ORIGIN AND APPLICATIONS OF THE ISODIFFERENTIAL CALCULUS AND ISODYNAMICAL EQUATIONS

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Let $F(n, \times, 1)$, be a field of characteristic zero with elements n (real, complex and quaternionic numbers), associative product $n \times m$ and multiplicative unit 1 with related axiom $1 \times n = n \times 1 \equiv n \forall n \in F$. Let L be an N-dimensional Lie algebra over F with Hermitean generators $J_k,\ k = 1, 2, ..., N$. universal enveloping associative algebra $\xi(L),\ L \approx [\xi(l)]^-$, associative product $J_i \times J_j$, Lie algebra $[J_i, J_j] = J_i \times J_j - J_j \times J_i = C^k_{ij} \times J_k$, and related Lie transformation groups. As it is well known, Lie's theory can only provide the invariant description of linear, local and Hamiltonian (variationally selfadjoint - SA) dynamical systems. In order to achieve the invariant description of non-linear, non-local and non-Hamiltonian (variationally nonselfadjoint - NSA) dynamical systems, the author proposed in 1978 [1] (see reviews [2,3]) an isotopic (axiom-preserving) lifting of the various branches of Lie's theory with: associativity-preserving product $J_i \hat{\times} J_j = J_i \times \hat{T} \times J_j$ where $\hat{T}$ (called the isotopic element) is a fixed positive-definite operator with otherwise an unrestricted functional dependence on local variables; isotopically lifted enveloping algebra $\hat{\xi}(\hat{L})$ characterizing the isotopies $\hat{L} \approx [\hat{\xi}(\hat{L})]^-$ of L; Lie-isotopic algebras $[J_i \hat{,} J_j] = J_i \hat{\times} J_j - J_j \hat{\times} J_i = \hat{C}^k_{ij} \hat{\times} J_k$; and isotopies of Lie's group and related symmetries.

In 1993, the author realized that, in order to achieve the invariance of non-linear, non-local and non-Hamiltonian/NSA systems, the isotopies of Lie's theory has to be formulated over compatible isotopies of the basic numeric field. Therefore, the author indicated in Ref. [4] (see also Refs. [4,5]) that the axioms of a numeric field are verified by the ring $\hat{F}(\hat{n}, \hat{\times} \hat{I})$ (called isofield) with elements $\hat{n} = n \times \hat{I}, n \in F$ (called isonumbers), isoproduct $\hat{n} \hat{\times} \hat{m} = (n \times m) \times \hat{I}$ and generalized multiplicative unit $\hat{I} = 1/\hat{T} > 0$ (called isounit), with isoaxiom $\hat{I} \hat{\times} \hat{n} = \hat{n} \hat{\times} \hat{I} \equiv \hat{n} \forall \hat{n} \in \hat{F}$. The isounit $\hat{I}$ can be either outside the original field (yielding isofields of first kind), or an element of the original, field (yielding isofields of the second kind) in which case $\hat{n} \in F$ with nontrivial implications for the number theory because, e.g., for the isofield of the second kind $\hat{F}(n, \hat{\times}, \hat{I}), \hat{I} = 3$, the number 4 is prime.

In 1996, the author realized that, despite the compatible formulation of the Lie-isotopic theory over isofields and the isotopies of functional analysis, vector spaces, and other methods, the desired invariance of NSA systems required the additional isotopies of the Newton-Leibnitz differential calculus with differential dr of a variable r and related derivative of a function of r, $\partial f(r)/\partial r$. Therefore, the author proposed in Ref. [7] (see the recent advances by Svetlin Georgiev [8]) the isotopies of differential calculus $\hat{d}\hat{r} = \hat{T} d(r \times \hat{I})$ and $\hat{\partial} \hat{f}(\hat{r})/\hat{\partial} \hat{r} = \hat{I} \times (\partial \hat{f}(\hat{r})/\partial \hat{r})$ (here suggested for referral as the Santilli-Georgiev IsoDifferential Calculus, GS-IDC) that coincides with the conventional calculus for $\hat{I}$ constant or independent from the variable r, perhaps explaining why the IDC remained unidentified for centuries.

The above advances finally permitted the formulation of IsoDynamical Systems (IDS) as non-linear, non-local and non-Hamiltonian/NSA systems under invariant representation by the Lie-isotopic theory, characterized by the first known structural lifting of

Newton's equation $m \times dv/dt = F^{SA}(t,r,v) + F^{NSA}(t,r,v)$ into the isodynamical form $\hat{m}\hat{\times}\hat{d}\hat{v}/\hat{d}\hat{t} = \hat{F}^{SA}(\hat{t},\hat{r},\hat{v})$ where all non-Hamiltonian/NSA forces are embedded in the isodifferential $\hat{d}\hat{v}/\hat{d}\hat{t}$, $\hat{v} = \hat{d}\hat{r}/\hat{d}\hat{t}$[7]. Realizations of the isounit of the type $\hat{I} = Diag.(n_1^2, n_2^2, n_3^2) \times e^{\Gamma(t,r,v)}$ then permit the lifting of Newton's concept of massive points into bodies with extended shapes (represented by the n^2s) under the most general known SA forces (represented by F^{SA}) and NSA forces (represented by the Γs). The embedding of shapes and NSA forces into the isodifferential. then permitted the isotopic lifting of action principles, Hamiltonian mechanics, quantum mechanics and quantum chemistry with numerous scientific and industrial applications [9-12].

Following a brief review of isomathematics, IDC and IDS, the author shall present recent applications, such as: the first known extension of the optimal control theory to NSA systems [7]; the first known studies on the connection between mechanics and thermodynamics [13]; the first known consistent representation of the synthesis of the neutron inside a star from a proton and an electron, with ensuing nuclear fusions without harmful radiations [14]; the experimental verification of the redshift of laser and Sun light due to loss of energy to cold media without relative motion [15]; and other applications. The author hopes to present the broader geno- and hyper-mathematics and their applicatioons at some future meeting of this series..

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