - (j-3)P_1, 4 \leq j \leq k. \tag{39}$$

$$\phi(\omega) = \prod_{2 \leq P} (P-1).$$

Let  $\hat{I} = 1$ . From (24) we have twin prime theorem

$$P_1 = P + 2 \tag{27}$$

Theorem 2. Goldbach isoprime theorem

$$\hat{N} = \hat{P}_1 + \hat{P}_2 \tag{28}$$

Jiang function is

$$J_2(\omega) = \prod_{3 \leq P} (P-2) \prod_{P|N} \frac{P-1}{P-2} \neq 0 \tag{29}$$

Since  $J_2(\omega) \neq 0$  every isoeven number  $\hat{N}$  greater than  $\hat{4}$  is the sum of two isoprimes.

We have

$$\pi_2(\hat{N}, 2) \sim \frac{J_2(\omega)}{\phi^2(\omega)} \frac{N}{\log^2 N}. \tag{30}$$

Let  $\hat{I} = 1$ . From (28) we have Goldbach theorem

$$N = P_1 + P_2 \tag{31}$$

Theorem 3. The isoprimes contain arbitrarily long arithmetic progressions. We define arithmetic progressions of isoprimes:

Jiang function is

$$J_4(\omega) = \prod_{3 \leq P} ((P-1)^3 - \chi(P)), \quad (40) \quad \prod_{j=4}^k [q_3 + (j-3)q_2 - (j-3)q_1] \equiv 0 \pmod{P}, \quad (41)$$

$\chi(P)$  denotes the number of solutions for the following congruence

where  $q_i = 1, 2, \dots, P-1, i = 1, 2, 3$ .  
From (41) we have

$$J_4(\omega) = \prod_{3 \leq P < (k-1)} (P-1)^2 \prod_{(k-1) \leq P} (P-1)[(P-1)^2 - (P-2)(k-3)] \neq 0. \quad (42)$$

We prove there exist infinitely many primes  $P_1, P_2$  and  $P_3$  such that  $P_4, \dots, P_k$  are all primes for all  $k \geq 4$ .

We have the best asymptotic formula

$$\begin{aligned} \pi_{k-2}(N, 4) &= |\{P_3 + (j-3)P_2 - (j-3)P_1 = \text{prime}, 4 \leq j \leq k, P_1, P_2, P_3 \leq N\}| \sim \frac{J_4(\omega)\omega^{k-3}}{6\phi^k(\omega)} \frac{N^3}{\log^k N} \\ &= \frac{1}{6} \prod_{2 \leq P < (k-1)} \frac{P^{k-3}}{(P-1)^{k-2}} \prod_{(k-1) \leq P} \frac{P^{k-3}[(P-1)^2 - (P-2)(k-3)]}{(P-1)^{k-1}} \frac{N^3}{\log^k N} \end{aligned} \quad (43)$$

The prime distribution is order rather than random. The arithmetic progressions in primes are not directly related to ergodic theory, harmonic analysis, discrete geometry and combinatorics. Using the ergodic theory Green and Tao prove there exist arbitrarily long arithmetic progressions of primes which is false [6,7,8,9,10].

where

$$X(P) = \begin{cases} (-\frac{2I}{P}) \\ -1 & \text{if } P | \hat{I} \end{cases}$$

Theorem 5. Isoprime equation

$$P_2 = \hat{P}_1 + 2 = P_1 \hat{I} + 2. \quad (44)$$

Let  $\hat{I}$  be the odd number. Jiang function is

$$J_2(\omega) = \prod_{3 \leq P} (P-2) \prod_{P | \hat{I}} \frac{P-1}{P-2} \neq 0. \quad (45)$$

If  $(\frac{-2I}{3}) = -1$ , there infinitely many primes  $P_1$  such that  $P_2$  is a prime. If  $(\frac{-2I}{3}) = 1, J_2(3) = 0$ , there exist finite primes  $P_1$  such that  $P_2$  is a prime.

Since  $J_2(\omega) \neq 0$ , there exist infinitely primes  $P_1$  such that  $P_2$  is a prime.

We have

$$\pi_2(N, 2) \sim \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{N}{\log^2 N}. \quad (46)$$

Theorem 6. Isoprime equation

$$P_2 = (\hat{P}_1)^2 + 2 = P_1^2 \hat{I} + 2. \quad (47)$$

Let  $\hat{I}$  be the odd number. Jiang function is

$$J_2(\omega) = \prod_{3 \leq P} (P-2 - X(P)), \quad (48)$$

$$a \hat{\div} b = (a/b)\hat{I}, a^{\hat{n}} = a \hat{\times} \dots \hat{\times} a \text{ (ntimes)} = a^n (\hat{T})^{n-1}, a^{\hat{1/2}} = a^{1/2} (\hat{I})^{1/2}. \quad (50)$$

Theorem 7. Isoprime equations

$$P_2 = P_1^2 + 6, P_3 = P_1^2 + 12, P_4 = P_1^2 + 18 \quad (51)$$

Santilli's isofields of the second kind  $\hat{F} = \hat{F}(a, +, \hat{\times})$  (that is,  $a \in F$  is not lifted to  $\hat{a} = a\hat{I}$ ) also verify all the axioms of a field.

The isomultiplication is defined by

$$a \hat{\times} b = a\hat{T}b. \quad (49)$$

We then have the isoquotient, isopower, isosquare root, etc.,

$$P_2 = P^2 + 6, P_3 = P_1^2 + 12, P_4 = P_1^2 + 18, \quad (52)$$

Jiang function is

Let  $T = 1$ . From (51) we have

$$J_2(\omega) = 2 \prod_{5 \leq P} (P - 4 - (\frac{-6}{P}) - (\frac{-3}{P}) - (\frac{-2}{P})) \neq 0, \quad (53)$$

where  $(\frac{-6}{P})$ ,  $(\frac{-3}{P})$  and  $(\frac{-2}{P})$  denote the Legendre symbols.

Since  $J_2(\omega) \neq 0$ , there exist infinitely many primes  $P_1$  such that  $P_2, P_3$  and  $P_4$  are primes.

$$\pi_4(N, 2) \sim \frac{J_2(\omega)\omega^3}{8\phi^4(\omega)} \frac{N}{\log^4 N} \quad (54)$$

Let  $\hat{T} = 5$ . From (51) we have

$$P_2 = 5P_1^2 + 6, P_3 = 5P_1^2 + 12, P_4 = 5P_1^2 + 18. \quad (55)$$

Jiang function is

$$J_2(\omega) = 8 \prod_{7 \leq P} (P - 4 - (\frac{-30}{P}) - (\frac{-15}{P}) - (\frac{-10}{P})) \neq 0. \quad (56)$$

Since  $J_2(\omega) \neq 0$ , there exist infinitely many primes  $P_1$  such that  $P_2, P_3$  and  $P_4$  are primes.

We have

$$\pi_4(N, 2) \sim \frac{J_2(\omega)\omega^3}{8\phi^4(\omega)} \frac{N}{\log^4 N}. \quad (57)$$

Let  $\hat{T} = 7$ . From (51) we have

$$P_2 = 7P_1^2 + 6, P_3 = 7P_1^2 + 12, P_4 = 7P_1^2 + 18. \quad (58)$$

We have Jiang function

$$J_2(5) = 0. \quad (59)$$

There exist finite primes  $P_1$  such that  $P_2, P_3$  and  $P_4$  are primes.

Theorem 8. Isoprime equations

$$P_2 = P_1^{\hat{2}} + 30, P_3 = P_1^{\hat{2}} + 60, P_4 = P_1^{\hat{2}} + 90, P_5 = P_1^{\hat{2}} + 120. \quad (60)$$

Let  $\hat{T} = 7$ . From (60) we have

$$P_2 = 7P_1^2 + 30, P_3 = 7P_1^2 + 60, P_4 = 7P_1^2 + 90, P_5 = 7P_1^2 + 120 \quad (61)$$

Jiang function is

$$\pi_2(N, 3) = |\{P_1, P_2 : P_1, P_2 \leq N, P_3 = \text{prime}\}| \sim \frac{J_3(\omega)\omega}{4\phi^3(\omega)} \frac{N^2}{\log^3 N}. \quad (69)$$

Theorem 10. Isoprime equation

$$P_3 = P_2 \hat{\times} (P_1^2 + b) - b \quad (70)$$

Let  $\hat{T} = 1$  Jiang function is

$$J_3(\omega) = \prod_{3 \leq P \leq P_1} (P^2 - 3P + 3 + \chi(P)) \neq 0 \quad (71)$$

$$J_2(\omega) = 48 \prod_{11 \leq P} (P - 5 - \sum_{j=1}^4 (\frac{-210j}{P})) \neq 0. \quad (62)$$

Since  $J_2(\omega) \neq 0$ , there exist infinitely many primes  $P_1$  such that  $P_2, P_3, P_4$  and  $P_5$  are primes.

We have

$$\pi_5(N, 2) \sim \frac{J_2(\omega)\omega^4}{16\phi^5(\omega)} \frac{N}{\log^5 N}. \quad (63)$$

Let  $\hat{T} \geq 7$  be the odd prime. From (60) we have

$$P_k = P_1^2 \hat{T} + 30(k-1), k = 2, 3, 4, 5. \quad (64)$$

Jiang function is

$$J_2(\omega) = 8 \prod_{7 \leq P} (P - 5 - \chi(P)) \neq 0. \quad (65)$$

If  $P | \hat{T}$ ,  $\chi(P) = 4$ ;  $\chi(P) = \sum_{j=1}^4 (\frac{-30\hat{T}j}{P})$  otherwise.

Since  $J_2(\omega) \neq 0$ , there exist infinitely many primes  $P_1$  such that  $P_2, P_3, P_4$  and  $P_5$  are primes.

We have

$$\pi_5(N, 2) \sim \frac{J_2(\omega)\omega^4}{16\phi^5(\omega)} \frac{N}{\log^5 N}. \quad (66)$$

Theorem 9. Isoprime equation

$$P_3 = P_2 \hat{\times} (P_1 + b) - b. \quad (67)$$

Let  $\hat{T} = 1$  Jiang function is

$$J_2(\omega) = \prod_{3 \leq P \leq P_1} (P^2 + 3P + 3 - \chi(P)) \neq 0, \quad (68)$$

where  $\chi(P) = -P + 2$  if  $P | b$ ;  $\chi(P) = 0$  otherwise.

Since  $J_3(\omega) \neq 0$ , there exist infinitely many primes  $P_1$  and  $P_2$  such that  $P_3$  is also a prime.

The best asymptotic formula is

where  $\chi(P) = P - 2$  if  $P | b$ ;  $\chi(P) = (\frac{-b}{P})$  otherwise.

Since  $J_3(\omega) \neq 0$ , there exist infinitely many primes  $P_1$  and  $P_2$  such that  $P_3$  is also a prime.

The best asymptotic formula is

$$\pi_2(N, 3) = |\{P_1, P_2 : P_1, P_2 \leq N; P_3 = \text{prime}\}| \sim \frac{J_3(\omega)\omega}{6\phi^3(\omega)} \frac{N^2}{\log^3 N}. \tag{72}$$

Theorem 11. Isoprime equation

$$P_3 = P_2^{\hat{2}}(P_1 + 1) - 1. \tag{73}$$

Let  $\hat{T} = 1$ . Jiang function is

$$J_2(\omega) = \prod_{3 \leq P \leq P_1} (P^2 - 3P + 4) \neq 0 \tag{74}$$

Since  $J_3(\omega) \neq 0$ , there exist infinitely many primes  $P_1$  and  $P_2$  such that  $P_3$  is also a prime.

The best asymptotic formula is

$$\begin{aligned} \pi_2(N, 3) &= |\{P_1, P_2 : P_1, P_2 \leq N; P_3 = \text{prime}\}| \\ &\sim \frac{J_3(\omega)\omega}{6\phi^3(\omega)} \frac{N^2}{\log^3 N}. \end{aligned} \tag{75}$$

### 4. Isoprime Theory in Santilli's New Isomathematics

Theorem 12. Isoprime equation

$$P_3 = P_1 \hat{+} P_2 = P_1 + P_2 + \hat{0}. \tag{76}$$

Suppose  $\hat{0} = 1$ . From (76) we have

$$P_3 = P_1 + P_2 + 1. \tag{77}$$

Jiang function is

$$J_3(\omega) = \prod_{3 \leq P} (P^2 - 3P + 3) \neq 0. \tag{78}$$

Since  $J_3(\omega) \neq 0$ , there exist infinitely many primes  $P_1$  and  $P_2$  such that  $P_3$  is also a prime.

We have the best asymptotic formula is

$$\hat{y} = a_1 \hat{\times} (b_1 \hat{+} c_1) \hat{+} a_2 \hat{\div} (b_2 \hat{-} c_2) = a_1 \hat{T} (b_1 + c_1 + \hat{0}) + \hat{0} + a_2 / \hat{T} (b_2 - c_2 - \hat{0}). \tag{85}$$

If  $\hat{T} = 1$  and  $\hat{0} = 0$ , then  $y = \hat{y}$ .

Let  $\hat{T} = 2$  and  $\hat{0} = 3$ . From (85) we have the isomathematical subequation

$$\hat{y}_1 = 2a_1(b_1 + c_1 + 3) + 3 + a_2 / 2(b_2 - c_2 - 3). \tag{86}$$

Let  $\hat{T} = 5$  and  $\hat{0} = 6$ . From (85) we have the isomathematical subequation

$$\hat{y}_2 = 5a_1(b_1 + c_1 + 6) + 6 + a_2 / 5(b_2 - c_2 - 6). \tag{87}$$

$$\pi_2(N, 3) \sim \frac{J_3(\omega)\omega}{2\phi^3(\omega)} \frac{N^2}{\log^3 N}. \tag{79}$$

Theorem 13. Isoprime equation

$$P_3 = (P_1 \hat{+} 2) \hat{\times} (P_1 \hat{-} 2) \hat{+} P_2 = \hat{T} [P_1^2 - (2 + \hat{0})^2] + P_2 + \hat{0} \tag{80}$$

Suppose  $\hat{T} = 6$  and  $\hat{0} = 4$ . From (80) we have

$$P_3 = 6(P_1^2 - 36) + P_2 + 4 \tag{81}$$

Jiang function is

$$J_3(\omega) = \prod_{3 \leq P} (P^2 - 3P + 2) \neq 0. \tag{82}$$

Since  $J_3(\omega) \neq 0$ , there exist infinitely many primes  $P_1$  and  $P_2$  such that  $P_3$  is also a prime.

We have the best asymptotic formula is

$$\pi_2(N, 3) \sim \frac{J_3(\omega)\omega}{4\phi^3(\omega)} \frac{N^2}{\log^3 N}. \tag{83}$$

### 5. An Example

We give an example to illustrate the Santilli's isomathematics.

Suppose that algebraic equation

$$y = a_1 \times (b_1 + c_1) + a_2 \div (b_2 - c_2) \tag{84}$$

We consider that (84) may be represented the mathematical system, physical system, biological system, IT system and another system. (84) may be written as the isomathematical equation

Let  $\hat{T} = 8$  and  $\hat{0} = 10$ . From (85) we have the isomathematical subequation

$$\hat{y}_3 = 8a_1(b_1 + c_1 + 10) + 10 + a_2 / 8(b_2 - c_2 - 10) \tag{88}$$

From (85) we have infinitely many isomathematical subequations. Using (85)-(88),  $\hat{T}$  and  $\hat{0}$  we study stability and optimum structures of algebraic equation (84).

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